Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.1-Sine/188-4.1.7.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**145**]. This is test number [188].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (145)	0.00 (0)
Mathematica	100.00 (145)	0.00 (0)
Maple	92.41 (134)	7.59 (11)
Fricas	83.45 (121)	16.55 (24)
Mupad	64.83 (94)	35.17 (51)
Giac	64.83 (94)	35.17 (51)
Maxima	53.10 (77)	46.90 (68)
Reduce	53.10 (77)	46.90 (68)
Sympy	48.97 (71)	51.03 (74)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

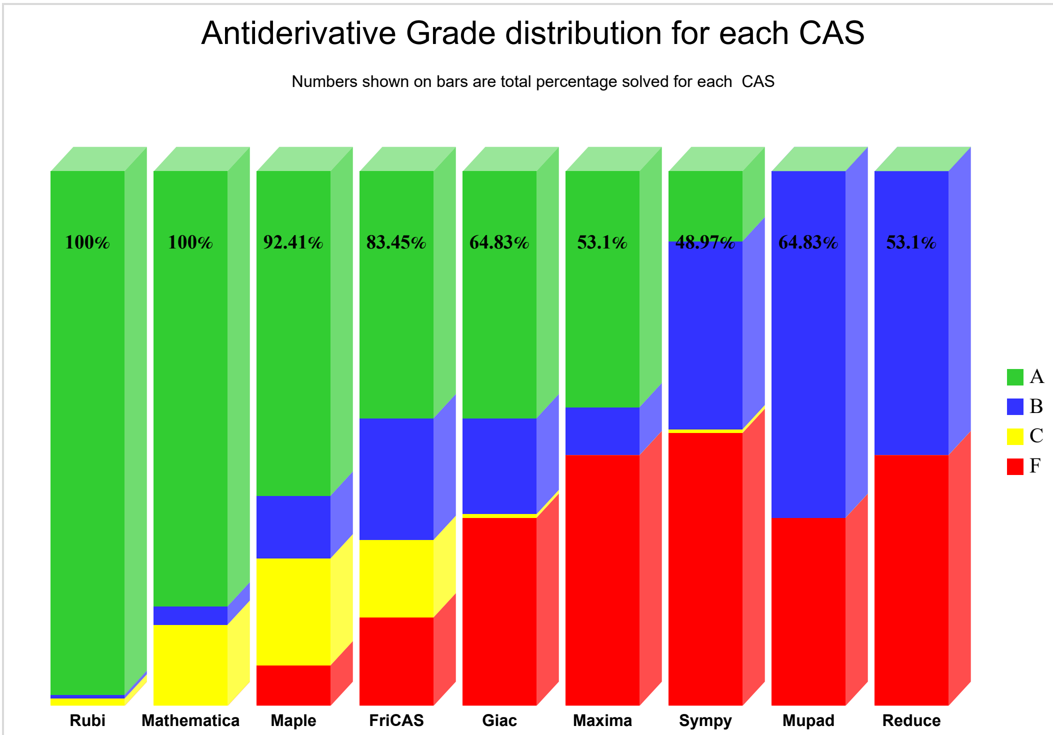
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

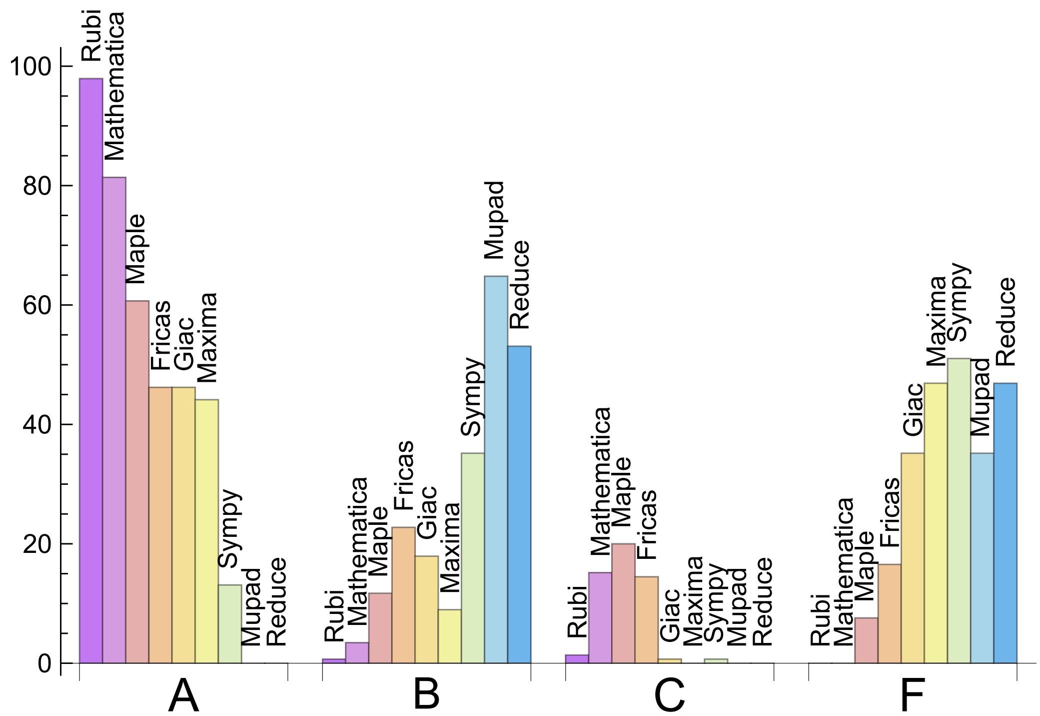
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.931	0.690	1.379	0.000
Mathematica	81.379	3.448	15.172	0.000
Maple	60.690	11.724	20.000	7.586
Fricas	46.207	22.759	14.483	16.552
Giac	46.207	17.931	0.690	35.172
Maxima	44.138	8.966	0.000	46.897
Sympy	13.103	35.172	0.690	51.034
Mupad	0.000	64.828	0.000	35.172
Reduce	0.000	53.103	0.000	46.897

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	11	100.00	0.00	0.00
Fricas	24	66.67	0.00	33.33
Mupad	51	0.00	100.00	0.00
Giac	51	94.12	0.00	5.88
Maxima	68	94.12	0.00	5.88
Reduce	68	100.00	0.00	0.00
Sympy	74	68.92	31.08	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Reduce	0.20
Fricas	0.24
Rubi	0.35
Giac	0.50
Mathematica	1.13
Maple	2.28
Sympy	3.06
Mupad	32.39

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	61.99	0.90	38.00	0.87
Maxima	64.30	1.37	22.00	0.76
Maple	64.47	1.09	37.50	0.80
Rubi	72.19	0.98	47.00	1.00
Giac	113.74	1.52	40.00	1.10
Reduce	212.77	3.20	37.00	1.02
Mupad	273.79	1.86	33.00	0.87
Sympy	1632.34	46.79	100.00	2.91
Fricas	11600.52	63.85	58.00	1.44

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

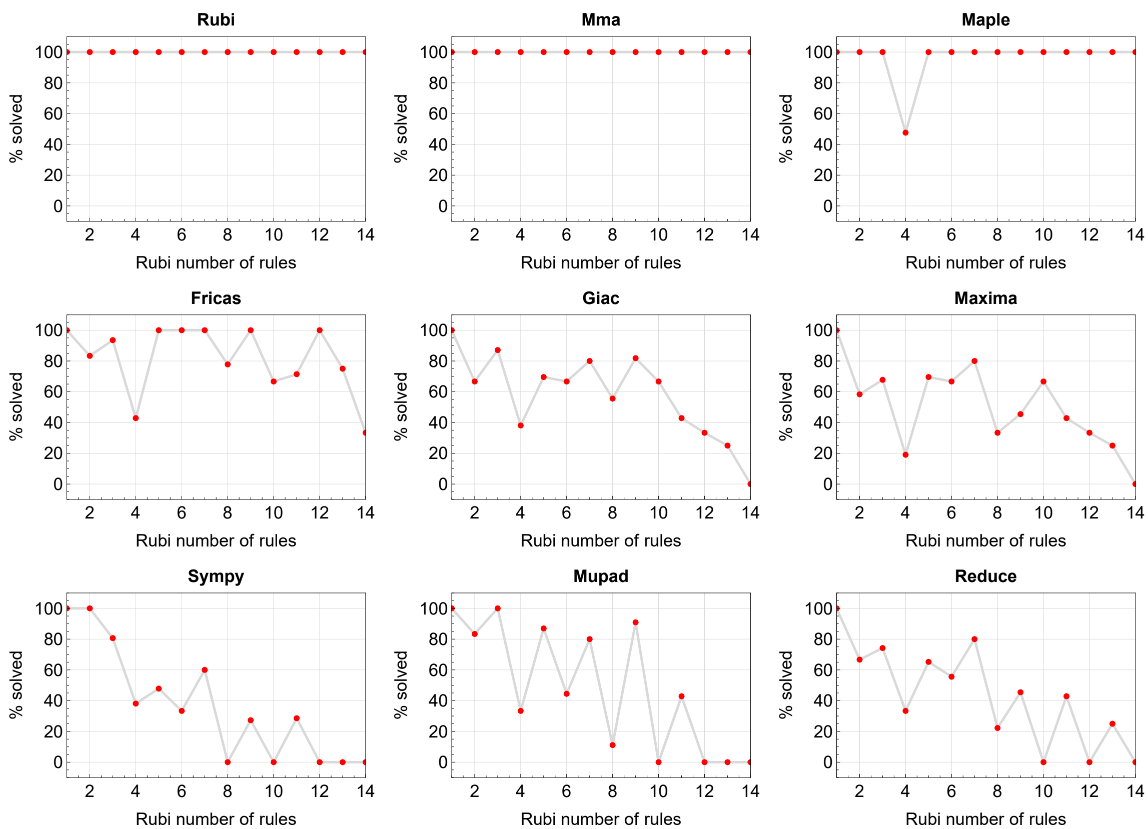


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

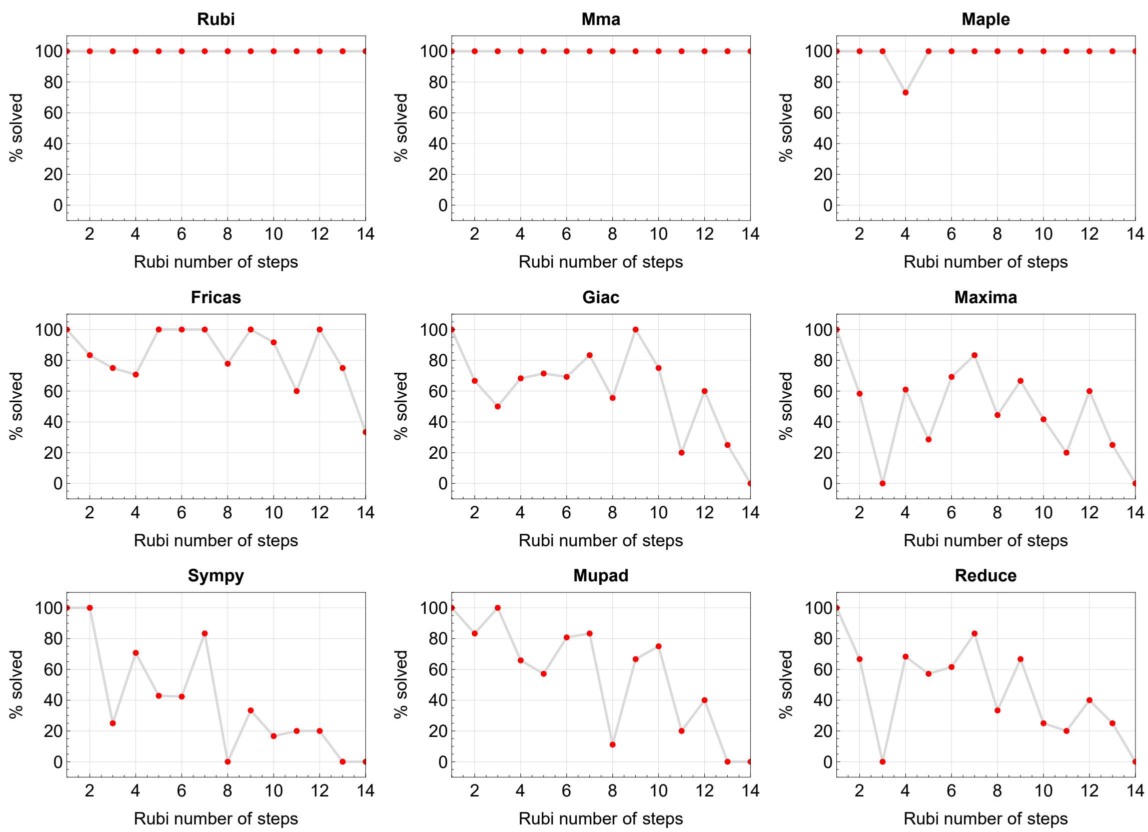


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

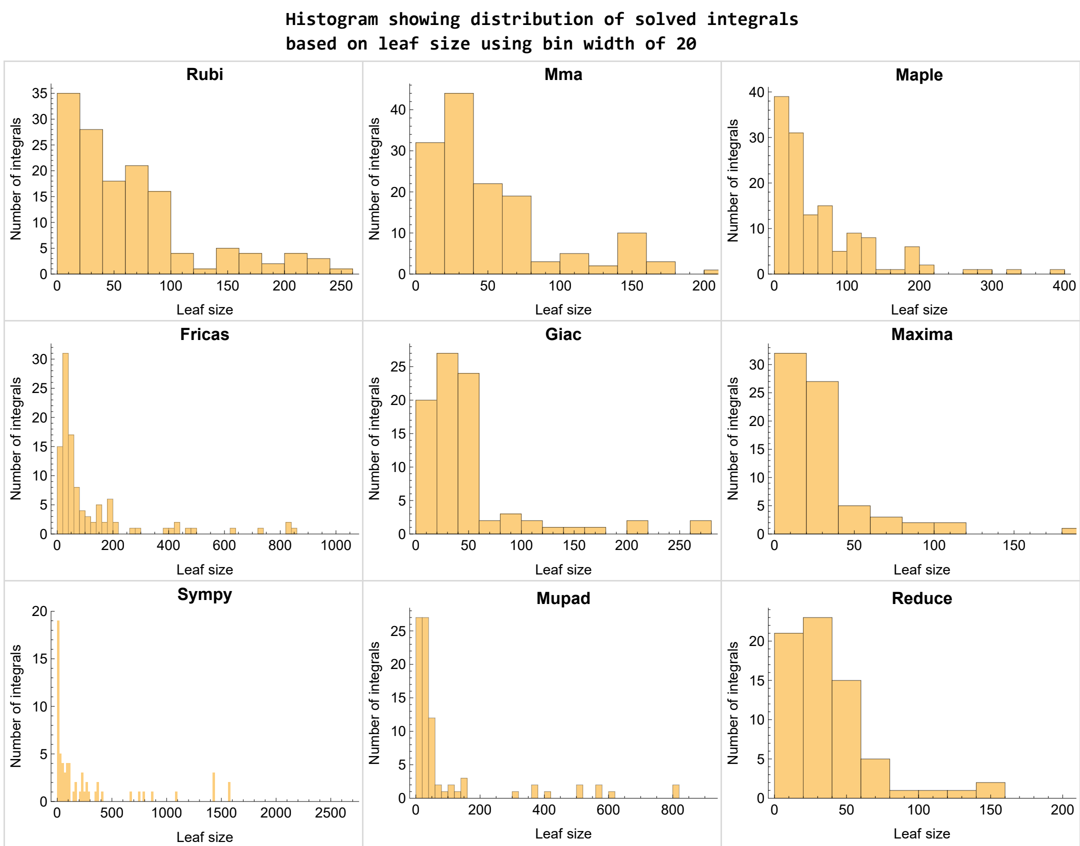


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

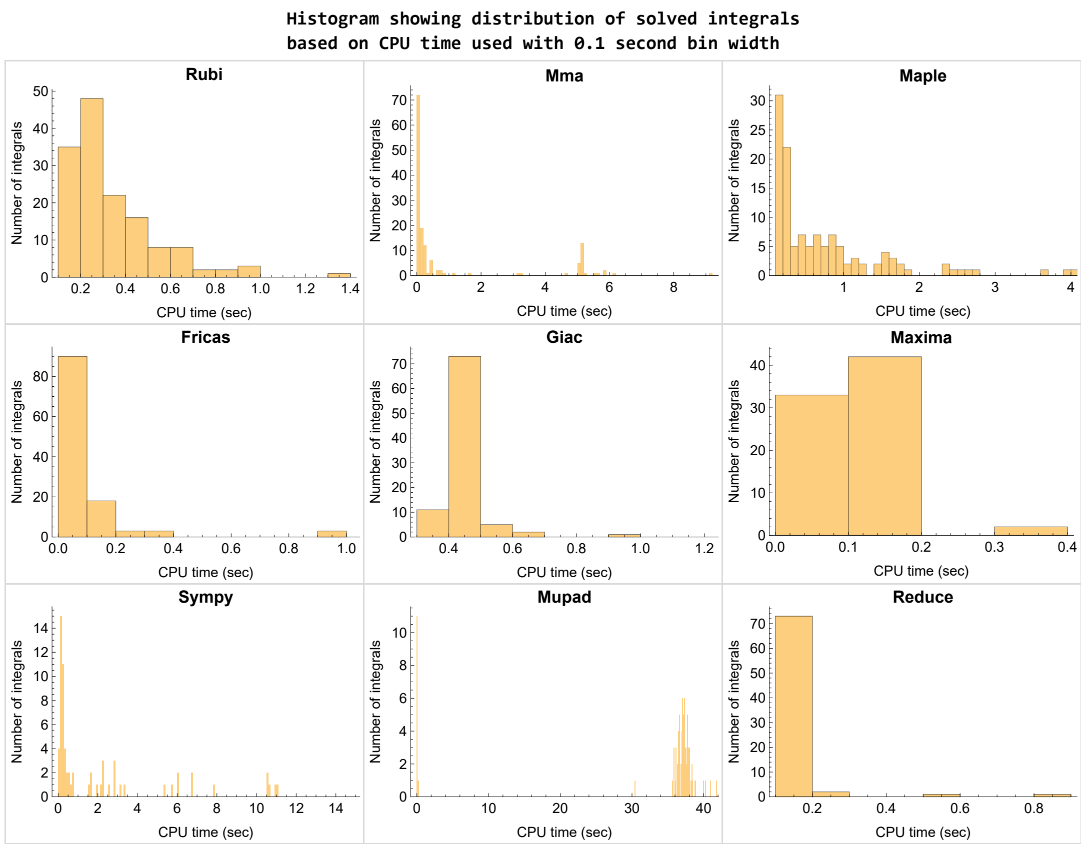


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

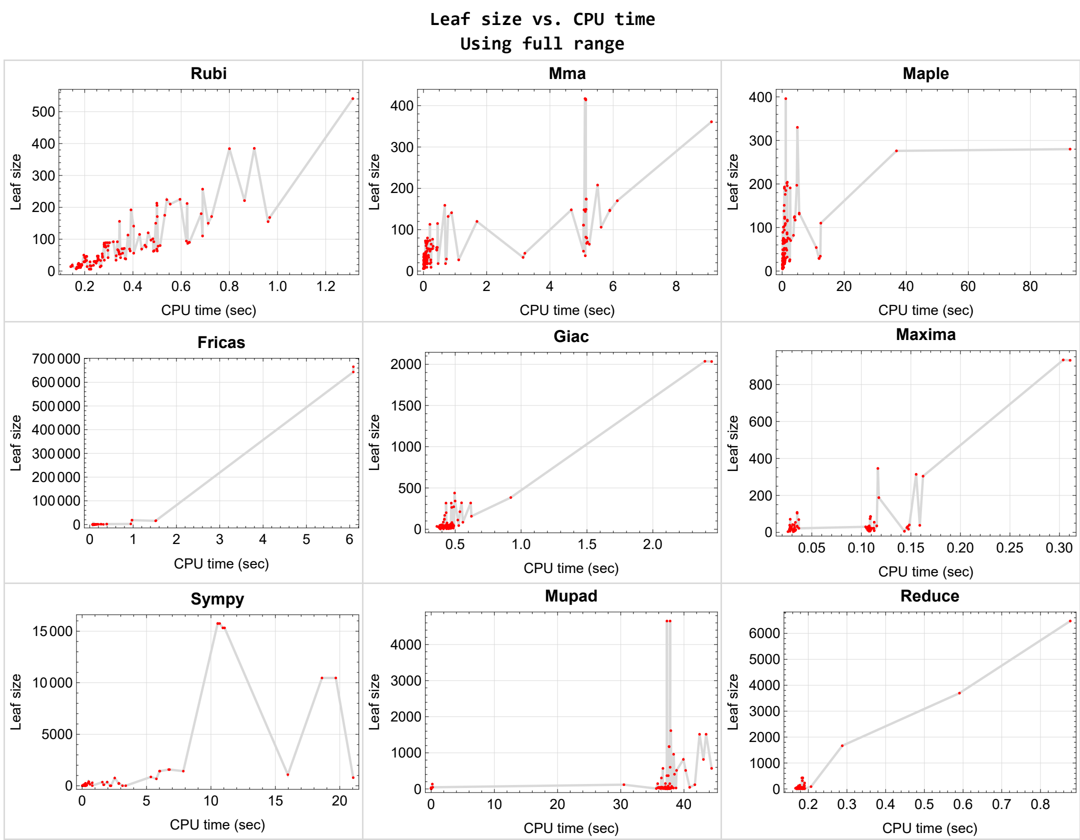


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{56, 59, 74, 77, 81, 84\}$

Maple $\{98\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

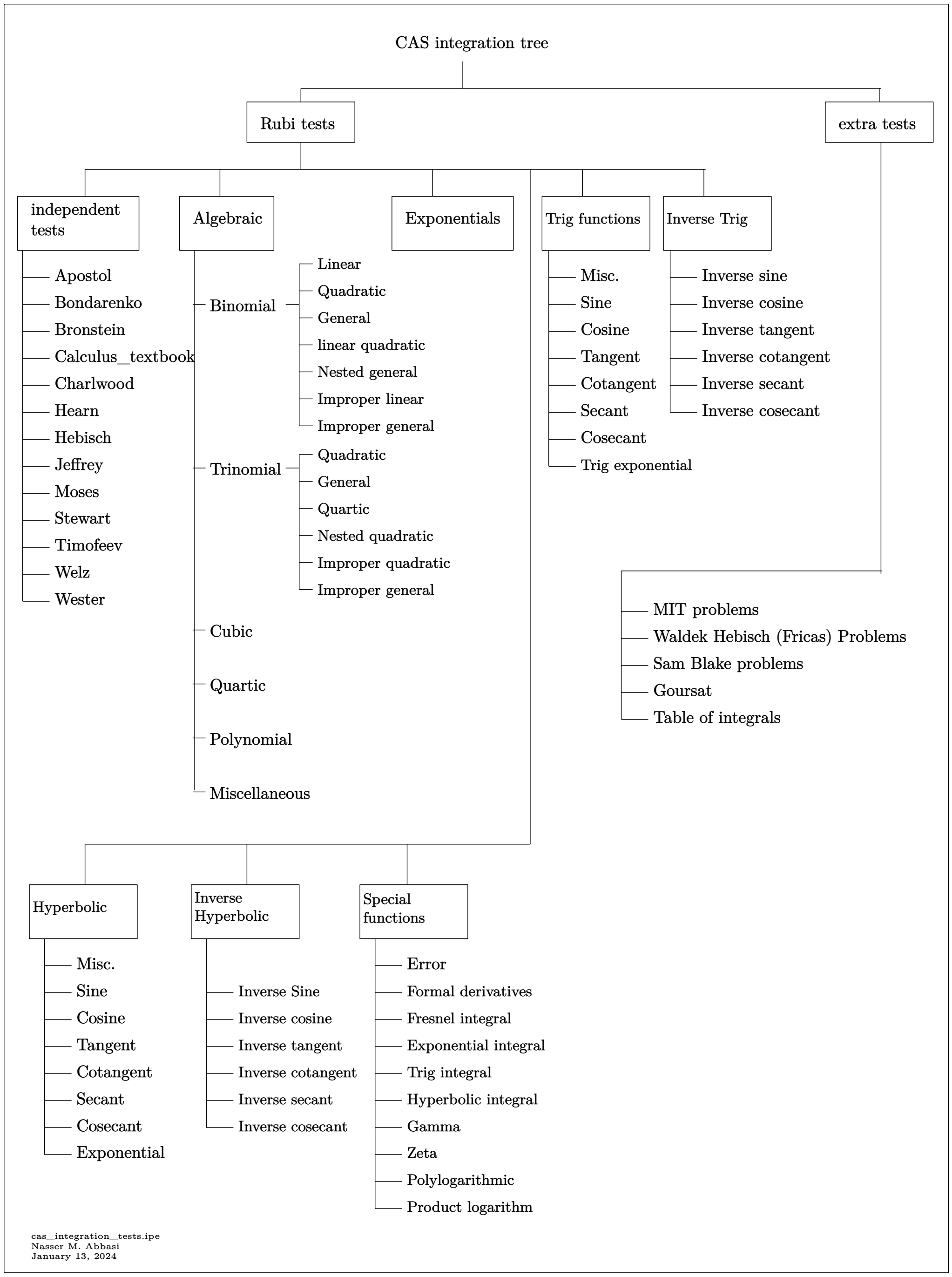
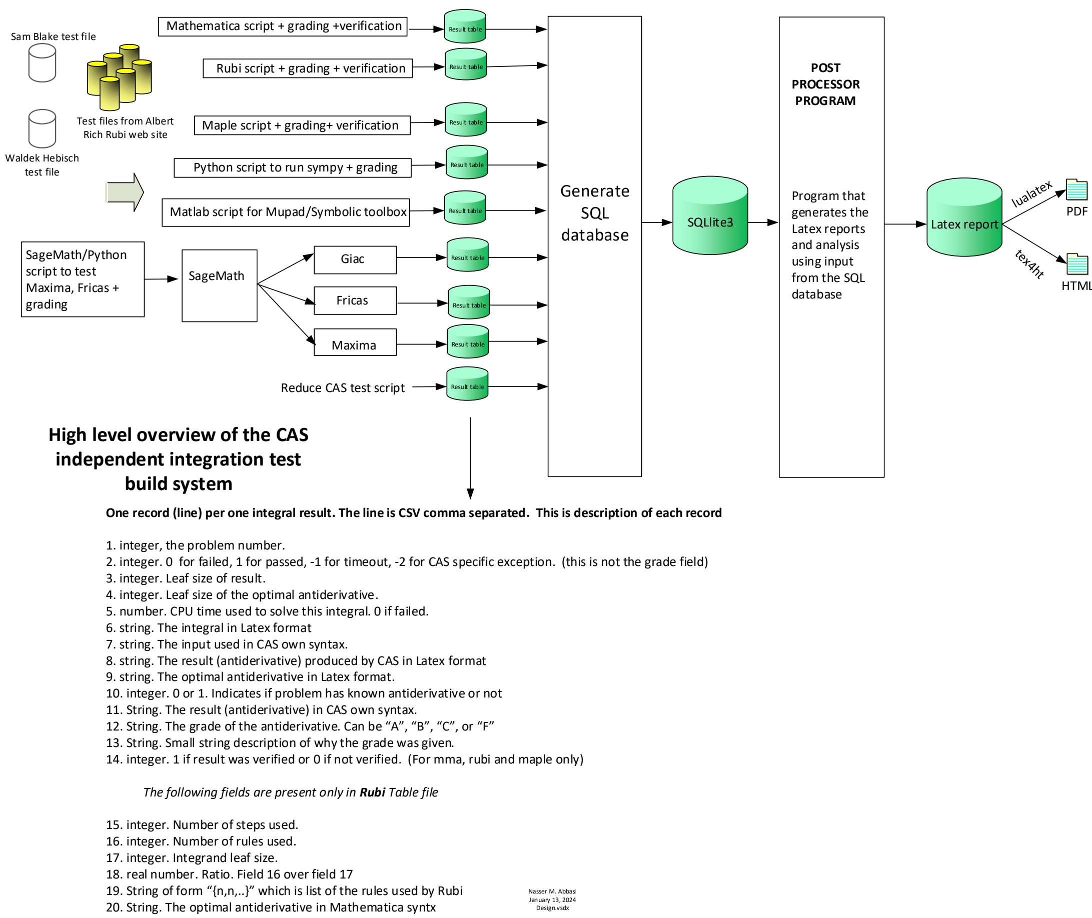


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { 79 }

C grade { 57, 60 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 66, 71, 72, 75, 78, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { 38, 43, 61, 68, 120 }

C grade { 1, 56, 57, 58, 59, 60, 62, 63, 65, 67, 69, 70, 73, 74, 76, 77, 79, 80, 81, 83, 84, 133 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 9, 16, 17, 18, 19, 20, 21, 32, 34, 35, 36, 37, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 61, 64, 66, 68, 71, 75, 78, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 118, 119, 121, 122, 123, 124, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 144 }

B grade { 7, 8, 38, 43, 109, 110, 111, 114, 115, 120, 125, 126, 127, 130, 131, 140, 145 }

C grade { 1, 10, 11, 12, 13, 14, 15, 56, 57, 58, 60, 62, 63, 65, 67, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 83, 84, 98, 143 }

F normal fail { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 16, 17, 18, 20, 21, 32, 34, 35, 36, 37, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 61, 64, 65, 66, 68, 71, 75, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 116, 117, 118, 119, 132, 133, 134, 135 }

B grade { 7, 19, 38, 39, 40, 41, 42, 43, 50, 56, 57, 58, 59, 60, 62, 63, 67, 69, 70, 72, 74, 78, 79, 81, 98, 120, 121, 122, 123, 136, 137, 138, 139 }

C grade { 10, 11, 12, 13, 14, 15, 73, 76, 80, 83, 112, 113, 114, 115, 128, 129, 130, 131, 143, 144, 145 }

F normal fail { 28, 29, 30, 31, 33, 108, 109, 110, 111, 124, 125, 126, 127, 140, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { 22, 23, 24, 25, 26, 27, 77, 84 }

Maxima

A grade { 2, 3, 6, 7, 16, 17, 18, 19, 20, 21, 34, 35, 36, 37, 39, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 61, 64, 66, 68, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 101, 102, 103, 104, 105, 106, 107, 116, 117, 118, 119, 121, 122, 123, 132, 133, 134, 135, 136, 137 }

B grade { 8, 9, 38, 40, 41, 43, 85, 98, 99, 100, 120, 138, 139 }

C grade { }

F normal fail { 1, 4, 5, 10, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 56, 57, 58, 59, 60, 62, 63, 65, 67, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 83, 84, 108, 109, 110, 111, 112, 113, 114, 115, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145 }

F(-1) timedout fail { }

F(-2) exception fail { 51, 71, 75, 82 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 16, 17, 18, 19, 20, 21, 34, 35, 36, 37, 44, 45, 46, 47, 48, 49, 52, 53, 54, 56, 59, 61, 64, 65, 66, 68, 75, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 104, 105, 106, 107, 116, 117, 118, 119, 121, 122, 123, 132, 133, 134, 135, 137, 138, 139 }

B grade { 8, 9, 32, 38, 39, 40, 41, 42, 43, 50, 51, 55, 57, 62, 63, 69, 70, 71, 72, 78, 79, 95, 96, 97, 120, 136 }

C grade { 7 }

F normal fail { 10, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 73, 74, 76, 77, 80, 81, 83, 84, 98, 108, 109, 110, 111, 112, 113, 114, 115, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145 }

F(-1) timedout fail { }

F(-2) exception fail { 58, 60, 67 }

Mupad

A grade { }

B grade { 1, 2, 3, 6, 19, 20, 21, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 101, 102, 103, 104, 105, 106, 107, 111, 116, 117, 118, 119, 120, 121, 122, 123, 127, 132, 133, 134, 135, 136, 137, 138, 139 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 94, 95, 96, 98, 99, 100, 108, 109, 110, 112, 113, 114, 115, 124, 125, 126, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 6, 35, 46, 53, 61, 68, 88, 104, 111, 112, 119, 127, 128, 135, 142, 143 }

B grade { 32, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 62, 64, 71, 75, 78, 82, 85, 86, 87, 89, 90, 91, 92, 93, 101, 102, 103, 105, 106, 107, 116, 117, 118, 120, 121, 122, 123, 132, 133, 134, 136 }

C grade { 69 }

F normal fail { 4, 5, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 58, 63, 67, 70, 73, 74, 76, 77, 80, 81, 83, 84, 96, 97, 98, 99, 100, 110, 113, 114, 126, 130, 141, 144, 145 }

F(-1) timedout fail { 10, 22, 56, 57, 59, 60, 65, 66, 72, 79, 94, 95, 108, 109, 115, 124, 125, 129, 131, 137, 138, 139, 140 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 16, 17, 18, 19, 20, 21, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 61, 64, 68, 71, 75, 78, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 101, 102, 103, 104, 105, 106, 107, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139 }

C grade { }

F normal fail { 10, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 56, 57, 58, 59, 60, 62, 63, 65, 66, 67, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 83, 84, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 114, 115, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	192	69	40	0	146	17	157	117	135
N.S.	1	1.25	0.45	0.26	0.00	0.95	0.11	1.03	0.76	0.88
time (sec)	N/A	0.392	0.072	0.217	0.000	0.076	0.791	0.623	0.190	0.187

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	18	14	23	27	27	46	12	26
N.S.	1	1.00	0.44	0.34	0.56	0.66	0.66	1.12	0.29	0.63
time (sec)	N/A	0.202	0.083	0.162	0.108	0.075	0.116	0.476	0.189	38.740

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	18	18	14	13	31	63	46	37	26
N.S.	1	0.46	0.46	0.36	0.33	0.79	1.62	1.18	0.95	0.67
time (sec)	N/A	0.184	0.009	0.240	0.107	0.079	0.256	0.454	0.176	38.333

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	36	32	0	43	0	32	29	0
N.S.	1	1.06	0.68	0.60	0.00	0.81	0.00	0.60	0.55	0.00
time (sec)	N/A	0.357	0.066	0.672	0.000	0.076	0.000	0.429	0.190	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	0	29	0	24	19	0
N.S.	1	1.00	0.76	0.71	0.00	0.85	0.00	0.71	0.56	0.00
time (sec)	N/A	0.281	0.098	0.174	0.000	0.074	0.000	0.434	0.182	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	13	19	17	17	6	40
N.S.	1	1.00	1.00	1.14	0.93	1.36	1.21	1.21	0.43	2.86
time (sec)	N/A	0.210	0.024	0.174	0.109	0.073	0.114	0.438	0.187	37.404

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	49	26	70	0	37	11	0
N.S.	1	1.00	1.00	2.88	1.53	4.12	0.00	2.18	0.65	0.00
time (sec)	N/A	0.222	0.008	0.198	0.146	0.085	0.000	0.425	0.178	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	55	70	314	58	0	81	26	0
N.S.	1	1.00	1.31	1.67	7.48	1.38	0.00	1.93	0.62	0.00
time (sec)	N/A	0.297	0.110	0.218	0.155	0.084	0.000	0.476	0.177	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	69	77	89	931	78	0	122	35	0
N.S.	1	1.13	1.26	1.46	15.26	1.28	0.00	2.00	0.57	0.00
time (sec)	N/A	0.386	0.288	0.237	0.311	0.088	0.000	0.419	0.188	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	101	65	117	0	103	0	0	16	0
N.S.	1	0.82	0.53	0.95	0.00	0.84	0.00	0.00	0.13	0.00
time (sec)	N/A	0.483	0.243	4.214	0.000	0.112	0.000	0.000	0.181	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	65	54	176	0	74	0	0	14	0
N.S.	1	0.89	0.74	2.41	0.00	1.01	0.00	0.00	0.19	0.00
time (sec)	N/A	0.339	0.166	0.996	0.000	0.088	0.000	0.000	0.185	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	41	94	0	63	0	0	11	0
N.S.	1	0.94	0.82	1.88	0.00	1.26	0.00	0.00	0.22	0.00
time (sec)	N/A	0.273	0.057	0.724	0.000	0.098	0.000	0.000	0.182	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	37	151	0	89	0	0	16	0
N.S.	1	0.90	0.77	3.15	0.00	1.85	0.00	0.00	0.33	0.00
time (sec)	N/A	0.274	0.080	0.678	0.000	0.080	0.000	0.000	0.181	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	67	48	105	0	120	0	0	16	0
N.S.	1	0.87	0.62	1.36	0.00	1.56	0.00	0.00	0.21	0.00
time (sec)	N/A	0.336	0.109	0.920	0.000	0.083	0.000	0.000	0.189	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	97	60	185	0	177	0	0	16	0
N.S.	1	0.79	0.49	1.50	0.00	1.44	0.00	0.00	0.13	0.00
time (sec)	N/A	0.475	0.303	1.194	0.000	0.098	0.000	0.000	0.183	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	92	53	54	85	82	0	39	49	0
N.S.	1	0.70	0.40	0.41	0.64	0.62	0.00	0.30	0.37	0.00
time (sec)	N/A	0.487	0.236	11.000	0.109	0.089	0.000	0.411	0.188	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	60	38	40	55	56	0	27	31	0
N.S.	1	0.77	0.49	0.51	0.71	0.72	0.00	0.35	0.40	0.00
time (sec)	N/A	0.341	0.157	0.415	0.113	0.091	0.000	0.412	0.176	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	29	25	24	22	36	0	15	12	0
N.S.	1	0.81	0.69	0.67	0.61	1.00	0.00	0.42	0.33	0.00
time (sec)	N/A	0.222	0.043	0.319	0.109	0.077	0.000	0.396	0.177	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	9	36	0	9	13	7
N.S.	1	1.00	1.00	1.12	0.56	2.25	0.00	0.56	0.81	0.44
time (sec)	N/A	0.227	0.032	0.290	0.113	0.068	0.000	0.454	0.189	37.625

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	38	34	37	23	74	0	23	27	44
N.S.	1	0.56	0.50	0.54	0.34	1.09	0.00	0.34	0.40	0.65
time (sec)	N/A	0.256	0.063	0.365	0.109	0.077	0.000	0.472	0.179	38.036

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	54	47	49	35	104	0	35	39	117
N.S.	1	0.46	0.40	0.42	0.30	0.88	0.00	0.30	0.33	0.99
time (sec)	N/A	0.273	0.092	0.477	0.116	0.083	0.000	0.447	0.189	41.723

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	18	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.303	0.205	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0	16	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.280	0.134	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	88	68	0	0	0	0	0	15	0
N.S.	1	1.19	0.92	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.281	0.081	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0	20	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.285	0.084	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	0	0	0	0	0	20	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.295	0.124	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	0	0	0	0	0	20	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.288	0.123	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	0	0	0	0	0	16	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.281	0.074	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	0	0	0	16	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.281	0.083	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	0	0	0	0	0	16	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.287	0.109	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	0	0	0	0	0	16	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.284	0.097	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	16	58	384	16	36
N.S.	1	1.00	1.00	1.16	0.00	0.64	2.32	15.36	0.64	1.44
time (sec)	N/A	0.248	0.060	0.562	0.000	0.079	0.362	0.922	0.191	37.302

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	0	0	21	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.297	0.061	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	21	21	14	13	31	80	50	33	26
N.S.	1	0.48	0.48	0.32	0.30	0.70	1.82	1.14	0.75	0.59
time (sec)	N/A	0.188	0.114	0.161	0.107	0.080	0.242	0.390	0.182	37.112

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	18	18	14	13	31	63	46	37	26
N.S.	1	0.44	0.44	0.34	0.32	0.76	1.54	1.12	0.90	0.63
time (sec)	N/A	0.188	0.013	0.263	0.107	0.077	0.271	0.414	0.178	37.173

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	11	9	8	7	21	41	20	25	16
N.S.	1	0.42	0.35	0.31	0.27	0.81	1.58	0.77	0.96	0.62
time (sec)	N/A	0.182	0.238	0.500	0.108	0.075	0.313	0.425	0.188	37.238

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	8	15	4	8	4
N.S.	1	1.00	1.00	0.83	0.67	1.33	2.50	0.67	1.33	0.67
time (sec)	N/A	0.222	0.007	0.151	0.026	0.067	0.284	0.412	0.181	37.273

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	27	16	15	34	32	17	57	7
N.S.	1	1.00	2.45	1.45	1.36	3.09	2.91	1.55	5.18	0.64
time (sec)	N/A	0.176	0.231	0.183	0.031	0.078	0.193	0.475	0.186	37.302

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	24	57	1426	31	53	13
N.S.	1	1.00	1.00	0.78	1.33	3.17	79.22	1.72	2.94	0.72
time (sec)	N/A	0.184	0.091	0.195	0.106	0.076	6.036	0.488	0.180	37.181

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	30	58	1576	31	59	13
N.S.	1	1.00	1.00	0.67	1.43	2.76	75.05	1.48	2.81	0.62
time (sec)	N/A	0.180	0.236	0.164	0.104	0.080	6.783	0.403	0.178	37.038

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	30	58	1576	31	59	13
N.S.	1	1.00	1.00	0.67	1.43	2.76	75.05	1.48	2.81	0.62
time (sec)	N/A	0.176	0.106	0.137	0.107	0.077	6.718	0.416	0.184	36.674

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	24	57	1426	31	53	13
N.S.	1	1.00	1.00	0.78	1.33	3.17	79.22	1.72	2.94	0.72
time (sec)	N/A	0.182	0.698	0.178	0.107	0.078	6.004	0.453	0.179	37.740

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	27	16	15	34	32	17	57	7
N.S.	1	1.00	2.45	1.45	1.36	3.09	2.91	1.55	5.18	0.64
time (sec)	N/A	0.172	0.115	0.161	0.028	0.085	0.178	0.472	0.184	37.440

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	8	14	4	8	4
N.S.	1	1.00	1.00	0.83	0.67	1.33	2.33	0.67	1.33	0.67
time (sec)	N/A	0.219	0.002	0.148	0.031	0.066	0.264	0.387	0.178	37.040

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	11	9	8	7	21	42	20	25	16
N.S.	1	0.42	0.35	0.31	0.27	0.81	1.62	0.77	0.96	0.62
time (sec)	N/A	0.173	0.106	0.270	0.108	0.081	0.317	0.417	0.177	36.916

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	18	18	14	13	31	65	46	37	26
N.S.	1	0.44	0.44	0.34	0.32	0.76	1.59	1.12	0.90	0.63
time (sec)	N/A	0.176	0.462	0.276	0.107	0.080	0.276	0.384	0.177	37.316

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	21	21	14	13	31	82	50	33	26
N.S.	1	0.48	0.48	0.32	0.30	0.70	1.86	1.14	0.75	0.59
time (sec)	N/A	0.172	0.069	0.141	0.106	0.083	0.238	0.457	0.172	37.059

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	18	18	16	15	34	233	49	52	15
N.S.	1	0.46	0.46	0.41	0.38	0.87	5.97	1.26	1.33	0.38
time (sec)	N/A	0.181	0.204	0.224	0.107	0.082	2.863	0.430	0.189	37.471

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	10	17	6	10	6
N.S.	1	1.00	1.00	1.17	1.00	1.67	2.83	1.00	1.67	1.00
time (sec)	N/A	0.225	0.003	0.148	0.032	0.063	0.171	0.437	0.171	37.709

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	22	190	15745	49	427	31
N.S.	1	1.00	1.00	0.79	0.76	6.55	542.93	1.69	14.72	1.07
time (sec)	N/A	0.196	0.726	0.214	0.112	0.092	10.679	0.445	0.183	38.265

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	197	15319	54	62	33
N.S.	1	1.00	1.00	0.88	0.00	5.97	464.21	1.64	1.88	1.00
time (sec)	N/A	0.208	0.163	0.213	0.000	0.091	11.058	0.401	0.178	37.704

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	18	18	13	12	31	219	46	49	25
N.S.	1	0.46	0.46	0.33	0.31	0.79	5.62	1.18	1.26	0.64
time (sec)	N/A	0.183	0.130	0.268	0.107	0.087	2.839	0.440	0.173	37.318

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	18	13	22	29	26	46	13	25
N.S.	1	1.00	0.42	0.30	0.51	0.67	0.60	1.07	0.30	0.58
time (sec)	N/A	0.200	0.114	0.182	0.105	0.080	0.121	0.475	0.185	38.014

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	10	17	6	10	6
N.S.	1	1.00	1.00	1.17	1.00	1.67	2.83	1.00	1.67	1.00
time (sec)	N/A	0.224	0.001	0.131	0.032	0.086	0.170	0.435	0.170	0.002

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	27	22	23	46	792	57	70	21
N.S.	1	0.94	0.87	0.71	0.74	1.48	25.55	1.84	2.26	0.68
time (sec)	N/A	0.198	1.114	0.526	0.109	0.084	21.041	0.484	0.173	37.738

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	80	120	66	0	490	0	212	15	146
N.S.	1	0.48	0.72	0.40	0.00	2.93	0.00	1.27	0.09	0.87
time (sec)	N/A	0.513	1.692	0.959	0.000	0.121	0.000	0.536	0.190	37.756

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	81	67	197	0	474	0	320	15	81
N.S.	1	0.47	0.39	1.13	0.00	2.72	0.00	1.84	0.09	0.47
time (sec)	N/A	0.450	5.161	4.730	0.000	0.158	0.000	0.549	0.210	37.505

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	150	417	82	0	2010	0	0	15	1176
N.S.	1	0.36	0.99	0.19	0.00	4.77	0.00	0.00	0.04	2.79
time (sec)	N/A	0.712	5.111	3.668	0.000	0.394	0.000	0.000	0.200	37.645

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	171	361	276	0	638	0	439	15	1615
N.S.	1	0.55	1.16	0.89	0.00	2.05	0.00	1.41	0.05	5.19
time (sec)	N/A	0.726	9.110	36.854	0.000	0.313	0.000	0.496	0.245	37.925

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	221	208	280	0	2470	0	0	15	961
N.S.	1	0.70	0.66	0.89	0.00	7.87	0.00	0.00	0.05	3.06
time (sec)	N/A	0.863	5.507	92.793	0.000	0.946	0.000	0.000	0.339	38.363

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	28	14	17	20	10	13	17	13
N.S.	1	1.00	2.33	1.17	1.42	1.67	0.83	1.08	1.42	1.08
time (sec)	N/A	0.183	0.025	0.144	0.028	0.080	0.119	0.486	0.175	37.896

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	147	71	0	439	10462	317	15	368
N.S.	1	1.00	1.28	0.62	0.00	3.82	90.97	2.76	0.13	3.20
time (sec)	N/A	0.428	5.891	0.371	0.000	0.095	19.690	0.470	0.188	37.521

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	416	121	0	1621	0	2036	15	4652
N.S.	1	1.00	1.96	0.57	0.00	7.65	0.00	9.60	0.07	21.94
time (sec)	N/A	0.625	5.132	0.845	0.000	0.265	0.000	2.395	0.185	37.806

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	18	18	16	15	34	233	49	52	15
N.S.	1	0.46	0.46	0.41	0.38	0.87	5.97	1.26	1.33	0.38
time (sec)	N/A	0.187	0.018	0.171	0.107	0.082	2.852	0.467	0.180	0.002

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	206	48	138	0	419	0	264	14	59
N.S.	1	0.71	0.17	0.48	0.00	1.44	0.00	0.91	0.05	0.20
time (sec)	N/A	0.500	5.062	0.603	0.000	0.122	0.000	0.474	0.179	37.054

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	92	82	76	86	141	0	200	14	72
N.S.	1	0.65	0.58	0.54	0.61	0.99	0.00	1.41	0.10	0.51
time (sec)	N/A	0.335	5.137	1.859	0.109	0.115	0.000	0.432	0.186	38.052

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	144	74	0	1977	0	0	14	1169
N.S.	1	1.00	1.02	0.52	0.00	14.02	0.00	0.00	0.10	8.29
time (sec)	N/A	0.403	5.120	2.359	0.000	0.389	0.000	0.000	0.206	37.653

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	29	14	16	22	10	13	17	13
N.S.	1	1.00	2.42	1.17	1.33	1.83	0.83	1.08	1.42	1.08
time (sec)	N/A	0.168	0.022	0.128	0.027	0.065	0.116	0.381	0.182	0.027

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	146	71	0	434	10462	317	14	368
N.S.	1	1.00	1.29	0.63	0.00	3.84	92.58	2.81	0.12	3.26
time (sec)	N/A	0.379	5.895	0.338	0.000	0.103	18.616	0.430	0.185	37.298

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	414	121	0	1612	0	2033	14	4652
N.S.	1	1.00	1.97	0.58	0.00	7.68	0.00	9.68	0.07	22.15
time (sec)	N/A	0.555	5.136	0.801	0.000	0.254	0.000	2.446	0.192	37.307

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	197	15319	54	62	33
N.S.	1	1.00	1.00	0.88	0.00	5.97	464.21	1.64	1.88	1.00
time (sec)	N/A	0.203	0.033	0.154	0.000	0.094	10.923	0.433	0.178	0.002

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	156	132	101	0	835	0	342	16	601
N.S.	1	1.88	1.59	1.22	0.00	10.06	0.00	4.12	0.19	7.24
time (sec)	N/A	0.344	0.780	0.421	0.000	0.175	0.000	0.500	0.191	37.812

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	148	71	0	16697	0	0	16	513
N.S.	1	1.00	0.85	0.41	0.00	95.41	0.00	0.00	0.09	2.93
time (sec)	N/A	0.532	5.080	1.532	0.000	1.528	0.000	0.000	0.202	38.828

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	213	174	88	0	643307	0	0	16	818
N.S.	1	1.18	0.96	0.49	0.00	3554.18	0.00	0.00	0.09	4.52
time (sec)	N/A	0.499	5.145	1.589	0.000	6.083	0.000	0.000	0.216	39.908

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	41	39	0	152	102	50	47	46
N.S.	1	1.12	1.00	0.95	0.00	3.71	2.49	1.22	1.15	1.12
time (sec)	N/A	0.206	0.061	0.169	0.000	0.094	1.693	0.397	0.174	36.624

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	111	75	0	18599	0	0	16	570
N.S.	1	1.00	0.49	0.33	0.00	82.66	0.00	0.00	0.07	2.53
time (sec)	N/A	0.595	5.062	0.426	0.000	0.972	0.000	0.000	0.183	36.672

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	149	109	0	0	0	0	16	1515
N.S.	1	1.00	0.39	0.28	0.00	0.00	0.00	0.00	0.04	3.94
time (sec)	N/A	0.904	5.138	1.068	0.000	0.000	0.000	0.000	0.195	43.499

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	22	190	15745	49	427	31
N.S.	1	1.00	1.00	0.79	0.76	6.55	542.93	1.69	14.72	1.07
time (sec)	N/A	0.201	0.035	0.175	0.112	0.090	10.521	0.465	0.186	0.002

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	541	148	101	0	823	0	318	12	407
N.S.	1	6.22	1.70	1.16	0.00	9.46	0.00	3.66	0.14	4.68
time (sec)	N/A	1.313	4.675	0.499	0.000	0.200	0.000	0.618	0.185	38.498

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	171	148	68	0	15501	0	0	12	513
N.S.	1	0.96	0.83	0.38	0.00	86.60	0.00	0.00	0.07	2.87
time (sec)	N/A	0.501	5.085	1.478	0.000	1.519	0.000	0.000	0.181	40.265

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	257	174	85	0	665483	0	0	12	816
N.S.	1	1.36	0.92	0.45	0.00	3521.07	0.00	0.00	0.06	4.32
time (sec)	N/A	0.689	5.150	1.583	0.000	6.084	0.000	0.000	0.217	43.080

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	40	39	0	148	100	48	45	45
N.S.	1	1.15	1.00	0.98	0.00	3.70	2.50	1.20	1.12	1.12
time (sec)	N/A	0.208	0.041	0.125	0.000	0.092	1.665	0.451	0.175	40.905

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	111	75	0	18599	0	0	12	570
N.S.	1	1.00	0.50	0.33	0.00	83.03	0.00	0.00	0.05	2.54
time (sec)	N/A	0.540	5.062	0.411	0.000	0.977	0.000	0.000	0.181	44.375

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	149	109	0	0	0	0	12	1515
N.S.	1	1.00	0.39	0.28	0.00	0.00	0.00	0.00	0.03	3.95
time (sec)	N/A	0.800	5.139	1.019	0.000	0.000	0.000	0.000	0.183	42.463

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	42	34	104	46	376	43	39	51
N.S.	1	1.07	0.71	0.58	1.76	0.78	6.37	0.73	0.66	0.86
time (sec)	N/A	0.389	0.015	12.324	0.035	0.088	0.528	0.490	0.171	36.440

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	34	26	69	37	233	34	31	42
N.S.	1	1.04	0.74	0.57	1.50	0.80	5.07	0.74	0.67	0.91
time (sec)	N/A	0.331	0.004	2.516	0.037	0.085	0.263	0.453	0.179	36.191

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	20	40	28	110	25	23	33
N.S.	1	1.00	0.79	0.61	1.21	0.85	3.33	0.76	0.70	1.00
time (sec)	N/A	0.266	0.004	0.889	0.032	0.078	0.133	0.480	0.174	35.892

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	17	12	15	17	10	11
N.S.	1	1.00	1.00	0.81	1.06	0.75	0.94	1.06	0.62	0.69
time (sec)	N/A	0.148	0.002	0.233	0.032	0.080	0.019	0.462	0.173	35.627

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	10	17	6	10	6
N.S.	1	1.00	1.00	1.17	1.00	1.67	2.83	1.00	1.67	1.00
time (sec)	N/A	0.221	0.003	0.122	0.028	0.067	0.168	0.451	0.179	0.002

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	15	14	14	19	144	14	27	13
N.S.	1	1.00	0.83	0.78	0.78	1.06	8.00	0.78	1.50	0.72
time (sec)	N/A	0.236	0.010	0.275	0.034	0.067	0.615	0.437	0.167	36.695

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	26	23	20	22	25	362	22	39	21
N.S.	1	0.90	0.79	0.69	0.76	0.86	12.48	0.76	1.34	0.72
time (sec)	N/A	0.247	0.011	0.515	0.037	0.070	1.945	0.467	0.185	36.620

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	32	27	24	28	31	675	28	51	33
N.S.	1	0.86	0.73	0.65	0.76	0.84	18.24	0.76	1.38	0.89
time (sec)	N/A	0.252	0.012	0.966	0.028	0.067	5.765	0.407	0.175	36.830

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	42	39	32	34	37	1083	34	63	43
N.S.	1	0.82	0.76	0.63	0.67	0.73	21.24	0.67	1.24	0.84
time (sec)	N/A	0.255	0.014	1.637	0.035	0.077	15.969	0.361	0.182	36.467

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	39	38	40	49	0	111	72	0
N.S.	1	1.08	0.54	0.53	0.56	0.68	0.00	1.54	1.00	0.00
time (sec)	N/A	0.504	0.038	0.698	0.148	0.078	0.000	0.520	0.193	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	33	32	31	40	0	84	52	0
N.S.	1	1.06	0.62	0.60	0.58	0.75	0.00	1.58	0.98	0.00
time (sec)	N/A	0.403	0.026	0.601	0.147	0.088	0.000	0.559	0.175	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	24	17	26	0	57	34	0
N.S.	1	1.00	0.68	0.71	0.50	0.76	0.00	1.68	1.00	0.00
time (sec)	N/A	0.330	0.015	0.194	0.148	0.073	0.000	0.473	0.182	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	6	15	0	27	14	46
N.S.	1	1.00	1.00	1.15	0.46	1.15	0.00	2.08	1.08	3.54
time (sec)	N/A	0.264	0.032	0.185	0.143	0.071	0.000	0.382	0.177	0.240

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	38	65	0	0	27	0
N.S.	1	1.00	1.00	1.38	2.38	4.06	0.00	0.00	1.69	0.00
time (sec)	N/A	0.268	0.013	0.242	0.159	0.098	0.000	0.000	0.169	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	26	41	304	40	0	47	32	0
N.S.	1	1.00	0.62	0.98	7.24	0.95	0.00	1.12	0.76	0.00
time (sec)	N/A	0.349	0.025	0.230	0.162	0.083	0.000	0.472	0.187	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	69	36	63	933	49	0	63	39	0
N.S.	1	1.13	0.59	1.03	15.30	0.80	0.00	1.03	0.64	0.00
time (sec)	N/A	0.436	0.060	0.243	0.304	0.087	0.000	0.466	0.174	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	70	38	29	30	31	272	28	34	36
N.S.	1	1.17	0.63	0.48	0.50	0.52	4.53	0.47	0.57	0.60
time (sec)	N/A	0.355	0.060	11.949	0.029	0.076	0.466	0.447	0.182	35.899

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	30	23	24	25	173	22	26	30
N.S.	1	1.12	0.71	0.55	0.57	0.60	4.12	0.52	0.62	0.71
time (sec)	N/A	0.256	0.097	2.420	0.028	0.075	0.247	0.452	0.172	36.100

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	87	16	18	24
N.S.	1	1.00	0.92	0.71	0.67	0.79	3.62	0.67	0.75	1.00
time (sec)	N/A	0.174	0.074	0.822	0.029	0.074	0.122	0.462	0.168	36.104

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.00	0.71	0.71	0.71
time (sec)	N/A	0.142	0.037	0.231	0.033	0.077	0.020	0.467	0.186	36.334

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	11	9	8	7	21	41	20	25	16
N.S.	1	0.42	0.35	0.31	0.27	0.81	1.58	0.77	0.96	0.62
time (sec)	N/A	0.172	0.010	0.299	0.109	0.080	0.315	0.453	0.170	0.002

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	30	33	21	20	48	357	32	74	30
N.S.	1	0.68	0.75	0.48	0.45	1.09	8.11	0.73	1.68	0.68
time (sec)	N/A	0.243	3.150	0.618	0.110	0.097	1.571	0.439	0.176	36.436

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	53	37	29	35	69	860	41	120	43
N.S.	1	0.85	0.60	0.47	0.56	1.11	13.87	0.66	1.94	0.69
time (sec)	N/A	0.343	5.118	1.148	0.108	0.083	5.336	0.450	0.175	36.020

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	54	131	0	0	0	0	68	0
N.S.	1	1.14	0.68	1.66	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.633	0.287	5.514	0.000	0.000	0.000	0.000	0.177	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	48	123	0	0	0	0	50	0
N.S.	1	1.09	0.84	2.16	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.485	0.207	3.915	0.000	0.000	0.000	0.000	0.186	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	39	37	115	0	0	0	0	32	0
N.S.	1	1.08	1.03	3.19	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.367	0.088	1.704	0.000	0.000	0.000	0.000	0.191	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	50	0	0	5	0	11	5
N.S.	1	1.00	1.00	7.14	0.00	0.00	0.71	0.00	1.57	0.71
time (sec)	N/A	0.165	0.032	1.276	0.000	0.000	2.231	0.000	0.178	0.019

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	41	5	0	24	0
N.S.	1	1.00	1.00	1.11	0.00	4.56	0.56	0.00	2.67	0.00
time (sec)	N/A	0.167	0.050	0.186	0.000	0.093	2.202	0.000	0.174	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	35	52	0	141	0	0	28	0
N.S.	1	1.00	1.17	1.73	0.00	4.70	0.00	0.00	0.93	0.00
time (sec)	N/A	0.243	0.078	0.700	0.000	0.096	0.000	0.000	0.173	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	69	51	190	0	180	0	0	36	0
N.S.	1	1.17	0.86	3.22	0.00	3.05	0.00	0.00	0.61	0.00
time (sec)	N/A	0.499	0.416	0.746	0.000	0.104	0.000	0.000	0.187	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	87	57	196	0	216	0	0	40	0
N.S.	1	1.10	0.72	2.48	0.00	2.73	0.00	0.00	0.51	0.00
time (sec)	N/A	0.627	0.439	1.658	0.000	0.104	0.000	0.000	0.181	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	70	38	29	30	31	274	28	34	36
N.S.	1	1.17	0.63	0.48	0.50	0.52	4.57	0.47	0.57	0.60
time (sec)	N/A	0.360	0.058	11.888	0.035	0.078	0.475	0.372	0.179	37.101

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	30	23	24	25	173	22	26	30
N.S.	1	1.12	0.71	0.55	0.57	0.60	4.12	0.52	0.62	0.71
time (sec)	N/A	0.252	0.094	2.389	0.029	0.085	0.243	0.415	0.177	37.276

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	87	16	18	24
N.S.	1	1.00	0.92	0.71	0.67	0.79	3.62	0.67	0.75	1.00
time (sec)	N/A	0.175	0.075	0.819	0.031	0.076	0.123	0.443	0.183	37.356

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.00	0.71	0.71	0.71
time (sec)	N/A	0.143	0.038	0.234	0.032	0.080	0.020	0.393	0.181	37.188

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	27	16	15	34	32	17	57	7
N.S.	1	1.00	2.45	1.45	1.36	3.09	2.91	1.55	5.18	0.64
time (sec)	N/A	0.170	0.008	0.177	0.030	0.075	0.180	0.437	0.170	0.002

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	43	32	27	68	292	29	146	22
N.S.	1	1.00	1.43	1.07	0.90	2.27	9.73	0.97	4.87	0.73
time (sec)	N/A	0.238	3.207	0.391	0.033	0.085	0.786	0.435	0.178	36.978

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	65	48	42	95	755	38	232	35
N.S.	1	1.10	1.35	1.00	0.88	1.98	15.73	0.79	4.83	0.73
time (sec)	N/A	0.348	5.246	0.533	0.031	0.100	2.532	0.430	0.191	36.940

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	76	80	64	54	119	1421	44	318	47
N.S.	1	1.15	1.21	0.97	0.82	1.80	21.53	0.67	4.82	0.71
time (sec)	N/A	0.454	5.165	0.643	0.033	0.096	7.854	0.527	0.185	36.976

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	52	133	0	0	0	0	68	0
N.S.	1	1.14	0.66	1.68	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.634	0.220	5.520	0.000	0.000	0.000	0.000	0.216	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	64	53	125	0	0	0	0	50	0
N.S.	1	1.19	0.98	2.31	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.489	0.152	4.010	0.000	0.000	0.000	0.000	0.202	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	38	37	117	0	0	0	0	32	0
N.S.	1	1.03	1.00	3.16	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.370	0.124	1.701	0.000	0.000	0.000	0.000	0.204	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	52	0	0	5	0	11	5
N.S.	1	1.00	1.00	7.43	0.00	0.00	0.71	0.00	1.57	0.71
time (sec)	N/A	0.165	0.033	1.287	0.000	0.000	2.236	0.000	0.187	0.013

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	45	5	0	24	0
N.S.	1	1.00	1.00	1.11	0.00	5.00	0.56	0.00	2.67	0.00
time (sec)	N/A	0.167	0.047	0.197	0.000	0.094	2.178	0.000	0.201	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	35	52	0	165	0	0	28	0
N.S.	1	1.00	1.17	1.73	0.00	5.50	0.00	0.00	0.93	0.00
time (sec)	N/A	0.243	0.080	0.732	0.000	0.107	0.000	0.000	0.195	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	51	193	0	216	0	0	36	0
N.S.	1	1.07	0.86	3.27	0.00	3.66	0.00	0.00	0.61	0.00
time (sec)	N/A	0.501	0.422	0.753	0.000	0.111	0.000	0.000	0.212	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	94	57	204	0	264	0	0	40	0
N.S.	1	1.19	0.72	2.58	0.00	3.34	0.00	0.00	0.51	0.00
time (sec)	N/A	0.622	0.419	1.685	0.000	0.131	0.000	0.000	0.197	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	113	110	108	123	410	118	147	147
N.S.	1	1.07	0.81	0.79	0.77	0.88	2.93	0.84	1.05	1.05
time (sec)	N/A	0.495	0.204	12.470	0.035	0.148	0.508	0.418	0.189	35.839

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	92	80	70	71	81	246	76	89	118
N.S.	1	1.06	0.92	0.80	0.82	0.93	2.83	0.87	1.02	1.36
time (sec)	N/A	0.319	0.126	2.767	0.028	0.080	0.262	0.438	0.207	30.498

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	41	39	47	110	42	44	44
N.S.	1	1.00	0.86	0.82	0.78	0.94	2.20	0.84	0.88	0.88
time (sec)	N/A	0.196	0.076	0.915	0.028	0.081	0.129	0.428	0.181	36.342

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	16	15	17	15	15
N.S.	1	1.00	1.00	0.84	0.89	0.84	0.79	0.89	0.79	0.79
time (sec)	N/A	0.148	0.036	0.255	0.029	0.074	0.019	0.451	0.189	36.876

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	22	190	15745	49	427	31
N.S.	1	1.00	1.00	0.79	0.76	6.55	542.93	1.69	14.72	1.07
time (sec)	N/A	0.197	0.033	0.170	0.113	0.095	10.544	0.396	0.186	0.002

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	67	74	383	0	86	1666	61
N.S.	1	1.00	1.08	1.03	1.14	5.89	0.00	1.32	25.63	0.94
time (sec)	N/A	0.296	5.190	0.417	0.109	0.098	0.000	0.410	0.288	37.075

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	120	106	124	188	737	0	168	3695	150
N.S.	1	1.12	0.99	1.16	1.76	6.89	0.00	1.57	34.53	1.40
time (sec)	N/A	0.463	5.619	0.819	0.118	0.115	0.000	0.421	0.591	37.068

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	180	170	202	346	1229	0	272	6476	305
N.S.	1	1.17	1.10	1.31	2.25	7.98	0.00	1.77	42.05	1.98
time (sec)	N/A	0.683	6.132	1.514	0.117	0.140	0.000	0.489	0.877	36.431

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	155	159	330	0	0	0	0	56	0
N.S.	1	1.02	1.05	2.17	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.961	0.675	4.931	0.000	0.000	0.000	0.000	0.199	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	110	115	191	0	0	0	0	32	0
N.S.	1	0.99	1.04	1.72	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.689	0.445	2.602	0.000	0.000	0.000	0.000	0.190	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	46	48	0	0	10	0	11	0
N.S.	1	0.97	1.24	1.30	0.00	0.00	0.27	0.00	0.30	0.00
time (sec)	N/A	0.258	0.069	1.496	0.000	0.000	3.133	0.000	0.182	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	45	37	0	286	10	0	22	0
N.S.	1	0.97	1.22	1.00	0.00	7.73	0.27	0.00	0.59	0.00
time (sec)	N/A	0.256	0.073	0.296	0.000	0.103	3.387	0.000	0.181	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	71	71	72	0	854	0	0	34	0
N.S.	1	0.99	0.99	1.00	0.00	11.86	0.00	0.00	0.47	0.00
time (sec)	N/A	0.364	0.128	0.807	0.000	0.134	0.000	0.000	0.190	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	168	141	396	0	1406	0	0	46	0
N.S.	1	1.02	0.85	2.40	0.00	8.52	0.00	0.00	0.28	0.00
time (sec)	N/A	0.968	0.888	1.181	0.000	0.175	0.000	0.000	0.187	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [16] had the largest ratio of [1.3000000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.25	27	0.074
2	A	2	2	1.00	8	0.250
3	A	4	3	0.46	10	0.300
4	A	8	8	1.06	10	0.800
5	A	6	6	1.00	10	0.600
6	A	4	4	1.00	10	0.400
7	A	4	4	1.00	10	0.400
8	A	6	6	1.00	10	0.600
9	A	8	8	1.13	10	0.800
10	A	12	12	0.82	10	1.200
11	A	8	8	0.89	10	0.800
12	A	6	6	0.94	10	0.600
13	A	6	6	0.90	10	0.600
14	A	8	8	0.87	10	0.800
15	A	12	12	0.79	10	1.200
16	A	13	13	0.70	10	1.300
17	A	9	9	0.77	10	0.900
18	A	5	5	0.81	10	0.500
19	A	6	5	1.00	10	0.500
20	A	6	5	0.56	10	0.500
21	A	6	5	0.46	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	4	1.00	14	0.286
23	A	4	4	1.00	14	0.286
24	A	4	4	1.19	14	0.286
25	A	4	4	1.00	14	0.286
26	A	4	4	1.00	14	0.286
27	A	4	4	1.00	14	0.286
28	A	4	4	1.00	12	0.333
29	A	4	4	1.00	12	0.333
30	A	4	4	1.00	12	0.333
31	A	4	4	1.00	12	0.333
32	A	4	4	1.00	14	0.286
33	A	4	4	1.00	14	0.286
34	A	4	3	0.48	10	0.300
35	A	4	3	0.44	10	0.300
36	A	4	3	0.42	10	0.300
37	A	6	5	1.00	10	0.500
38	A	4	3	1.00	10	0.300
39	A	4	3	1.00	10	0.300
40	A	4	3	1.00	10	0.300
41	A	4	3	1.00	10	0.300
42	A	4	3	1.00	10	0.300
43	A	4	3	1.00	10	0.300
44	A	6	5	1.00	10	0.500
45	A	4	3	0.42	10	0.300
46	A	4	3	0.44	10	0.300
47	A	4	3	0.48	8	0.375
48	A	4	3	0.46	10	0.300
49	A	6	5	1.00	11	0.455
50	A	4	3	1.00	10	0.300
51	A	4	3	1.00	11	0.273
52	A	4	3	0.46	10	0.300
53	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	5	1.00	11	0.455
55	A	6	5	0.94	11	0.455
56	A	10	9	0.48	11	0.818
57	C	10	9	0.47	11	0.818
58	A	10	9	0.36	11	0.818
59	A	10	9	0.55	11	0.818
60	C	10	9	0.70	11	0.818
61	A	2	2	1.00	9	0.222
62	A	3	3	1.00	11	0.273
63	A	3	3	1.00	11	0.273
64	A	4	3	0.46	10	0.300
65	A	9	8	0.71	10	0.800
66	A	6	5	0.65	10	0.500
67	A	6	5	1.00	10	0.500
68	A	2	2	1.00	8	0.250
69	A	3	3	1.00	10	0.300
70	A	3	3	1.00	10	0.300
71	A	4	3	1.00	11	0.273
72	A	5	4	1.88	11	0.364
73	A	6	5	1.00	11	0.455
74	A	6	5	1.18	11	0.455
75	A	5	4	1.12	9	0.444
76	A	3	3	1.00	11	0.273
77	A	3	3	1.00	11	0.273
78	A	4	3	1.00	10	0.300
79	B	10	9	6.22	10	0.900
80	A	6	5	0.96	10	0.500
81	A	6	5	1.36	10	0.500
82	A	5	4	1.15	8	0.500
83	A	3	3	1.00	10	0.300
84	A	3	3	1.00	10	0.300
85	A	11	11	1.07	11	1.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	9	1.04	11	0.818
87	A	7	7	1.00	11	0.636
88	A	1	1	1.00	9	0.111
89	A	6	5	1.00	11	0.455
90	A	6	5	1.00	11	0.455
91	A	6	5	0.90	11	0.455
92	A	6	5	0.86	11	0.455
93	A	6	5	0.82	11	0.455
94	A	12	12	1.08	13	0.923
95	A	10	10	1.06	13	0.769
96	A	8	8	1.00	13	0.615
97	A	6	6	1.00	13	0.462
98	A	6	6	1.00	13	0.462
99	A	8	8	1.00	13	0.615
100	A	10	10	1.13	13	0.769
101	A	7	7	1.17	10	0.700
102	A	5	5	1.12	10	0.500
103	A	2	2	1.00	10	0.200
104	A	1	1	1.00	8	0.125
105	A	4	3	0.42	10	0.300
106	A	7	6	0.68	10	0.600
107	A	10	9	0.85	10	0.900
108	A	14	14	1.14	12	1.167
109	A	11	11	1.09	12	0.917
110	A	8	8	1.08	12	0.667
111	A	2	2	1.00	12	0.167
112	A	2	2	1.00	12	0.167
113	A	5	5	1.00	12	0.417
114	A	11	11	1.17	12	0.917
115	A	13	13	1.10	12	1.083
116	A	7	7	1.17	10	0.700
117	A	4	4	1.12	10	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	10	0.200
119	A	1	1	1.00	8	0.125
120	A	4	3	1.00	10	0.300
121	A	7	6	1.00	10	0.600
122	A	10	9	1.10	10	0.900
123	A	12	11	1.15	10	1.100
124	A	14	14	1.14	12	1.167
125	A	10	10	1.19	12	0.833
126	A	8	8	1.03	12	0.667
127	A	2	2	1.00	12	0.167
128	A	2	2	1.00	12	0.167
129	A	5	5	1.00	12	0.417
130	A	11	11	1.07	12	0.917
131	A	13	13	1.19	12	1.083
132	A	6	6	1.07	10	0.600
133	A	4	4	1.06	10	0.400
134	A	2	2	1.00	10	0.200
135	A	1	1	1.00	8	0.125
136	A	4	3	1.00	10	0.300
137	A	8	7	1.00	10	0.700
138	A	10	9	1.12	10	0.900
139	A	12	11	1.17	10	1.100
140	A	13	13	1.02	12	1.083
141	A	11	11	0.99	12	0.917
142	A	4	4	0.97	12	0.333
143	A	4	4	0.97	12	0.333
144	A	7	7	0.99	12	0.583
145	A	14	14	1.02	12	1.167

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = \frac{x}{4} - \frac{1}{8}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2x}{\sqrt{1+\sqrt{2}}}\right) + \frac{\arctan\left(\sqrt{2}x\right)}{4\sqrt{2}} + \frac{1}{8}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}+2x}{\sqrt{1+\sqrt{2}}}\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x}{1+\sqrt{2}x^2}\right)$$

output

```
1/4*x-1/8*(1+2^(1/2))^(1/2)*arctan(((2^(1/2)-1)^(1/2)-2*x)/(1+2^(1/2))^(1/
2))+1/8*arctan(x*2^(1/2))*2^(1/2)+1/8*(1+2^(1/2))^(1/2)*arctan(((2^(1/2)-1
)^(1/2)+2*x)/(1+2^(1/2))^(1/2))+1/8*(2^(1/2)-1)^(1/2)*arctanh((-2+2*2^(1/2
))^(1/2)*x/(1+x^2*2^(1/2)))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = \frac{\sqrt{2}x+\sqrt{1-i}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}-\frac{i}{2}}}\right)+\sqrt{1+i}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}+\frac{i}{2}}}\right)+\arctan\left(\sqrt{2}x\right)}{4\sqrt{2}}$$

input
```
Integrate[(1 + x^2)^3/(1 + 4*x^2 + 6*x^4 + 4*x^6),x]
```

output
```
(Sqrt[2]*x + Sqrt[1 - I]*ArcTan[x/Sqrt[1/2 - I/2]] + Sqrt[1 + I]*ArcTan[x/
Sqrt[1/2 + I/2]] + ArcTan[Sqrt[2]*x])/(4*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^3}{4x^6+6x^4+4x^2+1}dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{1}{4(2x^2+1)}+\frac{x^2+1}{2(2x^4+2x^2+1)}+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt{2}-1}-2x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\arctan\left(\sqrt{2}x\right)}{4\sqrt{2}}+\frac{\arctan\left(\frac{2x+\sqrt{\sqrt{2}-1}}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(2x^2-2\sqrt{\sqrt{2}-1}x+\sqrt{2}\right)+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}x^2+\sqrt{2\left(\sqrt{2}-1\right)}x+1\right)+\frac{x}{4}$$

input
```
Int[(1 + x^2)^3/(1 + 4*x^2 + 6*x^4 + 4*x^6),x]
```

output
```
x/4 - ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*x)/Sqrt[1 + Sqrt[2]]]/(8*Sqrt[-1 + Sq
rt[2]]) + ArcTan[Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[(Sqrt[-1 + Sqrt[2]] + 2*x
)/Sqrt[1 + Sqrt[2]]]/(8*Sqrt[-1 + Sqrt[2]]) - (Sqrt[-1 + Sqrt[2]]*Log[Sqrt
[2] - 2*Sqrt[-1 + Sqrt[2]]*x + 2*x^2])/16 + (Sqrt[-1 + Sqrt[2]]*Log[1 + Sq
rt[2*(-1 + Sqrt[2])]*x + Sqrt[2]*x^2])/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.26

method	result
risch	$\frac{x}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^{4}+2\_Z^{2}+1\right)}\_R\ln\left(x+\_R\right)\right)}{8}+\frac{\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{8}$
default	$\frac{x}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\sqrt{2}+\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,x+2x^{2}\right)}{4}+\frac{\left(-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{4}+2\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4x}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}\right)}{8}+\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(-\sqrt{\right.\right.}{}}{}$

input
```
int((x^2+1)^3/(4*x^6+6*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sum(_R*ln(x+_R),_R=RootOf(2*_Z^4+2*_Z^2+1))+1/8*arctan(2^(1/2)*x
)*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$
\begin{aligned}
\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx
&= \frac{1}{8}\sqrt{2}\arctan\left(\sqrt{2}x\right) \\
&\quad + \frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(\left(2\sqrt{2}x+\left(\sqrt{2}-1\right)^{\frac{3}{2}}-2x\right)\sqrt{\sqrt{2}+1}\right) \\
&\quad - \frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(-\left(2\sqrt{2}x-\left(\sqrt{2}-1\right)^{\frac{3}{2}}-2x\right)\sqrt{\sqrt{2}+1}\right) \\
&\quad + \frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(2x^2+2x\sqrt{\sqrt{2}-1}+\sqrt{2}\right) \\
&\quad - \frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(2x^2-2x\sqrt{\sqrt{2}-1}+\sqrt{2}\right)+\frac{1}{4}x
\end{aligned}
$$

input
```
integrate((x^2+1)^3/(4*x^6+6*x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*arctan(sqrt(2)*x) + 1/8*sqrt(sqrt(2) + 1)*arctan((2*sqrt(2)*x
+ (sqrt(2) - 1)^(3/2) - 2*x)*sqrt(sqrt(2) + 1)) - 1/8*sqrt(sqrt(2) + 1)*ar
ctan(-(2*sqrt(2)*x - (sqrt(2) - 1)^(3/2) - 2*x)*sqrt(sqrt(2) + 1)) + 1/16*
sqrt(sqrt(2) - 1)*log(2*x^2 + 2*x*sqrt(sqrt(2) - 1) + sqrt(2)) - 1/16*sqrt
(sqrt(2) - 1)*log(2*x^2 - 2*x*sqrt(sqrt(2) - 1) + sqrt(2)) + 1/4*x
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.11

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = \frac{x}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{8}$$

input
```
integrate((x**2+1)**3/(4*x**6+6*x**4+4*x**2+1),x)
```

output
```
x/4 + sqrt(2)*atan(sqrt(2)*x)/8
```

Maxima [F]

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = \int \frac{(x^2+1)^3}{4\,x^6+6\,x^4+4\,x^2+1}\,dx$$

input
```
integrate((x^2+1)^3/(4*x^6+6*x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*arctan(sqrt(2)*x) + 1/4*x + 1/2*integrate((x^2 + 1)/(2*x^4 + 2
*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\sqrt{2}x\right) + \frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(2x+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(2x-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(x^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{\frac{1}{2}}\right) - \frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(x^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{\frac{1}{2}}\right) + \frac{1}{4}x$$

input
```
integrate((x^2+1)^3/(4*x^6+6*x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(sqrt(2)*x) + 1/8*sqrt(sqrt(2) + 1)*arctan(2*(1/2)^(3/4)
*(2*x + (1/2)^(1/4)*sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt
(2) + 1)*arctan(2*(1/2)^(3/4)*(2*x - (1/2)^(1/4)*sqrt(-sqrt(2) + 2))/sqrt(
sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) - 1)*log(x^2 + (1/2)^(1/4)*x*sqrt(-sqrt(
2) + 2) + sqrt(1/2)) - 1/16*sqrt(sqrt(2) - 1)*log(x^2 - (1/2)^(1/4)*x*sqrt
(-sqrt(2) + 2) + sqrt(1/2)) + 1/4*x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = \frac{x}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{8} + \mathrm{atan}\left(\left(\sqrt{2}\,x\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i} - \sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i} + \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right)\right) + \mathrm{atan}\left(\left(\sqrt{2}\,x\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i} + \sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i} - \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right)\right)$$

input
```
int((x^2 + 1)^3/(4*x^2 + 6*x^4 + 4*x^6 + 1),x)
```

output
```
x/4 + (2^(1/2)*atan(2^(1/2)*x))/8 + atan(2^(1/2)*x*(- 2^(1/2)/256 - 1/256)
^(1/2)*8i - 2^(1/2)*x*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)/256 - 1/
256)^(1/2)*2i + (2^(1/2)/256 - 1/256)^(1/2)*2i) + atan(2^(1/2)*x*(- 2^(1/2
)/256 - 1/256)^(1/2)*8i + 2^(1/2)*x*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^
(1/2)/256 - 1/256)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \frac{(1+x^2)^3}{1+4x^2+6x^4+4x^6}\,dx = -\frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}-2x}{\sqrt{\sqrt{2}+1}}\right)}{8} + \frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}+2x}{\sqrt{\sqrt{2}+1}}\right)}{8} + \frac{\sqrt{2}\,atan\left(\frac{2x}{\sqrt{2}}\right)}{8} - \frac{\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}\,x^2+1\right)}{16} + \frac{\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}\,x^2+1\right)}{16} + \frac{x}{4}$$

input
```
int((x^2+1)^3/(4*x^6+6*x^4+4*x^2+1),x)
```

output
```
( - 2*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1) - 2*x)/sqrt(sqrt(2) + 1))
+ 2*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1) + 2*x)/sqrt(sqrt(2) + 1)) +
2*sqrt(2)*atan((2*x)/sqrt(2)) - sqrt(sqrt(2) - 1)*log( - sqrt(sqrt(2) - 1)
*sqrt(2)*x + sqrt(2)*x**2 + 1) + sqrt(sqrt(2) - 1)*log(sqrt(sqrt(2) - 1)*s
qrt(2)*x + sqrt(2)*x**2 + 1) + 4*x)/16
```

3.2 $\int \frac{1}{3+\cos(2x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{x}{2\sqrt{2}} - \frac{\arctan\left(\frac{\sin(2x)}{3+2\sqrt{2}+\cos(2x)}\right)}{2\sqrt{2}}$$

output
```
1/4*x*2^(1/2)-1/4*arctan(sin(2*x)/(3+2*2^(1/2)+cos(2*x)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(3 + Cos[2*x])^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(2x)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(2x+\frac{\pi}{2}\right)+3}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{x}{2\sqrt{2}}-\frac{\arctan\left(\frac{\sin(2x)}{\cos(2x)+2\sqrt{2}+3}\right)}{2\sqrt{2}}$$

input
```
Int[(3 + Cos[2*x])^(-1),x]
```

output
```
x/(2*Sqrt[2]) - ArcTan[Sin[2*x]/(3 + 2*Sqrt[2] + Cos[2*x])]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.34

method	result	size
derivativedivides	$\frac{\sqrt{2}\, \arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{4}$	14
default	$\frac{\sqrt{2}\, \arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{4}$	14
risch	$\frac{i\sqrt{2}\, \ln\left(\mathrm{e}^{2ix}+3+2\sqrt{2}\right)}{8}-\frac{i\sqrt{2}\, \ln\left(\mathrm{e}^{2ix}+3-2\sqrt{2}\right)}{8}$	40

input
```
int(1/(3+cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{1}{3+\cos(2x)}\,dx = -\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(2\,x\right)+\sqrt{2}}{4\,\sin\left(2\,x\right)}\right)$$

input
```
integrate(1/(3+cos(2*x)),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(2*x) + sqrt(2))/sin(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan\left(x\right)}{2}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}$$

input
```
integrate(1/(3+cos(2*x)),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x)/2) + pi*floor((x - pi/2)/pi))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(2\,x\right)}{2\left(\cos\left(2\,x\right)+1\right)}\right)$$

input
```
integrate(1/(3+cos(2*x)),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*sin(2*x)/(cos(2*x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{1}{4}\,\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2\,x\right)-\sin\left(2\,x\right)}{\sqrt{2}\cos\left(2\,x\right)+\sqrt{2}-\cos\left(2\,x\right)+1}\right)\right)$$

input
```
integrate(1/(3+cos(2*x)),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 38.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}$$

input
```
int(1/(cos(2*x) + 3),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/4 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.29

$$\int \frac{1}{3+\cos(2x)}\,dx = \frac{\sqrt{2}\,atan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{4}$$

input
```
int(1/(3+cos(2*x)),x)
```

output
```
(sqrt(2)*atan(tan(x)/sqrt(2)))/4
```

3.3 $\int \frac{1}{2+2\cos^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{x}{2\sqrt{2}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{2\sqrt{2}}$$

output
```
1/4*x*2^(1/2)-1/4*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(2 + 2*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\cos^2(x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{2\sin\left(x+\frac{\pi}{2}\right)^2+2}dx$$

$$\Big\downarrow \text{3660}$$

$$-\int \frac{1}{4\cot^2(x)+2}d\cot(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{2\sqrt{2}}$$

input
```
Int[(2 + 2*Cos[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{4}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+3+2\sqrt{2}\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+3-2\sqrt{2}\right)}{8}$	40

input
```
int(1/(2+2*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{2+2\cos^2(x)}\,dx=-\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(2+2*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}$$

input
```
integrate(1/(2+2*cos(x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(2+2*cos(x)^2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$

input
```
integrate(1/(2+2*cos(x)^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 38.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{\sqrt{2}\,(x-\mathrm{atan}(\tan(x)))}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}$$

input
```
int(1/(2*cos(x)^2 + 2),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/4 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1}{2+2\cos^2(x)}\,dx = \frac{\sqrt{2}\left(-atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{4}$$

input
```
int(1/(2+2*cos(x)^2),x)
```

output
```
(sqrt(2)*( - atan((sqrt(2) - 2*tan(x/2))/sqrt(2)) + atan((sqrt(2) + 2*tan(
x/2))/sqrt(2))))/4
```

3.4 $\int (a\sin^2(x))^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int (a\sin^2(x))^{5/2}\,dx = -\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{4}{15}a\cot(x)\,(a\sin^2(x))^{3/2}-\frac{1}{5}\cot(x)\,(a\sin^2(x))^{5/2}$$

output
```
-8/15*a^2*cot(x)*(a*sin(x)^2)^(1/2)-4/15*a*cot(x)*(a*sin(x)^2)^(3/2)-1/5*c
ot(x)*(a*sin(x)^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int (a\sin^2(x))^{5/2}\,dx = -\frac{1}{240}a^2(150\cos(x)-25\cos(3x)+3\cos(5x))\csc(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[(a*Sin[x]^2)^(5/2),x]
```

output
```
-1/240*(a^2*(150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[a*Sin[x]^2
])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3682, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\sin^2(x)\right)^{3/2} dx - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \left(a\sin(x)^2\right)^{3/2} dx - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin^2(x)}dx - \frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right) - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin(x)^2}dx - \frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right) - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx - \frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right) - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx - \frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right) - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{4}{5}a\left(-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

input
```
Int[(a*Sin[x]^2)^(5/2),x]
```

output
```
-1/5*(Cot[x]*(a*Sin[x]^2)^(5/2)) + (4*a*((-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3
- (Cot[x]*(a*Sin[x]^2)^(3/2))/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$-\frac{a^{3} \sin(x) \cos(x) \left(3 \sin(x)^{4}+4 \sin(x)^{2}+8\right)}{15 \sqrt{a \sin(x)^{2}}}$
risch	$-\frac{i a^{2} e^{6 i x} \sqrt{-a(e^{2 i x}-1)^{2} e^{-2 i x}}}{160(e^{2 i x}-1)}-\frac{5 i a^{2} e^{2 i x} \sqrt{-a(e^{2 i x}-1)^{2} e^{-2 i x}}}{16(e^{2 i x}-1)}-\frac{5 i \sqrt{-a(e^{2 i x}-1)^{2} e^{-2 i x}}\, a^{2}}{16(e^{2 i x}-1)}+\frac{5 i a^{2} e^{-2 i x} \sqrt{-a(e^{2 i x}-1)^{2} e^{-2}}}{96(e^{2 i x}-1)}$

input
```
int((a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^3*sin(x)*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \left(a\sin^2(x)\right)^{5/2} dx = -\frac{\left(3\,a^2\cos\left(x\right)^5-10\,a^2\cos\left(x\right)^3+15\,a^2\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^2+a}}{15\,\sin\left(x\right)}$$

input
```
integrate((a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x))*sqrt(-a*cos(x)^2
+ a)/sin(x)
```

Sympy [F]

$$\int \left(a\sin^2(x)\right)^{5/2} dx = \int \left(a\sin^{2}\left(x\right)\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((a*sin(x)**2)**(5/2),x)
```

output
```
Integral((a*sin(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = \int \left(a\sin\left(x\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = -\frac{1}{15}\left(\left(3\cos\left(x\right)^5 - 10\cos\left(x\right)^3 + 15\cos\left(x\right)\right)\operatorname{sgn}(\sin\left(x\right)) - 8\operatorname{sgn}(\sin\left(x\right))\right)a^{\frac{5}{2}}$$

input
```
integrate((a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/15*((3*cos(x)^5 - 10*cos(x)^3 + 15*cos(x))*sgn(sin(x)) - 8*sgn(sin(x)))
*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx=\int \left(a\sin\left(x\right)^2\right)^{5/2}\,dx$$

input
```
int((a*sin(x)^2)^(5/2),x)
```

output
```
int((a*sin(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx=\frac{\sqrt{a}\,a^2\left(-3\cos\left(x\right)\sin\left(x\right)^4-4\cos\left(x\right)\sin\left(x\right)^2-8\cos\left(x\right)+8\right)}{15}$$

input
```
int((a*sin(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 3*cos(x)*sin(x)**4 - 4*cos(x)*sin(x)**2 - 8*cos(x) + 8))
/15
```

3.5 $\int \left(a\sin^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx=-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

output
```
-2/3*a*cot(x)*(a*sin(x)^2)^(1/2)-1/3*cot(x)*(a*sin(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx=\frac{1}{12}a(-9\cos(x)+\cos(3x))\csc(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[(a*Sin[x]^2)^(3/2),x]
```

output
```
(a*(-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[a*Sin[x]^2])/12
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\sin^2(x)}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\int \sqrt{a\sin(x)^2}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

input
```
Int[(a*Sin[x]^2)^(3/2),x]
```

output
```
(-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	s
default	$-\frac{a^{2} \sin \left(x \right) \cos \left(x \right) \left(\sin \left(x \right)^{2}+2\right)}{3 \sqrt{a \sin \left(x \right)^{2}}}$	2
risch	$\frac{i a \,{\mathrm e}^{4 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \,{\mathrm e}^{2 i x}-24}-\frac{3 i a \,{\mathrm e}^{2 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}-1\right)}-\frac{3 i \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}\, a}{8 \left({\mathrm e}^{2 i x}-1\right)}+\frac{i a \,{\mathrm e}^{-2 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \,{\mathrm e}^{2 i x}-24}$	1

input
```
int((a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*sin(x)*cos(x)*(sin(x)^2+2)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \frac{\left(a\cos\left(x\right)^3 - 3\,a\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^2 + a}}{3\,\sin\left(x\right)}$$

input
```
integrate((a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(x)^3 - 3*a*cos(x))*sqrt(-a*cos(x)^2 + a)/sin(x)
```

Sympy [F]

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \int \left(a\sin^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)**2)**(3/2),x)
```

output
```
Integral((a*sin(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \int \left(a\sin\left(x\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a\sin^2(x)\right)^{3/2} \, dx = \frac{1}{3}\left(\left(\cos\left(x\right)^3 - 3\,\cos\left(x\right)\right)\mathrm{sgn}(\sin\left(x\right)) + 2\,\mathrm{sgn}(\sin\left(x\right))\right)a^{\frac{3}{2}}$$

input
```
integrate((a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*((cos(x)^3 - 3*cos(x))*sgn(sin(x)) + 2*sgn(sin(x)))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^2(x)\right)^{3/2} \, dx = \int \left(a\sin\left(x\right)^2\right)^{3/2} \, dx$$

input
```
int((a*sin(x)^2)^(3/2),x)
```

output
```
int((a*sin(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \left(a\sin^2(x)\right)^{3/2} \, dx = \frac{\sqrt{a}\,a\left(-\cos\left(x\right)\sin\left(x\right)^2 - 2\cos\left(x\right) + 2\right)}{3}$$

input
```
int((a*sin(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*a*( - cos(x)*sin(x)**2 - 2*cos(x) + 2))/3
```

3.6 $\int \sqrt{a \sin^2(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{a \sin^2(x)}\, dx = -\cot(x)\sqrt{a \sin^2(x)}$$

output
```
-cot(x)*(a*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{a \sin^2(x)}\, dx = -\cot(x)\sqrt{a \sin^2(x)}$$

input
```
Integrate[Sqrt[a*Sin[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a \sin(x)^2}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\csc(x)\sqrt{a \sin^2(x)}\int \sin(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\csc(x)\sqrt{a \sin^2(x)}\int \sin(x)dx$$

$$\Big\downarrow \text{ 3118}$$

$$-\cot(x)\sqrt{a \sin^2(x)}$$

input
```
Int[Sqrt[a*Sin[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{a\cos(x)\sin(x)}{\sqrt{a\sin(x)^2}}$	16
risch	$-\frac{i\sqrt{-a(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}\,\mathrm{e}^{2ix}}{2(\mathrm{e}^{2ix}-1)}-\frac{i\sqrt{-a(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}{2(\mathrm{e}^{2ix}-1)}$	69

input
```
int((a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*sin(x)^2)^(1/2)*a*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{-a\cos\left(x\right)^2 + a}\cos\left(x\right)}{\sin\left(x\right)}$$

input
```
integrate((a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-a*cos(x)^2 + a)*cos(x)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{a\sin^2\left(x\right)}\cos\left(x\right)}{\sin\left(x\right)}$$

input
```
integrate((a*sin(x)**2)**(1/2),x)
```

output
```
-sqrt(a*sin(x)**2)*cos(x)/sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{a}}{\sqrt{\tan\left(x\right)^2 + 1}}$$

input
```
integrate((a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a)/sqrt(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \sqrt{a\sin^2(x)}\,dx = -(\cos(x)\,\mathrm{sgn}(\sin(x)) - \mathrm{sgn}(\sin(x)))\sqrt{a}$$

input
```
integrate((a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-(cos(x)*sgn(sin(x)) - sgn(sin(x)))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 37.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{2\sin(x)^2}\,\left(-\sin(x)^2 + \frac{\sin(2\,x)\,1\mathrm{i}}{2} + 1\right)}{\sin(x)^2\,2\mathrm{i} + \sin(2\,x)}$$

input
```
int((a*sin(x)^2)^(1/2),x)
```

output
```
-(2^(1/2)*a^(1/2)*(2*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(si
n(2*x) + sin(x)^2*2i)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.43

$$\int \sqrt{a\sin^2(x)}\,dx = -\sqrt{a}\,\cos(x)$$

input
```
int((a*sin(x)^2)^(1/2),x)
```

output
```
- sqrt(a)*cos(x)
```

3.7 $\int \frac{1}{\sqrt{a \sin^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{\sqrt{a \sin^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{a \sin^2(x)}}$$

output
```
-arctanh(cos(x))*sin(x)/(a*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \sin^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{a \sin^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Sin[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sin^2(x)}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a \sin(x)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x) \int \csc(x)dx}{\sqrt{a \sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x) \int \csc(x)dx}{\sqrt{a \sin^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{\sqrt{a \sin^2(x)}}$$

input
```
Int[1/Sqrt[a*Sin[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

method	result	size
default	$-\frac{\sin(x)\sqrt{a\cos(x)^2}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cos(x)^2}+2a}{\sin(x)}\right)}{\sqrt{a}\,\cos(x)\sqrt{a\sin(x)^2}}$	49
risch	$-\frac{2\ln(e^{ix}+1)\sin(x)}{\sqrt{-a(e^{2ix}-1)^2e^{-2ix}}}+\frac{2\ln(e^{ix}-1)\sin(x)}{\sqrt{-a(e^{2ix}-1)^2e^{-2ix}}}$	64

input
```
int(1/(a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)*(a*cos(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)/sin
(x))/cos(x)/(a*sin(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \left[\frac{\sqrt{-a\cos(x)^2+a}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{2\,a\sin(x)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos(x)^2+a}\sqrt{-a}\cos(x)}{a\sin(x)}\right)}{a}\right]$$

input
```
integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a*cos(x)^2 + a)*log(-(cos(x) - 1)/(cos(x) + 1))/(a*sin(x)), sqr
t(-a)*arctan(sqrt(-a*cos(x)^2 + a)*sqrt(-a)*cos(x)/(a*sin(x)))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sin^2(x)}}\,dx$$

input
```
integrate(1/(a*sin(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \frac{\sqrt{-a}(\arctan(\sin(x),\cos(x)+1)-\arctan(\sin(x),\cos(x)-1))}{a}$$

input
```
integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(\sin(x))}{\sqrt{-a}} + \frac{\arctan(i\cos(x))}{\sqrt{-a}\operatorname{sgn}(\sin(x))}$$

input
```
integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-arctan(sqrt(a)/sqrt(-a))*sgn(sin(x))/sqrt(-a) + arctan(I*cos(x))/(sqrt(-a
)*sgn(sin(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^2}}\,dx$$

input
```
int(1/(a*sin(x)^2)^(1/2),x)
```

output
```
int(1/(a*sin(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \frac{\sqrt{a}\log\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

input
```
int(1/(a*sin(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*log(tan(x/2)))/a
```

3.8 $\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}} - \frac{\operatorname{arctanh}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}$$

output
```
-1/2*cot(x)/a/(a*sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/a/(a*sin(x)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = -\frac{\left(\csc^2\left(\frac{x}{2}\right) + 4\log\left(\cos\left(\frac{x}{2}\right)\right) - 4\log\left(\sin\left(\frac{x}{2}\right)\right) - \sec^2\left(\frac{x}{2}\right)\right)\sin^3(x)}{8\left(a\sin^2(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sin[x]^2)^(-3/2),x]
```

output
```
-1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]
^3)/(a*Sin[x]^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{\int \frac{1}{\sqrt{a\sin^2(x)}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin(x)^2}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

input
```
Int[(a*Sin[x]^2)^(-3/2),x]
```

output
```
-1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Si
n[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\sqrt{a\cos(x)^{2}}\left(\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cos(x)^{2}}+2a}{\sin(x)}\right)\sin(x)^{2}a+\sqrt{a}\,\sqrt{a\cos(x)^{2}}\right)}{2a^{\frac{5}{2}}\sin(x)\cos(x)\sqrt{a\sin(x)^{2}}}$	70
risch	$-\frac{i\left(\mathrm{e}^{2ix}+1\right)}{a\left(\mathrm{e}^{2ix}-1\right)\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}}}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)\sin(x)}{a\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}}}+\frac{\ln\left(\mathrm{e}^{ix}-1\right)\sin(x)}{a\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}}}$	110

input
```
int(1/(a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(5/2)/sin(x)*(a*cos(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a
)/sin(x))*sin(x)^2*a+a^(1/2)*(a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a\sin^2(x))^{3/2}}\,dx = \frac{\sqrt{-a\cos(x)^2+a}\left((\cos(x)^2-1)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)+2\cos(x)\right)}{4\,(a^2\cos(x)^2-a^2)\sin(x)}$$

input
```
integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-a*cos(x)^2 + a)*((cos(x)^2 - 1)*log(-(cos(x) - 1)/(cos(x) + 1))
+ 2*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(a\sin^2(x))^{3/2}}\,dx = \int \frac{1}{(a\sin^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**2)**(3/2),x)
```

output
```
Integral((a*sin(x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(34) = 68$.

Time = 0.16 (sec) , antiderivative size = 314, normalized size of antiderivative = 7.48

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx =$$
$$-\frac{\left(\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2\right.\right.}{}$$

input
```
integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^
2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), c
os(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(si
n(x), cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))
*sin(4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*s
in(2*x) - 4*cos(2*x)*sin(x) + 2*sin(x))*sqrt(-a)/(a^2*cos(4*x)^2 + 4*a^2*c
os(2*x)^2 + a^2*sin(4*x)^2 - 4*a^2*sin(4*x)*sin(2*x) + 4*a^2*sin(2*x)^2 -
4*a^2*cos(2*x) + a^2 - 2*(2*a^2*cos(2*x) - a^2)*cos(4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(34) = 68$.

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.93

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = -\frac{\frac{\left(\frac{2\left(\cos(x)-1\right)}{\cos(x)+1}-1\right)\left(\cos(x)+1\right)}{\sqrt{a}\left(\cos(x)-1\right)\operatorname{sgn}\left(\sin(x)\right)} - \frac{2\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{\sqrt{a}\operatorname{sgn}\left(\sin(x)\right)} + \frac{\cos(x)-1}{\sqrt{a}\left(\cos(x)+1\right)\operatorname{sgn}\left(\sin(x)\right)}}{8a}$$

input
```
integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*((2*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(sqrt(a)*(cos(x) - 1)
*sgn(sin(x))) - 2*log(-(cos(x) - 1)/(cos(x) + 1))/(sqrt(a)*sgn(sin(x))) +
(cos(x) - 1)/(sqrt(a)*(cos(x) + 1)*sgn(sin(x))))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a*sin(x)^2)^(3/2),x)
```

output
```
int(1/(a*sin(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(-\cos\left(x\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)\sin\left(x\right)^2\right)}{2\sin\left(x\right)^2 a^2}$$

input
```
int(1/(a*sin(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*( - cos(x) + log(tan(x/2))*sin(x)**2))/(2*sin(x)**2*a**2)
```

3.9 $\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx=-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\text{arctanh}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}$$

output
```
-1/4*cot(x)/a/(a*sin(x)^2)^(3/2)-3/8*cot(x)/a^2/(a*sin(x)^2)^(1/2)-3/8*arc
tanh(cos(x))*sin(x)/a^2/(a*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = -\frac{\csc(x)\left(6\csc^2\left(\frac{x}{2}\right)+\csc^4\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-6\sec^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)\right)\sqrt{a\sin^2(x)}}{64a^3}$$

input
```
Integrate[(a*Sin[x]^2)^(-5/2),x]
```

output
```
-1/64*(Csc[x]*(6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2
]]) - 6*Sec[x/2]^2 - Sec[x/2]^4)*Sqrt[a*Sin[x]^2])/a^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sin^2(x)\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a \sin(x)^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 3683}$$
$$\frac{3 \int \frac{1}{\left(a \sin^2(x)\right)^{3/2}}dx}{4a}-\frac{\cot(x)}{4a\left(a \sin^2(x)\right)^{3/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{3 \int \frac{1}{\left(a \sin(x)^2\right)^{3/2}}dx}{4a}-\frac{\cot(x)}{4a\left(a \sin^2(x)\right)^{3/2}}$$
$$\Big\downarrow \text{ 3683}$$
$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a \sin^2(x)}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a \sin^2(x)\right)^{3/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a \sin(x)^2}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a \sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{3\left(\frac{\sin(x)\int\csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\sin(x)\int\csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3\left(-\frac{\sin(x)\text{arctanh}(\cos(x))}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

input
```
Int[(a*Sin[x]^2)^(-5/2),x]
```

output
```
-1/4*Cot[x]/(a*(a*Sin[x]^2)^(3/2)) + (3*(-1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2])
- (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin[x]^2])))/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

method	result	size
default	$-\frac{\sqrt{a\cos(x)^{2}}\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cos(x)^{2}}+2a}{\sin(x)}\right)a\sin(x)^{4}+3\sqrt{a\cos(x)^{2}}\,\sin(x)^{2}\sqrt{a}+2\sqrt{a}\,\sqrt{a\cos(x)^{2}}\right)}{8a^{\frac{7}{2}}\sin(x)^{3}\cos(x)\sqrt{a\sin(x)^{2}}}$	89
risch	$-\frac{i(3\,\mathrm{e}^{6ix}-11\,\mathrm{e}^{4ix}-11\,\mathrm{e}^{2ix}+3)}{4a^{2}(\mathrm{e}^{2ix}-1)^{3}\sqrt{-a(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}+\frac{3\ln(\mathrm{e}^{ix}-1)\sin(x)}{4a^{2}\sqrt{-a(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}-\frac{3\ln(\mathrm{e}^{ix}+1)\sin(x)}{4a^{2}\sqrt{-a(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}$	127

input
```
int(1/(a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^(7/2)/sin(x)^3*(a*cos(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/
2)+a)/sin(x))*a*sin(x)^4+3*(a*cos(x)^2)^(1/2)*sin(x)^2*a^(1/2)+2*a^(1/2)*(
a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{-a\cos\left(x\right)^{2}+a}\left(6\,\cos\left(x\right)^{3}+3\,\left(\cos\left(x\right)^{4}-2\,\cos\left(x\right)^{2}+1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)-10\,\mathrm{co}\right.}{16\,\left(a^{3}\cos\left(x\right)^{4}-2\,a^{3}\cos\left(x\right)^{2}+a^{3}\right)\sin\left(x\right)}$$

input
```
integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(-a*cos(x)^2 + a)*(6*cos(x)^3 + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log
(-(cos(x) - 1)/(cos(x) + 1)) - 10*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)^2
+ a^3)*sin(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**2)**(5/2),x)
```

output
```
Integral((a*sin(x)**2)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 931, normalized size of antiderivative = 15.26

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*(3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^
2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2
*x) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*si
n(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x
)^2 + 8*cos(2*x) - 1)*arctan2(sin(x), cos(x) + 1) - 3*(2*(4*cos(6*x) - 6*c
os(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*
x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4
*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x)
- sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*
sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*arctan
2(sin(x), cos(x) - 1) + 2*(3*sin(7*x) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(
x))*cos(8*x) + 12*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 8*(11*
sin(5*x) + 11*sin(3*x) - 3*sin(x))*cos(6*x) + 44*(3*sin(4*x) - 2*sin(2*x))
*cos(5*x) - 12*(11*sin(3*x) - 3*sin(x))*cos(4*x) - 2*(3*cos(7*x) - 11*cos(
5*x) - 11*cos(3*x) + 3*cos(x))*sin(8*x) - 6*(4*cos(6*x) - 6*cos(4*x) + 4*c
os(2*x) - 1)*sin(7*x) - 8*(11*cos(5*x) + 11*cos(3*x) - 3*cos(x))*sin(6*x)
- 22*(6*cos(4*x) - 4*cos(2*x) + 1)*sin(5*x) + 12*(11*cos(3*x) - 3*cos(x))*
sin(4*x) + 22*(4*cos(2*x) - 1)*sin(3*x) - 88*cos(3*x)*sin(2*x) + 24*cos(x)
*sin(2*x) - 24*cos(2*x)*sin(x) + 6*sin(x))*sqrt(-a)/(a^3*cos(8*x)^2 + 1...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(49) = 98$.

