Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.1-Sine/192-4.1.10

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**348**]. This is test number [192].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.43 (346)	0.57 (2)
Rubi	95.11 (331)	4.89 (17)
Fricas	92.53 (322)	7.47 (26)
Maple	75.86 (264)	24.14 (84)
Maxima	58.33 (203)	41.67 (145)
Giac	52.01 (181)	47.99 (167)
Mupad	41.09 (143)	58.91 (205)
Reduce	39.66 (138)	60.34 (210)
Sympy	33.62 (117)	66.38 (231)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

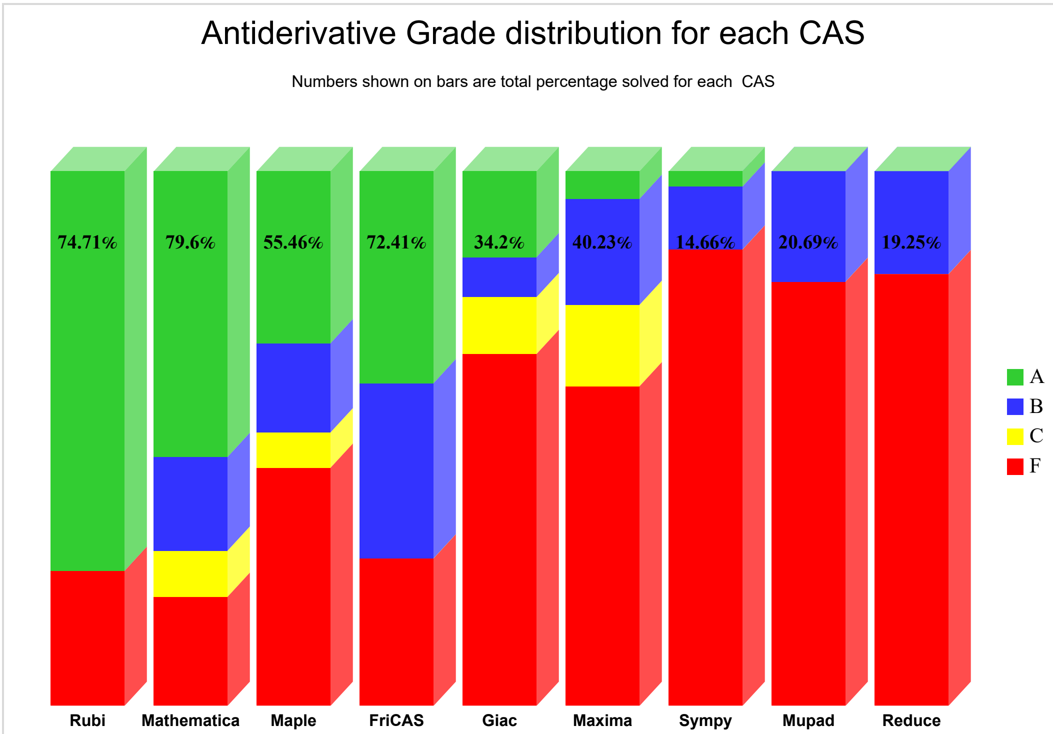
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

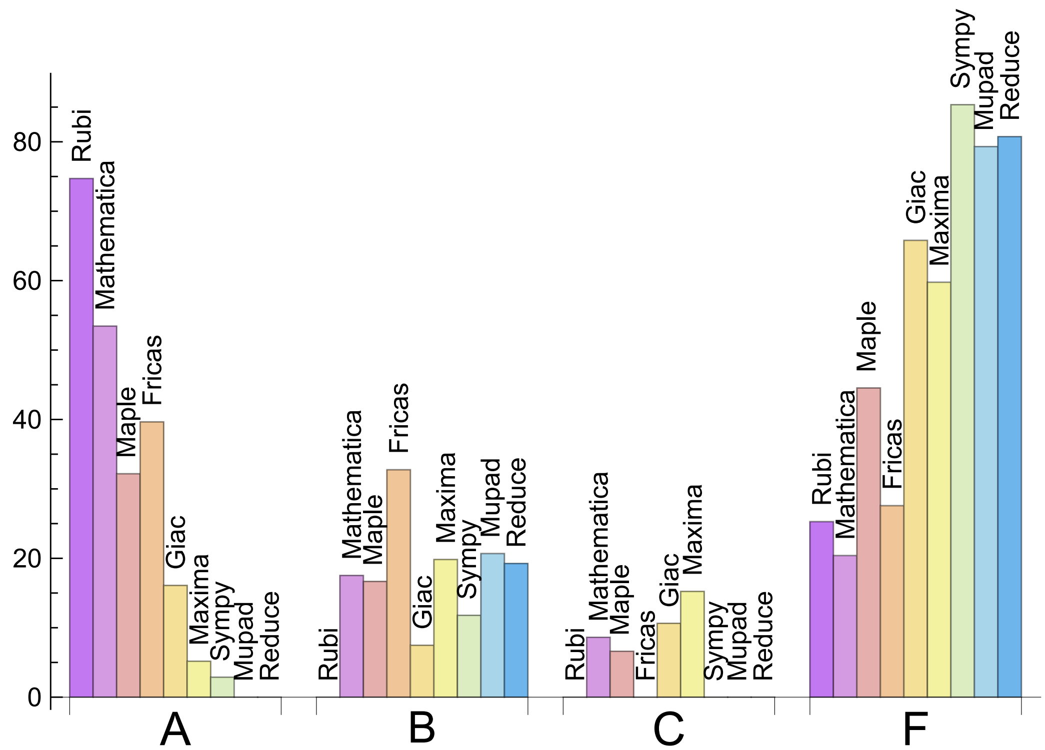
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.713	0.000	0.000	25.287
Mathematica	53.448	17.529	8.621	20.402
Fricas	39.655	32.759	0.000	27.586
Maple	32.184	16.667	6.609	44.540
Giac	16.092	7.471	10.632	65.805
Maxima	5.172	19.828	15.230	59.770
Sympy	2.874	11.782	0.000	85.345
Mupad	0.000	20.690	0.000	79.310
Reduce	0.000	19.253	0.000	80.747

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	2	0.00	100.00	0.00
Rubi	17	100.00	0.00	0.00
Fricas	26	23.08	0.00	76.92
Maple	84	100.00	0.00	0.00
Maxima	145	35.86	0.00	64.14
Giac	167	82.63	17.37	0.00
Mupad	205	0.00	100.00	0.00
Reduce	210	100.00	0.00	0.00
Sympy	231	80.95	17.32	1.73

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.15
Reduce	0.42
Rubi	0.87
Maxima	1.13
Maple	1.33
Giac	1.40
Mathematica	7.69
Sympy	9.80
Mupad	32.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	107.23	1.56	30.00	1.11
Rubi	193.05	1.01	113.00	1.00
Sympy	271.50	3.25	58.00	1.50
Maple	309.58	1.69	112.00	1.13
Mathematica	568.85	1.57	122.50	1.09
Maxima	716.07	13.19	239.00	2.15
Fricas	771.76	2.43	174.00	1.61
Reduce	1877.08	70.73	90.50	1.82
Giac	2915.03	16.40	94.00	1.24

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

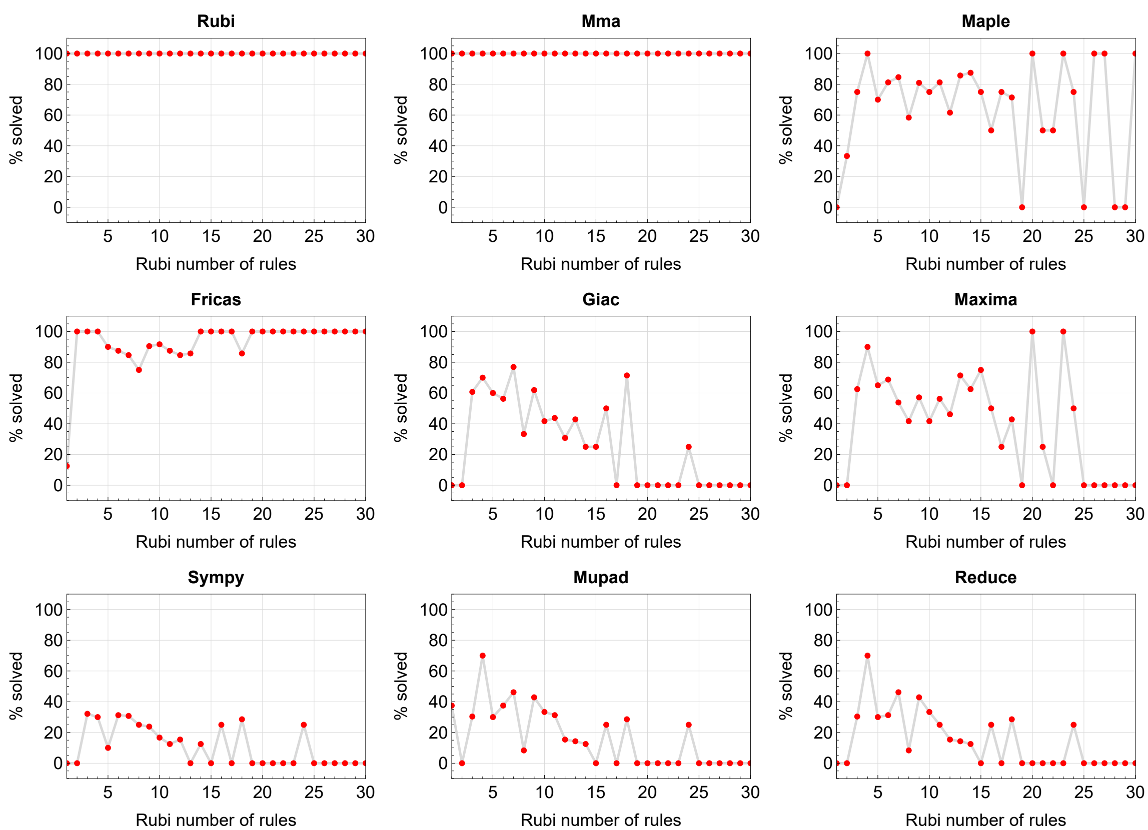


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

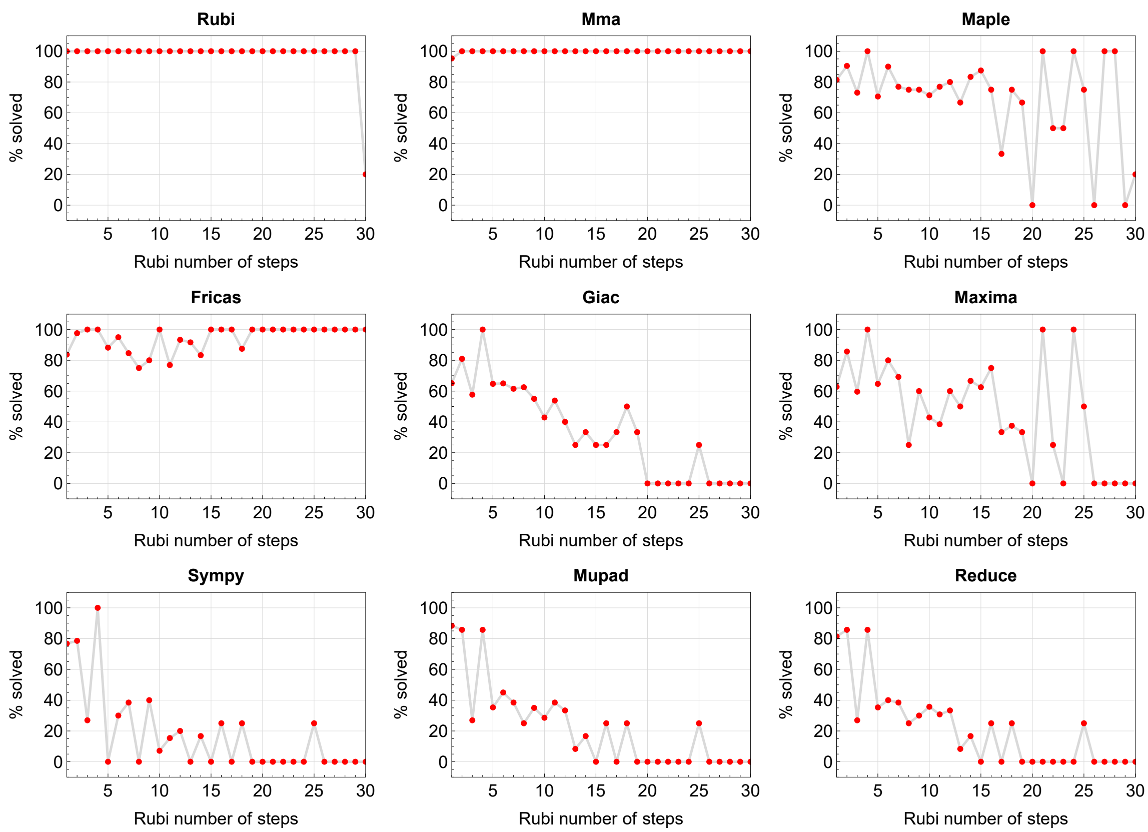


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

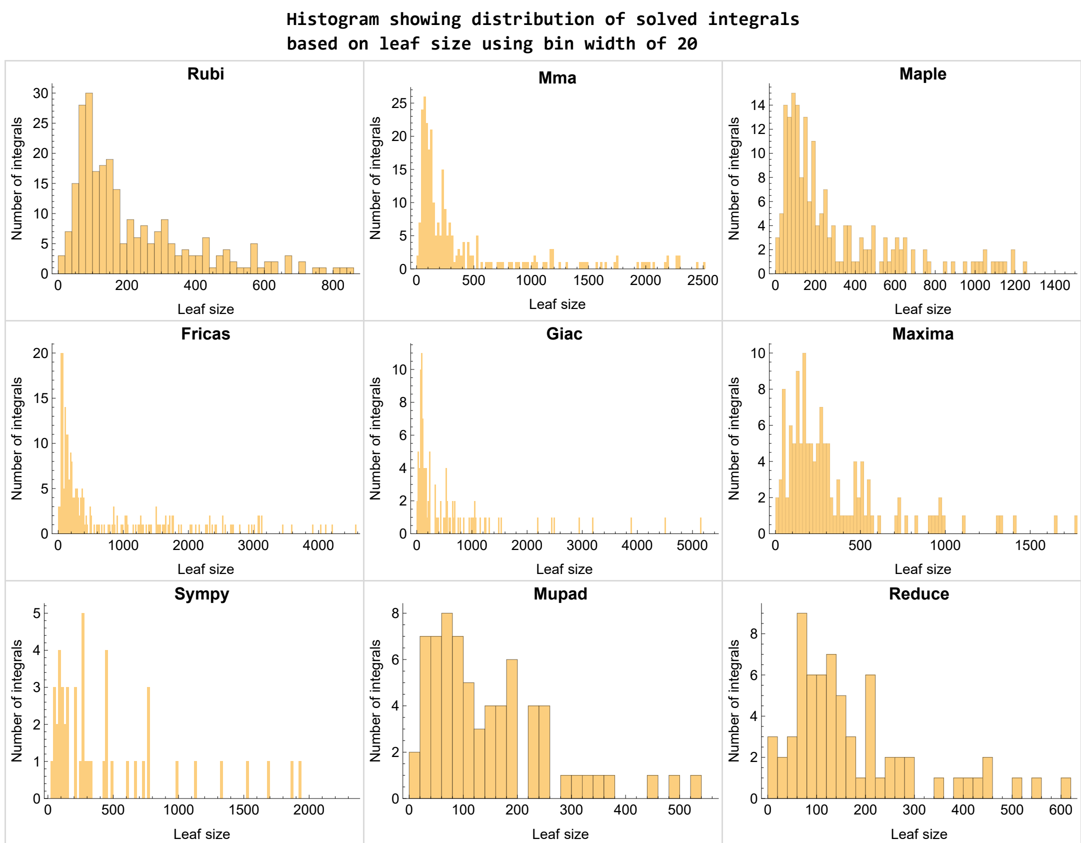


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

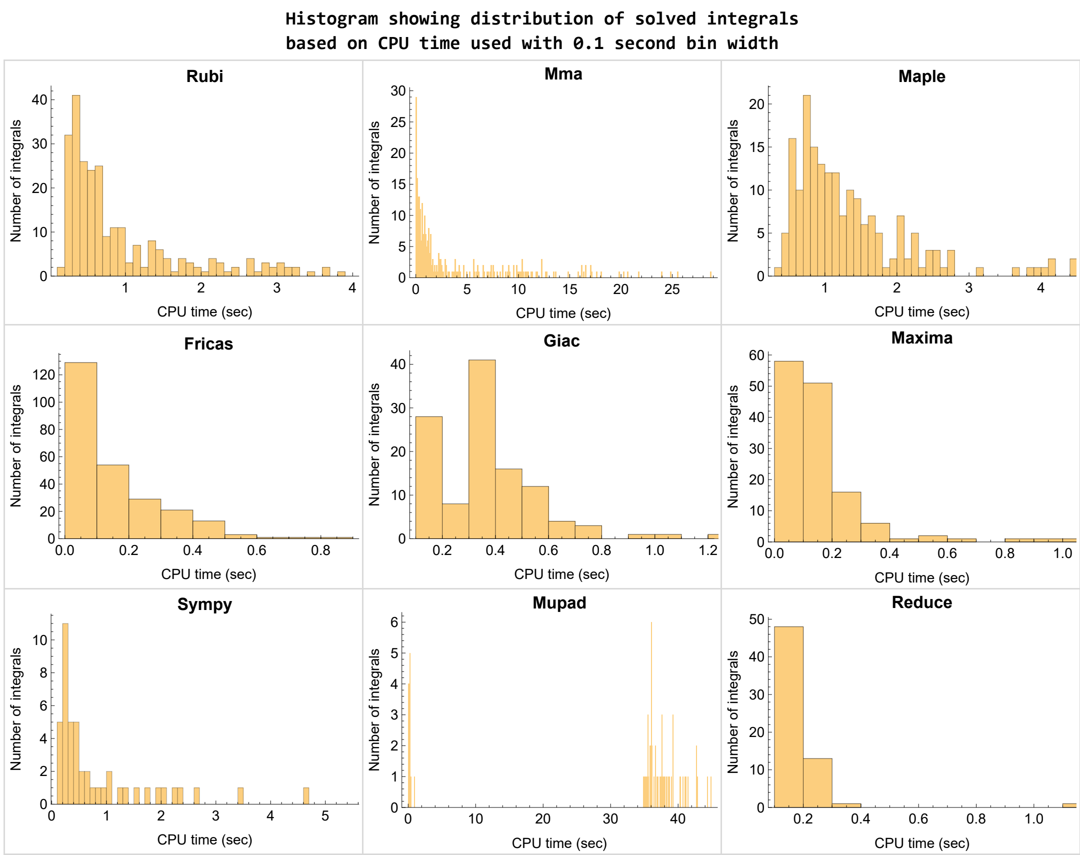


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

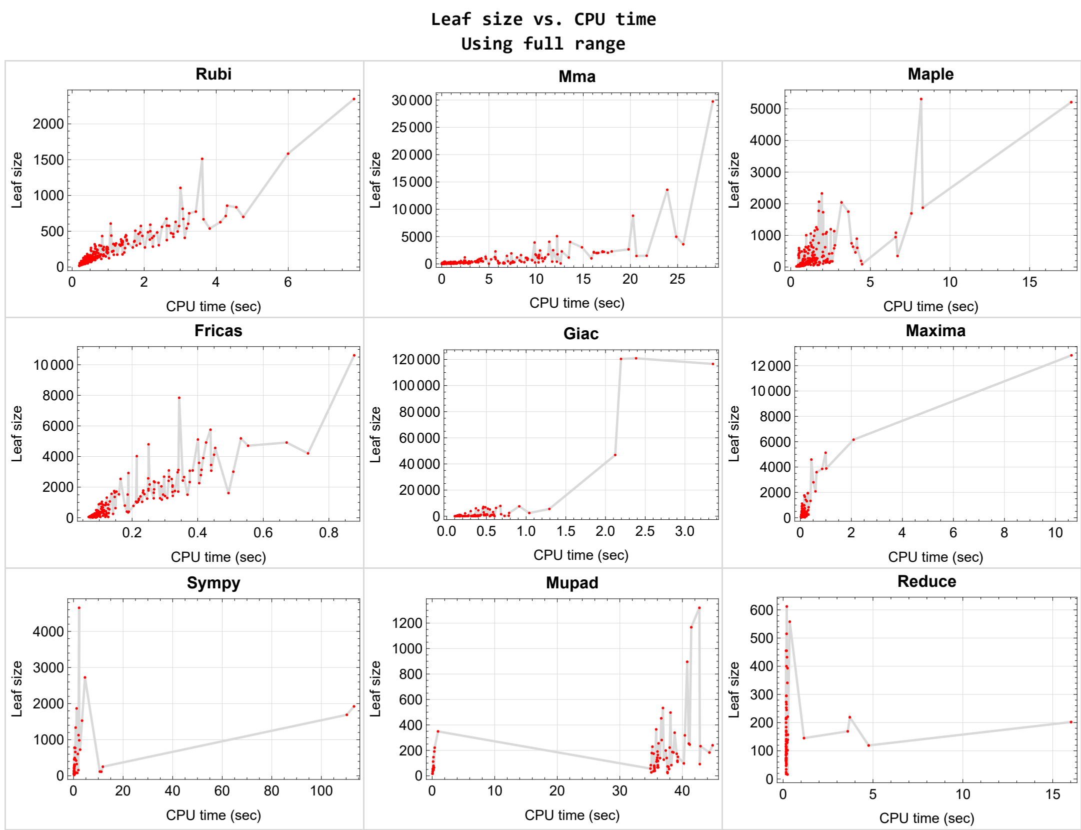


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 71, 75, 76, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 166, 167, 171, 172, 173, 177, 178, 183, 184, 189, 190, 195, 196, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 255, 256, 273, 274, 279, 280, 285, 286, 290, 291, 292, 293, 314, 315, 316, 317, 318}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {28, 29, 53, 54, 136, 141, 191, 197, 203, 204, 209, 210, 224, 226, 228, 229, 230, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 269, 270, 275, 276, 281, 282, 283, 300, 302, 303, 306, 307, 308, 310, 311, 312, 321, 323, 324, 325, 327, 329, 330, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347}

Maple {95, 96}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

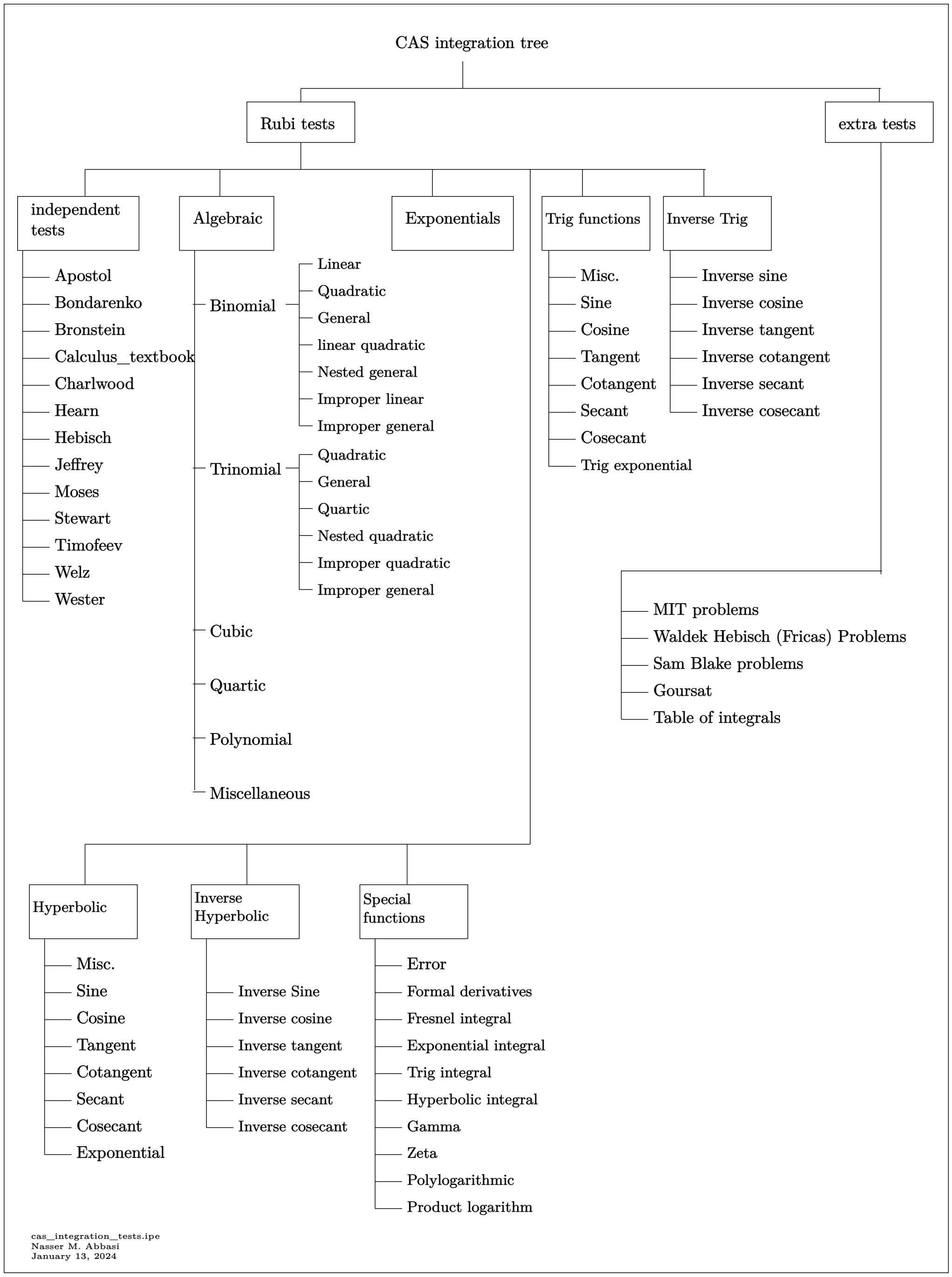
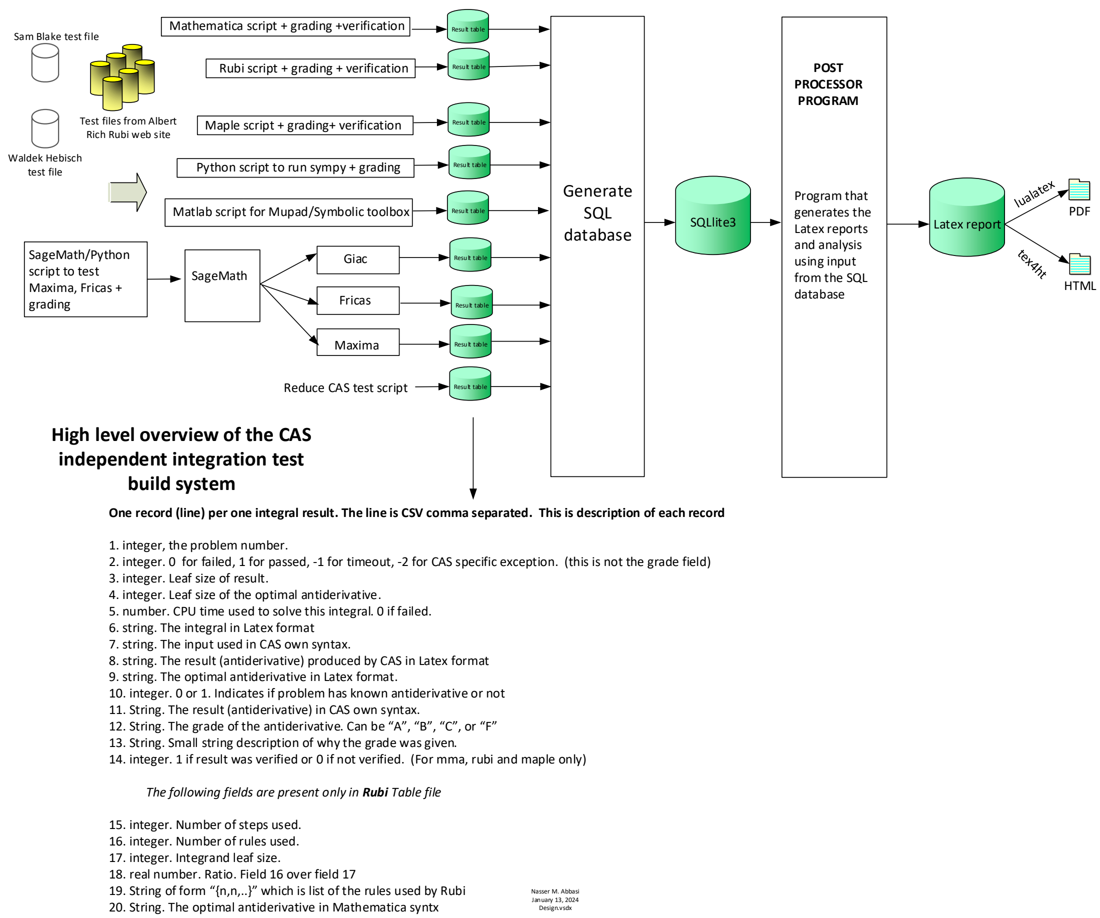


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 181, 182, 185, 186, 187, 188, 191, 192, 193, 194, 197, 198, 199, 200, 203, 204, 205, 206, 211, 212, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 282, 283, 284, 287, 288, 289, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 319, 320, 321, 322, 323, 324, 326, 327, 328, 330, 331, 332, 336, 340, 344, 348 }

B grade { }

C grade { }

F normal fail { 209, 210, 281, 325, 329, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 33, 67, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 139, 140, 141, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 186, 188, 191, 193, 194, 197, 198, 200, 206, 212, 220, 221, 222, 223, 225, 227, 229, 231, 232, 233, 235, 236, 237, 239, 251, 252, 254, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 272, 276, 277, 278, 284, 287, 288, 289, 294, 295, 296, 297, 298, 299, 305, 309, 310, 311, 313, 319, 320, 321, 322, 325, 326, 328, 332, 333, 334, 336, 340, 342, 344, 348 }

B grade { 28, 29, 34, 35, 181, 182, 185, 187, 192, 199, 203, 204, 205, 209, 210, 211, 224, 226, 228, 230, 234, 238, 245, 246, 247, 248, 249, 250, 253, 260, 269, 270, 271, 275, 281, 282, 283, 300, 301, 302, 303, 304, 306, 307, 308, 312, 323, 324, 327, 329, 330, 331, 335, 337, 338, 339, 341, 343, 345, 346, 347 }

C grade { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 132, 133 }

F normal fail { }

F(-1) timedout fail { 213, 214 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 30, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 187, 188, 200, 206, 212, 223, 227, 231, 235, 239, 245, 248, 254, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 272, 276, 284, 297, 301, 305, 309, 313, 328, 332, 336, 340, 344, 348 }

B grade { 23, 24, 25, 28, 29, 33, 34, 35, 107, 108, 112, 113, 117, 118, 165, 170, 179, 180, 185, 186, 191, 192, 197, 198, 199, 203, 204, 205, 209, 210, 211, 222, 226, 230, 234, 238, 251, 252, 253, 269, 270, 275, 281, 282, 283, 296, 300, 304, 308, 312, 320, 323, 327, 331, 335, 339, 343, 347 }

C grade { 12, 13, 77, 78, 79, 80, 81, 82, 83, 109, 114, 119, 122, 123, 124, 181, 182, 193, 194, 277, 278, 319, 322 }

F normal fail { 67, 68, 69, 70, 72, 73, 74, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 163, 164, 168, 169, 174, 175, 176, 220, 221, 224, 225, 228, 229, 232, 233, 236, 237, 246, 247, 249, 250, 287, 288, 289, 294, 295, 298, 299, 302, 303, 306, 307, 310, 311, 321, 324, 325, 326, 329, 330, 333, 334, 337, 338, 341, 342, 345, 346 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 30, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 114, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 174, 175, 176, 182, 188, 193, 194, 223, 227, 231, 235, 254, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 277, 278, 284, 287, 288, 289, 297, 301, 305, 309, 313, 319, 328, 332, 336, 340, 344, 348 }

B grade { 23, 24, 25, 28, 29, 33, 34, 35, 52, 107, 108, 109, 112, 113, 117, 118, 119, 163, 164, 165, 168, 169, 170, 179, 180, 181, 185, 186, 187, 191, 192, 197, 198, 199, 200, 203, 204, 205, 206, 209, 210, 211, 212, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 251, 252, 253, 269, 270, 271, 275, 276, 281, 282, 283, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

C grade { }

F normal fail { 134, 135, 136, 139, 140, 141 }

F(-1) timedout fail { }

F(-2) exception fail { 67, 68, 70, 91, 92, 93, 94, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 144 }

Maxima

A grade { 4, 11, 19, 159, 182, 200, 253, 254, 265, 266, 272, 284, 297, 305, 309, 332, 340, 348 }

B grade { 1, 2, 3, 8, 9, 10, 16, 17, 18, 23, 24, 25, 28, 29, 30, 33, 34, 35, 95, 96, 97, 101, 102, 103, 107, 108, 109, 112, 113, 114, 117, 118, 119, 151, 152, 153, 157, 158, 179, 180, 181, 185, 186, 187, 188, 194, 197, 198, 199, 206, 209, 210, 211, 212, 251, 252, 257, 258, 259, 260, 263, 264, 269, 270, 271, 275, 276, 277, 278 }

C grade { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 98, 99, 100, 104, 105, 106, 154, 155, 156, 160, 161, 162, 261, 262, 267, 268 }

F normal fail { 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 174, 175, 176, 287, 288, 289 }

F(-1) timedout fail { }

F(-2) exception fail { 163, 164, 165, 168, 169, 170, 191, 192, 193, 195, 196, 203, 204, 205, 207, 208, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 281, 282, 283, 285, 286, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 310, 311, 312, 313, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347 }

Giac

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 95, 96, 97, 101, 102, 103, 122, 123, 124, 130, 131, 151, 152, 153, 157, 158, 159, 182, 188, 194, 200, 206, 212, 223, 227, 231, 235, 239, 254, 260, 266, 272, 278, 284, 297, 301, 305, 309, 313, 328, 332, 336, 340, 348 }

B grade { 6, 13, 21, 30, 99, 105, 109, 114, 119, 128, 129, 132, 133, 155, 161, 181, 187, 193, 257, 258, 259, 263, 264, 265, 277, 344 }

C grade { 5, 7, 12, 14, 15, 20, 22, 38, 39, 40, 41, 45, 46, 47, 48, 53, 54, 55, 56, 60, 61, 62, 98, 100, 104, 106, 125, 126, 127, 154, 156, 160, 162, 261, 262, 267, 268 }

F normal fail { 23, 24, 25, 28, 29, 33, 34, 35, 42, 43, 44, 49, 50, 51, 52, 57, 58, 59, 63, 64, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 107, 108, 112, 113, 117, 118, 134, 135, 136, 141, 146, 147, 148, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 185, 186, 191, 192, 197, 199, 203, 204, 205, 210, 211, 220, 221, 222, 224, 225, 226, 228, 229, 230, 245, 246, 247, 248, 249, 250, 251, 252, 253, 269, 270, 271, 275, 276, 281, 282, 283, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 319, 320, 321, 322, 323, 324, 329, 330, 331, 337, 338, 339, 346, 347 }

F(-1) timedout fail { 37, 139, 140, 143, 198, 202, 207, 208, 209, 213, 214, 232, 233, 234, 236, 237, 238, 285, 286, 325, 326, 327, 333, 334, 335, 341, 342, 343, 345 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 67, 69, 70, 95, 96, 97, 101, 102, 103, 109, 114, 119, 122, 123, 124, 151, 152, 153, 157, 158, 159, 181, 182, 187, 188, 193, 194, 200, 206, 212, 223, 227, 231, 235, 239, 254, 257, 258, 259, 260, 263, 264, 265, 266, 272, 277, 278, 284, 297, 301, 305, 309, 313, 328, 332, 336, 340, 344, 348 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 104, 105, 106, 107, 108, 112, 113, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 154, 155, 156, 160, 161, 162, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 185, 186, 191, 192, 197, 198, 199, 203, 204, 205, 209, 210, 211, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 251, 252, 253, 261, 262, 267, 268, 269, 270, 271, 275, 276, 281, 282, 283, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

F(-2) exception fail { }

Sympy

A grade { 4, 19, 60, 61, 62, 63, 64, 97, 153, 254 }

B grade { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 95, 96, 101, 102, 103, 109, 114, 119, 151, 152, 157, 158, 159, 181, 182, 187, 188, 193, 194, 223, 227, 257, 258, 259, 260, 263, 264, 265, 266, 297, 301 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 104, 105, 106, 107, 108, 112, 113, 117, 118, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 154, 155, 156, 160, 161, 162, 163, 164, 165, 174, 175, 176, 179, 180, 185, 186, 191, 192, 197, 198, 199, 200, 203, 204, 205, 206, 209, 210, 211, 212, 220, 221, 222, 232, 233, 234, 235, 236, 237, 238, 239, 251, 252, 253, 261, 262, 269, 270, 271, 272, 275, 276, 277, 278, 281, 282, 283, 284, 289, 295, 296, 306, 307, 308, 309, 310, 311, 312, 313, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347 }

F(-1) timedout fail { 23, 52, 53, 145, 168, 169, 170, 172, 173, 196, 224, 225, 226, 228, 229, 230, 231, 245, 246, 247, 248, 249, 250, 268, 294, 298, 299, 300, 302, 303, 304, 305, 319, 320, 321, 322, 323, 324, 336, 348 }

F(-2) exception fail { 195, 267, 287, 288 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 95, 96, 97, 101, 102, 103, 109, 114, 119, 151, 152, 153, 157, 158, 159, 181, 182, 187, 188, 193, 194, 200, 206, 212, 223, 227, 231, 235, 239, 254, 257, 258, 259, 260, 263, 264, 265, 266, 272, 277, 278, 284, 297, 301, 305, 309, 313, 319, 322, 328, 332, 336, 340, 344 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 104, 105, 106, 107, 108, 112, 113, 117, 118, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 154, 155, 156, 160, 161, 162, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 185, 186, 191, 192, 197, 198, 199, 203, 204, 205, 209, 210, 211, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 251, 252, 253, 261, 262, 267, 268, 269, 270, 271, 275, 276, 281, 282, 283, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 320, 321, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347, 348 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	101	77	168	490	170	311	171	245	221
N.S.	1	1.10	0.84	1.83	5.33	1.85	3.38	1.86	2.66	2.40
time (sec)	N/A	0.556	0.343	0.968	0.067	0.083	0.360	0.342	0.195	0.402

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	62	108	285	110	202	111	155	147
N.S.	1	1.08	0.87	1.52	4.01	1.55	2.85	1.56	2.18	2.07
time (sec)	N/A	0.435	0.213	0.866	0.049	0.079	0.282	0.397	0.176	0.214

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	52	45	61	141	63	112	65	85	84
N.S.	1	1.04	0.90	1.22	2.82	1.26	2.24	1.30	1.70	1.68
time (sec)	N/A	0.326	0.186	0.819	0.040	0.074	0.192	0.388	0.163	34.956

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	53	30	46	31	34	35
N.S.	1	1.00	0.96	1.04	1.89	1.07	1.64	1.11	1.21	1.25
time (sec)	N/A	0.227	0.144	0.644	0.029	0.072	0.150	0.393	0.172	0.086

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	78	141	61	0	597	16	0
N.S.	1	1.00	0.96	1.53	2.76	1.20	0.00	11.71	0.31	0.00
time (sec)	N/A	0.385	0.128	0.721	0.082	0.079	0.000	0.420	0.165	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	66	112	164	97	0	521	27	0
N.S.	1	1.06	0.92	1.56	2.28	1.35	0.00	7.24	0.38	0.00
time (sec)	N/A	0.497	0.271	0.783	0.103	0.071	0.000	0.423	0.176	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	105	87	150	199	163	0	5727	38	0
N.S.	1	1.01	0.84	1.44	1.91	1.57	0.00	55.07	0.37	0.00
time (sec)	N/A	0.603	0.837	0.968	0.166	0.077	0.000	0.518	0.180	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	166	132	145	735	286	660	222	455	349
N.S.	1	1.03	0.82	0.90	4.57	1.78	4.10	1.38	2.83	2.17
time (sec)	N/A	0.499	0.622	1.591	0.064	0.086	0.703	0.376	0.166	0.935

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	106	121	442	189	456	153	295	229
N.S.	1	1.04	0.85	0.98	3.56	1.52	3.68	1.23	2.38	1.85
time (sec)	N/A	0.356	0.468	1.195	0.048	0.084	0.362	0.394	0.177	35.212

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	77	79	232	112	264	94	171	179
N.S.	1	1.02	0.81	0.83	2.44	1.18	2.78	0.99	1.80	1.88
time (sec)	N/A	0.319	0.348	1.093	0.042	0.079	0.268	0.361	0.173	35.101

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	46	96	54	126	48	72	57
N.S.	1	1.00	0.95	0.84	1.75	0.98	2.29	0.87	1.31	1.04
time (sec)	N/A	0.210	0.273	0.756	0.031	0.077	0.188	0.396	0.168	34.887

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	107	162	73	0	612	18	0
N.S.	1	1.00	0.83	1.37	2.08	0.94	0.00	7.85	0.23	0.00
time (sec)	N/A	0.358	0.125	0.957	0.086	0.075	0.000	0.418	0.175	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	75	155	171	102	0	535	29	0
N.S.	1	1.05	0.93	1.91	2.11	1.26	0.00	6.60	0.36	0.00
time (sec)	N/A	0.518	0.471	1.046	0.114	0.085	0.000	0.395	0.171	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	147	101	193	206	181	0	5141	40	0
N.S.	1	1.30	0.89	1.71	1.82	1.60	0.00	45.50	0.35	0.00
time (sec)	N/A	0.491	1.407	1.319	0.173	0.088	0.000	0.543	0.178	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	122	229	258	284	0	7832	51	0
N.S.	1	1.00	0.75	1.41	1.59	1.75	0.00	48.35	0.31	0.00
time (sec)	N/A	0.766	1.413	1.613	0.261	0.089	0.000	0.676	0.181	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	302	150	181	934	351	772	351	612	533
N.S.	1	1.34	0.67	0.80	4.15	1.56	3.43	1.56	2.72	2.37
time (sec)	N/A	1.788	1.047	2.516	0.088	0.100	0.667	0.354	0.205	36.895

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	218	127	142	541	227	495	231	393	365
N.S.	1	1.25	0.73	0.81	3.09	1.30	2.83	1.32	2.25	2.09
time (sec)	N/A	1.112	1.062	2.196	0.059	0.084	0.465	0.361	0.247	35.812

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	135	86	106	270	131	284	137	221	174
N.S.	1	1.10	0.70	0.86	2.20	1.07	2.31	1.11	1.80	1.41
time (sec)	N/A	0.617	0.609	2.027	0.044	0.078	0.337	0.394	0.258	35.402

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	63	104	62	126	69	91	79
N.S.	1	1.00	0.79	0.84	1.39	0.83	1.68	0.92	1.21	1.05
time (sec)	N/A	0.350	0.292	1.680	0.036	0.073	0.250	0.386	0.172	0.214

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	172	279	124	0	6296	18	0
N.S.	1	1.00	0.84	1.42	2.31	1.02	0.00	52.03	0.15	0.00
time (sec)	N/A	0.466	0.281	1.336	0.108	0.077	0.000	0.621	0.183	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	148	175	245	306	188	0	1000	29	0
N.S.	1	1.02	1.21	1.69	2.11	1.30	0.00	6.90	0.20	0.00
time (sec)	N/A	0.458	1.272	1.523	0.161	0.089	0.000	0.511	0.193	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	242	221	318	341	315	0	116534	40	0
N.S.	1	1.32	1.20	1.73	1.85	1.71	0.00	633.34	0.22	0.00
time (sec)	N/A	0.965	1.020	1.970	0.247	0.094	0.000	3.353	0.170	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	203	221	633	716	820	0	0	72	0
N.S.	1	1.10	1.19	3.42	3.87	4.43	0.00	0.00	0.39	0.00
time (sec)	N/A	0.695	0.495	0.939	0.137	0.118	0.000	0.000	0.186	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	148	361	398	504	0	0	51	0
N.S.	1	1.07	1.20	2.93	3.24	4.10	0.00	0.00	0.41	0.00
time (sec)	N/A	0.485	0.337	0.862	0.109	0.114	0.000	0.000	0.214	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	108	124	175	252	0	0	30	0
N.S.	1	1.00	1.61	1.85	2.61	3.76	0.00	0.00	0.45	0.00
time (sec)	N/A	0.287	0.078	0.608	0.106	0.093	0.000	0.000	0.172	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.29
time (sec)	N/A	0.201	7.095	0.437	0.206	0.074	0.340	0.433	0.173	34.916

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	27	18
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.93	1.29
time (sec)	N/A	0.200	8.255	0.444	0.465	0.075	0.524	1.784	0.167	34.942

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	150	486	541	1654	676	0	0	397	0
N.S.	1	1.33	4.30	4.79	14.64	5.98	0.00	0.00	3.51	0.00
time (sec)	N/A	0.693	7.020	1.165	0.178	0.111	0.000	0.000	0.183	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	106	181	276	552	379	0	0	255	0
N.S.	1	1.28	2.18	3.33	6.65	4.57	0.00	0.00	3.07	0.00
time (sec)	N/A	0.500	6.124	1.086	0.134	0.100	0.000	0.000	0.170	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	31	52	53	217	46	0	1027	75	55
N.S.	1	1.07	1.79	1.83	7.48	1.59	0.00	35.41	2.59	1.90
time (sec)	N/A	0.263	0.256	0.868	0.039	0.085	0.000	0.569	0.166	35.542

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	477	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	29.81	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.216	7.852	0.302	0.345	0.078	0.375	0.428	0.184	34.902

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	718	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	44.88	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.214	7.972	0.305	0.919	0.075	0.529	0.843	0.168	35.003

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	330	528	1056	3886	1744	0	0	131	0
N.S.	1	1.07	1.71	3.42	12.58	5.64	0.00	0.00	0.42	0.00
time (sec)	N/A	1.222	5.946	1.256	1.015	0.145	0.000	0.000	0.182	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	194	471	548	1938	972	0	0	100	0
N.S.	1	1.08	2.62	3.04	10.77	5.40	0.00	0.00	0.56	0.00
time (sec)	N/A	0.728	7.529	1.131	0.298	0.120	0.000	0.000	0.171	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	110	292	246	763	452	0	0	67	0
N.S.	1	1.01	2.68	2.26	7.00	4.15	0.00	0.00	0.61	0.00
time (sec)	N/A	0.402	2.348	0.905	0.169	0.097	0.000	0.000	0.172	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1791	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	111.94	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.218	38.118	0.299	2.555	0.085	0.396	2.764	0.187	34.982

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	2287	29	15	0	29	18
N.S.	1	1.00	1.12	1.00	142.94	1.81	0.94	0.00	1.81	1.12
time (sec)	N/A	0.217	40.025	0.319	9.005	0.096	0.573	0.000	0.176	35.102

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	204	127	233	263	190	0	1226	62	0
N.S.	1	1.05	0.65	1.19	1.35	0.97	0.00	6.29	0.32	0.00
time (sec)	N/A	0.996	0.073	0.850	0.066	0.087	0.000	0.502	0.188	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	174	125	188	242	156	0	758	36	0
N.S.	1	1.02	0.74	1.11	1.42	0.92	0.00	4.46	0.21	0.00
time (sec)	N/A	0.788	0.113	0.779	0.064	0.090	0.000	0.471	0.186	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	125	145	196	127	0	422	15	0
N.S.	1	1.04	0.88	1.02	1.38	0.89	0.00	2.97	0.11	0.00
time (sec)	N/A	0.648	0.067	0.772	0.051	0.081	0.000	0.345	0.174	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	99	159	107	0	166	17	0
N.S.	1	1.00	1.03	0.85	1.36	0.91	0.00	1.42	0.15	0.00
time (sec)	N/A	0.510	0.055	0.720	0.049	0.079	0.000	0.338	0.257	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	145	148	140	129	146	0	0	29	0
N.S.	1	1.04	1.06	1.01	0.93	1.05	0.00	0.00	0.21	0.00
time (sec)	N/A	0.644	0.333	0.752	0.231	0.087	0.000	0.000	0.286	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	175	162	180	129	208	0	0	46	0
N.S.	1	1.04	0.96	1.07	0.77	1.24	0.00	0.00	0.27	0.00
time (sec)	N/A	0.805	0.634	0.747	0.236	0.100	0.000	0.000	0.170	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	207	208	220	129	297	0	0	63	0
N.S.	1	1.07	1.08	1.14	0.67	1.54	0.00	0.00	0.33	0.00
time (sec)	N/A	0.957	0.566	0.756	0.225	0.098	0.000	0.000	0.169	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	238	150	242	295	258	0	1310	68	0
N.S.	1	1.03	0.65	1.05	1.28	1.12	0.00	5.67	0.29	0.00
time (sec)	N/A	0.706	0.891	1.226	0.143	0.100	0.000	0.593	0.222	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	210	150	197	274	195	0	797	40	0
N.S.	1	1.03	0.74	0.97	1.35	0.96	0.00	3.93	0.20	0.00
time (sec)	N/A	0.616	0.682	1.069	0.139	0.094	0.000	0.490	0.180	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	130	150	229	148	0	436	17	0
N.S.	1	1.00	0.82	0.95	1.45	0.94	0.00	2.76	0.11	0.00
time (sec)	N/A	0.471	0.836	1.109	0.137	0.095	0.000	0.413	0.179	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	128	108	187	114	0	167	19	0
N.S.	1	1.00	0.98	0.83	1.44	0.88	0.00	1.28	0.15	0.00
time (sec)	N/A	0.412	0.624	1.058	0.131	0.088	0.000	0.342	0.172	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	140	175	145	135	138	0	0	31	0
N.S.	1	1.04	1.30	1.07	1.00	1.02	0.00	0.00	0.23	0.00
time (sec)	N/A	0.689	0.783	1.188	0.233	0.091	0.000	0.000	0.195	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	213	188	189	136	209	0	0	48	0
N.S.	1	1.25	1.11	1.11	0.80	1.23	0.00	0.00	0.28	0.00
time (sec)	N/A	0.600	1.135	1.190	0.222	0.099	0.000	0.000	0.171	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	223	237	230	136	328	0	0	65	0
N.S.	1	1.03	1.10	1.06	0.63	1.52	0.00	0.00	0.30	0.00
time (sec)	N/A	0.962	1.162	1.220	0.240	0.107	0.000	0.000	0.179	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	296	277	273	136	422	0	0	82	0
N.S.	1	1.20	1.12	1.11	0.55	1.71	0.00	0.00	0.33	0.00
time (sec)	N/A	0.806	1.422	1.195	0.229	0.120	0.000	0.000	0.182	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	578	252	476	547	371	0	2453	68	0
N.S.	1	1.41	0.61	1.16	1.33	0.90	0.00	5.98	0.17	0.00
time (sec)	N/A	2.630	1.788	2.014	0.165	0.107	0.000	1.040	0.201	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	501	254	384	499	300	0	1521	40	0
N.S.	1	1.42	0.72	1.08	1.41	0.85	0.00	4.30	0.11	0.00
time (sec)	N/A	2.171	1.463	1.500	0.166	0.099	0.000	0.685	0.187	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	251	296	424	246	0	848	17	0
N.S.	1	1.00	0.83	0.97	1.39	0.81	0.00	2.79	0.06	0.00
time (sec)	N/A	0.756	1.166	1.466	0.154	0.092	0.000	0.557	0.170	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	233	210	377	212	0	332	19	0
N.S.	1	1.00	0.91	0.82	1.47	0.82	0.00	1.29	0.07	0.00
time (sec)	N/A	0.642	0.856	1.384	0.138	0.082	0.000	0.357	0.163	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	286	294	288	253	274	0	0	31	0
N.S.	1	1.06	1.09	1.07	0.94	1.01	0.00	0.00	0.11	0.00
time (sec)	N/A	0.650	1.827	1.378	0.316	0.092	0.000	0.000	0.186	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	444	374	368	253	388	0	0	48	0
N.S.	1	1.52	1.28	1.26	0.87	1.33	0.00	0.00	0.16	0.00
time (sec)	N/A	1.493	1.842	1.409	0.300	0.109	0.000	0.000	0.168	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	507	460	450	254	549	0	0	65	0
N.S.	1	1.42	1.29	1.26	0.71	1.54	0.00	0.00	0.18	0.00
time (sec)	N/A	1.754	2.879	1.369	0.313	0.131	0.000	0.000	0.170	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	60	73	106	72	117	212	40	0
N.S.	1	1.03	0.69	0.84	1.22	0.83	1.34	2.44	0.46	0.00
time (sec)	N/A	0.412	0.020	0.573	0.055	0.084	10.615	0.348	0.167	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	54	84	54	85	178	12	0
N.S.	1	1.00	0.94	0.83	1.29	0.83	1.31	2.74	0.18	0.00
time (sec)	N/A	0.305	0.019	0.540	0.044	0.080	0.996	0.354	0.166	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	59	33	67	38	54	134	16	0
N.S.	1	1.00	1.28	0.72	1.46	0.83	1.17	2.91	0.35	0.00
time (sec)	N/A	0.229	0.012	0.521	0.042	0.074	0.546	0.345	0.178	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	55	38	57	80	0	44	0
N.S.	1	1.00	1.00	0.86	0.59	0.89	1.25	0.00	0.69	0.00
time (sec)	N/A	0.311	0.029	0.507	0.174	0.081	1.518	0.000	0.170	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	91	111	73	38	69	114	0	22	0
N.S.	1	1.05	1.28	0.84	0.44	0.79	1.31	0.00	0.25	0.00
time (sec)	N/A	0.414	0.100	0.517	0.178	0.087	11.199	0.000	0.163	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	15	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	0.94	1.12
time (sec)	N/A	0.213	18.973	0.499	0.558	0.086	2.437	0.422	0.177	34.801

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	17	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.06	1.12
time (sec)	N/A	0.215	25.040	0.504	0.478	0.077	1.391	0.351	0.168	34.782

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	0	0	0	0	31	36
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.82	0.95
time (sec)	N/A	0.214	1.269	0.000	0.000	0.000	0.000	0.000	0.170	35.383

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	163	0	0	0	0	0	35	0
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.259	6.003	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	0	0	48	0	0	41	140
N.S.	1	1.00	0.83	0.00	0.00	1.14	0.00	0.00	0.98	3.33
time (sec)	N/A	0.211	1.042	0.000	0.000	0.078	0.000	0.000	0.162	37.490

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	0	0	0	0	0	33	253
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.40	3.05
time (sec)	N/A	0.250	1.532	0.000	0.000	0.000	0.000	0.000	0.171	41.025

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	22	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.22	1.11
time (sec)	N/A	0.216	1.194	0.312	0.520	0.097	7.945	1.526	0.169	35.117

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	251	0	0	188	0	0	18	0
N.S.	1	1.00	0.94	0.00	0.00	0.70	0.00	0.00	0.07	0.00
time (sec)	N/A	0.525	0.710	0.000	0.000	0.097	0.000	0.000	0.168	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	136	0	0	18	0
N.S.	1	1.00	0.93	0.00	0.00	0.84	0.00	0.00	0.11	0.00
time (sec)	N/A	0.428	0.799	0.000	0.000	0.098	0.000	0.000	0.177	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	121	0	0	94	0	0	659	0
N.S.	1	1.00	0.95	0.00	0.00	0.74	0.00	0.00	5.19	0.00
time (sec)	N/A	0.312	0.052	0.000	0.000	0.084	0.000	0.000	0.189	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.29
time (sec)	N/A	0.197	10.815	0.321	0.189	0.073	1.235	0.311	0.174	34.827

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.213	1.192	0.308	0.288	0.078	3.142	0.315	0.170	35.617

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	454	0	52	0	0	671	0
N.S.	1	1.00	1.00	5.75	0.00	0.66	0.00	0.00	8.49	0.00
time (sec)	N/A	0.276	0.025	0.556	0.000	0.077	0.000	0.000	0.183	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	353	0	52	0	0	184	0
N.S.	1	1.00	1.00	4.71	0.00	0.69	0.00	0.00	2.45	0.00
time (sec)	N/A	0.273	0.021	0.509	0.000	0.082	0.000	0.000	0.193	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	290	0	52	0	0	252	0
N.S.	1	1.00	1.00	3.67	0.00	0.66	0.00	0.00	3.19	0.00
time (sec)	N/A	0.270	0.021	0.514	0.000	0.082	0.000	0.000	0.245	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	378	0	48	0	0	12	0
N.S.	1	1.00	1.00	5.04	0.00	0.64	0.00	0.00	0.16	0.00
time (sec)	N/A	0.268	0.018	0.480	0.000	0.077	0.000	0.000	0.281	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	426	0	48	0	0	146	0
N.S.	1	1.00	0.91	6.17	0.00	0.70	0.00	0.00	2.12	0.00
time (sec)	N/A	0.267	0.026	0.510	0.000	0.082	0.000	0.000	0.162	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	529	0	52	0	0	15	0
N.S.	1	1.00	0.92	7.45	0.00	0.73	0.00	0.00	0.21	0.00
time (sec)	N/A	0.267	0.023	0.526	0.000	0.076	0.000	0.000	0.178	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	599	0	52	0	0	15	0
N.S.	1	1.00	1.00	7.58	0.00	0.66	0.00	0.00	0.19	0.00
time (sec)	N/A	0.272	0.021	0.516	0.000	0.080	0.000	0.000	0.186	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	118	0	0	77	0	0	822	0
N.S.	1	1.00	1.22	0.00	0.00	0.79	0.00	0.00	8.47	0.00
time (sec)	N/A	0.352	0.377	0.000	0.000	0.084	0.000	0.000	0.185	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	120	0	0	77	0	0	1573	0
N.S.	1	1.00	1.17	0.00	0.00	0.75	0.00	0.00	15.27	0.00
time (sec)	N/A	0.340	0.357	0.000	0.000	0.081	0.000	0.000	0.189	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	116	0	0	77	0	0	15	0
N.S.	1	1.00	1.17	0.00	0.00	0.78	0.00	0.00	0.15	0.00
time (sec)	N/A	0.331	0.350	0.000	0.000	0.085	0.000	0.000	0.202	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	120	0	0	69	0	0	14	0
N.S.	1	1.00	1.17	0.00	0.00	0.67	0.00	0.00	0.14	0.00
time (sec)	N/A	0.325	0.313	0.000	0.000	0.083	0.000	0.000	0.169	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	0	0	64	0	0	17	0
N.S.	1	1.00	0.92	0.00	0.00	0.77	0.00	0.00	0.20	0.00
time (sec)	N/A	0.316	0.350	0.000	0.000	0.079	0.000	0.000	0.168	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	0	0	77	0	0	17	0
N.S.	1	1.00	0.89	0.00	0.00	0.76	0.00	0.00	0.17	0.00
time (sec)	N/A	0.342	0.447	0.000	0.000	0.083	0.000	0.000	0.171	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	94	0	0	77	0	0	17	0
N.S.	1	1.00	0.97	0.00	0.00	0.79	0.00	0.00	0.18	0.00
time (sec)	N/A	0.340	0.453	0.000	0.000	0.082	0.000	0.000	0.166	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	0	0	0	0	0	33	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.273	0.896	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	87	0	0	0	0	0	37	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.390	0.931	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	0	0	0	0	0	41	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.245	0.944	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	0	0	0	33	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.282	6.044	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	123	107	462	168	264	155	200	191
N.S.	1	1.00	1.37	1.19	5.13	1.87	2.93	1.72	2.22	2.12
time (sec)	N/A	0.332	0.728	1.124	0.056	0.084	0.293	0.327	0.165	36.016

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	81	81	239	102	151	93	116	112
N.S.	1	1.00	1.19	1.19	3.51	1.50	2.22	1.37	1.71	1.65
time (sec)	N/A	0.285	0.553	0.962	0.047	0.085	0.220	0.332	0.160	0.154

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	51	42	93	51	68	45	52	54
N.S.	1	1.00	1.13	0.93	2.07	1.13	1.51	1.00	1.16	1.20
time (sec)	N/A	0.236	4.998	0.796	0.037	0.078	0.158	0.333	0.185	0.103

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	87	171	74	0	693	30	0
N.S.	1	1.00	0.84	1.36	2.67	1.16	0.00	10.83	0.47	0.00
time (sec)	N/A	0.348	0.477	0.920	0.114	0.081	0.000	0.344	0.175	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	110	122	196	105	0	533	76	0
N.S.	1	1.00	1.25	1.39	2.23	1.19	0.00	6.06	0.86	0.00
time (sec)	N/A	0.378	0.643	1.003	0.112	0.083	0.000	0.349	0.183	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	104	160	265	178	0	6033	161	0
N.S.	1	1.00	0.85	1.30	2.15	1.45	0.00	49.05	1.31	0.00
time (sec)	N/A	0.460	0.813	1.155	0.162	0.091	0.000	0.456	0.181	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	216	195	969	368	779	335	432	452
N.S.	1	1.00	0.96	0.87	4.33	1.64	3.48	1.50	1.93	2.02
time (sec)	N/A	0.532	1.388	2.295	0.085	0.093	0.424	0.324	0.214	36.620

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	182	138	508	212	456	203	253	255
N.S.	1	1.00	1.08	0.82	3.02	1.26	2.71	1.21	1.51	1.52
time (sec)	N/A	0.436	0.687	1.831	0.061	0.091	0.319	0.318	0.178	36.181

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	83	205	101	219	103	110	127
N.S.	1	1.00	0.82	0.85	2.09	1.03	2.23	1.05	1.12	1.30
time (sec)	N/A	0.322	12.604	1.478	0.042	0.079	0.214	0.309	0.173	35.933

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	139	114	198	337	145	0	6807	53	0
N.S.	1	0.96	0.79	1.37	2.32	1.00	0.00	46.94	0.37	0.00
time (sec)	N/A	0.576	0.578	1.527	0.119	0.081	0.000	0.487	0.177	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	165	206	274	372	219	0	1048	146	0
N.S.	1	1.02	1.27	1.69	2.30	1.35	0.00	6.47	0.90	0.00
time (sec)	N/A	0.569	0.678	1.665	0.169	0.094	0.000	0.431	0.177	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	312	353	347	477	373	0	120870	0	0
N.S.	1	1.39	1.57	1.54	2.12	1.66	0.00	537.20	0.00	0.00
time (sec)	N/A	1.396	1.279	2.069	0.259	0.104	0.000	2.385	0.230	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	175	126	484	979	915	0	0	768	0
N.S.	1	1.18	0.85	3.27	6.61	6.18	0.00	0.00	5.19	0.00
time (sec)	N/A	0.874	1.245	0.869	0.152	0.107	0.000	0.000	0.188	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	129	94	254	309	493	0	0	232	0
N.S.	1	1.14	0.83	2.25	2.73	4.36	0.00	0.00	2.05	0.00
time (sec)	N/A	0.640	0.825	0.776	0.140	0.093	0.000	0.000	0.188	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	51	73	169	100	272	548	134	66
N.S.	1	1.05	0.85	1.22	2.82	1.67	4.53	9.13	2.23	1.10
time (sec)	N/A	0.367	0.186	0.694	0.045	0.078	0.451	0.415	0.178	36.143

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	285	27	27	22	50	22
N.S.	1	1.00	1.10	1.00	14.25	1.35	1.35	1.10	2.50	1.10
time (sec)	N/A	0.243	8.548	0.290	0.393	0.068	1.046	0.348	0.232	35.877

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	442	51	58	22	148	22
N.S.	1	1.00	1.10	1.00	22.10	2.55	2.90	1.10	7.40	1.10
time (sec)	N/A	0.243	7.948	0.286	0.631	0.073	2.144	0.508	0.250	36.115

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	329	257	895	3593	1708	0	0	0	0
N.S.	1	1.06	0.83	2.90	11.63	5.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.390	2.277	4.147	0.634	0.152	0.000	0.000	0.252	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	247	175	463	827	876	0	0	478	0
N.S.	1	1.02	0.72	1.91	3.40	3.60	0.00	0.00	1.97	0.00
time (sec)	N/A	0.994	2.531	4.059	0.325	0.099	0.000	0.000	0.192	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	146	225	132	910	204	1336	2486	295	183
N.S.	1	0.99	1.52	0.89	6.15	1.38	9.03	16.80	1.99	1.24
time (sec)	N/A	0.551	2.694	2.427	0.057	0.080	0.842	0.786	0.184	44.327

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2914	60	54	22	134	22
N.S.	1	1.00	1.10	1.00	145.70	3.00	2.70	1.10	6.70	1.10
time (sec)	N/A	0.240	15.857	1.086	8.255	0.086	2.151	0.344	0.183	36.293

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	3521	102	105	22	40461	22
N.S.	1	1.00	1.10	1.00	176.05	5.10	5.25	1.10	2023.05	1.10
time (sec)	N/A	0.239	16.444	1.282	16.774	0.094	6.614	1.075	0.479	36.145

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	153	124	484	984	916	0	0	769	0
N.S.	1	1.04	0.84	3.29	6.69	6.23	0.00	0.00	5.23	0.00
time (sec)	N/A	0.865	1.279	0.942	0.143	0.102	0.000	0.000	0.198	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	92	254	311	496	0	0	234	0
N.S.	1	1.04	0.82	2.27	2.78	4.43	0.00	0.00	2.09	0.00
time (sec)	N/A	0.629	0.811	0.794	0.127	0.090	0.000	0.000	0.170	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	47	73	169	101	272	549	134	66
N.S.	1	1.03	0.80	1.24	2.86	1.71	4.61	9.31	2.27	1.12
time (sec)	N/A	0.364	0.189	0.809	0.043	0.079	0.427	0.408	0.173	36.021

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	285	28	29	25	53	23
N.S.	1	1.00	1.10	1.00	13.57	1.33	1.38	1.19	2.52	1.10
time (sec)	N/A	0.246	8.318	0.312	0.381	0.071	1.582	0.322	0.174	35.912

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	442	52	60	25	154	23
N.S.	1	1.00	1.10	1.00	21.05	2.48	2.86	1.19	7.33	1.10
time (sec)	N/A	0.244	8.095	0.310	0.622	0.069	3.668	0.476	0.185	36.148

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	145	108	145	0	0	0	117	18	82
N.S.	1	1.21	0.90	1.21	0.00	0.00	0.00	0.98	0.15	0.68
time (sec)	N/A	0.641	2.993	0.946	0.000	0.000	0.000	0.339	0.177	36.084

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	113	92	119	0	0	0	93	18	64
N.S.	1	1.15	0.94	1.21	0.00	0.00	0.00	0.95	0.18	0.65
time (sec)	N/A	0.503	1.133	0.721	0.000	0.000	0.000	0.336	0.169	0.299

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	81	76	93	0	0	0	69	16	47
N.S.	1	1.40	1.31	1.60	0.00	0.00	0.00	1.19	0.28	0.81
time (sec)	N/A	0.372	0.520	0.708	0.000	0.000	0.000	0.311	0.179	0.234

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	70	83	0	0	0	0	383	18	0
N.S.	1	0.69	0.82	0.00	0.00	0.00	0.00	3.79	0.18	0.00
time (sec)	N/A	0.501	0.441	0.000	0.000	0.000	0.000	0.395	0.170	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	103	117	0	0	0	0	1140	52	0
N.S.	1	0.79	0.90	0.00	0.00	0.00	0.00	8.77	0.40	0.00
time (sec)	N/A	0.606	0.714	0.000	0.000	0.000	0.000	0.470	0.175	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	130	153	0	0	0	0	1487	58	0
N.S.	1	0.75	0.88	0.00	0.00	0.00	0.00	8.55	0.33	0.00
time (sec)	N/A	0.687	0.805	0.000	0.000	0.000	0.000	0.507	0.183	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	363	231	0	0	0	0	989	41	0
N.S.	1	1.08	0.69	0.00	0.00	0.00	0.00	2.93	0.12	0.00
time (sec)	N/A	1.404	1.229	0.000	0.000	0.000	0.000	0.468	0.185	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	246	191	0	0	0	0	499	41	0
N.S.	1	0.91	0.70	0.00	0.00	0.00	0.00	1.84	0.15	0.00
time (sec)	N/A	0.855	0.948	0.000	0.000	0.000	0.000	0.396	0.188	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	161	113	0	0	0	0	226	37	0
N.S.	1	0.98	0.68	0.00	0.00	0.00	0.00	1.37	0.22	0.00
time (sec)	N/A	0.555	6.663	0.000	0.000	0.000	0.000	0.353	0.183	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	124	127	0	0	0	0	130	41	0
N.S.	1	0.56	0.57	0.00	0.00	0.00	0.00	0.59	0.19	0.00
time (sec)	N/A	0.537	2.373	0.000	0.000	0.000	0.000	0.353	0.171	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	153	226	0	0	0	0	505	145	0
N.S.	1	0.58	0.86	0.00	0.00	0.00	0.00	1.92	0.55	0.00
time (sec)	N/A	0.523	1.459	0.000	0.000	0.000	0.000	0.386	0.190	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	249	295	0	0	0	0	1256	85	0
N.S.	1	0.75	0.89	0.00	0.00	0.00	0.00	3.78	0.26	0.00
time (sec)	N/A	0.958	1.323	0.000	0.000	0.000	0.000	0.483	0.198	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	417	254	306	0	0	0	0	0	31	0
N.S.	1	0.61	0.73	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.855	1.195	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	178	245	0	0	0	0	0	31	0
N.S.	1	0.61	0.84	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.649	0.961	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	112	231	0	0	0	0	0	29	0
N.S.	1	0.64	1.32	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.435	2.185	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	30	17	18	30	18
N.S.	1	1.00	1.11	0.89	1.00	1.67	0.94	1.00	1.67	1.00
time (sec)	N/A	0.248	4.498	0.426	0.410	0.075	0.746	0.377	0.229	35.995

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	34	19	18	34	18
N.S.	1	1.00	1.11	0.89	1.00	1.89	1.06	1.00	1.89	1.00
time (sec)	N/A	0.245	1.126	0.433	0.427	0.076	1.046	0.362	0.180	36.068

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	691	419	455	0	0	0	0	0	41	0
N.S.	1	0.61	0.66	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.474	3.522	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	435	281	352	0	0	0	0	0	41	0
N.S.	1	0.65	0.81	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.965	2.467	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	188	308	0	0	0	0	0	39	0
N.S.	1	0.76	1.24	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.587	3.229	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	50	17	18	41	18
N.S.	1	1.00	1.11	0.89	1.00	2.78	0.94	1.00	2.28	1.00
time (sec)	N/A	0.253	38.309	0.529	0.576	0.100	2.429	43.024	0.176	36.601

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	56	19	0	47	18
N.S.	1	1.00	1.11	0.89	1.00	3.11	1.06	0.00	2.61	1.00
time (sec)	N/A	0.249	20.926	0.542	0.718	0.080	3.554	0.000	0.232	36.586

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	0	15	18	20	18
N.S.	1	1.00	1.11	0.89	1.00	0.00	0.83	1.00	1.11	1.00
time (sec)	N/A	0.228	4.651	0.275	0.370	0.000	0.795	0.314	0.171	35.853

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10	1.10
time (sec)	N/A	0.216	1.200	0.359	0.536	0.089	0.000	1.285	0.177	36.063

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	433	376	0	0	386	0	0	155	0
N.S.	1	0.96	0.84	0.00	0.00	0.86	0.00	0.00	0.35	0.00
time (sec)	N/A	0.833	0.954	0.000	0.000	0.107	0.000	0.000	0.184	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	293	260	0	0	270	0	0	112	0
N.S.	1	0.98	0.87	0.00	0.00	0.90	0.00	0.00	0.37	0.00
time (sec)	N/A	0.646	0.333	0.000	0.000	0.095	0.000	0.000	0.179	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	0	0	136	0	0	191	0
N.S.	1	1.00	0.93	0.00	0.00	0.92	0.00	0.00	1.29	0.00
time (sec)	N/A	0.370	0.458	0.000	0.000	0.085	0.000	0.000	0.183	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	154	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	7.70	1.10
time (sec)	N/A	0.228	1.149	0.365	0.214	0.076	0.997	0.279	0.182	35.934

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	42	29	22	34	22
N.S.	1	1.00	1.10	1.00	1.10	2.10	1.45	1.10	1.70	1.10
time (sec)	N/A	0.228	10.165	0.937	0.515	0.084	10.973	0.290	0.192	36.197

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	124	111	462	168	264	155	213	191
N.S.	1	1.00	1.38	1.23	5.13	1.87	2.93	1.72	2.37	2.12
time (sec)	N/A	0.332	0.413	1.066	0.052	0.077	0.288	0.274	0.162	0.344

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	84	85	239	102	151	93	124	112
N.S.	1	1.00	1.24	1.25	3.51	1.50	2.22	1.37	1.82	1.65
time (sec)	N/A	0.293	0.338	0.918	0.039	0.084	0.215	0.259	0.163	35.869

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	42	93	51	68	45	56	50
N.S.	1	1.00	0.96	0.93	2.07	1.13	1.51	1.00	1.24	1.11
time (sec)	N/A	0.239	0.163	0.763	0.035	0.086	0.158	0.544	0.175	0.103

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	87	171	74	0	693	32	0
N.S.	1	1.00	0.89	1.36	2.67	1.16	0.00	10.83	0.50	0.00
time (sec)	N/A	0.325	0.192	0.894	0.084	0.080	0.000	0.590	0.168	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	72	122	196	105	0	533	79	0
N.S.	1	1.00	0.82	1.39	2.23	1.19	0.00	6.06	0.90	0.00
time (sec)	N/A	0.380	0.431	0.959	0.104	0.079	0.000	0.556	0.165	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	94	160	265	178	0	6033	165	0
N.S.	1	1.00	0.76	1.30	2.15	1.45	0.00	49.05	1.34	0.00
time (sec)	N/A	0.404	0.951	1.115	0.149	0.090	0.000	0.368	0.174	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	232	213	959	382	779	367	558	497
N.S.	1	1.00	0.98	0.90	4.05	1.61	3.29	1.55	2.35	2.10
time (sec)	N/A	0.518	1.174	2.204	0.077	0.095	0.417	0.782	0.369	38.098

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	249	158	502	226	456	225	341	281
N.S.	1	1.00	1.37	0.87	2.76	1.24	2.51	1.24	1.87	1.54
time (sec)	N/A	0.419	0.744	1.786	0.051	0.091	0.300	0.728	0.236	36.691

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	96	105	202	109	219	115	157	143
N.S.	1	1.00	0.85	0.93	1.79	0.96	1.94	1.02	1.39	1.27
time (sec)	N/A	0.317	6.530	1.430	0.038	0.091	0.214	0.238	0.266	35.980

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	134	203	336	149	0	7139	59	0
N.S.	1	1.00	0.86	1.30	2.15	0.96	0.00	45.76	0.38	0.00
time (sec)	N/A	0.539	0.384	1.482	0.110	0.122	0.000	0.462	0.166	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	232	284	371	218	0	1050	157	0
N.S.	1	1.00	1.27	1.55	2.03	1.19	0.00	5.74	0.86	0.00
time (sec)	N/A	0.591	0.694	1.602	0.172	0.088	0.000	0.355	0.185	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	395	359	476	368	0	120406	0	0
N.S.	1	1.00	1.61	1.47	1.94	1.50	0.00	491.45	0.00	0.00
time (sec)	N/A	0.688	1.408	2.034	0.246	0.097	0.000	2.195	0.256	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	495	463	401	0	0	2173	0	0	219	0
N.S.	1	0.94	0.81	0.00	0.00	4.39	0.00	0.00	0.44	0.00
time (sec)	N/A	1.842	0.277	0.000	0.000	0.253	0.000	0.000	0.187	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	347	296	0	0	1543	0	0	159	0
N.S.	1	0.95	0.81	0.00	0.00	4.20	0.00	0.00	0.43	0.00
time (sec)	N/A	1.404	0.232	0.000	0.000	0.238	0.000	0.000	0.188	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	237	182	492	0	997	0	0	101	0
N.S.	1	1.01	0.78	2.10	0.00	4.26	0.00	0.00	0.43	0.00
time (sec)	N/A	0.873	0.052	0.688	0.000	0.215	0.000	0.000	0.206	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	27	17	22	56	22
N.S.	1	1.00	1.10	1.00	1.10	1.35	0.85	1.10	2.80	1.10
time (sec)	N/A	0.231	1.028	0.352	0.612	0.096	10.493	0.249	0.185	35.886

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	51	19	22	164	22
N.S.	1	1.00	1.10	1.00	1.10	2.55	0.95	1.10	8.20	1.10
time (sec)	N/A	0.229	0.887	0.366	0.992	0.074	64.498	0.507	0.190	36.182

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	925	857	742	0	0	5112	0	0	0	0
N.S.	1	0.93	0.80	0.00	0.00	5.53	0.00	0.00	0.00	0.00
time (sec)	N/A	4.303	3.837	0.000	0.000	0.401	0.000	0.000	0.265	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	671	631	530	0	0	3091	0	0	0	0
N.S.	1	0.94	0.79	0.00	0.00	4.61	0.00	0.00	0.00	0.00
time (sec)	N/A	2.873	2.038	0.000	0.000	0.312	0.000	0.000	0.206	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	317	236	641	0	1512	0	0	430	0
N.S.	1	1.04	0.77	2.10	0.00	4.96	0.00	0.00	1.41	0.00
time (sec)	N/A	1.170	1.272	3.909	0.000	0.295	0.000	0.000	0.193	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1601	64	19	22	173	22
N.S.	1	1.00	1.10	1.00	80.05	3.20	0.95	1.10	8.65	1.10
time (sec)	N/A	0.231	43.463	0.826	7.399	0.084	175.865	0.384	0.219	35.390

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2265	109	0	22	8816	22
N.S.	1	1.00	1.10	1.00	113.25	5.45	0.00	1.10	440.80	1.10
time (sec)	N/A	0.228	170.682	0.806	20.344	0.094	0.000	2.476	1.056	35.259

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10	1.10
time (sec)	N/A	0.224	1.423	0.296	0.576	0.091	0.000	0.807	0.262	35.430

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	415	0	0	436	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.72	0.00	0.00	0.00	0.00
time (sec)	N/A	1.070	11.710	0.000	0.000	0.115	0.000	0.000	0.342	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	268	0	0	278	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.627	10.261	0.000	0.000	0.098	0.000	0.000	0.264	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	138	0	0	136	0	0	198	0
N.S.	1	1.00	0.93	0.00	0.00	0.92	0.00	0.00	1.34	0.00
time (sec)	N/A	0.357	0.214	0.000	0.000	0.089	0.000	0.000	0.197	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	97	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	4.85	1.10
time (sec)	N/A	0.231	1.045	0.296	0.264	0.072	0.948	0.228	0.207	34.901

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	46	19	22	38	22
N.S.	1	1.00	1.10	1.00	1.10	2.30	0.95	1.10	1.90	1.10
time (sec)	N/A	0.231	10.958	0.701	0.892	0.080	10.969	0.256	0.220	35.196

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	193	261	606	1303	1044	0	0	892	0
N.S.	1	1.18	1.59	3.70	7.95	6.37	0.00	0.00	5.44	0.00
time (sec)	N/A	0.980	2.821	1.051	0.254	0.108	0.000	0.000	0.200	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	147	213	335	402	583	0	0	320	0
N.S.	1	1.14	1.65	2.60	3.12	4.52	0.00	0.00	2.48	0.00
time (sec)	N/A	0.742	2.008	0.858	0.220	0.106	0.000	0.000	0.194	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	199	88	273	151	456	656	184	80
N.S.	1	1.07	2.62	1.16	3.59	1.99	6.00	8.63	2.42	1.05
time (sec)	N/A	0.465	0.913	0.835	0.114	0.086	0.670	0.275	0.172	35.502

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	72	29	50	54	80	32	47	27
N.S.	1	1.00	2.57	1.04	1.79	1.93	2.86	1.14	1.68	0.96
time (sec)	N/A	0.262	0.083	0.475	0.112	0.085	0.579	0.199	0.167	35.036

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	373	34	32	28	37	28
N.S.	1	1.00	1.08	1.00	14.35	1.31	1.23	1.08	1.42	1.08
time (sec)	N/A	0.225	16.671	0.358	0.338	0.081	1.236	0.223	0.178	35.196

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	522	58	63	28	65	28
N.S.	1	1.00	1.08	1.00	20.08	2.23	2.42	1.08	2.50	1.08
time (sec)	N/A	0.224	12.464	0.342	0.692	0.070	4.262	0.500	0.180	35.331

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	274	1314	759	4592	1313	0	0	1284	0
N.S.	1	1.11	5.32	3.07	18.59	5.32	0.00	0.00	5.20	0.00
time (sec)	N/A	2.021	4.644	1.753	0.431	0.123	0.000	0.000	0.197	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	203	295	419	602	716	0	0	552	0
N.S.	1	1.08	1.57	2.23	3.20	3.81	0.00	0.00	2.94	0.00
time (sec)	N/A	1.447	3.830	2.408	0.307	0.107	0.000	0.000	0.171	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	113	236	127	1762	196	1867	2952	273	164
N.S.	1	1.03	2.15	1.15	16.02	1.78	16.97	26.84	2.48	1.49
time (sec)	N/A	0.811	7.316	1.540	0.154	0.116	1.246	0.466	0.171	36.082

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	42	85	48	129	69	422	77	74	69
N.S.	1	0.93	1.89	1.07	2.87	1.53	9.38	1.71	1.64	1.53
time (sec)	N/A	0.357	0.212	0.527	0.112	0.079	1.046	0.220	0.186	35.511

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1266	39	34	30	39	30
N.S.	1	1.00	1.07	1.00	45.21	1.39	1.21	1.07	1.39	1.07
time (sec)	N/A	0.250	10.445	1.094	0.487	0.075	2.656	0.251	0.223	35.439

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1388	63	65	30	67	30
N.S.	1	1.00	1.07	1.00	49.57	2.25	2.32	1.07	2.39	1.07
time (sec)	N/A	0.254	12.497	1.138	0.910	0.075	13.145	0.644	0.278	35.705

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	408	538	1054	0	1565	0	0	0	0
N.S.	1	1.11	1.46	2.86	0.00	4.24	0.00	0.00	0.00	0.00
time (sec)	N/A	3.128	4.140	2.204	0.000	0.137	0.000	0.000	0.250	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	305	830	591	0	846	0	0	0	0
N.S.	1	1.10	2.99	2.13	0.00	3.04	0.00	0.00	0.00	0.00
time (sec)	N/A	2.407	5.231	2.720	0.000	0.112	0.000	0.000	0.180	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	173	298	187	0	250	4653	7564	515	246
N.S.	1	1.12	1.92	1.21	0.00	1.61	30.02	48.80	3.32	1.59
time (sec)	N/A	1.253	8.755	2.069	0.000	0.142	2.245	0.914	0.194	41.172

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	117	81	212	92	1127	91	115	92
N.S.	1	1.00	1.56	1.08	2.83	1.23	15.03	1.21	1.53	1.23
time (sec)	N/A	0.322	0.514	0.671	0.109	0.076	2.013	0.215	0.181	42.790

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	45	0	30	39	30
N.S.	1	1.00	1.07	1.00	0.00	1.61	0.00	1.07	1.39	1.07
time (sec)	N/A	0.249	11.253	1.911	0.000	0.079	0.000	0.219	0.191	38.016

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	69	0	30	67	30
N.S.	1	1.00	1.07	1.00	0.00	2.46	0.00	1.07	2.39	1.07
time (sec)	N/A	0.250	6.788	1.991	0.000	0.075	0.000	0.992	0.222	37.861

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	383	443	1151	2796	2924	0	0	265	0
N.S.	1	1.09	1.26	3.27	7.94	8.31	0.00	0.00	0.75	0.00
time (sec)	N/A	2.157	4.364	1.503	0.511	0.189	0.000	0.000	0.207	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	265	330	643	1418	1642	0	0	194	0
N.S.	1	1.06	1.33	2.58	5.69	6.59	0.00	0.00	0.78	0.00
time (sec)	N/A	1.469	3.893	1.310	0.210	0.147	0.000	0.000	0.178	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	135	300	245	514	609	0	0	121	0
N.S.	1	1.01	2.24	1.83	3.84	4.54	0.00	0.00	0.90	0.00
time (sec)	N/A	0.692	7.682	1.282	0.141	0.127	0.000	0.000	0.182	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	34	51	97	0	38	62	39
N.S.	1	1.00	1.26	0.89	1.34	2.55	0.00	1.00	1.63	1.03
time (sec)	N/A	0.311	0.176	0.603	0.031	0.085	0.000	0.231	0.166	35.602

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	559	34	32	28	37	30
N.S.	1	1.00	1.08	1.00	21.50	1.31	1.23	1.08	1.42	1.15
time (sec)	N/A	0.223	15.607	0.586	0.575	0.082	1.047	15.462	0.197	36.023

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	915	58	63	0	65	30
N.S.	1	1.00	1.08	1.00	35.19	2.23	2.42	0.00	2.50	1.15
time (sec)	N/A	0.228	17.270	0.580	1.139	0.087	4.295	0.000	0.204	36.456

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	538	1052	1774	0	4799	0	0	359	0
N.S.	1	1.16	2.27	3.83	0.00	10.37	0.00	0.00	0.78	0.00
time (sec)	N/A	3.819	10.619	1.753	0.000	0.250	0.000	0.000	0.199	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	376	709	984	0	2539	0	0	273	0
N.S.	1	1.15	2.17	3.01	0.00	7.76	0.00	0.00	0.83	0.00
time (sec)	N/A	2.629	9.571	1.461	0.000	0.165	0.000	0.000	0.201	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	171	396	369	0	858	0	0	181	0
N.S.	1	1.01	2.34	2.18	0.00	5.08	0.00	0.00	1.07	0.00
time (sec)	N/A	1.158	8.435	1.347	0.000	0.127	0.000	0.000	0.247	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	57	59	112	156	0	88	102	83
N.S.	1	1.02	1.12	1.16	2.20	3.06	0.00	1.73	2.00	1.63
time (sec)	N/A	0.435	0.255	0.713	0.032	0.080	0.000	0.154	0.246	36.061

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	36	34	0	39	30
N.S.	1	1.00	1.07	1.00	0.00	1.29	1.21	0.00	1.39	1.07
time (sec)	N/A	0.249	35.547	0.651	0.000	0.091	1.144	0.000	0.197	36.677

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	60	65	0	67	30
N.S.	1	1.00	1.07	1.00	0.00	2.14	2.32	0.00	2.39	1.07
time (sec)	N/A	0.261	73.736	0.638	0.000	0.093	4.433	0.000	0.232	37.039

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	B	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	0	1493	2326	12815	7842	0	0	397	0
N.S.	1	0.00	2.49	3.88	21.36	13.07	0.00	0.00	0.66	0.00
time (sec)	N/A	0.000	21.723	1.954	10.626	0.344	0.000	0.000	0.195	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	B	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	0	1452	1257	6160	4026	0	0	309	0
N.S.	1	0.00	3.70	3.21	15.71	10.27	0.00	0.00	0.79	0.00
time (sec)	N/A	0.000	20.640	1.602	2.086	0.214	0.000	0.000	0.175	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	285	484	487	2080	1359	0	0	211	0
N.S.	1	1.32	2.24	2.25	9.63	6.29	0.00	0.00	0.98	0.00
time (sec)	N/A	2.242	10.726	1.431	0.593	0.147	0.000	0.000	0.181	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	83	85	87	157	232	0	112	128	116
N.S.	1	1.01	1.04	1.06	1.91	2.83	0.00	1.37	1.56	1.41
time (sec)	N/A	0.547	0.663	0.768	0.035	0.083	0.000	0.170	0.217	36.091

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	0	28	7381	36	34	0	39	30
N.S.	1	1.00	0.00	1.00	263.61	1.29	1.21	0.00	1.39	1.07
time (sec)	N/A	0.252	0.000	0.645	11.669	0.113	1.971	0.000	0.198	37.363

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	0	28	9726	60	65	0	67	30
N.S.	1	1.00	0.00	1.00	347.36	2.14	2.32	0.00	2.39	1.07
time (sec)	N/A	0.251	0.000	0.654	24.479	0.136	4.447	0.000	0.221	38.253

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	33	26	30	3499	30
N.S.	1	1.00	1.07	1.00	1.07	1.18	0.93	1.07	124.96	1.07
time (sec)	N/A	0.246	11.279	0.815	0.631	0.075	13.925	0.481	0.349	36.361

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	377	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	14.50	1.08
time (sec)	N/A	0.219	2.327	0.368	0.237	0.072	4.157	0.660	0.188	36.159

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	154	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	7.70	1.10
time (sec)	N/A	0.236	1.147	0.356	0.217	0.071	1.427	0.534	0.185	36.485

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	30	30
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.15	1.15
time (sec)	N/A	0.221	40.490	0.497	0.446	0.077	11.168	0.615	0.217	36.077

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	26	30	6170	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.93	1.07	220.36	1.07
time (sec)	N/A	0.241	45.991	0.516	0.949	0.079	46.048	0.598	0.336	36.126

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	544	485	956	0	0	2322	0	0	281	0
N.S.	1	0.89	1.76	0.00	0.00	4.27	0.00	0.00	0.52	0.00
time (sec)	N/A	1.889	4.689	0.000	0.000	0.269	0.000	0.000	0.247	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	408	369	445	0	0	1646	0	0	208	0
N.S.	1	0.90	1.09	0.00	0.00	4.03	0.00	0.00	0.51	0.00
time (sec)	N/A	1.516	2.912	0.000	0.000	0.233	0.000	0.000	0.272	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	259	299	548	0	1052	0	0	133	0
N.S.	1	0.97	1.12	2.05	0.00	3.94	0.00	0.00	0.50	0.00
time (sec)	N/A	0.956	2.293	0.849	0.000	0.212	0.000	0.000	0.184	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	59	68	0	237	253	77	72	139
N.S.	1	1.11	1.04	1.19	0.00	4.16	4.44	1.35	1.26	2.44
time (sec)	N/A	0.297	0.124	0.463	0.000	0.096	11.831	0.604	0.164	36.432

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	643	573	1617	0	0	2671	0	0	674	0
N.S.	1	0.89	2.51	0.00	0.00	4.15	0.00	0.00	1.05	0.00
time (sec)	N/A	2.970	8.402	0.000	0.000	0.313	0.000	0.000	0.194	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	432	531	0	0	1855	0	0	428	0
N.S.	1	0.90	1.11	0.00	0.00	3.87	0.00	0.00	0.89	0.00
time (sec)	N/A	2.268	4.257	0.000	0.000	0.267	0.000	0.000	0.166	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	298	1973	625	0	1154	0	0	224	0
N.S.	1	0.96	6.39	2.02	0.00	3.73	0.00	0.00	0.72	0.00
time (sec)	N/A	1.346	16.532	2.158	0.000	0.220	0.000	0.000	0.188	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	84	71	90	0	283	1690	99	97	127
N.S.	1	1.12	0.95	1.20	0.00	3.77	22.53	1.32	1.29	1.69
time (sec)	N/A	0.410	0.717	0.588	0.000	0.113	110.283	0.315	0.166	37.027

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	789	713	1923	0	0	3008	0	0	1199	0
N.S.	1	0.90	2.44	0.00	0.00	3.81	0.00	0.00	1.52	0.00
time (sec)	N/A	4.266	7.379	0.000	0.000	0.509	0.000	0.000	0.235	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	592	540	1166	0	0	2050	0	0	584	0
N.S.	1	0.91	1.97	0.00	0.00	3.46	0.00	0.00	0.99	0.00
time (sec)	N/A	3.178	5.684	0.000	0.000	0.289	0.000	0.000	0.199	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	364	2036	686	0	1247	0	0	1079	0
N.S.	1	0.97	5.41	1.82	0.00	3.32	0.00	0.00	2.87	0.00
time (sec)	N/A	1.734	17.611	2.773	0.000	0.246	0.000	0.000	0.201	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	124	97	146	0	359	0	151	167	199
N.S.	1	1.16	0.91	1.36	0.00	3.36	0.00	1.41	1.56	1.86
time (sec)	N/A	0.600	1.327	0.720	0.000	0.106	0.000	0.164	0.173	37.337

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	732	675	894	0	0	3584	0	0	297	0
N.S.	1	0.92	1.22	0.00	0.00	4.90	0.00	0.00	0.41	0.00
time (sec)	N/A	2.620	3.992	0.000	0.000	0.404	0.000	0.000	0.188	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	528	487	573	0	0	2412	0	0	224	0
N.S.	1	0.92	1.09	0.00	0.00	4.57	0.00	0.00	0.42	0.00
time (sec)	N/A	2.106	2.484	0.000	0.000	0.311	0.000	0.000	0.191	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	313	2016	651	0	1428	0	0	149	0
N.S.	1	0.96	6.20	2.00	0.00	4.39	0.00	0.00	0.46	0.00
time (sec)	N/A	1.313	16.586	1.116	0.000	0.324	0.000	0.000	0.181	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	73	77	67	0	297	0	83	88	173
N.S.	1	1.09	1.15	1.00	0.00	4.43	0.00	1.24	1.31	2.58
time (sec)	N/A	0.362	0.688	0.550	0.000	0.122	0.000	0.168	0.174	39.090

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	882	836	1735	0	0	4562	0	0	408	0
N.S.	1	0.95	1.97	0.00	0.00	5.17	0.00	0.00	0.46	0.00
time (sec)	N/A	4.557	11.293	0.000	0.000	0.453	0.000	0.000	0.218	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	639	604	933	0	0	2972	0	0	317	0
N.S.	1	0.95	1.46	0.00	0.00	4.65	0.00	0.00	0.50	0.00
time (sec)	N/A	3.222	10.896	0.000	0.000	0.339	0.000	0.000	0.210	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	355	2171	766	0	1686	0	0	220	0
N.S.	1	0.96	5.87	2.07	0.00	4.56	0.00	0.00	0.59	0.00
time (sec)	N/A	1.723	17.038	1.244	0.000	0.323	0.000	0.000	0.261	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	94	111	101	0	400	0	130	139	222
N.S.	1	1.13	1.34	1.22	0.00	4.82	0.00	1.57	1.67	2.67
time (sec)	N/A	0.493	1.958	0.618	0.000	0.184	0.000	0.145	0.258	37.974

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	33	26	30	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.18	0.93	1.07	1.07	1.07
time (sec)	N/A	0.242	18.726	0.718	0.685	0.073	16.961	0.170	0.187	37.863

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08	1.08
time (sec)	N/A	0.223	1.396	0.317	0.266	0.071	4.913	0.178	0.198	37.634

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	97	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	4.85	1.10
time (sec)	N/A	0.236	1.109	0.259	0.241	0.096	1.618	0.143	0.216	37.682

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	28	30
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08	1.15
time (sec)	N/A	0.221	37.087	0.412	1.890	0.074	11.922	0.142	0.195	37.800

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	26	30	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.93	1.07	1.07	1.07
time (sec)	N/A	0.247	46.066	0.424	10.141	0.082	50.201	0.158	0.232	37.771

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	2286	750	0	1506	0	0	466	0
N.S.	1	1.00	3.98	1.31	0.00	2.62	0.00	0.00	0.81	0.00
time (sec)	N/A	1.909	17.040	3.816	0.000	0.369	0.000	0.000	0.218	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1106	1106	3595	0	0	3122	0	0	0	0
N.S.	1	1.00	3.25	0.00	0.00	2.82	0.00	0.00	0.00	0.00
time (sec)	N/A	3.007	25.599	0.000	0.000	0.341	0.000	0.000	0.231	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1512	1512	4970	0	0	5184	0	0	0	0
N.S.	1	1.00	3.29	0.00	0.00	3.43	0.00	0.00	0.00	0.00
time (sec)	N/A	3.611	24.857	0.000	0.000	0.532	0.000	0.000	0.291	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	2666	1084	0	2429	0	0	1332	0
N.S.	1	1.00	3.55	1.44	0.00	3.23	0.00	0.00	1.77	0.00
time (sec)	N/A	3.251	19.802	6.603	0.000	0.355	0.000	0.000	0.202	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1584	1584	13567	0	0	5755	0	0	0	0
N.S.	1	1.00	8.57	0.00	0.00	3.63	0.00	0.00	0.00	0.00
time (sec)	N/A	5.997	23.904	0.000	0.000	0.439	0.000	0.000	0.386	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2348	2348	29732	0	0	10614	0	0	0	0
N.S.	1	1.00	12.66	0.00	0.00	4.52	0.00	0.00	0.00	0.00
time (sec)	N/A	7.826	28.735	0.000	0.000	0.877	0.000	0.000	1.091	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	156	276	691	519	492	0	0	175	0
N.S.	1	1.03	1.83	4.58	3.44	3.26	0.00	0.00	1.16	0.00
time (sec)	N/A	0.708	2.583	1.303	0.129	0.109	0.000	0.000	0.171	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	117	221	433	297	304	0	0	126	0
N.S.	1	1.03	1.94	3.80	2.61	2.67	0.00	0.00	1.11	0.00
time (sec)	N/A	0.561	1.646	1.086	0.103	0.092	0.000	0.000	0.200	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	81	246	203	116	156	0	0	77	0
N.S.	1	1.03	3.11	2.57	1.47	1.97	0.00	0.00	0.97	0.00
time (sec)	N/A	0.390	6.552	1.178	0.100	0.086	0.000	0.000	0.187	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	18	16	19	18	16	24	17	16	16
N.S.	1	1.12	1.00	1.19	1.12	1.00	1.50	1.06	1.00	1.00
time (sec)	N/A	0.200	0.010	0.357	0.033	0.081	0.242	0.112	0.243	0.057

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	32	28	51	28
N.S.	1	1.00	1.08	1.00	1.08	1.31	1.23	1.08	1.96	1.08
time (sec)	N/A	0.217	4.304	0.446	0.211	0.069	1.258	0.128	0.263	37.941

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	58	63	28	154	28
N.S.	1	1.00	1.08	1.00	1.08	2.23	2.42	1.08	5.92	1.08
time (sec)	N/A	0.213	7.742	0.478	0.334	0.095	4.545	0.552	0.174	38.152

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	102	108	534	157	984	1077	217	184
N.S.	1	1.01	1.03	1.09	5.39	1.59	9.94	10.88	2.19	1.86
time (sec)	N/A	0.580	1.608	0.972	0.150	0.084	2.305	0.158	0.169	38.569

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	88	309	96	605	656	131	110
N.S.	1	1.00	0.99	1.17	4.12	1.28	8.07	8.75	1.75	1.47
time (sec)	N/A	0.452	1.238	0.848	0.125	0.081	1.735	0.135	0.220	39.221

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	44	151	49	326	322	67	53
N.S.	1	1.00	1.04	0.86	2.96	0.96	6.39	6.31	1.31	1.04
time (sec)	N/A	0.329	2.805	0.722	0.120	0.127	1.312	0.135	0.162	37.876

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	97	19	52	17	88	34	18	29
N.S.	1	1.00	5.11	1.00	2.74	0.89	4.63	1.79	0.95	1.53
time (sec)	N/A	0.206	0.186	0.552	0.107	0.078	1.022	0.110	0.170	37.604

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	58	104	163	73	0	670	33	0
N.S.	1	0.97	0.81	1.44	2.26	1.01	0.00	9.31	0.46	0.00
time (sec)	N/A	0.528	0.498	0.878	0.094	0.084	0.000	0.181	0.214	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	80	137	172	101	0	3192	73	0
N.S.	1	1.02	0.84	1.44	1.81	1.06	0.00	33.60	0.77	0.00
time (sec)	N/A	0.641	0.591	1.010	0.121	0.083	0.000	0.584	0.181	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	214	132	159	572	270	2725	3893	400	339
N.S.	1	0.98	0.60	0.73	2.61	1.23	12.44	17.78	1.83	1.55
time (sec)	N/A	1.112	2.278	1.227	0.068	0.089	4.610	0.302	0.185	38.761

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	146	95	112	289	149	1528	2190	219	187
N.S.	1	0.98	0.64	0.75	1.94	1.00	10.26	14.70	1.47	1.26
time (sec)	N/A	0.773	1.633	1.028	0.050	0.079	3.439	0.232	3.710	38.391

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	52	69	114	67	724	947	90	84
N.S.	1	1.00	0.57	0.76	1.25	0.74	7.96	10.41	0.99	0.92
time (sec)	N/A	0.523	7.271	1.766	0.037	0.075	2.648	0.172	0.205	38.237

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	25	25	25	158	25	24	22
N.S.	1	1.00	1.04	1.09	1.09	1.09	6.87	1.09	1.04	0.96
time (sec)	N/A	0.217	0.053	0.434	0.034	0.091	1.959	0.102	0.167	37.636

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	123	105	163	282	127	0	4510	45	0
N.S.	1	0.96	0.82	1.27	2.20	0.99	0.00	35.23	0.35	0.00
time (sec)	N/A	1.024	0.679	1.042	0.122	0.075	0.000	0.398	0.182	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	176	203	229	309	188	0	46878	67	0
N.S.	1	1.01	1.16	1.31	1.77	1.07	0.00	267.87	0.38	0.00
time (sec)	N/A	1.399	0.835	1.311	0.180	0.082	0.000	2.122	28.062	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	496	1025	1196	3854	1884	0	0	0	0
N.S.	1	0.99	2.04	2.38	7.68	3.75	0.00	0.00	0.00	0.00
time (sec)	N/A	2.929	10.311	2.510	0.855	0.250	0.000	0.000	0.234	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	274	725	617	1932	1064	0	0	927	0
N.S.	1	0.99	2.61	2.22	6.95	3.83	0.00	0.00	3.33	0.00
time (sec)	N/A	1.684	9.076	1.415	0.287	0.132	0.000	0.000	0.186	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	163	655	254	725	508	0	0	362	0
N.S.	1	0.95	3.81	1.48	4.22	2.95	0.00	0.00	2.10	0.00
time (sec)	N/A	0.888	9.585	2.305	0.173	0.110	0.000	0.000	0.184	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	39	30	43	47	58	0	55	85	33
N.S.	1	1.05	0.81	1.16	1.27	1.57	0.00	1.49	2.30	0.89
time (sec)	N/A	0.252	0.044	0.751	0.032	0.081	0.000	0.114	0.164	0.087

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	1504	34	32	28	132	30
N.S.	1	1.00	1.08	1.00	57.85	1.31	1.23	1.08	5.08	1.15
time (sec)	N/A	0.223	17.541	0.727	2.681	0.097	1.325	2.251	0.204	37.862

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	2018	58	63	28	23269	30
N.S.	1	1.00	1.08	1.00	77.62	2.23	2.42	1.08	894.96	1.15
time (sec)	N/A	0.223	24.952	0.728	5.154	0.094	8.629	11.523	0.329	38.150

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	474	1173	1135	5130	1531	0	0	0	0
N.S.	1	1.00	2.47	2.39	10.80	3.22	0.00	0.00	0.00	0.00
time (sec)	N/A	2.745	10.162	2.727	0.991	0.160	0.000	0.000	0.216	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	326	637	568	1328	859	0	0	1046	0
N.S.	1	0.95	1.86	1.66	3.87	2.50	0.00	0.00	3.05	0.00
time (sec)	N/A	1.886	8.260	1.615	0.398	0.121	0.000	0.000	0.184	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	148	231	166	1115	156	0	5555	265	240
N.S.	1	0.97	1.52	1.09	7.34	1.03	0.00	36.55	1.74	1.58
time (sec)	N/A	0.934	7.515	2.472	0.066	0.093	0.000	1.293	0.187	44.834

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	51	129	49	0	67	68	71
N.S.	1	1.00	1.07	1.21	3.07	1.17	0.00	1.60	1.62	1.69
time (sec)	N/A	0.291	0.069	0.726	0.036	0.068	0.000	0.132	0.169	37.754

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	3760	36	34	30	39	30
N.S.	1	1.00	1.07	1.00	134.29	1.29	1.21	1.07	1.39	1.07
time (sec)	N/A	0.247	21.704	0.826	10.227	0.087	1.351	30.713	0.235	38.200

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	4597	60	65	30	57348	30
N.S.	1	1.00	1.07	1.00	164.18	2.14	2.32	1.07	2048.14	1.07
time (sec)	N/A	0.250	27.880	0.859	21.451	0.098	8.922	69.776	0.606	38.513

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	698	0	2278	2041	0	2572	0	0	0	0
N.S.	1	0.00	3.26	2.92	0.00	3.68	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	12.726	3.187	0.000	0.249	0.000	0.000	0.222	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	421	1579	1035	0	1517	0	0	0	0
N.S.	1	0.98	3.66	2.40	0.00	3.52	0.00	0.00	0.00	0.00
time (sec)	N/A	2.811	10.009	2.024	0.000	0.187	0.000	0.000	0.222	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	225	1171	434	0	792	0	0	352	0
N.S.	1	0.93	4.86	1.80	0.00	3.29	0.00	0.00	1.46	0.00
time (sec)	N/A	1.143	13.463	2.595	0.000	0.178	0.000	0.000	0.194	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	80	75	67	91	125	0	85	214	74
N.S.	1	0.96	0.90	0.81	1.10	1.51	0.00	1.02	2.58	0.89
time (sec)	N/A	0.299	0.113	1.252	0.028	0.091	0.000	0.140	0.181	37.626

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	36	34	0	39	30
N.S.	1	1.00	1.07	1.00	0.00	1.29	1.21	0.00	1.39	1.07
time (sec)	N/A	0.261	39.130	0.882	0.000	0.124	2.847	0.000	0.208	38.876

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	60	65	0	97475	30
N.S.	1	1.00	1.07	1.00	0.00	2.14	2.32	0.00	3481.25	1.07
time (sec)	N/A	0.251	51.846	0.944	0.000	0.157	9.348	0.000	0.615	38.895

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	439	405	0	0	340	0	0	32	0
N.S.	1	0.98	0.90	0.00	0.00	0.76	0.00	0.00	0.07	0.00
time (sec)	N/A	1.088	12.203	0.000	0.000	0.099	0.000	0.000	28.470	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	280	253	0	0	189	0	0	32	0
N.S.	1	1.01	0.91	0.00	0.00	0.68	0.00	0.00	0.12	0.00
time (sec)	N/A	0.883	9.893	0.000	0.000	0.092	0.000	0.000	0.689	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	220	0	0	130	0	0	128	0
N.S.	1	1.00	1.43	0.00	0.00	0.84	0.00	0.00	0.83	0.00
time (sec)	N/A	0.440	1.336	0.000	0.000	0.092	0.000	0.000	0.184	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	87	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	3.35	1.08
time (sec)	N/A	0.222	12.046	0.422	0.220	0.070	4.211	0.161	0.202	37.745

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	154	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	7.70	1.10
time (sec)	N/A	0.234	0.234	0.000	0.224	0.072	1.463	0.170	0.214	0.002

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	0	28	24	28	30	30
N.S.	1	1.00	1.08	1.00	0.00	1.08	0.92	1.08	1.15	1.15
time (sec)	N/A	0.218	141.407	0.562	0.000	0.104	25.218	0.183	0.303	37.745

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	0	30	26	30	6074	30
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.93	1.07	216.93	1.07
time (sec)	N/A	0.239	25.931	0.589	0.000	0.114	104.360	0.196	0.515	38.108

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	432	438	410	0	0	1777	0	0	386	0
N.S.	1	1.01	0.95	0.00	0.00	4.11	0.00	0.00	0.89	0.00
time (sec)	N/A	1.500	0.330	0.000	0.000	0.234	0.000	0.000	0.211	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	322	302	0	0	1235	0	0	271	0
N.S.	1	1.01	0.94	0.00	0.00	3.86	0.00	0.00	0.85	0.00
time (sec)	N/A	1.143	0.195	0.000	0.000	0.221	0.000	0.000	0.196	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	197	1006	0	773	0	0	47	0
N.S.	1	1.00	0.93	4.75	0.00	3.65	0.00	0.00	0.22	0.00
time (sec)	N/A	0.673	0.057	0.970	0.000	0.204	0.000	0.000	0.174	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	41	19	18	18
N.S.	1	1.00	1.00	1.06	1.00	1.00	2.28	1.06	1.00	1.00
time (sec)	N/A	0.198	0.007	0.426	0.032	0.089	0.314	0.166	0.189	0.057

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	618	576	1025	0	0	2331	0	0	0	0
N.S.	1	0.93	1.66	0.00	0.00	3.77	0.00	0.00	0.00	0.00
time (sec)	N/A	2.692	4.206	0.000	0.000	0.376	0.000	0.000	0.424	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	435	536	0	0	1636	0	0	1083	0
N.S.	1	0.95	1.17	0.00	0.00	3.56	0.00	0.00	2.35	0.00
time (sec)	N/A	1.950	3.701	0.000	0.000	0.266	0.000	0.000	0.266	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	301	1991	1105	0	1034	0	0	521	0
N.S.	1	1.01	6.70	3.72	0.00	3.48	0.00	0.00	1.75	0.00
time (sec)	N/A	1.195	16.394	2.254	0.000	0.218	0.000	0.000	0.204	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	81	361	96	0	214	1923	95	65	318
N.S.	1	1.16	5.16	1.37	0.00	3.06	27.47	1.36	0.93	4.54
time (sec)	N/A	0.425	1.420	0.671	0.000	0.102	113.176	0.170	0.165	40.399

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	737	667	2452	0	0	2684	0	0	30	0
N.S.	1	0.91	3.33	0.00	0.00	3.64	0.00	0.00	0.04	0.00
time (sec)	N/A	3.647	10.424	0.000	0.000	0.298	0.000	0.000	200.024	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	536	483	2283	0	0	1779	0	0	0	0
N.S.	1	0.90	4.26	0.00	0.00	3.32	0.00	0.00	0.00	0.00
time (sec)	N/A	2.361	5.694	0.000	0.000	0.250	0.000	0.000	10.348	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	318	816	1750	0	1037	0	0	1226	0
N.S.	1	0.91	2.32	4.99	0.00	2.95	0.00	0.00	3.49	0.00
time (sec)	N/A	1.371	4.065	3.612	0.000	0.231	0.000	0.000	0.222	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	54	54	55	53	0	60	141	55
N.S.	1	0.89	0.89	0.89	0.90	0.87	0.00	0.98	2.31	0.90
time (sec)	N/A	0.257	0.068	0.598	0.029	0.118	0.000	0.158	0.174	0.107

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	937	814	2506	0	0	3069	0	0	316	0
N.S.	1	0.87	2.67	0.00	0.00	3.28	0.00	0.00	0.34	0.00
time (sec)	N/A	3.068	11.804	0.000	0.000	0.376	0.000	0.000	0.226	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	667	591	1461	0	0	2035	0	0	244	0
N.S.	1	0.89	2.19	0.00	0.00	3.05	0.00	0.00	0.37	0.00
time (sec)	N/A	2.170	6.908	0.000	0.000	0.323	0.000	0.000	0.283	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	386	1185	846	0	1181	0	0	166	0
N.S.	1	0.93	2.87	2.05	0.00	2.86	0.00	0.00	0.40	0.00
time (sec)	N/A	1.358	10.967	1.395	0.000	0.291	0.000	0.000	0.282	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	64	71	64	62	0	76	104	69
N.S.	1	1.07	0.85	0.95	0.85	0.83	0.00	1.01	1.39	0.92
time (sec)	N/A	0.298	0.039	0.625	0.033	0.094	0.000	0.161	0.174	0.216

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	923	774	1438	0	0	4116	0	0	0	0
N.S.	1	0.84	1.56	0.00	0.00	4.46	0.00	0.00	0.00	0.00
time (sec)	N/A	3.436	10.313	0.000	0.000	0.450	0.000	0.000	0.472	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	659	561	1122	0	0	2659	0	0	394	0
N.S.	1	0.85	1.70	0.00	0.00	4.03	0.00	0.00	0.60	0.00
time (sec)	N/A	2.495	8.914	0.000	0.000	0.359	0.000	0.000	0.220	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	324	2180	2065	0	1267	0	0	251	0
N.S.	1	0.93	6.25	5.92	0.00	3.63	0.00	0.00	0.72	0.00
time (sec)	N/A	1.555	16.117	1.773	0.000	0.266	0.000	0.000	0.218	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	152	112	0	305	0	107	131	149
N.S.	1	1.07	1.81	1.33	0.00	3.63	0.00	1.27	1.56	1.77
time (sec)	N/A	0.357	0.229	0.647	0.000	0.129	0.000	0.172	0.218	39.284

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	26	30	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.93	1.07	1.07	1.07
time (sec)	N/A	0.239	14.771	0.680	0.669	0.081	16.913	0.175	0.208	38.927

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	226	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	8.69	1.08
time (sec)	N/A	0.226	10.163	0.346	0.339	0.072	4.995	0.153	0.218	39.222

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	97	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	4.85	1.10
time (sec)	N/A	0.230	0.066	0.015	0.244	0.113	1.667	0.178	0.223	0.002

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	24	28	28	30
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08	1.15
time (sec)	N/A	0.215	161.166	0.479	2.575	0.078	26.337	0.204	0.217	38.846

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	26	30	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.93	1.07	1.07	1.07
time (sec)	N/A	0.241	26.069	0.487	18.851	0.076	106.329	0.206	0.246	39.139

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	83	73	194	0	339	0	0	158	0
N.S.	1	1.08	0.95	2.52	0.00	4.40	0.00	0.00	2.05	0.00
time (sec)	N/A	0.316	1.507	4.411	0.000	0.094	0.000	0.000	0.182	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	272	311	606	0	1393	0	0	1574	0
N.S.	1	0.97	1.11	2.16	0.00	4.98	0.00	0.00	5.62	0.00
time (sec)	N/A	0.988	3.634	4.177	0.000	0.226	0.000	0.000	0.203	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	418	382	446	0	0	2284	0	0	0	0
N.S.	1	0.91	1.07	0.00	0.00	5.46	0.00	0.00	0.00	0.00
time (sec)	N/A	1.516	3.153	0.000	0.000	0.280	0.000	0.000	0.244	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	128	112	349	0	625	0	0	455	0
N.S.	1	1.10	0.97	3.01	0.00	5.39	0.00	0.00	3.92	0.00
time (sec)	N/A	0.437	2.298	6.703	0.000	0.149	0.000	0.000	0.193	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	355	1017	946	0	2375	0	0	1145	0
N.S.	1	0.99	2.85	2.65	0.00	6.65	0.00	0.00	3.21	0.00
time (sec)	N/A	1.370	15.867	6.582	0.000	0.267	0.000	0.000	0.233	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	753	672	8825	0	0	4917	0	0	0	0
N.S.	1	0.89	11.72	0.00	0.00	6.53	0.00	0.00	0.00	0.00
time (sec)	N/A	3.086	20.275	0.000	0.000	0.426	0.000	0.000	2.875	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	765	0	1194	0	0	3089	0	0	34	0
N.S.	1	0.00	1.56	0.00	0.00	4.04	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	2.448	0.000	0.000	0.385	0.000	0.000	200.021	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	557	627	607	0	0	2109	0	0	34	0
N.S.	1	1.13	1.09	0.00	0.00	3.79	0.00	0.00	0.06	0.00
time (sec)	N/A	4.114	2.390	0.000	0.000	0.323	0.000	0.000	200.023	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	406	2079	1189	0	1273	0	0	32	0
N.S.	1	1.16	5.94	3.40	0.00	3.64	0.00	0.00	0.09	0.00
time (sec)	N/A	2.221	16.125	1.668	0.000	0.285	0.000	0.000	200.024	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	87	90	94	0	262	0	94	67	896
N.S.	1	1.16	1.20	1.25	0.00	3.49	0.00	1.25	0.89	11.95
time (sec)	N/A	0.517	0.110	0.802	0.000	0.122	0.000	0.148	0.176	40.772

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	763	0	4052	0	0	3449	0	0	36	0
N.S.	1	0.00	5.31	0.00	0.00	4.52	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	11.397	0.000	0.000	0.441	0.000	0.000	200.017	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	566	701	1741	0	0	2260	0	0	36	0
N.S.	1	1.24	3.08	0.00	0.00	3.99	0.00	0.00	0.06	0.00
time (sec)	N/A	4.751	9.939	0.000	0.000	0.405	0.000	0.000	200.024	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	463	868	1694	0	1293	0	0	34	0
N.S.	1	1.22	2.29	4.47	0.00	3.41	0.00	0.00	0.09	0.00
time (sec)	N/A	2.804	5.711	7.575	0.000	0.307	0.000	0.000	200.017	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	53	55	54	55	0	61	119	98
N.S.	1	0.97	0.90	0.93	0.92	0.93	0.00	1.03	2.02	1.66
time (sec)	N/A	0.306	0.059	1.427	0.035	0.104	0.000	0.117	4.760	40.233

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	0	1181	0	0	4205	0	0	36	0
N.S.	1	0.00	1.05	0.00	0.00	3.74	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	7.979	0.000	0.000	0.736	0.000	0.000	200.024	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	825	0	1254	0	0	2787	0	0	36	0
N.S.	1	0.00	1.52	0.00	0.00	3.38	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	5.630	0.000	0.000	0.410	0.000	0.000	200.024	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	0	2194	1874	0	1611	0	0	34	0
N.S.	1	0.00	4.29	3.66	0.00	3.15	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	17.190	8.290	0.000	0.494	0.000	0.000	200.023	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	138	143	180	0	350	0	183	169	1320
N.S.	1	1.11	1.15	1.45	0.00	2.82	0.00	1.48	1.36	10.65
time (sec)	N/A	0.744	0.252	2.610	0.000	0.188	0.000	0.150	3.597	42.721

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	852	0	3039	0	0	3903	0	0	36	0
N.S.	1	0.00	3.57	0.00	0.00	4.58	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	14.866	0.000	0.000	0.417	0.000	0.000	200.021	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	616	0	1752	0	0	2529	0	0	36	0
N.S.	1	0.00	2.84	0.00	0.00	4.11	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	12.271	0.000	0.000	0.321	0.000	0.000	200.029	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	1111	1732	0	1419	0	0	34	0
N.S.	1	0.00	2.88	4.49	0.00	3.68	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	9.011	2.030	0.000	0.300	0.000	0.000	200.047	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	59	57	69	0	66	145	118
N.S.	1	1.00	0.90	0.98	0.95	1.15	0.00	1.10	2.42	1.97
time (sec)	N/A	0.321	0.068	0.893	0.032	0.103	0.000	0.129	1.162	39.207

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1144	0	3915	0	0	4706	0	0	38	0
N.S.	1	0.00	3.42	0.00	0.00	4.11	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	9.840	0.000	0.000	0.554	0.000	0.000	200.027	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	840	0	973	0	0	3075	0	0	38	0
N.S.	1	0.00	1.16	0.00	0.00	3.66	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	9.111	0.000	0.000	0.441	0.000	0.000	200.024	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	0	2279	5310	0	1751	0	0	36	0
N.S.	1	0.00	4.49	10.45	0.00	3.45	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	18.020	8.188	0.000	0.330	0.000	0.000	200.025	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	123	146	155	0	396	0	221	202	1167
N.S.	1	1.18	1.40	1.49	0.00	3.81	0.00	2.12	1.94	11.22
time (sec)	N/A	0.694	1.049	2.293	0.000	0.190	0.000	0.162	16.022	41.420

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1432	0	4009	0	0	4916	0	0	38	0
N.S.	1	0.00	2.80	0.00	0.00	3.43	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	13.604	0.000	0.000	0.671	0.000	0.000	200.025	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1014	0	5075	0	0	3131	0	0	38	0
N.S.	1	0.00	5.00	0.00	0.00	3.09	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	12.217	0.000	0.000	0.412	0.000	0.000	200.021	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	0	1650	5210	0	1707	0	0	36	0
N.S.	1	0.00	2.57	8.13	0.00	2.66	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	10.317	17.613	0.000	0.341	0.000	0.000	200.023	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	90	86	90	91	133	0	101	31	233
N.S.	1	0.94	0.90	0.94	0.95	1.39	0.00	1.05	0.32	2.43
time (sec)	N/A	0.352	0.164	4.477	0.034	0.111	0.000	0.145	200.018	42.898

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [16] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	12	1.10	14	0.857
2	A	9	9	1.08	14	0.643
3	A	7	7	1.04	14	0.500
4	A	4	4	1.00	12	0.333
5	A	5	5	1.00	14	0.357
6	A	7	7	1.06	14	0.500
7	A	10	10	1.01	14	0.714
8	A	9	9	1.03	16	0.562
9	A	6	6	1.04	16	0.375
10	A	6	6	1.02	16	0.375
11	A	3	3	1.00	14	0.214
12	A	3	3	1.00	16	0.188
13	A	8	8	1.05	16	0.500
14	A	6	6	1.30	16	0.375
15	A	11	11	1.00	16	0.688
16	A	25	24	1.34	16	1.500
17	A	16	16	1.25	16	1.000
18	A	12	11	1.10	16	0.688
19	A	6	6	1.00	14	0.429
20	A	3	3	1.00	16	0.188
21	A	3	3	1.02	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	9	9	1.32	16	0.562
23	A	7	6	1.10	14	0.429
24	A	6	5	1.07	14	0.357
25	A	5	4	1.00	12	0.333
26	N/A	2	0	1.00	14	0.000
27	N/A	2	0	1.00	14	0.000
28	A	10	9	1.33	16	0.562
29	A	9	8	1.28	16	0.500
30	A	5	5	1.07	14	0.357
31	N/A	2	0	1.00	16	0.000
32	N/A	2	0	1.00	16	0.000
33	A	11	10	1.07	16	0.625
34	A	9	8	1.08	16	0.500
35	A	7	6	1.01	14	0.429
36	N/A	2	0	1.00	16	0.000
37	N/A	2	0	1.00	16	0.000
38	A	15	14	1.05	16	0.875
39	A	13	12	1.02	16	0.750
40	A	10	9	1.04	16	0.562
41	A	8	7	1.00	16	0.438
42	A	10	9	1.04	16	0.562
43	A	13	12	1.04	16	0.750
44	A	15	14	1.07	16	0.875
45	A	6	6	1.03	18	0.333
46	A	6	6	1.03	18	0.333
47	A	3	3	1.00	18	0.167
48	A	3	3	1.00	18	0.167
49	A	11	10	1.04	18	0.556
50	A	6	6	1.25	18	0.333
51	A	14	13	1.03	18	0.722
52	A	9	9	1.20	18	0.500
53	A	19	18	1.41	18	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	17	16	1.42	18	0.889
55	A	3	3	1.00	18	0.167
56	A	3	3	1.00	18	0.167
57	A	3	3	1.06	18	0.167
58	A	12	11	1.52	18	0.611
59	A	14	13	1.42	18	0.722
60	A	9	8	1.03	12	0.667
61	A	6	5	1.00	12	0.417
62	A	4	3	1.00	12	0.250
63	A	6	5	1.00	12	0.417
64	A	9	8	1.05	12	0.667
65	N/A	2	0	1.00	16	0.000
66	N/A	2	0	1.00	16	0.000
67	A	1	1	1.00	25	0.040
68	A	1	1	1.00	29	0.034
69	A	1	1	1.00	28	0.036
70	A	1	1	1.00	28	0.036
71	N/A	2	0	1.00	18	0.000
72	A	3	3	1.00	16	0.188
73	A	3	3	1.00	16	0.188
74	A	3	3	1.00	14	0.214
75	N/A	2	0	1.00	14	0.000
76	N/A	2	0	1.00	16	0.000
77	A	3	3	1.00	12	0.250
78	A	3	3	1.00	12	0.250
79	A	3	3	1.00	12	0.250
80	A	3	3	1.00	10	0.300
81	A	3	3	1.00	12	0.250
82	A	3	3	1.00	12	0.250
83	A	3	3	1.00	12	0.250
84	A	3	3	1.00	14	0.214
85	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.00	14	0.214
87	A	3	3	1.00	12	0.250
88	A	3	3	1.00	14	0.214
89	A	3	3	1.00	14	0.214
90	A	3	3	1.00	14	0.214
91	A	1	1	1.00	28	0.036
92	A	1	1	1.00	32	0.031
93	A	1	1	1.00	28	0.036
94	A	1	1	1.00	28	0.036
95	A	3	3	1.00	18	0.167
96	A	3	3	1.00	18	0.167
97	A	3	3	1.00	16	0.188
98	A	3	3	1.00	18	0.167
99	A	3	3	1.00	18	0.167
100	A	3	3	1.00	18	0.167
101	A	3	3	1.00	20	0.150
102	A	3	3	1.00	20	0.150
103	A	3	3	1.00	18	0.167
104	A	5	5	0.96	20	0.250
105	A	5	5	1.02	20	0.250
106	A	15	15	1.39	20	0.750
107	A	12	11	1.18	20	0.550
108	A	11	10	1.14	20	0.500
109	A	7	7	1.05	18	0.389
110	N/A	2	0	1.00	20	0.000
111	N/A	2	0	1.00	20	0.000
112	A	15	14	1.06	20	0.700
113	A	15	14	1.02	20	0.700
114	A	9	9	0.99	18	0.500
115	N/A	2	0	1.00	20	0.000
116	N/A	2	0	1.00	20	0.000
117	A	13	12	1.04	21	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	12	11	1.04	21	0.524
119	A	7	7	1.03	19	0.368
120	N/A	2	0	1.00	21	0.000
121	N/A	2	0	1.00	21	0.000
122	A	11	11	1.21	18	0.611
123	A	9	9	1.15	18	0.500
124	A	6	6	1.40	16	0.375
125	A	7	7	0.69	18	0.389
126	A	9	9	0.79	18	0.500
127	A	12	12	0.75	18	0.667
128	A	18	18	1.08	18	1.000
129	A	14	13	0.91	18	0.722
130	A	8	8	0.98	16	0.500
131	A	5	5	0.56	18	0.278
132	A	5	5	0.58	18	0.278
133	A	11	11	0.75	18	0.611
134	A	9	8	0.61	18	0.444
135	A	8	7	0.61	18	0.389
136	A	7	6	0.64	16	0.375
137	N/A	2	0	1.00	18	0.000
138	N/A	2	0	1.00	18	0.000
139	A	13	12	0.61	18	0.667
140	A	11	10	0.65	18	0.556
141	A	9	8	0.76	16	0.500
142	N/A	2	0	1.00	18	0.000
143	N/A	2	0	1.00	18	0.000
144	N/A	2	0	1.00	18	0.000
145	N/A	2	0	1.00	20	0.000
146	A	5	5	0.96	20	0.250
147	A	5	5	0.98	20	0.250
148	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	N/A	2	0	1.00	20	0.000
150	N/A	2	0	1.00	20	0.000
151	A	3	3	1.00	18	0.167
152	A	3	3	1.00	18	0.167
153	A	3	3	1.00	16	0.188
154	A	3	3	1.00	18	0.167
155	A	3	3	1.00	18	0.167
156	A	3	3	1.00	18	0.167
157	A	3	3	1.00	20	0.150
158	A	3	3	1.00	20	0.150
159	A	3	3	1.00	18	0.167
160	A	3	3	1.00	20	0.150
161	A	3	3	1.00	20	0.150
162	A	3	3	1.00	20	0.150
163	A	10	9	0.94	20	0.450
164	A	9	8	0.95	20	0.400
165	A	8	7	1.01	18	0.389
166	N/A	2	0	1.00	20	0.000
167	N/A	2	0	1.00	20	0.000
168	A	17	16	0.93	20	0.800
169	A	15	14	0.94	20	0.700
170	A	12	11	1.04	18	0.611
171	N/A	2	0	1.00	20	0.000
172	N/A	2	0	1.00	20	0.000
173	N/A	2	0	1.00	20	0.000
174	A	3	3	1.00	20	0.150
175	A	3	3	1.00	20	0.150
176	A	3	3	1.00	18	0.167
177	N/A	2	0	1.00	20	0.000
178	N/A	2	0	1.00	20	0.000
179	A	14	13	1.18	26	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	13	12	1.14	26	0.462
181	A	9	9	1.07	24	0.375
182	A	4	4	1.00	19	0.211
183	N/A	1	0	1.00	26	0.000
184	N/A	1	0	1.00	26	0.000
185	A	24	23	1.11	28	0.821
186	A	21	20	1.08	28	0.714
187	A	14	14	1.03	26	0.538
188	A	7	7	0.93	21	0.333
189	N/A	1	0	1.00	28	0.000
190	N/A	1	0	1.00	28	0.000
191	A	31	30	1.11	28	1.071
192	A	28	27	1.10	28	0.964
193	A	18	18	1.12	26	0.692
194	A	4	4	1.00	21	0.190
195	N/A	1	0	1.00	28	0.000
196	N/A	1	0	1.00	28	0.000
197	A	18	17	1.09	26	0.654
198	A	16	15	1.06	26	0.577
199	A	12	11	1.01	24	0.458
200	A	5	5	1.00	19	0.263
201	N/A	1	0	1.00	26	0.000
202	N/A	1	0	1.00	26	0.000
203	A	27	26	1.16	28	0.929
204	A	25	24	1.15	28	0.857
205	A	18	17	1.01	26	0.654
206	A	10	9	1.02	21	0.429
207	N/A	1	0	1.00	28	0.000
208	N/A	1	0	1.00	28	0.000
209	F	0	0	N/A	0.000	N/A
210	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
211	A	25	24	1.32	26	0.923
212	A	12	11	1.01	21	0.524
213	N/A	1	0	1.00	28	0.000
214	N/A	1	0	1.00	28	0.000
215	N/A	1	0	1.00	28	0.000
216	N/A	1	0	1.00	26	0.000
217	N/A	2	0	1.00	20	0.000
218	N/A	1	0	1.00	26	0.000
219	N/A	1	0	1.00	28	0.000
220	A	12	11	0.89	26	0.423
221	A	11	10	0.90	26	0.385
222	A	10	9	0.97	24	0.375
223	A	7	6	1.11	19	0.316
224	A	22	21	0.89	28	0.750
225	A	19	18	0.90	28	0.643
226	A	15	14	0.96	26	0.538
227	A	11	10	1.12	21	0.476
228	A	29	28	0.90	28	1.000
229	A	26	25	0.91	28	0.893
230	A	19	18	0.97	26	0.692
231	A	11	10	1.16	21	0.476
232	A	13	12	0.92	26	0.462
233	A	13	12	0.92	26	0.462
234	A	12	11	0.96	24	0.458
235	A	8	7	1.09	19	0.368
236	A	23	22	0.95	28	0.786
237	A	22	21	0.95	28	0.750
238	A	18	17	0.96	26	0.654
239	A	12	11	1.13	21	0.524
240	N/A	1	0	1.00	28	0.000
241	N/A	1	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
242	N/A	2	0	1.00	20	0.000
243	N/A	1	0	1.00	26	0.000
244	N/A	1	0	1.00	28	0.000
245	A	2	2	1.00	24	0.083
246	A	2	2	1.00	26	0.077
247	A	2	2	1.00	26	0.077
248	A	2	2	1.00	24	0.083
249	A	2	2	1.00	26	0.077
250	A	2	2	1.00	26	0.077
251	A	7	6	1.03	26	0.231
252	A	6	5	1.03	26	0.192
253	A	5	4	1.03	24	0.167
254	A	4	3	1.12	19	0.158
255	N/A	1	0	1.00	26	0.000
256	N/A	1	0	1.00	26	0.000
257	A	11	11	1.01	28	0.393
258	A	9	9	1.00	28	0.321
259	A	6	6	1.00	26	0.231
260	A	3	3	1.00	21	0.143
261	A	7	7	0.97	28	0.250
262	A	9	9	1.02	28	0.321
263	A	18	18	0.98	28	0.643
264	A	12	12	0.98	28	0.429
265	A	10	10	1.00	26	0.385
266	A	4	3	1.00	21	0.143
267	A	13	13	0.96	28	0.464
268	A	18	18	1.01	28	0.643
269	A	21	20	0.99	26	0.769
270	A	16	15	0.99	26	0.577
271	A	12	11	0.95	24	0.458
272	A	5	4	1.05	19	0.211
273	N/A	1	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
274	N/A	1	0	1.00	26	0.000
275	A	22	21	1.00	28	0.750
276	A	21	20	0.95	28	0.714
277	A	13	13	0.97	26	0.500
278	A	6	5	1.00	21	0.238
279	N/A	1	0	1.00	28	0.000
280	N/A	1	0	1.00	28	0.000
281	F	0	0	N/A	0.000	N/A
282	A	23	22	0.98	28	0.786
283	A	14	13	0.93	26	0.500
284	A	5	4	0.96	21	0.190
285	N/A	1	0	1.00	28	0.000
286	N/A	1	0	1.00	28	0.000
287	A	6	6	0.98	28	0.214
288	A	10	10	1.01	28	0.357
289	A	5	5	1.00	28	0.179
290	N/A	1	0	1.00	26	0.000
291	N/A	2	0	1.00	20	0.000
292	N/A	1	0	1.00	26	0.000
293	N/A	1	0	1.00	28	0.000
294	A	7	6	1.01	26	0.231
295	A	6	5	1.01	26	0.192
296	A	5	4	1.00	24	0.167
297	A	4	3	1.00	19	0.158
298	A	20	19	0.93	28	0.679
299	A	17	16	0.95	28	0.571
300	A	13	12	1.01	26	0.462
301	A	9	8	1.16	21	0.381
302	A	25	24	0.91	28	0.857
303	A	18	17	0.90	28	0.607
304	A	15	14	0.91	26	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	5	4	0.89	21	0.190
306	A	10	9	0.87	26	0.346
307	A	9	8	0.89	26	0.308
308	A	8	7	0.93	24	0.292
309	A	5	4	1.07	19	0.211
310	A	13	12	0.84	28	0.429
311	A	12	11	0.85	28	0.393
312	A	11	10	0.93	26	0.385
313	A	8	7	1.07	21	0.333
314	N/A	1	0	1.00	28	0.000
315	N/A	1	0	1.00	26	0.000
316	N/A	2	0	1.00	20	0.000
317	N/A	1	0	1.00	26	0.000
318	N/A	1	0	1.00	28	0.000
319	A	6	5	1.08	24	0.208
320	A	9	8	0.97	26	0.308
321	A	10	9	0.91	26	0.346
322	A	10	9	1.10	24	0.375
323	A	13	12	0.99	26	0.462
324	A	16	15	0.89	26	0.577
325	F	0	0	N/A	0.000	N/A
326	A	29	28	1.13	32	0.875
327	A	22	21	1.16	30	0.700
328	A	10	9	1.16	25	0.360
329	F	0	0	N/A	0.000	N/A
330	A	30	29	1.24	34	0.853
331	A	27	26	1.22	32	0.812
332	A	6	5	0.97	27	0.185
333	F	0	0	N/A	0.000	N/A
334	F	0	0	N/A	0.000	N/A
335	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	11	10	1.11	27	0.370
337	F	0	0	N/A	0.000	N/A
338	F	0	0	N/A	0.000	N/A
339	F	0	0	N/A	0.000	N/A
340	A	6	5	1.00	27	0.185
341	F	0	0	N/A	0.000	N/A
342	F	0	0	N/A	0.000	N/A
343	F	0	0	N/A	0.000	N/A
344	A	10	9	1.18	29	0.310
345	F	0	0	N/A	0.000	N/A
346	F	0	0	N/A	0.000	N/A
347	F	0	0	N/A	0.000	N/A
348	A	6	5	0.94	29	0.172

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (c+dx)^4 \sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int (c+dx)^4 \sin(a+bx)\,dx = -\frac{24d^4\cos(a+bx)}{b^5} + \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} - \frac{(c+dx)^4\cos(a+bx)}{b} - \frac{24d^3(c+dx)\sin(a+bx)}{b^4} + \frac{4d(c+dx)^3\sin(a+bx)}{b^2}$$

output
```
-24*d^4*cos(b*x+a)/b^5+12*d^2*(d*x+c)^2*cos(b*x+a)/b^3-(d*x+c)^4*cos(b*x+a
)/b-24*d^3*(d*x+c)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int (c+dx)^4 \sin(a+bx)\,dx = \frac{-\left((24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)\right) + 4bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^5}$$

input
```
Integrate[(c + d*x)^4*Sin[a + b*x],x]
```

output
```
(-((24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]) + 4*b
*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b^5
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^4\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\int (c+dx)^3\cos(a+bx)dx}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\left(\frac{3d\int -(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$\Big\downarrow$ 3042

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$\Big\downarrow$ 3777

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$\Big\downarrow$ 25

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$\Big\downarrow$ 3042

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

$\Big\downarrow$ 3118

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^4*Sin[a + b*x],x]
```

output
```
-(((c + d*x)^4*Cos[a + b*x])/b) + (4*d*(((c + d*x)^3*Sin[a + b*x])/b - (3*
d*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*
x)*Sin[a + b*x])/b))/b))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4\right)\cos(bx+a)}{b^5}+\frac{4d\left(b^2d^3x^3\right.}{}$
parallelrisch	$\frac{\left(\left(d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+2c^4\right)b^4+\left(-12d^4x^2-24c\,d^3x-24c^2d^2\right)b^2+48d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+8(dx+c)db\left((dx+c\right.}{b^5\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)}$
orering	$\frac{8d\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-9b^2d^4x^2-18b^2c\,d^3x-9b^2c^2d^2+12d^4\right)\sin(bx+a)}{b^6(dx+c)}-\frac{\left(d^4x^4b^4+4b^4\right.}{}$
norman	$\frac{\frac{\left(2b^4c^4-24b^2c^2d^2+48d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b^5}+\frac{d^4x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}-\frac{d^4x^4}{b}-\frac{6d^2\left(b^2c^2-2d^2\right)x^2}{b^3}-\frac{4d^3c\,x^3}{b}+\frac{8d^4x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{4cd}{}}{}$
parts	$-\frac{\cos(bx+a)d^4x^4}{b}-\frac{4\cos(bx+a)c\,d^3x^3}{b}-\frac{6\cos(bx+a)c^2d^2x^2}{b}-\frac{4\cos(bx+a)c^3dx}{b}-\frac{\cos(bx+a)c^4}{b}+\frac{4d\left(-\frac{a^3}{}\right.}{}$
meijerg	$\frac{16d^4\sin(a)\sqrt{\pi}\left(-\frac{x\left(b^2\right)^{\frac{5}{2}}\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{10\sqrt{\pi}\,b^4}+\frac{\left(b^2\right)^{\frac{5}{2}}\left(\frac{5}{8}x^4b^4-\frac{15}{2}x^2b^2+15\right)\sin(bx)}{10\sqrt{\pi}\,b^5}\right)}{b^4\sqrt{b^2}}+\frac{16d^4\cos(a)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a^4d^4\cos(bx+a)}{b^4}+\frac{4a^3c\,d^3\cos(bx+a)}{b^3}-\frac{4a^3d^4(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^4}-\frac{6a^2c^2d^2\cos(bx+a)}{b^2}+\frac{12a^2c\,d^3(\sin(bx+a)-(bx+a}{b^3}}{}$
default	$\frac{-\frac{a^4d^4\cos(bx+a)}{b^4}+\frac{4a^3c\,d^3\cos(bx+a)}{b^3}-\frac{4a^3d^4(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^4}-\frac{6a^2c^2d^2\cos(bx+a)}{b^2}+\frac{12a^2c\,d^3(\sin(bx+a)-(bx+a}{b^3}}{}$

input
```
int((d*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(b^4*d^4*x^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+b^4*c^4-12*b
^2*d^4*x^2-24*b^2*c*d^3*x-12*b^2*c^2*d^2+24*d^4)/b^5*cos(b*x+a)+4/b^4*d*(b
^2*d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+
a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.85

$$\int (c+dx)^4\sin(a+bx)\,dx =$$
$$-\frac{(b^4d^4x^4+4\,b^4cd^3x^3+b^4c^4-12\,b^2c^2d^2+24\,d^4+6\,(b^4c^2d^2-2\,b^2d^4)x^2+4\,(b^4c^3d-6\,b^2cd^3)x)\cos{(bx}}{b^5}$$

input
```
integrate((d*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
-((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(
b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)
 - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d
^2 - 2*b*d^4)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(92) = 184.

Time = 0.36 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.38

$$\int (c+dx)^4\sin(a+bx)\,dx$$
$$=\begin{cases} -\frac{c^4\cos{(a+bx)}}{b}-\frac{4c^3dx\cos{(a+bx)}}{b}-\frac{6c^2d^2x^2\cos{(a+bx)}}{b}-\frac{4cd^3x^3\cos{(a+bx)}}{b}-\frac{d^4x^4\cos{(a+bx)}}{b}+\frac{4c^3d\sin{(a+bx)}}{b^2}+\frac{12c^2d^2x\sin}{b} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin{(a)} \end{cases}$$

input
```
integrate((d*x+c)**4*sin(b*x+a),x)
```

output
```
Piecewise((-c**4*cos(a + b*x)/b - 4*c**3*d*x*cos(a + b*x)/b - 6*c**2*d**2*
x**2*cos(a + b*x)/b - 4*c*d**3*x**3*cos(a + b*x)/b - d**4*x**4*cos(a + b*x
)/b + 4*c**3*d*sin(a + b*x)/b**2 + 12*c**2*d**2*x*sin(a + b*x)/b**2 + 12*c
*d**3*x**2*sin(a + b*x)/b**2 + 4*d**4*x**3*sin(a + b*x)/b**2 + 12*c**2*d**
2*cos(a + b*x)/b**3 + 24*c*d**3*x*cos(a + b*x)/b**3 + 12*d**4*x**2*cos(a +
 b*x)/b**3 - 24*c*d**3*sin(a + b*x)/b**4 - 24*d**4*x*sin(a + b*x)/b**4 - 2
4*d**4*cos(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**
2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(92) = 184$.

Time = 0.07 (sec) , antiderivative size = 490, normalized size of antiderivative = 5.33

$$\int (c+dx)^4 \sin(a+bx)\,dx = -\frac{c^4\cos(bx+a) - \frac{4\,ac^3d\cos(bx+a)}{b} + \frac{6\,a^2c^2d^2\cos(bx+a)}{b^2} - \frac{4\,a^3cd^3\cos(bx+a)}{b^3} + \frac{a^4d^4\cos(bx+a)}{b^4} + \frac{4\,((bx+a)\cos(bx+a)-\sin(}{b}}{}$$

input
```
integrate((d*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
-(c^4*cos(b*x + a) - 4*a*c^3*d*cos(b*x + a)/b + 6*a^2*c^2*d^2*cos(b*x + a)
/b^2 - 4*a^3*c*d^3*cos(b*x + a)/b^3 + a^4*d^4*cos(b*x + a)/b^4 + 4*((b*x +
 a)*cos(b*x + a) - sin(b*x + a))*c^3*d/b - 12*((b*x + a)*cos(b*x + a) - si
n(b*x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2
*c*d^3/b^3 - 4*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^3*d^4/b^4 + 6*(((
b*x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 12*
(((b*x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 +
6*(((b*x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4
+ 4*(((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*
x + a))*c*d^3/b^3 - 4*(((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 3*((b*x
+ a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + (((b*x + a)^4 - 12*(b*x + a)^2 + 24)
*cos(b*x + a) - 4*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

$$\int (c+dx)^4 \sin(a+bx)\,dx = -\frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4)\cos(b}{b^5} + \frac{4\,(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3)\sin(bx+a)}{b^5}$$

input
```
integrate((d*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
-(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x +
 a)/b^5 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d -
 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.40

$$\int (c+dx)^4\sin(a+bx)\,dx = \frac{4\,x\,\cos(a+b\,x)\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^3} - \frac{4\,\sin(a+b\,x)\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^4} - \frac{d^4\,x^4\,\cos(a+b\,x)}{b} - \frac{\cos(a+b\,x)\,\left(b^4\,c^4-12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5} + \frac{4\,d^4\,x^3\,\sin(a+b\,x)}{b^2} - \frac{12\,x\,\sin(a+b\,x)\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^4} + \frac{6\,x^2\,\cos(a+b\,x)\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^3} - \frac{4\,c\,d^3\,x^3\,\cos(a+b\,x)}{b} + \frac{12\,c\,d^3\,x^2\,\sin(a+b\,x)}{b^2}$$

input
```
int(sin(a + b*x)*(c + d*x)^4,x)
```

output
```
(4*x*cos(a + b*x)*(6*c*d^3 - b^2*c^3*d))/b^3 - (4*sin(a + b*x)*(6*c*d^3 -
b^2*c^3*d))/b^4 - (d^4*x^4*cos(a + b*x))/b - (cos(a + b*x)*(24*d^4 + b^4*c
^4 - 12*b^2*c^2*d^2))/b^5 + (4*d^4*x^3*sin(a + b*x))/b^2 - (12*x*sin(a + b
*x)*(2*d^4 - b^2*c^2*d^2))/b^4 + (6*x^2*cos(a + b*x)*(2*d^4 - b^2*c^2*d^2)
)/b^3 - (4*c*d^3*x^3*cos(a + b*x))/b + (12*c*d^3*x^2*sin(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\int (c+dx)^4\sin(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\,b^4c^4 - 4\cos(bx+a)\,b^4c^3dx - 6\cos(bx+a)\,b^4c^2d^2x^2 - 4\cos(bx+a)\,b^4c\,d^3x^3 - \cos(bx+}{}$$

input
```
int((d*x+c)^4*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)*b**4*c**4 - 4*cos(a + b*x)*b**4*c**3*d*x - 6*cos(a + b*x)
*b**4*c**2*d**2*x**2 - 4*cos(a + b*x)*b**4*c*d**3*x**3 - cos(a + b*x)*b**4
*d**4*x**4 + 12*cos(a + b*x)*b**2*c**2*d**2 + 24*cos(a + b*x)*b**2*c*d**3*
x + 12*cos(a + b*x)*b**2*d**4*x**2 - 24*cos(a + b*x)*d**4 + 4*sin(a + b*x)
*b**3*c**3*d + 12*sin(a + b*x)*b**3*c**2*d**2*x + 12*sin(a + b*x)*b**3*c*d
**3*x**2 + 4*sin(a + b*x)*b**3*d**4*x**3 - 24*sin(a + b*x)*b*c*d**3 - 24*s
in(a + b*x)*b*d**4*x)/b**5
```

3.2 $\int (c+dx)^3 \sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int (c+dx)^3\sin(a+bx)\,dx = \frac{6d^2(c+dx)\cos(a+bx)}{b^3}-\frac{(c+dx)^3\cos(a+bx)}{b} - \frac{6d^3\sin(a+bx)}{b^4}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}$$

output
```
6*d^2*(d*x+c)*cos(b*x+a)/b^3-(d*x+c)^3*cos(b*x+a)/b-6*d^3*sin(b*x+a)/b^4+3
*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int (c+dx)^3\sin(a+bx)\,dx = \frac{-b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+3d\left(-2d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x],x]
```

output
```
(-(b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x]) + 3*d*(-2*d^2 + b^
2*(c + d*x)^2)*Sin[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\int (c+dx)^2 \cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x],x]
```

output
```
-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (2*
d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(bx+a)}{b^3}+\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{b^4}$
orering	$\frac{6d(x^2d^2b^2+2b^2cdx+b^2c^2-4d^2)\sin(bx+a)}{b^4}-\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-6d^2)\left(3(dx+c)^2\sin(bx+a)d+(dx+c)^3b\cos(bx\right.}{(dx+c)^2b^4}$
parallelrisch	$\frac{3dbx\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+6d\left((dx+c)^2b^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\frac{dx}{2}+c\right)\left((x^2d^2+cdx+c^2)b^2-6\right.}{b^4\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)}$
parts	$-\frac{\cos(bx+a)d^3x^3}{b}-\frac{3\cos(bx+a)c\,d^2x^2}{b}-\frac{3\cos(bx+a)c^2dx}{b}-\frac{\cos(bx+a)c^3}{b}+\frac{3d\left(\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+c}{b}\right.}{}$
norman	$\frac{\frac{(2b^2c^3-12c\,d^2)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b^3}+\frac{d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}-\frac{d^3x^3}{b}-\frac{3d(b^2c^2-2d^2)x}{b^3}-\frac{3c\,d^2x^2}{b}+\frac{6d(b^2c^2-2d^2)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^4}+\frac{6d^3x^2}{}}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$
derivativedivides	$\frac{a^3d^3\cos(bx+a)}{b^3}-\frac{3a^2c\,d^2\cos(bx+a)}{b^2}+\frac{3a^2d^3(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^3}+\frac{3a\,c^2d\cos(bx+a)}{b}-\frac{6ac\,d^2(\sin(bx+a)-(bx+a)\cos(b}{b^2}$
default	$\frac{a^3d^3\cos(bx+a)}{b^3}-\frac{3a^2c\,d^2\cos(bx+a)}{b^2}+\frac{3a^2d^3(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^3}+\frac{3a\,c^2d\cos(bx+a)}{b}-\frac{6ac\,d^2(\sin(bx+a)-(bx+a)\cos(b}{b^2}$
meijerg	$\frac{8d^3\sin(a)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2b^2}{2}+3\right)\cos(bx)}{4\sqrt{\pi}}-\frac{xb\left(-\frac{x^2b^2}{2}+3\right)\sin(bx)}{4\sqrt{\pi}}\right)}{b^4}+\frac{8d^3\cos(a)\sqrt{\pi}\left(\frac{xb\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{20\sqrt{\pi}}-\frac{(-}{}\right.}{b^4}$

input
```
int((d*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(b^2*d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)
*cos(b*x+a)+3*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*sin(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.55

$$\int (c+dx)^3\sin(a+bx)\,dx =$$
$$-\frac{(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-6\,bcd^2+3\,(b^3c^2d-2\,bd^3)x)\cos(bx+a)-3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-}{b^4}$$

input
```
integrate((d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*
b*d^3)*x)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^
3)*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.85

$$\int (c+dx)^3 \sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\cos(a+bx)}{b} - \frac{3c^2dx\cos(a+bx)}{b} - \frac{3cd^2x^2\cos(a+bx)}{b} - \frac{d^3x^3\cos(a+bx)}{b} + \frac{3c^2d\sin(a+bx)}{b^2} + \frac{6cd^2x\sin(a+bx)}{b^2} + \frac{3d^3x^2\sin(a}{b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a) \end{cases}$$

input
```
integrate((d*x+c)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c**3*cos(a + b*x)/b - 3*c**2*d*x*cos(a + b*x)/b - 3*c*d**2*x**
2*cos(a + b*x)/b - d**3*x**3*cos(a + b*x)/b + 3*c**2*d*sin(a + b*x)/b**2 +
 6*c*d**2*x*sin(a + b*x)/b**2 + 3*d**3*x**2*sin(a + b*x)/b**2 + 6*c*d**2*c
os(a + b*x)/b**3 + 6*d**3*x*cos(a + b*x)/b**3 - 6*d**3*sin(a + b*x)/b**4,
Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(71) = 142$.

Time = 0.05 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.01

$$\int (c+dx)^3 \sin(a+bx)\,dx =$$

$$-\frac{c^3\cos(bx+a) - \frac{3\,ac^2d\cos(bx+a)}{b} + \frac{3\,a^2cd^2\cos(bx+a)}{b^2} - \frac{a^3d^3\cos(bx+a)}{b^3} + \frac{3\,((bx+a)\cos(bx+a)-\sin(bx+a))c^2d}{b} - \frac{6\,((bx+a}{}}{}$$

input
```
integrate((d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-(c^3*cos(b*x + a) - 3*a*c^2*d*cos(b*x + a)/b + 3*a^2*c*d^2*cos(b*x + a)/b
^2 - a^3*d^3*cos(b*x + a)/b^3 + 3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*
c^2*d/b - 6*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c*d^2/b^2 + 3*((b*x
+ a)*cos(b*x + a) - sin(b*x + a))*a^2*d^3/b^3 + 3*(((b*x + a)^2 - 2)*cos(b
*x + a) - 2*(b*x + a)*sin(b*x + a))*c*d^2/b^2 - 3*(((b*x + a)^2 - 2)*cos(b
*x + a) - 2*(b*x + a)*sin(b*x + a))*a*d^3/b^3 + (((b*x + a)^3 - 6*b*x - 6*
a)*cos(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int (c+dx)^3 \sin(a+bx)\,dx = -\frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2)\cos{(bx+a)}}{b^4} + \frac{3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3)\sin{(bx+a)}}{b^4}$$

input
```
integrate((d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*
b*c*d^2)*cos(b*x + a)/b^4 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2
*d^3)*sin(b*x + a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.07

$$\int (c+dx)^3\sin(a+bx)\,dx = \frac{\cos(a+b\,x)\,\left(6\,c\,d^2-b^2\,c^3\right)}{b^3} - \frac{3\,\sin(a+b\,x)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^4} - \frac{d^3\,x^3\,\cos(a+b\,x)}{b} + \frac{3\,d^3\,x^2\,\sin(a+b\,x)}{b^2} + \frac{3\,x\,\cos(a+b\,x)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^3} + \frac{6\,c\,d^2\,x\,\sin(a+b\,x)}{b^2} - \frac{3\,c\,d^2\,x^2\,\cos(a+b\,x)}{b}$$

input
```
int(sin(a + b*x)*(c + d*x)^3,x)
```

output
```
(cos(a + b*x)*(6*c*d^2 - b^2*c^3))/b^3 - (3*sin(a + b*x)*(2*d^3 - b^2*c^2*
d))/b^4 - (d^3*x^3*cos(a + b*x))/b + (3*d^3*x^2*sin(a + b*x))/b^2 + (3*x*c
os(a + b*x)*(2*d^3 - b^2*c^2*d))/b^3 + (6*c*d^2*x*sin(a + b*x))/b^2 - (3*c
*d^2*x^2*cos(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.18

$$\int (c+dx)^3\sin(a+bx)\,dx = \frac{-\cos(bx+a)\,b^3c^3-3\cos(bx+a)\,b^3c^2dx-3\cos(bx+a)\,b^3c\,d^2x^2-\cos(bx+a)\,b^3d^3x^3+6\cos(bx+a}{}$$

input
```
int((d*x+c)^3*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)*b**3*c**3 - 3*cos(a + b*x)*b**3*c**2*d*x - 3*cos(a + b*x)
*b**3*c*d**2*x**2 - cos(a + b*x)*b**3*d**3*x**3 + 6*cos(a + b*x)*b*c*d**2
+ 6*cos(a + b*x)*b*d**3*x + 3*sin(a + b*x)*b**2*c**2*d + 6*sin(a + b*x)*b*
*2*c*d**2*x + 3*sin(a + b*x)*b**2*d**3*x**2 - 6*sin(a + b*x)*d**3)/b**4
```

3.3 $\int (c+dx)^2 \sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int (c+dx)^2 \sin(a+bx)\,dx = \frac{2d^2\cos(a+bx)}{b^3} - \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2d(c+dx)\sin(a+bx)}{b^2}$$

output
```
2*d^2*cos(b*x+a)/b^3-(d*x+c)^2*cos(b*x+a)/b+2*d*(d*x+c)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \sin(a+bx)\,dx = \frac{-((-2d^2+b^2(c+dx)^2)\cos(a+bx)) + 2bd(c+dx)\sin(a+bx)}{b^3}$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x],x]
```

output
```
(-((-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x]) + 2*b*d*(c + d*x)*Sin[a + b*x]
)/b^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d\int (c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x],x]
```

output
```
-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*
Sin[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{b^3}+\frac{2d(dx+c)\sin(bx+a)}{b^2}$
parallelrisch	$\frac{2\left(\frac{dx}{2}+c\right)d\,b^2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+4bd(dx+c)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+(-x^2d^2-2cdx-2c^2)b^2+4d^2}{b^3\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)}$
parts	$-\frac{\cos(bx+a)x^2d^2}{b}-\frac{2\cos(bx+a)cdx}{b}-\frac{\cos(bx+a)c^2}{b}+\frac{2d\left(-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(b}{b}\right.}{b^2}$
orering	$\frac{4d(x^2d^2b^2+2b^2cdx+b^2c^2-d^2)\sin(bx+a)}{b^4(dx+c)}-\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\left(2(dx+c)\sin(bx+a)d+(dx+c)^2b\cos(bx+\right.}{b^4(dx+c)^2}$
norman	$\frac{\frac{-2b^2c^2+4d^2}{b^3}+\frac{d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}-\frac{d^2x^2}{b}-\frac{2cdx}{b}+\frac{4cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{4d^2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{2cdx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$
derivativedivides	$\frac{-\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2acd\cos(bx+a)}{b}-\frac{2a\,d^2(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^2}-c^2\cos(bx+a)+\frac{2cd(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}+}{b}$
default	$\frac{-\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2acd\cos(bx+a)}{b}-\frac{2a\,d^2(\sin(bx+a)-(bx+a)\cos(bx+a))}{b^2}-c^2\cos(bx+a)+\frac{2cd(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}+}{b}$
meijerg	$\frac{4d^2\sin(a)\sqrt{\pi}\left(\frac{x(b^2)^{\frac{3}{2}}\cos(bx)}{2\sqrt{\pi}\,b^2}-\frac{(b^2)^{\frac{3}{2}}\left(-\frac{3x^2b^2}{2}+3\right)\sin(bx)}{6\sqrt{\pi}\,b^3}\right)}{b^2\sqrt{b^2}}+\frac{4d^2\cos(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2b^2}{2}+1\right)\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(}{2\sqrt{}}\right.}{b^3}$

input
```
int((d*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^3*cos(b*x+a)+2*d*(d*x+c)*sin(b*
x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int (c+dx)^2\sin(a+bx)\,dx = -\frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)-2\,(bd^2x+bcd)\sin(bx+a)}{b^3}$$

input
```
integrate((d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(b*d^2*x
+ b*c*d)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

$$\int (c+dx)^2\sin(a+bx)\,dx = \begin{cases} -\frac{c^2\cos(a+bx)}{b} - \frac{2cdx\cos(a+bx)}{b} - \frac{d^2x^2\cos(a+bx)}{b} + \frac{2cd\sin(a+bx)}{b^2} + \frac{2d^2x\sin(a+bx)}{b^2} + \frac{2d^2\cos(a+bx)}{b^3} & \text{for } b\neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c**2*cos(a + b*x)/b - 2*c*d*x*cos(a + b*x)/b - d**2*x**2*cos(a
 + b*x)/b + 2*c*d*sin(a + b*x)/b**2 + 2*d**2*x*sin(a + b*x)/b**2 + 2*d**2*
cos(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.82

$$\int (c+dx)^2\sin(a+bx)\,dx =$$

$$-\frac{c^2\cos(bx+a) - \frac{2\,acd\cos(bx+a)}{b} + \frac{a^2d^2\cos(bx+a)}{b^2} + \frac{2\,((bx+a)\cos(bx+a)-\sin(bx+a))cd}{b} - \frac{2\,((bx+a)\cos(bx+a)-\sin(bx+a))a}{b^2}}{b}$$

input
```
integrate((d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-(c^2*cos(b*x + a) - 2*a*c*d*cos(b*x + a)/b + a^2*d^2*cos(b*x + a)/b^2 + 2
*((b*x + a)*cos(b*x + a) - sin(b*x + a))*c*d/b - 2*((b*x + a)*cos(b*x + a)
 - sin(b*x + a))*a*d^2/b^2 + (((b*x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)
*sin(b*x + a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (c+dx)^2 \sin(a+bx)\,dx = -\frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos{(bx+a)}}{b^3} + \frac{2\,(bd^2x+bcd)\sin{(bx+a)}}{b^3}$$

input
```
integrate((d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 + 2*(b*d^2
*x + b*c*d)*sin(b*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 34.96 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

$$\int (c+dx)^2 \sin(a+bx)\,dx = \frac{\cos{(a+b\,x)}\,\left(2\,d^2-b^2\,c^2\right)}{b^3} - \frac{d^2\,x^2\,\cos{(a+b\,x)}}{b} + \frac{2\,c\,d\,\sin{(a+b\,x)}}{b^2} + \frac{2\,d^2\,x\,\sin{(a+b\,x)}}{b^2} - \frac{2\,c\,d\,x\,\cos{(a+b\,x)}}{b}$$

input
```
int(sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(cos(a + b*x)*(2*d^2 - b^2*c^2))/b^3 - (d^2*x^2*cos(a + b*x))/b + (2*c*d*s
in(a + b*x))/b^2 + (2*d^2*x*sin(a + b*x))/b^2 - (2*c*d*x*cos(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.70

$$\int (c+dx)^2\sin(a+bx)\,dx$$
$$= \frac{-\cos\left(bx+a\right)b^2c^2-2\cos\left(bx+a\right)b^2cdx-\cos\left(bx+a\right)b^2d^2x^2+2\cos\left(bx+a\right)d^2+2\sin\left(bx+a\right)bcd+}{b^3}$$

input
```
int((d*x+c)^2*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)*b**2*c**2 - 2*cos(a + b*x)*b**2*c*d*x - cos(a + b*x)*b**2
*d**2*x**2 + 2*cos(a + b*x)*d**2 + 2*sin(a + b*x)*b*c*d + 2*sin(a + b*x)*b
*d**2*x)/b**3
```

3.4 $\int (c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (c+dx)\sin(a+bx)\,dx=-\frac{(c+dx)\cos(a+bx)}{b}+\frac{d\sin(a+bx)}{b^2}$$

output
```
-(d*x+c)*cos(b*x+a)/b+d*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (c+dx)\sin(a+bx)\,dx=\frac{-b(c+dx)\cos(a+bx)+d\sin(a+bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x],x]
```

output
```
(-(b*(c + d*x)*Cos[a + b*x]) + d*Sin[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{d\int \cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sin[a + b*x],x]
```

output
```
-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{(dx+c)\cos(bx+a)}{b}+\frac{d\sin(bx+a)}{b^2}$
parallelrisch	$\frac{-(dx+c)b\cos(bx+a)+bc+\sin(bx+a)d}{b^2}$
parts	$-\frac{\cos(bx+a)dx}{b}-\frac{\cos(bx+a)c}{b}+\frac{d\sin(bx+a)}{b^2}$
orering	$\frac{2d\sin(bx+a)}{b^2}-\frac{\sin(bx+a)d+(dx+c)b\cos(bx+a)}{b^2}$
derivativedivides	$\frac{\frac{da\cos(bx+a)}{b}-c\cos(bx+a)+\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}}{b}$
default	$\frac{\frac{da\cos(bx+a)}{b}-c\cos(bx+a)+\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}}{b}$
norman	$\frac{\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}+\frac{dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}+\frac{2d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{dx}{b}}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$
meijerg	$\frac{2d\sin(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{2d\cos(a)\sqrt{\pi}\left(-\frac{xb\cos(bx)}{2\sqrt{\pi}}+\frac{\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{c\sin(a)\sin(bx)}{b}+c\cos(a)\sqrt{\pi}$

input
```
int((d*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(d*x+c)*cos(b*x+a)/b+d*sin(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (c+dx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(bx+a) - d\sin(bx+a)}{b^2}$$

input
```
integrate((d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-((b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c+dx)\sin(a+bx)\,dx = \begin{cases} -\frac{c\cos(a+bx)}{b} - \frac{dx\cos(a+bx)}{b} + \frac{d\sin(a+bx)}{b^2} & \text{for } b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sin(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((-c*cos(a + b*x)/b - d*x*cos(a + b*x)/b + d*sin(a + b*x)/b**2, N
e(b, 0)), ((c*x + d*x**2/2)*sin(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int (c+dx)\sin(a+bx)\,dx = -\frac{c\cos(bx+a) - \frac{ad\cos(bx+a)}{b} + \frac{((bx+a)\cos(bx+a)-\sin(bx+a))d}{b}}{b}$$

input
```
integrate((d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-(c*cos(b*x + a) - a*d*cos(b*x + a)/b + ((b*x + a)*cos(b*x + a) - sin(b*x
+ a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int (c+dx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(bx+a)}{b^2} + \frac{d\sin(bx+a)}{b^2}$$

input
```
integrate((d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-(b*d*x + b*c)*cos(b*x + a)/b^2 + d*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (c+dx)\sin(a+bx)\,dx = \frac{d\,\sin(a+b\,x)}{b^2} - \frac{c\,\cos(a+b\,x) + d\,x\,\cos(a+b\,x)}{b}$$

input
```
int(sin(a + b*x)*(c + d*x),x)
```

output
```
(d*sin(a + b*x))/b^2 - (c*cos(a + b*x) + d*x*cos(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int (c+dx)\sin(a+bx)\,dx = \frac{-\cos(bx+a)\,bc - \cos(bx+a)\,bdx + \sin(bx+a)\,d}{b^2}$$

input
```
int((d*x+c)*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)*b*c - cos(a + b*x)*b*d*x + sin(a + b*x)*d)/b**2
```

3.5 $\int \frac{\sin(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sin(a+bx)}{c+dx}\,dx=\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

output
```
Ci(b*c/d+b*x)*sin(a-b*c/d)/d+cos(a-b*c/d)*Si(b*c/d+b*x)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\sin(a+bx)}{c+dx}\,dx=\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x),x]
```

output
```
(CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegra
l[(b*c)/d + b*x])/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3780}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[Sin[a + b*x]/(c + d*x),x]
```

output
```
(CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (Cos[a - (b*c)/d]*SinInt
egral[(b*c)/d + b*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}$	78
default	$-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}$	78
risch	$\frac{i e^{\frac{i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-ibx-ia-\frac{-iad+ibc}{d}\right)}{2d}-\frac{i e^{-\frac{i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-bc)}{d}\right)}{2d}$	98

input
```
int(sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-
a*d+b*c)/d)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = \frac{\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{d}$$

input
```
integrate(sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
(cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + cos(-(b*c - a*d)/d)*s
in_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)}{c+dx}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = -\frac{b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\right.\right.}{2\,bd}$$

input
```
integrate(sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_in
tegral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 597, normalized size of antiderivative = 11.71

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 -
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*si
n_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c
/d))*tan(1/2*a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2
*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(cos_integral(b*x
 + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)*tan(1/2*b*c/d) - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 +
 imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2*sin_integral((
b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 2*real_p
art(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-
b*x - b*c/d))*tan(1/2*b*c/d) + imag_part(cos_integral(b*x + b*c/d)) - imag
_part(cos_integral(-b*x - b*c/d)) + 2*sin_integral((b*d*x + b*c)/d))/(d*ta
n(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2 + d*tan(1/2*b*c/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x),x)
```

output
```
int(sin(a + b*x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(bx+a\right)}{dx+c}dx$$

input
```
int(sin(b*x+a)/(d*x+c),x)
```

output
```
int(sin(a + b*x)/(c + d*x),x)
```

3.6 $\int \frac{\sin(a+bx)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{\sin(a+bx)}{(c+dx)^2} \, dx = \frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{\sin(a+bx)}{d(c+dx)} - \frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

output
```
b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2-sin(b*x+a)/d/(d*x+c)-b*sin(a-b*c/d)*Si(b*
c/d+b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{\sin(a+bx)}{(c+dx)^2} \, dx = \frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \frac{d\sin(a+bx)}{c+dx} - b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x)^2,x]
```

output
```
(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - (d*Sin[a + b*x])/(c + d*x)
- b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{b\int \frac{\cos(a+bx)}{c+dx}dx}{d}-\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3780}$$

$$\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}$$

input
```
Int[Sin[a + b*x]/(c + d*x)^2,x]
```

output
```
-(Sin[a + b*x]/(d*(c + d*x))) + (b*((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d
+ b*x])/d - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

method	result
derivativedivides	$b\left(-\frac{\sin(bx+a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}}{d}\right)$
default	$b\left(-\frac{\sin(bx+a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}}{d}\right)$
risch	$-\frac{b\,\mathrm{e}^{\frac{i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-ibx-ia-\frac{-iad+ibc}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{-\frac{i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-bc)}{d}\right)}{2d^2}-\frac{(-2dxb-2b}{2d(dx+c)}$

input
```
int(sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-sin(b*x+a)/(-a*d+b*c+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+
b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = \frac{(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)-(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-d\sin\left(bx+a\right)}{d^3x+cd^2}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b*d*x + b*c)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - (b*d*x
+ b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - d*sin(b*x + a))
/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\big(\cdots}{2\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(
2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d + (b*
x + a)*d^2 - a*d^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(72) = 144$.

Time = 0.42 (sec) , antiderivative size = 521, normalized size of antiderivative = 7.24

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = \frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)+b^3c\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{(dx+c)}{}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*cos
_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) +
 b^3*c*cos(-(b*c - a*d)/d)*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*
d/(d*x + c)) + b*c - a*d)/d) - a*b^2*d*cos(-(b*c - a*d)/d)*cos_integral(((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + (d*x + c)*(
b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*sin(-(b*c - a*d)/d)*sin_integral(-(
(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^3*c*sin(
-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
)) + b*c - a*d)/d) - a*b^2*d*sin(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^2*d*sin(-(d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c
) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sin(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sin(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.7 $\int \frac{\sin(a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\sin(a+bx)}{(c+dx)^3} dx = -\frac{b\cos(a+bx)}{2d^2(c+dx)} - \frac{b^2 \operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{\sin(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

output
```
-1/2*b*cos(b*x+a)/d^2/(d*x+c)-1/2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3-1/2*s
in(b*x+a)/d/(d*x+c)^2-1/2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\sin(a+bx)}{(c+dx)^3} dx = -\frac{b^2 \operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right) + \frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2} + b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*(b^2*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[
a + b*x] + d*Sin[a + b*x]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral
[b*(c/d + x)])/d^3
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{b\left(\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(-\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\left(-\frac{b\int\frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3784

$$\frac{b\left(-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3042

$$\frac{b\left(-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3780

$$\frac{b\left(-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3783

$$\frac{b\left(-\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Sin[a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*Sin[a + b*x]/(d*(c + d*x)^2) + (b*(-(Cos[a + b*x]/(d*(c + d*x))) - (b
*((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d))/d))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44

method	result
derivativedivides	$b^2\left(-\frac{\sin(bx+a)}{2(-ad+bc+d(bx+a))^2 d}+\frac{-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\mathrm{s}}{d}}{d}}{2d}\right.$
default	$b^2\left(-\frac{\sin(bx+a)}{2(-ad+bc+d(bx+a))^2 d}+\frac{-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\mathrm{s}}{d}}{d}}{2d}\right.$
risch	$-\frac{ib^2 e^{\frac{i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-ibx-ia-\frac{-iad+ibc}{d}\right)}{4d^3}+\frac{ib^2 e^{-\frac{i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-bc)}{d}\right)}{4d^3}+\frac{i(2ib^3d^3}{}$

input
```
int(sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2*sin(b*x+a)/(-a*d+b*c+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+b*c+d
*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b
*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.57

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{d^2\sin(bx+a)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\right.}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^2*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integra
l((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + (b*d^2*x + b*c*d)
*cos(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{b^3\left(iE_3\left(\frac{ibc+i(bx+a)d-iad}{d}\right) - iE_3\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + b^3\left(E_3\left(\frac{ibc+i(bx+a)d-iad}{d}\right) + E_3\left(\cdots\right.\right.}{2\left(b^2c^2d - 2\,abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2\,(bcd^2 - ad^3)(bx+a)\right)b}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(
3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*d -
 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 5727, normalized size of antiderivative = 55.07

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*
d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(1/2*b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-b*x -
b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*
d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 +
 b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*
a)^2 + 4*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*t
an(1/2*a)*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c*d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx=\int \frac{\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx=\int \frac{\sin\left(bx+a\right)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(sin(b*x+a)/(d*x+c)^3,x)
```

output
```
int(sin(a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.8 $\int (c+dx)^4 \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 161

$$\int (c+dx)^4 \sin^2(a+bx)\,dx = \frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3} - \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} - \frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4} + \frac{d(c+dx)^3\sin^2(a+bx)}{b^2}$$

output
```
3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/4*d^4*cos(b*x+a)*sin(
b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3-1/2*(d*x+c)^4*cos(b
*x+a)*sin(b*x+a)/b-3/2*d^3*(d*x+c)*sin(b*x+a)^2/b^4+d*(d*x+c)^3*sin(b*x+a)
^2/b^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int (c+dx)^4 \sin^2(a+bx)\,dx$$
$$= \frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))-}{80b^5}$$

input
```
Integrate[(c + d*x)^4*Sin[a + b*x]^2,x]
```

output
```
(8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 2
0*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 10*(3*d^4
- 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3792, 17, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^2(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^4 \sin(a+bx)^2dx$$
$$\Big\downarrow \text{3792}$$
$$-\frac{3d^2\int (c+dx)^2\sin^2(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^4dx+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}$$
$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\int(c+dx)^2\sin^2(a+bx)dx}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$\Big\downarrow$ 3042

$$-\frac{3d^2\int(c+dx)^2\sin(a+bx)^2dx}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$\Big\downarrow$ 3792

$$-\frac{3d^2\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$\Big\downarrow$ 17

$$-\frac{3d^2\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$\Big\downarrow$ 3042

$$-\frac{3d^2\left(-\frac{d^2\int\sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$\Big\downarrow$ 3115

$$-\frac{3d^2\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$\Big\downarrow$ 24

$$-\frac{3d^2\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

input
```
Int[(c + d*x)^4*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^5/(10*d) - ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c
 + d*x)^3*Sin[a + b*x]^2)/b^2 - (3*d^2*((c + d*x)^3/(6*d) - ((c + d*x)^2*C
os[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2) - (
d^2*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/b^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(-2(dx+c)^4 b^4+6d^2(dx+c)^2 b^2-3d^4\right)\sin(2bx+2a)+4b\left(-(dx+c)d\left((dx+c)^2 b^2-\frac{3d^2}{2}\right)\cos(2bx+2a)+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3\right.\right.}{8b^5}$
risch	$\frac{d^4x^5}{10}+\frac{d^3c\,x^4}{2}+d^2c^2x^3+d\,c^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}-\frac{d(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3c\,d^2)\,c}{4b^4}$
orering	$\frac{(2b^6d^6x^7+14b^6c\,d^5x^6+42b^6c^2d^4x^5+70b^6c^3d^3x^4+70b^6c^4d^2x^3+20b^4d^6x^5+40b^6c^5d\,x^2+100b^4c\,d^5x^4+10b^6c^6x+200b^4}{10b}$
norman	$-\frac{d^4x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+d^3c\,x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\frac{d^4x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{b}+\frac{d^3c\,x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{2}+\frac{\left(2b^4c^4+18b^2c^2d^2-9d^4\right)x\tan\left(\frac{bx}{2}+\right.}{2b^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((-2*(d*x+c)^4*b^4+6*d^2*(d*x+c)^2*b^2-3*d^4)*sin(2*b*x+2*a)+4*b*(-(d*
x+c)*d*((d*x+c)^2*b^2-3/2*d^2)*cos(2*b*x+2*a)+x*(1/5*d^4*x^4+c*d^3*x^3+2*c
^2*d^2*x^2+2*c^3*d*x+c^4)*b^4+b^2*c^3*d-3/2*d^3*c))/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.78

$$\int (c+dx)^4 \sin^2(a+bx)\,dx$$
$$= \frac{2\,b^5d^4x^5+10\,b^5cd^3x^4+10\,(2\,b^5c^2d^2+b^3d^4)x^3+10\,(2\,b^5c^3d+3\,b^3cd^3)x^2-10\,(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b}{}$$

input
```
integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 + b^3*d^4)*x^3
+ 10*(2*b^5*c^3*d + 3*b^3*c*d^3)*x^2 - 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2
 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 -
 5*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 +
6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x
 + a)*sin(b*x + a) + 5*(2*b^5*c^4 + 6*b^3*c^2*d^2 - 3*b*d^4)*x)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.

Time = 0.70 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.10

$$\int (c+dx)^4 \sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x*
*2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a +
b*x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 +
 c*d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*
cos(a + b*x)**2/10 - c**4*sin(a + b*x)*cos(a + b*x)/(2*b) - 2*c**3*d*x*sin
(a + b*x)*cos(a + b*x)/b - 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b -
2*c*d**3*x**3*sin(a + b*x)*cos(a + b*x)/b - d**4*x**4*sin(a + b*x)*cos(a +
 b*x)/(2*b) + c**3*d*sin(a + b*x)**2/b**2 + 3*c**2*d**2*x*sin(a + b*x)**2/
(2*b**2) - 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) + 3*c*d**3*x**2*sin(a +
b*x)**2/(2*b**2) - 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) + d**4*x**3*sin(
a + b*x)**2/(2*b**2) - d**4*x**3*cos(a + b*x)**2/(2*b**2) + 3*c**2*d**2*si
n(a + b*x)*cos(a + b*x)/(2*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b*
*3 + 3*d**4*x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) - 3*c*d**3*sin(a + b*x
)**2/(2*b**4) - 3*d**4*x*sin(a + b*x)**2/(4*b**4) + 3*d**4*x*cos(a + b*x)*
*2/(4*b**4) - 3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*
*2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(147) = 294$.

Time = 0.06 (sec) , antiderivative size = 735, normalized size of antiderivative = 4.57

$$\int (c+dx)^4 \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/40*(10*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a - sin(2*b*
x + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2
- 40*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a - si
n(2*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x +
2*a) - cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b
*x + 2*a) - cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 - 2*(b*x +
 a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2
 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b
*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x
 + 2*a))*c^2*d^2/b^2 - 20*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) -
3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 -
6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2
*d^4/b^4 + 10*(2*(b*x + a)^4 - 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*
(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a
)^4 - 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x -
3*a)*sin(2*b*x + 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 - 10*(2*(b*x + a)^3 - 3*
b*x - 3*a)*cos(2*b*x + 2*a) - 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*
b*x + 2*a))*d^4/b^4)/b
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.38

$$\int (c+dx)^4 \sin^2(a+bx)\, dx = \frac{1}{10}\, d^4x^5 + \frac{1}{2}\, cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}\, c^4x - \frac{(2\, b^3d^4x^3 + 6\, b^3cd^3x^2 + 6\, b^3c^2d^2x + 2\, b^3c^3d - 3\, bd^4x - 3\, bcd^3)\cos(2\, bx + 2\, a)}{4\, b^5} - \frac{(2\, b^4d^4x^4 + 8\, b^4cd^3x^3 + 12\, b^4c^2d^2x^2 + 8\, b^4c^3dx + 2\, b^4c^4 - 6\, b^2d^4x^2 - 12\, b^2cd^3x - 6\, b^2c^2d^2 + 3\, d^4)\sin(2}{8\, b^5}$$

input
```
integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x - 1/4*(
2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*
x - 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 - 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3
 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2
*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.17

$$\int (c+dx)^4 \sin^2(a+bx)\,dx = -\frac{\frac{15\,d^4\,\sin(2\,a+2\,b\,x)}{2} - 10\,b^5\,c^4\,x + 5\,b^4\,c^4\,\sin\left(2\,a+2\,b\,x\right) - 2\,b^5\,d^4\,x^5 + 10\,b^3\,c^3\,d\,\cos\left(2\,a+2\,b\,x\right) - 20\,b^5\,c^3$$

input
```
int(sin(a + b*x)^2*(c + d*x)^4,x)
```

output
```
-((15*d^4*sin(2*a + 2*b*x))/2 - 10*b^5*c^4*x + 5*b^4*c^4*sin(2*a + 2*b*x)
- 2*b^5*d^4*x^5 + 10*b^3*c^3*d*cos(2*a + 2*b*x) - 20*b^5*c^3*d*x^2 - 10*b^
5*c*d^3*x^4 - 15*b^2*c^2*d^2*sin(2*a + 2*b*x) + 10*b^3*d^4*x^3*cos(2*a + 2
*b*x) - 20*b^5*c^2*d^2*x^3 - 15*b^2*d^4*x^2*sin(2*a + 2*b*x) + 5*b^4*d^4*x
^4*sin(2*a + 2*b*x) - 15*b*c*d^3*cos(2*a + 2*b*x) - 15*b*d^4*x*cos(2*a + 2
*b*x) + 30*b^4*c^2*d^2*x^2*sin(2*a + 2*b*x) - 30*b^2*c*d^3*x*sin(2*a + 2*b
*x) + 20*b^4*c^3*d*x*sin(2*a + 2*b*x) + 30*b^3*c^2*d^2*x*cos(2*a + 2*b*x)
+ 30*b^3*c*d^3*x^2*cos(2*a + 2*b*x) + 20*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/(
20*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.83

$$\int (c+dx)^4 \sin^2(a+bx)\,dx$$

$$= \frac{-10\cos(bx+a)\sin(bx+a)\,b^4d^4x^4 + 30\cos(bx+a)\sin(bx+a)\,b^2c^2d^2 + 30\cos(bx+a)\sin(bx+a)\,b^2}{}$$

input

```
int((d*x+c)^4*sin(b*x+a)^2,x)
```

output

```
( - 10*cos(a + b*x)*sin(a + b*x)*b**4*c**4 - 40*cos(a + b*x)*sin(a + b*x)*
b**4*c**3*d*x - 60*cos(a + b*x)*sin(a + b*x)*b**4*c**2*d**2*x**2 - 40*cos(
a + b*x)*sin(a + b*x)*b**4*c*d**3*x**3 - 10*cos(a + b*x)*sin(a + b*x)*b**4
*d**4*x**4 + 30*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d**2 + 60*cos(a + b*x)
*sin(a + b*x)*b**2*c*d**3*x + 30*cos(a + b*x)*sin(a + b*x)*b**2*d**4*x**2
- 15*cos(a + b*x)*sin(a + b*x)*d**4 + 20*sin(a + b*x)**2*b**3*c**3*d + 60*
sin(a + b*x)**2*b**3*c**2*d**2*x + 60*sin(a + b*x)**2*b**3*c*d**3*x**2 + 2
0*sin(a + b*x)**2*b**3*d**4*x**3 - 30*sin(a + b*x)**2*b*c*d**3 - 30*sin(a
+ b*x)**2*b*d**4*x + 10*a*b**4*c**4 + 90*a*b**2*c**2*d**2 - 45*a*d**4 + 10
*b**5*c**4*x + 20*b**5*c**3*d*x**2 + 20*b**5*c**2*d**2*x**3 + 10*b**5*c*d*
*3*x**4 + 2*b**5*d**4*x**5 - 40*b**3*c**3*d - 30*b**3*c**2*d**2*x - 30*b**
3*c*d**3*x**2 - 10*b**3*d**4*x**3 + 60*b*c*d**3 + 15*b*d**4*x)/(20*b**5)
```

3.9 $\int (c+dx)^3 \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (c+dx)^3 \sin^2(a+bx)\,dx = -\frac{3d(c+dx)^2}{8b^2} + \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} - \frac{3d^3\sin^2(a+bx)}{8b^4} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}$$

output
```
-3/8*d*(d*x+c)^2/b^2+1/8*(d*x+c)^4/d+3/4*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)
/b^3-1/2*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b-3/8*d^3*sin(b*x+a)^2/b^4+3/4*d*
(d*x+c)^2*sin(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int (c+dx)^3\sin^2(a+bx)\,dx$$
$$=\frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))-2b(c+dx)\left(-3d^2+2b^2(\right.}{16b^4}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]^2,x]
```

output
```
(2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 2*b^2*(
c + d*x)^2)*Cos[2*(a + b*x)] - 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*
Sin[2*(a + b*x)])/(16*b^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin^2(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3\sin(a+bx)^2dx$$
$$\Big\downarrow \text{3792}$$
$$-\frac{3d^2\int (c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^3dx+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}$$
$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$\Big\downarrow$ 3042

$$-\frac{3d^2\int(c+dx)\sin(a+bx)^2dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$\Big\downarrow$ 3791

$$-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$\Big\downarrow$ 17

$$-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^4/(8*d) - ((c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(
c + d*x)^2*Sin[a + b*x]^2)/(4*b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x
)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{-4(dx+c)\left((dx+c)^2b^2-\frac{3d^2}{2}\right)b\sin(2bx+2a)-6\left((dx+c)^2b^2-\frac{d^2}{2}\right)d\cos(2bx+2a)+2(d^3x^4+4c\,d^2x^3+6d\,c^2x^2+4c^3x)b^4+}{16b^4}$
risch	$\frac{d^3x^4}{8}+\frac{c\,d^2x^3}{2}+\frac{3d\,c^2x^2}{4}+\frac{c^3x}{2}+\frac{c^4}{8d}-\frac{3d(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2bx+2a)}{16b^4}-\frac{(2b^2d^3x^3+6b^2c\,d^2}{}$
orering	$\frac{(b^4d^5x^6+6b^4c\,d^4x^5+15b^4c^2d^3x^4+20b^4c^3d^2x^3+14b^4c^4d\,x^2+6b^2d^5x^4+4b^4c^5x+24b^2c\,d^4x^3+39b^2c^2d^3x^2+30b^2c^3d^2x+}{4b^4(dx+c)^2}$
norman	$\frac{c\,d^2x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\frac{d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{b}+\frac{d^3x^4}{8}+\frac{c\,d^2x^3}{2}+\frac{d^3x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{4}+\frac{d^3x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{8}+\frac{(6b^2c^2d-3d^3)\tan\left(\frac{b}{}\right.}{2b^4}}{}$
derivativedivides	$\frac{-\frac{a^3d^3\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{3a^2c\,d^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{3a^2d^3\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\right.\right.}{b^3}}{}$
default	$\frac{-\frac{a^3d^3\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{3a^2c\,d^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{3a^2d^3\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\right.\right.}{b^3}}{}$

input
```
int((d*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-4*(d*x+c)*((d*x+c)^2*b^2-3/2*d^2)*b*sin(2*b*x+2*a)-6*((d*x+c)^2*b^2
-1/2*d^2)*d*cos(2*b*x+2*a)+2*(d^3*x^4+4*c*d^2*x^3+6*c^2*d*x^2+4*c^3*x)*b^4
+6*b^2*c^2*d-3*d^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.52

$$\int (c+dx)^3\sin^2(a+bx)\,dx$$
$$= \frac{b^4d^3x^4+4\,b^4cd^2x^3+3\,(2\,b^4c^2d+b^2d^3)x^2-3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos{(bx+a)}^2-2\,(2\,b^3 \cdots}{8\,b \cdots}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d + b^2*d^3)*x^2 - 3*(2*
b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 2*(2*b^3
*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^
3)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 + 3*b^2*c*d^2)*x)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(119) = 238.