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a\sin^2(x))^{5/2}}\,dx =$$

$$-\frac{\frac{\frac{8\sqrt{a}(\cos(x)-1)\mathrm{sgn}(\sin(x))}{\cos(x)+1}-\frac{\sqrt{a}(\cos(x)-1)^2\mathrm{sgn}(\sin(x))}{(\cos(x)+1)^2}}{a}-\frac{\left(\frac{8(\cos(x)-1)}{\cos(x)+1}-\frac{18(\cos(x)-1)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)^2}{\sqrt{a}(\cos(x)-1)^2\mathrm{sgn}(\sin(x))}-\frac{12\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{\sqrt{a}\mathrm{sgn}(\sin(x))}}{64\,a^2}$$

input
```
integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/64*((8*sqrt(a)*(cos(x) - 1)*sgn(sin(x))/(cos(x) + 1) - sqrt(a)*(cos(x)
- 1)^2*sgn(sin(x))/(cos(x) + 1)^2)/a - (8*(cos(x) - 1)/(cos(x) + 1) - 18*(
cos(x) - 1)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^2/(sqrt(a)*(cos(x) - 1)^2*s
gn(sin(x))) - 12*log(-(cos(x) - 1)/(cos(x) + 1))/(sqrt(a)*sgn(sin(x))))/a^
2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin\left(x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a*sin(x)^2)^(5/2),x)
```

output
```
int(1/(a*sin(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(-3\cos\left(x\right)\sin\left(x\right)^2-2\cos\left(x\right)+3\log\left(\tan\left(\frac{x}{2}\right)\right)\sin\left(x\right)^4\right)}{8\sin\left(x\right)^4a^3}$$

input
```
int(1/(a*sin(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*( - 3*cos(x)*sin(x)**2 - 2*cos(x) + 3*log(tan(x/2))*sin(x)**4))/(
8*sin(x)**4*a**3)
```

3.10 $\int (a\sin^3(x))^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 123

$$\begin{aligned}\int (a\sin^3(x))^{5/2}\,dx &= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)}\\ &\quad-\frac{26a^2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)}-\frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)}\\ &\quad-\frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)}-\frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)}\end{aligned}$$

output
```
-26/77*a^2*cot(x)*(a*sin(x)^3)^(1/2)+26/77*a^2*InverseJacobiAM(-1/4*Pi+1/2
*x,2^(1/2))*(a*sin(x)^3)^(1/2)/sin(x)^(3/2)-78/385*a^2*cos(x)*sin(x)*(a*si
n(x)^3)^(1/2)-26/165*a^2*cos(x)*sin(x)^3*(a*sin(x)^3)^(1/2)-2/15*a^2*cos(x
)*sin(x)^5*(a*sin(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int (a\sin^3(x))^{5/2}\,dx = \frac{a\Big(-12480\,\text{EllipticF}\left(\frac{1}{4}(\pi-2x),2\right)+(-15465\cos(x)+3657\cos(3x)-749\cos(5x)}{36960\sin^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Sin[x]^3)^(5/2),x]
```

output
```
(a*(-12480*EllipticF[(Pi - 2*x)/4, 2] + (-15465*Cos[x] + 3657*Cos[3*x] - 7
49*Cos[5*x] + 77*Cos[7*x])*Sqrt[Sin[x]])*(a*Sin[x]^3)^(3/2))/(36960*Sin[x]
^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sin^3(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin(x)^3\right)^{5/2} dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a^2 \sqrt{a \sin^3(x)} \int \sin^{\frac{15}{2}}(x)dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \sqrt{a \sin^3(x)} \int \sin(x)^{15/2}dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2 \sqrt{a \sin^3(x)} \left(\frac{13}{15} \int \sin^{\frac{11}{2}}(x)dx - \frac{2}{15} \sin^{\frac{13}{2}}(x) \cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \sqrt{a \sin^3(x)} \left(\frac{13}{15} \int \sin(x)^{11/2}dx - \frac{2}{15} \sin^{\frac{13}{2}}(x) \cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3115

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int \sin^{\frac{7}{2}}(x)dx-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3042

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int \sin(x)^{7/2}dx-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3115

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(x)dx-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3042

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int \sin(x)^{3/2}dx-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3115

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}\right.}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3042

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}\right.}{\sin^{\frac{3}{2}}(x)}$$

$\big\downarrow$ 3120

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(-\frac{2}{3}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)-\frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)\right.}{\sin^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sin[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Sin[x]^3]*((-2*Cos[x]*Sin[x]^(13/2))/15 + (13*((-2*Cos[x]*Sin[
x]^(9/2))/11 + (9*((5*((-2*EllipticF[Pi/4 - x/2, 2])/3 - (2*Cos[x]*Sqrt[Si
n[x]])/3))/7 - (2*Cos[x]*Sin[x]^(5/2))/7))/11))/15))/Sin[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

method	result
default	$\frac{\sqrt{a \sin(x)^3}\, a^2 \left(\cot(x)\sqrt{2}\left(77\cos(x)^6-322\cos(x)^4+530\cos(x)^2-480\right)+i\csc(x)^2(195\cos(x)+195)\sqrt{1-i\cot(x)+i\csc(x)}\,\sqrt{i(\csc(x}{2310}$

input
```
int((a*sin(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2310*(a*sin(x)^3)^(1/2)*a^2*(cot(x)*2^(1/2)*(77*cos(x)^6-322*cos(x)^4+53
0*cos(x)^2-480)+I*csc(x)^2*(195*cos(x)+195)*(1-I*cot(x)+I*csc(x))^(1/2)*(I
*(csc(x)-cot(x)))^(1/2)*EllipticF((1+I*cot(x)-I*csc(x))^(1/2),1/2*2^(1/2))
*(1+I*cot(x)-I*csc(x))^(1/2))*8^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \frac{2\left(195\sqrt{-\frac{1}{2}i\,a}a^2\sin(x)\,\text{weierstrassPInverse}(4,0,\cos(x)+i\,\sin(x))+195\sqrt{\frac{1}{2}i\,a}a^2\right.}{}$$

input
```
integrate((a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
2/1155*(195*sqrt(-1/2*I*a)*a^2*sin(x)*weierstrassPInverse(4, 0, cos(x) + I
*sin(x)) + 195*sqrt(1/2*I*a)*a^2*sin(x)*weierstrassPInverse(4, 0, cos(x) -
 I*sin(x)) + (77*a^2*cos(x)^7 - 322*a^2*cos(x)^5 + 530*a^2*cos(x)^3 - 480*
a^2*cos(x))*sqrt(-(a*cos(x)^2 - a)*sin(x)))/sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(x)**3)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \int \left(a\sin\left(x\right)^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \int \left(a\sin\left(x\right)^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \int \left(a\sin\left(x\right)^3\right)^{5/2}\,dx$$

input
```
int((a*sin(x)^3)^(5/2),x)
```

output
```
int((a*sin(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(a\sin^3(x)\right)^{5/2} dx = \sqrt{a}\left(\int \sqrt{\sin(x)}\,\sin(x)^7\,dx\right)a^2$$

input
```
int((a*sin(x)^3)^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(sin(x))*sin(x)**7,x)*a**2
```

3.11 $\int (a\sin^3(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int (a\sin^3(x))^{3/2}\,dx = -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)} - \frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)} - \frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}$$

output
```
-14/45*a*cos(x)*(a*sin(x)^3)^(1/2)-14/15*a*EllipticE(cos(1/4*Pi+1/2*x),2^(
1/2))*(a*sin(x)^3)^(1/2)/sin(x)^(3/2)-2/9*a*cos(x)*sin(x)^2*(a*sin(x)^3)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int (a\sin^3(x))^{3/2}\,dx = \frac{(a\sin^3(x))^{3/2}\left(-168E\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\sqrt{\sin(x)}(-38\sin(2x)+5\sin(4x))\right)}{180\sin^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Sin[x]^3)^(3/2),x]
```

output
```
((a*Sin[x]^3)^(3/2)*(-168*EllipticE[(Pi - 2*x)/4, 2] + Sqrt[Sin[x]]*(-38*S
in[2*x] + 5*Sin[4*x])))/(180*Sin[x]^(9/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sin^3(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin(x)^3\right)^{3/2} dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a\sqrt{a \sin^3(x)} \int \sin^{\frac{9}{2}}(x)dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a \sin^3(x)} \int \sin(x)^{9/2}dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\sqrt{a \sin^3(x)} \left(\frac{7}{9} \int \sin^{\frac{5}{2}}(x)dx - \frac{2}{9} \sin^{\frac{7}{2}}(x) \cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a \sin^3(x)} \left(\frac{7}{9} \int \sin(x)^{5/2}dx - \frac{2}{9} \sin^{\frac{7}{2}}(x) \cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int\sqrt{\sin(x)}dx-\frac{2}{5}\sin^{\frac{3}{2}}(x)\cos(x)\right)-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int\sqrt{\sin(x)}dx-\frac{2}{5}\sin^{\frac{3}{2}}(x)\cos(x)\right)-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3119$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\left(-\frac{6}{5}E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)-\frac{2}{5}\sin^{\frac{3}{2}}(x)\cos(x)\right)-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sin[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Sin[x]^3]*((-2*Cos[x]*Sin[x]^(7/2))/9 + (7*((-6*EllipticE[Pi/4 -
 x/2, 2])/5 - (2*Cos[x]*Sin[x]^(3/2))/5))/9))/Sin[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.41

method	result
default	$-\frac{\csc(x)^2\left((-21\cos(x)-21)\operatorname{EllipticF}\left(\sqrt{1+i\cot(x)-i\csc(x)},\frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(x)+i\csc(x)}\,\sqrt{i(\csc(x)-\cot(x))}\,\sqrt{1+i\cot(x)-i\csc(x}\right.}{}$

input
```
int((a*sin(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/90*csc(x)^2*((-21*cos(x)-21)*EllipticF((1+I*cot(x)-I*csc(x))^(1/2),1/2*
2^(1/2))*(1-I*cot(x)+I*csc(x))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*(1+I*cot(x)
-I*csc(x))^(1/2)+(42*cos(x)+42)*(1-I*cot(x)+I*csc(x))^(1/2)*(I*(csc(x)-cot
(x)))^(1/2)*EllipticE((1+I*cot(x)-I*csc(x))^(1/2),1/2*2^(1/2))*(1+I*cot(x)
-I*csc(x))^(1/2)+(5*cos(x)^5-17*cos(x)^3+33*cos(x)-21)*2^(1/2))*(a*sin(x)^
3)^(1/2)*a*8^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \frac{14}{15}i\sqrt{-\frac{1}{2}i\,a}a\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(x)+i\sin(x))) - \frac{14}{15}i\sqrt{\frac{1}{2}i\,a}a\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(x)-i\sin(x))) + \frac{2}{45}\left(5\,a\cos(x)^3-12\,a\cos(x)\right)\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)}$$

input
```
integrate((a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
14/15*I*sqrt(-1/2*I*a)*a*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
os(x) + I*sin(x))) - 14/15*I*sqrt(1/2*I*a)*a*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(x) - I*sin(x))) + 2/45*(5*a*cos(x)^3 - 12*a*cos(x))
*sqrt(-(a*cos(x)^2 - a)*sin(x))
```

Sympy [F]

$$\int \left(a\sin^3(x)\right)^{3/2} dx = \int \left(a\sin^3{(x)}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sin(x)**3)**(3/2),x)
```

output
```
Integral((a*sin(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\sin^3(x)\right)^{3/2} dx = \int \left(a\sin{(x)}^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \int \left(a\sin\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \int \left(a\sin\left(x\right)^3\right)^{3/2}\,dx$$

input
```
int((a*sin(x)^3)^(3/2),x)
```

output
```
int((a*sin(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \sqrt{a}\left(\int \sqrt{\sin\left(x\right)}\,\sin\left(x\right)^4\,dx\right)a$$

input
```
int((a*sin(x)^3)^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(sin(x))*sin(x)**4,x)*a
```

3.12 $\int \sqrt{a\sin^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \sqrt{a\sin^3(x)}\,dx = -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)} - \frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

output
```
-2/3*cot(x)*(a*sin(x)^3)^(1/2)+2/3*InverseJacobiAM(-1/4*Pi+1/2*x,2^(1/2))*
(a*sin(x)^3)^(1/2)/sin(x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{a\sin^3(x)}\,dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(\pi-2x),2\right)+\cos(x)\sqrt{\sin(x)}\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sqrt[a*Sin[x]^3],x]
```

output
```
(-2*(EllipticF[(Pi - 2*x)/4, 2] + Cos[x]*Sqrt[Sin[x]])*Sqrt[a*Sin[x]^3])/(
3*Sin[x]^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3686, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin^3(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a \sin(x)^3}dx$$

$$\Big\downarrow \ 3686$$

$$\frac{\sqrt{a \sin^3(x)} \int \sin^{\frac{3}{2}}(x)dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a \sin^3(x)} \int \sin(x)^{3/2}dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3115$$

$$\frac{\sqrt{a \sin^3(x)} \left(\frac{1}{3} \int \frac{1}{\sqrt{\sin(x)}}dx - \frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a \sin^3(x)} \left(\frac{1}{3} \int \frac{1}{\sqrt{\sin(x)}}dx - \frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3120$$

$$\frac{\sqrt{a \sin^3(x)} \left(-\frac{2}{3}\text{EllipticF}\left(\frac{\pi}{4} - \frac{x}{2}, 2\right) - \frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Sin[x]^3],x]
```

output
```
(((-2*EllipticF[Pi/4 - x/2, 2])/3 - (2*Cos[x]*Sqrt[Sin[x]])/3)*Sqrt[a*Sin[
x]^3])/Sin[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{a\sin(x)^3}\left(i\csc(x)^2(-1-\cos(x))\sqrt{1-i\cot(x)+i\csc(x)}\,\sqrt{i(\csc(x)-\cot(x))}\ \text{EllipticF}\left(\sqrt{1+i\cot(x)-i\csc(x)},\frac{\sqrt{2}}{2}\right)\sqrt{1+i\cot(x)}\right.}{6}$

input
```
int((a*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*sin(x)^3)^(1/2)*(I*csc(x)^2*(-1-cos(x))*(1-I*cot(x)+I*csc(x))^(1/2
)*(I*(csc(x)-cot(x)))^(1/2)*EllipticF((1+I*cot(x)-I*csc(x))^(1/2),1/2*2^(1
/2))*(1+I*cot(x)-I*csc(x))^(1/2)+2^(1/2)*cot(x))*8^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \sqrt{a\sin^3(x)}\,dx = \frac{2\left(\sqrt{-\frac{1}{2}i\,a}\sin\left(x\right)\text{weierstrassPInverse}(4,0,\cos\left(x\right)+i\,\sin\left(x\right))+\sqrt{\frac{1}{2}i\,a}\sin\left(x\right)\text{weierstrassPInverse}(4,0}{3\,\sin\left(x\right)}$$

input
```
integrate((a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(-1/2*I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x) + I*sin(x)) +
sqrt(1/2*I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x) - I*sin(x)) - sqrt(-
(a*cos(x)^2 - a)*sin(x))*cos(x))/sin(x)
```

Sympy [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin^3\left(x\right)}\,dx$$

input
```
integrate((a*sin(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*sin(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin{(x)}^3}\,dx$$

input
```
integrate((a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin{(x)}^3}\,dx$$

input
```
integrate((a*sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin{(x)}^3}\,dx$$

input
```
int((a*sin(x)^3)^(1/2),x)
```

output
```
int((a*sin(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \sqrt{a}\left(\int \sqrt{\sin(x)}\,\sin(x)\,dx\right)$$

input

```
int((a*sin(x)^3)^(1/2),x)
```

output

```
sqrt(a)*int(sqrt(sin(x))*sin(x),x)
```

3.13 $\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}}+\frac{2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{\sqrt{a\sin^3(x)}}$$

output
```
-2*cos(x)*sin(x)/(a*sin(x)^3)^(1/2)+2*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))
*sin(x)^(3/2)/(a*sin(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \frac{2E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\sin^{\frac{3}{2}}(x)-\sin(2x)}{\sqrt{a\sin^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Sin[x]^3],x]
```

output
```
(2*EllipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2) - Sin[2*x])/Sqrt[a*Sin[x]^3]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3686, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sin^3(x)}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a \sin(x)^3}} dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x) \int \frac{1}{\sin^{\frac{3}{2}}(x)} dx}{\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x) \int \frac{1}{\sin(x)^{3/2}} dx}{\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x) \left(-\int \sqrt{\sin(x)} dx - \frac{2 \cos(x)}{\sqrt{\sin(x)}}\right)}{\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x) \left(-\int \sqrt{\sin(x)} dx - \frac{2 \cos(x)}{\sqrt{\sin(x)}}\right)}{\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{3}{2}}(x) \left(2 E\left(\frac{\pi}{4} - \frac{x}{2} \middle| 2\right) - \frac{2 \cos(x)}{\sqrt{\sin(x)}}\right)}{\sqrt{a \sin^3(x)}}$$

input
```
Int[1/Sqrt[a*Sin[x]^3],x]
```

output
```
((2*EllipticE[Pi/4 - x/2, 2] - (2*Cos[x])/Sqrt[Sin[x]])*Sin[x]^(3/2))/Sqrt
[a*Sin[x]^3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.15

method	result
default	$-\frac{\sin(x)\left(-2\sqrt{1+i(-\csc(x)+\cot(x))}\,\sqrt{1-i(-\csc(x)+\cot(x))}\,\sqrt{-i(-\csc(x)+\cot(x))}\,(\cos(x)+1)\,\text{EllipticE}\left(\sqrt{1+i\cot(x)-i\csc(x}\right.\right.}{}$

input
```
int(1/(a*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sin(x)*(-2*(1+I*(-csc(x)+cot(x)))^(1/2)*(1-I*(-csc(x)+cot(x)))^(1/2)*
(-I*(-csc(x)+cot(x)))^(1/2)*(cos(x)+1)*EllipticE((1+I*cot(x)-I*csc(x))^(1/
2),1/2*2^(1/2))+(1+I*(-csc(x)+cot(x)))^(1/2)*(1-I*(-csc(x)+cot(x)))^(1/2)*
(-I*(-csc(x)+cot(x)))^(1/2)*(cos(x)+1)*EllipticF((1+I*cot(x)-I*csc(x))^(1/
2),1/2*2^(1/2))+2^(1/2))/(a*sin(x)^3)^(1/2)*8^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx =$$
$$-\frac{2\left(\left(i\,\cos\left(x\right)^2-i\right)\sqrt{-\frac{1}{2}i\,a}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(x\right)+i\,\sin\left(x\right)))+(-i\,\mathrm{c}\right.}{}$$

input
```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
-2*((I*cos(x)^2 - I)*sqrt(-1/2*I*a)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(x) + I*sin(x))) + (-I*cos(x)^2 + I)*sqrt(1/2*I*a)*weierstras
sZeta(4, 0, weierstrassPInverse(4, 0, cos(x) - I*sin(x))) - sqrt(-(a*cos(x
)^2 - a)*sin(x))*cos(x))/(a*cos(x)^2 - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin^3{(x)}}}\,dx$$

input
```
integrate(1/(a*sin(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sin(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*sin(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

input
```
int(1/(a*sin(x)^3)^(1/2),x)
```

output
```
int(1/(a*sin(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(x)}}{\sin(x)^2}dx\right)}{a}$$

input
```
int(1/(a*sin(x)^3)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(x))/sin(x)**2,x))/a
```

3.14 $\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 77

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx=-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}-\frac{10\,\mathrm{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)\sin^{\frac{3}{2}}(x)}{21a\sqrt{a\sin^3(x)}}$$

output
```
-10/21*cos(x)/a/(a*sin(x)^3)^(1/2)-2/7*cot(x)*csc(x)/a/(a*sin(x)^3)^(1/2)+
10/21*InverseJacobiAM(-1/4*Pi+1/2*x,2^(1/2))*sin(x)^(3/2)/a/(a*sin(x)^3)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = -\frac{2\sin^2(x)\left(3\cot(x)+5\cos(x)\sin(x)+5\,\mathrm{EllipticF}\left(\frac{1}{4}(\pi-2x),2\right)\sin^{\frac{5}{2}}(x)\right)}{21\left(a\sin^3(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sin[x]^3)^(-3/2),x]
```

output
```
(-2*Sin[x]^2*(3*Cot[x] + 5*Cos[x]*Sin[x] + 5*EllipticF[(Pi - 2*x)/4, 2]*Si
n[x]^(5/2)))/(21*(a*Sin[x]^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sin^3(x)\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a \sin(x)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x) \int \frac{1}{\sin^{\frac{9}{2}}(x)}dx}{a\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x) \int \frac{1}{\sin(x)^{9/2}}dx}{a\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x) \left(\frac{5}{7} \int \frac{1}{\sin^{\frac{5}{2}}(x)}dx - \frac{2 \cos(x)}{7 \sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x) \left(\frac{5}{7} \int \frac{1}{\sin(x)^{5/2}}dx - \frac{2 \cos(x)}{7 \sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a \sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(x)}}dx-\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(x)}\right)-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(x)}}dx-\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(x)}\right)-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(-\frac{2}{3}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)-\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(x)}\right)-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

input
```
Int[(a*Sin[x]^3)^(-3/2),x]
```

output
```
(((5*((-2*EllipticF[Pi/4 - x/2, 2])/3 - (2*Cos[x])/(3*Sin[x]^(3/2))))/7 -
(2*Cos[x])/(7*Sin[x]^(7/2)))*Sin[x]^(3/2))/(a*Sqrt[a*Sin[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

method	result
default	$\frac{\left(i \sin(x)(5\cos(x)+5)\,\mathrm{EllipticF}\left(\sqrt{1+i\cot(x)-i\csc(x)}, \frac{\sqrt{2}}{2}\right)\sqrt{1+i\cot(x)-i\csc(x)}\,\sqrt{1-i\cot(x)+i\csc(x)}\,\sqrt{i(\csc(x)-\cot(x))}+\cot(\right.}{42\sqrt{a\sin(x)^3}\,a}$

input
```
int(1/(a*sin(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/42/(a*sin(x)^3)^(1/2)/a*(I*sin(x)*(5*cos(x)+5)*EllipticF((1+I*cot(x)-I*c
sc(x))^(1/2),1/2*2^(1/2))*(1+I*cot(x)-I*csc(x))^(1/2)*(1-I*cot(x)+I*csc(x)
)^(1/2)*(I*(csc(x)-cot(x)))^(1/2)+cot(x)*csc(x)*2^(1/2)*(5*cos(x)^2-8))*8^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \frac{2\left(5\left(\cos\left(x\right)^4-2\cos\left(x\right)^2+1\right)\sqrt{-\frac{1}{2}i\,a}\sin\left(x\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)+i\right.}{}$$

input
```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(5*(cos(x)^4 - 2*cos(x)^2 + 1)*sqrt(-1/2*I*a)*sin(x)*weierstrassPInve
rse(4, 0, cos(x) + I*sin(x)) + 5*(cos(x)^4 - 2*cos(x)^2 + 1)*sqrt(1/2*I*a)
*sin(x)*weierstrassPInverse(4, 0, cos(x) - I*sin(x)) + (5*cos(x)^3 - 8*cos
(x))*sqrt(-(a*cos(x)^2 - a)*sin(x)))/((a^2*cos(x)^4 - 2*a^2*cos(x)^2 + a^2
)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(a\sin^3(x))^{3/2}}\,dx = \int \frac{1}{(a\sin^3(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**3)**(3/2),x)
```

output
```
Integral((a*sin(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\sin^3(x))^{3/2}}\,dx = \int \frac{1}{(a\sin(x)^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(a*sin(x)^3)^(3/2),x)
```

output
```
int(1/(a*sin(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(x)}}{\sin(x)^5}dx\right)}{a^2}$$

input
```
int(1/(a*sin(x)^3)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(x))/sin(x)**5,x))/a**2
```

3.15 $\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 123

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} + \frac{154E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{195a^2\sqrt{a\sin^3(x)}}$$

output
```
-154/585*cot(x)/a^2/(a*sin(x)^3)^(1/2)-22/117*cot(x)*csc(x)^2/a^2/(a*sin(x
)^3)^(1/2)-2/13*cot(x)*csc(x)^4/a^2/(a*sin(x)^3)^(1/2)-154/195*cos(x)*sin(
x)/a^2/(a*sin(x)^3)^(1/2)+154/195*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin
(x)^(3/2)/a^2/(a*sin(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = -\frac{2\left(\cot(x)\left(77+55\csc^2(x)+45\csc^4(x)\right)+231\cos(x)\sin(x)-231E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\sin^{\frac{3}{2}}(x)\right)}{585a^2\sqrt{a\sin^3(x)}}$$

input
```
Integrate[(a*Sin[x]^3)^(-5/2),x]
```

output
```
(-2*(Cot[x]*(77 + 55*Csc[x]^2 + 45*Csc[x]^4) + 231*Cos[x]*Sin[x] - 231*Ell
ipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2)))/(585*a^2*Sqrt[a*Sin[x]^3])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin(x)^{15/2}}dx}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\sin^{\frac{11}{2}}(x)}dx-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\int\frac{1}{\sin(x)^{11/2}}dx-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\sin^{\frac{7}{2}}(x)}dx-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\sin(x)^{7/2}}dx-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sin^{\frac{3}{2}}(x)}dx-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sin(x)^{3/2}}dx-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\int\sqrt{\sin(x)}dx-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\int\sqrt{\sin(x)}dx-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

input
```
Int[(a*Sin[x]^3)^(-5/2),x]
```

output
```
(((11*((7*((3*(2*EllipticE[Pi/4 - x/2, 2] - (2*Cos[x])/Sqrt[Sin[x]]))/5 -
(2*Cos[x])/(5*Sin[x]^(5/2))))/9 - (2*Cos[x])/(9*Sin[x]^(9/2))))/13 - (2*Co
s[x])/(13*Sin[x]^(13/2)))*Sin[x]^(3/2))/(a^2*Sqrt[a*Sin[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.50

method	result
default	$\frac{\left(\sin(x)(462\cos(x)+462)\sqrt{1+i\cot(x)-i\csc(x)}\,\sqrt{1-i\cot(x)+i\csc(x)}\,\sqrt{i(\csc(x)-\cot(x))}\ \mathrm{EllipticE}\left(\sqrt{1+i\cot(x)-i\csc(x)},\frac{\sqrt{2}}{2}\right)+\right.}{}$

input
```
int(1/(a*sin(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1170/(a*sin(x)^3)^(1/2)/a^2*(sin(x)*(462*cos(x)+462)*(1+I*cot(x)-I*csc(x
))^(1/2)*(1-I*cot(x)+I*csc(x))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticE((
1+I*cot(x)-I*csc(x))^(1/2),1/2*2^(1/2))+sin(x)*(-231*cos(x)-231)*(1+I*cot(
x)-I*csc(x))^(1/2)*(1-I*cot(x)+I*csc(x))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*E
llipticF((1+I*cot(x)-I*csc(x))^(1/2),1/2*2^(1/2))+2^(1/2)*(-231*sin(x)-77*
cot(x)-55*cot(x)*csc(x)^2-45*cot(x)*csc(x)^4))*8^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx =$$

$$-\frac{2\left(231\left(i\cos(x)^8-4i\cos(x)^6+6i\cos(x)^4-4i\cos(x)^2+i\right)\sqrt{-\frac{1}{2}i\,a}\text{weierstrassZeta}(4,0,\text{weierstrassP}$$

input
```
integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
-2/585*(231*(I*cos(x)^8 - 4*I*cos(x)^6 + 6*I*cos(x)^4 - 4*I*cos(x)^2 + I)*
sqrt(-1/2*I*a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(x) + I*
sin(x))) + 231*(-I*cos(x)^8 + 4*I*cos(x)^6 - 6*I*cos(x)^4 + 4*I*cos(x)^2 -
 I)*sqrt(1/2*I*a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(x) -
 I*sin(x))) - (231*cos(x)^7 - 770*cos(x)^5 + 902*cos(x)^3 - 408*cos(x))*sq
rt(-(a*cos(x)^2 - a)*sin(x)))/(a^3*cos(x)^8 - 4*a^3*cos(x)^6 + 6*a^3*cos(x
)^4 - 4*a^3*cos(x)^2 + a^3)
```

Sympy [F]

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx = \int \frac{1}{(a\sin^3{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**3)**(5/2),x)
```

output
```
Integral((a*sin(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx = \int \frac{1}{\left(a\sin{(x)}^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx = \int \frac{1}{\left(a\sin{(x)}^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin{(x)}^3\right)^{5/2}}\,dx$$

input
```
int(1/(a*sin(x)^3)^(5/2),x)
```

output
```
int(1/(a*sin(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(x)}}{\sin(x)^8}dx\right)}{a^3}$$

input
```
int(1/(a*sin(x)^3)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(x))/sin(x)**8,x))/a**3
```

3.16 $\int \left(a\sin^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)}+\frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)} -\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} -\frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)}-\frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)}$$

output
```
-63/256*a^2*cot(x)*(a*sin(x)^4)^(1/2)+63/256*a^2*x*csc(x)^2*(a*sin(x)^4)^(
1/2)-21/128*a^2*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-21/160*a^2*cos(x)*sin(x)^
3*(a*sin(x)^4)^(1/2)-9/80*a^2*cos(x)*sin(x)^5*(a*sin(x)^4)^(1/2)-1/10*a^2*
cos(x)*sin(x)^7*(a*sin(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \frac{a\csc^6(x)\left(a\sin^4(x)\right)^{3/2}(2520x-2100\sin(2x)+600\sin(4x)-150\sin(6x)+25\sin(8}{10240}$$

input
```
Integrate[(a*Sin[x]^4)^(5/2),x]
```

output
```
(a*Csc[x]^6*(a*Sin[x]^4)^(3/2)*(2520*x - 2100*Sin[2*x] + 600*Sin[4*x] - 15
0*Sin[6*x] + 25*Sin[8*x] - 2*Sin[10*x]))/10240
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^4(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^4\right)^{5/2} dx$$

$$\Big\downarrow \text{3686}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\int \sin^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\int \sin(x)^{10}dx$$

$$\Big\downarrow \text{3115}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\int \sin^8(x)dx-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\int \sin(x)^8dx-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int \sin^6(x)dx-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int\sin(x)^6dx-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$\big\downarrow$ 3115

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int\sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$\big\downarrow$ 3042

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int\sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$\big\downarrow$ 3115

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)\right)-\right.$$

$\big\downarrow$ 3042

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)\right)-\right.$$

$\big\downarrow$ 3115

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin\right.\right.$$

$\big\downarrow$ 24

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\right.\right.$$

input
```
Int[(a*Sin[x]^4)^(5/2),x]
```

output
```
a^2*Csc[x]^2*Sqrt[a*Sin[x]^4]*(-1/10*(Cos[x]*Sin[x]^9) + (9*(-1/8*(Cos[x]*
Sin[x]^7) + (7*(-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin[x]^3) + (3*(
x/2 - (Cos[x]*Sin[x])/2))/4))/6))/8))/10)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 11.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

method	result
default	$-\frac{\sqrt{a\sin(x)^4}\,a^2\left(\cot(x)\left(128\cos(x)^8-656\cos(x)^6+1368\cos(x)^4-1490\cos(x)^2+965\right)-315\csc(x)^2x\right)\sqrt{16}}{5120}$
risch	$-\frac{63a^2e^{2ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}\,x}{256(e^{2ix}-1)^2}-\frac{ia^2e^{12ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{10240(e^{2ix}-1)^2}+\frac{5ia^2e^{10ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{4096(e^{2ix}-1)^2}-\frac{105ia^2e^{4ix}\sqrt{a(e^{2ix}-1)^4e^{-}}}{1024(e^{2ix}-1)^2}$

input
```
int((a*sin(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5120*(a*sin(x)^4)^(1/2)*a^2*(cot(x)*(128*cos(x)^8-656*cos(x)^6+1368*cos
(x)^4-1490*cos(x)^2+965)-315*csc(x)^2*x)*16^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.62

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx =$$
$$-\frac{\sqrt{a\cos(x)^4-2\,a\cos(x)^2+a}\,\big(315\,a^2x-\big(128\,a^2\cos(x)^9-656\,a^2\cos(x)^7+1368\,a^2\cos(x)^5-1490\,a^2\,\mathrm{c}}{1280\left(\cos(x)^2-1\right)}$$

input
```
integrate((a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/1280*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(315*a^2*x - (128*a^2*cos(x)^9
 - 656*a^2*cos(x)^7 + 1368*a^2*cos(x)^5 - 1490*a^2*cos(x)^3 + 965*a^2*cos(
x))*sin(x))/(cos(x)^2 - 1)
```

Sympy [F]

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \int \left(a\sin^4(x)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)**4)**(5/2),x)
```

output
```
Integral((a*sin(x)**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \frac{63}{256}\,a^{\frac{5}{2}}x$$
$$-\frac{965\,a^{\frac{5}{2}}\tan(x)^9+2370\,a^{\frac{5}{2}}\tan(x)^7+2688\,a^{\frac{5}{2}}\tan(x)^5+1470\,a^{\frac{5}{2}}\tan(x)^3+315\,a^{\frac{5}{2}}\tan(x)}{1280\left(\tan(x)^{10}+5\,\tan(x)^8+10\,\tan(x)^6+10\,\tan(x)^4+5\,\tan(x)^2+1\right)}$$

input
```
integrate((a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
63/256*a^(5/2)*x - 1/1280*(965*a^(5/2)*tan(x)^9 + 2370*a^(5/2)*tan(x)^7 +
2688*a^(5/2)*tan(x)^5 + 1470*a^(5/2)*tan(x)^3 + 315*a^(5/2)*tan(x))/(tan(x
)^10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.30

$$\int \left(a\sin^4(x)\right)^{5/2} dx = \frac{1}{10240}a^{\frac{5}{2}}(2520x - 2\sin(10x) + 25\sin(8x) - 150\sin(6x) + 600\sin(4x) - 2100$$

input
```
integrate((a*sin(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/10240*a^(5/2)*(2520*x - 2*sin(10*x) + 25*sin(8*x) - 150*sin(6*x) + 600*s
in(4*x) - 2100*sin(2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^4(x)\right)^{5/2} dx = \int \left(a\sin(x)^4\right)^{5/2} dx$$

input
```
int((a*sin(x)^4)^(5/2),x)
```

output
```
int((a*sin(x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \frac{\sqrt{a}\,a^2\left(-128\cos(x)\sin(x)^9-144\cos(x)\sin(x)^7-168\cos(x)\sin(x)^5-210\cos(x\right.}{1280}$$

input
```
int((a*sin(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 128*cos(x)*sin(x)**9 - 144*cos(x)*sin(x)**7 - 168*cos(x)
*sin(x)**5 - 210*cos(x)*sin(x)**3 - 315*cos(x)*sin(x) + 315*x))/1280
```

3.17 $\int (a\sin^4(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int (a\sin^4(x))^{3/2}\,dx = -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)} - \frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}$$

output
```
-5/16*a*cot(x)*(a*sin(x)^4)^(1/2)+5/16*a*x*csc(x)^2*(a*sin(x)^4)^(1/2)-5/2
4*a*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-1/6*a*cos(x)*sin(x)^3*(a*sin(x)^4)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int (a\sin^4(x))^{3/2}\,dx = -\frac{1}{192}\csc^6(x)\left(a\sin^4(x)\right)^{3/2}(-60x+45\sin(2x)-9\sin(4x)+\sin(6x))$$

input
```
Integrate[(a*Sin[x]^4)^(3/2),x]
```

output
```
-1/192*(Csc[x]^6*(a*Sin[x]^4)^(3/2)*(-60*x + 45*Sin[2*x] - 9*Sin[4*x] + Si
n[6*x]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^4\right)^{3/2}dx$$

$$\Big\downarrow \text{3686}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\int \sin^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\int \sin(x)^6dx$$

$$\Big\downarrow \text{3115}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\int \sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\int \sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int\sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

input
```
Int[(a*Sin[x]^4)^(3/2),x]
```

output
```
a*Csc[x]^2*Sqrt[a*Sin[x]^4]*(-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin
[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

method	result
default	$-\frac{\sqrt{a \sin(x)^{4}}\, a \left(\cot(x) \left(8 \cos(x)^{4}-26 \cos(x)^{2}+33\right)-15 \csc(x)^{2} x\right) \sqrt{16}}{192}$
risch	$-\frac{5 a\, \mathrm{e}^{2 i x} \sqrt{a \left(\mathrm{e}^{2 i x}-1\right)^{4} \mathrm{e}^{-4 i x}}\, x}{16 \left(\mathrm{e}^{2 i x}-1\right)^{2}}-\frac{i a\, \mathrm{e}^{8 i x} \sqrt{a \left(\mathrm{e}^{2 i x}-1\right)^{4} \mathrm{e}^{-4 i x}}}{384 \left(\mathrm{e}^{2 i x}-1\right)^{2}}+\frac{3 i a\, \mathrm{e}^{6 i x} \sqrt{a \left(\mathrm{e}^{2 i x}-1\right)^{4} \mathrm{e}^{-4 i x}}}{128 \left(\mathrm{e}^{2 i x}-1\right)^{2}}-\frac{15 i a\, \mathrm{e}^{4 i x} \sqrt{a \left(\mathrm{e}^{2 i x}-1\right)^{4} \mathrm{e}^{-4 i x}}}{128 \left(\mathrm{e}^{2 i x}-1\right)^{2}}+$

input
```
int((a*sin(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(a*sin(x)^4)^(1/2)*a*(cot(x)*(8*cos(x)^4-26*cos(x)^2+33)-15*csc(x)^
2*x)*16^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \left(a \sin^4(x)\right)^{3/2} \, dx = -\frac{\sqrt{a \cos\left(x\right)^{4} - 2 \, a \cos\left(x\right)^{2} + a} \left(15 \, a x - \left(8 \, a \cos\left(x\right)^{5} - 26 \, a \cos\left(x\right)^{3} + 33 \, a \cos\left(x\right)\right) \sin\left(x\right)\right)}{48 \, \left(\cos\left(x\right)^{2} - 1\right)}$$

input
```
integrate((a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/48*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(15*a*x - (8*a*cos(x)^5 - 26*a*c
os(x)^3 + 33*a*cos(x))*sin(x))/(cos(x)^2 - 1)
```

Sympy [F]

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx = \int \left(a\sin^4\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)**4)**(3/2),x)
```

output
```
Integral((a*sin(x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx = \frac{5}{16}\,a^{\frac{3}{2}}x - \frac{33\,a^{\frac{3}{2}}\tan\left(x\right)^5 + 40\,a^{\frac{3}{2}}\tan\left(x\right)^3 + 15\,a^{\frac{3}{2}}\tan\left(x\right)}{48\,\left(\tan\left(x\right)^6 + 3\,\tan\left(x\right)^4 + 3\,\tan\left(x\right)^2 + 1\right)}$$

input
```
integrate((a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
5/16*a^(3/2)*x - 1/48*(33*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 15*a^(3
/2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx = \frac{1}{192}\,a^{\frac{3}{2}}\left(60\,x - \sin\left(6\,x\right) + 9\,\sin\left(4\,x\right) - 45\,\sin\left(2\,x\right)\right)$$

input
```
integrate((a*sin(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/192*a^(3/2)*(60*x - sin(6*x) + 9*sin(4*x) - 45*sin(2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx=\int \left(a\sin\left(x\right)^4\right)^{3/2}\,dx$$

input
```
int((a*sin(x)^4)^(3/2),x)
```

output
```
int((a*sin(x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx=\frac{\sqrt{a}\,a\left(-8\cos\left(x\right)\sin\left(x\right)^5-10\cos\left(x\right)\sin\left(x\right)^3-15\cos\left(x\right)\sin\left(x\right)+15x\right)}{48}$$

input
```
int((a*sin(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*a*( - 8*cos(x)*sin(x)**5 - 10*cos(x)*sin(x)**3 - 15*cos(x)*sin(x)
 + 15*x))/48
```

3.18 $\int \sqrt{a \sin^4(x)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a \sin^4(x)} \, dx = -\frac{1}{2} \cot(x) \sqrt{a \sin^4(x)} + \frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)}$$

output
```
-1/2*cot(x)*(a*sin(x)^4)^(1/2)+1/2*x*csc(x)^2*(a*sin(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sqrt{a \sin^4(x)} \, dx = \frac{1}{2} \csc(x)(-\cos(x) + x \csc(x)) \sqrt{a \sin^4(x)}$$

input
```
Integrate[Sqrt[a*Sin[x]^4],x]
```

output
```
(Csc[x]*(-Cos[x] + x*Csc[x])*Sqrt[a*Sin[x]^4])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin^4(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\sin(x)^4}dx$$

$$\Big\downarrow \; 3686$$

$$\csc^2(x)\sqrt{a\sin^4(x)}\int \sin^2(x)dx$$

$$\Big\downarrow \; 3042$$

$$\csc^2(x)\sqrt{a\sin^4(x)}\int \sin(x)^2dx$$

$$\Big\downarrow \; 3115$$

$$\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 24$$

$$\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[Sqrt[a*Sin[x]^4],x]
```

output
```
Csc[x]^2*Sqrt[a*Sin[x]^4]*(x/2 - (Cos[x]*Sin[x])/2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{\sqrt{a\sin\left(x\right)^{4}}\,\left(\cot\left(x\right)-\csc\left(x\right)^{2}x\right)\sqrt{16}}{8}$	24
risch	$-\frac{\sqrt{a\left(\mathrm{e}^{2 i x}-1\right)^{4}\mathrm{e}^{-4 i x}}\,\mathrm{e}^{2 i x}x}{2\left(\mathrm{e}^{2 i x}-1\right)^{2}}-\frac{i\sqrt{a\left(\mathrm{e}^{2 i x}-1\right)^{4}\mathrm{e}^{-4 i x}}\,\mathrm{e}^{4 i x}}{8\left(\mathrm{e}^{2 i x}-1\right)^{2}}+\frac{i\sqrt{a\left(\mathrm{e}^{2 i x}-1\right)^{4}\mathrm{e}^{-4 i x}}}{8\left(\mathrm{e}^{2 i x}-1\right)^{2}}$	102

input
```
int((a*sin(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*sin(x)^4)^(1/2)*(cot(x)-csc(x)^2*x)*16^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{a\sin^4(x)}\,dx = \frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}(\cos(x)\sin(x) - x)}{2\left(\cos(x)^2 - 1\right)}$$

input
```
integrate((a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(cos(x)*sin(x) - x)/(cos(x)^2 - 1)
```

Sympy [F]

$$\int \sqrt{a\sin^4(x)}\,dx = \int \sqrt{a\sin^4(x)}\,dx$$

input
```
integrate((a*sin(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a*sin(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \sqrt{a\sin^4(x)}\,dx = \frac{1}{2}\sqrt{a}x - \frac{\sqrt{a}\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

input
```
integrate((a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*x - 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \sqrt{a \sin^4(x)}\, dx = \frac{1}{4}\sqrt{a}(2\,x - \sin{(2\,x)})$$

input
```
integrate((a*sin(x)^4)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a)*(2*x - sin(2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \sin^4(x)}\, dx = \int \sqrt{a \sin{(x)}^4}\, dx$$

input
```
int((a*sin(x)^4)^(1/2),x)
```

output
```
int((a*sin(x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.33

$$\int \sqrt{a \sin^4(x)}\, dx = \frac{\sqrt{a}\,(-\cos{(x)}\sin{(x)} + x)}{2}$$

input
```
int((a*sin(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*( - cos(x)*sin(x) + x))/2
```

3.19 $\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}$$

output
```
-cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Sin[x]^4],x]
```

output
```
-((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin^4(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a\sin(x)^4}}dx$$
$$\Big\downarrow \text{3686}$$
$$\frac{\sin^2(x)\int \csc^2(x)dx}{\sqrt{a\sin^4(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sin^2(x)\int \csc(x)^2dx}{\sqrt{a\sin^4(x)}}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\sin^2(x)\int 1d\cot(x)}{\sqrt{a\sin^4(x)}}$$
$$\Big\downarrow \text{24}$$
$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

input
```
Int[1/Sqrt[a*Sin[x]^4],x]
```

output
```
-((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\cos(x)\sin(x)\sqrt{16}}{4\sqrt{a\sin(x)^{4}}}$	18
risch	$\frac{2i\left(1-\mathrm{e}^{-2ix}\right)}{\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^{4}\mathrm{e}^{-4ix}}}$	31

input
```
int(1/(a*sin(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x)*sin(x)/(a*sin(x)^4)^(1/2)*16^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = \frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\cos(x)}{(a\cos(x)^2 - a)\sin(x)}$$

input
```
integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*cos(x)/((a*cos(x)^2 - a)*sin(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = \int \frac{1}{\sqrt{a\sin^4(x)}}\,dx$$

input
```
integrate(1/(a*sin(x)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{1}{\sqrt{a}\tan(x)}$$

input
```
integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(a)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{1}{\sqrt{a}\tan(x)}$$

input
```
integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/(sqrt(a)*tan(x))
```

Mupad [B] (verification not implemented)

Time = 37.63 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{\cot(x)}{\sqrt{a}}$$

input
```
int(1/(a*sin(x)^4)^(1/2),x)
```

output
```
-cot(x)/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{\sqrt{a}\,\cos(x)}{\sin(x)\,a}$$

input
```
int(1/(a*sin(x)^4)^(1/2),x)
```

output
```
( - sqrt(a)*cos(x))/(sin(x)*a)
```

3.20 $\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx=-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{\cos(x)\sin(x)}{a\sqrt{a\sin^4(x)}}$$

output
```
-2/3*cos(x)^2*cot(x)/a/(a*sin(x)^4)^(1/2)-1/5*cos(x)^2*cot(x)^3/a/(a*sin(x
)^4)^(1/2)-cos(x)*sin(x)/a/(a*sin(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx=-\frac{\cos(x)\left(8+4\csc^2(x)+3\csc^4(x)\right)\sin^5(x)}{15\left(a\sin^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sin[x]^4)^(-3/2),x]
```

output
```
-1/15*(Cos[x]*(8 + 4*Csc[x]^2 + 3*Csc[x]^4)*Sin[x]^5)/(a*Sin[x]^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^2(x)\int \csc^6(x)dx}{a\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(x)\int \csc(x)^6dx}{a\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sin^2(x)\int \left(\cot^4(x)+2\cot^2(x)+1\right)d\cot(x)}{a\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin^2(x)\left(\frac{\cot^5(x)}{5}+\frac{2\cot^3(x)}{3}+\cot(x)\right)}{a\sqrt{a\sin^4(x)}}$$

input
```
Int[(a*Sin[x]^4)^(-3/2),x]
```

output
```
-(((Cot[x] + (2*Cot[x]^3)/3 + Cot[x]^5/5)*Sin[x]^2)/(a*Sqrt[a*Sin[x]^4]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\cot(x)\csc(x)^{2}\left(8\cos(x)^{4}-20\cos(x)^{2}+15\right)\sqrt{16}}{60\sqrt{a\sin(x)^{4}}\,a}$	37
risch	$\frac{16i(-5+11\cos(2x)+9i\sin(2x))}{15a(\mathrm{e}^{2ix}-1)^{3}\sqrt{a(\mathrm{e}^{2ix}-1)^{4}\mathrm{e}^{-4ix}}}$	49

input
```
int(1/(a*sin(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*cot(x)*csc(x)^2*(8*cos(x)^4-20*cos(x)^2+15)/(a*sin(x)^4)^(1/2)/a*16^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \frac{\sqrt{a\cos\left(x\right)^4 - 2\,a\cos\left(x\right)^2 + a}\left(8\,\cos\left(x\right)^5 - 20\,\cos\left(x\right)^3 + 15\,\cos\left(x\right)\right)}{15\,\left(a^2\cos\left(x\right)^6 - 3\,a^2\cos\left(x\right)^4 + 3\,a^2\cos\left(x\right)^2 - a^2\right)\sin\left(x\right)}$$

input
```
integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/15*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(8*cos(x)^5 - 20*cos(x)^3 + 15*co
s(x))/((a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**4)**(3/2),x)
```

output
```
Integral((a*sin(x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = -\frac{15\,\tan\left(x\right)^4 + 10\,\tan\left(x\right)^2 + 3}{15\,a^{\frac{3}{2}}\tan\left(x\right)^5}$$

input
```
integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = -\frac{15\,\tan\left(x\right)^4 + 10\,\tan\left(x\right)^2 + 3}{15\,a^{\frac{3}{2}}\tan\left(x\right)^5}$$

input
```
integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

Mupad [B] (verification not implemented)

Time = 38.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \frac{\frac{8\mathrm{i}}{15\,a^{3/2}} - \frac{4\left(2\sin(2\,x)^3 - 9\,\sin(2\,x) + 3\,\sin(4\,x) + 2\mathrm{i}\right)}{15\,a^{3/2}}}{\left(\cos\left(2\,x\right) - 1\right)^3}$$

input
```
int(1/(a*sin(x)^4)^(3/2),x)
```

output
```
(8i/(15*a^(3/2)) - (4*(3*sin(4*x) - 9*sin(2*x) + 2*sin(2*x)^3 + 2i))/(15*a
^(3/2)))/(cos(2*x) - 1)^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,\cos\left(x\right)\left(-8\sin\left(x\right)^4 - 4\sin\left(x\right)^2 - 3\right)}{15\sin\left(x\right)^5 a^2}$$

input
```
int(1/(a*sin(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*cos(x)*( - 8*sin(x)**4 - 4*sin(x)**2 - 3))/(15*sin(x)**5*a**2)
```

3.21 $\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}} - \frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}} - \frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}} - \frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}} - \frac{\cos(x)\sin(x)}{a^2\sqrt{a\sin^4(x)}}$$

output
```
-4/3*cos(x)^2*cot(x)/a^2/(a*sin(x)^4)^(1/2)-6/5*cos(x)^2*cot(x)^3/a^2/(a*s
in(x)^4)^(1/2)-4/7*cos(x)^2*cot(x)^5/a^2/(a*sin(x)^4)^(1/2)-1/9*cos(x)^2*c
ot(x)^7/a^2/(a*sin(x)^4)^(1/2)-cos(x)*sin(x)/a^2/(a*sin(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{\cos(x)\left(128+64\csc^2(x)+48\csc^4(x)+40\csc^6(x)+35\csc^8(x)\right)\sin(x)}{315a^2\sqrt{a\sin^4(x)}}$$

input
```
Integrate[(a*Sin[x]^4)^(-5/2),x]
```

output
```
-1/315*(Cos[x]*(128 + 64*Csc[x]^2 + 48*Csc[x]^4 + 40*Csc[x]^6 + 35*Csc[x]^
8)*Sin[x])/(a^2*Sqrt[a*Sin[x]^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sin^4(x)\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a \sin(x)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\sin^2(x) \int \csc^{10}(x)dx}{a^2 \sqrt{a \sin^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin^2(x) \int \csc(x)^{10}dx}{a^2 \sqrt{a \sin^4(x)}}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\sin^2(x) \int \left(\cot^8(x)+4 \cot^6(x)+6 \cot^4(x)+4 \cot^2(x)+1\right)d\cot(x)}{a^2 \sqrt{a \sin^4(x)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sin^2(x) \left(\frac{\cot^9(x)}{9}+\frac{4 \cot^7(x)}{7}+\frac{6 \cot^5(x)}{5}+\frac{4 \cot^3(x)}{3}+\cot(x)\right)}{a^2 \sqrt{a \sin^4(x)}}$$

input
```
Int[(a*Sin[x]^4)^(-5/2),x]
```

output
```
-(((Cot[x] + (4*Cot[x]^3)/3 + (6*Cot[x]^5)/5 + (4*Cot[x]^7)/7 + Cot[x]^9/9
)*Sin[x]^2)/(a^2*Sqrt[a*Sin[x]^4]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
default	$-\frac{\cot\left(x\right)\csc\left(x\right)^{6}\left(128\cos\left(x\right)^{8}-576\cos\left(x\right)^{6}+1008\cos\left(x\right)^{4}-840\cos\left(x\right)^{2}+315\right)\sqrt{16}}{1260\sqrt{a\sin\left(x\right)^{4}}\,a^{2}}$	49
risch	$\frac{256i\left(126\,{\mathrm e}^{6ix}-84\,{\mathrm e}^{4ix}-9+37\cos\left(2x\right)+35i\sin\left(2x\right)\right)}{315a^{2}\left({\mathrm e}^{2ix}-1\right)^{7}\sqrt{a\left({\mathrm e}^{2ix}-1\right)^{4}{\mathrm e}^{-4ix}}}$	63

input
```
int(1/(a*sin(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1260*cot(x)*csc(x)^6*(128*cos(x)^8-576*cos(x)^6+1008*cos(x)^4-840*cos(x
)^2+315)/(a*sin(x)^4)^(1/2)/a^2*16^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \frac{\left(128\,\cos\left(x\right)^{9} - 576\,\cos\left(x\right)^{7} + 1008\,\cos\left(x\right)^{5} - 840\,\cos\left(x\right)^{3} + 315\,\cos\left(x\right)\right)\sqrt{a\cos\left(x\right.}}{315\,\left(a^{3}\cos\left(x\right)^{10} - 5\,a^{3}\cos\left(x\right)^{8} + 10\,a^{3}\cos\left(x\right)^{6} - 10\,a^{3}\cos\left(x\right)^{4} + 5\,a^{3}\cos\left(x\right)^{2}\right.}$$

input
```
integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(128*cos(x)^9 - 576*cos(x)^7 + 1008*cos(x)^5 - 840*cos(x)^3 + 315*co
s(x))*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)/((a^3*cos(x)^10 - 5*a^3*cos(x)^8
 + 10*a^3*cos(x)^6 - 10*a^3*cos(x)^4 + 5*a^3*cos(x)^2 - a^3)*sin(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin^{4}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**4)**(5/2),x)
```

output
```
Integral((a*sin(x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.30

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{315\,\tan\left(x\right)^{8}+420\,\tan\left(x\right)^{6}+378\,\tan\left(x\right)^{4}+180\,\tan\left(x\right)^{2}+35}{315\,a^{\frac{5}{2}}\,\tan\left(x\right)^{9}}$$

input
```
integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
-1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a
^(5/2)*tan(x)^9)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.30

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{315\,\tan\left(x\right)^{8}+420\,\tan\left(x\right)^{6}+378\,\tan\left(x\right)^{4}+180\,\tan\left(x\right)^{2}+35}{315\,a^{\frac{5}{2}}\,\tan\left(x\right)^{9}}$$

input
```
integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a
^(5/2)*tan(x)^9)
```

Mupad [B] (verification not implemented)

Time = 41.72 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \frac{256\,\left({\mathrm{e}}^{x\,46\mathrm{i}}\,1\mathrm{i}-{\mathrm{e}}^{x\,48\mathrm{i}}\,9\mathrm{i}+{\mathrm{e}}^{x\,50\mathrm{i}}\,36\mathrm{i}-{\mathrm{e}}^{x\,52\mathrm{i}}\,84\mathrm{i}+{\mathrm{e}}^{x\,54\mathrm{i}}\,126\mathrm{i}\right)}{315\,a^{5/2}\,\left({\mathrm{e}}^{x\,46\mathrm{i}}-9\,{\mathrm{e}}^{x\,48\mathrm{i}}+36\,{\mathrm{e}}^{x\,50\mathrm{i}}-84\,{\mathrm{e}}^{x\,52\mathrm{i}}+126\,{\mathrm{e}}^{x\,54\mathrm{i}}-126\,{\mathrm{e}}^{x\,56\mathrm{i}}+84\,{\mathrm{e}}^{x\,58\mathrm{i}}-36\,\mathrm{e}}$$

input
```
int(1/(a*sin(x)^4)^(5/2),x)
```

output
```
(256*(exp(x*46i)*1i - exp(x*48i)*9i + exp(x*50i)*36i - exp(x*52i)*84i + ex
p(x*54i)*126i))/(315*a^(5/2)*(exp(x*46i) - 9*exp(x*48i) + 36*exp(x*50i) -
84*exp(x*52i) + 126*exp(x*54i) - 126*exp(x*56i) + 84*exp(x*58i) - 36*exp(x
*60i) + 9*exp(x*62i) - exp(x*64i)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,\cos(x)\left(-128\sin(x)^8-64\sin(x)^6-48\sin(x)^4-40\sin(x)^2-35\right)}{315\sin(x)^9\,a^3}$$

input
```
int(1/(a*sin(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*cos(x)*( - 128*sin(x)**8 - 64*sin(x)**6 - 48*sin(x)**4 - 40*sin(x
)**2 - 35))/(315*sin(x)**9*a**3)
```

3.22 $\int (c\sin^m(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \frac{2c^2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\sin^2(a+bx)\right)\sin^{1+2m}(a+bx}{b(2+5m)\sqrt{\cos^2(a+bx)}}$$

output
```
2*c^2*cos(b*x+a)*hypergeom([1/2, 1/2+5/4*m],[3/2+5/4*m],sin(b*x+a)^2)*sin(
b*x+a)^(1+2*m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+5*m)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\sin^2(a+bx)\right)(c\sin^m(a+bx}{b(2+5m)}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(5/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, S
in[a + b*x]^2]*(c*Sin[a + b*x]^m)^(5/2)*Tan[a + b*x])/(b*(2 + 5*m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin^m(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx)^m)^{5/2}\,dx$$

$$\Big\downarrow \text{3687}$$

$$c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin^{\frac{5m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin(a+bx)^{5m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2c^2\cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5m+2),\frac{1}{4}(5m+6),\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(5/2),x]
```

output
```
(2*c^2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Sin[a
 + b*x]^2]*Sin[a + b*x]^(1 + 2*m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 5*m)*Sqr
t[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (c\sin (bx+a)^{m})^{\frac{5}{2}}\,dx$$

input
```
int((c*sin(b*x+a)^m)^(5/2),x)
```

output
```
int((c*sin(b*x+a)^m)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a)**m)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \int (c\sin{(bx+a)^m})^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(5/2), x)
```

Giac [F]

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \int (c\sin{(bx+a)^m})^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \int (c\sin(a+b\,x)^m)^{5/2}\,dx$$

input
```
int((c*sin(a + b*x)^m)^(5/2),x)
```

output
```
int((c*sin(a + b*x)^m)^(5/2), x)
```

Reduce [F]

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \sqrt{c}\left(\int \sin(bx+a)^{\frac{5m}{2}}\,dx\right)c^2$$

input
```
int((c*sin(b*x+a)^m)^(5/2),x)
```

output
```
sqrt(c)*int(sin(a + b*x)**((5*m)/2),x)*c**2
```

3.23 $\int (c\sin^m(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \frac{2c\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\sin^2(a+bx)\right)\sin^{1+m}(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+3m)\sqrt{\cos^2(a+bx)}}$$

output

```
2*c*cos(b*x+a)*hypergeom([1/2, 1/2+3/4*m],[3/2+3/4*m],sin(b*x+a)^2)*sin(b*
x+a)^(1+m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+3*m)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\sin^2(a+bx)\right)(c\sin^m(a+bx))^{3/2}}{b(2+3m)}$$

input

```
Integrate[(c*Sin[a + b*x]^m)^(3/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4,
 Sin[a + b*x]^2]*(c*Sin[a + b*x]^m)^(3/2)*Tan[a + b*x])/(b*(2 + 3*m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin^m(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c\sin(a+bx)^m)^{3/2}\,dx$$

$$\Big\downarrow \; 3687$$

$$c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin^{\frac{3m}{2}}(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin(a+bx)^{3m/2}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3m+2),\frac{3(m+2)}{4},\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(3/2),x]
```

output
```
(2*c*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4, Sin[a
 + b*x]^2]*Sin[a + b*x]^(1 + m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 3*m)*Sqrt[
Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (c\sin (bx+a)^m)^{\frac{3}{2}}\, dx$$

input
```
int((c*sin(b*x+a)^m)^(3/2),x)
```

output
```
int((c*sin(b*x+a)^m)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c\sin^m(a+bx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin^m{(a+bx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)**m)**(3/2),x)
```

output
```
Integral((c*sin(a + b*x)**m)**(3/2), x)
```

Maxima [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(3/2), x)
```

Giac [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int \left(c\sin\left(a+b\,x\right)^m\right)^{3/2}\,dx$$

input
```
int((c*sin(a + b*x)^m)^(3/2),x)
```

output
```
int((c*sin(a + b*x)^m)^(3/2), x)
```

Reduce [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \sqrt{c}\left(\int \sin\left(bx+a\right)^{\frac{3m}{2}}\,dx\right)c$$

input
```
int((c*sin(b*x+a)^m)^(3/2),x)
```

output
```
sqrt(c)*int(sin(a + b*x)**((3*m)/2),x)*c
```

3.24 $\int \sqrt{c\sin^m(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \sqrt{c\sin^m(a+bx)}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\sin^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+m)\sqrt{\cos^2(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2+1/4*m],[3/2+1/4*m],sin(b*x+a)^2)*sin(b*x+
a)*(c*sin(b*x+a)^m)^(1/2)/b/(2+m)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \sqrt{c\sin^m(a+bx)}\,dx = \frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\sin^2(a+bx)\right)\sqrt{c\sin^m(a+bx)}\tan(a+bx)}{b(2+m)}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a
 + b*x]^2]*Sqrt[c*Sin[a + b*x]^m]*Tan[a + b*x])/(b*(2 + m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c \sin^m(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c \sin(a+bx)^m}dx$$

$$\Big\downarrow \text{3687}$$

$$\sin^{-\frac{m}{2}}(a+bx)\sqrt{c \sin^m(a+bx)}\int \sin^{\frac{m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-\frac{m}{2}}(a+bx)\sqrt{c \sin^m(a+bx)}\int \sin(a+bx)^{m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\cos(a+bx)\sin^{\frac{m+2}{2}-\frac{m}{2}}(a+bx)\sqrt{c \sin^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{4},\frac{m+6}{4},\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a + b*x]^
2]*Sin[a + b*x]^(-1/2*m + (2 + m)/2)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + m)*Sq
rt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sqrt{c \sin \left(b x +a \right)^{m}}d x$$

input
```
int((c*sin(b*x+a)^m)^(1/2),x)
```

output
```
int((c*sin(b*x+a)^m)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c \sin^m(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{c \sin^m(a+bx)}\, dx = \int \sqrt{c \sin^m{(a+bx)}}\, dx$$

input
```
integrate((c*sin(b*x+a)**m)**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x)**m), x)
```

Maxima [F]

$$\int \sqrt{c \sin^m(a+bx)}\, dx = \int \sqrt{c \sin{(bx+a)}^m}\, dx$$

input
```
integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a)^m), x)
```

Giac [F]

$$\int \sqrt{c \sin^m(a+bx)}\, dx = \int \sqrt{c \sin{(bx+a)}^m}\, dx$$

input
```
integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c\sin^m(a+bx)}\,dx = \int \sqrt{c\sin\left(a+b\,x\right)^m}\,dx$$

input
```
int((c*sin(a + b*x)^m)^(1/2),x)
```

output
```
int((c*sin(a + b*x)^m)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c\sin^m(a+bx)}\,dx = \sqrt{c}\left(\int \sin\left(bx+a\right)^{\frac{m}{2}}dx\right)$$

input
```
int((c*sin(b*x+a)^m)^(1/2),x)
```

output
```
sqrt(c)*int(sin(a + b*x)**(m/2),x)
```

3.25 $\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx = \frac{2 \cos(a+bx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{4}, \frac{6-m}{4}, \sin^2(a+bx)\right) \sin(a+bx)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c \sin^m(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2-1/4*m],[3/2-1/4*m],sin(b*x+a)^2)*sin(b*x+
a)/b/(2-m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx = -\frac{2\sqrt{\cos^2(a+bx)} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{4}, \frac{6-m}{4}, \sin^2(a+bx)\right) \tan(a+bx)}{b(-2+m)\sqrt{c \sin^m(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(-2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[
a + b*x]^2]*Tan[a + b*x])/(b*(-2 + m)*Sqrt[c*Sin[a + b*x]^m])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c \sin^m(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c \sin(a+bx)^m}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{m}{2}}(a+bx)dx}{\sqrt{c \sin^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin(a+bx)^{-m/2}dx}{\sqrt{c \sin^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(a+bx)\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c \sin^m(a+bx)}}$$

input
```
Int[1/Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a + b*x]^
2]*Sin[a + b*x])/(b*(2 - m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]^m])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\sqrt{c \sin \left(b x+a \right)^{m}}}d x$$

input
```
int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

output
```
int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin^m\left(a+bx\right)}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)**m)**(1/2),x)
```

output
```
Integral(1/sqrt(c*sin(a + b*x)**m), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin\left(bx+a\right)^m}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin\left(bx+a\right)^m}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin\left(a+b\,x\right)^m}}\,dx$$

input
```
int(1/(c*sin(a + b*x)^m)^(1/2),x)
```

output
```
int(1/(c*sin(a + b*x)^m)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{1}{\sin(bx+a)^{\frac{m}{2}}}dx\right)}{c}$$

input
```
int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

output
```
(sqrt(c)*int(1/sin(a + b*x)**(m/2),x))/c
```

3.26 $\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\sin^2(a+bx)\right)\sin^{1-m}(}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2-3/4*m],[3/2-3/4*m],sin(b*x+a)^2)*sin(b*x+
a)^(1-m)/b/c/(2-3*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),-\frac{3}{4}(-2+m),\sin^2(a+bx)\right)}{\left(b-\frac{3bm}{2}\right)(c\sin^m(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(-3/2),x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4,
 Sin[a + b*x]^2]*Tan[a + b*x])/((b - (3*b*m)/2)*(c*Sin[a + b*x]^m)^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx)^m)^{3/2}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{3m}{2}}(a+bx)dx}{c\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin(a+bx)^{-3m/2}dx}{c\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(-3/2),x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4, Sin[a +
 b*x]^2]*Sin[a + b*x]^(1 - m))/(b*c*(2 - 3*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*
Sin[a + b*x]^m])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(c \sin \left(b x +a \right)^{m}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

output
```
int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin^m(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)**m)**(3/2),x)
```

output
```
Integral((c*sin(a + b*x)**m)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(bx+a)^m)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(bx+a)^m)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\sin\left(a+b\,x\right)^m\right)^{3/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x)^m)^(3/2),x)
```

output
```
int(1/(c*sin(a + b*x)^m)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{1}{\sin(bx+a)^{\frac{3m}{2}}}dx\right)}{c^2}$$

input
```
int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

output
```
(sqrt(c)*int(1/sin(a + b*x)**((3*m)/2),x))/c**2
```

3.27 $\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\sin^2(a+bx)\right)\sin^{1}}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2-5/4*m],[3/2-5/4*m],sin(b*x+a)^2)*sin(b*x+
a)^(1-2*m)/b/c^2/(2-5*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\sin^2(a+bx)\right)\text{ta}}{\left(b-\frac{5bm}{2}\right)(c\sin^m(a+bx))^{5/2}}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(-5/2),x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin
[a + b*x]^2]*Tan[a + b*x])/((b - (5*b*m)/2)*(c*Sin[a + b*x]^m)^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(c\sin(a+bx)^m)^{5/2}}dx$$

$$\Big\downarrow \; 3687$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{5m}{2}}(a+bx)dx}{c^2\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin(a+bx)^{-5m/2}dx}{c^2\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \; 3122$$

$$\frac{2\cos(a+bx)\sin^{1-2m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(-5/2),x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[a + b
*x]^2]*Sin[a + b*x]^(1 - 2*m))/(b*c^2*(2 - 5*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[
c*Sin[a + b*x]^m])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(c\sin\left(bx+a\right)^{m}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

output
```
int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin^m{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)**m)**(5/2),x)
```

output
```
Integral((c*sin(a + b*x)**m)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)}^m)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)}^m)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(c\sin\left(a+b\,x\right)^m\right)^{5/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x)^m)^(5/2),x)
```

output
```
int(1/(c*sin(a + b*x)^m)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{1}{\sin(bx+a)^{\frac{5m}{2}}}dx\right)}{c^3}$$

input
```
int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

output
```
(sqrt(c)*int(1/sin(a + b*x)**((5*m)/2),x))/c**3
```

3.28 $\int (b\sin^n(c+dx))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (b\sin^n(c+dx))^p \, dx = \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(c+dx)\right)\sin(c+dx)\,(b\sin^n(c+dx))^p}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

output
```
cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*
x+c)*(b*sin(d*x+c)^n)^p/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int (b\sin^n(c+dx))^p \, dx = \frac{\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(c+dx)\right)(b\sin^n(c+dx))^p\tan(c+dx)}{d(1+np)}$$

input
```
Integrate[(b*Sin[c + d*x]^n)^p,x]
```

output
```
(Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin
[c + d*x]^2]*(b*Sin[c + d*x]^n)^p*Tan[c + d*x])/(d*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin^n(c+dx))^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin(c+dx)^n)^p dx$$

$$\Big\downarrow \text{3687}$$

$$\sin^{-np}(c+dx)\left(b\sin^n(c+dx)\right)^p\int \sin^{np}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-np}(c+dx)\left(b\sin^n(c+dx)\right)^p\int \sin(c+dx)^{np}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(c+dx)\cos(c+dx)\left(b\sin^n(c+dx)\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(b*Sin[c + d*x]^n)^p,x]
```

output
```
(Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x
]^2]*Sin[c + d*x]*(b*Sin[c + d*x]^n)^p)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b\sin (dx+c)^n)^p\,dx$$

input
```
int((b*sin(d*x+c)^n)^p,x)
```

output
```
int((b*sin(d*x+c)^n)^p,x)
```

Fricas [F]

$$\int (b\sin^n(c+dx))^p\,dx = \int (b\sin (dx+c)^n)^p\,dx$$

input
```
integrate((b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n)^p, x)
```

Sympy [F]

$$\int (b\sin^n(c+dx))^p \, dx = \int (b\sin^n(c+dx))^p \, dx$$

input
```
integrate((b*sin(d*x+c)**n)**p,x)
```

output
```
Integral((b*sin(c + d*x)**n)**p, x)
```

Maxima [F]

$$\int (b\sin^n(c+dx))^p \, dx = \int (b\sin(dx+c)^n)^p \, dx$$

input
```
integrate((b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n)^p, x)
```

Giac [F]

$$\int (b\sin^n(c+dx))^p \, dx = \int (b\sin(dx+c)^n)^p \, dx$$

input
```
integrate((b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin^n(c+dx))^p\ dx = \int (b\sin(c+d\,x)^n)^p\,dx$$

input
```
int((b*sin(c + d*x)^n)^p,x)
```

output
```
int((b*sin(c + d*x)^n)^p, x)
```

Reduce [F]

$$\int (b\sin^n(c+dx))^p\ dx = b^p\left(\int \sin(dx+c)^{np}\,dx\right)$$

input
```
int((b*sin(d*x+c)^n)^p,x)
```

output
```
b**p*int(sin(c + d*x)**(n*p),x)
```

3.29 $\int (c\sin^2(a+bx))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (c\sin^2(a+bx))^p \, dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+2p),\frac{1}{2}(3+2p),\sin^2(a+bx)\right)\sin(a+bx)\,(c\sin^2(a+bx))^p}{b(1+2p)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+p],[3/2+p],sin(b*x+a)^2)*sin(b*x+a)*(c*sin(
b*x+a)^2)^p/b/(1+2*p)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int (c\sin^2(a+bx))^p \, dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+p,\frac{3}{2}+p,\sin^2(a+bx)\right)(c\sin^2(a+bx))^p\tan(a+bx)}{b+2bp}$$

input
```
Integrate[(c*Sin[a + b*x]^2)^p,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + p, 3/2 + p, Sin[a + b*x
]^2]*(c*Sin[a + b*x]^2)^p*Tan[a + b*x])/(b + 2*b*p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3686, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^2(a+bx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin(a+bx)^2\right)^p dx$$

$$\Big\downarrow \text{3686}$$

$$\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p\int \sin^{2p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p\int \sin(a+bx)^{2p}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2p+1),\frac{1}{2}(2p+3),\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^2)^p,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 2*p)/2, (3 + 2*p)/2, Sin[a + b*x
]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^2)^p)/(b*(1 + 2*p)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(c \sin \left(b x +a \right)^{2}\right)^{p}d x$$

input
```
int((c*sin(b*x+a)^2)^p,x)
```

output
```
int((c*sin(b*x+a)^2)^p,x)
```

Fricas [F]

$$\int \left(c\sin^2(a+bx)\right)^p\,dx=\int \left(c\sin\left(bx+a\right)^2\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^2)^p,x, algorithm="fricas")
```

output
```
integral((-c*cos(b*x + a)^2 + c)^p, x)
```

Sympy [F]

$$\int \left(c\sin^2(a+bx)\right)^p\,dx=\int \left(c\sin^2\left(a+bx\right)\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)**2)**p,x)
```

output
```
Integral((c*sin(a + b*x)**2)**p, x)
```

Maxima [F]

$$\int \left(c\sin^2(a+bx)\right)^p\,dx=\int \left(c\sin\left(bx+a\right)^2\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^2)^p,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^2)^p, x)
```

Giac [F]

$$\int \left(c\sin^2(a+bx)\right)^p\,dx=\int \left(c\sin\left(bx+a\right)^2\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^2)^p,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^2(a+bx)\right)^p\,dx = \int \left(c\sin\left(a+b\,x\right)^2\right)^p\,dx$$

input
```
int((c*sin(a + b*x)^2)^p,x)
```

output
```
int((c*sin(a + b*x)^2)^p, x)
```

Reduce [F]

$$\int \left(c\sin^2(a+bx)\right)^p\,dx = c^p\left(\int \sin\left(bx+a\right)^{2p}dx\right)$$

input
```
int((c*sin(b*x+a)^2)^p,x)
```

output
```
c**p*int(sin(a + b*x)**(2*p),x)
```

3.30 $\int (c\sin^3(a+bx))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c\sin^3(a+bx))^p \, dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+3p),\frac{3(1+p)}{2},\sin^2(a+bx)\right)\sin(a+bx)\,(c\sin^3(a+bx))^p}{b(1+3p)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+3/2*p],[3/2*p+3/2],sin(b*x+a)^2)*sin(b*x+a)
*(c*sin(b*x+a)^3)^p/b/(1+3*p)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int (c\sin^3(a+bx))^p \, dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+3p),\frac{3(1+p)}{2},\sin^2(a+bx)\right)(c\sin^3(a+bx))^p\tan(a+bx)}{b+3bp}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^p,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, S
in[a + b*x]^2]*(c*Sin[a + b*x]^3)^p*Tan[a + b*x])/(b + 3*b*p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3686, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^3(a+bx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin(a+bx)^3\right)^p\,dx$$

$$\Big\downarrow \text{3686}$$

$$\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\int \sin^{3p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\int \sin(a+bx)^{3p}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^3(a+bx)\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(3p+1),\frac{3(p+1)}{2},\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^3)^p,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Sin[a + b
*x]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^3)^p)/(b*(1 + 3*p)*Sqrt[Cos[a + b*x]^2
])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(c\sin\left(bx+a\right)^{3}\right)^{p}dx$$

input
```
int((c*sin(b*x+a)^3)^p,x)
```

output
```
int((c*sin(b*x+a)^3)^p,x)
```

Fricas [F]

$$\int \left(c\sin^3(a+bx)\right)^p\, dx = \int \left(c\sin\left(bx+a\right)^{3}\right)^{p}\, dx$$

input
```
integrate((c*sin(b*x+a)^3)^p,x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^p, x)
```

Sympy [F]

$$\int \left(c\sin^3(a+bx)\right)^p\,dx=\int \left(c\sin^3{(a+bx)}\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**p,x)
```

output
```
Integral((c*sin(a + b*x)**3)**p, x)
```

Maxima [F]

$$\int \left(c\sin^3(a+bx)\right)^p\,dx=\int \left(c\sin\left(bx+a\right)^3\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^p,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^3)^p, x)
```

Giac [F]

$$\int \left(c\sin^3(a+bx)\right)^p\,dx=\int \left(c\sin\left(bx+a\right)^3\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^p,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3(a+bx)\right)^p\,dx = \int \left(c\sin\left(a+b\,x\right)^3\right)^p\,dx$$

input
```
int((c*sin(a + b*x)^3)^p,x)
```

output
```
int((c*sin(a + b*x)^3)^p, x)
```

Reduce [F]

$$\int \left(c\sin^3(a+bx)\right)^p\,dx = c^p\left(\int \sin\left(bx+a\right)^{3p}dx\right)$$

input
```
int((c*sin(b*x+a)^3)^p,x)
```

output
```
c**p*int(sin(a + b*x)**(3*p),x)
```

3.31 $\int (c\sin^4(a+bx))^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (c\sin^4(a+bx))^p\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+4p),\frac{1}{2}(3+4p),\sin^2(a+bx)\right)\sin(a+bx)\,(c\sin^4(a+bx))^p}{b(1+4p)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+2*p],[3/2+2*p],sin(b*x+a)^2)*sin(b*x+a)*(c*
sin(b*x+a)^4)^p/b/(1+4*p)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int (c\sin^4(a+bx))^p\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+2p,\frac{3}{2}+2p,\sin^2(a+bx)\right)(c\sin^4(a+bx))^p\tan(a+bx)}{b+4bp}$$

input
```
Integrate[(c*Sin[a + b*x]^4)^p,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + 2*p, 3/2 + 2*p, Sin[a +
 b*x]^2]*(c*Sin[a + b*x]^4)^p*Tan[a + b*x])/(b + 4*b*p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3686, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^4(a+bx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin(a+bx)^4\right)^p\,dx$$

$$\Big\downarrow \text{3686}$$

$$\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\int \sin^{4p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\int \sin(a+bx)^{4p}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(4p+1),\frac{1}{2}(4p+3),\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^4)^p,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 4*p)/2, (3 + 4*p)/2, Sin[a + b*x
]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^4)^p)/(b*(1 + 4*p)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(c \sin \left(b x+a\right)^{4}\right)^{p} d x$$

input
```
int((c*sin(b*x+a)^4)^p,x)
```

output
```
int((c*sin(b*x+a)^4)^p,x)
```

Fricas [F]

$$\int \left(c \sin ^{4}(a+b x)\right)^{p} \, d x=\int \left(c \sin \left(b x+a\right)^{4}\right)^{p} \, d x$$

input
```
integrate((c*sin(b*x+a)^4)^p,x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a)^4 - 2*c*cos(b*x + a)^2 + c)^p, x)
```

Sympy [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin^4\left(a+bx\right)\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)**4)**p,x)
```

output
```
Integral((c*sin(a + b*x)**4)**p, x)
```

Maxima [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin\left(bx+a\right)^4\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^4)^p,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^4)^p, x)
```

Giac [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin\left(bx+a\right)^4\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^4)^p,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^4)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin\left(a+b\,x\right)^4\right)^p\,dx$$

input
```
int((c*sin(a + b*x)^4)^p,x)
```

output
```
int((c*sin(a + b*x)^4)^p, x)
```

Reduce [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = c^p\left(\int \sin\left(bx+a\right)^{4p}\,dx\right)$$

input
```
int((c*sin(b*x+a)^4)^p,x)
```

output
```
c**p*int(sin(a + b*x)**(4*p),x)
```

3.32 $\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

output
```
-cot(b*x+a)*(c*sin(b*x+a)^n)^(1/n)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

input
```
Integrate[(c*Sin[a + b*x]^n)^n^(-1),x]
```

output
```
-((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx)^n)^{\frac{1}{n}}\,dx$$

$$\Big\downarrow \text{3687}$$

$$\csc(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}\int \sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}\int \sin(a+bx)dx$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

input
```
Int[(c*Sin[a + b*x]^n)^n^(-1),x]
```

output
```
-((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
parallelrisch	$-\frac{\cot\left(\frac{b x}{2}+\frac{a}{2}\right) \left(c \sin \left(b x +a \right)^{n}\right)^{\frac{1}{n}}}{b}$	29

input
```
int((c*sin(b*x+a)^n)^(1/n),x,method=_RETURNVERBOSE)
```

output
```
-cot(1/2*b*x+1/2*a)*(c*sin(b*x+a)^n)^(1/n)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{c^{\left(\frac{1}{n}\right)}\cos(bx+a)}{b}$$

input
```
integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="fricas")
```

output
```
-c^(1/n)*cos(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(22) = 44.

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = \begin{cases} x(c\sin^n(a))^{\frac{1}{n}} & \text{for } b=0 \\ x(0^n c)^{\frac{1}{n}} & \text{for } a=-bx \vee a=-bx+\pi \\ -\frac{(c\sin^n(a+bx))^{\frac{1}{n}}\cos(a+bx)}{b\sin(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate((c*sin(b*x+a)**n)**(1/n),x)
```

output
```
Piecewise((x*(c*sin(a)**n)**(1/n), Eq(b, 0)), (x*(0**n*c)**(1/n), Eq(a, -b
*x) | Eq(a, -b*x + pi)), (-(c*sin(a + b*x)**n)**(1/n)*cos(a + b*x)/(b*sin(
a + b*x)), True))
```

Maxima [F]

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = \int (c\sin(bx+a)^n)^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^n)^(1/n), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(25) = 50.

Time = 0.92 (sec) , antiderivative size = 384, normalized size of antiderivative = 15.36

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = \text{Too large to display}$$

input
```
integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="giac")
```

output
```
(abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*
b*x + 1/2*a)^4 - 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/
4*pi/n)^2*tan(1/2*b*x + 1/2*a)^2 + 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/
4*pi*sgn(c)/n - 1/4*pi/n)*tan(1/2*b*x + 1/2*a)^3 - abs(c)^(1/n)*tan(1/2*b*
x + 1/2*a)^4 + abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi
/n)^2 - 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)*t
an(1/2*b*x + 1/2*a) + 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/n)
)/(b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2
*a)^4 + 2*b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*
x + 1/2*a)^2 + b*tan(1/2*b*x + 1/2*a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*s
gn(c)/n - 1/4*pi/n)^2 + 2*b*tan(1/2*b*x + 1/2*a)^2 + b)
```

Mupad [B] (verification not implemented)

Time = 37.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{\sin(2\,a+2\,b\,x)\,\left(c\sin(a+b\,x)^n\right)^{1/n}}{2\,b\sin(a+b\,x)^2}$$

input
```
int((c*sin(a + b*x)^n)^(1/n),x)
```

output
```
-(sin(2*a + 2*b*x)*(c*sin(a + b*x)^n)^(1/n))/(2*b*sin(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{c^{\frac{1}{n}}\cos(bx+a)}{b}$$

input
```
int((c*sin(b*x+a)^n)^(1/n),x)
```

output
```
( - c**(1/n)*cos(a + b*x))/b
```

3.33 $\int (a(b\sin(c+dx))^p)^n \, dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int (a(b\sin(c+dx))^p)^n \, dx = \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(c+dx)\right)\sin(c+dx)\,(a(b\sin(c+dx))^p)^n}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

output
```
cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*
x+c)*(a*(b*sin(d*x+c))^p)^n/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int (a(b\sin(c+dx))^p)^n \, dx = \frac{\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n\tan(c+}{d(1+np)}$$

input
```
Integrate[(a*(b*Sin[c + d*x])^p)^n,x]
```

output
```
(Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin
[c + d*x]^2]*(a*(b*Sin[c + d*x])^p)^n*Tan[c + d*x])/(d*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(b\sin(c+dx))^p)^n \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a(b\sin(c+dx))^p)^n dx$$

$$\Big\downarrow \text{ 3687}$$

$$(b\sin(c+dx))^{-np} (a(b\sin(c+dx))^p)^n \int (b\sin(c+dx))^{np} dx$$

$$\Big\downarrow \text{ 3042}$$

$$(b\sin(c+dx))^{-np} (a(b\sin(c+dx))^p)^n \int (b\sin(c+dx))^{np} dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\sin(c+dx)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(a*(b*Sin[c + d*x])^p)^n,x]
```

output
```
(Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x
]^2]*Sin[c + d*x]*(a*(b*Sin[c + d*x])^p)^n)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]
^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (a(b\sin (dx+c))^p)^n \, dx$$

input
```
int((a*(b*sin(d*x+c))^p)^n,x)
```

output
```
int((a*(b*sin(d*x+c))^p)^n,x)
```

Fricas [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int ((b\sin (dx+c))^p a)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*sin(d*x + c))^p*a)^n, x)
```

Sympy [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int (a(b\sin{(c+dx)})^p)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))**p)**n,x)
```

output
```
Integral((a*(b*sin(c + d*x))**p)**n, x)
```

Maxima [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int ((b\sin{(dx+c)})^p a)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*sin(d*x + c))^p*a)^n, x)
```

Giac [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int ((b\sin{(dx+c)})^p a)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*sin(d*x + c))^p*a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int (a\,(b\,\sin(c+d\,x))^p)^n \, dx$$

input
```
int((a*(b*sin(c + d*x))^p)^n,x)
```

output
```
int((a*(b*sin(c + d*x))^p)^n, x)
```

Reduce [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = b^{np}a^n\left(\int \sin(dx+c)^{np}\, dx\right)$$

input
```
int((a*(b*sin(d*x+c))^p)^n,x)
```

output
```
b**(n*p)*a**n*int(sin(c + d*x)**(n*p),x)
```

3.34 $\int \frac{1}{4-\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 44

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{x}{2\sqrt{3}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{2\left(2+\sqrt{3}\right)-\sin^2(x)}\right)}{2\sqrt{3}}$$

output
```
1/6*x*3^(1/2)-1/6*arctan(cos(x)*sin(x)/(4+2*3^(1/2)-sin(x)^2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{\arctan\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Integrate[(4 - Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[3]*Tan[x])/2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{4-\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$\int \frac{1}{3\tan^2(x)+4}d\tan(x)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Int[(4 - Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[3]*Tan[x])/2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\tan(x)}{2}\right)}{6}$	14
risch	$\frac{i\sqrt{3}\,\ln\left(e^{2ix}+4\sqrt{3}+7\right)}{12}-\frac{i\sqrt{3}\,\ln\left(e^{2ix}-4\sqrt{3}+7\right)}{12}$	40

input
```
int(1/(4-sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/2*3^(1/2)*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{4-\sin^2(x)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{7\,\sqrt{3}\cos{(x)}^2-3\,\sqrt{3}}{12\,\cos{(x)}\sin{(x)}}\right)$$

input
```
integrate(1/(4-sin(x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 3*sqrt(3))/(cos(x)*sin(x))
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}-\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(4-sin(x)**2),x)
```

output
```
sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 - sqrt(3)/3) + pi*floor((x/2 - pi/2)/pi
))/6 + sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 + sqrt(3)/3) + pi*floor((x/2 - p
i/2)/pi))/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\tan(x)\right)$$

input
```
integrate(1/(4-sin(x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/2*sqrt(3)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{2\sqrt{3}\sin(2x)-3\sin(2x)}{2\sqrt{3}\cos(2x)+2\sqrt{3}-3\cos(2x)+3}\right)\right)$$

input
```
integrate(1/(4-sin(x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(2*sqrt(3)*sin(2*x) - 3*sin(2*x))/(2*sqrt(3)*cos(
2*x) + 2*sqrt(3) - 3*cos(2*x) + 3)))
```

Mupad [B] (verification not implemented)

Time = 37.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{\sqrt{3}\,(x-\mathrm{atan}(\tan(x)))}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan(x)}{2}\right)}{6}$$

input
```
int(-1/(sin(x)^2 - 4),x)
```

output
```
(3^(1/2)*(x - atan(tan(x))))/6 + (3^(1/2)*atan((3^(1/2)*tan(x))/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{1}{4-\sin^2(x)}\,dx = \frac{\sqrt{3}\left(atan\left(\frac{2\tan\left(\frac{x}{2}\right)-1}{\sqrt{3}}\right)+atan\left(\frac{2\tan\left(\frac{x}{2}\right)+1}{\sqrt{3}}\right)\right)}{6}$$

input
```
int(1/(4-sin(x)^2),x)
```

output
```
(sqrt(3)*(atan((2*tan(x/2) - 1)/sqrt(3)) + atan((2*tan(x/2) + 1)/sqrt(3)))
)/6
```

3.35 $\int \frac{1}{4-2\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{x}{2\sqrt{2}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{2+\sqrt{2}-\sin^2(x)}\right)}{2\sqrt{2}}$$

output
```
1/4*x*2^(1/2)-1/4*arctan(cos(x)*sin(x)/(2+2^(1/2)-sin(x)^2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(4 - 2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-2\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{4-2\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$\int \frac{1}{2\tan^2(x)+4}d\tan(x)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[(4 - 2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{4}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+3+2\sqrt{2}\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+3-2\sqrt{2}\right)}{8}$	40

input
```
int(1/(4-2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{4-2\sin^2(x)}\,dx = -\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos{(x)}^2-\sqrt{2}}{4\,\cos{(x)}\sin{(x)}}\right)$$

input
```
integrate(1/(4-2*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}$$

input
```
integrate(1/(4-2*sin(x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(4-2*sin(x)^2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$

input
```
integrate(1/(4-2*sin(x)^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 37.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}$$

input
```
int(-1/(2*sin(x)^2 - 4),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/4 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{4-2\sin^2(x)}\,dx = \frac{\sqrt{2}\left(-atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{4}$$

input
```
int(1/(4-2*sin(x)^2),x)
```

output
```
(sqrt(2)*( - atan((sqrt(2) - 2*tan(x/2))/sqrt(2)) + atan((sqrt(2) + 2*tan(
x/2))/sqrt(2))))/4
```

3.36 $\int \frac{1}{4-3\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{x}{2}-\frac{1}{2}\arctan\left(\frac{\cos(x)\sin(x)}{2-\sin^2(x)}\right)$$

output
```
1/2*x-1/2*arctan(cos(x)*sin(x)/(2-sin(x)^2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.35

$$\int \frac{1}{4-3\sin^2(x)}\,dx = -\frac{1}{2}\arctan(2\cot(x))$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[2*Cot[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-3\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{4-3\sin(x)^2}dx$$
$$\Big\downarrow \text{ 3660}$$
$$\int \frac{1}{\tan^2(x)+4}d\tan(x)$$
$$\Big\downarrow \text{ 216}$$
$$\frac{1}{2}\arctan\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(4 - 3*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/2]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)}{2}\right)}{2}$	8
risch	$-\frac{i\ln\left(\mathrm{e}^{2ix}+\frac{1}{3}\right)}{4}+\frac{i\ln\left(\mathrm{e}^{2ix}+3\right)}{4}$	24
parallelrisch	$-\frac{i\left(\ln\left(-i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)-\ln\left(i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)\right)}{4}$	39

input
```
int(1/(4-3*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{4-3\sin^2(x)}\,dx = -\frac{1}{4}\arctan\left(\frac{5\cos(x)^2-1}{4\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(4-3*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*arctan(1/4*(5*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{2} + \frac{\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{2} + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor$$

input
```
integrate(1/(4-3*sin(x)**2),x)
```

output
```
atan(2*tan(x/2) - sqrt(3))/2 + atan(2*tan(x/2) + sqrt(3))/2 + pi*floor((x/
2 - pi/2)/pi)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.27

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\tan(x)\right)$$

input
```
integrate(1/(4-3*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*arctan(1/2*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{1}{2}\,x - \frac{1}{2}\arctan\left(\frac{\sin(2\,x)}{\cos(2\,x)+3}\right)$$

input
```
integrate(1/(4-3*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*x - 1/2*arctan(sin(2*x)/(cos(2*x) + 3))
```

Mupad [B] (verification not implemented)

Time = 37.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{x}{2} - \frac{\operatorname{atan}(\tan(x))}{2} + \frac{\operatorname{atan}\left(\frac{\tan(x)}{2}\right)}{2}$$

input
```
int(-1/(3*sin(x)^2 - 4),x)
```

output
```
x/2 - atan(tan(x))/2 + atan(tan(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{4-3\sin^2(x)}\,dx = -\frac{atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{atan\left(\sqrt{3}+2\tan\left(\frac{x}{2}\right)\right)}{2}$$

input
```
int(1/(4-3*sin(x)^2),x)
```

output
```
( - atan(sqrt(3) - 2*tan(x/2)) + atan(sqrt(3) + 2*tan(x/2)))/2
```

3.37 $\int \frac{1}{4-4\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{1}{4-4\sin^2(x)}\,dx = \frac{\tan(x)}{4}$$

output
```
1/4*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-4\sin^2(x)}\,dx = \frac{\tan(x)}{4}$$

input
```
Integrate[(4 - 4*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-4\sin^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{4-4\sin(x)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{1}{4}\int \sec^2(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{4}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{ 4254}$$
$$-\frac{1}{4}\int 1d(-\tan(x))$$
$$\Big\downarrow \text{ 24}$$
$$\frac{\tan(x)}{4}$$

input
```
Int[(4 - 4*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\tan(x)}{4}$	5
parallelrisch	$\frac{\sin(x)}{4\cos(x)}$	9
risch	$\frac{i}{2\,{\mathrm e}^{2ix}+2}$	13
norman	$-\frac{\tan\left(\frac{x}{2}\right)}{2\left(-1+\tan\left(\frac{x}{2}\right)^{2}\right)}$	17

input
```
int(1/(4-4*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{4-4\sin^2(x)}\,dx=\frac{\sin(x)}{4\cos(x)}$$

input
```
integrate(1/(4-4*sin(x)^2),x, algorithm="fricas")
```

output
```
1/4*sin(x)/cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{4-4\sin^2(x)}\,dx=-\frac{\tan\left(\frac{x}{2}\right)}{2\tan^2\left(\frac{x}{2}\right)-2}$$

input
```
integrate(1/(4-4*sin(x)**2),x)
```

output
```
-tan(x/2)/(2*tan(x/2)**2 - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-4\sin^2(x)}\,dx=\frac{1}{4}\tan(x)$$

input
```
integrate(1/(4-4*sin(x)^2),x, algorithm="maxima")
```

output
```
1/4*tan(x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-4\sin^2(x)}\,dx = \frac{1}{4}\tan(x)$$

input
```
integrate(1/(4-4*sin(x)^2),x, algorithm="giac")
```

output
```
1/4*tan(x)
```

Mupad [B] (verification not implemented)

Time = 37.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-4\sin^2(x)}\,dx = \frac{\tan(x)}{4}$$

input
```
int(-1/(4*sin(x)^2 - 4),x)
```

output
```
tan(x)/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{4-4\sin^2(x)}\,dx = \frac{\sin(x)}{4\cos(x)}$$

input
```
int(1/(4-4*sin(x)^2),x)
```

output
```
sin(x)/(4*cos(x))
```

3.38 $\int \frac{1}{4-5\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

output
```
1/2*arctanh(1/2*tan(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{4-5\sin^2(x)}\,dx = -\frac{1}{4}\log(2\cos(x)-\sin(x))+\frac{1}{4}\log(2\cos(x)+\sin(x))$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-1),x]
```

output
```
-1/4*Log[2*Cos[x] - Sin[x]] + Log[2*Cos[x] + Sin[x]]/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-5\sin^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{4-5\sin(x)^2}dx$$

$$\Big\downarrow \; 3660$$

$$\int \frac{1}{4-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[Tan[x]/2]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\ln(\tan(x)+2)}{4} - \frac{\ln(\tan(x)-2)}{4}$	16
risch	$-\frac{\ln\left(e^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)}{4} + \frac{\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{4}$	26
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)-1\right)}{4}$	34
parallelrisch	$\ln\left(\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}\right)+\ln\left(\frac{1}{\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}}\right)$	34

input
```
int(1/(4-5*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(tan(x)+2)-1/4*ln(tan(x)-2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(7) = 14$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{8}\log\left(\frac{3}{4}\cos(x)^2+\cos(x)\sin(x)+\frac{1}{4}\right) - \frac{1}{8}\log\left(\frac{3}{4}\cos(x)^2-\cos(x)\sin(x)+\frac{1}{4}\right)$$

input
```
integrate(1/(4-5*sin(x)^2),x, algorithm="fricas")
```

output
```
1/8*log(3/4*cos(x)^2 + cos(x)*sin(x) + 1/4) - 1/8*log(3/4*cos(x)^2 - cos(x
)*sin(x) + 1/4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(7) = 14$.

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{\log\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)-1\right)}{4}$$

input
```
integrate(1/(4-5*sin(x)**2),x)
```

output
```
log(tan(x/2)**2 - tan(x/2) - 1)/4 - log(tan(x/2)**2 + tan(x/2) - 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{4}\log\left(\tan\left(x\right)+2\right) - \frac{1}{4}\log\left(\tan\left(x\right)-2\right)$$

input
```
integrate(1/(4-5*sin(x)^2),x, algorithm="maxima")
```

output
```
1/4*log(tan(x) + 2) - 1/4*log(tan(x) - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{4}\log\left(|\tan(x)+2|\right) - \frac{1}{4}\log\left(|\tan(x)-2|\right)$$

input
```
integrate(1/(4-5*sin(x)^2),x, algorithm="giac")
```

output
```
1/4*log(abs(tan(x) + 2)) - 1/4*log(abs(tan(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 37.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{\operatorname{atanh}\left(\frac{\tan(x)}{2}\right)}{2}$$

input
```
int(-1/(5*sin(x)^2 - 4),x)
```

output
```
atanh(tan(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 5.18

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)}{4} + \frac{\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)}{4}$$

input
```
int(1/(4-5*sin(x)^2),x)
```

output
```
(log( - sqrt(5) + 2*tan(x/2) - 1) - log( - sqrt(5) + 2*tan(x/2) + 1) + log
(sqrt(5) + 2*tan(x/2) - 1) - log(sqrt(5) + 2*tan(x/2) + 1))/4
```

3.39 $\int \frac{1}{4-6\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 18

$$\int \frac{1}{4-6\sin^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
1/4*arctanh(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-6\sin^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(4 - 6*Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[Tan[x]/Sqrt[2]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-6\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{4-6\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$\int \frac{1}{4-2\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[(4 - 6*Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[Tan[x]/Sqrt[2]]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{4}$	14
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+\frac{2i\sqrt{2}}{3}+\frac{1}{3}\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\frac{2i\sqrt{2}}{3}+\frac{1}{3}\right)}{8}$	40

input
```
int(1/(4-6*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.17

$$\int \frac{1}{4-6\sin^2(x)}\,dx = \frac{1}{16}\sqrt{2}\log\left(-\frac{7\cos(x)^4-10\cos(x)^2-4\left(\sqrt{2}\cos(x)^3+\sqrt{2}\cos(x)\right)\sin(x)-1}{9\cos(x)^4-6\cos(x)^2+1}\right)$$

input
```
integrate(1/(4-6*sin(x)^2),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*log(-(7*cos(x)^4 - 10*cos(x)^2 - 4*(sqrt(2)*cos(x)^3 + sqrt(2
)*cos(x))*sin(x) - 1)/(9*cos(x)^4 - 6*cos(x)^2 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(17) = 34$.

Time = 6.04 (sec) , antiderivative size = 1426, normalized size of antiderivative = 79.22

$$\int \frac{1}{4-6\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4-6*sin(x)**2),x)
```

output
```
-10955043*sqrt(sqrt(3) + 2)*log(tan(x/2) - sqrt(2 - sqrt(3)))/(-33114930*s
qrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914*
sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 100239*sqrt(3)*sqrt(2 - sqr
t(3))*log(tan(x/2) - sqrt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(
sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(
3))*sqrt(sqrt(3) + 2)) + 173619*sqrt(2 - sqrt(3))*log(tan(x/2) - sqrt(2 -
sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3)
 + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 63248
97*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(x/2) - sqrt(2 - sqrt(3)))/(-33114930*
sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914
*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 8258730*sqrt(3)*sqrt(sqrt(
3) + 2)*log(tan(x/2) + sqrt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqr
t(3))*sqrt(sqrt(3) + 2)) - 1071114*sqrt(2 - sqrt(3))*log(tan(x/2) + sqrt(2
 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt
(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 61
8408*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(x/2) + sqrt(2 - sqrt(3)))/(-3311493
0*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 191189
14*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 14304540*sqrt(sqrt(3) +
2)*log(tan(x/2) + sqrt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1}{4-6\sin^2(x)}\,dx = -\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(x)}{\sqrt{2}+\tan(x)}\right)$$

input
```
integrate(1/(4-6*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*log(-(sqrt(2) - tan(x))/(sqrt(2) + tan(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{4-6\sin^2(x)}\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(x)\right|}{\left|2\sqrt{2}+2\tan(x)\right|}\right)$$

input
```
integrate(1/(4-6*sin(x)^2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(x))/abs(2*sqrt(2) + 2*tan(x)))
```

Mupad [B] (verification not implemented)

Time = 37.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{1}{4-6\sin^2(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}$$

input
```
int(-1/(6*sin(x)^2 - 4),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*tan(x))/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{1}{4-6\sin^2(x)}\,dx = \frac{\sqrt{2}\left(2atanh\left(\frac{2\tan\left(\frac{x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)+\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+\tan\left(\frac{x}{2}\right)\right)-\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+\tan\left(\frac{x}{2}\right)\right)\right)}{8}$$

input
```
int(1/(4-6*sin(x)^2),x)
```

output
```
(sqrt(2)*(2*atanh((2*tan(x/2))/(sqrt(6) - sqrt(2))) + log(( - sqrt(6) - sq
rt(2) + 2*tan(x/2))/2) - log((sqrt(6) + sqrt(2) + 2*tan(x/2))/2)))/8
```

3.40 $\int \frac{1}{4-7\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{4-7\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

output
```
1/6*arctanh(1/2*3^(1/2)*tan(x))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-7\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Integrate[(4 - 7*Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[3]*Tan[x])/2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-7\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{4-7\sin(x)^2}dx$$
$$\Big\downarrow \text{3660}$$
$$\int \frac{1}{4-3\tan^2(x)}d\tan(x)$$
$$\Big\downarrow \text{219}$$
$$\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Int[(4 - 7*Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[3]*Tan[x])/2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\,\tan(x)}{2}\right)\sqrt{3}}{6}$	14
risch	$\frac{\sqrt{3}\,\ln\left(e^{2ix}+\frac{4i\sqrt{3}}{7}+\frac{1}{7}\right)}{12}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\frac{4i\sqrt{3}}{7}+\frac{1}{7}\right)}{12}$	40

input
```
int(1/(4-7*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(1/2*3^(1/2)*tan(x))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{1}{4-7\sin^2(x)}\,dx$$
$$=\frac{1}{24}\sqrt{3}\log\left(-\frac{47\cos(x)^4-54\cos(x)^2-8\left(\sqrt{3}\cos(x)^3+3\sqrt{3}\cos(x)\right)\sin(x)-9}{49\cos(x)^4-42\cos(x)^2+9}\right)$$

input
```
integrate(1/(4-7*sin(x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(3)*log(-(47*cos(x)^4 - 54*cos(x)^2 - 8*(sqrt(3)*cos(x)^3 + 3*sqr
t(3)*cos(x))*sin(x) - 9)/(49*cos(x)^4 - 42*cos(x)^2 + 9))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(17) = 34$.

Time = 6.78 (sec) , antiderivative size = 1576, normalized size of antiderivative = 75.05

$$\int \frac{1}{4-7\sin^2(x)}\,dx = \text{Too large to display}$$

input

```
integrate(1/(4-7*sin(x)**2),x)
```

output

```
-117579*sqrt(2)*sqrt(sqrt(21) + 5)*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21
))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt
(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21) + 2811216) - 79939*sqr
t(42)*sqrt(5 - sqrt(21))*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21))/2)/(298
416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(
21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21) + 2811216) - 333165*sqrt(2)*sqrt
(5 - sqrt(21))*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5
 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(
sqrt(21) + 5) + 454512*sqrt(21) + 2811216) + 8569*sqrt(42)*sqrt(sqrt(21) +
 5)*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21)
)*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) +
5) + 454512*sqrt(21) + 2811216) - 8569*sqrt(42)*sqrt(sqrt(21) + 5)*log(tan
(x/2) + sqrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt
(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512
*sqrt(21) + 2811216) + 333165*sqrt(2)*sqrt(5 - sqrt(21))*log(tan(x/2) + sq
rt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5)
+ 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21)
+ 2811216) + 79939*sqrt(42)*sqrt(5 - sqrt(21))*log(tan(x/2) + sqrt(2)*sqrt
(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*s
qrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21) + 28112...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{1}{4-7\sin^2(x)}\,dx = -\frac{1}{12}\sqrt{3}\log\left(-\frac{2\sqrt{3}-3\tan(x)}{2\sqrt{3}+3\tan(x)}\right)$$

input
```
integrate(1/(4-7*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(-(2*sqrt(3) - 3*tan(x))/(2*sqrt(3) + 3*tan(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{4-7\sin^2(x)}\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+6\tan(x)\right|}{\left|4\sqrt{3}+6\tan(x)\right|}\right)$$

input
```
integrate(1/(4-7*sin(x)^2),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(abs(-4*sqrt(3) + 6*tan(x))/abs(4*sqrt(3) + 6*tan(x)))
```

Mupad [B] (verification not implemented)

Time = 37.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{4-7\sin^2(x)}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\tan(x)}{2}\right)}{6}$$

input
```
int(-1/(7*sin(x)^2 - 4),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*tan(x))/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{1}{4-7\sin^2(x)}\,dx$$
$$= \frac{\sqrt{3}\left(2atanh\left(\frac{2\tan\left(\frac{x}{2}\right)}{\sqrt{7}-\sqrt{3}}\right)+\log\left(-\frac{\sqrt{14}}{2}-\frac{\sqrt{6}}{2}+\sqrt{2}\tan\left(\frac{x}{2}\right)\right)-\log\left(\frac{\sqrt{14}}{2}+\frac{\sqrt{6}}{2}+\sqrt{2}\tan\left(\frac{x}{2}\right)\right)\right)}{12}$$

input
```
int(1/(4-7*sin(x)^2),x)
```

output
```
(sqrt(3)*(2*atanh((2*tan(x/2))/(sqrt(7) - sqrt(3))) + log(( - sqrt(14) - s
qrt(6) + 2*sqrt(2)*tan(x/2))/2) - log((sqrt(14) + sqrt(6) + 2*sqrt(2)*tan(
x/2))/2)))/12
```

3.41 $\int \frac{1}{-4+7\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/2*3^(1/2)*tan(x))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Integrate[(-4 + 7*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*Tan[x])/2]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{7\sin^2(x)-4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{7\sin(x)^2-4}dx$$

$$\Big\downarrow \; 3660$$

$$\int \frac{1}{3\tan^2(x)-4}d\tan(x)$$

$$\Big\downarrow \; 220$$

$$-\frac{\text{arctanh}\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Int[(-4 + 7*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*Tan[x])/2]/Sqrt[3]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\,\tan(x)}{2}\right)\sqrt{3}}{6}$	14
risch	$\frac{\sqrt{3}\,\ln\left(e^{2ix}-\frac{4i\sqrt{3}}{7}+\frac{1}{7}\right)}{12}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+\frac{4i\sqrt{3}}{7}+\frac{1}{7}\right)}{12}$	40

input
```
int(1/(-4+7*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctanh(1/2*3^(1/2)*tan(x))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{1}{-4+7\sin^2(x)}\,dx$$
$$=\frac{1}{24}\sqrt{3}\log\left(-\frac{47\cos(x)^4-54\cos(x)^2+8\left(\sqrt{3}\cos(x)^3+3\sqrt{3}\cos(x)\right)\sin(x)-9}{49\cos(x)^4-42\cos(x)^2+9}\right)$$

input
```
integrate(1/(-4+7*sin(x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(3)*log(-(47*cos(x)^4 - 54*cos(x)^2 + 8*(sqrt(3)*cos(x)^3 + 3*sqr
t(3)*cos(x))*sin(x) - 9)/(49*cos(x)^4 - 42*cos(x)^2 + 9))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(19) = 38$.

Time = 6.72 (sec) , antiderivative size = 1576, normalized size of antiderivative = 75.05

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-4+7*sin(x)**2),x)
```

output
```
-8569*sqrt(42)*sqrt(sqrt(21) + 5)*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21)
)/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(
5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21) + 2811216) + 333165*sqr
t(2)*sqrt(5 - sqrt(21))*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21))/2)/(2984
16*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(2
1))*sqrt(sqrt(21) + 5) + 454512*sqrt(21) + 2811216) + 79939*sqrt(42)*sqrt(
5 - sqrt(21))*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5
- sqrt(21))*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(s
qrt(21) + 5) + 454512*sqrt(21) + 2811216) + 117579*sqrt(2)*sqrt(sqrt(21) +
 5)*log(tan(x/2) - sqrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21)
)*sqrt(sqrt(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) +
5) + 454512*sqrt(21) + 2811216) - 117579*sqrt(2)*sqrt(sqrt(21) + 5)*log(ta
n(x/2) + sqrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqr
t(21) + 5) + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 45451
2*sqrt(21) + 2811216) - 79939*sqrt(42)*sqrt(5 - sqrt(21))*log(tan(x/2) + s
qrt(2)*sqrt(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5)
 + 143184*sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21)
 + 2811216) - 333165*sqrt(2)*sqrt(5 - sqrt(21))*log(tan(x/2) + sqrt(2)*sqr
t(5 - sqrt(21))/2)/(298416*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 143184*
sqrt(21)*sqrt(5 - sqrt(21))*sqrt(sqrt(21) + 5) + 454512*sqrt(21) + 2811...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{2\sqrt{3}-3\tan(x)}{2\sqrt{3}+3\tan(x)}\right)$$

input
```
integrate(1/(-4+7*sin(x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*log(-(2*sqrt(3) - 3*tan(x))/(2*sqrt(3) + 3*tan(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+6\tan(x)\right|}{\left|4\sqrt{3}+6\tan(x)\right|}\right)$$

input
```
integrate(1/(-4+7*sin(x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 6*tan(x))/abs(4*sqrt(3) + 6*tan(x)))
```

Mupad [B] (verification not implemented)

Time = 36.67 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = -\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\tan(x)}{2}\right)}{6}$$

input
```
int(1/(7*sin(x)^2 - 4),x)
```

output
```
-(3^(1/2)*atanh((3^(1/2)*tan(x))/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{1}{-4+7\sin^2(x)}\,dx = \frac{\sqrt{3}\left(-2atanh\left(\frac{2\tan\left(\frac{x}{2}\right)}{\sqrt{7}-\sqrt{3}}\right)-\log\left(-\frac{\sqrt{14}}{2}-\frac{\sqrt{6}}{2}+\sqrt{2}\tan\left(\frac{x}{2}\right)\right)+\log\left(\frac{\sqrt{14}}{2}+\frac{\sqrt{6}}{2}+\sqrt{2}\tan\left(\frac{x}{2}\right)\right)\right)}{12}$$

input
```
int(1/(-4+7*sin(x)^2),x)
```

output
```
(sqrt(3)*( - 2*atanh((2*tan(x/2))/(sqrt(7) - sqrt(3))) - log(( - sqrt(14)
- sqrt(6) + 2*sqrt(2)*tan(x/2))/2) + log((sqrt(14) + sqrt(6) + 2*sqrt(2)*t
an(x/2))/2)))/12
```

3.42 $\int \frac{1}{-4+6\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 18

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(-4 + 6*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTanh[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{6\sin^2(x)-4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{6\sin(x)^2-4}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{2\tan^2(x)-4}d\tan(x)$$

$$\Big\downarrow \text{220}$$

$$-\frac{\text{arctanh}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[(-4 + 6*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTanh[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{4}$	14
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}-\frac{2i\sqrt{2}}{3}+\frac{1}{3}\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+\frac{2i\sqrt{2}}{3}+\frac{1}{3}\right)}{8}$	40

input
```
int(1/(-4+6*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctanh(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.17

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = \frac{1}{16}\sqrt{2}\log\left(-\frac{7\cos(x)^4-10\cos(x)^2+4\left(\sqrt{2}\cos(x)^3+\sqrt{2}\cos(x)\right)\sin(x)-1}{9\cos(x)^4-6\cos(x)^2+1}\right)$$

input
```
integrate(1/(-4+6*sin(x)^2),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*log(-(7*cos(x)^4 - 10*cos(x)^2 + 4*(sqrt(2)*cos(x)^3 + sqrt(2
)*cos(x))*sin(x) - 1)/(9*cos(x)^4 - 6*cos(x)^2 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(19) = 38$.

Time = 6.00 (sec) , antiderivative size = 1426, normalized size of antiderivative = 79.22

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-4+6*sin(x)**2),x)
```

output
```
-6324897*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(x/2) - sqrt(2 - sqrt(3)))/(-331
14930*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19
118914*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 173619*sqrt(2 - sqrt
(3))*log(tan(x/2) - sqrt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(s
qrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(3
))*sqrt(sqrt(3) + 2)) + 100239*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(x/2) - sq
rt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*
sqrt(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
 + 10955043*sqrt(sqrt(3) + 2)*log(tan(x/2) - sqrt(2 - sqrt(3)))/(-33114930*
sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914
*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 14304540*sqrt(sqrt(3) + 2)
*log(tan(x/2) + sqrt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(sqrt(
3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(3))*s
qrt(sqrt(3) + 2)) - 618408*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(x/2) + sqrt(2
 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt
(3) + 2933682 + 19118914*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 10
71114*sqrt(2 - sqrt(3))*log(tan(x/2) + sqrt(2 - sqrt(3)))/(-33114930*sqrt(
2 - sqrt(3))*sqrt(sqrt(3) + 2) - 1693762*sqrt(3) + 2933682 + 19118914*sqrt
(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 8258730*sqrt(3)*sqrt(sqrt(3) +
2)*log(tan(x/2) + sqrt(2 - sqrt(3)))/(-33114930*sqrt(2 - sqrt(3))*sqrt(...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(x)}{\sqrt{2}+\tan(x)}\right)$$

input
```
integrate(1/(-4+6*sin(x)^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(-(sqrt(2) - tan(x))/(sqrt(2) + tan(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{|-2\sqrt{2}+2\tan(x)|}{|2\sqrt{2}+2\tan(x)|}\right)$$

input
```
integrate(1/(-4+6*sin(x)^2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(x))/abs(2*sqrt(2) + 2*tan(x)))
```

Mupad [B] (verification not implemented)

Time = 37.74 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}$$

input
```
int(1/(6*sin(x)^2 - 4),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(x))/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{1}{-4+6\sin^2(x)}\,dx = \frac{\sqrt{2}\left(-2atanh\left(\frac{2\tan\left(\frac{x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)-\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+\tan\left(\frac{x}{2}\right)\right)+\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+\tan\left(\frac{x}{2}\right)\right)\right)}{8}$$

input
```
int(1/(-4+6*sin(x)^2),x)
```

output
```
(sqrt(2)*( - 2*atanh((2*tan(x/2))/(sqrt(6) - sqrt(2))) - log(( - sqrt(6) -
 sqrt(2) + 2*tan(x/2))/2) + log((sqrt(6) + sqrt(2) + 2*tan(x/2))/2)))/8
```

3.43 $\int \frac{1}{-4+5\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

output
```
-1/2*arctanh(1/2*tan(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = \frac{1}{4}\log(2\cos(x)-\sin(x)) - \frac{1}{4}\log(2\cos(x)+\sin(x))$$

input
```
Integrate[(-4 + 5*Sin[x]^2)^(-1),x]
```

output
```
Log[2*Cos[x] - Sin[x]]/4 - Log[2*Cos[x] + Sin[x]]/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sin^2(x)-4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{5\sin(x)^2-4}dx$$

$$\Big\downarrow \ 3660$$

$$\int \frac{1}{\tan^2(x)-4}d\tan(x)$$

$$\Big\downarrow \ 220$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(-4 + 5*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTanh[Tan[x]/2]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\ln(\tan(x)-2)}{4}-\frac{\ln(\tan(x)+2)}{4}$	16
risch	$\frac{\ln\left(e^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)}{4}-\frac{\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{4}$	26
norman	$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)-1\right)}{4}$	34
parallelrisch	$\ln\left(\frac{1}{\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}}\right)+\ln\left(\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}\right)$	34

input
```
int(1/(-4+5*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(tan(x)-2)-1/4*ln(tan(x)+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(7) = 14$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{1}{8}\log\left(\frac{3}{4}\cos(x)^2+\cos(x)\sin(x)+\frac{1}{4}\right) + \frac{1}{8}\log\left(\frac{3}{4}\cos(x)^2-\cos(x)\sin(x)+\frac{1}{4}\right)$$

input
```
integrate(1/(-4+5*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/8*log(3/4*cos(x)^2 + cos(x)*sin(x) + 1/4) + 1/8*log(3/4*cos(x)^2 - cos(
x)*sin(x) + 1/4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{\log\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)-1\right)}{4}+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)-1\right)}{4}$$

input
```
integrate(1/(-4+5*sin(x)**2),x)
```

output
```
-log(tan(x/2)**2 - tan(x/2) - 1)/4 + log(tan(x/2)**2 + tan(x/2) - 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{1}{4}\log\left(\tan\left(x\right)+2\right)+\frac{1}{4}\log\left(\tan\left(x\right)-2\right)$$

input
```
integrate(1/(-4+5*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/4*log(tan(x) + 2) + 1/4*log(tan(x) - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{1}{4}\log\left(|\tan(x)+2|\right) + \frac{1}{4}\log\left(|\tan(x)-2|\right)$$

input
```
integrate(1/(-4+5*sin(x)^2),x, algorithm="giac")
```

output
```
-1/4*log(abs(tan(x) + 2)) + 1/4*log(abs(tan(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 37.44 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{\operatorname{atanh}\left(\frac{\tan(x)}{2}\right)}{2}$$

input
```
int(1/(5*sin(x)^2 - 4),x)
```

output
```
-atanh(tan(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 5.18

$$\int \frac{1}{-4+5\sin^2(x)}\,dx = -\frac{\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)}{4} + \frac{\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)}{4} - \frac{\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)}{4} + \frac{\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)}{4}$$

input
```
int(1/(-4+5*sin(x)^2),x)
```

output
```
( - log( - sqrt(5) + 2*tan(x/2) - 1) + log( - sqrt(5) + 2*tan(x/2) + 1) -
log(sqrt(5) + 2*tan(x/2) - 1) + log(sqrt(5) + 2*tan(x/2) + 1))/4
```

3.44 $\int \frac{1}{-4+4\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{\tan(x)}{4}$$

output
```
-1/4*tan(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{\tan(x)}{4}$$

input
```
Integrate[(-4 + 4*Sin[x]^2)^(-1),x]
```

output
```
-1/4*Tan[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\sin^2(x)-4}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{4\sin(x)^2-4}dx$$
$$\Big\downarrow \text{ 3654}$$
$$-\frac{1}{4}\int \sec^2(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$-\frac{1}{4}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{ 4254}$$
$$\frac{1}{4}\int 1d(-\tan(x))$$
$$\Big\downarrow \text{ 24}$$
$$-\frac{\tan(x)}{4}$$

input
```
Int[(-4 + 4*Sin[x]^2)^(-1),x]
```

output
```
-1/4*Tan[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\tan(x)}{4}$	5
parallelrisch	$-\frac{\sin(x)}{4\cos(x)}$	9
risch	$-\frac{i}{2(e^{2ix}+1)}$	13
norman	$\frac{\tan(\frac{x}{2})}{-2+2\tan(\frac{x}{2})^2}$	17

input
```
int(1/(-4+4*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{\sin(x)}{4\cos(x)}$$

input
```
integrate(1/(-4+4*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sin(x)/cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2\tan^2\left(\frac{x}{2}\right)-2}$$

input
```
integrate(1/(-4+4*sin(x)**2),x)
```

output
```
tan(x/2)/(2*tan(x/2)**2 - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{1}{4}\tan(x)$$

input
```
integrate(1/(-4+4*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/4*tan(x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{1}{4}\,\tan\left(x\right)$$

input
```
integrate(1/(-4+4*sin(x)^2),x, algorithm="giac")
```

output
```
-1/4*tan(x)
```

Mupad [B] (verification not implemented)

Time = 37.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{\tan(x)}{4}$$

input
```
int(1/(4*sin(x)^2 - 4),x)
```

output
```
-tan(x)/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{-4+4\sin^2(x)}\,dx = -\frac{\sin\left(x\right)}{4\cos\left(x\right)}$$

input
```
int(1/(-4+4*sin(x)^2),x)
```

output
```
( - sin(x))/(4*cos(x))
```

3.45 $\int \frac{1}{-4+3\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = -\frac{x}{2}+\frac{1}{2}\arctan\left(\frac{\cos(x)\sin(x)}{2-\sin^2(x)}\right)$$

output
```
-1/2*x+1/2*arctan(cos(x)*sin(x)/(2-sin(x)^2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.35

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = \frac{1}{2}\arctan(2\cot(x))$$

input
```
Integrate[(-4 + 3*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[2*Cot[x]]/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3660, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sin^2(x)-4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{3\sin(x)^2-4}dx$$

$$\Big\downarrow \text{ 3660}$$

$$\int \frac{1}{-\tan^2(x)-4}d\tan(x)$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{1}{2}\arctan\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(-4 + 3*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[Tan[x]/2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

method	result	size
default	$-\frac{\arctan\left(\frac{\tan(x)}{2}\right)}{2}$	8
risch	$\frac{i\ln\left(\mathrm{e}^{2ix}+\frac{1}{3}\right)}{4}-\frac{i\ln\left(\mathrm{e}^{2ix}+3\right)}{4}$	24
parallelrisch	$\frac{i\left(\ln\left(-i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)-\ln\left(i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)\right)}{4}$	39

input
```
int(1/(-4+3*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = \frac{1}{4}\arctan\left(\frac{5\cos(x)^2-1}{4\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(-4+3*sin(x)^2),x, algorithm="fricas")
```

output
```
1/4*arctan(1/4*(5*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = -\frac{\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{2} - \frac{\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{2} - \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor$$

input
```
integrate(1/(-4+3*sin(x)**2),x)
```

output
```
-atan(2*tan(x/2) - sqrt(3))/2 - atan(2*tan(x/2) + sqrt(3))/2 - pi*floor((x
/2 - pi/2)/pi)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.27

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}\tan(x)\right)$$

input
```
integrate(1/(-4+3*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/2*arctan(1/2*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = -\frac{1}{2}x + \frac{1}{2}\arctan\left(\frac{\sin(2x)}{\cos(2x)+3}\right)$$

input
```
integrate(1/(-4+3*sin(x)^2),x, algorithm="giac")
```

output
```
-1/2*x + 1/2*arctan(sin(2*x)/(cos(2*x) + 3))
```

Mupad [B] (verification not implemented)

Time = 36.92 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = \frac{\operatorname{atan}(\tan(x))}{2} - \frac{x}{2} - \frac{\operatorname{atan}\left(\frac{\tan(x)}{2}\right)}{2}$$

input
```
int(1/(3*sin(x)^2 - 4),x)
```

output
```
atan(tan(x))/2 - x/2 - atan(tan(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{-4+3\sin^2(x)}\,dx = \frac{atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{atan\left(\sqrt{3}+2\tan\left(\frac{x}{2}\right)\right)}{2}$$

input
```
int(1/(-4+3*sin(x)^2),x)
```

output
```
(atan(sqrt(3) - 2*tan(x/2)) - atan(sqrt(3) + 2*tan(x/2)))/2
```

3.46 $\int \frac{1}{-4+2\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = -\frac{x}{2\sqrt{2}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{2+\sqrt{2}-\sin^2(x)}\right)}{2\sqrt{2}}$$

output
```
-1/4*x*2^(1/2)+1/4*arctan(cos(x)*sin(x)/(2+2^(1/2)-sin(x)^2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = -\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(-4 + 2*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sin^2(x)-4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{2\sin(x)^2-4}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{-2\tan^2(x)-4}d\tan(x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[(-4 + 2*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.34

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{4}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+3-2\sqrt{2}\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+3+2\sqrt{2}\right)}{8}$	40

input
```
int(1/(-4+2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{-4+2\sin^2(x)}\,dx=\frac{1}{8}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(-4+2*sin(x)^2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = -\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} - \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}$$

input
```
integrate(1/(-4+2*sin(x)**2),x)
```

output
```
-sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 - sqrt
(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(-4+2*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = -\frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$

input
```
integrate(1/(-4+2*sin(x)^2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - cos(2*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 37.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = -\frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{4} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}$$

input
```
int(1/(2*sin(x)^2 - 4),x)
```

output
```
- (2^(1/2)*(x - atan(tan(x))))/4 - (2^(1/2)*atan((2^(1/2)*tan(x))/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{-4+2\sin^2(x)}\,dx = \frac{\sqrt{2}\left(atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{4}$$

input
```
int(1/(-4+2*sin(x)^2),x)
```

output
```
(sqrt(2)*(atan((sqrt(2) - 2*tan(x/2))/sqrt(2)) - atan((sqrt(2) + 2*tan(x/2
))/sqrt(2))))/4
```

3.47 $\int \frac{1}{-4+\sin^2(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{2\left(2+\sqrt{3}\right)-\sin^2(x)}\right)}{2\sqrt{3}}$$

output
```
-1/6*x*3^(1/2)+1/6*arctan(cos(x)*sin(x)/(4+2*3^(1/2)-sin(x)^2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{\arctan\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Integrate[(-4 + Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[(Sqrt[3]*Tan[x])/2]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3660, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^2(x)-4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^2-4}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{-3\tan^2(x)-4}d\tan(x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{1}{2}\sqrt{3}\tan(x)\right)}{2\sqrt{3}}$$

input
```
Int[(-4 + Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[(Sqrt[3]*Tan[x])/2]/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.32

method	result	size
default	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\tan(x)}{2}\right)}{6}$	14
risch	$\frac{i\sqrt{3}\,\ln\left(e^{2ix}-4\sqrt{3}+7\right)}{12}-\frac{i\sqrt{3}\,\ln\left(e^{2ix}+4\sqrt{3}+7\right)}{12}$	40

input
```
int(1/(-4+sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*3^(1/2)*arctan(1/2*3^(1/2)*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{-4+\sin^2(x)}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{7\,\sqrt{3}\cos{(x)}^2-3\,\sqrt{3}}{12\,\cos{(x)}\sin{(x)}}\right)$$

input
```
integrate(1/(-4+sin(x)^2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 3*sqrt(3))/(cos(x)*sin(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}-\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} - \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(-4+sin(x)**2),x)
```

output
```
-sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 - sqrt(3)/3) + pi*floor((x/2 - pi/2)/p
i))/6 - sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 + sqrt(3)/3) + pi*floor((x/2 -
pi/2)/pi))/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\tan(x)\right)$$

input
```
integrate(1/(-4+sin(x)^2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/2*sqrt(3)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{2\sqrt{3}\sin(2x)-3\sin(2x)}{2\sqrt{3}\cos(2x)+2\sqrt{3}-3\cos(2x)+3}\right)\right)$$

input
```
integrate(1/(-4+sin(x)^2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(x + arctan(-(2*sqrt(3)*sin(2*x) - 3*sin(2*x))/(2*sqrt(3)*cos
(2*x) + 2*sqrt(3) - 3*cos(2*x) + 3)))
```

Mupad [B] (verification not implemented)

Time = 37.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{\sqrt{3}\,(x-\operatorname{atan}(\tan(x)))}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\tan(x)}{2}\right)}{6}$$

input
```
int(1/(sin(x)^2 - 4),x)
```

output
```
- (3^(1/2)*(x - atan(tan(x))))/6 - (3^(1/2)*atan((3^(1/2)*tan(x))/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{1}{-4+\sin^2(x)}\,dx = -\frac{\sqrt{3}\left(atan\left(\frac{2\tan\left(\frac{x}{2}\right)-1}{\sqrt{3}}\right)+atan\left(\frac{2\tan\left(\frac{x}{2}\right)+1}{\sqrt{3}}\right)\right)}{6}$$

input
```
int(1/(-4+sin(x)^2),x)
```

output
```
( - sqrt(3)*(atan((2*tan(x/2) - 1)/sqrt(3)) + atan((2*tan(x/2) + 1)/sqrt(3
))))/6
```

3.48 $\int \frac{1}{a+a\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{x}{\sqrt{2}a} + \frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}a}$$

output
```
1/2*x*2^(1/2)/a+1/2*arctan(cos(x)*sin(x)/(1+2^(1/2)+sin(x)^2))*2^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}a}$$

input
```
Integrate[(a + a*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(Sqrt[2]*a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^2(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin(x)^2+a}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{2a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}a}$$

input
```
Int[(a + a*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(Sqrt[2]*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\arctan\left(\tan(x)\sqrt{2}\right)\sqrt{2}}{2a}$	16
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{4a}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{4a}$	46

input
```
int(1/(a+a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(tan(x)*2^(1/2))*2^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+a\sin^2(x)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)}{4a}$$

input
```
integrate(1/(a+a*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))/
a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(37) = 74$.

Time = 2.86 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.97

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a+39202a} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a+39202a} + \frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a+39202a} + \frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a+39202a}$$

input
```
integrate(1/(a+a*sin(x)**2),x)
```

output
```
47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi
*floor((x/2 - pi/2)/pi))/(27720*sqrt(2)*a + 39202*a) + 66922*sqrt(3 - 2*sq
rt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(2
7720*sqrt(2)*a + 39202*a) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2
)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(27720*sqrt(2)*a + 392
02*a) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi
*floor((x/2 - pi/2)/pi))/(27720*sqrt(2)*a + 39202*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)}{2\,a}$$

input
```
integrate(1/(a+a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*tan(x))/a
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+a\sin^2(x)}\,dx=\frac{\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)}{2a}$$

input
```
integrate(1/(a+a*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2)))/a
```

Mupad [B] (verification not implemented)

Time = 37.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{a+a\sin^2(x)}\,dx=\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{2a}$$

input
```
int(1/(a + a*sin(x)^2),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*tan(x)))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{1}{a+a\sin^2(x)}\,dx=\frac{\sqrt{2}\left(2atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2}+1}\right)-\log\left(-\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)+i\right)i+\log\left(\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)-i\right)i\right)}{4a}$$

input
```
int(1/(a+a*sin(x)^2),x)
```

output
```
(sqrt(2)*(2*atan(tan(x/2)/(sqrt(2) + 1)) - log( - sqrt(2)*i + tan(x/2) + i
)*i + log(sqrt(2)*i + tan(x/2) - i)*i))/(4*a)
```

3.49 $\int \frac{1}{a-a\sin^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{1}{a-a\sin^2(x)}\,dx=\frac{\tan(x)}{a}$$

output
```
tan(x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sin^2(x)}\,dx=\frac{\tan(x)}{a}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a-a\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sec^2(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\int 1d(-\tan(x))}{a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tan(x)}{a}$$

input
```
Int[(a - a*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\tan(x)}{a}$	7
parallelrisch	$\frac{\sin(x)}{\cos(x)a}$	11
risch	$\frac{2i}{a(e^{2ix}+1)}$	16
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(-1+\tan\left(\frac{x}{2}\right)^2\right)}$	20

input
```
int(1/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{a-a\sin^2(x)}\,dx = \frac{\sin(x)}{a\cos(x)}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
sin(x)/(a*cos(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{1}{a-a\sin^2(x)}\,dx = -\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)-a}$$

input
```
integrate(1/(a-a*sin(x)**2),x)
```

output
```
-2*tan(x/2)/(a*tan(x/2)**2 - a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
tan(x)/a
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
tan(x)/a
```

Mupad [B] (verification not implemented)

Time = 37.71 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
int(1/(a - a*sin(x)^2),x)
```

output
```
tan(x)/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\sin(x)}{\cos(x)\,a}$$

input
```
int(1/(a-a*sin(x)^2),x)
```

output
```
sin(x)/(cos(x)*a)
```

3.50 $\int \frac{1}{a+b\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{\sqrt{a(a+b)}}$	23
risch	$-\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}+\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ba}-b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}$	160

input
```
int(1/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.55

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2+2ab+}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right.}{4(a^2+ab)}\right.$$
$$\left.-\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*
a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 -
a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 +
a^2 + 2*a*b + b^2))/(a^2 + a*b), -1/2*arctan(1/2*((2*a + b)*cos(x)^2 - a -
 b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15745 vs. $2(27) = 54$.

Time = 10.68 (sec) , antiderivative size = 15745, normalized size of antiderivative = 542.93

$$\int \frac{1}{a + b\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)**2),x)
```

output
```
Piecewise((zoo*(tan(x/2)/2 - 1/(2*tan(x/2))), Eq(a, 0) & Eq(b, 0)), ((tan(
x/2)/2 - 1/(2*tan(x/2)))/b, Eq(a, 0)), (2*tan(x/2)/(b*tan(x/2)**2 - b), Eq
(a, -b)), (x/a, Eq(b, 0)), (6*a**3*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/
a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(x/2))/(10*a**4*b*sqr
t(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) - 2*a**4*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(
-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b**2*sqrt(-1 - 2*b/a - 2*sqrt
(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*b*sqrt(a
*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqr
t(a*b + b**2)/a) + 72*a**2*b**3*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sq
rt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a**2*b**2*sqrt(a*b + b**2)*sqrt
(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a
) + 32*a*b**4*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*
sqrt(a*b + b**2)/a) - 32*a*b**3*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(
a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)) - 6*a**3*b*sqrt(-1
 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) + tan(x/2))/(10*a**4*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1
- 2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a
- 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*
b**2*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{a+b\sin^2(x)}\,dx = \frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan((a + b)*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(21) = 42$.

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+b\sin^2(x)}\,dx = \frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a
^2 + a*b)))/sqrt(a^2 + a*b)
```

Mupad [B] (verification not implemented)

Time = 38.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+b\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(x)\,(2a+2b)}{2\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*sin(x)^2),x)
```

output
```
atan((tan(x)*(2*a + 2*b))/(2*(a*b + a^2)^(1/2)))/(a*b + a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 427, normalized size of antiderivative = 14.72

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{b}\,\sqrt{a+b}\,\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a}{\sqrt{a}\,\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}}\right)+2\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}\,atan\left(\frac{}{\sqrt{}}\right.\right.}{}$$

input
```
int(1/(a+b*sin(x)^2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(b)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*a
tan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b))) + 2*sqrt
(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b
)*sqrt(a + b) + a + 2*b)))*a + 2*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*ata
n((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*b - sqrt(b
)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log( - sqrt(2*sqrt(b)*
sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2)) + sqrt(b)*sqrt(a + b)*sqrt(2*sq
rt(b)*sqrt(a + b) - a - 2*b)*log(sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + s
qrt(a)*tan(x/2)) - sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log( - sqrt(2*sqr
t(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2))*a - sqrt(2*sqrt(b)*sqrt(a
+ b) - a - 2*b)*log( - sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan
(x/2))*b + sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log(sqrt(2*sqrt(b)*sqrt(a
 + b) - a - 2*b) + sqrt(a)*tan(x/2))*a + sqrt(2*sqrt(b)*sqrt(a + b) - a -
2*b)*log(sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2))*b))/(2*
a**2*(a + b))
```

3.51 $\int \frac{1}{a-b\sin^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{a-b\sin^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{-a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{-a+b}}$$

output
```
arctanh((-a+b)^(1/2)*tan(x)/a^(1/2))/a^(1/2)/(-a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-b\sin^2(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

input
```
Integrate[(a - b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(x)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{(a-b)\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

input
```
Int[(a - b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{a(a-b)}}\right)}{\sqrt{a(a-b)}}$	29
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2-2iab+2a\sqrt{-a^2+ba}-b\sqrt{-a^2+ba}}{b\sqrt{-a^2+ba}}\right)}{2\sqrt{-a^2+ba}}+\frac{\ln\left(e^{2ix}-\frac{2ia^2-2iab-2a\sqrt{-a^2+ba}+b\sqrt{-a^2+ba}}{b\sqrt{-a^2+ba}}\right)}{2\sqrt{-a^2+ba}}$	152

input
```
int(1/(a-b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a-b))^(1/2)*arctan((a-b)*tan(x)/(a*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.97

$$\int \frac{1}{a-b\sin^2(x)}\,dx$$

$$= \left[-\frac{\sqrt{-a^2+ab}\log\left(\frac{(8a^2-8ab+b^2)\cos(x)^4-2(4a^2-5ab+b^2)\cos(x)^2+4\left((2a-b)\cos(x)^3-(a-b)\cos(x)\right)\sqrt{-a^2+ab}\sin(x)+a^2-2ab+}{b^2\cos(x)^4+2(ab-b^2)\cos(x)^2+a^2-2ab+b^2}\right.}{4(a^2-ab)}\right.$$

$$\left. -\frac{\arctan\left(\frac{(2a-b)\cos(x)^2-a+b}{2\sqrt{a^2-ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2-ab}}\right]$$

input
```
integrate(1/(a-b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 + a*b)*log(((8*a^2 - 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 - 5*
a*b + b^2)*cos(x)^2 + 4*((2*a - b)*cos(x)^3 - (a - b)*cos(x))*sqrt(-a^2 +
a*b)*sin(x) + a^2 - 2*a*b + b^2)/(b^2*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 +
a^2 - 2*a*b + b^2))/(a^2 - a*b), -1/2*arctan(1/2*((2*a - b)*cos(x)^2 - a +
 b)/(sqrt(a^2 - a*b)*cos(x)*sin(x)))/sqrt(a^2 - a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15319 vs. $2(27) = 54$.

Time = 11.06 (sec) , antiderivative size = 15319, normalized size of antiderivative = 464.21

$$\int \frac{1}{a - b\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)**2),x)
```

output
```
Piecewise((zoo*(tan(x/2)/2 - 1/(2*tan(x/2))), Eq(a, 0) & Eq(b, 0)), (-(tan
(x/2)/2 - 1/(2*tan(x/2)))/b, Eq(a, 0)), (-2*tan(x/2)/(b*tan(x/2)**2 - b),
Eq(a, b)), (x/a, Eq(b, 0)), (6*a**3*b*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2
)/a)*log(-sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) + tan(x/2))/(10*a**4*b*
sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b
**2)/a) + 2*a**4*sqrt(-a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a
)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) - 50*a**3*b**2*sqrt(-1 + 2*b/a
- 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) - 26*a**
3*b*sqrt(-a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2
*b/a + 2*sqrt(-a*b + b**2)/a) + 72*a**2*b**3*sqrt(-1 + 2*b/a - 2*sqrt(-a*b
 + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) + 56*a**2*b**2*sqrt(-
a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*s
qrt(-a*b + b**2)/a) - 32*a*b**4*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*s
qrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) - 32*a*b**3*sqrt(-a*b + b**2)*sqrt
(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)
/a)) - 6*a**3*b*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(-1 + 2*b
/a + 2*sqrt(-a*b + b**2)/a) + tan(x/2))/(10*a**4*b*sqrt(-1 + 2*b/a - 2*sqr
t(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) + 2*a**4*sqrt(-
a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*s
qrt(-a*b + b**2)/a) - 50*a**3*b**2*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a - b\sin^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a-b*sin(x)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \frac{1}{a - b\sin^2(x)}\,dx = -\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan(x) - b\tan(x)}{\sqrt{a^2 - ab}}\right)}{\sqrt{a^2 - ab}}$$

input
```
integrate(1/(a-b*sin(x)^2),x, algorithm="giac")
```

output
```
-(pi*floor(x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(x) - b*tan(x))/sqr
t(a^2 - a*b)))/sqrt(a^2 - a*b)
```

Mupad [B] (verification not implemented)

Time = 37.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-b\sin^2(x)}\,dx=\frac{\operatorname{atan}\left(\frac{\tan(x)\,(2\,a-2\,b)}{2\,\sqrt{a^2-a\,b}}\right)}{\sqrt{a^2-a\,b}}$$

input
```
int(1/(a - b*sin(x)^2),x)
```

output
```
atan((tan(x)*(2*a - 2*b))/(2*(a^2 - a*b)^(1/2)))/(a^2 - a*b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \frac{1}{a-b\sin^2(x)}\,dx=\frac{\sqrt{a}\,\sqrt{a-b}\,\left(atan\left(\frac{\sqrt{a}\,\tan\left(\frac{x}{2}\right)-\sqrt{b}}{\sqrt{a-b}}\right)+atan\left(\frac{\sqrt{a}\,\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a-b}}\right)\right)}{a\,(a-b)}$$

input
```
int(1/(a-b*sin(x)^2),x)
```

output
```
(sqrt(a)*sqrt(a - b)*(atan((sqrt(a)*tan(x/2) - sqrt(b))/sqrt(a - b)) + ata
n((sqrt(a)*tan(x/2) + sqrt(b))/sqrt(a - b))))/(a*(a - b))
```

3.52 $\int \frac{1}{2+2\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{2+2\sin^2(x)}\,dx=\frac{x}{2\sqrt{2}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{2\sqrt{2}}$$

output
```
1/4*x*2^(1/2)+1/4*arctan(cos(x)*sin(x)/(1+2^(1/2)+sin(x)^2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+2\sin^2(x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[(2 + 2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sin^2(x)+2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{2\sin(x)^2+2}dx$$
$$\Big\downarrow \text{3660}$$
$$\int \frac{1}{4\tan^2(x)+2}d\tan(x)$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}$$

input
```
Int[(2 + 2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{4}$	13
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{8}$	40

input
```
int(1/(2+2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctan(tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{2+2\sin^2(x)}\,dx = -\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(x\right)^2-2\,\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)$$

input
```
integrate(1/(2+2*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(34) = 68$.

Time = 2.84 (sec) , antiderivative size = 219, normalized size of antiderivative = 5.62

$$\int \frac{1}{2+2\sin^2(x)}\,dx = \frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{55440\sqrt{2}+78404} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{55440\sqrt{2}+78404} + \frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{55440\sqrt{2}+78404} + \frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{55440\sqrt{2}+78404}$$

input
```
integrate(1/(2+2*sin(x)**2),x)
```

output
```
47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi
*floor((x/2 - pi/2)/pi))/(55440*sqrt(2) + 78404) + 66922*sqrt(3 - 2*sqrt(2
))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(55440
*sqrt(2) + 78404) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2
*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(55440*sqrt(2) + 78404) + 1148
2*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2
- pi/2)/pi))/(55440*sqrt(2) + 78404)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.31

$$\int \frac{1}{2+2\sin^2(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(2+2*sin(x)^2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{1}{2+2\sin^2(x)}\,dx = \frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)$$

input
```
integrate(1/(2+2*sin(x)^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2)))
```

Mupad [B] (verification not implemented)

Time = 37.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{2+2\sin^2(x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)}{4}$$

input
```
int(1/(2*sin(x)^2 + 2),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/4 + (2^(1/2)*atan(2^(1/2)*tan(x)))/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{2+2\sin^2(x)}\,dx = \frac{\sqrt{2}\left(2atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2}+1}\right)-\log\left(-\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)+i\right)i+\log\left(\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)-i\right)i\right)}{8}$$

input
```
int(1/(2+2*sin(x)^2),x)
```

output
```
(sqrt(2)*(2*atan(tan(x/2)/(sqrt(2) + 1)) - log( - sqrt(2)*i + tan(x/2) + i
)*i + log(sqrt(2)*i + tan(x/2) - i)*i))/8
```

3.53 $\int \frac{1}{3-\cos(2x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{1}{3-\cos(2x)}\,dx = \frac{x}{2\sqrt{2}} + \frac{\arctan\left(\frac{\sin(2x)}{3+2\sqrt{2}-\cos(2x)}\right)}{2\sqrt{2}}$$

output
```
1/4*x*2^(1/2)+1/4*arctan(sin(2*x)/(3+2*2^(1/2)-cos(2*x)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int \frac{1}{3-\cos(2x)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[(3 - Cos[2*x])^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3-\cos(2x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{3-\sin\left(2x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3136}$$
$$\frac{\arctan\left(\frac{\sin(2x)}{-\cos(2x)+2\sqrt{2}+3}\right)}{2\sqrt{2}}+\frac{x}{2\sqrt{2}}$$

input
```
Int[(3 - Cos[2*x])^(-1),x]
```

output
```
x/(2*Sqrt[2]) + ArcTan[Sin[2*x]/(3 + 2*Sqrt[2] - Cos[2*x])]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{4}$	13
default	$\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{4}$	13
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)}{8}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)}{8}$	40

input
```
int(1/(3-cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctan(tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{3-\cos(2x)}\,dx=-\frac{1}{8}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(2x)-\sqrt{2}}{4\sin(2x)}\right)$$

input
```
integrate(1/(3-cos(2*x)),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(2*x) - sqrt(2))/sin(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1}{3-\cos(2x)}\,dx=\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan(x)\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}$$

input
```
integrate(1/(3-cos(2*x)),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x)) + pi*floor((x - pi/2)/pi))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

$$\int \frac{1}{3-\cos(2x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(2x)}{\cos(2x)+1}\right)$$

input
```
integrate(1/(3-cos(2*x)),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*sin(2*x)/(cos(2*x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{3-\cos(2x)}\,dx = \frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)$$

input
```
integrate(1/(3-cos(2*x)),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2)))
```

Mupad [B] (verification not implemented)

Time = 38.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.58

$$\int \frac{1}{3-\cos(2x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)}{4}$$

input
```
int(-1/(cos(2*x) - 3),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/4 + (2^(1/2)*atan(2^(1/2)*tan(x)))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{1}{3-\cos(2x)}\,dx = \frac{\sqrt{2}\,atan\left(\frac{2\tan(x)}{\sqrt{2}}\right)}{4}$$

input
```
int(1/(3-cos(2*x)),x)
```

output
```
(sqrt(2)*atan((2*tan(x))/sqrt(2)))/4
```

3.54 $\int \frac{1}{a-a\sin^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{1}{a-a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

output
```
tan(x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a-a\sin(x)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \sec^2(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{a}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\int 1d(-\tan(x))}{a}$$
$$\Big\downarrow \text{24}$$
$$\frac{\tan(x)}{a}$$

input
```
Int[(a - a*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\tan(x)}{a}$	7
parallelrisch	$\frac{\sin(x)}{\cos(x)a}$	11
risch	$\frac{2i}{a(e^{2ix}+1)}$	16
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(-1+\tan\left(\frac{x}{2}\right)^{2}\right)}$	20

input
```
int(1/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{a-a\sin^2(x)}\,dx=\frac{\sin(x)}{a\cos(x)}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
sin(x)/(a*cos(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{1}{a-a\sin^2(x)}\,dx=-\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)-a}$$

input
```
integrate(1/(a-a*sin(x)**2),x)
```

output
```
-2*tan(x/2)/(a*tan(x/2)**2 - a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sin^2(x)}\,dx=\frac{\tan(x)}{a}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
tan(x)/a
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\tan{(x)}}{a}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
tan(x)/a
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
int(1/(a - a*sin(x)^2),x)
```

output
```
tan(x)/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\sin{(x)}}{\cos{(x)}\,a}$$

input
```
int(1/(a-a*sin(x)^2),x)
```

output
```
sin(x)/(cos(x)*a)
```

3.55 $\int \frac{1}{a-a\sin^4(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{a-a\sin^4(x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}a}+\frac{\tan(x)}{2a}$$

output
```
1/4*arctan(tan(x)*2^(1/2))*2^(1/2)/a+1/2*tan(x)/a
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{a-a\sin^4(x)}\,dx=\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+2\tan(x)}{4a}$$

input
```
Integrate[(a - a*Sin[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/(4*a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3688, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin(x)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\int \frac{\tan^2(x)+1}{a\left(2\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\tan^2(x)+1}{2\tan^2(x)+1}d\tan(x)}{a}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{2}\int \frac{1}{2\tan^2(x)+1}d\tan(x)+\frac{\tan(x)}{2}}{a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}}{a}$$

input
```
Int[(a - a*Sin[x]^4)^(-1),x]
```

output
```
(ArcTan[Sqrt[2]*Tan[x]]/(2*Sqrt[2]) + Tan[x]/2)/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\frac{\tan(x)}{2}+\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{4}}{a}$	22
risch	$\frac{i}{a(\mathrm{e}^{2ix}+1)}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)}{8a}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)}{8a}$	61

input
```
int(1/(a-a*sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*tan(x)+1/4*2^(1/2)*arctan(tan(x)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{1}{a-a\sin^4(x)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)\cos(x)-4\sin(x)}{8\,a\cos(x)}$$

input
```
integrate(1/(a-a*sin(x)^4),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))
*cos(x) - 4*sin(x))/(a*cos(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(24) = 48.

Time = 21.04 (sec) , antiderivative size = 792, normalized size of antiderivative = 25.55

$$\int \frac{1}{a-a\sin^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)**4),x)
```

output
```
54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) +
 pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*a*tan(x/2)**2 +
90478148*a*tan(x/2)**2 - 90478148*a - 63977712*sqrt(2)*a) + 77227930*sqrt(
3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)
/pi))*tan(x/2)**2/(63977712*sqrt(2)*a*tan(x/2)**2 + 90478148*a*tan(x/2)**2
 - 90478148*a - 63977712*sqrt(2)*a) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(t
an(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2
)*a*tan(x/2)**2 + 90478148*a*tan(x/2)**2 - 90478148*a - 63977712*sqrt(2)*a
) - 54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)
)) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*a*tan(x/2)**2 + 90478148
*a*tan(x/2)**2 - 90478148*a - 63977712*sqrt(2)*a) + 9369319*sqrt(2)*sqrt(2
*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/
pi))*tan(x/2)**2/(63977712*sqrt(2)*a*tan(x/2)**2 + 90478148*a*tan(x/2)**2
- 90478148*a - 63977712*sqrt(2)*a) + 13250218*sqrt(2*sqrt(2) + 3)*(atan(ta
n(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(6397
7712*sqrt(2)*a*tan(x/2)**2 + 90478148*a*tan(x/2)**2 - 90478148*a - 6397771
2*sqrt(2)*a) - 13250218*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2)
+ 3)) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*a*tan(x/2)**2 + 90478
148*a*tan(x/2)**2 - 90478148*a - 63977712*sqrt(2)*a) - 9369319*sqrt(2)*sqr
t(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - ...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{a - a\sin^4(x)}\,dx = \frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)}{4a} + \frac{\tan(x)}{2a}$$

input
```
integrate(1/(a-a*sin(x)^4),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*tan(x))/a + 1/2*tan(x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \frac{1}{a - a\sin^4(x)}\,dx = \frac{\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - 2\sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - 2\cos(2x) + 2}\right)\right)}{4a} + \frac{\tan(x)}{2a}$$

input
```
integrate(1/(a-a*sin(x)^4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2)))/a + 1/2*tan(x)/a
```

Mupad [B] (verification not implemented)

Time = 37.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{a - a\sin^4(x)}\,dx = \frac{2\tan(x) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{4a}$$

input
```
int(1/(a - a*sin(x)^4),x)
```

output
```
(2*tan(x) + 2^(1/2)*atan(2^(1/2)*tan(x)))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{1}{a - a\sin^4(x)}\,dx = \frac{2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2}+1}\right)\cos(x) - \sqrt{2}\cos(x)\log\left(-\sqrt{2}\,i + \tan\left(\frac{x}{2}\right) + i\right)i + \sqrt{2}\cos(x)\log\left(\sqrt{2}\,i + \tan\left(\frac{x}{2}\right) - i\right)}{8\cos(x)\,a}$$

input
```
int(1/(a-a*sin(x)^4),x)
```

output
```
(2*sqrt(2)*atan(tan(x/2)/(sqrt(2) + 1))*cos(x) - sqrt(2)*cos(x)*log( - sqr
t(2)*i + tan(x/2) + i)*i + sqrt(2)*cos(x)*log(sqrt(2)*i + tan(x/2) - i)*i
+ 4*sin(x))/(8*cos(x)*a)
```

3.56 $\int \frac{1}{a-a\sin^6(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 167

$$\int \frac{1}{a-a\sin^6(x)}\,dx=-\frac{\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\arctan\left(2-\sqrt{3}-2\sqrt{-3+2\sqrt{3}}\tan(x)\right)}{6a}+\frac{\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\arctan\left(2-\sqrt{3}+2\sqrt{-3+2\sqrt{3}}\tan(x)\right)}{6a}+\frac{\sqrt{\frac{1}{3}\left(-3+2\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\tan(x)}{1+\sqrt{3}\tan^2(x)}\right)}{6a}+\frac{\tan(x)}{3a}$$

output
```
1/18*(9+6*3^(1/2))^(1/2)*arctan(-2+3^(1/2)+2*(-3+2*3^(1/2))^(1/2)*tan(x))/
a-1/18*(9+6*3^(1/2))^(1/2)*arctan(-2+3^(1/2)-2*(-3+2*3^(1/2))^(1/2)*tan(x)
)/a+1/18*(-9+6*3^(1/2))^(1/2)*arctanh((-3+2*3^(1/2))^(1/2)*tan(x)/(1+3^(1/
2)*tan(x)^2))/a+1/3*tan(x)/a
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{1}{a-a\sin^6(x)}\,dx$$
$$= \frac{\cos(x)(15-8\cos(2x)+\cos(4x))\left(i\sqrt[4]{-3}(3i+\sqrt{3})\arctan\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}(-3i+\sqrt{3})\tan(x)\right)\cos(x)+\sqrt[4]{-3}\right.}{144a\left(-1+\sin^6(x)\right)}$$

input
```
Integrate[(a - a*Sin[x]^6)^(-1),x]
```

output
```
(Cos[x]*(15 - 8*Cos[2*x] + Cos[4*x])*(I*(-3)^(1/4)*(3*I + Sqrt[3])*ArcTan[
((-1/3)^(1/4)*(-3*I + Sqrt[3])*Tan[x])/2]*Cos[x] + (-3)^(1/4)*(-3*I + Sqrt
[3])*ArcTan[((-1)^(3/4)*(3*I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Cos[x] - 6*Si
n[x]))/(144*a*(-1 + Sin[x]^6))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^6(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a-a\sin(x)^6}dx$$
$$\Big\downarrow \text{ 3690}$$
$$\frac{\int \frac{1}{1-\sin^2(x)}dx}{3a}+\frac{\int \frac{1}{\sqrt[3]{-1}\sin^2(x)+1}dx}{3a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin^2(x)}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\sin(x)^2}dx}{3a}+\frac{\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx}{3a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx}{3a}$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx}{3a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx}{3a}+\frac{\int \sec^2(x)dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx}{3a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx}{3a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{\left(1+\sqrt[3]{-1}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1-(-1)^{2/3}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{3a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{3a}+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(x))}{3a}+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}a}+\frac{\tan(x)}{3a}$$

input
```
Int[(a - a*Sin[x]^6)^(-1),x]
```

output
```
ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(1/3)]*a) + ArcTan[Sq
rt[1 - (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(2/3)]*a) + Tan[x]/(3*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

method	result
risch	$\frac{2i}{3a(\mathrm{e}^{2ix}+1)}+\left(\sum_{\_R=\mathrm{RootOf}(3888a^4\_Z^4+108a^2\_Z^2+1)}\_R\ln\left(\mathrm{e}^{2ix}-1296ia^3\_R^3+216a^2\_R^2+1\right)\right)$
default	$\frac{\frac{\tan(x)}{3}+\frac{\sqrt{3}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\ln\left(\sqrt{3}+\sqrt{-3+2\sqrt{3}}\,\sqrt{3}\,\tan(x)+3\tan(x)^2\right)}{6}+\frac{2\left(-\frac{\left(-3+2\sqrt{3}\right)\sqrt{3}}{6}+2\right)\arctan\left(\frac{\sqrt{3}\,\sqrt{-3+2\sqrt{3}}+6\tan(x)}{\sqrt{9+6\sqrt{3}}}\right)}{\sqrt{9+6\sqrt{3}}}\right)}{6}+\frac{\sqrt{3}\left(-\frac{\sqrt{\cdots}}{}\right.}{}}{a}$

input
```
int(1/(a-a*sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/a/(exp(2*I*x)+1)+sum(_R*ln(exp(2*I*x)-1296*I*a^3*_R^3+216*a^2*_R^2+1
),_R=RootOf(3888*_Z^4*a^4+108*_Z^2*a^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(113) = 226.

Time = 0.12 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.93

$$\int \frac{1}{a-a\sin^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^6),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(1/2)*a*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4) + 1)/a^2)*cos(x)*log(
6*sqrt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*cos(x)*sin(x) + a*cos(x)*sin(x))*s
qrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4) + 1)/a^2) + 3*sqrt(1/3)*(2*a^2*cos(x)^2 -
 a^2)*sqrt(-1/a^4) + 4*cos(x)^2 - 3) - sqrt(1/2)*a*sqrt(-(sqrt(1/3)*a^2*sq
rt(-1/a^4) + 1)/a^2)*cos(x)*log(-6*sqrt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*c
os(x)*sin(x) + a*cos(x)*sin(x))*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4) + 1)/a^2
) + 3*sqrt(1/3)*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) + 4*cos(x)^2 - 3) + sq
rt(1/2)*a*sqrt((sqrt(1/3)*a^2*sqrt(-1/a^4) - 1)/a^2)*cos(x)*log(6*sqrt(1/2
)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*cos(x)*sin(x) - a*cos(x)*sin(x))*sqrt((sqrt(
1/3)*a^2*sqrt(-1/a^4) - 1)/a^2) + 3*sqrt(1/3)*(2*a^2*cos(x)^2 - a^2)*sqrt(
-1/a^4) - 4*cos(x)^2 + 3) - sqrt(1/2)*a*sqrt((sqrt(1/3)*a^2*sqrt(-1/a^4) -
 1)/a^2)*cos(x)*log(-6*sqrt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*cos(x)*sin(x)
 - a*cos(x)*sin(x))*sqrt((sqrt(1/3)*a^2*sqrt(-1/a^4) - 1)/a^2) + 3*sqrt(1/
3)*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) - 4*cos(x)^2 + 3) - 4*sin(x))/(a*co
s(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - a\sin^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-a*sin(x)**6),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a - a\sin^6(x)}\,dx = \int -\frac{1}{a\sin{(x)}^6 - a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^6),x, algorithm="maxima")
```

output
```
-1/3*(3*(a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*integrate(4/3*((c
os(6*x) - 10*cos(4*x) + cos(2*x))*cos(8*x) + (110*cos(4*x) - 16*cos(2*x) +
 1)*cos(6*x) - 8*cos(6*x)^2 + 10*(11*cos(2*x) - 1)*cos(4*x) - 300*cos(4*x)
^2 - 8*cos(2*x)^2 + (sin(6*x) - 10*sin(4*x) + sin(2*x))*sin(8*x) + 2*(55*s
in(4*x) - 8*sin(2*x))*sin(6*x) - 8*sin(6*x)^2 - 300*sin(4*x)^2 + 110*sin(4
*x)*sin(2*x) - 8*sin(2*x)^2 + cos(2*x))/(a*cos(8*x)^2 + 64*a*cos(6*x)^2 +
900*a*cos(4*x)^2 + 64*a*cos(2*x)^2 + a*sin(8*x)^2 + 64*a*sin(6*x)^2 + 900*
a*sin(4*x)^2 - 480*a*sin(4*x)*sin(2*x) + 64*a*sin(2*x)^2 - 2*(8*a*cos(6*x)
 - 30*a*cos(4*x) + 8*a*cos(2*x) - a)*cos(8*x) - 16*(30*a*cos(4*x) - 8*a*co
s(2*x) + a)*cos(6*x) - 60*(8*a*cos(2*x) - a)*cos(4*x) - 16*a*cos(2*x) - 4*
(4*a*sin(6*x) - 15*a*sin(4*x) + 4*a*sin(2*x))*sin(8*x) - 32*(15*a*sin(4*x)
 - 4*a*sin(2*x))*sin(6*x) + a), x) - 2*sin(2*x))/(a*cos(2*x)^2 + a*sin(2*x
)^2 + 2*a*cos(2*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.27

$$\int \frac{1}{a-a\sin^6(x)}\,dx$$
$$= \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-\arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan(x)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9}}{18a}$$
$$+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)-4\tan(x)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9}}{18a}$$
$$+\frac{\sqrt{6\sqrt{3}-9}\log\left(\frac{1}{2}\left(\sqrt{6}\left(\frac{1}{3}\right)^{\frac{1}{4}}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)\tan(x)+\tan(x)^2+\sqrt{\frac{1}{3}}\right)}{36a}$$
$$-\frac{\sqrt{6\sqrt{3}-9}\log\left(-\frac{1}{2}\left(\sqrt{6}\left(\frac{1}{3}\right)^{\frac{1}{4}}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)\tan(x)+\tan(x)^2+\sqrt{\frac{1}{3}}\right)}{36a}+\frac{\tan(x)}{3a}$$

input
```
integrate(1/(a-a*sin(x)^6),x, algorithm="giac")
```

output
```
1/18*(pi*floor(x/pi + 1/2) - arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) -
 sqrt(2)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9)/a + 1/18*(
pi*floor(x/pi + 1/2) + arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) - sqrt(
2)) - 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9)/a + 1/36*sqrt(6*
sqrt(3) - 9)*log(1/2*(sqrt(6)*(1/3)^(1/4) - sqrt(2)*(1/3)^(1/4))*tan(x) +
tan(x)^2 + sqrt(1/3))/a - 1/36*sqrt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*(1/3)
^(1/4) - sqrt(2)*(1/3)^(1/4))*tan(x) + tan(x)^2 + sqrt(1/3))/a + 1/3*tan(x
)/a
```

Mupad [B] (verification not implemented)

Time = 37.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{1}{a-a\sin^6(x)}\,dx$$
$$= \frac{\tan(x)}{3a} + \frac{\operatorname{atan}\left(\frac{a\tan(x)\sqrt{-\frac{18}{a^2}-\frac{\sqrt{3}\,6i}{a^2}}\,1i}{4}+\frac{\sqrt{3}\,a\tan(x)\sqrt{-\frac{18}{a^2}-\frac{\sqrt{3}\,6i}{a^2}}}{12}\right)\sqrt{-\frac{6\left(3+\sqrt{3}\,1i\right)}{a^2}}\,1i}{18}$$
$$+ \frac{\operatorname{atan}\left(\frac{a\tan(x)\sqrt{-\frac{18}{a^2}+\frac{\sqrt{3}\,6i}{a^2}}\,1i}{4}-\frac{\sqrt{3}\,a\tan(x)\sqrt{-\frac{18}{a^2}+\frac{\sqrt{3}\,6i}{a^2}}}{12}\right)\sqrt{\frac{6\left(-3+\sqrt{3}\,1i\right)}{a^2}}\,1i}{18}$$

input
```
int(1/(a - a*sin(x)^6),x)
```

output
```
tan(x)/(3*a) + (atan((a*tan(x)*(- (3^(1/2)*6i)/a^2 - 18/a^2)^(1/2)*1i)/4 +
 (3^(1/2)*a*tan(x)*(- (3^(1/2)*6i)/a^2 - 18/a^2)^(1/2))/12)*(-(6*(3^(1/2)*
1i + 3))/a^2)^(1/2)*1i)/18 + (atan((a*tan(x)*((3^(1/2)*6i)/a^2 - 18/a^2)^(
1/2)*1i)/4 - (3^(1/2)*a*tan(x)*((3^(1/2)*6i)/a^2 - 18/a^2)^(1/2))/12)*((6*
(3^(1/2)*1i - 3))/a^2)^(1/2)*1i)/18
```

Reduce [F]

$$\int \frac{1}{a - a\sin^6(x)}\,dx = -\frac{\int \frac{1}{\sin(x)^6 - 1}dx}{a}$$

input
```
int(1/(a-a*sin(x)^6),x)
```

output
```
( - int(1/(sin(x)**6 - 1),x))/a
```

3.57 $\int \frac{1}{a-a\sin^8(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 174

$$\int \frac{1}{a-a\sin^8(x)}\,dx = -\frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{\sqrt{1+\sqrt{2}}}\right)}{8a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}a}+\frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}+2\tan(x)}{\sqrt{1+\sqrt{2}}}\right)}{8a}+\frac{\sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\tan(x)}{1+\sqrt{2}\tan^2(x)}\right)}{8a}+\frac{\tan(x)}{4a}$$

output
```
-1/8*(1+2^(1/2))^(1/2)*arctan(((2^(1/2)-1)^(1/2)-2*tan(x))/(1+2^(1/2))^(1/
2))/a+1/8*arctan(tan(x)*2^(1/2))*2^(1/2)/a+1/8*(1+2^(1/2))^(1/2)*arctan(((
2^(1/2)-1)^(1/2)+2*tan(x))/(1+2^(1/2))^(1/2))/a+1/8*(2^(1/2)-1)^(1/2)*arct
anh((-2+2*2^(1/2))^(1/2)*tan(x)/(1+2^(1/2)*tan(x)^2))/a+1/4*tan(x)/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.39

$$\int \frac{1}{a - a\sin^8(x)}\,dx = \frac{\frac{2\arctan\left(\sqrt{1-i}\tan(x)\right)}{\sqrt{1-i}} + \frac{2\arctan\left(\sqrt{1+i}\tan(x)\right)}{\sqrt{1+i}} + \sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + 2\tan(x)}{8a}$$

input
```
Integrate[(a - a*Sin[x]^8)^(-1),x]
```

output
```
((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]
])/Sqrt[1 + I] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/(8*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a - a\sin^8(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a - a\sin(x)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\sin^2(x)}dx}{4a} + \frac{\int \frac{1}{1-i\sin^2(x)}dx}{4a} + \frac{\int \frac{1}{i\sin^2(x)+1}dx}{4a} + \frac{\int \frac{1}{\sin^2(x)+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\sin(x)^2}dx}{4a}+\frac{\int \frac{1}{1-i\sin(x)^2}dx}{4a}+\frac{\int \frac{1}{i\sin(x)^2+1}dx}{4a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{4a}$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \frac{1}{1-i\sin(x)^2}dx}{4a}+\frac{\int \frac{1}{i\sin(x)^2+1}dx}{4a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{4a}+\frac{\int \sec^2(x)dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-i\sin(x)^2}dx}{4a}+\frac{\int \frac{1}{i\sin(x)^2+1}dx}{4a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{4a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{(1-i)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{(1+i)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{2\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{4a}+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}a}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(x))}{4a}+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}a}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}a}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}a}+\frac{\tan(x)}{4a}$$

input
```
Int[(a - a*Sin[x]^8)^(-1),x]
```

output
```
ArcTan[Sqrt[1 - I]*Tan[x]]/(4*Sqrt[1 - I]*a) + ArcTan[Sqrt[1 + I]*Tan[x]]/
(4*Sqrt[1 + I]*a) + ArcTan[Sqrt[2]*Tan[x]]/(4*Sqrt[2]*a) + Tan[x]/(4*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.73 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$\frac{i}{2a(e^{2ix}+1)}+\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}+i\sqrt{-2-2i}-\sqrt{-2-2i}-1-2i\right)}{16a}-\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}-i\sqrt{-2-2i}+\sqrt{-2-2i}-1-2i\right)}{16a}+\frac{i\sqrt{2}\,\ln\left(e^{2i\cdots\right.}}{16\cdots}$
default	$\frac{\frac{\tan(x)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\sqrt{2}+\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)+2\tan(x)^{2}\right)}{4}+\frac{\left(-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{4}+2\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}\right)}{8}+\frac{\sqrt{2}\left(-\frac{\sqrt{-\cdots}}{\cdots}\right.}{\cdots}}{a}$

input
```
int(1/(a-a*sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a/(exp(2*I*x)+1)+1/16*(-2-2*I)^(1/2)/a*ln(exp(2*I*x)+I*(-2-2*I)^(1/2
)-(-2-2*I)^(1/2)-1-2*I)-1/16*(-2-2*I)^(1/2)/a*ln(exp(2*I*x)-I*(-2-2*I)^(1/
2)+(-2-2*I)^(1/2)-1-2*I)+1/16*I*2^(1/2)/a*ln(exp(2*I*x)-2*2^(1/2)-3)-1/16*
I*2^(1/2)/a*ln(exp(2*I*x)+2*2^(1/2)-3)+1/16*(-2+2*I)^(1/2)/a*ln(exp(2*I*x)
+I*(-2+2*I)^(1/2)+(-2+2*I)^(1/2)-1+2*I)-1/16*(-2+2*I)^(1/2)/a*ln(exp(2*I*x
)-I*(-2+2*I)^(1/2)-(-2+2*I)^(1/2)-1+2*I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(126) = 252.

Time = 0.16 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.72

$$\int \frac{1}{a-a\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^8),x, algorithm="fricas")
```

output
```
-1/16*(sqrt(1/2)*a*sqrt(-(a^2*sqrt(-1/a^4) + 1)/a^2)*cos(x)*log(1/2*sqrt(1
/2)*(a^3*sqrt(-1/a^4)*cos(x)*sin(x) + a*cos(x)*sin(x))*sqrt(-(a^2*sqrt(-1/
a^4) + 1)/a^2) + 1/4*cos(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) -
1/4) - sqrt(1/2)*a*sqrt(-(a^2*sqrt(-1/a^4) + 1)/a^2)*cos(x)*log(-1/2*sqrt(
1/2)*(a^3*sqrt(-1/a^4)*cos(x)*sin(x) + a*cos(x)*sin(x))*sqrt(-(a^2*sqrt(-1
/a^4) + 1)/a^2) + 1/4*cos(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) -
 1/4) + sqrt(1/2)*a*sqrt((a^2*sqrt(-1/a^4) - 1)/a^2)*cos(x)*log(1/2*sqrt(1
/2)*(a^3*sqrt(-1/a^4)*cos(x)*sin(x) - a*cos(x)*sin(x))*sqrt((a^2*sqrt(-1/a
^4) - 1)/a^2) - 1/4*cos(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) + 1
/4) - sqrt(1/2)*a*sqrt((a^2*sqrt(-1/a^4) - 1)/a^2)*cos(x)*log(-1/2*sqrt(1/
2)*(a^3*sqrt(-1/a^4)*cos(x)*sin(x) - a*cos(x)*sin(x))*sqrt((a^2*sqrt(-1/a^
4) - 1)/a^2) - 1/4*cos(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) + 1/
4) + sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))*
cos(x) - 4*sin(x))/(a*cos(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a-a\sin^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-a*sin(x)**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a-a\sin^8(x)}\,dx = \int -\frac{1}{a\sin{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^8),x, algorithm="maxima")
```

output

```
1/16*((sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt
(2))*arctan2(2*sqrt(2)*sin(x)/(2*(sqrt(2) + 1)*cos(x) + cos(x)^2 + sin(x)^
2 + 2*sqrt(2) + 3), (cos(x)^2 + sin(x)^2 + 2*cos(x) - 1)/(2*(sqrt(2) + 1)*
cos(x) + cos(x)^2 + sin(x)^2 + 2*sqrt(2) + 3)) - (sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(2))*arctan2(2*sqrt(2)*sin(x)/(
2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(2) + 3), (cos(x)^2 +
 sin(x)^2 - 2*cos(x) - 1)/(2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 -
2*sqrt(2) + 3)) + 128*(a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*int
egrate(-((4*cos(2*x) - 1)*cos(4*x) - cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*
x) - 22*cos(4*x)^2 - sin(8*x)*sin(4*x) + 4*sin(6*x)*sin(4*x) - 22*sin(4*x)
^2 + 4*sin(4*x)*sin(2*x))/(a*cos(8*x)^2 + 16*a*cos(6*x)^2 + 484*a*cos(4*x)
^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*sin(6*x)^2 + 484*a*sin(4*x)^2 -
 176*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 - 2*(4*a*cos(6*x) - 22*a*cos(4*
x) + 4*a*cos(2*x) - a)*cos(8*x) - 8*(22*a*cos(4*x) - 4*a*cos(2*x) + a)*cos
(6*x) - 44*(4*a*cos(2*x) - a)*cos(4*x) - 8*a*cos(2*x) - 4*(2*a*sin(6*x) -
11*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) - 16*(11*a*sin(4*x) - 2*a*sin(2*x))
*sin(6*x) + a), x) + 8*sin(2*x))/(a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*
x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(126) = 252$.

Time = 0.55 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.84

$$\int \frac{1}{a-a\sin^8(x)}\,dx = \frac{\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)}{8a} + \frac{\tan(x)}{4a}$$
$$+ \frac{\left(a^2\sqrt{2\sqrt{2}+2}+a\sqrt{2\sqrt{2}-2}|a|\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)}{16a^3}$$
$$+ \frac{\left(a^2\sqrt{2\sqrt{2}+2}+a\sqrt{2\sqrt{2}-2}|a|\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)}{16a^3}$$
$$- \frac{\left(a^2\sqrt{2\sqrt{2}-2}-a\sqrt{2\sqrt{2}+2}|a|\right)\log\left(\tan(x)^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)}{32a^3}$$
$$+ \frac{\left(a^2\sqrt{2\sqrt{2}-2}-a\sqrt{2\sqrt{2}+2}|a|\right)\log\left(\tan(x)^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)}{32a^3}$$

input
```
integrate(1/(a-a*sin(x)^8),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2)))/a + 1/4*tan(x)/a + 1/16*(a^2*sqrt(2*sqrt(2)
 + 2) + a*sqrt(2*sqrt(2) - 2)*abs(a))*(pi*floor(x/pi + 1/2) + arctan(2*(1/
2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))/a
^3 + 1/16*(a^2*sqrt(2*sqrt(2) + 2) + a*sqrt(2*sqrt(2) - 2)*abs(a))*(pi*flo
or(x/pi + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) - 2
*tan(x))/sqrt(sqrt(2) + 2)))/a^3 - 1/32*(a^2*sqrt(2*sqrt(2) - 2) - a*sqrt(
2*sqrt(2) + 2)*abs(a))*log(tan(x)^2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x
) + sqrt(1/2))/a^3 + 1/32*(a^2*sqrt(2*sqrt(2) - 2) - a*sqrt(2*sqrt(2) + 2)
*abs(a))*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2))
/a^3
```

Mupad [B] (verification not implemented)

Time = 37.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \frac{1}{a-a\sin^8(x)}\,dx = \frac{\tan(x)}{4\,a} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{8\,a} + \mathrm{atan}\left(a\tan(x)\,\sqrt{\frac{-\frac{1}{128}-\frac{1}{128}\mathrm{i}}{a^2}}\,(8+8\mathrm{i})\right)\sqrt{\frac{-\frac{1}{128}-\frac{1}{128}\mathrm{i}}{a^2}}\,2\mathrm{i} - \mathrm{atan}\left(a\tan(x)\,\sqrt{\frac{-\frac{1}{128}+\frac{1}{128}\mathrm{i}}{a^2}}\,(8-8\mathrm{i})\right)\sqrt{\frac{-\frac{1}{128}+\frac{1}{128}\mathrm{i}}{a^2}}\,2\mathrm{i}$$

input
```
int(1/(a - a*sin(x)^8),x)
```

output
```
tan(x)/(4*a) + atan(a*tan(x)*((- 1/128 - 1i/128)/a^2)^(1/2)*(8 + 8i))*((-
1/128 - 1i/128)/a^2)^(1/2)*2i - atan(a*tan(x)*((- 1/128 + 1i/128)/a^2)^(1/
2)*(8 - 8i))*((- 1/128 + 1i/128)/a^2)^(1/2)*2i + (2^(1/2)*atan(2^(1/2)*tan
(x)))/(8*a)
```

Reduce [F]

$$\int \frac{1}{a - a\sin^8(x)}\,dx = -\frac{\int \frac{1}{\sin(x)^8 - 1}dx}{a}$$

input
```
int(1/(a-a*sin(x)^8),x)
```

output
```
( - int(1/(sin(x)**8 - 1),x))/a
```

3.58 $\int \frac{1}{a-a\sin^{10}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 421

$$\begin{aligned}\int \frac{1}{a-a\sin^{10}(x)}\,dx &= \frac{\arctan\left(\frac{\sqrt[5]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}a}+\frac{\arctan\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}a}\\
&+\frac{\arctan\left(\frac{(-1)^{3/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}a}+\frac{\arctan\left(\frac{(-1)^{4/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}a}\\
&-\frac{\arctan\left(\frac{(-1)^{4/5}\left(1+\sqrt[5]{-1}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}a}\\
&-\frac{\arctan\left(\frac{(-1)^{3/5}\left(1+(-1)^{2/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}a}\\
&-\frac{\arctan\left(\frac{(-1)^{2/5}\left(1+(-1)^{3/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}a}\\
&-\frac{\arctan\left(\frac{\sqrt[5]{-1}\left(1+(-1)^{4/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}a}\\
&+\frac{\cos(x)}{10a(1-\sin(x))}-\frac{\cos(x)}{10a(1+\sin(x))}\end{aligned}$$

output
```
1/5*arctan(((-1)^(1/5)+tan(1/2*x))/(1-(-1)^(2/5))^(1/2))/(1-(-1)^(2/5))^(1
/2)/a+1/5*arctan(((-1)^(2/5)+tan(1/2*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/
5))^(1/2)/a+1/5*arctan(((-1)^(3/5)+tan(1/2*x))/(1+(-1)^(1/5))^(1/2))/(1+(-
1)^(1/5))^(1/2)/a+1/5*arctan(((-1)^(4/5)+tan(1/2*x))/(1+(-1)^(3/5))^(1/2))
/(1+(-1)^(3/5))^(1/2)/a-1/5*arctan((-1)^(4/5)*(1+(-1)^(1/5)*tan(1/2*x))/(1
+(-1)^(3/5))^(1/2))/(1+(-1)^(3/5))^(1/2)/a-1/5*arctan((-1)^(3/5)*(1+(-1)^(
2/5)*tan(1/2*x))/(1+(-1)^(1/5))^(1/2))/(1+(-1)^(1/5))^(1/2)/a-1/5*arctan((
-1)^(2/5)*(1+(-1)^(3/5)*tan(1/2*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(
1/2)/a-1/5*arctan((-1)^(1/5)*(1+(-1)^(4/5)*tan(1/2*x))/(1-(-1)^(2/5))^(1/2
))/(1-(-1)^(2/5))^(1/2)/a+1/10*cos(x)/a/(1-sin(x))-1/10*cos(x)/a/(1+sin(x)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.11 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.99

$$\int \frac{1}{a-a\sin^{10}(x)}\,dx = -\frac{\text{RootSum}\left[1-12\#1+68\#1^2-244\#1^3+630\#1^4-244\#1^5+68\#1^6-12\#1^7+\#1^8\&,\frac{2\arctan\left(\frac{\text{s}}{\text{cos(}}\right.}{}\right.}{}$$

input
```
Integrate[(a - a*Sin[x]^10)^(-1),x]
```

output
```
-1/20*(RootSum[1 - 12*#1 + 68*#1^2 - 244*#1^3 + 630*#1^4 - 244*#1^5 + 68*#
1^6 - 12*#1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)] - I*Log[1 - 2
*Cos[2*x]*#1 + #1^2] - 28*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1 + (14*I)*Log
[1 - 2*Cos[2*x]*#1 + #1^2]*#1 + 190*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2
- (95*I)*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^2 - 840*ArcTan[Sin[2*x]/(Cos[2*x
] - #1)]*#1^3 + (420*I)*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3 + 190*ArcTan[Si
n[2*x]/(Cos[2*x] - #1)]*#1^4 - (95*I)*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^4 -
 28*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^5 + (14*I)*Log[1 - 2*Cos[2*x]*#1 +
 #1^2]*#1^5 + 2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[2*
x]*#1 + #1^2]*#1^6)/(-3 + 34*#1 - 183*#1^2 + 630*#1^3 - 305*#1^4 + 102*#1^
5 - 21*#1^6 + 2*#1^7) & ] - 4*Tan[x])/a
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^{10}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a-a\sin(x)^{10}}dx$$

$$\Big\downarrow \text{ 3690}$$

$$\frac{\int \frac{1}{1-\sin^2(x)}dx}{5a}+\frac{\int \frac{1}{\sqrt[5]{-1}\sin^2(x)+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{2/5}\sin^2(x)}dx}{5a}+\frac{\int \frac{1}{(-1)^{3/5}\sin^2(x)+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{4/5}\sin^2(x)}dx}{5a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{1-\sin(x)^2}dx}{5a}+\frac{\int \frac{1}{\sqrt[5]{-1}\sin(x)^2+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{2/5}\sin(x)^2}dx}{5a}+\frac{\int \frac{1}{(-1)^{3/5}\sin(x)^2+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{4/5}\sin(x)^2}dx}{5a}$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \frac{1}{\sqrt[5]{-1}\sin(x)^2+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{2/5}\sin(x)^2}dx}{5a}+\frac{\int \frac{1}{(-1)^{3/5}\sin(x)^2+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{4/5}\sin(x)^2}dx}{5a}+\frac{\int \sec^2(x)dx}{5a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt[5]{-1}\sin(x)^2+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{2/5}\sin(x)^2}dx}{5a}+\frac{\int \frac{1}{(-1)^{3/5}\sin(x)^2+1}dx}{5a}+\frac{\int \frac{1}{1-(-1)^{4/5}\sin(x)^2}dx}{5a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{5a}$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{\int \frac{1}{\left(1+\sqrt[5]{-1}\right)\tan^2(x)+1}d\tan(x)}{5a}+\frac{\int \frac{1}{\left(1-(-1)^{2/5}\right)\tan^2(x)+1}d\tan(x)}{5a}+\frac{\int \frac{1}{\left(1+(-1)^{3/5}\right)\tan^2(x)+1}d\tan(x)}{5a}+\frac{\int \frac{1}{\left(1-(-1)^{4/5}\right)\tan^2(x)+1}d\tan(x)}{5a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{5a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{5a}+\frac{\arctan\left(\sqrt{1+\sqrt[5]{-1}}\tan(x)\right)}{5\sqrt{1+\sqrt[5]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/5}}\tan(x)\right)}{5\sqrt{1-(-1)^{2/5}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/5}}\tan(x)\right)}{5\sqrt{1+(-1)^{3/5}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{4/5}}\tan(x)\right)}{5\sqrt{1-(-1)^{4/5}}a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(x))}{5a}+\frac{\arctan\left(\sqrt{1+\sqrt[5]{-1}}\tan(x)\right)}{5\sqrt{1+\sqrt[5]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/5}}\tan(x)\right)}{5\sqrt{1-(-1)^{2/5}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/5}}\tan(x)\right)}{5\sqrt{1+(-1)^{3/5}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{4/5}}\tan(x)\right)}{5\sqrt{1-(-1)^{4/5}}a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\arctan\left(\sqrt{1+\sqrt[5]{-1}}\tan(x)\right)}{5\sqrt{1+\sqrt[5]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/5}}\tan(x)\right)}{5\sqrt{1-(-1)^{2/5}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/5}}\tan(x)\right)}{5\sqrt{1+(-1)^{3/5}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{4/5}}\tan(x)\right)}{5\sqrt{1-(-1)^{4/5}}a}+\frac{\tan(x)}{5a}$$

input
```
Int[(a - a*Sin[x]^10)^(-1),x]
```

output
```
ArcTan[Sqrt[1 + (-1)^(1/5)]*Tan[x]]/(5*Sqrt[1 + (-1)^(1/5)]*a) + ArcTan[Sq
rt[1 - (-1)^(2/5)]*Tan[x]]/(5*Sqrt[1 - (-1)^(2/5)]*a) + ArcTan[Sqrt[1 + (-
1)^(3/5)]*Tan[x]]/(5*Sqrt[1 + (-1)^(3/5)]*a) + ArcTan[Sqrt[1 - (-1)^(4/5)]
*Tan[x]]/(5*Sqrt[1 - (-1)^(4/5)]*a) + Tan[x]/(5*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.67 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.19

method	result
default	$\frac{\frac{\tan(x)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^8+10\_Z^6+10\_Z^4+5\_Z^2+1\right)}\frac{\left(10\_R^6+20\_R^4+15\_R^2+4\right)\ln\left(\tan(x)-\_R\right)}{4\_R^7+6\_R^5+4\_R^3+\_R}\right)}{50}}{a}$
risch	$\frac{2i}{5a(\mathrm{e}^{2ix}+1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(500000000a^8\_Z^8+10000000a^6\_Z^6+100000a^4\_Z^4+500a^2\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2ix}-1000000\right.\right.$

input
```
int(1/(a-a*sin(x)^10),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*tan(x)+1/50*sum((10*_R^6+20*_R^4+15*_R^2+4)/(4*_R^7+6*_R^5+4*_R^3
+_R)*ln(tan(x)-_R),_R=RootOf(5*_Z^8+10*_Z^6+10*_Z^4+5*_Z^2+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(295) = 590.

Time = 0.39 (sec) , antiderivative size = 2010, normalized size of antiderivative = 4.77

$$\int \frac{1}{a-a\sin^{10}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^10),x, algorithm="fricas")
```

output
```
-1/20*(sqrt(1/2)*a*sqrt(-(a^2*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4
) + 1)/a^2)*cos(x)*log(10*sqrt(1/2)*(sqrt(1/5)*a^5*sqrt(a^(-8))*cos(x)*sin
(x) - a*cos(x)*sin(x) + (sqrt(1/5)*a^7*sqrt(a^(-8))*cos(x)*sin(x) - a^3*co
s(x)*sin(x))*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4))*sqrt(-(a^2*sqr
t(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + 1)/a^2) + 5*sqrt(1/5)*(2*a^4*
cos(x)^2 - a^4)*sqrt(a^(-8)) - 6*cos(x)^2 - 5*(2*a^2*cos(x)^2 - a^2 - sqrt
(1/5)*(2*a^6*cos(x)^2 - a^6)*sqrt(a^(-8)))*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(
-8)) + 1)/a^4) + 5) - sqrt(1/2)*a*sqrt(-(a^2*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a
^(-8)) + 1)/a^4) + 1)/a^2)*cos(x)*log(-10*sqrt(1/2)*(sqrt(1/5)*a^5*sqrt(a^
(-8))*cos(x)*sin(x) - a*cos(x)*sin(x) + (sqrt(1/5)*a^7*sqrt(a^(-8))*cos(x)
*sin(x) - a^3*cos(x)*sin(x))*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4)
)*sqrt(-(a^2*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + 1)/a^2) + 5*s
qrt(1/5)*(2*a^4*cos(x)^2 - a^4)*sqrt(a^(-8)) - 6*cos(x)^2 - 5*(2*a^2*cos(x
)^2 - a^2 - sqrt(1/5)*(2*a^6*cos(x)^2 - a^6)*sqrt(a^(-8)))*sqrt(-(2*sqrt(1
/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + 5) - sqrt(1/2)*a*sqrt((a^2*sqrt(-(2*sqrt(
1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) - 1)/a^2)*cos(x)*log(10*sqrt(1/2)*(sqrt(1/
5)*a^5*sqrt(a^(-8))*cos(x)*sin(x) - a*cos(x)*sin(x) - (sqrt(1/5)*a^7*sqrt(
a^(-8))*cos(x)*sin(x) - a^3*cos(x)*sin(x))*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(
-8)) + 1)/a^4))*sqrt((a^2*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) -
1)/a^2) - 5*sqrt(1/5)*(2*a^4*cos(x)^2 - a^4)*sqrt(a^(-8)) + 6*cos(x)^2 ...
```

Sympy [F]

$$\int \frac{1}{a-a\sin^{10}(x)}\,dx=-\frac{\int \frac{1}{\sin^{10}(x)-1}\,dx}{a}$$

input
```
integrate(1/(a-a*sin(x)**10),x)
```

output
```
-Integral(1/(sin(x)**10 - 1), x)/a
```

Maxima [F]

$$\int \frac{1}{a - a\sin^{10}(x)}\,dx = \int -\frac{1}{a\sin{(x)}^{10} - a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^10),x, algorithm="maxima")
```

output
```
-1/5*(5*(a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*integrate(4/5*((c
os(14*x) - 14*cos(12*x) + 95*cos(10*x) - 420*cos(8*x) + 95*cos(6*x) - 14*c
os(4*x) + cos(2*x))*cos(16*x) + (236*cos(12*x) - 1384*cos(10*x) + 5670*cos
(8*x) - 1384*cos(6*x) + 236*cos(4*x) - 24*cos(2*x) + 1)*cos(14*x) - 12*cos
(14*x)^2 + 2*(4938*cos(10*x) - 18690*cos(8*x) + 4938*cos(6*x) - 952*cos(4*
x) + 118*cos(2*x) - 7)*cos(12*x) - 952*cos(12*x)^2 + (162330*cos(8*x) - 46
360*cos(6*x) + 9876*cos(4*x) - 1384*cos(2*x) + 95)*cos(10*x) - 23180*cos(1
0*x)^2 + 210*(773*cos(6*x) - 178*cos(4*x) + 27*cos(2*x) - 2)*cos(8*x) - 26
4600*cos(8*x)^2 + (9876*cos(4*x) - 1384*cos(2*x) + 95)*cos(6*x) - 23180*co
s(6*x)^2 + 2*(118*cos(2*x) - 7)*cos(4*x) - 952*cos(4*x)^2 - 12*cos(2*x)^2
+ (sin(14*x) - 14*sin(12*x) + 95*sin(10*x) - 420*sin(8*x) + 95*sin(6*x) -
14*sin(4*x) + sin(2*x))*sin(16*x) + 2*(118*sin(12*x) - 692*sin(10*x) + 283
5*sin(8*x) - 692*sin(6*x) + 118*sin(4*x) - 12*sin(2*x))*sin(14*x) - 12*sin
(14*x)^2 + 4*(2469*sin(10*x) - 9345*sin(8*x) + 2469*sin(6*x) - 476*sin(4*x
) + 59*sin(2*x))*sin(12*x) - 952*sin(12*x)^2 + 2*(81165*sin(8*x) - 23180*s
in(6*x) + 4938*sin(4*x) - 692*sin(2*x))*sin(10*x) - 23180*sin(10*x)^2 + 21
0*(773*sin(6*x) - 178*sin(4*x) + 27*sin(2*x))*sin(8*x) - 264600*sin(8*x)^2
 + 4*(2469*sin(4*x) - 346*sin(2*x))*sin(6*x) - 23180*sin(6*x)^2 - 952*sin(
4*x)^2 + 236*sin(4*x)*sin(2*x) - 12*sin(2*x)^2 + cos(2*x))/(a*cos(16*x)^2
+ 144*a*cos(14*x)^2 + 4624*a*cos(12*x)^2 + 59536*a*cos(10*x)^2 + 396900...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a - a\sin^{10}(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-a*sin(x)^10),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionUnable to transpose Error: Bad Argument ValueDegree mismat
ch inside
```

Mupad [B] (verification not implemented)

Time = 37.65 (sec) , antiderivative size = 1176, normalized size of antiderivative = 2.79

$$\int \frac{1}{a-a\sin^{10}(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - a*sin(x)^10),x)
```

output
```
tan(x)/(5*a) + atan((tan(x)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/(200*a^2) - 1/(
200*a^2))^(1/2)*5i)/((4*(- (2*5^(1/2))/5 - 1)^(1/2))/a - (2*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/a) - (5^(1/2)*tan(x)*(- (- (2*5^(1/2))/5 - 1)^(1/2)
/(200*a^2) - 1/(200*a^2))^(1/2)*5i)/((4*(- (2*5^(1/2))/5 - 1)^(1/2))/a - (
2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/a) + (tan(x)*(- (- (2*5^(1/2))/5 -
1)^(1/2)/(200*a^2) - 1/(200*a^2))^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*75i)/(
(4*(- (2*5^(1/2))/5 - 1)^(1/2))/a - (2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2)
)/a) - (5^(1/2)*tan(x)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/(200*a^2) - 1/(200*a
^2))^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*35i)/((4*(- (2*5^(1/2))/5 - 1)^(1/2
))/a - (2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/a))*(-((- (2*5^(1/2))/5 - 1
)^(1/2) + 1)/(200*a^2))^(1/2)*2i - atan((tan(x)*((- (2*5^(1/2))/5 - 1)^(1/
2)/(200*a^2) - 1/(200*a^2))^(1/2)*5i)/((4*(- (2*5^(1/2))/5 - 1)^(1/2))/a -
 (2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/a) - (5^(1/2)*tan(x)*((- (2*5^(1/
2))/5 - 1)^(1/2)/(200*a^2) - 1/(200*a^2))^(1/2)*5i)/((4*(- (2*5^(1/2))/5 -
 1)^(1/2))/a - (2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/a) - (tan(x)*((- (2
*5^(1/2))/5 - 1)^(1/2)/(200*a^2) - 1/(200*a^2))^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2)*75i)/((4*(- (2*5^(1/2))/5 - 1)^(1/2))/a - (2*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2))/a) + (5^(1/2)*tan(x)*((- (2*5^(1/2))/5 - 1)^(1/2)/(200*a^2
) - 1/(200*a^2))^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*35i)/((4*(- (2*5^(1/2))
/5 - 1)^(1/2))/a - (2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/a))*(((- (2*...
```

Reduce [F]

$$\int \frac{1}{a - a\sin^{10}(x)}\,dx = -\frac{\int \frac{1}{\sin(x)^{10}-1}dx}{a}$$

input
```
int(1/(a-a*sin(x)^10),x)
```

output
```
( - int(1/(sin(x)**10 - 1),x))/a
```

3.59 $\int \frac{1}{a-a\sin^{12}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 311

$$\begin{aligned}\int \frac{1}{a-a\sin^{12}(x)}\,dx =& \frac{x}{6\sqrt{2}a}+\frac{x}{2\sqrt{3}a}+\frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{4\sqrt{3}a}\\ &-\frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{4\sqrt{3}a}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{6\sqrt{2}a}\\ &-\frac{\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\arctan\left(2-\sqrt{3}-2\sqrt{-3+2\sqrt{3}}\tan(x)\right)}{12a}\\ &+\frac{\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\arctan\left(2-\sqrt{3}+2\sqrt{-3+2\sqrt{3}}\tan(x)\right)}{12a}\\ &+\frac{\operatorname{arctanh}(\cos(x)\sin(x))}{12a}\\ &+\frac{\sqrt{\frac{1}{3}\left(-3+2\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\tan(x)}{1+\sqrt{3}\tan^2(x)}\right)}{12a}+\frac{\tan(x)}{6a}\end{aligned}$$

output

```
1/12*x*2^(1/2)/a+1/6*x*3^(1/2)/a+1/12*arctan((1-2*cos(x)^2)/(2+3^(1/2)-2*c
os(x)*sin(x)))*3^(1/2)/a-1/12*arctan((1-2*cos(x)^2)/(2+3^(1/2)+2*cos(x)*si
n(x)))*3^(1/2)/a+1/12*arctan(cos(x)*sin(x)/(1+2^(1/2)+sin(x)^2))*2^(1/2)/a
+1/36*(9+6*3^(1/2))^(1/2)*arctan(-2+3^(1/2)+2*(-3+2*3^(1/2))^(1/2)*tan(x))
/a-1/36*(9+6*3^(1/2))^(1/2)*arctan(-2+3^(1/2)-2*(-3+2*3^(1/2))^(1/2)*tan(x
))/a+1/12*arctanh(cos(x)*sin(x))/a+1/36*(-9+6*3^(1/2))^(1/2)*arctanh((-3+2
*3^(1/2))^(1/2)*tan(x)/(1+3^(1/2)*tan(x)^2))/a+1/6*tan(x)/a
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.16

$$\int \frac{1}{a - a\sin^{12}(x)}\,dx$$

$$= \frac{12\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - 4(-1)^{3/4}\sqrt[4]{3}(3i+\sqrt{3})\arctan\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}(-3i+\sqrt{3})\tan(x)\right) + 4(-1)^{3/4}\sqrt[4]{3}}{}$$

input

```
Integrate[(a - a*Sin[x]^12)^(-1),x]
```

output

```
(12*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] - 4*(-1)^(3/4)*3^(1/4)*(3*I + Sqrt[3])*
ArcTan[((-1/3)^(1/4)*(-3*I + Sqrt[3])*Tan[x])/2] + 4*(-1)^(3/4)*3^(1/4)*(3
 + I*Sqrt[3])*ArcTan[((-1)^(3/4)*(3*I + Sqrt[3])*Tan[x])/(2*3^(1/4))] + 24
*Tan[x] + (3*(I*Sqrt[1 - I*Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((-I + Sqrt[3])
*Tan[x])/2] + Sqrt[1 + I*Sqrt[3]]*(3 + I*Sqrt[3])*ArcTan[((I + Sqrt[3])*Ta
n[x])/2])*(42 + 15*Cos[2*x] + 6*Cos[4*x] + Cos[6*x])*Sec[x]^2)/((-2 + Sin[
2*x])*(2 + Sin[2*x])*((-3*I)*Sqrt[1 - I*Sqrt[3]] - (3*I)*Sqrt[1 + I*Sqrt[3
]] + Sqrt[3 - (3*I)*Sqrt[3]] - Sqrt[3 + (3*I)*Sqrt[3]] + 2*(Sqrt[3 - (3*I)
*Sqrt[3]] - Sqrt[3 + (3*I)*Sqrt[3]])*Tan[x]^2)))/(144*a)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.55, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^{12}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin(x)^{12}}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\sin^2(x)}dx}{6a}+\frac{\int \frac{1}{\sin^2(x)+1}dx}{6a}+\frac{\int \frac{1}{1-\sqrt[3]{-1}\sin^2(x)}dx}{6a}+\frac{\int \frac{1}{\sqrt[3]{-1}\sin^2(x)+1}dx}{6a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin^2(x)}dx}{6a}+\frac{\int \frac{1}{(-1)^{2/3}\sin^2(x)+1}dx}{6a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{6a}+\frac{\int \frac{1}{1-\sqrt[3]{-1}\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx}{6a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{(-1)^{2/3}\sin(x)^2+1}dx}{6a}$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \frac{1}{\sin(x)^2+1}dx}{6a}+\frac{\int \frac{1}{1-\sqrt[3]{-1}\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx}{6a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{(-1)^{2/3}\sin(x)^2+1}dx}{6a}+\frac{\int \sec^2(x)dx}{6a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin(x)^2+1}dx}{6a}+\frac{\int \frac{1}{1-\sqrt[3]{-1}\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx}{6a}+\frac{\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx}{6a}+\frac{\int \frac{1}{(-1)^{2/3}\sin(x)^2+1}dx}{6a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{6a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{2\tan^2(x)+1}d\tan(x)}{6a}+\frac{\int \frac{1}{\left(1-\sqrt[3]{-1}\right)\tan^2(x)+1}d\tan(x)}{6a}+\frac{\int \frac{1}{\left(1+\sqrt[3]{-1}\right)\tan^2(x)+1}d\tan(x)}{6a}+\frac{\int \frac{1}{(1-(-1)^{2/3})\tan^2(x)+1}d\tan(x)}{6a}+\frac{\int \frac{1}{(1+(-1)^{2/3})\tan^2(x)+1}d\tan(x)}{6a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{6a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{6a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{6\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{6\sqrt{1-\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{6\sqrt{1+\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{6\sqrt{1-(-1)^{2/3}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{6\sqrt{1+(-1)^{2/3}}a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(x))}{6a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{6\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{6\sqrt{1-\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{6\sqrt{1+\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{6\sqrt{1-(-1)^{2/3}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{6\sqrt{1+(-1)^{2/3}}a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{6\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{6\sqrt{1-\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{6\sqrt{1+\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{6\sqrt{1-(-1)^{2/3}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{6\sqrt{1+(-1)^{2/3}}a}+\frac{\tan(x)}{6a}$$

input
```
Int[(a - a*Sin[x]^12)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(6*Sqrt[2]*a) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]
/(6*Sqrt[1 - (-1)^(1/3)]*a) + ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(6*Sqrt[
1 + (-1)^(1/3)]*a) + ArcTan[Sqrt[1 - (-1)^(2/3)]*Tan[x]]/(6*Sqrt[1 - (-1)^
(2/3)]*a) + ArcTan[Sqrt[1 + (-1)^(2/3)]*Tan[x]]/(6*Sqrt[1 + (-1)^(2/3)]*a)
 + Tan[x]/(6*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 36.85 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.89

method	result
default	$\frac{\tan(x)}{6}+\frac{\ln\left(\tan(x)^{2}+\tan(x)+1\right)}{24}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{12}-\frac{\ln\left(\tan(x)^{2}-\tan(x)+1\right)}{24}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{}}{3}\right.}{12}$
risch	$\frac{i}{3a(e^{2ix}+1)}-\frac{\ln\left(e^{2ix}-i\sqrt{3}-2i\right)}{24a}+\frac{i\ln\left(e^{2ix}-i\sqrt{3}-2i\right)\sqrt{3}}{24a}-\frac{\ln\left(e^{2ix}+i\sqrt{3}-2i\right)}{24a}-\frac{i\ln\left(e^{2ix}+i\sqrt{3}-2i\right)\sqrt{3}}{24a}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}\right.}{24a}$

input
```
int(1/(a-a*sin(x)^12),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/6*tan(x)+1/24*ln(tan(x)^2+tan(x)+1)+1/12*3^(1/2)*arctan(1/3*(2*tan(
x)+1)*3^(1/2))+1/12*2^(1/2)*arctan(tan(x)*2^(1/2))-1/24*ln(tan(x)^2-tan(x)
+1)+1/12*3^(1/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))+1/12*3^(1/2)*(-1/6*(-3+2
*3^(1/2))^(1/2)*ln(-(-3+2*3^(1/2))^(1/2)*3^(1/2)*tan(x)+3*tan(x)^2+3^(1/2)
)+2*(-1/6*(-3+2*3^(1/2))*3^(1/2)+2)/(9+6*3^(1/2))^(1/2)*arctan((-3^(1/2)*(
-3+2*3^(1/2))^(1/2)+6*tan(x))/(9+6*3^(1/2))^(1/2)))+1/12*3^(1/2)*(1/6*(-3+
2*3^(1/2))^(1/2)*ln(3^(1/2)+(-3+2*3^(1/2))^(1/2)*3^(1/2)*tan(x)+3*tan(x)^2
)+2*(-1/6*(-3+2*3^(1/2))*3^(1/2)+2)/(9+6*3^(1/2))^(1/2)*arctan((3^(1/2)*(-
3+2*3^(1/2))^(1/2)+6*tan(x))/(9+6*3^(1/2))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(232) = 464.

Time = 0.31 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.05

$$\int \frac{1}{a-a\sin^{12}(x)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^12),x, algorithm="fricas")
```

output
```
-1/48*(2*sqrt(1/2)*a*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4) + 1)/a^2)*cos(x)*lo
g(6*sqrt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*cos(x)*sin(x) + a*cos(x)*sin(x))
*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4) + 1)/a^2) + 3*sqrt(1/3)*(2*a^2*cos(x)^2
 - a^2)*sqrt(-1/a^4) + 4*cos(x)^2 - 3) - 2*sqrt(1/2)*a*sqrt(-(sqrt(1/3)*a^
2*sqrt(-1/a^4) + 1)/a^2)*cos(x)*log(-6*sqrt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^
4)*cos(x)*sin(x) + a*cos(x)*sin(x))*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4) + 1)
/a^2) + 3*sqrt(1/3)*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) + 4*cos(x)^2 - 3)
+ 2*sqrt(1/2)*a*sqrt((sqrt(1/3)*a^2*sqrt(-1/a^4) - 1)/a^2)*cos(x)*log(6*sq
rt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*cos(x)*sin(x) - a*cos(x)*sin(x))*sqrt(
(sqrt(1/3)*a^2*sqrt(-1/a^4) - 1)/a^2) + 3*sqrt(1/3)*(2*a^2*cos(x)^2 - a^2)
*sqrt(-1/a^4) - 4*cos(x)^2 + 3) - 2*sqrt(1/2)*a*sqrt((sqrt(1/3)*a^2*sqrt(-
1/a^4) - 1)/a^2)*cos(x)*log(-6*sqrt(1/2)*(sqrt(1/3)*a^3*sqrt(-1/a^4)*cos(x
)*sin(x) - a*cos(x)*sin(x))*sqrt((sqrt(1/3)*a^2*sqrt(-1/a^4) - 1)/a^2) + 3
*sqrt(1/3)*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) - 4*cos(x)^2 + 3) - 2*sqrt(
3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1))*cos(x)
 - 2*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^2 -
1))*cos(x) + 2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)
*sin(x)))*cos(x) - cos(x)*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1)
+ cos(x)*log(-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1) - 8*sin(x))/(a*co
s(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - a\sin^{12}(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-a*sin(x)**12),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a-a\sin^{12}(x)}\,dx=\int -\frac{1}{a\sin{(x)}^{12}-a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^12),x, algorithm="maxima")
```

output
```
1/24*((sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt
(2))*arctan2(2*sqrt(2)*sin(x)/(2*(sqrt(2) + 1)*cos(x) + cos(x)^2 + sin(x)^
2 + 2*sqrt(2) + 3), (cos(x)^2 + sin(x)^2 + 2*cos(x) - 1)/(2*(sqrt(2) + 1)*
cos(x) + cos(x)^2 + sin(x)^2 + 2*sqrt(2) + 3)) - (sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(2))*arctan2(2*sqrt(2)*sin(x)/(
2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(2) + 3), (cos(x)^2 +
 sin(x)^2 - 2*cos(x) - 1)/(2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 -
2*sqrt(2) + 3)) - 24*(a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*inte
grate(2/3*((cos(6*x) - 10*cos(4*x) + cos(2*x))*cos(8*x) + (110*cos(4*x) -
16*cos(2*x) + 1)*cos(6*x) - 8*cos(6*x)^2 + 10*(11*cos(2*x) - 1)*cos(4*x) -
 300*cos(4*x)^2 - 8*cos(2*x)^2 + (sin(6*x) - 10*sin(4*x) + sin(2*x))*sin(8
*x) + 2*(55*sin(4*x) - 8*sin(2*x))*sin(6*x) - 8*sin(6*x)^2 - 300*sin(4*x)^
2 + 110*sin(4*x)*sin(2*x) - 8*sin(2*x)^2 + cos(2*x))/(a*cos(8*x)^2 + 64*a*
cos(6*x)^2 + 900*a*cos(4*x)^2 + 64*a*cos(2*x)^2 + a*sin(8*x)^2 + 64*a*sin(
6*x)^2 + 900*a*sin(4*x)^2 - 480*a*sin(4*x)*sin(2*x) + 64*a*sin(2*x)^2 - 2*
(8*a*cos(6*x) - 30*a*cos(4*x) + 8*a*cos(2*x) - a)*cos(8*x) - 16*(30*a*cos(
4*x) - 8*a*cos(2*x) + a)*cos(6*x) - 60*(8*a*cos(2*x) - a)*cos(4*x) - 16*a*
cos(2*x) - 4*(4*a*sin(6*x) - 15*a*sin(4*x) + 4*a*sin(2*x))*sin(8*x) - 32*(
15*a*sin(4*x) - 4*a*sin(2*x))*sin(6*x) + a), x) + 24*(a*cos(2*x)^2 + a*sin
(2*x)^2 + 2*a*cos(2*x) + a)*integrate(1/6*((cos(3*x) + 4*cos(2*x) - cos...
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.41

$$\int \frac{1}{a-a\sin^{12}(x)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^12),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(
sqrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2)))/a + 1/12*sqrt(3)
*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(
2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2)))/a + 1/12*sqrt(2)*(x + arctan
(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x)
 + 2)))/a + 1/24*log(tan(x)^2 + tan(x) + 1)/a - 1/24*log(tan(x)^2 - tan(x)
 + 1)/a + 1/6*tan(x)/a + 1/72*(108^(1/4)*a^2 + 3*12^(1/4)*a*abs(a))*(pi*fl
oor(x/pi + 1/2) - arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) - sqrt(2)) +
 4*tan(x))/(sqrt(6) + sqrt(2))))/a^3 + 1/72*(108^(1/4)*a^2 + 3*12^(1/4)*a*
abs(a))*(pi*floor(x/pi + 1/2) + arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6
) - sqrt(2)) - 4*tan(x))/(sqrt(6) + sqrt(2))))/a^3 - 1/144*(108^(1/4)*a^2
- 3*12^(1/4)*a*abs(a))*log(1/2*(sqrt(6)*(1/3)^(1/4) - sqrt(2)*(1/3)^(1/4))
*tan(x) + tan(x)^2 + sqrt(1/3))/a^3 + 1/144*(108^(1/4)*a^2 - 3*12^(1/4)*a*
abs(a))*log(-1/2*(sqrt(6)*(1/3)^(1/4) - sqrt(2)*(1/3)^(1/4))*tan(x) + tan(
x)^2 + sqrt(1/3))/a^3
```

Mupad [B] (verification not implemented)