Time = 0.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.68

$$\int (c+dx)^3\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3x\sin^2{(a+bx)}}{2}+\frac{c^3x\cos^2{(a+bx)}}{2}+\frac{3c^2dx^2\sin^2{(a+bx)}}{4}+\frac{3c^2dx^2\cos^2{(a+bx)}}{4}+\frac{cd^2x^3\sin^2{(a+bx)}}{2}+\frac{cd^2x^3\cos^2{(a+bx)}}{2}+\frac{d^3x^4 \cdots}{\cdots} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*
x**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin
(a + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2
/8 + d**3*x**4*cos(a + b*x)**2/8 - c**3*sin(a + b*x)*cos(a + b*x)/(2*b) -
3*c**2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c*d**2*x**2*sin(a + b*x)*co
s(a + b*x)/(2*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c**2*d*si
n(a + b*x)**2/(4*b**2) + 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) - 3*c*d**2*x*
cos(a + b*x)**2/(4*b**2) + 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) - 3*d**3*x
**2*cos(a + b*x)**2/(8*b**2) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3)
 + 3*d**3*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3*sin(a + b*x)**2/(8
*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
*sin(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(112) = 224$.

Time = 0.05 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.56

$$\int (c + dx)^3 \sin^2(a + bx)\, dx$$

$$= \frac{4\,(2\,bx + 2\,a - \sin\,(2\,bx + 2\,a))c^3 - \frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))ac^2d}{b} + \frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2cd^2}{b^2} - \frac{4\,(2\,bx+2\,a-\sin(}{b^3}}{}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a - sin(2*b*x
 + 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4
*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 - 2*(b*x
+ a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 - 2*
(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a
)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*
(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 - 3*(2*(
b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b
*x + 2*a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int (c+dx)^3 \sin^2(a+bx)\,dx = \frac{1}{8}\,d^3x^4 + \frac{1}{2}\,cd^2x^3 + \frac{3}{4}\,c^2dx^2 + \frac{1}{2}\,c^3x - \frac{3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\cos(2\,bx + 2\,a)}{16\,b^4} - \frac{(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\sin(2\,bx + 2\,a)}{8\,b^4}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x - 3/16*(2*b^2*d^3*
x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 - 1/8*(2*b^3
*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c
*d^2)*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 35.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

$$\int (c+dx)^3 \sin^2(a+bx)\,dx = \frac{\frac{3\,d^3\,\cos(2\,a+2\,b\,x)}{2} + 4\,b^4\,c^3\,x - 2\,b^3\,c^3\,\sin(2\,a+2\,b\,x) + b^4\,d^3\,x^4 - 3\,b^2\,c^2\,d\,\cos(2\,a+2\,b\,x) + 6\,b^4\,c^2\,d\,x^2 + 4$$

input
```
int(sin(a + b*x)^2*(c + d*x)^3,x)
```

output
```
((3*d^3*cos(2*a + 2*b*x))/2 + 4*b^4*c^3*x - 2*b^3*c^3*sin(2*a + 2*b*x) + b
^4*d^3*x^4 - 3*b^2*c^2*d*cos(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2*
x^3 - 3*b^2*d^3*x^2*cos(2*a + 2*b*x) - 2*b^3*d^3*x^3*sin(2*a + 2*b*x) + 3*
b*c*d^2*sin(2*a + 2*b*x) + 3*b*d^3*x*sin(2*a + 2*b*x) - 6*b^2*c*d^2*x*cos(
2*a + 2*b*x) - 6*b^3*c^2*d*x*sin(2*a + 2*b*x) - 6*b^3*c*d^2*x^2*sin(2*a +
2*b*x))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.38

$$\int (c+dx)^3 \sin^2(a+bx)\,dx$$

$$= \frac{-4\cos(bx+a)\sin(bx+a)\,b^3c^3 - 12\cos(bx+a)\sin(bx+a)\,b^3c^2dx - 12\cos(bx+a)\sin(bx+a)\,b^3c\,d^2}{}$$

input
```
int((d*x+c)^3*sin(b*x+a)^2,x)
```

output
```
( - 4*cos(a + b*x)*sin(a + b*x)*b**3*c**3 - 12*cos(a + b*x)*sin(a + b*x)*b
**3*c**2*d*x - 12*cos(a + b*x)*sin(a + b*x)*b**3*c*d**2*x**2 - 4*cos(a + b
*x)*sin(a + b*x)*b**3*d**3*x**3 + 6*cos(a + b*x)*sin(a + b*x)*b*c*d**2 + 6
*cos(a + b*x)*sin(a + b*x)*b*d**3*x + 6*sin(a + b*x)**2*b**2*c**2*d + 12*s
in(a + b*x)**2*b**2*c*d**2*x + 6*sin(a + b*x)**2*b**2*d**3*x**2 - 3*sin(a
+ b*x)**2*d**3 + 4*a*b**3*c**3 + 18*a*b*c*d**2 + 4*b**4*c**3*x + 6*b**4*c*
*2*d*x**2 + 4*b**4*c*d**2*x**3 + b**4*d**3*x**4 - 12*b**2*c**2*d - 6*b**2*
c*d**2*x - 3*b**2*d**3*x**2 + 6*d**3)/(8*b**4)
```

3.10 $\int (c+dx)^2 \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int (c+dx)^2 \sin^2(a+bx)\,dx = -\frac{d^2x}{4b^2} + \frac{(c+dx)^3}{6d} + \frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b} + \frac{d(c+dx)\sin^2(a+bx)}{2b^2}$$

output
```
-1/4*d^2*x/b^2+1/6*(d*x+c)^3/d+1/4*d^2*cos(b*x+a)*sin(b*x+a)/b^3-1/2*(d*x+
c)^2*cos(b*x+a)*sin(b*x+a)/b+1/2*d*(d*x+c)*sin(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int (c+dx)^2 \sin^2(a+bx)\,dx = \frac{4b^3x(3c^2+3cdx+d^2x^2) - 6bd(c+dx)\cos(2(a+bx)) - 3(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{24b^3}$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x]^2,x]
```

output
```
(4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cos[2*(a + b*x)] -
3*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{24}$$

$$\frac{d(c+dx)\sin^2(a+bx)}{2b^2} - \frac{d^2\left(\frac{x}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2} - \frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^3}{6d}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^3/(6*d) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c
+ d*x)*Sin[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(
2*b)))/(2*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(-2(dx+c)^2b^2+d^2\right)\sin(2bx+2a)+4b\left(-\frac{d(dx+c)\cos(2bx+2a)}{2}+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2+\frac{cd}{2}\right)}{8b^3}$
risch	$\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}-\frac{d(dx+c)\cos(2bx+2a)}{4b^2}-\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{\frac{a^2d^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\right.\right.}{b^2}}{}$
default	$\frac{\frac{a^2d^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\right.\right.}{b^2}}{}$
norman	$\frac{cd\,x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\frac{d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{b}+\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{d^2x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{3}+\frac{d^2x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{6}-\frac{\left(2b^2c^2-d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right.}{2b^3}}{}$
orering	$\frac{(4b^4d^4x^5+20b^4c\,d^3x^4+40b^4c^2d^2x^3+36b^4c^3d\,x^2+12b^4c^4x+12b^2d^4x^3+42b^2c\,d^3x^2+48b^2c^2d^2x+12b^2c^3d-12d^4x-3d^3}{12(dx+c)^2b^4}$

input
```
int((d*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((-2*(d*x+c)^2*b^2+d^2)*sin(2*b*x+2*a)+4*b*(-1/2*d*(d*x+c)*cos(2*b*x+2
*a)+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2+1/2*c*d))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int (c+dx)^2\sin^2(a+bx)\,dx$$
$$=\frac{2\,b^3d^2x^3+6\,b^3cdx^2-6\,(bd^2x+bcd)\cos{(bx+a)}^2-3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(bx+a)}\sin{(}}{12\,b^3}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 -
 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x +
a) + 3*(2*b^3*c^2 + b*d^2)*x)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int (c+dx)^2\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2x\sin^2(a+bx)}{2} + \frac{c^2x\cos^2(a+bx)}{2} + \frac{cdx^2\sin^2(a+bx)}{2} + \frac{cdx^2\cos^2(a+bx)}{2} + \frac{d^2x^3\sin^2(a+bx)}{6} + \frac{d^2x^3\cos^2(a+bx)}{6} - \frac{c^2\sin(a+bx)}{2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*
sin(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2
/6 + d**2*x**3*cos(a + b*x)**2/6 - c**2*sin(a + b*x)*cos(a + b*x)/(2*b) -
c*d*x*sin(a + b*x)*cos(a + b*x)/b - d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2
*b) + c*d*sin(a + b*x)**2/(2*b**2) + d**2*x*sin(a + b*x)**2/(4*b**2) - d**
2*x*cos(a + b*x)**2/(4*b**2) + d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne
(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(85) = 170$.

Time = 0.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.44

$$\int (c+dx)^2\sin^2(a+bx)\,dx$$
$$= \frac{6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))c^2 - \frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))acd}{b} + \frac{6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2d^2}{b^2} + \frac{6\left(2\,(bx+a)^2-2\,(bx-\right.}{}}{}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/24*(6*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a - sin(2*b*x
 + 2*a))*a*c*d/b + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(
b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*c*d/b - 6*(2
*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a*d^2/b^2
+ (4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*si
n(2*b*x + 2*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int (c+dx)^2\sin^2(a+bx)\,dx = \frac{1}{6}\,d^2x^3+\frac{1}{2}\,cdx^2+\frac{1}{2}\,c^2x-\frac{(bd^2x+bcd)\cos(2\,bx+2\,a)}{4\,b^3} - \frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\sin(2\,bx+2\,a)}{8\,b^3}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x - 1/4*(b*d^2*x + b*c*d)*cos(2*b*x +
2*a)/b^3 - 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x +
 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 35.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\int (c+dx)^2\sin^2(a+bx)\,dx = x\left(\frac{c^2}{4}-\frac{d^2}{8\,b^2}\right)+x\left(\frac{c^2}{4}+\frac{d^2}{8\,b^2}\right)+\frac{d^2\,x^3}{6}+\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(d^2-2\,b^2\,c^2\right)}{8\,b^3}+\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}-\frac{d^2}{4\,b^2}\right)}{2}-\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}+\frac{d^2}{4\,b^2}\right)}{2}+\frac{c\,d\,x^2}{2}-\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}-\frac{c\,d\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{c\,d\,x\,\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

input
```
int(sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
x*(c^2/4 - d^2/(8*b^2)) + x*(c^2/4 + d^2/(8*b^2)) + (d^2*x^3)/6 + (sin(2*a
 + 2*b*x)*(d^2 - 2*b^2*c^2))/(8*b^3) + (x*cos(2*a + 2*b*x)*(c^2/2 - d^2/(4
*b^2)))/2 - (x*cos(2*a + 2*b*x)*(c^2/2 + d^2/(4*b^2)))/2 + (c*d*x^2)/2 - (
d^2*x^2*sin(2*a + 2*b*x))/(4*b) - (c*d*cos(2*a + 2*b*x))/(4*b^2) - (c*d*x*
sin(2*a + 2*b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.80

$$\int (c+dx)^2\sin^2(a+bx)\,dx = \frac{-6\cos(bx+a)\sin(bx+a)\,b^2c^2-12\cos(bx+a)\sin(bx+a)\,b^2cdx-6\cos(bx+a)\sin(bx+a)\,b^2d^2x^2\ \cdots}{}$$

input
```
int((d*x+c)^2*sin(b*x+a)^2,x)
```

output
```
( - 6*cos(a + b*x)*sin(a + b*x)*b**2*c**2 - 12*cos(a + b*x)*sin(a + b*x)*b
**2*c*d*x - 6*cos(a + b*x)*sin(a + b*x)*b**2*d**2*x**2 + 3*cos(a + b*x)*si
n(a + b*x)*d**2 + 6*sin(a + b*x)**2*b*c*d + 6*sin(a + b*x)**2*b*d**2*x + 6
*a*b**2*c**2 + 9*a*d**2 + 6*b**3*c**2*x + 6*b**3*c*d*x**2 + 2*b**3*d**2*x*
*3 - 12*b*c*d - 3*b*d**2*x)/(12*b**3)
```

3.11 $\int (c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int (c+dx)\sin^2(a+bx)\,dx=\frac{(c+dx)^2}{4d}-\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{d\sin^2(a+bx)}{4b^2}$$

output
```
1/4*(d*x+c)^2/d-1/2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b+1/4*d*sin(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (c+dx)\sin^2(a+bx)\,dx = \frac{-d\cos(2(a+bx))+2b(2ac+bx(2c+dx)-(c+dx)\sin(2(a+bx)))}{8b^2}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]^2,x]
```

output
```
(-(d*Cos[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) - (c + d*x)*Sin[2*(a
 + b*x)]))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{2}\int (c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}$$

input
```
Int[(c + d*x)*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a
 + b*x]^2)/(4*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result
risch	$\frac{d\,x^{2}}{4}+\frac{cx}{2}-\frac{d\cos(2bx+2a)}{8b^{2}}-\frac{(dx+c)\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{-\frac{da\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-(bx\right.}{b}}{b}$
default	$\frac{-\frac{da\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-(bx\right.}{b}}{b}$
norman	$\frac{\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{b}+cx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\frac{d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{b^{2}}+\frac{dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{b}+\frac{cx}{2}+\frac{d\,x^{2}}{4}-\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{cx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{2}+\frac{d\,x^{2}\,t}{}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)^{2}}$
orering	$\frac{(2b^{2}d^{3}x^{4}+8b^{2}c\,d^{2}x^{3}+10b^{2}c^{2}d\,x^{2}+4b^{2}c^{3}x+3d^{3}x^{2}+6c\,d^{2}x+2d\,c^{2})\sin(bx+a)^{2}}{4b^{2}(dx+c)^{2}}-\frac{(2x^{2}d^{2}+4cdx+c^{2})\left(d\sin(bx+a)^{2}+\right.}{4(dx+c}$

input
```
int((d*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d*x^2+1/2*c*x-1/8*d/b^2*cos(2*b*x+2*a)-1/4/b*(d*x+c)*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int (c+dx)\sin^2(a+bx)\,dx = \frac{b^2dx^2+2\,b^2cx-d\cos{(bx+a)}^2-2\,(bdx+bc)\cos{(bx+a)}\sin{(bx+a)}}{4\,b^2}$$

input
```
integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*d*x^2 + 2*b^2*c*x - d*cos(b*x + a)^2 - 2*(b*d*x + b*c)*cos(b*x +
a)*sin(b*x + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int (c+dx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{cx\sin^2(a+bx)}{2} + \frac{cx\cos^2(a+bx)}{2} + \frac{dx^2\sin^2(a+bx)}{4} + \frac{dx^2\cos^2(a+bx)}{4} - \frac{c\sin(a+bx)\cos(a+bx)}{2b} - \frac{dx\sin(a+bx)\cos(a+bx)}{2b} + \frac{d\,\mathrm{si}}{} \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a) \end{cases}$$

input
```
integrate((d*x+c)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a +
b*x)**2/4 + d*x**2*cos(a + b*x)**2/4 - c*sin(a + b*x)*cos(a + b*x)/(2*b) -
 d*x*sin(a + b*x)*cos(a + b*x)/(2*b) + d*sin(a + b*x)**2/(4*b**2), Ne(b, 0
)), ((c*x + d*x**2/2)*sin(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int (c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{2\,(2\,bx + 2\,a - \sin(2\,bx + 2\,a))c - \frac{2\,(2\,bx+2\,a-\sin(2\,bx+2\,a))ad}{b} + \frac{\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\right)d}{b}}{8\,b}$$

input
```
integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*(2*(2*b*x + 2*a - sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a - sin(2*b*x + 2
*a))*a*d/b + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int (c+dx)\sin^2(a+bx)\,dx = \frac{1}{4}dx^2 + \frac{1}{2}cx - \frac{d\cos(2bx+2a)}{8b^2} - \frac{(bdx+bc)\sin(2bx+2a)}{4b^2}$$

input
```
integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*d*x^2 + 1/2*c*x - 1/8*d*cos(2*b*x + 2*a)/b^2 - 1/4*(b*d*x + b*c)*sin(2
*b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 34.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int (c+dx)\sin^2(a+bx)\,dx = \frac{cx}{2} + \frac{dx^2}{4} - \frac{d\cos(2a+2bx)}{8b^2} - \frac{c\sin(2a+2bx)}{4b} - \frac{dx\sin(2a+2bx)}{4b}$$

input
```
int(sin(a + b*x)^2*(c + d*x),x)
```

output
```
(c*x)/2 + (d*x^2)/4 - (d*cos(2*a + 2*b*x))/(8*b^2) - (c*sin(2*a + 2*b*x))/
(4*b) - (d*x*sin(2*a + 2*b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int (c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)\sin(bx+a)\,bc-2\cos(bx+a)\sin(bx+a)\,bdx+\sin(bx+a)^2\,d+2abc+2b^2cx+b^2d\,x^2}{4b^2}$$

input
```
int((d*x+c)*sin(b*x+a)^2,x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)*b*c - 2*cos(a + b*x)*sin(a + b*x)*b*d*x +
sin(a + b*x)**2*d + 2*a*b*c + 2*b**2*c*x + b**2*d*x**2 - 2*d)/(4*b**2)
```

3.12 $\int \frac{\sin^2(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d} + \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output
```
-1/2*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d+1/2*ln(d*x+c)/d+1/2*sin(2*a-2*b*
c/d)*Si(2*b*c/d+2*b*x)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = \frac{-\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)+\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x),x]
```

output
```
(-(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d]) + Log[c + d*x] + S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2a+2bx)}{2(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x),x]
```

output
```
-1/2*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x
]/(2*d) + (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

method	result
risch	$\frac{e^{-\frac{2i(ad-bc)}{d}} \operatorname{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-bc)}{d}\right)}{4d}+\frac{e^{\frac{2i(ad-bc)}{d}} \operatorname{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{4d}+\frac{\ln(dx+}{2d}$
derivativedivides	$\frac{\frac{b\ln(-ad+bc+d(bx+a))}{2d}-\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{4}}{b}$
default	$\frac{\frac{b\ln(-ad+bc+d(bx+a))}{2d}-\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{4}}{b}$

input
```
int(sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*b*x+2*I*a-2*I*(a*d-b*c)/d)+1/4/d*exp(
2*I*(a*d-b*c)/d)*Ei(1,-2*I*b*x-2*I*a-2*(-I*a*d+I*b*c)/d)+1/2*ln(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\log\left(dx+c\right)}{2\,d}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - sin(-2*(b*c
- a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.08

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = \frac{b\left(E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right.\right.}{4\,bd}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*(b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
b*(I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integ
ral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) + 2
*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 612, normalized size of antiderivative = 7.85

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 - real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^
2*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - 2*i
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 2*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*sin_integral(2*(b*
d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 + real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(a)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)
 + 2*log(abs(d*x + c))*tan(b*c/d)^2 + real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*c/d)^2 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 2*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(a) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)
 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*c/d) + 2*log(abs(d*x + c)) - real_part(cos_integral(2*b*x + 2*b*c/
d)) - real_part(cos_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2
+ d*tan(a)^2 + d*tan(b*c/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx=\int \frac{\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x),x)
```

output
```
int(sin(a + b*x)^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)^2}{dx+c}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c),x)
```

output
```
int(sin(a + b*x)**2/(c + d*x),x)
```

3.13 $\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx = \frac{b\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{\sin^2(a+bx)}{d(c+dx)} + \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
b*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-sin(b*x+a)^2/d/(d*x+c)+b*cos(2*a-
2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx = \frac{b\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) - \frac{d\sin^2(a+bx)}{c+dx} + b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x)^2,x]
```

output
```
(b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - (d*Sin[a + b*x]^2
)/(c + d*x) + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3794, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$\frac{2b\int \frac{\sin(2a+2bx)}{2(c+dx)}dx}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3780}$$

$$\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$\frac{b\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^2,x]
```

output
```
-(Sin[a + b*x]^2/(d*(c + d*x))) + (b*((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[
2*a - (2*b*c)/d])/d + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x]
)/d))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.91

method	result
risch	$-\frac{ib\,\mathrm{e}^{-\frac{2i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-bc)}{d}\right)}{2d^2}+\frac{ib\,\mathrm{e}^{\frac{2i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^2}-\ldots$
derivativedivides	$\frac{-\frac{b^2}{2(-ad+bc+d(bx+a))d}-\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2a\ldots}{}\right)\ldots}{}\right)}{d}\right)}{4}}{b}$
default	$\frac{-\frac{b^2}{2(-ad+bc+d(bx+a))d}-\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2a\ldots}{}\right)\ldots}{}\right)}{d}\right)}{4}}{b}$

input
```
int(sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b/d^2*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*b*x+2*I*a-2*I*(a*d-b*c)/d)+1/2
*I*b/d^2*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*b*x-2*I*a-2*(-I*a*d+I*b*c)/d)-1/2/
d/(d*x+c)+1/4/d*(-2*b*d*x-2*b*c)/(-b*d*x-b*c)/(d*x+c)*cos(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx = \frac{d\cos(bx+a)^2 + (bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) + (bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right) -}{d^3x+cd^2}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
(d*cos(b*x + a)^2 + (b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(
b*c - a*d)/d) + (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x
+ b*c)/d) - d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{b^2\left(E_2\left(\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{2(-i\,bc-i\,(bx+a)d+i\,a}{d}\right.\right.}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d)
+ b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d)
 - 2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(81) = 162.

Time = 0.39 (sec) , antiderivative size = 535, normalized size of antiderivative = 6.60

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+2\,b^3c\,\mathrm{Ci}\left(\frac{2\left((dx+c)\right(}{}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(2*((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c -
 a*d)/d) + 2*b^3*c*cos_integral(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x
 + c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d)/d) - 2*a*b^2*d*cos_integral(2*((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c -
 a*d)/d) - 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-2*(b*c
 - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))
+ b*c - a*d)/d) - 2*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 2*a*b^2*d*cos(-2*(b
*c - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)
) + b*c - a*d)/d) + b^2*d*cos(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
 c))/d) - b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 +
 b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\sin\left(bx+a\right)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sin(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.14 $\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output
```
b^2*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^3-b*cos(b*x+a)*sin(b*x+a)/d^2/(d*
x+c)-1/2*sin(b*x+a)^2/d/(d*x+c)^2-b^2*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d
^3
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = -\frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(d\sin^2(a+bx)+b(c+dx)\sin(2(a+bx)))}{(c+dx)^2} + 2b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(\cdots}{}\right.}{2d^3}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x)^3,x]
```

output
```
-1/2*(-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d*S
in[a + b*x]^2 + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a
 - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3795, 16, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{2b^2\int \frac{\sin^2(a+bx)}{c+dx}dx}{d^2}+\frac{b^2\int \frac{1}{c+dx}dx}{d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\sin^2(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2b^2\int \frac{\sin^2(a+bx)}{c+dx}dx}{d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\sin^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b^2\int \frac{\sin(a+bx)^2}{c+dx}dx}{d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\sin^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{2b^2\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2a+2bx)}{2(c+dx)}\right)dx}{d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\sin^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2\left(-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\sin^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(b^2*Log[c + d*x])/d^3 - (b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*x)) - S
in[a + b*x]^2/(2*d*(c + d*x)^2) - (2*b^2*(-1/2*(Cos[2*a - (2*b*c)/d]*CosIn
tegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/(2*d) + (Sin[2*a - (2*b*c)/d]*
SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)))/d^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{b^3}{4(-ad+bc+d(bx+a))^2 d}-\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+bc+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2\cdots}{d}\right)}{d}\cdots}{d}\cdots\right)}{4}}{b}$
default	$\frac{-\frac{b^3}{4(-ad+bc+d(bx+a))^2 d}-\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+bc+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2\cdots}{d}\right)}{d}\cdots}{d}\cdots\right)}{4}}{b}$
risch	$-\frac{b^2 e^{-\frac{2i(ad-bc)}{d}}\operatorname{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-bc)}{d}\right)}{2d^3}-\frac{b^2 e^{\frac{2i(ad-bc)}{d}}\operatorname{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^3}-\cdots$

input
```
int(sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4*b^3/(-a*d+b*c+d*(b*x+a))^2/d-1/4*b^3*(-cos(2*b*x+2*a)/(-a*d+b*c+
d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-a*d+b*c+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2
*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*co
s(2*(-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.60

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{d^2\cos(bx+a)^2 + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) - 2\left(bd^2x + bcd\right)\cos(bx+ \cdots}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(d^2*cos(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(
b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 2*(b*d^2*x + b*c*d)*cos(b*
x + a)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c
- a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - d^2)/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.82

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = \frac{b^3\left(E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{2(-ibc-i(bx+a)d+ia\cdots}{d}\right)\cdots\right.}{4\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2(bcd^2-ad^3)(\cdots\right.}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d)
+ b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d)
 - b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 5141, normalized size of antiderivative = 45.50

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x
^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*
d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c
/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b
*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*
b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*
c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx=\int \frac{\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(bx+a)^2}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c)^3,x)
```

output
```
int(sin(a + b*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.15 $\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{b^2}{3d^3(c+dx)} - \frac{2b^3\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}$$

output
```
-1/3*b^2/d^3/(d*x+c)-2/3*b^3*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/3*b*
cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^2-1/3*sin(b*x+a)^2/d/(d*x+c)^3+2/3*b^2*s
in(b*x+a)^2/d^3/(d*x+c)-2/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{4b^3\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d\left(\left(-d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+d(d+b(c+dx)\sin(2(a+bx)))\right)}{(c+dx)^3}+4b^3\cos}{6d^4}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x)^4,x]
```

output
```
-1/6*(4*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-d^
2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(d + b*(c + d*x)*Sin[2*(a + b*
x)])))/(c + d*x)^3 + 4*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x)
)/d])/d^4
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3795, 17, 3042, 3794, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{2b^2\int \frac{\sin^2(a+bx)}{(c+dx)^2}dx}{3d^2}+\frac{b^2\int \frac{1}{(c+dx)^2}dx}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{2b^2\int \frac{\sin^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2b^2\int \frac{\sin(a+bx)^2}{(c+dx)^2}dx}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{ 3794}$$

$$-\frac{2b^2\left(\frac{2b\int\frac{\sin(2a+2bx)}{2(c+dx)}dx}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b^2\left(\frac{b\int\frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b^2\left(\frac{b\int\frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{2b^2\left(\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b^2\left(\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{2b^2\left(\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{2b^2\left(\frac{b\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}-\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^4,x]
```

output
```
-1/3*b^2/(d^3*(c + d*x)) - (b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^
2) - Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*(-(Sin[a + b*x]^2/(d*(c + d
*x))) + (b*((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (Cos
[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d))/d))/(3*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{-\frac{b^4}{6(-ad+bc+d(bx+a))^3 d}-\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3(-ad+bc+d(bx+a))^3 d}-\frac{2\left(-\frac{\sin(2bx+2a)}{(-ad+bc+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2b \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right)}{\cdots}\right)}{4 \cdots}}{b}$
default	$\frac{-\frac{b^4}{6(-ad+bc+d(bx+a))^3 d}-\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3(-ad+bc+d(bx+a))^3 d}-\frac{2\left(-\frac{\sin(2bx+2a)}{(-ad+bc+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2b \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right)}{\cdots}\right)}{4 \cdots}}{b}$
risch	$\frac{ib^3 e^{-\frac{2i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-bc)}{d}\right)}{3d^4}-\frac{ib^3 e^{\frac{2i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{3d^4}-\cdots$

input
```
int(sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/6*b^4/(-a*d+b*c+d*(b*x+a))^3/d-1/4*b^4*(-2/3*cos(2*b*x+2*a)/(-a*d+
b*c+d*(b*x+a))^3/d-2/3*(-sin(2*b*x+2*a)/(-a*d+b*c+d*(b*x+a))^2/d+(-2*cos(2
*b*x+2*a)/(-a*d+b*c+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2
*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/
d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.75

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = \frac{b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-d^3-\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-\left(bd^3x+bcd^2\right)\cos\left(\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - d^3 - (2*b^2*d^3*x^2 + 4*b^
2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - (b*d^3*x + b*c*d^2)*cos(b*
x + a)*sin(b*x + a) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*si
n_integral(2*(b*d*x + b*c)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*
d^4)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.59

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{3\,b^4\left(E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+3\,b^4\left(i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d}{d}\right.\right.}{12\,\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+\right.}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(3*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/
d) + 3*b^4*(I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*
exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*
d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3
*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 7832, normalized size of antiderivative = 48.35

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/3*(b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^3*d^3*x^3*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b
^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a)*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2
*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^
3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b
^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 -
 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^3
*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*
c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)
^2*tan(a)*tan(b*c/d) + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 6*b^3*c*d^2*x^2*real_part(cos_in...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^4,x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin{(bx+a)}^2}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx$$

input

```
int(sin(b*x+a)^2/(d*x+c)^4,x)
```

output

```
int(sin(a + b*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3
+ d**4*x**4),x)
```

3.16 $\int (c+dx)^4 \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 225

$$\begin{aligned}\int (c+dx)^4 \sin^3(a+bx)\,dx = &-\frac{488d^4\cos(a+bx)}{27b^5}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}\\ &-\frac{2(c+dx)^4\cos(a+bx)}{3b}+\frac{8d^4\cos^3(a+bx)}{81b^5}\\ &-\frac{160d^3(c+dx)\sin(a+bx)}{9b^4}+\frac{8d(c+dx)^3\sin(a+bx)}{3b^2}\\ &+\frac{4d^2(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{9b^3}\\ &-\frac{(c+dx)^4\cos(a+bx)\sin^2(a+bx)}{3b}\\ &-\frac{8d^3(c+dx)\sin^3(a+bx)}{27b^4}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}\end{aligned}$$

output
```
-488/27*d^4*cos(b*x+a)/b^5+80/9*d^2*(d*x+c)^2*cos(b*x+a)/b^3-2/3*(d*x+c)^4
*cos(b*x+a)/b+8/81*d^4*cos(b*x+a)^3/b^5-160/9*d^3*(d*x+c)*sin(b*x+a)/b^4+8
/3*d*(d*x+c)^3*sin(b*x+a)/b^2+4/9*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2/b^
3-1/3*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2/b-8/27*d^3*(d*x+c)*sin(b*x+a)^3/b^
4+4/9*d*(d*x+c)^3*sin(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int (c+dx)^4\sin^3(a+bx)\,dx$$
$$= \frac{-243(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)+(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cos}{32}$$

input
```
Integrate[(c + d*x)^4*Sin[a + b*x]^3,x]
```

output
```
(-243*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] + (
8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] - 24
*b*d*(c + d*x)*(242*d^2 - 39*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)^2
)*Cos[2*(a + b*x)])*Sin[a + b*x])/(324*b^5)
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.34, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\sin^3(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^4\sin(a+bx)^3dx$$

$$\Big\downarrow \ 3792$$

$$-\frac{4d^2\int (c+dx)^2\sin^3(a+bx)dx}{3b^2}+\frac{2}{3}\int (c+dx)^4\sin(a+bx)dx+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\sin(a+bx)dx+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\int(c+dx)^3\cos(a+bx)dx}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{3d\int-(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 25

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3118

$$-\frac{4d^2\int(c+dx)^2\sin(a+bx)^3dx}{3b^2}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3792

$$-\frac{4d^2\left(-\frac{2d^2\int\sin^3(a+bx)dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{4d^2\left(-\frac{2d^2\int\sin(a+bx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3113

$$-\frac{4d^2\left(\frac{2d^2\int\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 2009

$$-\frac{4d^2\left(\frac{2}{3}\int (c+dx)^2\sin(a+bx)dx+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 3777

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 3042

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin}{}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-$$

$$\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\big\downarrow$ 3777

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-$$

$$\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\big\downarrow$ 25

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\left(\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-$$

$$\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\left(\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-$$

$$\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^4\cos(a+bx)}{b}\right)-$$

$$\frac{4d^2\left(\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)-\frac{(c+dx}{}}{3b^2}$$

$$\frac{(c+dx)^4\sin^2(a+bx)\cos(a+bx)}{3b}$$

input
```
Int[(c + d*x)^4*Sin[a + b*x]^3,x]
```

output
```
-1/3*((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2)/b + (4*d*(c + d*x)^3*Sin[a
+ b*x]^3)/(9*b^2) - (4*d^2*((2*d^2*(Cos[a + b*x] - Cos[a + b*x]^3/3))/(9*b
^3) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (2*d*(c + d*x)*Sin
[a + b*x]^3)/(9*b^2) + (2*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[
a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/3))/(3*b^2) + (2*(-(((c +
 d*x)^4*Cos[a + b*x])/b) + (4*d*(((c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((
c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[
a + b*x])/b))/b))/b))/b))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\left(27(dx+c)^4b^4-36d^2(dx+c)^2b^2+8d^4\right)\cos(3bx+3a)-36(dx+c)db\left((dx+c)^2b^2-\frac{2d^2}{3}\right)\sin(3bx+3a)+\left(-243(dx+c)^4b^4+\right.$ … / 324 …
risch	$-\frac{3\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4\right)\cos(bx+a)}{4b^5}+$ $3d\left(b^2d^3x\right.$ …
orering	$16d\left(135b^6d^6x^6+810b^6c\,d^5x^5+2025b^6c^2d^4x^4+2700b^6c^3d^3x^3+2025b^6c^4d^2x^2-891b^4d^6x^4+810b^6c^5dx-3564b^4c\,d^5x^3+\right.$ … / $243b^8(dx$ …
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/324*((27*(d*x+c)^4*b^4-36*d^2*(d*x+c)^2*b^2+8*d^4)*cos(3*b*x+3*a)-36*(d*
x+c)*d*b*((d*x+c)^2*b^2-2/3*d^2)*sin(3*b*x+3*a)+(-243*(d*x+c)^4*b^4+2916*d
^2*(d*x+c)^2*b^2-5832*d^4)*cos(b*x+a)+972*(d*x+c)*d*b*((d*x+c)^2*b^2-6*d^2
)*sin(b*x+a)-216*b^4*c^4+2880*b^2*c^2*d^2-5824*d^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.56

$$\int (c+dx)^4 \sin^3(a+bx)\,dx$$
$$= \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 27\,b^4c^4 - 36\,b^2c^2d^2 + 8\,d^4 + 18\,(9\,b^4c^2d^2 - 2\,b^2d^4)x^2 + 36\,(3\,b^4c^3d - 2\,b^2cd^3)x}{}$$

input
```
integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3
)*x)*cos(b*x + a)^3 - 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 -
 252*b^2*c^2*d^2 + 488*d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 + 36*(3*b
^4*c^3*d - 14*b^2*c*d^3)*x)*cos(b*x + a) + 12*(21*b^3*d^4*x^3 + 63*b^3*c*d
^3*x^2 + 21*b^3*c^3*d - 122*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3
*b^3*c^3*d - 2*b*c*d^3 + (9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + (63
*b^3*c^2*d^2 - 122*b*d^4)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(226) = 452.

Time = 0.67 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.43

$$\int (c+dx)^4 \sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**4*cos(a + b*x)**3/(
3*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 8*c**3*d*x*cos(a + b*x)
**3/(3*b) - 6*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 4*c**2*d**2*
x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)/b - 8*
c*d**3*x**3*cos(a + b*x)**3/(3*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)
/b - 2*d**4*x**4*cos(a + b*x)**3/(3*b) + 28*c**3*d*sin(a + b*x)**3/(9*b**2
) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 28*c**2*d**2*x*sin(a
+ b*x)**3/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 28*
c*d**3*x**2*sin(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a +
b*x)**2/b**2 + 28*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 8*d**4*x**3*sin(a +
 b*x)*cos(a + b*x)**2/(3*b**2) + 28*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)
/(3*b**3) + 80*c**2*d**2*cos(a + b*x)**3/(9*b**3) + 56*c*d**3*x*sin(a + b*
x)**2*cos(a + b*x)/(3*b**3) + 160*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 28*d
**4*x**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 80*d**4*x**2*cos(a + b*x)
**3/(9*b**3) - 488*c*d**3*sin(a + b*x)**3/(27*b**4) - 160*c*d**3*sin(a + b
*x)*cos(a + b*x)**2/(9*b**4) - 488*d**4*x*sin(a + b*x)**3/(27*b**4) - 160*
d**4*x*sin(a + b*x)*cos(a + b*x)**2/(9*b**4) - 488*d**4*sin(a + b*x)**2*co
s(a + b*x)/(27*b**5) - 1456*d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c
**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(
a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(205) = 410$.

Time = 0.09 (sec) , antiderivative size = 934, normalized size of antiderivative = 4.15

$$\int (c + dx)^4 \sin^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/324*(108*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^4 - 432*(cos(b*x + a)^3 - 3
*cos(b*x + a))*a*c^3*d/b + 648*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*c^2*d
^2/b^2 - 432*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^3*c*d^3/b^3 + 108*(cos(b*
x + a)^3 - 3*cos(b*x + a))*a^4*d^4/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a)
- 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*c^3*d/b
- 108*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*
x + 3*a) + 27*sin(b*x + a))*a*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3
*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a^2*
c*d^3/b^3 - 36*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) -
 sin(3*b*x + 3*a) + 27*sin(b*x + a))*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)
*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*
b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 36*((9*(b*x + a)^2
- 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*si
n(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 18*((9*(b*x + a
)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a
)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 12*(3*(3*(b
*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x + a)^3 - 6*b*x - 6*a)
*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 243*((b*x + a)^2 -
2)*sin(b*x + a))*c*d^3/b^3 - 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x
 + 3*a) - 81*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 ...
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.56

$$\int (c+dx)^4 \sin^3(a+bx)\,dx$$
$$= \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 108\,b^4c^3dx + 27\,b^4c^4 - 36\,b^2d^4x^2 - 72\,b^2cd^3x - 36\,b^2c^2d^2 + 8\,\ldots}{324\,b^5}$$
$$- \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4)\cos(\ldots}{4\,b^5}$$
$$- \frac{(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 9\,b^3c^2d^2x + 3\,b^3c^3d - 2\,bd^4x - 2\,bcd^3)\sin(3\,bx + 3\,a)}{27\,b^5}$$
$$+ \frac{3\,(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3)\sin(bx + a)}{b^5}$$

input
```
integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*
c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 +
8*d^4)*cos(3*b*x + 3*a)/b^5 - 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c
^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 1
2*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 - 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d
^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x
+ 3*a)/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*
d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 36.90 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.37

$$\int (c+dx)^4\sin^3(a+bx)\,dx$$
$$= \frac{8\,x\cos\left(a+b\,x\right)^3\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^3} - \frac{2\cos\left(a+b\,x\right)^3\left(27\,b^4\,c^4-360\,b^2\,c^2\,d^2+728\,d^4\right)}{81\,b^5} - \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{27\,b^5} - \frac{8\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^4} - \frac{2\,d^4\,x^4\cos\left(a+b\,x\right)^3}{3\,b} - \frac{4\sin\left(a+b\,x\right)^3\left(122\,c\,d^3-21\,b^2\,c^3\,d\right)}{27\,b^4} + \frac{28\,d^4\,x^3\sin\left(a+b\,x\right)^3}{9\,b^2} - \frac{4\,x\sin\left(a+b\,x\right)^3\left(122\,d^4-63\,b^2\,c^2\,d^2\right)}{27\,b^4} + \frac{4\,x^2\cos\left(a+b\,x\right)^3\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^3} + \frac{2\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{3\,b^3} - \frac{8\,c\,d^3\,x^3\cos\left(a+b\,x\right)^3}{3\,b} - \frac{d^4\,x^4\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b} + \frac{8\,d^4\,x^3\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^2} + \frac{28\,c\,d^3\,x^2\sin\left(a+b\,x\right)^3}{3\,b^2} - \frac{8\,x\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^4} + \frac{4\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{3\,b^3} - \frac{4\,c\,d^3\,x^3\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b} + \frac{8\,c\,d^3\,x^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b^2}$$

input
```
int(sin(a + b*x)^3*(c + d*x)^4,x)
```

output
```
(8*x*cos(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2*cos(a + b*x)^3*
(728*d^4 + 27*b^4*c^4 - 360*b^2*c^2*d^2))/(81*b^5) - (cos(a + b*x)*sin(a +
 b*x)^2*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(27*b^5) - (8*cos(a + b*
x)^2*sin(a + b*x)*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (2*d^4*x^4*cos(a + b
*x)^3)/(3*b) - (4*sin(a + b*x)^3*(122*c*d^3 - 21*b^2*c^3*d))/(27*b^4) + (2
8*d^4*x^3*sin(a + b*x)^3)/(9*b^2) - (4*x*sin(a + b*x)^3*(122*d^4 - 63*b^2*
c^2*d^2))/(27*b^4) + (4*x^2*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(9*b^
3) + (2*x^2*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(3*b^3)
- (8*c*d^3*x^3*cos(a + b*x)^3)/(3*b) - (d^4*x^4*cos(a + b*x)*sin(a + b*x)^
2)/b + (8*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (28*c*d^3*x^2*sin
(a + b*x)^3)/(3*b^2) - (8*x*cos(a + b*x)^2*sin(a + b*x)*(20*d^4 - 9*b^2*c^
2*d^2))/(9*b^4) + (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*c*d^3 - 3*b^2*c^3*d
))/(3*b^3) - (4*c*d^3*x^3*cos(a + b*x)*sin(a + b*x)^2)/b + (8*c*d^3*x^2*co
s(a + b*x)^2*sin(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.72

$$\int (c + dx)^4 \sin^3(a + bx)\, dx$$
$$= \frac{36 \sin(bx + a)^3 b^3 c^3 d + 36 \sin(bx + a)^3 b^3 d^4 x^3 - 24 \sin(bx + a)^3 bc\, d^3 - 24 \sin(bx + a)^3 b\, d^4 x + 216 a\, b^3 c}{}$$

input
```
int((d*x+c)^4*sin(b*x+a)^3,x)
```

output

```
( - 27*cos(a + b*x)*sin(a + b*x)**2*b**4*c**4 - 108*cos(a + b*x)*sin(a + b
*x)**2*b**4*c**3*d*x - 162*cos(a + b*x)*sin(a + b*x)**2*b**4*c**2*d**2*x**
2 - 108*cos(a + b*x)*sin(a + b*x)**2*b**4*c*d**3*x**3 - 27*cos(a + b*x)*si
n(a + b*x)**2*b**4*d**4*x**4 + 36*cos(a + b*x)*sin(a + b*x)**2*b**2*c**2*d
**2 + 72*cos(a + b*x)*sin(a + b*x)**2*b**2*c*d**3*x + 36*cos(a + b*x)*sin(
a + b*x)**2*b**2*d**4*x**2 - 8*cos(a + b*x)*sin(a + b*x)**2*d**4 - 54*cos(
a + b*x)*b**4*c**4 - 216*cos(a + b*x)*b**4*c**3*d*x - 324*cos(a + b*x)*b**
4*c**2*d**2*x**2 - 216*cos(a + b*x)*b**4*c*d**3*x**3 - 54*cos(a + b*x)*b**
4*d**4*x**4 + 720*cos(a + b*x)*b**2*c**2*d**2 + 1440*cos(a + b*x)*b**2*c*d
**3*x + 720*cos(a + b*x)*b**2*d**4*x**2 - 1456*cos(a + b*x)*d**4 + 36*sin(
a + b*x)**3*b**3*c**3*d + 108*sin(a + b*x)**3*b**3*c**2*d**2*x + 108*sin(a
 + b*x)**3*b**3*c*d**3*x**2 + 36*sin(a + b*x)**3*b**3*d**4*x**3 - 24*sin(a
 + b*x)**3*b*c*d**3 - 24*sin(a + b*x)**3*b*d**4*x + 216*sin(a + b*x)*b**3*
c**3*d + 648*sin(a + b*x)*b**3*c**2*d**2*x + 648*sin(a + b*x)*b**3*c*d**3*
x**2 + 216*sin(a + b*x)*b**3*d**4*x**3 - 1440*sin(a + b*x)*b*c*d**3 - 1440
*sin(a + b*x)*b*d**4*x + 216*a*b**3*c**3*d - 576*a*b*c*d**3 - 54*b**4*c**4
 + 288*b**2*c**2*d**2 - 496*d**4)/(81*b**5)
```

3.17 $\int (c+dx)^3 \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\begin{aligned}\int (c+dx)^3 \sin^3(a+bx)\,dx = {}& \frac{40d^2(c+dx)\cos(a+bx)}{9b^3} - \frac{2(c+dx)^3\cos(a+bx)}{3b} \\ & - \frac{40d^3\sin(a+bx)}{9b^4} + \frac{2d(c+dx)^2\sin(a+bx)}{b^2} \\ & + \frac{2d^2(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^3} \\ & - \frac{(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{3b} \\ & - \frac{2d^3\sin^3(a+bx)}{27b^4} + \frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}\end{aligned}$$

output
```
40/9*d^2*(d*x+c)*cos(b*x+a)/b^3-2/3*(d*x+c)^3*cos(b*x+a)/b-40/9*d^3*sin(b*
x+a)/b^4+2*d*(d*x+c)^2*sin(b*x+a)/b^2+2/9*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a
)^2/b^3-1/3*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b-2/27*d^3*sin(b*x+a)^3/b^4+
1/3*d*(d*x+c)^2*sin(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

$$\int (c+dx)^3 \sin^3(a+bx)\, dx = \frac{-162b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+6b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))-4}{216b^4}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]^3,x]
```

output
```
(-162*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 6*b*(c + d*x)*
(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] - 4*d*(242*d^2 - 117*b^2*(c
+ d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(2
16*b^4)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^3(a+bx)\, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^3 \sin(a+bx)^3 dx$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{2d^2\int (c+dx)\sin^3(a+bx)dx}{3b^2}+\frac{2}{3}\int (c+dx)^3\sin(a+bx)dx+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin(a+bx)dx+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int(c+dx)^2\cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{2d\int-((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3117

$$-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3791

$$-\frac{2d^2\left(\frac{2}{3}\int (c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2d^2\left(\frac{2}{3}\int (c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3117}$$

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]^3,x]
```

output
```
-1/3*((c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/b + (d*(c + d*x)^2*Sin[a +
b*x]^3)/(3*b^2) - (2*d^2*(-1/3*((c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/b +
 (d*Sin[a + b*x]^3)/(9*b^2) + (2*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a
 + b*x])/b^2))/3))/(3*b^2) + (2*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*((
(c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin
[a + b*x])/b^2))/b))/b))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{3(dx+c)\left((dx+c)^2b^2-\frac{2d^2}{3}\right)b\cos(3bx+3a)-3d\left((dx+c)^2b^2-\frac{2d^2}{9}\right)\sin(3bx+3a)-27(dx+c)b\left((dx+c)^2b^2-6d^2\right)\cos(bx+}{36b^4}$
risch	$-\frac{3(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(bx+a)}{4b^3}+\frac{9d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^4}+$
derivativedivides	$\frac{a^3d^3\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3b^3}-\frac{a^2c\,d^2\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3}+\frac{\sin(bx+a)^3}{9}\right.}{b^3}$
default	$\frac{a^3d^3\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3b^3}-\frac{a^2c\,d^2\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3}+\frac{\sin(bx+a)^3}{9}\right.}{b^3}$
orering	$\frac{20d(9d^4x^4b^4+36b^4c\,d^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4-22b^2d^4x^2-44b^2c\,d^3x-22b^2c^2d^2-72d^4)\sin(bx+a)^3}{27b^6(dx+c)^2}-$
norman	$\frac{\frac{8c\,d^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b^3}+\frac{-12b^2c^3+80c\,d^2}{9b^3}-\frac{2d^3x^3}{3b}+\frac{\left(-12b^2c^3+56c\,d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{3b^3}-\frac{2c\,d^2x^2}{b}+\frac{4d\left(9b^2c^2-20d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{9b^4}+}{}$

input
```
int((d*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*(d*x+c)*((d*x+c)^2*b^2-2/3*d^2)*b*cos(3*b*x+3*a)-3*d*((d*x+c)^2*b^
2-2/9*d^2)*sin(3*b*x+3*a)-27*(d*x+c)*b*((d*x+c)^2*b^2-6*d^2)*cos(b*x+a)+81
*d*((d*x+c)^2*b^2-2*d^2)*sin(b*x+a)-24*b^3*c^3+160*c*d^2*b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int (c+dx)^3\sin^3(a+bx)\,dx$$
$$=\frac{3\left(3b^3d^3x^3+9b^3cd^2x^2+3b^3c^3-2bcd^2+(9b^3c^2d-2bd^3)x\right)\cos(bx+a)^3-9\left(3b^3d^3x^3+9b^3cd^2x^2+3\right.}{}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*
c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 +
3*b^3*c^3 - 14*b*c*d^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*cos(b*x + a) + (63*b^
2*d^3*x^2 + 126*b^2*c*d^2*x + 63*b^2*c^2*d - 122*d^3 - (9*b^2*d^3*x^2 + 18
*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 0.46 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.83

$$\int (c+dx)^3 \sin^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c^3\cos^3(a+bx)}{3b} - \frac{3c^2dx\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c^2dx\cos^3(a+bx)}{b} - \frac{3cd^2x^2\sin^2(a+bx)\cos(a+bx)}{b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**3*cos(a + b*x)**3/(
3*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**2*d*x*cos(a + b*x)
**3/b - 3*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*c*d**2*x**2*cos(a
 + b*x)**3/b - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)/b - 2*d**3*x**3*cos(
a + b*x)**3/(3*b) + 7*c**2*d*sin(a + b*x)**3/(3*b**2) + 2*c**2*d*sin(a + b
*x)*cos(a + b*x)**2/b**2 + 14*c*d**2*x*sin(a + b*x)**3/(3*b**2) + 4*c*d**2
*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 7*d**3*x**2*sin(a + b*x)**3/(3*b**2
) + 2*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2 + 14*c*d**2*sin(a + b*x)
**2*cos(a + b*x)/(3*b**3) + 40*c*d**2*cos(a + b*x)**3/(9*b**3) + 14*d**3*x
*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 40*d**3*x*cos(a + b*x)**3/(9*b**3
) - 122*d**3*sin(a + b*x)**3/(27*b**4) - 40*d**3*sin(a + b*x)*cos(a + b*x)
**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x
**4/4)*sin(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(161) = 322$.

Time = 0.06 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.09

$$\int (c+dx)^3 \sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/108*(36*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^3 - 108*(cos(b*x + a)^3 - 3*
cos(b*x + a))*a*c^2*d/b + 108*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*c*d^2/
b^2 - 36*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^3*d^3/b^3 + 9*(3*(b*x + a)*co
s(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x
 + a))*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x +
 a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos
(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x
+ a))*a^2*d^3/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a
)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b
*x + a))*c*d^2/b^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x +
a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(
b*x + a))*a*d^3/b^3 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) -
81*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*
x + 3*a) + 243*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.32

$$\begin{aligned}\int (c+dx)^3 \sin^3(a+bx)\,dx &= \frac{(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2)\cos(3\,bx+3\,a)}{36\,b^4}\\ &\quad-\frac{3\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\cos(bx+a)}{4\,b^4}\\ &\quad-\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\sin(3\,bx+3\,a)}{108\,b^4}\\ &\quad+\frac{9\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\sin(bx+a)}{4\,b^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^
3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 3/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 1/1
08*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)
/b^4 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/
b^4
```

Mupad [B] (verification not implemented)

Time = 35.81 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.09

$$\int (c+dx)^3\sin^3(a+bx)\,dx = \frac{2\cos(a+b\,x)^3\,(20\,c\,d^2-3\,b^2\,c^3)}{9\,b^3} - \frac{\sin(a+b\,x)^3\,(122\,d^3-63\,b^2\,c^2\,d)}{27\,b^4} + \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(14\,c\,d^2-3\,b^2\,c^3)}{3\,b^3} - \frac{2\cos(a+b\,x)^2\,\sin(a+b\,x)\,(20\,d^3-9\,b^2\,c^2\,d)}{9\,b^4} + \frac{2\,x\cos(a+b\,x)^3\,(20\,d^3-9\,b^2\,c^2\,d)}{9\,b^3} - \frac{2\,d^3\,x^3\cos(a+b\,x)^3}{3\,b} + \frac{7\,d^3\,x^2\sin(a+b\,x)^3}{3\,b^2} + \frac{14\,c\,d^2\,x\sin(a+b\,x)^3}{3\,b^2} + \frac{x\,\cos(a+b\,x)\,\sin(a+b\,x)^2\,(14\,d^3-9\,b^2\,c^2\,d)}{3\,b^3} - \frac{2\,c\,d^2\,x^2\cos(a+b\,x)^3}{b} - \frac{d^3\,x^3\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{b} + \frac{2\,d^3\,x^2\cos(a+b\,x)^2\,\sin(a+b\,x)}{b^2} - \frac{3\,c\,d^2\,x^2\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{b} + \frac{4\,c\,d^2\,x\cos(a+b\,x)^2\,\sin(a+b\,x)}{b^2}$$

input
```
int(sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
(2*cos(a + b*x)^3*(20*c*d^2 - 3*b^2*c^3))/(9*b^3) - (sin(a + b*x)^3*(122*d
^3 - 63*b^2*c^2*d))/(27*b^4) + (cos(a + b*x)*sin(a + b*x)^2*(14*c*d^2 - 3*
b^2*c^3))/(3*b^3) - (2*cos(a + b*x)^2*sin(a + b*x)*(20*d^3 - 9*b^2*c^2*d))
/(9*b^4) + (2*x*cos(a + b*x)^3*(20*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*d^3*x^
3*cos(a + b*x)^3)/(3*b) + (7*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (14*c*d^2*x
*sin(a + b*x)^3)/(3*b^2) + (x*cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*
c^2*d))/(3*b^3) - (2*c*d^2*x^2*cos(a + b*x)^3)/b - (d^3*x^3*cos(a + b*x)*s
in(a + b*x)^2)/b + (2*d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 - (3*c*d^2*
x^2*cos(a + b*x)*sin(a + b*x)^2)/b + (4*c*d^2*x*cos(a + b*x)^2*sin(a + b*x
))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.25

$$\int (c+dx)^3 \sin^3(a+bx)\,dx$$
$$= \frac{-9\cos(bx+a)\sin(bx+a)^2 b^3c^3 - 27\cos(bx+a)\sin(bx+a)^2 b^3c^2dx - 27\cos(bx+a)\sin(bx+a)^2 b^3 c}{}$$

input
```
int((d*x+c)^3*sin(b*x+a)^3,x)
```

output
```
( - 9*cos(a + b*x)*sin(a + b*x)**2*b**3*c**3 - 27*cos(a + b*x)*sin(a + b*x
)**2*b**3*c**2*d*x - 27*cos(a + b*x)*sin(a + b*x)**2*b**3*c*d**2*x**2 - 9*
cos(a + b*x)*sin(a + b*x)**2*b**3*d**3*x**3 + 6*cos(a + b*x)*sin(a + b*x)*
*2*b*c*d**2 + 6*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x - 18*cos(a + b*x)*b*
*3*c**3 - 54*cos(a + b*x)*b**3*c**2*d*x - 54*cos(a + b*x)*b**3*c*d**2*x**2
 - 18*cos(a + b*x)*b**3*d**3*x**3 + 120*cos(a + b*x)*b*c*d**2 + 120*cos(a
+ b*x)*b*d**3*x + 9*sin(a + b*x)**3*b**2*c**2*d + 18*sin(a + b*x)**3*b**2*
c*d**2*x + 9*sin(a + b*x)**3*b**2*d**3*x**2 - 2*sin(a + b*x)**3*d**3 + 54*
sin(a + b*x)*b**2*c**2*d + 108*sin(a + b*x)*b**2*c*d**2*x + 54*sin(a + b*x
)*b**2*d**3*x**2 - 120*sin(a + b*x)*d**3 + 54*a*b**2*c**2*d - 48*a*d**3 -
18*b**3*c**3 + 48*b*c*d**2)/(27*b**4)
```

3.18 $\int (c+dx)^2 \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int (c+dx)^2 \sin^3(a+bx)\,dx = \frac{14d^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^2\cos(a+bx)}{3b} - \frac{2d^2\cos^3(a+bx)}{27b^3} + \frac{4d(c+dx)\sin(a+bx)}{3b^2} - \frac{(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{2d(c+dx)\sin^3(a+bx)}{9b^2}$$

output

```
14/9*d^2*cos(b*x+a)/b^3-2/3*(d*x+c)^2*cos(b*x+a)/b-2/27*d^2*cos(b*x+a)^3/b
^3+4/3*d*(d*x+c)*sin(b*x+a)/b^2-1/3*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2/b+2/
9*d*(d*x+c)*sin(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int (c+dx)^2 \sin^3(a+bx)\, dx = \frac{-81(-2d^2+b^2(c+dx)^2)\cos(a+bx)+(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))-6bd(c+dx)(-27\sin(a$$
$$108b^3$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x]^3,x]
```

output
```
(-81*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d*x)^2
)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[a + b*x] + Sin[3*(a + b*x)])
)/(108*b^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^3(a+bx)\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \sin(a+bx)^3 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{2d^2\int \sin^3(a+bx)dx}{9b^2}+\frac{2}{3}\int (c+dx)^2 \sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int\sin(a+bx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 3113

$$\frac{2d^2\int\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 2009

$$\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 3777

$$\frac{2}{3}\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 3777

$$\frac{2}{3}\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\Big\downarrow$ 25

$$\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}$$

input

```
Int[(c + d*x)^2*Sin[a + b*x]^3,x]
```

output

```
(2*d^2*(Cos[a + b*x] - Cos[a + b*x]^3/3))/(9*b^3) - ((c + d*x)^2*Cos[a + b
*x]*Sin[a + b*x]^2)/(3*b) + (2*d*(c + d*x)*Sin[a + b*x]^3)/(9*b^2) + (2*(-
(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*S
in[a + b*x])/b))/b))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{\left(9(dx+c)^2b^2-2d^2\right)\cos(3bx+3a)-6bd(dx+c)\sin(3bx+3a)+\left(-81(dx+c)^2b^2+162d^2\right)\cos(bx+a)+162bd(dx+c)\sin(b \ldots}{108b^3}$
risch	$-\frac{3\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\cos(bx+a)}{4b^3}+\frac{3d(dx+c)\sin(bx+a)}{2b^2}+\frac{\left(9x^2d^2b^2+18b^2cdx+9b^2c^2-2d^2\right)\cos(3bx+3 \ldots}{108b^3}$
derivativedivides	$\frac{-\frac{a^2d^2\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3b^2}+\frac{2acd\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3}+\frac{\sin(bx+a)^3}{9}+ \ldots\right.}{b^2} \ldots}{\ldots}$
default	$\frac{-\frac{a^2d^2\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3b^2}+\frac{2acd\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{3}+\frac{\sin(bx+a)^3}{9}+ \ldots\right.}{b^2} \ldots}{\ldots}$
norman	$\frac{\frac{-36b^2c^2+80d^2}{27b^3}-\frac{2d^2x^2}{3b}+\frac{8d^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{3b^3}+\frac{\left(-36b^2c^2+56d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{9b^3}-\frac{4cdx}{3b}+\frac{8cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b^2}+\frac{64cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{9b^2}+ \ldots}{\ldots}$
orering	$\frac{40d\left(9d^4x^4b^4+36b^4c\,d^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4-b^2d^4x^2-2b^2c\,d^3x-b^2c^2d^2-12d^4\right)\sin(bx+a)^3}{81b^6(dx+c)^3}-\frac{2\left(45d^4x \ldots\right.}{\ldots}$

input
```
int((d*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*((9*(d*x+c)^2*b^2-2*d^2)*cos(3*b*x+3*a)-6*b*d*(d*x+c)*sin(3*b*x+3*a)
+(-81*(d*x+c)^2*b^2+162*d^2)*cos(b*x+a)+162*b*d*(d*x+c)*sin(b*x+a)-72*b^2*
c^2+160*d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int (c+dx)^2\sin^3(a+bx)\,dx$$
$$= \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(bx+a)^3-3\,(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-14\,d^2)\cos(bx+a) \ldots}{27\,b^3}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
3*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 14*d^2)*cos(b*x + a) + 6*(7*
b*d^2*x + 7*b*c*d - (b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.

Time = 0.34 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.31

$$\int (c+dx)^2 \sin^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c^2\cos^3(a+bx)}{3b} - \frac{2cdx\sin^2(a+bx)\cos(a+bx)}{b} - \frac{4cdx\cos^3(a+bx)}{3b} - \frac{d^2x^2\sin^2(a+bx)\cos(a+bx)}{b} - \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**2*cos(a + b*x)**3/(
3*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 4*c*d*x*cos(a + b*x)**3/(3
*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*d**2*x**2*cos(a + b*x)*
*3/(3*b) + 14*c*d*sin(a + b*x)**3/(9*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*
x)**2/(3*b**2) + 14*d**2*x*sin(a + b*x)**3/(9*b**2) + 4*d**2*x*sin(a + b*x
)*cos(a + b*x)**2/(3*b**2) + 14*d**2*sin(a + b*x)**2*cos(a + b*x)/(9*b**3)
 + 40*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**
2*x**3/3)*sin(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(111) = 222$.

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.20

$$\int (c+dx)^2 \sin^3(a+bx)\,dx$$
$$= \frac{36\left(\cos(bx+a)^3 - 3\cos(bx+a)\right)c^2 - \frac{72\left(\cos(bx+a)^3 - 3\cos(bx+a)\right)acd}{b} + \frac{36\left(\cos(bx+a)^3 - 3\cos(bx+a)\right)a^2d^2}{b^2} + \frac{6(3(bx}{}}{}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/108*(36*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^2 - 72*(cos(b*x + a)^3 - 3*c
os(b*x + a))*a*c*d/b + 36*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*d^2/b^2 +
6*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x +
3*a) + 27*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x
+ a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a*d^2/b^2 + ((9*(b
*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b
*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int (c+dx)^2\sin^3(a+bx)\,dx = \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(3\,bx+3\,a)}{108\,b^3} - \frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)}{4\,b^3} - \frac{(bd^2x+bcd)\sin(3\,bx+3\,a)}{18\,b^3} + \frac{3\,(bd^2x+bcd)\sin(bx+a)}{2\,b^3}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/
b^3 - 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 -
 1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*sin(b
*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 35.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.41

$$\int (c+dx)^2\sin^3(a+bx)\,dx$$
$$=\frac{\frac{3\,d^2\,x\,\sin(a+b\,x)}{2}-\frac{d^2\,x\,\sin(3\,a+3\,b\,x)}{18}+\frac{3\,c\,d\,\sin(a+b\,x)}{2}-\frac{c\,d\,\sin(3\,a+3\,b\,x)}{18}}{b^2}$$
$$-\frac{\frac{3\,c^2\,\cos(a+b\,x)}{4}-\frac{c^2\,\cos(3\,a+3\,b\,x)}{12}+\frac{3\,d^2\,x^2\,\cos(a+b\,x)}{4}-\frac{d^2\,x^2\,\cos(3\,a+3\,b\,x)}{12}-\frac{c\,d\,x\,\cos(3\,a+3\,b\,x)}{6}+\frac{3\,c\,d\,x\,\cos(a+b\,x)}{2}}{b}$$
$$+\frac{3\,d^2\,\cos\left(a+b\,x\right)}{2\,b^3}-\frac{d^2\,\cos\left(3\,a+3\,b\,x\right)}{54\,b^3}$$

input
```
int(sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
((3*d^2*x*sin(a + b*x))/2 - (d^2*x*sin(3*a + 3*b*x))/18 + (3*c*d*sin(a + b
*x))/2 - (c*d*sin(3*a + 3*b*x))/18)/b^2 - ((3*c^2*cos(a + b*x))/4 - (c^2*c
os(3*a + 3*b*x))/12 + (3*d^2*x^2*cos(a + b*x))/4 - (d^2*x^2*cos(3*a + 3*b*
x))/12 - (c*d*x*cos(3*a + 3*b*x))/6 + (3*c*d*x*cos(a + b*x))/2)/b + (3*d^2
*cos(a + b*x))/(2*b^3) - (d^2*cos(3*a + 3*b*x))/(54*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.80

$$\int (c+dx)^2\sin^3(a+bx)\,dx$$
$$=\frac{-9\cos\left(bx+a\right)\sin\left(bx+a\right)^2b^2c^2-18\cos\left(bx+a\right)\sin\left(bx+a\right)^2b^2cdx-9\cos\left(bx+a\right)\sin\left(bx+a\right)^2b^2d^2\cdots}{}$$

input
```
int((d*x+c)^2*sin(b*x+a)^3,x)
```

output
```
( - 9*cos(a + b*x)*sin(a + b*x)**2*b**2*c**2 - 18*cos(a + b*x)*sin(a + b*x
)**2*b**2*c*d*x - 9*cos(a + b*x)*sin(a + b*x)**2*b**2*d**2*x**2 + 2*cos(a
+ b*x)*sin(a + b*x)**2*d**2 - 18*cos(a + b*x)*b**2*c**2 - 36*cos(a + b*x)*
b**2*c*d*x - 18*cos(a + b*x)*b**2*d**2*x**2 + 40*cos(a + b*x)*d**2 + 6*sin
(a + b*x)**3*b*c*d + 6*sin(a + b*x)**3*b*d**2*x + 36*sin(a + b*x)*b*c*d +
36*sin(a + b*x)*b*d**2*x + 36*a*b*c*d - 18*b**2*c**2 + 16*d**2)/(27*b**3)
```

3.19 $\int (c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int (c+dx)\sin^3(a+bx)\,dx = -\frac{2(c+dx)\cos(a+bx)}{3b}+\frac{2d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sin^3(a+bx)}{9b^2}$$

output
```
-2/3*(d*x+c)*cos(b*x+a)/b+2/3*d*sin(b*x+a)/b^2-1/3*(d*x+c)*cos(b*x+a)*sin(
b*x+a)^2/b+1/9*d*sin(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int (c+dx)\sin^3(a+bx)\,dx = \frac{-27b(c+dx)\cos(a+bx)+3b(c+dx)\cos(3(a+bx))+d(27\sin(a+bx)-\sin(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]^3,x]
```

output
```
(-27*b*(c + d*x)*Cos[a + b*x] + 3*b*(c + d*x)*Cos[3*(a + b*x)] + d*(27*Sin
[a + b*x] - Sin[3*(a + b*x)]))/(36*b^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = $\{3042, 3791, 3042, 3777, 3042, 3117\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{2}{3}\int (c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int (c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{2}{3}\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Sin[a + b*x]^3,x]
```

output
```
-1/3*((c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/b + (d*Sin[a + b*x]^3)/(9*b^2
) + (2*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{3b(dx+c)\cos(3bx+3a)-d\sin(3bx+3a)-27(dx+c)b\cos(bx+a)-24bc+27\sin(bx+a)d}{36b^2}$
risch	$-\frac{3(dx+c)\cos(bx+a)}{4b}+\frac{3d\sin(bx+a)}{4b^2}+\frac{(dx+c)\cos(3bx+3a)}{12b}-\frac{d\sin(3bx+3a)}{36b^2}$
derivativedivides	$\frac{\frac{da(2+\sin(bx+a)^2)\cos(bx+a)}{3b}-\frac{c(2+\sin(bx+a)^2)\cos(bx+a)}{3}+\frac{d(-\frac{(bx+a)(2+\sin(bx+a)^2)\cos(bx+a)}{3}+\frac{\sin(bx+a)^3}{9}+\frac{2\sin(bx+a)}{3}}{b}}{b}$
default	$\frac{\frac{da(2+\sin(bx+a)^2)\cos(bx+a)}{3b}-\frac{c(2+\sin(bx+a)^2)\cos(bx+a)}{3}+\frac{d(-\frac{(bx+a)(2+\sin(bx+a)^2)\cos(bx+a)}{3}+\frac{\sin(bx+a)^3}{9}+\frac{2\sin(bx+a)}{3}}{b}}{b}$
norman	$\frac{-\frac{4c\tan(\frac{bx}{2}+\frac{a}{2})^2}{b}-\frac{4c}{3b}+\frac{4d\tan(\frac{bx}{2}+\frac{a}{2})}{3b^2}+\frac{32d\tan(\frac{bx}{2}+\frac{a}{2})^3}{9b^2}+\frac{4d\tan(\frac{bx}{2}+\frac{a}{2})^5}{3b^2}-\frac{2dx}{3b}-\frac{2dx\tan(\frac{bx}{2}+\frac{a}{2})^2}{b}+\frac{2dx\tan(\frac{bx}{2}+\frac{a}{2})^4}{b}}{(1+\tan(\frac{bx}{2}+\frac{a}{2})^2)^3}$
orering	$\frac{4d(5x^2d^2b^2+10b^2cdx+5b^2c^2+2d^2)\sin(bx+a)^3}{9b^4(dx+c)^2}-\frac{2(5x^2d^2b^2+10b^2cdx+5b^2c^2+4d^2)(d\sin(bx+a)^3+3(dx+c)\sin(bx}{9b^4(dx+c)^2}$

input
```
int((d*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b*(d*x+c)*cos(3*b*x+3*a)-d*sin(3*b*x+3*a)-27*(d*x+c)*b*cos(b*x+a)-
24*b*c+27*sin(b*x+a)*d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int (c+dx)\sin^3(a+bx)\,dx$$
$$=\frac{3\,(bdx+bc)\cos{(bx+a)}^3-9\,(bdx+bc)\cos{(bx+a)}-\left(d\cos{(bx+a)}^2-7\,d\right)\sin{(bx+a)}}{9\,b^2}$$

input
```
integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - 9*(b*d*x + b*c)*cos(b*x + a) - (d*co
s(b*x + a)^2 - 7*d)*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (c+dx)\sin^3(a+bx)\,dx$$

$$= \begin{cases} -\frac{c\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c\cos^3(a+bx)}{3b} - \frac{dx\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2dx\cos^3(a+bx)}{3b} + \frac{7d\sin^3(a+bx)}{9b^2} + \frac{2d\sin(a+bx)\cos^2 \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a) \end{cases}$$

input
```
integrate((d*x+c)*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)/b - 2*c*cos(a + b*x)**3/(3*b) -
 d*x*sin(a + b*x)**2*cos(a + b*x)/b - 2*d*x*cos(a + b*x)**3/(3*b) + 7*d*si
n(a + b*x)**3/(9*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b,
0)), ((c*x + d*x**2/2)*sin(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int (c+dx)\sin^3(a+bx)\,dx$$

$$= \frac{12\left(\cos(bx+a)^3 - 3\cos(bx+a)\right)c - \frac{12\left(\cos(bx+a)^3-3\cos(bx+a)\right)ad}{b} + \frac{(3(bx+a)\cos(3bx+3a)-27(bx+a)\cos(bx+a)-\sin}{b}}{36b}$$

input
```
integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36*(12*(cos(b*x + a)^3 - 3*cos(b*x + a))*c - 12*(cos(b*x + a)^3 - 3*cos(
b*x + a))*a*d/b + (3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a
) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int (c+dx)\sin^3(a+bx)\,dx = \frac{(bdx+bc)\cos(3bx+3a)}{12b^2} - \frac{3(bdx+bc)\cos(bx+a)}{4b^2} - \frac{d\sin(3bx+3a)}{36b^2} + \frac{3d\sin(bx+a)}{4b^2}$$

input
```
integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 3/4*(b*d*x + b*c)*cos(b*x + a)/b
^2 - 1/36*d*sin(3*b*x + 3*a)/b^2 + 3/4*d*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int (c+dx)\sin^3(a+bx)\,dx = \frac{7\,d\,\sin(a+b\,x)}{9\,b^2} - \frac{c\,\cos(a+b\,x) - \frac{c\cos(a+b\,x)^3}{3} + d\,x\,\cos(a+b\,x) - \frac{d\,x\cos(a+b\,x)^3}{3}}{b} - \frac{d\cos(a+b\,x)^2\,\sin(a+b\,x)}{9\,b^2}$$

input
```
int(sin(a + b*x)^3*(c + d*x),x)
```

output
```
(7*d*sin(a + b*x))/(9*b^2) - (c*cos(a + b*x) - (c*cos(a + b*x)^3)/3 + d*x*
cos(a + b*x) - (d*x*cos(a + b*x)^3)/3)/b - (d*cos(a + b*x)^2*sin(a + b*x))
/(9*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int (c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{-3\cos(bx+a)\sin(bx+a)^2 bc - 3\cos(bx+a)\sin(bx+a)^2 bdx - 6\cos(bx+a)bc - 6\cos(bx+a)bdx}{9b^2}$$

input
```
int((d*x+c)*sin(b*x+a)^3,x)
```

output
```
( - 3*cos(a + b*x)*sin(a + b*x)**2*b*c - 3*cos(a + b*x)*sin(a + b*x)**2*b*
d*x - 6*cos(a + b*x)*b*c - 6*cos(a + b*x)*b*d*x + sin(a + b*x)**3*d + 6*si
n(a + b*x)*d - 6*a*d + 6*b*c)/(9*b**2)
```

3.20 $\int \frac{\sin^3(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d} + \frac{3\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d} + \frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
-1/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+3/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
+3/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right) - 3\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right) - 3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}\right.\right.}{4d}$$

input
```
Integrate[Sin[a + b*x]^3/(c + d*x),x]
```

output
```
-1/4*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - 3*CosIntegral[
b*(c/d + x)]*Sin[a - (b*c)/d] - 3*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)
] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{3\sin(a+bx)}{4(c+dx)}-\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[Sin[a + b*x]^3/(c + d*x),x]
```

output
```
-1/4*(CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/d + (3*CosInteg
ral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (3*Cos[a - (b*c)/d]*SinIntegr
al[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3
*b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{12}+\frac{3b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+\cdots}{d}\right)\cdots}{\cdots}\right.}{\cdots}}{b}$
default	$\frac{-\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{12}+\frac{3b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+\cdots}{d}\right)\cdots}{\cdots}\right.}{\cdots}}{b}$
risch	$-\frac{i e^{\frac{3i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-3ibx-3ia-\frac{3(-iad+ibc)}{d}\right)}{8d}+\frac{i e^{-\frac{3i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-bc)}{d}\right)}{8d}-\frac{3i\cdots}{\cdots}$

input
```
int(sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(
3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+3/4*b*(-Si(-b*x-a-(-a*d+b
*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = \frac{3\,\text{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) - \text{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right) - \cos\left(-\frac{3\,(bc-ad)}{d}\right)\text{Si}\left(\frac{3\,(bdx+bc)}{d}\right) + 3\,\cos\left(-\frac{b}{}\right.}{4\,d}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(3*cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) - cos_integral(3*
(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) - cos(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) + 3*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d)
)/d
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.31

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = -\frac{3\,b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) - i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) - b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*(3*b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b
*(-I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integ
ral_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3
*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(
1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(exp_integ
ral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*
b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 6296, normalized size of antiderivative = 52.03

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
-1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*imag_part(
cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*
c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*sin_
integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 - 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*real_part(cos_integral(-b*x - b*c
/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx=\int \frac{\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x),x)
```

output
```
int(sin(a + b*x)^3/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin (bx+a)^3}{dx+c}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c),x)
```

output
```
int(sin(a + b*x)**3/(c + d*x),x)
```

3.21 $\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\sin^3(a+bx)}{d(c+dx)} - \frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output
```
3/4*b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2-3/4*b*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b
*x)/d^2-sin(b*x+a)^3/d/(d*x+c)-3/4*b*sin(a-b*c/d)*Si(b*c/d+b*x)/d^2+3/4*b*
sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.21

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = \frac{3b(c+dx)\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - 3b(c+dx)\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - 3d\,c}{}$$

input
```
Integrate[Sin[a + b*x]^3/(c + d*x)^2,x]
```

output
```
(3*b*(c + d*x)*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - 3*b*(c + d*x)*C
os[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - 3*d*Cos[b*x]*Sin[a] +
 d*Cos[3*b*x]*Sin[3*a] - 3*d*Cos[a]*Sin[b*x] + d*Cos[3*a]*Sin[3*b*x] - 3*b
*(c + d*x)*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*(c + d*x)*Sin[3
*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$\frac{3b\int \left(\frac{\cos(a+bx)}{4(c+dx)}-\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx}{d}-\frac{\sin^3(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{d}-\frac{\sin^3(a+bx)}{d(c+dx)}$$

input
```
Int[Sin[a + b*x]^3/(c + d*x)^2,x]
```

output
```
-(Sin[a + b*x]^3/(d*(c + d*x))) + (3*b*((Cos[a - (b*c)/d]*CosIntegral[(b*c
)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/
(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d) + (Sin[3*a - (
3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
 - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{-\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}}{d}\right)}{12}}{b}$
default	$\frac{-\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}}{d}\right)}{12}}{b}$
risch	$\frac{3b\,e^{\frac{3i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-3ibx-3ia-\frac{3(-iad+ibc)}{d}\right)}{8d^2}+\frac{3b\,e^{-\frac{3i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-bc)}{d}\right)}{8d^2}-$

input
```
int(sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/12*b^2*(-3*sin(3*b*x+3*a)/(-a*d+b*c+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3
*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d)/d)+3/4*b^2*(-sin(b*x+a)/(-a*d+b*c+d*(b*x+a))/d+(-Si(-
b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b
*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.30

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{3\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)-3\,(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)-3\,(bdx+bc)\sin\left(\right.}{4\,(d^3x}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d
) - 3*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 3*
(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 3*(b
*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 4*(d*cos(b
*x + a)^2 - d)*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.11

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{3\,b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(-i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(3*b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*ex
p_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
 b^2*(-I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_i
ntegral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
 + 3*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(e
xp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d +
(b*x + a)*d^2 - a*d^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(137) = 274.

Time = 0.51 (sec) , antiderivative size = 1000, normalized size of antiderivative = 6.90

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-3*(b*c - a*
d)/d)*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d) + 3*b^3*c*cos(-3*(b*c - a*d)/d)*cos_integral(3*((d*x + c)*(b - b
*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*a*b^2*d*cos(-3*(b*c - a*
d)/d)*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c
- a*d)/d)*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*
c - a*d)/d) - 3*b^3*c*cos(-(b*c - a*d)/d)*cos_integral(((d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-(b*c - a*d)/d
)*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*sin(-(b*c - a*d)
/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a
*d)/d) - 3*b^3*c*sin(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*
x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*sin(-(b*c - a*d)/d)*si
n_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)
 + 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*sin(-3*(b*c - a*d)/
d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
a*d)/d) + 3*b^3*c*sin(-3*(b*c - a*d)/d)*sin_integral(-3*((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*a*b^2*d*sin(-3*(b*c - a*d
)/d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^3/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(sin(a + b*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.22 $\int \frac{\sin^3(a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 184

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(c+dx)^3} dx = & \frac{9b^2 \text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} \\ & - \frac{3b^2 \text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^3} \\ & - \frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} \\ & - \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

output
```
9/8*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-3/8*b^2*Ci(b*c/d+b*x)*sin(a
-b*c/d)/d^3-3/2*b*cos(b*x+a)*sin(b*x+a)^2/d^2/(d*x+c)-1/2*sin(b*x+a)^3/d/(
d*x+c)^2-3/8*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+9/8*b^2*cos(3*a-3*b*c/d)*S
i(3*b*c/d+3*b*x)/d^3
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{-6d\cos(bx)(b(c+dx)\cos(a)+d\sin(a))+2d\cos(3bx)(3b(c+dx)\cos(3a)+d\sin(3a))+6d(-d\cos(a)}{}$$

input
```
Integrate[Sin[a + b*x]^3/(c + d*x)^3,x]
```

output
```
(-6*d*Cos[b*x]*(b*(c + d*x)*Cos[a] + d*Sin[a]) + 2*d*Cos[3*b*x]*(3*b*(c +
d*x)*Cos[3*a] + d*Sin[3*a]) + 6*d*(-(d*Cos[a]) + b*(c + d*x)*Sin[a])*Sin[b
*x] + 2*d*(d*Cos[3*a] - 3*b*(c + d*x)*Sin[3*a])*Sin[3*b*x] + 6*b^2*(c + d*
x)^2*(3*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - CosIntegral[
b*(c/d + x)]*Sin[a - (b*c)/d] - Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]
+ 3*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d*x
)^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3795, 3042, 3784, 3042, 3780, 3783, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(a+bx)^3}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3795}$$

$$-\frac{9b^2\int\frac{\sin^3(a+bx)}{c+dx}dx}{2d^2}+\frac{3b^2\int\frac{\sin(a+bx)}{c+dx}dx}{d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\int\frac{\sin(a+bx)}{c+dx}dx}{d^2}-\frac{9b^2\int\frac{\sin(a+bx)^3}{c+dx}dx}{2d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{9b^2\int\frac{\sin(a+bx)^3}{c+dx}dx}{2d^2}+\frac{3b^2\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9b^2\int\frac{\sin(a+bx)^3}{c+dx}dx}{2d^2}+\frac{3b^2\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{3b^2\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{9b^2\int\frac{\sin(a+bx)^3}{c+dx}dx}{2d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{9b^2\int\frac{\sin(a+bx)^3}{c+dx}dx}{2d^2}+\frac{3b^2\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{9b^2\int \left(\frac{3\sin(a+bx)}{4(c+dx)}-\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx}{2d^2}+\frac{3b^2\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^2\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{9b^2\left(-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3b\right.}{4d}\right.}{2d^2}-\frac{3b\sin^2(a+bx)\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin^3(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Sin[a + b*x]^3/(c + d*x)^3,x]
```

output
```
(-3*b*Cos[a + b*x]*Sin[a + b*x]^2)/(2*d^2*(c + d*x)) - Sin[a + b*x]^3/(2*d
*(c + d*x)^2) + (3*b^2*((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d +
(Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d))/d^2 - (9*b^2*(-1/4*(CosI
ntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/d + (3*CosIntegral[(b*c)/
d + b*x]*Sin[a - (b*c)/d])/(4*d) + (3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d
 + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*
d)))/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.73

method	result
derivativedivides	$-\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-ad+bc+d(bx+a))^2d}+\frac{-\frac{9\cos(3bx+3a)}{2(-ad+bc+d(bx+a))d}-\frac{9\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\right.}{}\right.}{2d}}{d}\right.}{12}$
default	$-\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-ad+bc+d(bx+a))^2d}+\frac{-\frac{9\cos(3bx+3a)}{2(-ad+bc+d(bx+a))d}-\frac{9\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\right.}{}\right.}{2d}}{d}\right.}{12}$
risch	$\frac{9ib^2e^{\frac{3i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(-3ibx-3ia-\frac{3(-iad+ibc)}{d}\right)}{16d^3}-\frac{9ib^2e^{-\frac{3i(ad-bc)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-bc)}{d}\right)}{16d^3}+$

input
```
int(sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/12*b^3*(-3/2*sin(3*b*x+3*a)/(-a*d+b*c+d*(b*x+a))^2/d+3/2*(-3*cos(3
*b*x+3*a)/(-a*d+b*c+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3
*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/
d)+3/4*b^3*(-1/2*sin(b*x+a)/(-a*d+b*c+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*
d+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+
(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.71

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = \frac{12\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-3\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+9\left(b^2d^2x^2+2\,b^2cdx\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(12*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 9*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d)*sin(-3*(b*c - a
*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin
_integral(3*(b*d*x + b*c)/d) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 12*(b*d^2*x + b*c*d)*cos(
b*x + a) + 4*(d^2*cos(b*x + a)^2 - d^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x
 + c^2*d^3)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.85

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3\,b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(-i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*ex
p_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
 b^3*(-I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_i
ntegral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
 + 3*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(e
xp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*
d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.35 (sec) , antiderivative size = 116534, normalized size of antiderivative = 633.34

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*(9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 - 3*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
3*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^
2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b
^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2
*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*real_part(c
os_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*real_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*d^2*x^2*real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*real_part(cos_integral(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^3/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(bx+a)^3}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c)^3,x)
```

output
```
int(sin(a + b*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.23 $\int (c+dx)^3 \csc(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 185

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx)\,dx = &-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\ &+\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\ &-\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\ &-\frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\ &+\frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\ &-\frac{6id^3 \operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3 \operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}\end{aligned}$$

output
```
-2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b
*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*po
lylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6
*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^
4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.19

$$\int (c+dx)^3\csc(a+bx)\,dx$$
$$= \frac{-2b^3(c+dx)^3\text{arctanh}(\cos(a+bx)+i\sin(a+bx))+3id(b^2(c+dx)^2\,\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx))\ldots}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x],x]
```

output
```
(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*
(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*
x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a +
b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x]
 + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a
 + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/b^4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\csc(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3\csc(a+bx)dx$$
$$\Big\downarrow \text{4671}$$
$$-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{7163}$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x],x]
```

output
```
(-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*PolyLog
[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, -E^(I*(a +
 b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))])/b^2))/b))/b - (3*d*((I*(c +
d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[
3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I*(a + b*x))])/b^2))/b))/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(167) = 334.

Time = 0.94 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.42

method	result
risch	$-\frac{6c\,d^{2}a^{2}\operatorname{arctanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b^{3}}-\frac{d^{3}\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^{3}}{b}+\frac{d^{3}\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^{3}}{b}+\frac{d^{3}\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^{3}}{b^{4}}-\frac{d^{3}\ln\left(\mathrm{e}^{i(bx+a)}+1\right)a^{3}}{b^{4}}$

input
```
int((d*x+c)^3*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-6/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+
a)))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/b
*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+
6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^
3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b^
4*d^3*ln(exp(I*(b*x+a))+1)*a^3-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^
3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6
/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))
+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/b
^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-3/b^3
*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+3
*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,exp(I*(b*x
+a)))*x^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+6*I*d^3*polylog(4,exp(I*
(b*x+a)))/b^4-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-2/b*c^3*arctanh(exp(I
*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(161) = 322.

Time = 0.12 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.43

$$\int (c+dx)^3 \csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4
, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*si
n(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^
2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x
+ a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x +
 a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*
dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*
x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(
b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin
(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x +
b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog
(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, ...
```

Sympy [F(-1)]

Timed out.

$$\int (c + dx)^3 \csc(a + bx) \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(161) = 322$.