Time = 37.93 (sec) , antiderivative size = 1615, normalized size of antiderivative = 5.19

$$\int \frac{1}{a - a\sin^{12}(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - a*sin(x)^12),x)
```

output
```
tan(x)/(6*a) + atan(a*tan(x)*(- (3^(1/2)*1i)/(864*a^2) - 1/(288*a^2))^(1/2
)*18i + 6*3^(1/2)*a*tan(x)*(- (3^(1/2)*1i)/(864*a^2) - 1/(288*a^2))^(1/2))
*(-(3^(1/2)*1i + 3)/(864*a^2))^(1/2)*2i + atan(a*tan(x)*((3^(1/2)*1i)/(864
*a^2) - 1/(288*a^2))^(1/2)*18i - 6*3^(1/2)*a*tan(x)*((3^(1/2)*1i)/(864*a^2
) - 1/(288*a^2))^(1/2))*((3^(1/2)*1i - 3)/(864*a^2))^(1/2)*2i + (atan(((((
422*tan(x))/a^8 - ((3^(1/2)*1i - 1)*(13080/a^7 - ((3^(1/2)*1i - 1)*((26640
0*tan(x))/a^6 - ((3^(1/2)*1i - 1)*(3335040/a^5 - ((3^(1/2)*1i - 1)*((69672
960*tan(x))/a^4 - ((3^(1/2)*1i - 1)*(418037760/a^3 - ((3^(1/2)*1i - 1)*((8
778792960*tan(x))/a^2 - ((3^(1/2)*1i - 1)*(30098718720/a - (27088846848*ta
n(x)*(3^(1/2)*1i - 1))/a))/(24*a)))/(24*a)))/(24*a)))/(24*a)))/(24*a)))/(2
4*a)))/(24*a))*(3^(1/2)*1i - 1)*1i)/(24*a) + (((422*tan(x))/a^8 + ((3^(1/2
)*1i - 1)*(13080/a^7 + ((3^(1/2)*1i - 1)*((266400*tan(x))/a^6 + ((3^(1/2)*
1i - 1)*(3335040/a^5 + ((3^(1/2)*1i - 1)*((69672960*tan(x))/a^4 + ((3^(1/2
)*1i - 1)*(418037760/a^3 + ((3^(1/2)*1i - 1)*((8778792960*tan(x))/a^2 + ((
3^(1/2)*1i - 1)*(30098718720/a + (27088846848*tan(x)*(3^(1/2)*1i - 1))/a))
/(24*a)))/(24*a)))/(24*a)))/(24*a)))/(24*a)))/(24*a)))/(24*a))*(3^(1/2)*1i
 - 1)*1i)/(24*a))/(42/a^9 - (((422*tan(x))/a^8 - ((3^(1/2)*1i - 1)*(13080/
a^7 - ((3^(1/2)*1i - 1)*((266400*tan(x))/a^6 - ((3^(1/2)*1i - 1)*(3335040/
a^5 - ((3^(1/2)*1i - 1)*((69672960*tan(x))/a^4 - ((3^(1/2)*1i - 1)*(418037
760/a^3 - ((3^(1/2)*1i - 1)*((8778792960*tan(x))/a^2 - ((3^(1/2)*1i - 1...
```

Reduce [F]

$$\int \frac{1}{a - a\sin^{12}(x)}\,dx = -\frac{\int \frac{1}{\sin(x)^{12}-1}dx}{a}$$

input
```
int(1/(a-a*sin(x)^12),x)
```

output
```
( - int(1/(sin(x)**12 - 1),x))/a
```

3.60 $\int \frac{1}{a-a\sin^{16}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 314

$$\begin{aligned}\int \frac{1}{a-a\sin^{16}(x)}\,dx =& -\frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{\sqrt{1+\sqrt{2}}}\right)}{16a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{8\sqrt{2}a}\\&+\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1+\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1-(-1)^{3/4}}a}\\&+\frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}+2\tan(x)}{\sqrt{1+\sqrt{2}}}\right)}{16a}\\&+\frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{-1+\sqrt[4]{-1}}a}\\&+\frac{\operatorname{arctanh}\left(\sqrt{-1-(-1)^{3/4}}\tan(x)\right)}{8\sqrt{-1-(-1)^{3/4}}a}\\&+\frac{\sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\tan(x)}{1+\sqrt{2}\tan^2(x)}\right)}{16a}+\frac{\tan(x)}{8a}\end{aligned}$$

output
```
-1/16*(1+2^(1/2))^(1/2)*arctan(((2^(1/2)-1)^(1/2)-2*tan(x))/(1+2^(1/2))^(1
/2))/a+1/16*arctan(tan(x)*2^(1/2))*2^(1/2)/a+1/8*arctan((1+(-1)^(1/4))^(1/
2)*tan(x))/(1+(-1)^(1/4))^(1/2)/a+1/8*arctan((1-(-1)^(3/4))^(1/2)*tan(x))/
(1-(-1)^(3/4))^(1/2)/a+1/16*(1+2^(1/2))^(1/2)*arctan(((2^(1/2)-1)^(1/2)+2*
tan(x))/(1+2^(1/2))^(1/2))/a+1/8*arctanh((-1+(-1)^(1/4))^(1/2)*tan(x))/(-1
+(-1)^(1/4))^(1/2)/a+1/8*arctanh((-1-(-1)^(3/4))^(1/2)*tan(x))/(-1-(-1)^(3
/4))^(1/2)/a+1/16*(2^(1/2)-1)^(1/2)*arctanh((-2+2*2^(1/2))^(1/2)*tan(x)/(1
+2^(1/2)*tan(x)^2))/a+1/8*tan(x)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.66

$$\int \frac{1}{a-a\sin^{16}(x)}\,dx$$

$$= \frac{\frac{2\arctan\left(\sqrt{1-i}\tan(x)\right)}{\sqrt{1-i}}+\frac{2\arctan\left(\sqrt{1+i}\tan(x)\right)}{\sqrt{1+i}}+\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+64\text{RootSum}\left[1-8\#1+28\#1^2-56\#\right.}{}$$

input
```
Integrate[(a - a*Sin[x]^16)^(-1),x]
```

output
```
((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]
])/Sqrt[1 + I] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 64*RootSum[1 - 8*#1 + 28
*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*#1^7 + #1^8 & , (2*ArcT
an[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/
(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 - 7*#1^6 + #1^7) & ] +
 2*Tan[x])/(16*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^{16}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin(x)^{16}}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\sin^2(x)}dx}{8a}+\frac{\int \frac{1}{1-i\sin^2(x)}dx}{8a}+\frac{\int \frac{1}{i\sin^2(x)+1}dx}{8a}+\frac{\int \frac{1}{\sin^2(x)+1}dx}{8a}+\frac{\int \frac{1}{1-\sqrt[4]{-1}\sin^2(x)}dx}{8a}+\frac{\int \frac{1}{\sqrt[4]{-1}\sin^2(x)+1}dx}{8a}+\frac{\int \frac{1}{1-(-1)^{3/4}\sin^2(x)}dx}{8a}+\frac{\int \frac{1}{(-1)^{3/4}\sin^2(x)+1}dx}{8a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{1-i\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{i\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{1-\sqrt[4]{-1}\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{\sqrt[4]{-1}\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{1-(-1)^{3/4}\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{(-1)^{3/4}\sin(x)^2+1}dx}{8a}$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \frac{1}{1-i\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{i\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{1-\sqrt[4]{-1}\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{\sqrt[4]{-1}\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{1-(-1)^{3/4}\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{(-1)^{3/4}\sin(x)^2+1}dx}{8a}+\frac{\int \sec^2(x)dx}{8a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-i\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{i\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{1-\sqrt[4]{-1}\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{\sqrt[4]{-1}\sin(x)^2+1}dx}{8a}+\frac{\int \frac{1}{1-(-1)^{3/4}\sin(x)^2}dx}{8a}+\frac{\int \frac{1}{(-1)^{3/4}\sin(x)^2+1}dx}{8a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{8a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{(1-i)\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \frac{1}{(1+i)\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \frac{1}{2\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \frac{1}{\left(1-\sqrt[4]{-1}\right)\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \frac{1}{\left(1+\sqrt[4]{-1}\right)\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \frac{1}{\left(1-(-1)^{3/4}\right)\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \frac{1}{\left(1+(-1)^{3/4}\right)\tan^2(x)+1}d\tan(x)}{8a}+\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{8a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{8a}+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{8\sqrt{1-i}a}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{8\sqrt{1+i}a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{8\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1-\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1+\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1-(-1)^{3/4}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1+(-1)^{3/4}}a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(x))}{8a}+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{8\sqrt{1-i}a}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{8\sqrt{1+i}a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{8\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1-\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1+\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1-(-1)^{3/4}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1+(-1)^{3/4}}a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{8\sqrt{1-i}a}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{8\sqrt{1+i}a}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{8\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1-\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{8\sqrt{1+\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1-(-1)^{3/4}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{8\sqrt{1+(-1)^{3/4}}a}+\frac{\tan(x)}{8a}$$

input
```
Int[(a - a*Sin[x]^16)^(-1),x]
```

output
```
ArcTan[Sqrt[1 - I]*Tan[x]]/(8*Sqrt[1 - I]*a) + ArcTan[Sqrt[1 + I]*Tan[x]]/
(8*Sqrt[1 + I]*a) + ArcTan[Sqrt[2]*Tan[x]]/(8*Sqrt[2]*a) + ArcTan[Sqrt[1 -
 (-1)^(1/4)]*Tan[x]]/(8*Sqrt[1 - (-1)^(1/4)]*a) + ArcTan[Sqrt[1 + (-1)^(1/
4)]*Tan[x]]/(8*Sqrt[1 + (-1)^(1/4)]*a) + ArcTan[Sqrt[1 - (-1)^(3/4)]*Tan[x
]]/(8*Sqrt[1 - (-1)^(3/4)]*a) + ArcTan[Sqrt[1 + (-1)^(3/4)]*Tan[x]]/(8*Sqr
t[1 + (-1)^(3/4)]*a) + Tan[x]/(8*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 92.79 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.89

method	result
default	$\frac{\tan(x)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+1\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{2\_R^7+3\_R^5+3\_R^3+\_R}\right)}{16}+\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{16}+\frac{\sqrt{2}\,(\ldots}{\ldots}$
risch	$\frac{i}{4a(\mathrm{e}^{2ix}+1)}+\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2+2i}+\sqrt{-2+2i}-1+2i\right)}{32a}-\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{-2+2i}-\sqrt{-2+2i}-1+2i\right)}{32a}+\left(\sum_{\_R=\mathrm{Roo}\ldots}\right.$

input
```
int(1/(a-a*sin(x)^16),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/8*tan(x)+1/16*sum((_R^6+3*_R^4+3*_R^2+1)/(2*_R^7+3*_R^5+3*_R^3+_R)*
ln(tan(x)-_R),_R=RootOf(2*_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+1))+1/16*2^(1/2)*arcta
n(tan(x)*2^(1/2))+1/16*2^(1/2)*(-1/4*(-2+2*2^(1/2))^(1/2)*ln(-(-2+2*2^(1/2
))^(1/2)*2^(1/2)*tan(x)+2*tan(x)^2+2^(1/2))+(-1/4*(-2+2*2^(1/2))*2^(1/2)+2
)/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1
+2^(1/2))^(1/2)))+1/16*2^(1/2)*(1/4*(-2+2*2^(1/2))^(1/2)*ln(2^(1/2)+(-2+2*
2^(1/2))^(1/2)*2^(1/2)*tan(x)+2*tan(x)^2)+(-1/4*(-2+2*2^(1/2))*2^(1/2)+2)/
(1+2^(1/2))^(1/2)*arctan(1/2*(2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^
(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2470 vs. $2(226) = 452$.

Time = 0.95 (sec) , antiderivative size = 2470, normalized size of antiderivative = 7.87

$$\int \frac{1}{a - a\sin^{16}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^16),x, algorithm="fricas")
```

output
```
-1/32*(sqrt(1/2)*a*sqrt(-(a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4
) + 1)/a^2)*cos(x)*log(1/2*sqrt(1/2)*(sqrt(1/2)*a^5*sqrt(a^(-8))*cos(x)*si
n(x) - a*cos(x)*sin(x) + (sqrt(1/2)*a^7*sqrt(a^(-8))*cos(x)*sin(x) - a^3*c
os(x)*sin(x))*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4))*sqrt(-(a^2*sq
rt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) + 1)/a^2) + 1/4*sqrt(1/2)*(2*a
^4*cos(x)^2 - a^4)*sqrt(a^(-8)) - 1/4*cos(x)^2 - 1/4*(2*a^2*cos(x)^2 - a^2
 - sqrt(1/2)*(2*a^6*cos(x)^2 - a^6)*sqrt(a^(-8)))*sqrt(-(4*sqrt(1/2)*a^4*s
qrt(a^(-8)) + 3)/a^4) + 1/4) - sqrt(1/2)*a*sqrt(-(a^2*sqrt(-(4*sqrt(1/2)*a
^4*sqrt(a^(-8)) + 3)/a^4) + 1)/a^2)*cos(x)*log(-1/2*sqrt(1/2)*(sqrt(1/2)*a
^5*sqrt(a^(-8))*cos(x)*sin(x) - a*cos(x)*sin(x) + (sqrt(1/2)*a^7*sqrt(a^(-
8))*cos(x)*sin(x) - a^3*cos(x)*sin(x))*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8))
 + 3)/a^4))*sqrt(-(a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) + 1)/
a^2) + 1/4*sqrt(1/2)*(2*a^4*cos(x)^2 - a^4)*sqrt(a^(-8)) - 1/4*cos(x)^2 -
1/4*(2*a^2*cos(x)^2 - a^2 - sqrt(1/2)*(2*a^6*cos(x)^2 - a^6)*sqrt(a^(-8)))
*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) + 1/4) - sqrt(1/2)*a*sqrt((
a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) - 1)/a^2)*cos(x)*log(1/2
*sqrt(1/2)*(sqrt(1/2)*a^5*sqrt(a^(-8))*cos(x)*sin(x) - a*cos(x)*sin(x) - (
sqrt(1/2)*a^7*sqrt(a^(-8))*cos(x)*sin(x) - a^3*cos(x)*sin(x))*sqrt(-(4*sqr
t(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4))*sqrt((a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a
^(-8)) + 3)/a^4) - 1)/a^2) - 1/4*sqrt(1/2)*(2*a^4*cos(x)^2 - a^4)*sqrt(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - a\sin^{16}(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-a*sin(x)**16),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a - a\sin^{16}(x)}\,dx = \int -\frac{1}{a\sin{(x)}^{16} - a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^16),x, algorithm="maxima")
```

output
```
1/32*((sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt
(2))*arctan2(2*sqrt(2)*sin(x)/(2*(sqrt(2) + 1)*cos(x) + cos(x)^2 + sin(x)^
2 + 2*sqrt(2) + 3), (cos(x)^2 + sin(x)^2 + 2*cos(x) - 1)/(2*(sqrt(2) + 1)*
cos(x) + cos(x)^2 + sin(x)^2 + 2*sqrt(2) + 3)) - (sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(2))*arctan2(2*sqrt(2)*sin(x)/(
2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(2) + 3), (cos(x)^2 +
 sin(x)^2 - 2*cos(x) - 1)/(2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 -
2*sqrt(2) + 3)) + 4096*(a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*in
tegrate(-((56*cos(6*x) - 28*cos(4*x) + 8*cos(2*x) - 1)*cos(8*x) - cos(16*x
)*cos(8*x) + 8*cos(14*x)*cos(8*x) - 28*cos(12*x)*cos(8*x) + 56*cos(10*x)*c
os(8*x) - 326*cos(8*x)^2 + 4*(14*sin(6*x) - 7*sin(4*x) + 2*sin(2*x))*sin(8
*x) - sin(16*x)*sin(8*x) + 8*sin(14*x)*sin(8*x) - 28*sin(12*x)*sin(8*x) +
56*sin(10*x)*sin(8*x) - 326*sin(8*x)^2)/(a*cos(16*x)^2 + 64*a*cos(14*x)^2
+ 784*a*cos(12*x)^2 + 3136*a*cos(10*x)^2 + 106276*a*cos(8*x)^2 + 3136*a*co
s(6*x)^2 + 784*a*cos(4*x)^2 + 64*a*cos(2*x)^2 + a*sin(16*x)^2 + 64*a*sin(1
4*x)^2 + 784*a*sin(12*x)^2 + 3136*a*sin(10*x)^2 + 106276*a*sin(8*x)^2 + 31
36*a*sin(6*x)^2 + 784*a*sin(4*x)^2 - 448*a*sin(4*x)*sin(2*x) + 64*a*sin(2*
x)^2 - 2*(8*a*cos(14*x) - 28*a*cos(12*x) + 56*a*cos(10*x) - 326*a*cos(8*x)
 + 56*a*cos(6*x) - 28*a*cos(4*x) + 8*a*cos(2*x) - a)*cos(16*x) - 16*(28*a*
cos(12*x) - 56*a*cos(10*x) + 326*a*cos(8*x) - 56*a*cos(6*x) + 28*a*cos(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a - a\sin^{16}(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-a*sin(x)^16),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 38.36 (sec) , antiderivative size = 961, normalized size of antiderivative = 3.06

$$\int \frac{1}{a-a\sin^{16}(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - a*sin(x)^16),x)
```

output
```
tan(x)/(8*a) + atan(a*tan(x)*((- 1/512 - 1i/512)/a^2)^(1/2)*(16 + 16i))*((
- 1/512 - 1i/512)/a^2)^(1/2)*2i - atan(a*tan(x)*((- 1/512 + 1i/512)/a^2)^(
1/2)*(16 - 16i))*((- 1/512 + 1i/512)/a^2)^(1/2)*2i - (atan((tan(x)*(- (2*(
- 2*2^(1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*17i)/(4*((7*2^(1/2))/(2*a) - 5/
a)) - (2^(1/2)*tan(x)*(- (2*(- 2*2^(1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*3i
)/((7*2^(1/2))/(2*a) - 5/a) - (tan(x)*(- (2*(- 2*2^(1/2) - 3)^(1/2))/a^2 -
 2/a^2)^(1/2)*(- 2*2^(1/2) - 3)^(1/2)*17i)/(4*((7*2^(1/2))/(2*a) - 5/a)) +
 (2^(1/2)*tan(x)*(- (2*(- 2*2^(1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*(- 2*2^
(1/2) - 3)^(1/2)*3i)/((7*2^(1/2))/(2*a) - 5/a))*(-(2*((- 2*2^(1/2) - 3)^(1
/2) + 1))/a^2)^(1/2)*1i)/16 - (atan((tan(x)*((2*(- 2*2^(1/2) - 3)^(1/2))/a
^2 - 2/a^2)^(1/2)*17i)/(4*((7*2^(1/2))/(2*a) - 5/a)) - (2^(1/2)*tan(x)*((2
*(- 2*2^(1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*3i)/((7*2^(1/2))/(2*a) - 5/a)
 + (tan(x)*((2*(- 2*2^(1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*(- 2*2^(1/2) -
3)^(1/2)*17i)/(4*((7*2^(1/2))/(2*a) - 5/a)) - (2^(1/2)*tan(x)*((2*(- 2*2^(
1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*(- 2*2^(1/2) - 3)^(1/2)*3i)/((7*2^(1/2
))/(2*a) - 5/a))*((2*((- 2*2^(1/2) - 3)^(1/2) - 1))/a^2)^(1/2)*1i)/16 + (a
tan((tan(x)*(- (2*(2*2^(1/2) - 3)^(1/2))/a^2 - 2/a^2)^(1/2)*17i)/(4*((7*2^
(1/2))/(2*a) + 5/a)) + (2^(1/2)*tan(x)*(- (2*(2*2^(1/2) - 3)^(1/2))/a^2 -
2/a^2)^(1/2)*3i)/((7*2^(1/2))/(2*a) + 5/a) - (tan(x)*(- (2*(2*2^(1/2) - 3)
^(1/2))/a^2 - 2/a^2)^(1/2)*(2*2^(1/2) - 3)^(1/2)*17i)/(4*((7*2^(1/2))/(...
```

Reduce [F]

$$\int \frac{1}{a-a\sin^{16}(x)}\,dx = -\frac{\int \frac{1}{\sin(x)^{16}-1}dx}{a}$$

input
```
int(1/(a-a*sin(x)^16),x)
```

output
```
( - int(1/(sin(x)**16 - 1),x))/a
```

3.61 $\int \frac{1}{a-a\sin(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{a-a\sin(x)}\,dx = \frac{\cos(x)}{a-a\sin(x)}$$

output
```
cos(x)/(a-a*sin(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{1}{a-a\sin(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{a\cos\left(\frac{x}{2}\right)-a\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(a - a*Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2])/(a*Cos[x/2] - a*Sin[x/2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin(x)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{a-a\sin(x)}dx$$
$$\Big\downarrow \ 3127$$
$$\frac{\cos(x)}{a-a\sin(x)}$$

input
```
Int[(a - a*Sin[x])^(-1),x]
```

output
```
Cos[x]/(a - a*Sin[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)-1\right)}$	14
norman	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)-1\right)}$	14
parallelrisch	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)-1\right)}$	14
risch	$\frac{2}{\left(e^{ix}-i\right)a}$	16

input
```
int(1/(a-a*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(tan(1/2*x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{a-a\sin(x)}\,dx = \frac{\cos(x)+\sin(x)+1}{a\cos(x)-a\sin(x)+a}$$

input
```
integrate(1/(a-a*sin(x)),x, algorithm="fricas")
```

output
```
(cos(x) + sin(x) + 1)/(a*cos(x) - a*sin(x) + a)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{a-a\sin(x)}\,dx = -\frac{2}{a\tan\left(\frac{x}{2}\right)-a}$$

input
```
integrate(1/(a-a*sin(x)),x)
```

output
```
-2/(a*tan(x/2) - a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{a - a\sin(x)}\,dx = \frac{2}{a - \frac{a\sin(x)}{\cos(x)+1}}$$

input
```
integrate(1/(a-a*sin(x)),x, algorithm="maxima")
```

output
```
2/(a - a*sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a - a\sin(x)}\,dx = -\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right) - 1\right)}$$

input
```
integrate(1/(a-a*sin(x)),x, algorithm="giac")
```

output
```
-2/(a*(tan(1/2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 37.90 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a - a\sin(x)}\,dx = -\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right) - 1\right)}$$

input
```
int(1/(a - a*sin(x)),x)
```

output
```
-2/(a*(tan(x/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{a-a\sin(x)}\,dx = -\frac{2\tan\left(\frac{x}{2}\right)}{a\left(\tan\left(\frac{x}{2}\right)-1\right)}$$

input
```
int(1/(a-a*sin(x)),x)
```

output
```
( - 2*tan(x/2))/(a*(tan(x/2) - 1))
```

3.62 $\int \frac{1}{a-a\sin^3(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{1}{a-a\sin^3(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt[3]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}a} - \frac{2\arctan\left(\frac{(-1)^{2/3}\left(1+\sqrt[3]{-1}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}a} + \frac{\cos(x)}{3a(1-\sin(x))}$$

output
```
2/3*arctan(((-1)^(1/3)+tan(1/2*x))/(1-(-1)^(2/3))^(1/2))/(1-(-1)^(2/3))^(1
/2)/a-2/3*arctan((-1)^(2/3)*(1+(-1)^(1/3)*tan(1/2*x))/(1+(-1)^(1/3))^(1/2)
)/(1+(-1)^(1/3))^(1/2)/a+1/3*cos(x)/a/(1-sin(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.89 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.28

$$\int \frac{1}{a-a\sin^3(x)}\,dx = \frac{i\sqrt{-18-6i\sqrt{3}}\arctan\left(\frac{2+\left(1-i\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-6-2i\sqrt{3}}}\right)-i\sqrt{6i\left(3i+\sqrt{3}\right)}\arctan\left(\frac{2+\left(1+i\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-6+2i\sqrt{3}}}\right)+\frac{6\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}}{9a}$$

input
```
Integrate[(a - a*Sin[x]^3)^(-1),x]
```

output
```
(I*Sqrt[-18 - (6*I)*Sqrt[3]]*ArcTan[(2 + (1 - I*Sqrt[3])*Tan[x/2])/Sqrt[-6
 - (2*I)*Sqrt[3]]] - I*Sqrt[(6*I)*(3*I + Sqrt[3])]*ArcTan[(2 + (1 + I*Sqrt
[3])*Tan[x/2])/Sqrt[-6 + (2*I)*Sqrt[3]]] + (6*Sin[x/2])/(Cos[x/2] - Sin[x/
2]))/(9*a)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^3(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a-a\sin(x)^3}dx$$
$$\Big\downarrow \text{ 3692}$$
$$\int \left(\frac{1}{3a\left(\sqrt[3]{-1}\sin(x)+1\right)}+\frac{1}{3a\left(1-(-1)^{2/3}\sin(x)\right)}+\frac{1}{3a(1-\sin(x))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[3]{-1}}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}a}-\frac{2\arctan\left(\frac{(-1)^{2/3}\left(\sqrt[3]{-1}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}a}+\frac{\cos(x)}{3a(1-\sin(x))}$$

input
```
Int[(a - a*Sin[x]^3)^(-1),x]
```

output
```
(2*ArcTan[((-1)^(1/3) + Tan[x/2])/Sqrt[1 - (-1)^(2/3)]])/(3*Sqrt[1 - (-1)^
(2/3)]*a) - (2*ArcTan[((-1)^(2/3)*(1 + (-1)^(1/3)*Tan[x/2]))/Sqrt[1 + (-1)
^(1/3)]])/(3*Sqrt[1 + (-1)^(1/3)]*a) + Cos[x]/(3*a*(1 - Sin[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

method	result
risch	$\frac{2}{3(e^{ix}-i)a}+\left(\sum_{\_R=\mathrm{RootOf}(243a^4\_Z^4+27a^2\_Z^2+1)}\_R\ln\left(e^{ix}+162a^3\_R^3+27ia^2\_R^2+9a\_R+2i\right)\right)$
default	$\frac{\frac{2\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^3+6\_Z^2+2\_Z+1)}\frac{(\_R^2+\_R+1)\ln(\tan(\frac{x}{2})-\_R)}{2\_R^3+3\_R^2+6\_R+1}\right)}{3}-\frac{2}{3(\tan(\frac{x}{2})-1)}}{a}$

input
```
int(1/(a-a*sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2/3/(exp(I*x)-I)/a+sum(_R*ln(exp(I*x)+162*a^3*_R^3+27*I*a^2*_R^2+9*a*_R+2*
I),_R=RootOf(243*_Z^4*a^4+27*_Z^2*a^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(81) = 162.

Time = 0.10 (sec) , antiderivative size = 439, normalized size of antiderivative = 3.82

$$\int \frac{1}{a-a\sin^3(x)}\,dx = -\frac{\sqrt{2}(a\cos(x)-a\sin(x)+a)\sqrt{-\frac{\sqrt{\frac{1}{3}}a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}\log\left(-3\sqrt{2}\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{\sqrt{\frac{1}{3}}a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}\sqrt{-\frac{1}{a^4}}\cos(x)+3\sqrt{\right.}{}$$

input
```
integrate(1/(a-a*sin(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2)*(a*cos(x) - a*sin(x) + a)*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4)
 + 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4)
+ 1)/a^2)*sqrt(-1/a^4)*cos(x) + 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) - sin(
x) - 2) - sqrt(2)*(a*cos(x) - a*sin(x) + a)*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a
^4) + 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^
4) + 1)/a^2)*sqrt(-1/a^4)*cos(x) - 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) + s
in(x) + 2) + sqrt(2)*(a*cos(x) - a*sin(x) + a)*sqrt((sqrt(1/3)*a^2*sqrt(-1
/a^4) - 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt((sqrt(1/3)*a^2*sqrt(-1/a
^4) - 1)/a^2)*sqrt(-1/a^4)*cos(x) + 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) +
sin(x) + 2) - sqrt(2)*(a*cos(x) - a*sin(x) + a)*sqrt((sqrt(1/3)*a^2*sqrt(-
1/a^4) - 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt((sqrt(1/3)*a^2*sqrt(-1/
a^4) - 1)/a^2)*sqrt(-1/a^4)*cos(x) - 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) -
 sin(x) - 2) - 4*cos(x) - 4*sin(x) - 4)/(a*cos(x) - a*sin(x) + a)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10462 vs. $2(105) = 210$.

Time = 19.69 (sec) , antiderivative size = 10462, normalized size of antiderivative = 90.97

$$\int \frac{1}{a - a\sin^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)**3),x)
```

output
```
-406593*sqrt(6)*sqrt(-3 + sqrt(3)*I)*log(tan(x/2) + 1/2 + sqrt(2)*sqrt(-3
- sqrt(3)*I)/2 - sqrt(3)*I/2)*tan(x/2)/(1219779*sqrt(3)*a*sqrt(-3 - sqrt(3
)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 4242534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt
(-3 + sqrt(3)*I)*tan(x/2) - 2698299*sqrt(6)*a*sqrt(-3 + sqrt(3)*I)*tan(x/2
) - 4130865*sqrt(2)*I*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 1219779*sqrt(3)*a*
sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I) + 4130865*sqrt(2)*I*a*sqrt(-3 +
sqrt(3)*I) + 2698299*sqrt(6)*a*sqrt(-3 + sqrt(3)*I) + 4242534*I*a*sqrt(-3
- sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)) - 238761*I*sqrt(-3 - sqrt(3)*I)*sqrt(-3
 + sqrt(3)*I)*log(tan(x/2) + 1/2 + sqrt(2)*sqrt(-3 - sqrt(3)*I)/2 - sqrt(3
)*I/2)*tan(x/2)/(1219779*sqrt(3)*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*
I)*tan(x/2) - 4242534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/
2) - 2698299*sqrt(6)*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 4130865*sqrt(2)*I*a
*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 1219779*sqrt(3)*a*sqrt(-3 - sqrt(3)*I)*sq
rt(-3 + sqrt(3)*I) + 4130865*sqrt(2)*I*a*sqrt(-3 + sqrt(3)*I) + 2698299*sq
rt(6)*a*sqrt(-3 + sqrt(3)*I) + 4242534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 +
sqrt(3)*I)) - 679209*sqrt(3)*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*log
(tan(x/2) + 1/2 + sqrt(2)*sqrt(-3 - sqrt(3)*I)/2 - sqrt(3)*I/2)*tan(x/2)/(
1219779*sqrt(3)*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 424
2534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 2698299*sqrt
(6)*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 4130865*sqrt(2)*I*a*sqrt(-3 + sqr...
```

Maxima [F]

$$\int \frac{1}{a-a\sin^3(x)}\,dx=\int -\frac{1}{a\sin{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^3),x, algorithm="maxima")
```

output
```
1/3*(3*(a*cos(x)^2 + a*sin(x)^2 - 2*a*sin(x) + a)*integrate(-2/3*((4*cos(2
*x) + sin(3*x) - sin(x))*cos(4*x) + 2*(2*cos(x) + 7*sin(2*x))*cos(3*x) - 2
*cos(3*x)^2 + 2*(7*sin(x) + 2)*cos(2*x) - 24*cos(2*x)^2 - 2*cos(x)^2 - (co
s(3*x) - cos(x) - 4*sin(2*x))*sin(4*x) - (14*cos(2*x) - 4*sin(x) - 1)*sin(
3*x) - 2*sin(3*x)^2 - 14*cos(x)*sin(2*x) - 24*sin(2*x)^2 - 2*sin(x)^2 - si
n(x))/(a*cos(4*x)^2 + 4*a*cos(3*x)^2 + 36*a*cos(2*x)^2 + 4*a*cos(x)^2 + a*
sin(4*x)^2 + 4*a*sin(3*x)^2 + 24*a*cos(x)*sin(2*x) + 36*a*sin(2*x)^2 + 4*a
*sin(x)^2 - 2*(6*a*cos(2*x) + 2*a*sin(3*x) - 2*a*sin(x) - a)*cos(4*x) - 8*
(a*cos(x) + 3*a*sin(2*x))*cos(3*x) - 12*(2*a*sin(x) + a)*cos(2*x) + 4*(a*c
os(3*x) - a*cos(x) - 3*a*sin(2*x))*sin(4*x) + 4*(6*a*cos(2*x) - 2*a*sin(x)
 - a)*sin(3*x) + 4*a*sin(x) + a), x) + 2*cos(x))/(a*cos(x)^2 + a*sin(x)^2
- 2*a*sin(x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(81) = 162$.

Time = 0.47 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.76

$$\int \frac{1}{a - a\sin^3(x)}\,dx =$$

$$-\frac{\sqrt{6\sqrt{3}-9}\log\left(196\left(2\sqrt{3}\sqrt{6\sqrt{3}-9}+3\sqrt{3}+3\sqrt{6\sqrt{3}-9}\right)^2+196\left(\sqrt{3}\sqrt{6\sqrt{3}-9}+6\tan\left(\frac{1}{2}x\right)\right.\right.}{}$$

$$-\frac{2}{3a\left(\tan\left(\frac{1}{2}x\right)-1\right)}$$

input
```
integrate(1/(a-a*sin(x)^3),x, algorithm="giac")
```

output
```
-1/18*(sqrt(6*sqrt(3) - 9)*log(196*(2*sqrt(3)*sqrt(6*sqrt(3) - 9) + 3*sqrt
(3) + 3*sqrt(6*sqrt(3) - 9))^2 + 196*(sqrt(3)*sqrt(6*sqrt(3) - 9) + 6*tan(
1/2*x) + 3)^2) - sqrt(6*sqrt(3) - 9)*log(196*(2*sqrt(3)*sqrt(6*sqrt(3) - 9
) - 3*sqrt(3) + 3*sqrt(6*sqrt(3) - 9))^2 + 196*(sqrt(3)*sqrt(6*sqrt(3) - 9
) - 6*tan(1/2*x) - 3)^2) - 2*sqrt(3)*sqrt(6*sqrt(3) - 9)*arctan(3*(sqrt(2*
sqrt(3) - 3) + 2*tan(1/2*x) + 1)/(2*sqrt(3)*sqrt(6*sqrt(3) - 9) + 3*sqrt(3
) + 3*sqrt(6*sqrt(3) - 9)))/(2*sqrt(3) - 3) - 2*sqrt(3)*sqrt(6*sqrt(3) - 9
)*arctan(-3*(sqrt(2*sqrt(3) - 3) - 2*tan(1/2*x) - 1)/(2*sqrt(3)*sqrt(6*sqr
t(3) - 9) - 3*sqrt(3) + 3*sqrt(6*sqrt(3) - 9)))/(2*sqrt(3) - 3))/a - 2/3/(
a*(tan(1/2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 37.52 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.20

$$\int \frac{1}{a-a\sin^3(x)}\,dx = \frac{2}{3\,\left(a-a\tan\left(\frac{x}{2}\right)\right)} + 2\,\mathrm{atanh}\left(\frac{5184\,a^3\sqrt{-\frac{1}{18\,a^2}-\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{3456\,a^2\tan\left(\frac{x}{2}\right)+864\,a^2-\sqrt{3}\,a^2\,864\mathrm{i}} + \frac{2592\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}-\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{3456\,a^2\tan\left(\frac{x}{2}\right)+864\,a^2-\sqrt{3}\,a^2\,864\mathrm{i}} + \frac{\sqrt{3}\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}-\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}\,7776\mathrm{i}}{3456\,a^2\tan\left(\frac{x}{2}\right)+864\,a^2-\sqrt{3}\,a^2\,864\mathrm{i}}\right)\sqrt{-\frac{3+\sqrt{3}\,1\mathrm{i}}{54\,a^2}} + 2\,\mathrm{atanh}\left(\frac{5184\,a^3\sqrt{-\frac{1}{18\,a^2}+\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{3456\,a^2\tan\left(\frac{x}{2}\right)+864\,a^2+\sqrt{3}\,a^2\,864\mathrm{i}} + \frac{2592\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}+\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{3456\,a^2\tan\left(\frac{x}{2}\right)+864\,a^2+\sqrt{3}\,a^2\,864\mathrm{i}} - \frac{\sqrt{3}\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}+\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}\,7776\mathrm{i}}{3456\,a^2\tan\left(\frac{x}{2}\right)+864\,a^2+\sqrt{3}\,a^2\,864\mathrm{i}}\right)\sqrt{\frac{-3+\sqrt{3}\,1\mathrm{i}}{54\,a^2}}$$

input
```
int(1/(a - a*sin(x)^3),x)
```

output
```
2/(3*(a - a*tan(x/2))) + 2*atanh((5184*a^3*(- (3^(1/2)*1i)/(54*a^2) - 1/(1
8*a^2))^(1/2))/(3456*a^2*tan(x/2) - 3^(1/2)*a^2*864i + 864*a^2) + (2592*a^
3*tan(x/2)*(- (3^(1/2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2))/(3456*a^2*tan(x/2
) - 3^(1/2)*a^2*864i + 864*a^2) + (3^(1/2)*a^3*tan(x/2)*(- (3^(1/2)*1i)/(5
4*a^2) - 1/(18*a^2))^(1/2)*7776i)/(3456*a^2*tan(x/2) - 3^(1/2)*a^2*864i +
864*a^2))*(-(3^(1/2)*1i + 3)/(54*a^2))^(1/2) + 2*atanh((5184*a^3*((3^(1/2)
*1i)/(54*a^2) - 1/(18*a^2))^(1/2))/(3456*a^2*tan(x/2) + 3^(1/2)*a^2*864i +
 864*a^2) + (2592*a^3*tan(x/2)*((3^(1/2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2))
/(3456*a^2*tan(x/2) + 3^(1/2)*a^2*864i + 864*a^2) - (3^(1/2)*a^3*tan(x/2)*
((3^(1/2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2)*7776i)/(3456*a^2*tan(x/2) + 3^(
1/2)*a^2*864i + 864*a^2))*((3^(1/2)*1i - 3)/(54*a^2))^(1/2)
```

Reduce [F]

$$\int \frac{1}{a - a\sin^3(x)}\, dx = -\frac{\int \frac{1}{\sin(x)^3 - 1} dx}{a}$$

input
```
int(1/(a-a*sin(x)^3),x)
```

output
```
( - int(1/(sin(x)**3 - 1),x))/a
```

3.63 $\int \frac{1}{a-a\sin^5(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 212

$$\int \frac{1}{a-a\sin^5(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt[5]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}a}+\frac{2\arctan\left(\frac{(-1)^{3/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}a} - \frac{2\arctan\left(\frac{(-1)^{4/5}\left(1+\sqrt[5]{-1}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}a} - \frac{2\arctan\left(\frac{(-1)^{2/5}\left(1+(-1)^{3/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}a}+\frac{\cos(x)}{5a(1-\sin(x))}$$

output
```
2/5*arctan(((-1)^(1/5)+tan(1/2*x))/(1-(-1)^(2/5))^(1/2))/(1-(-1)^(2/5))^(1
/2)/a+2/5*arctan(((-1)^(3/5)+tan(1/2*x))/(1+(-1)^(1/5))^(1/2))/(1+(-1)^(1/
5))^(1/2)/a-2/5*arctan((-1)^(4/5)*(1+(-1)^(1/5)*tan(1/2*x))/(1+(-1)^(3/5))
^(1/2))/(1+(-1)^(3/5))^(1/2)/a-2/5*arctan((-1)^(2/5)*(1+(-1)^(3/5)*tan(1/2
*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)/a+1/5*cos(x)/a/(1-sin(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.13 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.96

$$\int \frac{1}{a - a\sin^5(x)}\,dx$$
$$= \frac{i\text{RootSum}\left[1 - 2i\#1 - 8\#1^2 + 14i\#1^3 + 30\#1^4 - 14i\#1^5 - 8\#1^6 + 2i\#1^7 + \#1^8\&, \frac{-2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right.}{}\right.}{}$$

input
```
Integrate[(a - a*Sin[x]^5)^(-1),x]
```

output
```
(I*RootSum[1 - (2*I)*#1 - 8*#1^2 + (14*I)*#1^3 + 30*#1^4 - (14*I)*#1^5 - 8
*#1^6 + (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log[1 -
 2*Cos[x]*#1 + #1^2] + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + 4*Log[1 - 2
*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[
1 - 2*Cos[x]*#1 + #1^2]*#1^2 - (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 -
40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4
 + (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + (8*I)*ArcTan[Sin[x]/(Cos[x] -
 #1)]*#1^5 + 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x]
- #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-I - 8*#1 + (21*I)*#1^2
 + 60*#1^3 - (35*I)*#1^4 - 24*#1^5 + (7*I)*#1^6 + 4*#1^7) & ] + (4*Sin[x/2
])/(Cos[x/2] - Sin[x/2]))/(10*a)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a - a\sin^5(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin(x)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{5a\left(\sqrt[5]{-1}\sin(x)+1\right)}+\frac{1}{5a\left(1-(-1)^{2/5}\sin(x)\right)}+\frac{1}{5a\left((-1)^{3/5}\sin(x)+1\right)}+\frac{1}{5a\left(1-(-1)^{4/5}\sin(x)\right)}+\frac{}{5a(}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}a}+\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}a}-\frac{2\arctan\left(\frac{(-1)^{4/5}\left(\sqrt[5]{-1}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}a}-\frac{2\arctan\left(\frac{(-1)^{2/5}\left((-1)^{3/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}a}+\frac{\cos(x)}{5a(1-\sin(x))}$$

input
```
Int[(a - a*Sin[x]^5)^(-1),x]
```

output
```
(2*ArcTan[((-1)^(1/5) + Tan[x/2])/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^
(2/5)]*a) + (2*ArcTan[((-1)^(3/5) + Tan[x/2])/Sqrt[1 + (-1)^(1/5)]])/(5*Sq
rt[1 + (-1)^(1/5)]*a) - (2*ArcTan[((-1)^(4/5)*(1 + (-1)^(1/5)*Tan[x/2]))/S
qrt[1 + (-1)^(3/5)]])/(5*Sqrt[1 + (-1)^(3/5)]*a) - (2*ArcTan[((-1)^(2/5)*(
1 + (-1)^(3/5)*Tan[x/2]))/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^(4/5)]*a
) + Cos[x]/(5*a*(1 - Sin[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.57

method	result
risch	$\frac{2}{5(e^{ix}-i)a}+\left(\sum_{\_R=\operatorname{RootOf}(1953125a^8\_Z^8+156250a^6\_Z^6+6250a^4\_Z^4+125a^2\_Z^2+1)}\_R\ln\left(e^{ix}-2343750a^7\_R^7\right.\right.$
default	$\frac{-\frac{2}{5\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}(\_Z^8+2\_Z^7+8\_Z^6+14\_Z^5+30\_Z^4+14\_Z^3+8\_Z^2+2\_Z+1)}\frac{\left(2\_R^6+3\_R^5+10\_R^4+10\_R^3+\right.}{4\_R^7+7\_R^6+24\_R^5+35\_}\right.}{5}}{a}$

input
```
int(1/(a-a*sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
2/5/(exp(I*x)-I)/a+sum(_R*ln(exp(I*x)-2343750*a^7*_R^7-234375*I*a^6*_R^6-1
40625*a^5*_R^5-15625*I*a^4*_R^4-4375*a^3*_R^3-500*I*a^2*_R^2-50*a*_R-6*I),
_R=RootOf(1953125*_Z^8*a^8+156250*_Z^6*a^6+6250*_Z^4*a^4+125*_Z^2*a^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1621 vs. 2(148) = 296.

Time = 0.26 (sec) , antiderivative size = 1621, normalized size of antiderivative = 7.65

$$\int \frac{1}{a-a\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^5),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2)*(a*cos(x) - a*sin(x) + a)*sqrt(-(a^2*sqrt(-(2*sqrt(1/5)*a^4*
sqrt(a^(-8)) + 1)/a^4) + 1)/a^2)*log(5*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) -
 5*sqrt(2)*(3*sqrt(1/5)*a^7*sqrt(a^(-8))*cos(x) - a^3*cos(x))*sqrt(-(a^2*s
qrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + 1)/a^2)*sqrt(-(2*sqrt(1/5)*
a^4*sqrt(a^(-8)) + 1)/a^4) - 5*(3*sqrt(1/5)*a^6*sqrt(a^(-8))*sin(x) - a^2*
sin(x))*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + sin(x) + 4) - sqrt
(2)*(a*cos(x) - a*sin(x) + a)*sqrt((a^2*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)
) + 1)/a^4) - 1)/a^2)*log(5*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) - 5*sqrt(2)*
(3*sqrt(1/5)*a^7*sqrt(a^(-8))*cos(x) - a^3*cos(x))*sqrt((a^2*sqrt(-(2*sqrt
(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) - 1)/a^2)*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(
-8)) + 1)/a^4) + 5*(3*sqrt(1/5)*a^6*sqrt(a^(-8))*sin(x) - a^2*sin(x))*sqrt
(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + sin(x) + 4) + sqrt(2)*(a*cos(x
) - a*sin(x) + a)*sqrt(-(a^2*sqrt((2*sqrt(1/5)*a^4*sqrt(a^(-8)) - 1)/a^4)
+ 1)/a^2)*log(5*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) - 5*sqrt(2)*(3*sqrt(1/5)
*a^7*sqrt(a^(-8))*cos(x) + a^3*cos(x))*sqrt(-(a^2*sqrt((2*sqrt(1/5)*a^4*sq
rt(a^(-8)) - 1)/a^4) + 1)/a^2)*sqrt((2*sqrt(1/5)*a^4*sqrt(a^(-8)) - 1)/a^4
) - 5*(3*sqrt(1/5)*a^6*sqrt(a^(-8))*sin(x) + a^2*sin(x))*sqrt((2*sqrt(1/5)
*a^4*sqrt(a^(-8)) - 1)/a^4) - sin(x) - 4) - sqrt(2)*(a*cos(x) - a*sin(x) +
 a)*sqrt((a^2*sqrt((2*sqrt(1/5)*a^4*sqrt(a^(-8)) - 1)/a^4) - 1)/a^2)*log(5
*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) - 5*sqrt(2)*(3*sqrt(1/5)*a^7*sqrt(a^...
```

Sympy [F]

$$\int \frac{1}{a - a\sin^5(x)}\,dx = -\frac{\int \frac{1}{\sin^5(x)-1}\,dx}{a}$$

input
```
integrate(1/(a-a*sin(x)**5),x)
```

output
```
-Integral(1/(sin(x)**5 - 1), x)/a
```

Maxima [F]

$$\int \frac{1}{a-a\sin^5(x)}\,dx=\int -\frac{1}{a\sin{(x)}^5-a}\,dx$$

input
```
integrate(1/(a-a*sin(x)^5),x, algorithm="maxima")
```

output
```
1/5*(5*(a*cos(x)^2 + a*sin(x)^2 - 2*a*sin(x) + a)*integrate(-2/5*((4*cos(6
*x) - 40*cos(4*x) + 4*cos(2*x) + sin(7*x) - 15*sin(5*x) + 15*sin(3*x) - si
n(x))*cos(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 8*sin(6*x) - 55
*sin(4*x) + 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos
(2*x) + 44*sin(5*x) - 44*sin(3*x) + 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2
 + 2*(210*cos(3*x) - 22*cos(x) + 505*sin(4*x) - 88*sin(2*x))*cos(5*x) - 21
0*cos(5*x)^2 + 10*(44*cos(2*x) + 101*sin(3*x) - 11*sin(x) - 4)*cos(4*x) -
1200*cos(4*x)^2 + 44*(cos(x) + 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 + 4*(
4*sin(x) + 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 - (cos(7*x) - 15*cos(5
*x) + 15*cos(3*x) - cos(x) - 4*sin(6*x) + 40*sin(4*x) - 4*sin(2*x))*sin(8*
x) - (16*cos(6*x) - 110*cos(4*x) + 16*cos(2*x) - 44*sin(5*x) + 44*sin(3*x)
 - 4*sin(x) - 1)*sin(7*x) - 2*sin(7*x)^2 - 8*(22*cos(5*x) - 22*cos(3*x) +
2*cos(x) - 55*sin(4*x) + 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 - (1010*cos(
4*x) - 176*cos(2*x) - 420*sin(3*x) + 44*sin(x) + 15)*sin(5*x) - 210*sin(5*
x)^2 - 10*(101*cos(3*x) - 11*cos(x) - 44*sin(2*x))*sin(4*x) - 1200*sin(4*x
)^2 - (176*cos(2*x) - 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 - 16*cos(x
)*sin(2*x) - 32*sin(2*x)^2 - 2*sin(x)^2 - sin(x))/(a*cos(8*x)^2 + 4*a*cos(
7*x)^2 + 64*a*cos(6*x)^2 + 196*a*cos(5*x)^2 + 900*a*cos(4*x)^2 + 196*a*cos
(3*x)^2 + 64*a*cos(2*x)^2 + 4*a*cos(x)^2 + a*sin(8*x)^2 + 4*a*sin(7*x)^2 +
 64*a*sin(6*x)^2 + 196*a*sin(5*x)^2 + 900*a*sin(4*x)^2 + 196*a*sin(3*x)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2036 vs. $2(148) = 296$.

Time = 2.40 (sec) , antiderivative size = 2036, normalized size of antiderivative = 9.60

$$\int \frac{1}{a-a\sin^5(x)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^5),x, algorithm="giac")
```

output
```
-1/50*(2*sqrt(1/5)*sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)*
(arctan(1/2) - arctan(-1/2*(24209596193492233425*sqrt(5)*(sqrt(5) + 5) - 4
9645000851686087842*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) +
 50) - 25) + 24822500425843043921 0*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sq
rt(5) + 50) - 25) - 26032980235517655592 25*sqrt(5) - 992900017033721756840
0*tan(1/2*x) - 26032980235517655592 25)/(14744762480652671482 00*sqrt(5)*(sq
rt(5) + 5) + 14893500255505826352 6*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sq
rt(10*sqrt(5) + 50) - 25) + 12411250212921521960 5*sqrt(5)*sqrt(10*sqrt(5)
+ 50) + 74467501277529131763 0*sqrt(5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) -
 24822500425843043921 0*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50)
- 25) - 73723812403263357410 00*sqrt(5) - 62056251064607609802 5*sqrt(10*sqr
t(5) + 50) - 12411250212921521960 50*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 7
37238124032633574100 0)))/(2*sqrt(1/10)*sqrt(sqrt(5) + 5) - 1) - 2*sqrt(1/5
)*sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)*arctan(-1/2*(5878
49076675773567575*sqrt(5)*(sqrt(5) + 5) + 49645000851686087842*sqrt(5)*sqr
t(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 248225004258430439
210*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 54214954259
63172229975*sqrt(5) - 992900017033721756840 0*tan(1/2*x) - 5421495425963172
229975)/(17018668022061712105 50*sqrt(5)*(sqrt(5) + 5) + 148935002555058263
526*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - ...
```

Mupad [B] (verification not implemented)

Time = 37.81 (sec) , antiderivative size = 4652, normalized size of antiderivative = 21.94

$$\int \frac{1}{a - a\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - a*sin(x)^5),x)
```

output
```
2/(5*(a - a*tan(x/2))) - 2*atanh((241664000000000000*a^7*(- (- (2*5^(1/2))
/5 - 1)^(1/2)/(50*a^2) - 1/(50*a^2))^(1/2))/(4096000000000000*5^(1/2)*a^6
- 73728000000000000*a^6*tan(x/2) + 20480000000000000*a^6*(- (2*5^(1/2))/5
- 1)^(1/2) + 90112000000000000*a^6 - 23265280000000000*5^(1/2)*a^6*tan(x/2)
 + 63897600000000000*a^6*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2) + 2211840000
0000000*5^(1/2)*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 21299200000000000*5^(1/2
)*a^6*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)) + (495616000000000000*a^7*tan(
x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/(50*a^2) - 1/(50*a^2))^(1/2))/(4096000
000000000*5^(1/2)*a^6 - 73728000000000000*a^6*tan(x/2) + 20480000000000000
*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 90112000000000000*a^6 - 232652800000000000
0*5^(1/2)*a^6*tan(x/2) + 63897600000000000*a^6*tan(x/2)*(- (2*5^(1/2))/5 -
 1)^(1/2) + 22118400000000000*5^(1/2)*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 21
299200000000000*5^(1/2)*a^6*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)) + (79462
400000000000*5^(1/2)*a^7*(- (- (2*5^(1/2))/5 - 1)^(1/2)/(50*a^2) - 1/(50*a
^2))^(1/2))/(4096000000000000*5^(1/2)*a^6 - 73728000000000000*a^6*tan(x/2)
 + 20480000000000000*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 90112000000000000*a^
6 - 23265280000000000*5^(1/2)*a^6*tan(x/2) + 63897600000000000*a^6*tan(x/2
)*(- (2*5^(1/2))/5 - 1)^(1/2) + 22118400000000000*5^(1/2)*a^6*(- (2*5^(1/2
))/5 - 1)^(1/2) + 21299200000000000*5^(1/2)*a^6*tan(x/2)*(- (2*5^(1/2))/5
- 1)^(1/2)) - (135168000000000000*a^7*(- (- (2*5^(1/2))/5 - 1)^(1/2)/(5...
```

Reduce [F]

$$\int \frac{1}{a-a\sin^5(x)}\,dx=-\frac{\int \frac{1}{\sin(x)^5-1}dx}{a}$$

input
```
int(1/(a-a*sin(x)^5),x)
```

output
```
( - int(1/(sin(x)**5 - 1),x))/a
```

3.64 $\int \frac{1}{a+a\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{x}{\sqrt{2}a}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}a}$$

output
```
1/2*x*2^(1/2)/a+1/2*arctan(cos(x)*sin(x)/(1+2^(1/2)+sin(x)^2))*2^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}a}$$

input
```
Integrate[(a + a*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(Sqrt[2]*a)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^2(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin(x)^2+a}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{2a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}a}$$

input
```
Int[(a + a*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(Sqrt[2]*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\arctan\left(\tan(x)\sqrt{2}\right)\sqrt{2}}{2a}$	16
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)}{4a}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)}{4a}$	46

input
```
int(1/(a+a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(tan(x)*2^(1/2))*2^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+a\sin^2(x)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)}{4\,a}$$

input
```
integrate(1/(a+a*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))/
a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(37) = 74$.

Time = 2.85 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.97

$$\int \frac{1}{a + a\sin^2(x)}\,dx = \frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a + 39202a} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a + 39202a} + \frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a + 39202a} + \frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{27720\sqrt{2}a + 39202a}$$

input
```
integrate(1/(a+a*sin(x)**2),x)
```

output
```
47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi
*floor((x/2 - pi/2)/pi))/(27720*sqrt(2)*a + 39202*a) + 66922*sqrt(3 - 2*sq
rt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(2
7720*sqrt(2)*a + 39202*a) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2
)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(27720*sqrt(2)*a + 392
02*a) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi
*floor((x/2 - pi/2)/pi))/(27720*sqrt(2)*a + 39202*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{a + a\sin^2(x)}\,dx = \frac{\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)}{2\,a}$$

input
```
integrate(1/(a+a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*tan(x))/a
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)}{2a}$$

input
```
integrate(1/(a+a*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2)))/a
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{2a}$$

input
```
int(1/(a + a*sin(x)^2),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*tan(x)))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{1}{a+a\sin^2(x)}\,dx = \frac{\sqrt{2}\left(2atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2}+1}\right)-\log\left(-\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)+i\right)i+\log\left(\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)-i\right)i\right)}{4a}$$

input
```
int(1/(a+a*sin(x)^2),x)
```

output
```
(sqrt(2)*(2*atan(tan(x/2)/(sqrt(2) + 1)) - log( - sqrt(2)*i + tan(x/2) + i
)*i + log(sqrt(2)*i + tan(x/2) - i)*i))/(4*a)
```

3.65 $\int \frac{1}{a+a\sin^4(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 290

$$\int \frac{1}{a+a\sin^4(x)}\,dx = \frac{\sqrt{1+\sqrt{2}}x}{2a} + \frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)+\left(2-\sqrt{2}\right)\cos(x)\sin(x)}{2+\sqrt{1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cos^2(x)-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)}\right)}{4a} - \frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)-\left(2-\sqrt{2}\right)\cos(x)\sin(x)}{2+\sqrt{1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cos^2(x)+2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)}\right)}{4a} + \frac{\sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\tan(x)}{1+\sqrt{2}\tan^2(x)}\right)}{4a}$$

output
```
1/2*(1+2^(1/2))^(1/2)*x/a+1/4*(1+2^(1/2))^(1/2)*arctan(((2^(1/2)-1)^(1/2)-
2*(2^(1/2)-1)^(1/2)*cos(x)^2+(2-2^(1/2))*cos(x)*sin(x))/(2+(1+2^(1/2))^(1/
2)-(2-2^(1/2))*cos(x)^2-2*(2^(1/2)-1)^(1/2)*cos(x)*sin(x)))/a-1/4*(1+2^(1/
2))^(1/2)*arctan(((2^(1/2)-1)^(1/2)-2*(2^(1/2)-1)^(1/2)*cos(x)^2-(2-2^(1/2
))*cos(x)*sin(x))/(2+(1+2^(1/2))^(1/2)-(2-2^(1/2))*cos(x)^2+2*(2^(1/2)-1)^
(1/2)*cos(x)*sin(x)))/a+1/4*(2^(1/2)-1)^(1/2)*arctanh((-2+2*2^(1/2))^(1/2)
*tan(x)/(1+2^(1/2)*tan(x)^2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.17

$$\int \frac{1}{a+a\sin^4(x)}\,dx=\frac{\frac{2\arctan\left(\sqrt{1-i}\tan(x)\right)}{\sqrt{1-i}}+\frac{2\arctan\left(\sqrt{1+i}\tan(x)\right)}{\sqrt{1+i}}}{4a}$$

input
```
Integrate[(a + a*Sin[x]^4)^(-1),x]
```

output
```
((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]
])/Sqrt[1 + I])/(4*a)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.800$, Rules used = {3042, 3688, 1483, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^4(x)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a\sin(x)^4+a}dx$$

$$\Big\downarrow \ 3688$$

$$\int \frac{\tan^2(x)+1}{2a\tan^4(x)+2a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \ 1483$$

$$\frac{\int \frac{2\sqrt{-1+\sqrt{2}}-\left(2-\sqrt{2}\right)\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}+\frac{\int \frac{\left(2-\sqrt{2}\right)\tan(x)+2\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$

$$\Big\downarrow \; 1142$$

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)-\frac{1}{4}\left(2-\sqrt{2}\right)\int -\frac{2\left(\sqrt{-1+\sqrt{2}}-2\tan(x)\right)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{2\left(2\tan(x)+\sqrt{-1+\sqrt{2}}\right)}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\tan(x)+\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\tan(x)-2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\Big(4\tan(x)-}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$
$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\tan(x)+\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\tan(x)+2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\Big(4\tan(x)+}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{\arctan\left(\frac{4\tan(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}a}+\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\tan(x)+\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{\arctan\left(\frac{4\tan(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{4\tan(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}-\frac{1}{4}(2-\sqrt{2})\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\tan(x)+\sqrt{2}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}+\frac{\frac{\arctan\left(\frac{4\tan(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}+\frac{1}{4}(2-\sqrt{2})\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\tan(x)+1\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}a}$$

input
```
Int[(a + a*Sin[x]^4)^(-1),x]
```

output
```
(ArcTan[(-2*Sqrt[-1 + Sqrt[2]] + 4*Tan[x])/(2*Sqrt[1 + Sqrt[2]])]/Sqrt[2]
- ((2 - Sqrt[2])*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Tan[x] + 2*Tan[x]^2])/
4)/(2*Sqrt[2*(-1 + Sqrt[2])]*a) + (ArcTan[(2*Sqrt[-1 + Sqrt[2]] + 4*Tan[x]
)/(2*Sqrt[1 + Sqrt[2]])]/Sqrt[2] + ((2 - Sqrt[2])*Log[1 + Sqrt[2*(-1 + Sqr
t[2])]*Tan[x] + Sqrt[2]*Tan[x]^2])/4)/(2*Sqrt[2*(-1 + Sqrt[2])]*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\sqrt{-2-2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2-2i}-\sqrt{-2-2i}-1-2i\right)}{8a}-\frac{\sqrt{-2-2i}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{-2-2i}+\sqrt{-2-2i}-1-2i\right)}{8a}+\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2+2\cdots\right.}{8a}$
default	$\frac{\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)+2\tan(x)^2+\sqrt{2}\right)}{4}+\frac{\left(-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{4}+2\right)\arctan\left(\frac{-\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{\cdots}}}{\cdots}\right.}{\cdots}}{a}$

input
```
int(1/(a+a*sin(x)^4),x,method=_RETURNVERBOSE)
```

output

```
1/8*(-2-2*I)^(1/2)/a*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)-(-2-2*I)^(1/2)-1-2*I)-
1/8*(-2-2*I)^(1/2)/a*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+(-2-2*I)^(1/2)-1-2*I)+
1/8*(-2+2*I)^(1/2)/a*ln(exp(2*I*x)+I*(-2+2*I)^(1/2)+(-2+2*I)^(1/2)-1+2*I)-
1/8*(-2+2*I)^(1/2)/a*ln(exp(2*I*x)-I*(-2+2*I)^(1/2)-(-2+2*I)^(1/2)-1+2*I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+a\sin^4(x)}\,dx = -\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}\log\left(\frac{1}{2}\sqrt{\frac{1}{2}}\left(a^3\sqrt{-\frac{1}{a^4}}\cos(x)\sin(x)+a\cos(x)\sin(x)\right)\sqrt{-\frac{a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}+\frac{1}{4}\cos(x)^2+\frac{1}{4}\left(2a^2\cos(x)^2-a^2\right)\sqrt{-\frac{1}{a^4}}-\frac{1}{4}\right)$$
$$+\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}\log\left(-\frac{1}{2}\sqrt{\frac{1}{2}}\left(a^3\sqrt{-\frac{1}{a^4}}\cos(x)\sin(x)+a\cos(x)\sin(x)\right)\sqrt{-\frac{a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}+\frac{1}{4}\cos(x)^2+\frac{1}{4}\left(2a^2\cos(x)^2-a^2\right)\sqrt{-\frac{1}{a^4}}-\frac{1}{4}\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{\frac{a^2\sqrt{-\frac{1}{a^4}}-1}{a^2}}\log\left(\frac{1}{2}\sqrt{\frac{1}{2}}\left(a^3\sqrt{-\frac{1}{a^4}}\cos(x)\sin(x)-a\cos(x)\sin(x)\right)\sqrt{\frac{a^2\sqrt{-\frac{1}{a^4}}-1}{a^2}}-\frac{1}{4}\cos(x)^2+\frac{1}{4}\left(2a^2\cos(x)^2-a^2\right)\sqrt{-\frac{1}{a^4}}+\frac{1}{4}\right)$$
$$+\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{\frac{a^2\sqrt{-\frac{1}{a^4}}-1}{a^2}}\log\left(-\frac{1}{2}\sqrt{\frac{1}{2}}\left(a^3\sqrt{-\frac{1}{a^4}}\cos(x)\sin(x)-a\cos(x)\sin(x)\right)\sqrt{\frac{a^2\sqrt{-\frac{1}{a^4}}-1}{a^2}}-\frac{1}{4}\cos(x)^2+\frac{1}{4}\left(2a^2\cos(x)^2-a^2\right)\sqrt{-\frac{1}{a^4}}+\frac{1}{4}\right)$$

input
```
integrate(1/(a+a*sin(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(1/2)*sqrt(-(a^2*sqrt(-1/a^4) + 1)/a^2)*log(1/2*sqrt(1/2)*(a^3*sq
rt(-1/a^4)*cos(x)*sin(x) + a*cos(x)*sin(x))*sqrt(-(a^2*sqrt(-1/a^4) + 1)/a
^2) + 1/4*cos(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) - 1/4) + 1/8*
sqrt(1/2)*sqrt(-(a^2*sqrt(-1/a^4) + 1)/a^2)*log(-1/2*sqrt(1/2)*(a^3*sqrt(-
1/a^4)*cos(x)*sin(x) + a*cos(x)*sin(x))*sqrt(-(a^2*sqrt(-1/a^4) + 1)/a^2)
+ 1/4*cos(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) - 1/4) - 1/8*sqrt
(1/2)*sqrt((a^2*sqrt(-1/a^4) - 1)/a^2)*log(1/2*sqrt(1/2)*(a^3*sqrt(-1/a^4)
*cos(x)*sin(x) - a*cos(x)*sin(x))*sqrt((a^2*sqrt(-1/a^4) - 1)/a^2) - 1/4*c
os(x)^2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) + 1/4) + 1/8*sqrt(1/2)*s
qrt((a^2*sqrt(-1/a^4) - 1)/a^2)*log(-1/2*sqrt(1/2)*(a^3*sqrt(-1/a^4)*cos(x
)*sin(x) - a*cos(x)*sin(x))*sqrt((a^2*sqrt(-1/a^4) - 1)/a^2) - 1/4*cos(x)^
2 + 1/4*(2*a^2*cos(x)^2 - a^2)*sqrt(-1/a^4) + 1/4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + a\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sin(x)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a + a\sin^4(x)}\,dx = \int \frac{1}{a\sin{(x)}^4 + a}\,dx$$

input
```
integrate(1/(a+a*sin(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(a*sin(x)^4 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+a\sin^4(x)}\,dx$$
$$= \frac{\left(a^2\sqrt{2\sqrt{2}+2}+a\sqrt{2\sqrt{2}-2}|a|\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)}{8a^3}$$
$$+\frac{\left(a^2\sqrt{2\sqrt{2}+2}+a\sqrt{2\sqrt{2}-2}|a|\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)}{8a^3}$$
$$-\frac{\left(a^2\sqrt{2\sqrt{2}-2}-a\sqrt{2\sqrt{2}+2}|a|\right)\log\left(\tan(x)^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)}{16a^3}$$
$$+\frac{\left(a^2\sqrt{2\sqrt{2}-2}-a\sqrt{2\sqrt{2}+2}|a|\right)\log\left(\tan(x)^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)}{16a^3}$$

input
```
integrate(1/(a+a*sin(x)^4),x, algorithm="giac")
```

output
```
1/8*(a^2*sqrt(2*sqrt(2) + 2) + a*sqrt(2*sqrt(2) - 2)*abs(a))*(pi*floor(x/p
i + 1/2) + arctan(2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x)
)/sqrt(sqrt(2) + 2)))/a^3 + 1/8*(a^2*sqrt(2*sqrt(2) + 2) + a*sqrt(2*sqrt(2
) - 2)*abs(a))*(pi*floor(x/pi + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*
sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))/a^3 - 1/16*(a^2*sqrt(2*
sqrt(2) - 2) - a*sqrt(2*sqrt(2) + 2)*abs(a))*log(tan(x)^2 + (1/2)^(1/4)*sq
rt(-sqrt(2) + 2)*tan(x) + sqrt(1/2))/a^3 + 1/16*(a^2*sqrt(2*sqrt(2) - 2) -
 a*sqrt(2*sqrt(2) + 2)*abs(a))*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt(2) +
2)*tan(x) + sqrt(1/2))/a^3
```

Mupad [B] (verification not implemented)

Time = 37.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.20

$$\int \frac{1}{a+a\sin^4(x)}\,dx = \operatorname{atan}\left(a\tan(x)\sqrt{\frac{-\frac{1}{32}-\frac{1}{32}\mathrm{i}}{a^2}}\,(4+4\mathrm{i})\right)\sqrt{\frac{-\frac{1}{32}-\frac{1}{32}\mathrm{i}}{a^2}}\,2\mathrm{i} - \operatorname{atan}\left(a\tan(x)\sqrt{\frac{-\frac{1}{32}+\frac{1}{32}\mathrm{i}}{a^2}}\,(4-4\mathrm{i})\right)\sqrt{\frac{-\frac{1}{32}+\frac{1}{32}\mathrm{i}}{a^2}}\,2\mathrm{i}$$

input
```
int(1/(a + a*sin(x)^4),x)
```

output
```
atan(a*tan(x)*((- 1/32 - 1i/32)/a^2)^(1/2)*(4 + 4i))*((- 1/32 - 1i/32)/a^2
)^(1/2)*2i - atan(a*tan(x)*((- 1/32 + 1i/32)/a^2)^(1/2)*(4 - 4i))*((- 1/32
 + 1i/32)/a^2)^(1/2)*2i
```

Reduce [F]

$$\int \frac{1}{a+a\sin^4(x)}\,dx = \frac{\int \frac{1}{\sin(x)^4+1}dx}{a}$$

input
```
int(1/(a+a*sin(x)^4),x)
```

output
```
int(1/(sin(x)**4 + 1),x)/a
```

3.66 $\int \frac{1}{a+a\sin^6(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 142

$$\int \frac{1}{a+a\sin^6(x)}\,dx=\frac{x}{3\sqrt{2}a}+\frac{x}{\sqrt{3}a}+\frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{2\sqrt{3}a}-\frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}a}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{3\sqrt{2}a}+\frac{\operatorname{arctanh}(\cos(x)\sin(x))}{6a}$$

output
```
1/6*x*2^(1/2)/a+1/3*x*3^(1/2)/a+1/6*arctan((1-2*cos(x)^2)/(2+3^(1/2)-2*cos
(x)*sin(x)))*3^(1/2)/a-1/6*arctan((1-2*cos(x)^2)/(2+3^(1/2)+2*cos(x)*sin(x
)))*3^(1/2)/a+1/6*arctan(cos(x)*sin(x)/(1+2^(1/2)+sin(x)^2))*2^(1/2)/a+1/6
*arctanh(cos(x)*sin(x))/a
```

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+a\sin^6(x)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+2\sqrt{3}\arctan\left(\frac{1+2\tan(x)}{\sqrt{3}}\right)-\log(2-\sin(2x))+\log}{12a}$$

input
```
Integrate[(a + a*Sin[x]^6)^(-1),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Sqrt[2]*Tan[
x]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] - Log[2 - Sin[2*x]] + Log[2
 + Sin[2*x]])/(12*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^6(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin(x)^6+a}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{\sin^2(x)+1}dx}{3a}+\frac{\int \frac{1}{1-\sqrt[3]{-1}\sin^2(x)}dx}{3a}+\frac{\int \frac{1}{(-1)^{2/3}\sin^2(x)+1}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin(x)^2+1}dx}{3a}+\frac{\int \frac{1}{1-\sqrt[3]{-1}\sin(x)^2}dx}{3a}+\frac{\int \frac{1}{(-1)^{2/3}\sin(x)^2+1}dx}{3a}$$

$$\Big\downarrow \; 3660$$

$$\frac{\int \frac{1}{2\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1-\sqrt[3]{-1}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1+(-1)^{2/3}\right)\tan^2(x)+1}d\tan(x)}{3a}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{3\sqrt{2}a}+\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}a}$$

input
```
Int[(a + a*Sin[x]^6)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(3*Sqrt[2]*a) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]
/(3*Sqrt[1 - (-1)^(1/3)]*a) + ArcTan[Sqrt[1 + (-1)^(2/3)]*Tan[x]]/(3*Sqrt[
1 + (-1)^(2/3)]*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

method	result
default	$\frac{\frac{\ln\left(\tan(x)^2+\tan(x)+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{6}-\frac{\ln\left(\tan(x)^2-\tan(x)+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{6}}{a}$
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{12a}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{12a}-\frac{\ln\left(e^{2ix}-i\sqrt{3}-2i\right)}{12a}+\frac{i\ln\left(e^{2ix}-i\sqrt{3}-2i\right)\sqrt{3}}{12a}-\frac{\ln\left(e^{2ix}+i\sqrt{3}-2i\right)}{12a}-$

input
```
int(1/(a+a*sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1)*3^(1/2
))+1/6*2^(1/2)*arctan(tan(x)*2^(1/2))-1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/
2)*arctan(1/3*(2*tan(x)-1)*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{1}{a+a\sin^6(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)+2\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)-2\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)+}{24a}$$

input
```
integrate(1/(a+a*sin(x)^6),x, algorithm="fricas")
```

output
```
1/24*(2*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2
 - 1)) + 2*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x
)^2 - 1)) - 2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*
sin(x))) + log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) - log(-cos(x)^4
 + cos(x)^2 - 2*cos(x)*sin(x) + 1))/a
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+a\sin^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sin(x)**6),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

$$\int \frac{1}{a+a\sin^6(x)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right)}{6a} + \frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right)}{6a} + \frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)}{6a} + \frac{\log\left(\tan(x)^2+\tan(x)+1\right)}{12a} - \frac{\log\left(\tan(x)^2-\tan(x)+1\right)}{12a}$$

input
```
integrate(1/(a+a*sin(x)^6),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1))/a + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*tan(x) - 1))/a + 1/6*sqrt(2)*arctan(sqrt(2)*tan(x))/a + 1/12*lo
g(tan(x)^2 + tan(x) + 1)/a - 1/12*log(tan(x)^2 - tan(x) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.41

$$\int \frac{1}{a+a\sin^6(x)}\,dx = \frac{\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)+\cos(2x)-2\sin(2x)+1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)-\sin(2x)+2}\right)\right)}{6a} + \frac{\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right)}{6a} + \frac{\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)}{6a} + \frac{\log\left(\tan(x)^2+\tan(x)+1\right)}{12a} - \frac{\log\left(\tan(x)^2-\tan(x)+1\right)}{12a}$$

input
```
integrate(1/(a+a*sin(x)^6),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2)))/a + 1/6*sqrt(3)*(
x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*
x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2)))/a + 1/6*sqrt(2)*(x + arctan(-(
sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) +
2)))/a + 1/12*log(tan(x)^2 + tan(x) + 1)/a - 1/12*log(tan(x)^2 - tan(x) +
1)/a
```

Mupad [B] (verification not implemented)

Time = 38.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{1}{a+a\sin^6(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{6a} + \frac{\mathrm{atan}\left(-\frac{\sqrt{3}\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{6a} - \frac{\mathrm{atan}\left(\frac{\sqrt{3}\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{6a}$$

input
```
int(1/(a + a*sin(x)^6),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*tan(x)))/(6*a) + (atan((tan(x)*1i)/2 - (3^(1/2)*tan(
x))/2)*(3^(1/2)*1i - 1)*1i)/(6*a) - (atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))
/2)*(3^(1/2)*1i + 1)*1i)/(6*a)
```

Reduce [F]

$$\int \frac{1}{a + a\sin^6(x)}\,dx = \frac{\int \frac{1}{\sin(x)^6+1}dx}{a}$$

input
```
int(1/(a+a*sin(x)^6),x)
```

output
```
int(1/(sin(x)**6 + 1),x)/a
```

3.67 $\int \frac{1}{a+a\sin^8(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 141

$$\int \frac{1}{a+a\sin^8(x)}\,dx = \frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}a}+\frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{-1+\sqrt[4]{-1}}a}+\frac{\operatorname{arctanh}\left(\sqrt{-1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{-1-(-1)^{3/4}}a}$$

output
```
1/4*arctan((1+(-1)^(1/4))^(1/2)*tan(x))/(1+(-1)^(1/4))^(1/2)/a+1/4*arctan(
(1-(-1)^(3/4))^(1/2)*tan(x))/(1-(-1)^(3/4))^(1/2)/a+1/4*arctanh((-1+(-1)^(
1/4))^(1/2)*tan(x))/(-1+(-1)^(1/4))^(1/2)/a+1/4*arctanh((-1-(-1)^(3/4))^(1
/2)*tan(x))/(-1-(-1)^(3/4))^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+a\sin^8(x)}\,dx = \frac{8\text{RootSum}\left[1-8\#1+28\#1^2-56\#1^3+326\#1^4-56\#1^5+28\#1^6-8\#1^7+\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)}{-1+7\#1-21\#1^2+\cdots}\right.}{a}$$

input
```
Integrate[(a + a*Sin[x]^8)^(-1),x]
```

output
```
(8*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8
*#1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Co
s[2*x]*#1 + #1^2]*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^
5 - 7*#1^6 + #1^7) & ])/a
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^8(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin(x)^8+a}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\sqrt[4]{-1}\sin^2(x)}dx}{4a}+\frac{\int \frac{1}{\sqrt[4]{-1}\sin^2(x)+1}dx}{4a}+\frac{\int \frac{1}{1-(-1)^{3/4}\sin^2(x)}dx}{4a}+\frac{\int \frac{1}{(-1)^{3/4}\sin^2(x)+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\sqrt[4]{-1}\sin(x)^2}dx}{4a}+\frac{\int \frac{1}{\sqrt[4]{-1}\sin(x)^2+1}dx}{4a}+\frac{\int \frac{1}{1-(-1)^{3/4}\sin(x)^2}dx}{4a}+\frac{\int \frac{1}{(-1)^{3/4}\sin(x)^2+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{\left(1-\sqrt[4]{-1}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(1+\sqrt[4]{-1}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(1-(-1)^{3/4}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(1+(-1)^{3/4}\right)\tan^2(x)+1}d\tan(x)}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}a}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}a}+\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1+(-1)^{3/4}}a}$$

input
```
Int[(a + a*Sin[x]^8)^(-1),x]
```

output
```
ArcTan[Sqrt[1 - (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(1/4)]*a) + ArcTan[Sq
rt[1 + (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 + (-1)^(1/4)]*a) + ArcTan[Sqrt[1 - (-
1)^(3/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(3/4)]*a) + ArcTan[Sqrt[1 + (-1)^(3/4)]
*Tan[x]]/(4*Sqrt[1 + (-1)^(3/4)]*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(2\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+1\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{2\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(8192a^4\_Z^4+(-128ia^2+128a^2)\_Z^2+1-i\right)}\_R\ln\left(\mathrm{e}^{2ix}+(-1024ia^3-1024a^3)\_R^3+(-128ia\right.\right.$

input
```
int(1/(a+a*sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(2*_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_
R=RootOf(2*_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1977 vs. 2(101) = 202.

Time = 0.39 (sec) , antiderivative size = 1977, normalized size of antiderivative = 14.02

$$\int \frac{1}{a+a\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sin(x)^8),x, algorithm="fricas")
```

output
```
-1/16*sqrt(1/2)*sqrt(-(a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) +
 1)/a^2)*log(1/2*sqrt(1/2)*(sqrt(1/2)*a^5*sqrt(a^(-8))*cos(x)*sin(x) - a*c
os(x)*sin(x) + (sqrt(1/2)*a^7*sqrt(a^(-8))*cos(x)*sin(x) - a^3*cos(x)*sin(
x))*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4))*sqrt(-(a^2*sqrt(-(4*sqr
t(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) + 1)/a^2) + 1/4*sqrt(1/2)*(2*a^4*cos(x)^
2 - a^4)*sqrt(a^(-8)) - 1/4*cos(x)^2 - 1/4*(2*a^2*cos(x)^2 - a^2 - sqrt(1/
2)*(2*a^6*cos(x)^2 - a^6)*sqrt(a^(-8)))*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)
) + 3)/a^4) + 1/4) + 1/16*sqrt(1/2)*sqrt(-(a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt
(a^(-8)) + 3)/a^4) + 1)/a^2)*log(-1/2*sqrt(1/2)*(sqrt(1/2)*a^5*sqrt(a^(-8)
)*cos(x)*sin(x) - a*cos(x)*sin(x) + (sqrt(1/2)*a^7*sqrt(a^(-8))*cos(x)*sin
(x) - a^3*cos(x)*sin(x))*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4))*sq
rt(-(a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) + 1)/a^2) + 1/4*sqr
t(1/2)*(2*a^4*cos(x)^2 - a^4)*sqrt(a^(-8)) - 1/4*cos(x)^2 - 1/4*(2*a^2*cos
(x)^2 - a^2 - sqrt(1/2)*(2*a^6*cos(x)^2 - a^6)*sqrt(a^(-8)))*sqrt(-(4*sqrt
(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) + 1/4) + 1/16*sqrt(1/2)*sqrt((a^2*sqrt(-(
4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) - 1)/a^2)*log(1/2*sqrt(1/2)*(sqrt(1
/2)*a^5*sqrt(a^(-8))*cos(x)*sin(x) - a*cos(x)*sin(x) - (sqrt(1/2)*a^7*sqrt
(a^(-8))*cos(x)*sin(x) - a^3*cos(x)*sin(x))*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^
(-8)) + 3)/a^4))*sqrt((a^2*sqrt(-(4*sqrt(1/2)*a^4*sqrt(a^(-8)) + 3)/a^4) -
 1)/a^2) - 1/4*sqrt(1/2)*(2*a^4*cos(x)^2 - a^4)*sqrt(a^(-8)) + 1/4*cos(...
```

Sympy [F]

$$\int \frac{1}{a + a\sin^8(x)}\, dx = \frac{\int \frac{1}{\sin^8(x)+1}\, dx}{a}$$

input
```
integrate(1/(a+a*sin(x)**8),x)
```

output
```
Integral(1/(sin(x)**8 + 1), x)/a
```

Maxima [F]

$$\int \frac{1}{a + a\sin^8(x)}\,dx = \int \frac{1}{a\sin{(x)}^8 + a}\,dx$$

input
```
integrate(1/(a+a*sin(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(a*sin(x)^8 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a + a\sin^8(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sin(x)^8),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 37.65 (sec) , antiderivative size = 1169, normalized size of antiderivative = 8.29

$$\int \frac{1}{a + a\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + a*sin(x)^8),x)
```

output
```
atan((tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1/2)*32i
)/((16*(- 2*2^(1/2) - 3)^(1/2))/a - (12*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/a
) - (2^(1/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1
/2)*32i)/((16*(- 2*2^(1/2) - 3)^(1/2))/a - (12*2^(1/2)*(- 2*2^(1/2) - 3)^(
1/2))/a) + (tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1/
2)*(- 2*2^(1/2) - 3)^(1/2)*224i)/((16*(- 2*2^(1/2) - 3)^(1/2))/a - (12*2^(
1/2)*(- 2*2^(1/2) - 3)^(1/2))/a) - (2^(1/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1
/2)/(128*a^2) - 1/(128*a^2))^(1/2)*(- 2*2^(1/2) - 3)^(1/2)*160i)/((16*(- 2
*2^(1/2) - 3)^(1/2))/a - (12*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/a))*(-((- 2*
2^(1/2) - 3)^(1/2) + 1)/(128*a^2))^(1/2)*2i - atan((tan(x)*((- 2*2^(1/2) -
 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1/2)*32i)/((16*(- 2*2^(1/2) - 3)^(1/2)
)/a - (12*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/a) - (2^(1/2)*tan(x)*((- 2*2^(1
/2) - 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1/2)*32i)/((16*(- 2*2^(1/2) - 3)^
(1/2))/a - (12*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/a) - (tan(x)*((- 2*2^(1/2)
 - 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1/2)*(- 2*2^(1/2) - 3)^(1/2)*224i)/(
(16*(- 2*2^(1/2) - 3)^(1/2))/a - (12*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/a) +
 (2^(1/2)*tan(x)*((- 2*2^(1/2) - 3)^(1/2)/(128*a^2) - 1/(128*a^2))^(1/2)*(
- 2*2^(1/2) - 3)^(1/2)*160i)/((16*(- 2*2^(1/2) - 3)^(1/2))/a - (12*2^(1/2)
*(- 2*2^(1/2) - 3)^(1/2))/a))*(((- 2*2^(1/2) - 3)^(1/2) - 1)/(128*a^2))^(1
/2)*2i + atan((tan(x)*(- (2*2^(1/2) - 3)^(1/2)/(128*a^2) - 1/(128*a^2))...
```

Reduce [F]

$$\int \frac{1}{a+a\sin^8(x)}\,dx=\frac{\int \frac{1}{\sin(x)^8+1}dx}{a}$$

input
```
int(1/(a+a*sin(x)^8),x)
```

output
```
int(1/(sin(x)**8 + 1),x)/a
```

3.68 $\int \frac{1}{a+a\sin(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{a+a\sin(x)}\,dx=-\frac{\cos(x)}{a+a\sin(x)}$$

output
```
-cos(x)/(a+a*sin(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{a+a\sin(x)}\,dx=\frac{2\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{a+a\sin(x)}$$

input
```
Integrate[(a + a*Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2]*(Cos[x/2] + Sin[x/2]))/(a + a*Sin[x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin(x)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a\sin(x)+a}dx$$
$$\Big\downarrow \text{ 3127}$$
$$-\frac{\cos(x)}{a\sin(x)+a}$$

input
```
Int[(a + a*Sin[x])^(-1),x]
```

output
```
-(Cos[x]/(a + a*Sin[x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$	14
norman	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$	14
parallelrisch	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$	14
risch	$-\frac{2}{\left(\mathrm{e}^{ix}+i\right)a}$	16

input
```
int(1/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(tan(1/2*x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{1}{a+a\sin(x)}\,dx = -\frac{\cos\left(x\right)-\sin\left(x\right)+1}{a\cos\left(x\right)+a\sin\left(x\right)+a}$$

input
```
integrate(1/(a+a*sin(x)),x, algorithm="fricas")
```

output
```
-(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+a\sin(x)}\,dx = -\frac{2}{a\tan\left(\frac{x}{2}\right)+a}$$

input
```
integrate(1/(a+a*sin(x)),x)
```

output
```
-2/(a*tan(x/2) + a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{a + a\sin(x)}\,dx = -\frac{2}{a + \frac{a\sin(x)}{\cos(x)+1}}$$

input
```
integrate(1/(a+a*sin(x)),x, algorithm="maxima")
```

output
```
-2/(a + a*sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a + a\sin(x)}\,dx = -\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)}$$

input
```
integrate(1/(a+a*sin(x)),x, algorithm="giac")
```

output
```
-2/(a*(tan(1/2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a + a\sin(x)}\,dx = -\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

input
```
int(1/(a + a*sin(x)),x)
```

output
```
-2/(a*(tan(x/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{a + a\sin(x)}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

input
```
int(1/(a+a*sin(x)),x)
```

output
```
(2*tan(x/2))/(a*(tan(x/2) + 1))
```

3.69 $\int \frac{1}{a+a\sin^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int \frac{1}{a+a\sin^3(x)}\,dx=\frac{2\arctan\left(\frac{(-1)^{2/3}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}a}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left(1+(-1)^{2/3}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}a}-\frac{\cos(x)}{3a(1+\sin(x))}$$

output
```
2/3*arctan(((-1)^(2/3)+tan(1/2*x))/(1+(-1)^(1/3))^(1/2))/(1+(-1)^(1/3))^(1
/2)/a-2/3*arctan((-1)^(1/3)*(1+(-1)^(2/3)*tan(1/2*x))/(1-(-1)^(2/3))^(1/2)
)/(1-(-1)^(2/3))^(1/2)/a-1/3*cos(x)/a/(1+sin(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.89 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{1}{a+a\sin^3(x)}\,dx = \frac{i\sqrt{6i\left(3i+\sqrt{3}\right)}\arctan\left(\frac{2+\left(-1-i\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-6+2i\sqrt{3}}}\right)-i\sqrt{-18-6i\sqrt{3}}\arctan\left(\frac{2+i\left(i+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-6-2i\sqrt{3}}}\right)+\frac{6\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}}{9a}$$

input
```
Integrate[(a + a*Sin[x]^3)^(-1),x]
```

output
```
(I*Sqrt[(6*I)*(3*I + Sqrt[3])]*ArcTan[(2 + (-1 - I*Sqrt[3])*Tan[x/2])/Sqrt
[-6 + (2*I)*Sqrt[3]]] - I*Sqrt[-18 - (6*I)*Sqrt[3]]*ArcTan[(2 + I*(I + Sqr
t[3])*Tan[x/2])/Sqrt[-6 - (2*I)*Sqrt[3]]] + (6*Sin[x/2])/(Cos[x/2] + Sin[x
/2]))/(9*a)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^3(x)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a\sin(x)^3+a}dx$$

$$\Big\downarrow \; 3692$$

$$\int \left(-\frac{1}{3a\left(\sqrt[3]{-1}\sin(x)-1\right)}-\frac{1}{3a\left(-(-1)^{2/3}\sin(x)-1\right)}-\frac{1}{3a(-\sin(x)-1)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2\arctan\left(\frac{\tan(\frac{x}{2})+(-1)^{2/3}}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}a}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}((-1)^{2/3}\tan(\frac{x}{2})+1)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}a}-\frac{\cos(x)}{3a(\sin(x)+1)}$$

input
```
Int[(a + a*Sin[x]^3)^(-1),x]
```

output
```
(2*ArcTan[((-1)^(2/3) + Tan[x/2])/Sqrt[1 + (-1)^(1/3)]])/(3*Sqrt[1 + (-1)^
(1/3)]*a) - (2*ArcTan[((-1)^(1/3)*(1 + (-1)^(2/3)*Tan[x/2]))/Sqrt[1 - (-1)
^(2/3)]])/(3*Sqrt[1 - (-1)^(2/3)]*a) - Cos[x]/(3*a*(1 + Sin[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{2}{3(e^{ix}+i)a}+\left(\sum_{\_R=\mathrm{RootOf}(243a^4\_Z^4+27a^2\_Z^2+1)}\_R\ln\left(e^{ix}-162a^3\_R^3-27ia^2\_R^2-9a\_R-2i\right)\right.$
default	$\frac{\frac{2\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4-2\_Z^3+6\_Z^2-2\_Z+1)}\frac{(\_R^2-\_R+1)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3-3\_R^2+6\_R-1}\right)}{3}-\frac{2}{3\left(\tan\left(\frac{x}{2}\right)+1\right)}}{a}$

input
```
int(1/(a+a*sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(exp(I*x)+I)/a+sum(_R*ln(exp(I*x)-162*a^3*_R^3-27*I*a^2*_R^2-9*a*_R-2
*I),_R=RootOf(243*_Z^4*a^4+27*_Z^2*a^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(81) = 162.

Time = 0.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.84

$$\int \frac{1}{a+a\sin^3(x)}\,dx =$$

$$-\frac{\sqrt{2}(a\cos(x)+a\sin(x)+a)\sqrt{-\frac{\sqrt{\frac{1}{3}}a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}\log\left(-3\sqrt{2}\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{\sqrt{\frac{1}{3}}a^2\sqrt{-\frac{1}{a^4}}+1}{a^2}}\sqrt{-\frac{1}{a^4}}\cos(x)+3\sqrt{\right.}{}}{}$$

input
```
integrate(1/(a+a*sin(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2)*(a*cos(x) + a*sin(x) + a)*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4)
 + 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^4)
+ 1)/a^2)*sqrt(-1/a^4)*cos(x) + 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) - sin(
x) + 2) - sqrt(2)*(a*cos(x) + a*sin(x) + a)*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a
^4) + 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt(-(sqrt(1/3)*a^2*sqrt(-1/a^
4) + 1)/a^2)*sqrt(-1/a^4)*cos(x) - 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) + s
in(x) - 2) + sqrt(2)*(a*cos(x) + a*sin(x) + a)*sqrt((sqrt(1/3)*a^2*sqrt(-1
/a^4) - 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt((sqrt(1/3)*a^2*sqrt(-1/a
^4) - 1)/a^2)*sqrt(-1/a^4)*cos(x) + 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) +
sin(x) - 2) - sqrt(2)*(a*cos(x) + a*sin(x) + a)*sqrt((sqrt(1/3)*a^2*sqrt(-
1/a^4) - 1)/a^2)*log(-3*sqrt(2)*sqrt(1/3)*a^3*sqrt((sqrt(1/3)*a^2*sqrt(-1/
a^4) - 1)/a^2)*sqrt(-1/a^4)*cos(x) - 3*sqrt(1/3)*a^2*sqrt(-1/a^4)*sin(x) -
 sin(x) + 2) + 4*cos(x) - 4*sin(x) + 4)/(a*cos(x) + a*sin(x) + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.62 (sec) , antiderivative size = 10462, normalized size of antiderivative = 92.58

$$\int \frac{1}{a + a\sin^3(x)}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sin(x)**3),x)
```

output
```
-406593*sqrt(6)*sqrt(-3 + sqrt(3)*I)*log(tan(x/2) - 1/2 + sqrt(2)*sqrt(-3
- sqrt(3)*I)/2 + sqrt(3)*I/2)*tan(x/2)/(1219779*sqrt(3)*a*sqrt(-3 - sqrt(3
)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 4242534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt
(-3 + sqrt(3)*I)*tan(x/2) + 4130865*sqrt(2)*I*a*sqrt(-3 + sqrt(3)*I)*tan(x
/2) + 2698299*sqrt(6)*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) + 1219779*sqrt(3)*a*
sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I) - 4242534*I*a*sqrt(-3 - sqrt(3)*
I)*sqrt(-3 + sqrt(3)*I) + 4130865*sqrt(2)*I*a*sqrt(-3 + sqrt(3)*I) + 26982
99*sqrt(6)*a*sqrt(-3 + sqrt(3)*I)) + 679209*sqrt(3)*sqrt(-3 - sqrt(3)*I)*s
qrt(-3 + sqrt(3)*I)*log(tan(x/2) - 1/2 + sqrt(2)*sqrt(-3 - sqrt(3)*I)/2 +
sqrt(3)*I/2)*tan(x/2)/(1219779*sqrt(3)*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sq
rt(3)*I)*tan(x/2) - 4242534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*
tan(x/2) + 4130865*sqrt(2)*I*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) + 2698299*sqr
t(6)*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) + 1219779*sqrt(3)*a*sqrt(-3 - sqrt(3)
*I)*sqrt(-3 + sqrt(3)*I) - 4242534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt
(3)*I) + 4130865*sqrt(2)*I*a*sqrt(-3 + sqrt(3)*I) + 2698299*sqrt(6)*a*sqrt
(-3 + sqrt(3)*I)) + 238761*I*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*log
(tan(x/2) - 1/2 + sqrt(2)*sqrt(-3 - sqrt(3)*I)/2 + sqrt(3)*I/2)*tan(x/2)/(
1219779*sqrt(3)*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/2) - 424
2534*I*a*sqrt(-3 - sqrt(3)*I)*sqrt(-3 + sqrt(3)*I)*tan(x/2) + 4130865*sqrt
(2)*I*a*sqrt(-3 + sqrt(3)*I)*tan(x/2) + 2698299*sqrt(6)*a*sqrt(-3 + sqr...
```

Maxima [F]

$$\int \frac{1}{a+a\sin^3(x)}\,dx=\int \frac{1}{a\sin\left(x\right)^3+a}\,dx$$

input
```
integrate(1/(a+a*sin(x)^3),x, algorithm="maxima")
```

output
```
-1/3*(3*(a*cos(x)^2 + a*sin(x)^2 + 2*a*sin(x) + a)*integrate(2/3*((4*cos(2
*x) - sin(3*x) + sin(x))*cos(4*x) + 2*(2*cos(x) - 7*sin(2*x))*cos(3*x) - 2
*cos(3*x)^2 - 2*(7*sin(x) - 2)*cos(2*x) - 24*cos(2*x)^2 - 2*cos(x)^2 + (co
s(3*x) - cos(x) + 4*sin(2*x))*sin(4*x) + (14*cos(2*x) + 4*sin(x) - 1)*sin(
3*x) - 2*sin(3*x)^2 + 14*cos(x)*sin(2*x) - 24*sin(2*x)^2 - 2*sin(x)^2 + si
n(x))/(a*cos(4*x)^2 + 4*a*cos(3*x)^2 + 36*a*cos(2*x)^2 + 4*a*cos(x)^2 + a*
sin(4*x)^2 + 4*a*sin(3*x)^2 - 24*a*cos(x)*sin(2*x) + 36*a*sin(2*x)^2 + 4*a
*sin(x)^2 - 2*(6*a*cos(2*x) - 2*a*sin(3*x) + 2*a*sin(x) - a)*cos(4*x) - 8*
(a*cos(x) - 3*a*sin(2*x))*cos(3*x) + 12*(2*a*sin(x) - a)*cos(2*x) - 4*(a*c
os(3*x) - a*cos(x) + 3*a*sin(2*x))*sin(4*x) - 4*(6*a*cos(2*x) + 2*a*sin(x)
 - a)*sin(3*x) - 4*a*sin(x) + a), x) + 2*cos(x))/(a*cos(x)^2 + a*sin(x)^2
+ 2*a*sin(x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(81) = 162$.

Time = 0.43 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.81

$$\int \frac{1}{a + a\sin^3(x)}\,dx$$

$$= \frac{\sqrt{6\sqrt{3}-9}\log\left(196\left(2\sqrt{3}\sqrt{6\sqrt{3}-9}+3\sqrt{3}+3\sqrt{6\sqrt{3}-9}\right)^2+196\left(\sqrt{3}\sqrt{6\sqrt{3}-9}-6\tan\left(\frac{1}{2}x\right)+\right.\right.}{}$$

$$-\frac{2}{3\,a\left(\tan\left(\frac{1}{2}x\right)+1\right)}$$

input
```
integrate(1/(a+a*sin(x)^3),x, algorithm="giac")
```

output
```
1/18*(sqrt(6*sqrt(3) - 9)*log(196*(2*sqrt(3)*sqrt(6*sqrt(3) - 9) + 3*sqrt(
3) + 3*sqrt(6*sqrt(3) - 9))^2 + 196*(sqrt(3)*sqrt(6*sqrt(3) - 9) - 6*tan(1
/2*x) + 3)^2) - sqrt(6*sqrt(3) - 9)*log(196*(2*sqrt(3)*sqrt(6*sqrt(3) - 9)
 - 3*sqrt(3) + 3*sqrt(6*sqrt(3) - 9))^2 + 196*(sqrt(3)*sqrt(6*sqrt(3) - 9)
 + 6*tan(1/2*x) - 3)^2) + 2*sqrt(3)*sqrt(6*sqrt(3) - 9)*arctan(3*(sqrt(2*s
qrt(3) - 3) + 2*tan(1/2*x) - 1)/(2*sqrt(3)*sqrt(6*sqrt(3) - 9) - 3*sqrt(3)
 + 3*sqrt(6*sqrt(3) - 9)))/(2*sqrt(3) - 3) + 2*sqrt(3)*sqrt(6*sqrt(3) - 9)
*arctan(-3*(sqrt(2*sqrt(3) - 3) - 2*tan(1/2*x) + 1)/(2*sqrt(3)*sqrt(6*sqrt
(3) - 9) + 3*sqrt(3) + 3*sqrt(6*sqrt(3) - 9)))/(2*sqrt(3) - 3))/a - 2/3/(a
*(tan(1/2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 37.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+a\sin^3(x)}\,dx = -\frac{2}{3\,\left(a+a\tan\left(\frac{x}{2}\right)\right)} - 2\,\mathrm{atanh}\left(-\frac{5184\,a^3\sqrt{-\frac{1}{18\,a^2}-\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{3456\,a^2\tan\left(\frac{x}{2}\right)-864\,a^2+\sqrt{3}\,a^2\,864\mathrm{i}} + \frac{2592\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}-\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{3456\,a^2\tan\left(\frac{x}{2}\right)-864\,a^2+\sqrt{3}\,a^2\,864\mathrm{i}} + \frac{\sqrt{3}\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}-\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}\,7776\mathrm{i}}{3456\,a^2\tan\left(\frac{x}{2}\right)-864\,a^2+\sqrt{3}\,a^2\,864\mathrm{i}}\right)\sqrt{-\frac{3+\sqrt{3}\,1\mathrm{i}}{54\,a^2}} - 2\,\mathrm{atanh}\left(\frac{5184\,a^3\sqrt{-\frac{1}{18\,a^2}+\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{864\,a^2-3456\,a^2\tan\left(\frac{x}{2}\right)+\sqrt{3}\,a^2\,864\mathrm{i}} - \frac{2592\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}+\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}}{864\,a^2-3456\,a^2\tan\left(\frac{x}{2}\right)+\sqrt{3}\,a^2\,864\mathrm{i}} + \frac{\sqrt{3}\,a^3\tan\left(\frac{x}{2}\right)\sqrt{-\frac{1}{18\,a^2}+\frac{\sqrt{3}\,1\mathrm{i}}{54\,a^2}}\,7776\mathrm{i}}{864\,a^2-3456\,a^2\tan\left(\frac{x}{2}\right)+\sqrt{3}\,a^2\,864\mathrm{i}}\right)\sqrt{\frac{-3+\sqrt{3}\,1\mathrm{i}}{54\,a^2}}$$

input
```
int(1/(a + a*sin(x)^3),x)
```

output
```
- 2/(3*(a + a*tan(x/2))) - 2*atanh((2592*a^3*tan(x/2)*(- (3^(1/2)*1i)/(54*
a^2) - 1/(18*a^2))^(1/2))/(3456*a^2*tan(x/2) + 3^(1/2)*a^2*864i - 864*a^2)
 - (5184*a^3*(- (3^(1/2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2))/(3456*a^2*tan(x
/2) + 3^(1/2)*a^2*864i - 864*a^2) + (3^(1/2)*a^3*tan(x/2)*(- (3^(1/2)*1i)/
(54*a^2) - 1/(18*a^2))^(1/2)*7776i)/(3456*a^2*tan(x/2) + 3^(1/2)*a^2*864i
- 864*a^2))*(-(3^(1/2)*1i + 3)/(54*a^2))^(1/2) - 2*atanh((5184*a^3*((3^(1/
2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2))/(3^(1/2)*a^2*864i - 3456*a^2*tan(x/2)
 + 864*a^2) - (2592*a^3*tan(x/2)*((3^(1/2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2
))/(3^(1/2)*a^2*864i - 3456*a^2*tan(x/2) + 864*a^2) + (3^(1/2)*a^3*tan(x/2
)*((3^(1/2)*1i)/(54*a^2) - 1/(18*a^2))^(1/2)*7776i)/(3^(1/2)*a^2*864i - 34
56*a^2*tan(x/2) + 864*a^2))*((3^(1/2)*1i - 3)/(54*a^2))^(1/2)
```

Reduce [F]

$$\int \frac{1}{a+a\sin^3(x)}\,dx=\frac{\int \frac{1}{\sin(x)^3+1}dx}{a}$$

input
```
int(1/(a+a*sin(x)^3),x)
```

output
```
int(1/(sin(x)**3 + 1),x)/a
```

3.70 $\int \frac{1}{a+a\sin^5(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 210

$$\int \frac{1}{a+a\sin^5(x)}\,dx = \frac{2\arctan\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}a}+\frac{2\arctan\left(\frac{(-1)^{4/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}a}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left(1+(-1)^{2/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}a}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left(1+(-1)^{4/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}a}-\frac{\cos(x)}{5a(1+\sin(x))}$$

output

```
2/5*arctan(((-1)^(2/5)+tan(1/2*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(1
/2)/a+2/5*arctan(((-1)^(4/5)+tan(1/2*x))/(1+(-1)^(3/5))^(1/2))/(1+(-1)^(3/
5))^(1/2)/a-2/5*arctan((-1)^(3/5)*(1+(-1)^(2/5)*tan(1/2*x))/(1+(-1)^(1/5))
^(1/2))/(1+(-1)^(1/5))^(1/2)/a-2/5*arctan((-1)^(1/5)*(1+(-1)^(4/5)*tan(1/2
*x))/(1-(-1)^(2/5))^(1/2))/(1-(-1)^(2/5))^(1/2)/a-1/5*cos(x)/a/(1+sin(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.97

$$\int \frac{1}{a+a\sin^5(x)}\,dx$$

$$= \frac{-i\text{RootSum}\left[1+2i\#1-8\#1^2-14i\#1^3+30\#1^4+14i\#1^5-8\#1^6-2i\#1^7+\#1^8\&,\frac{-2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + a*Sin[x]^5)^(-1),x]
```

output
```
((-I)*RootSum[1 + (2*I)*#1 - 8*#1^2 - (14*I)*#1^3 + 30*#1^4 + (14*I)*#1^5
 - 8*#1^6 - (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log[
1 - 2*Cos[x]*#1 + #1^2] - (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 4*Log[1
 - 2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*L
og[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3
 + 40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#
1^4 + (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 - (8*I)*ArcTan[Sin[x]/(Cos[x
] - #1)]*#1^5 - 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[
x] - #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(I - 8*#1 - (21*I)*#1
^2 + 60*#1^3 + (35*I)*#1^4 - 24*#1^5 - (7*I)*#1^6 + 4*#1^7) & ] + (4*Sin[x
/2])/(Cos[x/2] + Sin[x/2]))/(10*a)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin^5(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin(x)^5+a}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5a\left(\sqrt[5]{-1}\sin(x)-1\right)}-\frac{1}{5a\left(-(-1)^{2/5}\sin(x)-1\right)}-\frac{1}{5a\left((-1)^{3/5}\sin(x)-1\right)}-\frac{1}{5a\left(-(-1)^{4/5}\sin(x)-1\right)}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}a}+\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}a}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}a}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}a}-\frac{\cos(x)}{5a(\sin(x)+1)}$$

input
```
Int[(a + a*Sin[x]^5)^(-1),x]
```

output
```
(2*ArcTan[((-1)^(2/5) + Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^
(4/5)]*a) + (2*ArcTan[((-1)^(4/5) + Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sq
rt[1 + (-1)^(3/5)]*a) - (2*ArcTan[((-1)^(3/5)*(1 + (-1)^(2/5)*Tan[x/2]))/S
qrt[1 + (-1)^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]*a) - (2*ArcTan[((-1)^(1/5)*(
1 + (-1)^(4/5)*Tan[x/2]))/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^(2/5)]*a
) - Cos[x]/(5*a*(1 + Sin[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{2}{5(e^{ix}+i)a}+\left(\sum_{\_R=\text{RootOf}(1953125a^8\_Z^8+156250a^6\_Z^6+6250a^4\_Z^4+125a^2\_Z^2+1)}\_R\ln\left(e^{ix}+2343750a^7\_\right.\right.$
default	$\frac{\frac{2\left(\sum_{\_R=\text{RootOf}(\_Z^8-2\_Z^7+8\_Z^6-14\_Z^5+30\_Z^4-14\_Z^3+8\_Z^2-2\_Z+1)}\frac{(2\_R^6-3\_R^5+10\_R^4-10\_R^3+10\_R^2-3\_R+}{4\_R^7-7\_R^6+24\_R^5-35\_R^4+60\_R^3-21\_}\right.}{5}}{a}$

input
```
int(1/(a+a*sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(exp(I*x)+I)/a+sum(_R*ln(exp(I*x)+2343750*a^7*_R^7+234375*I*a^6*_R^6+
140625*a^5*_R^5+15625*I*a^4*_R^4+4375*a^3*_R^3+500*I*a^2*_R^2+50*a*_R+6*I)
,_R=RootOf(1953125*_Z^8*a^8+156250*_Z^6*a^6+6250*_Z^4*a^4+125*_Z^2*a^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(148) = 296.

Time = 0.25 (sec) , antiderivative size = 1612, normalized size of antiderivative = 7.68

$$\int \frac{1}{a+a\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sin(x)^5),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2)*(a*cos(x) + a*sin(x) + a)*sqrt(-(a^2*sqrt(-(2*sqrt(1/5)*a^4*
sqrt(a^(-8)) + 1)/a^4) + 1)/a^2)*log(5*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) -
 5*sqrt(2)*(3*sqrt(1/5)*a^7*sqrt(a^(-8))*cos(x) - a^3*cos(x))*sqrt(-(a^2*s
qrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + 1)/a^2)*sqrt(-(2*sqrt(1/5)*
a^4*sqrt(a^(-8)) + 1)/a^4) - 5*(3*sqrt(1/5)*a^6*sqrt(a^(-8))*sin(x) - a^2*
sin(x))*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + sin(x) - 4) - sqrt
(2)*(a*cos(x) + a*sin(x) + a)*sqrt((a^2*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)
) + 1)/a^4) - 1)/a^2)*log(5*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) - 5*sqrt(2)*
(3*sqrt(1/5)*a^7*sqrt(a^(-8))*cos(x) - a^3*cos(x))*sqrt((a^2*sqrt(-(2*sqrt
(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) - 1)/a^2)*sqrt(-(2*sqrt(1/5)*a^4*sqrt(a^(
-8)) + 1)/a^4) + 5*(3*sqrt(1/5)*a^6*sqrt(a^(-8))*sin(x) - a^2*sin(x))*sqrt
(-(2*sqrt(1/5)*a^4*sqrt(a^(-8)) + 1)/a^4) + sin(x) - 4) + sqrt(2)*(a*cos(x
) + a*sin(x) + a)*sqrt(-(a^2*sqrt((2*sqrt(1/5)*a^4*sqrt(a^(-8)) - 1)/a^4)
+ 1)/a^2)*log(5*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) - 5*sqrt(2)*(3*sqrt(1/5)
*a^7*sqrt(a^(-8))*cos(x) + a^3*cos(x))*sqrt(-(a^2*sqrt((2*sqrt(1/5)*a^4*sq
rt(a^(-8)) - 1)/a^4) + 1)/a^2)*sqrt((2*sqrt(1/5)*a^4*sqrt(a^(-8)) - 1)/a^4
) - 5*(3*sqrt(1/5)*a^6*sqrt(a^(-8))*sin(x) + a^2*sin(x))*sqrt((2*sqrt(1/5)
*a^4*sqrt(a^(-8)) - 1)/a^4) - sin(x) + 4) - sqrt(2)*(a*cos(x) + a*sin(x) +
 a)*sqrt((a^2*sqrt((2*sqrt(1/5)*a^4*sqrt(a^(-8)) - 1)/a^4) - 1)/a^2)*log(5
*sqrt(1/5)*a^4*sqrt(a^(-8))*sin(x) - 5*sqrt(2)*(3*sqrt(1/5)*a^7*sqrt(a^...
```

Sympy [F]

$$\int \frac{1}{a + a\sin^5(x)}\,dx = \frac{\int \frac{1}{\sin^5(x)+1}\,dx}{a}$$

input
```
integrate(1/(a+a*sin(x)**5),x)
```

output
```
Integral(1/(sin(x)**5 + 1), x)/a
```

Maxima [F]

$$\int \frac{1}{a + a\sin^5(x)}\,dx = \int \frac{1}{a\sin(x)^5 + a}\,dx$$

input
```
integrate(1/(a+a*sin(x)^5),x, algorithm="maxima")
```

output
```
-1/5*(5*(a*cos(x)^2 + a*sin(x)^2 + 2*a*sin(x) + a)*integrate(2/5*((4*cos(6
*x) - 40*cos(4*x) + 4*cos(2*x) - sin(7*x) + 15*sin(5*x) - 15*sin(3*x) + si
n(x))*cos(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 8*sin(6*x) + 55
*sin(4*x) - 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos
(2*x) - 44*sin(5*x) + 44*sin(3*x) - 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2
 + 2*(210*cos(3*x) - 22*cos(x) - 505*sin(4*x) + 88*sin(2*x))*cos(5*x) - 21
0*cos(5*x)^2 + 10*(44*cos(2*x) - 101*sin(3*x) + 11*sin(x) - 4)*cos(4*x) -
1200*cos(4*x)^2 + 44*(cos(x) - 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 - 4*(
4*sin(x) - 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (cos(7*x) - 15*cos(5
*x) + 15*cos(3*x) - cos(x) + 4*sin(6*x) - 40*sin(4*x) + 4*sin(2*x))*sin(8*
x) + (16*cos(6*x) - 110*cos(4*x) + 16*cos(2*x) + 44*sin(5*x) - 44*sin(3*x)
 + 4*sin(x) - 1)*sin(7*x) - 2*sin(7*x)^2 + 8*(22*cos(5*x) - 22*cos(3*x) +
2*cos(x) + 55*sin(4*x) - 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 + (1010*cos(
4*x) - 176*cos(2*x) + 420*sin(3*x) - 44*sin(x) + 15)*sin(5*x) - 210*sin(5*
x)^2 + 10*(101*cos(3*x) - 11*cos(x) + 44*sin(2*x))*sin(4*x) - 1200*sin(4*x
)^2 + (176*cos(2*x) + 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 + 16*cos(x
)*sin(2*x) - 32*sin(2*x)^2 - 2*sin(x)^2 + sin(x))/(a*cos(8*x)^2 + 4*a*cos(
7*x)^2 + 64*a*cos(6*x)^2 + 196*a*cos(5*x)^2 + 900*a*cos(4*x)^2 + 196*a*cos
(3*x)^2 + 64*a*cos(2*x)^2 + 4*a*cos(x)^2 + a*sin(8*x)^2 + 4*a*sin(7*x)^2 +
 64*a*sin(6*x)^2 + 196*a*sin(5*x)^2 + 900*a*sin(4*x)^2 + 196*a*sin(3*x)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2033 vs. $2(148) = 296$.

Time = 2.45 (sec) , antiderivative size = 2033, normalized size of antiderivative = 9.68

$$\int \frac{1}{a + a\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sin(x)^5),x, algorithm="giac")
```

output
```
1/50*(2*sqrt(1/5)*sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)*(
arctan(1/2) + arctan(1/2*(242095961934922334 25*sqrt(5)*(sqrt(5) + 5) - 496
45000851686087842*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 5
0) - 25) + 248225004258430439210*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt
(5) + 50) - 25) - 260329802355176555922 5*sqrt(5) + 992900017033721756840 0*
tan(1/2*x) - 260329802355176555922 5)/(147447624806526714820 0*sqrt(5)*(sqrt
(5) + 5) + 148935002555058263526*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt
(10*sqrt(5) + 50) - 25) + 124112502129215219605*sqrt(5)*sqrt(10*sqrt(5) +
50) + 744675012775291317630*sqrt(5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 2
48225004258430439210*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) -
25) - 7372381240326335741000*sqrt(5) - 620562510646076098025*sqrt(10*sqrt(
5) + 50) - 1241125021292152196050*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 737
2381240326335741000)))/(2*sqrt(1/10)*sqrt(sqrt(5) + 5) - 1) + 2*sqrt(1/5)*
sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)*arctan(1/2*(5878490
76675773567575*sqrt(5)*(sqrt(5) + 5) + 49645000851686087842*sqrt(5)*sqrt(1
0*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 248225004258430439210
*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 54214954259631
72229975*sqrt(5) + 9929000170337217568400*tan(1/2*x) - 5421495425963172229
975)/(1701866802206171210550*sqrt(5)*(sqrt(5) + 5) + 148935002555058263526
*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 124...
```

Mupad [B] (verification not implemented)

Time = 37.31 (sec) , antiderivative size = 4652, normalized size of antiderivative = 22.15

$$\int \frac{1}{a+a\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + a*sin(x)^5),x)
```

output
```
2*atanh((241664000000000000*a^7*((- (2*5^(1/2))/5 - 1)^(1/2)/(50*a^2) - 1/
(50*a^2))^(1/2))/(73728000000000000*a^6*tan(x/2) + 4096000000000000*5^(1/2
)*a^6 - 20480000000000000*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 90112000000000
00*a^6 + 23265280000000000*5^(1/2)*a^6*tan(x/2) + 63897600000000000*a^6*ta
n(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2) - 22118400000000000*5^(1/2)*a^6*(- (2*5
^(1/2))/5 - 1)^(1/2) + 21299200000000000*5^(1/2)*a^6*tan(x/2)*(- (2*5^(1/2
))/5 - 1)^(1/2)) - (495616000000000000*a^7*tan(x/2)*((- (2*5^(1/2))/5 - 1)
^(1/2)/(50*a^2) - 1/(50*a^2))^(1/2))/(73728000000000000*a^6*tan(x/2) + 409
6000000000000*5^(1/2)*a^6 - 20480000000000000*a^6*(- (2*5^(1/2))/5 - 1)^(1
/2) + 9011200000000000*a^6 + 23265280000000000*5^(1/2)*a^6*tan(x/2) + 6389
7600000000000*a^6*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2) - 22118400000000000
*5^(1/2)*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 21299200000000000*5^(1/2)*a^6*t
an(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)) + (79462400000000000*5^(1/2)*a^7*((-
(2*5^(1/2))/5 - 1)^(1/2)/(50*a^2) - 1/(50*a^2))^(1/2))/(73728000000000000*
a^6*tan(x/2) + 4096000000000000*5^(1/2)*a^6 - 20480000000000000*a^6*(- (2*
5^(1/2))/5 - 1)^(1/2) + 9011200000000000*a^6 + 23265280000000000*5^(1/2)*a
^6*tan(x/2) + 63897600000000000*a^6*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2) -
 22118400000000000*5^(1/2)*a^6*(- (2*5^(1/2))/5 - 1)^(1/2) + 2129920000000
0000*5^(1/2)*a^6*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)) + (1351680000000000
00*a^7*((- (2*5^(1/2))/5 - 1)^(1/2)/(50*a^2) - 1/(50*a^2))^(1/2)*(- (2*...
```

Reduce [F]

$$\int \frac{1}{a + a \sin^5(x)}\, dx = \frac{\int \frac{1}{\sin(x)^5 + 1} dx}{a}$$

input
```
int(1/(a+a*sin(x)^5),x)
```

output
```
int(1/(sin(x)**5 + 1),x)/a
```

3.71 $\int \frac{1}{a-b\sin^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{a-b\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{-a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{-a+b}}$$

output
```
arctanh((-a+b)^(1/2)*tan(x)/a^(1/2))/a^(1/2)/(-a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

input
```
Integrate[(a - b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a-b\sin(x)^2}dx$$
$$\Big\downarrow \text{ 3660}$$
$$\int \frac{1}{(a-b)\tan^2(x)+a}d\tan(x)$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

input
```
Int[(a - b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{a(a-b)}}\right)}{\sqrt{a(a-b)}}$	29
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2-2iab+2a\sqrt{-a^2+ba}-b\sqrt{-a^2+ba}}{b\sqrt{-a^2+ba}}\right)}{2\sqrt{-a^2+ba}}+\frac{\ln\left(e^{2ix}-\frac{2ia^2-2iab-2a\sqrt{-a^2+ba}+b\sqrt{-a^2+ba}}{b\sqrt{-a^2+ba}}\right)}{2\sqrt{-a^2+ba}}$	152

input
```
int(1/(a-b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a-b))^(1/2)*arctan((a-b)*tan(x)/(a*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.97

$$\int \frac{1}{a-b\sin^2(x)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+ab}\log\left(\frac{(8a^2-8ab+b^2)\cos(x)^4-2(4a^2-5ab+b^2)\cos(x)^2+4\left((2a-b)\cos(x)^3-(a-b)\cos(x)\right)\sqrt{-a^2+ab}\sin(x)+a^2-2ab+}{b^2\cos(x)^4+2(ab-b^2)\cos(x)^2+a^2-2ab+b^2}\right.}{4(a^2-ab)}\right.$$
$$\left.-\frac{\arctan\left(\frac{(2a-b)\cos(x)^2-a+b}{2\sqrt{a^2-ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2-ab}}\right]$$

input
```
integrate(1/(a-b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 + a*b)*log(((8*a^2 - 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 - 5*
a*b + b^2)*cos(x)^2 + 4*((2*a - b)*cos(x)^3 - (a - b)*cos(x))*sqrt(-a^2 +
a*b)*sin(x) + a^2 - 2*a*b + b^2)/(b^2*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 +
a^2 - 2*a*b + b^2))/(a^2 - a*b), -1/2*arctan(1/2*((2*a - b)*cos(x)^2 - a +
 b)/(sqrt(a^2 - a*b)*cos(x)*sin(x)))/sqrt(a^2 - a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15319 vs. $2(27) = 54$.

Time = 10.92 (sec) , antiderivative size = 15319, normalized size of antiderivative = 464.21

$$\int \frac{1}{a - b\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)**2),x)
```

output
```
Piecewise((zoo*(tan(x/2)/2 - 1/(2*tan(x/2))), Eq(a, 0) & Eq(b, 0)), (-(tan
(x/2)/2 - 1/(2*tan(x/2)))/b, Eq(a, 0)), (-2*tan(x/2)/(b*tan(x/2)**2 - b),
Eq(a, b)), (x/a, Eq(b, 0)), (6*a**3*b*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2
)/a)*log(-sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) + tan(x/2))/(10*a**4*b*
sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b
**2)/a) + 2*a**4*sqrt(-a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a
)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) - 50*a**3*b**2*sqrt(-1 + 2*b/a
- 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) - 26*a**
3*b*sqrt(-a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2
*b/a + 2*sqrt(-a*b + b**2)/a) + 72*a**2*b**3*sqrt(-1 + 2*b/a - 2*sqrt(-a*b
 + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) + 56*a**2*b**2*sqrt(-
a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*s
qrt(-a*b + b**2)/a) - 32*a*b**4*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*s
qrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) - 32*a*b**3*sqrt(-a*b + b**2)*sqrt
(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)
/a)) - 6*a**3*b*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(-1 + 2*b
/a + 2*sqrt(-a*b + b**2)/a) + tan(x/2))/(10*a**4*b*sqrt(-1 + 2*b/a - 2*sqr
t(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*sqrt(-a*b + b**2)/a) + 2*a**4*sqrt(-
a*b + b**2)*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(-1 + 2*b/a + 2*s
qrt(-a*b + b**2)/a) - 50*a**3*b**2*sqrt(-1 + 2*b/a - 2*sqrt(-a*b + b**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\sin^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a-b*sin(x)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \frac{1}{a-b\sin^2(x)}\,dx = -\frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan(x)-b\tan(x)}{\sqrt{a^2-ab}}\right)}{\sqrt{a^2-ab}}$$

input
```
integrate(1/(a-b*sin(x)^2),x, algorithm="giac")
```

output
```
-(pi*floor(x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(x) - b*tan(x))/sqr
t(a^2 - a*b)))/sqrt(a^2 - a*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-b\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(x)\,(2\,a-2\,b)}{2\,\sqrt{a^2-a\,b}}\right)}{\sqrt{a^2-a\,b}}$$

input
```
int(1/(a - b*sin(x)^2),x)
```

output
```
atan((tan(x)*(2*a - 2*b))/(2*(a^2 - a*b)^(1/2)))/(a^2 - a*b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \frac{1}{a-b\sin^2(x)}\,dx = \frac{\sqrt{a}\,\sqrt{a-b}\,\left(atan\left(\frac{\sqrt{a}\,\tan\left(\frac{x}{2}\right)-\sqrt{b}}{\sqrt{a-b}}\right)+atan\left(\frac{\sqrt{a}\,\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a-b}}\right)\right)}{a\,(a-b)}$$

input
```
int(1/(a-b*sin(x)^2),x)
```

output
```
(sqrt(a)*sqrt(a - b)*(atan((sqrt(a)*tan(x/2) - sqrt(b))/sqrt(a - b)) + ata
n((sqrt(a)*tan(x/2) + sqrt(b))/sqrt(a - b))))/(a*(a - b))
```

3.72 $\int \frac{1}{a-b\sin^4(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 83

$$\int \frac{1}{a-b\sin^4(x)}\,dx=\frac{\arctan\left(\sqrt{1+\sqrt{\frac{b}{a}}}\tan(x)\right)}{2a\sqrt{1+\sqrt{\frac{b}{a}}}}+\frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt{\frac{b}{a}}}\tan(x)\right)}{2a\sqrt{-1+\sqrt{\frac{b}{a}}}}$$

output
```
1/2*arctan((1+(b/a)^(1/2))^(1/2)*tan(x))/a/(1+(b/a)^(1/2))^(1/2)+1/2*arcta
nh((-1+(b/a)^(1/2))^(1/2)*tan(x))/a/(-1+(b/a)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.59

$$\int \frac{1}{a-b\sin^4(x)}\,dx$$
$$=\frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}\arctan\left(\frac{\sqrt{a+\sqrt{a}\sqrt{b}}\tan(x)}{\sqrt{a}}\right)-\left(\sqrt{a}+\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}\operatorname{arctanh}\left(\frac{\sqrt{-a+\sqrt{a}\sqrt{b}}\tan(x)}{\sqrt{a}}\right)}{2a(a-b)}$$

input
```
Integrate[(a - b*Sin[x]^4)^(-1),x]
```

output
```
((Sqrt[a] - Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]*ArcTan[(Sqrt[a + Sqrt[a]*Sq
rt[b]]*Tan[x])/Sqrt[a]] - (Sqrt[a] + Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]]*A
rcTanh[(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Tan[x])/Sqrt[a]])/(2*a*(a - b))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3688, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(x)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\int \frac{\tan^2(x)+1}{(a-b)\tan^4(x)+2a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{(a-b)\tan^2(x)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(x)+\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{(a-b)\tan^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[(a - b*Sin[x]^4)^(-1),x]
```

output
```
((1 + Sqrt[b]/Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[x])/a^(1/4)])/(
2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) + ((1 - Sqrt[b]/Sqr
t[a])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a
] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(256a^4-256a^3b\right)\_Z^4+32a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{128ia^4}{b}-128ia^3\right)\_R^3+\left(-\frac{32a^3}{b}+32a^2\right)\_$
default	$(a-b)\left(\frac{\left(b+\sqrt{ba}\right)\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{\left(\sqrt{ba}+a\right)(a-b)}}\right)}{2\sqrt{ba}\,(a-b)\sqrt{\left(\sqrt{ba}+a\right)(a-b)}}+\frac{\left(\sqrt{ba}-b\right)\mathrm{arctanh}\left(\frac{(-a+b)\tan(x)}{\sqrt{\left(\sqrt{ba}-a\right)(a-b)}}\right)}{2\sqrt{ba}\,(a-b)\sqrt{\left(\sqrt{ba}-a\right)(a-b)}}\right)$

input
```
int(1/(a-b*sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*x)+(128*I/b*a^4-128*I*a^3)*_R^3+(-32/b*a^3+32*a^2)*_R^2+
(8*I/b*a^2+8*I*a)*_R-2/b*a-1),_R=RootOf(1+(256*a^4-256*a^3*b)*_Z^4+32*a^2*
_Z^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(63) = 126.

Time = 0.18 (sec) , antiderivative size = 835, normalized size of antiderivative = 10.06

$$\int \frac{1}{a-b\sin^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)
)*log(1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) - (a^4 - a^3*b)*sqrt(b/(a^5
- 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^
4*b + a^3*b^2)) + 1)/(a^2 - a*b)) + 1/4*(a^3 - a^2*b - 2*(a^3 - a^2*b)*cos
(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(-((a^2 - a*b)
*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b))*log(1/4*b*cos(x)^2 -
1/2*(a*b*cos(x)*sin(x) - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*c
os(x)*sin(x))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a
^2 - a*b)) + 1/4*(a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*
a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b +
 a^3*b^2)) - 1)/(a^2 - a*b))*log(-1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x)
+ (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^
2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + 1/4*(a^3 -
a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1/4*
b) - 1/8*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a
*b))*log(-1/4*b*cos(x)^2 - 1/2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(
a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2
*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + 1/4*(a^3 - a^2*b - 2*(a^3 - a^2*b)*
cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1/4*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - b\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*sin(x)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a-b\sin^4(x)}\,dx=\int -\frac{1}{b\sin{(x)}^4-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^4),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^4 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(63) = 126.

Time = 0.50 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.12

$$\int \frac{1}{a-b\sin^4(x)}\,dx$$
$$=\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\right.}{2\,(3\,a^5-12\,a^4b+14\,a^3b^2-4\,a^2b^3-ab^4)}$$
$$+\frac{\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}a^2-6\sqrt{a^2-ab-\sqrt{ab}(a-b)}ab-\sqrt{a^2-ab-\sqrt{ab}(a-b)}b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right.\right.}{2\,(3\,a^5-12\,a^4b+14\,a^3b^2-4\,a^2b^3-ab^4)}$$

input
```
integrate(1/(a-b*sin(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^2)*(pi*floor(x/pi
 + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a - b)*a + 16*a^2))/(a - b
))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3 - a*b^4) + 1/2*
(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^2)*(pi*floor(x/pi + 1
/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))
*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3 - a*b^4)
```

Mupad [B] (verification not implemented)

Time = 37.81 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.24

$$\int \frac{1}{a-b\sin^4(x)}\,dx$$
$$=\operatorname{atan}\left(\frac{a^3\tan(x)\,\sqrt{-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}}\,4\mathrm{i}+a^5\tan(x)\,\left(-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}\right)^{3/2}64\mathrm{i}+a^2\,b\tan(x)\,\sqrt{-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}}\,4\mathrm{i}\right.$$

input
```
int(1/(a - b*sin(x)^4),x)
```

output
```
atan((a^3*tan(x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i + a^5*tan(x)*(-
1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*64i + a^2*b*tan(x)*(-1/(16*a^2 + 16*(
a^3*b)^(1/2)))^(1/2)*4i - a^4*b*tan(x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3
/2)*64i + a*tan(x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i
 + b*tan(x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i + a^3*
tan(x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i - a^2*b*ta
n(x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b + (a^3
*b)^(1/2)))*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*2i + atan((a^3*tan(x)*(
-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*4i + a^5*tan(x)*(-1/(16*a^2 - 16*(a^
3*b)^(1/2)))^(3/2)*64i + a^2*b*tan(x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/
2)*4i - a^4*b*tan(x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*64i - a*tan(x)
*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i - b*tan(x)*(-1/(1
6*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i - a^3*tan(x)*(-1/(16*a^2
 - 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i + a^2*b*tan(x)*(-1/(16*a^2 -
 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b - (a^3*b)^(1/2)))*(-1/(1
6*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*2i
```

Reduce [F]

$$\int \frac{1}{a-b\sin^4(x)}\,dx=-\left(\int \frac{1}{\sin(x)^4 b-a}dx\right)$$

input
```
int(1/(a-b*sin(x)^4),x)
```

output
```
- int(1/(sin(x)**4*b - a),x)
```

3.73 $\int \frac{1}{a-b\sin^6(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 175

$$\int \frac{1}{a-b\sin^6(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{-\sqrt[3]{a}+\sqrt[3]{b}}}$$

output
```
1/3*arctan((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(
1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/
2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)+1/3*arctanh(
(-a^(1/3)+b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(-a^(1/3)+b^(1/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \frac{8}{3}\text{RootSum}\left[b-6b\#1+15b\#1^2+64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5 + b\#1^6\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{-b+5b\#1+32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

input
```
Integrate[(a - b*Sin[x]^6)^(-1),x]
```

output
```
(8*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2
*Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^
3 - 5*b*#1^4 + b*#1^5) & ])/3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^6(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a-b\sin(x)^6}dx$$

$$\Big\downarrow \; 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}dx}{3a}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}}dx}{3a}$$

$\Big\downarrow$ 3660

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\tan^2(x)+1}d\tan(x)}{3a}$$

$\Big\downarrow$ 216

$$\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Sin[x]^6)^(-1),x]
```

output
```
ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
 b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(
3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1
)^(2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(
1/3)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a-\_R^5b+2\_R^3a+a\_R}\right)}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6-46656a^5b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{15552ia^6}{b}+15552ia^5\right)\_R^5\right.$

input
```
int(1/(a-b*sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((_R^4+2*_R^2+1)/(_R^5*a-_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Roo
tOf((a-b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 16697, normalized size of antiderivative = 95.41

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \int \frac{1}{a-b\sin^6(x)}\,dx$$

input
```
integrate(1/(a-b*sin(x)**6),x)
```

output
```
Integral(1/(a - b*sin(x)**6), x)
```

Maxima [F]

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \int -\frac{1}{b\sin(x)^6-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^6),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^6 - a), x)
```

Giac [F]

$$\int \frac{1}{a-b\sin^6(x)}\,dx=\int -\frac{1}{b\sin(x)^6-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^6),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^6 - a), x)
```

Mupad [B] (verification not implemented)

Time = 38.83 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.93

$$\int \frac{1}{a-b\sin^6(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{b^3\,(a-b)\,\left(\cot(x)-\mathrm{root}(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k)\;a\,8+\right.}{\ldots-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k)}\right.$$

input
```
int(1/(a - b*sin(x)^6),x)
```

output

```
symsum(log(-(3*b^3*(a - b)*(cot(x) - 8*root(46656*a^5*b*d^6 - 46656*a^6*d^
6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a + 2*root(46656*a^5*b*d^6 - 466
56*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*b - 504*root(46656*a^5*
b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^3*a^3 - 7776
*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d,
 k)^5*a^5 - 144*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*
a^2*d^2 - 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3
888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^5*a^4*b + 60*root(46656*a^5*b*d^6 - 4
6656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2*cot(x) + 864*ro
ot(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)
^4*a^4*cot(x) - 864*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 -
108*a^2*d^2 - 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 - 46656*a^
6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a*b*cot(x)))/cot(x))*root(
46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k
, 1, 6)
```

Reduce [F]

$$\int \frac{1}{a - b\sin^6(x)}\,dx = -\left(\int \frac{1}{\sin(x)^6 b - a}dx\right)$$

input

```
int(1/(a-b*sin(x)^6),x)
```

output

```
- int(1/(sin(x)**6*b - a),x)
```

3.74 $\int \frac{1}{a-b\sin^8(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 181

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \frac{\arctan\left(\sqrt{1-i\sqrt[4]{\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{1-i\sqrt[4]{\frac{b}{a}}}} + \frac{\arctan\left(\sqrt{1+\sqrt[4]{\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{1+\sqrt[4]{\frac{b}{a}}}} + \frac{\operatorname{arctanh}\left(\sqrt{-1-i\sqrt[4]{\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{-1-i\sqrt[4]{\frac{b}{a}}}} + \frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt[4]{\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{-1+\sqrt[4]{\frac{b}{a}}}}$$

output
```
1/4*arctan((1-I*(b/a)^(1/4))^(1/2)*tan(x))/a/(1-I*(b/a)^(1/4))^(1/2)+1/4*a
rctan((1+(b/a)^(1/4))^(1/2)*tan(x))/a/(1+(b/a)^(1/4))^(1/2)+1/4*arctanh((-
1-I*(b/a)^(1/4))^(1/2)*tan(x))/a/(-1-I*(b/a)^(1/4))^(1/2)+1/4*arctanh((-1+
(b/a)^(1/4))^(1/2)*tan(x))/a/(-1+(b/a)^(1/4))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96

$$\int \frac{1}{a-b\sin^8(x)}\,dx = -8\text{RootSum}\Bigg[b-8b\#1+28b\#1^2-56b\#1^3-256a\#1^4+70b\#1^4 -56b\#1^5+28b\#1^6-8b\#1^7 +b\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-b+7b\#1-21b\#1^2-128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\Bigg]$$

input
```
Integrate[(a - b*Sin[x]^8)^(-1),x]
```

output
```
-8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 - 5
6*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x]
- #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1
^2 - 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) &
 ]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^8(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a-b\sin(x)^8}dx$$

$$\Big\downarrow \; 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \; 3660$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

input
```
Int[(a - b*Sin[x]^8)^(-1),x]
```

output
```
ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
 b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)
*Sqrt[a^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Tan[x])/a^
(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + b^(1
/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{8}+4a\_Z^{6}+6a\_Z^{4}+4a\_Z^{2}+a\right)}\frac{\left(\_R^{6}+3\_R^{4}+3\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{7}a-\_R^{7}b+3\_R^{5}a+3\_R^{3}a+a\_R}\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^{8}-16777216a^{7}b)\_Z^{8}+1048576a^{6}\_Z^{6}+24576a^{4}\_Z^{4}+256a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{4194304ia^{8}}{b}\right.\right.$

input
```
int(1/(a-b*sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a-_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(t
an(x)-_R),_R=RootOf((a-b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643307 vs. $2(133) = 266$.

Time = 6.08 (sec) , antiderivative size = 643307, normalized size of antiderivative = 3554.18

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \int \frac{1}{a-b\sin^8(x)}\,dx$$

input
```
integrate(1/(a-b*sin(x)**8),x)
```

output
```
Integral(1/(a - b*sin(x)**8), x)
```

Maxima [F]

$$\int \frac{1}{a - b\sin^8(x)}\,dx = \int -\frac{1}{b\sin(x)^8 - a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^8),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^8 - a), x)
```

Giac [F]

$$\int \frac{1}{a - b\sin^8(x)}\,dx = \int -\frac{1}{b\sin(x)^8 - a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^8),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^8 - a), x)
```

Mupad [B] (verification not implemented)

Time = 39.91 (sec) , antiderivative size = 818, normalized size of antiderivative = 4.52

$$\int \frac{1}{a - b\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*sin(x)^8),x)
```

output
```
symsum(log(-2*b^5*(a - b)*(4*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 -
1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*b*tan(x) - 43008*
root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d
^4 - 256*a^2*d^2 - 1, d, k)^4*a^4 - 786432*root(16777216*a^7*b*d^8 - 16777
216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a
^6 - 800*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24
576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^2*a^2 - 6144*root(16777216*a^7*b*d^8
- 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d,
 k)^4*a^3*b + 786432*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*
a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a^5*b + 9984*root(16777
216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a
^2*d^2 - 1, d, k)^3*a^3*tan(x) + 557056*root(16777216*a^7*b*d^8 - 16777216
*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^5*a^5*
tan(x) + 10485760*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6
*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^7*a^7*tan(x) + 32*root(16777
216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a
^2*d^2 - 1, d, k)^2*a*b + 60*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 -
1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 768*ro
ot(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4
 - 256*a^2*d^2 - 1, d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^...
```

Reduce [F]

$$\int \frac{1}{a-b\sin^8(x)}\,dx=-\left(\int \frac{1}{\sin(x)^8 b-a}dx\right)$$

input
```
int(1/(a-b*sin(x)^8),x)
```

output
```
 - int(1/(sin(x)**8*b - a),x)
```

3.75 $\int \frac{1}{a-b\sin(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \frac{1}{a-b\sin(x)}\,dx = -\frac{2\arctan\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
-2*arctan((b-a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-b\sin(x)}\,dx = -\frac{2\arctan\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Integrate[(a - b*Sin[x])^(-1),x]
```

output
```
(-2*ArcTan[(b - a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a-b\sin(x)}dx$$
$$\Big\downarrow \text{3139}$$
$$2\int \frac{1}{a\tan^2\left(\frac{x}{2}\right)-2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{1083}$$
$$-4\int \frac{1}{-\left(2a\tan\left(\frac{x}{2}\right)-2b\right)^2-4\left(a^2-b^2\right)}d\left(2a\tan\left(\frac{x}{2}\right)-2b\right)$$
$$\Big\downarrow \text{217}$$
$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Int[(a - b*Sin[x])^(-1),x]
```

output
```
(2*ArcTan[(-2*b + 2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}$	39
risch	$-\frac{\ln\left({\mathrm e}^{i x}-\frac{i\sqrt{-a^{2}+b^{2}}\, a-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\, b}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left({\mathrm e}^{i x}-\frac{i\sqrt{-a^{2}+b^{2}}\, a+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\, b}\right)}{\sqrt{-a^{2}+b^{2}}}$	121

input
```
int(1/(a-b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)-2*b)/(a^2-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.71

$$\int \frac{1}{a-b\sin(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(x)^2+2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)-b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2+2ab\sin(x)-a^2-b^2}\right)}{2(a^2-b^2)},\; -\frac{\arctan\left(-\frac{a\sin(x)-b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a-b*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 + 2*a*b*sin(x) - a^2 -
 b^2 + 2*(a*cos(x)*sin(x) - b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 + 2*
a*b*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) - b)/(sqrt(a^2 -
b^2)*cos(x)))/sqrt(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(32) = 64$.

Time = 1.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \frac{1}{a-b\sin(x)}\,dx = \begin{cases} \tilde{\infty}\log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a=0 \wedge b=0 \\ -\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ \frac{2}{b\tan\left(\frac{x}{2}\right)+b} & \text{for } a=-b \\ -\frac{2}{b\tan\left(\frac{x}{2}\right)-b} & \text{for } a=b \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a-b*sin(x)),x)
```

output
```
Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (-log(tan(x/2))/b, Eq(
a, 0)), (2/(b*tan(x/2) + b), Eq(a, -b)), (-2/(b*tan(x/2) - b), Eq(a, b)),
(log(tan(x/2) - b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x
/2) - b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a - b\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a-b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{1}{a - b\sin(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right) - b}{\sqrt{a^2 - b^2}}\right)\right)}{\sqrt{a^2 - b^2}}$$

input
```
integrate(1/(a-b*sin(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) - b)/sqrt(a^2 -
b^2)))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 36.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{a-b\sin(x)}\,dx=-\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}-\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a - b*sin(x)),x)
```

output
```
-(2*atan(b/(a^2 - b^2)^(1/2) - (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{1}{a-b\sin(x)}\,dx=\frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-b}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

input
```
int(1/(a-b*sin(x)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan(x/2)*a - b)/sqrt(a**2 - b**2)))/(a**2 - b**
2)
```

3.76 $\int \frac{1}{a-b\sin^3(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 225

$$\int \frac{1}{a-b\sin^3(x)}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\arctan\left(\frac{(-1)^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

output
```
-2/3*arctan((b^(1/3)-a^(1/3)*tan(1/2*x))/(a^(2/3)-b^(2/3))^(1/2))/a^(2/3)/
(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(1/3)*b^(1/3)+a^(1/3)*tan(1/2*x))
/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-(-1)^(2/3)*b^(2/3))^
(1/2)-2/3*arctan((-1)^(2/3)*(b^(1/3)+(-1)^(1/3)*a^(1/3)*tan(1/2*x))/(a^(2/
3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.49

$$\int \frac{1}{a-b\sin^3(x)}\,dx = \frac{2}{3}i\text{RootSum}\left[ib-3ib\#1^2+8a\#1^3+3ib\#1^4 - ib\#1^6\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1}{b+4ia\#1-2b\#1^2+b\#1^4}\&\right]$$

input
```
Integrate[(a - b*Sin[x]^3)^(-1),x]
```

output
```
((2*I)/3)*RootSum[I*b - (3*I)*b*#1^2 + 8*a*#1^3 + (3*I)*b*#1^4 - I*b*#1^6
& , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1)
/(b + (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(x)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-\sqrt[3]{b}\sin(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(x)\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\arctan\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{x}{2}\right)+\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\arctan\left(\frac{(-1)^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{x}{2}\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

input
```
Int[(a - b*Sin[x]^3)^(-1),x]
```

output
```
(-2*ArcTan[(b^(1/3) - a^(1/3)*Tan[x/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - b^(2/3)]) + (2*ArcTan[((-1)^(1/3)*b^(1/3) + a^(1/3)*Tan[
x/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^
(2/3)*b^(2/3)]) - (2*ArcTan[((-1)^(2/3)*(b^(1/3) + (-1)^(1/3)*a^(1/3)*Tan[
x/2]))/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)
^(1/3)*b^(2/3)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4-8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a-4\_R^2b+a\_R}\right)}{3}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(729a^6-729a^4b^2\right)\_Z^6+243a^4\_Z^4+27a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(\frac{486a^6}{b}-486a^4b\right)\_R^5+\left(\frac{81ia^5}{b}-\right.\right.$

input
```
int(1/(a-b*sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a-4*_R^2*b+_R*a)*ln(tan(1/2*x)-_R),
_R=RootOf(_Z^6*a+3*_Z^4*a-8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 18599, normalized size of antiderivative = 82.66

$$\int \frac{1}{a-b\sin^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a-b\sin^3(x)}\,dx=\int \frac{1}{a-b\sin^3{(x)}}\,dx$$

input
```
integrate(1/(a-b*sin(x)**3),x)
```

output
```
Integral(1/(a - b*sin(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{a-b\sin^3(x)}\,dx=\int -\frac{1}{b\sin{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^3),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^3 - a), x)
```

Giac [F]

$$\int \frac{1}{a-b\sin^3(x)}\,dx=\int -\frac{1}{b\sin{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^3),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^3 - a), x)
```

Mupad [B] (verification not implemented)

Time = 36.67 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.53

$$\int \frac{1}{a-b\sin^3(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{8192\,a\,b^3\left(729\,a^5+243\tan\left(\frac{x}{2}\right)\,a^4\,b+324\tan\left(\frac{x}{2}\right)\,a^4\operatorname{root}(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a\right.}{}\right.$$
$$-729\,a^6\,d^6-243\,a^4\,d^4-27\,a^2\,d^2-1,d,k)$$

input
```
int(1/(a - b*sin(x)^3),x)
```

output
```
symsum(log(-(8192*a*b^3*(4*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 +
729*a^4*(a^2 - b^2), d, k)^5 + 9*a*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 7
29*a^4*(a^2 - b^2), d, k)^4 + 729*a^5 - 972*a^3*b^2 + 162*a^3*root(d^6 + 2
7*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 + 72*a^2*tan(x/2)*r
oot(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3 + 243*a^
4*b*tan(x/2) + 324*tan(x/2)*a^4*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*
a^4*(a^2 - b^2), d, k) + 24*b*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2
 + 729*a^4*(a^2 - b^2), d, k)^4 + 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d
^2 + 729*a^4*(a^2 - b^2), d, k)^3 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) - 648*tan(x/2)*a^2*b^2*root(d^6 + 27*
a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(x/2)*ro
ot(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^
6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*
b^2*d^6 - 729*a^6*d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{1}{a-b\sin^3(x)}\,dx=-\left(\int \frac{1}{\sin(x)^3 b-a}dx\right)$$

input
```
int(1/(a-b*sin(x)^3),x)
```

output
```
 - int(1/(sin(x)**3*b - a),x)
```

3.77 $\int \frac{1}{a-b\sin^5(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 385

$$\int \frac{1}{a-b\sin^5(x)}\,dx = -\frac{2\arctan\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{-1}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$
$$+\frac{2\arctan\left(\frac{(-1)^{3/5}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}$$
$$-\frac{2\arctan\left(\frac{(-1)^{4/5}\left(\sqrt[5]{b}+\sqrt[5]{-1}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}$$
$$-\frac{2\arctan\left(\frac{(-1)^{2/5}\left(\sqrt[5]{b}+(-1)^{3/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}$$

output
```
-2/5*arctan((b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/
(a^(2/5)-b^(2/5))^(1/2)+2/5*arctan(((-1)^(1/5)*b^(1/5)+a^(1/5)*tan(1/2*x))
/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^
(1/2)+2/5*arctan(((-1)^(3/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(1/
5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2)-2/5*arctan((
-1)^(4/5)*(b^(1/5)+(-1)^(1/5)*a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5)*b^(2
/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)-2/5*arctan((-1)^(2/
5)*(b^(1/5)+(-1)^(3/5)*a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1
/2))/a^(4/5)/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.39

$$\int \frac{1}{a-b\sin^5(x)}\,dx$$
$$= -\frac{8}{5}i\text{RootSum}\left[-ib+5ib\#1^2-10ib\#1^4+32a\#1^5+10ib\#1^6-5ib\#1^8\right.$$
$$\left.+ib\#1^{10}\&,\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b-4b\#1^2-16ia\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a - b*Sin[x]^5)^(-1),x]
```

output
```
((-8*I)/5)*RootSum[(-I)*b + (5*I)*b*#1^2 - (10*I)*b*#1^4 + 32*a*#1^5 + (10
*I)*b*#1^6 - (5*I)*b*#1^8 + I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*
#1^3 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 - (16*I)*a*#1^3 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^5(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(x)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{a}\right.}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{3/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{2/5}\left((-1)^{3/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}$$

input

```
Int[(a - b*Sin[x]^5)^(-1),x]
```

output
```
(-2*ArcTan[(b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/
5)*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(1/5)*b^(1/5) + a^(1/5)*Tan[
x/2])/Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^
(2/5)*b^(2/5)]) + (2*ArcTan[((-1)^(3/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a
^(2/5) + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5
)]) - (2*ArcTan[((-1)^(4/5)*(b^(1/5) + (-1)^(1/5)*a^(1/5)*Tan[x/2]))/Sqrt[
a^(2/5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/
5)]) - (2*ArcTan[((-1)^(2/5)*(b^(1/5) + (-1)^(3/5)*a^(1/5)*Tan[x/2]))/Sqrt
[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/5)*b^(2
/5)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.28

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}-32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a-16\_R^{4}b+4\_R^{3}a+a\_R}\right)}{5}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+\left(9765625a^{10}-9765625a^{8}b^{2}\right)\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(e^{ix}+\left(\right.\right.$

input
```
int(1/(a-b*sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a-16*_R^4*b+
4*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a-32*_
Z^5*b+10*_Z^4*a+5*_Z^2*a+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\sin^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-b*sin(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]

$$\int \frac{1}{a-b\sin^5(x)}\,dx = \int \frac{1}{a-b\sin^5{(x)}}\,dx$$

input
```
integrate(1/(a-b*sin(x)**5),x)
```

output
```
Integral(1/(a - b*sin(x)**5), x)
```

Maxima [F]

$$\int \frac{1}{a - b\sin^5(x)}\,dx = \int -\frac{1}{b\sin(x)^5 - a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^5),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^5 - a), x)
```

Giac [F]

$$\int \frac{1}{a - b\sin^5(x)}\,dx = \int -\frac{1}{b\sin(x)^5 - a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^5),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^5 - a), x)
```

Mupad [B] (verification not implemented)

Time = 43.50 (sec) , antiderivative size = 1515, normalized size of antiderivative = 3.94

$$\int \frac{1}{a - b\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*sin(x)^5),x)
```

output
```
symsum(log(10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
125*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d
^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d
, k)^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 195312
5*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 +
 3171875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^
6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a
^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b^
2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d
^4 - 125*a^2*d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*d
^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4
- 125*a^2*d^2 - 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
125*a^2*d^2 - 1, d, k)^8*a^8*tan(x/2) - 14000*root(9765625*a^8*b^2*d^10 -
9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*
a^2*d^2 - 1, d, k)^4*a^3*b - 175000*root(9765625*a^8*b^2*d^10 - 9765625...
```

Reduce [F]

$$\int \frac{1}{a - b\sin^5(x)}\,dx = -\left(\int \frac{1}{\sin(x)^5 b - a}dx\right)$$

input
```
int(1/(a-b*sin(x)^5),x)
```

output
```
 - int(1/(sin(x)**5*b - a),x)
```

3.78 $\int \frac{1}{a+b\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sin(x)^2}dx$$
$$\Big\downarrow \text{3660}$$
$$\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{\sqrt{a(a+b)}}$	23
risch	$-\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}+\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ba}-b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}$	160

input
```
int(1/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.55

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$

$$= \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2+2ab+}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right.}{4(a^2+ab)}\right.$$

$$\left.-\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*
a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 -
a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 +
a^2 + 2*a*b + b^2))/(a^2 + a*b), -1/2*arctan(1/2*((2*a + b)*cos(x)^2 - a -
 b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15745 vs. $2(27) = 54$.

Time = 10.52 (sec) , antiderivative size = 15745, normalized size of antiderivative = 542.93

$$\int \frac{1}{a + b\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)**2),x)
```

output
```
Piecewise((zoo*(tan(x/2)/2 - 1/(2*tan(x/2))), Eq(a, 0) & Eq(b, 0)), ((tan(
x/2)/2 - 1/(2*tan(x/2)))/b, Eq(a, 0)), (2*tan(x/2)/(b*tan(x/2)**2 - b), Eq
(a, -b)), (x/a, Eq(b, 0)), (6*a**3*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/
a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(x/2))/(10*a**4*b*sqr
t(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) - 2*a**4*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(
-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b**2*sqrt(-1 - 2*b/a - 2*sqrt
(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*b*sqrt(a
*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqr
t(a*b + b**2)/a) + 72*a**2*b**3*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sq
rt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a**2*b**2*sqrt(a*b + b**2)*sqrt
(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a
) + 32*a*b**4*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*
sqrt(a*b + b**2)/a) - 32*a*b**3*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(
a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)) - 6*a**3*b*sqrt(-1
 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) + tan(x/2))/(10*a**4*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1
- 2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a
- 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*
b**2*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{a+b\sin^2(x)}\,dx = \frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan((a + b)*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+b\sin^2(x)}\,dx = \frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a
^2 + a*b)))/sqrt(a^2 + a*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+b\sin^2(x)}\,dx = \frac{\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*sin(x)^2),x)
```

output
```
atan((tan(x)*(2*a + 2*b))/(2*(a*b + a^2)^(1/2)))/(a*b + a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 427, normalized size of antiderivative = 14.72

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{b}\,\sqrt{a+b}\,\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a}{\sqrt{a}\,\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}}\right)+2\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}\,atan\left(\frac{}{\sqrt{}}\right.\right.}{}$$

input
```
int(1/(a+b*sin(x)^2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(b)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*a
tan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b))) + 2*sqrt
(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b
)*sqrt(a + b) + a + 2*b)))*a + 2*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*ata
n((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*b - sqrt(b
)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log( - sqrt(2*sqrt(b)*
sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2)) + sqrt(b)*sqrt(a + b)*sqrt(2*sq
rt(b)*sqrt(a + b) - a - 2*b)*log(sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + s
qrt(a)*tan(x/2)) - sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log( - sqrt(2*sqr
t(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2))*a - sqrt(2*sqrt(b)*sqrt(a
+ b) - a - 2*b)*log( - sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan
(x/2))*b + sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log(sqrt(2*sqrt(b)*sqrt(a
 + b) - a - 2*b) + sqrt(a)*tan(x/2))*a + sqrt(2*sqrt(b)*sqrt(a + b) - a -
2*b)*log(sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2))*b))/(2*
a**2*(a + b))
```

3.79 $\int \frac{1}{a+b\sin^4(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{1}{a+b\sin^4(x)}\,dx=\frac{\arctan\left(\sqrt{1+\sqrt{-\frac{b}{a}}}\tan(x)\right)}{2a\sqrt{1+\sqrt{-\frac{b}{a}}}}+\frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt{-\frac{b}{a}}}\tan(x)\right)}{2a\sqrt{-1+\sqrt{-\frac{b}{a}}}}$$

output
```
1/2*arctan((1+(-b/a)^(1/2))^(1/2)*tan(x))/a/(1+(-b/a)^(1/2))^(1/2)+1/2*arc
tanh((-1+(-b/a)^(1/2))^(1/2)*tan(x))/a/(-1+(-b/a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\int \frac{1}{a+b\sin^4(x)}\,dx$$
$$=\frac{\left(\sqrt{a}-i\sqrt{b}\right)\sqrt{a+i\sqrt{a}\sqrt{b}}\arctan\left(\frac{\sqrt{a+i\sqrt{a}\sqrt{b}}\tan(x)}{\sqrt{a}}\right)-\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{-a+i\sqrt{a}\sqrt{b}}\operatorname{arctanh}\left(\frac{\sqrt{-a+i\sqrt{a}\sqrt{}}}{\sqrt{a}}\right.}{2a(a+b)}$$

input
```
Integrate[(a + b*Sin[x]^4)^(-1),x]
```

output
```
((Sqrt[a] - I*Sqrt[b])*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*ArcTan[(Sqrt[a + I*Sqrt
[a]*Sqrt[b]]*Tan[x])/Sqrt[a]] - (Sqrt[a] + I*Sqrt[b])*Sqrt[-a + I*Sqrt[a]*
Sqrt[b]]*ArcTanh[(Sqrt[-a + I*Sqrt[a]*Sqrt[b]]*Tan[x])/Sqrt[a]])/(2*a*(a +
 b))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 541 vs. $2(87) = 174$.

Time = 1.31 (sec) , antiderivative size = 541, normalized size of antiderivative = 6.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3688, 1483, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin(x)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\int \frac{\tan^2(x)+1}{(a+b)\tan^4(x)+2a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{1483}$$

$$\frac{\sqrt[4]{a+b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\tan(x)}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\sqrt[4]{a+b}\int \frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\tan(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$\Big\downarrow$ 1142

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int -\frac{\sqrt{2}\left(\sqrt[4]{\cdots}\right.\cdots}{\tan^2(x)-\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\tan\cdots\right.}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\Big\downarrow$ 25

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\tan\cdots\right.}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\Big\downarrow$ 27

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{1}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a}\cdots}}{(a+b)^{3/4}}\cdots}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(\cdots}\cdots}\cdots}{\sqrt{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{1}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\sqrt{2}\tan(x)+\frac{\sqrt[4]{\cdots}}{\cdots}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(\cdots}\cdots}\cdots}{\sqrt{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{\cdots}{-\left(2\tan(x)-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\cdots\right.}\cdots}{\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\sqrt{2}\tan(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{\cdots}{-\left(2\tan(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\cdots\right.}\cdots}{\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \; 217$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\tan(x)\cdots\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\right.}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\tan(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\right.}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\tan(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}\cdots\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}+2\tan(x)\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}\cdots\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

input

```
Int[(a + b*Sin[x]^4)^(-1),x]
```

output
```
((a + b)^(1/4)*(((Sqrt[a] + Sqrt[a + b])*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
*ArcTan[((a + b)^(3/4)*(-((Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b
]])/(a + b)^(3/4)) + 2*Tan[x]))/(Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt
[a + b]])])/(Sqrt[a + b]*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((1 - Sqrt[a
]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b - Sq
rt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/2))/(2*Sqrt[2]*a^(3/4
)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) + ((a + b)^(1/4)*(((Sqrt[a] + Sqrt[a
+ b])*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*ArcTan[((a + b)^(3/4)*((Sqrt[2]*a^
(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])/(a + b)^(3/4) + 2*Tan[x]))/(Sqrt[
2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b +
Sqrt[a]*Sqrt[a + b]]) + ((1 - Sqrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/
4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3
/4)*Tan[x]^2])/2))/(2*Sqrt[2]*a^(3/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

method	result
risch	$\sum_{\_R=\text{RootOf}\left(1+\left(256a^4+256a^3b\right)\_Z^4+32a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{128ia^4}{b}-128ia^3\right)\_R^3+\left(\frac{32a^3}{b}+32a^2\right)\_$
default	$\frac{\frac{\left(-a^{\frac{5}{2}}\sqrt{2\sqrt{a^2+ba}-2a}-a^{\frac{3}{2}}\sqrt{a^2+ba}\,\sqrt{2\sqrt{a^2+ba}-2a}+\sqrt{a+b}\,\sqrt{a^2+ba}\,\sqrt{2\sqrt{a^2+ba}-2a}\,a+\sqrt{a+b}\,\sqrt{2\sqrt{a^2+ba}-2a}\,a^2\right)\ln\left(\sqrt{a+b}\,\tan(x)^2+\tan(x)\sqrt{\right.}{2\sqrt{a+b}}}{}$

input
```
int(1/(a+b*sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*x)+(-128*I/b*a^4-128*I*a^3)*_R^3+(32/b*a^3+32*a^2)*_R^2+
(-8*I/b*a^2+8*I*a)*_R+2/b*a-1),_R=RootOf(1+(256*a^4+256*a^3*b)*_Z^4+32*a^2
*_Z^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(67) = 134$.

Time = 0.20 (sec) , antiderivative size = 823, normalized size of antiderivative = 9.46

$$\int \frac{1}{a + b\sin^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b
))*log(1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^
5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2
*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*
cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(-((a^2 +
a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(1/4*b*cos(x)
^2 - 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b
^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))
+ 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(
a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 +
2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 + 1/2*(a*b*cos(x
)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))
*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) -
1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*
b^2)) + 1/4*b) - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))
- 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 - 1/2*(a*b*cos(x)*sin(x) - (a^4 + a^
3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*s
qrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2
*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(x)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \int \frac{1}{b\sin(x)^4+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(67) = 134$.

Time = 0.62 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.66

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \frac{\left(3\sqrt{a^2+ab+\sqrt{-ab}(a+b)}a^2+6\sqrt{a^2+ab+\sqrt{-ab}(a+b)}ab-\sqrt{a^2+ab+\sqrt{-ab}(a+b)}b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}\right.\right.}{2\left(3a^5+12a^4b+14a^3b^2+4a^2b^3-ab^4\right)} + \frac{\left(3\sqrt{a^2+ab-\sqrt{-ab}(a+b)}a^2+6\sqrt{a^2+ab-\sqrt{-ab}(a+b)}ab-\sqrt{a^2+ab-\sqrt{-ab}(a+b)}b^2\right)\left(\pi\left\lfloor\right.\right.}{2\left(3a^5+12a^4b+14a^3b^2+4a^2b^3-ab^4\right)}$$

input
```
integrate(1/(a+b*sin(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b + sqrt(
-a*b)*(a + b))*a*b - sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x
/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)*a + 16*a^2))/(a
+ b))))*abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4) + 1
/2*(3*sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b - sqrt(-
a*b)*(a + b))*a*b - sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x/
pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/(a +
 b))))*abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4)
```

Mupad [B] (verification not implemented)

Time = 38.50 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.68

$$\int \frac{1}{a+b\sin^4(x)}\,dx$$
$$= \operatorname{atan}\left(\frac{a^3\tan(x)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\tan(x)\,\left(-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)^{3/2}64\mathrm{i}-a^2\,b\tan(x)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^4}{\sqrt{-a^3\,b}}\right.$$
$$-\operatorname{atan}\left(\frac{a^3\tan(x)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\tan(x)\,\left(-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)^{3/2}64\mathrm{i}-a^2\,b\tan(x)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+}{\sqrt{-a^3\,b}}\right.$$

input
```
int(1/(a + b*sin(x)^4),x)
```

output
```
atan((a^3*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i +
a^5*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b
*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan
(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2
))*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*2i - atan((a^3*tan(
x)*(-(a^2 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^5*tan(x)*(-(
a^2 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b*tan(x)*(-(a^2
 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan(x)*(-(a^2 + (
-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2))*(-(a^2 + (-
a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*2i
```

Reduce [F]

$$\int \frac{1}{a+b\sin^4(x)}\,dx=\int \frac{1}{\sin(x)^4 b+a}dx$$

input
```
int(1/(a+b*sin(x)^4),x)
```

output
```
int(1/(sin(x)**4*b + a),x)
```

3.80 $\int \frac{1}{a+b\sin^6(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 179

$$\int \frac{1}{a+b\sin^6(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
1/3*arctan((a^(1/3)+b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/3
))^(1/2)+1/3*arctanh((-a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a
^(5/6)/(-a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh((-a^(1/3)-(-1)^(2/3
)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(-a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\sin^6(x)}\,dx$$
$$= -\frac{8}{3}\text{RootSum}\left[b-6b\#1+15b\#1^2-64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5 + b\#1^6\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{-b+5b\#1-32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

input
```
Integrate[(a + b*Sin[x]^6)^(-1),x]
```

output
```
(-8*RootSum[b - 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6
*b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 -
2*Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 - 32*a*#1^2 - 10*b*#1^2 + 10*b*#1
^3 - 5*b*#1^4 + b*#1^5) & ])/3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^6(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sin(x)^6}dx$$
$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{\int \frac{1}{\left(\frac{\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{3a}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a + b*Sin[x]^6)^(-1),x]
```

output
```
ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
 b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(
3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1
)^(2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(
1/3)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.38

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a+\_R^5b+2\_R^3a+a\_R}\right)}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6+46656a^5b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{15552ia^6}{b}+15552ia^5\right)\_R^5+\right.$

input
```
int(1/(a+b*sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Roo
tOf((a+b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 15501, normalized size of antiderivative = 86.60

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \int \frac{1}{a+b\sin^6{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**6),x)
```

output
```
Integral(1/(a + b*sin(x)**6), x)
```

Maxima [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \int \frac{1}{b\sin{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^6),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^6 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \int \frac{1}{b\sin{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^6),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^6 + a), x)
```

Mupad [B] (verification not implemented)

Time = 40.27 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.87

$$\int \frac{1}{a+b\sin^6(x)}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{b^3\,(a+b)\,\left(-\cot(x)+\mathrm{root}(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k)\;a\,8-\right.}{}\right.$$
$$+\,46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k)$$

input
```
int(1/(a + b*sin(x)^6),x)
```

output
```
symsum(log(-(3*b^3*(a + b)*(8*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*
a^4*d^4 + 108*a^2*d^2 + 1, d, k)*a - cot(x) + 2*root(46656*a^5*b*d^6 + 466
56*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*b + 504*root(46656*a^5*
b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^3*a^3 + 7776
*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d,
 k)^5*a^5 - 144*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*
a^2*d^2 + 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3
888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^5*a^4*b - 60*root(46656*a^5*b*d^6 + 4
6656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2*cot(x) - 864*ro
ot(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)
^4*a^4*cot(x) - 864*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 +
108*a^2*d^2 + 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 + 46656*a^
6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a*b*cot(x)))/cot(x))*root(
46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k
, 1, 6)
```

Reduce [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx=\int \frac{1}{\sin(x)^6 b+a}dx$$

input
```
int(1/(a+b*sin(x)^6),x)
```

output
```
int(1/(sin(x)**6*b + a),x)
```

3.81 $\int \frac{1}{a+b\sin^8(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 189

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \frac{\arctan\left(\sqrt{1-i\sqrt[4]{-\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{1-i\sqrt[4]{-\frac{b}{a}}}} + \frac{\arctan\left(\sqrt{1+\sqrt[4]{-\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{1+\sqrt[4]{-\frac{b}{a}}}} + \frac{\operatorname{arctanh}\left(\sqrt{-1-i\sqrt[4]{-\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{-1-i\sqrt[4]{-\frac{b}{a}}}} + \frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt[4]{-\frac{b}{a}}}\tan(x)\right)}{4a\sqrt{-1+\sqrt[4]{-\frac{b}{a}}}}$$

output
```
1/4*arctan((1-I*(-b/a)^(1/4))^(1/2)*tan(x))/a/(1-I*(-b/a)^(1/4))^(1/2)+1/4
*arctan((1+(-b/a)^(1/4))^(1/2)*tan(x))/a/(1+(-b/a)^(1/4))^(1/2)+1/4*arctan
h((-1-I*(-b/a)^(1/4))^(1/2)*tan(x))/a/(-1-I*(-b/a)^(1/4))^(1/2)+1/4*arctan
h((-1+(-b/a)^(1/4))^(1/2)*tan(x))/a/(-1+(-b/a)^(1/4))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+b\sin^8(x)}\,dx = 8\text{RootSum}\Big[b-8b\#1+28b\#1^2-56b\#1^3+256a\#1^4+70b\#1^4 -56b\#1^5+28b\#1^6-8b\#1^7 +b\#1^8\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-b+7b\#1-21b\#1^2+128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\Big]$$

input
```
Integrate[(a + b*Sin[x]^8)^(-1),x]
```

output
```
8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
 #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^
2 + 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) &
]
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^8(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\sin(x)^8}dx$$

$$\Big\downarrow \; 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \; 3660$$

$$\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}a}{(-a)^{5/4}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{(-a)^{5/8}\arctan\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\tan(x)}{(-a)^{5/8}}\right)}{4a\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

input
```
Int[(a + b*Sin[x]^8)^(-1),x]
```

output
```
((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tan[x])/(-a)^(1/8)])/(4*a
*Sqrt[(-a)^(1/4) - I*b^(1/4)]) + ((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) + I*b
^(1/4)]*Tan[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)^(1/4) + I*b^(1/4)]) + ((-a)^(1
/8)*ArcTan[(Sqrt[(-a)^(1/4) + b^(1/4)]*Tan[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)
^(1/4) + b^(1/4)]) + ((-a)^(5/8)*ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tan[
x])/(-a)^(5/8)])/(4*a*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.45

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7a+\_R^7b+3\_R^5a+3\_R^3a+a\_R}\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8+16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{4194304ia^8}{b}\right.\right.$

input
```
int(1/(a+b*sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a+_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(t
an(x)-_R),_R=RootOf((a+b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665483 vs. 2(141) = 282.

Time = 6.08 (sec) , antiderivative size = 665483, normalized size of antiderivative = 3521.07

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx=\int \frac{1}{a+b\sin^8{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**8),x)
```

output
```
Integral(1/(a + b*sin(x)**8), x)
```

Maxima [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx=\int \frac{1}{b\sin{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^8 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx=\int \frac{1}{b\sin{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^8),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^8 + a), x)
```

Mupad [B] (verification not implemented)

Time = 43.08 (sec) , antiderivative size = 816, normalized size of antiderivative = 4.32

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*sin(x)^8),x)
```

output
```
symsum(log(-2*b^5*(a + b)*(800*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8
+ 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^2*a^2 + 43008*r
oot(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^
4 + 256*a^2*d^2 + 1, d, k)^4*a^4 + 786432*root(16777216*a^7*b*d^8 + 167772
16*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^
6 + 4*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576
*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*b*tan(x) - 6144*root(16777216*a^7*b*d^8
+ 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d,
 k)^4*a^3*b + 786432*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*
a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^5*b - 9984*root(16777
216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a
^2*d^2 + 1, d, k)^3*a^3*tan(x) - 557056*root(16777216*a^7*b*d^8 + 16777216
*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^5*a^5*
tan(x) - 10485760*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6
*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^7*a^7*tan(x) + 32*root(16777
216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a
^2*d^2 + 1, d, k)^2*a*b - 60*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 +
1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 768*ro
ot(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4
 + 256*a^2*d^2 + 1, d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^...
```

Reduce [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \int \frac{1}{\sin(x)^8 b+a}dx$$

input
```
int(1/(a+b*sin(x)^8),x)
```

output

```
int(1/(sin(x)**8*b + a),x)
```

3.82 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Integrate[(a + b*Sin[x])^(-1),x]
```

output
```
(2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{a+b\sin(x)}dx$$
$$\Big\downarrow \ 3139$$
$$2\int \frac{1}{a\tan^2\left(\frac{x}{2}\right)+2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \ 1083$$
$$-4\int \frac{1}{-\left(2b+2a\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{x}{2}\right)\right)$$
$$\Big\downarrow \ 217$$
$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Int[(a + b*Sin[x])^(-1),x]
```

output
```
(2*ArcTan[(2*b + 2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}$	39
risch	$-\frac{\ln\left({\mathrm e}^{i x}+\frac{i\sqrt{-a^{2}+b^{2}}\,a-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left({\mathrm e}^{i x}+\frac{i\sqrt{-a^{2}+b^{2}}\,a+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}}$	119

input
```
int(1/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.70

$$\int \frac{1}{a+b\sin(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\,(a^2-b^2)},\ -\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 -
b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a
*b*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b
^2)*cos(x)))/sqrt(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(31) = 62$.

Time = 1.66 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.50

$$\int \frac{1}{a+b\sin(x)}\,dx = \begin{cases} \tilde{\infty}\log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ \frac{2}{b\tan\left(\frac{x}{2}\right)-b} & \text{for } a=-b \\ -\frac{2}{b\tan\left(\frac{x}{2}\right)+b} & \text{for } a=b \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*sin(x)),x)
```

output
```
Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a
, 0)), (2/(b*tan(x/2) - b), Eq(a, -b)), (-2/(b*tan(x/2) + b), Eq(a, b)), (
log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/
2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 40.90 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b*sin(x)),x)
```

output
```
(2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

input
```
int(1/(a+b*sin(x)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan(x/2)*a + b)/sqrt(a**2 - b**2)))/(a**2 - b**
2)
```

3.83 $\int \frac{1}{a+b\sin^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 224

$$\int \frac{1}{a+b\sin^3(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

output

```
2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*x))/(a^(2/3)-b^(2/3))^(1/2))/a^(2/3)/(
a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*x))/
(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(
1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*x))/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.50

$$\int \frac{1}{a+b\sin^3(x)}\,dx = -\frac{2}{3}i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]$$

input
```
Integrate[(a + b*Sin[x]^3)^(-1),x]
```

output
```
((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - I*Log[1 - 2*Cos[x]*#1 + #1^2]
*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^3(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+b\sin(x)^3}dx$$

$$\Big\downarrow \ 3692$$

$$\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(x)\right)}-\frac{1}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}\sin(x)-\sqrt[3]{a}\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(x)\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{x}{2}\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{x}{2}\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

input
```
Int[(a + b*Sin[x]^3)^(-1),x]
```

output
```
(2*ArcTan[(b^(1/3) + a^(1/3)*Tan[x/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*a^(2/3
)*Sqrt[a^(2/3) - b^(2/3)]) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[x
/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(
1/3)*b^(2/3)]) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[x
/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^
(2/3)*b^(2/3)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+a\_R}\right)}{3}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(729a^6-729a^4b^2\right)\_Z^6+243a^4\_Z^4+27a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(-\frac{486a^6}{b}+486a^4b\right)\_R^5+\left(-\frac{81ia^5}{b}\right.\right.$

input
```
int(1/(a+b*sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*x)-_R),
_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 18599, normalized size of antiderivative = 83.03

$$\int \frac{1}{a+b\sin^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\sin^3(x)}\,dx=\int \frac{1}{a+b\sin^3{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**3),x)
```

output
```
Integral(1/(a + b*sin(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{a+b\sin^3(x)}\,dx=\int \frac{1}{b\sin{(x)}^3+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^3 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\sin^3(x)}\,dx=\int \frac{1}{b\sin{(x)}^3+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^3),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 44.38 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.54

$$\int \frac{1}{a+b\sin^3(x)}\,dx$$

$$= \sum_{k=1}^{6} \ln\left(-\frac{8192\,a\,b^3\left(-729\,a^5+243\tan\left(\frac{x}{2}\right)\,a^4\,b-324\tan\left(\frac{x}{2}\right)\,a^4\operatorname{root}(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\right.}{}\right.$$

$$-729\,a^6\,d^6-243\,a^4\,d^4-27\,a^2\,d^2-1,d,k\Big)$$

input
```
int(1/(a + b*sin(x)^3),x)
```

output
```
symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 9*a*root(d^6 + 27*a^2*d^4 + 243*a^4
*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 729*a^5 - 4*tan(x/2)*root(d^6 + 27*a
^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 - 162*a^3*root(d^6 + 2
7*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 72*a^2*tan(x/2)*r
oot(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3 + 243*a^
4*b*tan(x/2) - 324*tan(x/2)*a^4*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*
a^4*(a^2 - b^2), d, k) + 24*b*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2
 + 729*a^4*(a^2 - b^2), d, k)^4 + 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d
^2 + 729*a^4*(a^2 - b^2), d, k)^3 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 648*tan(x/2)*a^2*b^2*root(d^6 + 27*
a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(x/2)*ro
ot(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^
6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*
b^2*d^6 - 729*a^6*d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{1}{a+b\sin^3(x)}\,dx = \int \frac{1}{\sin(x)^3\,b+a}dx$$

input
```
int(1/(a+b*sin(x)^3),x)
```

output
```
int(1/(sin(x)**3*b + a),x)
```

3.84 $\int \frac{1}{a+b\sin^5(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 384

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{(-1)^{2/5}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}$$
$$+\frac{2\arctan\left(\frac{(-1)^{4/5}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}$$
$$-\frac{2\arctan\left(\frac{(-1)^{3/5}\left(\sqrt[5]{b}+(-1)^{2/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}$$
$$-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left(\sqrt[5]{b}+(-1)^{4/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

output
```
2/5*arctan((b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(
a^(2/5)-b^(2/5))^(1/2)+2/5*arctan(((-1)^(2/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/
(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(
1/2)+2/5*arctan(((-1)^(4/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5
)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)-2/5*arctan((-
1)^(3/5)*(b^(1/5)+(-1)^(2/5)*a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(1/5)*b^(2/
5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2)-2/5*arctan((-1)^(1/5
)*(b^(1/5)+(-1)^(4/5)*a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/
2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.39

$$\int \frac{1}{a+b\sin^5(x)}\,dx$$
$$= \frac{8}{5}i\text{RootSum}\left[ib-5ib\#1^2+10ib\#1^4+32a\#1^5-10ib\#1^6+5ib\#1^8 - ib\#1^{10}\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b-4b\#1^2+16ia\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a + b*Sin[x]^5)^(-1),x]
```

output
```
((8*I)/5)*RootSum[I*b - (5*I)*b*#1^2 + (10*I)*b*#1^4 + 32*a*#1^5 - (10*I)*
b*#1^6 + (5*I)*b*#1^8 - I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3
 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 + (16*I)*a*#1^3 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^5(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin(x)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}-\frac{1}{5a^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{b}\sin(x)-\sqrt[5]{a}\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)\right)}-\frac{}{5a^{4/5}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

input
```
Int[(a + b*Sin[x]^5)^(-1),x]
```

output
```
(2*ArcTan[(b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5
)*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(2/5)*b^(1/5) + a^(1/5)*Tan[x
/2])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(
4/5)*b^(2/5)]) + (2*ArcTan[((-1)^(4/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^
(2/5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)
]) - (2*ArcTan[((-1)^(3/5)*(b^(1/5) + (-1)^(2/5)*a^(1/5)*Tan[x/2]))/Sqrt[a
^(2/5) + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5
)]) - (2*ArcTan[((-1)^(1/5)*(b^(1/5) + (-1)^(4/5)*a^(1/5)*Tan[x/2]))/Sqrt[
a^(2/5) - (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/
5)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.28

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}+32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a+16\_R^{4}b+4\_R^{3}a+a\_R}\right)}{5}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+\left(9765625a^{10}-9765625a^{8}b^{2}\right)\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(e^{ix}+\left(\right.\right.$

input
```
int(1/(a+b*sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a+16*_R^4*b+
4*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a+32*_
Z^5*b+10*_Z^4*a+5*_Z^2*a+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*sin(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \int \frac{1}{a+b\sin^5{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**5),x)
```

output
```
Integral(1/(a + b*sin(x)**5), x)
```

Maxima [F]

$$\int \frac{1}{a+b\sin^5(x)}\,dx=\int \frac{1}{b\sin{(x)}^5+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^5),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^5 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\sin^5(x)}\,dx=\int \frac{1}{b\sin{(x)}^5+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^5),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^5 + a), x)
```

Mupad [B] (verification not implemented)

Time = 42.46 (sec) , antiderivative size = 1515, normalized size of antiderivative = 3.95

$$\int \frac{1}{a+b\sin^5(x)}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*sin(x)^5),x)
```

output
```
symsum(log(-10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^
10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
 125*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*
d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 19531
25*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5
+ 3171875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
- 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*
root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a
^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*
a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b
^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*
d^4 - 125*a^2*d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*
d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4
 - 125*a^2*d^2 - 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^
10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
 125*a^2*d^2 - 1, d, k)^8*a^8*tan(x/2) + 14000*root(9765625*a^8*b^2*d^10 -
 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125
*a^2*d^2 - 1, d, k)^4*a^3*b + 175000*root(9765625*a^8*b^2*d^10 - 976562...
```

Reduce [F]

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \int \frac{1}{\sin(x)^5 b + a}dx$$

input
```
int(1/(a+b*sin(x)^5),x)
```

output
```
int(1/(sin(x)**5*b + a),x)
```

3.85 $\int \left(a-a\sin^2(x)\right)^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \left(a-a\sin^2(x)\right)^4\,dx=\frac{35a^4x}{128}+\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)$$

output
```
35/128*a^4*x+35/128*a^4*cos(x)*sin(x)+35/192*a^4*cos(x)^3*sin(x)+7/48*a^4*
cos(x)^5*sin(x)+1/8*a^4*cos(x)^7*sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \left(a-a\sin^2(x)\right)^4\,dx=a^4\left(\frac{35x}{128}+\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)+\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}\right)$$

input
```
Integrate[(a - a*Sin[x]^2)^4,x]
```

output
```
a^4*((35*x)/128 + (7*Sin[2*x])/32 + (7*Sin[4*x])/128 + Sin[6*x]/96 + Sin[8
*x]/1024)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^4 dx$$

$$\Big\downarrow \text{3654}$$

$$a^4\int \cos^8(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \sin\left(x+\frac{\pi}{2}\right)^8 dx$$

$$\Big\downarrow \text{3115}$$

$$a^4\left(\frac{7}{8}\int \cos^6(x)dx+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{7}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6 dx+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4 dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a^4\left(\frac{1}{8}\sin(x)\cos^7(x)+\frac{7}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^4,x]
```

output
```
a^4*((Cos[x]^7*Sin[x])/8 + (7*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])
/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4))/6))/8)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 12.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{a^4(672\sin(2x)+840x+3\sin(8x)+32\sin(6x)+168\sin(4x))}{3072}$
risch	$\frac{35a^4x}{128}+\frac{a^4\sin(8x)}{1024}+\frac{a^4\sin(6x)}{96}+\frac{7a^4\sin(4x)}{128}+\frac{7a^4\sin(2x)}{32}$
default	$a^4\left(-\frac{\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)-4a^4\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\right.$
parts	$a^4\left(-\frac{\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)-4a^4\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\right.$
norman	$\frac{\frac{35a^4x}{128}+\frac{93a^4\tan\left(\frac{x}{2}\right)}{64}+\frac{91a^4\tan\left(\frac{x}{2}\right)^3}{192}+\frac{1799a^4\tan\left(\frac{x}{2}\right)^5}{192}-\frac{1085a^4\tan\left(\frac{x}{2}\right)^7}{192}+\frac{1085a^4\tan\left(\frac{x}{2}\right)^9}{192}-\frac{1799a^4\tan\left(\frac{x}{2}\right)^{11}}{192}-\frac{91a^4\tan\left(\frac{x}{2}\right)^{13}}{192}-}{}$

input
```
int((a-a*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3072*a^4*(672*sin(2*x)+840*x+3*sin(8*x)+32*sin(6*x)+168*sin(4*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \left(a-a\sin^2(x)\right)^4\,dx$$
$$=\frac{35}{128}\,a^4x$$
$$+\frac{1}{384}\left(48\,a^4\cos\left(x\right)^7+56\,a^4\cos\left(x\right)^5+70\,a^4\cos\left(x\right)^3+105\,a^4\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate((a-a*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
35/128*a^4*x + 1/384*(48*a^4*cos(x)^7 + 56*a^4*cos(x)^5 + 70*a^4*cos(x)^3
+ 105*a^4*cos(x))*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(65) = 130$.

Time = 0.53 (sec) , antiderivative size = 376, normalized size of antiderivative = 6.37

$$\int \left(a-a\sin^2(x)\right)^4\,dx = \frac{35a^4x\sin^8(x)}{128} + \frac{35a^4x\sin^6(x)\cos^2(x)}{32} - \frac{5a^4x\sin^6(x)}{4} + \frac{105a^4x\sin^4(x)\cos^4(x)}{64} - \frac{15a^4x\sin^4(x)\cos^2(x)}{4} + \frac{9a^4x\sin^4(x)}{4} + \frac{35a^4x\sin^2(x)\cos^6(x)}{32} - \frac{15a^4x\sin^2(x)\cos^4(x)}{4} + \frac{9a^4x\sin^2(x)\cos^2(x)}{2} - 2a^4x\sin^2(x) + \frac{35a^4x\cos^8(x)}{128} - \frac{5a^4x\cos^6(x)}{4} + \frac{9a^4x\cos^4(x)}{4} - 2a^4x\cos^2(x) + a^4x - \frac{93a^4\sin^7(x)\cos(x)}{128} - \frac{511a^4\sin^5(x)\cos^3(x)}{384} + \frac{11a^4\sin^5(x)\cos(x)}{4} - \frac{385a^4\sin^3(x)\cos^5(x)}{384} + \frac{10a^4\sin^3(x)\cos^3(x)}{3} - \frac{15a^4\sin^3(x)\cos(x)}{4} - \frac{35a^4\sin(x)\cos^7(x)}{128} + \frac{5a^4\sin(x)\cos^5(x)}{4} - \frac{9a^4\sin(x)\cos^3(x)}{4} + 2a^4\sin(x)\cos(x)$$

input
```
integrate((a-a*sin(x)**2)**4,x)
```

output
```
35*a**4*x*sin(x)**8/128 + 35*a**4*x*sin(x)**6*cos(x)**2/32 - 5*a**4*x*sin(
x)**6/4 + 105*a**4*x*sin(x)**4*cos(x)**4/64 - 15*a**4*x*sin(x)**4*cos(x)**
2/4 + 9*a**4*x*sin(x)**4/4 + 35*a**4*x*sin(x)**2*cos(x)**6/32 - 15*a**4*x*
sin(x)**2*cos(x)**4/4 + 9*a**4*x*sin(x)**2*cos(x)**2/2 - 2*a**4*x*sin(x)**
2 + 35*a**4*x*cos(x)**8/128 - 5*a**4*x*cos(x)**6/4 + 9*a**4*x*cos(x)**4/4
- 2*a**4*x*cos(x)**2 + a**4*x - 93*a**4*sin(x)**7*cos(x)/128 - 511*a**4*si
n(x)**5*cos(x)**3/384 + 11*a**4*sin(x)**5*cos(x)/4 - 385*a**4*sin(x)**3*co
s(x)**5/384 + 10*a**4*sin(x)**3*cos(x)**3/3 - 15*a**4*sin(x)**3*cos(x)/4 -
 35*a**4*sin(x)*cos(x)**7/128 + 5*a**4*sin(x)*cos(x)**5/4 - 9*a**4*sin(x)*
cos(x)**3/4 + 2*a**4*sin(x)*cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \left(a - a\sin^2(x)\right)^4 \, dx &= \frac{1}{3072}\left(128\,\sin\left(2\,x\right)^3 + 840\,x + 3\,\sin\left(8\,x\right) + 168\,\sin\left(4\,x\right) - 768\,\sin\left(2\,x\right)\right)a^4 \\ &\quad - \frac{1}{48}\left(4\,\sin\left(2\,x\right)^3 + 60\,x + 9\,\sin\left(4\,x\right) - 48\,\sin\left(2\,x\right)\right)a^4 \\ &\quad + \frac{3}{16}\,a^4(12\,x + \sin\left(4\,x\right) - 8\,\sin\left(2\,x\right)) - a^4(2\,x - \sin\left(2\,x\right)) + a^4x\end{aligned}$$

input
```
integrate((a-a*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/3072*(128*sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))
*a^4 - 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^4 + 3/16*a^
4*(12*x + sin(4*x) - 8*sin(2*x)) - a^4*(2*x - sin(2*x)) + a^4*x
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \left(a-a\sin^2(x)\right)^4\,dx=\frac{35}{128}\,a^4x+\frac{1}{1024}\,a^4\sin\left(8\,x\right)+\frac{1}{96}\,a^4\sin\left(6\,x\right)+\frac{7}{128}\,a^4\sin\left(4\,x\right)+\frac{7}{32}\,a^4\sin\left(2\,x\right)$$

input
```
integrate((a-a*sin(x)^2)^4,x, algorithm="giac")
```

output
```
35/128*a^4*x + 1/1024*a^4*sin(8*x) + 1/96*a^4*sin(6*x) + 7/128*a^4*sin(4*x
) + 7/32*a^4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 36.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \left(a-a\sin^2(x)\right)^4\,dx=\frac{\frac{35\,a^4\,\tan(x)^7}{128}+\frac{385\,a^4\,\tan(x)^5}{384}+\frac{511\,a^4\,\tan(x)^3}{384}+\frac{93\,a^4\,\tan(x)}{128}}{\left(\tan\left(x\right)^2+1\right)^4}+\frac{35\,a^4\,x}{128}$$

input
```
int((a - a*sin(x)^2)^4,x)
```

output
```
((93*a^4*tan(x))/128 + (511*a^4*tan(x)^3)/384 + (385*a^4*tan(x)^5)/384 + (
35*a^4*tan(x)^7)/128)/(tan(x)^2 + 1)^4 + (35*a^4*x)/128
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \left(a-a\sin^2(x)\right)^4\,dx=\frac{a^4\left(-48\cos\left(x\right)\sin\left(x\right)^7+200\cos\left(x\right)\sin\left(x\right)^5-326\cos\left(x\right)\sin\left(x\right)^3+279\cos\left(x\right)\sin\left(x\right)+105x\right)}{384}$$

input
```
int((a-a*sin(x)^2)^4,x)
```

output
```
(a**4*( - 48*cos(x)*sin(x)**7 + 200*cos(x)*sin(x)**5 - 326*cos(x)*sin(x)**
3 + 279*cos(x)*sin(x) + 105*x))/384
```

3.86 $\int (a - a\sin^2(x))^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a - a\sin^2(x))^3 \, dx = \frac{5a^3x}{16} + \frac{5}{16}a^3\cos(x)\sin(x) + \frac{5}{24}a^3\cos^3(x)\sin(x) + \frac{1}{6}a^3\cos^5(x)\sin(x)$$

output
```
5/16*a^3*x+5/16*a^3*cos(x)*sin(x)+5/24*a^3*cos(x)^3*sin(x)+1/6*a^3*cos(x)^
5*sin(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int (a - a\sin^2(x))^3 \, dx = a^3\left(\frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)\right)$$

input
```
Integrate[(a - a*Sin[x]^2)^3,x]
```

output
```
a^3*((5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^3dx$$

$$\Big\downarrow \text{3654}$$

$$a^3\int \cos^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^3\int \sin\left(x+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$a^3\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^3\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a^3\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^3,x]
```

output
```
a^3*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin
[x])/2))/4))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{a^{3}\left(45 \sin \left(2 x\right)+60 x+\sin \left(6 x\right)+9 \sin \left(4 x\right)\right)}{192}$
risch	$\frac{5 a^{3} x}{16}+\frac{a^{3} \sin \left(6 x\right)}{192}+\frac{3 a^{3} \sin \left(4 x\right)}{64}+\frac{15 a^{3} \sin \left(2 x\right)}{64}$
default	$-a^{3} \left(-\frac{\left(\sin \left(x\right)^{5}+\frac{5 \sin \left(x\right)^{3}}{4}+\frac{15 \sin \left(x\right)}{8}\right) \cos \left(x\right)}{6}+\frac{5 x}{16}\right)+3 a^{3} \left(-\frac{\left(\sin \left(x\right)^{3}+\frac{3 \sin \left(x\right)}{2}\right) \cos \left(x\right)}{4}+\frac{3 x}{8}\right)-3 a^{3} \left(-\frac{\mathrm{c}}{}\right.$
parts	$-a^{3} \left(-\frac{\left(\sin \left(x\right)^{5}+\frac{5 \sin \left(x\right)^{3}}{4}+\frac{15 \sin \left(x\right)}{8}\right) \cos \left(x\right)}{6}+\frac{5 x}{16}\right)+3 a^{3} \left(-\frac{\left(\sin \left(x\right)^{3}+\frac{3 \sin \left(x\right)}{2}\right) \cos \left(x\right)}{4}+\frac{3 x}{8}\right)-3 a^{3} \left(-\frac{\mathrm{c}}{}\right.$
norman	$\frac{\frac{5 a^{3} x}{16}+\frac{11 a^{3} \tan \left(\frac{x}{2}\right)}{8}-\frac{5 a^{3} \tan \left(\frac{x}{2}\right)^{3}}{24}+\frac{15 a^{3} \tan \left(\frac{x}{2}\right)^{5}}{4}-\frac{15 a^{3} \tan \left(\frac{x}{2}\right)^{7}}{4}+\frac{5 a^{3} \tan \left(\frac{x}{2}\right)^{9}}{24}-\frac{11 a^{3} \tan \left(\frac{x}{2}\right)^{11}}{8}+\frac{15 a^{3} x \tan \left(\frac{x}{2}\right)^{2}}{8}+\frac{75 a^{3} x \tan (}{16}}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right)^{6}}$
orering	$\left(\frac{x}{2}+\frac{5}{75072}\right) \left(a -a \sin \left(x \right)^{2}\right)^{3}+\frac{18863 \left(a -a \sin \left(x\right)^{2}\right)^{2} a \cos \left(x\right) \sin \left(x\right)}{12512}+\left(\frac{49 x}{288}+\frac{5}{75072}\right) \left(24 \left(a -a \sin \left(x\right.\right.\right.$

input
```
int((a-a*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*a^3*(45*sin(2*x)+60*x+sin(6*x)+9*sin(4*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a-a\sin^2(x)\right)^3\,dx=\frac{5}{16}\,a^3x+\frac{1}{48}\left(8\,a^3\cos\left(x\right)^5+10\,a^3\cos\left(x\right)^3+15\,a^3\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate((a-a*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
5/16*a^3*x + 1/48*(8*a^3*cos(x)^5 + 10*a^3*cos(x)^3 + 15*a^3*cos(x))*sin(x
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.07

$$\int \left(a - a\sin^2(x)\right)^3 \, dx = -\frac{5a^3x\sin^6(x)}{16} - \frac{15a^3x\sin^4(x)\cos^2(x)}{16} + \frac{9a^3x\sin^4(x)}{8} - \frac{15a^3x\sin^2(x)\cos^4(x)}{16} + \frac{9a^3x\sin^2(x)\cos^2(x)}{4} - \frac{3a^3x\sin^2(x)}{2} - \frac{5a^3x\cos^6(x)}{16} + \frac{9a^3x\cos^4(x)}{8} - \frac{3a^3x\cos^2(x)}{2} + a^3x + \frac{11a^3\sin^5(x)\cos(x)}{16} + \frac{5a^3\sin^3(x)\cos^3(x)}{6} - \frac{15a^3\sin^3(x)\cos(x)}{8} + \frac{5a^3\sin(x)\cos^5(x)}{16} - \frac{9a^3\sin(x)\cos^3(x)}{8} + \frac{3a^3\sin(x)\cos(x)}{2}$$

input
```
integrate((a-a*sin(x)**2)**3,x)
```

output
```
-5*a**3*x*sin(x)**6/16 - 15*a**3*x*sin(x)**4*cos(x)**2/16 + 9*a**3*x*sin(x
)**4/8 - 15*a**3*x*sin(x)**2*cos(x)**4/16 + 9*a**3*x*sin(x)**2*cos(x)**2/4
 - 3*a**3*x*sin(x)**2/2 - 5*a**3*x*cos(x)**6/16 + 9*a**3*x*cos(x)**4/8 - 3
*a**3*x*cos(x)**2/2 + a**3*x + 11*a**3*sin(x)**5*cos(x)/16 + 5*a**3*sin(x)
**3*cos(x)**3/6 - 15*a**3*sin(x)**3*cos(x)/8 + 5*a**3*sin(x)*cos(x)**5/16
- 9*a**3*sin(x)*cos(x)**3/8 + 3*a**3*sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \left(a - a\sin^2(x)\right)^3 \, dx = -\frac{1}{192}\left(4\sin(2x)^3 + 60x + 9\sin(4x) - 48\sin(2x)\right)a^3 + \frac{3}{32}a^3(12x + \sin(4x) - 8\sin(2x)) - \frac{3}{4}a^3(2x - \sin(2x)) + a^3x$$

input
```
integrate((a-a*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^3 + 3/32*a^3*(12
*x + sin(4*x) - 8*sin(2*x)) - 3/4*a^3*(2*x - sin(2*x)) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(a-a\sin^2(x)\right)^3 \, dx = \frac{5}{16}\,a^3x+\frac{1}{192}\,a^3\sin\left(6\,x\right)+\frac{3}{64}\,a^3\sin\left(4\,x\right)+\frac{15}{64}\,a^3\sin\left(2\,x\right)$$

input
```
integrate((a-a*sin(x)^2)^3,x, algorithm="giac")
```

output
```
5/16*a^3*x + 1/192*a^3*sin(6*x) + 3/64*a^3*sin(4*x) + 15/64*a^3*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 36.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a-a\sin^2(x)\right)^3 \, dx = \frac{11\,a^3\cos\left(x\right)^5\,\sin\left(x\right)}{16}+\frac{5\,a^3\cos\left(x\right)^3\sin\left(x\right)^3}{6}+\frac{5\,a^3\,\cos\left(x\right)\,\sin\left(x\right)^5}{16}+\frac{5\,x\,a^3}{16}$$

input
```
int((a - a*sin(x)^2)^3,x)
```

output
```
(5*a^3*x)/16 + (5*a^3*cos(x)*sin(x)^5)/16 + (11*a^3*cos(x)^5*sin(x))/16 +
(5*a^3*cos(x)^3*sin(x)^3)/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \left(a - a\sin^2(x)\right)^3 \, dx = \frac{a^3 \left(8\cos(x)\sin(x)^5 - 26\cos(x)\sin(x)^3 + 33\cos(x)\sin(x) + 15x\right)}{48}$$

input
```
int((a-a*sin(x)^2)^3,x)
```

output
```
(a**3*(8*cos(x)*sin(x)**5 - 26*cos(x)*sin(x)**3 + 33*cos(x)*sin(x) + 15*x)
)/48
```

3.87 $\int (a-a\sin^2(x))^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (a-a\sin^2(x))^2\,dx=\frac{3a^2x}{8}+\frac{3}{8}a^2\cos(x)\sin(x)+\frac{1}{4}a^2\cos^3(x)\sin(x)$$

output
```
3/8*a^2*x+3/8*a^2*cos(x)*sin(x)+1/4*a^2*cos(x)^3*sin(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (a-a\sin^2(x))^2\,dx=a^2\left(\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)\right)$$

input
```
Integrate[(a - a*Sin[x]^2)^2,x]
```

output
```
a^2*((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^2\,dx$$

$$\Big\downarrow \text{3654}$$

$$a^2\int \cos^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \sin\left(x+\frac{\pi}{2}\right)^4\,dx$$

$$\Big\downarrow \text{3115}$$

$$a^2\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2\,dx+\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a^2\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^2,x]
```

output
```
a^2*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{a^2(8\sin(2x)+12x+\sin(4x))}{32}$
risch	$\frac{3a^2x}{8}+\frac{a^2\sin(4x)}{32}+\frac{a^2\sin(2x)}{4}$
default	$a^2\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-2a^2\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
parts	$a^2\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-2a^2\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
norman	$\frac{\frac{3a^2x}{8}+\frac{5a^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3a^2\tan\left(\frac{x}{2}\right)^3}{4}+\frac{3a^2\tan\left(\frac{x}{2}\right)^5}{4}-\frac{5a^2\tan\left(\frac{x}{2}\right)^7}{4}+\frac{3a^2x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{9a^2x\tan\left(\frac{x}{2}\right)^4}{4}+\frac{3a^2x\tan\left(\frac{x}{2}\right)^6}{2}+\frac{3a^2x\tan\left(\frac{x}{2}\right)^8}{8}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$
orering	$x\left(a-a\sin(x)^2\right)^2+\left(a-a\sin(x)^2\right)a\cos(x)\sin(x)+\frac{5x\left(8\cos(x)^2\sin(x)^2a^2+4\left(a-a\sin(x)^2\right)a\sin(x)\right.}{16}$

input
```
int((a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*a^2*(8*sin(2*x)+12*x+sin(4*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \left(a - a\sin^2(x)\right)^2 \, dx = \frac{3}{8}\, a^2 x + \frac{1}{8}\left(2\, a^2 \cos\left(x\right)^3 + 3\, a^2 \cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate((a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
3/8*a^2*x + 1/8*(2*a^2*cos(x)^3 + 3*a^2*cos(x))*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.33

$$\begin{aligned}\int \left(a - a\sin^2(x)\right)^2 \, dx = {} & \frac{3a^2 x \sin^4\left(x\right)}{8} + \frac{3a^2 x \sin^2\left(x\right)\cos^2\left(x\right)}{4} - a^2 x \sin^2\left(x\right) \\ & + \frac{3a^2 x \cos^4\left(x\right)}{8} - a^2 x \cos^2\left(x\right) + a^2 x - \frac{5a^2 \sin^3\left(x\right)\cos\left(x\right)}{8} \\ & - \frac{3a^2 \sin\left(x\right)\cos^3\left(x\right)}{8} + a^2 \sin\left(x\right)\cos\left(x\right)\end{aligned}$$

input
```
integrate((a-a*sin(x)**2)**2,x)
```

output
```
3*a**2*x*sin(x)**4/8 + 3*a**2*x*sin(x)**2*cos(x)**2/4 - a**2*x*sin(x)**2 +
 3*a**2*x*cos(x)**4/8 - a**2*x*cos(x)**2 + a**2*x - 5*a**2*sin(x)**3*cos(x
)/8 - 3*a**2*sin(x)*cos(x)**3/8 + a**2*sin(x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \left(a - a\sin^2(x)\right)^2 \, dx = \frac{1}{32}a^2(12x + \sin(4x) - 8\sin(2x)) - \frac{1}{2}a^2(2x - \sin(2x)) + a^2x$$

input
```
integrate((a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/32*a^2*(12*x + sin(4*x) - 8*sin(2*x)) - 1/2*a^2*(2*x - sin(2*x)) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(a - a\sin^2(x)\right)^2 \, dx = \frac{3}{8}a^2x + \frac{1}{32}a^2\sin(4x) + \frac{1}{4}a^2\sin(2x)$$

input
```
integrate((a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
3/8*a^2*x + 1/32*a^2*sin(4*x) + 1/4*a^2*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 35.89 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(a - a\sin^2(x)\right)^2 \, dx = \frac{\frac{3a^2\tan(x)^3}{8} + \frac{5a^2\tan(x)}{8}}{\left(\tan(x)^2 + 1\right)^2} + \frac{3a^2x}{8}$$

input
```
int((a - a*sin(x)^2)^2,x)
```

output
```
((5*a^2*tan(x))/8 + (3*a^2*tan(x)^3)/8)/(tan(x)^2 + 1)^2 + (3*a^2*x)/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \left(a-a\sin^2(x)\right)^2\,dx=\frac{a^2\left(-2\cos(x)\sin(x)^3+5\cos(x)\sin(x)+3x\right)}{8}$$

input
```
int((a-a*sin(x)^2)^2,x)
```

output
```
(a**2*( - 2*cos(x)*sin(x)**3 + 5*cos(x)*sin(x) + 3*x))/8
```

3.88 $\int (a - a\sin^2(x))\, dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a - a\sin^2(x))\, dx = \frac{ax}{2} + \frac{1}{2}a\cos(x)\sin(x)$$

output
```
1/2*a*x+1/2*a*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a - a\sin^2(x))\, dx = a\left(\frac{x}{2} + \frac{1}{4}\sin(2x)\right)$$

input
```
Integrate[a - a*Sin[x]^2,x]
```

output
```
a*(x/2 + Sin[2*x]/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - a \sin^2(x)\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax}{2} + \frac{1}{2} a \sin(x) \cos(x)$$

input
```
Int[a - a*Sin[x]^2,x]
```

output
```
(a*x)/2 + (a*Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{ax}{2}+\frac{a\sin(2x)}{4}$	13
default	$ax-a\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	18
parallelrisch	$-a\left(-\frac{\sin(2x)}{4}+\frac{x}{2}\right)+ax$	18
parts	$ax-a\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	18
orering	$x\left(a-a\sin\left(x\right)^{2}\right)+\frac{a\cos(x)\sin(x)}{2}+\frac{x\left(2a\sin(x)^{2}-2a\cos(x)^{2}\right)}{4}$	38
norman	$\frac{a\tan\left(\frac{x}{2}\right)+ax\tan\left(\frac{x}{2}\right)^{2}+\frac{ax}{2}-a\tan\left(\frac{x}{2}\right)^{3}+\frac{ax\tan\left(\frac{x}{2}\right)^{4}}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	51

input
```
int(a-a*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+1/4*a*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(a-a\sin^2(x)\right)\,dx=\frac{1}{2}\,a\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,ax$$

input
```
integrate(a-a*sin(x)^2,x, algorithm="fricas")
```

output
```
1/2*a*cos(x)*sin(x) + 1/2*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \left(a - a\sin^2(x)\right)\,dx = ax - a\left(\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}\right)$$

input
```
integrate(a-a*sin(x)**2,x)
```

output
```
a*x - a*(x/2 - sin(x)*cos(x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(a - a\sin^2(x)\right)\,dx = -\frac{1}{4}\,a(2\,x - \sin(2\,x)) + ax$$

input
```
integrate(a-a*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x - sin(2*x)) + a*x
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(a - a\sin^2(x)\right)\,dx = -\frac{1}{4}\,a(2\,x - \sin(2\,x)) + ax$$

input
```
integrate(a-a*sin(x)^2,x, algorithm="giac")
```

output
```
-1/4*a*(2*x - sin(2*x)) + a*x
```

Mupad [B] (verification not implemented)

Time = 35.63 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \left(a-a\sin^2(x)\right)\,dx=\frac{a\,(2\,x+\sin\left(2\,x\right))}{4}$$

input
```
int(a - a*sin(x)^2,x)
```

output
```
(a*(2*x + sin(2*x)))/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \left(a-a\sin^2(x)\right)\,dx=\frac{a(\cos\left(x\right)\sin\left(x\right)+x)}{2}$$

input
```
int(a-a*sin(x)^2,x)
```

output
```
(a*(cos(x)*sin(x) + x))/2
```

3.89 $\int \frac{1}{a-a\sin^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{1}{a-a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

output
```
tan(x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a-a\sin(x)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \sec^2(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{a}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\int 1d(-\tan(x))}{a}$$
$$\Big\downarrow \text{24}$$
$$\frac{\tan(x)}{a}$$

input
```
Int[(a - a*Sin[x]^2)^(-1),x]
```

output
```
Tan[x]/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\tan(x)}{a}$	7
parallelrisch	$\frac{\sin(x)}{\cos(x)a}$	11
risch	$\frac{2i}{a(e^{2ix}+1)}$	16
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(-1+\tan\left(\frac{x}{2}\right)^{2}\right)}$	20

input
```
int(1/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{a - a\sin^2(x)}\, dx = \frac{\sin(x)}{a\cos(x)}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
sin(x)/(a*cos(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{1}{a - a\sin^2(x)}\, dx = -\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right) - a}$$

input
```
integrate(1/(a-a*sin(x)**2),x)
```

output
```
-2*tan(x/2)/(a*tan(x/2)**2 - a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\, dx = \frac{\tan(x)}{a}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
tan(x)/a
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
integrate(1/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
tan(x)/a
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\tan(x)}{a}$$

input
```
int(1/(a - a*sin(x)^2),x)
```

output
```
tan(x)/a
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{a - a\sin^2(x)}\,dx = \frac{\sin(x)}{\cos(x)\,a}$$

input
```
int(1/(a-a*sin(x)^2),x)
```

output
```
sin(x)/(cos(x)*a)
```

3.90 $\int \frac{1}{(a-a\sin^2(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{(a-a\sin^2(x))^2}\,dx = \frac{\tan(x)}{a^2} + \frac{\tan^3(x)}{3a^2}$$

output
```
tan(x)/a^2+1/3*tan(x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-a\sin^2(x))^2}\,dx = \frac{\tan(x) + \frac{\tan^3(x)}{3}}{a^2}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-2),x]
```

output
```
(Tan[x] + Tan[x]^3/3)/a^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \ 3654$$

$$\frac{\int \sec^4(x)dx}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^4dx}{a^2}$$

$$\Big\downarrow \ 4254$$

$$-\frac{\int \left(\tan^2(x)+1\right)d(-\tan(x))}{a^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{1}{3}\tan^3(x)-\tan(x)}{a^2}$$

input
```
Int[(a - a*Sin[x]^2)^(-2),x]
```

output
```
-((-Tan[x] - Tan[x]^3/3)/a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\frac{\tan(x)^3}{3}+\tan(x)}{a^2}$	14
risch	$\frac{4i(3\,e^{2ix}+1)}{3(e^{2ix}+1)^3 a^2}$	25
parallelrisch	$\frac{2\sin(3x)+6\sin(x)}{3a^2(\cos(3x)+3\cos(x))}$	28
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\tan\left(\frac{x}{2}\right)^3}{3a}-\frac{2\tan\left(\frac{x}{2}\right)^5}{a}}{a\left(-1+\tan\left(\frac{x}{2}\right)^2\right)^3}$	47

input
```
int(1/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/3*tan(x)^3+tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\left(2\cos\left(x\right)^2+1\right)\sin\left(x\right)}{3\,a^2\cos\left(x\right)^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(2*cos(x)^2 + 1)*sin(x)/(a^2*cos(x)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(15) = 30.

Time = 0.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 8.00

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^2}\,dx=-\frac{6\tan^5\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)-9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)-3a^2}+\frac{4\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)-9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)-3a^2}-\frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)-9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)-3a^2}$$

input
```
integrate(1/(a-a*sin(x)**2)**2,x)
```

output
```
-6*tan(x/2)**5/(3*a**2*tan(x/2)**6 - 9*a**2*tan(x/2)**4 + 9*a**2*tan(x/2)*
*2 - 3*a**2) + 4*tan(x/2)**3/(3*a**2*tan(x/2)**6 - 9*a**2*tan(x/2)**4 + 9*
a**2*tan(x/2)**2 - 3*a**2) - 6*tan(x/2)/(3*a**2*tan(x/2)**6 - 9*a**2*tan(x
/2)**4 + 9*a**2*tan(x/2)**2 - 3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\tan\left(x\right)^3+3\,\tan\left(x\right)}{3\,a^2}$$

input
```
integrate(1/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(tan(x)^3 + 3*tan(x))/a^2
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\tan\left(x\right)^3+3\,\tan\left(x\right)}{3\,a^2}$$

input
```
integrate(1/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/3*(tan(x)^3 + 3*tan(x))/a^2
```

Mupad [B] (verification not implemented)

Time = 36.69 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\tan(x)\,\left(\tan(x)^2+3\right)}{3\,a^2}$$

input
```
int(1/(a - a*sin(x)^2)^2,x)
```

output
```
(tan(x)*(tan(x)^2 + 3))/(3*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x)\left(2\sin(x)^2-3\right)}{3\cos(x)\,a^2\left(\sin(x)^2-1\right)}$$

input
```
int(1/(a-a*sin(x)^2)^2,x)
```

output
```
(sin(x)*(2*sin(x)**2 - 3))/(3*cos(x)*a**2*(sin(x)**2 - 1))
```

3.91 $\int \frac{1}{(a-a\sin^2(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx = \frac{\tan(x)}{a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan^5(x)}{5a^3}$$

output
```
tan(x)/a^3+2/3*tan(x)^3/a^3+1/5*tan(x)^5/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx = \frac{\tan(x)+\frac{2\tan^3(x)}{3}+\frac{\tan^5(x)}{5}}{a^3}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-3),x]
```

output
```
(Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5)/a^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sin(x)^2)^3}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^6(x)dx}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^6dx}{a^3}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^4(x)+2\tan^2(x)+1\right)d(-\tan(x))}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\tan^5(x)-\frac{2\tan^3(x)}{3}-\tan(x)}{a^3}$$

input
```
Int[(a - a*Sin[x]^2)^(-3),x]
```

output
```
-((-Tan[x] - (2*Tan[x]^3)/3 - Tan[x]^5/5)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{\tan(x)^5}{5}+\frac{2\tan(x)^3}{3}+\tan(x)}{a^3}$	20
risch	$\frac{16i\left(10\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}+1\right)^5 a^3}$	32
parallelrisch	$\frac{\frac{8\sin(3x)}{3}+\frac{16\sin(x)}{3}+\frac{8\sin(5x)}{15}}{a^3\left(\cos(5x)+5\cos(3x)+10\cos(x)\right)}$	38
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{8\tan\left(\frac{x}{2}\right)^3}{3a}-\frac{116\tan\left(\frac{x}{2}\right)^5}{15a}+\frac{8\tan\left(\frac{x}{2}\right)^7}{3a}-\frac{2\tan\left(\frac{x}{2}\right)^9}{a}}{a^2\left(-1+\tan\left(\frac{x}{2}\right)^2\right)^5}$	69

input
```
int(1/(a-a*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^3}\, dx = \frac{\left(8\cos\left(x\right)^4 + 4\cos\left(x\right)^2 + 3\right)\sin\left(x\right)}{15\,a^3\cos\left(x\right)^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^3*cos(x)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(27) = 54.

Time = 1.95 (sec) , antiderivative size = 362, normalized size of antiderivative = 12.48

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^3}\, dx = - \frac{30\tan^9\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} + \frac{40\tan^7\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} - \frac{116\tan^5\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} + \frac{40\tan^3\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} - \frac{30\tan\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3}$$

input
```
integrate(1/(a-a*sin(x)**2)**3,x)
```

output
```
-30*tan(x/2)**9/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan
(x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(
x/2)**7/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6
 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 116*tan(x/2)**5
/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*
a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/2)**3/(15*a**
3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan
(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 30*tan(x/2)/(15*a**3*tan(x/2)*
*10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**4 +
75*a**3*tan(x/2)**2 - 15*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx = \frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx = \frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^3,x, algorithm="giac")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 36.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx=\frac{\tan(x)\,\left(3\tan(x)^4+10\tan(x)^2+15\right)}{15\,a^3}$$

input
```
int(1/(a - a*sin(x)^2)^3,x)
```

output
```
(tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx=\frac{\sin(x)\left(8\sin(x)^4-20\sin(x)^2+15\right)}{15\cos(x)\,a^3\left(\sin(x)^4-2\sin(x)^2+1\right)}$$

input
```
int(1/(a-a*sin(x)^2)^3,x)
```

output
```
(sin(x)*(8*sin(x)**4 - 20*sin(x)**2 + 15))/(15*cos(x)*a**3*(sin(x)**4 - 2*
sin(x)**2 + 1))
```

3.92 $\int \frac{1}{(a-a\sin^2(x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{(a-a\sin^2(x))^4}\,dx = \frac{\tan(x)}{a^4}+\frac{\tan^3(x)}{a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^7(x)}{7a^4}$$

output
```
tan(x)/a^4+tan(x)^3/a^4+3/5*tan(x)^5/a^4+1/7*tan(x)^7/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-a\sin^2(x))^4}\,dx = \frac{\tan(x)+\tan^3(x)+\frac{3\tan^5(x)}{5}+\frac{\tan^7(x)}{7}}{a^4}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-4),x]
```

output
```
(Tan[x] + Tan[x]^3 + (3*Tan[x]^5)/5 + Tan[x]^7/7)/a^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin(x)^2\right)^4}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^8(x)dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^8dx}{a^4}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^6(x)+3\tan^4(x)+3\tan^2(x)+1\right)d(-\tan(x))}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{7}\tan^7(x)-\frac{3\tan^5(x)}{5}-\tan^3(x)-\tan(x)}{a^4}$$

input
```
Int[(a - a*Sin[x]^2)^(-4),x]
```

output
```
-((-Tan[x] - Tan[x]^3 - (3*Tan[x]^5)/5 - Tan[x]^7/7)/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\frac{\tan(x)^7}{7}+\frac{3\tan(x)^5}{5}+\tan(x)^3+\tan(x)}{a^4}$	24
risch	$\frac{32i(35\,\mathrm{e}^{6ix}+21\,\mathrm{e}^{4ix}+7\,\mathrm{e}^{2ix}+1)}{35(\mathrm{e}^{2ix}+1)^7a^4}$	39
parallelrisch	$\frac{\frac{48\sin(3x)}{5}+16\sin(x)+\frac{16\sin(7x)}{35}+\frac{16\sin(5x)}{5}}{a^4(\cos(7x)+7\cos(5x)+21\cos(3x)+35\cos(x))}$	50
norman	$\frac{-\frac{2\tan(\frac{x}{2})}{a}+\frac{4\tan(\frac{x}{2})^3}{a}-\frac{86\tan(\frac{x}{2})^5}{5a}+\frac{424\tan(\frac{x}{2})^7}{35a}-\frac{86\tan(\frac{x}{2})^9}{5a}+\frac{4\tan(\frac{x}{2})^{11}}{a}-\frac{2\tan(\frac{x}{2})^{13}}{a}}{a^3\left(-1+\tan(\frac{x}{2})^2\right)^7}$	91

input
```
int(1/(a-a*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^4}\,dx = \frac{\left(16\,\cos\left(x\right)^6 + 8\,\cos\left(x\right)^4 + 6\,\cos\left(x\right)^2 + 5\right)\sin\left(x\right)}{35\,a^4\cos\left(x\right)^7}$$

input
```
integrate(1/(a-a*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^4*cos(x)^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(36) = 72$.

Time = 5.77 (sec) , antiderivative size = 675, normalized size of antiderivative = 18.24

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)**2)**4,x)
```

output
```
-70*tan(x/2)**13/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*
tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*ta
n(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(x/2)**11/(35*a**4*ta
n(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan
(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2
)**2 - 35*a**4) - 602*tan(x/2)**9/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2
)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)
**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 424*tan(x/2
)**7/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10
 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 +
245*a**4*tan(x/2)**2 - 35*a**4) - 602*tan(x/2)**5/(35*a**4*tan(x/2)**14 -
245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 12
25*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**
4) + 140*tan(x/2)**3/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a
**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**
4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) - 70*tan(x/2)/(35*a**4*tan
(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(
x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)
**2 - 35*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^4}\,dx = \frac{5\,\tan\left(x\right)^7 + 21\,\tan\left(x\right)^5 + 35\,\tan\left(x\right)^3 + 35\,\tan\left(x\right)}{35\,a^4}$$

input
```
integrate(1/(a-a*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx=\frac{5\,\tan\left(x\right)^7+21\,\tan\left(x\right)^5+35\,\tan\left(x\right)^3+35\,\tan\left(x\right)}{35\,a^4}$$

input
```
integrate(1/(a-a*sin(x)^2)^4,x, algorithm="giac")
```

output
```
1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

Mupad [B] (verification not implemented)

Time = 36.83 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx=\frac{\tan(x)}{a^4}+\frac{\tan(x)^3}{a^4}+\frac{3\tan(x)^5}{5\,a^4}+\frac{\tan(x)^7}{7\,a^4}$$

input
```
int(1/(a - a*sin(x)^2)^4,x)
```

output
```
tan(x)/a^4 + tan(x)^3/a^4 + (3*tan(x)^5)/(5*a^4) + tan(x)^7/(7*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx=\frac{\sin\left(x\right)\left(16\sin\left(x\right)^6-56\sin\left(x\right)^4+70\sin\left(x\right)^2-35\right)}{35\cos\left(x\right)a^4\left(\sin\left(x\right)^6-3\sin\left(x\right)^4+3\sin\left(x\right)^2-1\right)}$$

input
```
int(1/(a-a*sin(x)^2)^4,x)
```

output
```
(sin(x)*(16*sin(x)**6 - 56*sin(x)**4 + 70*sin(x)**2 - 35))/(35*cos(x)*a**4
*(sin(x)**6 - 3*sin(x)**4 + 3*sin(x)**2 - 1))
```

3.93 $\int \frac{1}{(a-a\sin^2(x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{1}{(a-a\sin^2(x))^5}\,dx=\frac{\tan(x)}{a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{\tan^9(x)}{9a^5}$$

output
```
tan(x)/a^5+4/3*tan(x)^3/a^5+6/5*tan(x)^5/a^5+4/7*tan(x)^7/a^5+1/9*tan(x)^9
/a^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a-a\sin^2(x))^5}\,dx=\frac{\tan(x)+\frac{4\tan^3(x)}{3}+\frac{6\tan^5(x)}{5}+\frac{4\tan^7(x)}{7}+\frac{\tan^9(x)}{9}}{a^5}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-5),x]
```

output
```
(Tan[x] + (4*Tan[x]^3)/3 + (6*Tan[x]^5)/5 + (4*Tan[x]^7)/7 + Tan[x]^9/9)/a
^5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a-a\sin(x)^2)^5}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \sec^{10}(x)dx}{a^5}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^{10}dx}{a^5}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\int \left(\tan^8(x)+4\tan^6(x)+6\tan^4(x)+4\tan^2(x)+1\right)d(-\tan(x))}{a^5}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{-\frac{1}{9}\tan^9(x)-\frac{4\tan^7(x)}{7}-\frac{6\tan^5(x)}{5}-\frac{4\tan^3(x)}{3}-\tan(x)}{a^5}$$

input
```
Int[(a - a*Sin[x]^2)^(-5),x]
```

output
```
-((-Tan[x] - (4*Tan[x]^3)/3 - (6*Tan[x]^5)/5 - (4*Tan[x]^7)/7 - Tan[x]^9/9
)/a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\frac{\tan(x)^9}{9}+\frac{4\tan(x)^7}{7}+\frac{6\tan(x)^5}{5}+\frac{4\tan(x)^3}{3}+\tan(x)}{a^5}$	32
risch	$\frac{256i\left(126\,\mathrm{e}^{8ix}+84\,\mathrm{e}^{6ix}+36\,\mathrm{e}^{4ix}+9\,\mathrm{e}^{2ix}+1\right)}{315\left(\mathrm{e}^{2ix}+1\right)^9 a^5}$	46
parallelrisch	$\frac{\frac{512\sin(3x)}{15}+\frac{256\sin(x)}{5}+\frac{128\sin(7x)}{35}+\frac{512\sin(5x)}{35}+\frac{128\sin(9x)}{315}}{a^5\left(\cos(9x)+9\cos(7x)+36\cos(5x)+84\cos(3x)+126\cos(x)\right)}$	62

input
```
int(1/(a-a*sin(x)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/9*tan(x)^9+4/7*tan(x)^7+6/5*tan(x)^5+4/3*tan(x)^3+tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx = \frac{\left(128\,\cos\left(x\right)^{8}+64\,\cos\left(x\right)^{6}+48\,\cos\left(x\right)^{4}+40\,\cos\left(x\right)^{2}+35\right)\sin\left(x\right)}{315\,a^{5}\cos\left(x\right)^{9}}$$

input
```
integrate(1/(a-a*sin(x)^2)^5,x, algorithm="fricas")
```

output
```
1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sin(x)
/(a^5*cos(x)^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(51) = 102.