Time = 0.14 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.87

$$\int (c+dx)^3 \csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*c^3*log(cot(b*x + a) + csc(b*x + a)) - 6*a*c^2*d*log(cot(b*x + a)
+ csc(b*x + a))/b + 6*a^2*c*d^2*log(cot(b*x + a) + csc(b*x + a))/b^2 - 2*a
^3*d^3*log(cot(b*x + a) + csc(b*x + a))/b^3 + (12*I*d^3*polylog(4, -e^(I*b
*x + I*a)) - 12*I*d^3*polylog(4, e^(I*b*x + I*a)) - 2*(-I*(b*x + a)^3*d^3
+ 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 -
 I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*
x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*
I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
1) - 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I
*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d +
 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*
(b*x + a))*dilog(e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^
2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b
*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

Giac [F]

$$\int (c+dx)^3 \csc(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^3/sin(a + b*x),x)
```

output
```
int((c + d*x)^3/sin(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3 \csc(a+bx)\,dx$$
$$= \frac{\left(\int \csc(bx+a)\,x^3dx\right) b\,d^3 + 3\left(\int \csc(bx+a)\,x^2dx\right) bc\,d^2 + 3\left(\int \csc(bx+a)\,xdx\right) b\,c^2d + \log\left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right.\right.}{b}$$

input
```
int((d*x+c)^3*csc(b*x+a),x)
```

output
```
(int(csc(a + b*x)*x**3,x)*b*d**3 + 3*int(csc(a + b*x)*x**2,x)*b*c*d**2 + 3
*int(csc(a + b*x)*x,x)*b*c**2*d + log(tan((a + b*x)/2))*c**3)/b
```

3.24 $\int (c+dx)^2 \csc(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 123

$$\int (c+dx)^2 \csc(a+bx)\,dx = -\frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{2d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

output
```
-2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x
+a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp
(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.20

$$\int (c+dx)^2\csc(a+bx)\,dx$$
$$=\frac{(c+dx)^2\log\left(1-e^{i(a+bx)}\right)-(c+dx)^2\log\left(1+e^{i(a+bx)}\right)+\frac{2id(b(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})+id\operatorname{PolyLog}(3,-e^{i(a+bx)}}{b^2}}{b}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x],x]
```

output
```
((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a + b*x)
)] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -
E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))
] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2)/b
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\csc(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2\csc(a+bx)dx$$
$$\Big\downarrow \text{4671}$$
$$-\frac{2d\int (c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$
$$\Big\downarrow \text{3011}$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x],x]
```

output
```
(-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2
, -E^(I*(a + b*x))])/b - (d*PolyLog[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*(
(I*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x)
)])/b^2))/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(111) = 222$.

Time = 0.86 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.93

method	result
risch	$-\frac{2d^2a^2 \operatorname{arctanh}(e^{i(bx+a)})}{b^3} - \frac{2id^2 \operatorname{polylog}(2,e^{i(bx+a)})x}{b^2} + \frac{2cd \ln(1-e^{i(bx+a)})a}{b^2} + \frac{2id^2 \operatorname{polylog}(2,-e^{i(bx+a)})x}{b^2} - \frac{2cd \ln(e^{i(b}}{}$

input
```
int((d*x+c)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)
))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)
))*x-2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+1/b^3*d
^2*ln(exp(I*(b*x+a))+1)*a^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2/b^2*c*d*l
n(exp(I*(b*x+a))+1)*a+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*p
olylog(2,exp(I*(b*x+a)))+4/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-2/b*c^2*arcta
nh(exp(I*(b*x+a)))-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2*d^2*polylog(3,-exp(I
*(b*x+a)))/b^3+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+2*d^2*polylog(3,exp(I*(b*x
+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 504, normalized size of antiderivative = 4.10

$$\int (c+dx)^2 \csc(a+bx)\,dx = \frac{2\,d^2\mathrm{polylog}(3,\cos(bx+a)+i\,\sin(bx+a))+2\,d^2\mathrm{polylog}(3,\cos(bx+a)-i\,\sin(bx+a))-2\,d^2\mathrm{polylog}}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x +
I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b
*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
+ I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x
+ a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1))/b^3
```

Sympy [F]

$$\int (c+dx)^2\csc(a+bx)\,dx=\int (c+dx)^2\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.24

$$\int (c+dx)^2\csc(a+bx)\,dx=$$

$$-\frac{2\,c^2\log\left(\cot\left(bx+a\right)+\csc\left(bx+a\right)\right)-\frac{4\,acd\log(\cot(bx+a)+\csc(bx+a))}{b}+\frac{2\,a^2d^2\log(\cot(bx+a)+\csc(bx+a))}{b^2}+\frac{4\,d^2\mathrm{Li}_3(-}{}}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*c^2*log(cot(b*x + a) + csc(b*x + a)) - 4*a*c*d*log(cot(b*x + a) +
csc(b*x + a))/b + 2*a^2*d^2*log(cot(b*x + a) + csc(b*x + a))/b^2 + (4*d^2*
polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) - 2*(-I*(
b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), c
os(b*x + a) + 1) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*b*c*d + I*(b*x + a)*d^2
 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*
d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*cos(b*x + a) + 1))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2\csc(a+bx)\,dx=\int (dx+c)^2\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\csc(a+bx)\,dx=\int \frac{(c+d\,x)^2}{\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^2/sin(a + b*x),x)
```

output
```
int((c + d*x)^2/sin(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2\csc(a+bx)\,dx=\frac{\left(\int \csc(bx+a)\,x^2dx\right)b\,d^2+2\left(\int \csc(bx+a)\,xdx\right)bcd+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c^2}{b}$$

input
```
int((d*x+c)^2*csc(b*x+a),x)
```

output
```
(int(csc(a + b*x)*x**2,x)*b*d**2 + 2*int(csc(a + b*x)*x,x)*b*c*d + log(tan
((a + b*x)/2))*c**2)/b
```

3.25 $\int (c+dx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int (c+dx)\csc(a+bx)\,dx = -\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}$$

output
```
-2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*
d*polylog(2,exp(I*(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int (c+dx)\csc(a+bx)\,dx = -\frac{c\operatorname{arctanh}(\cos(a+bx))}{b} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)\right.\right.}{b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x],x]
```

output
```
-((c*ArcTanh[Cos[a + b*x]])/b) + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] -
 Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I
*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)\csc(a+bx)dx$$

$$\Big\downarrow \ 4671$$

$$-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \ 2715$$

$$\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \ 2838$$

$$-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}$$

input
```
Int[(c + d*x)*Csc[a + b*x],x]
```

output
```
(-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.

Time = 0.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{da\ln(\csc(bx+a)-\cot(bx+a))}{b}+c\ln(\csc(bx+a)-\cot(bx+a))+\frac{d\left((bx+a)\ln\left(1-e^{i(bx+a)}\right)-(bx+a)\ln\left(e^{i(bx+a)}+1\right)+i\,\mathrm{dilog}\left(e^{i(\cdots}\right.\right.}{b}\cdots}{b}$
default	$\frac{-\frac{da\ln(\csc(bx+a)-\cot(bx+a))}{b}+c\ln(\csc(bx+a)-\cot(bx+a))+\frac{d\left((bx+a)\ln\left(1-e^{i(bx+a)}\right)-(bx+a)\ln\left(e^{i(bx+a)}+1\right)+i\,\mathrm{dilog}\left(e^{i(\cdots}\right.\right.}{b}\cdots}{b}$
risch	$-\frac{2c\,\mathrm{arctanh}\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b}-\frac{d\ln\left(e^{i(bx+a)}+1\right)a}{b^2}+\frac{id\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2}+\frac{d\ln\left(1-e^{i(bx+\cdots}\right.}{b}\cdots$

input
```
int((d*x+c)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a*ln(csc(b*x+a)-cot(b*x+a))+c*ln(csc(b*x+a)-cot(b*x+a))+1/b*d*
((b*x+a)*ln(1-exp(I*(b*x+a)))-(b*x+a)*ln(exp(I*(b*x+a))+1)+I*dilog(exp(I*(
b*x+a))+1)-I*dilog(1-exp(I*(b*x+a)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.76

$$\int (c + dx)\csc(a + bx)\,dx$$
$$= \frac{-i\,d\mathrm{Li}_2(\cos{(bx+a)} + i\,\sin{(bx+a)}) + i\,d\mathrm{Li}_2(\cos{(bx+a)} - i\,\sin{(bx+a)}) - i\,d\mathrm{Li}_2(-\cos{(bx+a)} + i}{}$$

input
```
integrate((d*x+c)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) -
I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(-c
os(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c -
 a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(
-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x
 + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1))/b^2
```

Sympy [F]

$$\int (c + dx)\csc(a + bx)\,dx = \int (c + dx)\csc{(a + bx)}\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.61

$$\int (c+dx)\csc(a+bx)\,dx =$$
$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,b\right.}{}$$

input
```
integrate((d*x+c)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2
(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(b*x +
a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x + I*a)) + 2*I*d*dilog(e^(I*b
*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x
 + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
 + a) + 1))/b^2
```

Giac [F]

$$\int (c+dx)\csc(a+bx)\,dx = \int (dx+c)\csc\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\,dx=\int \frac{c+d\,x}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)/sin(a + b*x),x)
```

output
```
int((c + d*x)/sin(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)\csc(a+bx)\,dx=\frac{\left(\int \csc\left(bx+a\right)xdx\right)bd+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c}{b}$$

input
```
int((d*x+c)*csc(b*x+a),x)
```

output
```
(int(csc(a + b*x)*x,x)*b*d + log(tan((a + b*x)/2))*c)/b
```

3.26 $\int \frac{\csc(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(csc(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 7.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Csc[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Csc[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(a+bx)}{c+dx}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\csc(a+bx)}{c+dx}dx$$

input
```
Int[Csc[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\csc (bx+a)}{dx+c}dx$$

input
```
int(csc(b*x+a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \int \frac{\csc (bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \int \frac{\csc (a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{c+dx}\,dx=\int \frac{\csc\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{c+dx}\,dx=\int \frac{\csc\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 34.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\csc(a+bx)}{c+dx}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(sin(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)}{dx+c}dx$$

input
```
int(csc(b*x+a)/(d*x+c),x)
```

output
```
int(csc(a + b*x)/(c + d*x),x)
```

3.27 $\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx=\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(csc(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx=\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Csc[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csc[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)}{(dx+c)^2}dx$$

input
```
int(csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 34.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\csc(a+bx)}{(c+dx)^2} \, dx = \int \frac{1}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \,d x$$

input
```
int(1/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\csc(a+bx)}{(c+dx)^2} \, dx = \int \frac{\csc \left(b x +a \right)}{d^{2} x^{2}+2 c d x +c^{2}}d x$$

input
```
int(csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.28 $\int (c+dx)^3 \csc^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int (c+dx)^3 \csc^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}$$

output

```
-I*(d*x+c)^3/b-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))
/b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,exp
(2*I*(b*x+a)))/b^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 486 vs. $2(113) = 226$.

Time = 7.02 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.30

$$\int (c+dx)^3 \csc^2(a+bx)\,dx =$$

$$-\frac{d^3e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2\left(1-e^{-2ia}\right)x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2\left(1-e^{-2ia}\right)x^2\log\left(1+e^{-i(a+bx)}\right)\right.}{}$$

$$+\frac{3c^2d\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}$$

$$+\frac{\csc(a)\csc(a+bx)\left(c^3\sin(bx)+3c^2dx\sin(bx)+3cd^2x^2\sin(bx)+d^3x^3\sin(bx)\right)}{b}$$

$$-\frac{3cd^2\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+}}{}\right.}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)\right.}}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2,x]
```

output
```
-1/2*(d^3*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)
*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (
6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-
2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^4 + (3*c^2*d*Csc[a]*(-(b*x*Cos
[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Si
n[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin[b*x] + 3*c*d
^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b - (3*c*d^2*Csc[a]*Sec[a]*(b^2*E^(I*
ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2
*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]
]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^
2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc(a+bx)^2 dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{3d\int (c+dx)^2 \cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d\int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^2,x]
```

output
```
-(((c + d*x)^3*Cot[a + b*x])/b) - (3*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-
1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*(((I/2)*(c
+ d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a
 + Pi + 2*b*x))])/(4*b^2)))/b)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(103) = 206.

Time = 1.16 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.79

method	result
risch	$-\frac{12 i d^2 c a x}{b^2}-\frac{6 d c^2 \ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{3 d c^2 \ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}-\frac{2 i\left(d^3 x^3+3 c d^2 x^2+3 c^2 d x+c^3\right)}{b\left(\mathrm{e}^{2 i(bx+a)}-1\right)}-\frac{6 d^3 a^2 \ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}+\frac{3 d^3 a^2 \ln}{}$

input
```
int((d*x+c)^3*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))-1)-12*I/b^2*d
^2*c*a*x-6/b^4*d^3*a^2*ln(exp(I*(b*x+a)))+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-
1)-3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2+3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2-2
*I/b*d^3*x^3+4*I/b^4*d^3*a^3+12/b^3*d^2*c*a*ln(exp(I*(b*x+a)))+3/b^2*d^3*l
n(1-exp(I*(b*x+a)))*x^2+3/b^2*d*c^2*ln(exp(I*(b*x+a))+1)-6/b^2*d*c^2*ln(ex
p(I*(b*x+a)))+3/b^2*d*c^2*ln(exp(I*(b*x+a))-1)-6/b^3*d^2*c*a*ln(exp(I*(b*x
+a))-1)+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a+6/b^2*d^2*c*ln(exp(I*(b*x+a))+1
)*x+6/b^2*d^2*c*ln(1-exp(I*(b*x+a)))*x-6*I/b*d^2*c*x^2-6*I/b^3*d^2*c*a^2-6
*I/b^3*d^2*c*polylog(2,-exp(I*(b*x+a)))-6*I/b^3*d^2*c*polylog(2,exp(I*(b*x
+a)))+6*I/b^3*d^3*a^2*x-6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^3*d
^3*polylog(2,exp(I*(b*x+a)))*x+6/b^4*d^3*polylog(3,exp(I*(b*x+a)))+6/b^4*d
^3*polylog(3,-exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 676, normalized size of antiderivative = 5.98

$$\int (c+dx)^3 \csc^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*
polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3,
-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x +
 a) - I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*dilog(cos(b
*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*d
ilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^3*x + I*b*c*d
^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x
+ a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*si
n(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-
1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*
sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log
(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*
sin(b*x + a) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*cos(b*x + a))/(b^4*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)^3 \csc^2(a+bx) \, dx = \int (c+dx)^3 \csc^2{(a+bx)} \, dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(100) = 200$.

Time = 0.18 (sec) , antiderivative size = 1654, normalized size of antiderivative = 14.64

$$\int (c + dx)^3 \csc^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x +
2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*
d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) -
 6*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/
((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) +
 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/
((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) -
 2*c^3/tan(b*x + a) + 6*a*c^2*d/(b*tan(b*x + a)) - 6*a^2*c*d^2/(b^2*tan(b*
x + a)) + 2*a^3*d^3/(b^3*tan(b*x + a)) - 2*(6*((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(2*b*x + 2*a) + (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*((b*x +...
```

Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^3/sin(a + b*x)^2,x)
```

output
```
int((c + d*x)^3/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^3 \csc^2(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)b\,c^3 - 6\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)b\,c^2dx - 3\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)bc\,d^2x}{}$$

input
```
int((d*x+c)^3*csc(b*x+a)^2,x)
```

output
```
( - 2*cos(a + b*x)*tan((a + b*x)/2)*b*c**3 - 6*cos(a + b*x)*tan((a + b*x)/
2)*b*c**2*d*x - 3*cos(a + b*x)*tan((a + b*x)/2)*b*c*d**2*x**2 - cos(a + b*
x)*tan((a + b*x)/2)*b*d**3*x**3 + 3*int(x**2/tan((a + b*x)/2),x)*sin(a + b
*x)*tan((a + b*x)/2)*b*d**3 + 6*int(x/tan((a + b*x)/2),x)*sin(a + b*x)*tan
((a + b*x)/2)*b*c*d**2 - 3*int(tan((a + b*x)/2)*x**2,x)*sin(a + b*x)*tan((
a + b*x)/2)*b*d**3 - 6*int(tan((a + b*x)/2)*x,x)*sin(a + b*x)*tan((a + b*x
)/2)*b*c*d**2 - 6*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*tan((a + b*x)/
2)*c**2*d + 6*log(tan((a + b*x)/2))*sin(a + b*x)*tan((a + b*x)/2)*c**2*d -
 3*sin(a + b*x)*b*c*d**2*x**2 - sin(a + b*x)*b*d**3*x**3 + 3*tan((a + b*x)
/2)*b*c*d**2*x**2 + tan((a + b*x)/2)*b*d**3*x**3)/(2*sin(a + b*x)*tan((a +
 b*x)/2)*b**2)
```

3.29 $\int (c+dx)^2 \csc^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int (c+dx)^2 \csc^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}$$

output
```
-I*(d*x+c)^2/b-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b
^2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 181 vs. 2(83) = 166.

Time = 6.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.18

$$\int (c+dx)^2 \csc^2(a+bx)\,dx$$

$$= \frac{\csc(a)\left(-2bcd(bx\cos(a)-\log(\sin(a+bx))\sin(a))+d^2\left(-b^2e^{i\arctan(\tan(a))}x^2\cos(a)\sqrt{\sec^2(a)}-(-ibx(\pi\right.\right.}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2,x]
```

output
```
(Csc[a]*(-2*b*c*d*(b*x*Cos[a] - Log[Sin[a + b*x]]*Sin[a]) + d^2*(-(b^2*E^(
I*ArcTan[Tan[a]])*x^2*Cos[a]*Sqrt[Sec[a]^2]) - ((-I)*b*x*(Pi - 2*ArcTan[Ta
n[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^(
(2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[S
in[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))]
)*Sin[a]) + b^2*(c + d*x)^2*Csc[a + b*x]*Sin[b*x]))/b^3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \csc(a+bx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{2d\int (c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2d\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}$$

$\Big\downarrow$ 2620

$$-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

$\Big\downarrow$ 2715

$$-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

$\Big\downarrow$ 2838

$$-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^2,x]
```

output
```
-(((c + d*x)^2*Cot[a + b*x])/b) - (2*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-
1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(
I*(2*a + Pi + 2*b*x))])/(4*b^2))))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(77) = 154$.

Time = 1.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.33

method	result
risch	$-\frac{2i(x^2d^2+2cdx+c^2)}{b(\mathrm{e}^{2i(bx+a)}-1)}+\frac{2dc\ln(\mathrm{e}^{i(bx+a)}+1)}{b^2}-\frac{4dc\ln(\mathrm{e}^{i(bx+a)})}{b^2}+\frac{2dc\ln(\mathrm{e}^{i(bx+a)}-1)}{b^2}-\frac{2id^2x^2}{b}-\frac{4id^2ax}{b^2}-\frac{2id^2a^2}{b^3}+$

input
```
int((d*x+c)^2*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)+2/b^2*d*c*ln(exp(I*(b*x+
a))+1)-4/b^2*d*c*ln(exp(I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))-1)-2*I/b*d
^2*x^2-4*I/b^2*d^2*a*x-2*I/b^3*d^2*a^2+2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x-2*
I/b^3*d^2*polylog(2,-exp(I*(b*x+a)))+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^
3*d^2*ln(1-exp(I*(b*x+a)))*a-2*I/b^3*d^2*polylog(2,exp(I*(b*x+a)))+4/b^3*d
^2*a*ln(exp(I*(b*x+a)))-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(74) = 148.

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.57

$$\int (c+dx)^2 \csc^2(a+bx)\,dx$$
$$= \frac{-i\,d^2\mathrm{Li}_2(\cos(bx+a)+i\sin(bx+a))\sin(bx+a)+i\,d^2\mathrm{Li}_2(\cos(bx+a)-i\sin(bx+a))\sin(bx+a)+}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
(-I*d^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(co
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*
sin(b*x + a))*sin(b*x + a) - I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*s
in(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin
(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b
*x + a) + (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
)*sin(b*x + a) + (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a
) + 1/2)*sin(b*x + a) + (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x +
a) + 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a
) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a))/
(b^3*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)^2 \csc^2(a+bx)\,dx = \int (c+dx)^2 \csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(74) = 148$.

Time = 0.13 (sec) , antiderivative size = 552, normalized size of antiderivative = 6.65

$$\int (c+dx)^2 \csc^2(a+bx)\,dx =$$
$$-\frac{2\,b^2c^2 + 2\,(bd^2x + bcd - (bd^2x + bcd)\cos(2\,bx + 2\,a) + (-i\,bd^2x - i\,bcd)\sin(2\,bx + 2\,a))\arctan(\sin(\ldots}{\ldots}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*b^2*c^2 + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-
I*b*d^2*x - I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a)
+ 1) - 2*(b*c*d*cos(2*b*x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arcta
n2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2
*x*sin(2*b*x + 2*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
2*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) + 2*(d^2*cos(2*b*x + 2*a) +
 I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) + 2*(d^2*cos(2*b*x
+ 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x
 + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*s
in(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2
*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x)*sin(2*b*x + 2*a))/(-I*b^
3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^2 \,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^2/sin(a + b*x)^2,x)
```

output
```
int((c + d*x)^2/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \csc^2(a+bx)\,dx$$

$$= \frac{-2\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)b\,c^2 - 4\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)bcdx - \cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)b\,d^2x^2 + 2}{}$$

input
```
int((d*x+c)^2*csc(b*x+a)^2,x)
```

output
```
( - 2*cos(a + b*x)*tan((a + b*x)/2)*b*c**2 - 4*cos(a + b*x)*tan((a + b*x)/
2)*b*c*d*x - cos(a + b*x)*tan((a + b*x)/2)*b*d**2*x**2 + 2*int(x/tan((a +
b*x)/2),x)*sin(a + b*x)*tan((a + b*x)/2)*b*d**2 - 2*int(tan((a + b*x)/2)*x
,x)*sin(a + b*x)*tan((a + b*x)/2)*b*d**2 - 4*log(tan((a + b*x)/2)**2 + 1)*
sin(a + b*x)*tan((a + b*x)/2)*c*d + 4*log(tan((a + b*x)/2))*sin(a + b*x)*t
an((a + b*x)/2)*c*d - sin(a + b*x)*b*d**2*x**2 + tan((a + b*x)/2)*b*d**2*x
**2)/(2*sin(a + b*x)*tan((a + b*x)/2)*b**2)
```

3.30 $\int (c+dx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (c+dx)\csc^2(a+bx)\,dx=-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}$$

output
```
-(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int (c+dx)\csc^2(a+bx)\,dx=-\frac{dx\cot(a)}{b}-\frac{c\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}+\frac{dx\csc(a)\csc(a+bx)\sin(bx)}{b}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2,x]
```

output
```
-((d*x*Cot[a])/b) - (c*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2 + (d*x*
Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)\csc(a+bx)^2dx$$

$$\Big\downarrow \; 4672$$

$$\frac{d\int \cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{d\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{d\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}$$

$$\Big\downarrow \; 3956$$

$$\frac{d\log(-\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2,x]
```

output
```
-(((c + d*x)*Cot[a + b*x])/b) + (d*Log[-Sin[a + b*x]])/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

method	result	size
derivativedivides	$\frac{\frac{da\cot(bx+a)}{b}-c\cot(bx+a)+\frac{d(-(bx+a)\cot(bx+a)+\ln(\sin(bx+a)))}{b}}{b}$	53
default	$\frac{\frac{da\cot(bx+a)}{b}-c\cot(bx+a)+\frac{d(-(bx+a)\cot(bx+a)+\ln(\sin(bx+a)))}{b}}{b}$	53
risch	$-\frac{2idx}{b}-\frac{2ida}{b^2}-\frac{2i(dx+c)}{b(\mathrm{e}^{2i(bx+a)}-1)}+\frac{d\ln(\mathrm{e}^{2i(bx+a)}-1)}{b^2}$	59
parallelrisch	$\frac{-2\ln\left(\sec\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)d+2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-b\left(\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(dx+c)}{2b^2}$	64
norman	$\frac{-\frac{c}{2b}-\frac{dx}{2b}+\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{2b}+\frac{dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}-\frac{d\ln\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)}{b^2}$	98

input
```
int((d*x+c)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/b*d*a*cot(b*x+a)-c*cot(b*x+a)+1/b*d*(-(b*x+a)*cot(b*x+a)+ln(sin(b*x
+a))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int (c+dx)\csc^2(a+bx)\,dx = \frac{d\log\left(\frac{1}{2}\sin(bx+a)\right)\sin(bx+a)-(bdx+bc)\cos(bx+a)}{b^2\sin(bx+a)}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
(d*log(1/2*sin(b*x + a))*sin(b*x + a) - (b*d*x + b*c)*cos(b*x + a))/(b^2*s
in(b*x + a))
```

Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\,dx = \int (c+dx)\csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 7.48

$$\int (c+dx)\csc^2(a+bx)\,dx = \frac{\frac{\left(\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)+\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\,c\right.}{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right.}}{2b}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 2*c/t
an(b*x + a) + 2*a*d/(b*tan(b*x + a)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(29) = 58$.

Time = 0.57 (sec) , antiderivative size = 1027, normalized size of antiderivative = 35.41

$$\int (c + dx)\csc^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 -
 b*d*x*tan(1/2*b*x)^2 - 4*b*d*x*tan(1/2*b*x)*tan(1/2*a) + d*log(16*(tan(1/
2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan
(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1/2*a)^2 -
2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
 tan(1/2*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^
2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(
1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a) - b*d*x*tan(1/2*a)^2 + d*log(16*(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*tan
(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*ta
n(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(t
an(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x
)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/
2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^2
 - b*c*tan(1/2*b*x)^2 - 4*b*c*tan(1/2*b*x)*tan(1/2*a) - b*c*tan(1/2*a)^2 +
 b*d*x - d*log(16*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*
a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1
/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*
tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + ...
```

Mupad [B] (verification not implemented)

Time = 35.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int (c+dx)\csc^2(a+bx)\,dx = \frac{d\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}-1\right)}{b^2}-\frac{(c+d\,x)\,2i}{b\,\left(e^{a\,2i+b\,x\,2i}-1\right)}-\frac{d\,x\,2i}{b}$$

input
```
int((c + d*x)/sin(a + b*x)^2,x)
```

output
```
(d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - ((c + d*x)*2i)/(b*(exp(a*2i + b*x
*2i) - 1)) - (d*x*2i)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int (c+dx)\csc^2(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\,bc-\cos(bx+a)\,bdx-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a)\,d+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx}{\sin(bx+a)\,b^2}$$

input
```
int((d*x+c)*csc(b*x+a)^2,x)
```

output
```
( - cos(a + b*x)*b*c - cos(a + b*x)*b*d*x - log(tan((a + b*x)/2)**2 + 1)*s
in(a + b*x)*d + log(tan((a + b*x)/2))*sin(a + b*x)*d)/(sin(a + b*x)*b**2)
```

3.31 $\int \frac{\csc^2(a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\csc^2(a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\csc^2(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 7.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{c+dx} \, dx = \int \frac{\csc^2(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Csc[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Csc[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)^2}{c+dx}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\csc^2(a+bx)}{c+dx}dx$$

input
```
Int[Csc[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 477, normalized size of antiderivative = 29.81

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\csc{(bx+a)}^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(si
n(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^2*x + b*c*d + (b*d^2*
x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(
b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b
*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cos(b*x + a)), x) - 2*sin(2*b*x + 2*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x
+ 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*
b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\csc{(bx+a)}^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 34.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(sin(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx=\int \frac{\csc\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(a + b*x)**2/(c + d*x),x)
```

3.32 $\int \frac{\csc^2(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\csc^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^2(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Csc[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Csc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc(a+bx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csc[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc \left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 718, normalized size of antiderivative = 44.88

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d
)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a
)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(si
n(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 +
 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b
*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x
^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*
x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d
^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*
sin(b*x + a)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b
*x + a)), x) - sin(2*b*x + 2*a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^
2)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)
)
```

Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 35.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/(sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(sin(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.33 $\int (c+dx)^3 \csc^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 309

$$\int (c+dx)^3 \csc^3(a+bx)\,dx = -\frac{6d^2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b} + \frac{3id^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} - \frac{3id^3\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{3id^3\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} + \frac{3id^3\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}$$

output
```
-6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3-(d*x+c)^3*arctanh(exp(I*(b*x+a)
))/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+
3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*
(b*x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*poly
log(2,exp(I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3+3*d
^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(4,-exp(I*(b*x+a))
)/b^4+3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 5.95 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.71

$$\int (c+dx)^3 \csc^3(a+bx)\, dx =$$
$$-\frac{b^2(c+dx)^2(3d+b(c+dx)\cot(a+bx))\csc(a+bx)-b^3c^3\log\left(1-e^{i(a+bx)}\right)-6bcd^2\log\left(1-e^{i(a+bx)}\right)}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^3,x]
```

output
```
-1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] - b^3*
c^3*Log[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] - 3*b^3*
c^2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] - 3*
b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - E^(I*(a + b*x
))] + b^3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))
] + 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b
*x))] + 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + E^(
I*(a + b*x))] - (3*I)*d*(2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*
x))] + (3*I)*d*(2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x)
)] - 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^(I*(a
+ b*x))] + (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, E
^(I*(a + b*x))])/b^4
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 4674, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\csc(a+bx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$\frac{3d^2\int (c+dx)\csc(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\csc(a+bx)dx-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int (c+dx)\csc(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\csc(a+bx)dx-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

$$\frac{3d^2\left(-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3d^2\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 2838

$$\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)+\frac{3d^2\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 3011

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\text{I}}{}\right.}{b}\right.$$
$$\frac{3d^2\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 7163

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\text{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}}{b}\right.}{}\right.$$
$$\frac{3d^2\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(\cdots}{}\right.}{}\right.$$

$$\frac{3d^2\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}-$$

$$\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3d^2\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+\cdots}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^3,x]
```

output
```
(-3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]*Csc[a
+ b*x])/(2*b) + (3*d^2*((-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*P
olyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2))
/b^2 + ((-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2
*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, -E
^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))])/b^2))/b))/b - (3*d*(
(I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*
PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I*(a + b*x))])/b^2))/b))
/b)/2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(275) = 550$.

Time = 1.26 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.42

method	result	size
risch	Expression too large to display	1056

input
```
int((d*x+c)^3*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a+1/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/2/b^
4*d^3*ln(1-exp(I*(b*x+a)))*a^3-3/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+3/b^
3*d^3*polylog(3,exp(I*(b*x+a)))*x-3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d
^3*ln(1-exp(I*(b*x+a)))*x-3/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+3/b^3*c*d
^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*arctanh(exp(I*(b*x+a)))+3/2/b^2*c
^2*d*ln(1-exp(I*(b*x+a)))*a-3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/2/b*c*d
^2*ln(1-exp(I*(b*x+a)))*x^2-3/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/2/b^2*c^2
*d*ln(exp(I*(b*x+a))+1)*a+3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/b^3*c*d^2*a
^2*arctanh(exp(I*(b*x+a)))+3/2/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-3/2/b^3*
c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+3/2*I
/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3/2*I/b^2*d^3*polylog(2,exp(I*(b*x
+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polyl
og(2,exp(I*(b*x+a)))+3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+3*I*d^3*polylo
g(4,exp(I*(b*x+a)))/b^4+1/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^4*d^3*a*
arctanh(exp(I*(b*x+a)))-1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/2/b^4*d^3*ln(
exp(I*(b*x+a))+1)*a^3-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a+3*I/b^2*c*d^2*polyl
og(2,-exp(I*(b*x+a)))*x-3*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-1/b*c^3*
arctanh(exp(I*(b*x+a)))-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3*I*d^3*poly
log(4,-exp(I*(b*x+a)))/b^4+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I
*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1744 vs. $2(265) = 530$.

Time = 0.15 (sec) , antiderivative size = 1744, normalized size of antiderivative = 5.64

$$\int (c+dx)^3 \csc^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
 a) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3 + (I*b^2
*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^
2*c^2*d + 2*I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*
d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(-I*b^2*d^3*
x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c
*d^2*x + I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3 +
 (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2
)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 + 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^
2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)
*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 + 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^
2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)
*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^
2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*
b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*s
in(b*x + a) + 1/2) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (...
```

Sympy [F]

$$\int (c+dx)^3 \csc^3(a+bx)\,dx = \int (c+dx)^3 \csc^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3886 vs. $2(265) = 530$.

Time = 1.02 (sec) , antiderivative size = 3886, normalized size of antiderivative = 12.58

$$\int (c + dx)^3 \csc^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - lo
g(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x +
a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
^2 - a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1)
+ log(cos(b*x + a) - 1))/b^3 - 4*(2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2
)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a
*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))
*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2
- a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x +
a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(
-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 - 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 + 6*I*b
*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d
- 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) + 1) - 12*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*
cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 - I*a
*d^3)*sin(4*b*x + 4*a) + 2*(-I*b*c*d^2 + I*a*d^3)*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a...
```

Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/sin(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc^3(a+bx)\,dx = \frac{-\cos(bx+a)\,c^3 + 2\left(\int \csc(bx+a)^3 x^3 dx\right)\sin(bx+a)^2 b\,d^3 + 6\left(\int \csc(bx+a)^3 x^2 dx\right)\sin(bx+a)^2 bc\,d^2}{2\sin(bx+a)^2 b}$$

input
```
int((d*x+c)^3*csc(b*x+a)^3,x)
```

output
```
( - cos(a + b*x)*c**3 + 2*int(csc(a + b*x)**3*x**3,x)*sin(a + b*x)**2*b*d*
*3 + 6*int(csc(a + b*x)**3*x**2,x)*sin(a + b*x)**2*b*c*d**2 + 6*int(csc(a
+ b*x)**3*x,x)*sin(a + b*x)**2*b*c**2*d + log(tan((a + b*x)/2))*sin(a + b*
x)**2*c**3)/(2*sin(a + b*x)**2*b)
```

3.34 $\int (c+dx)^2 \csc^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 180

$$\begin{aligned}\int (c+dx)^2 \csc^3(a+bx)\,dx = &-\frac{(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\ &-\frac{d^2\operatorname{arctanh}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2} \\ &-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b} \\ &+\frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\ &-\frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\ &-\frac{d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}\end{aligned}$$

output
```
-(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)
*csc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b+I*d*(d*x+c)*polylog(
2,-exp(I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-d^2*polyl
og(3,-exp(I*(b*x+a)))/b^3+d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 471 vs. $2(180) = 360$.

Time = 7.53 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.62

$$\int (c+dx)^2 \csc^3(a+bx)\,dx = -\frac{d(c+dx)\csc(a)}{b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$

$$+ \, b^2c^2\log\left(1-e^{i(a+bx)}\right) + 2d^2\log\left(1-e^{i(a+bx)}\right) + 2b^2cdx\log\left(1-e^{i(a+bx)}\right) + b^2d^2x^2\log\left(1-e^{i(a+bx)}\right) - \cdots$$

$$+ \frac{(c^2+2cdx+d^2x^2)\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$

$$+ \frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$

$$+ \frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^3,x]
```

output
```
-((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/
2]^2)/(8*b) + (b^2*c^2*Log[1 - E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a +
b*x))] + 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - E^(I*(
a + b*x))] - b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*
x))] - 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + E^(I*(a
+ b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c
 + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] +
 2*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*S
ec[a/2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)
/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Si
n[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \csc(a+bx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$\frac{d^2\int \csc(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\csc(a+bx)dx-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \csc(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\csc(a+bx)dx-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\int (c+dx)^2\csc(a+bx)dx-\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

$$\frac{1}{2}\left(-\frac{2d\int (c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{id\int \operatorname{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{id\int \operatorname{PolyLog}(2,e^{i(a+bx)}}{b}\right.}{b}\right.$$

$$\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$\big\downarrow$ 2720

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-e^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\int e^{-i(}}{}\right.}{b}\right.$$

$$\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$\big\downarrow$ 7143

$$-\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e}{b}\right.}{}\right.$$

$$\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^3,x]
```

output
```
-((d^2*ArcTanh[Cos[a + b*x]])/b^3) - (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c
+ d*x)^2*Cot[a + b*x]*Csc[a + b*x])/(2*b) + ((-2*(c + d*x)^2*ArcTanh[E^(I*
(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*P
olyLog[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*
(a + b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b)/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(166) = 332.

Time = 1.13 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.04

method	result
risch	$\frac{x^2 d^2 b\,e^{3i(bx+a)}+2cdxb\,e^{3i(bx+a)}+b\,c^2 e^{3i(bx+a)}+x^2 d^2 b\,e^{i(bx+a)}+2cdxb\,e^{i(bx+a)}-2id^2 x\,e^{3i(bx+a)}+b\,c^2 e^{i(bx+a)}-2idc\,e^{3i(bx+a)}+2}{b^2\left(e^{2i(bx+a)}-1\right)^2}$

input
```
int((d*x+c)^2*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(x^2*d^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I
*(b*x+a))+b*c^2*exp(3*I*(b*x+a))+x^2*d^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*
(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+b*c^2*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(
b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))-1/2/b*d^2*ln(exp(
I*(b*x+a))+1)*x^2+I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/2/b*d^2*ln(1-ex
p(I*(b*x+a)))*x^2+I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-I/b^2*d^2*polylog(2
,exp(I*(b*x+a)))*x-I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-1/b*c*d*ln(exp(I*(b
*x+a))+1)*x-1/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+1/b*c*d*ln(1-exp(I*(b*x+a)))*
x+1/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-1/
2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+2/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-d^2
*polylog(3,-exp(I*(b*x+a)))/b^3+d^2*polylog(3,exp(I*(b*x+a)))/b^3-1/b*c^2*
arctanh(exp(I*(b*x+a)))-1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-2/b^3*d^2*ar
ctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(162) = 324$.

Time = 0.12 (sec) , antiderivative size = 972, normalized size of antiderivative = 5.40

$$\int (c + dx)^2 \csc^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) - 2*(-I*b*d^2*x
- I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*s
in(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a
)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b
*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) -
2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos
(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cos(b*x + a)^2 + 2*d^2)*log(cos(b
*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cos(b*x + a)^2 + 2*d^2)*log(cos
(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2 - (
b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log
(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 ...
```

Sympy [F]

$$\int (c+dx)^2 \csc^3(a+bx)\,dx = \int (c+dx)^2 \csc^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(162) = 324$.

Time = 0.30 (sec) , antiderivative size = 1938, normalized size of antiderivative = 10.77

$$\int (c+dx)^2 \csc^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(
cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(co
s(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 - 4
*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-I*(b*x
+ a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a)
- 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*
b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(d^2*cos(4*b*x + 4
*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x
+ 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(-I*(b*
x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(-I*b*c*d + (I*a - 1)*d^2)*(b*x + a))
*cos(3*b*x + 3*a) - 4*(-I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(-I*b*c*
d + (I*a + 1)*d^2)*(b*x + a))*cos(b*x + a) - 4*(b*c*d + (b*x + a)*d^2 -...
```

Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/sin(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc^3(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\,c^2 + 2\left(\int \csc(bx+a)^3 x^2 dx\right)\sin(bx+a)^2 b\,d^2 + 4\left(\int \csc(bx+a)^3 x dx\right)\sin(bx+a)^2 bcd +}{2\sin(bx+a)^2 b}$$

input
```
int((d*x+c)^2*csc(b*x+a)^3,x)
```

output
```
( - cos(a + b*x)*c**2 + 2*int(csc(a + b*x)**3*x**2,x)*sin(a + b*x)**2*b*d*
*2 + 4*int(csc(a + b*x)**3*x,x)*sin(a + b*x)**2*b*c*d + log(tan((a + b*x)/
2))*sin(a + b*x)**2*c**2)/(2*sin(a + b*x)**2*b)
```

3.35 $\int (c+dx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int (c+dx)\csc^3(a+bx)\,dx = -\frac{(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} + \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} - \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}$$

output
```
-(d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*
x+a)*csc(b*x+a)/b+1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/2*I*d*polylog(2
,exp(I*(b*x+a)))/b^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 292 vs. $2(109) = 218$.

Time = 2.35 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.68

$$\int (c+dx)\csc^3(a+bx)\,dx = -\frac{dx\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right.\right.\right.}{2b^2} + \frac{dx\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{d\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2} - \frac{d\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3,x]
```

output
```
-1/8*(d*x*Csc[a/2 + (b*x)/2]^2)/b - (c*Csc[(a + b*x)/2]^2)/(8*b) - (c*Log[
Cos[(a + b*x)/2]])/(2*b) + (c*Log[Sin[(a + b*x)/2]])/(2*b) + (d*((a + b*x)
*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a + b*
x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/
(2*b^2) + (d*x*Sec[a/2 + (b*x)/2]^2)/(8*b) + (c*Sec[(a + b*x)/2]^2)/(8*b)
+ (d*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) - (d*Sec[a/2]*Sec[a
/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)\csc(a+bx)^3dx$$

$$\Big\downarrow \text{4673}$$

$$\frac{1}{2}\int (c+dx)\csc(a+bx)dx-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int (c+dx)\csc(a+bx)dx-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

$$\frac{1}{2}\left(-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a\right.}}{b}\right.$$
$$\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3,x]
```

output
```
-1/2*(d*Csc[a + b*x])/b^2 - ((c + d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) +
((-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2)/2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(93) = 186$.

Time = 0.90 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.26

method	result
risch	$\frac{dxb\,e^{3i(bx+a)}+bc\,e^{3i(bx+a)}+dxb\,e^{i(bx+a)}+bc\,e^{i(bx+a)}-id\,e^{3i(bx+a)}+id\,e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2}-\frac{c\,\mathrm{arctanh}\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{2b}-$

input
```
int((d*x+c)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))+
d*x*b*exp(I*(b*x+a))+b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*
x+a)))-1/b*c*arctanh(exp(I*(b*x+a)))-1/2/b*d*ln(exp(I*(b*x+a))+1)*x-1/2/b^
2*d*ln(exp(I*(b*x+a))+1)*a+1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/2/b*d*
ln(1-exp(I*(b*x+a)))*x+1/2/b^2*d*ln(1-exp(I*(b*x+a)))*a-1/2*I*d*polylog(2,
exp(I*(b*x+a)))/b^2+1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 452, normalized size of antiderivative = 4.15

$$\int (c+dx)\csc^3(a+bx)\,dx = \frac{2\,(bdx+bc)\cos(bx+a) + \left(-i\,d\cos(bx+a)^2 + i\,d\right)\mathrm{Li}_2(\cos(bx+a) + i\,\sin(bx+a)) + \left(i\,d\cos(bx+a)\right.}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(
b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*x + a)
 - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) + I*s
in(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) + (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) + (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*l
og(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + ((b*c - a*d)*cos(b*x +
a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b*d
*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a
) + 1) - (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int (c+dx)\csc^3(a+bx)\,dx = \int (c+dx)\csc^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(89) = 178$.

Time = 0.17 (sec) , antiderivative size = 763, normalized size of antiderivative = 7.00

$$\int (c+dx)\csc^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin
(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x
+ 4*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b
*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*
b*x + 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d
*x*sin(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*
(-I*b*d*x - I*b*c - d)*cos(3*b*x + 3*a) - 4*(-I*b*d*x - I*b*c + d)*cos(b*x
 + a) - 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a
) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x +
 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*
a) + d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*co
s(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin
(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*c
os(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin
(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*d*x + b*c - I*d)*sin(3*b*x + 3*a)
 - 4*(b*d*x + b*c + I*d)*sin(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^
2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - ...
```

Giac [F]

$$\int (c+dx)\csc^3(a+bx)\,dx=\int (dx+c)\csc{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/sin(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^3(a+bx)\,dx = \frac{-\cos(bx+a)\,c+2\left(\int \csc(bx+a)^3\,xdx\right)\sin(bx+a)^2\,bd+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2\,c}{2\sin(bx+a)^2\,b}$$

input
```
int((d*x+c)*csc(b*x+a)^3,x)
```

output
```
( - cos(a + b*x)*c + 2*int(csc(a + b*x)**3*x,x)*sin(a + b*x)**2*b*d + log(
tan((a + b*x)/2))*sin(a + b*x)**2*c)/(2*sin(a + b*x)**2*b)
```

3.36 $\int \frac{\csc^3(a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\csc^3(a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 38.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{c+dx} \, dx = \int \frac{\csc^3(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Csc[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Csc[a + b*x]^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)^3}{c+dx} dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\csc^3(a+bx)}{c+dx} dx$$

input
```
Int[Csc[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc{(bx+a)}^3}{dx+c}dx$$

input
```
int(csc(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx = \int \frac{\csc{(bx+a)}^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^3{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 1791, normalized size of antiderivative = 111.94

$$\int \frac{\csc^3(a+bx)}{c+dx} \, dx = \int \frac{\csc(bx+a)^3}{dx+c} \, dx$$

input
```
integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
(((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*
x + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*
c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x +
 b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(
b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*
a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
+ 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + 2*d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x
+ a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x
 + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(
b*x + a)), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x
 + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*si...
```

Giac [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 34.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(sin(a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3}{dx+c}dx$$

input
```
int(csc(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(a + b*x)**3/(c + d*x),x)
```

3.37 $\int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} \, dx = \text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 40.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} \, dx = \int \frac{\csc^3(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Csc[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Csc[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)^3}{(c+dx)^2} dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$$

input
```
Int[Csc[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 9.00 (sec) , antiderivative size = 2287, normalized size of antiderivative = 142.94

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*
b*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x
+ b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d
*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c
)*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*
a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
n(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4
+ 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^
3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2
*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (b^2*d^3*x^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 35.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(sin(a + b*x)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(a + b*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.38 $\int (c+dx)^{5/2}\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 195

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{7/2}}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2}$$

output
```
15/4*d^2*(d*x+c)^(1/2)*cos(b*x+a)/b^3-(d*x+c)^(5/2)*cos(b*x+a)/b-15/8*d^(5
/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))/b^(7/2)+15/8*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)+5/2*d*(d*x+c)^(
3/2)*sin(b*x+a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = \frac{id^3e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sin[a + b*x],x]
```

output
```
((I/2)*d^3*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((-I)*b*(c +
 d*x))/d] - E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, (I*b*(c +
 d*x))/d]))/(b^4*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{5/2}\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{5d\int (c+dx)^{3/2}\cos(a+bx)dx}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5d\int(c+dx)^{3/2}\sin\left(a+bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{5d\left(\frac{3d\int-\sqrt{c+dx}\sin(a+bx)dx}{2b}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sin(a+bx)dx}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sin(a+bx)dx}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3787}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3785}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3833

$$\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^(5/2)*Sin[a + b*x],x]
```

output
```
-(((c + d*x)^(5/2)*Cos[a + b*x])/b) + (5*d*((-3*d*(-((Sqrt[c + d*x]*Cos[a
+ b*x])/b) + (d*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b
]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))
/(2*b)))/(2*b) + ((c + d*x)^(3/2)*Sin[a + b*x])/b))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\cdots}{\cdots}\right)}{\cdots}\right)}{b}}{d}$
default	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\cdots}{\cdots}\right)}{\cdots}\right)}{b}}{d}$

input
```
int((d*x+c)^(5/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/2/b*d*(d*x+c)^(5/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+5/2/b*d*(1/2/b*d*
(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/2)
*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx =$$

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+}{8\,b^4}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fres
nel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt
(d*x + c)*((4*b^3*d^2*x^2 + 8*b^3*c*d*x + 4*b^3*c^2 - 15*b*d^2)*cos(b*x +
a) - 10*(b^2*d^2*x + b^2*c*d)*sin(b*x + a)))/b^4
```

Sympy [F]

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sin(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sin(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = \frac{\sqrt{2}\left(40\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-4\left(4\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{(dx\cdot}{}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32*sqrt(2)*(40*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin((((d*x + c)*b - b*c + a*
d)/d) - 4*(4*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2)
*cos((((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1
/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(pi)*d^3*(b^2/d
^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.29

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16*(8*(sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)))*c^3 - d^3*((sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2
*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)
*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sq
rt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^3)/d^3 + (sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b
*c*d^2 + 15*I*d^3)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
+ 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d
+ 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x +
c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)
/b^3)/d^3) - 12*(sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*er
f(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx=\int \sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\sin(bx+a)\,x^2dx\right)d^2 + 2\left(\int \sqrt{dx+c}\,\sin(bx+a)\,xdx\right)cd + \left(\int \sqrt{dx+c}\,\sin(bx+a)\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*sin(a +
b*x)*x,x)*c*d + int(sqrt(c + d*x)*sin(a + b*x),x)*c**2
```

3.39 $\int (c+dx)^{3/2} \sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)}{b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}$$

output

```
-(d*x+c)^(3/2)*cos(b*x+a)/b-3/4*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/4*d^(3/2)*2
^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*s
in(a-b*c/d)/b^(5/2)+3/2*d*(d*x+c)^(1/2)*sin(b*x+a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx=-\frac{ide^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^(3/2)*Sin[a + b*x],x]
```

output
```
((-1/2*I)*d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\int \sqrt{c+dx}\cos(a+bx)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\left(\frac{d\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3787}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3785}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3833}$$

$$\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}$$

input

```
Int[(c + d*x)^(3/2)*Sin[a + b*x],x]
```

output

```
-(((c + d*x)^(3/2)*Cos[a + b*x])/b) + (3*d*(-1/2*(d*((Sqrt[2*Pi]*Cos[a - (
b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqr
t[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sin[a + b*x])/b))/
(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{b}}{d}$
default	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{b}}{d}$

input
```
int((d*x+c)^(3/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/2/b*d*(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*
(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx =$$

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\,(}{4\,b^3}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d
*sin(b*x + a) - 2*(b^2*d*x + b^2*c)*cos(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx=$$

$$-\frac{\sqrt{2}\left(8\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-12\sqrt{2}\sqrt{dx+c}bd\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+3\left((i+1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*(8*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(((d*x + c)*b - b*c + a*d)
/d) - 12*sqrt(2)*sqrt(d*x + c)*b*d*sin(((d*x + c)*b - b*c + a*d)/d) + 3*((
I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) +
 3*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sq
rt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*
b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 758, normalized size of antiderivative = 4.46

$$\int (c+dx)^{3/2}\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/8*(4*(sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)))*c^2 - 4*(sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(2*b*c
- I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*
sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*
d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + sqrt(2)*sqrt(pi)*(4*b^2*c^
2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b^2) + sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/
2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*(d*x +
c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b^2 + 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \sin(a+bx) \, dx = \int \sin(a+b\,x) \, (c+d\,x)^{3/2} \, dx$$

input
```
int(sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \sin(a+bx) \, dx = \left(\int \sqrt{dx+c}\,\sin(bx+a)\,xdx\right)d + \left(\int \sqrt{dx+c}\,\sin(bx+a)\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*sin(a + b*x),x)*
c
```

3.40 $\int \sqrt{c+dx}\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 142

$$\int \sqrt{c+dx}\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(a+bx)}{b} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{b^{3/2}}$$

output
```
-(d*x+c)^(1/2)*cos(b*x+a)/b+1/2*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*Fres
nelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/2*d^(1/2)*2
^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*s
in(a-b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \sqrt{c+dx}\sin(a+bx)\,dx$$
$$= \frac{ide^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^2\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x]*Sin[a + b*x],x]
```

output
```
((I/2)*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c +
 d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c
+ d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{c+dx}\sin(a+bx)dx$$
$$\Big\downarrow \text{3777}$$
$$\frac{d\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3787

$$\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3042

$$\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3785

$$\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3786

$$\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int \sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3832

$$\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

$\Big\downarrow$ 3833

$$\frac{d\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

input
```
Int[Sqrt[c + d*x]*Sin[a + b*x],x]
```

output
```
-((Sqrt[c + d*x]*Cos[a + b*x])/b) + (d*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*Fresn
elC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt
[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/
d])/(Sqrt[b]*Sqrt[d])))/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2b\sqrt{\frac{b}{d}}}}{d}$

input
```
int((d*x+c)^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/2/b*d*(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \sqrt{c+dx}\sin(a+bx)\,dx$$
$$=\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx+}}{2\,b^2}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*s
qrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*b
*cos(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.38

$$\int \sqrt{c+dx}\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{dx+c}b\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left((i-1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x + c)*b*cos(((d*x + c)*b - b*c + a*d)/d) +
 ((I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi
)*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) +
(-(I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi
)*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/
b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.97

$$\int \sqrt{c+dx}\sin(a+bx)\,dx =$$

$$\frac{\sqrt{2}\sqrt{\pi}(2\,bc+i\,d)d\,\mathrm{erf}\left(\frac{i\,\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}(2\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{i\,\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b} -$$

$$-\overline{\qquad\qquad}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 2*(sqrt(2)*sqrt(pi)*d*er
f(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 2*sqrt
(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^
((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sin(a+bx)\,dx = \int \sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sin(a+bx)\,dx=\int \sqrt{dx+c}\,\sin\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x),x)
```

3.41 $\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx=\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))/b^(1/2)/d^(1/2)+2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

input

```
Integrate[Sin[a + b*x]/Sqrt[c + d*x],x]
```

output

```
-1/2*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*x))
/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))
/d])/(b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3787}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3785}$$

$$\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3786}$$

$$\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[Sin[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/S
qrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	99
default	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	99

input
```
int(sin(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(sin(a + b*x)/sqrt(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.36

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{2}\left(\left(-(i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+\left((i\right.\right.}{4b}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*((-(I + 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I
- 1)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
I*b/d)) + ((I - 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*
sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/
d)))/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.42

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{2\,d}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin\left(a+b\,x\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin\left(bx+a\right)}{\sqrt{dx+c}}dx$$

input
```
int(sin(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(sin(a + b*x)/sqrt(c + d*x),x)
```

3.42 $\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}} - \frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

output
```
2*b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/d^(3/2)-2*b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/d^(3/2)-2*sin(b*x+a)
/d/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+2ie^{\frac{i(bc+ad)}{d}}\right.}{d\sqrt{c+dx}}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(I*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*x))
/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x)
)/d] + (2*I)*E^((I*(b*c + a*d))/d)*Sin[a + b*x]))/(d*E^((I*(b*c + a*d))/d)
*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3778, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{2b\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3787

$$\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3042

$$\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3785

$$\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3786

$$\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int \sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3832

$$\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3833

$$\frac{2b\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[Sin[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(2*b*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/d - (2*Sin
[a + b*x])/(d*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	140
default	$\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	140

input
```
int(sin(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{2\left(\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\right.\right.}{d^2x+cd}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi
*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d)
 - sqrt(d*x + c)*sin(b*x + a))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Integral(sin(a + b*x)/(c + d*x)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{i\,(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{i\,(dx+c)}{d}\right.\right.}{4\sqrt{dx+c}d}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(((I - 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamm
a(-1/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-1
/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(
-(b*c - a*d)/d))*sqrt((d*x + c)*b/d)/(sqrt(d*x + c)*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin(a+b\,x)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x)^(3/2),x)
```

output
```
int(sin(a + b*x)/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\,dx$$

input
```
int(sin(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(sin(a + b*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.43 $\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 168

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{3d^{5/2}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
-4/3*b*cos(b*x+a)/d^2/(d*x+c)^(1/2)-4/3*b^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c
/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)-4/3*b
^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(a-b*c/d)/d^(5/2)-2/3*sin(b*x+a)/d/(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2\left(-b(c+dx)\left(-e^{i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{-i(a+bx)}\left(1+e^{2i(a+bx)}-e^{\frac{ib(c+}{d}}\right.\right.\right.}{3d^2(c+dx)^{3/2}}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(2*(-(b*(c + d*x)*(-(E^(I*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[
1/2, ((-I)*b*(c + d*x))/d]) + (1 + E^((2*I)*(a + b*x)) - E^((I*b*(c + d*x)
)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*(a + b*x)
))) - d*Sin[a + b*x]))/(3*d^2*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3778, 3042, 3778, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{2b\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3778}$$

$$\frac{2b\left(\frac{2b\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\left(-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\left(-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3787}$$

$$\frac{2b\left(-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\left(-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2b\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2b\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2b\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2b\left(-\frac{2b\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Int[Sin[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(2*b*((-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2*b*((Sqrt[2*Pi]*Cos[a - (b*c
)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d
]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[
a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/d))/(3*d) - (2*Sin[a + b*x])/(3*d*(c + d
*x)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\text{Fresne}}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$
default	$\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\text{Fresne}}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$

input
```
int(sin(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/3/(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(
1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\left(2\sqrt{2}(\pi bd^2x^2+2\pi bcdx+\pi bc^2)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+2\sqrt{2}(\pi bd^2x^2+2\pi bcdx+\pi\right.}{3\left(d^4x^2+2\,cd^3\right.}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 2*sq
rt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*
(2*(b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^
2*d^2)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx=\int \frac{\sin(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)**(5/2),x)
```

output
```
Integral(sin(a + b*x)/(c + d*x)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx=$$

$$-\frac{\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{i(dx-}{d}\right.\right.}{4(dx+c)^{\frac{3}{2}}d}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*((-(I + 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gam
ma(-3/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-
3/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin
(-(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2)/((d*x + c)^(3/2)*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin(a+b\,x)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x)^(5/2),x)
```

output
```
int(sin(a + b*x)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(sin(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
int(sin(a + b*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*
x)*d**2*x**2),x)
```

3.44 $\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 193

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{15d^{7/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}}$$

output
```
-4/15*b*cos(b*x+a)/d^2/(d*x+c)^(3/2)-8/15*b^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b
*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+8/1
5*b^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(a-b*c/d)/d^(7/2)-2/5*sin(b*x+a)/d/(d*x+c)^(5/2)+8/15*b^2*sin
(b*x+a)/d^3/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx =$$
$$-\frac{i\left(b(c+dx)\left(2e^{i\left(a-\frac{bc}{d}\right)}\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)-ie^{-i(a+bx)}\left(\right.\right.\right.}{15d^3(c+dx)^{5/2}}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
((-1/15*I)*(b*(c + d*x)*(2*E^(I*(a - (b*c)/d))*(E^((I*b*(c + d*x))/d)*((-I
)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-
I)*b*(c + d*x))/d]) - (I*(2*d - (4*I)*b*(c + d*x) + 4*d*E^((I*b*(c + d*x))
/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d]))/E^(I*(a + b*
x))) - (6*I)*d^2*Sin[a + b*x]))/(d^3*(c + d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}dx$$
$$\Big\downarrow \ 3778$$

$$\frac{2b\int\frac{\cos(a+bx)}{(c+dx)^{5/2}}dx}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{2b\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{5/2}}dx}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 3778

$$\frac{2b\left(\frac{2b\int-\frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 25

$$\frac{2b\left(-\frac{2b\int\frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{2b\left(-\frac{2b\int\frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 3778

$$\frac{2b\left(-\frac{2b\left(\frac{2b\int\frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{2b\left(-\frac{2b\left(\frac{2b\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\big\downarrow$ 3787

$$\frac{2b\left(-\frac{2b\left(\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\downarrow$ 3042

$$\frac{2b\left(-\frac{2b\left(\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\downarrow$ 3785

$$\frac{2b\left(-\frac{2b\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\downarrow$ 3786

$$\frac{2b\left(-\frac{2b\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\Big\downarrow$ 3832

$$\frac{2b\left(-\frac{2b\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

$\Big\downarrow$ 3833

$$\frac{2b\left(-\frac{2b\left(\frac{2b\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}}$$

input
```
Int[Sin[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(-2*Sin[a + b*x])/(5*d*(c + d*x)^(5/2)) + (2*b*((-2*Cos[a + b*x])/(3*d*(c
+ d*x)^(3/2)) - (2*b*((2*b*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*Fresn
elS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]
*Sqrt[d])))/d - (2*Sin[a + b*x])/(d*Sqrt[c + d*x])))/(3*d)))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelC}\left(\sqrt{\right.\right.}}{}\right)}{3d}\right)}{5d}}{d}$
default	$\frac{-\frac{2\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelC}\left(\sqrt{\right.\right.}}{}\right)}{3d}\right)}{5d}}{d}$

input
```
int(sin(b*x+a)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/5/(d*x+c)^(5/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+2/5*b/d*(-1/3/(d*x+c)
^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(b*(d*x+c
)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.54

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{2\left(4\sqrt{2}(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\sqrt{2}(\pi b^2 \cdots\right.}{}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x +
 pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 +
 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(2*(b*d^2*x + b
*c*d)*cos(b*x + a) - (4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 3*d^2)*sin
(b*x + a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)**(7/2),x)
```

output
```
Integral(sin(a + b*x)/(c + d*x)**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.67

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1\right.\right.}{4\,(dx+c)}$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/4*(((I - 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma
(-5/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-5/
2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-
(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2)/((d*x + c)^(5/2)*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x)^(7/2),x)
```

output
```
int(sin(a + b*x)/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(sin(b*x+a)/(d*x+c)^(7/2),x)
```

output
```
int(sin(a + b*x)/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sqrt(c
 + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.45 $\int (c+dx)^{5/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 231

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} - \frac{15d^{5/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}+\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}$$

output
```
-5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d-15/128*d^(5/2)*Pi^(1/2)*cos(
2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(7/2)-15
/128*d^(5/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*s
in(2*a-2*b*c/d)/b^(7/2)-1/2*(d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)/b+5/8*d*(d
*x+c)^(3/2)*sin(b*x+a)^2/b^2+15/64*d^2*(d*x+c)^(1/2)*sin(2*b*x+2*a)/b^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx=\frac{64(c+dx)^4+\frac{7\sqrt{2}d^4e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)}{b^4}+\frac{7\sqrt{2}d^4e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)}{b^4}}{448d\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sin[a + b*x]^2,x]
```

output
```
(64*(c + d*x)^4 + (7*Sqrt[2]*d^4*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c +
 d*x))/d]*Gamma[7/2, ((-2*I)*b*(c + d*x))/d])/b^4 + (7*Sqrt[2]*d^4*Sqrt[(I
*b*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d])/(b^4*E^((2*I)*(a - (b*
c)/d))))/(448*d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3792, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{5/2}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{15d^2\int\sqrt{c+dx}\sin^2(a+bx)dx}{16b^2}+\frac{1}{2}\int(c+dx)^{5/2}dx+\frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{15d^2\int\sqrt{c+dx}\sin^2(a+bx)dx}{16b^2}+\frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{15d^2\int\sqrt{c+dx}\sin(a+bx)^2dx}{16b^2}+\frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 3793$$

$$-\frac{15d^2\int\left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)dx}{16b^2}+\frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 2009$$

$$\frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2}-\frac{15d^2\left(\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}\right)}{16b^2}-\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

input
```
Int[(c + d*x)^(5/2)*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(7/2)/(7*d) - ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x])/(2*b)
+ (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^2)/(8*b^2) - (15*d^2*((c + d*x)^(3/2)/
(3*d) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c
+ d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(8*b^(3/2)
) - (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)))/(16*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+2\right.}{4b}\right.}{}\right.}{}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+2\right.}{4b}\right.}{}\right.}{}}{d}$

input
```
int((d*x+c)^(5/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/14*(d*x+c)^(7/2)-1/8/b*d*(d*x+c)^(5/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c
)/d)+5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+3/4/
b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/
2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.12

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx =$$

$$-\frac{105\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4}{}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt
(d*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(32*b^4*d^3*x^3 + 96*b^4
*c*d^2*x^2 + 32*b^4*c^3 + 70*b^2*c*d^2 - 140*(b^2*d^3*x + b^2*c*d^2)*cos(b
*x + a)^2 - 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*
cos(b*x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d + 35*b^2*d^3)*x)*sqrt(d*x + c)
)/(b^4*d)
```

Sympy [F]

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sin^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(5/2)*sin(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.28

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d} - 1120\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right) + 105\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 1120*sqrt(2)*(d*x + c)
^(3/2)*b^2*d*cos(2*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*4^(1/4)*sq
rt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/
d)) + 105*((I - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)
/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 1
5*sqrt(2)*sqrt(d*x + c)*b*d^2)*sin(2*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 1310, normalized size of antiderivative = 5.67

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8960*(2240*(I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)) - I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*s
qrt(d*x + c))*c^3 - d^3*(256*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(-I*sqrt(pi)*(64*b^
3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-16*I*(d*x + c)^(5/2)*b^2*d + 48*I*(d*x
+ c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d
^2 - 36*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 35*(I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c
^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 2*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*
d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x
+ c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*
d)/d)/b^3)/d^3) + 560*(-3*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx=\int \sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\sin(bx+a)^2\,x^2dx\right)d^2 + 2\left(\int \sqrt{dx+c}\,\sin(bx+a)^2\,xdx\right)cd + \left(\int \sqrt{dx+c}\,\sin(bx+a)^2\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*sin(a
 + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*sin(a + b*x)**2,x)*c**2
```

3.46 $\int (c+dx)^{3/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 203

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}+\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3d^{3/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}}-\frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}$$

output
```
-3/16*d*(d*x+c)^(1/2)/b^2+1/5*(d*x+c)^(5/2)/d+3/32*d^(3/2)*Pi^(1/2)*cos(2*
a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)-3/32
*d^(3/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2
*a-2*b*c/d)/b^(5/2)-1/2*(d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)/b+3/8*d*(d*x+c
)^(1/2)*sin(b*x+a)^2/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx=\frac{\sqrt{c+dx}\left(32(c+dx)^2-\frac{5\sqrt{2}d^2e^{2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{5\sqrt{2}d^2e^{-2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{160d}$$

input
```
Integrate[(c + d*x)^(3/2)*Sin[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(32*(c + d*x)^2 - (5*Sqrt[2]*d^2*E^((2*I)*(a - (b*c)/d))*Ga
mma[5/2, ((-2*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) - (5*Sq
rt[2]*d^2*Gamma[5/2, ((2*I)*b*(c + d*x))/d])/(b^2*E^((2*I)*(a - (b*c)/d))*
Sqrt[(I*b*(c + d*x))/d])))/(160*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3042, 3792, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^{3/2}\sin(a+bx)^2dx$$

$$\Big\downarrow \ 3792$$

$$-\frac{3d^2\int\frac{\sin^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}+\frac{1}{2}\int(c+dx)^{3/2}dx+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{3d^2\int\frac{\sin^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3d^2\int\frac{\sin(a+bx)^2}{\sqrt{c+dx}}dx}{16b^2}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \; 3793$$

$$-\frac{3d^2\int\left(\frac{1}{2\sqrt{c+dx}}-\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{16b^2}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{3d^2\left(-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{16b^2}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

input
```
Int[(c + d*x)^(3/2)*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(5*d) - (3*d^2*(Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos[2*a - (2*b
*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*
Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/(16*b^2) - ((c + d*x)^(3/2)*
Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]^2)/(8*b
^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelC}\left(\cdots\right.\right.}{\cdots}\right.}{4b}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelC}\left(\cdots\right.\right.}{\cdots}\right.}{4b}}{d}$

input
```
int((d*x+c)^(3/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/10*(d*x+c)^(5/2)-1/8/b*d*(d*x+c)^(3/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c
)/d)+3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+1/8/
b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = \frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad}{d}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 2*(16*b^3*d^2*x^2 + 32*b^3*c*
d*x + 16*b^3*c^2 - 30*b*d^2*cos(b*x + a)^2 + 15*b*d^2 - 40*(b^2*d^2*x + b^
2*c*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.35

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{128\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d} - 160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right) - 120\sqrt{2}\sqrt{dx+c}\,bd\cos\left(\frac{2((dx+c)\cdots}{d}\cdots\right.\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 160*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(2*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b
*d*cos(2*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*
(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 15*((
I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1
)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-2*I*b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.93

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/960*(240*(I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1))
 - I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqr
t(d*x + c))*c^2 - 192*(d*x + c)^(5/2) + 640*(d*x + c)^(3/2)*c - 960*sqrt(d
*x + c)*c^2 + 40*(-3*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)
*c + 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*
sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + 15*I*sqrt
(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 15*I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(
I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 30*(4*I*(d*x + c)^(3/
2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)
*b - I*b*c + I*a*d)/d)/b^2 + 30*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx=\int \sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{3/2}\,dx$$

input
```
int(sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\sin(bx+a)^2\,xdx\right)d + \left(\int \sqrt{dx+c}\,\sin(bx+a)^2\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*x)*sin(a + b*x)*
*2,x)*c
```

3.47 $\int \sqrt{c+dx}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 158

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx=\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}$$

output
```
1/3*(d*x+c)^(3/2)/d+1/8*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)+1/8*d^(1/2)*Pi^(1/2)*FresnelC(2
*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)-1/4*(d*x
+c)^(1/2)*sin(2*b*x+2*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.82

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx = \frac{(c+dx)^{3/2}\left(16+\frac{3\sqrt{2}e^{2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}+\frac{3\sqrt{2}e^{-2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{48d}$$

input
```
Integrate[Sqrt[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^(3/2)*(16 + (3*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*Gamma[3/2, ((-2*
I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2) + (3*Sqrt[2]*Gamma[3/2, (
(2*I)*b*(c + d*x))/d])/(E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^(3/2))
))/(48*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{c+dx}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{ 3793}$$

$$\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

input
```
Int[Sqrt[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(3*d) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2
*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(8*b^(3/2)) - (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\text{FresnelC}\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\text{FresnelC}\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$

input
```
int((d*x+c)^(1/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/6*(d*x+c)^(3/2)-1/8/b*d*(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)
/d)+1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/
(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int\sqrt{c+dx}\sin^2(a+bx)\,dx$$
$$=\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\,(2\,b}{24\,b^2d}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(2*b^2*d*x - 3*b*d*cos(b*x + a
)*sin(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.45

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx$$
$$= \frac{\sqrt{2}\Bigg(\frac{32\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d} - 24\sqrt{2}\sqrt{dx+c}\,b\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right) + 3\Bigg((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right) -}{}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(2*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*4^(1/4)*sqrt(pi)*d*(b^2
/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(
1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)
*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4
)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-2*I*b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx =$$

$$-\frac{12\left(\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}d\operatorname{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-4\sqrt{dx}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/48*(12*(I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) -
 I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(
d*x + c))*c - 3*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) + 3*I*sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c +
6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(
d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\,\sin{(bx+a)}^2\,dx$$

input
```
int((d*x+c)^(1/2)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)**2,x)
```

3.48 $\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}}{d} - \frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
(d*x+c)^(1/2)/d-1/2*Pi^(1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2)/Pi^(1/2))/b^(1/2)/d^(1/2)+1/2*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}\left(8+\frac{\sqrt{2}e^{2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{\sqrt{2}e^{-2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8d}$$

input
```
Integrate[Sin[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
(Sqrt[c + d*x]*(8 + (Sqrt[2]*E^((2*I)*(a - (b*c)/d))*Gamma[1/2, ((-2*I)*b*
(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d] + (Sqrt[2]*Gamma[1/2, ((2*I)*b*(
c + d*x))/d])/(E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/(8*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

input
```
Int[Sin[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2
*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[
b]*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{dx+c}-\frac{\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108
default	$\frac{\sqrt{dx+c}-\frac{\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108

input
```
int(sin(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/2*(d*x+c)^(1/2)-1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresne
lC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\sqrt{dx+}}{2\,bd}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b
/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin^2{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**(1/2),x)
```

output
```
Integral(sin(a + b*x)**2/sqrt(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$$
$$=\frac{\sqrt{2}\left(\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2i\,b}{d}}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*(((I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)
/d) + (I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*
cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + 8*sqrt(2)*sqrt(d*x + c)*
b/d)/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - \frac{i\sqrt{\pi}d\operatorname{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - 4\sqrt{dx+c}}{4\,d}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sq
rt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x +
 c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\sin\left(bx+a\right)^2}{\sqrt{dx+c}}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
int(sin(a + b*x)**2/sqrt(c + d*x),x)
```

3.49 $\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 135

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}} \, dx = \frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} + \frac{2\sqrt{b}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}} - \frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

output
```
2*b^(1/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))/d^(3/2)+2*b^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/d^(3/2)-2*sin(b*x+a)^2/d/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}} \, dx = \frac{e^{-\frac{2i(ad+b(c+dx))}{d}}\left(-\sqrt{2}e^{2i(2a+bx)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\left(-1+e^{2i(a+bx)}\right)^2 -\right.\right.}{2d\sqrt{c+dx}}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(-(Sqrt[2]*E^((2*I)*(2*a + b*x))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-
2*I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*((-1 + E^((2*I)*(a + b*x)))^2 -
Sqrt[2]*E^(((2*I)*b*(c + d*x))/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, ((2*I
)*b*(c + d*x))/d]))/(2*d*E^(((2*I)*(a*d + b*(c + d*x)))/d)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3794, 27, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3794}$$

$$\frac{4b\int \frac{\sin(2a+2bx)}{2\sqrt{c+dx}}dx}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3787}$$

$$\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3042

$$\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3785

$$\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3786

$$\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3832

$$\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

$\Big\downarrow$ 3833

$$\frac{2b\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(2*b*((Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(S
qrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[
c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/d
- (2*Sin[a + b*x]^2)/(d*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{1}{\sqrt{dx+c}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{-\frac{1}{\sqrt{dx+c}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$

input
```
int(sin(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/2/(d*x+c)^(1/2)+1/2/(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)
+b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx\right.\right.}{d^2x+cd}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt
(d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*s
qrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*(cos(b*
x + a)^2 - 1))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**(3/2),x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\Gamma\left(\cdots\right)\right.\cdots}{8\sqrt{dx+c}\,d}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) + (I - 1)*
sqrt(2)*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + ((I - 1)*
sqrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*
(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) + 8)/(sqrt(d*x
+ c)*d)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c)^(3/2),x)
```

output
```
int(sin(a + b*x)**2/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.50 $\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 170

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}} \, dx = \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} - \frac{8b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
8/3*b^(3/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))/d^(5/2)-8/3*b^(3/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/
2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/d^(5/2)-8/3*b*cos(b*x+a)*sin(b*x+a)/
d^2/(d*x+c)^(1/2)-2/3*sin(b*x+a)^2/d/(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{-2d+e^{2i\left(a-\frac{bc}{d}\right)}\left(e^{\frac{2ib(c+dx)}{d}}(d+4ib(c+dx))+4\sqrt{2}d\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)\right)}{6d^2(c+dx)}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(-2*d + E^((2*I)*(a - (b*c)/d))*(E^(((2*I)*b*(c + d*x))/d)*(d + (4*I)*b*(c
 + d*x)) + 4*Sqrt[2]*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(
c + d*x))/d]) + (d - (4*I)*b*(c + d*x) + 4*Sqrt[2]*d*E^(((2*I)*b*(c + d*x)
)/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/E^((2*I)
*(a + b*x)))/(6*d^2*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3795, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{16b^2\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 17$$

$$-\frac{16b^2\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{16b^2\int \frac{\sin(a+bx)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \; 3793$$

$$-\frac{16b^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{16b^2\left(-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(16*b^2*Sqrt[c + d*x])/(3*d^3) - (16*b^2*(Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(
2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]
*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/(3*d^2) - (8*b*Cos
[a + b*x]*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^2)/(3*d*(c
 + d*x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\cdots\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$
default	$\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\cdots\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$

input
```
int(sin(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/6/(d*x+c)^(3/2)+1/6/(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)
+2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)
/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.23

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2\left(4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\left(\pi bd^2x^2\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*c
 - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2 +
 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*cos
(b*x + a)*sin(b*x + a) - d)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin^2\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**(5/2),x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{2i\,(dx+c)b}{d}\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{2i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\right.\right.}{12\,(dx+c)^{\frac{3}{2}}d}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(3*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I +
1)*sqrt(2)*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I +
 1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -
2*I*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) + 4)/((d*
x + c)^(3/2)*d)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
int(sin(a + b*x)**2/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c +
 d*x)*d**2*x**2),x)
```

3.51 $\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 216

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} - \frac{32b^{5/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{15d^{7/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}}$$

output

```
-16/15*b^2/d^3/(d*x+c)^(1/2)-32/15*b^(5/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*Fresn
elS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/d^(7/2)-32/15*b^(5/2)*Pi^(1/
2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/d^(
7/2)-8/15*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sin(b*x+a)^2/d/(d*
x+c)^(5/2)+32/15*b^2*sin(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.10

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{-6d^2 + e^{2ia}\left(3d^2e^{2ibx} - 4be^{-\frac{2ibc}{d}}(c+dx)\left(e^{\frac{2ib(c+dx)}{d}}(-id+4b(c+dx)) - 4i\sqrt{2}d\left(-\frac{ib(\cdots}{}\right.\right.\right.}{}$$

input

```
Integrate[Sin[a + b*x]^2/(c + d*x)^(7/2),x]
```

output

```
(-6*d^2 + E^((2*I)*a)*(3*d^2*E^((2*I)*b*x) - (4*b*(c + d*x)*(E^(((2*I)*b*(
c + d*x))/d)*((-I)*d + 4*b*(c + d*x)) - (4*I)*Sqrt[2]*d*(((-I)*b*(c + d*x)
)/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d]))/E^(((2*I)*b*c)/d)) + (3*d^
2 + (2*I)*b*(c + d*x)*(-2*d + (8*I)*b*(c + d*x) - 8*Sqrt[2]*d*E^(((2*I)*b*
(c + d*x))/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d])
)/E^((2*I)*(a + b*x)))/(30*d^3*(c + d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3042, 3795, 17, 3042, 3794, 27, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2} + \frac{8b^2\int \frac{1}{(c+dx)^{3/2}}dx}{15d^2} - \frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 17$$

$$-\frac{16b^2\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{16b^2\int \frac{\sin(a+bx)^2}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3794$$

$$-\frac{16b^2\left(\frac{4b\int \frac{\sin(2a+2bx)}{2\sqrt{c+dx}}dx}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 27$$

$$-\frac{16b^2\left(\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{16b^2\left(\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3787$$

$$-\frac{16b^2\left(\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{16b^2\left(\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3785

$$-\frac{16b^2\left(\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3786

$$-\frac{16b^2\left(\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3832

$$-\frac{16b^2\left(\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 3833

$$-\frac{16b^2\left(\frac{2b\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (8*b*Cos[a + b*x]*Sin[a + b*x])/(15*d^2
*(c + d*x)^(3/2)) - (2*Sin[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) - (16*b^2*((2
*b*((Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c
+ d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/d -
(2*Sin[a + b*x]^2)/(d*Sqrt[c + d*x])))/(15*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2\cdots}{}\right.\right.}{}\right.}{}\right.}{5d}}{d}$
default	$\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2\cdots}{}\right.\right.}{}\right.}{}\right.}{5d}}{d}$

input
```
int(sin(b*x+a)^2/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/10/(d*x+c)^(5/2)+1/10/(d*x+c)^(5/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/
d)+2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+4/3*b/d*(-
1/(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2
)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+s
in(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.52

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{2\left(16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+16\left(\pi b^2d^3x\cdots\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2
*c^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqr
t(b/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x
+ pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-2*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2
*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x
+ b*c*d)*cos(b*x + a)*sin(b*x + a) - 3*d^2)*sqrt(d*x + c))/(d^6*x^3 + 3*c*
d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin^2(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**(7/2),x)
```

output
```
Integral(sin(a + b*x)**2/(c + d*x)**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.63

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{5\sqrt{2}\left(\left((i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{2i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i-1)\sqrt{2}\Gamma\right.\right.}{10\,(dx+c)^{\frac{5}{2}}d}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/10*(5*sqrt(2)*(((I + 1)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) - (I - 1
)*sqrt(2)*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I -
1)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-5/2, -2
*I*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) + 2)/((d*x
 + c)^(5/2)*d)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin{(bx+a)}^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin{(a+b\,x)}^2}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^(7/2),x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin(bx+a)^2}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(sin(b*x+a)^2/(d*x+c)^(7/2),x)
```

output
```
int(sin(a + b*x)**2/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sqr
t(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.52 $\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 247

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} + \frac{128b^{7/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{105d^{9/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\sin^2(a+bx)}{105d^3(c+dx)^{3/2}}$$

output
```
-16/105*b^2/d^3/(d*x+c)^(3/2)-128/105*b^(7/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*Fr
esnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/d^(9/2)+128/105*b^(7/2)*P
i^(1/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d
)/d^(9/2)-8/35*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(5/2)+128/105*b^3*cos(b
*x+a)*sin(b*x+a)/d^4/(d*x+c)^(1/2)-2/7*sin(b*x+a)^2/d/(d*x+c)^(7/2)+32/105
*b^2*sin(b*x+a)^2/d^3/(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.12

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{-30d^3+e^{2ia}\left(15d^3e^{2ibx}-4ib(c+dx)\left(-3d^2e^{2ibx}+4be^{-\frac{2ibc}{d}}(c+dx)\left(e^{\frac{2ib(c+dx)}{d}}(-id\right.\right.\right.}{}$$

input
```
Integrate[Sin[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(-30*d^3 + E^((2*I)*a)*(15*d^3*E^((2*I)*b*x) - (4*I)*b*(c + d*x)*(-3*d^2*E
^((2*I)*b*x) + (4*b*(c + d*x)*(E^(((2*I)*b*(c + d*x))/d)*((-I)*d + 4*b*(c
+ d*x)) - (4*I)*Sqrt[2]*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*
b*(c + d*x))/d]))/E^(((2*I)*b*c)/d))) + (15*d^3 + (4*I)*b*(c + d*x)*(-3*d^
2 - (2*I)*b*(c + d*x)*(-2*d + (8*I)*b*(c + d*x) - 8*Sqrt[2]*d*E^(((2*I)*b*
(c + d*x))/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d])
))/E^((2*I)*(a + b*x)))/(210*d^4*(c + d*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3795, 17, 3042, 3795, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(a+bx)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \; 3795$$

$$-\frac{16b^2\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b^2\int \frac{1}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{16b^2\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{16b^2\int \frac{\sin(a+bx)^2}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}\right)}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\sin(a+bx)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16b^2\left(-\frac{16b^2\left(-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\,\text{s}}{3d}\right.}{35d^2}$$

$$\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

input
```
Int[Sin[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/(35*
d^2*(c + d*x)^(5/2)) - (2*Sin[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) - (16*b^2*
((16*b^2*Sqrt[c + d*x])/(3*d^3) - (16*b^2*(Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos
[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d
]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/(3*d^2) - (8*b*Co
s[a + b*x]*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^2)/(3*d*(
c + d*x)^(3/2))))/(35*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{d}$
default	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}+\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}+\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{d}$

input
```
int(sin(b*x+a)^2/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/14/(d*x+c)^(7/2)+1/14/(d*x+c)^(7/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/
d)+2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+4/5*b/d*(-
1/3/(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/
2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d
-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c
)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(195) = 390$.

Time = 0.12 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.71

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{2\left(64\left(\pi b^3d^4x^4+4\,\pi b^3cd^3x^3+6\,\pi b^3c^2d^2x^2+4\,\pi b^3c^3dx+\pi b^3c^4\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4
*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel
_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3
*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b
^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 15*d^3 - (16*b^2*d^3*x^2 + 32*
b^2*c*d^2*x + 16*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 +
48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*co
s(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6
*x^2 + 4*c^3*d^5*x + c^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.55

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{7\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2}, \frac{2i(dx+c)b}{d}\right) + (i+1)\sqrt{2}\Gamma\left(-\frac{7}{2}, -\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad}{d}\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/7*(7*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I + 1)
*sqrt(2)*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1
)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-7/2, -2*
I*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(7/2) - 1)/((d*x
+ c)^(7/2)*d)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(c + d*x)^(9/2),x)
```

output
```
int(sin(a + b*x)^2/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sqrt{dx+c}\,c^4+4\sqrt{dx+c}\,c^3dx+6\sqrt{dx+c}\,c^2d^2x^2+4\sqrt{dx+c}\,c\,d^3x^3+\sqrt{dx+c}\,d^4x}$$

input
```
int(sin(b*x+a)^2/(d*x+c)^(9/2),x)
```

output
```
int(sin(a + b*x)**2/(sqrt(c + d*x)*c**4 + 4*sqrt(c + d*x)*c**3*d*x + 6*sqr
t(c + d*x)*c**2*d**2*x**2 + 4*sqrt(c + d*x)*c*d**3*x**3 + sqrt(c + d*x)*d*
*4*x**4),x)
```

3.53 $\int (c+dx)^{5/2}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 410

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \frac{45d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} - \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} + \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{3b^2} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}$$

output
```
45/16*d^2*(d*x+c)^(1/2)*cos(b*x+a)/b^3-2/3*(d*x+c)^(5/2)*cos(b*x+a)/b-5/14
4*d^2*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^3-45/32*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(
a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)+
5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*Fres
nelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7
/2)+45/32*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)+5/3*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2-
1/3*(d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2/b+5/18*d*(d*x+c)^(3/2)*sin(b*x+a
)^3/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.61

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{5/2}\left(243e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+243e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},\frac{ib(c\cdots}{d}\right.\right.}{648b\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/\cdots}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^(5/2)*(243*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Ga
mma[7/2, ((-I)*b*(c + d*x))/d] + 243*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I
)*b*(c + d*x))/d]*Gamma[7/2, (I*b*(c + d*x))/d] - Sqrt[3]*(E^((6*I)*a)*Sqr
t[(I*b*(c + d*x))/d]*Gamma[7/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((3*I)*b*(c + d*x))/d])))/(648*b*E
^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.41, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{5/2}\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin^3(a+bx)dx}{12b^2}+\frac{2}{3}\int (c+dx)^{5/2}\sin(a+bx)dx+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\int (c+dx)^{5/2}\sin(a+bx)dx+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\int (c+dx)^{3/2}\cos(a+bx)dx}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\int(c+dx)^{3/2}\sin\left(a+bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\left(\frac{3d\int-\sqrt{c+dx}\sin(a+bx)dx}{2b}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sin(a+bx)dx}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sin(a+bx)dx}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\cos(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3787}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}\text{co}}{b}\right. -\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/2}}{}\right. -\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\frac{(c+d}{}\right. -\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\right. -\frac{5d^2\int\sqrt{c+dx}\sin(a+bx)^3dx}{12b^2}+$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{5d^2\int\left(\frac{3}{4}\sqrt{c+dx}\sin(a+bx)-\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx}{12b^2}+\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\right.$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}-\right.$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-$$

$$\frac{5d^2\left(\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\ldots\right.}{12b^{3/2}}}{12b^2}$$

$$\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)}{2b}\right.$$

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-$$

$$\frac{5d^2\left(\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\cdots\right)}{12b^{3/2}}\right.}{12b^2}$$

$$\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3833

$$\frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2}-$$

$$\frac{5d^2\left(\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\right.}{12b^{3/2}}}{12b^2}$$

$$\frac{2}{3}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx}{b}\right.}{2b}\right.}{2b}\right.$$

$$\frac{(c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

input
```
Int[(c + d*x)^(5/2)*Sin[a + b*x]^3,x]
```

output
```
(-5*d^2*((-3*Sqrt[c + d*x]*Cos[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cos[3*a +
3*b*x])/(12*b) + (3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*
Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[
3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12
*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*Fres
nelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3
/2))))/(12*b^2) - ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (5
*d*(c + d*x)^(3/2)*Sin[a + b*x]^3)/(18*b^2) + (2*(-(((c + d*x)^(5/2)*Cos[a
 + b*x])/b) + (5*d*((-3*d*(-((Sqrt[c + d*x]*Cos[a + b*x])/b) + (d*((Sqrt[2
*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]
)/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/(2*b)))/(2*b) + ((c + d
*x)^(3/2)*Sin[a + b*x])/b))/(2*b)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.16

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{}}{}\right.}{}\right.}{4b}$
default	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{}}{}\right.}{}\right.}{4b}$

input
```
int((d*x+c)^(5/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/8/b*d*(d*x+c)^(5/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+15/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/2
)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/
24/b*d*(d*x+c)^(5/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-5/24/b*d*(1/6/b*d*(d
*x+c)^(3/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.90

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 1215\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx\right.}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 12
15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin
(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*((12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^3 - 3*(12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2)*cos(b*x + a) + 10*(7*b^2*
d^2*x + 7*b^2*c*d - (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sq
rt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.33

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) - 6480
*(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(
5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) + 6
48*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b
*c + a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(
1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 1215*((I -
1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sq
rt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(I*b/d)) - 1215*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos
(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(-3*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 2453, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/1728*(72*(9*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 - d^3*(81*(sqrt(2)*sqrt(p
i)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*
b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*
e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - (sqrt(6)*sqrt(pi)*(72*b^
3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*I*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*I*(-12*I*(d*x + c)^(5/2)*b^2*d
 + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x +
 c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx=\int \sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2}\,dx$$

input
```
int(sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\sin(bx+a)^3\,x^2dx\right)d^2 + 2\left(\int \sqrt{dx+c}\,\sin(bx+a)^3\,xdx\right)cd + \left(\int \sqrt{dx+c}\,\sin(bx+a)^3\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*sin(a
 + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*sin(a + b*x)**3,x)*c**2
```

3.54 $\int (c+dx)^{3/2} \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 354

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx=-\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}-\frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}}-\frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}}+\frac{d\sqrt{c+dx}\sin(a+bx)}{b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}$$

output
```
-2/3*(d*x+c)^(3/2)*cos(b*x+a)/b-9/16*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)
*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/144*d^
(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(
1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(5/2)-9/16
*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*sin(a-b*c/d)/b^(5/2)+d*(d*x+c)^(1/2)*sin(b*x+a)/b^2-1/3*(d*x+c)^(
3/2)*cos(b*x+a)*sin(b*x+a)^2/b+1/6*d*(d*x+c)^(1/2)*sin(b*x+a)^3/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.72

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx = \frac{ie^{-\frac{3i(bc+ad)}{d}}(c+dx)^{5/2}\left(-81e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)+81e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c\cdots\right.\right.}{216d\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/\cdots}}$$

input
```
Integrate[(c + d*x)^(3/2)*Sin[a + b*x]^3,x]
```

output
```
((I/216)*(c + d*x)^(5/2)*(-81*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x
))/d]*Gamma[5/2, ((-I)*b*(c + d*x))/d] + 81*E^((2*I)*a + ((4*I)*b*c)/d)*Sq
rt[((-I)*b*(c + d*x))/d]*Gamma[5/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)
*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[5/2, ((-3*I)*b*(c + d*x))/d] - E^(((6*I)
*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((3*I)*b*(c + d*x))/d])))/(
d*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^{3/2}\sin(a+bx)^3dx$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{d^2\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin(a+bx)dx+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin(a+bx)dx+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\int \sqrt{c+dx}\cos(a+bx)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{d\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{d^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)+$$

$$-\frac{d^2\int\frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3785

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)-$$

$$-\frac{d^2\int\frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3786

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+b$$

$$-\frac{d^2\int\frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3793

$$-\frac{d^2\int\left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}}-\frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{12b^2}+$$

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+b\right.$$

$$\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 2009

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+b\right.$$

$$\frac{d^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+\right.}{2\sqrt{b}\sqrt{d}}\right.}{12b^2}$$

$$\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3832

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a}{b}\right.$$

$$\frac{d^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right.}{2\sqrt{b}\sqrt{d}}}{12b^2}$$

$$\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}-\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

$\downarrow$ 3833

$$-\frac{d^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{}}{\sqrt{d}}\right.}{2\sqrt{b}\sqrt{d}}}{12b^2}$$

$$\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}+$$

$$\frac{2}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\text{co}}{b}\right.$$

$$\frac{(c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)}{3b}$$

input
```
Int[(c + d*x)^(3/2)*Sin[a + b*x]^3,x]
```

output
```
-1/12*(d^2*((3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)
/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[
d]) - (Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin
[3*a - (3*b*c)/d])/(2*Sqrt[b]*Sqrt[d]) + (3*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/
b^2 - ((c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sqrt[c + d*
x]*Sin[a + b*x]^3)/(6*b^2) + (2*(-(((c + d*x)^(3/2)*Cos[a + b*x])/b) + (3*
d*(-1/2*(d*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/b +
(Sqrt[c + d*x]*Sin[a + b*x])/b))/(2*b)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

input
```
int((d*x+c)^(3/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/8/b*d*(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+9/8/b*d*(1/2/b*d*
(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))+1/24/b*d*(d*x+c)^(3/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/8/b*d*(
1/6/b*d*(d*x+c)^(1/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi
^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.85