Time = 15.97 (sec) , antiderivative size = 1083, normalized size of antiderivative = 21.24

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)**2)**5,x)
```

output
```
-630*tan(x/2)**17/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*
a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39
690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2
835*a**5*tan(x/2)**2 - 315*a**5) + 1680*tan(x/2)**15/(315*a**5*tan(x/2)**1
8 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)
**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x
/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 9576
*tan(x/2)**13/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5
*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*
a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*
a**5*tan(x/2)**2 - 315*a**5) + 10224*tan(x/2)**11/(315*a**5*tan(x/2)**18 -
 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**1
2 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)
**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 21316*t
an(x/2)**9/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*ta
n(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**
5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**
5*tan(x/2)**2 - 315*a**5) + 10224*tan(x/2)**7/(315*a**5*tan(x/2)**18 - 283
5*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 +
39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a - a\sin^2(x))^5}\,dx = \frac{35\tan(x)^9 + 180\tan(x)^7 + 378\tan(x)^5 + 420\tan(x)^3 + 315\tan(x)}{315\,a^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^5,x, algorithm="maxima")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))/a^5
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-a\sin^2(x))^5}\,dx = \frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315\,a^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^5,x, algorithm="giac")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))/a^5
```

Mupad [B] (verification not implemented)

Time = 36.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-a\sin^2(x))^5}\,dx = \frac{\tan(x)}{a^5}+\frac{4\tan(x)^3}{3\,a^5}+\frac{6\tan(x)^5}{5\,a^5}+\frac{4\tan(x)^7}{7\,a^5}+\frac{\tan(x)^9}{9\,a^5}$$

input
```
int(1/(a - a*sin(x)^2)^5,x)
```

output
```
tan(x)/a^5 + (4*tan(x)^3)/(3*a^5) + (6*tan(x)^5)/(5*a^5) + (4*tan(x)^7)/(7
*a^5) + tan(x)^9/(9*a^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-a\sin^2(x))^5}\,dx = \frac{\sin(x)\left(128\sin(x)^8-576\sin(x)^6+1008\sin(x)^4-840\sin(x)^2+315\right)}{315\cos(x)\,a^5\left(\sin(x)^8-4\sin(x)^6+6\sin(x)^4-4\sin(x)^2+1\right)}$$

input
```
int(1/(a-a*sin(x)^2)^5,x)
```

output
```
(sin(x)*(128*sin(x)**8 - 576*sin(x)**6 + 1008*sin(x)**4 - 840*sin(x)**2 +
315))/(315*cos(x)*a**5*(sin(x)**8 - 4*sin(x)**6 + 6*sin(x)**4 - 4*sin(x)**
2 + 1))
```

3.94 $\int (a-a\sin^2(x))^{7/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int (a-a\sin^2(x))^{7/2}\,dx = \frac{16}{35}a^3\sqrt{a\cos^2(x)}\tan(x) + \frac{8}{35}a^2\left(a\cos^2(x)\right)^{3/2}\tan(x) + \frac{6}{35}a\left(a\cos^2(x)\right)^{5/2}\tan(x) + \frac{1}{7}\left(a\cos^2(x)\right)^{7/2}\tan(x)$$

output
```
16/35*a^3*(a*cos(x)^2)^(1/2)*tan(x)+8/35*a^2*(a*cos(x)^2)^(3/2)*tan(x)+6/3
5*a*(a*cos(x)^2)^(5/2)*tan(x)+1/7*(a*cos(x)^2)^(7/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int (a-a\sin^2(x))^{7/2}\,dx = -\frac{1}{35}a^3\sqrt{a\cos^2(x)}\left(-35+35\sin^2(x)-21\sin^4(x)+5\sin^6(x)\right)\tan(x)$$

input
```
Integrate[(a - a*Sin[x]^2)^(7/2),x]
```

output
```
-1/35*(a^3*Sqrt[a*Cos[x]^2]*(-35 + 35*Sin[x]^2 - 21*Sin[x]^4 + 5*Sin[x]^6)
*Tan[x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 3655, 3042, 3682, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\cos^2(x)\right)^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\int \left(a\cos^2(x)\right)^{5/2}dx+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}a\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}dx+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\int \left(a\cos^2(x)\right)^{3/2}dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}\right)+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}\right)+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}\right)+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}\right)+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}\right)+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}\right)+\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{7}\tan(x)\left(a\cos^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^(7/2),x]
```

output
```
((a*Cos[x]^2)^(7/2)*Tan[x])/7 + (6*a*(((a*Cos[x]^2)^(5/2)*Tan[x])/5 + (4*a
*((2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.53

method	result
default	$-\frac{\cos(x) a^{4} \sin(x) \left(-5 \cos(x)^{6}-6 \cos(x)^{4}-8 \cos(x)^{2}-16\right)}{35 \sqrt{a \cos(x)^{2}}}$
risch	$-\frac{i a^{3} \mathrm{e}^{8 i x} \sqrt{a \left(\mathrm{e}^{2 i x}+1\right)^{2} \mathrm{e}^{-2 i x}}}{896 \left(\mathrm{e}^{2 i x}+1\right)}-\frac{35 i a^{3} \mathrm{e}^{2 i x} \sqrt{a \left(\mathrm{e}^{2 i x}+1\right)^{2} \mathrm{e}^{-2 i x}}}{128 \left(\mathrm{e}^{2 i x}+1\right)}+\frac{35 i a^{3} \sqrt{a \left(\mathrm{e}^{2 i x}+1\right)^{2} \mathrm{e}^{-2 i x}}}{128 \left(\mathrm{e}^{2 i x}+1\right)}+\frac{7 i a^{3} \mathrm{e}^{-2 i x} \sqrt{a \left(\mathrm{e}^{2 i x}+1\right)^{2} \mathrm{e}^{-2 i x}}}{128 \left(\mathrm{e}^{2 i x}+1\right)}-$

input
```
int((a-a*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*cos(x)*a^4*sin(x)*(-5*cos(x)^6-6*cos(x)^4-8*cos(x)^2-16)/(a*cos(x)^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int (a-a\sin^2(x))^{7/2}\, dx = \frac{(5\,a^3\cos(x)^6+6\,a^3\cos(x)^4+8\,a^3\cos(x)^2+16\,a^3)\sqrt{a\cos(x)^2}\sin(x)}{35\,\cos(x)}$$

input
```
integrate((a-a*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/35*(5*a^3*cos(x)^6 + 6*a^3*cos(x)^4 + 8*a^3*cos(x)^2 + 16*a^3)*sqrt(a*co
s(x)^2)*sin(x)/cos(x)
```

Sympy [F(-1)]

Timed out.

$$\int (a-a\sin^2(x))^{7/2}\, dx = \text{Timed out}$$

input
```
integrate((a-a*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int (a-a\sin^2(x))^{7/2}\,dx = \frac{1}{2240}\left(5\,a^3\sin(7\,x)+49\,a^3\sin(5\,x)+245\,a^3\sin(3\,x)+1225\,a^3\sin(x)\right)\sqrt{a}$$

input
```
integrate((a-a*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2240*(5*a^3*sin(7*x) + 49*a^3*sin(5*x) + 245*a^3*sin(3*x) + 1225*a^3*sin
(x))*sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int (a-a\sin^2(x))^{7/2}\,dx = -\frac{2\left(35\,a^{\frac{7}{2}}\left(\frac{1}{\tan(\frac{1}{2}\,x)}+\tan\left(\frac{1}{2}\,x\right)\right)^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)-140\,a^{\frac{7}{2}}\left(\frac{1}{\tan(\frac{1}{2}\,x)}+\tan\left(\frac{1}{2}\,x\right)\right)^4\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)\cdots}{35\left(\frac{1}{\tan(\frac{1}{2}\,x)}+\tan\left(\frac{1}{2}\,x\right)\cdots}$$

input
```
integrate((a-a*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
-2/35*(35*a^(7/2)*(1/tan(1/2*x) + tan(1/2*x))^6*sgn(tan(1/2*x)^4 - 1) - 14
0*a^(7/2)*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(tan(1/2*x)^4 - 1) + 336*a^(7/2
)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) - 320*a^(7/2)*sgn(ta
n(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^7
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{7/2}\,dx=\int \left(a-a\sin\left(x\right)^2\right)^{7/2}\,dx$$

input
```
int((a - a*sin(x)^2)^(7/2),x)
```

output
```
int((a - a*sin(x)^2)^(7/2), x)
```

Reduce [F]

$$\int \left(a-a\sin^2(x)\right)^{7/2}\,dx=\sqrt{a}\,a^3\left(\int \sqrt{-\sin\left(x\right)^2+1}dx-\left(\int \sqrt{-\sin\left(x\right)^2+1}\,\sin\left(x\right)^6dx\right)+3\left(\int \sqrt{-\sin\left(x\right)^2+1}\,\sin\left(x\right)^4dx\right)-3\left(\int \sqrt{-\sin\left(x\right)^2+1}\,\sin\left(x\right)^2dx\right)\right)$$

input
```
int((a-a*sin(x)^2)^(7/2),x)
```

output
```
sqrt(a)*a**3*(int(sqrt( - sin(x)**2 + 1),x) - int(sqrt( - sin(x)**2 + 1)*s
in(x)**6,x) + 3*int(sqrt( - sin(x)**2 + 1)*sin(x)**4,x) - 3*int(sqrt( - si
n(x)**2 + 1)*sin(x)**2,x))
```

3.95 $\int \left(a-a\sin^2(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x)+\frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)$$

output
```
8/15*a^2*(a*cos(x)^2)^(1/2)*tan(x)+4/15*a*(a*cos(x)^2)^(3/2)*tan(x)+1/5*(a
*cos(x)^2)^(5/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{1}{15}a^2\sqrt{a\cos^2(x)}\left(15-10\sin^2(x)+3\sin^4(x)\right)\tan(x)$$

input
```
Integrate[(a - a*Sin[x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cos[x]^2]*(15 - 10*Sin[x]^2 + 3*Sin[x]^4)*Tan[x])/15
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3655, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\cos^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\cos^2(x)\right)^{3/2} dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2} dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int\cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int\sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}\right)$$

input
```
Int[(a - a*Sin[x]^2)^(5/2),x]
```

output
```
((a*Cos[x]^2)^(5/2)*Tan[x])/5 + (4*a*((2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((
a*Cos[x]^2)^(3/2)*Tan[x])/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{\cos(x)a^3\sin(x)\left(3\cos(x)^4+4\cos(x)^2+8\right)}{15\sqrt{a\cos(x)^2}}$
risch	$-\frac{ia^2e^{6ix}\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{160(e^{2ix}+1)}-\frac{5ia^2e^{2ix}\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{16(e^{2ix}+1)}+\frac{5ia^2\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{16(e^{2ix}+1)}+\frac{5ia^2e^{-2ix}\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{96(e^{2ix}+1)}-$

input
```
int((a-a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*cos(x)*a^3*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{\left(3\,a^2\cos\left(x\right)^4+4\,a^2\cos\left(x\right)^2+8\,a^2\right)\sqrt{a\cos\left(x\right)^2}\sin\left(x\right)}{15\,\cos\left(x\right)}$$

input
```
integrate((a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos
(x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a-a*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \left(a-a\sin^2(x)\right)^{5/2} dx = \frac{1}{240}\left(3\,a^2\sin(5\,x)+25\,a^2\sin(3\,x)+150\,a^2\sin(x)\right)\sqrt{a}$$

input
```
integrate((a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \left(a-a\sin^2(x)\right)^{5/2} dx = -\frac{2\left(15\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)-40\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)\right.}{15\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^5}$$

input
```
integrate((a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-2/15*(15*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(tan(1/2*x)^4 - 1) - 40
*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) + 48*a^(5/2)*
sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\int \left(a-a\sin\left(x\right)^2\right)^{5/2}dx$$

input
```
int((a - a*sin(x)^2)^(5/2),x)
```

output
```
int((a - a*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\sqrt{a}\,a^2\left(\int\sqrt{-\sin\left(x\right)^2+1}dx+\int\sqrt{-\sin\left(x\right)^2+1}\,\sin\left(x\right)^4dx-2\left(\int\sqrt{-\sin\left(x\right)^2+1}\,\sin\left(x\right)^2dx\right)\right)$$

input
```
int((a-a*sin(x)^2)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt( - sin(x)**2 + 1),x) + int(sqrt( - sin(x)**2 + 1)*s
in(x)**4,x) - 2*int(sqrt( - sin(x)**2 + 1)*sin(x)**2,x))
```

3.96 $\int (a-a\sin^2(x))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int (a-a\sin^2(x))^{3/2}\,dx=\frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x)+\frac{1}{3}(a\cos^2(x))^{3/2}\tan(x)$$

output
```
2/3*a*(a*cos(x)^2)^(1/2)*tan(x)+1/3*(a*cos(x)^2)^(3/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (a-a\sin^2(x))^{3/2}\,dx=-\frac{1}{3}a\sqrt{a\cos^2(x)}(-3+\sin^2(x))\tan(x)$$

input
```
Integrate[(a - a*Sin[x]^2)^(3/2),x]
```

output
```
-1/3*(a*Sqrt[a*Cos[x]^2]*(-3 + Sin[x]^2)*Tan[x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\cos^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

input
```
Int[(a - a*Sin[x]^2)^(3/2),x]
```

output
```
(2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\cos(x)a^{2}\sin(x)\left(\sin(x)^{2}-3\right)}{3\sqrt{a\cos(x)^{2}}}$	24
risch	$-\frac{ia\,\mathrm{e}^{4ix}\sqrt{a(\mathrm{e}^{2ix}+1)^{2}\mathrm{e}^{-2ix}}}{24(\mathrm{e}^{2ix}+1)}-\frac{3ia\,\mathrm{e}^{2ix}\sqrt{a(\mathrm{e}^{2ix}+1)^{2}\mathrm{e}^{-2ix}}}{8(\mathrm{e}^{2ix}+1)}+\frac{3ia\sqrt{a(\mathrm{e}^{2ix}+1)^{2}\mathrm{e}^{-2ix}}}{8(\mathrm{e}^{2ix}+1)}+\frac{ia\,\mathrm{e}^{-2ix}\sqrt{a(\mathrm{e}^{2ix}+1)^{2}\mathrm{e}^{-2ix}}}{24\,\mathrm{e}^{2ix}+24}$	141

input
```
int((a-a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(x)*a^2*sin(x)*(sin(x)^2-3)/(a*cos(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a-a\sin^{2}(x)\right)^{3/2}\,dx=\frac{\left(a\cos\left(x\right)^{2}+2\,a\right)\sqrt{a\cos\left(x\right)^{2}}\sin\left(x\right)}{3\,\cos\left(x\right)}$$

input
```
integrate((a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F]

$$\int \left(a-a\sin^{2}(x)\right)^{3/2}\,dx=\int \left(-a\sin^{2}\left(x\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*sin(x)**2)**(3/2),x)
```

output
```
Integral((-a*sin(x)**2 + a)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \left(a - a\sin^2(x)\right)^{3/2} dx = \frac{1}{12}\left(a\sin\left(3x\right) + 9a\sin\left(x\right)\right)\sqrt{a}$$

input
```
integrate((a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \left(a - a\sin^2(x)\right)^{3/2} dx = -\frac{2\left(3a^{\frac{3}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)\right)^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4 - 1\right) - 4a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4 - 1\right)\right)}{3\left(\frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)\right)^3}$$

input
```
integrate((a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) - 4*a^
(3/2)*sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=\int \left(a-a\sin{(x)}^2\right)^{3/2}\,dx$$

input
```
int((a - a*sin(x)^2)^(3/2),x)
```

output
```
int((a - a*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{-\sin{(x)}^2+1}dx-\left(\int \sqrt{-\sin{(x)}^2+1}\,\sin{(x)}^2\,dx\right)\right)$$

input
```
int((a-a*sin(x)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt( - sin(x)**2 + 1),x) - int(sqrt( - sin(x)**2 + 1)*sin(
x)**2,x))
```

3.97 $\int \sqrt{a-a\sin^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \sqrt{a-a\sin^2(x)}\,dx=\sqrt{a\cos^2(x)}\tan(x)$$

output
```
(a*cos(x)^2)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\sin^2(x)}\,dx=\sqrt{a\cos^2(x)}\tan(x)$$

input
```
Integrate[Sqrt[a - a*Sin[x]^2],x]
```

output
```
Sqrt[a*Cos[x]^2]*Tan[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3655, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\sin(x)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sqrt{a\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\tan(x)\sqrt{a\cos^2(x)}$$

input
```
Int[Sqrt[a - a*Sin[x]^2],x]
```

output
```
Sqrt[a*Cos[x]^2]*Tan[x]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a\cos\left(x\right)\sin\left(x\right)}{\sqrt{a\cos\left(x\right)^{2}}}$	15
risch	$-\frac{i\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-2ix}}\,\mathrm{e}^{2ix}}{2\left(\mathrm{e}^{2ix}+1\right)}+\frac{i\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-2ix}}}{2\,\mathrm{e}^{2ix}+2}$	67

input
```
int((a-a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*cos(x)*sin(x)/(a*cos(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a - a\sin^2(x)}\,dx = \frac{\sqrt{a\cos(x)^2}\sin(x)}{\cos(x)}$$

input
```
integrate((a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F]

$$\int \sqrt{a - a\sin^2(x)}\,dx = \int \sqrt{-a\sin^2(x) + a}\,dx$$

input
```
integrate((a-a*sin(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*sin(x)**2 + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \sqrt{a - a\sin^2(x)}\,dx = \sqrt{a}\sin(x)$$

input
```
integrate((a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \sqrt{a - a\sin^2(x)}\,dx = -\frac{2\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4 - 1\right)}{\frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)}$$

input
```
integrate((a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*sgn(tan(1/2*x)^4 - 1)/(1/tan(1/2*x) + tan(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \sqrt{a - a\sin^2(x)}\,dx = \frac{\sqrt{2}\sqrt{a}\sqrt{\cos(2x) + 1}\,(\cos(2x) - 1 + \sin(2x)\ 1\mathrm{i})}{2\ (\cos(2x)\ 1\mathrm{i} - \sin(2x) + 1\mathrm{i})}$$

input
```
int((a - a*sin(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*a^(1/2)*(cos(2*x) + 1)^(1/2)*(cos(2*x) + sin(2*x)*1i - 1))/(2*(co
s(2*x)*1i - sin(2*x) + 1i))
```

Reduce [F]

$$\int \sqrt{a - a\sin^2(x)}\,dx = \sqrt{a}\left(\int \sqrt{-\sin(x)^2 + 1}dx\right)$$

input
```
int((a-a*sin(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - sin(x)**2 + 1),x)
```

3.98 $\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx=\frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

output
```
arctanh(sin(x))*cos(x)/(a*cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx=\frac{\coth^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

input
```
Integrate[1/Sqrt[a - a*Sin[x]^2],x]
```

output
```
(ArcCoth[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3655, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sin(x)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sqrt{a\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(x)\int \sec(x)dx}{\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(x)\text{arctanh}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

input
```
Int[1/Sqrt[a - a*Sin[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\operatorname{InverseJacobiAM}(x,1)}{\sec(x)\sqrt{a\cos(x)^{2}}\ \operatorname{csgn}(\cos(x))}$	22
risch	$\frac{2\ln(e^{ix}+i)\cos(x)}{\sqrt{a(e^{2ix}+1)^{2}e^{-2ix}}}-\frac{2\ln(e^{ix}-i)\cos(x)}{\sqrt{a(e^{2ix}+1)^{2}e^{-2ix}}}$	64

input
```
int(1/(a-a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/sec(x)/(a*cos(x)^2)^(1/2)/csgn(cos(x))*InverseJacobiAM(x,1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \left[-\frac{\sqrt{a\cos(x)^2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos(x)},\ -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(x)^2}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

input
```
integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-
a)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-a\sin^2(x)+a}}\,dx$$

input
```
integrate(1/(a-a*sin(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sin(x)**2 + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \frac{\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\sin\left(x\right)+1\right)-\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\sin\left(x\right)+1\right)}{2\,\sqrt{a}}$$

input
```
integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2
*sin(x) + 1))/sqrt(a)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-a\sin\left(x\right)^2+a}}\,dx$$

input
```
integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-a*sin(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a-a\sin\left(x\right)^2}}\,dx$$

input
```
int(1/(a - a*sin(x)^2)^(1/2),x)
```

output
```
int(1/(a - a*sin(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sin(x)^2+1}}{\sin(x)^2-1}dx\right)}{a}$$

input
```
int(1/(a-a*sin(x)^2)^(1/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - sin(x)**2 + 1)/(sin(x)**2 - 1),x))/a
```

3.99 $\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

output
```
1/2*arctanh(sin(x))*cos(x)/a/(a*cos(x)^2)^(1/2)+1/2*tan(x)/a/(a*cos(x)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}(\sin(x))\cos(x)+\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-3/2),x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x] + Tan[x])/(2*a*Sqrt[a*Cos[x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 3655, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(x)\int \sec(x)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(x)\text{arctanh}(\sin(x))}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

input
```
Int[(a - a*Sin[x]^2)^(-3/2),x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x
]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{(-\ln(1+\sin(x))+\ln(\sin(x)-1))\cos(x)^2-2\sin(x)}{4a\cos(x)\sqrt{a\cos(x)^2}}$	41
risch	$-\frac{i(\mathrm{e}^{2ix}-1)}{a(\mathrm{e}^{2ix}+1)\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}-\frac{\ln(\mathrm{e}^{ix}-i)\cos(x)}{a\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}+\frac{\ln(\mathrm{e}^{ix}+i)\cos(x)}{a\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}$	109

input
```
int(1/(a-a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*((-ln(1+sin(x))+ln(sin(x)-1))*cos(x)^2-2*sin(x))/cos(x)/(a*cos(x)^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx = -\frac{\sqrt{a\cos(x)^2}\left(\cos(x)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\sin(x)\right)}{4a^2\cos(x)^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x)
)/(a^2*cos(x)^3)
```

Sympy [F]

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-a\sin^2(x) + a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*sin(x)**2)**(3/2),x)
```

output
```
Integral((-a*sin(x)**2 + a)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(34) = 68.

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 7.24

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^{3/2}}\,dx = \frac{4\left(\sin(3x) - \sin(x)\right)\cos(4x) + \left(2\left(2\cos(2x) + 1\right)\cos(4x) + \cos(4x)^2 + 4\,\mathrm{co}\right.}{}$$

input
```
integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4
*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4
*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) +
1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
 + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x
)*sin(2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x
)^2 + 4*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)
^2 + 2*(2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx=-\frac{\frac{\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)}{\sqrt{a}}-\frac{\sqrt{a\tan(x)^2+a}\tan(x)}{a}}{2a}$$

input
```
integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - sqrt(a*ta
n(x)^2 + a)*tan(x)/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(a-a\sin\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a - a*sin(x)^2)^(3/2),x)
```

output
```
int(1/(a - a*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sin(x)^2+1}}{\sin(x)^4-2\sin(x)^2+1}dx\right)}{a^2}$$

input
```
int(1/(a-a*sin(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt( - sin(x)**2 + 1)/(sin(x)**4 - 2*sin(x)**2 + 1),x))/a**2
```

3.100 $\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx=\frac{3\text{arctanh}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

output
```
3/8*arctanh(sin(x))*cos(x)/a^2/(a*cos(x)^2)^(1/2)+1/4*tan(x)/a/(a*cos(x)^2
)^(3/2)+3/8*tan(x)/a^2/(a*cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx=\frac{3\text{arctanh}(\sin(x))\cos(x)+\left(3+2\sec^2(x)\right)\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-5/2),x]
```

output
```
(3*ArcTanh[Sin[x]]*Cos[x] + (3 + 2*Sec[x]^2)*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^
2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3655, 3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sin(x)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}dx}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3686

$$\frac{3\left(\frac{\cos(x)\int \sec(x)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{3\left(\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 4257

$$\frac{3\left(\frac{\cos(x)\text{arctanh}(\sin(x))}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

input
```
Int[(a - a*Sin[x]^2)^(-5/2),x]
```

output
```
Tan[x]/(4*a*(a*Cos[x]^2)^(3/2)) + (3*((ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a
*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]^2])))/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{(3\ln(1+\sin(x))-3\ln(\sin(x)-1))\cos(x)^4+6\cos(x)^2\sin(x)+4\sin(x)}{16a^2(1+\sin(x))(\sin(x)-1)\cos(x)\sqrt{a\cos(x)^2}}$	63
risch	$-\frac{i(3\,e^{6ix}+11\,e^{4ix}-11\,e^{2ix}-3)}{4a^2(e^{2ix}+1)^3\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}}-\frac{3\ln(e^{ix}-i)\cos(x)}{4a^2\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}}+\frac{3\ln(e^{ix}+i)\cos(x)}{4a^2\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}}$	126

input
```
int(1/(a-a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^2*((3*ln(1+sin(x))-3*ln(sin(x)-1))*cos(x)^4+6*cos(x)^2*sin(x)+4*si
n(x))/(1+sin(x))/(sin(x)-1)/cos(x)/(a*cos(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx = -\frac{\left(3\,\cos\left(x\right)^4\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(3\,\cos\left(x\right)^2+2\right)\sin\left(x\right)\right)\sqrt{a\cos\left(x\right)^2}}{16\,a^3\cos\left(x\right)^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin
(x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(-a\sin^2\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*sin(x)**2)**(5/2),x)
```

output
```
Integral((-a*sin(x)**2 + a)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 933, normalized size of antiderivative = 15.30

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*
(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin
(3*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(1
1*sin(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*
x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) +
 16*cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x
)^2 + 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*
(3*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*si
n(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x)
 + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 +
 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6
*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*
sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x)
+ 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(3*cos(7*x) + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*c
os(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos
(3*x) - 3*cos(x))*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 2
4*(11*cos(3*x) + 3*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*c
os(3*x)*sin(2*x) + 48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))...
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx = -\frac{5\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^3-\frac{12}{\tan\left(\frac{1}{2}x\right)}-12\tan\left(\frac{1}{2}x\right)}{4\left(\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2-4\right)^2a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}$$

input
```
integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(5*(1/tan(1/2*x) + tan(1/2*x))^3 - 12/tan(1/2*x) - 12*tan(1/2*x))/(((
1/tan(1/2*x) + tan(1/2*x))^2 - 4)^2*a^(5/2)*sgn(tan(1/2*x)^4 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx=\int \frac{1}{\left(a-a\sin(x)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a - a*sin(x)^2)^(5/2),x)
```

output
```
int(1/(a - a*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx=-\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sin(x)^2+1}}{\sin(x)^6-3\sin(x)^4+3\sin(x)^2-1}dx\right)}{a^3}$$

input
```
int(1/(a-a*sin(x)^2)^(5/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - sin(x)**2 + 1)/(sin(x)**6 - 3*sin(x)**4 + 3*sin(x)
**2 - 1),x))/a**3
```

3.101 $\int (4-3\sin^2(x))^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int (4-3\sin^2(x))^4 \, dx = \frac{10643x}{128} + \frac{14573}{128}\cos(x)\sin(x) - \frac{2193}{64}\cos(x)\sin^3(x) + \frac{35}{16}\cos(x)\sin(x)\left(4-3\sin^2(x)\right)^2 + \frac{3}{8}\cos(x)\sin(x)\left(4-3\sin^2(x)\right)^3$$

output
```
10643/128*x+14573/128*cos(x)*sin(x)-2193/64*cos(x)*sin(x)^3+35/16*cos(x)*s
in(x)*(4-3*sin(x)^2)^2+3/8*cos(x)*sin(x)*(4-3*sin(x)^2)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

$$\int (4-3\sin^2(x))^4 \, dx = \frac{10643x}{128} + \frac{1905}{32}\sin(2x) + \frac{1431}{128}\sin(4x) + \frac{45}{32}\sin(6x) + \frac{81\sin(8x)}{1024}$$

input
```
Integrate[(4 - 3*Sin[x]^2)^4,x]
```

output
```
(10643*x)/128 + (1905*Sin[2*x])/32 + (1431*Sin[4*x])/128 + (45*Sin[6*x])/3
2 + (81*Sin[8*x])/1024
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3659, 3042, 3649, 27, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3\sin^2(x)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-3\sin(x)^2\right)^4 dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{8}\int \left(116-105\sin^2(x)\right)\left(4-3\sin^2(x)\right)^2 dx+\frac{3}{8}\sin(x)\left(4-3\sin^2(x)\right)^3\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \left(116-105\sin(x)^2\right)\left(4-3\sin(x)^2\right)^2 dx+\frac{3}{8}\sin(x)\left(4-3\sin^2(x)\right)^3\cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{1}{8}\left(\frac{1}{6}\int 3\left(788-731\sin^2(x)\right)\left(4-3\sin^2(x)\right)dx+\frac{35}{2}\sin(x)\left(4-3\sin^2(x)\right)^2\cos(x)\right)+\frac{3}{8}\sin(x)\left(4-3\sin^2(x)\right)^3\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \left(788-731\sin^2(x)\right)\left(4-3\sin^2(x)\right)dx+\frac{35}{2}\sin(x)\left(4-3\sin^2(x)\right)^2\cos(x)\right)+\frac{3}{8}\sin(x)\left(4-3\sin^2(x)\right)^3\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \left(788-731\sin(x)^2\right)\left(4-3\sin(x)^2\right)dx+\frac{35}{2}\sin(x)\left(4-3\sin^2(x)\right)^2\cos(x)\right)+\frac{3}{8}\sin(x)\left(4-3\sin^2(x)\right)^3\cos(x)$$

$$\Big\downarrow \text{3648}$$

$$\frac{3}{8}\sin(x)\left(4-3\sin^2(x)\right)^3\cos(x)+\frac{1}{8}\left(\frac{1}{2}\left(\frac{10643x}{8}-\frac{2193}{4}\sin^3(x)\cos(x)+\frac{14573}{8}\sin(x)\cos(x)\right)+\frac{35}{2}\sin(x)\left(4-3\sin^2(x)\right)^2\cos(x)\right)$$

input
```
Int[(4 - 3*Sin[x]^2)^4,x]
```

output
```
(3*Cos[x]*Sin[x]*(4 - 3*Sin[x]^2)^3)/8 + ((35*Cos[x]*Sin[x]*(4 - 3*Sin[x]^
2)^2)/2 + ((10643*x)/8 + (14573*Cos[x]*Sin[x])/8 - (2193*Cos[x]*Sin[x]^3)/
4)/2)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 11.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

method	result
risch	$\frac{10643x}{128}+\frac{81\sin(8x)}{1024}+\frac{45\sin(6x)}{32}+\frac{1431\sin(4x)}{128}+\frac{1905\sin(2x)}{32}$
parallelrisch	$\frac{10643x}{128}+\frac{81\sin(8x)}{1024}+\frac{45\sin(6x)}{32}+\frac{1431\sin(4x)}{128}+\frac{1905\sin(2x)}{32}$
default	$-\frac{81\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{10643x}{128}+72\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x$
parts	$-\frac{81\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{10643x}{128}+72\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x$
norman	$\frac{10643x}{128}+\frac{38553\tan\left(\frac{x}{2}\right)^3}{64}+\frac{106173\tan\left(\frac{x}{2}\right)^5}{64}+\frac{6801\tan\left(\frac{x}{2}\right)^7}{64}-\frac{6801\tan\left(\frac{x}{2}\right)^9}{64}-\frac{106173\tan\left(\frac{x}{2}\right)^{11}}{64}-\frac{38553\tan\left(\frac{x}{2}\right)^{13}}{64}-\frac{22125\tan\left(\frac{x}{2}\right)^{15}}{64}$

input
```
int((4-3*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
10643/128*x+81/1024*sin(8*x)+45/32*sin(6*x)+1431/128*sin(4*x)+1905/32*sin(
2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \left(4-3\sin^2(x)\right)^4\,dx$$
$$=\frac{3}{128}\left(432\cos\left(x\right)^7+1272\cos\left(x\right)^5+2166\cos\left(x\right)^3+3505\cos\left(x\right)\right)\sin\left(x\right)+\frac{10643}{128}x$$

input
```
integrate((4-3*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
3/128*(432*cos(x)^7 + 1272*cos(x)^5 + 2166*cos(x)^3 + 3505*cos(x))*sin(x)
+ 10643/128*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(66) = 132.

Time = 0.47 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.53

$$\begin{aligned}\int \left(4-3\sin^2(x)\right)^4\,dx=&\ \frac{2835x\sin^8\left(x\right)}{128}+\frac{2835x\sin^6\left(x\right)\cos^2\left(x\right)}{32}-135x\sin^6\left(x\right)\\&+\frac{8505x\sin^4\left(x\right)\cos^4\left(x\right)}{64}-405x\sin^4\left(x\right)\cos^2\left(x\right)\\&+324x\sin^4\left(x\right)+\frac{2835x\sin^2\left(x\right)\cos^6\left(x\right)}{32}\\&-405x\sin^2\left(x\right)\cos^4\left(x\right)+648x\sin^2\left(x\right)\cos^2\left(x\right)\\&-384x\sin^2\left(x\right)+\frac{2835x\cos^8\left(x\right)}{128}-135x\cos^6\left(x\right)\\&+324x\cos^4\left(x\right)-384x\cos^2\left(x\right)+256x-\frac{7533\sin^7\left(x\right)\cos\left(x\right)}{128}\\&-\frac{13797\sin^5\left(x\right)\cos^3\left(x\right)}{128}+297\sin^5\left(x\right)\cos\left(x\right)\\&-\frac{10395\sin^3\left(x\right)\cos^5\left(x\right)}{128}+360\sin^3\left(x\right)\cos^3\left(x\right)\\&-540\sin^3\left(x\right)\cos\left(x\right)-\frac{2835\sin\left(x\right)\cos^7\left(x\right)}{128}\\&+135\sin\left(x\right)\cos^5\left(x\right)-324\sin\left(x\right)\cos^3\left(x\right)+384\sin\left(x\right)\cos\left(x\right)\end{aligned}$$

input
```
integrate((4-3*sin(x)**2)**4,x)
```

output
```
2835*x*sin(x)**8/128 + 2835*x*sin(x)**6*cos(x)**2/32 - 135*x*sin(x)**6 + 8
505*x*sin(x)**4*cos(x)**4/64 - 405*x*sin(x)**4*cos(x)**2 + 324*x*sin(x)**4
 + 2835*x*sin(x)**2*cos(x)**6/32 - 405*x*sin(x)**2*cos(x)**4 + 648*x*sin(x
)**2*cos(x)**2 - 384*x*sin(x)**2 + 2835*x*cos(x)**8/128 - 135*x*cos(x)**6
+ 324*x*cos(x)**4 - 384*x*cos(x)**2 + 256*x - 7533*sin(x)**7*cos(x)/128 -
13797*sin(x)**5*cos(x)**3/128 + 297*sin(x)**5*cos(x) - 10395*sin(x)**3*cos
(x)**5/128 + 360*sin(x)**3*cos(x)**3 - 540*sin(x)**3*cos(x) - 2835*sin(x)*
cos(x)**7/128 + 135*sin(x)*cos(x)**5 - 324*sin(x)*cos(x)**3 + 384*sin(x)*c
os(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int \left(4-3\sin^2(x)\right)^4\,dx = -\frac{45}{8}\sin(2x)^3+\frac{10643}{128}x+\frac{81}{1024}\sin(8x)+\frac{1431}{128}\sin(4x)+\frac{255}{4}\sin(2x)$$

input
```
integrate((4-3*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
-45/8*sin(2*x)^3 + 10643/128*x + 81/1024*sin(8*x) + 1431/128*sin(4*x) + 25
5/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \left(4-3\sin^2(x)\right)^4\,dx = \frac{10643}{128}x+\frac{81}{1024}\sin(8x)+\frac{45}{32}\sin(6x)+\frac{1431}{128}\sin(4x)+\frac{1905}{32}\sin(2x)$$

input
```
integrate((4-3*sin(x)^2)^4,x, algorithm="giac")
```

output
```
10643/128*x + 81/1024*sin(8*x) + 45/32*sin(6*x) + 1431/128*sin(4*x) + 1905
/32*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 35.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \left(4-3\sin^2(x)\right)^4\,dx=\frac{10643\,x}{128}+\frac{\frac{10515\tan(x)^7}{128}+\frac{38043\tan(x)^5}{128}+\frac{48357\tan(x)^3}{128}+\frac{22125\tan(x)}{128}}{\left(\tan(x)^2+1\right)^4}$$

input
```
int((3*sin(x)^2 - 4)^4,x)
```

output
```
(10643*x)/128 + ((22125*tan(x))/128 + (48357*tan(x)^3)/128 + (38043*tan(x)
^5)/128 + (10515*tan(x)^7)/128)/(tan(x)^2 + 1)^4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \left(4-3\sin^2(x)\right)^4\,dx=-\frac{81\cos(x)\sin(x)^7}{8}+\frac{963\cos(x)\sin(x)^5}{16}-\frac{9009\cos(x)\sin(x)^3}{64}+\frac{22125\cos(x)\sin(x)}{128}+\frac{10643x}{128}$$

input
```
int((4-3*sin(x)^2)^4,x)
```

output
```
( - 1296*cos(x)*sin(x)**7 + 7704*cos(x)*sin(x)**5 - 18018*cos(x)*sin(x)**3
 + 22125*cos(x)*sin(x) + 10643*x)/128
```

3.102 $\int (4-3\sin^2(x))^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int (4-3\sin^2(x))^3 \, dx = \frac{385x}{16}+\frac{511}{16}\cos(x)\sin(x)-\frac{75}{8}\cos(x)\sin^3(x) + \frac{1}{2}\cos(x)\sin(x)\left(4-3\sin^2(x)\right)^2$$

output
```
385/16*x+511/16*cos(x)*sin(x)-75/8*cos(x)*sin(x)^3+1/2*cos(x)*sin(x)*(4-3*
sin(x)^2)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int (4-3\sin^2(x))^3 \, dx = \frac{385x}{16}+\frac{981}{64}\sin(2x)+\frac{135}{64}\sin(4x)+\frac{9}{64}\sin(6x)$$

input
```
Integrate[(4 - 3*Sin[x]^2)^3,x]
```

output
```
(385*x)/16 + (981*Sin[2*x])/64 + (135*Sin[4*x])/64 + (9*Sin[6*x])/64
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3659, 27, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3 \sin ^2(x)\right)^3 \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(4-3 \sin (x)^2\right)^3 dx$$
$$\Big\downarrow \text{3659}$$
$$\frac{1}{6} \int 3 \left(28-25 \sin ^2(x)\right) \left(4-3 \sin ^2(x)\right) dx+\frac{1}{2} \sin (x) \left(4-3 \sin ^2(x)\right)^2 \cos (x)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2} \int \left(28-25 \sin ^2(x)\right) \left(4-3 \sin ^2(x)\right) dx+\frac{1}{2} \sin (x) \left(4-3 \sin ^2(x)\right)^2 \cos (x)$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2} \int \left(28-25 \sin (x)^2\right) \left(4-3 \sin (x)^2\right) dx+\frac{1}{2} \sin (x) \left(4-3 \sin ^2(x)\right)^2 \cos (x)$$
$$\Big\downarrow \text{3648}$$
$$\frac{1}{2} \left(\frac{385 x}{8}-\frac{75}{4} \sin ^3(x) \cos (x)+\frac{511}{8} \sin (x) \cos (x)\right)+\frac{1}{2} \sin (x) \left(4-3 \sin ^2(x)\right)^2 \cos (x)$$

input
```
Int[(4 - 3*Sin[x]^2)^3,x]
```

output
```
(Cos[x]*Sin[x]*(4 - 3*Sin[x]^2)^2)/2 + ((385*x)/8 + (511*Cos[x]*Sin[x])/8
- (75*Cos[x]*Sin[x]^3)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

method	result
risch	$\frac{385x}{16}+\frac{9\sin(6x)}{64}+\frac{135\sin(4x)}{64}+\frac{981\sin(2x)}{64}$
parallelrisch	$\frac{385x}{16}+\frac{9\sin(6x)}{64}+\frac{135\sin(4x)}{64}+\frac{981\sin(2x)}{64}$
default	$\frac{385x}{16}+72\cos(x)\sin(x)-27\left(\sin(x)^{3}+\frac{3\sin(x)}{2}\right)\cos(x)+\frac{9\left(\sin(x)^{5}+\frac{5\sin(x)^{3}}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{2}$
parts	$\frac{385x}{16}+72\cos(x)\sin(x)-27\left(\sin(x)^{3}+\frac{3\sin(x)}{2}\right)\cos(x)+\frac{9\left(\sin(x)^{5}+\frac{5\sin(x)^{3}}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{2}$
norman	$\frac{\frac{385x}{16}+\frac{549\tan(\frac{x}{2})^{3}}{8}+\frac{531\tan(\frac{x}{2})^{5}}{4}-\frac{531\tan(\frac{x}{2})^{7}}{4}-\frac{549\tan(\frac{x}{2})^{9}}{8}-\frac{639\tan(\frac{x}{2})^{11}}{8}+\frac{1155x\tan(\frac{x}{2})^{2}}{8}+\frac{5775x\tan(\frac{x}{2})^{4}}{16}+\frac{1925x\tan(\frac{x}{2})}{4}}{\left(1+\tan(\frac{x}{2})^{2}\right)^{6}}$
orering	$\left(\frac{x}{2}+\frac{385}{2539008}\right)\left(4-3\sin(x)^{2}\right)^{3}+\frac{33793\cos(x)\sin(x)\left(4-3\sin(x)^{2}\right)^{2}}{7424}+\left(\frac{49x}{288}+\frac{385}{2539008}\right)\left(18\sin(x)^{2}\right.$

input
```
int((4-3*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
385/16*x+9/64*sin(6*x)+135/64*sin(4*x)+981/64*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \left(4-3\sin^2(x)\right)^3\,dx=\frac{9}{16}\left(8\cos(x)^{5}+22\cos(x)^{3}+41\cos(x)\right)\sin(x)+\frac{385}{16}\,x$$

input
```
integrate((4-3*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
9/16*(8*cos(x)^5 + 22*cos(x)^3 + 41*cos(x))*sin(x) + 385/16*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(44) = 88.

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 4.12

$$\int \left(4-3\sin^2(x)\right)^3\,dx = -\frac{135x\sin^6(x)}{16}-\frac{405x\sin^4(x)\cos^2(x)}{16}+\frac{81x\sin^4(x)}{2}-\frac{405x\sin^2(x)\cos^4(x)}{16}+81x\sin^2(x)\cos^2(x)-72x\sin^2(x)-\frac{135x\cos^6(x)}{16}+\frac{81x\cos^4(x)}{2}-72x\cos^2(x)+64x+\frac{297\sin^5(x)\cos(x)}{16}+\frac{45\sin^3(x)\cos^3(x)}{2}-\frac{135\sin^3(x)\cos(x)}{2}+\frac{135\sin(x)\cos^5(x)}{16}-\frac{81\sin(x)\cos^3(x)}{2}+72\sin(x)\cos(x)$$

input
```
integrate((4-3*sin(x)**2)**3,x)
```

output
```
-135*x*sin(x)**6/16 - 405*x*sin(x)**4*cos(x)**2/16 + 81*x*sin(x)**4/2 - 40
5*x*sin(x)**2*cos(x)**4/16 + 81*x*sin(x)**2*cos(x)**2 - 72*x*sin(x)**2 - 1
35*x*cos(x)**6/16 + 81*x*cos(x)**4/2 - 72*x*cos(x)**2 + 64*x + 297*sin(x)*
*5*cos(x)/16 + 45*sin(x)**3*cos(x)**3/2 - 135*sin(x)**3*cos(x)/2 + 135*sin
(x)*cos(x)**5/16 - 81*sin(x)*cos(x)**3/2 + 72*sin(x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \left(4-3\sin^2(x)\right)^3\,dx = -\frac{9}{16}\,\sin\left(2\,x\right)^3+\frac{385}{16}\,x+\frac{135}{64}\,\sin\left(4\,x\right)+\frac{63}{4}\,\sin\left(2\,x\right)$$

input
```
integrate((4-3*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
-9/16*sin(2*x)^3 + 385/16*x + 135/64*sin(4*x) + 63/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \left(4-3\sin^2(x)\right)^3\,dx=\frac{385}{16}\,x+\frac{9}{64}\,\sin(6\,x)+\frac{135}{64}\,\sin(4\,x)+\frac{981}{64}\,\sin(2\,x)$$

input
```
integrate((4-3*sin(x)^2)^3,x, algorithm="giac")
```

output
```
385/16*x + 9/64*sin(6*x) + 135/64*sin(4*x) + 981/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 36.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \left(4-3\sin^2(x)\right)^3\,dx=\frac{639\cos(x)^5\,\sin(x)}{16}+\frac{117\cos(x)^3\sin(x)^3}{2}+\frac{369\,\cos(x)\,\sin(x)^5}{16}+\frac{385\,x}{16}$$

input
```
int(-(3*sin(x)^2 - 4)^3,x)
```

output
```
(385*x)/16 + (369*cos(x)*sin(x)^5)/16 + (639*cos(x)^5*sin(x))/16 + (117*co
s(x)^3*sin(x)^3)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \left(4-3\sin^2(x)\right)^3\,dx=\frac{9\cos(x)\sin(x)^5}{2}-\frac{171\cos(x)\sin(x)^3}{8}+\frac{639\cos(x)\sin(x)}{16}+\frac{385x}{16}$$

input
```
int((4-3*sin(x)^2)^3,x)
```

output
```
(72*cos(x)*sin(x)**5 - 342*cos(x)*sin(x)**3 + 639*cos(x)*sin(x) + 385*x)/1
6
```

3.103 $\int (4-3\sin^2(x))^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int (4-3\sin^2(x))^2 \, dx = \frac{59x}{8} + \frac{69}{8}\cos(x)\sin(x) - \frac{9}{4}\cos(x)\sin^3(x)$$

output
```
59/8*x+69/8*cos(x)*sin(x)-9/4*cos(x)*sin(x)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (4-3\sin^2(x))^2 \, dx = \frac{59x}{8} + \frac{15}{4}\sin(2x) + \frac{9}{32}\sin(4x)$$

input
```
Integrate[(4 - 3*Sin[x]^2)^2,x]
```

output
```
(59*x)/8 + (15*Sin[2*x])/4 + (9*Sin[4*x])/32
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3\sin^2(x)\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-3\sin(x)^2\right)^2dx$$

$$\Big\downarrow \text{3658}$$

$$\frac{59x}{8}-\frac{9}{4}\sin^3(x)\cos(x)+\frac{69}{8}\sin(x)\cos(x)$$

input
```
Int[(4 - 3*Sin[x]^2)^2,x]
```

output
```
(59*x)/8 + (69*Cos[x]*Sin[x])/8 - (9*Cos[x]*Sin[x]^3)/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result
risch	$\frac{59x}{8}+\frac{9\sin(4x)}{32}+\frac{15\sin(2x)}{4}$
parallelrisch	$\frac{59x}{8}+\frac{9\sin(4x)}{32}+\frac{15\sin(2x)}{4}$
default	$-\frac{9\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{59x}{8}+12\cos\left(x\right)\sin\left(x\right)$
parts	$-\frac{9\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{59x}{8}+12\cos\left(x\right)\sin\left(x\right)$
norman	$\frac{\frac{59x}{8}-\frac{3\tan\left(\frac{x}{2}\right)^3}{4}+\frac{3\tan\left(\frac{x}{2}\right)^5}{4}-\frac{69\tan\left(\frac{x}{2}\right)^7}{4}+\frac{59x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{177x\tan\left(\frac{x}{2}\right)^4}{4}+\frac{59x\tan\left(\frac{x}{2}\right)^6}{2}+\frac{59x\tan\left(\frac{x}{2}\right)^8}{8}+\frac{69\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$
orering	$x\left(4-3\sin\left(x\right)^2\right)^2+3\left(4-3\sin\left(x\right)^2\right)\cos\left(x\right)\sin\left(x\right)+\frac{5x\left(72\cos(x)^2\sin(x)^2+12\left(4-3\sin(x)^2\right)\sin(x)^2-\right.}{16}$

input
```
int((4-3*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
59/8*x+9/32*sin(4*x)+15/4*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \left(4-3\sin^2(x)\right)^2\,dx=\frac{3}{8}\,\left(6\,\cos\left(x\right)^3+17\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{59}{8}\,x$$

input
```
integrate((4-3*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
3/8*(6*cos(x)^3 + 17*cos(x))*sin(x) + 59/8*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.62

$$\int \left(4-3\sin^2(x)\right)^2\,dx = \frac{27x\sin^4(x)}{8} + \frac{27x\sin^2(x)\cos^2(x)}{4} - 12x\sin^2(x) + \frac{27x\cos^4(x)}{8} - 12x\cos^2(x) + 16x - \frac{45\sin^3(x)\cos(x)}{8} - \frac{27\sin(x)\cos^3(x)}{8} + 12\sin(x)\cos(x)$$

input
```
integrate((4-3*sin(x)**2)**2,x)
```

output
```
27*x*sin(x)**4/8 + 27*x*sin(x)**2*cos(x)**2/4 - 12*x*sin(x)**2 + 27*x*cos(
x)**4/8 - 12*x*cos(x)**2 + 16*x - 45*sin(x)**3*cos(x)/8 - 27*sin(x)*cos(x)
**3/8 + 12*sin(x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(4-3\sin^2(x)\right)^2\,dx = \frac{59}{8}\,x + \frac{9}{32}\,\sin(4\,x) + \frac{15}{4}\,\sin(2\,x)$$

input
```
integrate((4-3*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
59/8*x + 9/32*sin(4*x) + 15/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(4-3\sin^2(x)\right)^2\,dx=\frac{59}{8}\,x+\frac{9}{32}\,\sin\left(4\,x\right)+\frac{15}{4}\,\sin\left(2\,x\right)$$

input
```
integrate((4-3*sin(x)^2)^2,x, algorithm="giac")
```

output
```
59/8*x + 9/32*sin(4*x) + 15/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 36.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(4-3\sin^2(x)\right)^2\,dx=\frac{59\,x}{8}+\frac{\frac{51\,\tan(x)^3}{8}+\frac{69\,\tan(x)}{8}}{\left(\tan\left(x\right)^2+1\right)^2}$$

input
```
int((3*sin(x)^2 - 4)^2,x)
```

output
```
(59*x)/8 + ((69*tan(x))/8 + (51*tan(x)^3)/8)/(tan(x)^2 + 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \left(4-3\sin^2(x)\right)^2\,dx=-\frac{9\cos\left(x\right)\sin\left(x\right)^3}{4}+\frac{69\cos\left(x\right)\sin\left(x\right)}{8}+\frac{59x}{8}$$

input
```
int((4-3*sin(x)^2)^2,x)
```

output
```
( - 18*cos(x)*sin(x)**3 + 69*cos(x)*sin(x) + 59*x)/8
```

3.104 $\int (4-3\sin^2(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int (4-3\sin^2(x))\,dx=\frac{5x}{2}+\frac{3}{2}\cos(x)\sin(x)$$

output
```
5/2*x+3/2*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (4-3\sin^2(x))\,dx=\frac{5x}{2}+\frac{3}{4}\sin(2x)$$

input
```
Integrate[4 - 3*Sin[x]^2,x]
```

output
```
(5*x)/2 + (3*Sin[2*x])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3\sin^2(x)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{5x}{2}+\frac{3}{2}\sin(x)\cos(x)$$

input
```
Int[4 - 3*Sin[x]^2,x]
```

output
```
(5*x)/2 + (3*Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{5x}{2}+\frac{3\cos(x)\sin(x)}{2}$	11
risch	$\frac{5x}{2}+\frac{3\sin(2x)}{4}$	11
parallelrisch	$\frac{5x}{2}+\frac{3\sin(2x)}{4}$	11
parts	$\frac{5x}{2}+\frac{3\cos(x)\sin(x)}{2}$	11
orering	$x\left(4-3\sin\left(x\right)^{2}\right)+\frac{3\cos(x)\sin(x)}{2}+\frac{x\left(6\sin(x)^{2}-6\cos(x)^{2}\right)}{4}$	34
norman	$\frac{\frac{5x}{2}-3\tan\left(\frac{x}{2}\right)^{3}+5x\tan\left(\frac{x}{2}\right)^{2}+\frac{5x\tan\left(\frac{x}{2}\right)^{4}}{2}+3\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	48

input
```
int(4-3*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
5/2*x+3/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-3\sin^2(x)\right)\,dx=\frac{3}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{5}{2}\,x$$

input
```
integrate(4-3*sin(x)^2,x, algorithm="fricas")
```

output
```
3/2*cos(x)*sin(x) + 5/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(4-3\sin^2(x)\right)\,dx=\frac{5x}{2}+\frac{3\sin{(x)}\cos{(x)}}{2}$$

input
```
integrate(4-3*sin(x)**2,x)
```

output
```
5*x/2 + 3*sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-3\sin^2(x)\right)\,dx=\frac{5}{2}\,x+\frac{3}{4}\,\sin{(2\,x)}$$

input
```
integrate(4-3*sin(x)^2,x, algorithm="maxima")
```

output
```
5/2*x + 3/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-3\sin^2(x)\right)\,dx=\frac{5}{2}\,x+\frac{3}{4}\,\sin{(2\,x)}$$

input
```
integrate(4-3*sin(x)^2,x, algorithm="giac")
```

output
```
5/2*x + 3/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 36.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-3\sin^2(x)\right)\,dx=\frac{5\,x}{2}+\frac{3\,\sin\left(2\,x\right)}{4}$$

input
```
int(4 - 3*sin(x)^2,x)
```

output
```
(5*x)/2 + (3*sin(2*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-3\sin^2(x)\right)\,dx=\frac{3\cos\left(x\right)\sin\left(x\right)}{2}+\frac{5x}{2}$$

input
```
int(4-3*sin(x)^2,x)
```

output
```
(3*cos(x)*sin(x) + 5*x)/2
```

3.105 $\int \frac{1}{4-3\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{x}{2} - \frac{1}{2}\arctan\left(\frac{\cos(x)\sin(x)}{2-\sin^2(x)}\right)$$

output
```
1/2*x-1/2*arctan(cos(x)*sin(x)/(2-sin(x)^2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.35

$$\int \frac{1}{4-3\sin^2(x)}\,dx = -\frac{1}{2}\arctan(2\cot(x))$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[2*Cot[x]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-3\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{4-3\sin(x)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{\tan^2(x)+4}d\tan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(4 - 3*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/2]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)}{2}\right)}{2}$	8
risch	$-\frac{i\ln\left(e^{2ix}+\frac{1}{3}\right)}{4}+\frac{i\ln\left(e^{2ix}+3\right)}{4}$	24
parallelrisch	$-\frac{i\left(\ln\left(-i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)-\ln\left(i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)\right)}{4}$	39

input
```
int(1/(4-3*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{4-3\sin^2(x)}\,dx = -\frac{1}{4}\arctan\left(\frac{5\cos(x)^2-1}{4\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(4-3*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*arctan(1/4*(5*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{2} + \frac{\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{2} + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor$$

input
```
integrate(1/(4-3*sin(x)**2),x)
```

output
```
atan(2*tan(x/2) - sqrt(3))/2 + atan(2*tan(x/2) + sqrt(3))/2 + pi*floor((x/
2 - pi/2)/pi)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.27

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\tan(x)\right)$$

input
```
integrate(1/(4-3*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*arctan(1/2*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{1}{2}x - \frac{1}{2}\arctan\left(\frac{\sin(2x)}{\cos(2x)+3}\right)$$

input
```
integrate(1/(4-3*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*x - 1/2*arctan(sin(2*x)/(cos(2*x) + 3))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{4-3\sin^2(x)}\,dx = \frac{x}{2} - \frac{\operatorname{atan}(\tan(x))}{2} + \frac{\operatorname{atan}\left(\frac{\tan(x)}{2}\right)}{2}$$

input
```
int(-1/(3*sin(x)^2 - 4),x)
```

output
```
x/2 - atan(tan(x))/2 + atan(tan(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{4-3\sin^2(x)}\,dx = -\frac{atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{atan\left(\sqrt{3}+2\tan\left(\frac{x}{2}\right)\right)}{2}$$

input
```
int(1/(4-3*sin(x)^2),x)
```

output
```
( - atan(sqrt(3) - 2*tan(x/2)) + atan(sqrt(3) + 2*tan(x/2)))/2
```

3.106 $\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 44

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx = \frac{5x}{16}-\frac{5}{16}\arctan\left(\frac{\cos(x)\sin(x)}{2-\sin^2(x)}\right)-\frac{3\cos(x)\sin(x)}{8\left(4-3\sin^2(x)\right)}$$

output
```
5/16*x-5/16*arctan(cos(x)*sin(x)/(2-sin(x)^2))-3*cos(x)*sin(x)/(32-24*sin(
x)^2)
```

Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx = \frac{-5\arctan(2\cot(x))(5+3\cos(2x))-6\sin(2x)}{80+48\cos(2x)}$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-2),x]
```

output
```
(-5*ArcTan[2*Cot[x]]*(5 + 3*Cos[2*x]) - 6*Sin[2*x])/(80 + 48*Cos[2*x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3663, 27, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(4-3\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3663}$$

$$-\frac{1}{8}\int -\frac{5}{4-3\sin^2(x)}dx-\frac{3\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{8}\int \frac{1}{4-3\sin^2(x)}dx-\frac{3\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{8}\int \frac{1}{4-3\sin(x)^2}dx-\frac{3\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{5}{8}\int \frac{1}{\tan^2(x)+4}d\tan(x)-\frac{3\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{5}{16}\arctan\left(\frac{\tan(x)}{2}\right)-\frac{3\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}$$

input
```
Int[(4 - 3*Sin[x]^2)^(-2),x]
```

output
```
(5*ArcTan[Tan[x]/2])/16 - (3*Cos[x]*Sin[x])/(8*(4 - 3*Sin[x]^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

method	result	size
default	$-\frac{3\tan(x)}{8\left(\tan(x)^{2}+4\right)}+\frac{5\arctan\left(\frac{\tan(x)}{2}\right)}{16}$	21
risch	$-\frac{i\left(5\,\mathrm{e}^{2ix}+3\right)}{4\left(3\,\mathrm{e}^{4ix}+10\,\mathrm{e}^{2ix}+3\right)}-\frac{5i\ln\left(\mathrm{e}^{2ix}+\frac{1}{3}\right)}{32}+\frac{5i\ln\left(\mathrm{e}^{2ix}+3\right)}{32}$	54
parallelrisch	$\frac{5i(-5-3\cos(2x))\ln\left(-i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)+5i(5+3\cos(2x))\ln\left(i\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)-12\sin(2x)}{160+96\cos(2x)}$	74

input
```
int(1/(4-3*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8*tan(x)/(tan(x)^2+4)+5/16*arctan(1/2*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx = -\frac{5\left(3\cos(x)^2+1\right)\arctan\left(\frac{5\cos(x)^2-1}{4\cos(x)\sin(x)}\right)+12\cos(x)\sin(x)}{32\left(3\cos(x)^2+1\right)}$$

input
```
integrate(1/(4-3*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/32*(5*(3*cos(x)^2 + 1)*arctan(1/4*(5*cos(x)^2 - 1)/(cos(x)*sin(x))) + 1
2*cos(x)*sin(x))/(3*cos(x)^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(39) = 78$.

Time = 1.57 (sec) , antiderivative size = 357, normalized size of antiderivative = 8.11

$$\int \frac{1}{(4-3\sin^2(x))^2}\,dx = \frac{5\left(\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} - \frac{5\left(\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{x}{2}\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} + \frac{5\left(\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} + \frac{5\left(\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} - \frac{5\left(\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{x}{2}\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} + \frac{5\left(\operatorname{atan}\left(2\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} + \frac{3\tan^3\left(\frac{x}{2}\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16} - \frac{3\tan\left(\frac{x}{2}\right)}{16\tan^4\left(\frac{x}{2}\right)-16\tan^2\left(\frac{x}{2}\right)+16}$$

input
```
integrate(1/(4-3*sin(x)**2)**2,x)
```

output
```
5*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(16
*tan(x/2)**4 - 16*tan(x/2)**2 + 16) - 5*(atan(2*tan(x/2) - sqrt(3)) + pi*f
loor((x/2 - pi/2)/pi))*tan(x/2)**2/(16*tan(x/2)**4 - 16*tan(x/2)**2 + 16)
+ 5*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/pi))/(16*tan(x/2)*
*4 - 16*tan(x/2)**2 + 16) + 5*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x/2
- pi/2)/pi))*tan(x/2)**4/(16*tan(x/2)**4 - 16*tan(x/2)**2 + 16) - 5*(atan(
2*tan(x/2) + sqrt(3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(16*tan(x/2
)**4 - 16*tan(x/2)**2 + 16) + 5*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x/
2 - pi/2)/pi))/(16*tan(x/2)**4 - 16*tan(x/2)**2 + 16) + 3*tan(x/2)**3/(16*
tan(x/2)**4 - 16*tan(x/2)**2 + 16) - 3*tan(x/2)/(16*tan(x/2)**4 - 16*tan(x
/2)**2 + 16)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx = -\frac{3\,\tan(x)}{8\left(\tan(x)^2+4\right)} + \frac{5}{16}\arctan\left(\frac{1}{2}\tan(x)\right)$$

input
```
integrate(1/(4-3*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
-3/8*tan(x)/(tan(x)^2 + 4) + 5/16*arctan(1/2*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx = \frac{5}{16}x - \frac{3\,\tan(x)}{8\left(\tan(x)^2+4\right)} - \frac{5}{16}\arctan\left(\frac{\sin(2x)}{\cos(2x)+3}\right)$$

input
```
integrate(1/(4-3*sin(x)^2)^2,x, algorithm="giac")
```

output
```
5/16*x - 3/8*tan(x)/(tan(x)^2 + 4) - 5/16*arctan(sin(2*x)/(cos(2*x) + 3))
```

Mupad [B] (verification not implemented)

Time = 36.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx = \frac{5\,x}{16} - \frac{5\,\mathrm{atan}(\tan(x))}{16} + \frac{5\,\mathrm{atan}\left(\frac{\tan(x)}{2}\right)}{16} - \frac{3\tan(x)}{8\,\left(\tan(x)^2+4\right)}$$

input
```
int(1/(3*sin(x)^2 - 4)^2,x)
```

output
```
(5*x)/16 - (5*atan(tan(x)))/16 + (5*atan(tan(x)/2))/16 - (3*tan(x))/(8*(ta
n(x)^2 + 4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^2}\,dx$$
$$= \frac{-15atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)\sin\left(x\right)^2+20atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)+15atan\left(\sqrt{3}+2\tan\left(\frac{x}{2}\right)\right)\sin\left(x\right)^2-20at}{48\sin\left(x\right)^2-64}$$

input
```
int(1/(4-3*sin(x)^2)^2,x)
```

output
```
( - 15*atan(sqrt(3) - 2*tan(x/2))*sin(x)**2 + 20*atan(sqrt(3) - 2*tan(x/2)
) + 15*atan(sqrt(3) + 2*tan(x/2))*sin(x)**2 - 20*atan(sqrt(3) + 2*tan(x/2)
) + 6*cos(x)*sin(x))/(16*(3*sin(x)**2 - 4))
```

3.107 $\int \frac{1}{(4-3\sin^2(x))^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{1}{(4-3\sin^2(x))^3}\,dx = \frac{59x}{256} - \frac{59}{256}\arctan\left(\frac{\cos(x)\sin(x)}{2-\sin^2(x)}\right) - \frac{3\cos(x)\sin(x)}{16(4-3\sin^2(x))^2} - \frac{45\cos(x)\sin(x)}{128(4-3\sin^2(x))}$$

output
```
59/256*x-59/256*arctan(cos(x)*sin(x)/(2-sin(x)^2))-3/16*cos(x)*sin(x)/(4-3
*sin(x)^2)^2-45*cos(x)*sin(x)/(512-384*sin(x)^2)
```

Mathematica [A] (verified)

Time = 5.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \frac{1}{(4-3\sin^2(x))^3}\,dx = \frac{1}{256}\left(-59\arctan(2\cot(x)) - \frac{3(182\sin(2x)+45\sin(4x))}{(5+3\cos(2x))^2}\right)$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-3),x]
```

output
```
(-59*ArcTan[2*Cot[x]] - (3*(182*Sin[2*x] + 45*Sin[4*x]))/(5 + 3*Cos[2*x])^
2)/256
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(4-3\sin(x)^2)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{16}\int -\frac{6\sin^2(x)+7}{\left(4-3\sin^2(x)\right)^2}dx-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\int \frac{6\sin^2(x)+7}{\left(4-3\sin^2(x)\right)^2}dx-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \frac{6\sin(x)^2+7}{(4-3\sin(x)^2)^2}dx-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{16}\left(\frac{1}{8}\int \frac{59}{4-3\sin^2(x)}dx-\frac{45\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}\right)-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{59}{8}\int \frac{1}{4-3\sin^2(x)}dx-\frac{45\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}\right)-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{16}\left(\frac{59}{8}\int \frac{1}{4-3\sin(x)^2}dx-\frac{45\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}\right)-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \ 3660$$

$$\frac{1}{16}\left(\frac{59}{8}\int \frac{1}{\tan^2(x)+4}d\tan(x)-\frac{45\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}\right)-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{16}\left(\frac{59}{16}\arctan\left(\frac{\tan(x)}{2}\right)-\frac{45\sin(x)\cos(x)}{8\left(4-3\sin^2(x)\right)}\right)-\frac{3\sin(x)\cos(x)}{16\left(4-3\sin^2(x)\right)^2}$$

input
```
Int[(4 - 3*Sin[x]^2)^(-3),x]
```

output
```
(-3*Cos[x]*Sin[x])/(16*(4 - 3*Sin[x]^2)^2) + ((59*ArcTan[Tan[x]/2])/16 - (
45*Cos[x]*Sin[x])/(8*(4 - 3*Sin[x]^2)))/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.47

method	result
default	$\frac{-\frac{69 \tan(x)^{3}}{128}-\frac{51 \tan(x)}{32}}{\left(\tan(x)^{2}+4\right)^{2}}+\frac{59 \arctan\left(\frac{\tan(x)}{2}\right)}{256}$
risch	$-\frac{3 i \left(59 \,\mathrm{e}^{6 i x}+295 \,\mathrm{e}^{4 i x}+241 \,\mathrm{e}^{2 i x}+45\right)}{64 \left(3 \,\mathrm{e}^{4 i x}+10 \,\mathrm{e}^{2 i x}+3\right)^{2}}+\frac{59 i \ln\left(\mathrm{e}^{2 i x}+3\right)}{512}-\frac{59 i \ln\left(\mathrm{e}^{2 i x}+\frac{1}{3}\right)}{512}$
parallelrisch	$\frac{59 i \left(-59-9 \cos(4 x)-60 \cos(2 x)\right) \ln\left(-i \tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)+59 i \left(9 \cos(4 x)+59+60 \cos(2 x)\right) \ln\left(i \tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{2}-1\right)-2}{4608 \cos(4 x)+30208+30720 \cos(2 x)}$

input
```
int(1/(4-3*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-69/128*tan(x)^3-51/32*tan(x))/(tan(x)^2+4)^2+59/256*arctan(1/2*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{1}{(4-3\sin^2(x))^3}\,dx = -\frac{59\left(9\cos(x)^4+6\cos(x)^2+1\right)\arctan\left(\frac{5\cos(x)^2-1}{4\cos(x)\sin(x)}\right)+12\left(45\cos(x)^3+23\cos(x)\right)\sin(x)}{512\left(9\cos(x)^4+6\cos(x)^2+1\right)}$$

input
```
integrate(1/(4-3*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/512*(59*(9*cos(x)^4 + 6*cos(x)^2 + 1)*arctan(1/4*(5*cos(x)^2 - 1)/(cos(
x)*sin(x))) + 12*(45*cos(x)^3 + 23*cos(x))*sin(x))/(9*cos(x)^4 + 6*cos(x)^
2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. 2(60) = 120.

Time = 5.34 (sec) , antiderivative size = 860, normalized size of antiderivative = 13.87

$$\int \frac{1}{(4-3\sin^2(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4-3*sin(x)**2)**3,x)
```

output
```
59*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**8/(2
56*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*tan(x/2)**2 + 256
) - 118*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)*
*6/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*tan(x/2)**2
+ 256) + 177*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/pi))*tan(
x/2)**4/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*tan(x/2
)**2 + 256) - 118*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/pi))
*tan(x/2)**2/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*ta
n(x/2)**2 + 256) + 59*(atan(2*tan(x/2) - sqrt(3)) + pi*floor((x/2 - pi/2)/
pi))/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*tan(x/2)**
2 + 256) + 59*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x/2 - pi/2)/pi))*tan
(x/2)**8/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*tan(x/
2)**2 + 256) - 118*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x/2 - pi/2)/pi)
)*tan(x/2)**6/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - 512*t
an(x/2)**2 + 256) + 177*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x/2 - pi/2
)/pi))*tan(x/2)**4/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 -
512*tan(x/2)**2 + 256) - 118*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x/2 -
 pi/2)/pi))*tan(x/2)**2/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)*
*4 - 512*tan(x/2)**2 + 256) + 59*(atan(2*tan(x/2) + sqrt(3)) + pi*floor((x
/2 - pi/2)/pi))/(256*tan(x/2)**8 - 512*tan(x/2)**6 + 768*tan(x/2)**4 - ...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^3}\,dx = -\frac{3\left(23\,\tan\left(x\right)^3+68\,\tan\left(x\right)\right)}{128\left(\tan\left(x\right)^4+8\,\tan\left(x\right)^2+16\right)}+\frac{59}{256}\arctan\left(\frac{1}{2}\tan\left(x\right)\right)$$

input
```
integrate(1/(4-3*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
-3/128*(23*tan(x)^3 + 68*tan(x))/(tan(x)^4 + 8*tan(x)^2 + 16) + 59/256*arc
tan(1/2*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \frac{1}{(4-3\sin^2(x))^3}\,dx = \frac{59}{256}\,x - \frac{3\left(23\,\tan\left(x\right)^3+68\,\tan\left(x\right)\right)}{128\left(\tan\left(x\right)^2+4\right)^2} - \frac{59}{256}\,\arctan\left(\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)+3}\right)$$

input
```
integrate(1/(4-3*sin(x)^2)^3,x, algorithm="giac")
```

output
```
59/256*x - 3/128*(23*tan(x)^3 + 68*tan(x))/(tan(x)^2 + 4)^2 - 59/256*arcta
n(sin(2*x)/(cos(2*x) + 3))
```

Mupad [B] (verification not implemented)

Time = 36.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int \frac{1}{(4-3\sin^2(x))^3}\,dx = \frac{59\,x}{256} - \frac{59\,\mathrm{atan}(\tan(x))}{256} + \frac{59\,\mathrm{atan}\left(\frac{\tan(x)}{2}\right)}{256} - \frac{\frac{69\tan(x)^3}{128}+\frac{51\tan(x)}{32}}{\tan\left(x\right)^4+8\tan\left(x\right)^2+16}$$

input
```
int(-1/(3*sin(x)^2 - 4)^3,x)
```

output
```
(59*x)/256 - (59*atan(tan(x)))/256 + (59*atan(tan(x)/2))/256 - ((51*tan(x)
)/32 + (69*tan(x)^3)/128)/(8*tan(x)^2 + tan(x)^4 + 16)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^3}\,dx$$
$$= \frac{-531 atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)\sin(x)^4+1416 atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)\sin(x)^2-944 atan\left(\sqrt{3}-2\tan\left(\frac{x}{2}\right)\right)+}{}$$

input
```
int(1/(4-3*sin(x)^2)^3,x)
```

output
```
( - 531*atan(sqrt(3) - 2*tan(x/2))*sin(x)**4 + 1416*atan(sqrt(3) - 2*tan(x
/2))*sin(x)**2 - 944*atan(sqrt(3) - 2*tan(x/2)) + 531*atan(sqrt(3) + 2*tan
(x/2))*sin(x)**4 - 1416*atan(sqrt(3) + 2*tan(x/2))*sin(x)**2 + 944*atan(sq
rt(3) + 2*tan(x/2)) + 270*cos(x)*sin(x)**3 - 408*cos(x)*sin(x))/(256*(9*si
n(x)**4 - 24*sin(x)**2 + 16))
```

3.108 $\int \left(4-3\sin^2(x)\right)^{7/2} dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \left(4-3\sin^2(x)\right)^{7/2} dx = \frac{224E\left(x\middle|\frac{3}{4}\right)}{3} - \frac{200\,\text{EllipticF}\left(x,\frac{3}{4}\right)}{21} + \frac{100}{7}\cos(x)\sin(x)\sqrt{4-3\sin^2(x)} + \frac{18}{7}\cos(x)\sin(x)\left(4-3\sin^2(x)\right)^{3/2} + \frac{3}{7}\cos(x)\sin(x)\left(4-3\sin^2(x)\right)^{5/2}$$

output
```
224/3*EllipticE(sin(x),1/2*3^(1/2))-200/21*InverseJacobiAM(x,1/2*3^(1/2))+
100/7*cos(x)*sin(x)*(4-3*sin(x)^2)^(1/2)+18/7*cos(x)*sin(x)*(4-3*sin(x)^2)
^(3/2)+3/7*cos(x)*sin(x)*(4-3*sin(x)^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \left(4-3\sin^2(x)\right)^{7/2} dx = \frac{224E\left(x\middle|\frac{3}{4}\right)}{3} - \frac{200\,\text{EllipticF}\left(x,\frac{3}{4}\right)}{21} + \frac{1}{448}\sqrt{10+6\cos(2x)}(2647\sin(2x)+396\sin(4x)+27\sin(6x))$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(7/2),x]
```

output
```
(224*EllipticE[x, 3/4])/3 - (200*EllipticF[x, 3/4])/21 + (Sqrt[10 + 6*Cos[
2*x]]*(2647*Sin[2*x] + 396*Sin[4*x] + 27*Sin[6*x]))/448
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3659, 27, 3042, 3649, 27, 3042, 3649, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3 \sin^2(x)\right)^{7/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-3 \sin(x)^2\right)^{7/2} dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{7} \int 10\left(10-9 \sin^2(x)\right) \left(4-3 \sin^2(x)\right)^{3/2} dx+\frac{3}{7} \sin(x) \left(4-3 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{7} \int \left(10-9 \sin^2(x)\right) \left(4-3 \sin^2(x)\right)^{3/2} dx+\frac{3}{7} \sin(x) \left(4-3 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7} \int \left(10-9 \sin(x)^2\right) \left(4-3 \sin(x)^2\right)^{3/2} dx+\frac{3}{7} \sin(x) \left(4-3 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{10}{7} \left(\frac{1}{5} \int 2\left(82-75 \sin^2(x)\right) \sqrt{4-3 \sin^2(x)}dx+\frac{9}{5} \sin(x) \left(4-3 \sin^2(x)\right)^{3/2} \cos(x)\right)+\frac{3}{7} \sin(x) \left(4-3 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{7}\left(\frac{2}{5}\int \left(82-75\sin^2(x)\right)\sqrt{4-3\sin^2(x)}dx+\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\left(\frac{2}{5}\int \left(82-75\sin(x)^2\right)\sqrt{4-3\sin(x)^2}dx+\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{10}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{12\left(57-49\sin^2(x)\right)}{\sqrt{4-3\sin^2(x)}}dx+25\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{7}\left(\frac{2}{5}\left(4\int \frac{57-49\sin^2(x)}{\sqrt{4-3\sin^2(x)}}dx+25\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\left(\frac{2}{5}\left(4\int \frac{57-49\sin(x)^2}{\sqrt{4-3\sin(x)^2}}dx+25\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3651}$$

$$\frac{10}{7}\left(\frac{2}{5}\left(4\left(\frac{49}{3}\int \sqrt{4-3\sin^2(x)}dx-\frac{25}{3}\int \frac{1}{\sqrt{4-3\sin^2(x)}}dx\right)+25\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{9}{5}\sin(x)\ (\text{...}\right.$$
$$\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\left(\frac{2}{5}\left(4\left(\frac{49}{3}\int\sqrt{4-3\sin(x)^2}dx-\frac{25}{3}\int\frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)+25\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{9}{5}\sin(x)\left(4\right.\right.$$
$$\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \; 3656$$

$$\frac{10}{7}\left(\frac{2}{5}\left(4\left(\frac{98E\left(x\middle|\frac{3}{4}\right)}{3}-\frac{25}{3}\int\frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)+25\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)\right.$$
$$\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \; 3661$$

$$\frac{3}{7}\sin(x)\left(4-3\sin^2(x)\right)^{5/2}\cos(x)+$$
$$\frac{10}{7}\left(\frac{9}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)+\frac{2}{5}\left(25\sin(x)\sqrt{4-3\sin^2(x)}\cos(x)+4\left(\frac{98E\left(x\middle|\frac{3}{4}\right)}{3}-\frac{25\,\text{EllipticF}\left(x,\frac{3}{4}\right)}{6}\right.\right.\right.$$

input
```
Int[(4 - 3*Sin[x]^2)^(7/2),x]
```

output
```
(3*Cos[x]*Sin[x]*(4 - 3*Sin[x]^2)^(5/2))/7 + (10*((9*Cos[x]*Sin[x]*(4 - 3*
Sin[x]^2)^(3/2))/5 + (2*(4*((98*EllipticE[x, 3/4])/3 - (25*EllipticF[x, 3/
4])/6) + 25*Cos[x]*Sin[x]*Sqrt[4 - 3*Sin[x]^2]))/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [A] (verified)

Time = 5.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{-\left(-4+3\sin(x)^2\right)\cos(x)^2}\,\left(-243\cos(x)^8\sin(x)-729\cos(x)^6\sin(x)-1305\cos(x)^4\sin(x)+200\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(x)^2+1}\ \text{Ell}}{21\sqrt{3\cos(x)^4+\cos(x)^2}\,\cos(x)\sqrt{4-3}}$

input
```
int((4-3*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)*(-243*cos(x)^8*sin(x)-729*cos(x)^6
*sin(x)-1305*cos(x)^4*sin(x)+200*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*Ell
ipticF(sin(x),1/2*3^(1/2))-1568*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*Elli
pticE(sin(x),1/2*3^(1/2))-363*cos(x)^2*sin(x))/(3*cos(x)^4+cos(x)^2)^(1/2)
/cos(x)/(4-3*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(4-3\sin^2(x)\right)^{7/2} dx = \int \left(-3\, \sin\left(x\right)^{2}+4\right)^{\frac{7}{2}} dx$$

input
```
integrate((4-3*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
integral((27*cos(x)^6 + 27*cos(x)^4 + 9*cos(x)^2 + 1)*sqrt(3*cos(x)^2 + 1)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(4-3\sin^2(x)\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((4-3*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(4-3\sin^2(x)\right)^{7/2}\,dx=\int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-3*sin(x)^2 + 4)^(7/2), x)
```

Giac [F]

$$\int \left(4-3\sin^2(x)\right)^{7/2}\,dx=\int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((-3*sin(x)^2 + 4)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-3\sin^2(x)\right)^{7/2}\,dx=\int \left(4-3\sin\left(x\right)^2\right)^{7/2}\,dx$$

input
```
int((4 - 3*sin(x)^2)^(7/2),x)
```

output
```
int((4 - 3*sin(x)^2)^(7/2), x)
```

Reduce [F]

$$\int \left(4-3\sin^2(x)\right)^{7/2}\,dx = 64\left(\int \sqrt{-3\sin(x)^2+4}dx\right) - 27\left(\int \sqrt{-3\sin(x)^2+4}\,\sin(x)^6\,dx\right) + 108\left(\int \sqrt{-3\sin(x)^2+4}\,\sin(x)^4\,dx\right) - 144\left(\int \sqrt{-3\sin(x)^2+4}\,\sin(x)^2\,dx\right)$$

input
```
int((4-3*sin(x)^2)^(7/2),x)
```

output
```
64*int(sqrt( - 3*sin(x)**2 + 4),x) - 27*int(sqrt( - 3*sin(x)**2 + 4)*sin(x
)**6,x) + 108*int(sqrt( - 3*sin(x)**2 + 4)*sin(x)**4,x) - 144*int(sqrt( -
3*sin(x)**2 + 4)*sin(x)**2,x)
```

3.109 $\int (4-3\sin^2(x))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (4-3\sin^2(x))^{5/2}\,dx=\frac{328E\left(x\middle|\frac{3}{4}\right)}{15}-\frac{8\operatorname{EllipticF}\left(x,\frac{3}{4}\right)}{3}+4\cos(x)\sin(x)\sqrt{4-3\sin^2(x)}+\frac{3}{5}\cos(x)\sin(x)\left(4-3\sin^2(x)\right)^{3/2}$$

output
```
328/15*EllipticE(sin(x),1/2*3^(1/2))-8/3*InverseJacobiAM(x,1/2*3^(1/2))+4*
cos(x)*sin(x)*(4-3*sin(x)^2)^(1/2)+3/5*cos(x)*sin(x)*(4-3*sin(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int (4-3\sin^2(x))^{5/2}\,dx=\frac{328E\left(x\middle|\frac{3}{4}\right)}{15}-\frac{8\operatorname{EllipticF}\left(x,\frac{3}{4}\right)}{3}+\frac{1}{80}\sqrt{10+6\cos(2x)}(110\sin(2x)+9\sin(4x))$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(5/2),x]
```

output
```
(328*EllipticE[x, 3/4])/15 - (8*EllipticF[x, 3/4])/3 + (Sqrt[10 + 6*Cos[2*
x]]*(110*Sin[2*x] + 9*Sin[4*x]))/80
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3659, 27, 3042, 3649, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3\sin^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-3\sin(x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{5}\int 4\left(17-15\sin^2(x)\right)\sqrt{4-3\sin^2(x)}dx+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{5}\int \left(17-15\sin^2(x)\right)\sqrt{4-3\sin^2(x)}dx+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \left(17-15\sin(x)^2\right)\sqrt{4-3\sin(x)^2}dx+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{4}{5}\left(\frac{1}{3}\int \frac{3\left(48-41\sin^2(x)\right)}{\sqrt{4-3\sin^2(x)}}dx+5\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{5}\left(\int \frac{48-41\sin^2(x)}{\sqrt{4-3\sin^2(x)}}dx+5\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\big\downarrow \; 3042$$

$$\frac{4}{5}\left(\int \frac{48-41\sin(x)^2}{\sqrt{4-3\sin(x)^2}}dx+5\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\big\downarrow \; 3651$$

$$\frac{4}{5}\left(-\frac{20}{3}\int \frac{1}{\sqrt{4-3\sin^2(x)}}dx+\frac{41}{3}\int \sqrt{4-3\sin^2(x)}dx+5\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\big\downarrow \; 3042$$

$$\frac{4}{5}\left(-\frac{20}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx+\frac{41}{3}\int \sqrt{4-3\sin(x)^2}dx+5\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)\right)+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\big\downarrow \; 3656$$

$$\frac{4}{5}\left(-\frac{20}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx+\frac{82E\left(x\middle|\frac{3}{4}\right)}{3}+5\sin(x)\sqrt{4-3\sin^2(x)}\cos(x)\right)+\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)$$

$$\big\downarrow \; 3661$$

$$\frac{3}{5}\sin(x)\left(4-3\sin^2(x)\right)^{3/2}\cos(x)+\frac{4}{5}\left(-\frac{10\,\text{EllipticF}\left(x,\frac{3}{4}\right)}{3}+\frac{82E\left(x\middle|\frac{3}{4}\right)}{3}+5\sin(x)\sqrt{4-3\sin^2(x)}\cos(x)\right)$$

input
```
Int[(4 - 3*Sin[x]^2)^(5/2),x]
```

output
```
(3*Cos[x]*Sin[x]*(4 - 3*Sin[x]^2)^(3/2))/5 + (4*((82*EllipticE[x, 3/4])/3
- (10*EllipticF[x, 3/4])/3 + 5*Cos[x]*Sin[x]*Sqrt[4 - 3*Sin[x]^2]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(52) = 104$.

Time = 3.92 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.16

method	result
default	$-\frac{\sqrt{-\left(-4+3\sin(x)^2\right)\cos(x)^2}\,\left(-81\cos(x)^6\sin(x)-234\cos(x)^4\sin(x)+40\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(x)^2+1}\ \text{EllipticF}\left(\sin(x),\frac{\sqrt{3}}{2}\right)-32}{15\sqrt{3\cos(x)^4+\cos(x)^2}\,\cos(x)\sqrt{4-3\sin(x)^2}}$

input
```
int((4-3*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)*(-81*cos(x)^6*sin(x)-234*cos(x)^4*
sin(x)+40*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*EllipticF(sin(x),1/2*3^(1/
2))-328*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*EllipticE(sin(x),1/2*3^(1/2)
)-69*cos(x)^2*sin(x))/(3*cos(x)^4+cos(x)^2)^(1/2)/cos(x)/(4-3*sin(x)^2)^(1
/2)
```

Fricas [F]

$$\int \left(4-3\sin^2(x)\right)^{5/2}\,dx = \int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((9*cos(x)^4 + 6*cos(x)^2 + 1)*sqrt(3*cos(x)^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(4-3\sin^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((4-3*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(4-3\sin^2(x)\right)^{5/2}\,dx=\int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-3*sin(x)^2 + 4)^(5/2), x)
```

Giac [F]

$$\int \left(4-3\sin^2(x)\right)^{5/2}\,dx=\int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-3*sin(x)^2 + 4)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-3\sin^2(x)\right)^{5/2} dx = \int \left(4-3\sin(x)^2\right)^{5/2} dx$$

input
```
int((4 - 3*sin(x)^2)^(5/2),x)
```

output
```
int((4 - 3*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(4-3\sin^2(x)\right)^{5/2} dx = 16\left(\int \sqrt{-3\sin(x)^2+4}dx\right) + 9\left(\int \sqrt{-3\sin(x)^2+4}\,\sin(x)^4\,dx\right) - 24\left(\int \sqrt{-3\sin(x)^2+4}\,\sin(x)^2\,dx\right)$$

input
```
int((4-3*sin(x)^2)^(5/2),x)
```

output
```
16*int(sqrt( - 3*sin(x)**2 + 4),x) + 9*int(sqrt( - 3*sin(x)**2 + 4)*sin(x)
**4,x) - 24*int(sqrt( - 3*sin(x)**2 + 4)*sin(x)**2,x)
```

3.110 $\int (4-3\sin^2(x))^{3/2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int (4-3\sin^2(x))^{3/2}\, dx = \frac{20E\left(x\middle|\frac{3}{4}\right)}{3} - \frac{2\,\text{EllipticF}\left(x,\frac{3}{4}\right)}{3} + \cos(x)\sin(x)\sqrt{4-3\sin^2(x)}$$

output
```
20/3*EllipticE(sin(x),1/2*3^(1/2))-2/3*InverseJacobiAM(x,1/2*3^(1/2))+cos(
x)*sin(x)*(4-3*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int (4-3\sin^2(x))^{3/2}\, dx = \frac{1}{12}\left(80E\left(x\middle|\frac{3}{4}\right) - 8\,\text{EllipticF}\left(x,\frac{3}{4}\right) + 3\sqrt{10+6\cos(2x)}\sin(2x)\right)$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(3/2),x]
```

output
```
(80*EllipticE[x, 3/4] - 8*EllipticF[x, 3/4] + 3*Sqrt[10 + 6*Cos[2*x]]*Sin[
2*x])/12
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-3\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{6\left(6-5\sin^2(x)\right)}{\sqrt{4-3\sin^2(x)}}dx+\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{6-5\sin^2(x)}{\sqrt{4-3\sin^2(x)}}dx+\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{6-5\sin(x)^2}{\sqrt{4-3\sin(x)^2}}dx+\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3651}$$

$$2\left(\frac{5}{3}\int \sqrt{4-3\sin^2(x)}dx-\frac{2}{3}\int \frac{1}{\sqrt{4-3\sin^2(x)}}dx\right)+\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{3}\int \sqrt{4-3\sin(x)^2}dx-\frac{2}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)+\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3656}$$

$$2\left(\frac{10E\left(x\middle|\frac{3}{4}\right)}{3}-\frac{2}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)+\sqrt{4-3\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3661}$$

$$\sin(x)\sqrt{4-3\sin^2(x)}\cos(x)+2\left(\frac{10E\left(x\middle|\frac{3}{4}\right)}{3}-\frac{\text{EllipticF}\left(x,\frac{3}{4}\right)}{3}\right)$$

input
```
Int[(4 - 3*Sin[x]^2)^(3/2),x]
```

output
```
2*((10*EllipticE[x, 3/4])/3 - EllipticF[x, 3/4]/3) + Cos[x]*Sin[x]*Sqrt[4
- 3*Sin[x]^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(35) = 70$.

Time = 1.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.19

method	result
default	$-\frac{\sqrt{-\left(-4+3\sin(x)^2\right)\cos(x)^2}\,\left(-9\cos(x)^4\sin(x)+2\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(x)^2+1}\ \mathrm{EllipticF}\left(\sin(x),\frac{\sqrt{3}}{2}\right)-20\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(}}{3\sqrt{3\cos(x)^4+\cos(x)^2}\,\cos(x)\sqrt{4-3\sin(x)^2}}$

input
```
int((4-3*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)*(-9*cos(x)^4*sin(x)+2*(cos(x)^2)^(1
/2)*(3*cos(x)^2+1)^(1/2)*EllipticF(sin(x),1/2*3^(1/2))-20*(cos(x)^2)^(1/2)
*(3*cos(x)^2+1)^(1/2)*EllipticE(sin(x),1/2*3^(1/2))-3*cos(x)^2*sin(x))/(3*
cos(x)^4+cos(x)^2)^(1/2)/cos(x)/(4-3*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx = \int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((3*cos(x)^2 + 1)^(3/2), x)
```

Sympy [F]

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx=\int \left(4-3\sin^2{(x)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-3*sin(x)**2)**(3/2),x)
```

output
```
Integral((4 - 3*sin(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx=\int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*sin(x)^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx=\int \left(-3\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*sin(x)^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx=\int \left(4-3\sin\left(x\right)^2\right)^{3/2}\,dx$$

input
```
int((4 - 3*sin(x)^2)^(3/2),x)
```

output
```
int((4 - 3*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(4-3\sin^2(x)\right)^{3/2}\,dx=4\left(\int \sqrt{-3\sin\left(x\right)^2+4}dx\right)\\-3\left(\int \sqrt{-3\sin\left(x\right)^2+4}\,\sin\left(x\right)^2dx\right)$$

input
```
int((4-3*sin(x)^2)^(3/2),x)
```

output
```
4*int(sqrt( - 3*sin(x)**2 + 4),x) - 3*int(sqrt( - 3*sin(x)**2 + 4)*sin(x)*
*2,x)
```

3.111 $\int \sqrt{4-3\sin^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 7

$$\int \sqrt{4-3\sin^2(x)}\,dx = 2E\left(x\middle|\frac{3}{4}\right)$$

output
```
2*EllipticE(sin(x),1/2*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \sqrt{4-3\sin^2(x)}\,dx = 2E\left(x\middle|\frac{3}{4}\right)$$

input
```
Integrate[Sqrt[4 - 3*Sin[x]^2],x]
```

output
```
2*EllipticE[x, 3/4]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4-3\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{4-3\sin(x)^2}dx$$

$$\Big\downarrow \text{3656}$$

$$2E\left(x\left|\frac{3}{4}\right.\right)$$

input
```
Int[Sqrt[4 - 3*Sin[x]^2],x]
```

output
```
2*EllipticE[x, 3/4]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(10) = 20$.

Time = 1.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 7.14

method	result	size
default	$\frac{2\sqrt{-\left(-4+3\sin\left(x\right)^{2}\right)\cos\left(x\right)^{2}}\,\sqrt{\frac{\cos\left(2x\right)}{2}+\frac{1}{2}}\,\operatorname{EllipticE}\left(\sin\left(x\right),\frac{\sqrt{3}}{2}\right)}{\sqrt{3\cos\left(x\right)^{4}+\cos\left(x\right)^{2}}\,\cos\left(x\right)}$	50

input
```
int((4-3*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)*(cos(x)^2)^(1/2)*EllipticE(sin(x),1/2*
3^(1/2))/(3*cos(x)^4+cos(x)^2)^(1/2)/cos(x)
```

Fricas [F]

$$\int \sqrt{4-3\sin^2(x)}\,dx = \int \sqrt{-3\,\sin\left(x\right)^{2}+4}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(3*cos(x)^2 + 1), x)
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \sqrt{4-3\sin^2(x)}\,dx = 2E\left(x\middle|\frac{3}{4}\right)$$

input
```
integrate((4-3*sin(x)**2)**(1/2),x)
```

output
```
2*elliptic_e(x, 3/4)
```

Maxima [F]

$$\int \sqrt{4-3\sin^2(x)}\,dx = \int \sqrt{-3\,\sin\left(x\right)^2+4}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*sin(x)^2 + 4), x)
```

Giac [F]

$$\int \sqrt{4-3\sin^2(x)}\,dx = \int \sqrt{-3\,\sin\left(x\right)^2+4}\,dx$$

input
```
integrate((4-3*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-3*sin(x)^2 + 4), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \sqrt{4-3\sin^2(x)}\,dx = 2\,\mathrm{E}\left(x\,\middle|\,\frac{3}{4}\right)$$

input
```
int((4 - 3*sin(x)^2)^(1/2),x)
```

output
```
2*ellipticE(x, 3/4)
```

Reduce [F]

$$\int \sqrt{4-3\sin^2(x)}\,dx = \int \sqrt{-3\sin(x)^2+4}dx$$

input
```
int((4-3*sin(x)^2)^(1/2),x)
```

output
```
int(sqrt( - 3*sin(x)**2 + 4),x)
```

3.112 $\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx = \frac{\operatorname{EllipticF}\left(x,\frac{3}{4}\right)}{2}$$

output
```
1/2*InverseJacobiAM(x,1/2*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx = \frac{\operatorname{EllipticF}\left(x,\frac{3}{4}\right)}{2}$$

input
```
Integrate[1/Sqrt[4 - 3*Sin[x]^2],x]
```

output
```
EllipticF[x, 3/4]/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx$$
$$\Big\downarrow \text{3661}$$
$$\frac{\text{EllipticF}\left(x,\frac{3}{4}\right)}{2}$$

input
```
Int[1/Sqrt[4 - 3*Sin[x]^2],x]
```

output
```
EllipticF[x, 3/4]/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\text{InverseJacobiAM}\left(x,\frac{\sqrt{3}}{2}\right)}{2}$	10

input
```
int(1/(4-3*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*InverseJacobiAM(x,1/2*3^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.56

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx = -F(\arcsin\left(\frac{1}{3}i\sqrt{3}\cos(x)-\frac{1}{3}\sqrt{3}\sin(x)\right)|9) - F(\arcsin\left(-\frac{1}{3}i\sqrt{3}\cos(x)-\frac{1}{3}\sqrt{3}\sin(x)\right)|9)$$

input
```
integrate(1/(4-3*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-elliptic_f(arcsin(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) - ellipt
ic_f(arcsin(-1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9)
```

Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx = \frac{F\left(x\middle|\frac{3}{4}\right)}{2}$$

input
```
integrate(1/(4-3*sin(x)**2)**(1/2),x)
```

output
```
elliptic_f(x, 3/4)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-3\,\sin\left(x\right)^2+4}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*sin(x)^2 + 4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-3\,\sin\left(x\right)^2+4}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*sin(x)^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx=\int \frac{1}{\sqrt{4-3\sin\left(x\right)^2}}\,dx$$

input
```
int(1/(4 - 3*sin(x)^2)^(1/2),x)
```

output
```
int(1/(4 - 3*sin(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{4-3\sin^2(x)}}\,dx=-\left(\int \frac{\sqrt{-3\sin\left(x\right)^2+4}}{3\sin\left(x\right)^2-4}dx\right)$$

input
```
int(1/(4-3*sin(x)^2)^(1/2),x)
```

output
```
- int(sqrt( - 3*sin(x)**2 + 4)/(3*sin(x)**2 - 4),x)
```

3.113 $\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx = \frac{E\left(x\middle|\frac{3}{4}\right)}{2} - \frac{3\cos(x)\sin(x)}{4\sqrt{4-3\sin^2(x)}}$$

output
```
1/2*EllipticE(sin(x),1/2*3^(1/2))-3/4*cos(x)*sin(x)/(4-3*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx = \frac{E\left(x\middle|\frac{3}{4}\right)}{2} - \frac{3\sin(2x)}{4\sqrt{2}\sqrt{5+3\cos(2x)}}$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-3/2),x]
```

output
```
EllipticE[x, 3/4]/2 - (3*Sin[2*x])/(4*Sqrt[2]*Sqrt[5 + 3*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3663, 25, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(4-3\sin(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{4}\int -\sqrt{4-3\sin^2(x)}dx-\frac{3\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \sqrt{4-3\sin^2(x)}dx-\frac{3\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sqrt{4-3\sin(x)^2}dx-\frac{3\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{E\left(x\left|\frac{3}{4}\right.\right)}{2}-\frac{3\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}$$

input
```
Int[(4 - 3*Sin[x]^2)^(-3/2),x]
```

output
```
EllipticE[x, 3/4]/2 - (3*Cos[x]*Sin[x])/(4*Sqrt[4 - 3*Sin[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

method	result	size
default	$\frac{2\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(x)^{2}+1}\ \mathrm{EllipticE}\left(\sin(x),\frac{\sqrt{3}}{2}\right)-3\cos(x)^{2}\sin(x)}{4\cos(x)\sqrt{4-3\sin(x)^{2}}}$	52

input
```
int(1/(4-3*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*EllipticE(sin(x),1/2*3^(1/2))
-3*cos(x)^2*sin(x))/cos(x)/(4-3*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.70

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{3/2}}\,dx =$$

$$-\frac{18\sqrt{3\cos(x)^2+1}\cos(x)\sin(x) - \left(3\cos(x)^2+1\right)E\left(\arcsin\left(\frac{1}{3}i\sqrt{3}\cos(x)-\frac{1}{3}\sqrt{3}\sin(x)\right)\,|\,9\right) - \left(3\cos\right.}{}$$

input
```
integrate(1/(4-3*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(18*sqrt(3*cos(x)^2 + 1)*cos(x)*sin(x) - (3*cos(x)^2 + 1)*elliptic_e
(arcsin(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) - (3*cos(x)^2 + 1)*
elliptic_e(arcsin(-1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) + 16*(3*
cos(x)^2 + 1)*elliptic_f(arcsin(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x))
, 9) + 16*(3*cos(x)^2 + 1)*elliptic_f(arcsin(-1/3*I*sqrt(3)*cos(x) - 1/3*s
qrt(3)*sin(x)), 9))/(3*cos(x)^2 + 1)
```

Sympy [F]

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(4-3\sin^2{\left(x \right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)**2)**(3/2),x)
```

output
```
Integral((4 - 3*sin(x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-3\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*sin(x)^2 + 4)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-3\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*sin(x)^2 + 4)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(4-3\sin\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(4 - 3*sin(x)^2)^(3/2),x)
```

output
```
int(1/(4 - 3*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(4-3\sin^2(x))^{3/2}}\,dx=\int \frac{\sqrt{-3\sin(x)^2+4}}{9\sin(x)^4-24\sin(x)^2+16}dx$$

input

```
int(1/(4-3*sin(x)^2)^(3/2),x)
```

output

```
int(sqrt( - 3*sin(x)**2 + 4)/(9*sin(x)**4 - 24*sin(x)**2 + 16),x)
```

3.114 $\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx = \frac{5E\left(x\middle|\frac{3}{4}\right)}{12}-\frac{\text{EllipticF}\left(x,\frac{3}{4}\right)}{24} - \frac{\cos(x)\sin(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}-\frac{5\cos(x)\sin(x)}{8\sqrt{4-3\sin^2(x)}}$$

output
```
5/12*EllipticE(sin(x),1/2*3^(1/2))-1/24*InverseJacobiAM(x,1/2*3^(1/2))-1/4
*cos(x)*sin(x)/(4-3*sin(x)^2)^(3/2)-5/8*cos(x)*sin(x)/(4-3*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx = \frac{1}{96}\left(40E\left(x\middle|\frac{3}{4}\right) - 4\,\text{EllipticF}\left(x,\frac{3}{4}\right)-\frac{3\sqrt{2}(58\sin(2x)+15\sin(4x))}{(5+3\cos(2x))^{3/2}}\right)$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-5/2),x]
```

output
```
(40*EllipticE[x, 3/4] - 4*EllipticF[x, 3/4] - (3*Sqrt[2]*(58*Sin[2*x] + 15
*Sin[4*x]))/(5 + 3*Cos[2*x])^(3/2))/96
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3663, 27, 3042, 3652, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(4-3\sin(x)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{12}\int -\frac{3\left(\sin^2(x)+2\right)}{\left(4-3\sin^2(x)\right)^{3/2}}dx-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\sin^2(x)+2}{\left(4-3\sin^2(x)\right)^{3/2}}dx-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sin(x)^2+2}{\left(4-3\sin(x)^2\right)^{3/2}}dx-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{2\left(6-5\sin^2(x)\right)}{\sqrt{4-3\sin^2(x)}}dx-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{6-5\sin^2(x)}{\sqrt{4-3\sin^2(x)}}dx-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{6-5\sin(x)^2}{\sqrt{4-3\sin(x)^2}}dx-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{5}{3}\int \sqrt{4-3\sin^2(x)}dx-\frac{2}{3}\int \frac{1}{\sqrt{4-3\sin^2(x)}}dx\right)-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{5}{3}\int \sqrt{4-3\sin(x)^2}dx-\frac{2}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{10E\left(x\left|\frac{3}{4}\right.\right)}{3}-\frac{2}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{10E\left(x\left|\frac{3}{4}\right.\right)}{3}-\frac{\text{EllipticF}\left(x,\frac{3}{4}\right)}{3}\right)-\frac{5\sin(x)\cos(x)}{2\sqrt{4-3\sin^2(x)}}\right)-\frac{\sin(x)\cos(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}$$

input
```
Int[(4 - 3*Sin[x]^2)^(-5/2),x]
```

output
```
-1/4*(Cos[x]*Sin[x])/(4 - 3*Sin[x]^2)^(3/2) + (((10*EllipticE[x, 3/4])/3 -
 EllipticF[x, 3/4]/3)/2 - (5*Cos[x]*Sin[x])/(2*Sqrt[4 - 3*Sin[x]^2]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(52) = 104.