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx = \frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-81\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(
-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqr
t(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)
))*sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^
2*c)*cos(b*x + a)^3 - 6*(b^2*d*x + b^2*c)*cos(b*x + a) - (b*d*cos(b*x + a)
^2 - 7*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d - 432*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b
^2*sin(3*((d*x + c)*b - b*c + a*d)/d) + 648*sqrt(d*x + c)*b^2*sin(((d*x +
c)*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 81*(-(I +
 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + 81*((I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*
b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d
)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 1521, normalized size of antiderivative = 4.30

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/288*(12*(9*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 - 4*(27*sqrt(2)*sqrt(pi)*(
2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
) - sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 27*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b
*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b) + 54*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d
*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx=\int \sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2}\,dx$$

input
```
int(sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\sin(bx+a)^3\,xdx\right)d + \left(\int \sqrt{dx+c}\,\sin(bx+a)^3\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*x)*sin(a + b*x)*
*3,x)*c
```

3.55 $\int \sqrt{c+dx}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 304

$$\begin{aligned}\int \sqrt{c+dx}\sin^3(a+bx)\,dx = & -\frac{3\sqrt{c+dx}\cos(a+bx)}{4b}+\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} \\ & +\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \\ & -\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} \\ & +\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} \\ & -\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}}\end{aligned}$$

output
```
-3/4*(d*x+c)^(1/2)*cos(b*x+a)/b+1/12*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b+3/8*d^
(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))/b^(3/2)-1/72*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*
FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/72*d^(1
/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*sin(3*a-3*b*c/d)/b^(3/2)-3/8*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{3i(bc+ad)}{d}}\sqrt{c+dx}\left(-27e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)-27e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\right.}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-27*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamm
a[3/2, ((-I)*b*(c + d*x))/d] - 27*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I)*b
*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[(
I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*S
qrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(72*b*E^(((
3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+dx}\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{3}{4}\sqrt{c+dx}\sin(a+bx)-\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{c+dx}\cos(a+bx)}{4b}+\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
(-3*Sqrt[c + d*x]*Cos[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(
12*b) + (3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*
b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2))
 + (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]
]*Sin[3*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}+\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}+$
default	$-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}+\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}+$

input
```
int((d*x+c)^(1/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/8/b*d*(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+3/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))+1/24/b*d*(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c
)/d)-1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx =$$

$$-\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*
c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)
*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-(b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^3 - 3*b*
cos(b*x + a))*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx$$
$$= \frac{\left(\frac{24\sqrt{dx+c}b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{216\sqrt{dx+c}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d - 216*sqr
t(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(3*I*b/d)) - 27*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(I*b/d)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)
^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-3*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/144*(27*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2
*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*sqrt(2)*sqrt(p
i)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b) - sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*(9*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*I
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*
erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(
pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c +
54*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x +
c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx=\int \sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\,\sin(bx+a)^3\,dx$$

input
```
int((d*x+c)^(1/2)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*sin(a + b*x)**3,x)
```

3.56 $\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 257

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{2\sqrt{b}\sqrt{d}} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
3/4*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)-1/12*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*Fr
esnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)-1/1
2*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*sin(3*a-3*b*c/d)/b^(1/2)/d^(1/2)+3/4*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{e^{-\frac{3i(bc+ad)}{d}}\left(-9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)-9e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{6ia}\sqrt{-\frac{ib}{}}\right.\right.}{24b\sqrt{c+dx}}$$

input

```
Integrate[Sin[a + b*x]^3/Sqrt[c + d*x],x]
```

output

```
(-9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*
b*(c + d*x))/d] - 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Ga
mma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[((-I)*b*(c + d*x))
/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d]))/(24*b*E^(((3*I)*(b*c + a*d))/
d)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 3793$$

$$\int \left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}}-\frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

input
```
Int[Sin[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
(3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])
/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS
[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[
Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(2*Sqrt[b]*Sqrt[d]) + (3*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(\cos\left(\frac{3ad-3bc}{d}\right)\mathrm{FresnelS}\left(\right.\right.}{}}{d}$
default	$\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(\cos\left(\frac{3ad-3bc}{d}\right)\mathrm{FresnelS}\left(\right.\right.}{}}{d}$

input
```
int(sin(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/8*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx =$$

$$-\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{}}{12\,b}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a
*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sq
rt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c -
 a*d)/d) + sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqr
t(b/(pi*d)))*sin(-3*(b*c - a*d)/d))/b
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin^3{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c)**(1/2),x)
```

output
```
Integral(sin(a + b*x)**3/sqrt(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.47

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\left(\left(-\frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)}{d}+\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)}{d}\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-9\left(-\frac{(i+\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/48*((-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a
*d)/d)/d + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c
- a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((
I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c
)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(
-3*(b*c - a*d)/d)/d - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*s
in(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.29

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{9\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)} - \frac{\sqrt{6}\sqrt{\pi}d\operatorname{erf}\left(-\frac{i\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)} + \frac{9\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\cdots\right.}{\cdots}}{24d}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*(9*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin(a+b\,x)^3}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{dx+c}}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
int(sin(a + b*x)**3/sqrt(c + d*x),x)
```

3.57 $\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 270

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{3/2}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}} - \frac{2\sin^3(a+bx)}{d\sqrt{c+dx}}$$

output
```
3/2*b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)-1/2*b^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*
c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+1/2*
b^(1/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*sin(3*a-3*b*c/d)/d^(3/2)-3/2*b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/d^(3/2)-2*sin(b*
x+a)^3/d/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.09

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{ie^{-3ia}\left(-e^{-3ibx}+3e^{2ia-ibx}+e^{3i(2a+bx)}-3e^{i(4a+bx)}+3e^{4ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)-3e^{i\left(2a+\frac{bc}{d}\right)}\sqrt{\cdots}\right.}{4d\sqrt{c+\cdots}}$$

input

```
Integrate[Sin[a + b*x]^3/(c + d*x)^(3/2),x]
```

output

```
((-1/4*I)*(-E^((-3*I)*b*x) + 3*E^((2*I)*a - I*b*x) + E^((3*I)*(2*a + b*x))
 - 3*E^(I*(4*a + b*x)) + 3*E^((4*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))
/d]*Gamma[1/2, ((-I)*b*(c + d*x))/d] - 3*E^(I*(2*a + (b*c)/d))*Sqrt[(I*b*(
c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d] - Sqrt[3]*E^((6*I)*a - ((3*I)*b
*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] + Sqr
t[3]*E^(((3*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*
x))/d]))/(d*E^((3*I)*a)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \; 3794$$

$$\frac{6b\int \left(\frac{\cos(a+bx)}{4\sqrt{c+dx}}-\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}-\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 2009$$

$$\frac{6b\left(\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{d}$$

$$\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[Sin[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(6*b*((Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (S
qrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/d - (2*Sin
[a + b*x]^3)/(d*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2\sqrt{dx+c}}+\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2d\sqrt{\frac{b}{d}}}+\frac{\sin\left(\frac{3b(}{}\right.}{}}{d}$
default	$\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2\sqrt{dx+c}}+\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2d\sqrt{\frac{b}{d}}}+\frac{\sin\left(\frac{3b(}{}\right.}{}}{d}$

input
```
int(sin(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/4/(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+3/4*b/d*2^(1/2)*Pi^(1
/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d))+1/4/(d*x+c)^(1/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/4*b
/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*Fres
nelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.01

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{\sqrt{6}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\,\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{c\right.}}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel
_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*d*x + pi*c)*sqr
t(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - sqrt(6)*(pi*d*x + pi*c)*s
qrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*
c - a*d)/d) - 4*sqrt(d*x + c)*(cos(b*x + a)^2 - 1)*sin(b*x + a))/(d^2*x +
c*d)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin^3{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c)**(3/2),x)
```

output
```
Integral(sin(a + b*x)**3/(c + d*x)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.94

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{3}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/16*(sqrt(3)*(((I - 1)*sqrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I + 1)*s
qrt(2)*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1)*s
qrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -3*I*(
d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 3*(((I - 1)*sq
rt(2)*gamma(-1/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x +
 c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b
/d) - (I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*
sqrt((d*x + c)*b/d))/(sqrt(d*x + c)*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^3/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c)^(3/2),x)
```

output
```
int(sin(a + b*x)**3/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.58 $\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 292

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}} \, dx = & -\frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} \\ & + \frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} \\ & + \frac{b^{3/2}\sqrt{6\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{5/2}} \\ & - \frac{b^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} \\ & - \frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}\end{aligned}$$

output
```
-b^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))/d^(5/2)+b^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*Fr
esnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+b^(3/2)*6^(
1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin
(3*a-3*b*c/d)/d^(5/2)-b^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/d^(5/2)-4*b*cos(b*x+a)*sin(b*x+
a)^2/d^2/(d*x+c)^(1/2)-2/3*sin(b*x+a)^3/d/(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.28

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-3ia}\left(6e^{4ia-\frac{ibc}{d}}\left(e^{\frac{ib(c+dx)}{d}}(id-2b(c+dx))+2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)+3i\right.}{}$$

input
```
Integrate[Sin[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(6*E^((4*I)*a - (I*b*c)/d)*(E^((I*b*(c + d*x))/d)*(I*d - 2*b*(c + d*x)) +
(2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)*b*(c + d*x))/d]) + (
3*I)*E^((2*I)*a - I*b*x)*(-2*d + (4*I)*b*(c + d*x) - 4*d*E^((I*b*(c + d*x)
)/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d]) + 2*E^((6*I)
*a - ((3*I)*b*c)/d)*(E^(((3*I)*b*(c + d*x))/d)*((-I)*d + 6*b*(c + d*x)) -
(6*I)*Sqrt[3]*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(c + d*x
))/d]) + (2*(I*d + 6*b*(c + d*x) + (6*I)*Sqrt[3]*d*E^(((3*I)*b*(c + d*x))/
d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d]))/E^((3*I)*
b*x))/(24*d^2*E^((3*I)*a)*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.52, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 3795, 3042, 3787, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{12b^2\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8b^2\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{12b^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{12b^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{12b^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{12b^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 3786

$$-\frac{12b^2\int \frac{\sin(a+bx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 3793

$$-\frac{12b^2\int\left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}}-\frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2}+\frac{8b^2\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{8b^2\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d^2}-\frac{12b^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 3832

$$\frac{8b^2\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d^2}-$$

$$\frac{12b^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{d^2}$$

$$\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{12b^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{d^2}$$

$$\frac{8b^2\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d^2}-$$

$$\frac{4b\sin^2(a+bx)\cos(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Int[Sin[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(-12*b^2*((3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]
) - (Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(2*Sqrt[b]*Sqrt[d]) + (3*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/d^
2 + (8*b^2*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/d^2
- (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^
3)/(3*d*(c + d*x)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}}+\frac{b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\cdots\right.}{d\sqrt{\frac{b}{d}}}\right.}{d}\cdots}{\cdots}$
default	$\frac{-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}}+\frac{b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\operatorname{FresnelC}\cdots\right.}{d\sqrt{\frac{b}{d}}}\right.}{d}\cdots}{\cdots}$

input
```
int(sin(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/4/(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+1/2*b/d*(-1/(d*x+c)^(
1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/12
/(d*x+c)^(3/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*
cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.33

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{3\sqrt{6}(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}(\pi b}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*s
qrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c -
 a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi
*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*s
qrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*b*d^2*x^2
 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(6*(b*d*x + b*c)*cos(b*x + a)
^3 - 6*(b*d*x + b*c)*cos(b*x + a) + (d*cos(b*x + a)^2 - d)*sin(b*x + a))*s
qrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin^3{(a+bx)}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c)**(5/2),x)
```

output
```
Integral(sin(a + b*x)**3/(c + d*x)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.87

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{3\left(\sqrt{3}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{3i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-}{d}\right.\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
3/16*(sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) + (I - 1)*
sqrt(2)*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)*
sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-3/2, -3*I*
(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - ((-(I + 1)*
sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x
 + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)
*b/d) - (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d)
)*((d*x + c)*b/d)^(3/2))/((d*x + c)^(3/2)*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^3/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
int(sin(a + b*x)**3/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c +
 d*x)*d**2*x**2),x)
```

3.59 $\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 356

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx =& -\frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+ \frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &- \frac{6b^{5/2}\sqrt{6\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{5d^{7/2}} \\ &+ \frac{2b^{5/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{5d^{7/2}} - \frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} \\ &- \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}}\end{aligned}$$

output
```
-2/5*b^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+6/5*b^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b
*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)-6/5
*b^(5/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*sin(3*a-3*b*c/d)/d^(7/2)+2/5*b^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^
(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/d^(7/2)-16/5*b^
2*sin(b*x+a)/d^3/(d*x+c)^(1/2)-4/5*b*cos(b*x+a)*sin(b*x+a)^2/d^2/(d*x+c)^(
3/2)-2/5*sin(b*x+a)^3/d/(d*x+c)^(5/2)+24/5*b^2*sin(b*x+a)^3/d^3/(d*x+c)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.29

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{i\left(2e^{ia}\left(-3d^2e^{ibx}+2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)\right)\right)}{}$$

input
```
Integrate[Sin[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
((-1/40*I)*(2*E^(I*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x
))/d)*((-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamm
a[1/2, ((-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) - 2*E^((3*I)*a)*(-(d^2*E^((3
*I)*b*x)) + (2*b*(c + d*x)*(E^(((3*I)*b*(c + d*x))/d)*((-I)*d + 6*b*(c + d
*x)) - (6*I)*Sqrt[3]*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(
c + d*x))/d]))/E^(((3*I)*b*c)/d)) + (2*(-d^2 - I*b*(c + d*x)*(-2*d + (12*I
)*b*(c + d*x) - 12*Sqrt[3]*d*E^(((3*I)*b*(c + d*x))/d)*((I*b*(c + d*x))/d)
^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d])))/E^((3*I)*(a + b*x)) - (-6*d^2
+ (4*I)*b*d*(c + d*x) + 8*b^2*(c + d*x)^2 + 8*d^2*((I*b*(c + d*x))/d)^(5/2
)*Gamma[1/2, (I*b*(c + d*x))/d]*(Cos[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(
Cos[a + b*x] - I*Sin[a + b*x])))/(d^3*(c + d*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3042, 3795, 3042, 3778, 3042, 3787, 3042, 3785, 3786, 3794, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{12b^2\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8b^2\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{12b^2\int \frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{12b^2\int \frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{12b^2\int \frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3787}$$

$$-\frac{12b^2\int\frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{12b^2\int\frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{12b^2\int\frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{12b^2\int\frac{\sin(a+bx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 3794$$

$$-\frac{12b^2\left(\frac{6b\int\left(\frac{\cos(a+bx)}{4\sqrt{c+dx}}-\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}-\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{8b^2\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{8b^2\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{12b^2\left(\frac{6b\left(\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\cdots\right)}{d}\cdots\right)}{5d^2}-\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 3832$$

$$\frac{8b^2\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-$$

$$\frac{12b^2\left(\frac{6b\left(\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\sqrt{}\right.}{d}\right.}{5d^2}$$

$$\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

$\Big\downarrow$ 3833

$$-\frac{12b^2\left(\frac{6b\left(\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\right.}{d}\right.}{5d^2}$$

$$\frac{8b^2\left(\frac{2b\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-$$

$$\frac{4b\sin^2(a+bx)\cos(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}}$$

input
```
Int[Sin[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
(-4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(5*d^2*(c + d*x)^(3/2)) - (2*Sin[a + b*
x]^3)/(5*d*(c + d*x)^(5/2)) + (8*b^2*((2*b*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*F
resnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (
Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b
*c)/d])/(Sqrt[b]*Sqrt[d])))/d - (2*Sin[a + b*x])/(d*Sqrt[c + d*x])))/(5*d^
2) - (12*b^2*((6*b*((Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (
3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]
*Sqrt[d]) + (Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d
]]*Sin[3*a - (3*b*c)/d])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/2]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d]
)))/d - (2*Sin[a + b*x]^3)/(d*Sqrt[c + d*x])))/(5*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}+\frac{3b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{\cdots}\right.\right.}{\cdots}\right)}{3d}\right)}{5d}}{\cdots}$
default	$\frac{-\frac{3\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}+\frac{3b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{\cdots}\right.\right.}{\cdots}\right)}{3d}\right)}{5d}}{\cdots}$

input

```
int(sin(b*x+a)^3/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
2/d*(-3/20/(d*x+c)^(5/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+3/10*b/d*(-1/3/(d*x+
c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(b*(d*x
+c)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*Fres
nelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/20/(d*x+c)^(5/2)
*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*cos(3*b*(d*
x+c)/d+3*(a*d-b*c)/d)-2*b/d*(-1/(d*x+c)^(1/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c
)/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d
)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.54

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(3\sqrt{6}(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx\right.}\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d
*x + c)*sqrt(b/(pi*d))) - sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3
*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_c
os(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*
b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_sin(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*
b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(
pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + (2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 2*(b*d^2*x + b*c*d)*cos(b*x +
 a) + (4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - (12*b^2*d^2*x^2 + 24*b^2*
c*d*x + 12*b^2*c^2 - d^2)*cos(b*x + a)^2 - d^2)*sin(b*x + a))*sqrt(d*x + c
))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin^3{(a+bx)}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**3/(d*x+c)**(7/2),x)
```

output
```
Integral(sin(a + b*x)**3/(c + d*x)**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{3\left(3\sqrt{3}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i+1)\sqrt{2}\right.\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-3/16*(3*sqrt(3)*(((I - 1)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I + 1
)*sqrt(2)*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1
)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -3*
I*(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - (((I - 1)
*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*
x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-5/2, I*(d*x + c
)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d
))*((d*x + c)*b/d)^(5/2))/((d*x + c)^(5/2)*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin(bx+a)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(c + d*x)^(7/2),x)
```

output
```
int(sin(a + b*x)^3/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(sin(b*x+a)^3/(d*x+c)^(7/2),x)
```

output
```
int(sin(a + b*x)**3/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sqr
t(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.60 $\int (dx)^{3/2} \sin(fx)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int (dx)^{3/2}\sin(fx)\,dx = -\frac{(dx)^{3/2}\cos(fx)}{f} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}} + \frac{3d\sqrt{dx}\sin(fx)}{2f^2}$$

output
```
-(d*x)^(3/2)*cos(f*x)/f-3/4*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(f^(1/2)*2^(1
/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))/f^(5/2)+3/2*d*(d*x)^(1/2)*sin(f*x)/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int (dx)^{3/2}\sin(fx)\,dx = \frac{d^2\left(\sqrt{-ifx}\,\Gamma\left(\frac{5}{2},-ifx\right)+\sqrt{ifx}\,\Gamma\left(\frac{5}{2},ifx\right)\right)}{2f^3\sqrt{dx}}$$

input
```
Integrate[(d*x)^(3/2)*Sin[f*x],x]
```

output
```
(d^2*(Sqrt[(-I)*f*x]*Gamma[5/2, (-I)*f*x] + Sqrt[I*f*x]*Gamma[5/2, I*f*x])
)/(2*f^3*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\sin(fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (dx)^{3/2}\sin(fx)dx$$

$$\Big\downarrow \; 3777$$

$$\frac{3d\int \sqrt{dx}\cos(fx)dx}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

$$\Big\downarrow \; 3042$$

$$\frac{3d\int \sqrt{dx}\sin\left(fx+\frac{\pi}{2}\right)dx}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

$$\Big\downarrow \; 3777$$

$$\frac{3d\left(\frac{d\int -\frac{\sin(fx)}{\sqrt{dx}}dx}{2f}+\frac{\sqrt{dx}\sin(fx)}{f}\right)}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{3d\left(\frac{\sqrt{dx}\sin(fx)}{f}-\frac{d\int \frac{\sin(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

$$\Big\downarrow \; 3042$$

$$\frac{3d\left(\frac{\sqrt{dx}\sin(fx)}{f}-\frac{d\int\frac{\sin(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

$$\Big\downarrow \textbf{3786}$$

$$\frac{3d\left(\frac{\sqrt{dx}\sin(fx)}{f}-\frac{\int\sin(fx)d\sqrt{dx}}{f}\right)}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

$$\Big\downarrow \textbf{3832}$$

$$\frac{3d\left(\frac{\sqrt{dx}\sin(fx)}{f}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\,\text{FresnelS}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}\right)}{2f}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

input
```
Int[(d*x)^(3/2)*Sin[f*x],x]
```

output
```
-(((d*x)^(3/2)*Cos[f*x])/f) + (3*d*(-((Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[f
]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/f^(3/2)) + (Sqrt[d*x]*Sin[f*x])/f))/(2*f
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{2(dx)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\left(-\frac{x^{\frac{3}{2}}\sqrt{2}\,f^{\frac{3}{2}}\cos(fx)}{4\sqrt{\pi}}+\frac{3\sqrt{x}\,\sqrt{2}\,\sqrt{f}\,\sin(fx)}{8\sqrt{\pi}}-\frac{3\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)}{8}\right)}{x^{\frac{3}{2}}f^{\frac{5}{2}}}$	73
derivativedivides	$\frac{-\frac{d(dx)^{\frac{3}{2}}\cos(fx)}{f}+\frac{3d\left(\frac{d\sqrt{dx}\,\sin(fx)}{2f}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{4f\sqrt{\frac{f}{d}}}\right)}{f}}{d}$	87
default	$\frac{-\frac{d(dx)^{\frac{3}{2}}\cos(fx)}{f}+\frac{3d\left(\frac{d\sqrt{dx}\,\sin(fx)}{2f}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{4f\sqrt{\frac{f}{d}}}\right)}{f}}{d}$	87

input
```
int((d*x)^(3/2)*sin(f*x),x,method=_RETURNVERBOSE)
```

output
```
2*(d*x)^(3/2)/x^(3/2)*2^(1/2)/f^(5/2)*Pi^(1/2)*(-1/4/Pi^(1/2)*x^(3/2)*2^(1
/2)*f^(3/2)*cos(f*x)+3/8/Pi^(1/2)*x^(1/2)*2^(1/2)*f^(1/2)*sin(f*x)-3/8*Fre
snelS(1/Pi^(1/2)*2^(1/2)*x^(1/2)*f^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int (dx)^{3/2}\sin(fx)\,dx = -\frac{3\sqrt{2}\pi d^2\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)+2\left(2\,df^2x\cos\left(fx\right)-3\,df\sin\left(fx\right)\right)\sqrt{dx}}{4\,f^3}$$

input
```
integrate((d*x)^(3/2)*sin(f*x),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt(2)*pi*d^2*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqrt(f
/(pi*d))) + 2*(2*d*f^2*x*cos(f*x) - 3*d*f*sin(f*x))*sqrt(d*x))/f^3
```

Sympy [A] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int (dx)^{3/2}\sin(fx)\,dx = -\frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cos\left(fx\right)\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)} + \frac{21d^{\frac{3}{2}}\sqrt{x}\sin\left(fx\right)\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)} - \frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((d*x)**(3/2)*sin(f*x),x)
```

output
```
-7*d**(3/2)*x**(3/2)*cos(f*x)*gamma(7/4)/(4*f*gamma(11/4)) + 21*d**(3/2)*s
qrt(x)*sin(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) - 21*sqrt(2)*sqrt(pi)*d**(
3/2)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(7/4)/(16*f**(5/2)*ga
mma(11/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int (dx)^{3/2}\sin(fx)\,dx =$$

$$-\frac{\sqrt{2}\left(8\sqrt{2}(dx)^{\frac{3}{2}}f^2\cos(fx)-12\sqrt{2}\sqrt{dx}df\sin(fx)+(3i+3)\sqrt{\pi}d^2\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{if}{d}}\right)-(3i-3)\sqrt{}\right.}{16f^3}$$

input
```
integrate((d*x)^(3/2)*sin(f*x),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*(8*sqrt(2)*(d*x)^(3/2)*f^2*cos(f*x) - 12*sqrt(2)*sqrt(d*x)*d
*f*sin(f*x) + (3*I + 3)*sqrt(pi)*d^2*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(I*
f/d)) - (3*I - 3)*sqrt(pi)*d^2*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(-I*f/d))
)/f^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.44

$$\int (dx)^{3/2}\sin(fx)\,dx =$$

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{df}\sqrt{dx}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)f^2}+\frac{3\sqrt{2}\sqrt{\pi}d^3\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{df}\sqrt{dx}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)f^2}-\frac{2i\left(2i\sqrt{dx}d^2fx-3\sqrt{dx}d^2\right)e^{(ifx)}}{f^2}-\frac{2i\left(2i\right.}{}}{8d}$$

input
```
integrate((d*x)^(3/2)*sin(f*x),x, algorithm="giac")
```

output
```
-1/8*(3*sqrt(2)*sqrt(pi)*d^3*erf(-1/2*I*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f
/sqrt(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)*f^2) + 3*sqrt(
2)*sqrt(pi)*d^3*erf(1/2*I*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2
) + 1)/d)/(sqrt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)*f^2) - 2*I*(2*I*sqrt(d*x)*
d^2*f*x - 3*sqrt(d*x)*d^2)*e^(I*f*x)/f^2 - 2*I*(2*I*sqrt(d*x)*d^2*f*x + 3*
sqrt(d*x)*d^2)*e^(-I*f*x)/f^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sin(fx)\,dx = \int \sin(f\,x)\,(d\,x)^{3/2}\,dx$$

input
```
int(sin(f*x)*(d*x)^(3/2),x)
```

output
```
int(sin(f*x)*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}\sin(fx)\,dx = \frac{\sqrt{d}\,d\left(-4\sqrt{x}\,\cos(fx)\,fx+6\sqrt{x}\,\sin(fx)-3\left(\int \frac{\sin(fx)}{\sqrt{x}}dx\right)\right)}{4f^2}$$

input
```
int((d*x)^(3/2)*sin(f*x),x)
```

output
```
(sqrt(d)*d*( - 4*sqrt(x)*cos(f*x)*f*x + 6*sqrt(x)*sin(f*x) - 3*int(sin(f*x
)/sqrt(x),x)))/(4*f**2)
```

3.61 $\int \sqrt{dx}\sin(fx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \sqrt{dx}\sin(fx)\,dx=-\frac{\sqrt{dx}\cos(fx)}{f}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}$$

output
```
-(d*x)^(1/2)*cos(f*x)/f+1/2*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(f^(1/2)*2^(1
/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))/f^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \sqrt{dx}\sin(fx)\,dx=-\frac{id\left(\sqrt{-ifx}\Gamma\left(\frac{3}{2},-ifx\right)-\sqrt{ifx}\Gamma\left(\frac{3}{2},ifx\right)\right)}{2f^2\sqrt{dx}}$$

input
```
Integrate[Sqrt[d*x]*Sin[f*x],x]
```

output
```
((-1/2*I)*d*(Sqrt[(-I)*f*x]*Gamma[3/2, (-I)*f*x] - Sqrt[I*f*x]*Gamma[3/2,
I*f*x]))/(f^2*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3777, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\sin(fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{dx}\sin(fx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{d\int \frac{\cos(fx)}{\sqrt{dx}}dx}{2f}-\frac{\sqrt{dx}\cos(fx)}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{\sin\left(fx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{2f}-\frac{\sqrt{dx}\cos(fx)}{f}$$

$$\Big\downarrow \text{3785}$$

$$\frac{\int \cos(fx)d\sqrt{dx}}{f}-\frac{\sqrt{dx}\cos(fx)}{f}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\,\text{FresnelC}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}-\frac{\sqrt{dx}\cos(fx)}{f}$$

input
```
Int[Sqrt[d*x]*Sin[f*x],x]
```

output
```
-((Sqrt[d*x]*Cos[f*x])/f) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[f]*Sqrt[2/P
i]*Sqrt[d*x])/Sqrt[d]])/f^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
meijerg	$\frac{\sqrt{dx}\,\sqrt{2}\,\sqrt{\pi}\,\left(-\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{f}\,\cos(fx)}{2\sqrt{\pi}}+\frac{\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)}{2}\right)}{\sqrt{x}\,f^{\frac{3}{2}}}$	54
derivativedivides	$\frac{-\frac{d\sqrt{dx}\,\cos(fx)}{f}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{2f\sqrt{\frac{f}{d}}}}{d}$	65
default	$\frac{-\frac{d\sqrt{dx}\,\cos(fx)}{f}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{2f\sqrt{\frac{f}{d}}}}{d}$	65

input
```
int((d*x)^(1/2)*sin(f*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^(1/2)/x^(1/2)*2^(1/2)/f^(3/2)*Pi^(1/2)*(-1/2/Pi^(1/2)*x^(1/2)*2^(1/2
)*f^(1/2)*cos(f*x)+1/2*FresnelC(1/Pi^(1/2)*2^(1/2)*x^(1/2)*f^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \sqrt{dx}\sin(fx)\,dx = \frac{\sqrt{2}\pi d\sqrt{\frac{f}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right) - 2\sqrt{dx}f\cos(fx)}{2f^2}$$

input
```
integrate((d*x)^(1/2)*sin(f*x),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*pi*d*sqrt(f/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*
d))) - 2*sqrt(d*x)*f*cos(f*x))/f^2
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \sqrt{dx}\sin(fx)\,dx = -\frac{5\sqrt{d}\sqrt{x}\cos(fx)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)} + \frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x)**(1/2)*sin(f*x),x)
```

output
```
-5*sqrt(d)*sqrt(x)*cos(f*x)*gamma(5/4)/(4*f*gamma(9/4)) + 5*sqrt(2)*sqrt(p
i)*sqrt(d)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(5/4)/(8*f**(3/
2)*gamma(9/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \sqrt{dx}\sin(fx)\,dx =$$

$$-\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{dx}f\cos\left(fx\right)+(i-1)\sqrt{\pi}d\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{if}{d}}\right)-(i+1)\sqrt{\pi}d\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{if}{d}}\right.\right.}{8f^2}$$

input
```
integrate((d*x)^(1/2)*sin(f*x),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x)*f*cos(f*x) + (I - 1)*sqrt(pi)*d*(f^2/d^2
)^(1/4)*erf(sqrt(d*x)*sqrt(I*f/d)) - (I + 1)*sqrt(pi)*d*(f^2/d^2)^(1/4)*er
f(sqrt(d*x)*sqrt(-I*f/d)))/f^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.74

$$\int \sqrt{dx}\sin(fx)\,dx =$$

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{df}\sqrt{dx}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)f}+\frac{i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{df}\sqrt{dx}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)f}+\frac{2\sqrt{dx}de^{(ifx)}}{f}+\frac{2\sqrt{dx}de^{(-ifx)}}{f}}{4d}$$

input
```
integrate((d*x)^(1/2)*sin(f*x),x, algorithm="giac")
```

output
```
-1/4*(-I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*I*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*
f/sqrt(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)*f) + I*sqrt(2
)*sqrt(pi)*d^2*erf(1/2*I*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2)
 + 1)/d)/(sqrt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)*f) + 2*sqrt(d*x)*d*e^(I*f*x
)/f + 2*sqrt(d*x)*d*e^(-I*f*x)/f)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\sin(fx)\,dx = \int \sin(f\,x)\,\sqrt{d\,x}\,dx$$

input
```
int(sin(f*x)*(d*x)^(1/2),x)
```

output
```
int(sin(f*x)*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}\sin(fx)\,dx = \sqrt{d}\left(\int \sqrt{x}\,\sin(fx)\,dx\right)$$

input
```
int((d*x)^(1/2)*sin(f*x),x)
```

output
```
sqrt(d)*int(sqrt(x)*sin(f*x),x)
```

3.62 $\int \frac{\sin(fx)}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{f}}$$

output
```
2^(1/2)*Pi^(1/2)*FresnelS(f^(1/2)*2^(1/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))/d^
(1/2)/f^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{-\sqrt{-ifx}\,\Gamma\left(\frac{1}{2},-ifx\right)-\sqrt{ifx}\,\Gamma\left(\frac{1}{2},ifx\right)}{2f\sqrt{dx}}$$

input
```
Integrate[Sin[f*x]/Sqrt[d*x],x]
```

output
```
(-(Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x]) - Sqrt[I*f*x]*Gamma[1/2, I*f*x])/(
2*f*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(fx)}{\sqrt{dx}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(fx)}{\sqrt{dx}}dx$$
$$\Big\downarrow \text{3786}$$
$$\frac{2\int \sin(fx)d\sqrt{dx}}{d}$$
$$\Big\downarrow \text{3832}$$
$$\frac{\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{f}}$$

input
```
Int[Sin[f*x]/Sqrt[d*x],x]
```

output
```
(Sqrt[2*Pi]*FresnelS[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqr
t[f])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{f}}{\sqrt{\pi}}\right)}{\sqrt{dx}\,\sqrt{f}}$	33
derivativedivides	$\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{d\sqrt{\frac{f}{d}}}$	42
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,f\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,d}\right)}{d\sqrt{\frac{f}{d}}}$	42

input
```
int(sin(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
Pi^(1/2)/(d*x)^(1/2)*x^(1/2)/f^(1/2)*2^(1/2)*FresnelS(1/Pi^(1/2)*2^(1/2)*x
^(1/2)*f^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)}{f}$$

input
```
integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*pi*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d)))/f
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{3\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(sin(f*x)/(d*x)**(1/2),x)
```

output
```
3*sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(3/4)/(
4*sqrt(d)*sqrt(f)*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{\sqrt{2}\left((i+1)\,\sqrt{\pi}\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{if}{d}}\right) - (i-1)\,\sqrt{\pi}\left(\frac{f^2}{d^2}\right)^{\frac{1}{4}}\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{if}{d}}\right)\right)}{4\,f}$$

input
```
integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*((I + 1)*sqrt(pi)*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(I*f/d)) -
 (I - 1)*sqrt(pi)*(f^2/d^2)^(1/4)*erf(sqrt(d*x)*sqrt(-I*f/d)))/f
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.91

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{df}\sqrt{dx}\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\left(\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{df}\sqrt{dx}\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}{2\,d}\right)}{\sqrt{df}\left(-\frac{i\,df}{\sqrt{d^2f^2}}+1\right)}}{2\,d}$$

input
```
integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f/sqrt
(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)) + sqrt(2)*sqrt(pi)
*d*erf(1/2*I*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2) + 1)/d)/(sq
rt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \int \frac{\sin(f\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int(sin(f*x)/(d*x)^(1/2),x)
```

output
```
int(sin(f*x)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{\int \frac{\sin(fx)}{\sqrt{x}}dx}{\sqrt{d}}$$

input
```
int(sin(f*x)/(d*x)^(1/2),x)
```

output
```
int(sin(f*x)/sqrt(x),x)/sqrt(d)
```

3.63 $\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \frac{2\sqrt{f}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sin(fx)}{d\sqrt{dx}}$$

output
```
2*f^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(f^(1/2)*2^(1/2)/Pi^(1/2)*(d*x)^(1/2)/d
^(1/2))/d^(3/2)-2*sin(f*x)/d/(d*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \frac{x\left(-i\sqrt{-ifx}\,\Gamma\left(\frac{1}{2},-ifx\right) + i\sqrt{ifx}\,\Gamma\left(\frac{1}{2},ifx\right) - 2\sin(fx)\right)}{(dx)^{3/2}}$$

input
```
Integrate[Sin[f*x]/(d*x)^(3/2),x]
```

output
```
(x*((-I)*Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x] + I*Sqrt[I*f*x]*Gamma[1/2, I*
f*x] - 2*Sin[f*x]))/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3778, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(fx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{2f\int \frac{\cos(fx)}{\sqrt{dx}}dx}{d}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2f\int \frac{\sin\left(fx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{d}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{4f\int \cos(fx)d\sqrt{dx}}{d^2}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2\sqrt{2\pi}\sqrt{f}\,\text{FresnelC}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

input
```
Int[Sin[f*x]/(d*x)^(3/2),x]
```

output
```
(2*Sqrt[f]*Sqrt[2*Pi]*FresnelC[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/d^
(3/2) - (2*Sin[f*x])/(d*Sqrt[d*x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result	size
meijerg	$\frac{\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{2}\, \sqrt{f}\, \left(-\frac{4 \sqrt{2}\, \sin \left(f x \right)}{\sqrt{\pi}\, \sqrt{x}\, \sqrt{f}}+8 \,\operatorname{FresnelC}\left(\frac{\sqrt{2}\, \sqrt{x}\, \sqrt{f}}{\sqrt{\pi}}\right)\right)}{4 \left(d x \right)^{\frac{3}{2}}}$	55
derivativedivides	$\frac{-\frac{2 \sin \left(f x \right)}{\sqrt{d x}}+\frac{2 f \sqrt{2}\, \sqrt{\pi}\, \operatorname{FresnelC}\left(\frac{\sqrt{2}\, f \sqrt{d x}}{\sqrt{\pi}\, \sqrt{\frac{f}{d}}\, d}\right)}{d \sqrt{\frac{f}{d}}}}{d}$	60
default	$\frac{-\frac{2 \sin \left(f x \right)}{\sqrt{d x}}+\frac{2 f \sqrt{2}\, \sqrt{\pi}\, \operatorname{FresnelC}\left(\frac{\sqrt{2}\, f \sqrt{d x}}{\sqrt{\pi}\, \sqrt{\frac{f}{d}}\, d}\right)}{d \sqrt{\frac{f}{d}}}}{d}$	60

input
```
int(sin(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*f^(1/2)*(-4/Pi^(1/2)*2^(1/2)/x^(1
/2)/f^(1/2)*sin(f*x)+8*FresnelC(1/Pi^(1/2)*2^(1/2)*x^(1/2)*f^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \frac{2\left(\sqrt{2}\pi dx\sqrt{\frac{f}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right) - \sqrt{dx}\sin(fx)\right)}{d^2x}$$

input
```
integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*pi*d*x*sqrt(f/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*
d))) - sqrt(d*x)*sin(f*x))/(d^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\pi}\sqrt{f}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} - \frac{\sin(fx)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(sin(f*x)/(d*x)**(3/2),x)
```

output
```
sqrt(2)*sqrt(pi)*sqrt(f)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(
1/4)/(2*d**(3/2)*gamma(5/4)) - sin(f*x)*gamma(1/4)/(2*d**(3/2)*sqrt(x)*gam
ma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = -\frac{\sqrt{fx}\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,fx\right) - (i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, -i\,fx\right)\right)}{4\sqrt{dx}d}$$

input
```
integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(f*x)*((I - 1)*sqrt(2)*gamma(-1/2, I*f*x) - (I + 1)*sqrt(2)*gamma
(-1/2, -I*f*x))/(sqrt(d*x)*d)
```

Giac [F]

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \int \frac{\sin(fx)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \int \frac{\sin(f\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int(sin(f*x)/(d*x)^(3/2),x)
```

output
```
int(sin(f*x)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\left(2\sqrt{x}\left(\int \frac{1}{\sqrt{x}\tan\left(\frac{fx}{2}\right)^2+\sqrt{x}}dx\right)f-\sin\left(fx\right)-2fx\right)}{\sqrt{x}\,d^2}$$

input
```
int(sin(f*x)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*(2*sqrt(x)*int(1/(sqrt(x)*tan((f*x)/2)**2 + sqrt(x)),x)*f - sin
(f*x) - 2*f*x))/(sqrt(x)*d**2)
```

3.64 $\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = -\frac{4f\cos(fx)}{3d^2\sqrt{dx}} - \frac{4f^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sin(fx)}{3d(dx)^{3/2}}$$

output
```
-4/3*f*cos(f*x)/d^2/(d*x)^(1/2)-4/3*f^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(f^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*sin(f*x)/d/(d*x)^(3/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = \frac{2fx^{5/2}\left(-\frac{e^{ifx}-\sqrt{-ifx}\,\Gamma\left(\frac{1}{2},-ifx\right)}{\sqrt{x}} + \frac{-e^{-ifx}+\sqrt{ifx}\,\Gamma\left(\frac{1}{2},ifx\right)}{\sqrt{x}}\right)}{3(dx)^{5/2}} - \frac{2x\sin(fx)}{3(dx)^{5/2}}$$

input
```
Integrate[Sin[f*x]/(d*x)^(5/2),x]
```

output
```
(2*f*x^(5/2)*(-((E^(I*f*x) - Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x])/Sqrt[x])
 + (-E^((-I)*f*x) + Sqrt[I*f*x]*Gamma[1/2, I*f*x])/Sqrt[x]))/(3*(d*x)^(5/2
)) - (2*x*Sin[f*x])/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3778, 3042, 3778, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(fx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{2f\int \frac{\cos(fx)}{(dx)^{3/2}}dx}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2f\int \frac{\sin\left(fx+\frac{\pi}{2}\right)}{(dx)^{3/2}}dx}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{3778}$$

$$\frac{2f\left(\frac{2f\int -\frac{\sin(fx)}{\sqrt{dx}}dx}{d}-\frac{2\cos(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2f\left(-\frac{2f\int \frac{\sin(fx)}{\sqrt{dx}}dx}{d}-\frac{2\cos(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2f\left(-\frac{2f\int\frac{\sin(fx)}{\sqrt{dx}}dx}{d}-\frac{2\cos(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2f\left(-\frac{4f\int\sin(fx)d\sqrt{dx}}{d^2}-\frac{2\cos(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2f\left(-\frac{2\sqrt{2\pi}\sqrt{f}\,\text{FresnelS}\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cos(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

input
```
Int[Sin[f*x]/(d*x)^(5/2),x]
```

output
```
(2*f*((-2*Cos[f*x])/(d*Sqrt[d*x]) - (2*Sqrt[f]*Sqrt[2*Pi]*FresnelS[(Sqrt[f
]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/d^(3/2)))/(3*d) - (2*Sin[f*x])/(3*d*(d*x
)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{\sqrt{\pi}\, x^{\frac{5}{2}} \sqrt{2}\, f^{\frac{3}{2}} \left(-\frac{16 \sqrt{2}\, \cos \left(f x \right)}{3 \sqrt{\pi}\, \sqrt{x}\, \sqrt{f}}-\frac{8 \sqrt{2}\, \sin \left(f x \right)}{3 \sqrt{\pi}\, x^{\frac{3}{2}} f^{\frac{3}{2}}}-\frac{32\, \operatorname{FresnelS}\left(\frac{\sqrt{2}\, \sqrt{x}\, \sqrt{f}}{\sqrt{\pi}}\right)}{3}\right)}{8 \left(d x \right)^{\frac{5}{2}}}$	73
derivativedivides	$\frac{-\frac{2 \sin \left(f x \right)}{3 \left(d x \right)^{\frac{3}{2}}}+\frac{4 f \left(-\frac{\cos \left(f x \right)}{\sqrt{d x}}-\frac{f \sqrt{2}\, \sqrt{\pi}\, \operatorname{FresnelS}\left(\frac{\sqrt{2}\, f \sqrt{d x}}{\sqrt{\pi}\, \sqrt{\frac{f}{d}}\, d}\right)}{d \sqrt{\frac{f}{d}}}\right)}{3 d}}{d}$	79
default	$\frac{-\frac{2 \sin \left(f x \right)}{3 \left(d x \right)^{\frac{3}{2}}}+\frac{4 f \left(-\frac{\cos \left(f x \right)}{\sqrt{d x}}-\frac{f \sqrt{2}\, \sqrt{\pi}\, \operatorname{FresnelS}\left(\frac{\sqrt{2}\, f \sqrt{d x}}{\sqrt{\pi}\, \sqrt{\frac{f}{d}}\, d}\right)}{d \sqrt{\frac{f}{d}}}\right)}{3 d}}{d}$	79

input
```
int(sin(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*f^(3/2)*(-16/3/Pi^(1/2)/x^(1/2)*2
^(1/2)/f^(1/2)*cos(f*x)-8/3/Pi^(1/2)/x^(3/2)*2^(1/2)/f^(3/2)*sin(f*x)-32/3
*FresnelS(1/Pi^(1/2)*2^(1/2)*x^(1/2)*f^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = -\frac{2\left(2\sqrt{2}\pi dfx^2\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right) + (2\,fx\cos(fx) + \sin(fx))\sqrt{dx}\right)}{3\,d^3x^2}$$

input
```
integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(2)*pi*d*f*x^2*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sq
rt(f/(pi*d))) + (2*f*x*cos(f*x) + sin(f*x))*sqrt(d*x))/(d^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{f\cos(fx)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{\sin(fx)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(sin(f*x)/(d*x)**(5/2),x)
```

output
```
sqrt(2)*sqrt(pi)*f**(3/2)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma
(-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cos(f*x)*gamma(-1/4)/(3*d**(5/2)*sqrt(x
)*gamma(3/4)) + sin(f*x)*gamma(-1/4)/(6*d**(5/2)*x**(3/2)*gamma(3/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = -\frac{(fx)^{\frac{3}{2}}\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2}, i\,fx\right) + (i-1)\sqrt{2}\Gamma\left(-\frac{3}{2}, -i\,fx\right)\right)}{4\,(dx)^{\frac{3}{2}}\,d}$$

input
```
integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(f*x)^(3/2)*(-(I + 1)*sqrt(2)*gamma(-3/2, I*f*x) + (I - 1)*sqrt(2)*ga
mma(-3/2, -I*f*x))/((d*x)^(3/2)*d)
```

Giac [F]

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = \int \frac{\sin(fx)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = \int \frac{\sin(f\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int(sin(f*x)/(d*x)^(5/2),x)
```

output
```
int(sin(f*x)/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx = \frac{\int \frac{\sin(fx)}{\sqrt{x}\,x^2}dx}{\sqrt{d}\,d^2}$$

input
```
int(sin(f*x)/(d*x)^(5/2),x)
```

output
```
int(sin(f*x)/(sqrt(x)*x**2),x)/(sqrt(d)*d**2)
```

3.65 $\int \sqrt{c+dx}\csc(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx}\csc(a+bx)\,dx=\text{Int}\left(\sqrt{c+dx}\csc(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^(1/2)*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 18.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\csc(a+bx)\,dx=\int \sqrt{c+dx}\csc(a+bx)\,dx$$

input
```
Integrate[Sqrt[c + d*x]*Csc[a + b*x],x]
```

output
```
Integrate[Sqrt[c + d*x]*Csc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+dx}\csc(a+bx)dx$$

$$\Big\downarrow \text{4680}$$

$$\int \sqrt{c+dx}\csc(a+bx)dx$$

input
```
Int[Sqrt[c + d*x]*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{dx+c}\,\csc(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*csc(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*csc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\csc(a+bx)\,dx = \int \sqrt{dx+c}\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\csc(a+bx)\,dx = \int \sqrt{c+dx}\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*csc(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\csc(a+bx)\,dx = \int \sqrt{dx+c}\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\csc(a+bx)\,dx = \int \sqrt{dx+c}\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 34.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\csc(a+bx)\,dx = \int \frac{\sqrt{c+d\,x}}{\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^(1/2)/sin(a + b*x),x)
```

output
```
int((c + d*x)^(1/2)/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\csc(a+bx)\,dx = \int \sqrt{dx+c}\,\csc(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*csc(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*csc(a + b*x),x)
```

3.66 $\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \operatorname{Int}\left(\frac{\csc(a+bx)}{\sqrt{c+dx}},x\right)$$

output
```
Defer(Int)(csc(b*x+a)/(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 25.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
Integrate[Csc[a + b*x]/Sqrt[c + d*x],x]
```

output
```
Integrate[Csc[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{ 4680}$$
$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}dx$$

input
```
Int[Csc[a + b*x]/Sqrt[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\csc (bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(csc(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(csc(b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\csc (bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)/sqrt(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\csc (a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(csc(a + b*x)/sqrt(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\csc(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sqrt(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\csc(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sqrt(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 34.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/(sin(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(csc(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(csc(a + b*x)/sqrt(c + d*x),x)
```

3.67 $\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right) dx = -\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{4\sqrt{\sin(e+fx)}}{f^2}$$

output
```
-2*x*cos(f*x+e)/f/sin(f*x+e)^(1/2)+4*sin(f*x+e)^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right) dx = \frac{-2fx\cos(e+fx)+4\sin(e+fx)}{f^2\sqrt{\sin(e+fx)}}$$

input
```
Integrate[x/Sin[e + f*x]^(3/2) + x*Sqrt[Sin[e + f*x]],x]
```

output
```
(-2*f*x*Cos[e + f*x] + 4*Sin[e + f*x])/(f^2*Sqrt[Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4\sqrt{\sin(e+fx)}}{f^2}-\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

input
```
Int[x/Sin[e + f*x]^(3/2) + x*Sqrt[Sin[e + f*x]],x]
```

output
```
(-2*x*Cos[e + f*x])/(f*Sqrt[Sin[e + f*x]]) + (4*Sqrt[Sin[e + f*x]])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\sin(fx+e)^{\frac{3}{2}}}+x\sqrt{\sin(fx+e)}\right)dx$$

input
```
int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

output
```
int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int\left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int\left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)dx=\int\frac{x(\sin^2(e+fx)+1)}{\sin^{\frac{3}{2}}(e+fx)}\,dx$$

input
```
integrate(x/sin(f*x+e)**(3/2)+x*sin(f*x+e)**(1/2),x)
```

output
```
Integral(x*(sin(e + f*x)**2 + 1)/sin(e + f*x)**(3/2), x)
```

Maxima [F]

$$\int\left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)dx=\int x\sqrt{\sin(fx+e)}+\frac{x}{\sin(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)\,dx=\int x\sqrt{\sin\left(fx+e\right)}+\frac{x}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 35.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)\,dx=\frac{4\sin\left(e+f\,x\right)^2-f\,x\,\sin\left(2\,e+2\,f\,x\right)}{f^2\sin\left(e+f\,x\right)^{3/2}}$$

input
```
int(x*sin(e + f*x)^(1/2) + x/sin(e + f*x)^(3/2),x)
```

output
```
(4*sin(e + f*x)^2 - f*x*sin(2*e + 2*f*x))/(f^2*sin(e + f*x)^(3/2))
```

Reduce [F]

$$\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right)\,dx=\int \frac{\sqrt{\sin\left(fx+e\right)}\,x}{\sin\left(fx+e\right)^{2}}dx+\int \sqrt{\sin\left(fx+e\right)}\,xdx$$

input
```
int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

output
```
int((sqrt(sin(e + f*x))*x)/sin(e + f*x)**2,x) + int(sqrt(sin(e + f*x))*x,x
)
```

3.68 $\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 62

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx=-\frac{16E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f^3}-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}$$

output
```
16*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f^3-2*x^2*cos(f*x+e)/f/sin
(f*x+e)^(1/2)+8*x*sin(f*x+e)^(1/2)/f^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.00 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.63

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx$$
$$=\frac{-((8+f^2x^2)\cos(fx)\sec(e))-(-8+f^2x^2)\cos(2e+fx)\sec(e)+8fx\sin(e+fx)+8\sqrt{\csc^2(e)}\csc(fx}{}$$

input
```
Integrate[x^2/Sin[e + f*x]^(3/2) + x^2*Sqrt[Sin[e + f*x]],x]
```

output
```
(-((8 + f^2*x^2)*Cos[f*x]*Sec[e]) - (-8 + f^2*x^2)*Cos[2*e + f*x]*Sec[e] +
 8*f*x*Sin[e + f*x] + 8*Sqrt[Csc[e]^2]*Csc[f*x - ArcTan[Cot[e]]]*Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[f*x - ArcTan[Cot[e]]]^2]*Sin[e]*Sqrt[Sin
[f*x - ArcTan[Cot[e]]]^2] + (4*Csc[e]*Sec[e]*(Sin[e + f*x - ArcTan[Cot[e]]
] + 3*Sin[e - f*x + ArcTan[Cot[e]]]))/Sqrt[Csc[e]^2])/(f^3*Sqrt[Sin[e + f*
x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f^3}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

input
```
Int[x^2/Sin[e + f*x]^(3/2) + x^2*Sqrt[Sin[e + f*x]],x]
```

output
```
(-16*EllipticE[(e - Pi/2 + f*x)/2, 2])/f^3 - (2*x^2*Cos[e + f*x])/(f*Sqrt[
Sin[e + f*x]]) + (8*x*Sqrt[Sin[e + f*x]])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\sin(fx+e)^{\frac{3}{2}}} + x^2\sqrt{\sin(fx+e)}\right) dx$$

input
```
int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

output
```
int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)} + x^2\sqrt{\sin(e+fx)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)} + x^2\sqrt{\sin(e+fx)}\right) dx = \int \frac{x^2(\sin^2(e+fx)+1)}{\sin^{\frac{3}{2}}(e+fx)}\, dx$$

input
```
integrate(x**2/sin(f*x+e)**(3/2)+x**2*sin(f*x+e)**(1/2),x)
```

output
```
Integral(x**2*(sin(e + f*x)**2 + 1)/sin(e + f*x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx=\int x^2\sqrt{\sin\left(fx+e\right)}+\frac{x^2}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(sin(f*x + e)) + x^2/sin(f*x + e)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx=\int x^2\sqrt{\sin\left(fx+e\right)}+\frac{x^2}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(sin(f*x + e)) + x^2/sin(f*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)\,dx=\int x^2\,\sqrt{\sin\left(e+f\,x\right)}+\frac{x^2}{{\sin\left(e+f\,x\right)}^{3/2}}\,dx$$

input
```
int(x^2*sin(e + f*x)^(1/2) + x^2/sin(e + f*x)^(3/2),x)
```

output
```
int(x^2*sin(e + f*x)^(1/2) + x^2/sin(e + f*x)^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)} + x^2\sqrt{\sin(e+fx)}\right) dx = \int \frac{\sqrt{\sin(fx+e)}\,x^2}{\sin(fx+e)^2}dx + \int \sqrt{\sin(fx+e)}\,x^2dx$$

input
```
int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

output
```
int((sqrt(sin(e + f*x))*x**2)/sin(e + f*x)**2,x) + int(sqrt(sin(e + f*x))*
x**2,x)
```

3.69 $\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)} - \frac{x}{3\sqrt{\sin(e+fx)}}\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 42

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)} - \frac{x}{3\sqrt{\sin(e+fx)}}\right) dx = -\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)} - \frac{4}{3f^2\sqrt{\sin(e+fx)}}$$

output
```
-2/3*x*cos(f*x+e)/f/sin(f*x+e)^(3/2)-4/3/f^2/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)} - \frac{x}{3\sqrt{\sin(e+fx)}}\right) dx = -\frac{2(fx\cos(e+fx)+2\sin(e+fx))}{3f^2\sin^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[x/Sin[e + f*x]^(5/2) - x/(3*Sqrt[Sin[e + f*x]]),x]
```

output
```
(-2*(f*x*Cos[e + f*x] + 2*Sin[e + f*x]))/(3*f^2*Sin[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4}{3f^2\sqrt{\sin(e+fx)}}-\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}$$

input
```
Int[x/Sin[e + f*x]^(5/2) - x/(3*Sqrt[Sin[e + f*x]]),x]
```

output
```
(-2*x*Cos[e + f*x])/(3*f*Sin[e + f*x]^(3/2)) - 4/(3*f^2*Sqrt[Sin[e + f*x]]
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\sin\left(fx+e\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\sin\left(fx+e\right)}}\right)dx$$

input
```
int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

output
```
int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)dx = \frac{2\left(fx\cos\left(fx+e\right)+2\sin\left(fx+e\right)\right)\sqrt{\sin\left(fx+e\right)}}{3\left(f^2\cos\left(fx+e\right)^2-f^2\right)}$$

input
```
integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(f*x*cos(f*x + e) + 2*sin(f*x + e))*sqrt(sin(f*x + e))/(f^2*cos(f*x +
e)^2 - f^2)
```

Sympy [F]

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)dx = -\frac{\int\left(-\frac{3x}{\sin^{\frac{5}{2}}(e+fx)}\right)dx+\int\frac{x}{\sqrt{\sin(e+fx)}}\,dx}{3}$$

input
```
integrate(x/sin(f*x+e)**(5/2)-1/3*x/sin(f*x+e)**(1/2),x)
```

output
```
-(Integral(-3*x/sin(e + f*x)**(5/2), x) + Integral(x/sqrt(sin(e + f*x)), x
))/3
```

Maxima [F]

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)dx = \int -\frac{x}{3\sqrt{\sin\left(fx+e\right)}}+\frac{x}{\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x/sqrt(sin(f*x + e)) + x/sin(f*x + e)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)\,dx=\int -\frac{x}{3\sqrt{\sin\left(fx+e\right)}}+\frac{x}{\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x/sqrt(sin(f*x + e)) + x/sin(f*x + e)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 37.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.33

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)\,dx=$$
$$-\frac{4\sqrt{\sin\left(e+f\,x\right)}\,\left(20\,\sin\left(e+f\,x\right)-10\,\sin\left(3\,e+3\,f\,x\right)+2\,\sin\left(5\,e+5\,f\,x\right)-2\,f\,x\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right.\right.}{3\,f^2\,\left(30\sin\left(e+f\,x\right)^2-12\sin\left(2\,e+2\,f\,x\right)^2+2\,\mathrm{s}\right.}$$

input
```
int(x/sin(e + f*x)^(5/2) - x/(3*sin(e + f*x)^(1/2)),x)
```

output
```
-(4*sin(e + f*x)^(1/2)*(20*sin(e + f*x) - 10*sin(3*e + 3*f*x) + 2*sin(5*e
+ 5*f*x) - 2*f*x*(2*sin(e/2 + (f*x)/2)^2 - 1) + 3*f*x*(2*sin((3*e)/2 + (3*
f*x)/2)^2 - 1) - f*x*(2*sin((5*e)/2 + (5*f*x)/2)^2 - 1)))/(3*f^2*(2*sin(3*
e + 3*f*x)^2 - 12*sin(2*e + 2*f*x)^2 + 30*sin(e + f*x)^2))
```

Reduce [F]

$$\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right)dx=\int \frac{\sqrt{\sin(fx+e)}\,x}{\sin(fx+e)^3}dx-\frac{\left(\int \frac{\sqrt{\sin(fx+e)}\,x}{\sin(fx+e)}dx\right)}{3}$$

input
```
int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

output
```
(3*int((sqrt(sin(e + f*x))*x)/sin(e + f*x)**3,x) - int((sqrt(sin(e + f*x))
*x)/sin(e + f*x),x))/3
```

3.70 $\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx=-\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}+\frac{12\sqrt{\sin(e+fx)}}{5f^2}$$

output
```
-2/5*x*cos(f*x+e)/f/sin(f*x+e)^(5/2)-4/15/f^2/sin(f*x+e)^(3/2)-6/5*x*cos(f
*x+e)/f/sin(f*x+e)^(1/2)+12/5*sin(f*x+e)^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx=\frac{-21fx\cos(e+fx)+9fx\cos(3(e+fx))+46\sin(e+fx)-18\sin(3(e+fx))}{30f^2\sin^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[x/Sin[e + f*x]^(7/2) + (3*x*Sqrt[Sin[e + f*x]])/5,x]
```

output
```
(-21*f*x*Cos[e + f*x] + 9*f*x*Cos[3*(e + f*x)] + 46*Sin[e + f*x] - 18*Sin[
3*(e + f*x)])/(30*f^2*Sin[e + f*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}+\frac{12\sqrt{\sin(e+fx)}}{5f^2}-\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}$$

input
```
Int[x/Sin[e + f*x]^(7/2) + (3*x*Sqrt[Sin[e + f*x]])/5,x]
```

output
```
(-2*x*Cos[e + f*x])/(5*f*Sin[e + f*x]^(5/2)) - 4/(15*f^2*Sin[e + f*x]^(3/2
)) - (6*x*Cos[e + f*x])/(5*f*Sqrt[Sin[e + f*x]]) + (12*Sqrt[Sin[e + f*x]])
/(5*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\sin (fx+e)^{\frac{7}{2}}}+\frac{3x\sqrt{\sin (fx+e)}}{5}\right)dx$$

input
```
int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

output
```
int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)dx = \frac{\int \frac{5x}{\sin^{\frac{7}{2}}(e+fx)}\,dx+\int 3x\sqrt{\sin (e+fx)}\,dx}{5}$$

input
```
integrate(x/sin(f*x+e)**(7/2)+3/5*x*sin(f*x+e)**(1/2),x)
```

output
```
(Integral(5*x/sin(e + f*x)**(7/2), x) + Integral(3*x*sqrt(sin(e + f*x)), x
))/5
```

Maxima [F]

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\sin\left(fx+e\right)}+\frac{x}{\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\sin\left(fx+e\right)}+\frac{x}{\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 41.02 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.05

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx$$
$$=\left(\frac{12}{5\,f^2}+\frac{x\,6\mathrm{i}}{5\,f}\right)\,\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}$$
$$-\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\frac{x\,3\mathrm{i}}{5\,f}-\frac{32+f\,x\,66\mathrm{i}}{30\,f^2}\right)}{{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^2}$$
$$-\frac{x\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}\,12\mathrm{i}}{5\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}$$
$$+\frac{x\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}}\,16\mathrm{i}}{5\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3}$$

input
```
int((3*x*sin(e + f*x)^(1/2))/5 + x/sin(e + f*x)^(7/2),x)
```

output
```
((x*6i)/(5*f) + 12/(5*f^2))*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^(1/2) - (exp(e*2i + f*x*2i)*((exp(- e*1i - f*x*1i)*1i)/2 - (ex
p(e*1i + f*x*1i)*1i)/2)^(1/2)*((x*3i)/(5*f) - (f*x*66i + 32)/(30*f^2)))/(e
xp(e*2i + f*x*2i) - 1)^2 - (x*exp(e*2i + f*x*2i)*((exp(- e*1i - f*x*1i)*1i
)/2 - (exp(e*1i + f*x*1i)*1i)/2)^(1/2)*12i)/(5*f*(exp(e*2i + f*x*2i) - 1))
 + (x*exp(e*2i + f*x*2i)*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i
)*1i)/2)^(1/2)*16i)/(5*f*(exp(e*2i + f*x*2i) - 1)^3)
```

Reduce [F]

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)\,dx=\int \frac{\sqrt{\sin(fx+e)}\,x}{\sin(fx+e)^4}dx+\frac{3\left(\int \sqrt{\sin(fx+e)}\,xdx\right)}{5}$$

input
```
int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

output
```
(5*int((sqrt(sin(e + f*x))*x)/sin(e + f*x)**4,x) + 3*int(sqrt(sin(e + f*x)
)*x,x))/5
```

3.71 $\int (c+dx)^m (b\sin(e+fx))^n \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx = \operatorname{Int}((c+dx)^m (b\sin(e+fx))^n, x)$$

output
```
Defer(Int)((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx = \int (c+dx)^m (b\sin(e+fx))^n \, dx$$

input
```
Integrate[(c + d*x)^m*(b*Sin[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(b*Sin[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(b\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m(b\sin(e+fx))^ndx$$

$$\Big\downarrow \text{ 3807}$$

$$\int (c+dx)^m(b\sin(e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(b*Sin[e + f*x])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx + c)^m (b \sin (fx + e))^n \, dx$$

input
```
int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx = \int (dx + c)^m (b \sin (fx + e))^n \, dx$$

input
```
integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 7.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx = \int (b \sin (e + fx))^n (c + dx)^m \, dx$$

input
```
integrate((d*x+c)**m*(b*sin(f*x+e))**n,x)
```

output
```
Integral((b*sin(e + f*x))**n*(c + d*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sin(e+fx))^n\,dx = \int (dx+c)^m(b\sin(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sin(e+fx))^n\,dx = \int (dx+c)^m(b\sin(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 35.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\sin(e+fx))^n\,dx = \int (b\,\sin(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((b*sin(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((b*sin(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (c+dx)^m (b\sin(e+fx))^n \, dx = b^n \left(\int \sin\left(fx+e\right)^n \left(dx+c\right)^m dx \right)$$

input
```
int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

output
```
b**n*int(sin(e + f*x)**n*(c + d*x)**m,x)
```

3.72 $\int (c+dx)^m \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 267

$$\int (c+dx)^m \sin^3(a+bx)\,dx = -\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{3e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b} + \frac{3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b} + \frac{3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}$$

output
```
-3/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-3/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)+1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x
+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+
c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \sin^3(a+bx)\,dx$$
$$= \frac{3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-3^{2+m}e^{2ia+}\right.}{}$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]^3,x]
```

output
```
(3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b*(c +
d*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((2*I)*a + ((
4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d] + E^
((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(
((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d
]))/(8*b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(a+bx)^3(c+dx)^m dx$$
$$\Big\downarrow \text{ 3793}$$
$$\int \left(\frac{3}{4}\sin(a+bx)(c+dx)^m-\frac{1}{4}\sin(3a+3bx)(c+dx)^m\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-\frac{3e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]^3,x]
```

output
```
(-3*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8
*b*(((-I)*b*(c + d*x))/d)^m) - (3*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))
/d])/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (3^(-1 - m)*E^((3*I
)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*((
(-I)*b*(c + d*x))/d)^m) + (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c
 + d*x))/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^m \sin(bx+a)^3\, dx$$

input
```
int((d*x+c)^m*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*sin(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \sin^3(a+bx)\, dx = -\frac{9\, e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)+9\,e^{\left(-\frac{dm}{}\right.}}{24\,b}$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/24*(9*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I
*b*c)/d) - e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3*
(I*b*d*x + I*b*c)/d) + 9*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m
+ 1, (-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)
*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \sin^3(a+bx)\, dx = \int (c+dx)^m \sin^3(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^m \sin^3(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)^3, x)
```

Giac [F]

$$\int (c+dx)^m \sin^3(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sin^3(a+bx)\,dx = \int \sin(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \sin^3(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*sin(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*sin(a + b*x)**3,x)
```

3.73 $\int (c+dx)^m \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}$$

output
```
1/2*(d*x+c)^(1+m)/d/(1+m)+I*2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+
m,-2*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-I*2^(-3-m)*(d*x+c)^m*GAMMA(1+m,
2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \frac{1}{8}(c+dx)^m \left(\frac{4c+4dx}{d+dm} + \frac{i2^{-m}e^{2i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{i2^{-m}e^{-2i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}\right)$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) + (I*E^((2*I)*(a - (b*c)/d))*Gamma[1
 + m, ((-2*I)*b*(c + d*x))/d])/(2^m*b*(((-I)*b*(c + d*x))/d)^m) - (I*Gamma
[1 + m, ((2*I)*b*(c + d*x))/d])/(2^m*b*E^((2*I)*(a - (b*c)/d))*((I*b*(c +
d*x))/d)^m)))/8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(c+dx)^m dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}(c+dx)^m-\frac{1}{2}\cos(2a+2bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(1 + m)/(2*d*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
 + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^
m) - (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^(
(2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^m \sin(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int (c+dx)^m \sin^2(a+bx)\, dx = \frac{(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)+(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m\right.}{8\,(bdm+bd)}$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*((I*d*m + I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m
+ 1, -2*(I*b*d*x + I*b*c)/d) + (-I*d*m - I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*
b*c + 2*I*a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + 4*(b*d*x + b*c)*
(d*x + c)^m)/(b*d*m + b*d)
```

Sympy [F]

$$\int (c+dx)^m \sin^2(a+bx)\, dx = \int (c+dx)^m \sin^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) - e^(m*log(d*x
+ c) + log(d*x + c)))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*sin(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*sin(a + b*x)**2,x)
```

3.74 $\int (c+dx)^m \sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 127

$$\int (c+dx)^m \sin(a+bx)\,dx = -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}$$

output
```
-1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int (c+dx)^m \sin(a+bx)\,dx = \frac{e^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x],x]
```

output
```
((c + d*x)^m*(-((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I)*b*
(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])/((I
*b*(c + d*x))/d)^m))/(2*b*E^((I*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)(c+dx)^m dx$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m dx-\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x],x]
```

output
```
-1/2*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d
])/(2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int (dx+c)^m \sin(bx+a)\, dx$$

input
```
int((d*x+c)^m*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*sin(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.74

$$\int (c+dx)^m \sin(a+bx)\,dx = -\frac{e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+1,\frac{-ibdx-ibc}{d}\right)}{2b}$$

input
```
integrate((d*x+c)^m*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*
c)/d) + e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x -
I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \sin(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^m \sin(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m \sin(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sin(a+bx)\,dx = \int \sin(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \sin(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*sin(b*x+a),x)
```

output

```
((c + d*x)**m*tan((a + b*x)/2)**2*b*c*m + (c + d*x)**m*tan((a + b*x)/2)**2
*b*d*m*x + 2*(c + d*x)**m*tan((a + b*x)/2)*d*m**2 + 2*(c + d*x)**m*tan((a
+ b*x)/2)*d*m - (c + d*x)**m*b*c*m - 2*(c + d*x)**m*b*c + (c + d*x)**m*b*d
*m*x - 2*int(((c + d*x)**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*c + tan(
(a + b*x)/2)**2*d*x + c + d*x),x)*tan((a + b*x)/2)**2*d**2*m**3 - 2*int(((
c + d*x)**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*c + tan((a + b*x)/2)**2
*d*x + c + d*x),x)*tan((a + b*x)/2)**2*d**2*m**2 - 2*int(((c + d*x)**m*tan
((a + b*x)/2))/(tan((a + b*x)/2)**2*c + tan((a + b*x)/2)**2*d*x + c + d*x)
,x)*d**2*m**3 - 2*int(((c + d*x)**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2
*c + tan((a + b*x)/2)**2*d*x + c + d*x),x)*d**2*m**2 - 2*int(((c + d*x)**m
*x)/(tan((a + b*x)/2)**2*c + tan((a + b*x)/2)**2*d*x + c + d*x),x)*tan((a
+ b*x)/2)**2*b*d**2*m**2 - 2*int(((c + d*x)**m*x)/(tan((a + b*x)/2)**2*c +
 tan((a + b*x)/2)**2*d*x + c + d*x),x)*tan((a + b*x)/2)**2*b*d**2*m - 2*in
t(((c + d*x)**m*x)/(tan((a + b*x)/2)**2*c + tan((a + b*x)/2)**2*d*x + c +
d*x),x)*b*d**2*m**2 - 2*int(((c + d*x)**m*x)/(tan((a + b*x)/2)**2*c + tan(
(a + b*x)/2)**2*d*x + c + d*x),x)*b*d**2*m)/(b**2*c*(tan((a + b*x)/2)**2*m
 + tan((a + b*x)/2)**2 + m + 1))
```

3.75 $\int (c+dx)^m \csc(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \csc(a+bx)\,dx = \text{Int}((c+dx)^m \csc(a+bx), x)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 10.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \csc(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc(a+bx)(c+dx)^m dx$$
$$\Big\downarrow \text{4680}$$
$$\int \csc(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc (bx+a)\, dx$$

input
```
int((d*x+c)^m*csc(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \csc(a+bx)\, dx = \int (dx+c)^m \csc (bx+a)\, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \csc(a+bx)\, dx = \int (c+dx)^m \csc (a+bx)\, dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \csc(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \csc(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 34.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (c+dx)^m \csc(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/sin(a + b*x),x)
```

output
```
int((c + d*x)^m/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \csc(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a),x)
```

output
```
int((c + d*x)**m*csc(a + b*x),x)
```

3.76 $\int (c+dx)^m \csc^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \text{Int}\left((c+dx)^m \csc^2(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^2(c+dx)^m dx$$

$$\Big\downarrow \text{4680}$$

$$\int \csc^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \csc^2(a+bx) \, dx = \int (dx+c)^m \csc(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \csc^2(a+bx) \, dx = \int (c+dx)^m \csc^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 35.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/sin(a + b*x)^2,x)
```

output
```
int((c + d*x)^m/sin(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**2,x)
```

3.77 $\int x^{3+m} \sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int x^{3+m}\sin(a+bx)\,dx=\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

output
```
1/2*I*exp(I*a)*x^m*GAMMA(4+m,-I*b*x)/b^4/((-I*b*x)^m)-1/2*I*x^m*GAMMA(4+m,
I*b*x)/b^4/exp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^{3+m}\sin(a+bx)\,dx=\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

input
```
Integrate[x^(3 + m)*Sin[a + b*x],x]
```

output
```
((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m
*Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+3}\sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{2}i\int e^{-i(a+bx)}x^{m+3}dx-\frac{1}{2}i\int e^{i(a+bx)}x^{m+3}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(m+4,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(m+4,ibx)}{2b^4}$$

input
```
Int[x^(3 + m)*Sin[a + b*x],x]
```

output
```
((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m
*Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.75

method	result
meijerg	$\frac{2^{3+m}\left(b^{2}\right)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-4-m}x^{3+m}b^{3}\left(b^{2}\right)^{\frac{m}{2}}\left(\frac{8}{3}+\frac{2m}{3}\right)\sin\left(bx\right)}{\sqrt{\pi}\,\left(4+m\right)}-\frac{2^{-3-m}x^{1+m}b\left(b^{2}\right)^{\frac{m}{2}}\left(-m^{2}-7m-12\right)\left(\cos\left(bx\right)xb-\sin\left(bx\right)\right)}{\sqrt{\pi}\,\left(4+m\right)}+\frac{2^{-3-m}x^{2+m}}{}\right.}{}$

input
```
int(x^(3+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(3+m)/b^4*(b^2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(4+m)*x^(3+m)*b^3
*(b^2)^(1/2*m)*(8/3+2/3*m)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(4+m)*x^(1+m)*b*(b^2
)^(1/2*m)*(-m^2-7*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(4+m)*x^
(2+m)*b^2*(b^2)^(1/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(m+3/2
,3/2,b*x)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)
*(3+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin
(a)+2^(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^
2+7*m+10)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x
))-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1
(m+1/2,3/2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*
x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int x^{3+m}\sin(a+bx)\,dx = -\frac{e^{(-(m+3)\log(ib)-ia)}\Gamma(m+4,ibx)+e^{(-(m+3)\log(-ib)+ia)}\Gamma(m+4,-ibx)}{2b}$$

input
```
integrate(x^(3+m)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) + e^(-(m + 3)*log(-I
*b) + I*a)*gamma(m + 4, -I*b*x))/b
```

Sympy [F]

$$\int x^{3+m}\sin(a+bx)\,dx = \int x^{m+3}\sin(a+bx)\,dx$$

input
```
integrate(x**(3+m)*sin(b*x+a),x)
```

output
```
Integral(x**(m + 3)*sin(a + b*x), x)
```

Maxima [F]

$$\int x^{3+m}\sin(a+bx)\,dx = \int x^{m+3}\sin(bx+a)\,dx$$

input
```
integrate(x^(3+m)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m + 3)*sin(b*x + a), x)
```

Giac [F]

$$\int x^{3+m}\sin(a+bx)\,dx=\int x^{m+3}\sin(bx+a)\,dx$$

input
```
integrate(x^(3+m)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\sin(a+bx)\,dx=\int x^{m+3}\sin(a+b\,x)\,dx$$

input
```
int(x^(m + 3)*sin(a + b*x),x)
```

output
```
int(x^(m + 3)*sin(a + b*x), x)
```

Reduce [F]

$$\int x^{3+m}\sin(a+bx)\,dx=\text{Too large to display}$$

input
```
int(x^(3+m)*sin(b*x+a),x)
```

output
```
( - x**m*cos(a + b*x)*tan((a + b*x)/2)**2*b**3*x**3 + x**m*cos(a + b*x)*ta
n((a + b*x)/2)**2*b*m**2*x + 5*x**m*cos(a + b*x)*tan((a + b*x)/2)**2*b*m*x
 + 6*x**m*cos(a + b*x)*tan((a + b*x)/2)**2*b*x - x**m*cos(a + b*x)*b**3*x*
*3 + x**m*cos(a + b*x)*b*m**2*x + 5*x**m*cos(a + b*x)*b*m*x + 6*x**m*cos(a
 + b*x)*b*x + x**m*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*m*x**2 + 3*x**m*s
in(a + b*x)*tan((a + b*x)/2)**2*b**2*x**2 + x**m*sin(a + b*x)*b**2*m*x**2
+ 3*x**m*sin(a + b*x)*b**2*x**2 - 2*x**m*tan((a + b*x)/2)*m**3 - 12*x**m*t
an((a + b*x)/2)*m**2 - 22*x**m*tan((a + b*x)/2)*m - 12*x**m*tan((a + b*x)/
2) + 2*int((x**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*tan((a +
 b*x)/2)**2*m**4 + 12*int((x**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*x +
 x),x)*tan((a + b*x)/2)**2*m**3 + 22*int((x**m*tan((a + b*x)/2))/(tan((a +
 b*x)/2)**2*x + x),x)*tan((a + b*x)/2)**2*m**2 + 12*int((x**m*tan((a + b*x
)/2))/(tan((a + b*x)/2)**2*x + x),x)*tan((a + b*x)/2)**2*m + 2*int((x**m*t
an((a + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*m**4 + 12*int((x**m*tan((a
 + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*m**3 + 22*int((x**m*tan((a + b*
x)/2))/(tan((a + b*x)/2)**2*x + x),x)*m**2 + 12*int((x**m*tan((a + b*x)/2)
)/(tan((a + b*x)/2)**2*x + x),x)*m)/(b**4*(tan((a + b*x)/2)**2 + 1))
```

3.78 $\int x^{2+m} \sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{2+m}\sin(a+bx)\,dx=\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

output
```
1/2*exp(I*a)*x^m*GAMMA(3+m,-I*b*x)/b^3/((-I*b*x)^m)+1/2*x^m*GAMMA(3+m,I*b*
x)/b^3/exp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{2+m}\sin(a+bx)\,dx=\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

input
```
Integrate[x^(2 + m)*Sin[a + b*x],x]
```

output
```
(E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 +
 m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+2}\sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{2}i\int e^{-i(a+bx)}x^{m+2}dx-\frac{1}{2}i\int e^{i(a+bx)}x^{m+2}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+3,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+3,ibx)}{2b^3}$$

input
```
Int[x^(2 + m)*Sin[a + b*x],x]
```

output
```
(E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 +
 m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.71

method	result
meijerg	$\frac{2^{2+m} \left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}} \sqrt{\pi} \left(\frac{3\, 2^{-3-m} x^{2+m} \left(b^2\right)^{\frac{3}{2}+\frac{m}{2}} \left(2+\frac{2m}{3}\right) \sin(bx)}{\sqrt{\pi}\,(3+m) b}-\frac{2^{-2-m} x^{2+m} \left(b^2\right)^{\frac{3}{2}+\frac{m}{2}} (2+m) m (bx)^{-\frac{3}{2}-m} \operatorname{LommelS1}\left(m+\frac{1}{2},\frac{3}{2},bx\right) \mathrm{s}}{\sqrt{\pi}\, b}\right.}{b^2}$

input
```
int(x^(2+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+m)
*(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(3
/2+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(-2-
m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b
-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)+2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(
-2-m)/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-2-m)/Pi^(1/2)/(
4+m)*x^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*si
n(b*x)+2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)*(1+m)*(b*x)^(-5/2-m)*(cos(b
*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int x^{2+m}\sin(a+bx)\,dx$$
$$= -\frac{e^{(-(m+2)\log(ib)-ia)}\Gamma(m+3, i\,bx) + e^{(-(m+2)\log(-ib)+ia)}\Gamma(m+3, -i\,bx)}{2\,b}$$

input
```
integrate(x^(2+m)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) + e^(-(m + 2)*log(-I
*b) + I*a)*gamma(m + 3, -I*b*x))/b
```

Sympy [F]

$$\int x^{2+m}\sin(a+bx)\,dx = \int x^{m+2}\sin(a+bx)\,dx$$

input
```
integrate(x**(2+m)*sin(b*x+a),x)
```

output
```
Integral(x**(m + 2)*sin(a + b*x), x)
```

Maxima [F]

$$\int x^{2+m}\sin(a+bx)\,dx = \int x^{m+2}\sin(bx+a)\,dx$$

input
```
integrate(x^(2+m)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m + 2)*sin(b*x + a), x)
```

Giac [F]

$$\int x^{2+m}\sin(a+bx)\,dx=\int x^{m+2}\sin(bx+a)\,dx$$

input
```
integrate(x^(2+m)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\sin(a+bx)\,dx=\int x^{m+2}\,\sin(a+b\,x)\,dx$$

input
```
int(x^(m + 2)*sin(a + b*x),x)
```

output
```
int(x^(m + 2)*sin(a + b*x), x)
```

Reduce [F]

$$\int x^{2+m}\sin(a+bx)\,dx$$

$$=\frac{-x^m\cos(bx+a)\,b^2x^2+x^m\cos(bx+a)\,m^2+3x^m\cos(bx+a)\,m+2x^m\cos(bx+a)+x^m\sin(bx+a)\,b}{}$$

input
```
int(x^(2+m)*sin(b*x+a),x)
```

output
```
( - x**m*cos(a + b*x)*b**2*x**2 + x**m*cos(a + b*x)*m**2 + 3*x**m*cos(a +
b*x)*m + 2*x**m*cos(a + b*x) + x**m*sin(a + b*x)*b*m*x + 2*x**m*sin(a + b*
x)*b*x + x**m*m**2 + 3*x**m*m + 2*x**m - 2*int(x**m/(tan((a + b*x)/2)**2*x
 + x),x)*m**3 - 6*int(x**m/(tan((a + b*x)/2)**2*x + x),x)*m**2 - 4*int(x**
m/(tan((a + b*x)/2)**2*x + x),x)*m)/b**3
```

3.79 $\int x^{1+m} \sin(a+bx)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int x^{1+m} \sin(a+bx)\, dx = -\frac{ie^{ia} x^m (-ibx)^{-m} \Gamma(2+m,-ibx)}{2b^2} + \frac{ie^{-ia} x^m (ibx)^{-m} \Gamma(2+m,ibx)}{2b^2}$$

output
```
-1/2*I*exp(I*a)*x^m*GAMMA(2+m,-I*b*x)/b^2/((-I*b*x)^m)+1/2*I*x^m*GAMMA(2+m
,I*b*x)/b^2/exp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^{1+m} \sin(a+bx)\, dx = -\frac{ie^{ia} x^m (-ibx)^{-m} \Gamma(2+m,-ibx)}{2b^2} + \frac{ie^{-ia} x^m (ibx)^{-m} \Gamma(2+m,ibx)}{2b^2}$$

input
```
Integrate[x^(1 + m)*Sin[a + b*x],x]
```

output
```
((-1/2*I)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*
x^m*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^{m+1}\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3789}$$

$$\frac{1}{2}i\int e^{-i(a+bx)}x^{m+1}dx-\frac{1}{2}i\int e^{i(a+bx)}x^{m+1}dx$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(m+2,ibx)}{2b^2}-\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(m+2,-ibx)}{2b^2}$$

input
```
Int[x^(1 + m)*Sin[a + b*x],x]
```

output
```
((-1/2*I)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*
x^m*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.67

method	result
meijerg	$\frac{2^{1+m}\left(b^2\right)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{2^{-1-m}x^{1+m}b\left(b^2\right)^{\frac{m}{2}}\sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{3\,2^{-2-m}x^{2+m}b^2\left(b^2\right)^{\frac{m}{2}}\left(\frac{2}{3}+\frac{2m}{3}\right)(bx)^{-\frac{3}{2}-m}\operatorname{LommelS1}\left(m+\frac{3}{2},\frac{3}{2},bx\right)\sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{2^{-1-m}\cdots}{\cdots}\right.}{b^2}$

input
```
int(x^(1+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(1+m)/b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^
2)^(1/2*m)*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2
/3+2/3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2
)*x^(2+m)*b^2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*L
ommelS1(m+1/2,1/2,b*x))*sin(a)+2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2
)*x^(2+m)*b^(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-1-
m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS
1(m+3/2,1/2,b*x))*cos(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int x^{1+m}\sin(a+bx)\,dx = -\frac{e^{(-(m+1)\log(i\,b)-i\,a)}\Gamma(m+2,i\,bx)+e^{(-(m+1)\log(-i\,b)+i\,a)}\Gamma(m+2,-i\,bx)}{2\,b}$$

input
```
integrate(x^(1+m)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) + e^(-(m + 1)*log(-I
*b) + I*a)*gamma(m + 2, -I*b*x))/b
```

Sympy [F]

$$\int x^{1+m}\sin(a+bx)\,dx = \int x^{m+1}\sin(a+bx)\,dx$$

input
```
integrate(x**(1+m)*sin(b*x+a),x)
```

output
```
Integral(x**(m + 1)*sin(a + b*x), x)
```

Maxima [F]

$$\int x^{1+m}\sin(a+bx)\,dx = \int x^{m+1}\sin(bx+a)\,dx$$

input
```
integrate(x^(1+m)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m + 1)*sin(b*x + a), x)
```

Giac [F]

$$\int x^{1+m}\sin(a+bx)\,dx=\int x^{m+1}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x^(1+m)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\sin(a+bx)\,dx=\int x^{m+1}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x^(m + 1)*sin(a + b*x),x)
```

output
```
int(x^(m + 1)*sin(a + b*x), x)
```

Reduce [F]

$$\int x^{1+m}\sin(a+bx)\,dx$$

$$=\frac{-x^m\cos\left(bx+a\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2bx-x^m\cos\left(bx+a\right)bx+2x^m\tan\left(\frac{bx}{2}+\frac{a}{2}\right)m+2x^m\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-2\Bigg(}{}$$

input
```
int(x^(1+m)*sin(b*x+a),x)
```

output
```
( - x**m*cos(a + b*x)*tan((a + b*x)/2)**2*b*x - x**m*cos(a + b*x)*b*x + 2*
x**m*tan((a + b*x)/2)*m + 2*x**m*tan((a + b*x)/2) - 2*int((x**m*tan((a + b
*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*tan((a + b*x)/2)**2*m**2 - 2*int((x
**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*tan((a + b*x)/2)**2*m
 - 2*int((x**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*m**2 - 2*i
nt((x**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*m)/(b**2*(tan((a
 + b*x)/2)**2 + 1))
```

3.80 $\int x^m \sin(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int x^m \sin(a+bx)\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

output
```
-1/2*exp(I*a)*x^m*GAMMA(1+m,-I*b*x)/b/((-I*b*x)^m)-1/2*x^m*GAMMA(1+m,I*b*x
)/b/exp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^m \sin(a+bx)\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

input
```
Integrate[x^m*Sin[a + b*x],x]
```

output
```
-1/2*(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) - (x^m*Gamma[1
+ m, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int x^m \sin(a+bx)dx$$
$$\Big\downarrow \text{ 3789}$$
$$\frac{1}{2}i\int e^{-i(a+bx)}x^m dx-\frac{1}{2}i\int e^{i(a+bx)}x^m dx$$
$$\Big\downarrow \text{ 2612}$$
$$-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+1,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+1,ibx)}{2b}$$

input
```
Int[x^m*Sin[a + b*x],x]
```

output
```
-1/2*(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) - (x^m*Gamma[1
+ m, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 378, normalized size of antiderivative = 5.04

method	result
meijerg	$2^{m}\left(b^{2}\right)^{-\frac{1}{2}-\frac{m}{2}} \sqrt{\pi}\,\left(\frac{3\,2^{-1-m} \left(b^{2}\right)^{\frac{1}{2}+\frac{m}{2}} x^{m} (6+2m) \sin (bx)}{\sqrt{\pi}\,(1+m)(9+3m) b}+\frac{\left(b^{2}\right)^{\frac{1}{2}+\frac{m}{2}} x^{m} 2^{-m} (\cos (bx) x b-\sin (bx))}{\sqrt{\pi}\,(1+m) b}+\frac{2^{-m} x^{2+m} \left(b^{2}\right)^{\frac{1}{2}+\cdots}}{\cdots}\right.$

input
```
int(x^m*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^m*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*
m)*x^m*(6+2*m)/(9+3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2
^(-m)/b*(cos(b*x)*x*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1
/2*m)*b*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/
(1+m)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*L
ommelS1(m+3/2,1/2,b*x))*sin(a)+2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(
1+m)*b^(1+m)*2^(-m)*sin(b*x)-2^(-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(
-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)
*b^(2+m)*(4/3+2/3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2
,1/2,b*x))*cos(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int x^m \sin(a+bx)\,dx = -\frac{e^{(-m\log(ib)-ia)}\Gamma(m+1,ibx)+e^{(-m\log(-ib)+ia)}\Gamma(m+1,-ibx)}{2b}$$

input
```
integrate(x^m*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*g
amma(m + 1, -I*b*x))/b
```

Sympy [F]

$$\int x^m \sin(a+bx)\,dx = \int x^m \sin(a+bx)\,dx$$

input
```
integrate(x**m*sin(b*x+a),x)
```

output
```
Integral(x**m*sin(a + b*x), x)
```

Maxima [F]

$$\int x^m \sin(a+bx)\,dx = \int x^m \sin(bx+a)\,dx$$

input
```
integrate(x^m*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*sin(b*x + a), x)
```

Giac [F]

$$\int x^m \sin(a+bx)\,dx = \int x^m \sin(bx+a)\,dx$$

input
```
integrate(x^m*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin(a+bx)\,dx = \int x^m \sin(a+b\,x)\,dx$$

input
```
int(x^m*sin(a + b*x),x)
```

output
```
int(x^m*sin(a + b*x), x)
```

Reduce [F]

$$\int x^m \sin(a+bx)\,dx = \int x^m \sin(bx+a)\,dx$$

input
```
int(x^m*sin(b*x+a),x)
```

output
```
int(x**m*sin(a + b*x),x)
```

3.81 $\int x^{-1+m} \sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int x^{-1+m}\sin(a+bx)\,dx=\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

output
```
1/2*I*exp(I*a)*x^m*GAMMA(m,-I*b*x)/((-I*b*x)^m)-1/2*I*x^m*GAMMA(m,I*b*x)/e
xp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int x^{-1+m}\sin(a+bx)\,dx=\frac{1}{2}ie^{-ia}x^m\left(e^{2ia}(-ibx)^{-m}\Gamma(m,-ibx)-(ibx)^{-m}\Gamma(m,ibx)\right)$$

input
```
Integrate[x^(-1 + m)*Sin[a + b*x],x]
```

output
```
((I/2)*x^m*((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - Gamma[m, I*b*x
]/(I*b*x)^m))/E^(I*a)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-1}\sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{2}i\int e^{-i(a+bx)}x^{m-1}dx-\frac{1}{2}i\int e^{i(a+bx)}x^{m-1}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

input
```
Int[x^(-1 + m)*Sin[a + b*x],x]
```

output
```
((I/2)*E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*x^m*Gamma[m,
I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 426, normalized size of antiderivative = 6.17

method	result
meijerg	$2^{-1+m}(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3x^{-1+m}2^{-m}(b^2)^{\frac{m}{2}}(2x^2b^2+2m+4)\sin(bx)}{\sqrt{\pi}\,m(6+3m)b}+\frac{2^{1-m}x^{-1+m}(b^2)^{\frac{m}{2}}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,mb}-\frac{3x^{2+m}2^{1}}{}\right.$

input
```
int(x^(-1+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^(-1+m)*2^(-m)*(b^2)^(1/2*
m)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi^(1/2)/m*x^(-1+m)*(b^2)^
(1/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*
m)*b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-1/Pi^(1/2)/
m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))
*LommelS1(m+1/2,1/2,b*x))*sin(a)+2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2
)/(1+m)*x^m*b^m*sin(b*x)-2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-si
n(b*x))-1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(m
+1/2,3/2,b*x)*sin(b*x)+1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-
5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int x^{-1+m}\sin(a+bx)\,dx = -\frac{e^{(-(m-1)\log(i\,b)-i\,a)}\Gamma(m,i\,bx)+e^{(-(m-1)\log(-i\,b)+i\,a)}\Gamma(m,-i\,bx)}{2\,b}$$

input
```
integrate(x^(-1+m)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) + e^(-(m - 1)*log(-I*b)
+ I*a)*gamma(m, -I*b*x))/b
```

Sympy [F]

$$\int x^{-1+m}\sin(a+bx)\,dx = \int x^{m-1}\sin\left(a+bx\right)\,dx$$

input
```
integrate(x**(-1+m)*sin(b*x+a),x)
```

output
```
Integral(x**(m - 1)*sin(a + b*x), x)
```

Maxima [F]

$$\int x^{-1+m}\sin(a+bx)\,dx = \int x^{m-1}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m - 1)*sin(b*x + a), x)
```