Time = 0.75 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.22

method	result
default	$-\frac{\sqrt{-\left(-4+3\sin(x)^2\right)\cos(x)^2}\,\left(3\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(x)^2+1}\ \mathrm{EllipticF}\left(\sin(x),\frac{\sqrt{3}}{2}\right)\cos(x)^2-30\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{3\cos(x)^2+1}\ \mathrm{Ellip}}{}$

input
```
int(1/(4-3*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)/(9*cos(x)^4+6*cos(x)^2+1)/cos(x)^3
*(3*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*EllipticF(sin(x),1/2*3^(1/2))*co
s(x)^2-30*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*EllipticE(sin(x),1/2*3^(1/
2))*cos(x)^2+45*cos(x)^4*sin(x)+(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*Elli
pticF(sin(x),1/2*3^(1/2))-10*(cos(x)^2)^(1/2)*(3*cos(x)^2+1)^(1/2)*Ellipti
cE(sin(x),1/2*3^(1/2))+21*cos(x)^2*sin(x))*(3*cos(x)^4+cos(x)^2)^(1/2)/(4-
3*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{18\left(15\cos(x)^3+7\cos(x)\right)\sqrt{3\cos(x)^2+1}\sin(x)-5\left(9\cos(x)^4+6\cos(x)^2+1\right)E(\arcsin\left(\frac{1}{3}i\sqrt{3}\cos\right.}{}$$

input
```
integrate(1/(4-3*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/144*(18*(15*cos(x)^3 + 7*cos(x))*sqrt(3*cos(x)^2 + 1)*sin(x) - 5*(9*cos
(x)^4 + 6*cos(x)^2 + 1)*elliptic_e(arcsin(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(
3)*sin(x)), 9) - 5*(9*cos(x)^4 + 6*cos(x)^2 + 1)*elliptic_e(arcsin(-1/3*I*
sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) + 68*(9*cos(x)^4 + 6*cos(x)^2 + 1
)*elliptic_f(arcsin(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) + 68*(9
*cos(x)^4 + 6*cos(x)^2 + 1)*elliptic_f(arcsin(-1/3*I*sqrt(3)*cos(x) - 1/3*
sqrt(3)*sin(x)), 9))/(9*cos(x)^4 + 6*cos(x)^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(4-3\sin^2(x))^{5/2}}\,dx = \int \frac{1}{(4-3\sin^2{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)**2)**(5/2),x)
```

output
```
Integral((4 - 3*sin(x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(4-3\sin^2(x))^{5/2}}\,dx = \int \frac{1}{(-3\,\sin{(x)}^2+4)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-3*sin(x)^2 + 4)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(-3\,\sin\left(x\right)^{2}+4\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-3*sin(x)^2 + 4)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(4-3\sin\left(x\right)^{2}\right)^{5/2}}\,dx$$

input
```
int(1/(4 - 3*sin(x)^2)^(5/2),x)
```

output
```
int(1/(4 - 3*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{5/2}}\,dx=-\left(\int \frac{\sqrt{-3\sin\left(x\right)^{2}+4}}{27\sin\left(x\right)^{6}-108\sin\left(x\right)^{4}+144\sin\left(x\right)^{2}-64}dx\right)$$

input
```
int(1/(4-3*sin(x)^2)^(5/2),x)
```

output
```
 - int(sqrt( - 3*sin(x)**2 + 4)/(27*sin(x)**6 - 108*sin(x)**4 + 144*sin(x)
**2 - 64),x)
```

3.115 $\int \frac{1}{\left(4-3\sin^2(x)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{7/2}}\,dx=\frac{41E\left(x\middle|\frac{3}{4}\right)}{120}-\frac{\text{EllipticF}\left(x,\frac{3}{4}\right)}{24}-\frac{3\cos(x)\sin(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}-\frac{\cos(x)\sin(x)}{4\left(4-3\sin^2(x)\right)^{3/2}}-\frac{41\cos(x)\sin(x)}{80\sqrt{4-3\sin^2(x)}}$$

output
```
41/120*EllipticE(sin(x),1/2*3^(1/2))-1/24*InverseJacobiAM(x,1/2*3^(1/2))-3
/20*cos(x)*sin(x)/(4-3*sin(x)^2)^(5/2)-1/4*cos(x)*sin(x)/(4-3*sin(x)^2)^(3
/2)-41/80*cos(x)*sin(x)/(4-3*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{7/2}}\,dx=\frac{656E\left(x\middle|\frac{3}{4}\right)-80\,\text{EllipticF}\left(x,\frac{3}{4}\right)-\frac{3\sqrt{2}(5461\sin(2x)+2700\sin(4x)+369\sin(6x))}{(5+3\cos(2x))^{5/2}}}{1920}$$

input
```
Integrate[(4 - 3*Sin[x]^2)^(-7/2),x]
```

output
```
(656*EllipticE[x, 3/4] - 80*EllipticF[x, 3/4] - (3*Sqrt[2]*(5461*Sin[2*x]
+ 2700*Sin[4*x] + 369*Sin[6*x]))/(5 + 3*Cos[2*x])^(5/2))/1920
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3652, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(4-3\sin(x)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{20}\int -\frac{9\sin^2(x)+8}{\left(4-3\sin^2(x)\right)^{5/2}}dx-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{20}\int \frac{9\sin^2(x)+8}{\left(4-3\sin^2(x)\right)^{5/2}}dx-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{20}\int \frac{9\sin(x)^2+8}{\left(4-3\sin(x)^2\right)^{5/2}}dx-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{20}\left(\frac{1}{12}\int \frac{12\left(5\sin^2(x)+7\right)}{\left(4-3\sin^2(x)\right)^{3/2}}dx-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}\left(\int \frac{5\sin^2(x)+7}{\left(4-3\sin^2(x)\right)^{3/2}}dx-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{20}\left(\int \frac{5\sin(x)^2+7}{\left(4-3\sin(x)^2\right)^{3/2}}dx-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{20}\left(\frac{1}{4}\int \frac{48-41\sin^2(x)}{\sqrt{4-3\sin^2(x)}}dx-\frac{41\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{20}\left(\frac{1}{4}\int \frac{48-41\sin(x)^2}{\sqrt{4-3\sin(x)^2}}dx-\frac{41\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{20}\left(\frac{1}{4}\left(\frac{41}{3}\int \sqrt{4-3\sin^2(x)}dx-\frac{20}{3}\int \frac{1}{\sqrt{4-3\sin^2(x)}}dx\right)-\frac{41\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{20}\left(\frac{1}{4}\left(\frac{41}{3}\int \sqrt{4-3\sin(x)^2}dx-\frac{20}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)-\frac{41\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{20}\left(\frac{1}{4}\left(\frac{82E\left(x\middle|\frac{3}{4}\right)}{3}-\frac{20}{3}\int \frac{1}{\sqrt{4-3\sin(x)^2}}dx\right)-\frac{41\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{20}\left(-\frac{41\sin(x)\cos(x)}{4\sqrt{4-3\sin^2(x)}}-\frac{5\sin(x)\cos(x)}{\left(4-3\sin^2(x)\right)^{3/2}}+\frac{1}{4}\left(\frac{82E\left(x\middle|\frac{3}{4}\right)}{3}-\frac{10\,\text{EllipticF}\left(x,\frac{3}{4}\right)}{3}\right)\right)-\frac{3\sin(x)\cos(x)}{20\left(4-3\sin^2(x)\right)^{5/2}}$$

input
```
Int[(4 - 3*Sin[x]^2)^(-7/2),x]
```

output
```
(-3*Cos[x]*Sin[x])/(20*(4 - 3*Sin[x]^2)^(5/2)) + (((82*EllipticE[x, 3/4])/
3 - (10*EllipticF[x, 3/4])/3)/4 - (5*Cos[x]*Sin[x])/(4 - 3*Sin[x]^2)^(3/2)
 - (41*Cos[x]*Sin[x])/(4*Sqrt[4 - 3*Sin[x]^2]))/20
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(68) = 136$.

Time = 1.66 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.48

method	result
default	$\frac{\sqrt{-\left(-4+3\sin(x)^2\right)\cos(x)^2}\left(\frac{\sin(x)\sqrt{3\cos(x)^4+\cos(x)^2}}{180\left(\sin(x)^2-\frac{4}{3}\right)^3}-\frac{\sin(x)\sqrt{3\cos(x)^4+\cos(x)^2}}{36\left(\sin(x)^2-\frac{4}{3}\right)^2}-\frac{41\cos(x)^2\sin(x)}{80\sqrt{-\left(-4+3\sin(x)^2\right)\cos(x)^2}}+\frac{3\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{4-}}{10\sqrt{3}}\right.}{\cos(x)\sqrt{4-3\sin(x)^2}}$

input
```
int(1/(4-3*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)*(1/180*sin(x)*(3*cos(x)^4+cos(x)^2)^(1/2
)/(sin(x)^2-4/3)^3-1/36*sin(x)*(3*cos(x)^4+cos(x)^2)^(1/2)/(sin(x)^2-4/3)^
2-41/80*cos(x)^2*sin(x)/(-(-4+3*sin(x)^2)*cos(x)^2)^(1/2)+3/10*(cos(x)^2)^
(1/2)*(4-3*sin(x)^2)^(1/2)/(3*cos(x)^4+cos(x)^2)^(1/2)*EllipticF(sin(x),1/
2*3^(1/2))-41/120*(cos(x)^2)^(1/2)*(4-3*sin(x)^2)^(1/2)/(3*cos(x)^4+cos(x)
^2)^(1/2)*(EllipticF(sin(x),1/2*3^(1/2))-EllipticE(sin(x),1/2*3^(1/2))))/c
os(x)/(4-3*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.73

$$\int \frac{1}{(4-3\sin^2(x))^{7/2}}\,dx =$$

$$-\frac{18\left(369\cos(x)^5+306\cos(x)^3+73\cos(x)\right)\sqrt{3\cos(x)^2+1}\sin(x)-41\left(27\cos(x)^6+27\cos(x)^4+9\right.}{}$$

input
```
integrate(1/(4-3*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/1440*(18*(369*cos(x)^5 + 306*cos(x)^3 + 73*cos(x))*sqrt(3*cos(x)^2 + 1)
*sin(x) - 41*(27*cos(x)^6 + 27*cos(x)^4 + 9*cos(x)^2 + 1)*elliptic_e(arcsi
n(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) - 41*(27*cos(x)^6 + 27*co
s(x)^4 + 9*cos(x)^2 + 1)*elliptic_e(arcsin(-1/3*I*sqrt(3)*cos(x) - 1/3*sqr
t(3)*sin(x)), 9) + 536*(27*cos(x)^6 + 27*cos(x)^4 + 9*cos(x)^2 + 1)*ellipt
ic_f(arcsin(1/3*I*sqrt(3)*cos(x) - 1/3*sqrt(3)*sin(x)), 9) + 536*(27*cos(x
)^6 + 27*cos(x)^4 + 9*cos(x)^2 + 1)*elliptic_f(arcsin(-1/3*I*sqrt(3)*cos(x
) - 1/3*sqrt(3)*sin(x)), 9))/(27*cos(x)^6 + 27*cos(x)^4 + 9*cos(x)^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(4-3\sin^2(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(4-3*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(4-3\sin^2(x))^{7/2}}\,dx = \int \frac{1}{\left(-3\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-3*sin(x)^2 + 4)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(4-3\sin^2(x))^{7/2}}\,dx = \int \frac{1}{\left(-3\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(4-3*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((-3*sin(x)^2 + 4)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4-3\sin^2(x))^{7/2}}\,dx = \int \frac{1}{\left(4-3\sin\left(x\right)^2\right)^{7/2}}\,dx$$

input
```
int(1/(4 - 3*sin(x)^2)^(7/2),x)
```

output
```
int(1/(4 - 3*sin(x)^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(4-3\sin^2(x)\right)^{7/2}}\,dx = \int \frac{\sqrt{-3\sin(x)^2+4}}{81\sin(x)^8-432\sin(x)^6+864\sin(x)^4-768\sin(x)^2+256}dx$$

input
```
int(1/(4-3*sin(x)^2)^(7/2),x)
```

output
```
int(sqrt( - 3*sin(x)**2 + 4)/(81*sin(x)**8 - 432*sin(x)**6 + 864*sin(x)**4
 - 768*sin(x)**2 + 256),x)
```

3.116 $\int (4-5\sin^2(x))^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \left(4-5\sin^2(x)\right)^4 \, dx = \frac{7923x}{128} + \frac{15245}{128}\cos(x)\sin(x) - \frac{3825}{64}\cos(x)\sin^3(x) + \frac{35}{16}\cos(x)\sin(x)\left(4-5\sin^2(x)\right)^2 + \frac{5}{8}\cos(x)\sin(x)\left(4-5\sin^2(x)\right)^3$$

output
```
7923/128*x+15245/128*cos(x)*sin(x)-3825/64*cos(x)*sin(x)^3+35/16*cos(x)*si
n(x)*(4-5*sin(x)^2)^2+5/8*cos(x)*sin(x)*(4-5*sin(x)^2)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

$$\int \left(4-5\sin^2(x)\right)^4 \, dx = \frac{7923x}{128} + \frac{1665}{32}\sin(2x) + \frac{1975}{128}\sin(4x) + \frac{125}{32}\sin(6x) + \frac{625\sin(8x)}{1024}$$

input
```
Integrate[(4 - 5*Sin[x]^2)^4,x]
```

output
```
(7923*x)/128 + (1665*Sin[2*x])/32 + (1975*Sin[4*x])/128 + (125*Sin[6*x])/3
2 + (625*Sin[8*x])/1024
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3659, 27, 3042, 3649, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sin^2(x)\right)^4 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-5 \sin(x)^2\right)^4 dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{8} \int 3\left(36-35 \sin^2(x)\right) \left(4-5 \sin^2(x)\right)^2 dx+\frac{5}{8} \sin(x) \left(4-5 \sin^2(x)\right)^3 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8} \int \left(36-35 \sin^2(x)\right) \left(4-5 \sin^2(x)\right)^2 dx+\frac{5}{8} \sin(x) \left(4-5 \sin^2(x)\right)^3 \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8} \int \left(36-35 \sin(x)^2\right) \left(4-5 \sin(x)^2\right)^2 dx+\frac{5}{8} \sin(x) \left(4-5 \sin^2(x)\right)^3 \cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{3}{8} \left(\frac{1}{6} \int \left(724-765 \sin^2(x)\right) \left(4-5 \sin^2(x)\right) dx+\frac{35}{6} \sin(x) \left(4-5 \sin^2(x)\right)^2 \cos(x)\right)+\frac{5}{8} \sin(x) \left(4-5 \sin^2(x)\right)^3 \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8} \left(\frac{1}{6} \int \left(724-765 \sin(x)^2\right) \left(4-5 \sin(x)^2\right) dx+\frac{35}{6} \sin(x) \left(4-5 \sin^2(x)\right)^2 \cos(x)\right)+\frac{5}{8} \sin(x) \left(4-5 \sin^2(x)\right)^3 \cos(x)$$

$$\Big\downarrow \text{3648}$$

$$\frac{5}{8}\sin(x)\left(4-5\sin^2(x)\right)^3\cos(x)+$$
$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{7923x}{8}-\frac{3825}{4}\sin^3(x)\cos(x)+\frac{15245}{8}\sin(x)\cos(x)\right)+\frac{35}{6}\sin(x)\left(4-5\sin^2(x)\right)^2\cos(x)\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^4,x]
```

output
```
(5*Cos[x]*Sin[x]*(4 - 5*Sin[x]^2)^3)/8 + (3*((35*Cos[x]*Sin[x]*(4 - 5*Sin[
x]^2)^2)/6 + ((7923*x)/8 + (15245*Cos[x]*Sin[x])/8 - (3825*Cos[x]*Sin[x]^3
)/4)/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 11.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

method	result
risch	$\frac{7923x}{128}+\frac{625\sin(8x)}{1024}+\frac{125\sin(6x)}{32}+\frac{1975\sin(4x)}{128}+\frac{1665\sin(2x)}{32}$
parallelrisch	$\frac{7923x}{128}+\frac{625\sin(8x)}{1024}+\frac{125\sin(6x)}{32}+\frac{1975\sin(4x)}{128}+\frac{1665\sin(2x)}{32}$
default	$-\frac{625\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{7923x}{128}+\frac{1000\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{3}-600($
parts	$-\frac{625\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{7923x}{128}+\frac{1000\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{3}-600($
norman	$\frac{\frac{7923x}{128}-\frac{27975\tan\left(\frac{x}{2}\right)^3}{64}+\frac{263005\tan\left(\frac{x}{2}\right)^5}{64}-\frac{324175\tan\left(\frac{x}{2}\right)^7}{64}+\frac{324175\tan\left(\frac{x}{2}\right)^9}{64}-\frac{263005\tan\left(\frac{x}{2}\right)^{11}}{64}+\frac{27975\tan\left(\frac{x}{2}\right)^{13}}{64}-\frac{24845\tan\left(\frac{x}{2}\right)}{64}}{}$

input
```
int((4-5*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
7923/128*x+625/1024*sin(8*x)+125/32*sin(6*x)+1975/128*sin(4*x)+1665/32*sin
(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \left(4-5\sin^2(x)\right)^4\,dx = \frac{5}{128}\left(2000\,\cos\left(x\right)^7+200\,\cos\left(x\right)^5+1210\,\cos\left(x\right)^3+1559\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{7923}{128}\,x$$

input
```
integrate((4-5*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
5/128*(2000*cos(x)^7 + 200*cos(x)^5 + 1210*cos(x)^3 + 1559*cos(x))*sin(x)
+ 7923/128*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(66) = 132$.

Time = 0.47 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.57

$$\begin{aligned}\int \left(4-5\sin^2(x)\right)^4\,dx = {} & \frac{21875x\sin^8(x)}{128} + \frac{21875x\sin^6(x)\cos^2(x)}{32} - 625x\sin^6(x) \\ & + \frac{65625x\sin^4(x)\cos^4(x)}{64} - 1875x\sin^4(x)\cos^2(x) \\ & + 900x\sin^4(x) + \frac{21875x\sin^2(x)\cos^6(x)}{32} \\ & - 1875x\sin^2(x)\cos^4(x) + 1800x\sin^2(x)\cos^2(x) \\ & - 640x\sin^2(x) + \frac{21875x\cos^8(x)}{128} - 625x\cos^6(x) \\ & + 900x\cos^4(x) - 640x\cos^2(x) + 256x - \frac{58125\sin^7(x)\cos(x)}{128} \\ & - \frac{319375\sin^5(x)\cos^3(x)}{384} + 1375\sin^5(x)\cos(x) \\ & - \frac{240625\sin^3(x)\cos^5(x)}{384} + \frac{5000\sin^3(x)\cos^3(x)}{3} \\ & - 1500\sin^3(x)\cos(x) - \frac{21875\sin(x)\cos^7(x)}{128} \\ & + 625\sin(x)\cos^5(x) - 900\sin(x)\cos^3(x) + 640\sin(x)\cos(x)\end{aligned}$$

input
```
integrate((4-5*sin(x)**2)**4,x)
```

output
```
21875*x*sin(x)**8/128 + 21875*x*sin(x)**6*cos(x)**2/32 - 625*x*sin(x)**6 +
 65625*x*sin(x)**4*cos(x)**4/64 - 1875*x*sin(x)**4*cos(x)**2 + 900*x*sin(x
)**4 + 21875*x*sin(x)**2*cos(x)**6/32 - 1875*x*sin(x)**2*cos(x)**4 + 1800*
x*sin(x)**2*cos(x)**2 - 640*x*sin(x)**2 + 21875*x*cos(x)**8/128 - 625*x*co
s(x)**6 + 900*x*cos(x)**4 - 640*x*cos(x)**2 + 256*x - 58125*sin(x)**7*cos(
x)/128 - 319375*sin(x)**5*cos(x)**3/384 + 1375*sin(x)**5*cos(x) - 240625*s
in(x)**3*cos(x)**5/384 + 5000*sin(x)**3*cos(x)**3/3 - 1500*sin(x)**3*cos(x
) - 21875*sin(x)*cos(x)**7/128 + 625*sin(x)*cos(x)**5 - 900*sin(x)*cos(x)*
*3 + 640*sin(x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int \left(4-5\sin^2(x)\right)^4 \, dx = -\frac{125}{8}\sin\left(2\,x\right)^3 + \frac{7923}{128}\,x + \frac{625}{1024}\sin\left(8\,x\right) + \frac{1975}{128}\sin\left(4\,x\right) + \frac{255}{4}\sin\left(2\,x\right)$$

input
```
integrate((4-5*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
-125/8*sin(2*x)^3 + 7923/128*x + 625/1024*sin(8*x) + 1975/128*sin(4*x) + 2
55/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \left(4-5\sin^2(x)\right)^4 \, dx = \frac{7923}{128}\,x + \frac{625}{1024}\sin\left(8\,x\right) + \frac{125}{32}\sin\left(6\,x\right) + \frac{1975}{128}\sin\left(4\,x\right) + \frac{1665}{32}\sin\left(2\,x\right)$$

input
```
integrate((4-5*sin(x)^2)^4,x, algorithm="giac")
```

output
```
7923/128*x + 625/1024*sin(8*x) + 125/32*sin(6*x) + 1975/128*sin(4*x) + 166
5/32*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 37.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \left(4-5\sin^2(x)\right)^4\,dx=\frac{7923\,x}{128}+\frac{\frac{7795\,\tan(x)^7}{128}+\frac{29435\,\tan(x)^5}{128}+\frac{36485\,\tan(x)^3}{128}+\frac{24845\,\tan(x)}{128}}{\left(\tan\left(x\right)^2+1\right)^4}$$

input
```
int((5*sin(x)^2 - 4)^4,x)
```

output
```
(7923*x)/128 + ((24845*tan(x))/128 + (36485*tan(x)^3)/128 + (29435*tan(x)^
5)/128 + (7795*tan(x)^7)/128)/(tan(x)^2 + 1)^4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \left(4-5\sin^2(x)\right)^4\,dx=-\frac{625\cos\left(x\right)\sin\left(x\right)^7}{8}+\frac{3875\cos\left(x\right)\sin\left(x\right)^5}{16}-\frac{19025\cos\left(x\right)\sin\left(x\right)^3}{64}+\frac{24845\cos\left(x\right)\sin\left(x\right)}{128}+\frac{7923x}{128}$$

input
```
int((4-5*sin(x)^2)^4,x)
```

output
```
( - 10000*cos(x)*sin(x)**7 + 31000*cos(x)*sin(x)**5 - 38050*cos(x)*sin(x)*
*3 + 24845*cos(x)*sin(x) + 7923*x)/128
```

3.117 $\int (4-5\sin^2(x))^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int (4-5\sin^2(x))^3 \, dx = \frac{279x}{16} + \frac{1595}{48}\cos(x)\sin(x) - \frac{125}{8}\cos(x)\sin^3(x) + \frac{5}{6}\cos(x)\sin(x)\left(4-5\sin^2(x)\right)^2$$

output
```
279/16*x+1595/48*cos(x)*sin(x)-125/8*cos(x)*sin(x)^3+5/6*cos(x)*sin(x)*(4-
5*sin(x)^2)^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int (4-5\sin^2(x))^3 \, dx = \frac{279x}{16} + \frac{915}{64}\sin(2x) + \frac{225}{64}\sin(4x) + \frac{125}{192}\sin(6x)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^3,x]
```

output
```
(279*x)/16 + (915*Sin[2*x])/64 + (225*Sin[4*x])/64 + (125*Sin[6*x])/192
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3659, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sin ^2(x)\right)^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-5 \sin (x)^2\right)^3 \, dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{6} \int \left(76-75 \sin ^2(x)\right) \left(4-5 \sin ^2(x)\right) \, dx+\frac{5}{6} \sin (x) \left(4-5 \sin ^2(x)\right)^2 \cos (x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6} \int \left(76-75 \sin (x)^2\right) \left(4-5 \sin (x)^2\right) \, dx+\frac{5}{6} \sin (x) \left(4-5 \sin ^2(x)\right)^2 \cos (x)$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{6} \left(\frac{837 x}{8}-\frac{375}{4} \sin ^3(x) \cos (x)+\frac{1595}{8} \sin (x) \cos (x)\right)+\frac{5}{6} \sin (x) \left(4-5 \sin ^2(x)\right)^2 \cos (x)$$

input
```
Int[(4 - 5*Sin[x]^2)^3,x]
```

output
```
(5*Cos[x]*Sin[x]*(4 - 5*Sin[x]^2)^2)/6 + ((837*x)/8 + (1595*Cos[x]*Sin[x])
/8 - (375*Cos[x]*Sin[x]^3)/4)/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

method	result
risch	$\frac{279x}{16}+\frac{125\sin(6x)}{192}+\frac{225\sin(4x)}{64}+\frac{915\sin(2x)}{64}$
parallelrisch	$\frac{279x}{16}+\frac{125\sin(6x)}{192}+\frac{225\sin(4x)}{64}+\frac{915\sin(2x)}{64}$
default	$\frac{279x}{16}+120\cos\left(x\right)\sin\left(x\right)-75\left(\sin\left(x\right)^{3}+\frac{3\sin(x)}{2}\right)\cos\left(x\right)+\frac{125\left(\sin(x)^{5}+\frac{5\sin(x)^{3}}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}$
parts	$\frac{279x}{16}+120\cos\left(x\right)\sin\left(x\right)-75\left(\sin\left(x\right)^{3}+\frac{3\sin(x)}{2}\right)\cos\left(x\right)+\frac{125\left(\sin(x)^{5}+\frac{5\sin(x)^{3}}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}$
norman	$\frac{\frac{279x}{16}-\frac{2695\tan\left(\frac{x}{2}\right)^{3}}{24}+\frac{1845\tan\left(\frac{x}{2}\right)^{5}}{4}-\frac{1845\tan\left(\frac{x}{2}\right)^{7}}{4}+\frac{2695\tan\left(\frac{x}{2}\right)^{9}}{24}-\frac{745\tan\left(\frac{x}{2}\right)^{11}}{8}+\frac{837x\tan\left(\frac{x}{2}\right)^{2}}{8}+\frac{4185x\tan\left(\frac{x}{2}\right)^{4}}{16}+\frac{1395x\tan(}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{6}}$
orering	$\left(\frac{x}{2}+\frac{93}{2670080}\right)\left(4-5\sin\left(x\right)^{2}\right)^{3}+\frac{2007861\cos(x)\sin(x)\left(4-5\sin(x)^{2}\right)^{2}}{267008}+\left(\frac{49x}{288}+\frac{93}{2670080}\right)\left(30\sin\left(x\right)^{2}\right.$

input
```
int((4-5*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
279/16*x+125/192*sin(6*x)+225/64*sin(4*x)+915/64*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \left(4-5\sin^2(x)\right)^3\,dx=\frac{5}{48}\left(200\cos\left(x\right)^5+70\cos\left(x\right)^3+177\cos\left(x\right)\right)\sin\left(x\right)+\frac{279}{16}\,x$$

input
```
integrate((4-5*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
5/48*(200*cos(x)^5 + 70*cos(x)^3 + 177*cos(x))*sin(x) + 279/16*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(46) = 92.

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 4.12

$$\begin{aligned}\int \left(4-5\sin^2(x)\right)^3\,dx=&-\frac{625x\sin^6(x)}{16}-\frac{1875x\sin^4(x)\cos^2(x)}{16}+\frac{225x\sin^4(x)}{2}\\&-\frac{1875x\sin^2(x)\cos^4(x)}{16}+225x\sin^2(x)\cos^2(x)\\&-120x\sin^2(x)-\frac{625x\cos^6(x)}{16}+\frac{225x\cos^4(x)}{2}\\&-120x\cos^2(x)+64x+\frac{1375\sin^5(x)\cos(x)}{16}\\&+\frac{625\sin^3(x)\cos^3(x)}{6}-\frac{375\sin^3(x)\cos(x)}{2}\\&+\frac{625\sin(x)\cos^5(x)}{16}-\frac{225\sin(x)\cos^3(x)}{2}+120\sin(x)\cos(x)\end{aligned}$$

input
```
integrate((4-5*sin(x)**2)**3,x)
```

output
```
-625*x*sin(x)**6/16 - 1875*x*sin(x)**4*cos(x)**2/16 + 225*x*sin(x)**4/2 -
1875*x*sin(x)**2*cos(x)**4/16 + 225*x*sin(x)**2*cos(x)**2 - 120*x*sin(x)**
2 - 625*x*cos(x)**6/16 + 225*x*cos(x)**4/2 - 120*x*cos(x)**2 + 64*x + 1375
*sin(x)**5*cos(x)/16 + 625*sin(x)**3*cos(x)**3/6 - 375*sin(x)**3*cos(x)/2
+ 625*sin(x)*cos(x)**5/16 - 225*sin(x)*cos(x)**3/2 + 120*sin(x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \left(4-5\sin^2(x)\right)^3 \, dx = -\frac{125}{48}\sin(2x)^3 + \frac{279}{16}x + \frac{225}{64}\sin(4x) + \frac{65}{4}\sin(2x)$$

input
```
integrate((4-5*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
-125/48*sin(2*x)^3 + 279/16*x + 225/64*sin(4*x) + 65/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \left(4-5\sin^2(x)\right)^3 \, dx = \frac{279}{16}x + \frac{125}{192}\sin(6x) + \frac{225}{64}\sin(4x) + \frac{915}{64}\sin(2x)$$

input
```
integrate((4-5*sin(x)^2)^3,x, algorithm="giac")
```

output
```
279/16*x + 125/192*sin(6*x) + 225/64*sin(4*x) + 915/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 37.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \left(4-5\sin^2(x)\right)^3\,dx=\frac{745\cos\left(x\right)^5\,\sin\left(x\right)}{16}+\frac{265\cos\left(x\right)^3\sin\left(x\right)^3}{6}+\frac{295\,\cos\left(x\right)\,\sin\left(x\right)^5}{16}+\frac{279\,x}{16}$$

input
```
int(-(5*sin(x)^2 - 4)^3,x)
```

output
```
(279*x)/16 + (295*cos(x)*sin(x)^5)/16 + (745*cos(x)^5*sin(x))/16 + (265*co
s(x)^3*sin(x)^3)/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \left(4-5\sin^2(x)\right)^3\,dx=\frac{125\cos\left(x\right)\sin\left(x\right)^5}{6}-\frac{1175\cos\left(x\right)\sin\left(x\right)^3}{24}+\frac{745\cos\left(x\right)\sin\left(x\right)}{16}+\frac{279x}{16}$$

input
```
int((4-5*sin(x)^2)^3,x)
```

output
```
(1000*cos(x)*sin(x)**5 - 2350*cos(x)*sin(x)**3 + 2235*cos(x)*sin(x) + 837*
x)/48
```

3.118 $\int \left(4-5 \sin^2(x)\right)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \left(4-5 \sin^2(x)\right)^2 \, dx=\frac{43x}{8}+\frac{85}{8}\cos(x)\sin(x)-\frac{25}{4}\cos(x)\sin^3(x)$$

output
```
43/8*x+85/8*cos(x)*sin(x)-25/4*cos(x)*sin(x)^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(4-5 \sin^2(x)\right)^2 \, dx=\frac{43x}{8}+\frac{15}{4}\sin(2x)+\frac{25}{32}\sin(4x)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^2,x]
```

output
```
(43*x)/8 + (15*Sin[2*x])/4 + (25*Sin[4*x])/32
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sin ^2(x)\right)^2 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(4-5 \sin (x)^2\right)^2dx$$

$$\Big\downarrow \text{ 3658}$$

$$\frac{43 x}{8}-\frac{25}{4} \sin ^3(x) \cos (x)+\frac{85}{8} \sin (x) \cos (x)$$

input
```
Int[(4 - 5*Sin[x]^2)^2,x]
```

output
```
(43*x)/8 + (85*Cos[x]*Sin[x])/8 - (25*Cos[x]*Sin[x]^3)/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result
risch	$\frac{43x}{8}+\frac{25\sin(4x)}{32}+\frac{15\sin(2x)}{4}$
parallelrisch	$\frac{43x}{8}+\frac{25\sin(4x)}{32}+\frac{15\sin(2x)}{4}$
default	$-\frac{25\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{43x}{8}+20\cos\left(x\right)\sin\left(x\right)$
parts	$-\frac{25\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{43x}{8}+20\cos\left(x\right)\sin\left(x\right)$
norman	$\frac{\frac{43x}{8}-\frac{115\tan\left(\frac{x}{2}\right)^3}{4}+\frac{115\tan\left(\frac{x}{2}\right)^5}{4}-\frac{85\tan\left(\frac{x}{2}\right)^7}{4}+\frac{43x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{129x\tan\left(\frac{x}{2}\right)^4}{4}+\frac{43x\tan\left(\frac{x}{2}\right)^6}{2}+\frac{43x\tan\left(\frac{x}{2}\right)^8}{8}+\frac{85\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$
orering	$x\left(4-5\sin\left(x\right)^2\right)^2+5\left(4-5\sin\left(x\right)^2\right)\cos\left(x\right)\sin\left(x\right)+\frac{5x\left(200\cos(x)^2\sin(x)^2+20\left(4-5\sin(x)^2\right)\sin(x)^2\right.}{16}$

input
```
int((4-5*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
43/8*x+25/32*sin(4*x)+15/4*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \left(4-5\sin^2(x)\right)^2\,dx=\frac{5}{8}\left(10\,\cos\left(x\right)^3+7\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{43}{8}\,x$$

input
```
integrate((4-5*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
5/8*(10*cos(x)^3 + 7*cos(x))*sin(x) + 43/8*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.62

$$\int \left(4-5\sin^2(x)\right)^2 dx = \frac{75x\sin^4(x)}{8} + \frac{75x\sin^2(x)\cos^2(x)}{4} - 20x\sin^2(x) + \frac{75x\cos^4(x)}{8} - 20x\cos^2(x) + 16x - \frac{125\sin^3(x)\cos(x)}{8} - \frac{75\sin(x)\cos^3(x)}{8} + 20\sin(x)\cos(x)$$

input
```
integrate((4-5*sin(x)**2)**2,x)
```

output
```
75*x*sin(x)**4/8 + 75*x*sin(x)**2*cos(x)**2/4 - 20*x*sin(x)**2 + 75*x*cos(
x)**4/8 - 20*x*cos(x)**2 + 16*x - 125*sin(x)**3*cos(x)/8 - 75*sin(x)*cos(x
)**3/8 + 20*sin(x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(4-5\sin^2(x)\right)^2 dx = \frac{43}{8}x + \frac{25}{32}\sin(4x) + \frac{15}{4}\sin(2x)$$

input
```
integrate((4-5*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
43/8*x + 25/32*sin(4*x) + 15/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(4-5\sin^2(x)\right)^2\,dx=\frac{43}{8}\,x+\frac{25}{32}\,\sin\left(4\,x\right)+\frac{15}{4}\,\sin\left(2\,x\right)$$

input
```
integrate((4-5*sin(x)^2)^2,x, algorithm="giac")
```

output
```
43/8*x + 25/32*sin(4*x) + 15/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 37.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(4-5\sin^2(x)\right)^2\,dx=\frac{43\,x}{8}+\frac{\frac{35\,\tan(x)^3}{8}+\frac{85\,\tan(x)}{8}}{\left(\tan\left(x\right)^2+1\right)^2}$$

input
```
int((5*sin(x)^2 - 4)^2,x)
```

output
```
(43*x)/8 + ((85*tan(x))/8 + (35*tan(x)^3)/8)/(tan(x)^2 + 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \left(4-5\sin^2(x)\right)^2\,dx=-\frac{25\cos\left(x\right)\sin\left(x\right)^3}{4}+\frac{85\cos\left(x\right)\sin\left(x\right)}{8}+\frac{43x}{8}$$

input
```
int((4-5*sin(x)^2)^2,x)
```

output
```
( - 50*cos(x)*sin(x)**3 + 85*cos(x)*sin(x) + 43*x)/8
```

3.119 $\int (4-5\sin^2(x)) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int (4-5\sin^2(x)) \, dx = \frac{3x}{2} + \frac{5}{2}\cos(x)\sin(x)$$

output
```
3/2*x+5/2*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (4-5\sin^2(x)) \, dx = \frac{3x}{2} + \frac{5}{4}\sin(2x)$$

input
```
Integrate[4 - 5*Sin[x]^2,x]
```

output
```
(3*x)/2 + (5*Sin[2*x])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sin^2(x)\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{3x}{2}+\frac{5}{2}\sin(x)\cos(x)$$

input
```
Int[4 - 5*Sin[x]^2,x]
```

output
```
(3*x)/2 + (5*Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3x}{2}+\frac{5\cos(x)\sin(x)}{2}$	11
risch	$\frac{3x}{2}+\frac{5\sin(2x)}{4}$	11
parallelrisch	$\frac{3x}{2}+\frac{5\sin(2x)}{4}$	11
parts	$\frac{3x}{2}+\frac{5\cos(x)\sin(x)}{2}$	11
orering	$x\left(4-5\sin\left(x\right)^{2}\right)+\frac{5\cos(x)\sin(x)}{2}+\frac{x\left(10\sin(x)^{2}-10\cos(x)^{2}\right)}{4}$	34
norman	$\frac{\frac{3x}{2}-5\tan\left(\frac{x}{2}\right)^{3}+3x\tan\left(\frac{x}{2}\right)^{2}+\frac{3x\tan\left(\frac{x}{2}\right)^{4}}{2}+5\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	48

input
```
int(4-5*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*x+5/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-5\sin^2(x)\right)\,dx=\frac{5}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{3}{2}\,x$$

input
```
integrate(4-5*sin(x)^2,x, algorithm="fricas")
```

output
```
5/2*cos(x)*sin(x) + 3/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(4-5\sin^2(x)\right)\,dx=\frac{3x}{2}+\frac{5\sin\left(x\right)\cos\left(x\right)}{2}$$

input
```
integrate(4-5*sin(x)**2,x)
```

output
```
3*x/2 + 5*sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-5\sin^2(x)\right)\,dx=\frac{3}{2}\,x+\frac{5}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(4-5*sin(x)^2,x, algorithm="maxima")
```

output
```
3/2*x + 5/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-5\sin^2(x)\right)\,dx=\frac{3}{2}\,x+\frac{5}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(4-5*sin(x)^2,x, algorithm="giac")
```

output
```
3/2*x + 5/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 37.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-5\sin^2(x)\right)\,dx=\frac{3\,x}{2}+\frac{5\,\sin\left(2\,x\right)}{4}$$

input
```
int(4 - 5*sin(x)^2,x)
```

output
```
(3*x)/2 + (5*sin(2*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(4-5\sin^2(x)\right)\,dx=\frac{5\cos\left(x\right)\sin\left(x\right)}{2}+\frac{3x}{2}$$

input
```
int(4-5*sin(x)^2,x)
```

output
```
(5*cos(x)*sin(x) + 3*x)/2
```

3.120 $\int \frac{1}{4-5\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

output
```
1/2*arctanh(1/2*tan(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{4-5\sin^2(x)}\,dx = -\frac{1}{4}\log(2\cos(x)-\sin(x))+\frac{1}{4}\log(2\cos(x)+\sin(x))$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-1),x]
```

output
```
-1/4*Log[2*Cos[x] - Sin[x]] + Log[2*Cos[x] + Sin[x]]/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-5\sin^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{4-5\sin(x)^2}dx$$

$$\Big\downarrow \; 3660$$

$$\int \frac{1}{4-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[Tan[x]/2]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\ln(\tan(x)-2)}{4}+\frac{\ln(\tan(x)+2)}{4}$	16
risch	$-\frac{\ln\left(e^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)}{4}+\frac{\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{4}$	26
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)^{2}-\tan\left(\frac{x}{2}\right)-1\right)}{4}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^{2}+\tan\left(\frac{x}{2}\right)-1\right)}{4}$	34
parallelrisch	$\ln\left(\left(\tan\left(\frac{x}{2}\right)^{2}-\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}\right)+\ln\left(\frac{1}{\left(\tan\left(\frac{x}{2}\right)^{2}+\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}}\right)$	34

input
```
int(1/(4-5*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(tan(x)-2)+1/4*ln(tan(x)+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(7) = 14$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{8}\log\left(\frac{3}{4}\cos(x)^2+\cos(x)\sin(x)+\frac{1}{4}\right) - \frac{1}{8}\log\left(\frac{3}{4}\cos(x)^2-\cos(x)\sin(x)+\frac{1}{4}\right)$$

input
```
integrate(1/(4-5*sin(x)^2),x, algorithm="fricas")
```

output
```
1/8*log(3/4*cos(x)^2 + cos(x)*sin(x) + 1/4) - 1/8*log(3/4*cos(x)^2 - cos(x
)*sin(x) + 1/4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{\log\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)-1\right)}{4}$$

input
```
integrate(1/(4-5*sin(x)**2),x)
```

output
```
log(tan(x/2)**2 - tan(x/2) - 1)/4 - log(tan(x/2)**2 + tan(x/2) - 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{4}\log\left(\tan\left(x\right)+2\right) - \frac{1}{4}\log\left(\tan\left(x\right)-2\right)$$

input
```
integrate(1/(4-5*sin(x)^2),x, algorithm="maxima")
```

output
```
1/4*log(tan(x) + 2) - 1/4*log(tan(x) - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{1}{4}\log\left(|\tan\left(x\right)+2|\right) - \frac{1}{4}\log\left(|\tan\left(x\right)-2|\right)$$

input
```
integrate(1/(4-5*sin(x)^2),x, algorithm="giac")
```

output
```
1/4*log(abs(tan(x) + 2)) - 1/4*log(abs(tan(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{\operatorname{atanh}\left(\frac{\tan(x)}{2}\right)}{2}$$

input
```
int(-1/(5*sin(x)^2 - 4),x)
```

output
```
atanh(tan(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 5.18

$$\int \frac{1}{4-5\sin^2(x)}\,dx = \frac{\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)}{4} + \frac{\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)}{4} - \frac{\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)}{4}$$

input
```
int(1/(4-5*sin(x)^2),x)
```

output
```
(log( - sqrt(5) + 2*tan(x/2) - 1) - log( - sqrt(5) + 2*tan(x/2) + 1) + log
(sqrt(5) + 2*tan(x/2) - 1) - log(sqrt(5) + 2*tan(x/2) + 1))/4
```

3.121 $\int \frac{1}{(4-5\sin^2(x))^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{(4-5\sin^2(x))^2}\,dx = -\frac{3}{16}\operatorname{arctanh}\left(\frac{\tan(x)}{2}\right)+\frac{5\cos(x)\sin(x)}{8\left(4-5\sin^2(x)\right)}$$

output
```
-3/16*arctanh(1/2*tan(x))+5*cos(x)*sin(x)/(32-40*sin(x)^2)
```

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{1}{(4-5\sin^2(x))^2}\,dx = \frac{1}{32}\left(3\log(2\cos(x)-\sin(x))-3\log(2\cos(x)+\sin(x)) + \frac{20\sin(2x)}{3+5\cos(2x)}\right)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-2),x]
```

output
```
(3*Log[2*Cos[x] - Sin[x]] - 3*Log[2*Cos[x] + Sin[x]] + (20*Sin[2*x])/(3 +
5*Cos[2*x]))/32
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3663, 27, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(4-5\sin(x)^2)^2}dx$$

$$\Big\downarrow \text{ 3663}$$

$$\frac{1}{8}\int -\frac{3}{4-5\sin^2(x)}dx+\frac{5\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{3}{8}\int \frac{1}{4-5\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{3}{8}\int \frac{1}{4-5\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{5\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{3}{8}\int \frac{1}{4-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{3}{16}\text{arctanh}\left(\frac{\tan(x)}{2}\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(-2),x]
```

output
```
(-3*ArcTanh[Tan[x]/2])/16 + (5*Cos[x]*Sin[x])/(8*(4 - 5*Sin[x]^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{5}{16(\tan(x)-2)}+\frac{3\ln(\tan(x)-2)}{32}-\frac{5}{16(\tan(x)+2)}-\frac{3\ln(\tan(x)+2)}{32}$	32
risch	$\frac{i(3\,e^{2ix}+5)}{20\,e^{4ix}+24\,e^{2ix}+20}-\frac{3\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{32}+\frac{3\ln\left(e^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)}{32}$	56
parallelrisch	$\frac{(-9-15\cos(2x))\ln\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)-1\right)+(9+15\cos(2x))\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)-1\right)+20\sin(2x)}{96+160\cos(2x)}$	65
norman	$\frac{-\frac{5\tan\left(\frac{x}{2}\right)^3}{16}+\frac{5\tan\left(\frac{x}{2}\right)}{16}}{\tan\left(\frac{x}{2}\right)^4-3\tan\left(\frac{x}{2}\right)^2+1}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)-1\right)}{32}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)-1\right)}{32}$	68

input
```
int(1/(4-5*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/16/(tan(x)-2)+3/32*ln(tan(x)-2)-5/16/(tan(x)+2)-3/32*ln(tan(x)+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^2}\,dx = -\frac{3\left(5\cos(x)^2-1\right)\log\left(\frac{3}{4}\cos(x)^2+\cos(x)\sin(x)+\frac{1}{4}\right)-3\left(5\cos(x)^2-1\right)\log\left(\frac{3}{4}\cos(x)^2-\cos(x)\,\mathrm{si}\right.}{64\left(5\cos(x)^2-1\right)}$$

input
```
integrate(1/(4-5*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/64*(3*(5*cos(x)^2 - 1)*log(3/4*cos(x)^2 + cos(x)*sin(x) + 1/4) - 3*(5*c
os(x)^2 - 1)*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 40*cos(x)*sin(x))/(
5*cos(x)^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(26) = 52$.

Time = 0.79 (sec) , antiderivative size = 292, normalized size of antiderivative = 9.73

$$\int \frac{1}{(4-5\sin^2(x))^2}\,dx = -\frac{3\log\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)-1\right)\tan^4\left(\frac{x}{2}\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} + \frac{9\log\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} - \frac{3\log\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)-1\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} + \frac{3\log\left(\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)-1\right)\tan^4\left(\frac{x}{2}\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} - \frac{9\log\left(\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} + \frac{3\log\left(\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)-1\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} - \frac{10\tan^3\left(\frac{x}{2}\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32} + \frac{10\tan\left(\frac{x}{2}\right)}{32\tan^4\left(\frac{x}{2}\right)-96\tan^2\left(\frac{x}{2}\right)+32}$$

input

```
integrate(1/(4-5*sin(x)**2)**2,x)
```

output

```
-3*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**4/(32*tan(x/2)**4 - 96*tan(x/
2)**2 + 32) + 9*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**2/(32*tan(x/2)**
4 - 96*tan(x/2)**2 + 32) - 3*log(tan(x/2)**2 - tan(x/2) - 1)/(32*tan(x/2)*
*4 - 96*tan(x/2)**2 + 32) + 3*log(tan(x/2)**2 + tan(x/2) - 1)*tan(x/2)**4/
(32*tan(x/2)**4 - 96*tan(x/2)**2 + 32) - 9*log(tan(x/2)**2 + tan(x/2) - 1)
*tan(x/2)**2/(32*tan(x/2)**4 - 96*tan(x/2)**2 + 32) + 3*log(tan(x/2)**2 +
tan(x/2) - 1)/(32*tan(x/2)**4 - 96*tan(x/2)**2 + 32) - 10*tan(x/2)**3/(32*
tan(x/2)**4 - 96*tan(x/2)**2 + 32) + 10*tan(x/2)/(32*tan(x/2)**4 - 96*tan(
x/2)**2 + 32)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^2}\,dx=-\frac{5\,\tan(x)}{8\left(\tan(x)^2-4\right)}-\frac{3}{32}\,\log\left(\tan(x)+2\right)+\frac{3}{32}\,\log\left(\tan(x)-2\right)$$

input
```
integrate(1/(4-5*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
-5/8*tan(x)/(tan(x)^2 - 4) - 3/32*log(tan(x) + 2) + 3/32*log(tan(x) - 2)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int \frac{1}{\left(4-5\sin^2(x)\right)^2}\,dx\\&=-\frac{5\,\tan(x)}{8\left(\tan(x)^2-4\right)}-\frac{3}{32}\,\log\left(|\tan(x)+2|\right)+\frac{3}{32}\,\log\left(|\tan(x)-2|\right)\end{aligned}$$

input
```
integrate(1/(4-5*sin(x)^2)^2,x, algorithm="giac")
```

output
```
-5/8*tan(x)/(tan(x)^2 - 4) - 3/32*log(abs(tan(x) + 2)) + 3/32*log(abs(tan(
x) - 2))
```

Mupad [B] (verification not implemented)

Time = 36.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^2}\,dx=-\frac{3\,\mathrm{atanh}\left(\frac{\tan(x)}{2}\right)}{16}-\frac{5\tan(x)}{8\,\left(\tan(x)^2-4\right)}$$

input
```
int(1/(5*sin(x)^2 - 4)^2,x)
```

output
```
- (3*atanh(tan(x)/2))/16 - (5*tan(x))/(8*(tan(x)^2 - 4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.87

$$\int \frac{1}{(4-5\sin^2(x))^2} dx$$
$$= \frac{-20\cos(x)\sin(x) - 15\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^2 + 12\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right) + 15\log\left(-\sqrt{}\right.}{}$$

input
```
int(1/(4-5*sin(x)^2)^2,x)
```

output
```
( - 20*cos(x)*sin(x) - 15*log( - sqrt(5) + 2*tan(x/2) - 1)*sin(x)**2 + 12*
log( - sqrt(5) + 2*tan(x/2) - 1) + 15*log( - sqrt(5) + 2*tan(x/2) + 1)*sin
(x)**2 - 12*log( - sqrt(5) + 2*tan(x/2) + 1) - 15*log(sqrt(5) + 2*tan(x/2)
 - 1)*sin(x)**2 + 12*log(sqrt(5) + 2*tan(x/2) - 1) + 15*log(sqrt(5) + 2*ta
n(x/2) + 1)*sin(x)**2 - 12*log(sqrt(5) + 2*tan(x/2) + 1))/(32*(5*sin(x)**2
 - 4))
```

3.122 $\int \frac{1}{(4-5\sin^2(x))^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \frac{1}{(4-5\sin^2(x))^3}\,dx = \frac{43}{256}\operatorname{arctanh}\left(\frac{\tan(x)}{2}\right)+\frac{5\cos(x)\sin(x)}{16\,(4-5\sin^2(x))^2}-\frac{45\cos(x)\sin(x)}{128\,(4-5\sin^2(x))}$$

output
```
43/256*arctanh(1/2*tan(x))+5/16*cos(x)*sin(x)/(4-5*sin(x)^2)^2-45*cos(x)*s
in(x)/(512-640*sin(x)^2)
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int \frac{1}{(4-5\sin^2(x))^3}\,dx = \frac{1}{512}\left(-43\log(2\cos(x)-\sin(x))+43\log(2\cos(x)+\sin(x)) + \frac{20}{(-2\cos(x)+\sin(x))^2}-\frac{20}{(2\cos(x)+\sin(x))^2} - \frac{180\sin(2x)}{3+5\cos(2x)}\right)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-3),x]
```

output
```
(-43*Log[2*Cos[x] - Sin[x]] + 43*Log[2*Cos[x] + Sin[x]] + 20/(-2*Cos[x] +
Sin[x])^2 - 20/(2*Cos[x] + Sin[x])^2 - (180*Sin[2*x])/(3 + 5*Cos[2*x]))/51
2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(4-5\sin(x)^2)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{1}{16}\int -\frac{10\sin^2(x)+1}{\left(4-5\sin^2(x)\right)^2}dx+\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}-\frac{1}{16}\int \frac{10\sin^2(x)+1}{\left(4-5\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}-\frac{1}{16}\int \frac{10\sin(x)^2+1}{(4-5\sin(x)^2)^2}dx$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{16}\left(\frac{1}{8}\int \frac{43}{4-5\sin^2(x)}dx-\frac{45\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}\right)+\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{43}{8}\int \frac{1}{4-5\sin^2(x)}dx-\frac{45\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}\right)+\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{43}{8}\int \frac{1}{4-5\sin(x)^2}dx-\frac{45\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}\right)+\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{16}\left(\frac{43}{8}\int \frac{1}{4-\tan^2(x)}d\tan(x)-\frac{45\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}\right)+\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{16}\left(\frac{43}{16}\text{arctanh}\left(\frac{\tan(x)}{2}\right)-\frac{45\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}\right)+\frac{5\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}$$

input
```
Int[(4 - 5*Sin[x]^2)^(-3),x]
```

output
```
(5*Cos[x]*Sin[x])/(16*(4 - 5*Sin[x]^2)^2) + ((43*ArcTanh[Tan[x]/2])/16 - (
45*Cos[x]*Sin[x])/(8*(4 - 5*Sin[x]^2)))/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result
default	$-\frac{25}{128(\tan(x)+2)^{2}}+\frac{85}{256(\tan(x)+2)}+\frac{43\ln(\tan(x)+2)}{512}+\frac{25}{128(\tan(x)-2)^{2}}+\frac{85}{256(\tan(x)-2)}-\frac{43\ln(\tan(x)-2)}{512}$
risch	$-\frac{i(215\,e^{6ix}+387\,e^{4ix}+325\,e^{2ix}+225)}{64(5\,e^{4ix}+6\,e^{2ix}+5)^{2}}+\frac{43\ln(e^{2ix}+\frac{3}{5}+\frac{4i}{5})}{512}-\frac{43\ln(e^{2ix}+\frac{3}{5}-\frac{4i}{5})}{512}$
norman	$\frac{\frac{95\tan(\frac{x}{2})^{3}}{128}-\frac{95\tan(\frac{x}{2})^{5}}{128}+\frac{35\tan(\frac{x}{2})^{7}}{256}-\frac{35\tan(\frac{x}{2})}{256}}{\left(\tan(\frac{x}{2})^{4}-3\tan(\frac{x}{2})^{2}+1\right)^{2}}+\frac{43\ln\left(\tan(\frac{x}{2})^{2}-\tan(\frac{x}{2})-1\right)}{512}-\frac{43\ln\left(\tan(\frac{x}{2})^{2}+\tan(\frac{x}{2})-1\right)}{512}$
parallelrisch	$\frac{(1849+1075\cos(4x)+2580\cos(2x))\ln\left(\tan(\frac{x}{2})^{2}-\tan(\frac{x}{2})-1\right)+(-1849-1075\cos(4x)-2580\cos(2x))\ln\left(\tan(\frac{x}{2})^{2}+\tan(\frac{x}{2})-\right.}{12800\cos(4x)+22016+30720\cos(2x)}$

input
```
int(1/(4-5*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-25/128/(tan(x)+2)^2+85/256/(tan(x)+2)+43/512*ln(tan(x)+2)+25/128/(tan(x)-
2)^2+85/256/(tan(x)-2)-43/512*ln(tan(x)-2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^3}\,dx = \frac{43\left(25\cos(x)^4-10\cos(x)^2+1\right)\log\left(\frac{3}{4}\cos(x)^2+\cos(x)\sin(x)+\frac{1}{4}\right)-43\left(25\cos(x)^4-10\cos(x)^2+\right.}{1024\left(25\cos(x)^4-10\cos(x)^2\right.}$$

input
```
integrate(1/(4-5*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
1/1024*(43*(25*cos(x)^4 - 10*cos(x)^2 + 1)*log(3/4*cos(x)^2 + cos(x)*sin(x
) + 1/4) - 43*(25*cos(x)^4 - 10*cos(x)^2 + 1)*log(3/4*cos(x)^2 - cos(x)*si
n(x) + 1/4) - 40*(45*cos(x)^3 - 17*cos(x))*sin(x))/(25*cos(x)^4 - 10*cos(x
)^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(46) = 92$.

Time = 2.53 (sec) , antiderivative size = 755, normalized size of antiderivative = 15.73

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4-5*sin(x)**2)**3,x)
```

output
```
43*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**8/(512*tan(x/2)**8 - 3072*tan
(x/2)**6 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 512) - 258*log(tan(x/2)**
2 - tan(x/2) - 1)*tan(x/2)**6/(512*tan(x/2)**8 - 3072*tan(x/2)**6 + 5632*t
an(x/2)**4 - 3072*tan(x/2)**2 + 512) + 473*log(tan(x/2)**2 - tan(x/2) - 1)
*tan(x/2)**4/(512*tan(x/2)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)**4 - 3072
*tan(x/2)**2 + 512) - 258*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**2/(512
*tan(x/2)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 51
2) + 43*log(tan(x/2)**2 - tan(x/2) - 1)/(512*tan(x/2)**8 - 3072*tan(x/2)**
6 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 512) - 43*log(tan(x/2)**2 + tan(
x/2) - 1)*tan(x/2)**8/(512*tan(x/2)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)*
*4 - 3072*tan(x/2)**2 + 512) + 258*log(tan(x/2)**2 + tan(x/2) - 1)*tan(x/2
)**6/(512*tan(x/2)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)**4 - 3072*tan(x/2
)**2 + 512) - 473*log(tan(x/2)**2 + tan(x/2) - 1)*tan(x/2)**4/(512*tan(x/2
)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 512) + 258
*log(tan(x/2)**2 + tan(x/2) - 1)*tan(x/2)**2/(512*tan(x/2)**8 - 3072*tan(x
/2)**6 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 512) - 43*log(tan(x/2)**2 +
 tan(x/2) - 1)/(512*tan(x/2)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)**4 - 30
72*tan(x/2)**2 + 512) + 70*tan(x/2)**7/(512*tan(x/2)**8 - 3072*tan(x/2)**6
 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 512) - 380*tan(x/2)**5/(512*tan(x
/2)**8 - 3072*tan(x/2)**6 + 5632*tan(x/2)**4 - 3072*tan(x/2)**2 + 512) ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1}{(4-5\sin^2(x))^3}\,dx = \frac{5\left(17\,\tan\left(x\right)^3-28\,\tan\left(x\right)\right)}{128\left(\tan\left(x\right)^4-8\,\tan\left(x\right)^2+16\right)} + \frac{43}{512}\,\log\left(\tan\left(x\right)+2\right)-\frac{43}{512}\,\log\left(\tan\left(x\right)-2\right)$$

input
```
integrate(1/(4-5*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
5/128*(17*tan(x)^3 - 28*tan(x))/(tan(x)^4 - 8*tan(x)^2 + 16) + 43/512*log(
tan(x) + 2) - 43/512*log(tan(x) - 2)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^3}\,dx = \frac{5\left(17\tan(x)^3-28\tan(x)\right)}{128\left(\tan(x)^2-4\right)^2} + \frac{43}{512}\log\left(\left|\tan(x)+2\right|\right) - \frac{43}{512}\log\left(\left|\tan(x)-2\right|\right)$$

input
```
integrate(1/(4-5*sin(x)^2)^3,x, algorithm="giac")
```

output
```
5/128*(17*tan(x)^3 - 28*tan(x))/(tan(x)^2 - 4)^2 + 43/512*log(abs(tan(x) +
 2)) - 43/512*log(abs(tan(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 36.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^3}\,dx = \frac{43\,\mathrm{atanh}\left(\frac{\tan(x)}{2}\right)}{256} - \frac{\frac{35\tan(x)}{32}-\frac{85\tan(x)^3}{128}}{\tan(x)^4-8\tan(x)^2+16}$$

input
```
int(-1/(5*sin(x)^2 - 4)^3,x)
```

output
```
(43*atanh(tan(x)/2))/256 - ((35*tan(x))/32 - (85*tan(x)^3)/128)/(tan(x)^4
- 8*tan(x)^2 + 16)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.83

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^3}\,dx = \frac{900\cos(x)\sin(x)^3-560\cos(x)\sin(x)+1075\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^4-1720\log\left(-\sqrt{5}+2\,\mathrm{t}\right.}{}$$

input
```
int(1/(4-5*sin(x)^2)^3,x)
```

output
```
(900*cos(x)*sin(x)**3 - 560*cos(x)*sin(x) + 1075*log( - sqrt(5) + 2*tan(x/
2) - 1)*sin(x)**4 - 1720*log( - sqrt(5) + 2*tan(x/2) - 1)*sin(x)**2 + 688*
log( - sqrt(5) + 2*tan(x/2) - 1) - 1075*log( - sqrt(5) + 2*tan(x/2) + 1)*s
in(x)**4 + 1720*log( - sqrt(5) + 2*tan(x/2) + 1)*sin(x)**2 - 688*log( - sq
rt(5) + 2*tan(x/2) + 1) + 1075*log(sqrt(5) + 2*tan(x/2) - 1)*sin(x)**4 - 1
720*log(sqrt(5) + 2*tan(x/2) - 1)*sin(x)**2 + 688*log(sqrt(5) + 2*tan(x/2)
 - 1) - 1075*log(sqrt(5) + 2*tan(x/2) + 1)*sin(x)**4 + 1720*log(sqrt(5) +
2*tan(x/2) + 1)*sin(x)**2 - 688*log(sqrt(5) + 2*tan(x/2) + 1))/(512*(25*si
n(x)**4 - 40*sin(x)**2 + 16))
```

3.123 $\int \frac{1}{(4-5\sin^2(x))^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{1}{(4-5\sin^2(x))^4}\,dx = -\frac{279\text{arctanh}\left(\frac{\tan(x)}{2}\right)}{2048} + \frac{5\cos(x)\sin(x)}{24\,(4-5\sin^2(x))^3} - \frac{25\cos(x)\sin(x)}{128\,(4-5\sin^2(x))^2} + \frac{995\cos(x)\sin(x)}{3072\,(4-5\sin^2(x))}$$

output
```
-279/2048*arctanh(1/2*tan(x))+5/24*cos(x)*sin(x)/(4-5*sin(x)^2)^3-25/128*c
os(x)*sin(x)/(4-5*sin(x)^2)^2+995*cos(x)*sin(x)/(12288-15360*sin(x)^2)
```

Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{1}{(4-5\sin^2(x))^4}\,dx = \frac{837\log(2\cos(x)-\sin(x)) - 837\log(2\cos(x)+\sin(x)) + \frac{10(31454\cos(x)+17075\cos(3x)+4975\cos(5x))\sin(x)}{(3+5\cos(2x))^3} - \frac{\cdots}{(-2\cos\cdots}}{12288}$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-4),x]
```

output
```
(837*Log[2*Cos[x] - Sin[x]] - 837*Log[2*Cos[x] + Sin[x]] + (10*(31454*Cos[
x] + 17075*Cos[3*x] + 4975*Cos[5*x])*Sin[x])/(3 + 5*Cos[2*x])^3 - 320/(-2*
Cos[x] + Sin[x])^2 + 320/(2*Cos[x] + Sin[x])^2)/12288
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {3042, 3663, 3042, 3652, 25, 3042, 3652, 27, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-5 \sin^2(x)\right)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(4-5 \sin(x)^2\right)^4}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{1}{24} \int \frac{1-20 \sin^2(x)}{\left(4-5 \sin^2(x)\right)^3}dx+\frac{5 \sin(x) \cos(x)}{24 \left(4-5 \sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{24} \int \frac{1-20 \sin(x)^2}{\left(4-5 \sin(x)^2\right)^3}dx+\frac{5 \sin(x) \cos(x)}{24 \left(4-5 \sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{24} \left(-\frac{1}{16} \int -\frac{150 \sin^2(x)+79}{\left(4-5 \sin^2(x)\right)^2}dx-\frac{75 \sin(x) \cos(x)}{16 \left(4-5 \sin^2(x)\right)^2}\right)+\frac{5 \sin(x) \cos(x)}{24 \left(4-5 \sin^2(x)\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{24} \left(\frac{1}{16} \int \frac{150 \sin^2(x)+79}{\left(4-5 \sin^2(x)\right)^2}dx-\frac{75 \sin(x) \cos(x)}{16 \left(4-5 \sin^2(x)\right)^2}\right)+\frac{5 \sin(x) \cos(x)}{24 \left(4-5 \sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{24}\left(\frac{1}{16}\int \frac{150\sin(x)^2+79}{(4-5\sin(x)^2)^2}dx-\frac{75\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}\right)+\frac{5\sin(x)\cos(x)}{24\left(4-5\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{1}{24}\left(\frac{1}{16}\left(\frac{995\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{1}{8}\int \frac{837}{4-5\sin^2(x)}dx\right)-\frac{75\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}\right)+\frac{5\sin(x)\cos(x)}{24\left(4-5\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{24}\left(\frac{1}{16}\left(\frac{995\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{837}{8}\int \frac{1}{4-5\sin^2(x)}dx\right)-\frac{75\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}\right)+\frac{5\sin(x)\cos(x)}{24\left(4-5\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{24}\left(\frac{1}{16}\left(\frac{995\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{837}{8}\int \frac{1}{4-5\sin(x)^2}dx\right)-\frac{75\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}\right)+\frac{5\sin(x)\cos(x)}{24\left(4-5\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{1}{24}\left(\frac{1}{16}\left(\frac{995\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{837}{8}\int \frac{1}{4-\tan^2(x)}d\tan(x)\right)-\frac{75\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}\right)+\frac{5\sin(x)\cos(x)}{24\left(4-5\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{24}\left(\frac{1}{16}\left(\frac{995\sin(x)\cos(x)}{8\left(4-5\sin^2(x)\right)}-\frac{837}{16}\text{arctanh}\left(\frac{\tan(x)}{2}\right)\right)-\frac{75\sin(x)\cos(x)}{16\left(4-5\sin^2(x)\right)^2}\right)+\frac{5\sin(x)\cos(x)}{24\left(4-5\sin^2(x)\right)^3}$$

input
```
Int[(4 - 5*Sin[x]^2)^(-4),x]
```

output
```
(5*Cos[x]*Sin[x])/(24*(4 - 5*Sin[x]^2)^3) + ((-75*Cos[x]*Sin[x])/(16*(4 -
5*Sin[x]^2)^2) + ((-837*ArcTanh[Tan[x]/2])/16 + (995*Cos[x]*Sin[x])/(8*(4
- 5*Sin[x]^2)))/16)/24
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

method	result
default	$-\frac{125}{768(\tan(x)+2)^3}+\frac{175}{512(\tan(x)+2)^2}-\frac{745}{2048(\tan(x)+2)}-\frac{279\ln(\tan(x)+2)}{4096}-\frac{125}{768(\tan(x)-2)^3}-\frac{175}{512(\tan(x)-2)^2}$
risch	$\frac{i(20925\,e^{10ix}+62775\,e^{8ix}+111042\,e^{6ix}+119310\,e^{4ix}+68625\,e^{2ix}+24875)}{1536(5\,e^{4ix}+6\,e^{2ix}+5)^3}-\frac{279\ln(e^{2ix}+\frac{3}{5}+\frac{4i}{5})}{4096}+\frac{279\ln(e^{2ix}+\frac{3}{5}-\frac{4i}{5})}{4096}$
norman	$\frac{-\frac{6545\tan(\frac{x}{2})^3}{6144}+\frac{5815\tan(\frac{x}{2})^5}{2048}-\frac{5815\tan(\frac{x}{2})^7}{2048}+\frac{6545\tan(\frac{x}{2})^9}{6144}-\frac{295\tan(\frac{x}{2})^{11}}{2048}+\frac{295\tan(\frac{x}{2})}{2048}}{\left(\tan(\frac{x}{2})^4-3\tan(\frac{x}{2})^2+1\right)^3}-\frac{279\ln\left(\tan(\frac{x}{2})^2-\tan(\frac{x}{2})-1\right)}{4096}$
parallelrisch	$\frac{(-467046-104625\cos(6x)-376650\cos(4x)-765855\cos(2x))\ln\left(\tan(\frac{x}{2})^2-\tan(\frac{x}{2})-1\right)+(467046+104625\cos(6x)+376650\cos}{1536000\cos(6x)+11243520\cos(2x)+552960}$

input
```
int(1/(4-5*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
-125/768/(tan(x)+2)^3+175/512/(tan(x)+2)^2-745/2048/(tan(x)+2)-279/4096*ln
(tan(x)+2)-125/768/(tan(x)-2)^3-175/512/(tan(x)-2)^2-745/2048/(tan(x)-2)+2
79/4096*ln(tan(x)-2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(56) = 112.

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{(4-5\sin^2(x))^4}\,dx = -\frac{837\left(125\cos(x)^6-75\cos(x)^4+15\cos(x)^2-1\right)\log\left(\frac{3}{4}\cos(x)^2+\cos(x)\sin(x)+\frac{1}{4}\right)-837\left(125\cos\right.}{24576\left(12\right.}$$

input
```
integrate(1/(4-5*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
-1/24576*(837*(125*cos(x)^6 - 75*cos(x)^4 + 15*cos(x)^2 - 1)*log(3/4*cos(x
)^2 + cos(x)*sin(x) + 1/4) - 837*(125*cos(x)^6 - 75*cos(x)^4 + 15*cos(x)^2
 - 1)*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 40*(4975*cos(x)^5 - 2590*c
os(x)^3 + 447*cos(x))*sin(x))/(125*cos(x)^6 - 75*cos(x)^4 + 15*cos(x)^2 -
1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(66) = 132$.

Time = 7.85 (sec) , antiderivative size = 1421, normalized size of antiderivative = 21.53

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4-5*sin(x)**2)**4,x)
```

output
```
-837*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**12/(12288*tan(x/2)**12 - 11
0592*tan(x/2)**10 + 368640*tan(x/2)**8 - 552960*tan(x/2)**6 + 368640*tan(x
/2)**4 - 110592*tan(x/2)**2 + 12288) + 7533*log(tan(x/2)**2 - tan(x/2) - 1
)*tan(x/2)**10/(12288*tan(x/2)**12 - 110592*tan(x/2)**10 + 368640*tan(x/2)
**8 - 552960*tan(x/2)**6 + 368640*tan(x/2)**4 - 110592*tan(x/2)**2 + 12288
) - 25110*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**8/(12288*tan(x/2)**12
- 110592*tan(x/2)**10 + 368640*tan(x/2)**8 - 552960*tan(x/2)**6 + 368640*t
an(x/2)**4 - 110592*tan(x/2)**2 + 12288) + 37665*log(tan(x/2)**2 - tan(x/2
) - 1)*tan(x/2)**6/(12288*tan(x/2)**12 - 110592*tan(x/2)**10 + 368640*tan(
x/2)**8 - 552960*tan(x/2)**6 + 368640*tan(x/2)**4 - 110592*tan(x/2)**2 + 1
2288) - 25110*log(tan(x/2)**2 - tan(x/2) - 1)*tan(x/2)**4/(12288*tan(x/2)*
*12 - 110592*tan(x/2)**10 + 368640*tan(x/2)**8 - 552960*tan(x/2)**6 + 3686
40*tan(x/2)**4 - 110592*tan(x/2)**2 + 12288) + 7533*log(tan(x/2)**2 - tan(
x/2) - 1)*tan(x/2)**2/(12288*tan(x/2)**12 - 110592*tan(x/2)**10 + 368640*t
an(x/2)**8 - 552960*tan(x/2)**6 + 368640*tan(x/2)**4 - 110592*tan(x/2)**2
+ 12288) - 837*log(tan(x/2)**2 - tan(x/2) - 1)/(12288*tan(x/2)**12 - 11059
2*tan(x/2)**10 + 368640*tan(x/2)**8 - 552960*tan(x/2)**6 + 368640*tan(x/2)
**4 - 110592*tan(x/2)**2 + 12288) + 837*log(tan(x/2)**2 + tan(x/2) - 1)*ta
n(x/2)**12/(12288*tan(x/2)**12 - 110592*tan(x/2)**10 + 368640*tan(x/2)**8
- 552960*tan(x/2)**6 + 368640*tan(x/2)**4 - 110592*tan(x/2)**2 + 12288)...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{1}{(4-5\sin^2(x))^4}\,dx = -\frac{5\left(447\,\tan(x)^5-1696\,\tan(x)^3+2832\,\tan(x)\right)}{3072\left(\tan(x)^6-12\,\tan(x)^4+48\,\tan(x)^2-64\right)} - \frac{279}{4096}\,\log(\tan(x)+2) + \frac{279}{4096}\,\log(\tan(x)-2)$$

input
```
integrate(1/(4-5*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
-5/3072*(447*tan(x)^5 - 1696*tan(x)^3 + 2832*tan(x))/(tan(x)^6 - 12*tan(x)
^4 + 48*tan(x)^2 - 64) - 279/4096*log(tan(x) + 2) + 279/4096*log(tan(x) -
2)
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^4}\,dx = -\frac{5\left(447\tan(x)^5-1696\tan(x)^3+2832\tan(x)\right)}{3072\left(\tan(x)^2-4\right)^3} - \frac{279}{4096}\log\left(\left|\tan(x)+2\right|\right)+\frac{279}{4096}\log\left(\left|\tan(x)-2\right|\right)$$

input
```
integrate(1/(4-5*sin(x)^2)^4,x, algorithm="giac")
```

output
```
-5/3072*(447*tan(x)^5 - 1696*tan(x)^3 + 2832*tan(x))/(tan(x)^2 - 4)^3 - 27
9/4096*log(abs(tan(x) + 2)) + 279/4096*log(abs(tan(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 36.98 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^4}\,dx = -\frac{279\,\mathrm{atanh}\left(\frac{\tan(x)}{2}\right)}{2048} - \frac{\frac{745\tan(x)^5}{1024}-\frac{265\tan(x)^3}{96}+\frac{295\tan(x)}{64}}{\tan(x)^6-12\tan(x)^4+48\tan(x)^2-64}$$

input
```
int(1/(5*sin(x)^2 - 4)^4,x)
```

output
```
- (279*atanh(tan(x)/2))/2048 - ((295*tan(x))/64 - (265*tan(x)^3)/96 + (745
*tan(x)^5)/1024)/(48*tan(x)^2 - 12*tan(x)^4 + tan(x)^6 - 64)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.82

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^4}\,dx = \frac{-99500\cos(x)\sin(x)^5+147200\cos(x)\sin(x)^3-56640\cos(x)\sin(x)-104625\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)\right.}{}$$

input
```
int(1/(4-5*sin(x)^2)^4,x)
```

output
```
( - 99500*cos(x)*sin(x)**5 + 147200*cos(x)*sin(x)**3 - 56640*cos(x)*sin(x)
 - 104625*log( - sqrt(5) + 2*tan(x/2) - 1)*sin(x)**6 + 251100*log( - sqrt(
5) + 2*tan(x/2) - 1)*sin(x)**4 - 200880*log( - sqrt(5) + 2*tan(x/2) - 1)*s
in(x)**2 + 53568*log( - sqrt(5) + 2*tan(x/2) - 1) + 104625*log( - sqrt(5)
+ 2*tan(x/2) + 1)*sin(x)**6 - 251100*log( - sqrt(5) + 2*tan(x/2) + 1)*sin(
x)**4 + 200880*log( - sqrt(5) + 2*tan(x/2) + 1)*sin(x)**2 - 53568*log( - s
qrt(5) + 2*tan(x/2) + 1) - 104625*log(sqrt(5) + 2*tan(x/2) - 1)*sin(x)**6
+ 251100*log(sqrt(5) + 2*tan(x/2) - 1)*sin(x)**4 - 200880*log(sqrt(5) + 2*
tan(x/2) - 1)*sin(x)**2 + 53568*log(sqrt(5) + 2*tan(x/2) - 1) + 104625*log
(sqrt(5) + 2*tan(x/2) + 1)*sin(x)**6 - 251100*log(sqrt(5) + 2*tan(x/2) + 1
)*sin(x)**4 + 200880*log(sqrt(5) + 2*tan(x/2) + 1)*sin(x)**2 - 53568*log(s
qrt(5) + 2*tan(x/2) + 1))/(12288*(125*sin(x)**6 - 300*sin(x)**4 + 240*sin(
x)**2 - 64))
```

3.124 $\int (4-5\sin^2(x))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int (4-5\sin^2(x))^{7/2}\,dx=\frac{1696E\left(x\middle|\frac{5}{4}\right)}{35}+\frac{632\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{105}+\frac{316}{21}\cos(x)\sin(x)\sqrt{4-5\sin^2(x)}+\frac{18}{7}\cos(x)\sin(x)\left(4-5\sin^2(x)\right)^{3/2}+\frac{5}{7}\cos(x)\sin(x)\left(4-5\sin^2(x)\right)^{5/2}$$

output
```
1696/35*EllipticE(sin(x),1/2*5^(1/2))+632/105*InverseJacobiAM(x,1/2*5^(1/2
))+316/21*cos(x)*sin(x)*(4-5*sin(x)^2)^(1/2)+18/7*cos(x)*sin(x)*(4-5*sin(x
)^2)^(3/2)+5/7*cos(x)*sin(x)*(4-5*sin(x)^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int (4-5\sin^2(x))^{7/2}\,dx=\frac{325632E\left(x\middle|\frac{5}{4}\right)+40448\,\text{EllipticF}\left(x,\frac{5}{4}\right)+5\sqrt{6+10\cos(2x)}(7267\sin(2x)+1980\sin(}{6720}$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(7/2),x]
```

output
```
(325632*EllipticE[x, 5/4] + 40448*EllipticF[x, 5/4] + 5*Sqrt[6 + 10*Cos[2*
x]]*(7267*Sin[2*x] + 1980*Sin[4*x] + 375*Sin[6*x]))/6720
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3659, 27, 3042, 3649, 27, 3042, 3649, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sin^2(x)\right)^{7/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-5 \sin(x)^2\right)^{7/2} dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{7} \int 2 \left(46-45 \sin^2(x)\right) \left(4-5 \sin^2(x)\right)^{3/2} dx+\frac{5}{7} \sin(x) \left(4-5 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7} \int \left(46-45 \sin^2(x)\right) \left(4-5 \sin^2(x)\right)^{3/2} dx+\frac{5}{7} \sin(x) \left(4-5 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7} \int \left(46-45 \sin(x)^2\right) \left(4-5 \sin(x)^2\right)^{3/2} dx+\frac{5}{7} \sin(x) \left(4-5 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{2}{7} \left(\frac{1}{5} \int 10 \left(74-79 \sin^2(x)\right) \sqrt{4-5 \sin^2(x)} dx+9 \sin(x) \left(4-5 \sin^2(x)\right)^{3/2} \cos(x)\right)+ \frac{5}{7} \sin(x) \left(4-5 \sin^2(x)\right)^{5/2} \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\left(2\int\left(74-79\sin^2(x)\right)\sqrt{4-5\sin^2(x)}dx+9\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(2\int\left(74-79\sin(x)^2\right)\sqrt{4-5\sin(x)^2}dx+9\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3649}$$

$$\frac{2}{7}\left(2\left(\frac{1}{3}\int\frac{4\left(143-159\sin^2(x)\right)}{\sqrt{4-5\sin^2(x)}}dx+\frac{79}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+9\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\left(2\left(\frac{4}{3}\int\frac{143-159\sin^2(x)}{\sqrt{4-5\sin^2(x)}}dx+\frac{79}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+9\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(2\left(\frac{4}{3}\int\frac{143-159\sin(x)^2}{\sqrt{4-5\sin(x)^2}}dx+\frac{79}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+9\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)\right)+\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3651}$$

$$\frac{2}{7}\left(2\left(\frac{4}{3}\left(\frac{79}{5}\int\frac{1}{\sqrt{4-5\sin^2(x)}}dx+\frac{159}{5}\int\sqrt{4-5\sin^2(x)}dx\right)+\frac{79}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+9\sin(x)\left(\right.\right.\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(2\left(\frac{4}{3}\left(\frac{79}{5}\int\frac{1}{\sqrt{4-5\sin(x)^2}}dx+\frac{159}{5}\int\sqrt{4-5\sin(x)^2}dx\right)+\frac{79}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+9\sin(x)\left(4\right.\right.$$
$$\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$\Big\downarrow$ 3656

$$\frac{2}{7}\left(2\left(\frac{4}{3}\left(\frac{79}{5}\int\frac{1}{\sqrt{4-5\sin(x)^2}}dx+\frac{318E\left(x\middle|\frac{5}{4}\right)}{5}\right)+\frac{79}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+9\sin(x)\left(4-5\sin^2(x)\right)\right.$$
$$\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)$$

$\Big\downarrow$ 3661

$$\frac{5}{7}\sin(x)\left(4-5\sin^2(x)\right)^{5/2}\cos(x)+$$
$$\frac{2}{7}\left(9\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)+2\left(\frac{79}{3}\sin(x)\sqrt{4-5\sin^2(x)}\cos(x)+\frac{4}{3}\left(\frac{79\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{10}+\frac{318E\left(x\middle|\frac{5}{4}\right)}{5}\right.\right.\right.$$

input
```
Int[(4 - 5*Sin[x]^2)^(7/2),x]
```

output
```
(5*Cos[x]*Sin[x]*(4 - 5*Sin[x]^2)^(5/2))/7 + (2*(9*Cos[x]*Sin[x]*(4 - 5*Si
n[x]^2)^(3/2) + 2*((4*((318*EllipticE[x, 5/4])/5 + (79*EllipticF[x, 5/4])/
10))/3 + (79*Cos[x]*Sin[x]*Sqrt[4 - 5*Sin[x]^2])/3)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}\,\left(9375\cos(x)^8\sin(x)+1125\cos(x)^6\sin(x)+6325\cos(x)^4\sin(x)+632\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{5\cos(x)^2-1}\ \text{Ellip}\right.}{105\sqrt{5\cos(x)^4-\cos(x)^2}\,\cos(x)\sqrt{4-5\text{s}}}$

input
```
int((4-5*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)*(9375*cos(x)^8*sin(x)+1125*cos(x)^
6*sin(x)+6325*cos(x)^4*sin(x)+632*(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*El
lipticF(sin(x),1/2*5^(1/2))+5088*(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*Ell
ipticE(sin(x),1/2*5^(1/2))-1385*cos(x)^2*sin(x))/(5*cos(x)^4-cos(x)^2)^(1/
2)/cos(x)/(4-5*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(4-5\sin^2(x)\right)^{7/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
integral((125*cos(x)^6 - 75*cos(x)^4 + 15*cos(x)^2 - 1)*sqrt(5*cos(x)^2 -
1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(4-5\sin^2(x)\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((4-5*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(4-5\sin^2(x)\right)^{7/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-5*sin(x)^2 + 4)^(7/2), x)
```

Giac [F]

$$\int \left(4-5\sin^2(x)\right)^{7/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((-5*sin(x)^2 + 4)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-5\sin^2(x)\right)^{7/2}\,dx=\int {\left(4-5\sin\left(x\right)^2\right)}^{7/2}\,dx$$

input
```
int((4 - 5*sin(x)^2)^(7/2),x)
```

output
```
int((4 - 5*sin(x)^2)^(7/2), x)
```

Reduce [F]

$$\int \left(4-5\sin^2(x)\right)^{7/2}\,dx = 64\left(\int \sqrt{-5\sin(x)^2+4}dx\right) - 125\left(\int \sqrt{-5\sin(x)^2+4}\,\sin(x)^6\,dx\right) + 300\left(\int \sqrt{-5\sin(x)^2+4}\,\sin(x)^4\,dx\right) - 240\left(\int \sqrt{-5\sin(x)^2+4}\,\sin(x)^2\,dx\right)$$

input
```
int((4-5*sin(x)^2)^(7/2),x)
```

output
```
64*int(sqrt( - 5*sin(x)**2 + 4),x) - 125*int(sqrt( - 5*sin(x)**2 + 4)*sin(
x)**6,x) + 300*int(sqrt( - 5*sin(x)**2 + 4)*sin(x)**4,x) - 240*int(sqrt( -
 5*sin(x)**2 + 4)*sin(x)**2,x)
```

3.125 $\int (4-5\sin^2(x))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int (4-5\sin^2(x))^{5/2}\,dx = \frac{72E\left(x\middle|\frac{5}{4}\right)}{5}+\frac{8\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{5} + 4\cos(x)\sin(x)\sqrt{4-5\sin^2(x)}+\cos(x)\sin(x)\left(4-5\sin^2(x)\right)^{3/2}$$

output
```
72/5*EllipticE(sin(x),1/2*5^(1/2))+8/5*InverseJacobiAM(x,1/2*5^(1/2))+4*co
s(x)*sin(x)*(4-5*sin(x)^2)^(1/2)+cos(x)*sin(x)*(4-5*sin(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int (4-5\sin^2(x))^{5/2}\,dx = \frac{72E\left(x\middle|\frac{5}{4}\right)}{5}+\frac{8\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{5} + \frac{\sqrt{3+5\cos(2x)}(22\sin(2x)+5\sin(4x))}{8\sqrt{2}}$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(5/2),x]
```

output
```
(72*EllipticE[x, 5/4])/5 + (8*EllipticF[x, 5/4])/5 + (Sqrt[3 + 5*Cos[2*x]]
*(22*Sin[2*x] + 5*Sin[4*x]))/(8*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 3659, 27, 3042, 3649, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sin ^2(x)\right)^{5/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-5 \sin (x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{5} \int 60 \sqrt{4-5 \sin ^2(x)} \left(1-\sin ^2(x)\right)dx+\sin (x) \left(4-5 \sin ^2(x)\right)^{3/2} \cos (x)$$

$$\Big\downarrow \text{27}$$

$$12 \int \sqrt{4-5 \sin ^2(x)} \left(1-\sin ^2(x)\right)dx+\sin (x) \left(4-5 \sin ^2(x)\right)^{3/2} \cos (x)$$

$$\Big\downarrow \text{3042}$$

$$12 \int \sqrt{4-5 \sin (x)^2} \left(1-\sin (x)^2\right)dx+\sin (x) \left(4-5 \sin ^2(x)\right)^{3/2} \cos (x)$$

$$\Big\downarrow \text{3649}$$

$$12 \left(\frac{1}{3} \int \frac{8-9 \sin ^2(x)}{\sqrt{4-5 \sin ^2(x)}}dx+\frac{1}{3} \sqrt{4-5 \sin ^2(x)} \sin (x) \cos (x)\right)+\sin (x) \left(4-5 \sin ^2(x)\right)^{3/2} \cos (x)$$

$$\Big\downarrow \text{3042}$$

$$12 \left(\frac{1}{3} \int \frac{8-9 \sin (x)^2}{\sqrt{4-5 \sin (x)^2}}dx+\frac{1}{3} \sqrt{4-5 \sin ^2(x)} \sin (x) \cos (x)\right)+\sin (x) \left(4-5 \sin ^2(x)\right)^{3/2} \cos (x)$$

$$\Big\downarrow \text{3651}$$

$$12\left(\frac{1}{3}\left(\frac{4}{5}\int \frac{1}{\sqrt{4-5\sin^2(x)}}dx+\frac{9}{5}\int \sqrt{4-5\sin^2(x)}dx\right)+\frac{1}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$12\left(\frac{1}{3}\left(\frac{4}{5}\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx+\frac{9}{5}\int \sqrt{4-5\sin(x)^2}dx\right)+\frac{1}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{3656}$$

$$12\left(\frac{1}{3}\left(\frac{4}{5}\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx+\frac{18E\left(x\middle|\frac{5}{4}\right)}{5}\right)+\frac{1}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)\right)+\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)$$

$$\Big\downarrow \text{3661}$$

$$\sin(x)\left(4-5\sin^2(x)\right)^{3/2}\cos(x)+12\left(\frac{1}{3}\sin(x)\sqrt{4-5\sin^2(x)}\cos(x)+\frac{1}{3}\left(\frac{2\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{5}+\frac{18E\left(x\middle|\frac{5}{4}\right)}{5}\right)\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(5/2),x]
```

output
```
Cos[x]*Sin[x]*(4 - 5*Sin[x]^2)^(3/2) + 12*(((18*EllipticE[x, 5/4])/5 + (2*
EllipticF[x, 5/4])/5)/3 + (Cos[x]*Sin[x]*Sqrt[4 - 5*Sin[x]^2])/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(51) = 102$.

Time = 4.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.31

method	result
default	$\frac{\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}\left(125\cos(x)^6\sin(x)+50\cos(x)^4\sin(x)+8\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{5\cos(x)^2-1}\,\text{EllipticF}\left(\sin(x),\frac{\sqrt{5}}{2}\right)+72\sqrt{\text{cos}}\right.}{5\sqrt{5\cos(x)^4-\cos(x)^2}\,\cos(x)\sqrt{4-5\sin(x)^2}}$

input
```
int((4-5*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)*(125*cos(x)^6*sin(x)+50*cos(x)^4*sin
(x)+8*(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*EllipticF(sin(x),1/2*5^(1/2))+
72*(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*EllipticE(sin(x),1/2*5^(1/2))-15*
cos(x)^2*sin(x))/(5*cos(x)^4-cos(x)^2)^(1/2)/cos(x)/(4-5*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(4-5\sin^2(x)\right)^{5/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((25*cos(x)^4 - 10*cos(x)^2 + 1)*sqrt(5*cos(x)^2 - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(4-5\sin^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((4-5*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(4-5\sin^2(x)\right)^{5/2} dx = \int \left(-5\sin(x)^2+4\right)^{\frac{5}{2}} dx$$

input
```
integrate((4-5*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-5*sin(x)^2 + 4)^(5/2), x)
```

Giac [F]

$$\int \left(4-5\sin^2(x)\right)^{5/2} dx = \int \left(-5\sin(x)^2+4\right)^{\frac{5}{2}} dx$$

input
```
integrate((4-5*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-5*sin(x)^2 + 4)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-5\sin^2(x)\right)^{5/2} dx = \int \left(4-5\sin(x)^2\right)^{5/2} dx$$

input
```
int((4 - 5*sin(x)^2)^(5/2),x)
```

output
```
int((4 - 5*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(4-5\sin^2(x)\right)^{5/2}\,dx = 16\left(\int \sqrt{-5\sin(x)^2+4}dx\right) + 25\left(\int \sqrt{-5\sin(x)^2+4}\,\sin(x)^4\,dx\right) - 40\left(\int \sqrt{-5\sin(x)^2+4}\,\sin(x)^2\,dx\right)$$

input
```
int((4-5*sin(x)^2)^(5/2),x)
```

output
```
16*int(sqrt( - 5*sin(x)**2 + 4),x) + 25*int(sqrt( - 5*sin(x)**2 + 4)*sin(x
)**4,x) - 40*int(sqrt( - 5*sin(x)**2 + 4)*sin(x)**2,x)
```

3.126 $\int (4-5\sin^2(x))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int (4-5\sin^2(x))^{3/2}\,dx = 4E\left(x\left|\frac{5}{4}\right.\right)+\frac{2\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{3}+\frac{5}{3}\cos(x)\sin(x)\sqrt{4-5\sin^2(x)}$$

output
```
4*EllipticE(sin(x),1/2*5^(1/2))+2/3*InverseJacobiAM(x,1/2*5^(1/2))+5/3*cos
(x)*sin(x)*(4-5*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (4-5\sin^2(x))^{3/2}\,dx = 4E\left(x\left|\frac{5}{4}\right.\right)+\frac{2\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{3}+\frac{5}{6}\cos(x)\sqrt{6+10\cos(2x)}\sin(x)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(3/2),x]
```

output
```
4*EllipticE[x, 5/4] + (2*EllipticF[x, 5/4])/3 + (5*Cos[x]*Sqrt[6 + 10*Cos[
2*x]]*Sin[x])/6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-5\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2\left(14-15\sin^2(x)\right)}{\sqrt{4-5\sin^2(x)}}dx+\frac{5}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{14-15\sin^2(x)}{\sqrt{4-5\sin^2(x)}}dx+\frac{5}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{14-15\sin(x)^2}{\sqrt{4-5\sin(x)^2}}dx+\frac{5}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3651}$$

$$\frac{2}{3}\left(2\int \frac{1}{\sqrt{4-5\sin^2(x)}}dx+3\int \sqrt{4-5\sin^2(x)}dx\right)+\frac{5}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(2\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx+3\int \sqrt{4-5\sin(x)^2}dx\right)+\frac{5}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3656}$$

$$\frac{2}{3}\left(2\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx+6E\left(x\left|\frac{5}{4}\right.\right)\right)+\frac{5}{3}\sqrt{4-5\sin^2(x)}\sin(x)\cos(x)$$

$$\Big\downarrow \text{3661}$$

$$\frac{5}{3}\sin(x)\sqrt{4-5\sin^2(x)}\cos(x)+\frac{2}{3}\left(\text{EllipticF}\left(x,\frac{5}{4}\right)+6E\left(x\left|\frac{5}{4}\right.\right)\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(2*(6*EllipticE[x, 5/4] + EllipticF[x, 5/4]))/3 + (5*Cos[x]*Sin[x]*Sqrt[4
- 5*Sin[x]^2])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(36) = 72$.

Time = 1.70 (sec), antiderivative size = 117, normalized size of antiderivative = 3.16

method	result
default	$\frac{\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}\left(25\cos(x)^4\sin(x)+2\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\sqrt{5\cos(x)^2-1}\ \mathrm{EllipticF}\left(\sin(x),\frac{\sqrt{5}}{2}\right)+12\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\sqrt{5\cos(x)^2}}{3\sqrt{5\cos(x)^4-\cos(x)^2}\cos(x)\sqrt{4-5\sin(x)^2}}$

input
```
int((4-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)*(25*cos(x)^4*sin(x)+2*(cos(x)^2)^(1/
2)*(5*cos(x)^2-1)^(1/2)*EllipticF(sin(x),1/2*5^(1/2))+12*(cos(x)^2)^(1/2)*
(5*cos(x)^2-1)^(1/2)*EllipticE(sin(x),1/2*5^(1/2))-5*cos(x)^2*sin(x))/(5*c
os(x)^4-cos(x)^2)^(1/2)/cos(x)/(4-5*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((5*cos(x)^2 - 1)^(3/2), x)
```

Sympy [F]

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx=\int \left(4-5\sin^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sin(x)**2)**(3/2),x)
```

output
```
Integral((4 - 5*sin(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*sin(x)^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*sin(x)^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx=\int \left(4-5\sin\left(x\right)^2\right)^{3/2}\,dx$$

input
```
int((4 - 5*sin(x)^2)^(3/2),x)
```

output
```
int((4 - 5*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(4-5\sin^2(x)\right)^{3/2}\,dx=4\left(\int \sqrt{-5\sin\left(x\right)^2+4}dx\right)-5\left(\int \sqrt{-5\sin\left(x\right)^2+4}\,\sin\left(x\right)^2dx\right)$$

input
```
int((4-5*sin(x)^2)^(3/2),x)
```

output
```
4*int(sqrt( - 5*sin(x)**2 + 4),x) - 5*int(sqrt( - 5*sin(x)**2 + 4)*sin(x)*
*2,x)
```

3.127 $\int \sqrt{4-5 \sin^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 7

$$\int \sqrt{4-5 \sin^2(x)}\,dx = 2E\left(x\left|\frac{5}{4}\right.\right)$$

output
```
2*EllipticE(sin(x),1/2*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \sqrt{4-5 \sin^2(x)}\,dx = 2E\left(x\left|\frac{5}{4}\right.\right)$$

input
```
Integrate[Sqrt[4 - 5*Sin[x]^2],x]
```

output
```
2*EllipticE[x, 5/4]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4-5\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{4-5\sin(x)^2}dx$$

$$\Big\downarrow \text{3656}$$

$$2E\left(x\left|\frac{5}{4}\right.\right)$$

input
```
Int[Sqrt[4 - 5*Sin[x]^2],x]
```

output
```
2*EllipticE[x, 5/4]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(10) = 20.

Time = 1.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 7.43

method	result	size
default	$\frac{2\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}\,\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\ \mathrm{EllipticE}\left(\sin(x),\frac{\sqrt{5}}{2}\right)}{\sqrt{5\cos(x)^4-\cos(x)^2}\,\cos(x)}$	52

input
```
int((4-5*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)*(cos(x)^2)^(1/2)*EllipticE(sin(x),1/2*
5^(1/2))/(5*cos(x)^4-cos(x)^2)^(1/2)/cos(x)
```

Fricas [F]

$$\int\sqrt{4-5\sin^2(x)}\,dx=\int\sqrt{-5\,\sin\left(x\right)^2+4}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*cos(x)^2 - 1), x)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int\sqrt{4-5\sin^2(x)}\,dx=2E\left(x\middle|\frac{5}{4}\right)$$

input
```
integrate((4-5*sin(x)**2)**(1/2),x)
```

output
```
2*elliptic_e(x, 5/4)
```

Maxima [F]

$$\int \sqrt{4-5\sin^2(x)}\,dx = \int \sqrt{-5\,\sin{(x)}^2+4}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-5*sin(x)^2 + 4), x)
```

Giac [F]

$$\int \sqrt{4-5\sin^2(x)}\,dx = \int \sqrt{-5\,\sin{(x)}^2+4}\,dx$$

input
```
integrate((4-5*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-5*sin(x)^2 + 4), x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \sqrt{4-5\sin^2(x)}\,dx = 2\,\mathrm{E}\left(x\,\middle|\,\frac{5}{4}\right)$$

input
```
int((4 - 5*sin(x)^2)^(1/2),x)
```

output
```
2*ellipticE(x, 5/4)
```

Reduce [F]

$$\int \sqrt{4-5\sin^2(x)}\,dx = \int \sqrt{-5\sin{(x)}^2+4}dx$$

input
```
int((4-5*sin(x)^2)^(1/2),x)
```

output
```
int(sqrt( - 5*sin(x)**2 + 4),x)
```

3.128 $\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx=\frac{\operatorname{EllipticF}\left(x,\frac{5}{4}\right)}{2}$$

output
```
1/2*InverseJacobiAM(x,1/2*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx=\frac{\operatorname{EllipticF}\left(x,\frac{5}{4}\right)}{2}$$

input
```
Integrate[1/Sqrt[4 - 5*Sin[x]^2],x]
```

output
```
EllipticF[x, 5/4]/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx$$

$$\Big\downarrow \text{ 3661}$$

$$\frac{\text{EllipticF}\left(x,\frac{5}{4}\right)}{2}$$

input
```
Int[1/Sqrt[4 - 5*Sin[x]^2],x]
```

output
```
EllipticF[x, 5/4]/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\mathrm{InverseJacobiAM}\left(x, \frac{\sqrt{5}}{2}\right)}{2}$	10

input
```
int(1/(4-5*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*InverseJacobiAM(x,1/2*5^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx = \left(\frac{3}{25}i-\frac{4}{25}\right)\sqrt{5}\sqrt{\frac{4}{5}i-\frac{3}{5}}F(\arcsin\left(\sqrt{\frac{4}{5}i-\frac{3}{5}}(\cos(x)+i\sin(x))\right)|\frac{24}{25}i-\frac{7}{25}) - \left(\frac{3}{25}i+\frac{4}{25}\right)\sqrt{5}\sqrt{-\frac{4}{5}i-\frac{3}{5}}F(\arcsin\left(\sqrt{-\frac{4}{5}i-\frac{3}{5}}(\cos(x)-i\sin(x))\right)|-\frac{24}{25}i-\frac{7}{25})$$

input
```
integrate(1/(4-5*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(3/25*I - 4/25)*sqrt(5)*sqrt(4/5*I - 3/5)*elliptic_f(arcsin(sqrt(4/5*I - 3
/5)*(cos(x) + I*sin(x))), 24/25*I - 7/25) - (3/25*I + 4/25)*sqrt(5)*sqrt(-
4/5*I - 3/5)*elliptic_f(arcsin(sqrt(-4/5*I - 3/5)*(cos(x) - I*sin(x))), -2
4/25*I - 7/25)
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx = \frac{F\left(x\middle|\frac{5}{4}\right)}{2}$$

input
```
integrate(1/(4-5*sin(x)**2)**(1/2),x)
```

output
```
elliptic_f(x, 5/4)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-5\,\sin\left(x\right)^2+4}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-5*sin(x)^2 + 4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-5\,\sin\left(x\right)^2+4}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-5*sin(x)^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{4-5\sin{(x)}^2}}\,dx$$

input
```
int(1/(4 - 5*sin(x)^2)^(1/2),x)
```

output
```
int(1/(4 - 5*sin(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{4-5\sin^2(x)}}\,dx = -\left(\int \frac{\sqrt{-5\sin{(x)}^2+4}}{5\sin{(x)}^2-4}dx\right)$$

input
```
int(1/(4-5*sin(x)^2)^(1/2),x)
```

output
```
- int(sqrt( - 5*sin(x)**2 + 4)/(5*sin(x)**2 - 4),x)
```

3.129 $\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = -\frac{E\left(x\middle|\frac{5}{4}\right)}{2} + \frac{5\cos(x)\sin(x)}{4\sqrt{4-5\sin^2(x)}}$$

output
```
-1/2*EllipticE(sin(x),1/2*5^(1/2))+5/4*cos(x)*sin(x)/(4-5*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = -\frac{E\left(x\middle|\frac{5}{4}\right)}{2} + \frac{5\sin(2x)}{4\sqrt{2}\sqrt{3+5\cos(2x)}}$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-3/2),x]
```

output
```
-1/2*EllipticE[x, 5/4] + (5*Sin[2*x])/(4*Sqrt[2]*Sqrt[3 + 5*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3663, 25, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-5 \sin ^2(x)\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(4-5 \sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{1}{4}\int -\sqrt{4-5 \sin^2(x)}dx+\frac{5 \sin(x) \cos(x)}{4 \sqrt{4-5 \sin^2(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{5 \sin(x) \cos(x)}{4 \sqrt{4-5 \sin^2(x)}}-\frac{1}{4}\int \sqrt{4-5 \sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5 \sin(x) \cos(x)}{4 \sqrt{4-5 \sin^2(x)}}-\frac{1}{4}\int \sqrt{4-5 \sin(x)^2}dx$$

$$\Big\downarrow \text{3656}$$

$$\frac{5 \sin(x) \cos(x)}{4 \sqrt{4-5 \sin^2(x)}}-\frac{E\left(x\left|\frac{5}{4}\right.\right)}{2}$$

input
```
Int[(4 - 5*Sin[x]^2)^(-3/2),x]
```

output
```
-1/2*EllipticE[x, 5/4] + (5*Cos[x]*Sin[x])/(4*Sqrt[4 - 5*Sin[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

method	result	size
default	$-\frac{2\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{5\cos(x)^{2}-1}\,\operatorname{EllipticE}\left(\sin(x),\frac{\sqrt{5}}{2}\right)-5\cos(x)^{2}\sin(x)}{4\cos(x)\sqrt{4-5\sin(x)^{2}}}$	52

input
```
int(1/(4-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*EllipticE(sin(x),1/2*5^(1/2)
)-5*cos(x)^2*sin(x))/cos(x)/(4-5*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.50

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = -\frac{5\sqrt{\frac{4}{5}i-\frac{3}{5}}\left(-(15i+20)\sqrt{5}\cos(x)^2+(3i+4)\sqrt{5}\right)E\left(\arcsin\left(\sqrt{\frac{4}{5}i-\frac{3}{5}}(\cos(x)+i\sin(x))\right)\mid\frac{24}{25}i-\frac{7}{25}\right)}{}$$

input
```
integrate(1/(4-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/200*(5*sqrt(4/5*I - 3/5)*(-(15*I + 20)*sqrt(5)*cos(x)^2 + (3*I + 4)*sqr
t(5))*elliptic_e(arcsin(sqrt(4/5*I - 3/5)*(cos(x) + I*sin(x))), 24/25*I -
7/25) + 5*sqrt(-4/5*I - 3/5)*((15*I - 20)*sqrt(5)*cos(x)^2 - (3*I - 4)*sqr
t(5))*elliptic_e(arcsin(sqrt(-4/5*I - 3/5)*(cos(x) - I*sin(x))), -24/25*I
- 7/25) + 8*sqrt(4/5*I - 3/5)*((15*I + 5)*sqrt(5)*cos(x)^2 - (3*I + 1)*sqr
t(5))*elliptic_f(arcsin(sqrt(4/5*I - 3/5)*(cos(x) + I*sin(x))), 24/25*I -
7/25) + 8*sqrt(-4/5*I - 3/5)*(-(15*I - 5)*sqrt(5)*cos(x)^2 + (3*I - 1)*sqr
t(5))*elliptic_f(arcsin(sqrt(-4/5*I - 3/5)*(cos(x) - I*sin(x))), -24/25*I
- 7/25) - 250*sqrt(5*cos(x)^2 - 1)*cos(x)*sin(x))/(5*cos(x)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(4-5*sin(x)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-5\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*sin(x)^2 + 4)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-5\,\sin\left(x\right)^2+4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*sin(x)^2 + 4)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(4-5\sin\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(4 - 5*sin(x)^2)^(3/2),x)
```

output
```
int(1/(4 - 5*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(4-5\sin^2(x))^{3/2}}\,dx = \int \frac{\sqrt{-5\sin(x)^2+4}}{25\sin(x)^4-40\sin(x)^2+16}dx$$

input
```
int(1/(4-5*sin(x)^2)^(3/2),x)
```

output
```
int(sqrt( - 5*sin(x)**2 + 4)/(25*sin(x)**4 - 40*sin(x)**2 + 16),x)
```

3.130 $\int \frac{1}{\left(4-5\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{5/2}}\,dx = \frac{E\left(x\middle|\frac{5}{4}\right)}{4}+\frac{\text{EllipticF}\left(x,\frac{5}{4}\right)}{24}+\frac{5\cos(x)\sin(x)}{12\left(4-5\sin^2(x)\right)^{3/2}}-\frac{5\cos(x)\sin(x)}{8\sqrt{4-5\sin^2(x)}}$$

output
```
1/4*EllipticE(sin(x),1/2*5^(1/2))+1/24*InverseJacobiAM(x,1/2*5^(1/2))+5/12
*cos(x)*sin(x)/(4-5*sin(x)^2)^(3/2)-5/8*cos(x)*sin(x)/(4-5*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{5/2}}\,dx = \frac{1}{96}\left(24E\left(x\middle|\frac{5}{4}\right)+4\,\text{EllipticF}\left(x,\frac{5}{4}\right)-\frac{25\sqrt{2}(2\sin(2x)+3\sin(4x))}{(3+5\cos(2x))^{3/2}}\right)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-5/2),x]
```

output
```
(24*EllipticE[x, 5/4] + 4*EllipticF[x, 5/4] - (25*Sqrt[2]*(2*Sin[2*x] + 3*
Sin[4*x]))/(3 + 5*Cos[2*x])^(3/2))/96
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4-5\sin^2(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(4-5\sin(x)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{1}{12}\int -\frac{5\sin^2(x)+2}{(4-5\sin^2(x))^{3/2}}dx+\frac{5\sin(x)\cos(x)}{12\,(4-5\sin^2(x))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\sin(x)\cos(x)}{12\,(4-5\sin^2(x))^{3/2}}-\frac{1}{12}\int \frac{5\sin^2(x)+2}{(4-5\sin^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sin(x)\cos(x)}{12\,(4-5\sin^2(x))^{3/2}}-\frac{1}{12}\int \frac{5\sin(x)^2+2}{(4-5\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{12}\left(\frac{1}{4}\int \frac{2(14-15\sin^2(x))}{\sqrt{4-5\sin^2(x)}}dx-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)+\frac{5\sin(x)\cos(x)}{12\,(4-5\sin^2(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\frac{1}{2}\int \frac{14-15\sin^2(x)}{\sqrt{4-5\sin^2(x)}}dx-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)+\frac{5\sin(x)\cos(x)}{12\,(4-5\sin^2(x))^{3/2}}$$

$$\big\downarrow \; 3042$$

$$\frac{1}{12}\left(\frac{1}{2}\int \frac{14-15\sin(x)^2}{\sqrt{4-5\sin(x)^2}}dx-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)+\frac{5\sin(x)\cos(x)}{12\left(4-5\sin^2(x)\right)^{3/2}}$$

$$\big\downarrow \; 3651$$

$$\frac{1}{12}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{4-5\sin^2(x)}}dx+3\int \sqrt{4-5\sin^2(x)}dx\right)-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)+\frac{5\sin(x)\cos(x)}{12\left(4-5\sin^2(x)\right)^{3/2}}$$

$$\big\downarrow \; 3042$$

$$\frac{1}{12}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx+3\int \sqrt{4-5\sin(x)^2}dx\right)-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)+\frac{5\sin(x)\cos(x)}{12\left(4-5\sin^2(x)\right)^{3/2}}$$

$$\big\downarrow \; 3656$$

$$\frac{1}{12}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{4-5\sin(x)^2}}dx+6E\left(x\left|\frac{5}{4}\right.\right)\right)-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)+\frac{5\sin(x)\cos(x)}{12\left(4-5\sin^2(x)\right)^{3/2}}$$

$$\big\downarrow \; 3661$$

$$\frac{5\sin(x)\cos(x)}{12\left(4-5\sin^2(x)\right)^{3/2}}+\frac{1}{12}\left(\frac{1}{2}\left(\text{EllipticF}\left(x,\frac{5}{4}\right)+6E\left(x\left|\frac{5}{4}\right.\right)\right)-\frac{15\sin(x)\cos(x)}{2\sqrt{4-5\sin^2(x)}}\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(-5/2),x]
```

output
```
(5*Cos[x]*Sin[x])/(12*(4 - 5*Sin[x]^2)^(3/2)) + ((6*EllipticE[x, 5/4] + El
lipticF[x, 5/4])/2 - (15*Cos[x]*Sin[x])/(2*Sqrt[4 - 5*Sin[x]^2]))/12
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(52) = 104.