Giac [F]

$$\int x^{-1+m}\sin(a+bx)\,dx=\int x^{m-1}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\sin(a+bx)\,dx=\int x^{m-1}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x^(m - 1)*sin(a + b*x),x)
```

output
```
int(x^(m - 1)*sin(a + b*x), x)
```

Reduce [F]

$$\int x^{-1+m}\sin(a+bx)\,dx$$
$$=\frac{x^m\sin\left(bx+a\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+x^m\sin\left(bx+a\right)-2x^m\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2\left(\int\frac{x^m\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2x+x}dx\right)\tan\left(\frac{bx}{2}+\right.}{m\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)}$$

input
```
int(x^(-1+m)*sin(b*x+a),x)
```

output
```
(x**m*sin(a + b*x)*tan((a + b*x)/2)**2 + x**m*sin(a + b*x) - 2*x**m*tan((a
 + b*x)/2) + 2*int((x**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2*x + x),x)*
tan((a + b*x)/2)**2*m + 2*int((x**m*tan((a + b*x)/2))/(tan((a + b*x)/2)**2
*x + x),x)*m)/(m*(tan((a + b*x)/2)**2 + 1))
```

3.82 $\int x^{-2+m}\sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int x^{-2+m}\sin(a+bx)\,dx = \frac{1}{2}be^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) + \frac{1}{2}be^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)$$

output
```
1/2*b*exp(I*a)*x^m*GAMMA(-1+m,-I*b*x)/((-I*b*x)^m)+1/2*b*x^m*GAMMA(-1+m,I*
b*x)/exp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x^{-2+m}\sin(a+bx)\,dx = \frac{1}{2}be^{-ia}x^m\left(e^{2ia}(-ibx)^{-m}\Gamma(-1+m,-ibx) + (ibx)^{-m}\Gamma(-1+m,ibx)\right)$$

input
```
Integrate[x^(-2 + m)*Sin[a + b*x],x]
```

output
```
(b*x^m*((E^((2*I)*a)*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m + Gamma[-1 + m,
 I*b*x]/(I*b*x)^m))/(2*E^(I*a))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-2}\sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{2}i\int e^{-i(a+bx)}x^{m-2}dx-\frac{1}{2}i\int e^{i(a+bx)}x^{m-2}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{2}e^{ia}bx^m(-ibx)^{-m}\Gamma(m-1,-ibx)+\frac{1}{2}e^{-ia}bx^m(ibx)^{-m}\Gamma(m-1,ibx)$$

input
```
Int[x^(-2 + m)*Sin[a + b*x],x]
```

output
```
(b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b*x^m*Gamma[-1
 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 529, normalized size of antiderivative = 7.45

method	result
meijerg	$2^{-2+m}b^2(b^2)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{1-m}x^{-2+m}(b^2)^{-\frac{1}{2}+\frac{m}{2}}(2x^2b^2+2m+2)\sin(bx)}{\sqrt{\pi}\,(-1+m)(3+3m)b}-\frac{2^{2-m}x^{-2+m}(b^2)^{-\frac{1}{2}+\frac{m}{2}}(x^2b^2-m^2-m)(c}{\sqrt{\pi}\,(-1+m)b(1+m)m}\right.$

input
```
int(x^(-2+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(-2+m)*b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(1-m)/Pi^(1/2)/(-1+m)*x^(-2+
m)*(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)-2^(2-m)/Pi^(1/2
)/(-1+m)*x^(-2+m)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/(1+m)/m*(cos(b*x)*x
*b-sin(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(3+3
*m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(2-m)/Pi^(1/2)/(-1+m
)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(1+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(
b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)+2^(-2+m)*b^(1-m)*Pi^(1/2)*(2^(1-m)/P
i^(1/2)/m*x^(-1+m)*b^(-1+m)*(-2*b^2*x^2+2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)
-3*2^(2-m)/Pi^(1/2)/m*x^(-1+m)*b^(-1+m)/(-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2
^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3
/2,3/2,b*x)*sin(b*x)+3*2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(
-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int x^{-2+m}\sin(a+bx)\,dx = -\frac{e^{(-(m-2)\log(ib)-ia)}\Gamma(m-1,ibx)+e^{(-(m-2)\log(-ib)+ia)}\Gamma(m-1,-ibx)}{2b}$$

input
```
integrate(x^(-2+m)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) + e^(-(m - 2)*log(-I
*b) + I*a)*gamma(m - 1, -I*b*x))/b
```

Sympy [F]

$$\int x^{-2+m}\sin(a+bx)\,dx = \int x^{m-2}\sin{(a+bx)}\,dx$$

input
```
integrate(x**(-2+m)*sin(b*x+a),x)
```

output
```
Integral(x**(m - 2)*sin(a + b*x), x)
```

Maxima [F]

$$\int x^{-2+m}\sin(a+bx)\,dx = \int x^{m-2}\sin(bx+a)\,dx$$

input
```
integrate(x^(-2+m)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m - 2)*sin(b*x + a), x)
```

Giac [F]

$$\int x^{-2+m}\sin(a+bx)\,dx=\int x^{m-2}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x^(-2+m)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\sin(a+bx)\,dx=\int x^{m-2}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x^(m - 2)*sin(a + b*x),x)
```

output
```
int(x^(m - 2)*sin(a + b*x), x)
```

Reduce [F]

$$\int x^{-2+m}\sin(a+bx)\,dx=\int\frac{x^m\sin\left(bx+a\right)}{x^2}dx$$

input
```
int(x^(-2+m)*sin(b*x+a),x)
```

output
```
int((x**m*sin(a + b*x))/x**2,x)
```

3.83 $\int x^{-3+m} \sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int x^{-3+m}\sin(a+bx)\,dx = -\frac{1}{2}ib^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx) + \frac{1}{2}ib^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

output
```
-1/2*I*b^2*exp(I*a)*x^m*GAMMA(-2+m,-I*b*x)/((-I*b*x)^m)+1/2*I*b^2*x^m*GAMM
A(-2+m,I*b*x)/exp(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^{-3+m}\sin(a+bx)\,dx = -\frac{1}{2}ib^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx) + \frac{1}{2}ib^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

input
```
Integrate[x^(-3 + m)*Sin[a + b*x],x]
```

output
```
((-1/2*I)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b
^2*x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3} \sin(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-3} \sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{2}i \int e^{-i(a+bx)} x^{m-3} dx - \frac{1}{2}i \int e^{i(a+bx)} x^{m-3} dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{2}ie^{-ia}b^2x^m(ibx)^{-m}\Gamma(m-2,ibx) - \frac{1}{2}ie^{ia}b^2x^m(-ibx)^{-m}\Gamma(m-2,-ibx)$$

input
```
Int[x^(-3 + m)*Sin[a + b*x],x]
```

output

```
((-1/2*I)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b
^2*x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 599, normalized size of antiderivative = 7.58

method	result
meijerg	$2^{-3+m} b^{2} \left(b^{2}\right)^{-\frac{m}{2}} \sqrt{\pi}\, \left(\frac{2^{2-m} x^{-3+m} \left(b^{2}\right)^{\frac{m}{2}} \left(-2 x^{4} b^{4}+2 x^{2} b^{2} m^{2}+2 x^{2} b^{2} m-4 x^{2} b^{2}+2 m^{3}+2 m^{2}-4 m\right) \sin \left(b x\right)}{\sqrt{\pi}\, \left(-2+m\right) b^{3} m \left(2+m\right) \left(-1+m\right)}-\frac{2^{-m+3} x^{-3+}}{}\right.$

input
```
int(x^(-3+m)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(-3+m)*b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3
*(b^2)^(1/2*m)*(-2*b^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2
-4*m)/m/(2+m)/(-1+m)*sin(b*x)-2^(-m+3)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3*(b^2)^
(1/2*m)*(b^2*x^2-m^2+m)/m/(-1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)
/(-2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m
+3/2,3/2,b*x)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/
m/(-1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*s
in(a)+2^(-3+m)*b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(-2+m)*b^(-2+m)
*(-2*b^2*x^2+2*m^2-2*m-4)/(1+m)/(-2+m)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-1+m)*x
^(-2+m)*b^(-2+m)*(b^2*x^2-m^2-m)/(1+m)/(-2+m)/m*(cos(b*x)*x*b-sin(b*x))+2^
(-m+3)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)*(b*x)^(-3/2-m)*LommelS
1(m+1/2,3/2,b*x)*sin(b*x)-2^(-m+3)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(
-2+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*co
s(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int x^{-3+m}\sin(a+bx)\,dx$$
$$= -\frac{e^{\left(-(m-3)\log(i\,b)-i\,a\right)}\Gamma(m-2,i\,bx)+e^{\left(-(m-3)\log(-i\,b)+i\,a\right)}\Gamma(m-2,-i\,bx)}{2\,b}$$

input
```
integrate(x^(-3+m)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) + e^(-(m - 3)*log(-I
*b) + I*a)*gamma(m - 2, -I*b*x))/b
```

Sympy [F]

$$\int x^{-3+m}\sin(a+bx)\,dx=\int x^{m-3}\sin{(a+bx)}\,dx$$

input
```
integrate(x**(-3+m)*sin(b*x+a),x)
```

output
```
Integral(x**(m - 3)*sin(a + b*x), x)
```

Maxima [F]

$$\int x^{-3+m}\sin(a+bx)\,dx=\int x^{m-3}\sin{(bx+a)}\,dx$$

input
```
integrate(x^(-3+m)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m - 3)*sin(b*x + a), x)
```

Giac [F]

$$\int x^{-3+m}\sin(a+bx)\,dx=\int x^{m-3}\sin{(bx+a)}\,dx$$

input
```
integrate(x^(-3+m)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\sin(a+bx)\,dx=\int x^{m-3}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x^(m - 3)*sin(a + b*x),x)
```

output
```
int(x^(m - 3)*sin(a + b*x), x)
```

Reduce [F]

$$\int x^{-3+m}\sin(a+bx)\,dx=\int\frac{x^m\sin\left(bx+a\right)}{x^3}dx$$

input
```
int(x^(-3+m)*sin(b*x+a),x)
```

output
```
int((x**m*sin(a + b*x))/x**3,x)
```

3.84 $\int x^{3+m} \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x^{3+m}\sin^2(a+bx)\,dx = \frac{x^{4+m}}{2(4+m)} + \frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4} + \frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}$$

output
```
x^(4+m)/(8+2*m)+2^(-6-m)*exp(2*I*a)*x^m*GAMMA(4+m,-2*I*b*x)/b^4/((-I*b*x)^
m)+2^(-6-m)*x^m*GAMMA(4+m,2*I*b*x)/b^4/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int x^{3+m}\sin^2(a+bx)\,dx$$
$$= \frac{2^{-6-m}x^m(b^2x^2)^{-m}\left(2^{5+m}b^4x^4(b^2x^2)^m + (4+m)(-ibx)^m\Gamma(4+m,2ibx)(\cos(a)-i\sin(a))^2 + (4+m)(ib\right.}{b^4(4+m)}$$

input
```
Integrate[x^(3 + m)*Sin[a + b*x]^2,x]
```

output
```
(2^(-6 - m)*x^m*(2^(5 + m)*b^4*x^4*(b^2*x^2)^m + (4 + m)*((-I)*b*x)^m*Gamm
a[4 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + (4 + m)*(I*b*x)^m*Gamma[4 + m,
 (-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b^4*(4 + m)*(b^2*x^2)^m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+3}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^{m+3}}{2}-\frac{1}{2}x^{m+3}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\Gamma(m+4,-2ibx)}{b^4}+\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\Gamma(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

input
```
Int[x^(3 + m)*Sin[a + b*x]^2,x]
```

output
```
x^(4 + m)/(2*(4 + m)) + (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*
x])/(b^4*((-I)*b*x)^m) + (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^(
(2*I)*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{3+m} \sin(bx+a)^2 \, dx$$

input
```
int(x^(3+m)*sin(b*x+a)^2,x)
```

output
```
int(x^(3+m)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int x^{3+m} \sin^2(a+bx) \, dx = \frac{4\,bxx^{m+3} + (-i\,m-4i)e^{(-(m+3)\log(2i\,b)-2i\,a)}\Gamma(m+4, 2i\,bx) + (i\,m+4i)e^{(-(m+3)\log(-2i\,b)+2i\,a)}\Gamma(m+4, -2}{8\,(bm+4\,b)}$$

input
```
integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m + 3) + (-I*m - 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(
m + 4, 2*I*b*x) + (I*m + 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4
, -2*I*b*x))/(b*m + 4*b)
```

Sympy [F]

$$\int x^{3+m}\sin^2(a+bx)\,dx=\int x^{m+3}\sin^2{(a+bx)}\,dx$$

input
```
integrate(x**(3+m)*sin(b*x+a)**2,x)
```

output
```
Integral(x**(m + 3)*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{3+m}\sin^2(a+bx)\,dx=\int x^{m+3}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((m + 4)*integrate(x^3*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 4*log
(x)))/(m + 4)
```

Giac [F]

$$\int x^{3+m}\sin^2(a+bx)\,dx=\int x^{m+3}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\sin^2(a+bx)\,dx=\int x^{m+3}\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m + 3)*sin(a + b*x)^2,x)
```

output
```
int(x^(m + 3)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{3+m}\sin^2(a+bx)\,dx=\text{Too large to display}$$

input
```
int(x^(3+m)*sin(b*x+a)^2,x)
```

output

```
( - 6*x**m*cos(a + b*x)*sin(a + b*x)*b**3*m*x**3 - 24*x**m*cos(a + b*x)*si
n(a + b*x)*b**3*x**3 + 2*x**m*cos(a + b*x)*sin(a + b*x)*b*m**3*x + 18*x**m
*cos(a + b*x)*sin(a + b*x)*b*m**2*x + 52*x**m*cos(a + b*x)*sin(a + b*x)*b*
m*x + 48*x**m*cos(a + b*x)*sin(a + b*x)*b*x + 2*x**m*cos(a + b*x)*m**4 + 2
0*x**m*cos(a + b*x)*m**3 + 70*x**m*cos(a + b*x)*m**2 + 100*x**m*cos(a + b*
x)*m + 48*x**m*cos(a + b*x) + 3*x**m*sin(a + b*x)**2*b**2*m**2*x**2 + 21*x
**m*sin(a + b*x)**2*b**2*m*x**2 + 36*x**m*sin(a + b*x)**2*b**2*x**2 - x**m
*sin(a + b*x)**2*m**4 - 10*x**m*sin(a + b*x)**2*m**3 - 35*x**m*sin(a + b*x
)**2*m**2 - 50*x**m*sin(a + b*x)**2*m - 24*x**m*sin(a + b*x)**2 + 2*x**m*s
in(a + b*x)*b*m**3*x + 18*x**m*sin(a + b*x)*b*m**2*x + 52*x**m*sin(a + b*x
)*b*m*x + 48*x**m*sin(a + b*x)*b*x + 6*x**m*b**4*x**4 + 2*x**m*m**4 + 20*x
**m*m**3 + 70*x**m*m**2 + 100*x**m*m + 48*x**m - 4*int(x**m/(tan((a + b*x)
/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)*m**5 - 40*int(x**m/(tan((a + b*
x)/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)*m**4 - 140*int(x**m/(tan((a +
 b*x)/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)*m**3 - 200*int(x**m/(tan((
a + b*x)/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)*m**2 - 96*int(x**m/(tan
((a + b*x)/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)*m - 4*int((x**m*x)/(t
an((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b**2*m**3 - 36*int((x**
m*x)/(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b**2*m**2 - 104*
int((x**m*x)/(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b**2*...
```

3.85 $\int x^{2+m}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 103

$$\int x^{2+m}\sin^2(a+bx)\,dx = \frac{x^{3+m}}{2(3+m)} - \frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3} + \frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}$$

output
```
x^(3+m)/(6+2*m)-I*2^(-5-m)*exp(2*I*a)*x^m*GAMMA(3+m,-2*I*b*x)/b^3/((-I*b*x
)^m)+I*2^(-5-m)*x^m*GAMMA(3+m,2*I*b*x)/b^3/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

$$\int x^{2+m}\sin^2(a+bx)\,dx = \frac{2^{-5-m}x^m(b^2x^2)^{-m}\left(2^{4+m}bx(b^2x^2)^{1+m} + (3+m)(ibx)^m\Gamma(3+m,-2ibx)(-i\cos(2a)+\sin(2a)) + (3+m\right.}{b^3(3+m)}$$

input
```
Integrate[x^(2 + m)*Sin[a + b*x]^2,x]
```

output
```
(2^(-5 - m)*x^m*(2^(4 + m)*b*x*(b^2*x^2)^(1 + m) + (3 + m)*(I*b*x)^m*Gamma
[3 + m, (-2*I)*b*x]*((-I)*Cos[2*a] + Sin[2*a]) + (3 + m)*((-I)*b*x)^m*Gamm
a[3 + m, (2*I)*b*x]*(I*Cos[2*a] + Sin[2*a])))/(b^3*(3 + m)*(b^2*x^2)^m)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+2}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^{m+2}}{2}-\frac{1}{2}x^{m+2}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\Gamma(m+3,-2ibx)}{b^3}+\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\Gamma(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

input
```
Int[x^(2 + m)*Sin[a + b*x]^2,x]
```

output
```
x^(3 + m)/(2*(3 + m)) - (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*
b*x])/(b^3*((-I)*b*x)^m) + (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3
*E^((2*I)*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{2+m} \sin(bx+a)^2 \, dx$$

input
```
int(x^(2+m)*sin(b*x+a)^2,x)
```

output
```
int(x^(2+m)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int x^{2+m} \sin^2(a+bx) \, dx = \frac{4\,bxx^{m+2} + (-i\,m-3i)e^{(-(m+2)\log(2i\,b)-2i\,a)}\Gamma(m+3, 2i\,bx) + (i\,m+3i)e^{(-(m+2)\log(-2i\,b)+2i\,a)}\Gamma(m+3, -2}{8\,(bm+3\,b)}$$

input
```
integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m + 2) + (-I*m - 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(
m + 3, 2*I*b*x) + (I*m + 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3
, -2*I*b*x))/(b*m + 3*b)
```

Sympy [F]

$$\int x^{2+m}\sin^2(a+bx)\,dx = \int x^{m+2}\sin^2{(a+bx)}\,dx$$

input
```
integrate(x**(2+m)*sin(b*x+a)**2,x)
```

output
```
Integral(x**(m + 2)*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{2+m}\sin^2(a+bx)\,dx = \int x^{m+2}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 3*log
(x)))/(m + 3)
```

Giac [F]

$$\int x^{2+m}\sin^2(a+bx)\,dx = \int x^{m+2}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\sin^2(a+bx)\,dx=\int x^{m+2}\sin{(a+b\,x)}^2\,dx$$

input
```
int(x^(m + 2)*sin(a + b*x)^2,x)
```

output
```
int(x^(m + 2)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{2+m}\sin^2(a+bx)\,dx=\text{Too large to display}$$

input
```
int(x^(2+m)*sin(b*x+a)^2,x)
```

output

```
(3*x**m*tan((a + b*x)/2)**4*b**3*x**3 + 6*x**m*tan((a + b*x)/2)**3*b**2*m*
x**2 + 18*x**m*tan((a + b*x)/2)**3*b**2*x**2 + 6*x**m*tan((a + b*x)/2)**2*
b**3*x**3 + 6*x**m*tan((a + b*x)/2)**2*b*m**2*x + 30*x**m*tan((a + b*x)/2)
**2*b*m*x + 36*x**m*tan((a + b*x)/2)**2*b*x - 6*x**m*tan((a + b*x)/2)*b**2
*m*x**2 - 18*x**m*tan((a + b*x)/2)*b**2*x**2 + 4*x**m*tan((a + b*x)/2)*m**
3 + 24*x**m*tan((a + b*x)/2)*m**2 + 44*x**m*tan((a + b*x)/2)*m + 24*x**m*t
an((a + b*x)/2) + 3*x**m*b**3*x**3 - 2*int(x**m/(tan((a + b*x)/2)**4 + 2*t
an((a + b*x)/2)**2 + 1),x)*tan((a + b*x)/2)**4*b*m**3 - 12*int(x**m/(tan((
a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*tan((a + b*x)/2)**4*b*m**2
- 22*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*tan((a
+ b*x)/2)**4*b*m - 12*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**
2 + 1),x)*tan((a + b*x)/2)**4*b - 4*int(x**m/(tan((a + b*x)/2)**4 + 2*tan(
(a + b*x)/2)**2 + 1),x)*tan((a + b*x)/2)**2*b*m**3 - 24*int(x**m/(tan((a +
 b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*tan((a + b*x)/2)**2*b*m**2 - 4
4*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*tan((a + b
*x)/2)**2*b*m - 24*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 +
 1),x)*tan((a + b*x)/2)**2*b - 2*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a
+ b*x)/2)**2 + 1),x)*b*m**3 - 12*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a
+ b*x)/2)**2 + 1),x)*b*m**2 - 22*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a
+ b*x)/2)**2 + 1),x)*b*m - 12*int(x**m/(tan((a + b*x)/2)**4 + 2*tan((a ...
```

3.86 $\int x^{1+m}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^{1+m}\sin^2(a+bx)\,dx = \frac{x^{2+m}}{2(2+m)} - \frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2} - \frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}$$

output
```
x^(2+m)/(4+2*m)-2^(-4-m)*exp(2*I*a)*x^m*GAMMA(2+m,-2*I*b*x)/b^2/((-I*b*x)^
m)-2^(-4-m)*x^m*GAMMA(2+m,2*I*b*x)/b^2/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int x^{1+m}\sin^2(a+bx)\,dx = \frac{2^{-4-m}x^m(b^2x^2)^{-m}\left(2^{3+m}(b^2x^2)^{1+m} - (2+m)(-ibx)^m\Gamma(2+m,2ibx)(\cos(a)-i\sin(a))^2 - (2+m)(ibx\right.}{b^2(2+m)}$$

input
```
Integrate[x^(1 + m)*Sin[a + b*x]^2,x]
```

output
```
(2^(-4 - m)*x^m*(2^(3 + m)*(b^2*x^2)^(1 + m) - (2 + m)*((-I)*b*x)^m*Gamma[
2 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 - (2 + m)*(I*b*x)^m*Gamma[2 + m, (
-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b^2*(2 + m)*(b^2*x^2)^m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+1}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^{m+1}}{2}-\frac{1}{2}x^{m+1}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\Gamma(m+2,-2ibx)}{b^2}-\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\Gamma(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

input
```
Int[x^(1 + m)*Sin[a + b*x]^2,x]
```

output
```
x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*
x])/(b^2*((-I)*b*x)^m) - (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^(
(2*I)*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{1+m} \sin (bx+a)^2 \, dx$$

input
```
int(x^(1+m)*sin(b*x+a)^2,x)
```

output
```
int(x^(1+m)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int x^{1+m} \sin^2(a+bx) \, dx = \frac{4\,bxx^{m+1} + (-i\,m-2i)e^{(-(m+1)\log(2i\,b)-2i\,a)}\Gamma(m+2,2i\,bx) + (i\,m+2i)e^{(-(m+1)\log(-2i\,b)+2i\,a)}\Gamma(m+2,-2}{8\,(bm+2\,b)}$$

input
```
integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m + 1) + (-I*m - 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(
m + 2, 2*I*b*x) + (I*m + 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2
, -2*I*b*x))/(b*m + 2*b)
```

Sympy [F]

$$\int x^{1+m}\sin^2(a+bx)\,dx=\int x^{m+1}\sin^2{(a+bx)}\,dx$$

input
```
integrate(x**(1+m)*sin(b*x+a)**2,x)
```

output
```
Integral(x**(m + 1)*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{1+m}\sin^2(a+bx)\,dx=\int x^{m+1}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 2*log(x
)))/(m + 2)
```

Giac [F]

$$\int x^{1+m}\sin^2(a+bx)\,dx=\int x^{m+1}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\sin^2(a+bx)\,dx=\int x^{m+1}\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m + 1)*sin(a + b*x)^2,x)
```

output
```
int(x^(m + 1)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{1+m}\sin^2(a+bx)\,dx=\int x^m\sin\left(bx+a\right)^2xdx$$

input
```
int(x^(1+m)*sin(b*x+a)^2,x)
```

output
```
int(x**m*sin(a + b*x)**2*x,x)
```

3.87 $\int x^m \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 103

$$\int x^m \sin^2(a+bx)\,dx = \frac{x^{1+m}}{2(1+m)} + \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b} - \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}$$

output
```
x^(1+m)/(2+2*m)+I*2^(-3-m)*exp(2*I*a)*x^m*GAMMA(1+m,-2*I*b*x)/b/((-I*b*x)^
m)-I*2^(-3-m)*x^m*GAMMA(1+m,2*I*b*x)/b/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

$$\int x^m \sin^2(a+bx)\,dx = \frac{2^{-3-m}x^m(b^2x^2)^{-m}\left(2^{2+m}bx(b^2x^2)^m - i(1+m)(-ibx)^m\Gamma(1+m,2ibx)(\cos(a)-i\sin(a))^2 + i(1+m)(ib\right.}{b(1+m)}$$

input
```
Integrate[x^m*Sin[a + b*x]^2,x]
```

output
```
(2^(-3 - m)*x^m*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I)*b*x)^m*Gamma[
1 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)^m*Gamma[1 + m,
 (-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b*(1 + m)*(b^2*x^2)^m)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\Gamma(m+1,-2ibx)}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\Gamma(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

input
```
Int[x^m*Sin[a + b*x]^2,x]
```

output
```
x^(1 + m)/(2*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*
b*x])/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^(
(2*I)*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^m \sin(bx+a)^2 \, dx$$

input
```
int(x^m*sin(b*x+a)^2,x)
```

output
```
int(x^m*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67

$$\int x^m \sin^2(a+bx) \, dx = \frac{4\,bxx^m + (-i\,m-i)e^{(-m\log(2i\,b)-2i\,a)}\Gamma(m+1,2i\,bx) + (i\,m+i)e^{(-m\log(-2i\,b)+2i\,a)}\Gamma(m+1,-2i\,bx)}{8\,(bm+b)}$$

input
```
integrate(x^m*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^m + (-I*m - I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x
) + (I*m + I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

Sympy [F]

$$\int x^m \sin^2(a+bx)\,dx = \int x^m \sin^2{(a+bx)}\,dx$$

input
```
integrate(x**m*sin(b*x+a)**2,x)
```

output
```
Integral(x**m*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^m \sin^2(a+bx)\,dx = \int x^m \sin{(bx+a)}^2\,dx$$

input
```
integrate(x^m*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))/
(m + 1)
```

Giac [F]

$$\int x^m \sin^2(a+bx)\,dx = \int x^m \sin{(bx+a)}^2\,dx$$

input
```
integrate(x^m*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin^2(a+bx)\,dx = \int x^m \sin{(a+b\,x)}^2\,dx$$

input
```
int(x^m*sin(a + b*x)^2,x)
```

output
```
int(x^m*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^m \sin^2(a+bx)\,dx = \int x^m \sin{(bx+a)}^2\,dx$$

input
```
int(x^m*sin(b*x+a)^2,x)
```

output
```
int(x**m*sin(a + b*x)**2,x)
```

3.88 $\int x^{-1+m}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\frac{x^m}{2m}+2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx)+2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)$$

output
```
1/2*x^m/m+2^(-2-m)*exp(2*I*a)*x^m*GAMMA(m,-2*I*b*x)/((-I*b*x)^m)+2^(-2-m)*
x^m*GAMMA(m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\frac{x^m\left(2+2^{-m}e^{2ia}m(-ibx)^{-m}\Gamma(m,-2ibx)+2^{-m}e^{-2ia}m(ibx)^{-m}\Gamma(m,2ibx)\right)}{4m}$$

input
```
Integrate[x^(-1 + m)*Sin[a + b*x]^2,x]
```

output
```
(x^m*(2 + (E^((2*I)*a)*m*Gamma[m, (-2*I)*b*x])/(2^m*((-I)*b*x)^m) + (m*Gam
ma[m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*b*x)^m)))/(4*m)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-1}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^{m-1}}{2}-\frac{1}{2}x^{m-1}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2ia}2^{-m-2}x^m(-ibx)^{-m}\Gamma(m,-2ibx)+e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\Gamma(m,2ibx)+\frac{x^m}{2m}$$

input
```
Int[x^(-1 + m)*Sin[a + b*x]^2,x]
```

output
```
x^m/(2*m) + (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
 + (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-1+m} \sin(bx+a)^2 \, dx$$

input
```
int(x^(-1+m)*sin(b*x+a)^2,x)
```

output
```
int(x^(-1+m)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int x^{-1+m} \sin^2(a+bx) \, dx = \frac{4\, bxx^{m-1} - i\, m e^{(-(m-1)\log(2i\, b)-2i\, a)}\Gamma(m, 2i\, bx) + i\, m e^{(-(m-1)\log(-2i\, b)+2i\, a)}\Gamma(m, -2i\, bx)}{8\, bm}$$

input
```
integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m - 1) - I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*
x) + I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

Sympy [F]

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\int x^{m-1}\sin^2{(a+bx)}\,dx$$

input
```
integrate(x**(-1+m)*sin(b*x+a)**2,x)
```

output
```
Integral(x**(m - 1)*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\int x^{m-1}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) - x^m)/m
```

Giac [F]

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\int x^{m-1}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\int x^{m-1}\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m - 1)*sin(a + b*x)^2,x)
```

output
```
int(x^(m - 1)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-1+m}\sin^2(a+bx)\,dx=\int\frac{x^m\sin\left(bx+a\right)^2}{x}dx$$

input
```
int(x^(-1+m)*sin(b*x+a)^2,x)
```

output
```
int((x**m*sin(a + b*x)**2)/x,x)
```

3.89 $\int x^{-2+m} \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x^{-2+m}\sin^2(a+bx)\,dx = -\frac{x^{-1+m}}{2(1-m)} - i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx) + i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx)$$

output
```
-1/2*x^(-1+m)/(1-m)-I*2^(-1-m)*b*exp(2*I*a)*x^m*GAMMA(-1+m,-2*I*b*x)/((-I*
b*x)^m)+I*2^(-1-m)*b*x^m*GAMMA(-1+m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int x^{-2+m}\sin^2(a+bx)\,dx = -\frac{1}{2}x^m\left(\frac{1}{x-mx} + i2^{-m}be^{2ia}(-ibx)^{-m}\Gamma(-1+m,-2ibx) - i2^{-m}be^{-2ia}(ibx)^{-m}\Gamma(-1+m,2ibx)\right)$$

input
```
Integrate[x^(-2 + m)*Sin[a + b*x]^2,x]
```

output
```
-1/2*(x^m*((x - m*x)^(-1) + (I*b*E^((2*I)*a)*Gamma[-1 + m, (-2*I)*b*x])/(2
^m*((-I)*b*x)^m) - (I*b*Gamma[-1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*b*x)
^m)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-2}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^{m-2}}{2}-\frac{1}{2}x^{m-2}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\Gamma(m-1,-2ibx)+ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\Gamma(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

input
```
Int[x^(-2 + m)*Sin[a + b*x]^2,x]
```

output
```
-1/2*x^(-1 + m)/(1 - m) - (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-
2*I)*b*x])/((-I)*b*x)^m + (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E
^((2*I)*a)*(I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-2+m}\sin(bx+a)^2\,dx$$

input
```
int(x^(-2+m)*sin(b*x+a)^2,x)
```

output
```
int(x^(-2+m)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int x^{-2+m}\sin^2(a+bx)\,dx = \frac{4\,bxx^{m-2}+(-i\,m+i)e^{(-(m-2)\log(2i\,b)-2i\,a)}\Gamma(m-1,2i\,bx)+(i\,m-i)e^{(-(m-2)\log(-2i\,b)+2i\,a)}\Gamma(m-1,-2i\,b}{8\,(bm-b)}$$

input
```
integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m - 2) + (-I*m + I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m
- 1, 2*I*b*x) + (I*m - I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2
*I*b*x))/(b*m - b)
```

Sympy [F]

$$\int x^{-2+m} \sin^2(a+bx)\,dx = \int x^{m-2} \sin^2{(a+bx)}\,dx$$

input
```
integrate(x**(-2+m)*sin(b*x+a)**2,x)
```

output
```
Integral(x**(m - 2)*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{-2+m} \sin^2(a+bx)\,dx = \int x^{m-2} \sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) - x^m)/((m - 1)*x)
```

Giac [F]

$$\int x^{-2+m} \sin^2(a+bx)\,dx = \int x^{m-2} \sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\sin^2(a+bx)\,dx=\int x^{m-2}\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m - 2)*sin(a + b*x)^2,x)
```

output
```
int(x^(m - 2)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-2+m}\sin^2(a+bx)\,dx=\int\frac{x^m\sin\left(bx+a\right)^2}{x^2}dx$$

input
```
int(x^(-2+m)*sin(b*x+a)^2,x)
```

output
```
int((x**m*sin(a + b*x)**2)/x**2,x)
```

3.90 $\int x^{-3+m} \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x^{-3+m}\sin^2(a+bx)\,dx = -\frac{x^{-2+m}}{2(2-m)} - 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) - 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)$$

output
```
-1/2*x^(-2+m)/(2-m)-b^2*exp(2*I*a)*x^m*GAMMA(-2+m,-2*I*b*x)/(2^m)/((-I*b*x
)^m)-b^2*x^m*GAMMA(-2+m,2*I*b*x)/(2^m)/exp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int x^{-3+m}\sin^2(a+bx)\,dx = -\frac{1}{2}x^m\left(-\frac{1}{(-2+m)x^2} + 2^{1-m}b^2e^{2ia}(-ibx)^{-m}\Gamma(-2+m,-2ibx) + 2^{1-m}b^2e^{-2ia}(ibx)^{-m}\Gamma(-2+m,2ibx)\right)$$

input
```
Integrate[x^(-3 + m)*Sin[a + b*x]^2,x]
```

output
```
-1/2*(x^m*(-(1/((-2 + m)*x^2)) + (2^(1 - m)*b^2*E^((2*I)*a)*Gamma[-2 + m,
(-2*I)*b*x])/((-I)*b*x)^m + (2^(1 - m)*b^2*Gamma[-2 + m, (2*I)*b*x])/(E^((
2*I)*a)*(I*b*x)^m)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-3}\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{x^{m-3}}{2}-\frac{1}{2}x^{m-3}\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-e^{2ia}b^22^{-m}x^m(-ibx)^{-m}\Gamma(m-2,-2ibx)-e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\Gamma(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

input
```
Int[x^(-3 + m)*Sin[a + b*x]^2,x]
```

output
```
-1/2*x^(-2 + m)/(2 - m) - (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/
(2^m*((-I)*b*x)^m) - (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(
I*b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-3+m} \sin (bx+a)^2 \, dx$$

input
```
int(x^(-3+m)*sin(b*x+a)^2,x)
```

output
```
int(x^(-3+m)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int x^{-3+m} \sin^2(a+bx) \, dx = \frac{4\,bxx^{m-3} + (-i\,m+2i)e^{(-(m-3)\log(2i\,b)-2i\,a)}\Gamma(m-2,2i\,bx) + (i\,m-2i)e^{(-(m-3)\log(-2i\,b)+2i\,a)}\Gamma(m-2,-2}{8\,(bm-2\,b)}$$

input
```
integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m - 3) + (-I*m + 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(
m - 2, 2*I*b*x) + (I*m - 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2
, -2*I*b*x))/(b*m - 2*b)
```

Sympy [F]

$$\int x^{-3+m}\sin^2(a+bx)\,dx = \int x^{m-3}\sin^2{(a+bx)}\,dx$$

input
```
integrate(x**(-3+m)*sin(b*x+a)**2,x)
```

output
```
Integral(x**(m - 3)*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{-3+m}\sin^2(a+bx)\,dx = \int x^{m-3}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) - x^m)/((m - 2)*x
^2)
```

Giac [F]

$$\int x^{-3+m}\sin^2(a+bx)\,dx = \int x^{m-3}\sin{(bx+a)}^2\,dx$$

input
```
integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\sin^2(a+bx)\,dx=\int x^{m-3}\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m - 3)*sin(a + b*x)^2,x)
```

output
```
int(x^(m - 3)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-3+m}\sin^2(a+bx)\,dx=\int \frac{x^m\sin\left(bx+a\right)^2}{x^3}dx$$

input
```
int(x^(-3+m)*sin(b*x+a)^2,x)
```

output
```
int((x**m*sin(a + b*x)**2)/x**3,x)
```

3.91 $\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)} - \frac{1}{3}x\sqrt{\csc(e+fx)}\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 42

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)} - \frac{1}{3}x\sqrt{\csc(e+fx)}\right) dx = \frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

output
```
4/9/f^2/csc(f*x+e)^(3/2)-2/3*x*cos(f*x+e)/f/csc(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)} - \frac{1}{3}x\sqrt{\csc(e+fx)}\right) dx = -\frac{2(-2+3fx\cot(e+fx))}{9f^2\csc^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[x/Csc[e + f*x]^(3/2) - (x*Sqrt[Csc[e + f*x]])/3,x]
```

output
```
(-2*(-2 + 3*f*x*Cot[e + f*x]))/(9*f^2*Csc[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

input
```
Int[x/Csc[e + f*x]^(3/2) - (x*Sqrt[Csc[e + f*x]])/3,x]
```

output
```
4/(9*f^2*Csc[e + f*x]^(3/2)) - (2*x*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\csc\left(fx+e\right)^{\frac{3}{2}}}-\frac{x\sqrt{\csc\left(fx+e\right)}}{3}\right)dx$$

input
```
int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
```

output
```
int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)\,dx = -\frac{\int \left(-\frac{3x}{\csc^{\frac{3}{2}}(e+fx)}\right)\,dx+\int x\sqrt{\csc(e+fx)}\,dx}{3}$$

input
```
integrate(x/csc(f*x+e)**(3/2)-1/3*x*csc(f*x+e)**(1/2),x)
```

output
```
-(Integral(-3*x/csc(e + f*x)**(3/2), x) + Integral(x*sqrt(csc(e + f*x)), x
))/3
```

Maxima [F]

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)\,dx = \int -\frac{1}{3}\,x\sqrt{\csc(fx+e)}+\frac{x}{\csc(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)\,dx=\int -\frac{1}{3}\,x\sqrt{\csc\left(fx+e\right)}+\frac{x}{\csc\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)\,dx=\int \frac{x}{\left(\frac{1}{\sin(e+f\,x)}\right)^{3/2}}-\frac{x\,\sqrt{\frac{1}{\sin(e+f\,x)}}}{3}\,dx$$

input
```
int(x/(1/sin(e + f*x))^(3/2) - (x*(1/sin(e + f*x))^(1/2))/3,x)
```

output
```
int(x/(1/sin(e + f*x))^(3/2) - (x*(1/sin(e + f*x))^(1/2))/3, x)
```

Reduce [F]

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)\,dx=\int \frac{\sqrt{\csc\left(fx+e\right)}\,x}{\csc\left(fx+e\right)^{2}}dx-\frac{\left(\int \sqrt{\csc\left(fx+e\right)}\,xdx\right)}{3}$$

input
```
int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
```

output
```
(3*int((sqrt(csc(e + f*x))*x)/csc(e + f*x)**2,x) - int(sqrt(csc(e + f*x))*
x,x))/3
```

3.92 $\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 111

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)\,dx = \frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}} - \frac{16\sqrt{\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{27f^3}$$

output
```
8/9*x/f^2/csc(f*x+e)^(3/2)+16/27*cos(f*x+e)/f^3/csc(f*x+e)^(1/2)-2/3*x^2*c
os(f*x+e)/f/csc(f*x+e)^(1/2)-16/27*csc(f*x+e)^(1/2)*InverseJacobiAM(1/2*e-
1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/f^3
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx =$$
$$-\frac{\sqrt{\csc(e+fx)}\left(-12fx+12fx\cos(2(e+fx))-16\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}-\right.}{27f^3}$$

input
```
Integrate[x^2/Csc[e + f*x]^(3/2) - (x^2*Sqrt[Csc[e + f*x]])/3,x]
```

output
```
-1/27*(Sqrt[Csc[e + f*x]]*(-12*f*x + 12*f*x*Cos[2*(e + f*x)] - 16*Elliptic
F[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]] - 8*Sin[2*(e + f*x)] + 9*f^
2*x^2*Sin[2*(e + f*x)]))/f^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{16\sqrt{\sin(e+fx)}\sqrt{\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)}{27f^3}+\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

input
```
Int[x^2/Csc[e + f*x]^(3/2) - (x^2*Sqrt[Csc[e + f*x]])/3,x]
```

output
```
(8*x)/(9*f^2*Csc[e + f*x]^(3/2)) + (16*Cos[e + f*x])/(27*f^3*Sqrt[Csc[e +
f*x]]) - (2*x^2*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]]) - (16*Sqrt[Csc[e +
f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(27*f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\csc\left(fx+e\right)^{\frac{3}{2}}}-\frac{x^2\sqrt{\csc\left(fx+e\right)}}{3}\right)dx$$

input
```
int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

output
```
int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="fric
as")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx = -\frac{\int \left(-\frac{3x^2}{\csc^{\frac{3}{2}}(e+fx)}\right)dx+\int x^2\sqrt{\csc(e+fx)}\,dx}{3}$$

input
```
integrate(x**2/csc(f*x+e)**(3/2)-1/3*x**2*csc(f*x+e)**(1/2),x)
```

output
```
-(Integral(-3*x**2/csc(e + f*x)**(3/2), x) + Integral(x**2*sqrt(csc(e + f*
x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx = \int -\frac{1}{3}x^2\sqrt{\csc(fx+e)}+\frac{x^2}{\csc(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(-1/3*x^2*sqrt(csc(f*x + e)) + x^2/csc(f*x + e)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx = \int -\frac{1}{3}x^2\sqrt{\csc(fx+e)}+\frac{x^2}{\csc(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="giac
")
```

output
```
integrate(-1/3*x^2*sqrt(csc(f*x + e)) + x^2/csc(f*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)\,dx=\int \frac{x^2}{\left(\frac{1}{\sin(e+f\,x)}\right)^{3/2}}-\frac{x^2\,\sqrt{\frac{1}{\sin(e+f\,x)}}}{3}\,dx$$

input
```
int(x^2/(1/sin(e + f*x))^(3/2) - (x^2*(1/sin(e + f*x))^(1/2))/3,x)
```

output
```
int(x^2/(1/sin(e + f*x))^(3/2) - (x^2*(1/sin(e + f*x))^(1/2))/3, x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)\,dx$$
$$=\int \frac{\sqrt{\csc(fx+e)}\,x^2}{\csc(fx+e)^2}dx-\frac{\left(\int \sqrt{\csc(fx+e)}\,x^2dx\right)}{3}$$

input
```
int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

output
```
(3*int((sqrt(csc(e + f*x))*x**2)/csc(e + f*x)**2,x) - int(sqrt(csc(e + f*x
))*x**2,x))/3
```

3.93 $\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 42

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=\frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}$$

output
```
4/25/f^2/csc(f*x+e)^(5/2)-2/5*x*cos(f*x+e)/f/csc(f*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=-\frac{2(-2+5fx\cot(e+fx))}{25f^2\csc^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[x/Csc[e + f*x]^(5/2) - (3*x)/(5*Sqrt[Csc[e + f*x]]),x]
```

output
```
(-2*(-2 + 5*f*x*Cot[e + f*x]))/(25*f^2*Csc[e + f*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}$$

input
```
Int[x/Csc[e + f*x]^(5/2) - (3*x)/(5*Sqrt[Csc[e + f*x]]),x]
```

output
```
4/(25*f^2*Csc[e + f*x]^(5/2)) - (2*x*Cos[e + f*x])/(5*f*Csc[e + f*x]^(3/2)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\csc(fx+e)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\csc(fx+e)}}\right)dx$$

input
```
int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
```

output
```
int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=-\frac{\int \left(-\frac{5x}{\csc^{\frac{5}{2}}(e+fx)}\right)dx+\int \frac{3x}{\sqrt{\csc(e+fx)}}\,dx}{5}$$

input
```
integrate(x/csc(f*x+e)**(5/2)-3/5*x/csc(f*x+e)**(1/2),x)
```

output
```
-(Integral(-5*x/csc(e + f*x)**(5/2), x) + Integral(3*x/sqrt(csc(e + f*x)),
 x))/5
```

Maxima [F]

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=\int -\frac{3\,x}{5\,\sqrt{\csc\left(fx+e\right)}}+\frac{x}{\csc\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-3/5*x/sqrt(csc(f*x + e)) + x/csc(f*x + e)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=\int -\frac{3\,x}{5\,\sqrt{\csc\left(fx+e\right)}}+\frac{x}{\csc\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(-3/5*x/sqrt(csc(f*x + e)) + x/csc(f*x + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx=\int \frac{x}{\left(\frac{1}{\sin(e+f\,x)}\right)^{5/2}}-\frac{3\,x}{5\,\sqrt{\frac{1}{\sin(e+f\,x)}}}\,dx$$

input
```
int(x/(1/sin(e + f*x))^(5/2) - (3*x)/(5*(1/sin(e + f*x))^(1/2)),x)
```

output
```
int(x/(1/sin(e + f*x))^(5/2) - (3*x)/(5*(1/sin(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)\,dx$$
$$=-\frac{3\left(\int \frac{\sqrt{\csc(fx+e)}\,x}{\csc(fx+e)}dx\right)}{5}+\int \frac{\sqrt{\csc\left(fx+e\right)}\,x}{\csc\left(fx+e\right)^{3}}dx$$

input
```
int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
```

output
```
( - 3*int((sqrt(csc(e + f*x))*x)/csc(e + f*x),x) + 5*int((sqrt(csc(e + f*x
))*x)/csc(e + f*x)**3,x))/5
```

3.94 $\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)} - \frac{5}{21}x\sqrt{\csc(e+fx)}\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)} - \frac{5}{21}x\sqrt{\csc(e+fx)}\right) dx = \frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)} + \frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)} - \frac{10x\cos(e+fx)}{21f\sqrt{\csc(e+fx)}}$$

output
```
4/49/f^2/csc(f*x+e)^(7/2)-2/7*x*cos(f*x+e)/f/csc(f*x+e)^(5/2)+20/63/f^2/cs
c(f*x+e)^(3/2)-10/21*x*cos(f*x+e)/f/csc(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 6.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)} - \frac{5}{21}x\sqrt{\csc(e+fx)}\right) dx = \frac{316 - 36\cos(2(e+fx)) - 483fx\cot(e+fx) + 63fx\cos(3(e+fx))\csc(e+fx)}{882f^2\csc^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[x/Csc[e + f*x]^(7/2) - (5*x*Sqrt[Csc[e + f*x]])/21,x]
```

output
```
(316 - 36*Cos[2*(e + f*x)] - 483*f*x*Cot[e + f*x] + 63*f*x*Cos[3*(e + f*x)
]*Csc[e + f*x])/(882*f^2*Csc[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)}-\frac{10x\cos(e+fx)}{21f\sqrt{\csc(e+fx)}}$$

input
```
Int[x/Csc[e + f*x]^(7/2) - (5*x*Sqrt[Csc[e + f*x]])/21,x]
```

output
```
4/(49*f^2*Csc[e + f*x]^(7/2)) - (2*x*Cos[e + f*x])/(7*f*Csc[e + f*x]^(5/2)
) + 20/(63*f^2*Csc[e + f*x]^(3/2)) - (10*x*Cos[e + f*x])/(21*f*Sqrt[Csc[e
+ f*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\csc\left(fx+e\right)^{\frac{7}{2}}}-\frac{5x\sqrt{\csc\left(fx+e\right)}}{21}\right)dx$$

input
```
int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
```

output
```
int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int\left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)dx = -\frac{\int\left(-\frac{21x}{\csc^{\frac{7}{2}}(e+fx)}\right)dx+\int 5x\sqrt{\csc(e+fx)}\,dx}{21}$$

input
```
integrate(x/csc(f*x+e)**(7/2)-5/21*x*csc(f*x+e)**(1/2),x)
```

output
```
-(Integral(-21*x/csc(e + f*x)**(7/2), x) + Integral(5*x*sqrt(csc(e + f*x))
, x))/21
```

Maxima [F]

$$\int\left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)dx = \int-\frac{5}{21}x\sqrt{\csc(fx+e)}+\frac{x}{\csc(fx+e)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(-5/21*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)\,dx$$
$$=\int -\frac{5}{21}\,x\sqrt{\csc\left(fx+e\right)}+\frac{x}{\csc\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(-5/21*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)\,dx=\int \frac{x}{\left(\frac{1}{\sin(e+f\,x)}\right)^{7/2}}-\frac{5\,x\,\sqrt{\frac{1}{\sin(e+f\,x)}}}{21}\,dx$$

input
```
int(x/(1/sin(e + f*x))^(7/2) - (5*x*(1/sin(e + f*x))^(1/2))/21,x)
```

output
```
int(x/(1/sin(e + f*x))^(7/2) - (5*x*(1/sin(e + f*x))^(1/2))/21, x)
```

Reduce [F]

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)\,dx$$
$$=\int \frac{\sqrt{\csc\left(fx+e\right)}\,x}{\csc\left(fx+e\right)^{4}}dx-\frac{5\left(\int \sqrt{\csc\left(fx+e\right)}\,xdx\right)}{21}$$

input
```
int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
```

output
```
(21*int((sqrt(csc(e + f*x))*x)/csc(e + f*x)**4,x) - 5*int(sqrt(csc(e + f*x
))*x,x))/21
```

3.95 $\int (c+dx)^3(a+a\sin(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx = \frac{a(c+dx)^4}{4d}+\frac{6ad^2(c+dx)\cos(e+fx)}{f^3} - \frac{a(c+dx)^3\cos(e+fx)}{f}-\frac{6ad^3\sin(e+fx)}{f^4} + \frac{3ad(c+dx)^2\sin(e+fx)}{f^2}$$

output
```
1/4*a*(d*x+c)^4/d+6*a*d^2*(d*x+c)*cos(f*x+e)/f^3-a*(d*x+c)^3*cos(f*x+e)/f-
6*a*d^3*sin(f*x+e)/f^4+3*a*d*(d*x+c)^2*sin(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx = a\left(\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\cos(e+fx)}{f^3} + \frac{3d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\sin(e+fx)}{f^4}\right)$$

input
```
Integrate[(c + d*x)^3*(a + a*Sin[e + f*x]),x]
```

output
```
a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - ((c + d*x)*(c^2*f^2
 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x])/f^3 + (3*d*(c^2*f^2 + 2
*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a\sin(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a\sin(e+fx)+a)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^3\sin(e+fx)+a(c+dx)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6ad^2(c+dx)\cos(e+fx)}{f^3}+\frac{3ad(c+dx)^2\sin(e+fx)}{f^2}-\frac{a(c+dx)^3\cos(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\sin(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + a*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) + (6*a*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (a*(c + d*x
)^3*Cos[e + f*x])/f - (6*a*d^3*Sin[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Sin[
e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{a(-(((dx+c)^2 f^2-6d^2) f(dx+c)\cos(fx+e)+3((dx+c)^2 f^2-2d^2) d\sin(fx+e)+(x(\frac{dx}{2}+c)(\frac{1}{2}x^2d^2+cdx+c^2)f^3+c^3f^2 \ldots}{f^4}$
risch	$\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}-\frac{a(d^3f^2x^3+3c\,d^2f^2x^2+3c^2d\,f^2x+c^3f^2-6d^3x-6c\,d^2)\cos(f \ldots}{f^3}$
norman	$\frac{\frac{(2a\,c^3f^2-12ac\,d^2)\tan(\frac{fx}{2}+\frac{e}{2})^2}{f^3}+\frac{a(c^3f^3-3c^2d\,f^2+6d^3)x}{f^3}+\frac{a(c^3f^3+3c^2d\,f^2-6d^3)x\tan(\frac{fx}{2}+\frac{e}{2})^2}{f^3}+\frac{d^2a(cf-d)x^3}{f}+\frac{d^2a(cf \ldots}{\ldots}}{\ldots}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{a(\frac{d^3(-(fx+e)^3\cos(fx+e)+3(fx+e)^2\sin(fx+e)-6\sin(fx+e)+6(fx+e)\cos(fx+e))}{f^3}+\frac{3c\,d^2(-(fx+e)^2\cos(fx+e \ldots}{\ldots}}{\ldots}$
orering	$\frac{(d^5f^4x^6+6c\,d^4f^4x^5+15c^2d^3f^4x^4+20c^3d^2f^4x^3+14c^4d\,f^4x^2+24d^5f^2x^4+4c^5f^4x+96c\,d^4f^2x^3+156c^2d^3f^2x^2+120c^3 \ldots}{4f^4(dx+c)^2}$
derivativedivides	$\frac{-c^3a\cos(fx+e)+\frac{3a\,c^2de\cos(fx+e)}{f}+\frac{3a\,c^2d(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{3ac\,d^2e^2\cos(fx+e)}{f^2}-\frac{6ac\,d^2e(\sin(fx+e)-(fx+e \ldots}{f^2}}{\ldots}$
default	$\frac{-c^3a\cos(fx+e)+\frac{3a\,c^2de\cos(fx+e)}{f}+\frac{3a\,c^2d(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{3ac\,d^2e^2\cos(fx+e)}{f^2}-\frac{6ac\,d^2e(\sin(fx+e)-(fx+e \ldots}{f^2}}{\ldots}$

input
```
int((d*x+c)^3*(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a*(-((d*x+c)^2*f^2-6*d^2)*f*(d*x+c)*cos(f*x+e)+3*((d*x+c)^2*f^2-2*d^2)*d*s
in(f*x+e)+(x*(1/2*d*x+c)*(1/2*x^2*d^2+c*d*x+c^2)*f^3+c^3*f^2-6*c*d^2)*f)/f
^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx$$
$$= \frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x-4\left(ad^3f^3x^3+3\,acd^2f^3x^2+ac^3f^3-6\,acd^2f+3\left(ac^2df^3\right.\right.}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 - 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a*c
^2*d*f^3 - 2*a*d^3*f)*x)*cos(f*x + e) + 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*
x + a*c^2*d*f^2 - 2*a*d^3)*sin(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.93

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx$$
$$= \begin{cases} ac^3x-\frac{ac^3\cos(e+fx)}{f}+\frac{3ac^2dx^2}{2}-\frac{3ac^2dx\cos(e+fx)}{f}+\frac{3ac^2d\sin(e+fx)}{f^2}+acd^2x^3-\frac{3acd^2x^2\cos(e+fx)}{f}+\frac{6acd^2x\sin(e+\right.}{f^2} \\ (a\sin(e)+a)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x - a*c**3*cos(e + f*x)/f + 3*a*c**2*d*x**2/2 - 3*a*c**2
*d*x*cos(e + f*x)/f + 3*a*c**2*d*sin(e + f*x)/f**2 + a*c*d**2*x**3 - 3*a*c
*d**2*x**2*cos(e + f*x)/f + 6*a*c*d**2*x*sin(e + f*x)/f**2 + 6*a*c*d**2*co
s(e + f*x)/f**3 + a*d**3*x**4/4 - a*d**3*x**3*cos(e + f*x)/f + 3*a*d**3*x*
*2*sin(e + f*x)/f**2 + 6*a*d**3*x*cos(e + f*x)/f**3 - 6*a*d**3*sin(e + f*x
)/f**4, Ne(f, 0)), ((a*sin(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(88) = 176$.

Time = 0.06 (sec) , antiderivative size = 462, normalized size of antiderivative = 5.13

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx$$

$$= \frac{4\,(fx+e)ac^3 + \frac{(fx+e)^4ad^3}{f^3} - \frac{4\,(fx+e)^3ad^3e}{f^3} + \frac{6\,(fx+e)^2ad^3e^2}{f^3} - \frac{4\,(fx+e)ad^3e^3}{f^3} + \frac{4\,(fx+e)^3acd^2}{f^2} - \frac{12\,(fx+e)^2acd^2e}{f^2} + 12}{}$$

input
```
integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
 + 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3
*a*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2
 + 6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f - 4*a*c^3*cos(f*x +
e) + 4*a*d^3*e^3*cos(f*x + e)/f^3 - 12*a*c*d^2*e^2*cos(f*x + e)/f^2 + 12*a
*c^2*d*e*cos(f*x + e)/f - 12*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*d^3
*e^2/f^3 + 24*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*c*d^2*e/f^2 - 12*(
(f*x + e)*cos(f*x + e) - sin(f*x + e))*a*c^2*d/f + 12*(((f*x + e)^2 - 2)*c
os(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*d^3*e/f^3 - 12*(((f*x + e)^2 - 2
)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*c*d^2/f^2 - 4*(((f*x + e)^3 -
 6*f*x - 6*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*a*d^3/f^3)/
f
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x - \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + ac^3f^3 - 6\,ad^3fx - 6\,acd^2f)\cos(fx+e)}{f^4} + \frac{3\,(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3)\sin(fx+e)}{f^4}$$

input
```
integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (a*d^3*f^3*x^3 +
 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*c*d^2
*f)*cos(f*x + e)/f^4 + 3*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 -
2*a*d^3)*sin(f*x + e)/f^4
```

Mupad [B] (verification not implemented)

Time = 36.02 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx = \frac{a\,d^3\,x^4}{4} - \frac{3\,\sin(e+f\,x)\,\left(2\,a\,d^3 - a\,c^2\,d\,f^2\right)}{f^4} - \frac{\cos(e+f\,x)\,\left(a\,c^3\,f^2 - 6\,a\,c\,d^2\right)}{f^3} + a\,c^3\,x + \frac{3\,x\,\cos(e+f\,x)\,\left(2\,a\,d^3 - a\,c^2\,d\,f^2\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{a\,d^3\,x^3\,\cos(e+f\,x)}{f} + \frac{3\,a\,d^3\,x^2\,\sin(e+f\,x)}{f^2} + \frac{6\,a\,c\,d^2\,x\,\sin(e+f\,x)}{f^2} - \frac{3\,a\,c\,d^2\,x^2\,\cos(e+f\,x)}{f}$$

input
```
int((a + a*sin(e + f*x))*(c + d*x)^3,x)
```

output
```
(a*d^3*x^4)/4 - (3*sin(e + f*x)*(2*a*d^3 - a*c^2*d*f^2))/f^4 - (cos(e + f*
x)*(a*c^3*f^2 - 6*a*c*d^2))/f^3 + a*c^3*x + (3*x*cos(e + f*x)*(2*a*d^3 - a
*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (a*d^3*x^3*cos(e + f*
x))/f + (3*a*d^3*x^2*sin(e + f*x))/f^2 + (6*a*c*d^2*x*sin(e + f*x))/f^2 -
(3*a*c*d^2*x^2*cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.22

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx$$
$$= \frac{a(-4\cos(fx+e)\,c^3f^3 - 12\cos(fx+e)\,c^2d\,f^3x - 12\cos(fx+e)\,c\,d^2f^3x^2 + 24\cos(fx+e)\,c\,d^2f - 4\,c}{}$$

input
```
int((d*x+c)^3*(a+a*sin(f*x+e)),x)
```

output
```
(a*( - 4*cos(e + f*x)*c**3*f**3 - 12*cos(e + f*x)*c**2*d*f**3*x - 12*cos(e
 + f*x)*c*d**2*f**3*x**2 + 24*cos(e + f*x)*c*d**2*f - 4*cos(e + f*x)*d**3*
f**3*x**3 + 24*cos(e + f*x)*d**3*f*x + 12*sin(e + f*x)*c**2*d*f**2 + 24*si
n(e + f*x)*c*d**2*f**2*x + 12*sin(e + f*x)*d**3*f**2*x**2 - 24*sin(e + f*x
)*d**3 + 4*c**3*f**4*x + 6*c**2*d*f**4*x**2 + 4*c*d**2*f**4*x**3 + d**3*f*
*4*x**4))/(4*f**4)
```

3.96 $\int (c+dx)^2(a+a\sin(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx = \frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{2ad(c+dx)\sin(e+fx)}{f^2}$$

output
```
1/3*a*(d*x+c)^3/d+2*a*d^2*cos(f*x+e)/f^3-a*(d*x+c)^2*cos(f*x+e)/f+2*a*d*(d
*x+c)*sin(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx = a\left(c^2x+cdx^2+\frac{d^2x^3}{3}-\frac{(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^3}+\frac{2d(c+dx)\sin(e+fx)}{f^2}\right)$$

input
```
Integrate[(c + d*x)^2*(a + a*Sin[e + f*x]),x]
```

output
```
a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 - ((c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2
*x^2))*Cos[e + f*x])/f^3 + (2*d*(c + d*x)*Sin[e + f*x])/f^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a\sin(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a\sin(e+fx)+a)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2\sin(e+fx)+a(c+dx)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ad(c+dx)\sin(e+fx)}{f^2}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + a*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + (2*a*d^2*Cos[e + f*x])/f^3 - (a*(c + d*x)^2*Cos[e
+ f*x])/f + (2*a*d*(c + d*x)*Sin[e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{a\left(\left(-(dx+c)^2 f^2+2d^2\right)\cos(fx+e)+2df(dx+c)\sin(fx+e)+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)f^3-c^2f^2+2d^2\right)}{f^3}$
risch	$\frac{d^2 a x^3}{3}+adc\,x^2+a\,c^2x+\frac{a\,c^3}{3d}-\frac{a\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\cos(fx+e)}{f^3}+\frac{2ad(dx+c)\sin(fx+e)}{f^2}$
parts	$\frac{a(dx+c)^3}{3d}+\frac{a\Big(\frac{d^2\left(-(fx+e)^2\cos(fx+e)+2\cos(fx+e)+2(fx+e)\sin(fx+e)\right)}{f^2}+\frac{2cd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{2d^2e(\sin(f}{}}{f}$
norman	$\frac{\frac{\left(2a\,c^2f^2-4d^2a\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f^3}+\frac{ac(cf-2d)x}{f}+\frac{ad(cf-d)x^2}{f}+\frac{ac(cf+2d)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}+\frac{ad(cf+d)x^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}+\frac{d^2a\,x^3}{3}}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}$
derivativedivides	$-a\,c^2\cos(fx+e)+\frac{2acde\cos(fx+e)}{f}+\frac{2acd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{a\,d^2e^2\cos(fx+e)}{f^2}-\frac{2a\,d^2e(\sin(fx+e)-(fx+e)\cos(f}{f^2}$
default	$-a\,c^2\cos(fx+e)+\frac{2acde\cos(fx+e)}{f}+\frac{2acd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{a\,d^2e^2\cos(fx+e)}{f^2}-\frac{2a\,d^2e(\sin(fx+e)-(fx+e)\cos(f}{f^2}$
orering	$\frac{(d^4f^4x^5+5c\,d^3f^4x^4+10c^2d^2f^4x^3+9c^3d\,f^4x^2+3c^4f^4x+12d^4f^2x^3+42c\,d^3f^2x^2+48c^2d^2f^2x+12c^3d\,f^2-48d^4x-12d^3}{3f^4(dx+c)^2}$

input
```
int((d*x+c)^2*(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a*((-(d*x+c)^2*f^2+2*d^2)*cos(f*x+e)+2*d*f*(d*x+c)*sin(f*x+e)+x*(1/3*x^2*d
^2+c*d*x+c^2)*f^3-c^2*f^2+2*d^2)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx$$
$$= \frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x - 3\,(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2)\cos(fx+e) + 6\,(ad^2fx + ac}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 3*(a*d^2*f^2*x^2 +
2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*cos(f*x + e) + 6*(a*d^2*f*x + a*c*d*f
)*sin(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.22

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx$$
$$= \begin{cases} ac^2x - \frac{ac^2\cos(e+fx)}{f} + acdx^2 - \frac{2acdx\cos(e+fx)}{f} + \frac{2acd\sin(e+fx)}{f^2} + \frac{ad^2x^3}{3} - \frac{ad^2x^2\cos(e+fx)}{f} + \frac{2ad^2x\sin(e+fx)}{f^2} + \\ (a\sin(e)+a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x - a*c**2*cos(e + f*x)/f + a*c*d*x**2 - 2*a*c*d*x*cos(e
 + f*x)/f + 2*a*c*d*sin(e + f*x)/f**2 + a*d**2*x**3/3 - a*d**2*x**2*cos(e
+ f*x)/f + 2*a*d**2*x*sin(e + f*x)/f**2 + 2*a*d**2*cos(e + f*x)/f**3, Ne(f
, 0)), ((a*sin(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(66) = 132$.

Time = 0.05 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.51

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx$$
$$= \frac{3\,(fx+e)ac^2 + \frac{(fx+e)^3ad^2}{f^2} - \frac{3\,(fx+e)^2ad^2e}{f^2} + \frac{3\,(fx+e)ad^2e^2}{f^2} + \frac{3\,(fx+e)^2acd}{f} - \frac{6\,(fx+e)acde}{f} - 3\,ac^2\cos(fx+e) - }{}$$

input
```
integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
 + 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e
/f - 3*a*c^2*cos(f*x + e) - 3*a*d^2*e^2*cos(f*x + e)/f^2 + 6*a*c*d*e*cos(f
*x + e)/f + 6*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*d^2*e/f^2 - 6*((f*
x + e)*cos(f*x + e) - sin(f*x + e))*a*c*d/f - 3*(((f*x + e)^2 - 2)*cos(f*x
 + e) - 2*(f*x + e)*sin(f*x + e))*a*d^2/f^2)/f
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx$$
$$= \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x - \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2)\cos(fx+e)}{f^3} + \frac{2\,(ad^2fx + acdf)\sin(fx+e)}{f^3}$$

input
```
integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c
^2*f^2 - 2*a*d^2)*cos(f*x + e)/f^3 + 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e)/
f^3
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx = \frac{a\,d^2\,x^3}{3}+\frac{\cos\left(e+f\,x\right)\,\left(2\,a\,d^2-a\,c^2\,f^2\right)}{f^3} + a\,c^2\,x+a\,c\,d\,x^2+\frac{2\,a\,d^2\,x\,\sin\left(e+f\,x\right)}{f^2} - \frac{a\,d^2\,x^2\,\cos\left(e+f\,x\right)}{f}+\frac{2\,a\,c\,d\,\sin\left(e+f\,x\right)}{f^2} - \frac{2\,a\,c\,d\,x\,\cos\left(e+f\,x\right)}{f}$$

input
```
int((a + a*sin(e + f*x))*(c + d*x)^2,x)
```

output
```
(a*d^2*x^3)/3 + (cos(e + f*x)*(2*a*d^2 - a*c^2*f^2))/f^3 + a*c^2*x + a*c*d
*x^2 + (2*a*d^2*x*sin(e + f*x))/f^2 - (a*d^2*x^2*cos(e + f*x))/f + (2*a*c*
d*sin(e + f*x))/f^2 - (2*a*c*d*x*cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx = \frac{a(-3\cos\left(fx+e\right)c^2f^2-6\cos\left(fx+e\right)cd\,f^2x-3\cos\left(fx+e\right)d^2f^2x^2+6\cos\left(fx+e\right)d^2+6\sin\left(fx+\right.}{3f^3}$$

input
```
int((d*x+c)^2*(a+a*sin(f*x+e)),x)
```

output
```
(a*( - 3*cos(e + f*x)*c**2*f**2 - 6*cos(e + f*x)*c*d*f**2*x - 3*cos(e + f*
x)*d**2*f**2*x**2 + 6*cos(e + f*x)*d**2 + 6*sin(e + f*x)*c*d*f + 6*sin(e +
 f*x)*d**2*f*x + 3*c**2*f**3*x + 3*c*d*f**3*x**2 + d**2*f**3*x**3))/(3*f**
3)
```

3.97 $\int (c+dx)(a+a\sin(e+fx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int (c+dx)(a+a\sin(e+fx))\,dx=\frac{a(c+dx)^2}{2d}-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{ad\sin(e+fx)}{f^2}$$

output
```
1/2*a*(d*x+c)^2/d-a*(d*x+c)*cos(f*x+e)/f+a*d*sin(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+a\sin(e+fx))\,dx \\ =-\frac{a((e+fx)(de-2cf-dfx)+2f(c+dx)\cos(e+fx)-2d\sin(e+fx))}{2f^2}$$

input
```
Integrate[(c + d*x)*(a + a*Sin[e + f*x]),x]
```

output
```
-1/2*(a*((e + f*x)*(d*e - 2*c*f - d*f*x) + 2*f*(c + d*x)*Cos[e + f*x] - 2*
d*Sin[e + f*x]))/f^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a\sin(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a\sin(e+fx)+a)dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)\sin(e+fx)+a(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\sin(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + a*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (a*(c + d*x)*Cos[e + f*x])/f + (a*d*Sin[e + f*x])/
f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
risch	$\frac{a d\,x^{2}}{2}+a c x -\frac{a \left(d x +c \right) \cos \left(f x +e \right)}{f}+\frac{a d \sin \left(f x +e \right)}{f^{2}}$
parallelrisch	$\frac{a \left(-f \left(d x +c \right) \cos \left(f x +e \right)+d \sin \left(f x +e \right)+f \left(x \left(\frac{d x}{2}+c \right) f -c \right)\right)}{f^{2}}$
parts	$a \left(\frac{1}{2} d\,x^{2}+c x \right)+\frac{a \left(\frac{d \left(\sin \left(f x +e \right)-\left(f x +e \right) \cos \left(f x +e \right)\right)}{f}-c \cos \left(f x +e \right)+\frac{d e \cos \left(f x +e \right)}{f}\right)}{f}$
derivativedivides	$\frac{-a c \cos \left(f x +e \right)+\frac{a d e \cos \left(f x +e \right)}{f}+\frac{a d \left(\sin \left(f x +e \right)-\left(f x +e \right) \cos \left(f x +e \right)\right)}{f}+a c \left(f x +e \right)-\frac{a d e \left(f x +e \right)}{f}+\frac{a d \left(f x +e \right)^{2}}{2 f}}{f}$
default	$\frac{-a c \cos \left(f x +e \right)+\frac{a d e \cos \left(f x +e \right)}{f}+\frac{a d \left(\sin \left(f x +e \right)-\left(f x +e \right) \cos \left(f x +e \right)\right)}{f}+a c \left(f x +e \right)-\frac{a d e \left(f x +e \right)}{f}+\frac{a d \left(f x +e \right)^{2}}{2 f}}{f}$
norman	$\frac{\frac{2 a c \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{f}+\frac{a \left(c f -d \right) x}{f}+\frac{a \left(c f +d \right) x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{f}+\frac{a d\,x^{2}}{2}+\frac{2 a d \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{f^{2}}+\frac{a d\,x^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2}}{1+\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}$
orering	$\frac{\left(d^{3} f^{2} x^{4}+4 c\,d^{2} f^{2} x^{3}+5 c^{2} d\,f^{2} x^{2}+2 c^{3} f^{2} x +6 d^{3} x^{2}+12 c\,d^{2} x +4 c^{2} d \right) \left(a +a \sin \left(f x +e \right)\right)}{2 f^{2} \left(d x +c \right)^{2}}-\frac{\left(2 x^{2} d^{2}+4 c d x +c^{2}\right) \left(d \left(a +a \sin \right.\right.}{\left(d x +\right.}$

input
```
int((d*x+c)*(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d*x^2+a*c*x-a*(d*x+c)*cos(f*x+e)/f+a*d*sin(f*x+e)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+a\sin(e+fx))\,dx = \frac{adf^2x^2+2\,acf^2x+2\,ad\sin(fx+e)-2\,(adfx+acf)\cos(fx+e)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*sin(f*x + e) - 2*(a*d*f*x + a*c*f)*
cos(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int (c+dx)(a+a\sin(e+fx))\,dx = \begin{cases} acx - \frac{ac\cos(e+fx)}{f} + \frac{adx^2}{2} - \frac{adx\cos(e+fx)}{f} + \frac{ad\sin(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a\sin(e)+a)\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((a*c*x - a*c*cos(e + f*x)/f + a*d*x**2/2 - a*d*x*cos(e + f*x)/f
+ a*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a*sin(e) + a)*(c*x + d*x**2/2), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(43) = 86$.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.07

$$\int (c+dx)(a+a\sin(e+fx))\,dx = \frac{2\,(fx+e)ac + \frac{(fx+e)^2ad}{f} - \frac{2\,(fx+e)ade}{f} - 2\,ac\cos\left(fx+e\right) + \frac{2\,ade\cos(fx+e)}{f} - \frac{2\,((fx+e)\cos(fx+e)-\sin(fx+e))ad}{f}}{2\,f}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f - 2*a*c*cos
(f*x + e) + 2*a*d*e*cos(f*x + e)/f - 2*((f*x + e)*cos(f*x + e) - sin(f*x +
 e))*a*d/f)/f
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (c+dx)(a+a\sin(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{ad\sin\left(fx+e\right)}{f^2} - \frac{(adfx+acf)\cos\left(fx+e\right)}{f^2}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + a*d*sin(f*x + e)/f^2 - (a*d*f*x + a*c*f)*cos(f*x + e
)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int (c+dx)(a+a\sin(e+fx))\,dx = \frac{a\,(d\,x^2+2\,c\,x)}{2} - \frac{\frac{a\,f\,(2\,c\,\cos(e+f\,x)+2\,d\,x\,\cos(e+f\,x))}{2} - a\,d\,\sin\left(e+f\,x\right)}{f^2}$$

input
```
int((a + a*sin(e + f*x))*(c + d*x),x)
```

output
```
(a*(2*c*x + d*x^2))/2 - ((a*f*(2*c*cos(e + f*x) + 2*d*x*cos(e + f*x)))/2 -
 a*d*sin(e + f*x))/f^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+a\sin(e+fx))\,dx = \frac{a(-2\cos\left(fx+e\right)cf-2\cos\left(fx+e\right)dfx+2\sin\left(fx+e\right)d+2cf^2x+d\,f^2x^2)}{2f^2}$$

input
```
int((d*x+c)*(a+a*sin(f*x+e)),x)
```

output
```
(a*( - 2*cos(e + f*x)*c*f - 2*cos(e + f*x)*d*f*x + 2*sin(e + f*x)*d + 2*c*
f**2*x + d*f**2*x**2))/(2*f**2)
```

3.98 $\int \frac{a+a\sin(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a+a\sin(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)}{d}+\frac{a\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*ln(d*x+c)/d-a*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d+a*cos(-e+c*f/d)*Si(c*f/d+f*x
)/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{a+a\sin(e+fx)}{c+dx}\,dx=\frac{a\left(\log(c+dx)+\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

input
```
Integrate[(a + a*Sin[e + f*x])/(c + d*x),x]
```

output
```
(a*(Log[c + d*x] + CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + Cos[e - (c*
f)/d]*SinIntegral[f*(c/d + x)]))/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sin(e+fx)+a}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(e+fx)+a}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\sin(e+fx)}{c+dx}+\frac{a}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

input
```
Int[(a + a*Sin[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x])/d + (a*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (
a*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

method	result
parts	$\frac{a\ln(dx+c)}{d}+a\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)$
derivativedivides	$\frac{af\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$
default	$\frac{af\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$
risch	$\frac{a\ln(dx+c)}{d}-\frac{ia\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}+\frac{ia\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$

input
```
int((a+a*sin(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+a*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*
e)/d)*sin((c*f-d*e)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{a+a\sin(e+fx)}{c+dx}\,dx = -\frac{a\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)-a\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)-a\log\left(dx+c\right)}{d}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
-(a*cos_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) - a*cos(-(d*e - c*f)
/d)*sin_integral((d*f*x + c*f)/d) - a*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{a+a\sin(e+fx)}{c+dx}\,dx = a\left(\int \frac{\sin\left(e+fx\right)}{c+dx}\,dx+\int \frac{1}{c+dx}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c),x)
```

output
```
a*(Integral(sin(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.67

$$\int \frac{a+a\sin(e+fx)}{c+dx}\,dx = \frac{\frac{2\,af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{\left(f\left(-i\,E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_1\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_1\left(\right.\right.}{d}}{2\,f}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(-I*exp_integral_e(1, (I*
(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d - I*
d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d
 - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/
d))*sin(-(d*e - c*f)/d))*a/d)/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 693, normalized size of antiderivative = 10.83

$$\int \frac{a + a\sin(e + fx)}{c + dx} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2
- a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 +
2*a*log(abs(d*x + c))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 2*a*sin_integral((d*
f*x + c*f)/d)*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 2*a*real_part(cos_integral(f
*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) + 2*a*real_part(cos_integral(-f*x
 - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) - 2*a*real_part(cos_integral(f*x +
c*f/d))*tan(1/2*e)*tan(1/2*c*f/d)^2 - 2*a*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*e)*tan(1/2*c*f/d)^2 - a*imag_part(cos_integral(f*x + c*f/d))
*tan(1/2*e)^2 + a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 + 2*a
*log(abs(d*x + c))*tan(1/2*e)^2 - 2*a*sin_integral((d*f*x + c*f)/d)*tan(1/
2*e)^2 + 4*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(1/2*c*f/d
) - 4*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(1/2*c*f/d) +
8*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(1/2*c*f/d) - a*imag_part(
cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + a*imag_part(cos_integral(-f*
x - c*f/d))*tan(1/2*c*f/d)^2 + 2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2 - 2*
a*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 + 2*a*real_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*e) + 2*a*real_part(cos_integral(-f*x - c*f/d))*t
an(1/2*e) - 2*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*a*
real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + a*imag_part(cos_int
egral(f*x + c*f/d)) - a*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*log...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sin(e + fx)}{c + dx}\,dx = \int \frac{a + a\,\sin\left(e + f\,x\right)}{c + d\,x}\,dx$$

input
```
int((a + a*sin(e + f*x))/(c + d*x),x)
```

output
```
int((a + a*sin(e + f*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a + a\sin(e + fx)}{c + dx}\,dx = \frac{a\left(\left(\int \frac{\sin(fx+e)}{dx+c}dx\right)d + \log(dx + c)\right)}{d}$$

input

```
int((a+a*sin(f*x+e))/(d*x+c),x)
```

output

```
(a*(int(sin(e + f*x)/(c + d*x),x)*d + log(c + d*x)))/d
```

3.99 $\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)}+\frac{af\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d^2} -\frac{a\sin(e+fx)}{d(c+dx)}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)+a*f*cos(-e+c*f/d)*Ci(c*f/d+f*x)/d^2-a*sin(f*x+e)/d/(d*x+c)+a*
f*sin(-e+c*f/d)*Si(c*f/d+f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx = \frac{a(1+\sin(e+fx))\left(f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)-d(1+\sin(e+fx))-f(c+dx)\sin\right.}{d^2(c+dx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[(a + a*Sin[e + f*x])/(c + d*x)^2,x]
```

output
```
(a*(1 + Sin[e + f*x])*(f*(c + d*x)*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x
)] - d*(1 + Sin[e + f*x]) - f*(c + d*x)*Sin[e - (c*f)/d]*SinIntegral[f*(c/
d + x)]))/(d^2*(c + d*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sin(e+fx)+a}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(e+fx)+a}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\sin(e+fx)}{(c+dx)^2}+\frac{a}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{af\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

input
```
Int[(a + a*Sin[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) + (a*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
 - (a*Sin[e + f*x])/(d*(c + d*x)) - (a*f*Sin[e - (c*f)/d]*SinIntegral[(c*f
)/d + f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{a}{d(dx+c)}+af\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)$
derivativedivides	$\frac{af^{2}\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{a f^{2}}{(cf-de+d(fx+e))d}}{f}$
default	$\frac{af^{2}\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{a f^{2}}{(cf-de+d(fx+e))d}}{f}$
risch	$-\frac{a}{d(dx+c)}-\frac{fa\,\mathrm{e}^{\frac{i(cf-de)}{d}}\mathrm{expIntegral}_{1}\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^{2}}-\frac{af\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\mathrm{expIntegral}_{1}\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d^{2}}$

input
```
int((a+a*sin(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+a*f*(-sin(f*x+e)/(c*f-d*e+d*(f*x+e))/d+(Si(f*x+e+(c*f-d*e)/d)
*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx = \frac{(adfx+acf)\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)-ad\sin\left(fx+e\right)+(adfx+acf)\sin\left(-\frac{de-cf}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)-ad}{d^3x+cd^2}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((a*d*f*x + a*c*f)*cos(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) - a*d
*sin(f*x + e) + (a*d*f*x + a*c*f)*sin(-(d*e - c*f)/d)*sin_integral((d*f*x
+ c*f)/d) - a*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx = a\left(\int \frac{\sin\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)**2,x)
```

output
```
a*(Integral(sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c*
*2 + 2*c*d*x + d**2*x**2), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx =$$

$$-\frac{\frac{2\,af^2}{(fx+e)d^2-d^2e+cdf}-\frac{\left(f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_2\left(\right.\right.}{(fx+e)d^2-d^2e+cdf}}{2\,f}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - (f^2*(-I*exp_integral_e(2,
 (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d
- I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(f*x
+ e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I
*c*f)/d))*sin(-(d*e - c*f)/d))*a/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(89) = 178$.