Time = 0.75 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.27

method	result
default	$\frac{\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}\left(5\sqrt{5\cos(x)^2-1}\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin(x),\frac{\sqrt{5}}{2}\right)\cos(x)^2+30\sqrt{5\cos(x)^2-1}\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\ \text{Elliptic}}{}$

input
```
int(1/(4-5*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)/(25*cos(x)^4-10*cos(x)^2+1)/cos(x)^
3*(5*(5*cos(x)^2-1)^(1/2)*(cos(x)^2)^(1/2)*EllipticF(sin(x),1/2*5^(1/2))*c
os(x)^2+30*(5*cos(x)^2-1)^(1/2)*(cos(x)^2)^(1/2)*EllipticE(sin(x),1/2*5^(1
/2))*cos(x)^2-75*cos(x)^4*sin(x)-(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*Ell
ipticF(sin(x),1/2*5^(1/2))-6*(cos(x)^2)^(1/2)*(5*cos(x)^2-1)^(1/2)*Ellipti
cE(sin(x),1/2*5^(1/2))+25*cos(x)^2*sin(x))*(5*cos(x)^4-cos(x)^2)^(1/2)/(4-
5*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.66

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{15\sqrt{\frac{4}{5}i-\frac{3}{5}}\left((75i+100)\sqrt{5}\cos(x)^4-(30i+40)\sqrt{5}\cos(x)^2+(3i+4)\sqrt{5}\right)E\left(\arcsin\left(\sqrt{\frac{4}{5}i-\frac{3}{5}}(\cos(x\right.\right.}{}$$

input
```
integrate(1/(4-5*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/1200*(15*sqrt(4/5*I - 3/5)*((75*I + 100)*sqrt(5)*cos(x)^4 - (30*I + 40)
*sqrt(5)*cos(x)^2 + (3*I + 4)*sqrt(5))*elliptic_e(arcsin(sqrt(4/5*I - 3/5)
*(cos(x) + I*sin(x))), 24/25*I - 7/25) + 15*sqrt(-4/5*I - 3/5)*(-(75*I - 1
00)*sqrt(5)*cos(x)^4 + (30*I - 40)*sqrt(5)*cos(x)^2 - (3*I - 4)*sqrt(5))*e
lliptic_e(arcsin(sqrt(-4/5*I - 3/5)*(cos(x) - I*sin(x))), -24/25*I - 7/25)
 + 4*sqrt(4/5*I - 3/5)*(-(525*I + 50)*sqrt(5)*cos(x)^4 + (210*I + 20)*sqrt
(5)*cos(x)^2 - (21*I + 2)*sqrt(5))*elliptic_f(arcsin(sqrt(4/5*I - 3/5)*(co
s(x) + I*sin(x))), 24/25*I - 7/25) + 4*sqrt(-4/5*I - 3/5)*((525*I - 50)*sq
rt(5)*cos(x)^4 - (210*I - 20)*sqrt(5)*cos(x)^2 + (21*I - 2)*sqrt(5))*ellip
tic_f(arcsin(sqrt(-4/5*I - 3/5)*(cos(x) - I*sin(x))), -24/25*I - 7/25) + 1
250*(3*cos(x)^3 - cos(x))*sqrt(5*cos(x)^2 - 1)*sin(x))/(25*cos(x)^4 - 10*c
os(x)^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(4-5\sin^2(x))^{5/2}}\,dx = \int \frac{1}{(4-5\sin^2{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)**2)**(5/2),x)
```

output
```
Integral((4 - 5*sin(x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(4-5\sin^2(x))^{5/2}}\,dx = \int \frac{1}{(-5\sin{(x)}^2+4)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-5*sin(x)^2 + 4)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(4-5\sin^2(x))^{5/2}}\,dx = \int \frac{1}{\left(-5\,\sin\left(x\right)^2+4\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-5*sin(x)^2 + 4)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4-5\sin^2(x))^{5/2}}\,dx = \int \frac{1}{\left(4-5\sin\left(x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(4 - 5*sin(x)^2)^(5/2),x)
```

output
```
int(1/(4 - 5*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(4-5\sin^2(x))^{5/2}}\,dx = -\left(\int \frac{\sqrt{-5\sin\left(x\right)^2+4}}{125\sin\left(x\right)^6-300\sin\left(x\right)^4+240\sin\left(x\right)^2-64}dx\right)$$

input
```
int(1/(4-5*sin(x)^2)^(5/2),x)
```

output
```
- int(sqrt( - 5*sin(x)**2 + 4)/(125*sin(x)**6 - 300*sin(x)**4 + 240*sin(x
)**2 - 64),x)
```

3.131 $\int \frac{1}{(4-5\sin^2(x))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(4-5\sin^2(x))^{7/2}}\,dx = -\frac{9E\left(x\middle|\frac{5}{4}\right)}{40} - \frac{\operatorname{EllipticF}\left(x,\frac{5}{4}\right)}{40} + \frac{\cos(x)\sin(x)}{4\,(4-5\sin^2(x))^{5/2}} - \frac{\cos(x)\sin(x)}{4\,(4-5\sin^2(x))^{3/2}} + \frac{9\cos(x)\sin(x)}{16\sqrt{4-5\sin^2(x)}}$$

output
```
-9/40*EllipticE(sin(x),1/2*5^(1/2))-1/40*InverseJacobiAM(x,1/2*5^(1/2))+1/
4*cos(x)*sin(x)/(4-5*sin(x)^2)^(5/2)-1/4*cos(x)*sin(x)/(4-5*sin(x)^2)^(3/2
)+9/16*cos(x)*sin(x)/(4-5*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{1}{(4-5\sin^2(x))^{7/2}}\,dx = \frac{1}{640}\left(-144E\left(x\middle|\frac{5}{4}\right) - 16\operatorname{EllipticF}\left(x,\frac{5}{4}\right) + \frac{5\sqrt{2}(517\sin(2x)+460\sin(4x)+225\sin(6x))}{(3+5\cos(2x))^{5/2}}\right)$$

input
```
Integrate[(4 - 5*Sin[x]^2)^(-7/2),x]
```

output
```
(-144*EllipticE[x, 5/4] - 16*EllipticF[x, 5/4] + (5*Sqrt[2]*(517*Sin[2*x]
+ 460*Sin[4*x] + 225*Sin[6*x]))/(3 + 5*Cos[2*x])^(5/2))/640
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3663, 27, 3042, 3652, 27, 3042, 3652, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(4-5\sin(x)^2\right)^{7/2}}dx$$
$$\Big\downarrow \text{3663}$$
$$\frac{1}{20}\int -\frac{15\sin^2(x)}{\left(4-5\sin^2(x)\right)^{5/2}}dx+\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\int \frac{\sin^2(x)}{\left(4-5\sin^2(x)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\int \frac{\sin(x)^2}{\left(4-5\sin(x)^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{3652}$$
$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}-\frac{1}{12}\int \frac{4\left(\sin^2(x)+1\right)}{\left(4-5\sin^2(x)\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}-\frac{1}{3}\int\frac{\sin^2(x)+1}{\left(4-5\sin^2(x)\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}-\frac{1}{3}\int\frac{\sin(x)^2+1}{\left(4-5\sin(x)^2\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{3652}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{4}\int\frac{8-9\sin^2(x)}{\sqrt{4-5\sin^2(x)}}dx-\frac{9\sin(x)\cos(x)}{4\sqrt{4-5\sin^2(x)}}\right)+\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{4}\int\frac{8-9\sin(x)^2}{\sqrt{4-5\sin(x)^2}}dx-\frac{9\sin(x)\cos(x)}{4\sqrt{4-5\sin^2(x)}}\right)+\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3651}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{4}\left(\frac{4}{5}\int\frac{1}{\sqrt{4-5\sin^2(x)}}dx+\frac{9}{5}\int\sqrt{4-5\sin^2(x)}dx\right)-\frac{9\sin(x)\cos(x)}{4\sqrt{4-5\sin^2(x)}}\right)+\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{4}\left(\frac{4}{5}\int\frac{1}{\sqrt{4-5\sin(x)^2}}dx+\frac{9}{5}\int\sqrt{4-5\sin(x)^2}dx\right)-\frac{9\sin(x)\cos(x)}{4\sqrt{4-5\sin^2(x)}}\right)+\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{4}\left(\frac{4}{5}\int\frac{1}{\sqrt{4-5\sin(x)^2}}dx+\frac{18E\left(x\middle|\frac{5}{4}\right)}{5}\right)-\frac{9\sin(x)\cos(x)}{4\sqrt{4-5\sin^2(x)}}\right)+\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sin(x)\cos(x)}{4\left(4-5\sin^2(x)\right)^{5/2}}-\frac{3}{4}\left(\frac{\sin(x)\cos(x)}{3\left(4-5\sin^2(x)\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{4}\left(\frac{2\,\text{EllipticF}\left(x,\frac{5}{4}\right)}{5}+\frac{18E\left(x\middle|\frac{5}{4}\right)}{5}\right)-\frac{9\sin(x)\cos(x)}{4\sqrt{4-5\sin^2(x)}}\right)\right)$$

input
```
Int[(4 - 5*Sin[x]^2)^(-7/2),x]
```

output
```
(Cos[x]*Sin[x])/(4*(4 - 5*Sin[x]^2)^(5/2)) - (3*((Cos[x]*Sin[x])/(3*(4 - 5
*Sin[x]^2)^(3/2)) + (((18*EllipticE[x, 5/4])/5 + (2*EllipticF[x, 5/4])/5)/
4 - (9*Cos[x]*Sin[x])/(4*Sqrt[4 - 5*Sin[x]^2]))/3))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(68) = 136$.

Time = 1.68 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.58

method	result
default	$\frac{\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}\left(-\frac{\sin(x)\sqrt{5\cos(x)^4-\cos(x)^2}}{500\left(\sin(x)^2-\frac{4}{5}\right)^3}-\frac{\sin(x)\sqrt{5\cos(x)^4-\cos(x)^2}}{100\left(\sin(x)^2-\frac{4}{5}\right)^2}+\frac{9\cos(x)^2\sin(x)}{16\sqrt{-\left(-4+5\sin(x)^2\right)\cos(x)^2}}-\frac{\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\sqrt{4}}{4\sqrt{}}\right.}{\cos(x)\sqrt{4-5\sin(x)^2}}$

input
```
int(1/(4-5*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)*(-1/500*sin(x)*(5*cos(x)^4-cos(x)^2)^(1/
2)/(sin(x)^2-4/5)^3-1/100*sin(x)*(5*cos(x)^4-cos(x)^2)^(1/2)/(sin(x)^2-4/5
)^2+9/16*cos(x)^2*sin(x)/(-(-4+5*sin(x)^2)*cos(x)^2)^(1/2)-1/4*(cos(x)^2)^
(1/2)*(4-5*sin(x)^2)^(1/2)/(5*cos(x)^4-cos(x)^2)^(1/2)*EllipticF(sin(x),1/
2*5^(1/2))+9/40*(cos(x)^2)^(1/2)*(4-5*sin(x)^2)^(1/2)/(5*cos(x)^4-cos(x)^2
)^(1/2)*(EllipticF(sin(x),1/2*5^(1/2))-EllipticE(sin(x),1/2*5^(1/2))))/cos
(x)/(4-5*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.34

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4-5*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/800*(9*sqrt(4/5*I - 3/5)*(-(375*I + 500)*sqrt(5)*cos(x)^6 + (225*I + 30
0)*sqrt(5)*cos(x)^4 - (45*I + 60)*sqrt(5)*cos(x)^2 + (3*I + 4)*sqrt(5))*el
liptic_e(arcsin(sqrt(4/5*I - 3/5)*(cos(x) + I*sin(x))), 24/25*I - 7/25) +
9*sqrt(-4/5*I - 3/5)*((375*I - 500)*sqrt(5)*cos(x)^6 - (225*I - 300)*sqrt(
5)*cos(x)^4 + (45*I - 60)*sqrt(5)*cos(x)^2 - (3*I - 4)*sqrt(5))*elliptic_e
(arcsin(sqrt(-4/5*I - 3/5)*(cos(x) - I*sin(x))), -24/25*I - 7/25) + 8*sqrt
(4/5*I - 3/5)*((750*I + 125)*sqrt(5)*cos(x)^6 - (450*I + 75)*sqrt(5)*cos(x
)^4 + (90*I + 15)*sqrt(5)*cos(x)^2 - (6*I + 1)*sqrt(5))*elliptic_f(arcsin(
sqrt(4/5*I - 3/5)*(cos(x) + I*sin(x))), 24/25*I - 7/25) + 8*sqrt(-4/5*I -
3/5)*(-(750*I - 125)*sqrt(5)*cos(x)^6 + (450*I - 75)*sqrt(5)*cos(x)^4 - (9
0*I - 15)*sqrt(5)*cos(x)^2 + (6*I - 1)*sqrt(5))*elliptic_f(arcsin(sqrt(-4/
5*I - 3/5)*(cos(x) - I*sin(x))), -24/25*I - 7/25) - 50*(225*cos(x)^5 - 110
*cos(x)^3 + 17*cos(x))*sqrt(5*cos(x)^2 - 1)*sin(x))/(125*cos(x)^6 - 75*cos
(x)^4 + 15*cos(x)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(4-5*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(4-5\sin^2(x))^{7/2}}\,dx = \int \frac{1}{\left(-5\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-5*sin(x)^2 + 4)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(4-5\sin^2(x))^{7/2}}\,dx = \int \frac{1}{\left(-5\,\sin\left(x\right)^2+4\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(4-5*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((-5*sin(x)^2 + 4)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4-5\sin^2(x))^{7/2}}\,dx = \int \frac{1}{\left(4-5\sin\left(x\right)^2\right)^{7/2}}\,dx$$

input
```
int(1/(4 - 5*sin(x)^2)^(7/2),x)
```

output
```
int(1/(4 - 5*sin(x)^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(4-5\sin^2(x)\right)^{7/2}}\,dx = \int \frac{\sqrt{-5\sin\left(x\right)^{2}+4}}{625\sin\left(x\right)^{8}-2000\sin\left(x\right)^{6}+2400\sin\left(x\right)^{4}-1280\sin\left(x\right)^{2}+256}dx$$

input
```
int(1/(4-5*sin(x)^2)^(7/2),x)
```

output
```
int(sqrt( - 5*sin(x)**2 + 4)/(625*sin(x)**8 - 2000*sin(x)**6 + 2400*sin(x)
**4 - 1280*sin(x)**2 + 256),x)
```

3.132 $\int \left(a+b\sin^2(x)\right)^4\,dx$

Optimal result

Integrand size = 10, antiderivative size = 140

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^4\,dx &= \frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x\\ &\quad -\frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\cos(x)\sin(x)\\ &\quad -\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\cos(x)\sin^3(x)\\ &\quad -\frac{7}{48}b(2a+b)\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2\\ &\quad -\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3\end{aligned}$$

output
```
1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*x-1/384*b*(608*a^3+
808*a^2*b+480*a*b^2+105*b^3)*cos(x)*sin(x)-1/192*b^2*(104*a^2+104*a*b+35*b
^2)*cos(x)*sin(x)^3-7/48*b*(2*a+b)*cos(x)*sin(x)*(a+b*sin(x)^2)^2-1/8*b*co
s(x)*sin(x)*(a+b*sin(x)^2)^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int \left(a+b\sin^2(x)\right)^4\,dx$$
$$=\frac{24(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\,x-96b(2a+b)\left(16a^2+16ab+7b^2\right)\sin(2x)+24b^2(24a}{3072}$$

input
```
Integrate[(a + b*Sin[x]^2)^4,x]
```

output
```
(24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x - 96*b*(2*a
 + b)*(16*a^2 + 16*a*b + 7*b^2)*Sin[2*x] + 24*b^2*(24*a^2 + 24*a*b + 7*b^2
)*Sin[4*x] - 32*b^3*(2*a + b)*Sin[6*x] + 3*b^4*Sin[8*x])/3072
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3659, 3042, 3649, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\sin(x)^2\right)^4dx$$
$$\Big\downarrow \text{3659}$$
$$\frac{1}{8}\int \left(b\sin^2(x)+a\right)^2\left(7b(2a+b)\sin^2(x)+a(8a+b)\right)dx-\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{8}\int \left(b\sin(x)^2+a\right)^2\left(7b(2a+b)\sin(x)^2+a(8a+b)\right)dx-\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

$$\Big\downarrow \text{3649}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \left(b\sin^2(x)+a\right)\left(b\left(104a^2+104ba+35b^2\right)\sin^2(x)+a\left(48a^2+20ba+7b^2\right)\right)dx-\frac{7}{6}b(2a+b)\sin(x)\cos(x)\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \left(b\sin(x)^2+a\right)\left(b\left(104a^2+104ba+35b^2\right)\sin(x)^2+a\left(48a^2+20ba+7b^2\right)\right)dx-\frac{7}{6}b(2a+b)\sin(x)\cos(x)\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(-\frac{1}{4}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{8}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\sin(x)\cos(x)+\frac{3}{8}x(\right.\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

input
```
Int[(a + b*Sin[x]^2)^4,x]
```

output
```
-1/8*(b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^3) + ((-7*b*(2*a + b)*Cos[x]*Sin[x]
*(a + b*Sin[x]^2)^2)/6 + ((3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^
3 + 35*b^4)*x)/8 - (b*(608*a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*S
in[x])/8 - (b^2*(104*a^2 + 104*a*b + 35*b^2)*Cos[x]*Sin[x]^3)/4)/6)/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 12.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result
default	$b^4\left(-\frac{\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}\right.$
parts	$b^4\left(-\frac{\left(\sin(x)^7+\frac{7\sin(x)^5}{6}+\frac{35\sin(x)^3}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}\right.$
parallelrisch	$\frac{(-32a^3b-48a^2b^2-30a\,b^3-7b^4)\sin(2x)}{32}+\frac{(24a^2b^2+24a\,b^3+7b^4)\sin(4x)}{128}+\frac{(-2a\,b^3-b^4)\sin(6x)}{96}+\frac{b^4\sin(8x)}{1024}+(a^4$
risch	$a^4x+2a^3bx+\frac{9a^2b^2x}{4}+\frac{5a\,b^3x}{4}+\frac{35b^4x}{128}+\frac{b^4\sin(8x)}{1024}-\frac{\sin(6x)a\,b^3}{48}-\frac{\sin(6x)b^4}{96}+\frac{3\sin(4x)a^2b^2}{16}+\frac{3\sin(4x}{16}$
norman	$\left(-36a^3b-\frac{153}{2}a^2b^2-\frac{383}{6}a\,b^3-\frac{2681}{192}b^4\right)\tan\left(\frac{x}{2}\right)^5+\left(-20a^3b-\frac{93}{2}a^2b^2-\frac{283}{6}a\,b^3-\frac{5053}{192}b^4\right)\tan\left(\frac{x}{2}\right)^7+\left(-20a^3b-\frac{69}{2}a^2b^2-\frac{115}{6}a\,b^3\right.$

input
```
int((a+b*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
b^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/12
8*x)+4*a*b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^
2*b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+4*a^3*b*(-1/2*cos(x)*sin(x
)+1/2*x)+a^4*x
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin^2(x)\right)^4\,dx = \frac{1}{128}\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x + \frac{1}{384}\left(48\,b^4\cos\left(x\right)^7-8\left(32\,ab^3+25\,b^4\right)\cos\left(x\right)^5+2\left(288\,a^2b^2+416\,ab^3+163\,b^4\right)\cos\left(x\right)^3-3\left(256\,a^3b\right.$$

input
```
integrate((a+b*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 1/384*(
48*b^4*cos(x)^7 - 8*(32*a*b^3 + 25*b^4)*cos(x)^5 + 2*(288*a^2*b^2 + 416*a*
b^3 + 163*b^4)*cos(x)^3 - 3*(256*a^3*b + 480*a^2*b^2 + 352*a*b^3 + 93*b^4)
*cos(x))*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(146) = 292$.

Time = 0.51 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^4\,dx = {} & a^4x+2a^3bx\sin^2\left(x\right)+2a^3bx\cos^2\left(x\right)-2a^3b\sin\left(x\right)\cos\left(x\right) \\ & +\frac{9a^2b^2x\sin^4\left(x\right)}{4}+\frac{9a^2b^2x\sin^2\left(x\right)\cos^2\left(x\right)}{2}+\frac{9a^2b^2x\cos^4\left(x\right)}{4} \\ & -\frac{15a^2b^2\sin^3\left(x\right)\cos\left(x\right)}{4}-\frac{9a^2b^2\sin\left(x\right)\cos^3\left(x\right)}{4} \\ & +\frac{5ab^3x\sin^6\left(x\right)}{4}+\frac{15ab^3x\sin^4\left(x\right)\cos^2\left(x\right)}{4} \\ & +\frac{15ab^3x\sin^2\left(x\right)\cos^4\left(x\right)}{4}+\frac{5ab^3x\cos^6\left(x\right)}{4} \\ & -\frac{11ab^3\sin^5\left(x\right)\cos\left(x\right)}{4}-\frac{10ab^3\sin^3\left(x\right)\cos^3\left(x\right)}{3} \\ & -\frac{5ab^3\sin\left(x\right)\cos^5\left(x\right)}{4}+\frac{35b^4x\sin^8\left(x\right)}{128}+\frac{35b^4x\sin^6\left(x\right)\cos^2\left(x\right)}{32} \\ & +\frac{105b^4x\sin^4\left(x\right)\cos^4\left(x\right)}{64}+\frac{35b^4x\sin^2\left(x\right)\cos^6\left(x\right)}{32} \\ & +\frac{35b^4x\cos^8\left(x\right)}{128}-\frac{93b^4\sin^7\left(x\right)\cos\left(x\right)}{128}-\frac{511b^4\sin^5\left(x\right)\cos^3\left(x\right)}{384} \\ & -\frac{385b^4\sin^3\left(x\right)\cos^5\left(x\right)}{384}-\frac{35b^4\sin\left(x\right)\cos^7\left(x\right)}{128}\end{aligned}$$

input
```
integrate((a+b*sin(x)**2)**4,x)
```

output
```
a**4*x + 2*a**3*b*x*sin(x)**2 + 2*a**3*b*x*cos(x)**2 - 2*a**3*b*sin(x)*cos
(x) + 9*a**2*b**2*x*sin(x)**4/4 + 9*a**2*b**2*x*sin(x)**2*cos(x)**2/2 + 9*
a**2*b**2*x*cos(x)**4/4 - 15*a**2*b**2*sin(x)**3*cos(x)/4 - 9*a**2*b**2*si
n(x)*cos(x)**3/4 + 5*a*b**3*x*sin(x)**6/4 + 15*a*b**3*x*sin(x)**4*cos(x)**
2/4 + 15*a*b**3*x*sin(x)**2*cos(x)**4/4 + 5*a*b**3*x*cos(x)**6/4 - 11*a*b*
*3*sin(x)**5*cos(x)/4 - 10*a*b**3*sin(x)**3*cos(x)**3/3 - 5*a*b**3*sin(x)*
cos(x)**5/4 + 35*b**4*x*sin(x)**8/128 + 35*b**4*x*sin(x)**6*cos(x)**2/32 +
 105*b**4*x*sin(x)**4*cos(x)**4/64 + 35*b**4*x*sin(x)**2*cos(x)**6/32 + 35
*b**4*x*cos(x)**8/128 - 93*b**4*sin(x)**7*cos(x)/128 - 511*b**4*sin(x)**5*
cos(x)**3/384 - 385*b**4*sin(x)**3*cos(x)**5/384 - 35*b**4*sin(x)*cos(x)**
7/128
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^4\,dx &= \frac{1}{48}\left(4\,\sin\left(2\,x\right)^3+60\,x+9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)ab^3\\&\quad+\frac{1}{3072}\left(128\,\sin\left(2\,x\right)^3+840\,x+3\,\sin\left(8\,x\right)+168\,\sin\left(4\,x\right)-768\,\sin\left(2\,x\right)\right)b^4\\&\quad+\frac{3}{16}\,a^2b^2(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))+a^3b(2\,x-\sin\left(2\,x\right))+a^4x\end{aligned}$$

input
```
integrate((a+b*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a*b^3 + 1/3072*(128*
sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*b^4 + 3/16*
a^2*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + a^3*b*(2*x - sin(2*x)) + a^4*x
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin^2(x)\right)^4\,dx=\frac{1}{1024}\,b^4\sin\left(8\,x\right)+\frac{1}{128}\,\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x-\frac{1}{96}\,\left(2\,ab^3+b^4\right)\sin\left(6\,x\right)+\frac{1}{128}\,\left(24\,a^2b^2+24\,ab^3+7\,b^4\right)\sin\left(4\,x\right)-\frac{1}{32}\,\left(32\,a^3b+48\,a^2b^2+30\,ab^3+7\,b^4\right)\sin\left(2\,x\right)$$

input
```
integrate((a+b*sin(x)^2)^4,x, algorithm="giac")
```

output
```
1/1024*b^4*sin(8*x) + 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3
 + 35*b^4)*x - 1/96*(2*a*b^3 + b^4)*sin(6*x) + 1/128*(24*a^2*b^2 + 24*a*b^
3 + 7*b^4)*sin(4*x) - 1/32*(32*a^3*b + 48*a^2*b^2 + 30*a*b^3 + 7*b^4)*sin(
2*x)
```

Mupad [B] (verification not implemented)

Time = 35.84 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \left(a+b\sin^2(x)\right)^4\,dx=x\,a^4-2\,\sin\left(x\right)\,a^3\,b\,\cos\left(x\right)+2\,x\,a^3\,b+\frac{3\,\sin\left(x\right)\,a^2\,b^2\,{\cos\left(x\right)}^3}{2}-\frac{15\,\sin\left(x\right)\,a^2\,b^2\,\cos\left(x\right)}{4}+\frac{9\,x\,a^2\,b^2}{4}-\frac{2\,\sin\left(x\right)\,a\,b^3\,{\cos\left(x\right)}^5}{3}+\frac{13\,\sin\left(x\right)\,a\,b^3\,{\cos\left(x\right)}^3}{6}-\frac{11\,\sin\left(x\right)\,a\,b^3\,\cos\left(x\right)}{4}+\frac{5\,x\,a\,b^3}{4}+\frac{\sin\left(x\right)\,b^4\,{\cos\left(x\right)}^7}{8}-\frac{25\,\sin\left(x\right)\,b^4\,{\cos\left(x\right)}^5}{48}+\frac{163\,\sin\left(x\right)\,b^4\,{\cos\left(x\right)}^3}{192}-\frac{93\,\sin\left(x\right)\,b^4\,\cos\left(x\right)}{128}+\frac{35\,x\,b^4}{128}$$

input
```
int((a + b*sin(x)^2)^4,x)
```

output
```
a^4*x + (35*b^4*x)/128 + (163*b^4*cos(x)^3*sin(x))/192 - (25*b^4*cos(x)^5*
sin(x))/48 + (b^4*cos(x)^7*sin(x))/8 + (9*a^2*b^2*x)/4 - (93*b^4*cos(x)*si
n(x))/128 + (5*a*b^3*x)/4 + 2*a^3*b*x + (3*a^2*b^2*cos(x)^3*sin(x))/2 - (1
1*a*b^3*cos(x)*sin(x))/4 - 2*a^3*b*cos(x)*sin(x) - (15*a^2*b^2*cos(x)*sin(
x))/4 + (13*a*b^3*cos(x)^3*sin(x))/6 - (2*a*b^3*cos(x)^5*sin(x))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \left(a+b\sin^2(x)\right)^4 \,dx = -\frac{\cos(x)\sin(x)^7 b^4}{8} - \frac{2\cos(x)\sin(x)^5 a b^3}{3} - \frac{7\cos(x)\sin(x)^5 b^4}{48} - \frac{3\cos(x)\sin(x)^3 a^2b^2}{2} - \frac{5\cos(x)\sin(x)^3 a b^3}{6} - \frac{35\cos(x)\sin(x)^3 b^4}{192} - 2\cos(x)\sin(x) a^3 b - \frac{9\cos(x)\sin(x) a^2b^2}{4} - \frac{5\cos(x)\sin(x) a b^3}{4} - \frac{35\cos(x)\sin(x) b^4}{128} + a^4x + 2a^3bx + \frac{9a^2b^2x}{4} + \frac{5a b^3x}{4} + \frac{35b^4x}{128}$$

input
```
int((a+b*sin(x)^2)^4,x)
```

output
```
( - 48*cos(x)*sin(x)**7*b**4 - 256*cos(x)*sin(x)**5*a*b**3 - 56*cos(x)*sin
(x)**5*b**4 - 576*cos(x)*sin(x)**3*a**2*b**2 - 320*cos(x)*sin(x)**3*a*b**3
 - 70*cos(x)*sin(x)**3*b**4 - 768*cos(x)*sin(x)*a**3*b - 864*cos(x)*sin(x)
*a**2*b**2 - 480*cos(x)*sin(x)*a*b**3 - 105*cos(x)*sin(x)*b**4 + 384*a**4*
x + 768*a**3*b*x + 864*a**2*b**2*x + 480*a*b**3*x + 105*b**4*x)/384
```

3.133 $\int (a+b\sin^2(x))^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\begin{aligned}\int (a+b\sin^2(x))^3 \, dx &= \frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x \\ &\quad - \frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x) \\ &\quad - \frac{5}{24}b^2(2a+b)\cos(x)\sin^3(x) - \frac{1}{6}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2\end{aligned}$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*x-1/48*b*(64*a^2+54*a*b+15*b^2)*cos(x)*si
n(x)-5/24*b^2*(2*a+b)*cos(x)*sin(x)^3-1/6*b*cos(x)*sin(x)*(a+b*sin(x)^2)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\begin{aligned}\int (a+b\sin^2(x))^3 \, dx &= \frac{1}{192}\big(12(2a+b)\left(8a^2+8ab+5b^2\right)x \\ &\quad + 9ib(4ia+(1+2i)b)(4a+(2+i)b)\sin(2x) \\ &\quad + 9b^2(2a+b)\sin(4x) - b^3\sin(6x)\big)\end{aligned}$$

input
```
Integrate[(a + b*Sin[x]^2)^3,x]
```

output
```
(12*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x + (9*I)*b*((4*I)*a + (1 + 2*I)*b)*
(4*a + (2 + I)*b)*Sin[2*x] + 9*b^2*(2*a + b)*Sin[4*x] - b^3*Sin[6*x])/192
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3659, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(x)^2\right)^3 dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{6}\int \left(b\sin^2(x)+a\right)\left(5b(2a+b)\sin^2(x)+a(6a+b)\right)dx-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \left(b\sin(x)^2+a\right)\left(5b(2a+b)\sin(x)^2+a(6a+b)\right)dx-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{6}\left(\frac{3}{8}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{8}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{4}b^2(2a+b)\sin^3(x)\cos(x)\right)-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

input
```
Int[(a + b*Sin[x]^2)^3,x]
```

output
```
-1/6*(b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b +
 5*b^2)*x)/8 - (b*(64*a^2 + 54*a*b + 15*b^2)*Cos[x]*Sin[x])/8 - (5*b^2*(2*
a + b)*Cos[x]*Sin[x]^3)/4)/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{3(-16a^2b-16b^2a-5b^3)\sin(2x)}{64}+\frac{3(2b^2a+b^3)\sin(4x)}{64}-\frac{b^3\sin(6x)}{192}+\left(a^2+ba+\frac{5}{8}b^2\right)x\left(a+\frac{b}{2}\right)$
default	$b^3\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3b^2a\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+3a^2b\left(-\frac{\mathrm{co}}{}\right.$
parts	$b^3\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3b^2a\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+3a^2b\left(-\frac{\mathrm{co}}{}\right.$
risch	$a^3x+\frac{3a^2bx}{2}+\frac{9a\,b^2x}{8}+\frac{5b^3x}{16}-\frac{b^3\sin(6x)}{192}+\frac{3\sin(4x)a\,b^2}{32}+\frac{3\sin(4x)b^3}{64}-\frac{3\sin(2x)a^2b}{4}-\frac{3\sin(2x)a\,b^2}{4}-15$
norman	$\frac{\left(-9a^2b-\frac{51}{4}b^2a-\frac{85}{24}b^3\right)\tan\left(\frac{x}{2}\right)^3+\left(-6a^2b-\frac{21}{2}b^2a-\frac{33}{4}b^3\right)\tan\left(\frac{x}{2}\right)^5+\left(-3a^2b-\frac{9}{4}b^2a-\frac{5}{8}b^3\right)\tan\left(\frac{x}{2}\right)+\left(3a^2b+\frac{9}{4}b^2a+\frac{5}{8}b^3\right)\tan(}{}$
orering	Expression too large to display

input
```
int((a+b*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*(-16*a^2*b-16*a*b^2-5*b^3)*sin(2*x)+3/64*(2*a*b^2+b^3)*sin(4*x)-1/192
*b^3*sin(6*x)+(a^2+b*a+5/8*b^2)*x*(a+1/2*b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \left(a+b\sin^2(x)\right)^3\,dx=\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x$$
$$-\frac{1}{48}\left(8\,b^3\cos\left(x\right)^5-2\left(18\,ab^2+13\,b^3\right)\cos\left(x\right)^3+3\left(24\,a^2b+30\,ab^2+11\,b^3\right)\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate((a+b*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x - 1/48*(8*b^3*cos(x)^5 - 2*(
18*a*b^2 + 13*b^3)*cos(x)^3 + 3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cos(x))*sin
(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.83

$$\int \left(a + b\sin^2(x)\right)^3 \, dx = a^3x + \frac{3a^2bx\sin^2(x)}{2} + \frac{3a^2bx\cos^2(x)}{2} - \frac{3a^2b\sin(x)\cos(x)}{2} + \frac{9ab^2x\sin^4(x)}{8} + \frac{9ab^2x\sin^2(x)\cos^2(x)}{4} + \frac{9ab^2x\cos^4(x)}{8} - \frac{15ab^2\sin^3(x)\cos(x)}{8} - \frac{9ab^2\sin(x)\cos^3(x)}{8} + \frac{5b^3x\sin^6(x)}{16} + \frac{15b^3x\sin^4(x)\cos^2(x)}{16} + \frac{15b^3x\sin^2(x)\cos^4(x)}{16} + \frac{5b^3x\cos^6(x)}{16} - \frac{11b^3\sin^5(x)\cos(x)}{16} - \frac{5b^3\sin^3(x)\cos^3(x)}{6} - \frac{5b^3\sin(x)\cos^5(x)}{16}$$

input
```
integrate((a+b*sin(x)**2)**3,x)
```

output
```
a**3*x + 3*a**2*b*x*sin(x)**2/2 + 3*a**2*b*x*cos(x)**2/2 - 3*a**2*b*sin(x)
*cos(x)/2 + 9*a*b**2*x*sin(x)**4/8 + 9*a*b**2*x*sin(x)**2*cos(x)**2/4 + 9*
a*b**2*x*cos(x)**4/8 - 15*a*b**2*sin(x)**3*cos(x)/8 - 9*a*b**2*sin(x)*cos(
x)**3/8 + 5*b**3*x*sin(x)**6/16 + 15*b**3*x*sin(x)**4*cos(x)**2/16 + 15*b*
*3*x*sin(x)**2*cos(x)**4/16 + 5*b**3*x*cos(x)**6/16 - 11*b**3*sin(x)**5*co
s(x)/16 - 5*b**3*sin(x)**3*cos(x)**3/6 - 5*b**3*sin(x)*cos(x)**5/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \left(a + b\sin^2(x)\right)^3 \, dx = \frac{1}{192} \left(4 \sin(2x)^3 + 60x + 9 \sin(4x) - 48 \sin(2x)\right)b^3 + \frac{3}{32} ab^2(12x + \sin(4x) - 8 \sin(2x)) + \frac{3}{4} a^2b(2x - \sin(2x)) + a^3x$$

input
```
integrate((a+b*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*b^3 + 3/32*a*b^2*(1
2*x + sin(4*x) - 8*sin(2*x)) + 3/4*a^2*b*(2*x - sin(2*x)) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \left(a+b\sin^2(x)\right)^3\,dx = -\frac{1}{192}\,b^3\sin(6\,x)+\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x+\frac{3}{64}\left(2\,ab^2+b^3\right)\sin(4\,x)-\frac{3}{64}\left(16\,a^2b+16\,ab^2+5\,b^3\right)\sin(2\,x)$$

input
```
integrate((a+b*sin(x)^2)^3,x, algorithm="giac")
```

output
```
-1/192*b^3*sin(6*x) + 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 3/64
*(2*a*b^2 + b^3)*sin(4*x) - 3/64*(16*a^2*b + 16*a*b^2 + 5*b^3)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 30.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \left(a+b\sin^2(x)\right)^3\,dx = a^3\,x+\frac{5\,b^3\,x}{16}-\frac{\left(72\,a^2\,b+90\,a\,b^2+33\,b^3\right)\,\tan(x)^5+\left(144\,a^2\,b+144\,a\,b^2+40\,b^3\right)\,\tan(x)^3+\left(72\,a^2\,b+54\,a\,b^2+15\,b^3\right)}{48\tan(x)^6+144\tan(x)^4+144\tan(x)^2+48}+\frac{9\,a\,b^2\,x}{8}+\frac{3\,a^2\,b\,x}{2}$$

input
```
int((a + b*sin(x)^2)^3,x)
```

output
```
a^3*x + (5*b^3*x)/16 - (tan(x)^5*(90*a*b^2 + 72*a^2*b + 33*b^3) + tan(x)^3
*(144*a*b^2 + 144*a^2*b + 40*b^3) + tan(x)*(54*a*b^2 + 72*a^2*b + 15*b^3))
/(144*tan(x)^2 + 144*tan(x)^4 + 48*tan(x)^6 + 48) + (9*a*b^2*x)/8 + (3*a^2
*b*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \left(a+b\sin^2(x)\right)^3 \, dx = -\frac{\cos(x)\sin(x)^5 b^3}{6} - \frac{3\cos(x)\sin(x)^3 a\, b^2}{4} - \frac{5\cos(x)\sin(x)^3 b^3}{24} - \frac{3\cos(x)\sin(x)\, a^2 b}{2} - \frac{9\cos(x)\sin(x)\, a\, b^2}{8} - \frac{5\cos(x)\sin(x)\, b^3}{16} + a^3 x + \frac{3a^2 b x}{2} + \frac{9a\, b^2 x}{8} + \frac{5b^3 x}{16}$$

input
```
int((a+b*sin(x)^2)^3,x)
```

output
```
( - 8*cos(x)*sin(x)**5*b**3 - 36*cos(x)*sin(x)**3*a*b**2 - 10*cos(x)*sin(x
)**3*b**3 - 72*cos(x)*sin(x)*a**2*b - 54*cos(x)*sin(x)*a*b**2 - 15*cos(x)*
sin(x)*b**3 + 48*a**3*x + 72*a**2*b*x + 54*a*b**2*x + 15*b**3*x)/48
```

3.134 $\int (a+b\sin^2(x))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (a+b\sin^2(x))^2\,dx=\frac{1}{8}(8a^2+8ab+3b^2)\,x-\frac{1}{8}b(8a+3b)\cos(x)\sin(x)-\frac{1}{4}b^2\cos(x)\sin^3(x)$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(x)*sin(x)-1/4*b^2*cos(x)*sin
(x)^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int (a+b\sin^2(x))^2\,dx=\frac{1}{32}(4(8a^2+8ab+3b^2)\,x-8b(2a+b)\sin(2x)+b^2\sin(4x))$$

input
```
Integrate[(a + b*Sin[x]^2)^2,x]
```

output
```
(4*(8*a^2 + 8*a*b + 3*b^2)*x - 8*b*(2*a + b)*Sin[2*x] + b^2*Sin[4*x])/32
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(x)^2\right)^2 dx$$

$$\Big\downarrow \text{3658}$$

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

input
```
Int[(a + b*Sin[x]^2)^2,x]
```

output
```
((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 - (b^2*Cos
[x]*Sin[x]^3)/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{(-2ba-b^2)\sin(2x)}{4}+\frac{b^2\sin(4x)}{32}+\left(a^2+ba+\frac{3}{8}b^2\right)x$
default	$b^2\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+2ba\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
parts	$b^2\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+2ba\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
risch	$a^2x+abx+\frac{3b^2x}{8}+\frac{b^2\sin(4x)}{32}-\frac{ab\sin(2x)}{2}-\frac{b^2\sin(2x)}{4}$
orering	$x\left(a+b\sin(x)^2\right)^2-\left(a+b\sin(x)^2\right)b\cos(x)\sin(x)+\frac{5x\left(8b^2\cos(x)^2\sin(x)^2-4\left(a+b\sin(x)^2\right)b\sin(x)^2\right.}{16}$
norman	$\frac{\left(-2ba-\frac{11}{4}b^2\right)\tan\left(\frac{x}{2}\right)^3+\left(-2ba-\frac{3}{4}b^2\right)\tan\left(\frac{x}{2}\right)+\left(2ba+\frac{3}{4}b^2\right)\tan\left(\frac{x}{2}\right)^7+\left(2ba+\frac{11}{4}b^2\right)\tan\left(\frac{x}{2}\right)^5+\left(a^2+ba+\frac{3}{8}b^2\right)x+\left(a^2+ba+\frac{3}{8}b^2\right.}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$

input
```
int((a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*a*b-b^2)*sin(2*x)+1/32*b^2*sin(4*x)+(a^2+b*a+3/8*b^2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \left(a+b\sin^2(x)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x+\frac{1}{8}\left(2b^2\cos(x)^3-\left(8ab+5b^2\right)\cos(x)\right)\sin(x)$$

input
```
integrate((a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/8*(2*b^2*cos(x)^3 - (8*a*b + 5*b^2)*cos(
x))*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(44) = 88.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \left(a+b\sin^2(x)\right)^2\,dx = a^2x + abx\sin^2(x) + abx\cos^2(x) - ab\sin(x)\cos(x) + \frac{3b^2x\sin^4(x)}{8} + \frac{3b^2x\sin^2(x)\cos^2(x)}{4} + \frac{3b^2x\cos^4(x)}{8} - \frac{5b^2\sin^3(x)\cos(x)}{8} - \frac{3b^2\sin(x)\cos^3(x)}{8}$$

input
```
integrate((a+b*sin(x)**2)**2,x)
```

output
```
a**2*x + a*b*x*sin(x)**2 + a*b*x*cos(x)**2 - a*b*sin(x)*cos(x) + 3*b**2*x*
sin(x)**4/8 + 3*b**2*x*sin(x)**2*cos(x)**2/4 + 3*b**2*x*cos(x)**4/8 - 5*b*
*2*sin(x)**3*cos(x)/8 - 3*b**2*sin(x)*cos(x)**3/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a+b\sin^2(x)\right)^2\,dx = \frac{1}{32}b^2(12\,x+\sin(4\,x)-8\,\sin(2\,x)) + \frac{1}{2}ab(2\,x-\sin(2\,x)) + a^2x$$

input
```
integrate((a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/32*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + 1/2*a*b*(2*x - sin(2*x)) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin^2(x)\right)^2\,dx=\frac{1}{32}\,b^2\sin\left(4\,x\right)+\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x-\frac{1}{4}\left(2\,ab+b^2\right)\sin\left(2\,x\right)$$

input
```
integrate((a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/32*b^2*sin(4*x) + 1/8*(8*a^2 + 8*a*b + 3*b^2)*x - 1/4*(2*a*b + b^2)*sin(
2*x)
```

Mupad [B] (verification not implemented)

Time = 36.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin^2(x)\right)^2\,dx=x\,a^2-\sin\left(x\right)\,a\,b\,\cos\left(x\right)+x\,a\,b+\frac{\sin\left(x\right)\,b^2\cos\left(x\right)^3}{4}-\frac{5\,\sin\left(x\right)\,b^2\,\cos\left(x\right)}{8}+\frac{3\,x\,b^2}{8}$$

input
```
int((a + b*sin(x)^2)^2,x)
```

output
```
a^2*x + (3*b^2*x)/8 + (b^2*cos(x)^3*sin(x))/4 + a*b*x - (5*b^2*cos(x)*sin(
x))/8 - a*b*cos(x)*sin(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin^2(x)\right)^2\,dx=-\frac{\cos\left(x\right)\sin\left(x\right)^3b^2}{4}-\cos\left(x\right)\sin\left(x\right)ab-\frac{3\cos\left(x\right)\sin\left(x\right)b^2}{8}+a^2x+abx+\frac{3b^2x}{8}$$

input
```
int((a+b*sin(x)^2)^2,x)
```

output

```
( - 2*cos(x)*sin(x)**3*b**2 - 8*cos(x)*sin(x)*a*b - 3*cos(x)*sin(x)*b**2 +
 8*a**2*x + 8*a*b*x + 3*b**2*x)/8
```

3.135 $\int (a+b\sin^2(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int (a+b\sin^2(x))\,dx = ax+\frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)$$

output
```
a*x+1/2*b*x-1/2*b*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+b\sin^2(x))\,dx = ax+\frac{bx}{2}-\frac{1}{4}b\sin(2x)$$

input
```
Integrate[a + b*Sin[x]^2,x]
```

output
```
a*x + (b*x)/2 - (b*Sin[2*x])/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

input
```
Int[a + b*Sin[x]^2,x]
```

output
```
a*x + (b*x)/2 - (b*Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
risch	$ax+\frac{bx}{2}-\frac{b\sin(2x)}{4}$	16
default	$ax+b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	17
parallelrisch	$b\left(-\frac{\sin(2x)}{4}+\frac{x}{2}\right)+ax$	17
parts	$ax+b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	17
orering	$x\left(a+b\sin\left(x\right)^{2}\right)-\frac{b\cos(x)\sin(x)}{2}+\frac{x\left(-2b\sin(x)^{2}+2b\cos(x)^{2}\right)}{4}$	37
norman	$\frac{b\tan\left(\frac{x}{2}\right)^{3}+\left(a+\frac{b}{2}\right)x+\left(a+\frac{b}{2}\right)x\tan\left(\frac{x}{2}\right)^{4}+(2a+b)x\tan\left(\frac{x}{2}\right)^{2}-b\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	61

input

```
int(a+b*sin(x)^2,x,method=_RETURNVERBOSE)
```

output

```
a*x+1/2*b*x-1/4*b*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin^2(x)\right)\,dx=-\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,(2\,a+b)x$$

input

```
integrate(a+b*sin(x)^2,x, algorithm="fricas")
```

output

```
-1/2*b*cos(x)*sin(x) + 1/2*(2*a + b)*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sin^2(x)\right)\,dx = ax + b\left(\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}\right)$$

input
```
integrate(a+b*sin(x)**2,x)
```

output
```
a*x + b*(x/2 - sin(x)*cos(x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a+b\sin^2(x)\right)\,dx = \frac{1}{4}b(2x - \sin(2x)) + ax$$

input
```
integrate(a+b*sin(x)^2,x, algorithm="maxima")
```

output
```
1/4*b*(2*x - sin(2*x)) + a*x
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a+b\sin^2(x)\right)\,dx = \frac{1}{4}b(2x - \sin(2x)) + ax$$

input
```
integrate(a+b*sin(x)^2,x, algorithm="giac")
```

output
```
1/4*b*(2*x - sin(2*x)) + a*x
```

Mupad [B] (verification not implemented)

Time = 36.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sin^2(x)\right)\,dx=x\left(a+\frac{b}{2}\right)-\frac{b\,\sin\left(2\,x\right)}{4}$$

input
```
int(a + b*sin(x)^2,x)
```

output
```
x*(a + b/2) - (b*sin(2*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sin^2(x)\right)\,dx=-\frac{\cos\left(x\right)\sin\left(x\right)b}{2}+ax+\frac{bx}{2}$$

input
```
int(a+b*sin(x)^2,x)
```

output
```
( - cos(x)*sin(x)*b + 2*a*x + b*x)/2
```

3.136 $\int \frac{1}{a+b\sin^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{\sqrt{a(a+b)}}$	23
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ba}-b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}$	160

input
```
int(1/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.55

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2+2ab+}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right.}{4(a^2+ab)}\right.$$
$$\left.-\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*
a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 -
a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 +
a^2 + 2*a*b + b^2))/(a^2 + a*b), -1/2*arctan(1/2*((2*a + b)*cos(x)^2 - a -
 b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15745 vs. $2(27) = 54$.

Time = 10.54 (sec) , antiderivative size = 15745, normalized size of antiderivative = 542.93

$$\int \frac{1}{a + b\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)**2),x)
```

output
```
Piecewise((zoo*(tan(x/2)/2 - 1/(2*tan(x/2))), Eq(a, 0) & Eq(b, 0)), ((tan(
x/2)/2 - 1/(2*tan(x/2)))/b, Eq(a, 0)), (2*tan(x/2)/(b*tan(x/2)**2 - b), Eq
(a, -b)), (x/a, Eq(b, 0)), (6*a**3*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/
a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(x/2))/(10*a**4*b*sqr
t(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) - 2*a**4*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(
-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b**2*sqrt(-1 - 2*b/a - 2*sqrt
(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*b*sqrt(a
*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqr
t(a*b + b**2)/a) + 72*a**2*b**3*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sq
rt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a**2*b**2*sqrt(a*b + b**2)*sqrt
(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a
) + 32*a*b**4*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*
sqrt(a*b + b**2)/a) - 32*a*b**3*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(
a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)) - 6*a**3*b*sqrt(-1
 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) + tan(x/2))/(10*a**4*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1
- 2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a
- 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*
b**2*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan((a + b)*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a
^2 + a*b)))/sqrt(a^2 + a*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+b\sin^2(x)}\,dx=\frac{\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*sin(x)^2),x)
```

output
```
atan((tan(x)*(2*a + 2*b))/(2*(a*b + a^2)^(1/2)))/(a*b + a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 427, normalized size of antiderivative = 14.72

$$\int \frac{1}{a+b\sin^2(x)}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{b}\,\sqrt{a+b}\,\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a}{\sqrt{a}\,\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}}\right)+2\sqrt{2\sqrt{b}\,\sqrt{a+b}+a+2b}\,atan\left(\frac{}{\sqrt{}}\right.\right.}{}$$

input
```
int(1/(a+b*sin(x)^2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(b)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*a
tan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b))) + 2*sqrt
(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b
)*sqrt(a + b) + a + 2*b)))*a + 2*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*ata
n((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*b - sqrt(b
)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log( - sqrt(2*sqrt(b)*
sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2)) + sqrt(b)*sqrt(a + b)*sqrt(2*sq
rt(b)*sqrt(a + b) - a - 2*b)*log(sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + s
qrt(a)*tan(x/2)) - sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log( - sqrt(2*sqr
t(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2))*a - sqrt(2*sqrt(b)*sqrt(a
+ b) - a - 2*b)*log( - sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan
(x/2))*b + sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b)*log(sqrt(2*sqrt(b)*sqrt(a
 + b) - a - 2*b) + sqrt(a)*tan(x/2))*a + sqrt(2*sqrt(b)*sqrt(a + b) - a -
2*b)*log(sqrt(2*sqrt(b)*sqrt(a + b) - a - 2*b) + sqrt(a)*tan(x/2))*b))/(2*
a**2*(a + b))
```

3.137 $\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx=\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

output
```
1/2*(2*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(3/2)+1/2*b*c
os(x)*sin(x)/a/(a+b)/(a+b*sin(x)^2)
```

Mathematica [A] (verified)

Time = 5.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx=\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(2x)}{2a(a+b)(-2a-b+b\cos(2x))}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-2),x]
```

output
```
((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2))
 - (b*Sin[2*x])/(2*a*(a + b)*(-2*a - b + b*Cos[2*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3663, 25, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}-\frac{\int -\frac{2a+b}{b\sin^2(x)+a}dx}{2a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a+b}{b\sin^2(x)+a}dx}{2a(a+b)}+\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2a+b)\int \frac{1}{b\sin^2(x)+a}dx}{2a(a+b)}+\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2a+b)\int \frac{1}{b\sin(x)^2+a}dx}{2a(a+b)}+\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{(2a+b)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{2a(a+b)}+\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

input
```
Int[(a + b*Sin[x]^2)^(-2),x]
```

output
```
((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2))
 + (b*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

method	result
default	$\frac{b\tan(x)}{2a(a+b)\left(\tan(x)^2a+b\tan(x)^2+a\right)}+\frac{(2a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}$
risch	$-\frac{i(2\,\mathrm{e}^{2ix}a+\mathrm{e}^{2ix}b-b)}{a(a+b)(-\mathrm{e}^{4ix}b+4\,\mathrm{e}^{2ix}a+2\,\mathrm{e}^{2ix}b-b)}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}\,(a+b)}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{}}{b\sqrt{-a^2-ba}}\right.}{4\sqrt{-a^2-ba}\,(a+b)a}$

input
```
int(1/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b/a/(a+b)*tan(x)/(tan(x)^2*a+b*tan(x)^2+a)+1/2*(2*a+b)/a/(a+b)/(a*(a+b
))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(53) = 106.

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 5.89

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx$$
$$=\left[\frac{4\left(a^2b+ab^2\right)\cos(x)\sin(x)+\left(\left(2\,ab+b^2\right)\cos(x)^2-2\,a^2-3\,ab-b^2\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)}{}\right.}{8\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3-\left(a^4b+2\,a^3b^2+\right.\right.}\right.$$

input
```
integrate(1/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2*b + a*b^2)*cos(x)*sin(x) + ((2*a*b + b^2)*cos(x)^2 - 2*a^2 -
3*a*b - b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a
^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(
-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(
x)^2 + a^2 + 2*a*b + b^2)))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^4*b
+ 2*a^3*b^2 + a^2*b^3)*cos(x)^2), 1/4*(2*(a^2*b + a*b^2)*cos(x)*sin(x) + (
(2*a*b + b^2)*cos(x)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(a^2 + a*b)*arctan(1/2*(
(2*a + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^5 + 3*a^4
*b + 3*a^3*b^2 + a^2*b^3 - (a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(x)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{b\tan(x)}{2\left(a^3+a^2b+\left(a^3+2a^2b+ab^2\right)\tan(x)^2\right)} + \frac{(2a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\left(a^2+ab\right)}$$

input
```
integrate(1/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*tan(x)/(a^3 + a^2*b + (a^3 + 2*a^2*b + a*b^2)*tan(x)^2) + 1/2*(2*a +
 b)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + a*b))
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(2a+b)}{2\left(a^2+ab\right)^{\frac{3}{2}}} + \frac{b\tan(x)}{2\left(a\tan(x)^2+b\tan(x)^2+a\right)\left(a^2+ab\right)}$$

input
```
integrate(1/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sq
rt(a^2 + a*b)))*(2*a + b)/(a^2 + a*b)^(3/2) + 1/2*b*tan(x)/((a*tan(x)^2 +
b*tan(x)^2 + a)*(a^2 + a*b))
```

Mupad [B] (verification not implemented)

Time = 37.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(x)\,(2a+2b)}{2\sqrt{a}\,\sqrt{a+b}}\right)\,(2a+b)}{2a^{3/2}\,(a+b)^{3/2}} + \frac{b\tan(x)}{2a\,(a+b)\,\left((a+b)\,\tan(x)^2+a\right)}$$

input
```
int(1/(a + b*sin(x)^2)^2,x)
```

output
```
(atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))*(2*a + b))/(2*a^(3/2
)*(a + b)^(3/2)) + (b*tan(x))/(2*a*(a + b)*(a + tan(x)^2*(a + b)))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1666, normalized size of antiderivative = 25.63

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sin(x)^2)^2,x)
```

output
```
( - 4*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*at
an((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**2
*a*b - 2*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)
*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)
**2*b**2 - 4*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a +
2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*a*
*2 - 2*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*a
tan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*a*b + 4*
sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*s
qrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**2*a**2*b + 6*sqrt(a)*sqrt(2
*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*
sqrt(a + b) + a + 2*b)))*sin(x)**2*a*b**2 + 2*sqrt(a)*sqrt(2*sqrt(b)*sqrt(
a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) +
a + 2*b)))*sin(x)**2*b**3 + 4*sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b
)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*a**3
+ 6*sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(
a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*a**2*b + 2*sqrt(a)*sqrt(2*sqrt(
b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a
 + b) + a + 2*b)))*a*b**2 + 2*cos(x)*sin(x)*a**3*b + 2*cos(x)*sin(x)*a**2*
b**2 - 2*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) - a - 2...
```

3.138 $\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 107

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}} + \frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2} + \frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\sin^2(x)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(5/2)/(a+b)^(
5/2)+1/4*b*cos(x)*sin(x)/a/(a+b)/(a+b*sin(x)^2)^2+3/8*b*(2*a+b)*cos(x)*sin
(x)/a^2/(a+b)^2/(a+b*sin(x)^2)
```

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{a}b(16a^2+16ab+3b^2-3b(2a+b)\cos(2x))\sin(2x)}{(a+b)^2(2a+b-b\cos(2x))^2}}{8a^{5/2}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-3),x]
```

output
```
(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a + b)^(5
/2) + (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 - 3*b*(2*a + b)*Cos[2*x])*Sin[2*
x])/((a + b)^2*(2*a + b - b*Cos[2*x])^2))/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(x)^2)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}-\frac{\int -\frac{-2b\sin^2(x)+4a+3b}{\left(b\sin^2(x)+a\right)^2}dx}{4a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2b\sin^2(x)+4a+3b}{\left(b\sin^2(x)+a\right)^2}dx}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-2b\sin(x)^2+4a+3b}{\left(b\sin(x)^2+a\right)^2}dx}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{8a^2+8ba+3b^2}{b\sin^2(x)+a}dx}{2a(a+b)}+\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin^2(x)+a}dx}{2a(a+b)}+\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin(x)^2+a}dx}{2a(a+b)}+\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{2a(a+b)}+\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}$$

input
```
Int[(a + b*Sin[x]^2)^(-3),x]
```

output
```
(b*Cos[x]*Sin[x])/(4*a*(a + b)*(a + b*Sin[x]^2)^2) + (((8*a^2 + 8*a*b + 3*
b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)) + (3*
b*(2*a + b)*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2)))/(4*a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

method	result
default	$\frac{\frac{(8a+3b)b\tan(x)^3}{8a^2(a+b)}+\frac{b(8a+5b)\tan(x)}{8a\left(a^2+2ba+b^2\right)}}{\left(\tan(x)^2a+b\tan(x)^2+a\right)^2}+\frac{(8a^2+8ba+3b^2)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{8a^2(a^2+2ba+b^2)\sqrt{a(a+b)}}$
risch	$-\frac{i(-8a^2b\,\mathrm{e}^{6ix}-8a\,b^2\mathrm{e}^{6ix}-3b^3\mathrm{e}^{6ix}+48a^3\mathrm{e}^{4ix}+72a^2b\,\mathrm{e}^{4ix}+42a\,b^2\mathrm{e}^{4ix}+9b^3\mathrm{e}^{4ix}-40a^2b\,\mathrm{e}^{2ix}-40a\,b^2\mathrm{e}^{2ix}-9b^3\mathrm{e}^{2ix}+6b^2a+3b^3)}{4a^2(a+b)^2(-\mathrm{e}^{4ix}b+4\,\mathrm{e}^{2ix}a+2\,\mathrm{e}^{2ix}b-b)^2}-$

input
```
int(1/(a+b*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(8*a+3*b)/a^2*b/(a+b)*tan(x)^3+1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tan(
x))/(tan(x)^2*a+b*tan(x)^2+a)^2+1/8*(8*a^2+8*a*b+3*b^2)/a^2/(a^2+2*a*b+b^2
)/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(93) = 186$.

Time = 0.12 (sec) , antiderivative size = 737, normalized size of antiderivative = 6.89

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(x)^4 + 8*a^4 + 24*a^3*b + 27*a^
2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b + 16*a^2*b^2 + 11*a*b^3 + 3*b^4)*cos
(x)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5
*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 -
 a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 +
 a^2 + 2*a*b + b^2)) + 4*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 - (8*
a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(x))*sin(x))/(a^8 + 5*a^7*b
+ 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + (a^6*b^2 + 3*a^5*b^3 + 3
*a^4*b^4 + a^3*b^5)*cos(x)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^
4 + a^3*b^5)*cos(x)^2), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(x)^4 + 8
*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b + 16*a^2*b^2
+ 11*a*b^3 + 3*b^4)*cos(x)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)
^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*cos(x)^3 - (8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(x))
*sin(x))/(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 +
(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 - 2*(a^7*b + 4*a^6*b^
2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*cos(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(x)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(93) = 186$.

Time $= 0.12$ (sec) , antiderivative size $= 188$, normalized size of antiderivative $= 1.76$

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \frac{(8\,a^2+8\,ab+3\,b^2)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{8\,(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)a}} + \frac{(8\,a^2b+11\,ab^2+3\,b^3)\tan(x)^3+(8\,a^2b+5\,ab^2)\tan(x)}{8\,(a^6+2\,a^5b+a^4b^2+(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\tan(x)^4+2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\tan(x)^2)}$$

input
```
integrate(1/(a+b*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/((a^4 +
 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)) + 1/8*((8*a^2*b + 11*a*b^2 + 3*b^3)*t
an(x)^3 + (8*a^2*b + 5*a*b^2)*tan(x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6 + 4*
a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*tan(x)^4 + 2*(a^6 + 3*a^5*b + 3*a
^4*b^2 + a^3*b^3)*tan(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(8a^2+8ab+3b^2)}{8(a^4+2a^3b+a^2b^2)\sqrt{a^2+ab}} + \frac{8a^2b\tan(x)^3+11ab^2\tan(x)^3+3b^3\tan(x)^3+8a^2b\tan(x)+5ab^2\tan(x)}{8(a^4+2a^3b+a^2b^2)\left(a\tan(x)^2+b\tan(x)^2+a\right)^2}$$

input
```
integrate(1/(a+b*sin(x)^2)^3,x, algorithm="giac")
```

output
```
1/8*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sq
rt(a^2 + a*b)))*(8*a^2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a^
2 + a*b)) + 1/8*(8*a^2*b*tan(x)^3 + 11*a*b^2*tan(x)^3 + 3*b^3*tan(x)^3 + 8
*a^2*b*tan(x) + 5*a*b^2*tan(x))/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(x)^2 + b
*tan(x)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 37.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^3}\,dx = \frac{\frac{\tan(x)\,(5b^2+8ab)}{8a\,(a^2+2ab+b^2)}+\frac{\tan(x)^3\,(3b^2+8ab)}{8a^2\,(a+b)}}{\tan(x)^2\,(2a^2+2ba)+\tan(x)^4\,(a^2+2ab+b^2)+a^2} + \frac{\operatorname{atan}\left(\frac{\tan(x)\,(2a+2b)\,(a^2+2ab+b^2)}{2\sqrt{a}\,(a+b)^{5/2}}\right)\,(8a^2+8ab+3b^2)}{8a^{5/2}\,(a+b)^{5/2}}$$

input
```
int(1/(a + b*sin(x)^2)^3,x)
```

output
```
((tan(x)*(8*a*b + 5*b^2))/(8*a*(2*a*b + a^2 + b^2)) + (tan(x)^3*(8*a*b + 3
*b^2))/(8*a^2*(a + b)))/(tan(x)^2*(2*a*b + 2*a^2) + tan(x)^4*(2*a*b + a^2
+ b^2) + a^2) + (atan((tan(x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*
(a + b)^(5/2)))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 3695, normalized size of antiderivative = 34.53

$$\int \frac{1}{(a+b\sin^2(x))^3} dx = \text{Too large to display}$$

input

```
int(1/(a+b*sin(x)^2)^3,x)
```

output

```
( - 16*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*a
tan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**
4*a**2*b**2 - 16*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) +
a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b))
)*sin(x)**4*a*b**3 - 6*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a +
 b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a +
 2*b)))*sin(x)**4*b**4 - 32*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqr
t(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b)
+ a + 2*b)))*sin(x)**2*a**3*b - 32*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt
(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(
a + b) + a + 2*b)))*sin(x)**2*a**2*b**2 - 12*sqrt(b)*sqrt(a)*sqrt(a + b)*s
qrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqr
t(b)*sqrt(a + b) + a + 2*b)))*sin(x)**2*a*b**3 - 16*sqrt(b)*sqrt(a)*sqrt(a
 + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqr
t(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*a**4 - 16*sqrt(b)*sqrt(a)*sqrt(a + b)
*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*s
qrt(b)*sqrt(a + b) + a + 2*b)))*a**3*b - 6*sqrt(b)*sqrt(a)*sqrt(a + b)*sqr
t(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(
b)*sqrt(a + b) + a + 2*b)))*a**2*b**2 + 16*sqrt(a)*sqrt(2*sqrt(b)*sqrt(a +
 b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + ...
```

3.139 $\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 154

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx = \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}} + \frac{b\cos(x)\sin(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3} + \frac{5b(2a+b)\cos(x)\sin(x)}{24a^2(a+b)^2\left(a+b\sin^2(x)\right)^2} + \frac{b\left(44a^2+44ab+15b^2\right)\cos(x)\sin(x)}{48a^3(a+b)^3\left(a+b\sin^2(x)\right)}$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(7/2
)/(a+b)^(7/2)+1/6*b*cos(x)*sin(x)/a/(a+b)/(a+b*sin(x)^2)^3+5/24*b*(2*a+b)*
cos(x)*sin(x)/a^2/(a+b)^2/(a+b*sin(x)^2)^2+1/48*b*(44*a^2+44*a*b+15*b^2)*c
os(x)*sin(x)/a^3/(a+b)^3/(a+b*sin(x)^2)
```

Mathematica [A] (verified)

Time = 6.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx$$
$$=\frac{\frac{3(16a^3+24a^2b+18ab^2+5b^3)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{(a+b)^{7/2}}+\frac{32a^{5/2}b\sin(2x)}{(a+b)(2a+b-b\cos(2x))^3}+\frac{20a^{3/2}b(2a+b)\sin(2x)}{(a+b)^2(2a+b-b\cos(2x))^2}+\frac{\sqrt{a}b(44a^2+44ab+15b^2)\sin}{(a+b)^3(2a+b-b\cos(2x)}}{48a^{7/2}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-4),x]
```

output
```
((3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqr
t[a]])/(a + b)^(7/2) + (32*a^(5/2)*b*Sin[2*x])/((a + b)*(2*a + b - b*Cos[2
*x])^3) + (20*a^(3/2)*b*(2*a + b)*Sin[2*x])/((a + b)^2*(2*a + b - b*Cos[2*
x])^2) + (Sqrt[a]*b*(44*a^2 + 44*a*b + 15*b^2)*Sin[2*x])/((a + b)^3*(2*a +
 b - b*Cos[2*x])))/(48*a^(7/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a+b\sin(x)^2\right)^4}dx$$
$$\Big\downarrow \text{3663}$$
$$\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}-\frac{\int -\frac{-4b\sin^2(x)+6a+5b}{\left(b\sin^2(x)+a\right)^3}dx}{6a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-4b\sin^2(x)+6a+5b}{\left(b\sin^2(x)+a\right)^3}dx}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-4b\sin(x)^2+6a+5b}{\left(b\sin(x)^2+a\right)^3}dx}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{24a^2+34ba+15b^2-10b(2a+b)\sin^2(x)}{\left(b\sin^2(x)+a\right)^2}dx}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{24a^2+34ba+15b^2-10b(2a+b)\sin(x)^2}{\left(b\sin(x)^2+a\right)^2}dx}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\frac{\int \frac{3(2a+b)\left(8a^2+8ba+5b^2\right)}{b\sin^2(x)+a}dx}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{b\sin^2(x)+a}dx}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{b\sin(x)^2+a}dx}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}}{4a(a+b)}+\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)\left(a+b\sin^2(x)\right)^2}}{6a(a+b)}+\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\sin^2(x)\right)^3}$$

input
```
Int[(a + b*Sin[x]^2)^(-4),x]
```

output
```
(b*Cos[x]*Sin[x])/(6*a*(a + b)*(a + b*Sin[x]^2)^3) + ((5*b*(2*a + b)*Cos[x
]*Sin[x])/(4*a*(a + b)*(a + b*Sin[x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b
+ 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)) +
 (b*(44*a^2 + 44*a*b + 15*b^2)*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2
)))/(4*a*(a + b)))/(6*a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.31

method	result
default	$\frac{\frac{\left(24a^2+18ba+5b^2\right)b\tan(x)^5}{16a^3(a+b)}+\frac{\left(18a^2+18ba+5b^2\right)b\tan(x)^3}{6a^2\left(a^2+2ba+b^2\right)}+\frac{b\left(24a^2+30ba+11b^2\right)\tan(x)}{16a\left(a^3+3a^2b+3b^2a+b^3\right)}}{\left(\tan(x)^2a+b\tan(x)^2+a\right)^3}+\frac{(16a^3+24a^2b+18b^2a+5b^3)\arctan\left(\frac{(a+b)\tan}{\sqrt{a(a+b}}\right.}{16a^3(a^3+3a^2b+3b^2a+b^3)\sqrt{a(a+b)}}$
risch	$-\frac{i(-44a^2b^3-44a\,b^4-15b^5+4176a^3b^2e^{6ix}+2744a^2b^3e^{6ix}-3264a^3b^2e^{4ix}-2592a^2b^3e^{4ix}+980a\,b^4e^{6ix}-1632a^4b\,e^{4ix}-960a\,b^4e^{4ix}}{}$

input
```
int(1/(a+b*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(24*a^2+18*a*b+5*b^2)/a^3*b/(a+b)*tan(x)^5+1/6*(18*a^2+18*a*b+5*b^2)
/a^2*b/(a^2+2*a*b+b^2)*tan(x)^3+1/16*b*(24*a^2+30*a*b+11*b^2)/a/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(x))/(tan(x)^2*a+b*tan(x)^2+a)^3+1/16*(16*a^3+24*a^2*b+
18*a*b^2+5*b^3)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arctan((a+b)
*tan(x)/(a*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(138) = 276$.

Time = 0.14 (sec) , antiderivative size = 1229, normalized size of antiderivative = 7.98

$$\int \frac{1}{\left(a + b\sin^2(x)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
[1/192*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(x)^6 - 16*a^6
- 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*b^6 - 3
*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(x)^4 + 3*(1
6*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^4 + 28*a*b^5 + 5*b^6)*cos(x)^
2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b
 + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b
)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2
 + 2*a*b + b^2)) + 4*((44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*co
s(x)^5 - 2*(54*a^5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6
)*cos(x)^3 + 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*
b^5 + 5*a*b^6)*cos(x))*sin(x))/(a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3
+ 35*a^7*b^4 + 21*a^6*b^5 + 7*a^5*b^6 + a^4*b^7 - (a^8*b^3 + 4*a^7*b^4 + 6
*a^6*b^5 + 4*a^5*b^6 + a^4*b^7)*cos(x)^6 + 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7
*b^4 + 10*a^6*b^5 + 5*a^5*b^6 + a^4*b^7)*cos(x)^4 - 3*(a^10*b + 6*a^9*b^2
+ 15*a^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*cos(x)^2), 1
/96*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(x)^6 - 16*a^6 - 7
2*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*b^6 - 3*(1
6*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(x)^4 + 3*(16*a
^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^4 + 28*a*b^5 + 5*b^6)*cos(x)^2)*
sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sin(x)**2)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(138) = 276.

Time = 0.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx = \frac{\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{16\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{(a+b)a}} + \frac{3\left(24\,a^4b+66\,a^3b^2+65\,a^2b^3+28\,ab^4+5\,b^5\right)\tan(x)^5+8\left(18\,a^4b+3\right.}{48\left(a^9+3\,a^8b+3\,a^7b^2+a^6b^3+\left(a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6\right)\tan(x)^6+3\right.}$$

input
```
integrate(1/(a+b*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*arctan((a + b)*tan(x)/sqrt((a
+ b)*a))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*a)) + 1/48*(3
*(24*a^4*b + 66*a^3*b^2 + 65*a^2*b^3 + 28*a*b^4 + 5*b^5)*tan(x)^5 + 8*(18*
a^4*b + 36*a^3*b^2 + 23*a^2*b^3 + 5*a*b^4)*tan(x)^3 + 3*(24*a^4*b + 30*a^3
*b^2 + 11*a^2*b^3)*tan(x))/(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3 + (a^9 + 6
*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*tan(x
)^6 + 3*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*ta
n(x)^4 + 3*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*tan(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx = \frac{\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{16\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{a^2+ab}} + \frac{72\,a^4b\tan(x)^5+198\,a^3b^2\tan(x)^5+195\,a^2b^3\tan(x)^5+84\,ab^4\tan(x)^5+15\,b^5\tan(x)^5+144\,a^4b\tan\left(\right.}{48\left(a^6+3\,a^5b+3\,a^4b^2+\right.}$$

input
```
integrate(1/(a+b*sin(x)^2)^4,x, algorithm="giac")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*(pi*floor(x/pi + 1/2)*sgn(2*a
+ 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/((a^6 + 3*a^5*b +
3*a^4*b^2 + a^3*b^3)*sqrt(a^2 + a*b)) + 1/48*(72*a^4*b*tan(x)^5 + 198*a^3*
b^2*tan(x)^5 + 195*a^2*b^3*tan(x)^5 + 84*a*b^4*tan(x)^5 + 15*b^5*tan(x)^5
+ 144*a^4*b*tan(x)^3 + 288*a^3*b^2*tan(x)^3 + 184*a^2*b^3*tan(x)^3 + 40*a*
b^4*tan(x)^3 + 72*a^4*b*tan(x) + 90*a^3*b^2*tan(x) + 33*a^2*b^3*tan(x))/((
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*tan(x)^2 + b*tan(x)^2 + a)^3)
```

Mupad [B] (verification not implemented)

Time = 36.43 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.98

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx = \frac{\frac{\tan(x)^3\,\left(18\,a^2\,b+18\,a\,b^2+5\,b^3\right)}{6\,a^2\,\left(a^2+2\,a\,b+b^2\right)}+\frac{\tan(x)^5\,\left(24\,a^2\,b+18\,a\,b^2+5\,b^3\right)}{16\,a^3\,\left(a+b\right)}+\frac{\tan(x)\,\left(24\,a^2\,b+30\,a\,b^2+11\,b^3\right)}{16\,a\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}}{\tan\left(x\right)^4\,\left(3\,a^3+6\,a^2\,b+3\,a\,b^2\right)+\tan\left(x\right)^6\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)+\tan\left(x\right)^2\,\left(3\,a^3+3\,b\,a^2\right)+a^3} + \frac{\mathrm{atan}\left(\frac{\tan(x)\,\left(2\,a+b\right)\,\left(2\,a+2\,b\right)\,\left(8\,a^2+8\,a\,b+5\,b^2\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,\sqrt{a}\,{\left(a+b\right)}^{7/2}\,\left(16\,a^3+24\,a^2\,b+18\,a\,b^2+5\,b^3\right)}\right)\,\left(2\,a+b\right)\,\left(8\,a^2+8\,a\,b+5\,b^2\right)}{16\,a^{7/2}\,{\left(a+b\right)}^{7/2}}$$

input
```
int(1/(a + b*sin(x)^2)^4,x)
```

output
```
((tan(x)^3*(18*a*b^2 + 18*a^2*b + 5*b^3))/(6*a^2*(2*a*b + a^2 + b^2)) + (t
an(x)^5*(18*a*b^2 + 24*a^2*b + 5*b^3))/(16*a^3*(a + b)) + (tan(x)*(30*a*b^
2 + 24*a^2*b + 11*b^3))/(16*a*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))/(tan(x)^4*
(3*a*b^2 + 6*a^2*b + 3*a^3) + tan(x)^6*(3*a*b^2 + 3*a^2*b + a^3 + b^3) + t
an(x)^2*(3*a^2*b + 3*a^3) + a^3) + (atan((tan(x)*(2*a + b)*(2*a + 2*b)*(8*
a*b + 8*a^2 + 5*b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(
7/2)*(18*a*b^2 + 24*a^2*b + 16*a^3 + 5*b^3)))*(2*a + b)*(8*a*b + 8*a^2 + 5
*b^2))/(16*a^(7/2)*(a + b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 6476, normalized size of antiderivative = 42.05

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^4}\,dx = \text{Too large to display}$$

input

```
int(1/(a+b*sin(x)^2)^4,x)
```

output

```
( - 96*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*a
tan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**
6*a**3*b**3 - 144*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) +
 a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)
))*sin(x)**6*a**2*b**4 - 108*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sq
rt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b)
 + a + 2*b)))*sin(x)**6*a*b**5 - 30*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqr
t(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt
(a + b) + a + 2*b)))*sin(x)**6*b**6 - 288*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt
(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b
)*sqrt(a + b) + a + 2*b)))*sin(x)**4*a**4*b**2 - 432*sqrt(b)*sqrt(a)*sqrt(
a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/(sqrt(a)*sq
rt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**4*a**3*b**3 - 324*sqrt(b)*sq
rt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((tan(x/2)*a)/
(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**4*a**2*b**4 - 90*
sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*atan((ta
n(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)**4*a*b**
5 - 288*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)*
atan((tan(x/2)*a)/(sqrt(a)*sqrt(2*sqrt(b)*sqrt(a + b) + a + 2*b)))*sin(x)*
*2*a**5*b - 432*sqrt(b)*sqrt(a)*sqrt(a + b)*sqrt(2*sqrt(b)*sqrt(a + b) ...
```

3.140 $\int \left(a+b\sin^2(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 152

$$\int \left(a+b\sin^2(x)\right)^{5/2}\,dx = -\frac{4}{15}b(2a+b)\cos(x)\sin(x)\sqrt{a+b\sin^2(x)}-\frac{1}{5}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^{3/2} + \frac{(23a^2+23ab+8b^2)\,E\left(x\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(x)}}{15\sqrt{\frac{a+b\sin^2(x)}{a}}} - \frac{4a(a+b)(2a+b)\,\text{EllipticF}\left(x,-\frac{b}{a}\right)\sqrt{\frac{a+b\sin^2(x)}{a}}}{15\sqrt{a+b\sin^2(x)}}$$

output
```
-4/15*b*(2*a+b)*cos(x)*sin(x)*(a+b*sin(x)^2)^(1/2)-1/5*b*cos(x)*sin(x)*(a+
b*sin(x)^2)^(3/2)+1/15*(23*a^2+23*a*b+8*b^2)*EllipticE(sin(x),(-b/a)^(1/2)
)*(a+b*sin(x)^2)^(1/2)/((a+b*sin(x)^2)/a)^(1/2)-4/15*a*(a+b)*(2*a+b)*Inver
seJacobiAM(x,(-b/a)^(1/2))*((a+b*sin(x)^2)/a)^(1/2)/(a+b*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int (a+b\sin^2(x))^{5/2}\,dx = \frac{16a(23a^2+23ab+8b^2)\sqrt{\frac{2a+b-b\cos(2x)}{a}}E\left(x\middle|-\frac{b}{a}\right)-64a(2a^2+3ab+b^2)\sqrt{\frac{2a+b-b\cos(2x)}{a}}}{240\sqrt{2a+}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(5/2),x]
```

output
```
(16*a*(23*a^2 + 23*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*x])/a]*EllipticE[x
, -(b/a)] - 64*a*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b - b*Cos[2*x])/a]*Elli
pticF[x, -(b/a)] - Sqrt[2]*b*(88*a^2 + 88*a*b + 25*b^2 - 28*b*(2*a + b)*Co
s[2*x] + 3*b^2*Cos[4*x])*Sin[2*x])/(240*Sqrt[2*a + b - b*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3659, 3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin^2(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(x)^2)^{5/2}\,dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{5}\int \sqrt{b\sin^2(x)+a}(4b(2a+b)\sin^2(x)+a(5a+b))\,dx-\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{b\sin(x)^2+a}\left(4b(2a+b)\sin(x)^2+a(5a+b)\right)dx-\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3649

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(23a^2+23ba+8b^2\right)\sin^2(x)+a\left(15a^2+11ba+4b^2\right)}{\sqrt{b\sin^2(x)+a}}dx-\frac{4}{3}b(2a+b)\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}\right)-\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(23a^2+23ba+8b^2\right)\sin(x)^2+a\left(15a^2+11ba+4b^2\right)}{\sqrt{b\sin(x)^2+a}}dx-\frac{4}{3}b(2a+b)\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}\right)-\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3651

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+23ab+8b^2\right)\int \sqrt{b\sin^2(x)+a}dx-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin^2(x)+a}}dx\right)-\frac{4}{3}b(2a+b)\sin(x)\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+23ab+8b^2\right)\int \sqrt{b\sin(x)^2+a}dx-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{4}{3}b(2a+b)\sin(x)\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3657

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin^2(x)}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{4}{3}b(\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(x)}\int\sqrt{\frac{b\sin(x)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-4a(a+b)(2a+b)\int\frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{4}{3}b(\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3656

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-4a(a+b)(2a+b)\int\frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{4}{3}b(2a+b)\mathrm{s}\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3662

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(x)}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}\right)-\frac{4}{3}b(\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(x)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}\right)-\frac{4}{3}b(\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

$\big\downarrow$ 3661

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\,\text{EllipticF}\left(x,-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(x)}}\right)-\frac{4}{3}\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^{3/2}$$

input
```
Int[(a + b*Sin[x]^2)^(5/2),x]
```

output
```
-1/5*(b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^(3/2)) + ((-4*b*(2*a + b)*Cos[x]*Si
n[x]*Sqrt[a + b*Sin[x]^2])/3 + (((23*a^2 + 23*a*b + 8*b^2)*EllipticE[x, -(
b/a)]*Sqrt[a + b*Sin[x]^2])/Sqrt[1 + (b*Sin[x]^2)/a] - (4*a*(a + b)*(2*a +
 b)*EllipticF[x, -(b/a)]*Sqrt[1 + (b*Sin[x]^2)/a])/Sqrt[a + b*Sin[x]^2])/3
)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(137) = 274$.

Time = 4.93 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.17

method	result
default	$\frac{-\frac{b^{3}\cos(x)^{6}\sin(x)}{5}+\frac{\left(14b^{2}a+10b^{3}\right)\cos(x)^{4}\sin(x)}{15}+\frac{\left(-11a^{2}b-18b^{2}a-7b^{3}\right)\cos(x)^{2}\sin(x)}{15}-\frac{8a^{3}\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\cos(x)^{2}}{a}+\frac{a+b}{a}}\,\operatorname{EllipticF}\left(\sin\right.}{15}}{}$

input
```
int((a+b*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/5*b^3*cos(x)^6*sin(x)+1/15*(14*a*b^2+10*b^3)*cos(x)^4*sin(x)+1/15*(-11
*a^2*b-18*a*b^2-7*b^3)*cos(x)^2*sin(x)-8/15*a^3*(cos(x)^2)^(1/2)*(-b/a*cos
(x)^2+(a+b)/a)^(1/2)*EllipticF(sin(x),(-b/a)^(1/2))-4/5*a^2*(cos(x)^2)^(1/
2)*(-b/a*cos(x)^2+(a+b)/a)^(1/2)*EllipticF(sin(x),(-b/a)^(1/2))*b-4/15*a*(
cos(x)^2)^(1/2)*(-b/a*cos(x)^2+(a+b)/a)^(1/2)*EllipticF(sin(x),(-b/a)^(1/2
))*b^2+23/15*(cos(x)^2)^(1/2)*(-b/a*cos(x)^2+(a+b)/a)^(1/2)*EllipticE(sin(
x),(-b/a)^(1/2))*a^3+23/15*(cos(x)^2)^(1/2)*(-b/a*cos(x)^2+(a+b)/a)^(1/2)*
EllipticE(sin(x),(-b/a)^(1/2))*a^2*b+8/15*(cos(x)^2)^(1/2)*(-b/a*cos(x)^2+
(a+b)/a)^(1/2)*EllipticE(sin(x),(-b/a)^(1/2))*a*b^2)/cos(x)/(a+b*sin(x)^2)
^(1/2)
```

Fricas [F]

$$\int \left(a+b\sin^2(x)\right)^{5/2}\,dx=\int \left(b\sin\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)*sqrt(
-b*cos(x)^2 + a + b), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sin^2(x)\right)^{5/2} dx = \int \left(b\sin{(x)}^2+a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(x)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \left(a+b\sin^2(x)\right)^{5/2} dx = \int \left(b\sin{(x)}^2+a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sin(x)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(x)\right)^{5/2} dx = \int \left(b\sin{(x)}^2+a\right)^{5/2} dx$$

input
```
int((a + b*sin(x)^2)^(5/2),x)
```

output
```
int((a + b*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\sin^2(x)\right)^{5/2}\,dx = \left(\int \sqrt{\sin(x)^2 b+a}dx\right)a^2 + \left(\int \sqrt{\sin(x)^2 b+a}\,\sin(x)^4\,dx\right)b^2 + 2\left(\int \sqrt{\sin(x)^2 b+a}\,\sin(x)^2\,dx\right)ab$$

input

```
int((a+b*sin(x)^2)^(5/2),x)
```

output

```
int(sqrt(sin(x)**2*b + a),x)*a**2 + int(sqrt(sin(x)**2*b + a)*sin(x)**4,x)
*b**2 + 2*int(sqrt(sin(x)**2*b + a)*sin(x)**2,x)*a*b
```

3.141 $\int \left(a+b\sin^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 111

$$\int \left(a+b\sin^2(x)\right)^{3/2}\,dx=-\frac{1}{3}b\cos(x)\sin(x)\sqrt{a+b\sin^2(x)}+\frac{2(2a+b)E\left(x\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(x)}}{3\sqrt{\frac{a+b\sin^2(x)}{a}}}-\frac{a(a+b)\operatorname{EllipticF}\left(x,-\frac{b}{a}\right)\sqrt{\frac{a+b\sin^2(x)}{a}}}{3\sqrt{a+b\sin^2(x)}}$$

output

```
-1/3*b*cos(x)*sin(x)*(a+b*sin(x)^2)^(1/2)+2/3*(2*a+b)*EllipticE(sin(x),(-b
/a)^(1/2))*(a+b*sin(x)^2)^(1/2)/((a+b*sin(x)^2)/a)^(1/2)-1/3*a*(a+b)*Inver
seJacobiAM(x,(-b/a)^(1/2))*((a+b*sin(x)^2)/a)^(1/2)/(a+b*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

$$\int \left(a+b\sin^2(x)\right)^{3/2}\,dx=\frac{8a(2a+b)\sqrt{\frac{2a+b-b\cos(2x)}{a}}E\left(x\middle|-\frac{b}{a}\right)-4a(a+b)\sqrt{\frac{2a+b-b\cos(2x)}{a}}\operatorname{EllipticF}\left(x,-\frac{b}{a}\right)+\sqrt{2}b\cdots}{12\sqrt{2a+b-b\cos(2x)}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(3/2),x]
```

output
```
(8*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*x])/a]*EllipticE[x, -(b/a)] - 4*a*(
a + b)*Sqrt[(2*a + b - b*Cos[2*x])/a]*EllipticF[x, -(b/a)] + Sqrt[2]*b*(-2
*a - b + b*Cos[2*x])*Sin[2*x])/(12*Sqrt[2*a + b - b*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin^2(x)+a(3a+b)}{\sqrt{b\sin^2(x)+a}}dx-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin(x)^2+a(3a+b)}{\sqrt{b\sin(x)^2+a}}dx-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin^2(x)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(x)+a}}dx\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin(x)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$\big\downarrow$ 3657

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin^2(x)}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin(x)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$\big\downarrow$ 3656

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\left|-\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$\big\downarrow$ 3662

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\left|-\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(x)}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\left|-\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(x)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

$\big\downarrow$ 3661

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\,\text{EllipticF}\left(x,-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(x)}}\right)-\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\sin^2(x)}$$

input
```
Int[(a + b*Sin[x]^2)^(3/2),x]
```

output
```
-1/3*(b*Cos[x]*Sin[x]*Sqrt[a + b*Sin[x]^2]) + ((2*(2*a + b)*EllipticE[x, -
(b/a)]*Sqrt[a + b*Sin[x]^2])/Sqrt[1 + (b*Sin[x]^2)/a] - (a*(a + b)*Ellipti
cF[x, -(b/a)]*Sqrt[1 + (b*Sin[x]^2)/a])/Sqrt[a + b*Sin[x]^2])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.72

method	result
default	$\frac{-\frac{a^{2}\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\sin(x)^{2}}{a}}\,\operatorname{EllipticF}\left(\sin(x),\sqrt{-\frac{b}{a}}\right)}{3}-\frac{a\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\sin(x)^{2}}{a}}\,\operatorname{EllipticF}\left(\sin(x),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\sin}{a}}}{}}{\cos(x)\sqrt{a+}}$

input
```
int((a+b*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a^2*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*EllipticF(sin(x),(-b/a
)^(1/2))-1/3*a*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*EllipticF(sin(x),
(-b/a)^(1/2))*b+4/3*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*EllipticE(si
n(x),(-b/a)^(1/2))*a^2+2/3*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*Ellip
ticE(sin(x),(-b/a)^(1/2))*a*b+1/3*b^2*sin(x)^5+1/3*a*b*sin(x)^3-1/3*b^2*si
n(x)^3-1/3*a*b*sin(x))/cos(x)/(a+b*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(a+b\sin^2(x)\right)^{3/2} dx = \int \left(b\sin\left(x\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((-b*cos(x)^2 + a + b)^(3/2), x)
```

Sympy [F]

$$\int \left(a+b\sin^2(x)\right)^{3/2} dx = \int \left(a+b\sin^2\left(x\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(x)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sin^2(x)\right)^{3/2} dx = \int \left(b\sin\left(x\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(x)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sin^2(x)\right)^{3/2} dx = \int \left(b\sin\left(x\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(x)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(x)\right)^{3/2} dx = \int \left(b\sin\left(x\right)^2+a\right)^{3/2} dx$$

input
```
int((a + b*sin(x)^2)^(3/2),x)
```

output
```
int((a + b*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sin^2(x)\right)^{3/2} dx = \left(\int \sqrt{\sin\left(x\right)^2 b+a}dx\right)a+\left(\int \sqrt{\sin\left(x\right)^2 b+a}\,\sin\left(x\right)^2 dx\right)b$$

input
```
int((a+b*sin(x)^2)^(3/2),x)
```

output
```
int(sqrt(sin(x)**2*b + a),x)*a + int(sqrt(sin(x)**2*b + a)*sin(x)**2,x)*b
```

3.142 $\int \sqrt{a+b\sin^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \sqrt{a+b\sin^2(x)}\,dx=\frac{E\left(x\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(x)}}{\sqrt{\frac{a+b\sin^2(x)}{a}}}$$

output
```
EllipticE(sin(x),(-b/a)^(1/2))*(a+b*sin(x)^2)^(1/2)/((a+b*sin(x)^2)/a)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \sqrt{a+b\sin^2(x)}\,dx=\frac{a\sqrt{\frac{2a+b-b\cos(2x)}{a}}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{2a+b-b\cos(2x)}}$$

input
```
Integrate[Sqrt[a + b*Sin[x]^2],x]
```

output
```
(a*Sqrt[(2*a + b - b*Cos[2*x])/a]*EllipticE[x, -(b/a)])/Sqrt[2*a + b - b*C
os[2*x]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3657}$$

$$\frac{\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin^2(x)}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin(x)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}$$

$$\Big\downarrow \text{ 3656}$$

$$\frac{\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sin[x]^2],x]
```

output
```
(EllipticE[x, -(b/a)]*Sqrt[a + b*Sin[x]^2])/Sqrt[1 + (b*Sin[x]^2)/a]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\sin(x)^2}{a}}\,\operatorname{EllipticE}\left(\sin(x),\sqrt{-\frac{b}{a}}\right)}{\cos(x)\sqrt{a+b\sin(x)^2}}$	48

input
```
int((a+b*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*EllipticE(sin(x),(-b/a)^(1/2))
/cos(x)/(a+b*sin(x)^2)^(1/2)
```

Fricas [F]

$$\int \sqrt{a+b\sin^2(x)}\,dx = \int \sqrt{b\sin{(x)}^2+a}\,dx$$

input
```
integrate((a+b*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(x)^2 + a + b), x)
```

Sympy [A] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.27

$$\int \sqrt{a+b\sin^2(x)}\,dx=\sqrt{a}E\left(x\middle|-\frac{b}{a}\right)$$

input
```
integrate((a+b*sin(x)**2)**(1/2),x)
```

output
```
sqrt(a)*elliptic_e(x, -b/a)
```

Maxima [F]

$$\int \sqrt{a+b\sin^2(x)}\,dx=\int \sqrt{b\sin{(x)}^2+a}\,dx$$

input
```
integrate((a+b*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(x)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sin^2(x)}\,dx=\int \sqrt{b\sin{(x)}^2+a}\,dx$$

input
```
integrate((a+b*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(x)}\,dx = \begin{cases} \sqrt{a}\,\mathrm{E}\left(x\middle|-\frac{b}{a}\right) & \text{if } 0<a \\ \int \sqrt{b\sin(x)^2+a}\,dx & \text{if } \neg 0<a \end{cases}$$

input
```
int((a + b*sin(x)^2)^(1/2),x)
```

output
```
piecewise(0 < a, a^(1/2)*ellipticE(x, -b/a), ~0 < a, int((a + b*sin(x)^2)^
(1/2), x))
```

Reduce [F]

$$\int \sqrt{a+b\sin^2(x)}\,dx = \int \sqrt{\sin(x)^2 b+a}dx$$

input
```
int((a+b*sin(x)^2)^(1/2),x)
```

output
```
int(sqrt(sin(x)**2*b + a),x)
```

3.143 $\int \frac{1}{\sqrt{a+b\sin^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{1}{\sqrt{a+b\sin^2(x)}}\,dx=\frac{\text{EllipticF}\left(x,-\frac{b}{a}\right)\sqrt{\frac{a+b\sin^2(x)}{a}}}{\sqrt{a+b\sin^2(x)}}$$

output
```
InverseJacobiAM(x,(-b/a)^(1/2))*((a+b*sin(x)^2)/a)^(1/2)/(a+b*sin(x)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{a+b\sin^2(x)}}\,dx=\frac{\sqrt{\frac{2a+b-b\cos(2x)}{a}}\,\text{EllipticF}\left(x,-\frac{b}{a}\right)}{\sqrt{2a+b-b\cos(2x)}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[x]^2],x]
```

output
```
(Sqrt[(2*a + b - b*Cos[2*x])/a]*EllipticF[x, -(b/a)])/Sqrt[2*a + b - b*Cos
[2*x]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(x)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\sin^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(x)}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\sin^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(x)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\frac{b\sin^2(x)}{a}+1}\,\text{EllipticF}\left(x,-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(x)}}$$

input
```
Int[1/Sqrt[a + b*Sin[x]^2],x]
```

output
```
(EllipticF[x, -(b/a)]*Sqrt[1 + (b*Sin[x]^2)/a])/Sqrt[a + b*Sin[x]^2]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{\frac{a+b\sin(x)^2}{a}}\ \mathrm{InverseJacobiAM}\left(x, \frac{i\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a+b\sin(x)^2}}$	37

input
```
int(1/(a+b*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b*sin(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*InverseJacobiAM(x,I/a^(1/2
)*b^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 286, normalized size of antiderivative = 7.73

$$\int \frac{1}{\sqrt{a+b\sin^2(x)}}\,dx = -\frac{\left(2i\sqrt{-b}b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}(\cos(x)+i\,\sin(x\right.\right.}{}$$

input
```
integrate(1/(a+b*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqrt((a^2 +
 a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*
b)/b^2))/b^2))/b^2
```

Sympy [A] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.27

$$\int \frac{1}{\sqrt{a+b\sin^2(x)}}\,dx = \frac{F\left(x\middle|-\frac{b}{a}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b*sin(x)**2)**(1/2),x)
```

output
```
elliptic_f(x, -b/a)/sqrt(a)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{b\sin{(x)}^2 + a}}\,dx$$

input
```
integrate(1/(a+b*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(x)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{b\sin{(x)}^2 + a}}\,dx$$

input
```
integrate(1/(a+b*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sin(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{b\sin{(x)}^2 + a}}\,dx$$

input
```
int(1/(a + b*sin(x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sin(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(x)}}\,dx = \int \frac{\sqrt{\sin(x)^2 b + a}}{\sin(x)^2 b + a}dx$$

input
```
int(1/(a+b*sin(x)^2)^(1/2),x)
```

output
```
int(sqrt(sin(x)**2*b + a)/(sin(x)**2*b + a),x)
```

3.144 $\int \frac{1}{\left(a+b\sin^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{3/2}}\,dx = \frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\sin^2(x)}} + \frac{E\left(x\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(x)}}{a(a+b)\sqrt{\frac{a+b\sin^2(x)}{a}}}$$

output
```
b*cos(x)*sin(x)/a/(a+b)/(a+b*sin(x)^2)^(1/2)+EllipticE(sin(x),(-b/a)^(1/2)
)*(a+b*sin(x)^2)^(1/2)/a/(a+b)/((a+b*sin(x)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{3/2}}\,dx = \frac{2a\sqrt{\frac{2a+b-b\cos(2x)}{a}}E\left(x\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2x)}{2a(a+b)\sqrt{2a+b-b\cos(2x)}}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-3/2),x]
```

output
```
(2*a*Sqrt[(2*a + b - b*Cos[2*x])/a]*EllipticE[x, -(b/a)] + Sqrt[2]*b*Sin[2
*x])/(2*a*(a + b)*Sqrt[2*a + b - b*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}-\frac{\int -\sqrt{b\sin^2(x)+a}dx}{a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\sin^2(x)+a}dx}{a(a+b)}+\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin(x)^2+a}dx}{a(a+b)}+\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin^2(x)}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}}+\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin(x)^2}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}}+\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}+\frac{\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}}$$

input
```
Int[(a + b*Sin[x]^2)^(-3/2),x]
```

output
```
(b*Cos[x]*Sin[x])/(a*(a + b)*Sqrt[a + b*Sin[x]^2]) + (EllipticE[x, -(b/a)]
*Sqrt[a + b*Sin[x]^2])/(a*(a + b)*Sqrt[1 + (b*Sin[x]^2)/a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\cos(x)^2}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\sin(x),\sqrt{-\frac{b}{a}}\right)+b\cos(x)^2\sin(x)}{a(a+b)\cos(x)\sqrt{a+b\sin(x)^2}}$	72

input
```
int(1/(a+b*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((cos(x)^2)^(1/2)*(-b/a*cos(x)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(x),(-b/a)^
(1/2))+b*cos(x)^2*sin(x))/a/(a+b)/cos(x)/(a+b*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 854, normalized size of antiderivative = 11.86

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(-b*cos(x)^2 + a + b)*b^3*cos(x)*sin(x) - (2*(I*b^3*cos(x)^2 -
I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (2*I*a^2*b + 3*I*a*b^2 +
 I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(x)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*s
qrt((a^2 + a*b)/b^2))/b^2) - (2*(-I*b^3*cos(x)^2 + I*a*b^2 + I*b^3)*sqrt(-
b)*sqrt((a^2 + a*b)/b^2) - (-2*I*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 +
I*b^3)*cos(x)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*e
lliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(x) - I
*sin(x))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b
^2) + 2*(2*(-I*a^2*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(x)^2)*sqr
t(-b)*sqrt((a^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b
 - I*a*b^2)*cos(x)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(x
) + I*sin(x))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^
2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2 - I*b^3)*cos(x)^2
)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*a^2*b - I*a*b^2 + (2*I*
a^2*b + I*a*b^2)*cos(x)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 +...
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\sin^{2}{\left(x \right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sin^2(x))^{3/2}}\,dx = \int \frac{1}{(b\sin(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(x)^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sin^2(x))^{3/2}}\,dx = \int \frac{1}{(b\sin(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(x)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin^2(x))^{3/2}}\,dx = \int \frac{1}{(b\sin(x)^2+a)^{3/2}}\,dx$$

input
```
int(1/(a + b*sin(x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sin(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sin\left(x\right)^2 b+a}}{\sin\left(x\right)^4 b^2+2\sin\left(x\right)^2 ab+a^2}dx$$

input
```
int(1/(a+b*sin(x)^2)^(3/2),x)
```

output
```
int(sqrt(sin(x)**2*b + a)/(sin(x)**4*b**2 + 2*sin(x)**2*a*b + a**2),x)
```

3.145 $\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 165

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx = \frac{b\cos(x)\sin(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}+\frac{2b(2a+b)\cos(x)\sin(x)}{3a^2(a+b)^2\sqrt{a+b\sin^2(x)}}+\frac{2(2a+b)E\left(x\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(x)}}{3a^2(a+b)^2\sqrt{\frac{a+b\sin^2(x)}{a}}}-\frac{\text{EllipticF}\left(x,-\frac{b}{a}\right)\sqrt{\frac{a+b\sin^2(x)}{a}}}{3a(a+b)\sqrt{a+b\sin^2(x)}}$$

output
```
1/3*b*cos(x)*sin(x)/a/(a+b)/(a+b*sin(x)^2)^(3/2)+2/3*b*(2*a+b)*cos(x)*sin(
x)/a^2/(a+b)^2/(a+b*sin(x)^2)^(1/2)+2/3*(2*a+b)*EllipticE(sin(x),(-b/a)^(1
/2))*(a+b*sin(x)^2)^(1/2)/a^2/(a+b)^2/((a+b*sin(x)^2)/a)^(1/2)-1/3*Inverse
JacobiAM(x,(-b/a)^(1/2))*((a+b*sin(x)^2)/a)^(1/2)/a/(a+b)/(a+b*sin(x)^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx = \frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2x)}{a}\right)^{3/2} E\left(x\middle|-\frac{b}{a}\right) - a^2(a+b)\left(\frac{2a+b-b\cos(2x)}{a}\right)^{3/2} \text{EllipticF}}{3a^2(a+b)^2(2a+b-b\cos}$$

input
```
Integrate[(a + b*Sin[x]^2)^(-5/2),x]
```

output
```
(2*a^2*(2*a + b)*((2*a + b - b*Cos[2*x])/a)^(3/2)*EllipticE[x, -(b/a)] - a
^2*(a + b)*((2*a + b - b*Cos[2*x])/a)^(3/2)*EllipticF[x, -(b/a)] - Sqrt[2]
*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*x])*Sin[2*x])/(3*a^2*(a + b)^
2*(2*a + b - b*Cos[2*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\sin(x)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3663}$$

$$\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}-\frac{\int -\frac{-b\sin^2(x)+3a+2b}{\left(b\sin^2(x)+a\right)^{3/2}}dx}{3a(a+b)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-b\sin^2(x)+3a+2b}{\left(b\sin^2(x)+a\right)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{-b\sin(x)^2+3a+2b}{\left(b\sin(x)^2+a\right)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3652

$$\frac{\frac{\int \frac{2b(2a+b)\sin^2(x)+a(3a+b)}{\sqrt{b\sin^2(x)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{2b(2a+b)\sin(x)^2+a(3a+b)}{\sqrt{b\sin(x)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3651

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin^2(x)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(x)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin(x)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3657

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(x)}\int \sqrt{\frac{b\sin^2(x)}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(x)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(x)}\int\sqrt{\frac{b\sin(x)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(x)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3656$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(x)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3662$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(x)}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(x)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(x)}}}{a(a+b)}+\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}}{3a(a+b)}+\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3661$$

$$\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\sin^2(x)\right)^{3/2}}+\frac{\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\sin^2(x)}}+\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(x)}E\left(x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(x)}{a}+1}\,\text{EllipticF}\left(x,-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(x)}}}{a(a+b)}}{3a(a+b)}$$

input
```
Int[(a + b*Sin[x]^2)^(-5/2),x]
```

output
```
(b*Cos[x]*Sin[x])/(3*a*(a + b)*(a + b*Sin[x]^2)^(3/2)) + ((2*b*(2*a + b)*C
os[x]*Sin[x])/(a*(a + b)*Sqrt[a + b*Sin[x]^2]) + ((2*(2*a + b)*EllipticE[x
, -(b/a)]*Sqrt[a + b*Sin[x]^2])/Sqrt[1 + (b*Sin[x]^2)/a] - (a*(a + b)*Elli
pticF[x, -(b/a)]*Sqrt[1 + (b*Sin[x]^2)/a])/Sqrt[a + b*Sin[x]^2])/(a*(a + b
)))/(3*a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(150) = 300$.

Time = 1.18 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.40

method	result
default	$-\frac{\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\sin(x)^2}{a}}\;\text{EllipticF}\left(\sin(x),\sqrt{-\frac{b}{a}}\right)a^2 b\sin(x)^2+\sqrt{\frac{\cos(2x)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\sin(x)^2}{a}}\;\text{EllipticF}\left(\sin(x),\sqrt{-\frac{b}{a}}\right)a\,b^2\sin(x}{}$

input
```
int(1/(a+b*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*EllipticF(sin(x),(-b/a)^(1
/2))*a^2*b*sin(x)^2+(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*EllipticF(si
n(x),(-b/a)^(1/2))*a*b^2*sin(x)^2-4*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1
/2)*EllipticE(sin(x),(-b/a)^(1/2))*a^2*b*sin(x)^2-2*(cos(x)^2)^(1/2)*((a+b
*sin(x)^2)/a)^(1/2)*EllipticE(sin(x),(-b/a)^(1/2))*a*b^2*sin(x)^2+4*a*b^2*
sin(x)^5+2*b^3*sin(x)^5+(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*Elliptic
F(sin(x),(-b/a)^(1/2))*a^3+(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2)*Ellip
ticF(sin(x),(-b/a)^(1/2))*a^2*b-4*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)^(1/2
)*EllipticE(sin(x),(-b/a)^(1/2))*a^3-2*(cos(x)^2)^(1/2)*((a+b*sin(x)^2)/a)
^(1/2)*EllipticE(sin(x),(-b/a)^(1/2))*a^2*b+5*a^2*b*sin(x)^3-a*b^2*sin(x)^
3-2*b^3*sin(x)^3-5*sin(x)*b*a^2-3*sin(x)*b^2*a)/(a+b*sin(x)^2)^(3/2)/(a+b)
^2/a^2/cos(x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1406, normalized size of antiderivative = 8.52

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^
5)*cos(x)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(x)^2)*sqrt(-b)*sqrt(
(a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 - 6*I*a*b^4 -
 I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(x)^4 + 2*(4*I*a^3*b^2 + 8*
I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(x)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*
sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I*a^3*b^2 - 5*I*a^2*b^3 - 4*I*a*b^4 -
 I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(x)^4 - 2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b
^5)*cos(x)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 +
 13*I*a^2*b^3 + 6*I*a*b^4 + I*b^5 + (4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(
x)^4 + 2*(-4*I*a^3*b^2 - 8*I*a^2*b^3 - 5*I*a*b^4 - I*b^5)*cos(x)^2)*sqrt(-
b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(x) - I*sin(x))), (8*a^2 + 8*a
*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b -
11*I*a^3*b^2 - 15*I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*
b^4 - 2*I*b^5)*cos(x)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 - 2*I*
b^5)*cos(x)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I*a^4*b - 1
7*I*a^3*b^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2*b^3 - I*a*b^
4)*cos(x)^4 + 2*(6*I*a^4*b + 11*I*a^3*b^2 + 6*I*a^2*b^3 + I*a*b^4)*cos(...
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\sin^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**2)**(5/2),x)
```

output
```
Integral((a + b*sin(x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sin\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(x)^2 + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sin\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sin(x)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sin\left(x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sin(x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sin(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + b\sin^2(x))^{5/2}} \, dx = \int \frac{\sqrt{\sin(x)^2 b + a}}{\sin(x)^6 b^3 + 3\sin(x)^4 a b^2 + 3\sin(x)^2 a^2 b + a^3} dx$$

input
```
int(1/(a+b*sin(x)^2)^(5/2),x)
```

output
```
int(sqrt(sin(x)**2*b + a)/(sin(x)**6*b**3 + 3*sin(x)**4*a*b**2 + 3*sin(x)*
*2*a**2*b + a**3),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file