Time = 0.35 (sec) , antiderivative size = 533, normalized size of antiderivative = 6.06

$$\int \frac{a + a\sin(e + fx)}{(c + dx)^2}\,dx = \frac{\left((dx + c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right)f^2\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right) - de + cf}{d}\right) - def^2\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{(d}{}\right.\right.}{}$$

$$- \frac{a}{(dx + c)d}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c*f)/d)*cos
_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) -
 d*e*f^2*cos(-(d*e - c*f)/d)*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f) - d*e + c*f)/d) + c*f^3*cos(-(d*e - c*f)/d)*cos_integral(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + (d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*sin(-(d*e - c*f)/d)*sin_integral(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - d*e*f^2*sin
(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d*e + c*f)/d) + c*f^3*sin(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*
e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + d*f^2*sin(-(d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f)/d))*a*d^2/(((d*x + c)*d^4*(d*e/(d*x + c)
 - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx=\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + a*sin(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + a*sin(e + f*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx=\frac{a\left(\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)c^2+\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)cdx+x\right)}{c\,(dx+c)}$$

input
```
int((a+a*sin(f*x+e))/(d*x+c)^2,x)
```

output
```
(a*(int(sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + int(sin(e + f*
x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + x))/(c*(c + d*x))
```

3.100 $\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)} - \frac{af^2\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3} - \frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{af^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*a*f*cos(f*x+e)/d^2/(d*x+c)+1/2*a*f^2*Ci(c*f/d+f*x)*
sin(-e+c*f/d)/d^3-1/2*a*sin(f*x+e)/d/(d*x+c)^2-1/2*a*f^2*cos(-e+c*f/d)*Si(
c*f/d+f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx = -\frac{a\left(f^2(c+dx)^2\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+d(f(c+dx)\cos(e+fx)+d(1+\sin(e+fx)))\right. -}{2d^3(c+dx)^2}$$

input
```
Integrate[(a + a*Sin[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*(a*(f^2*(c + d*x)^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + d*(f*
(c + d*x)*Cos[e + f*x] + d*(1 + Sin[e + f*x])) + f^2*(c + d*x)^2*Cos[e - (
c*f)/d]*SinIntegral[f*(c/d + x)]))/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sin(e+fx)+a}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(e+fx)+a}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\sin(e+fx)}{(c+dx)^3}+\frac{a}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{af^2\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{af^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

input
```
Int[(a + a*Sin[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (a*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (a*f^2*Cos
Integral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (a*Sin[e + f*x])/(2*d*
(c + d*x)^2) - (a*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

method	result
parts	$-\frac{a}{2d(dx+c)^2}+a\,f^2\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{Ci}\left(fx+e+\right.}{d}}{d}}{2d}\right.$
derivativedivides	$\frac{a\,f^3\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}}{2d}\right)-}{f}$
default	$\frac{a\,f^3\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}}{2d}\right)-}{f}$
risch	$-\frac{a}{2d(dx+c)^2}+\frac{i f^2 a\,e^{\frac{i(cf-de)}{d}}\operatorname{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{4d^3}-\frac{i f^2 a\,e^{-\frac{i(cf-de)}{d}}\operatorname{expIntegral}_1\left(-ifx-ie-\frac{icf}{}\right.}{4d^3}$

input
```
int((a+a*sin(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2+a*f^2*(-1/2*sin(f*x+e)/(c*f-d*e+d*(f*x+e))^2/d+1/2*(-co
s(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-C
i(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45

$$\int \frac{a + a\sin(e + fx)}{(c + dx)^3}\,dx = -\frac{ad^2\sin(fx + e) + ad^2 - (ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2)\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right) + (ad^2f^2x^2 + 2\,acdf^2}{2\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d^2*sin(f*x + e) + a*d^2 - (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*
f^2)*cos_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) + (a*d^2*f^2*x^2 +
2*a*c*d*f^2*x + a*c^2*f^2)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/
d) + (a*d^2*f*x + a*c*d*f)*cos(f*x + e))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{a + a\sin(e + fx)}{(c + dx)^3}\,dx = a\left(\int \frac{\sin(e + fx)}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\,dx + \int \frac{1}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)**3,x)
```

output
```
a*(Integral(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.15

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx = -\frac{\frac{af^3}{(fx+e)^2d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\,(d^3e-cd^2f)(fx+e)} - \frac{\left(f^3\left(-i\,E_3\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_3\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)}{(fx+e)^2d^3+d^3e^2-2\,cd^2ef+}}{2\,f}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*
e - c*d^2*f)*(f*x + e)) - (f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d - I*d*
e + I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*c
os(-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e + I*c*f
)/d) + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(d*e -
c*f)/d))*a/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e
 - c*d^2*f)*(f*x + e)))/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 6033, normalized size of antiderivative = 49.05

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*ta
n(1/2*e)^2*tan(1/2*c*f/d)^2 - a*d^2*f^2*x^2*imag_part(cos_integral(-f*x -
c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 2*a*d^2*f^2*x^2*sin
_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 +
2*a*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/
2*e)^2*tan(1/2*c*f/d) + 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/
d))*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d) - 2*a*d^2*f^2*x^2*real_part
(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)*tan(1/2*c*f/d)^2 - 2
*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/
2*e)*tan(1/2*c*f/d)^2 + 2*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))
*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 - 2*a*c*d*f^2*x*imag_part(co
s_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 4
*a*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2*tan
(1/2*c*f/d)^2 - a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*f*x)^2*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))
*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/
d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*a*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)*tan(1/2*c*f/d) - 4*a*d^2*f^2*x^2*ima
g_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)*tan(1/2*c*f/d
) + 8*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx=\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a + a*sin(e + f*x))/(c + d*x)^3,x)
```

output
```
int((a + a*sin(e + f*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a + a\sin(e + fx)}{(c + dx)^3}\,dx$$
$$= \frac{a\left(2\left(\int \frac{\sin(fx+e)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c^2d + 4\left(\int \frac{\sin(fx+e)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^2x + 2\left(\int \frac{\sin(fx+e)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)d^3}{2d\,(d^2x^2 + 2cdx + c^2)}$$

input
```
int((a+a*sin(f*x+e))/(d*x+c)^3,x)
```

output
```
(a*(2*int(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*
c**2*d + 4*int(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x)*c*d**2*x + 2*int(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*d**3*x**2 - 1))/(2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.101 $\int (c+dx)^3(a+a\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx = -\frac{3a^2d(c+dx)^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d} + \frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3} - \frac{2a^2(c+dx)^3\cos(e+fx)}{f}-\frac{12a^2d^3\sin(e+fx)}{f^4} + \frac{6a^2d(c+dx)^2\sin(e+fx)}{f^2} + \frac{3a^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3} - \frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f} - \frac{3a^2d^3\sin^2(e+fx)}{8f^4}+\frac{3a^2d(c+dx)^2\sin^2(e+fx)}{4f^2}$$

output
```
-3/8*a^2*d*(d*x+c)^2/f^2+3/8*a^2*(d*x+c)^4/d+12*a^2*d^2*(d*x+c)*cos(f*x+e)
/f^3-2*a^2*(d*x+c)^3*cos(f*x+e)/f-12*a^2*d^3*sin(f*x+e)/f^4+6*a^2*d*(d*x+c
)^2*sin(f*x+e)/f^2+3/4*a^2*d^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f^3-1/2*a^2*(
d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f-3/8*a^2*d^3*sin(f*x+e)^2/f^4+3/4*a^2*d*(d
*x+c)^2*sin(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx$$
$$=\frac{a^2(6f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-32f(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\cos(e+fx)-3}{}$$

input
```
Integrate[(c + d*x)^3*(a + a*Sin[e + f*x])^2,x]
```

output
```
(a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 32*f*(c + d*x)
*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x] - 3*d*(2*c^2*f^
2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 96*d*(c^2*f^2 +
 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x] - 2*f*(c + d*x)*(2*c^2*f^2
 + 4*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)]))/(16*f^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a\sin(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a\sin(e+fx)+a)^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)^3\sin^2(e+fx)+2a^2(c+dx)^3\sin(e+fx)+a^2(c+dx)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}+\frac{3a^2d(c+dx)^2\sin^2(e+fx)}{4f^2}+\frac{6a^2d(c+dx)^2\sin(e+fx)}{f^2}-\frac{2a^2(c+dx)^3\cos(e+fx)}{f}-\frac{a^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f}-\frac{3a^2d(c+dx)^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{3a^2d^3\sin^2(e+fx)}{8f^4}-\frac{12a^2d^3\sin(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + a*Sin[e + f*x])^2,x]
```

output
```
(-3*a^2*d*(c + d*x)^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*d) + (12*a^2*d^2*(
c + d*x)*Cos[e + f*x])/f^3 - (2*a^2*(c + d*x)^3*Cos[e + f*x])/f - (12*a^2*
d^3*Sin[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Sin[e + f*x])/f^2 + (3*a^2*d^
2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c + d*x)^3*Cos[e +
f*x]*Sin[e + f*x])/(2*f) - (3*a^2*d^3*Sin[e + f*x]^2)/(8*f^4) + (3*a^2*d*(
c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.87

method	result
parallelrisch	$-\frac{a^2\left(\left(\left((dx+c)^2f^2-\frac{3d^2}{2}\right)f(dx+c)\sin(2fx+2e)+\frac{3d\left((dx+c)^2f^2-\frac{d^2}{2}\right)\cos(2fx+2e)}{2}+8\left((dx+c)^2f^2-6d^2\right)f(dx+c)\cos(\right.\right.}{4}$
risch	$\frac{3d^3a^2x^4}{8}+\frac{3a^2cd^2x^3}{2}+\frac{9a^2dc^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}-\frac{2a^2(d^3f^2x^3+3cd^2f^2x^2+3c^2df^2x+c^3f^2-6d^3x-6c}{f^3}$
norman	$\frac{\frac{d^3a^2x^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{4a^2c^3f^2-24a^2cd^2}{f^3}+\frac{3d^3a^2x^4}{8}-\frac{\left(8a^2c^3f^3-6a^2c^2df^2-48a^2cd^2f+3d^3a^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2f^4}-\frac{a^2\left(2c^3f^3-\right.}{}}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*((((d*x+c)^2*f^2-3/2*d^2)*f*(d*x+c)*sin(2*f*x+2*e)+3/2*d*((d*x+c)^
2*f^2-1/2*d^2)*cos(2*f*x+2*e)+8*((d*x+c)^2*f^2-6*d^2)*f*(d*x+c)*cos(f*x+e)
-24*((d*x+c)^2*f^2-2*d^2)*d*sin(f*x+e)+(-6*x^3*c*d^2-9*x^2*c^2*d-3/2*d^3*x
^4-6*c^3*x)*f^4+8*c^3*f^3-3/2*c^2*d*f^2-48*c*d^2*f+3/4*d^3)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.64

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx$$
$$=\frac{3\,a^2d^3f^4x^4+12\,a^2cd^2f^4x^3+3\,(6\,a^2c^2df^4+a^2d^3f^2)x^2-3\,(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3)}{}$$

input
```
integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 + a^2*d
^3*f^2)*x^2 - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 -
 a^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 + a^2*c*d^2*f^2)*x - 16*(a^2*d
^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + a^2*c^3*f^3 - 6*a^2*c*d^2*f + 3*(a^2*c^
2*d*f^3 - 2*a^2*d^3*f)*x)*cos(f*x + e) + 2*(24*a^2*d^3*f^2*x^2 + 48*a^2*c*
d^2*f^2*x + 24*a^2*c^2*d*f^2 - 48*a^2*d^3 - (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d
^2*f^3*x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*
f)*x)*cos(f*x + e))*sin(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(228) = 456$.

Time = 0.42 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.48

$$\int (c + dx)^3 (a + a\sin(e + fx))^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
 a**2*c**3*x - a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**3*cos
(e + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*co
s(e + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 - 3*a**2*c**2*d*x*sin(e + f*x)*cos(
e + f*x)/(2*f) - 6*a**2*c**2*d*x*cos(e + f*x)/f + 3*a**2*c**2*d*sin(e + f*
x)**2/(4*f**2) + 6*a**2*c**2*d*sin(e + f*x)/f**2 + a**2*c*d**2*x**3*sin(e
+ f*x)**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 - 3*a*
*2*c*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a**2*c*d**2*x**2*cos(e
+ f*x)/f + 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 12*a**2*c*d**2*x*sin
(e + f*x)/f**2 - 3*a**2*c*d**2*x*cos(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*
sin(e + f*x)*cos(e + f*x)/(4*f**3) + 12*a**2*c*d**2*cos(e + f*x)/f**3 + a*
*2*d**3*x**4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d
**3*x**4/4 - a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*d**3*
x**3*cos(e + f*x)/f + 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 6*a**2*d
**3*x**2*sin(e + f*x)/f**2 - 3*a**2*d**3*x**2*cos(e + f*x)**2/(8*f**2) + 3
*a**2*d**3*x*sin(e + f*x)*cos(e + f*x)/(4*f**3) + 12*a**2*d**3*x*cos(e + f
*x)/f**3 - 3*a**2*d**3*sin(e + f*x)**2/(8*f**4) - 12*a**2*d**3*sin(e + f*x
)/f**4, Ne(f, 0)), ((a*sin(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x
**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(212) = 424$.

Time = 0.08 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.33

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*(4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 +
4*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*
a^2*d^3*e^2/f^3 - 4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 16*
(f*x + e)*a^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*
a^2*c*d^2*e/f^2 + 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 +
48*(f*x + e)*a^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 12*(2*f*x +
2*e - sin(2*f*x + 2*e))*a^2*c^2*d*e/f - 48*(f*x + e)*a^2*c^2*d*e/f - 32*a^
2*c^3*cos(f*x + e) + 32*a^2*d^3*e^3*cos(f*x + e)/f^3 - 96*a^2*c*d^2*e^2*co
s(f*x + e)/f^2 + 96*a^2*c^2*d*e*cos(f*x + e)/f + 6*(2*(f*x + e)^2 - 2*(f*x
 + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 - 96*((f*x + e)
*cos(f*x + e) - sin(f*x + e))*a^2*d^3*e^2/f^3 - 12*(2*(f*x + e)^2 - 2*(f*x
 + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 + 192*((f*x + e
)*cos(f*x + e) - sin(f*x + e))*a^2*c*d^2*e/f^2 + 6*(2*(f*x + e)^2 - 2*(f*x
 + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*c^2*d/f - 96*((f*x + e)*cos
(f*x + e) - sin(f*x + e))*a^2*c^2*d/f - 2*(4*(f*x + e)^3 - 6*(f*x + e)*cos
(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^3*e/f^3 + 96
*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a^2*d^3*e/f^3
 + 2*(4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)
*sin(2*f*x + 2*e))*a^2*c*d^2/f^2 - 96*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*
(f*x + e)*sin(f*x + e))*a^2*c*d^2/f^2 + (2*(f*x + e)^4 - 3*(2*(f*x + e)...
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.50

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx = \frac{3}{8}\,a^2d^3x^4 + \frac{3}{2}\,a^2cd^2x^3 + \frac{9}{4}\,a^2c^2dx^2 + \frac{3}{2}\,a^2c^3x - \frac{3\,(2\,a^2d^3f^2x^2 + 4\,a^2cd^2f^2x + 2\,a^2c^2df^2 - a^2d^3)\cos(2\,fx+2\,e)}{16\,f^4} - \frac{2\,(a^2d^3f^3x^3 + 3\,a^2cd^2f^3x^2 + 3\,a^2c^2df^3x + a^2c^3f^3 - 6\,a^2d^3fx - 6\,a^2cd^2f)\cos(fx+e)}{f^4} - \frac{(2\,a^2d^3f^3x^3 + 6\,a^2cd^2f^3x^2 + 6\,a^2c^2df^3x + 2\,a^2c^3f^3 - 3\,a^2d^3fx - 3\,a^2cd^2f)\sin(2\,fx+2\,e)}{8\,f^4} + \frac{6\,(a^2d^3f^2x^2 + 2\,a^2cd^2f^2x + a^2c^2df^2 - 2\,a^2d^3)\sin(fx+e)}{f^4}$$

input
```
integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x -
3/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*c
os(2*f*x + 2*e)/f^4 - 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2
*d*f^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*cos(f*x + e)/f^4 -
 1/8*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^2 + 6*a^2*c^2*d*f^3*x + 2*a^2*
c^3*f^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)*sin(2*f*x + 2*e)/f^4 + 6*(a^2*d^3
*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*sin(f*x + e)/f^4
```

Mupad [B] (verification not implemented)

Time = 36.62 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.02

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx =$$

$$-\frac{96\,a^2\,d^3\,\sin\left(e+f\,x\right)-\frac{3\,a^2\,d^3\,\cos(2\,e+2\,f\,x)}{2}+16\,a^2\,c^3\,f^3\,\cos\left(e+f\,x\right)-12\,a^2\,c^3\,f^4\,x+2\,a^2\,c^3\,f^3\,\sin\left(2\,e\right.}{}$$

input
```
int((a + a*sin(e + f*x))^2*(c + d*x)^3,x)
```

output
```
-(96*a^2*d^3*sin(e + f*x) - (3*a^2*d^3*cos(2*e + 2*f*x))/2 + 16*a^2*c^3*f^
3*cos(e + f*x) - 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sin(2*e + 2*f*x) - 3*a^2
*d^3*f^4*x^4 - 96*a^2*c*d^2*f*cos(e + f*x) - 96*a^2*d^3*f*x*cos(e + f*x) +
 3*a^2*d^3*f^2*x^2*cos(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sin(2*e + 2*f*x) -
 3*a^2*c*d^2*f*sin(2*e + 2*f*x) - 48*a^2*c^2*d*f^2*sin(e + f*x) - 3*a^2*d^
3*f*x*sin(2*e + 2*f*x) + 3*a^2*c^2*d*f^2*cos(2*e + 2*f*x) - 18*a^2*c^2*d*f
^4*x^2 - 12*a^2*c*d^2*f^4*x^3 + 16*a^2*d^3*f^3*x^3*cos(e + f*x) - 48*a^2*d
^3*f^2*x^2*sin(e + f*x) + 6*a^2*c*d^2*f^2*x*cos(2*e + 2*f*x) + 48*a^2*c*d^
2*f^3*x^2*cos(e + f*x) + 6*a^2*c^2*d*f^3*x*sin(2*e + 2*f*x) + 6*a^2*c*d^2*
f^3*x^2*sin(2*e + 2*f*x) + 48*a^2*c^2*d*f^3*x*cos(e + f*x) - 96*a^2*c*d^2*
f^2*x*sin(e + f*x))/(8*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.93

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx$$
$$= \frac{a^2\left(-12\cos(fx+e)\sin(fx+e)\,c^2d\,f^3x-12\cos(fx+e)\sin(fx+e)\,c\,d^2f^3x^2-4\cos(fx+e)\sin(fx\right.}{}$$

input
```
int((d*x+c)^3*(a+a*sin(f*x+e))^2,x)
```

output
```
(a**2*( - 4*cos(e + f*x)*sin(e + f*x)*c**3*f**3 - 12*cos(e + f*x)*sin(e +
f*x)*c**2*d*f**3*x - 12*cos(e + f*x)*sin(e + f*x)*c*d**2*f**3*x**2 + 6*cos
(e + f*x)*sin(e + f*x)*c*d**2*f - 4*cos(e + f*x)*sin(e + f*x)*d**3*f**3*x*
*3 + 6*cos(e + f*x)*sin(e + f*x)*d**3*f*x - 16*cos(e + f*x)*c**3*f**3 - 48
*cos(e + f*x)*c**2*d*f**3*x - 48*cos(e + f*x)*c*d**2*f**3*x**2 + 96*cos(e
+ f*x)*c*d**2*f - 16*cos(e + f*x)*d**3*f**3*x**3 + 96*cos(e + f*x)*d**3*f*
x + 6*sin(e + f*x)**2*c**2*d*f**2 + 12*sin(e + f*x)**2*c*d**2*f**2*x + 6*s
in(e + f*x)**2*d**3*f**2*x**2 - 3*sin(e + f*x)**2*d**3 + 48*sin(e + f*x)*c
**2*d*f**2 + 96*sin(e + f*x)*c*d**2*f**2*x + 48*sin(e + f*x)*d**3*f**2*x**
2 - 96*sin(e + f*x)*d**3 + 12*c**3*f**4*x + 18*c**2*d*f**4*x**2 - 12*c**2*
d*f**2 + 12*c*d**2*f**4*x**3 - 6*c*d**2*f**2*x + 3*d**3*f**4*x**4 - 3*d**3
*f**2*x**2 + 6*d**3))/(8*f**4)
```

3.102 $\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\begin{aligned}\int (c+dx)^2(a+a\sin(e+fx))^2\,dx = & -\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\cos(e+fx)}{f^3} \\ & -\frac{2a^2(c+dx)^2\cos(e+fx)}{f} \\ & +\frac{4a^2d(c+dx)\sin(e+fx)}{f^2} \\ & +\frac{a^2d^2\cos(e+fx)\sin(e+fx)}{4f^3} \\ & -\frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f} \\ & +\frac{a^2d(c+dx)\sin^2(e+fx)}{2f^2}\end{aligned}$$

output
```
-1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*a^2*d^2*cos(f*x+e)/f^3-2*a^2*(d*x
+c)^2*cos(f*x+e)/f+4*a^2*d*(d*x+c)*sin(f*x+e)/f^2+1/4*a^2*d^2*cos(f*x+e)*s
in(f*x+e)/f^3-1/2*a^2*(d*x+c)^2*cos(f*x+e)*sin(f*x+e)/f+1/2*a^2*d*(d*x+c)*
sin(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$$
$$= \frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3-16(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)-2df(c+dx)\cos(}{}$$

input
```
Integrate[(c + d*x)^2*(a + a*Sin[e + f*x])^2,x]
```

output
```
(a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 - 16*(c^2*f^2 + 2*c*d*
f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] - 2*d*f*(c + d*x)*Cos[2*(e + f*x)
] + 32*c*d*f*Sin[e + f*x] + 32*d^2*f*x*Sin[e + f*x] + d^2*Sin[2*(e + f*x)]
 - 2*c^2*f^2*Sin[2*(e + f*x)] - 4*c*d*f^2*x*Sin[2*(e + f*x)] - 2*d^2*f^2*x
^2*Sin[2*(e + f*x)]))/(8*f^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a\sin(e+fx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2(a\sin(e+fx)+a)^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^2\sin^2(e+fx)+2a^2(c+dx)^2\sin(e+fx)+a^2(c+dx)^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2d(c+dx)\sin^2(e+fx)}{2f^2}+\frac{4a^2d(c+dx)\sin(e+fx)}{f^2}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}-\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\cos(e+fx)}{f^3}+\frac{a^2d^2\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{a^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/4*(a^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d^2*Cos[e + f*x])/
f^3 - (2*a^2*(c + d*x)^2*Cos[e + f*x])/f + (4*a^2*d*(c + d*x)*Sin[e + f*x]
)/f^2 + (a^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c + d*x)^2*Cos
[e + f*x]*Sin[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sin[e + f*x]^2)/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

method	result
parallelrisch	$-\frac{a^2\left(\left(\left((dx+c)^2f^2-\frac{d^2}{2}\right)\sin(2fx+2e)+df(dx+c)\cos(2fx+2e)+\left(8(dx+c)^2f^2-16d^2\right)\cos(fx+e)-16df(dx+c)\sin(fx \cdots\right.\right.}{4f^3}$
risch	$\frac{a^2d^2x^3}{2}+\frac{3a^2cd\,x^2}{2}+\frac{3a^2c^2x}{2}+\frac{a^2c^3}{2d}-\frac{2a^2\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\cos(fx+e)}{f^3}+\frac{4a^2d(dx+c)\sin(fx \cdots}{f^2}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{a^2\left(\frac{d^2\left((fx+e)^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx+e)\cos(fx+e)^2}{2}+\frac{\sin(fx+e)\cos(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}-\frac{(fx \cdots}{3}\right.}{f^2} \cdots\right.}{\cdots}$
norman	$\frac{a^2d^2x^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{a^2d^2x^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{4a^2c^2f^2+2a^2cdf-8a^2d^2}{2f^3}+\frac{a^2d^2x^3}{2}+\frac{\left(4a^2c^2f^2-2a^2cdf-8a^2d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right) \cdots}{2f^3}}{\cdots}$
derivativedivides	$\frac{a^2c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cde\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a^2cd\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+ \cdots\right.\right.}{f}}{\cdots}$
default	$\frac{a^2c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cde\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a^2cd\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+ \cdots\right.\right.}{f}}{\cdots}$
orering	Expression too large to display

input
```
int((d*x+c)^2*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*(((d*x+c)^2*f^2-1/2*d^2)*sin(2*f*x+2*e)+d*f*(d*x+c)*cos(2*f*x+2*e
)+(8*(d*x+c)^2*f^2-16*d^2)*cos(f*x+e)-16*d*f*(d*x+c)*sin(f*x+e)+(-2*d^2*x^
3-6*c*d*x^2-6*c^2*x)*f^3+8*c^2*f^2-d*f*c-16*d^2)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.26

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$$
$$=\frac{2\,a^2d^2f^3x^3+6\,a^2cdf^3x^2-2\left(a^2d^2fx+a^2cdf\right)\cos\left(fx+e\right)^2+\left(6\,a^2c^2f^3+a^2d^2f\right)x-8\left(a^2d^2f^2x^2+2\,a^2 \cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 - 2*(a^2*d^2*f*x + a^2*c*d*f)*c
os(f*x + e)^2 + (6*a^2*c^2*f^3 + a^2*d^2*f)*x - 8*(a^2*d^2*f^2*x^2 + 2*a^2
*c*d*f^2*x + a^2*c^2*f^2 - 2*a^2*d^2)*cos(f*x + e) + (16*a^2*d^2*f*x + 16*
a^2*c*d*f - (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2
)*cos(f*x + e))*sin(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(163) = 326$.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.71

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^2x\sin^2(e+fx)}{2} + \frac{a^2c^2x\cos^2(e+fx)}{2} + a^2c^2x - \frac{a^2c^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^2c^2\cos(e+fx)}{f} + \frac{a^2cdx^2\sin^2(e+fx)}{2} + \frac{a^2cd}{} \\ (a\sin(e)+a)^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
 a**2*c**2*x - a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**2*cos
(e + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)
**2/2 + a**2*c*d*x**2 - a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f - 4*a**2*c*
d*x*cos(e + f*x)/f + a**2*c*d*sin(e + f*x)**2/(2*f**2) + 4*a**2*c*d*sin(e
+ f*x)/f**2 + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*
x)**2/6 + a**2*d**2*x**3/3 - a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f
) - 2*a**2*d**2*x**2*cos(e + f*x)/f + a**2*d**2*x*sin(e + f*x)**2/(4*f**2)
 + 4*a**2*d**2*x*sin(e + f*x)/f**2 - a**2*d**2*x*cos(e + f*x)**2/(4*f**2)
+ a**2*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3) + 4*a**2*d**2*cos(e + f*x)/
f**3, Ne(f, 0)), ((a*sin(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(158) = 316$.

Time = 0.06 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.02

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$$

$$= \frac{6\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))a^2c^2+24\,(fx+e)a^2c^2+\frac{8\,(fx+e)^3a^2d^2}{f^2}-\frac{24\,(fx+e)^2a^2d^2e}{f^2}+\frac{6\,(2\,fx+2\,e-\sin(2\,fx+}{f^2}}{}$$

input
```
integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/24*(6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 +
8*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 6*(2*f*x + 2*e
- sin(2*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 + 24*(f
*x + e)^2*a^2*c*d/f - 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d*e/f - 48
*(f*x + e)*a^2*c*d*e/f - 48*a^2*c^2*cos(f*x + e) - 48*a^2*d^2*e^2*cos(f*x
+ e)/f^2 + 96*a^2*c*d*e*cos(f*x + e)/f - 6*(2*(f*x + e)^2 - 2*(f*x + e)*si
n(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d^2*e/f^2 + 96*((f*x + e)*cos(f*x +
 e) - sin(f*x + e))*a^2*d^2*e/f^2 + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f
*x + 2*e) - cos(2*f*x + 2*e))*a^2*c*d/f - 96*((f*x + e)*cos(f*x + e) - sin
(f*x + e))*a^2*c*d/f + (4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(
2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 - 48*(((f*x + e)^2 - 2)*c
os(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a^2*d^2/f^2)/f
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\begin{aligned}\int (c+dx)^2(a+a\sin(e+fx))^2\,dx &= \frac{1}{2}\,a^2d^2x^3+\frac{3}{2}\,a^2cdx^2+\frac{3}{2}\,a^2c^2x-\frac{(a^2d^2fx+a^2cdf)\cos\left(2\,fx+2\,e\right)}{4\,f^3} \\ &\quad -\frac{2\,(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2-2\,a^2d^2)\cos\left(fx+e\right)}{f^3} \\ &\quad -\frac{(2\,a^2d^2f^2x^2+4\,a^2cdf^2x+2\,a^2c^2f^2-a^2d^2)\sin\left(2\,fx+2\,e\right)}{8\,f^3} \\ &\quad +\frac{4\,(a^2d^2fx+a^2cdf)\sin\left(fx+e\right)}{f^3}\end{aligned}$$

input
```
integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/4*(a^2*d^2*f*x + a^2
*c*d*f)*cos(2*f*x + 2*e)/f^3 - 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*
c^2*f^2 - 2*a^2*d^2)*cos(f*x + e)/f^3 - 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d
*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*sin(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x +
a^2*c*d*f)*sin(f*x + e)/f^3
```

Mupad [B] (verification not implemented)

Time = 36.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx =$$

$$-\frac{8\,a^2\,c^2\,f^2\,\cos\left(e+f\,x\right)-\frac{a^2\,d^2\,\sin(2\,e+2\,f\,x)}{2}-16\,a^2\,d^2\,\cos\left(e+f\,x\right)-6\,a^2\,c^2\,f^3\,x+a^2\,c^2\,f^2\,\sin\left(2\,e+2\,f\right.}{}$$

input
```
int((a + a*sin(e + f*x))^2*(c + d*x)^2,x)
```

output
```
-(8*a^2*c^2*f^2*cos(e + f*x) - (a^2*d^2*sin(2*e + 2*f*x))/2 - 16*a^2*d^2*c
os(e + f*x) - 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sin(2*e + 2*f*x) - 2*a^2*d^2*f
^3*x^3 + a^2*c*d*f*cos(2*e + 2*f*x) - 16*a^2*d^2*f*x*sin(e + f*x) + a^2*d^
2*f^2*x^2*sin(2*e + 2*f*x) - 6*a^2*c*d*f^3*x^2 + a^2*d^2*f*x*cos(2*e + 2*f
*x) - 16*a^2*c*d*f*sin(e + f*x) + 8*a^2*d^2*f^2*x^2*cos(e + f*x) + 16*a^2*
c*d*f^2*x*cos(e + f*x) + 2*a^2*c*d*f^2*x*sin(2*e + 2*f*x))/(4*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.51

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx$$

$$=\frac{a^2\left(-2\cos\left(fx+e\right)\sin\left(fx+e\right)c^2f^2-4\cos\left(fx+e\right)\sin\left(fx+e\right)cd\,f^2x-2\cos\left(fx+e\right)\sin\left(fx+e\right)d^2\right.}{}$$

input
```
int((d*x+c)^2*(a+a*sin(f*x+e))^2,x)
```

output
```
(a**2*( - 2*cos(e + f*x)*sin(e + f*x)*c**2*f**2 - 4*cos(e + f*x)*sin(e + f
*x)*c*d*f**2*x - 2*cos(e + f*x)*sin(e + f*x)*d**2*f**2*x**2 + cos(e + f*x)
*sin(e + f*x)*d**2 - 8*cos(e + f*x)*c**2*f**2 - 16*cos(e + f*x)*c*d*f**2*x
 - 8*cos(e + f*x)*d**2*f**2*x**2 + 16*cos(e + f*x)*d**2 + 2*sin(e + f*x)**
2*c*d*f + 2*sin(e + f*x)**2*d**2*f*x + 16*sin(e + f*x)*c*d*f + 16*sin(e +
f*x)*d**2*f*x + 2*c**2*e*f**2 + 6*c**2*f**3*x + 6*c*d*f**3*x**2 - 4*c*d*f
+ 3*d**2*e + 2*d**2*f**3*x**3 - d**2*f*x))/(4*f**3)
```

3.103 $\int (c+dx)(a+a\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx = \frac{3a^2(c+dx)^2}{4d} - \frac{2a^2(c+dx)\cos(e+fx)}{f} + \frac{2a^2d\sin(e+fx)}{f^2} - \frac{a^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f} + \frac{a^2d\sin^2(e+fx)}{4f^2}$$

output
```
3/4*a^2*(d*x+c)^2/d-2*a^2*(d*x+c)*cos(f*x+e)/f+2*a^2*d*sin(f*x+e)/f^2-1/2*
a^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f+1/4*a^2*d*sin(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 12.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx = -\frac{a^2(6(e+fx)(-2cf+d(e-fx))+16f(c+dx)\cos(e+fx)+d\cos(2(e+fx))-16d\sin(e+fx)+}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/8*(a^2*(6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*f*(c + d*x)*Cos[e + f*x
] + d*Cos[2*(e + f*x)] - 16*d*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)
]))/f^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a\sin(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a\sin(e+fx)+a)^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)\sin^2(e+fx)+2a^2(c+dx)\sin(e+fx)+a^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2(c+dx)\cos(e+fx)}{f}-\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2(c+dx)^2}{4d}+\frac{a^2d\sin^2(e+fx)}{4f^2}+\frac{2a^2d\sin(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + a*Sin[e + f*x])^2,x]
```

output
```
(3*a^2*(c + d*x)^2)/(4*d) - (2*a^2*(c + d*x)*Cos[e + f*x])/f + (2*a^2*d*Si
n[e + f*x])/f^2 - (a^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (a^2*d
*Sin[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

method	result
parallelrisch	$-\frac{a^{2}\left(f(dx+c)\sin(2fx+2e)+\frac{d\cos(2fx+2e)}{2}+8f(dx+c)\cos(fx+e)-8d\sin(fx+e)+(-3d\,x^{2}-6cx)f^{2}+8cf-\frac{d}{2}\right)}{4f^{2}}$
risch	$\frac{3a^{2}d\,x^{2}}{4}+\frac{3a^{2}cx}{2}-\frac{2a^{2}(dx+c)\cos(fx+e)}{f}+\frac{2a^{2}d\sin(fx+e)}{f^{2}}-\frac{a^{2}d\cos(2fx+2e)}{8f^{2}}-\frac{a^{2}(dx+c)\sin(2fx+2e)}{4f}$
parts	$a^{2}\left(\frac{1}{2}d\,x^{2}+cx\right)+\frac{a^{2}\left(\frac{d\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx+e)^{2}}{4}+\frac{\sin(fx+e)^{2}}{4}\right)}{f}+c\left(-\frac{\sin(fx+e)\cos(fx+}{2}\right.\right.}{f}$
derivativedivides	$\frac{a^{2}c\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^{2}de\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^{2}d\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{}\right.\right.}{f}}{}$
default	$\frac{a^{2}c\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^{2}de\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^{2}d\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{}\right.\right.}{f}}{}$
norman	$\frac{\frac{a^{2}(cf+4d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{f^{2}}+\frac{a^{2}dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{f}-\frac{4a^{2}c}{f}+\frac{3a^{2}d\,x^{2}}{4}-\frac{\left(4a^{2}cf-a^{2}d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{f^{2}}+3a^{2}cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+\frac{3a^{2}d}{}}{\left(1+\tan\right.}$
orering	$\frac{(2d^{5}f^{4}x^{6}+12c\,d^{4}f^{4}x^{5}+28c^{2}d^{3}f^{4}x^{4}+32c^{3}d^{2}f^{4}x^{3}+18c^{4}d\,f^{4}x^{2}+15d^{5}f^{2}x^{4}+4c^{5}f^{4}x+60c\,d^{4}f^{2}x^{3}+85c^{2}d^{3}f^{2}x^{2}+50c^{3}}{4f^{4}(dx+c)^{4}}$

input
```
int((d*x+c)*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*(f*(d*x+c)*sin(2*f*x+2*e)+1/2*d*cos(2*f*x+2*e)+8*f*(d*x+c)*cos(f*
x+e)-8*d*sin(f*x+e)+(-3*d*x^2-6*c*x)*f^2+8*c*f-1/2*d)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int(c+dx)(a+a\sin(e+fx))^{2}\,dx$$
$$=\frac{3\,a^{2}df^{2}x^{2}+6\,a^{2}cf^{2}x-a^{2}d\cos\left(fx+e\right)^{2}-8\left(a^{2}dfx+a^{2}cf\right)\cos\left(fx+e\right)+2\left(4\,a^{2}d-\left(a^{2}dfx+a^{2}cf\right)\mathrm{co}\right.}{4\,f^{2}}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x - a^2*d*cos(f*x + e)^2 - 8*(a^2*d*f*x
 + a^2*c*f)*cos(f*x + e) + 2*(4*a^2*d - (a^2*d*f*x + a^2*c*f)*cos(f*x + e)
)*sin(f*x + e))/f^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(94) = 188$.

Time = 0.21 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.23

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx$$
$$= \begin{cases} \frac{a^2cx\sin^2(e+fx)}{2} + \frac{a^2cx\cos^2(e+fx)}{2} + a^2cx - \frac{a^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^2c\cos(e+fx)}{f} + \frac{a^2dx^2\sin^2(e+fx)}{4} + \frac{a^2dx^2\cos^2}{4} \\ (a\sin(e)+a)^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*
c*x - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c*cos(e + f*x)/f + a
**2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2
/2 - a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*d*x*cos(e + f*x)/f
+ a**2*d*sin(e + f*x)**2/(4*f**2) + 2*a**2*d*sin(e + f*x)/f**2, Ne(f, 0)),
 ((a*sin(e) + a)**2*(c*x + d*x**2/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.09

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx$$
$$= \frac{2\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c+8\,(fx+e)a^2c+\frac{4\,(fx+e)^2a^2d}{f}-\frac{2\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2de}{f}-\frac{8\,(fx+e)a^2d}{f}}{}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/8*(2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
 + e)^2*a^2*d/f - 2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d*e/f - 8*(f*x +
e)*a^2*d*e/f - 16*a^2*c*cos(f*x + e) + 16*a^2*d*e*cos(f*x + e)/f + (2*(f*x
 + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d/f - 16*((
f*x + e)*cos(f*x + e) - sin(f*x + e))*a^2*d/f)/f
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx = \frac{3}{4}\,a^2dx^2 + \frac{3}{2}\,a^2cx - \frac{a^2d\cos\left(2\,fx+2\,e\right)}{8\,f^2} + \frac{2\,a^2d\sin\left(fx+e\right)}{f^2} - \frac{2\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)}{f^2} - \frac{\left(a^2dfx+a^2cf\right)\sin\left(2\,fx+2\,e\right)}{4\,f^2}$$

input
```
integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*sin
(f*x + e)/f^2 - 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)/f^2 - 1/4*(a^2*d*f*x
+ a^2*c*f)*sin(2*f*x + 2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 35.93 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx = \frac{a^2\,d\sin\left(e+f\,x\right)^2+8\,a^2\,d\,\sin\left(e+f\,x\right)+16\,a^2\,c\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\,d\,f^2\,x^2-a^2\,c\,f\,\sin\left(2\,e+2\,f\,x\right)-}{4\,f^2}$$

input
```
int((a + a*sin(e + f*x))^2*(c + d*x),x)
```

output
```
(a^2*d*sin(e + f*x)^2 + 8*a^2*d*sin(e + f*x) + 16*a^2*c*f*sin(e/2 + (f*x)/
2)^2 + 3*a^2*d*f^2*x^2 - a^2*c*f*sin(2*e + 2*f*x) + 6*a^2*c*f^2*x - a^2*d*
f*x*sin(2*e + 2*f*x) + 8*a^2*d*f*x*(2*sin(e/2 + (f*x)/2)^2 - 1))/(4*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx = \frac{a^2(-2\cos(fx+e)\sin(fx+e)\,cf-2\cos(fx+e)\sin(fx+e)\,dfx-8\cos(fx+e)\,cf-8\cos(fx+e)}{4f^2}$$

input
```
int((d*x+c)*(a+a*sin(f*x+e))^2,x)
```

output
```
(a**2*( - 2*cos(e + f*x)*sin(e + f*x)*c*f - 2*cos(e + f*x)*sin(e + f*x)*d*
f*x - 8*cos(e + f*x)*c*f - 8*cos(e + f*x)*d*f*x + sin(e + f*x)**2*d + 8*si
n(e + f*x)*d + 2*c*e*f + 6*c*f**2*x - 8*d*e + 3*d*f**2*x**2 - 2*d))/(4*f**
2)
```

3.104 $\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx = -\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{3a^2\log(c+dx)}{2d} + \frac{2a^2\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d} + \frac{2a^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
-1/2*a^2*cos(-2*e+2*c*f/d)*Ci(2*c*f/d+2*f*x)/d+3/2*a^2*ln(d*x+c)/d-2*a^2*C
i(c*f/d+f*x)*sin(-e+c*f/d)/d+2*a^2*cos(-e+c*f/d)*Si(c*f/d+f*x)/d-1/2*a^2*s
in(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx = \frac{a^2\left(-\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)+4\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4}{2d}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2/(c + d*x),x]
```

output
```
(a^2*(-(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x))/d]) + 3*Log[c + d
*x] + 4*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*Cos[e - (c*f)/d]*Sin
Integral[f*(c/d + x)] + Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d
]))/(2*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2}{c+dx}dx$$

$$\Big\downarrow \text{3799}$$

$$4a^2\int \frac{\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2 \int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx$$

$$\Big\downarrow \text{ 3793}$$

$$4a^2 \int \left(-\frac{\cos(2e+2fx)}{8(c+dx)}+\frac{\sin(e+fx)}{2(c+dx)}+\frac{3}{8(c+dx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$4a^2\left(\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{2d}-\frac{\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{8d}+\frac{\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf\right.}{8d}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2/(c + d*x),x]
```

output
```
4*a^2*(-1/8*(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d + (3*L
og[c + d*x])/(8*d) + (CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d) +
 (Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d) + (Sin[2*e - (2*c*f)/
d]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{3a^2 f\ln(cf-de+d(fx+e))}{2d}-\frac{a^2 f\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+2a^2 f\left(\right.}{f}$
default	$\frac{\frac{3a^2 f\ln(cf-de+d(fx+e))}{2d}-\frac{a^2 f\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+2a^2 f\left(\right.}{f}$
parts	$\frac{a^2\ln(dx+c)}{d}+\frac{a^2\ln(cf-de+d(fx+e))}{2d}-\frac{a^2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{2d}-\frac{a^2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2d}$
risch	$-\frac{ia^2 e^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d}+\frac{3a^2\ln(dx+c)}{2d}+\frac{a^2 e^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(2ifx+2ie+\frac{2i(cf-}{d}\right.}{4d}$

input
```
int((a+a*sin(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/f*(3/2*a^2*f*ln(c*f-d*e+d*(f*x+e))/d-1/4*a^2*f*(2*Si(2*f*x+2*e+2*(c*f-d*
e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d
)/d)+2*a^2*f*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/
d)*sin((c*f-d*e)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx = -\frac{a^2\cos\left(-\frac{2(de-cf)}{d}\right)\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)+4a^2\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)+a^2\sin\left(-\frac{2(de-cf)}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)-}{2d}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(a^2*cos(-2*(d*e - c*f)/d)*cos_integral(2*(d*f*x + c*f)/d) + 4*a^2*co
s_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) + a^2*sin(-2*(d*e - c*f)/d
)*sin_integral(2*(d*f*x + c*f)/d) - 4*a^2*cos(-(d*e - c*f)/d)*sin_integral
((d*f*x + c*f)/d) - 3*a^2*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx = a^2\left(\int \frac{2\sin(e+fx)}{c+dx}\,dx+\int \frac{\sin^2(e+fx)}{c+dx}\,dx+\int \frac{1}{c+dx}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2/(d*x+c),x)
```

output
```
a**2*(Integral(2*sin(e + f*x)/(c + d*x), x) + Integral(sin(e + f*x)**2/(c
+ d*x), x) + Integral(1/(c + d*x), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.32

$$\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx$$
$$= \frac{\frac{4\,a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{4\left(f\left(-i\,E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_1\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E\right.\right.}{d}}{}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d + 4*(f*(-I*exp_integral_e(1,
 (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d
- I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x +
e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c
*f)/d))*sin(-(d*e - c*f)/d))*a^2/d + (f*(exp_integral_e(1, 2*(-I*(f*x + e)
*d + I*d*e - I*c*f)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*
c*f)/d))*cos(-2*(d*e - c*f)/d) + f*(-I*exp_integral_e(1, 2*(-I*(f*x + e)*d
 + I*d*e - I*c*f)/d) + I*exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*
c*f)/d))*sin(-2*(d*e - c*f)/d) + 2*f*log((f*x + e)*d - d*e + c*f))*a^2/d)/
f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 6807, normalized size of antiderivative = 46.94

$$\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(
c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 6*a^2*log(abs(d*x + c)
)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*real_part(cos_
integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/
d)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2
*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(1
/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 8*a^2*real_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 8*a^
2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2
*tan(1/2*c*f/d) + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e
)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(-2
*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*a^2
*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*
c*f/d)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan
(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 2*a^2*imag_part(cos_integral(-2*f*x -
2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^2*sin_in
tegral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^
2 - 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f
/d)^2*tan(1/2*c*f/d)^2 - 8*a^2*real_part(cos_integral(-f*x - c*f/d))*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2}{c + dx}\,dx = \int \frac{(a + a\,\sin(e + f\,x))^2}{c + d\,x}\,dx$$

input
```
int((a + a*sin(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + a*sin(e + f*x))^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a + a\sin(e + fx))^2}{c + dx}\,dx = \frac{a^2\left(\left(\int \frac{\sin(fx+e)^2}{dx+c}dx\right)d + 2\left(\int \frac{\sin(fx+e)}{dx+c}dx\right)d + \log(dx + c)\right)}{d}$$

input
```
int((a+a*sin(f*x+e))^2/(d*x+c),x)
```

output
```
(a**2*(int(sin(e + f*x)**2/(c + d*x),x)*d + 2*int(sin(e + f*x)/(c + d*x),x
)*d + log(c + d*x)))/d
```

3.105 $\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx = \frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{a^2f\,\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{2a^2f\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{a^2f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

output
```
2*a^2*f*cos(-e+c*f/d)*Ci(c*f/d+f*x)/d^2-a^2*f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2
*c*f/d)/d^2-4*a^2*sin(1/2*e+1/4*Pi+1/2*f*x)^4/d/(d*x+c)+2*a^2*f*sin(-e+c*f
/d)*Si(c*f/d+f*x)/d^2+a^2*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx = \frac{a^2\left(-3d+d\cos(2(e+fx))+4f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+2f(c+dx)\,\text{CosIntegral}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(a^2*(-3*d + d*Cos[2*(e + f*x)] + 4*f*(c + d*x)*Cos[e - (c*f)/d]*CosIntegr
al[f*(c/d + x)] + 2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*Sin[2*e - (
2*c*f)/d] - 4*d*Sin[e + f*x] - 4*c*f*Sin[e - (c*f)/d]*SinIntegral[f*(c/d +
 x)] - 4*d*f*x*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2*e -
 (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)/d]*
SinIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3799, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3799}$$

$$4a^2\int \frac{\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3042$$

$$4a^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3794$$

$$4a^2\left(\frac{2f\int\left(\frac{\cos(e+fx)}{4(c+dx)}+\frac{\sin(2e+2fx)}{8(c+dx)}\right)dx}{d}-\frac{\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d(c+dx)}\right)$$

$$\Big\downarrow \; 2009$$

$$4a^2\left(\frac{2f\left(\frac{\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{8d}+\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{4d}-\frac{\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{4d}+\frac{\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf\right.}{8d}\right.}{d}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2/(c + d*x)^2,x]
```

output
```
4*a^2*(-(Sin[e/2 + Pi/4 + (f*x)/2]^4/(d*(c + d*x))) + (2*f*((Cos[e - (c*f)
/d]*CosIntegral[(c*f)/d + f*x])/(4*d) + (CosIntegral[(2*c*f)/d + 2*f*x]*Si
n[2*e - (2*c*f)/d])/(8*d) - (Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/
(4*d) + (Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*d)))/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{-\frac{3a^2f^2}{2(cf-de+d(fx+e))d}-\frac{a^2f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\mathrm{s}}{d}\right.}{d}\right.}{4}}{f}$
default	$\frac{-\frac{3a^2f^2}{2(cf-de+d(fx+e))d}-\frac{a^2f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\mathrm{s}}{d}\right.}{d}\right.}{4}}{f}$
parts	$-\frac{a^2}{d(dx+c)}+\frac{a^2\left(-\frac{f^2}{2(cf-de+d(fx+e))d}-\frac{f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}(2}{}\right.}{d}\right.}{4}\right.}{f}$
risch	$-\frac{f\,a^2\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d^2}-\frac{3a^2}{2d(dx+c)}-\frac{ia^2f\,\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(2ifx+2ie+\frac{2i(cf-}{d}\right.}{2d^2}$

input
```
int((a+a*sin(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-3/2*a^2*f^2/(c*f-d*e+d*(f*x+e))/d-1/4*a^2*f^2*(-2*cos(2*f*x+2*e)/(c*
f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2
*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d)+2*a^2*f^2*(-sin(f*x+
e)/(c*f-d*e+d*(f*x+e))/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+
e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.35

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx = \frac{a^2d\cos\left(fx+e\right)^2 - 2\,a^2d\sin\left(fx+e\right) - 2\,a^2d + 2\left(a^2dfx+a^2cf\right)\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right) - \left(a^2dfx+a^2\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
(a^2*d*cos(f*x + e)^2 - 2*a^2*d*sin(f*x + e) - 2*a^2*d + 2*(a^2*d*f*x + a^
2*c*f)*cos(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) - (a^2*d*f*x + a^
2*c*f)*cos_integral(2*(d*f*x + c*f)/d)*sin(-2*(d*e - c*f)/d) + (a^2*d*f*x
+ a^2*c*f)*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*(a^2*
d*f*x + a^2*c*f)*sin(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d))/(d^3*x
 + c*d^2)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx = a^2\left(\int \frac{2\sin\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{\sin^2\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2/(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(
sin(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*
d*x + d**2*x**2), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{4a^2f^2}{(fx+e)d^2-d^2e+cdf} - \frac{4\left(f^2\left(-iE_2\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_2\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_2\right.}{(fx+e)d^2-d^2e+cdf}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - 4*(f^2*(-I*exp_integral_
e(2, (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e
)*d - I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
 + I*c*f)/d))*sin(-(d*e - c*f)/d))*a^2/((f*x + e)*d^2 - d^2*e + c*d*f) - (
f^2*(exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + exp_integra
l_e(2, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*cos(-2*(d*e - c*f)/d) - f^2
*(I*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - I*exp_integr
al_e(2, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*sin(-2*(d*e - c*f)/d) - 2*
f^2)*a^2/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. 2(159) = 318.

Time = 0.43 (sec) , antiderivative size = 1048, normalized size of antiderivative = 6.47

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(4*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c
*f)/d)*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d) - 4*a^2*d*e*f^2*cos(-(d*e - c*f)/d)*cos_integral(((d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + 4*a^2*c*f^3*cos(-(d*e - c
*f)/d)*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_int
egral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin
(-2*(d*e - c*f)/d) + 2*a^2*d*e*f^2*cos_integral(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-2*(d*e - c*f)/d) - 2*a^2*c*f^3
*cos_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f
)/d)*sin(-2*(d*e - c*f)/d) + 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c
) + f)*f^2*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d) - 2*a^2*d*e*f^2*cos(-2*(d*e - c*f)/d)
*sin_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f
)/d) + 2*a^2*c*f^3*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + 4*(d*x + c)*a^2*(d*e/(d*x +
 c) - c*f/(d*x + c) + f)*f^2*sin(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - 4*a^2*d*e*f^2*sin(-(d
*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
 d*e + c*f)/d) + 4*a^2*c*f^3*sin(-(d*e - c*f)/d)*sin_integral(((d*x + c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + a*sin(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + a*sin(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a + a\sin(e + fx))^2}{(c + dx)^2}\,dx$$
$$= \frac{a^2\left(\left(\left(\int \frac{\sin(fx+e)^2}{d^2x^2+2cdx+c^2}dx\right)c^2 + \left(\int \frac{\sin(fx+e)^2}{d^2x^2+2cdx+c^2}dx\right)cdx + 2\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)c^2 + 2\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)cdx\right.\right.}{c\,(dx + c)}$$

input
```
int((a+a*sin(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(a**2*(int(sin(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + int(sin(
e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + 2*int(sin(e + f*x)/(c*
*2 + 2*c*d*x + d**2*x**2),x)*c**2 + 2*int(sin(e + f*x)/(c**2 + 2*c*d*x + d
**2*x**2),x)*c*d*x + x))/(c*(c + d*x))
```

3.106 $\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 225

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx = \frac{a^2f^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{d^3} - \frac{a^2f^2\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3} - \frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{a^2f^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^3} - \frac{a^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{d^3}$$

output
```
a^2*f^2*cos(-2*e+2*c*f/d)*Ci(2*c*f/d+2*f*x)/d^3+a^2*f^2*Ci(c*f/d+f*x)*sin(
-e+c*f/d)/d^3-4*a^2*f*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*x)^
3/d^2/(d*x+c)-2*a^2*sin(1/2*e+1/4*Pi+1/2*f*x)^4/d/(d*x+c)^2-a^2*f^2*cos(-e
+c*f/d)*Si(c*f/d+f*x)/d^3+a^2*f^2*sin(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d^3
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.57

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx =$$
$$-\frac{a^2\left(3d^2+4cdf\cos(e+fx)+4d^2fx\cos(e+fx)-d^2\cos(2(e+fx))-4f^2(c+dx)^2\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Co}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/4*(a^2*(3*d^2 + 4*c*d*f*Cos[e + f*x] + 4*d^2*f*x*Cos[e + f*x] - d^2*Cos
[2*(e + f*x)] - 4*f^2*(c + d*x)^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c
 + d*x))/d] + 4*f^2*(c + d*x)^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d]
+ 4*d^2*Sin[e + f*x] + 2*c*d*f*Sin[2*(e + f*x)] + 2*d^2*f*x*Sin[2*(e + f*x
)] + 4*c^2*f^2*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 8*c*d*f^2*x*Cos
[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 4*d^2*f^2*x^2*Cos[e - (c*f)/d]*Si
nIntegral[f*(c/d + x)] + 4*c^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(
c + d*x))/d] + 8*c*d*f^2*x*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x)
)/d] + 4*d^2*f^2*x^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))
/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.39, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3799, 3042, 3795, 3042, 3790, 16, 25, 3042, 3784, 3042, 3780, 3783, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2}{(c+dx)^3}dx$$

$\Big\downarrow$ 3799

$$4a^2\int \frac{\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{(c+dx)^3}dx$$

$\Big\downarrow$ 3042

$$4a^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{(c+dx)^3}dx$$

$\Big\downarrow$ 3795

$$4a^2\left(-\frac{2f^2\int \frac{\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{c+dx}dx}{d^2}+\frac{3f^2\int \frac{\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{c+dx}dx}{2d^2}-\frac{f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}-\frac{\sin^4\left(\frac{e}{2}+\frac{f}{2}\right.}{2d(c+d}\right.$$

$\Big\downarrow$ 3042

$$4a^2\left(\frac{3f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2}{c+dx}dx}{2d^2}-\frac{2f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}-\frac{f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}-\frac{\sin^4\left(\frac{e}{2}+\frac{fx}{2}\right.}{2d(c+dx}\right.$$

$\Big\downarrow$ 3790

$$4a^2\left(-\frac{2f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\int \frac{1}{c+dx}dx-\frac{1}{2}\int -\frac{\sin(e+fx)}{c+dx}dx\right)}{2d^2}-\frac{f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}\right.$$

$\Big\downarrow$ 16

$$4a^2\left(-\frac{2f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{\log(c+dx)}{2d}-\frac{1}{2}\int -\frac{\sin(e+fx)}{c+dx}dx\right)}{2d^2}-\frac{f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}-\right.$$

$\Big\downarrow$ 25

$$4a^2\left(-\frac{2f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\int \frac{\sin(e+fx)}{c+dx}dx+\frac{\log(c+dx)}{2d}\right)}{2d^2}-\frac{f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}-\right.$$

$$\Big\downarrow \text{3042}$$

$$4a^2\left(-\frac{2f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\int\frac{\sin(e+fx)}{c+dx}dx+\frac{\log(c+dx)}{2d}\right)}{2d^2}-\frac{f\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{d^2(c+dx)}-\right.$$

$$\Big\downarrow \text{3784}$$

$$4a^2\left(-\frac{2f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\left(\sin\left(e-\frac{cf}{d}\right)\int\frac{\cos\left(xf+\frac{cf}{d}\right)}{c+dx}dx+\cos\left(e-\frac{cf}{d}\right)\int\frac{\sin\left(xf+\frac{cf}{d}\right)}{c+dx}dx\right)+\frac{\log(c+}{2d}\right.}{2d^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$4a^2\left(-\frac{2f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\left(\sin\left(e-\frac{cf}{d}\right)\int\frac{\sin\left(xf+\frac{cf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(e-\frac{cf}{d}\right)\int\frac{\sin\left(xf+\frac{cf}{d}\right)}{c+dx}dx\right)+\log(\right.}{2d^2}\right.$$

$$\Big\downarrow \text{3780}$$

$$4a^2\left(\frac{3f^2\left(\frac{1}{2}\left(\sin\left(e-\frac{cf}{d}\right)\int\frac{\sin\left(xf+\frac{cf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}\right)+\frac{\log(c+dx)}{2d}\right)}{2d^2}-\frac{2f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}-\right.$$

$$\Big\downarrow \text{3783}$$

$$4a^2\left(-\frac{2f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\left(\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}\right)+\frac{\log(c+dx)}{2d}\right)}{2d^2}-\frac{f\,\text{s}}{}\right.$$

$$\Big\downarrow \text{3793}$$

$$4a^2\left(-\frac{2f^2\int\left(-\frac{\cos(2e+2fx)}{8(c+dx)}+\frac{\sin(e+fx)}{2(c+dx)}+\frac{3}{8(c+dx)}\right)dx}{d^2}+\frac{3f^2\left(\frac{1}{2}\left(\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}\right.\right.}{2d^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\left(\frac{3f^2\left(\frac{1}{2}\left(\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}\right)+\frac{\log(c+dx)}{2d}\right)}{2d^2}-\frac{2f^2\left(\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{2d}\right.}{}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2/(c + d*x)^3,x]
```

output
```
4*a^2*(-((f*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(d^2*(c
 + d*x))) - Sin[e/2 + Pi/4 + (f*x)/2]^4/(2*d*(c + d*x)^2) + (3*f^2*(Log[c
+ d*x]/(2*d) + ((CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (Cos[e -
 (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d)/2))/(2*d^2) - (2*f^2*(-1/8*(Cos[2
*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d + (3*Log[c + d*x])/(8*d)
 + (CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d) + (Cos[e - (c*f)/d]
*SinIntegral[(c*f)/d + f*x])/(2*d) + (Sin[2*e - (2*c*f)/d]*SinIntegral[(2*
c*f)/d + 2*f*x])/(8*d)))/d^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{3a^2f^3}{4(cf-de+d(fx+e))^2d}-\frac{a^2f^3\left(-\frac{\cos(2fx+2e)}{(cf-de+d(fx+e))^2d}-\frac{-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{\frac{4\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{C}}{d}}{d}}{}\right)}{4}$
default	$-\frac{3a^2f^3}{4(cf-de+d(fx+e))^2d}-\frac{a^2f^3\left(-\frac{\cos(2fx+2e)}{(cf-de+d(fx+e))^2d}-\frac{-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{\frac{4\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{C}}{d}}{d}}{}\right)}{4}$
parts	$-\frac{a^2}{2d(dx+c)^2}+\frac{a^2\left(-\frac{f^3}{4(cf-de+d(fx+e))^2d}-\frac{f^3\left(-\frac{\cos(2fx+2e)}{(cf-de+d(fx+e))^2d}-\frac{-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{4\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right.}{d}}{}\right)}{4}\right)}{f}$
risch	$\frac{if^2a^2\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^3}-\frac{3a^2}{4d(dx+c)^2}-\frac{a^2f^2\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(2ifx+2ie+\frac{2i(cf\right.}{d}\right.}{2d^3}$

input
```
int((a+a*sin(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-3/4*a^2*f^3/(c*f-d*e+d*(f*x+e))^2/d-1/4*a^2*f^3*(-cos(2*f*x+2*e)/(c*
f-d*e+d*(f*x+e))^2/d-(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*
x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*co
s(2*(c*f-d*e)/d)/d)/d)/d)+2*a^2*f^3*(-1/2*sin(f*x+e)/(c*f-d*e+d*(f*x+e))^2
/d+1/2*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-
d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.66

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx = \frac{a^2d^2\cos\left(fx+e\right)^2-2\,a^2d^2+2\left(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2\right)\cos\left(-\frac{2\,(de-cf)}{d}\right)\operatorname{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+2\,(a^2d}{}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*d^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f
^2*x + a^2*c^2*f^2)*cos(-2*(d*e - c*f)/d)*cos_integral(2*(d*f*x + c*f)/d)
+ 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos_integral((d*f*x
+ c*f)/d)*sin(-(d*e - c*f)/d) + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2
*c^2*f^2)*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) - 2*(a^2*d
^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos(-(d*e - c*f)/d)*sin_integr
al((d*f*x + c*f)/d) - 2*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e) - 2*(a^2*d^
2 + (a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e))*sin(f*x + e))/(d^5*x^2 + 2*c*d
^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx = a^2\left(\int \frac{2\sin\left(e+fx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{\sin^2\left(e+fx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2/(d*x+c)**3,x)
```

output
```
a**2*(Integral(2*sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3), x) + Integral(sin(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3), x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
 x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.12

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx =$$

$$-\frac{2a^2f^3}{(fx+e)^2d^3+d^3e^2-2cd^2ef+c^2df^2-2(d^3e-cd^2f)(fx+e)} - \frac{4\left(f^3\left(-iE_3\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_3\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-c}{d}\right.\right.}{(fx+e)^2d^3+d^3e^2-2cd^2ef}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(
d^3*e - c*d^2*f)*(f*x + e)) - 4*(f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d
- I*d*e + I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)
/d))*cos(-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e +
 I*c*f)/d) + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(
d*e - c*f)/d))*a^2/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 -
2*(d^3*e - c*d^2*f)*(f*x + e)) - (f^3*(exp_integral_e(3, 2*(-I*(f*x + e)*d
 + I*d*e - I*c*f)/d) + exp_integral_e(3, -2*(-I*(f*x + e)*d + I*d*e - I*c*
f)/d))*cos(-2*(d*e - c*f)/d) - f^3*(I*exp_integral_e(3, 2*(-I*(f*x + e)*d
+ I*d*e - I*c*f)/d) - I*exp_integral_e(3, -2*(-I*(f*x + e)*d + I*d*e - I*c
*f)/d))*sin(-2*(d*e - c*f)/d) - f^3)*a^2/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*
d^2*e*f + c^2*d*f^2 - 2*(d^3*e - c*d^2*f)*(f*x + e)))/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.39 (sec) , antiderivative size = 120870, normalized size of antiderivative = 537.20

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(a^2*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(
1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*d^2*f
^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan
(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*d^2*f^2*x^2*real_pa
rt(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*t
an(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*d^2*f^2*x^2*real_part(cos_inte
gral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*ta
n(c*f/d)^2*tan(1/2*c*f/d)^2 + 2*a^2*d^2*f^2*x^2*sin_integral((d*f*x + c*f)
/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c
*f/d)^2 + 2*a^2*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(f*x)^
2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 2*a^2
*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)
^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 2*a^2*d^2*f^2*x^2*i
mag_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*
e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*a^2*d^2*f^2*x^2*imag_part(co
s_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e
)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*a^2*d^2*f^2*x^2*sin_integral(2*(d*f*x
+ c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1
/2*c*f/d)^2 - 2*a^2*d^2*f^2*x^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx=\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a + a*sin(e + f*x))^2/(c + d*x)^3,x)
```

output
```
int((a + a*sin(e + f*x))^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx=\text{too large to display}$$

input
```
int((a+a*sin(f*x+e))^2/(d*x+c)^3,x)
```

output
```
(a**2*(2*cos(e + f*x)*sin(e + f*x)*c**2*d*f + 2*cos(e + f*x)*sin(e + f*x)*
c*d**2*f*x - 2*cos(e + f*x)*sin(e + f*x)*c*d**2 - 8*cos(e + f*x)*c*d**2 -
16*int(x**2/(tan((e + f*x)/2)**4*c**3 + 3*tan((e + f*x)/2)**4*c**2*d*x + 3
*tan((e + f*x)/2)**4*c*d**2*x**2 + tan((e + f*x)/2)**4*d**3*x**3 + 2*tan((
e + f*x)/2)**2*c**3 + 6*tan((e + f*x)/2)**2*c**2*d*x + 6*tan((e + f*x)/2)*
*2*c*d**2*x**2 + 2*tan((e + f*x)/2)**2*d**3*x**3 + c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3),x)*c**3*d**3*f**2 - 32*int(x**2/(tan((e + f*x)/2)*
*4*c**3 + 3*tan((e + f*x)/2)**4*c**2*d*x + 3*tan((e + f*x)/2)**4*c*d**2*x*
*2 + tan((e + f*x)/2)**4*d**3*x**3 + 2*tan((e + f*x)/2)**2*c**3 + 6*tan((e
 + f*x)/2)**2*c**2*d*x + 6*tan((e + f*x)/2)**2*c*d**2*x**2 + 2*tan((e + f*
x)/2)**2*d**3*x**3 + c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*c**
2*d**4*f**2*x - 16*int(x**2/(tan((e + f*x)/2)**4*c**3 + 3*tan((e + f*x)/2)
**4*c**2*d*x + 3*tan((e + f*x)/2)**4*c*d**2*x**2 + tan((e + f*x)/2)**4*d**
3*x**3 + 2*tan((e + f*x)/2)**2*c**3 + 6*tan((e + f*x)/2)**2*c**2*d*x + 6*t
an((e + f*x)/2)**2*c*d**2*x**2 + 2*tan((e + f*x)/2)**2*d**3*x**3 + c**3 +
3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*c*d**5*f**2*x**2 - 16*int(tan((
e + f*x)/2)/(tan((e + f*x)/2)**4*c**3 + 3*tan((e + f*x)/2)**4*c**2*d*x + 3
*tan((e + f*x)/2)**4*c*d**2*x**2 + tan((e + f*x)/2)**4*d**3*x**3 + 2*tan((
e + f*x)/2)**2*c**3 + 6*tan((e + f*x)/2)**2*c**2*d*x + 6*tan((e + f*x)/2)*
*2*c*d**2*x**2 + 2*tan((e + f*x)/2)**2*d**3*x**3 + c**3 + 3*c**2*d*x + ...
```

3.107 $\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3} + \frac{12d^3\,\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{af^4}$$

output
```
-I*(d*x+c)^3/a/f-(d*x+c)^3*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f+6*d*(d*x+c)^2*ln(
1-I*exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*polylog(2,I*exp(I*(f*x+e)))/a/f
^3+12*d^3*polylog(3,I*exp(I*(f*x+e)))/a/f^4
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{-12id^2f(c+dx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)+12d^3\operatorname{PolyLog}\left(3,ie^{i(e+fx)}\right)+f^2(c+dx)^2\left(-if(c+dx)+6d\log\right.}{af^4}$$

input
```
Integrate[(c + d*x)^3/(a + a*Sin[e + f*x]),x]
```

output
```
((-12*I)*d^2*f*(c + d*x)*PolyLog[2, I*E^(I*(e + f*x))] + 12*d^3*PolyLog[3,
 I*E^(I*(e + f*x))] + f^2*(c + d*x)^2*((-I)*f*(c + d*x) + 6*d*Log[1 - I*E^
(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(a*f^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a\sin(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^3}{a\sin(e+fx)+a}dx$$
$$\Big\downarrow \text{3799}$$
$$\frac{\int (c+dx)^3\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^3 \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2 dx}{2a}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{6d\int (c+dx)^2 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{6d\int -(c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{6d\int (c+dx)^2 \tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 4202$$

$$\frac{-\frac{2(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{\frac{1}{2}i(2e+2fx+3\pi)}(c+dx)^2}{1+e^{\frac{1}{2}i(2e+2fx+3\pi)}}dx\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{2(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\int (c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 3011$$

$$\frac{-\frac{2(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{id\int \text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)dx}{f}\right)}{f}-\frac{i(c+dx)^2 \cdots}{\cdots}\right.\right.}{f}}{2a}$$

$$\Big\downarrow \; 2720$$

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{d\int e^{-\frac{1}{2}i(2e+2fx+3\pi)}\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right.}{f^2}\right.}{f}\right.\right.}{f}}{2a}$$

$\downarrow$ 7143

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{d\,\text{PolyLog}\left(3,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f^2}\right)}{f}-\frac{i(c+dx)^2\log\left(\right.}{}\right.\right.}{f}}{2a}$$

input
```
Int[(c + d*x)^3/(a + a*Sin[e + f*x]),x]
```

output
```
((-2*(c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/f - (6*d*(((I/3)*(c + d*x)^3)/
d - (2*I)*(((-I)*(c + d*x)^2*Log[1 + E^((I/2)*(2*e + 3*Pi + 2*f*x))])/f +
((2*I)*d*((I*(c + d*x)*PolyLog[2, -E^((I/2)*(2*e + 3*Pi + 2*f*x))])/f - (d
*PolyLog[3, -E^((I/2)*(2*e + 3*Pi + 2*f*x))])/f^2))/f)))/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(131) = 262$.

Time = 0.87 (sec) , antiderivative size = 484, normalized size of antiderivative = 3.27

method	result
risch	$-\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa(\mathrm{e}^{i(fx+e)}+i)}-\frac{12ic\,d^2\,\mathrm{polylog}(2,i\mathrm{e}^{i(fx+e)})}{a\,f^3}+\frac{12d^3\,\mathrm{polylog}(3,i\mathrm{e}^{i(fx+e)})}{a\,f^4}-\frac{6e^2d^3\ln(1-i\mathrm{e}^{i(fx+e)})}{a\,f^4}+$ 12e

input
```
int((d*x+c)^3/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+I)-12*I/a/f^3*c
*d^2*polylog(2,I*exp(I*(f*x+e)))+12*d^3*polylog(3,I*exp(I*(f*x+e)))/a/f^4-
6/a/f^4*e^2*d^3*ln(1-I*exp(I*(f*x+e)))+12/a/f^3*e*c*d^2*ln(exp(I*(f*x+e)))
-2*I/a/f*d^3*x^3+4*I/a/f^4*d^3*e^3+6/a/f^2*ln(exp(I*(f*x+e))+I)*c^2*d-6/a/
f^2*ln(exp(I*(f*x+e)))*c^2*d-6*I/a/f^3*c*d^2*e^2-12*I/a/f^2*c*d^2*e*x-6/a/
f^4*e^2*d^3*ln(exp(I*(f*x+e)))+6/a/f^2*d^3*ln(1-I*exp(I*(f*x+e)))*x^2+12/a
/f^2*c*d^2*ln(1-I*exp(I*(f*x+e)))*x+12/a/f^3*c*d^2*ln(1-I*exp(I*(f*x+e)))*
e+6/a/f^4*e^2*d^3*ln(exp(I*(f*x+e))+I)+6*I/a/f^3*d^3*e^2*x-12/a/f^3*e*c*d^
2*ln(exp(I*(f*x+e))+I)-6*I/a/f*c*d^2*x^2-12*I/a/f^3*d^3*polylog(2,I*exp(I*
(f*x+e)))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(125) = 250$.

Time = 0.11 (sec) , antiderivative size = 915, normalized size of antiderivative = 6.18

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 + (d^3*f^3*x^3 +
 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*cos(f*x + e) + 6*(I*d^3*f*x +
I*c*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e) + (I*d^3*f*x + I*c*d^2*f)
*sin(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) + 6*(-I*d^3*f*x - I*c*
d^2*f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e) + (-I*d^3*f*x - I*c*d^2*f)*s
in(f*x + e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) - 3*(d^3*e^2 - 2*c*d^2*
e*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x + e) + (d^3*
e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sin(f*x + e))*log(cos(f*x + e) + I*sin(f*x
+ e) + I) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f + (d^3*
f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e) + (d^3*f^2*x
^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e))*log(I*cos(f*x +
e) + sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^
2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e)
 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e))*log
(-I*cos(f*x + e) + sin(f*x + e) + 1) - 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^
2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x + e) + (d^3*e^2 - 2*c*d^2*
e*f + c^2*d*f^2)*sin(f*x + e))*log(-cos(f*x + e) + I*sin(f*x + e) + I) - 6
*(d^3*cos(f*x + e) + d^3*sin(f*x + e) + d^3)*polylog(3, I*cos(f*x + e) - s
in(f*x + e)) - 6*(d^3*cos(f*x + e) + d^3*sin(f*x + e) + d^3)*polylog(3, -I
*cos(f*x + e) - sin(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{c^3}{\sin(e+fx)+1}\,dx + \int \frac{d^3x^3}{\sin(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\sin(e+fx)+1}\,dx + \int \frac{3c^2dx}{\sin(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a+a*sin(f*x+e)),x)
```

output
```
(Integral(c**3/(sin(e + f*x) + 1), x) + Integral(d**3*x**3/(sin(e + f*x) +
 1), x) + Integral(3*c*d**2*x**2/(sin(e + f*x) + 1), x) + Integral(3*c**2*
d*x/(sin(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(125) = 250$.

Time = 0.15 (sec) , antiderivative size = 979, normalized size of antiderivative = 6.61

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
(6*(2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*
x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*c*d
^2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*sin(f*x + e) +
 a*f^2) - 6*c*d^2*e^2/(a*f^2 + a*f^2*sin(f*x + e)/(cos(f*x + e) + 1)) - 3*
(2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x +
 e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*c^2*d/
(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*sin(f*x + e) + a*f) + 6*c
^2*d*e/(a*f + a*f*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*c^3/(a + a*sin(f*x
+ e)/(cos(f*x + e) + 1)) + (-2*I*d^3*e^3 + 6*(d^3*e^2*cos(f*x + e) + I*d^3
*e^2*sin(f*x + e) + I*d^3*e^2)*arctan2(sin(f*x + e) + 1, cos(f*x + e)) - 6
*(I*(f*x + e)^2*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e) + ((f*x + e)^2*d^
3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) + (I*(f*x + e)^2*d^3 + 2*(
-I*d^3*e + I*c*d^2*f)*(f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), sin(f
*x + e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^2
*f)*(f*x + e)^2)*cos(f*x + e) - 12*(I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f
+ ((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e) + (I*(f*x + e)*d^3 - I*d^
3*e + I*c*d^2*f)*sin(f*x + e))*dilog(I*e^(I*f*x + I*e)) + 3*((f*x + e)^2*d
^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e) - (I*(f*x + e)^2*d^3 + I*d^3*
e^2 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*cos(f*x + e) + ((f*x + e)^2*d^3
+ d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f*x + e))*log(cos(f*x + ...
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \int \frac{(dx+c)^3}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(a*sin(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a + a*sin(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + a*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3/(a+a*sin(f*x+e)),x)
```

output
```
(6*int(x**2/(tan((e + f*x)/2)**2 + 2*tan((e + f*x)/2) + 1),x)*tan((e + f*x
)/2)*d**3*f**3 + 6*int(x**2/(tan((e + f*x)/2)**2 + 2*tan((e + f*x)/2) + 1)
,x)*d**3*f**3 + 12*int(x/(tan((e + f*x)/2)**2 + 2*tan((e + f*x)/2) + 1),x)
*tan((e + f*x)/2)*c*d**2*f**3 - 12*int(x/(tan((e + f*x)/2)**2 + 2*tan((e +
 f*x)/2) + 1),x)*tan((e + f*x)/2)*d**3*f**2 + 12*int(x/(tan((e + f*x)/2)**
2 + 2*tan((e + f*x)/2) + 1),x)*c*d**2*f**3 - 12*int(x/(tan((e + f*x)/2)**2
 + 2*tan((e + f*x)/2) + 1),x)*d**3*f**2 - 3*log(tan((e + f*x)/2)**2 + 1)*t
an((e + f*x)/2)*c**2*d*f**2 + 6*log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)
/2)*c*d**2*f - 6*log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*d**3 - 3*lo
g(tan((e + f*x)/2)**2 + 1)*c**2*d*f**2 + 6*log(tan((e + f*x)/2)**2 + 1)*c*
d**2*f - 6*log(tan((e + f*x)/2)**2 + 1)*d**3 + 6*log(tan((e + f*x)/2) + 1)
*tan((e + f*x)/2)*c**2*d*f**2 - 12*log(tan((e + f*x)/2) + 1)*tan((e + f*x)
/2)*c*d**2*f + 12*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)*d**3 + 6*log(
tan((e + f*x)/2) + 1)*c**2*d*f**2 - 12*log(tan((e + f*x)/2) + 1)*c*d**2*f
+ 12*log(tan((e + f*x)/2) + 1)*d**3 + 2*tan((e + f*x)/2)*c**3*f**3 + 3*tan
((e + f*x)/2)*c**2*d*f**3*x + 3*tan((e + f*x)/2)*c*d**2*f**3*x**2 - 6*tan(
(e + f*x)/2)*c*d**2*f**2*x + tan((e + f*x)/2)*d**3*f**3*x**3 - 3*tan((e +
f*x)/2)*d**3*f**2*x**2 + 6*tan((e + f*x)/2)*d**3*f*x - 3*c**2*d*f**3*x - 3
*c*d**2*f**3*x**2 + 6*c*d**2*f**2*x - d**3*f**3*x**3 + 3*d**3*f**2*x**2 -
6*d**3*f*x)/(a*f**4*(tan((e + f*x)/2) + 1))
```

3.108 $\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx=-\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{4id^2\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}$$

output
```
-I*(d*x+c)^2/a/f-(d*x+c)^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f+4*d*(d*x+c)*ln(1-
I*exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a/f^3
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx$$
$$=\frac{-4id^2\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)+f(c+dx)\left(-if(c+dx)+4d\log\left(1-ie^{i(e+fx)}\right)+f(c+dx)\tan\left(\frac{1}{4}(2e-\right.\right.}{af^3}$$

input
```
Integrate[(c + d*x)^2/(a + a*Sin[e + f*x]),x]
```

output
```
((-4*I)*d^2*PolyLog[2, I*E^(I*(e + f*x))] + f*(c + d*x)*((-I)*f*(c + d*x)
+ 4*d*Log[1 - I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))
/(a*f^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a\sin(e+fx)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(c+dx)^2}{a\sin(e+fx)+a}dx$$
$$\Big\downarrow \text{ 3799}$$
$$\frac{\int (c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$
$$\Big\downarrow \text{ 4672}$$
$$\frac{\frac{4d\int (c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\frac{4d\int -\left((c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{4d\int (c+dx)\tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 4202$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{\frac{1}{2}i(2e+2fx+3\pi)}(c+dx)}{1+e^{\frac{1}{2}i(2e+2fx+3\pi)}}dx\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)dx}{f}-\frac{i(c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-\frac{1}{2}i(2e+2fx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)de^{\frac{1}{2}i(2e+2fx+3\pi)}}{f^2}-\frac{i(c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{i(c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{d\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f^2}\right)\right)}{f}}{2a}$$

input

```
Int[(c + d*x)^2/(a + a*Sin[e + f*x]),x]
```

output

```
((-2*(c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/f - (4*d*(((I/2)*(c + d*x)^2)/
d - (2*I)*(((-I)*(c + d*x)*Log[1 + E^((I/2)*(2*e + 3*Pi + 2*f*x))])/f - (d
*PolyLog[2, -E^((I/2)*(2*e + 3*Pi + 2*f*x))])/f^2)))/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.

Time = 0.78 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.25

method	result
risch	$-\frac{2(x^2d^2+2cdx+c^2)}{fa(e^{i(fx+e)}+i)}+\frac{4\ln(e^{i(fx+e)}+i)cd}{a\,f^2}-\frac{4\ln(e^{i(fx+e)})cd}{a\,f^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{a\,f^2}-\frac{2id^2e^2}{a\,f^3}+\frac{4d^2\ln(1-ie^{i(fx+e)})x}{a\,f^2}+$

input
```
int((d*x+c)^2/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+I)+4/a/f^2*ln(exp(I*(f*x+e))+
I)*c*d-4/a/f^2*ln(exp(I*(f*x+e)))*c*d-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*
I/a/f^3*d^2*e^2+4/a/f^2*d^2*ln(1-I*exp(I*(f*x+e)))*x+4/a/f^3*d^2*ln(1-I*ex
p(I*(f*x+e)))*e-4*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a/f^3-4/a/f^3*e*d^2*ln
(exp(I*(f*x+e))+I)+4/a/f^3*e*d^2*ln(exp(I*(f*x+e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(94) = 188$.

Time = 0.09 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.36

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx = -\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+(d^2f^2x^2+2\,cdf^2x+c^2f^2)\cos(fx+e)+2\,(i\,d^2\cos(fx+e)+i\,d^2\sin(fx$$

input
```
integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f
^2)*cos(f*x + e) + 2*(I*d^2*cos(f*x + e) + I*d^2*sin(f*x + e) + I*d^2)*dil
og(I*cos(f*x + e) - sin(f*x + e)) + 2*(-I*d^2*cos(f*x + e) - I*d^2*sin(f*x
 + e) - I*d^2)*dilog(-I*cos(f*x + e) - sin(f*x + e)) + 2*(d^2*e - c*d*f +
(d^2*e - c*d*f)*cos(f*x + e) + (d^2*e - c*d*f)*sin(f*x + e))*log(cos(f*x +
 e) + I*sin(f*x + e) + I) - 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x
 + e) + (d^2*f*x + d^2*e)*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e)
+ 1) - 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) + (d^2*f*x + d^
2*e)*sin(f*x + e))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) + 2*(d^2*e - c*
d*f + (d^2*e - c*d*f)*cos(f*x + e) + (d^2*e - c*d*f)*sin(f*x + e))*log(-co
s(f*x + e) + I*sin(f*x + e) + I) - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*s
in(f*x + e))/(a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + a*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{c^2}{\sin(e+fx)+1}\,dx + \int \frac{d^2x^2}{\sin(e+fx)+1}\,dx + \int \frac{2cdx}{\sin(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a+a*sin(f*x+e)),x)
```

output
```
(Integral(c**2/(sin(e + f*x) + 1), x) + Integral(d**2*x**2/(sin(e + f*x) +
 1), x) + Integral(2*c*d*x/(sin(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(94) = 188.

Time = 0.14 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.73

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{2\left(-i\,c^2f^2 - 2\left(cdf\cos\left(fx+e\right) + i\,cdf\sin\left(fx+e\right) + i\,cdf\right)\arctan\left(\sin\left(fx+e\right)+1,\cos\left(fx+e\right)\right) + \cdots\right)}{\cdots}$$

input
```
integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-2*(-I*c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) + I*c*d*f)*a
rctan2(sin(f*x + e) + 1, cos(f*x + e)) + 2*(d^2*f*x*cos(f*x + e) + I*d^2*f
*x*sin(f*x + e) + I*d^2*f*x)*arctan2(cos(f*x + e), sin(f*x + e) + 1) + (d^
2*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*
x + e) + I*d^2)*dilog(I*e^(I*f*x + I*e)) - (d^2*f*x + c*d*f - (I*d^2*f*x +
 I*c*d*f)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^
2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)
*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + a*f^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx = \int \frac{(dx+c)^2}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(a*sin(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a + a*sin(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + a*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx$$

$$= \frac{-4\left(\int \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)x}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}dx\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2f-4\left(\int \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)x}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}dx\right)d^2f-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\tan\left(\frac{f}{2}\right.}{}$$

input
```
int((d*x+c)^2/(a+a*sin(f*x+e)),x)
```

output
```
(2*( - 2*int((tan((e + f*x)/2)*x)/(tan((e + f*x)/2) + 1),x)*tan((e + f*x)/
2)*d**2*f - 2*int((tan((e + f*x)/2)*x)/(tan((e + f*x)/2) + 1),x)*d**2*f -
log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*c*d - log(tan((e + f*x)/2)**
2 + 1)*c*d + 2*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)*c*d + 2*log(tan(
(e + f*x)/2) + 1)*c*d + tan((e + f*x)/2)*c**2*f + tan((e + f*x)/2)*c*d*f*x
 + tan((e + f*x)/2)*d**2*f*x**2 - c*d*f*x))/(a*f**2*(tan((e + f*x)/2) + 1)
)
```

3.109 $\int \frac{c+dx}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx=-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}$$

output
```
-(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f+2*d*ln(sin(1/2*e+1/4*Pi+1/2*f*x))/a
/f^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx = \frac{2d\log\left(\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+f(c+dx)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{af^2}$$

input
```
Integrate[(c + d*x)/(a + a*Sin[e + f*x]),x]
```

output
```
(2*d*Log[Cos[(2*e - Pi + 2*f*x)/4]] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4
])/(a*f^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 3799, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2d\int \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2d\int \tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{4d\log\left(-\cos\left(\frac{e}{2}+\frac{fx}{2}-\frac{\pi}{4}\right)\right)}{f^2}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)/(a + a*Sin[e + f*x]),x]
```

output
```
((-2*(c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/f + (4*d*Log[-Cos[e/2 - Pi/4 + (
f*x)/2]])/f^2)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result	siz
risch	$-\frac{2idx}{af}-\frac{2ide}{a\,f^{2}}-\frac{2(dx+c)}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2d\ln\left(e^{i(fx+e)}+i\right)}{a\,f^{2}}$	73
parallelrisch	$\frac{-d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\ln\left(\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)+2d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2\left(-\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+\frac{dx}{2}+c\right)f}{a\,f^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	96
norman	$\frac{\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}-\frac{dx}{fa}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{a\,f^{2}}-\frac{d\ln\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{a\,f^{2}}$	10

input
```
int((d*x+c)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*d/a/f*x-2*I*d/a/f^2*e-2*(d*x+c)/f/a/(exp(I*(f*x+e))+I)+2*d/a/f^2*ln(e
xp(I*(f*x+e))+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx = -\frac{dfx+cf+(dfx+cf)\cos(fx+e)-(d\cos(fx+e)+d\sin(fx+e)+d)\log(\sin(fx+e)+1)-(d}{af^{2}\cos(fx+e)+af^{2}\sin(fx+e)+af^{2}}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d*f*x + c*f + (d*f*x + c*f)*cos(f*x + e) - (d*cos(f*x + e) + d*sin(f*x +
 e) + d)*log(sin(f*x + e) + 1) - (d*f*x + c*f)*sin(f*x + e))/(a*f^2*cos(f*
x + e) + a*f^2*sin(f*x + e) + a*f^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(46) = 92$.

Time = 0.45 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.53

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx$$
$$= \begin{cases} -\frac{2cf}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} + \frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} - \frac{dfx}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^2} + \frac{2d\log\left(\tan\right.}{af^2\tan\left(\right.} \\ \frac{cx+\frac{dx^2}{2}}{a\sin(e)+a} \end{cases}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-2*c*f/(a*f**2*tan(e/2 + f*x/2) + a*f**2) + d*f*x*tan(e/2 + f*x
/2)/(a*f**2*tan(e/2 + f*x/2) + a*f**2) - d*f*x/(a*f**2*tan(e/2 + f*x/2) +
a*f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 +
 f*x/2) + a*f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)/(a*f**2*tan(e/2 + f*x/2)
 + a*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e
/2 + f*x/2) + a*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)/(a*f**2*tan(e/2 + f
*x/2) + a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(48) = 96$.

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.82

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{\frac{\left(2\,(fx+e)\cos(fx+e)-\left(\cos(fx+e)^2+\sin(fx+e)^2+2\,\sin(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2+2\,\sin(fx+e)+1\right)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2+2\,af\sin(fx+e)+af} - \frac{2\,de}{af+\frac{af\sin(fx+}{\cos(fx+e)+}}}{f}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-((2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
 + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*d/(a
*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*sin(f*x + e) + a*f) - 2*d*e
/(a*f + a*f*sin(f*x + e)/(cos(f*x + e) + 1)) + 2*c/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(48) = 96$.

Time = 0.41 (sec) , antiderivative size = 548, normalized size of antiderivative = 9.13

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
-(d*f*x*tan(1/2*f*x)*tan(1/2*e) + d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) +
c*f*tan(1/2*f*x)*tan(1/2*e) - d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan
(1/2*f*x)^2*tan(1/2*e) - 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + ta
n(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)
^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) - d*f*x +
 c*f*tan(1/2*f*x) + d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^
2*tan(1/2*e) - 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2
 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1
/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x) + c*f*tan(1/2*e) + d*log(2*(ta
n(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) - 2*tan(1/2*f*x)*t
an(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e
) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*
tan(1/2*e) - c*f + d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2
*tan(1/2*e) - 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2
+ 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/
2*f*x)^2 + tan(1/2*e)^2 + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) - a*f^2*tan(
1/2*f*x) - a*f^2*tan(1/2*e) - a*f^2)
```

Mupad [B] (verification not implemented)

Time = 36.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,f^2} - \frac{2\,(c+d\,x)}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)} - \frac{d\,x\,2\mathrm{i}}{a\,f}$$

input
```
int((c + d*x)/(a + a*sin(e + f*x)),x)
```

output
```
(2*d*log(exp(e*1i)*exp(f*x*1i) + 1i))/(a*f^2) - (2*(c + d*x))/(a*f*(exp(e*
1i + f*x*1i) + 1i)) - (d*x*2i)/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.23

$$\int \frac{c+dx}{a+a\sin(e+fx)}\,dx = \frac{-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)d+2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}\right.}{a\,f^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

input
```
int((d*x+c)/(a+a*sin(f*x+e)),x)
```

output
```
( - log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*d - log(tan((e + f*x)/2)
**2 + 1)*d + 2*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)*d + 2*log(tan((e
 + f*x)/2) + 1)*d + 2*tan((e + f*x)/2)*c*f + tan((e + f*x)/2)*d*f*x - d*f*
x)/(a*f**2*(tan((e + f*x)/2) + 1))
```

3.110 $\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+a\sin(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\sin(e+fx)+a)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(c+dx)(a\sin(e+fx)+a)}dx$$
$$\Big\downarrow \; 3807$$
$$\int \frac{1}{(c+dx)(a\sin(e+fx)+a)}dx$$

input
```
Int[1/((c + d*x)*(a + a*Sin[e + f*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+a\sin{(fx+e)})}dx$$

input
```
int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\sin{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (a*d*x + a*c)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \frac{\int \frac{1}{c\sin{(e+fx)}+c+dx\sin{(e+fx)}+dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

output
```
Integral(1/(c*sin(e + f*x) + c + d*x*sin(e + f*x) + d*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 14.25

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f
*x + a*c*d*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e))*integ
rate(cos(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*
a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f
)*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin(f*x + e)),
x) + cos(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (
a*d*f*x + a*c*f)*sin(f*x + e)^2 + 2*(a*d*f*x + a*c*f)*sin(f*x + e))
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(a*sin(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))} \, dx = \int \frac{1}{(a+a\,\sin(e+f\,x))\,(c+d\,x)} \, dx$$

input
```
int(1/((a + a*sin(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + a*sin(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))} \, dx = \frac{-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)c+\sin(fx+e)dx+c+dx}dx\right)d+\log(dx+c)}{ad}$$

input
```
int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

output
```
( - int(sin(e + f*x)/(sin(e + f*x)*c + sin(e + f*x)*d*x + c + d*x),x)*d +
log(c + d*x))/(a*d)
```

3.111 $\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\sin(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 7.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\sin(e+fx)+a)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2(a\sin(e+fx)+a)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a\sin(e+fx)+a)}dx$$

input
```
Int[1/((c + d*x)^2*(a + a*Sin[e + f*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+a\sin(fx+e))}dx$$

input
```
int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2
)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \frac{\int \frac{1}{c^2\sin(e+fx)+c^2+2cdx\sin(e+fx)+2cdx+d^2x^2\sin(e+fx)+d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a+a*sin(f*x+e)),x)
```

output
```
Integral(1/(c**2*sin(e + f*x) + c**2 + 2*c*d*x*sin(e + f*x) + 2*c*d*x + d*
*2*x**2*sin(e + f*x) + d**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 442, normalized size of antiderivative = 22.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*
f*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f
)*sin(f*x + e)^2 + 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*sin(f*x + e
))*integrate(cos(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x +
 a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f
*x + e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(
f*x + e)^2 + 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*s
in(f*x + e)), x) + cos(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a
*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*
d*f*x + a*c^2*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*
sin(f*x + e))
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(a*sin(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 36.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx=\int \frac{1}{(a+a\,\sin\left(e+f\,x\right))\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*sin(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + a*sin(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 7.40

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \frac{-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)c^2+2\sin(fx+e)cdx+\sin(fx+e)d^2x^2+c^2+2cdx+d^2x^2}dx\right)c^2-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)c^2+2\sin(fx+e)cdx+\sin(fx+e)d^2x^2+c^2+}\right.}{ac\,(dx+c)}$$

input
```
int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

output
```
( - int(sin(e + f*x)/(sin(e + f*x)*c**2 + 2*sin(e + f*x)*c*d*x + sin(e + f
*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*c**2 - int(sin(e + f*x)/(si
n(e + f*x)*c**2 + 2*sin(e + f*x)*c*d*x + sin(e + f*x)*d**2*x**2 + c**2 + 2
*c*d*x + d**2*x**2),x)*c*d*x + x)/(a*c*(c + d*x))
```

3.112 $\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 309

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx = & -\frac{i(c+dx)^3}{3a^2f}-\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3}\\ & -\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}\\ & -\frac{d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2}\\ & -\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f}\\ & +\frac{2d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{a^2f^2}\\ & +\frac{4d^3\log\left(\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4}\\ & -\frac{4id^2(c+dx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)}{a^2f^3}\\ & +\frac{4d^3\operatorname{PolyLog}\left(3,ie^{i(e+fx)}\right)}{a^2f^4}\end{aligned}$$

output
```
-1/3*I*(d*x+c)^3/a^2/f-2*d^2*(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f^3-1/3
*(d*x+c)^3*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f-1/2*d*(d*x+c)^2*csc(1/2*e+1/4*P
i+1/2*f*x)^2/a^2/f^2-1/6*(d*x+c)^3*cot(1/2*e+1/4*Pi+1/2*f*x)*csc(1/2*e+1/4
*Pi+1/2*f*x)^2/a^2/f+2*d*(d*x+c)^2*ln(1-I*exp(I*(f*x+e)))/a^2/f^2+4*d^3*ln
(sin(1/2*e+1/4*Pi+1/2*f*x))/a^2/f^4-4*I*d^2*(d*x+c)*polylog(2,I*exp(I*(f*x
+e)))/a^2/f^3+4*d^3*polylog(3,I*exp(I*(f*x+e)))/a^2/f^4
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2} dx$$
$$= \frac{-2if(c+dx)^3 + 12d(c+dx)^2 \log\left(1-ie^{i(e+fx)}\right) + \frac{24d^3 \log\left(\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{f^2} + \frac{24d^2\left(-if(c+dx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)\right.}{f^2}}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Sin[e + f*x])^2,x]
```

output
```
((-2*I)*f*(c + d*x)^3 + 12*d*(c + d*x)^2*Log[1 - I*E^(I*(e + f*x))] + (24*
d^3*Log[Cos[(2*e - Pi + 2*f*x)/4]])/f^2 + (24*d^2*((-I)*f*(c + d*x)*PolyLo
g[2, I*E^(I*(e + f*x))] + d*PolyLog[3, I*E^(I*(e + f*x))]))/f^2 - 3*d*(c +
 d*x)^2*Sec[(2*e - Pi + 2*f*x)/4]^2 + (12*d^2*(c + d*x)*Tan[(2*e - Pi + 2*
f*x)/4])/f + 2*f*(c + d*x)^3*Tan[(2*e - Pi + 2*f*x)/4] + f*(c + d*x)^3*Sec
[(2*e - Pi + 2*f*x)/4]^2*Tan[(2*e - Pi + 2*f*x)/4])/(6*a^2*f^2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3799, 3042, 4674, 3042, 4672, 3042, 25, 3956, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\sin(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+dx)^3}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \; 3799$$

$$\frac{\int (c+dx)^3\csc^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (c+dx)^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4dx}{4a^2}$$

$$\Big\downarrow \; 4674$$

$$\frac{\frac{4d^2\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{2}{3}\int (c+dx)^3\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{2d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right.}{3f}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4d^2\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx}{f^2}+\frac{2}{3}\int (c+dx)^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx-\frac{2d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right.}{3f}}{4a^2}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{4d^2\left(\frac{2d\int \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{6d\int (c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{2d(c+dx)^2\,\mathrm{c}}{}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4d^2\left(\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{6d\int -(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{2d(c+}{}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{4d^2\left(-\frac{2d\int \tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(-\frac{6d\int (c+dx)^2\tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-}{4a^2}$$

$\downarrow$ 3956

$$\frac{\frac{2}{3}\left(-\frac{6d\int (c+dx)^2\tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{4d^2\left(\frac{4d\log\left(-\cos\left(\frac{e}{2}+\frac{fx}{2}-\frac{\pi}{4}\right)\right)}{f^2}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{f^2}-\frac{2c}{}}{4a^2}$$

$\downarrow$ 4202

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{\frac{1}{2}i(2e+2fx+3\pi)}(c+dx)^2}{1+e^{\frac{1}{2}i(2e+2fx+3\pi)}}dx\right)}{f}\right)+\frac{4d^2\left(\frac{4d\log\left(-\cos\left(\frac{e}{2}+\frac{fx}{2}-\frac{\pi}{4}\right)\right)}{f^2}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.}{f^2}}{4a^2}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\int (c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}\right)\right)}{f}\right)+\frac{4d^2}{}}{4a^2}$$

$\downarrow$ 3011

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)dx}{f}\right)}{f}-\frac{i(c-}{}\right.\right.}{f}\right.}{}$$

$\downarrow$ 2720

$$\frac{2}{3}\left(-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{d\int e^{-\frac{1}{2}i(2e+2fx+3\pi)}\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2f}\right.}{f^2}\right.}{f}\right.\right.}{f}\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{4d^2\left(\frac{4d\log\left(-\cos\left(\frac{e}{2}+\frac{fx}{2}-\frac{\pi}{4}\right)\right)}{f^2}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e\right.}{f}\right.}{}\right.\right.}{}\right.$$

input

```
Int[(c + d*x)^3/(a + a*Sin[e + f*x])^2,x]
```

output

```
((-2*d*(c + d*x)^2*Csc[e/2 + Pi/4 + (f*x)/2]^2)/f^2 - (2*(c + d*x)^3*Cot[e
/2 + Pi/4 + (f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(3*f) + (4*d^2*((-2*(c +
 d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/f + (4*d*Log[-Cos[e/2 - Pi/4 + (f*x)/2]])
/f^2))/f^2 + (2*((-2*(c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/f - (6*d*(((I/
3)*(c + d*x)^3)/d - (2*I)*(((-I)*(c + d*x)^2*Log[1 + E^((I/2)*(2*e + 3*Pi
+ 2*f*x))])/f + ((2*I)*d*((I*(c + d*x)*PolyLog[2, -E^((I/2)*(2*e + 3*Pi +
2*f*x))])/f - (d*PolyLog[3, -E^((I/2)*(2*e + 3*Pi + 2*f*x))])/f^2))/f)))/f
))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(254) = 508$.

Time = 4.15 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.90

method	result
risch	$\frac{2d^3\ln\left(1-ie^{i(fx+e)}\right)x^2}{a^2f^2}-\frac{2e^2d^3\ln\left(1-ie^{i(fx+e)}\right)}{a^2f^4}-\frac{2id^3x^3}{3a^2f}-\frac{4id^3\arctan\left(e^{i(fx+e)}\right)}{a^2f^4}+\frac{4id^3e^3}{3a^2f^4}-\frac{2i\left(6ifc\,d^2x\,e^{i(fx+e)}+3d^3\right.}{}$

input
```
int((d*x+c)^3/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/a^2/f^2*d^3*ln(1-I*exp(I*(f*x+e)))*x^2+1/a^2/f^2*c^2*d*ln(exp(2*I*(f*x+e
))+1)-2/a^2/f^2*c^2*d*ln(exp(I*(f*x+e)))-2/a^2/f^4*d^3*e^2*ln(exp(I*(f*x+e
)))+1/a^2/f^4*d^3*e^2*ln(exp(2*I*(f*x+e))+1)-2/a^2/f^4*e^2*d^3*ln(1-I*exp(
I*(f*x+e)))-2/3*I/a^2/f*d^3*x^3-4*I/a^2/f^4*d^3*arctan(exp(I*(f*x+e)))+4/3
*I/a^2/f^4*d^3*e^3+4/a^2/f^2*c*d^2*ln(1-I*exp(I*(f*x+e)))*x+4/a^2/f^3*c*d^
2*ln(1-I*exp(I*(f*x+e)))*e+4/a^2/f^3*c*d^2*e*ln(exp(I*(f*x+e)))-2/a^2/f^3*
c*d^2*e*ln(exp(2*I*(f*x+e))+1)-2/3*I*(6*I*f*c*d^2*x*exp(I*(f*x+e))+3*d^3*f
^2*x^3*exp(I*(f*x+e))+6*I*c*d^2+I*d^3*f^2*x^3+9*c*d^2*f^2*x^2*exp(I*(f*x+e
))+3*f*d^3*x^2*exp(2*I*(f*x+e))+3*I*c^2*d*f^2*x-6*I*c*d^2*exp(2*I*(f*x+e))
+I*c^3*f^2+9*c^2*d*f^2*x*exp(I*(f*x+e))+6*f*c*d^2*x*exp(2*I*(f*x+e))-6*I*d
^3*x*exp(2*I*(f*x+e))+6*I*d^3*x+3*I*f*d^3*x^2*exp(I*(f*x+e))+3*c^3*f^2*exp
(I*(f*x+e))+3*f*c^2*d*exp(2*I*(f*x+e))+3*I*c*d^2*f^2*x^2+12*d^3*x*exp(I*(f
*x+e))+3*I*f*c^2*d*exp(I*(f*x+e))+12*c*d^2*exp(I*(f*x+e)))/(exp(I*(f*x+e))
+I)^3/f^3/a^2-2*I/a^2/f*c*d^2*x^2-2*I/a^2/f^4*d^3*e^2*arctan(exp(I*(f*x+e)
))+2*I/a^2/f^3*d^3*e^2*x-4*I/a^2/f^3*d^3*polylog(2,I*exp(I*(f*x+e)))*x-2*I
/a^2/f^2*c^2*d*arctan(exp(I*(f*x+e)))-4*I/a^2/f^3*c*d^2*polylog(2,I*exp(I*
(f*x+e)))-2*I/a^2/f^3*c*d^2*e^2+4*I/a^2/f^3*c*d^2*e*arctan(exp(I*(f*x+e)))
-4*I/a^2/f^2*c*d^2*e*x+4*d^3*polylog(3,I*exp(I*(f*x+e)))/a^2/f^4+2/a^2/f^4
*d^3*ln(exp(2*I*(f*x+e))+1)-4/a^2/f^4*d^3*ln(exp(I*(f*x+e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1708 vs. $2(248) = 496$.

Time = 0.15 (sec) , antiderivative size = 1708, normalized size of antiderivative = 5.53

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output

```
1/3*(d^3*f^3*x^3 + c^3*f^3 + 3*c^2*d*f^2 + 3*(c*d^2*f^3 + d^3*f^2)*x^2 + (
d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f + 3*(c^2*d*f^3 + 2*d^3
*f)*x)*cos(f*x + e)^2 + 3*(c^2*d*f^3 + 2*c*d^2*f^2)*x + (2*d^3*f^3*x^3 + 2
*c^3*f^3 + 3*c^2*d*f^2 + 6*c*d^2*f + 3*(2*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(c^
2*d*f^3 + c*d^2*f^2 + d^3*f)*x)*cos(f*x + e) + 6*(2*I*d^3*f*x + 2*I*c*d^2*
f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e)^2 + (I*d^3*f*x + I*c*d^2*f)*cos(
f*x + e) + (2*I*d^3*f*x + 2*I*c*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x +
e))*sin(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) + 6*(-2*I*d^3*f*x -
 2*I*c*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e)^2 + (-I*d^3*f*x - I*c*
d^2*f)*cos(f*x + e) + (-2*I*d^3*f*x - 2*I*c*d^2*f + (-I*d^3*f*x - I*c*d^2*
f)*cos(f*x + e))*sin(f*x + e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) - 3*(
2*d^3*e^2 - 4*c*d^2*e*f + 2*c^2*d*f^2 + 4*d^3 - (d^3*e^2 - 2*c*d^2*e*f + c
^2*d*f^2 + 2*d^3)*cos(f*x + e)^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + 2*
d^3)*cos(f*x + e) + (2*d^3*e^2 - 4*c*d^2*e*f + 2*c^2*d*f^2 + 4*d^3 + (d^3*
e^2 - 2*c*d^2*e*f + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*sin(f*x + e))*log(cos
(f*x + e) + I*sin(f*x + e) + I) - 3*(2*d^3*f^2*x^2 + 4*c*d^2*f^2*x - 2*d^3
*e^2 + 4*c*d^2*e*f - (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)
*cos(f*x + e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*co
s(f*x + e) + (2*d^3*f^2*x^2 + 4*c*d^2*f^2*x - 2*d^3*e^2 + 4*c*d^2*e*f + (d
^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e))*sin(f...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{c^3}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{d^3x^3}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{3c^2dx}{\sin^2(e+fx)+2\sin$$

input

```
integrate((d*x+c)**3/(a+a*sin(f*x+e))**2,x)
```

output

```
(Integral(c**3/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(d**3*
x**3/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(
sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(3*c**2*d*x/(sin(e + f
*x)**2 + 2*sin(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3593 vs. $2(248) = 496$.

Time = 0.63 (sec) , antiderivative size = 3593, normalized size of antiderivative = 11.63

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/3*(6*c*d^2*e^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a^2*f^2 + 3*a^2*f^2*sin(f*x + e)/(cos(f*x + e) +
1) + 3*a^2*f^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*f^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + 6*(2*(f*x + 3*(f*x + e)*sin(f*x + e) + e + cos(f
*x + e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 2*(9*(f*x + e)*cos(f*x + e)
 - 6*sin(f*x + e) - 1)*cos(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)^2 - 6*cos(f*x
 + e)^2 - (6*(cos(f*x + e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - cos(3*f*
x + 3*e)^2 + 6*(3*sin(f*x + e) + 1)*cos(2*f*x + 2*e) - 9*cos(2*f*x + 2*e)^
2 - 9*cos(f*x + e)^2 - 2*(3*cos(2*f*x + 2*e) - 3*sin(f*x + e) - 1)*sin(3*f
*x + 3*e) - sin(3*f*x + 3*e)^2 - 18*cos(f*x + e)*sin(2*f*x + 2*e) - 9*sin(
2*f*x + 2*e)^2 - 9*sin(f*x + e)^2 - 6*sin(f*x + e) - 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 2*(3*(f*x + e)*cos(f*x + e) + co
s(2*f*x + 2*e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*sin
(f*x + e) + e + 2*cos(f*x + e))*sin(2*f*x + 2*e) - 6*sin(2*f*x + 2*e)^2 -
6*sin(f*x + e)^2 - 2*sin(f*x + e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9
*a^2*f^2*cos(2*f*x + 2*e)^2 + 9*a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x
 + 3*e)^2 + 18*a^2*f^2*cos(f*x + e)*sin(2*f*x + 2*e) + 9*a^2*f^2*sin(2*f*x
 + 2*e)^2 + 9*a^2*f^2*sin(f*x + e)^2 + 6*a^2*f^2*sin(f*x + e) + a^2*f^2 -
6*(a^2*f^2*cos(f*x + e) + a^2*f^2*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(
3*a^2*f^2*sin(f*x + e) + a^2*f^2)*cos(2*f*x + 2*e) + 2*(3*a^2*f^2*cos(2...
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(dx+c)^3}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(a*sin(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx=\text{Hanged}$$

input
```
int((c + d*x)^3/(a + a*sin(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx=\text{too large to display}$$

input
```
int((d*x+c)^3/(a+a*sin(f*x+e))^2,x)
```

output
```
(336*int(x**3/(tan((e + f*x)/2)**4 + 4*tan((e + f*x)/2)**3 + 6*tan((e + f*
x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((e + f*x)/2)**3*d**3*f**4 + 1008
*int(x**3/(tan((e + f*x)/2)**4 + 4*tan((e + f*x)/2)**3 + 6*tan((e + f*x)/2
)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((e + f*x)/2)**2*d**3*f**4 + 1008*int
(x**3/(tan((e + f*x)/2)**4 + 4*tan((e + f*x)/2)**3 + 6*tan((e + f*x)/2)**2
 + 4*tan((e + f*x)/2) + 1),x)*tan((e + f*x)/2)*d**3*f**4 + 336*int(x**3/(t
an((e + f*x)/2)**4 + 4*tan((e + f*x)/2)**3 + 6*tan((e + f*x)/2)**2 + 4*tan
((e + f*x)/2) + 1),x)*d**3*f**4 + 1008*int(x**2/(tan((e + f*x)/2)**4 + 4*t
an((e + f*x)/2)**3 + 6*tan((e + f*x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*ta
n((e + f*x)/2)**3*c*d**2*f**4 - 864*int(x**2/(tan((e + f*x)/2)**4 + 4*tan(
(e + f*x)/2)**3 + 6*tan((e + f*x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((
e + f*x)/2)**3*d**3*f**3 + 3024*int(x**2/(tan((e + f*x)/2)**4 + 4*tan((e +
 f*x)/2)**3 + 6*tan((e + f*x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((e +
f*x)/2)**2*c*d**2*f**4 - 2592*int(x**2/(tan((e + f*x)/2)**4 + 4*tan((e + f
*x)/2)**3 + 6*tan((e + f*x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((e + f*
x)/2)**2*d**3*f**3 + 3024*int(x**2/(tan((e + f*x)/2)**4 + 4*tan((e + f*x)/
2)**3 + 6*tan((e + f*x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((e + f*x)/2
)*c*d**2*f**4 - 2592*int(x**2/(tan((e + f*x)/2)**4 + 4*tan((e + f*x)/2)**3
 + 6*tan((e + f*x)/2)**2 + 4*tan((e + f*x)/2) + 1),x)*tan((e + f*x)/2)*d**
3*f**3 + 1008*int(x**2/(tan((e + f*x)/2)**4 + 4*tan((e + f*x)/2)**3 + 6...
```

3.113 $\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 243

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = &-\frac{i(c+dx)^2}{3a^2f}-\frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3}\\&-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}\\&-\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2}\\&-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f}\\&+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{3a^2f^2}\\&-\frac{4id^2\,\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{3a^2f^3}\end{aligned}$$

output
```
-1/3*I*(d*x+c)^2/a^2/f-2/3*d^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f^3-1/3*(d*x+
c)^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f-1/3*d*(d*x+c)*csc(1/2*e+1/4*Pi+1/2*f*
x)^2/a^2/f^2-1/6*(d*x+c)^2*cot(1/2*e+1/4*Pi+1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*
f*x)^2/a^2/f+4/3*d*(d*x+c)*ln(1-I*exp(I*(f*x+e)))/a^2/f^2-4/3*I*d^2*polylo
g(2,I*exp(I*(f*x+e)))/a^2/f^3
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{-2if(c+dx)\left(f(c+dx)+4id\log\left(1-ie^{i(e+fx)}\right)\right)-8id^2\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)+2(c^2f^2+2cdf^2x+d^2(}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Sin[e + f*x])^2,x]
```

output
```
((-2*I)*f*(c + d*x)*(f*(c + d*x) + (4*I)*d*Log[1 - I*E^(I*(e + f*x))]) - (
8*I)*d^2*PolyLog[2, I*E^(I*(e + f*x))] + 2*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2
 + f^2*x^2))*Tan[(2*e - Pi + 2*f*x)/4] + f*(c + d*x)*Sec[(2*e - Pi + 2*f*x
)/4]^2*(-2*d + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(6*a^2*f^3)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3799, 3042, 4674, 3042, 4254, 24, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^2\csc^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^2 \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4 dx}{4a^2}$$

$\Big\downarrow$ 4674

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4d^2\int \csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{3f^2}-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}\right.}{3f}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx+\frac{4d^2\int \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx}{3f^2}-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}\right.}{3f}}{4a^2}$$

$\Big\downarrow$ 4254

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx-\frac{8d^2\int 1d\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^3}-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}\right.}{3f}}{4a^2}$$

$\Big\downarrow$ 24

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{8d^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right.}{3f^3}}{4a^2}$$

$\Big\downarrow$ 4672

$$\frac{\frac{2}{3}\left(\frac{4d\int (c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right.}{3f}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{4d\int -\left((c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}\right.}{3f}}{4a^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{2}{3}\left(-\frac{4d\int (c+dx)\tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\right.}{3f}}{4a^2}$$

$\downarrow$ 4202

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{\frac{1}{2}i(2e+2fx+3\pi)}(c+dx)}{1+e^{\frac{1}{2}i(2e+2fx+3\pi)}}dx\right)}{f}\right)-\frac{4d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right.}{3}}{4a^2}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)dx}{f}-\frac{i(c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}\right)\right)}{f}\right)-\frac{4d(c+dx)\,c}{}}{4a^2}$$

$\downarrow$ 2715

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-\frac{1}{2}i(2e+2fx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)de^{\frac{1}{2}i(2e+2fx+3\pi)}}{f^2}-\frac{i(c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2\right.}}{f}\right.\right.}{f}\right.}{4a^2}$$

$\downarrow$ 2838

$$\frac{\frac{2}{3}\left(-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{i(c+dx)\log\left(1+e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f}-\frac{d\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2e+2fx+3\pi)}\right)}{f^2}\right)\right)}{f}\right)-\frac{4d(c+dx}{}}{4a^2}$$

input
```
Int[(c + d*x)^2/(a + a*Sin[e + f*x])^2,x]
```

output
```
((-8*d^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*f^3) - (4*d*(c + d*x)*Csc[e/2 + Pi/
4 + (f*x)/2]^2)/(3*f^2) - (2*(c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2]*Csc[e/2
 + Pi/4 + (f*x)/2]^2)/(3*f) + (2*((-2*(c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2
])/f - (4*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-I)*(c + d*x)*Log[1 + E^((I/
2)*(2*e + 3*Pi + 2*f*x))])/f - (d*PolyLog[2, -E^((I/2)*(2*e + 3*Pi + 2*f*x
))])/f^2)))/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(191) = 382.

Time = 4.06 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.91

method	result
risch	$-\frac{2i\left(id^2x^2f^2+3d^2f^2x^2\mathrm{e}^{i(fx+e)}+2icd\,f^2x+2if\,d^2x\,\mathrm{e}^{i(fx+e)}+6cd\,f^2x\,\mathrm{e}^{i(fx+e)}+2f\,d^2x\,\mathrm{e}^{2i(fx+e)}+ic^2f^2+2ifcd\,\mathrm{e}^{i(fx+e)}-2id^2\mathrm{e}^{2i}\right.}{3\left(\mathrm{e}^{i(fx+e)}+i\right)^3f^3a^2}$

input
```
int((d*x+c)^2/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(I*d^2*x^2*f^2+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f^2*x+2*I*f*d^2
*x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+2*f*d^2*x*exp(2*I*(f*x+e))+I*
c^2*f^2+2*I*f*c*d*exp(I*(f*x+e))-2*I*d^2*exp(2*I*(f*x+e))+3*c^2*f^2*exp(I*
(f*x+e))+2*f*c*d*exp(2*I*(f*x+e))+2*I*d^2+4*d^2*exp(I*(f*x+e)))/(exp(I*(f*
x+e))+I)^3/f^3/a^2-4/3/a^2/f^2*c*d*ln(exp(I*(f*x+e)))+2/3/a^2/f^2*c*d*ln(e
xp(2*I*(f*x+e))+1)-4/3*I/a^2/f^2*c*d*arctan(exp(I*(f*x+e)))-2/3*I/a^2/f*d^
2*x^2-4/3*I/a^2/f^2*d^2*e*x-2/3*I/a^2/f^3*d^2*e^2+4/3/a^2/f^2*d^2*ln(1-I*e
xp(I*(f*x+e)))*x+4/3/a^2/f^3*d^2*ln(1-I*exp(I*(f*x+e)))*e-4/3*I*d^2*polylo
g(2,I*exp(I*(f*x+e)))/a^2/f^3+4/3/a^2/f^3*e*d^2*ln(exp(I*(f*x+e)))-2/3/a^2
/f^3*e*d^2*ln(exp(2*I*(f*x+e))+1)+4/3*I/a^2/f^3*e*d^2*arctan(exp(I*(f*x+e)
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 876 vs. 2(186) = 372.

Time = 0.10 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.60

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(d^2*f^2*x^2 + c^2*f^2 + 2*c*d*f + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^
2 + 2*d^2)*cos(f*x + e)^2 + 2*(c*d*f^2 + d^2*f)*x + 2*(d^2*f^2*x^2 + c^2*f
^2 + c*d*f + d^2 + (2*c*d*f^2 + d^2*f)*x)*cos(f*x + e) + 2*(-I*d^2*cos(f*x
 + e)^2 + I*d^2*cos(f*x + e) + 2*I*d^2 + (I*d^2*cos(f*x + e) + 2*I*d^2)*si
n(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) + 2*(I*d^2*cos(f*x + e)^2
 - I*d^2*cos(f*x + e) - 2*I*d^2 + (-I*d^2*cos(f*x + e) - 2*I*d^2)*sin(f*x
+ e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) + 2*(2*d^2*e - 2*c*d*f - (d^2*
e - c*d*f)*cos(f*x + e)^2 + (d^2*e - c*d*f)*cos(f*x + e) + (2*d^2*e - 2*c*
d*f + (d^2*e - c*d*f)*cos(f*x + e))*sin(f*x + e))*log(cos(f*x + e) + I*sin
(f*x + e) + I) - 2*(2*d^2*f*x + 2*d^2*e - (d^2*f*x + d^2*e)*cos(f*x + e)^2
 + (d^2*f*x + d^2*e)*cos(f*x + e) + (2*d^2*f*x + 2*d^2*e + (d^2*f*x + d^2*
e)*cos(f*x + e))*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e) + 1) - 2*
(2*d^2*f*x + 2*d^2*e - (d^2*f*x + d^2*e)*cos(f*x + e)^2 + (d^2*f*x + d^2*e
)*cos(f*x + e) + (2*d^2*f*x + 2*d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e))*si
n(f*x + e))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) + 2*(2*d^2*e - 2*c*d*f
 - (d^2*e - c*d*f)*cos(f*x + e)^2 + (d^2*e - c*d*f)*cos(f*x + e) + (2*d^2*
e - 2*c*d*f + (d^2*e - c*d*f)*cos(f*x + e))*sin(f*x + e))*log(-cos(f*x + e
) + I*sin(f*x + e) + I) - (d^2*f^2*x^2 + c^2*f^2 - 2*c*d*f + 2*(c*d*f^2 -
d^2*f)*x - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e))*sin
(f*x + e))/(a^2*f^3*cos(f*x + e)^2 - a^2*f^3*cos(f*x + e) - 2*a^2*f^3 -...
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{c^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{d^2x^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{2cdx}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**2/(a+a*sin(f*x+e))**2,x)
```

output
```
(Integral(c**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(d**2*
x**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(2*c*d*x/(sin(e
+ f*x)**2 + 2*sin(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(186) = 372$.

Time = 0.32 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.40

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(I*c^2*f^2 + 2*I*d^2 - 2*(c*d*f*cos(3*f*x + 3*e) + 3*I*c*d*f*cos(2*f*x
+ 2*e) - 3*c*d*f*cos(f*x + e) + I*c*d*f*sin(3*f*x + 3*e) - 3*c*d*f*sin(2*f
*x + 2*e) - 3*I*c*d*f*sin(f*x + e) - I*c*d*f)*arctan2(sin(f*x + e) + 1, co
s(f*x + e)) + 2*(d^2*f*x*cos(3*f*x + 3*e) + 3*I*d^2*f*x*cos(2*f*x + 2*e) -
 3*d^2*f*x*cos(f*x + e) + I*d^2*f*x*sin(3*f*x + 3*e) - 3*d^2*f*x*sin(2*f*x
 + 2*e) - 3*I*d^2*f*x*sin(f*x + e) - I*d^2*f*x)*arctan2(cos(f*x + e), sin(
f*x + e) + 1) + (d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x + 3*e) + (3*I*d^2*f^
2*x^2 + 2*c*d*f - 2*I*d^2 + 2*(3*I*c*d*f^2 + d^2*f)*x)*cos(2*f*x + 2*e) +
(3*c^2*f^2 + 2*I*d^2*f*x + 2*I*c*d*f + 4*d^2)*cos(f*x + e) + 2*(d^2*cos(3*
f*x + 3*e) + 3*I*d^2*cos(2*f*x + 2*e) - 3*d^2*cos(f*x + e) + I*d^2*sin(3*f
*x + 3*e) - 3*d^2*sin(2*f*x + 2*e) - 3*I*d^2*sin(f*x + e) - I*d^2)*dilog(I
*e^(I*f*x + I*e)) + (d^2*f*x + c*d*f + (I*d^2*f*x + I*c*d*f)*cos(3*f*x + 3
*e) - 3*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e) + 3*(-I*d^2*f*x - I*c*d*f)*cos(
f*x + e) - (d^2*f*x + c*d*f)*sin(3*f*x + 3*e) + 3*(-I*d^2*f*x - I*c*d*f)*s
in(2*f*x + 2*e) + 3*(d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + s
in(f*x + e)^2 + 2*sin(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(
3*f*x + 3*e) - (3*d^2*f^2*x^2 - 2*I*c*d*f - 2*d^2 + 2*(3*c*d*f^2 - I*d^2*f
)*x)*sin(2*f*x + 2*e) + (3*I*c^2*f^2 - 2*d^2*f*x - 2*c*d*f + 4*I*d^2)*sin(
f*x + e))/(-3*I*a^2*f^3*cos(3*f*x + 3*e) + 9*a^2*f^3*cos(2*f*x + 2*e) + 9*
I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*I*a^2*f^3*sin(2...
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(a*sin(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(a + a*sin(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{9\left(\int \frac{x^2}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 d^2f^2 + 27\left(\int \frac{x^2}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d^2f^2 + \cdots}{\cdots}$$

input
```
int((d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

output

```
(9*int(x**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*tan((e + f*x)/2)**3*
d**2*f**2 + 27*int(x**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*tan((e +
 f*x)/2)**2*d**2*f**2 + 27*int(x**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1)
,x)*tan((e + f*x)/2)*d**2*f**2 + 9*int(x**2/(sin(e + f*x)**2 + 2*sin(e + f
*x) + 1),x)*d**2*f**2 - 6*log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)**3
*c*d - 18*log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)**2*c*d - 18*log(ta
n((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*c*d - 6*log(tan((e + f*x)/2)**2 +
1)*c*d + 12*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**3*c*d + 36*log(tan
((e + f*x)/2) + 1)*tan((e + f*x)/2)**2*c*d + 36*log(tan((e + f*x)/2) + 1)*
tan((e + f*x)/2)*c*d + 12*log(tan((e + f*x)/2) + 1)*c*d + 6*tan((e + f*x)/
2)**3*c**2*f + 12*tan((e + f*x)/2)**3*c*d*f*x - 4*tan((e + f*x)/2)**3*c*d
- 6*c**2*f - 12*c*d*f*x - 4*c*d)/(9*a**2*f**2*(tan((e + f*x)/2)**3 + 3*tan
((e + f*x)/2)**2 + 3*tan((e + f*x)/2) + 1))
```

3.114 $\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} - \frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f} + \frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

output
```
-1/3*(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)/a^2/f-1/6*d*csc(1/2*e+1/4*Pi+1/2*f*
x)^2/a^2/f^2-1/6*(d*x+c)*cot(1/2*e+1/4*Pi+1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*
x)^2/a^2/f+2/3*d*ln(sin(1/2*e+1/4*Pi+1/2*f*x))/a^2/f^2
```

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.52

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(2+3e+3fx-6\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/6*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(2 + 3*e +
 3*f*x - 6*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Cos[(3*(e + f*x))/2
]*(-(d*e) + 2*c*f + d*f*x + 2*d*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
+ 2*(d + 2*d*e - 3*c*f - d*f*x + d*Cos[e + f*x]*(e + f*x - 2*Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]) - 4*d*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
])*Sin[(e + f*x)/2]))/(a^2*f^2*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3799, 3042, 4673, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\csc^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{2d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^2dx-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{2d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \ 4672$$

$$\frac{\frac{2}{3}\left(\frac{2d\int \cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{2d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2}{3}\left(\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{2d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2}{3}\left(-\frac{2d\int \tan\left(\frac{1}{4}(2e+3\pi)+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{2d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \ 3956$$

$$\frac{\frac{2}{3}\left(\frac{4d\log\left(-\cos\left(\frac{e}{2}+\frac{fx}{2}-\frac{\pi}{4}\right)\right)}{f^2}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}-\frac{2d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}}{4a^2}$$

input

```
Int[(c + d*x)/(a + a*Sin[e + f*x])^2,x]
```

output

```
((-2*d*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(3*f^2) - (2*(c + d*x)*Cot[e/2 + Pi/4
+ (f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(3*f) + (2*((-2*(c + d*x)*Cot[e/2
+ Pi/4 + (f*x)/2])/f + (4*d*Log[-Cos[e/2 - Pi/4 + (f*x)/2]])/f^2))/3)/(4*a
^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{2idx}{3a^2f}-\frac{2ide}{3a^2f^2}-\frac{2i\left(idfx+3dfx\,e^{i(fx+e)}+icf+id\,e^{i(fx+e)}+3cf\,e^{i(fx+e)}+d\,e^{2i(fx+e)}\right)}{3f^2\left(e^{i(fx+e)}+i\right)^3a^2}+\frac{2d\ln\left(e^{i(fx+e)}+i\right)}{3a^2f^2}$
parallelrisch	$\frac{-d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\ln\left(\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)+2d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3df+(-6cf+2d)\,\mathrm{ta}}{3f^2a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$\frac{-\frac{4c}{3fa}-\frac{2dx}{3fa}+\frac{(-6cf+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3a\,f^2}+\frac{(-6cf+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3a\,f^2}+\frac{2dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3fa}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{3a^2f^2}-\frac{d\ln\left(1+\mathrm{ta}\right.}{3a}$
default	$-\frac{2\left(-\frac{c\left(-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}\right)}{2f}+\frac{\frac{dx}{3f}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f^2}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3f^2}-\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}\right.}{a^2}$

input
```
int((d*x+c)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*d/a^2/f*x-2/3*I*d/a^2/f^2*e-2/3*I*(I*d*f*x+3*d*f*x*exp(I*(f*x+e))+I
*c*f+I*d*exp(I*(f*x+e))+3*c*f*exp(I*(f*x+e))+d*exp(2*I*(f*x+e)))/f^2/(exp(
I*(f*x+e))+I)^3/a^2+2/3*d/a^2/f^2*ln(exp(I*(f*x+e))+I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.38

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{dfx+(dfx+cf)\cos(fx+e)^2+cf+(2\,dfx+2\,cf+d)\cos(fx+e)+\left(d\cos(fx+e)^2-d\cos(fx+e\right.}{3\left(a^2f^2\cos(fx+e)^2-a^2f^2\cos(fx\right.}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(d*f*x + (d*f*x + c*f)*cos(f*x + e)^2 + c*f + (2*d*f*x + 2*c*f + d)*co
s(f*x + e) + (d*cos(f*x + e)^2 - d*cos(f*x + e) - (d*cos(f*x + e) + 2*d)*s
in(f*x + e) - 2*d)*log(sin(f*x + e) + 1) - (d*f*x + c*f - (d*f*x + c*f)*co
s(f*x + e) - d)*sin(f*x + e) + d)/(a^2*f^2*cos(f*x + e)^2 - a^2*f^2*cos(f*
x + e) - 2*a^2*f^2 - (a^2*f^2*cos(f*x + e) + 2*a^2*f^2)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(122) = 244$.

Time = 0.84 (sec) , antiderivative size = 1336, normalized size of antiderivative = 9.03

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*c*f*tan(e/2 + f*x/2)**2/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9
*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**
2) - 6*c*f*tan(e/2 + f*x/2)/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2
*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) - 4*c*f
/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a*
*2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) + 2*d*f*x*tan(e/2 + f*x/2)**3/(3*a
**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f*
*2*tan(e/2 + f*x/2) + 3*a**2*f**2) - 2*d*f*x/(3*a**2*f**2*tan(e/2 + f*x/2)
**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a
**2*f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**3/(3*a**2*f**2
*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e
/2 + f*x/2) + 3*a**2*f**2) + 6*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2
)**2/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f*x/2)**2 +
9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) + 6*d*log(tan(e/2 + f*x/2) + 1
)*tan(e/2 + f*x/2)/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2
+ f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) + 2*d*log(tan(e/
2 + f*x/2) + 1)/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**2*f**2*tan(e/2 + f
*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2) - d*log(tan(e/2 + f
*x/2)**2 + 1)*tan(e/2 + f*x/2)**3/(3*a**2*f**2*tan(e/2 + f*x/2)**3 + 9*a**
2*f**2*tan(e/2 + f*x/2)**2 + 9*a**2*f**2*tan(e/2 + f*x/2) + 3*a**2*f**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(110) = 220$.

Time = 0.06 (sec) , antiderivative size = 910, normalized size of antiderivative = 6.15

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(2*d*e*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 2)/(a^2*f + 3*a^2*f*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*
f*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*f*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3) + (2*(f*x + 3*(f*x + e)*sin(f*x + e) + e + cos(f*x + e) + sin(2*f
*x + 2*e))*cos(3*f*x + 3*e) - 2*(9*(f*x + e)*cos(f*x + e) - 6*sin(f*x + e)
 - 1)*cos(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)^2 - 6*cos(f*x + e)^2 - (6*(cos
(f*x + e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - cos(3*f*x + 3*e)^2 + 6*(3
*sin(f*x + e) + 1)*cos(2*f*x + 2*e) - 9*cos(2*f*x + 2*e)^2 - 9*cos(f*x + e
)^2 - 2*(3*cos(2*f*x + 2*e) - 3*sin(f*x + e) - 1)*sin(3*f*x + 3*e) - sin(3
*f*x + 3*e)^2 - 18*cos(f*x + e)*sin(2*f*x + 2*e) - 9*sin(2*f*x + 2*e)^2 -
9*sin(f*x + e)^2 - 6*sin(f*x + e) - 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2
 + 2*sin(f*x + e) + 1) - 2*(3*(f*x + e)*cos(f*x + e) + cos(2*f*x + 2*e) -
sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*sin(f*x + e) + e + 2
*cos(f*x + e))*sin(2*f*x + 2*e) - 6*sin(2*f*x + 2*e)^2 - 6*sin(f*x + e)^2
- 2*sin(f*x + e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2
 + 9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 18*a^2*f*cos(f*x +
e)*sin(2*f*x + 2*e) + 9*a^2*f*sin(2*f*x + 2*e)^2 + 9*a^2*f*sin(f*x + e)^2
+ 6*a^2*f*sin(f*x + e) + a^2*f - 6*(a^2*f*cos(f*x + e) + a^2*f*sin(2*f*x +
 2*e))*cos(3*f*x + 3*e) - 6*(3*a^2*f*sin(f*x + e) + a^2*f)*cos(2*f*x + 2*e
) + 2*(3*a^2*f*cos(2*f*x + 2*e) - 3*a^2*f*sin(f*x + e) - a^2*f)*sin(3*f...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2486 vs. $2(110) = 220$.

Time = 0.79 (sec) , antiderivative size = 2486, normalized size of antiderivative = 16.80

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/3*(2*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^3 + 2*c*f*tan(1/2*f*x)^3*tan(1/2*e
)^3 - d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) -
 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f
*x) + 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + ta
n(1/2*e)^2 + 1))*tan(1/2*f*x)^3*tan(1/2*e)^3 - 6*d*f*x*tan(1/2*f*x)^2*tan(
1/2*e)^2 + 3*d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1
/2*e) - 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*ta
n(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)
^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*d*log(2*(tan(1/2*f
*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) - 2*tan(1/2*f*x)*tan(1/2*
e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/
(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2
*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1/2*e)^3 + 2*d*f*x*tan(1/2*f*x
)^3 + 6*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*log(2*(tan(1/2*f*x)^2*tan(1/
2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) - 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1
/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1)/(tan(1/2*f*x
)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)^3*tan(
1/2*e) + 6*d*f*x*tan(1/2*f*x)*tan(1/2*e)^2 - 6*c*f*tan(1/2*f*x)^2*tan(1/2*
e)^2 - 3*d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e
) - 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan...
```

Mupad [B] (verification not implemented)

Time = 44.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{3\,a^2\,f^2} - \frac{(c\,f+d\,f\,x-d\,1\mathrm{i})\,2\mathrm{i}}{3\,a^2\,f^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}\right)} - \frac{d\,x\,2\mathrm{i}}{3\,a^2\,f} - \frac{d\,2\mathrm{i}}{3\,a^2\,f^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)} + \frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,(c+d\,x)\,4\mathrm{i}}{3\,a^2\,f\,\left(3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}-\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}+1\mathrm{i}\right)}$$

input
```
int((c + d*x)/(a + a*sin(e + f*x))^2,x)
```

output
```
(2*d*log(exp(e*1i)*exp(f*x*1i) + 1i))/(3*a^2*f^2) - ((c*f - d*1i + d*f*x)*
2i)/(3*a^2*f^2*(exp(e*1i + f*x*1i)*2i + exp(e*2i + f*x*2i) - 1)) - (d*x*2i
)/(3*a^2*f) - (d*2i)/(3*a^2*f^2*(exp(e*1i + f*x*1i) + 1i)) + (exp(e*1i + f
*x*1i)*(c + d*x)*4i)/(3*a^2*f*(3*exp(e*1i + f*x*1i) - exp(e*2i + f*x*2i)*3
i - exp(e*3i + f*x*3i) + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.99

$$\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx = \frac{-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 d - 9\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d - 9\log\left(\tan\left(\frac{fx}{2}\right.\right.}{}$$

input
```
int((d*x+c)/(a+a*sin(f*x+e))^2,x)
```

output
```
( - 3*log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)**3*d - 9*log(tan((e +
f*x)/2)**2 + 1)*tan((e + f*x)/2)**2*d - 9*log(tan((e + f*x)/2)**2 + 1)*tan
((e + f*x)/2)*d - 3*log(tan((e + f*x)/2)**2 + 1)*d + 6*log(tan((e + f*x)/2
) + 1)*tan((e + f*x)/2)**3*d + 18*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/
2)**2*d + 18*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)*d + 6*log(tan((e +
 f*x)/2) + 1)*d + 6*tan((e + f*x)/2)**3*c*f + 6*tan((e + f*x)/2)**3*d*f*x
- 2*tan((e + f*x)/2)**3*d - 6*c*f - 6*d*f*x - 2*d)/(9*a**2*f**2*(tan((e +
f*x)/2)**3 + 3*tan((e + f*x)/2)**2 + 3*tan((e + f*x)/2) + 1))
```

3.115 $\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(c+dx)(a+a\sin(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 15.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\sin(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a\sin(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a\sin(e+fx)+a)^2}dx$$

input
```
Int[1/((c + d*x)*(a + a*Sin[e + f*x])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+a\sin\left(fx+e\right)\right)^{2}}dx$$

input
```
int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.00

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\sin\left(fx+e\right)+a)^{2}}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(2*a^2*d*x + 2*a^2*c - (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^
2*d*x + a^2*c)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{1}{c\sin^{2}\left(e+fx\right)+2c\sin\left(e+fx\right)+c+dx\sin^{2}\left(e+fx\right)+2dx\sin\left(e+fx\right)+dx}\,dx}{a^{2}}$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e))**2,x)
```

output
```
Integral(1/(c*sin(e + f*x)**2 + 2*c*sin(e + f*x) + c + d*x*sin(e + f*x)**2
 + 2*d*x*sin(e + f*x) + d*x), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 8.25 (sec) , antiderivative size = 2914, normalized size of antiderivative = 145.70

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 - 4*d^2*cos(f*x + e) + 6*(d^2*
f*x + c*d*f)*cos(f*x + e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 6*(
d^2*f*x + c*d*f)*sin(f*x + e)^2 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 -
 2*d^2*cos(2*f*x + 2*e) + 2*d^2 - (d^2*f*x + c*d*f)*cos(f*x + e) - (d^2*f*
x + c*d*f)*sin(2*f*x + 2*e) + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4
*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) - 2*(d^2*f*x + c*d*f + 3*(3*d^2*f^2*x
^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*cos(f*x + e) + 6*(d^2*f*x + c*d*f)*s
in(f*x + e))*cos(2*f*x + 2*e) - 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 +
 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2
+ 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3
 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)
^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^
3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2
*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c
*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)*sin(2*f*x + 2
*e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c
^3*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*
a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 - 6*((a^2*d^3*f^3*x^3 + 3*a^
2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) + (a^2*d^3
*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2...
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(a*sin(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 36.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(a+a\sin(e+fx))^2\,(c+dx)}\,dx$$

input
```
int(1/((a + a*sin(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + a*sin(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 6.70

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\sin(fx+e)^2}{\sin(fx+e)^2c+\sin(fx+e)^2dx+2\sin(fx+e)c+2\sin(fx+e)dx+c+dx}dx\right)d-2\left(\int \frac{\sin(fx+e)}{\sin(fx+e)^2c+\sin(fx+e)^2dx+2\sin(fx+e)c+2\sin}\right.}{a^2d}$$

input
```
int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

output
```
( - int(sin(e + f*x)**2/(sin(e + f*x)**2*c + sin(e + f*x)**2*d*x + 2*sin(e
 + f*x)*c + 2*sin(e + f*x)*d*x + c + d*x),x)*d - 2*int(sin(e + f*x)/(sin(e
 + f*x)**2*c + sin(e + f*x)**2*d*x + 2*sin(e + f*x)*c + 2*sin(e + f*x)*d*x
 + c + d*x),x)*d + log(c + d*x))/(a**2*d)
```

3.116 $\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx=\text{Int}\left(\frac{1}{(c+dx)^2(a+a\sin(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\sin(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a\sin(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a\sin(e+fx)+a)^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+a\sin(fx+e))^2} dx$$

input
```
int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2} dx = \int \frac{1}{(dx+c)^2(a\sin(fx+e)+a)^2} dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(2*a^2*d^2*x^2 + 4*a^2*c*d*x + 2*a^2*c^2 - (a^2*d^2*x^2 + 2*a^2
*c*d*x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)
*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 6.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2} dx = \frac{\int \frac{1}{c^2\sin^2(e+fx)+2c^2\sin(e+fx)+c^2+2cdx\sin^2(e+fx)+4cdx\sin(e+fx)+2cdx+d^2x^2\sin^2(e+fx)+2d^2x^2\sin(e+fx)+d^2x^2} dx}{a^2}$$

input
```
integrate(1/(d*x+c)**2/(a+a*sin(f*x+e))**2,x)
```

output
```
Integral(1/(c**2*sin(e + f*x)**2 + 2*c**2*sin(e + f*x) + c**2 + 2*c*d*x*si
n(e + f*x)**2 + 4*c*d*x*sin(e + f*x) + 2*c*d*x + d**2*x**2*sin(e + f*x)**2
 + 2*d**2*x**2*sin(e + f*x) + d**2*x**2), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 3521, normalized size of antiderivative = 176.05

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 - 12*d^2*cos(f*x + e) + 12*(d
^2*f*x + c*d*f)*cos(f*x + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 +
 12*(d^2*f*x + c*d*f)*sin(f*x + e)^2 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*
f^2 - 6*d^2*cos(2*f*x + 2*e) + 6*d^2 - 2*(d^2*f*x + c*d*f)*cos(f*x + e) -
2*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*
f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) - 2*(2*d^2*f*x + 2*c*d*f + 9*(
d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e) + 12*(d^2*f*x +
c*d*f)*sin(f*x + e))*cos(2*f*x + 2*e) - 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f
^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^
4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*
x + a^2*c^4*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3
*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x
+ 2*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^
2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4
*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2
*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)*sin(2*f*x
 + 2*e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2
 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*x^
4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a...
```

Giac [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx=\int \frac{1}{(dx+c)^2(a\sin\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(a*sin(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 36.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*sin(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*sin(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 40461, normalized size of antiderivative = 2023.05

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

output
```
( - 60*cos(e + f*x)*tan((e + f*x)/2)**3*c*d - 180*cos(e + f*x)*tan((e + f*
x)/2)**2*c*d - 180*cos(e + f*x)*tan((e + f*x)/2)*c*d - 60*cos(e + f*x)*c*d
 + 72*int(cos(e + f*x)/(sin(e + f*x)**2*c**2 + 2*sin(e + f*x)**2*c*d*x + s
in(e + f*x)**2*d**2*x**2 + 2*sin(e + f*x)*c**2 + 4*sin(e + f*x)*c*d*x + 2*
sin(e + f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*sin(e + f*x)*tan((
e + f*x)/2)**3*c**3*d*f + 72*int(cos(e + f*x)/(sin(e + f*x)**2*c**2 + 2*si
n(e + f*x)**2*c*d*x + sin(e + f*x)**2*d**2*x**2 + 2*sin(e + f*x)*c**2 + 4*
sin(e + f*x)*c*d*x + 2*sin(e + f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2
),x)*sin(e + f*x)*tan((e + f*x)/2)**3*c**2*d**2*f*x + 80*int(cos(e + f*x)/
(sin(e + f*x)**2*c**2 + 2*sin(e + f*x)**2*c*d*x + sin(e + f*x)**2*d**2*x**
2 + 2*sin(e + f*x)*c**2 + 4*sin(e + f*x)*c*d*x + 2*sin(e + f*x)*d**2*x**2
+ c**2 + 2*c*d*x + d**2*x**2),x)*sin(e + f*x)*tan((e + f*x)/2)**3*c**2*d**
2 + 80*int(cos(e + f*x)/(sin(e + f*x)**2*c**2 + 2*sin(e + f*x)**2*c*d*x +
sin(e + f*x)**2*d**2*x**2 + 2*sin(e + f*x)*c**2 + 4*sin(e + f*x)*c*d*x + 2
*sin(e + f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*sin(e + f*x)*tan(
(e + f*x)/2)**3*c*d**3*x + 216*int(cos(e + f*x)/(sin(e + f*x)**2*c**2 + 2*
sin(e + f*x)**2*c*d*x + sin(e + f*x)**2*d**2*x**2 + 2*sin(e + f*x)*c**2 +
4*sin(e + f*x)*c*d*x + 2*sin(e + f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x*
*2),x)*sin(e + f*x)*tan((e + f*x)/2)**2*c**3*d*f + 216*int(cos(e + f*x)/(s
in(e + f*x)**2*c**2 + 2*sin(e + f*x)**2*c*d*x + sin(e + f*x)**2*d**2*x*...
```

3.117 $\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = -\frac{i(c+dx)^3}{af}+\frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3} + \frac{12d^3\,\mathrm{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{af^4}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
-I*(d*x+c)^3/a/f+6*d*(d*x+c)^2*ln(1+I*exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+
c)*polylog(2,-I*exp(I*(f*x+e)))/a/f^3+12*d^3*polylog(3,-I*exp(I*(f*x+e)))/
a/f^4+(d*x+c)^3*tan(1/2*e+1/4*Pi+1/2*f*x)/a/f
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \frac{-12id^2f(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)+12d^3\,\mathrm{PolyLog}\left(3,-ie^{i(e+fx)}\right)+f^2(c+dx)^2\left(-if(c+dx)+6\right.}{af^4}$$

input
```
Integrate[(c + d*x)^3/(a - a*Sin[e + f*x]),x]
```

output
```
((-12*I)*d^2*f*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))] + 12*d^3*PolyLog
[3, (-I)*E^(I*(e + f*x))] + f^2*(c + d*x)^2*((-I)*f*(c + d*x) + 6*d*Log[1
+ I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4]))/(a*f^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3799, 3042, 4672, 25, 3042, 4200, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{6d\int -(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\int(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{4200}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{ie^{i(e+fx)}(c+dx)^2}{1+ie^{i(e+fx)}}dx\right)}{f}}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(2\int\frac{e^{i(e+fx)}(c+dx)^2}{1+ie^{i(e+fx)}}dx+\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(2\left(\frac{2d\int(c+dx)\log\left(1+ie^{i(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{f}\right)+\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(2\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f}-\frac{id\int\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)dx}{f}\right)}{f}-\frac{(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{f}\right)+\frac{i(c+dx)^3}{3d}\cdots}{f}}{2a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(2\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)de^{i(e+fx)}}{f^2}\right)}{f}-\frac{(c+dx)^2\log\left(1+ie^{i(e+\cdots}{f}\cdots\right.\right.}{f}}{2a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{6d\left(2\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^2}\right)}{f}-\frac{(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{f}\right)+\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^3/(a - a*Sin[e + f*x]),x]
```

output
```
((-6*d*(((I/3)*(c + d*x)^3)/d + 2*(-(((c + d*x)^2*Log[1 + I*E^(I*(e + f*x)
)])/f) + (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))])/f - (d*PolyL
og[3, (-I)*E^(I*(e + f*x))])/f^2))/f)))/f + (2*(c + d*x)^3*Tan[e/2 + Pi/4
+ (f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(130) = 260.

Time = 0.94 (sec) , antiderivative size = 484, normalized size of antiderivative = 3.29

method	result
risch	$\frac{2d^3x^3+6c\,d^2x^2+6c^2dx+2c^3}{fa\left(\mathrm{e}^{i(fx+e)}-i\right)}-\frac{12ic\,d^2ex}{a\,f^2}+\frac{6\ln\left(\mathrm{e}^{i(fx+e)}-i\right)c^2d}{a\,f^2}-\frac{12id^3\,\mathrm{polylog}\left(2,-i\mathrm{e}^{i(fx+e)}\right)x}{a\,f^3}+\frac{12ec\,d^2\ln\left(\mathrm{e}^{i(fx+e)}\right)}{a\,f^3}-$

input
```
int((d*x+c)^3/(a-a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-I)-12*I/a/f^2*c*
d^2*e*x+6/a/f^2*ln(exp(I*(f*x+e))-I)*c^2*d-12*I/a/f^3*d^3*polylog(2,-I*exp
(I*(f*x+e)))*x+12/a/f^3*e*c*d^2*ln(exp(I*(f*x+e)))-12*I/a/f^3*c*d^2*polylo
g(2,-I*exp(I*(f*x+e)))-6*I/a/f*c*d^2*x^2-2*I/a/f*d^3*x^3+12/a/f^2*c*d^2*ln
(1+I*exp(I*(f*x+e)))*x-12/a/f^3*e*c*d^2*ln(exp(I*(f*x+e))-I)+6/a/f^4*e^2*d
^3*ln(exp(I*(f*x+e))-I)-6/a/f^4*e^2*d^3*ln(1+I*exp(I*(f*x+e)))+12/a/f^3*c*
d^2*ln(1+I*exp(I*(f*x+e)))*e-6/a/f^4*e^2*d^3*ln(exp(I*(f*x+e)))+12*d^3*pol
ylog(3,-I*exp(I*(f*x+e)))/a/f^4-6*I/a/f^3*c*d^2*e^2+6/a/f^2*d^3*ln(1+I*exp
(I*(f*x+e)))*x^2-6/a/f^2*ln(exp(I*(f*x+e)))*c^2*d+6*I/a/f^3*d^3*e^2*x+4*I/
a/f^4*d^3*e^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(124) = 248$.

Time = 0.10 (sec) , antiderivative size = 916, normalized size of antiderivative = 6.23

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 + (d^3*f^3*x^3 +
3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*cos(f*x + e) + 6*(I*d^3*f*x + I
*c*d^2*f + (I*d^3*f*x + I*c*d^2*f)*cos(f*x + e) + (-I*d^3*f*x - I*c*d^2*f)
*sin(f*x + e))*dilog(I*cos(f*x + e) + sin(f*x + e)) + 6*(-I*d^3*f*x - I*c*
d^2*f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e) + (I*d^3*f*x + I*c*d^2*f)*si
n(f*x + e))*dilog(-I*cos(f*x + e) + sin(f*x + e)) + 3*(d^3*e^2 - 2*c*d^2*e
*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x + e) - (d^3*e
^2 - 2*c*d^2*e*f + c^2*d*f^2)*sin(f*x + e))*log(cos(f*x + e) - I*sin(f*x +
 e) + I) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f + (d^3*f
^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e) - (d^3*f^2*x^
2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e))*log(I*cos(f*x + e
) - sin(f*x + e) + 1) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2
*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e)
- (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e))*log(
-I*cos(f*x + e) - sin(f*x + e) + 1) + 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2
 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x + e) - (d^3*e^2 - 2*c*d^2*e
*f + c^2*d*f^2)*sin(f*x + e))*log(-cos(f*x + e) - I*sin(f*x + e) + I) + 6*
(d^3*cos(f*x + e) - d^3*sin(f*x + e) + d^3)*polylog(3, I*cos(f*x + e) + si
n(f*x + e)) + 6*(d^3*cos(f*x + e) - d^3*sin(f*x + e) + d^3)*polylog(3, -I*
cos(f*x + e) + sin(f*x + e)) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = -\frac{\int \frac{c^3}{\sin(e+fx)-1}\,dx + \int \frac{d^3x^3}{\sin(e+fx)-1}\,dx + \int \frac{3cd^2x^2}{\sin(e+fx)-1}\,dx + \int \frac{3c^2dx}{\sin(e+fx)-1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a-a*sin(f*x+e)),x)
```

output
```
-(Integral(c**3/(sin(e + f*x) - 1), x) + Integral(d**3*x**3/(sin(e + f*x)
- 1), x) + Integral(3*c*d**2*x**2/(sin(e + f*x) - 1), x) + Integral(3*c**2
*d*x/(sin(e + f*x) - 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(124) = 248$.

Time = 0.14 (sec) , antiderivative size = 984, normalized size of antiderivative = 6.69

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-(6*(2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f
*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*c*
d^2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*sin(f*x + e)
+ a*f^2) - 6*c*d^2*e^2/(a*f^2 - a*f^2*sin(f*x + e)/(cos(f*x + e) + 1)) - 3
*(2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x
+ e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*c^2*d
/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*sin(f*x + e) + a*f) + 6*
c^2*d*e/(a*f - a*f*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*c^3/(a - a*sin(f*x
 + e)/(cos(f*x + e) + 1)) - (2*I*d^3*e^3 + 6*(d^3*e^2*cos(f*x + e) + I*d^3
*e^2*sin(f*x + e) - I*d^3*e^2)*arctan2(sin(f*x + e) - 1, cos(f*x + e)) - 6
*(I*(f*x + e)^2*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e) - ((f*x + e)^2*d^
3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) + (-I*(f*x + e)^2*d^3 + 2*
(I*d^3*e - I*c*d^2*f)*(f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), -sin(
f*x + e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^
2*f)*(f*x + e)^2)*cos(f*x + e) - 12*(-I*(f*x + e)*d^3 + I*d^3*e - I*c*d^2*
f + ((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e) + (I*(f*x + e)*d^3 - I*
d^3*e + I*c*d^2*f)*sin(f*x + e))*dilog(-I*e^(I*f*x + I*e)) - 3*((f*x + e)^
2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e) + (I*(f*x + e)^2*d^3 + I*d
^3*e^2 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*cos(f*x + e) - ((f*x + e)^2*d
^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f*x + e))*log(cos(f*x...
```

Giac [F]

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \int -\frac{(dx+c)^3}{a\sin(fx+e)-a}\,dx$$

input
```
integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^3/(a*sin(f*x + e) - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a-a\,\sin(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a - a*sin(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a - a*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3/(a-a*sin(f*x+e)),x)
```

output
```
( - 6*int(x**2/(tan((e + f*x)/2)**2 - 2*tan((e + f*x)/2) + 1),x)*tan((e +
f*x)/2)*d**3*f**3 + 6*int(x**2/(tan((e + f*x)/2)**2 - 2*tan((e + f*x)/2) +
 1),x)*d**3*f**3 - 12*int(x/(tan((e + f*x)/2)**2 - 2*tan((e + f*x)/2) + 1)
,x)*tan((e + f*x)/2)*c*d**2*f**3 - 12*int(x/(tan((e + f*x)/2)**2 - 2*tan((
e + f*x)/2) + 1),x)*tan((e + f*x)/2)*d**3*f**2 + 12*int(x/(tan((e + f*x)/2
)**2 - 2*tan((e + f*x)/2) + 1),x)*c*d**2*f**3 + 12*int(x/(tan((e + f*x)/2)
**2 - 2*tan((e + f*x)/2) + 1),x)*d**3*f**2 - 3*log(tan((e + f*x)/2)**2 + 1
)*tan((e + f*x)/2)*c**2*d*f**2 - 6*log(tan((e + f*x)/2)**2 + 1)*tan((e + f
*x)/2)*c*d**2*f - 6*log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*d**3 + 3
*log(tan((e + f*x)/2)**2 + 1)*c**2*d*f**2 + 6*log(tan((e + f*x)/2)**2 + 1)
*c*d**2*f + 6*log(tan((e + f*x)/2)**2 + 1)*d**3 + 6*log(tan((e + f*x)/2) -
 1)*tan((e + f*x)/2)*c**2*d*f**2 + 12*log(tan((e + f*x)/2) - 1)*tan((e + f
*x)/2)*c*d**2*f + 12*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)*d**3 - 6*l
og(tan((e + f*x)/2) - 1)*c**2*d*f**2 - 12*log(tan((e + f*x)/2) - 1)*c*d**2
*f - 12*log(tan((e + f*x)/2) - 1)*d**3 - 2*tan((e + f*x)/2)*c**3*f**3 - 3*
tan((e + f*x)/2)*c**2*d*f**3*x - 3*tan((e + f*x)/2)*c*d**2*f**3*x**2 - 6*t
an((e + f*x)/2)*c*d**2*f**2*x - tan((e + f*x)/2)*d**3*f**3*x**3 - 3*tan((e
 + f*x)/2)*d**3*f**2*x**2 - 6*tan((e + f*x)/2)*d**3*f*x - 3*c**2*d*f**3*x
- 3*c*d**2*f**3*x**2 - 6*c*d**2*f**2*x - d**3*f**3*x**3 - 3*d**3*f**2*x**2
 - 6*d**3*f*x)/(a*f**4*(tan((e + f*x)/2) - 1))
```

3.118 $\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx = -\frac{i(c+dx)^2}{af}+\frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{4id^2\,\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
-I*(d*x+c)^2/a/f+4*d*(d*x+c)*ln(1+I*exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(
2,-I*exp(I*(f*x+e)))/a/f^3+(d*x+c)^2*tan(1/2*e+1/4*Pi+1/2*f*x)/a/f
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx = \frac{-4id^2\,\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)+f(c+dx)\left(-if(c+dx)+4d\log\left(1+ie^{i(e+fx)}\right)+f(c+dx)\tan\left(\frac{1}{4}(2e-\right.\right.}{af^3}$$

input
```
Integrate[(c + d*x)^2/(a - a*Sin[e + f*x]),x]
```

output
```
((-4*I)*d^2*PolyLog[2, (-I)*E^(I*(e + f*x))] + f*(c + d*x)*((-I)*f*(c + d*
x) + 4*d*Log[1 + I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4
]))/(a*f^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3799, 3042, 4672, 25, 3042, 4200, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{4d\int -\left((c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)dx}{f}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\int(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}}{2a}$$

$\Big\downarrow$ 4200

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{ie^{i(e+fx)}(c+dx)}{1+ie^{i(e+fx)}}dx\right)}{f}}{2a}$$

$\Big\downarrow$ 26

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(2\int\frac{e^{i(e+fx)}(c+dx)}{1+ie^{i(e+fx)}}dx+\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

$\Big\downarrow$ 2620

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(2\left(\frac{d\int\log\left(1+ie^{i(e+fx)}\right)dx}{f}-\frac{(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{f}\right)+\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

$\Big\downarrow$ 2715

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(2\left(-\frac{id\int e^{-i(e+fx)}\log\left(1+ie^{i(e+fx)}\right)de^{i(e+fx)}}{f^2}-\frac{(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{f}\right)+\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

$\Big\downarrow$ 2838

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4d\left(2\left(\frac{id\,\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2}-\frac{(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{f}\right)+\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^2/(a - a*Sin[e + f*x]),x]
```

output
```
((-4*d*(((I/2)*(c + d*x)^2)/d + 2*(-(((c + d*x)*Log[1 + I*E^(I*(e + f*x))]
)/f) + (I*d*PolyLog[2, (-I)*E^(I*(e + f*x))])/f^2)))/f + (2*(c + d*x)^2*Ta
n[e/2 + Pi/4 + (f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(98) = 196$.

Time = 0.79 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.27

method	result
risch	$\frac{2x^2d^2+4cdx+2c^2}{fa\left(\mathrm{e}^{i(fx+e)}-i\right)}-\frac{4\ln\left(\mathrm{e}^{i(fx+e)}\right)cd}{a\,f^2}+\frac{4\ln\left(\mathrm{e}^{i(fx+e)}-i\right)cd}{a\,f^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{a\,f^2}-\frac{2id^2e^2}{a\,f^3}+\frac{4d^2\ln\left(1+i\mathrm{e}^{i(fx+e)}\right)x}{a\,f^2}+4a$

input
```
int((d*x+c)^2/(a-a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-I)-4/a/f^2*ln(exp(I*(f*x+e)))*
c*d+4/a/f^2*ln(exp(I*(f*x+e))-I)*c*d-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*I
/a/f^3*d^2*e^2+4/a/f^2*d^2*ln(1+I*exp(I*(f*x+e)))*x+4/a/f^3*d^2*ln(1+I*exp
(I*(f*x+e)))*e-4*I*d^2*polylog(2,-I*exp(I*(f*x+e)))/a/f^3+4/a/f^3*e*d^2*ln
(exp(I*(f*x+e)))-4/a/f^3*e*d^2*ln(exp(I*(f*x+e))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(93) = 186$.

Time = 0.09 (sec) , antiderivative size = 496, normalized size of antiderivative = 4.43

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx$$
$$=\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)\cos\left(fx+e\right)+2\left(i\,d^2\cos\left(fx+e\right)-i\,d^2\sin\left(fx+\right.\right.}{}$$

input
```
integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^
2)*cos(f*x + e) + 2*(I*d^2*cos(f*x + e) - I*d^2*sin(f*x + e) + I*d^2)*dilo
g(I*cos(f*x + e) + sin(f*x + e)) + 2*(-I*d^2*cos(f*x + e) + I*d^2*sin(f*x
+ e) - I*d^2)*dilog(-I*cos(f*x + e) + sin(f*x + e)) - 2*(d^2*e - c*d*f + (
d^2*e - c*d*f)*cos(f*x + e) - (d^2*e - c*d*f)*sin(f*x + e))*log(cos(f*x +
e) - I*sin(f*x + e) + I) + 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x
+ e) - (d^2*f*x + d^2*e)*sin(f*x + e))*log(I*cos(f*x + e) - sin(f*x + e) +
 1) + 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) - (d^2*f*x + d^2
*e)*sin(f*x + e))*log(-I*cos(f*x + e) - sin(f*x + e) + 1) - 2*(d^2*e - c*d
*f + (d^2*e - c*d*f)*cos(f*x + e) - (d^2*e - c*d*f)*sin(f*x + e))*log(-cos
(f*x + e) - I*sin(f*x + e) + I) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*si
n(f*x + e))/(a*f^3*cos(f*x + e) - a*f^3*sin(f*x + e) + a*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx = -\frac{\int \frac{c^2}{\sin(e+fx)-1}\,dx + \int \frac{d^2x^2}{\sin(e+fx)-1}\,dx + \int \frac{2cdx}{\sin(e+fx)-1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a-a*sin(f*x+e)),x)
```

output
```
-(Integral(c**2/(sin(e + f*x) - 1), x) + Integral(d**2*x**2/(sin(e + f*x)
- 1), x) + Integral(2*c*d*x/(sin(e + f*x) - 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(93) = 186.

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.78

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx = -\frac{2\left(ic^2f^2 - 2\left(cdf\cos(fx+e) + icdf\sin(fx+e) - icdf\right)\arctan\left(\sin(fx+e)-1,\cos(fx+e)\right) - 2\left(\right.}{}$$

input
```
integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-2*(I*c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) - I*c*d*f)*ar
ctan2(sin(f*x + e) - 1, cos(f*x + e)) - 2*(d^2*f*x*cos(f*x + e) + I*d^2*f*
x*sin(f*x + e) - I*d^2*f*x)*arctan2(cos(f*x + e), -sin(f*x + e) + 1) + (d^
2*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*
x + e) - I*d^2)*dilog(-I*e^(I*f*x + I*e)) + (d^2*f*x + c*d*f + (I*d^2*f*x
+ I*c*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)
^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x
)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - a*f^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx=\int -\frac{(dx+c)^2}{a\sin(fx+e)-a}\,dx$$

input
```
integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^2/(a*sin(f*x + e) - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx=\int \frac{(c+d\,x)^2}{a-a\,\sin(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a - a*sin(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a - a*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx$$
$$= \frac{4\left(\int \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)x}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}dx\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2f-4\left(\int \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)x}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}dx\right)d^2f-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\tan\left(\frac{fx}{2}\right.}{}$$

input
```
int((d*x+c)^2/(a-a*sin(f*x+e)),x)
```

output
```
(2*(2*int((tan((e + f*x)/2)*x)/(tan((e + f*x)/2) - 1),x)*tan((e + f*x)/2)*
d**2*f - 2*int((tan((e + f*x)/2)*x)/(tan((e + f*x)/2) - 1),x)*d**2*f - log
(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*c*d + log(tan((e + f*x)/2)**2 +
 1)*c*d + 2*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)*c*d - 2*log(tan((e
+ f*x)/2) - 1)*c*d - tan((e + f*x)/2)*c**2*f - tan((e + f*x)/2)*c*d*f*x -
tan((e + f*x)/2)*d**2*f*x**2 - c*d*f*x))/(a*f**2*(tan((e + f*x)/2) - 1))
```

3.119 $\int \frac{c+dx}{a-a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx=\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
2*d*ln(cos(1/2*e+1/4*Pi+1/2*f*x))/a/f^2+(d*x+c)*tan(1/2*e+1/4*Pi+1/2*f*x)/
a/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx =\frac{2d\log\left(\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+f(c+dx)\tan\left(\frac{1}{4}(2e+\pi+2fx)\right)}{af^2}$$

input
```
Integrate[(c + d*x)/(a - a*Sin[e + f*x]),x]
```

output
```
(2*d*Log[Cos[(2*e + Pi + 2*f*x)/4]] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4
])/(a*f^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3799, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{c+dx}{a-a\sin(e+fx)}dx$$

$$\Big\downarrow \ 3799$$

$$\frac{\int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \ 4672$$

$$\frac{\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{2d\int \tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{2d\int \tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \ 3956$$

$$\frac{\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}+\frac{4d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{f^2}}{2a}$$

input
```
Int[(c + d*x)/(a - a*Sin[e + f*x]),x]
```

output
```
((4*d*Log[Cos[e/2 + Pi/4 + (f*x)/2]])/f^2 + (2*(c + d*x)*Tan[e/2 + Pi/4 +
(f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{2idx}{af}-\frac{2ide}{a\,f^{2}}+\frac{2dx+2c}{fa\left(\mathrm{e}^{i(fx+e)}-i\right)}+\frac{2d\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a\,f^{2}}$	73
parallelrisch	$\frac{-d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\ln\left(\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)+2d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-2\left(\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+\frac{dx}{2}+c\right)f}{f^{2}a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	96
norman	$\frac{-\frac{2c}{fa}-\frac{dx}{fa}-\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{a\,f^{2}}-\frac{d\ln\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{a\,f^{2}}$	99

input
```
int((d*x+c)/(a-a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*d/a/f*x-2*I*d/a/f^2*e+2*(d*x+c)/f/a/(exp(I*(f*x+e))-I)+2*d/a/f^2*ln(e
xp(I*(f*x+e))-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(47) = 94$.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.71

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx = \frac{dfx+cf+(dfx+cf)\cos(fx+e)+(d\cos(fx+e)-d\sin(fx+e)+d)\log(-\sin(fx+e)+1)+(dj}{af^2\cos(fx+e)-af^2\sin(fx+e)+af^2}$$

input
```
integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
(d*f*x + c*f + (d*f*x + c*f)*cos(f*x + e) + (d*cos(f*x + e) - d*sin(f*x +
e) + d)*log(-sin(f*x + e) + 1) + (d*f*x + c*f)*sin(f*x + e))/(a*f^2*cos(f*
x + e) - a*f^2*sin(f*x + e) + a*f^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(44) = 88$.

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.61

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx = \begin{cases} -\frac{2cf}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{dfx}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^2} - \frac{2d\log\left(\tan\right.}{af^2\tan\left(\right.} \\ \frac{cx+\frac{dx^2}{2}}{-a\sin(e)+a} \end{cases}$$

input
```
integrate((d*x+c)/(a-a*sin(f*x+e)),x)
```

output
```
Piecewise((-2*c*f/(a*f**2*tan(e/2 + f*x/2) - a*f**2) - d*f*x*tan(e/2 + f*x
/2)/(a*f**2*tan(e/2 + f*x/2) - a*f**2) - d*f*x/(a*f**2*tan(e/2 + f*x/2) -
a*f**2) + 2*d*log(tan(e/2 + f*x/2) - 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 +
 f*x/2) - a*f**2) - 2*d*log(tan(e/2 + f*x/2) - 1)/(a*f**2*tan(e/2 + f*x/2)
 - a*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e
/2 + f*x/2) - a*f**2) + d*log(tan(e/2 + f*x/2)**2 + 1)/(a*f**2*tan(e/2 + f
*x/2) - a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(-a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx = \frac{\frac{\left(2(fx+e)\cos(fx+e)+\left(\cos(fx+e)^2+\sin(fx+e)^2-2\sin(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\sin(fx+e)+1\right)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2-2af\sin(fx+e)+af} - \frac{2de}{af-\frac{af\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

input
```
integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
((2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x
+ e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*d/(a*
f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*sin(f*x + e) + a*f) - 2*d*e/
(a*f - a*f*sin(f*x + e)/(cos(f*x + e) + 1)) + 2*c/(a - a*sin(f*x + e)/(cos
(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(47) = 94$.

Time = 0.41 (sec) , antiderivative size = 549, normalized size of antiderivative = 9.31

$$\int \frac{c + dx}{a - a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="giac")
```

output
```
(d*f*x*tan(1/2*f*x)*tan(1/2*e) - d*f*x*tan(1/2*f*x) - d*f*x*tan(1/2*e) + c
*f*tan(1/2*f*x)*tan(1/2*e) + d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(
1/2*f*x)^2*tan(1/2*e) + 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan
(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^
2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) - d*f*x -
c*f*tan(1/2*f*x) + d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2
*tan(1/2*e) + 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2
- 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/
2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x) - c*f*tan(1/2*e) + d*log(2*(tan
(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + 2*tan(1/2*f*x)*ta
n(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e)
 + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*t
an(1/2*e) - c*f - d*log(2*(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*
tan(1/2*e) + 2*tan(1/2*f*x)*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 -
 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2
*f*x)^2 + tan(1/2*e)^2 + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) + a*f^2*tan(1
/2*f*x) + a*f^2*tan(1/2*e) - a*f^2)
```

Mupad [B] (verification not implemented)

Time = 36.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx = \frac{2\,d\,\ln\left(e^{e\,1i}\,e^{f\,x\,1i}-i\right)}{a\,f^2}+\frac{2\,(c+d\,x)}{a\,f\,\left(e^{e\,1i+f\,x\,1i}-i\right)}-\frac{d\,x\,2i}{a\,f}$$

input
```
int((c + d*x)/(a - a*sin(e + f*x)),x)
```

output
```
(2*d*log(exp(e*1i)*exp(f*x*1i) - 1i))/(a*f^2) + (2*(c + d*x))/(a*f*(exp(e*
1i + f*x*1i) - 1i)) - (d*x*2i)/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.27

$$\int \frac{c+dx}{a-a\sin(e+fx)}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)d+2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}\right.}{a\,f^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$$

input
```
int((d*x+c)/(a-a*sin(f*x+e)),x)
```

output
```
( - log(tan((e + f*x)/2)**2 + 1)*tan((e + f*x)/2)*d + log(tan((e + f*x)/2)
**2 + 1)*d + 2*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)*d - 2*log(tan((e
 + f*x)/2) - 1)*d - 2*tan((e + f*x)/2)*c*f - tan((e + f*x)/2)*d*f*x - d*f*
x)/(a*f**2*(tan((e + f*x)/2) - 1))
```

3.120 $\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a-a\sin(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a - a*Sin[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a - a*Sin[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a - a*Sin[e + f*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a-a\sin{(fx+e)})}dx$$

input
```
int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\sin{(fx+e)}-a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c - (a*d*x + a*c)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = -\frac{\int \frac{1}{c\sin{(e+fx)}-c+dx\sin{(e+fx)}-dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

output
```
-Integral(1/(c*sin(e + f*x) - c + d*x*sin(e + f*x) - d*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 13.57

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\sin(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*
x + a*c*d*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e))*integr
ate(cos(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a
*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)
*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin(f*x + e)), x
) + cos(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a
*d*f*x + a*c*f)*sin(f*x + e)^2 - 2*(a*d*f*x + a*c*f)*sin(f*x + e))
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\sin(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-1/((d*x + c)*(a*sin(f*x + e) - a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int \frac{1}{(a-a\,\sin\left(e+f\,x\right))\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a - a*sin(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a - a*sin(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \frac{-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)c+\sin(fx+e)dx-c-dx}dx\right)d+\log(dx+c)}{ad}$$

input
```
int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

output
```
( - int(sin(e + f*x)/(sin(e + f*x)*c + sin(e + f*x)*d*x - c - d*x),x)*d +
log(c + d*x))/(a*d)
```

3.121 $\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))} dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a-a\sin(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))} dx = \int \frac{1}{(c+dx)^2(a-a\sin(e+fx))} dx$$

input
```
Integrate[1/((c + d*x)^2*(a - a*Sin[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a - a*Sin[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a - a*Sin[e + f*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(dx+c\right)^{2}\left(a-a\sin\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx=\int -\frac{1}{\left(dx+c\right)^{2}\left(a\sin\left(fx+e\right)-a\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2
)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$
$$=-\frac{\int \frac{1}{c^2\sin(e+fx)-c^2+2cdx\sin(e+fx)-2cdx+d^2x^2\sin(e+fx)-d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a-a*sin(f*x+e)),x)
```

output
```
-Integral(1/(c**2*sin(e + f*x) - c**2 + 2*c*d*x*sin(e + f*x) - 2*c*d*x + d
**2*x**2*sin(e + f*x) - d**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 442, normalized size of antiderivative = 21.05

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\sin(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f
*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)
*sin(f*x + e)^2 - 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*sin(f*x + e)
)*integrate(cos(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x +
a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*
x + e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f
*x + e)^2 - 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*si
n(f*x + e)), x) + cos(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*
d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d
*f*x + a*c^2*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*s
in(f*x + e))
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\sin(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-1/((d*x + c)^2*(a*sin(f*x + e) - a)), x)
```

Mupad [N/A]

Not integrable

Time = 36.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx=\int \frac{1}{\left(a-a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/((a - a*sin(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a - a*sin(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.33

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$
$$=\frac{-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)c^2+2\sin(fx+e)cdx+\sin(fx+e)d^2x^2-c^2-2cdx-d^2x^2}dx\right)c^2-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)c^2+2\sin(fx+e)cdx+\sin(fx+e)d^2x^2-c^2-}\right.}{ac\,(dx+c)}$$

input
```
int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

output
```
( - int(sin(e + f*x)/(sin(e + f*x)*c**2 + 2*sin(e + f*x)*c*d*x + sin(e + f
*x)*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*c**2 - int(sin(e + f*x)/(si
n(e + f*x)*c**2 + 2*sin(e + f*x)*c*d*x + sin(e + f*x)*d**2*x**2 - c**2 - 2
*c*d*x - d**2*x**2),x)*c*d*x + x)/(a*c*(c + d*x))
```

3.122 $\int x^3\sqrt{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx=-\frac{96\sqrt{a+a\sin(c+dx)}}{d^4}+\frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2}+\frac{48x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3}-\frac{2x^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}$$

output

```
-96*(a+a*sin(d*x+c))^(1/2)/d^4+12*x^2*(a+a*sin(d*x+c))^(1/2)/d^2+48*x*cot(
1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c))^(1/2)/d^3-2*x^3*cot(1/2*c+1/4*Pi+1/
2*d*x)*(a+a*sin(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\left((48-24dx-6d^2x^2+d^3x^3)\cos\left(\frac{1}{2}(c+dx)\right)+(48+24dx-6d^2x^2-d^3x^3)\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1-}{d^4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[x^3*Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(-2*((48 - 24*d*x - 6*d^2*x^2 + d^3*x^3)*Cos[(c + d*x)/2] + (48 + 24*d*x -
 6*d^2*x^2 - d^3*x^3)*Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d^4*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 3800, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sqrt{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int x^3\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int x^3\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\int x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2x^3\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\int x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2x^3\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 3777

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\left(\frac{4\int -x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}+\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)}{d}-\frac{2x^3\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 25

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}\right)}{d}-\frac{2x^3\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}\right)}{d}-\frac{2x^3\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 3777

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4\left(\frac{2\int\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)}{d}\right)}{d}-\frac{2x^3\cos}{}\right.$$

$\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4\left(\frac{2\int\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)}{d}\right)}{d}-\frac{2x^3\cos}{}\right.$$

$$\Big\downarrow \; 3118$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4\left(\frac{4\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d^2}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)}{d}\right)}{d}-\frac{2x^3\cos\left(\frac{c}{2}\right.}{d}\right.$$

input
```
Int[x^3*Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
Csc[c/2 + Pi/4 + (d*x)/2]*((-2*x^3*Cos[c/2 + Pi/4 + (d*x)/2])/d + (6*((2*x
^2*Sin[c/2 + Pi/4 + (d*x)/2])/d - (4*((-2*x*Cos[c/2 + Pi/4 + (d*x)/2])/d +
 (4*Sin[c/2 + Pi/4 + (d*x)/2])/d^2))/d))/d)*Sqrt[a + a*Sin[c + d*x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{i\sqrt{2}\,\sqrt{-a(-2-2\sin(dx+c))}\,(-ix^3d^3+d^3x^3\mathrm{e}^{i(dx+c)}+6id^2x^2\mathrm{e}^{i(dx+c)}-6x^2d^2+24idx-24dx\,\mathrm{e}^{i(dx+c)}-48i\mathrm{e}^{i(dx+c)}+48)\,(\mathrm{e}^{i(dx+c)}}{(\mathrm{e}^{2i(dx+c)}-1+2i\mathrm{e}^{i(dx+c)})\,d^4}$

input
```
int(x^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))-1+2*I*exp(I*(d*x
+c)))*(-I*x^3*d^3+d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-6*x^2*
d^2+24*I*d*x-24*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+48)*(exp(I*(d*x+c))
+I)/d^4
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx=\int x^3\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x**3*(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*(sin(c + d*x) + 1)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx=\int\sqrt{a\sin\left(dx+c\right)+a}x^3\,dx$$

input
```
integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a)*x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx$$
$$=2\sqrt{2}\sqrt{a}\left(\frac{6\left(d^2x^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-8\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\cos\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)}{d^4}\right.$$

input
```
integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*(6*(d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 8*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c)))*cos(1/4*pi - 1/2*d*x - 1/2*c)/d^4 - (d^3*
x^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 24*d*x*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c)))*sin(1/4*pi - 1/2*d*x - 1/2*c)/d^4)
```

Mupad [B] (verification not implemented)

Time = 36.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx =$$
$$-\frac{2\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(48\,\sin\left(c+d\,x\right)-6\,d^2\,x^2+d^3\,x^3\,\cos\left(c+d\,x\right)-6\,d^2\,x^2\,\sin\left(c+d\,x\right)-24\,d\,x}{d^4\,\left(\sin\left(c+d\,x\right)+1\right)}$$

input
```
int(x^3*(a + a*sin(c + d*x))^(1/2),x)
```

output
```
-(2*(a*(sin(c + d*x) + 1))^(1/2)*(48*sin(c + d*x) - 6*d^2*x^2 + d^3*x^3*co
s(c + d*x) - 6*d^2*x^2*sin(c + d*x) - 24*d*x*cos(c + d*x) + 48))/(d^4*(sin
(c + d*x) + 1))
```

Reduce [F]

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx = \sqrt{a}\left(\int\sqrt{\sin\left(dx+c\right)+1}\,x^3dx\right)$$

input
```
int(x^3*(a+a*sin(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sin(c + d*x) + 1)*x**3,x)
```

3.123 $\int x^2 \sqrt{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx = \frac{8x\sqrt{a+a\sin(c+dx)}}{d^2} + \frac{16\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3} - \frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}$$

output
```
8*x*(a+a*sin(d*x+c))^(1/2)/d^2+16*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c
))^(1/2)/d^3-2*x^2*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\left((-8-4dx+d^2x^2)\cos\left(\frac{1}{2}(c+dx)\right)-(-8+4dx+d^2x^2)\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[x^2*Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(-2*((-8 - 4*d*x + d^2*x^2)*Cos[(c + d*x)/2] - (-8 + 4*d*x + d^2*x^2)*Sin[
(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d^3*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a \sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sqrt{a \sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a \sin(c+dx)+a}\int x^2 \sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a \sin(c+dx)+a}\int x^2 \sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a \sin(c+dx)+a}\left(\frac{4\int x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a \sin(c+dx)+a}\left(\frac{4\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{4\left(\frac{2\int -\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}+\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 25

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{4\left(\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{2\int \sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{4\left(\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{2\int \sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$\downarrow$ 3118

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{4\left(\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d^2}+\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

input
```
Int[x^2*Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
Csc[c/2 + Pi/4 + (d*x)/2]*((-2*x^2*Cos[c/2 + Pi/4 + (d*x)/2])/d + (4*((4*C
os[c/2 + Pi/4 + (d*x)/2])/d^2 + (2*x*Sin[c/2 + Pi/4 + (d*x)/2])/d))/d)*Sqr
t[a + a*Sin[c + d*x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a(-2-2\sin(dx+c))}\,\left(-id^{2}x^{2}+d^{2}x^{2}\mathrm{e}^{i(dx+c)}+4idx\,\mathrm{e}^{i(dx+c)}-4dx+8i-8\,\mathrm{e}^{i(dx+c)}\right)\left(\mathrm{e}^{i(dx+c)}+i\right)}{\left(\mathrm{e}^{2i(dx+c)}-1+2i\mathrm{e}^{i(dx+c)}\right)d^{3}}$	119

input
```
int(x^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))-1+2*I*exp(I*(d*x
+c)))*(-I*d^2*x^2+d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-4*d*x+8*I-
8*exp(I*(d*x+c)))*(exp(I*(d*x+c))+I)/d^3
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + a \sin(c + dx)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2 \sqrt{a + a \sin(c + dx)} \, dx = \int x^2 \sqrt{a (\sin(c + dx) + 1)} \, dx$$

input
```
integrate(x**2*(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*(sin(c + d*x) + 1)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx = \int \sqrt{a\sin(dx+c)+a}x^2\,dx$$

input
```
integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a)*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx$$
$$=2\sqrt{2}\sqrt{a}\left(\frac{4\,x\cos\left(\frac{1}{4}\,\pi-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d^2}-\frac{\left(d^2x^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right.}{}\right.$$

input
```
integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*(4*x*cos(1/4*pi - 1/2*d*x - 1/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d^2 - (d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 8*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c)))*sin(1/4*pi - 1/2*d*x - 1/2*c)/d^3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx$$
$$=\frac{2\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\left(8\,\cos\left(c+d\,x\right)+4\,d\,x-d^2\,x^2\,\cos\left(c+d\,x\right)+4\,d\,x\,\sin\left(c+d\,x\right)\right)}{d^3\,\left(\sin\left(c+d\,x\right)+1\right)}$$

input
```
int(x^2*(a + a*sin(c + d*x))^(1/2),x)
```

output
```
(2*(a*(sin(c + d*x) + 1))^(1/2)*(8*cos(c + d*x) + 4*d*x - d^2*x^2*cos(c +
d*x) + 4*d*x*sin(c + d*x)))/(d^3*(sin(c + d*x) + 1))
```

Reduce [F]

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx=\sqrt{a}\left(\int\sqrt{\sin(dx+c)+1}\,x^2dx\right)$$

input
```
int(x^2*(a+a*sin(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sin(c + d*x) + 1)*x**2,x)
```

3.124 $\int x\sqrt{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x\sqrt{a+a\sin(c+dx)}\,dx=\frac{4\sqrt{a+a\sin(c+dx)}}{d^2}-\frac{2x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}$$

output
```
4*(a+a*sin(d*x+c))^(1/2)/d^2-2*x*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c)
)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int x\sqrt{a+a\sin(c+dx)}\,dx=-\frac{2\left((-2+dx)\cos\left(\frac{1}{2}(c+dx)\right)-(2+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[x*Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(-2*((-2 + d*x)*Cos[(c + d*x)/2] - (2 + d*x)*Sin[(c + d*x)/2])*Sqrt[a*(1 +
 Sin[c + d*x])])/(d^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3800, 3042, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{2\int\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{2\int\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

$$\Big\downarrow \text{3118}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{4\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d^2}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}\right)$$

input
```
Int[x*Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
Csc[c/2 + Pi/4 + (d*x)/2]*((-2*x*Cos[c/2 + Pi/4 + (d*x)/2])/d + (4*Sin[c/2
 + Pi/4 + (d*x)/2])/d^2)*Sqrt[a + a*Sin[c + d*x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(-2-2\sin\left(dx+c\right)\right)}\,\left(-idx+dx\,{\mathrm e}^{i\left(dx+c\right)}+2i{\mathrm e}^{i\left(dx+c\right)}-2\right)\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{\left({\mathrm e}^{2i\left(dx+c\right)}-1+2i{\mathrm e}^{i\left(dx+c\right)}\right)d^{2}}$	93

input
```
int(x*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))-1+2*I*exp(I*(d*x
+c)))*(-I*d*x+d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-2)*(exp(I*(d*x+c))+I)/
d^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\sin(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+a\sin(c+dx)}\,dx = \int x\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x*(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(x*sqrt(a*(sin(c + d*x) + 1)), x)
```

Maxima [F]

$$\int x\sqrt{a+a\sin(c+dx)}\,dx=\int\sqrt{a\sin(dx+c)+a}x\,dx$$

input
```
integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a)*x, x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int x\sqrt{a+a\sin(c+dx)}\,dx = -2\sqrt{2}\left(\frac{x\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)}{d}-\frac{2\cos\left(\frac{1}{4}\pi-\frac{1}{2}dx-\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi\right.\right.}{d^2}\right.$$

input
```
integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*(x*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi - 1/2*d*x - 1
/2*c)/d - 2*cos(1/4*pi - 1/2*d*x - 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))/d^2)*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int x\sqrt{a+a\sin(c+dx)}\,dx = \frac{2\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\sin\left(c+d\,x\right)-d\,x\,\cos\left(c+d\,x\right)+2\right)}{d^2\,\left(\sin\left(c+d\,x\right)+1\right)}$$

input
```
int(x*(a + a*sin(c + d*x))^(1/2),x)
```

output
```
(2*(a*(sin(c + d*x) + 1))^(1/2)*(2*sin(c + d*x) - d*x*cos(c + d*x) + 2))/(
d^2*(sin(c + d*x) + 1))
```

Reduce [F]

$$\int x\sqrt{a+a\sin(c+dx)}\,dx=\sqrt{a}\left(\int\sqrt{\sin\left(dx+c\right)+1}\,xdx\right)$$

input
```
int(x*(a+a*sin(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sin(c + d*x) + 1)*x,x)
```

3.125 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x} \, dx = \text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c+\pi)\right)\sqrt{a+a\sin(c+dx)}+\cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\text{Si}\left(\frac{dx}{2}\right)$$

output
```
Ci(1/2*d*x)*csc(1/2*c+1/4*Pi+1/2*d*x)*sin(1/2*c+1/4*Pi)*(a+a*sin(d*x+c))^(
1/2)+cos(1/2*c+1/4*Pi)*csc(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d*x+c))^(1/2)*Si
(1/2*d*x)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx = \frac{\sqrt{a(1+\sin(c+dx))}\left(\text{CosIntegral}\left(\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)+\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\text{Si}\left(\frac{dx}{2}\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}$$

input
```
Integrate[Sqrt[a + a*Sin[c + d*x]]/x,x]
```

output
```
(Sqrt[a*(1 + Sin[c + d*x])]*(CosIntegral[(d*x)/2]*(Cos[c/2] + Sin[c/2]) +
(Cos[c/2] - Sin[c/2])*SinIntegral[(d*x)/2]))/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3800, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(c+dx)+a}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(c+dx)+a}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\cos\left(\frac{dx}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{3780}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

$$\Big\downarrow \text{3783}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

input
```
Int[Sqrt[a + a*Sin[c + d*x]]/x,x]
```

output
```
Csc[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]]*(CosIntegral[(d*x)/2]*S
in[(2*c + Pi)/4] + Cos[(2*c + Pi)/4]*SinIntegral[(d*x)/2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + a\sin{(dx + c)}}}{x}dx$$

input
```
int((a+a*sin(d*x+c))^(1/2)/x,x)
```

output
```
int((a+a*sin(d*x+c))^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x}\, dx = \int \frac{\sqrt{a\,(\sin{(c + dx)} + 1)}}{x}\, dx$$

input
```
integrate((a+a*sin(d*x+c))**(1/2)/x,x)
```

output
```
Integral(sqrt(a*(sin(c + d*x) + 1))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x}\, dx = \int \frac{\sqrt{a\sin{(dx + c)} + a}}{x}\, dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a)/x, x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*tan(1/4*c)^2 - imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + real_part(cos_integral(1/2*d*x))*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + real_part(cos_integral(-1/2*d
*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*c)^2 + 2*imag_part(cos_i
ntegral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 2*imag_
part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c
) - 2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/4*c) - 2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*tan(1/4*c) + 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integra
l(1/2*d*x)*tan(1/4*c) - imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c)) + imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) - real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) - real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c)) - 2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x))
*sqrt(a)/(sqrt(2)*tan(1/4*c)^2 + sqrt(2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx=\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{x}\,dx$$

input
```
int((a + a*sin(c + d*x))^(1/2)/x,x)
```

output
```
int((a + a*sin(c + d*x))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sin\left(dx+c\right)+1}}{x}dx\right)$$

input
```
int((a+a*sin(d*x+c))^(1/2)/x,x)
```

output
```
sqrt(a)*int(sqrt(sin(c + d*x) + 1)/x,x)
```

3.126 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = -\frac{\sqrt{a+a\sin(c+dx)}}{x} - \frac{1}{2}d\,\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c-\pi)\right)\sqrt{a+a\sin(c+dx)} - \frac{1}{2}d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c+\pi)\right)\sqrt{a+a\sin(c+dx)}\,\text{Si}\left(\frac{dx}{2}\right)$$

output
```
-(a+a*sin(d*x+c))^(1/2)/x+1/2*d*Ci(1/2*d*x)*csc(1/2*c+1/4*Pi+1/2*d*x)*cos(
1/2*c+1/4*Pi)*(a+a*sin(d*x+c))^(1/2)-1/2*d*csc(1/2*c+1/4*Pi+1/2*d*x)*sin(1
/2*c+1/4*Pi)*(a+a*sin(d*x+c))^(1/2)*Si(1/2*d*x)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = \frac{\sqrt{a(1+\sin(c+dx))}\left(dx\,\text{CosIntegral}\left(\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)-2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{2x\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[Sqrt[a + a*Sin[c + d*x]]/x^2,x]
```

output
```
(Sqrt[a*(1 + Sin[c + d*x])]*(d*x*CosIntegral[(d*x)/2]*(Cos[c/2] - Sin[c/2]
) - 2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - d*x*(Cos[c/2] + Sin[c/2])*Si
nIntegral[(d*x)/2]))/(2*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(c+dx)+a}}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(c+dx)+a}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{2}d\int \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{2}d\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)}{x}dx-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}\right)$$

$\Big\downarrow$ 3784

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{2}d\left(-\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}dx-\sin\left(\frac{1}{4}(2c-\pi)\right)\int\frac{\cos\left(\frac{dx}{2}\right)}{x}dx\right.\right.$$

$\Big\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{2}d\left(-\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}dx-\sin\left(\frac{1}{4}(2c-\pi)\right)\int\frac{\sin\left(\frac{dx}{2}+\right.}{x}\right.\right.$$

$\Big\downarrow$ 3780

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{2}d\left(-\sin\left(\frac{1}{4}(2c-\pi)\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx-\sin\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\right.\right.$$

$\Big\downarrow$ 3783

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{2}d\left(\sin\left(\frac{1}{4}(2c-\pi)\right)\left(-\text{CosIntegral}\left(\frac{dx}{2}\right)\right)-\sin\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\right.\right.\right.$$

input
```
Int[Sqrt[a + a*Sin[c + d*x]]/x^2,x]
```

output
```
Csc[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]]*(-(Sin[c/2 + Pi/4 + (d*
x)/2]/x) + (d*(-(CosIntegral[(d*x)/2]*Sin[(2*c - Pi)/4]) - Sin[(2*c + Pi)/
4]*SinIntegral[(d*x)/2]))/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + a\sin(dx + c)}}{x^2} dx$$

input
```
int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

output
```
int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = \int \frac{\sqrt{a\,(\sin{(c+dx)}+1)}}{x^2}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*(sin(c + d*x) + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = \int \frac{\sqrt{a\sin{(dx+c)}+a}}{x^2}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a)/x^2, x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 1140, normalized size of antiderivative = 8.77

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x^2}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*
d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*
x*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/4*d*x)^2*tan(1/4*c)^2 - d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d*x*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c)^2
 - 2*d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/4*d*x)^2*tan(1/4*c) + 2*d*x*imag_part(cos_integral(-1/2*d*x))*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d*x*real_p
art(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
)^2*tan(1/4*c) - 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 4*d*x*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c) - d*x*imag_
part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*
x)^2 + d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*tan(1/4*d*x)^2 + d*x*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d*x*real_part(cos_integral(-1/2*d*
x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d*x*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2 + d*x*ima...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx=\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{x^2}\,dx$$

input
```
int((a + a*sin(c + d*x))^(1/2)/x^2,x)
```

output
```
int((a + a*sin(c + d*x))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\sin\left(dx+c\right)+1}+\left(\int \frac{\sqrt{\sin(dx+c)+1}\,\cos(dx+c)}{\sin(dx+c)x+x}dx\right)dx\right)}{2x}$$

input
```
int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

output
```
(sqrt(a)*( - 2*sqrt(sin(c + d*x) + 1) + int((sqrt(sin(c + d*x) + 1)*cos(c
+ d*x))/(sin(c + d*x)*x + x),x)*d*x))/(2*x)
```

3.127 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx = -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2} - \frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x} - \frac{1}{8}d^2\,\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c+\pi)\right)\sqrt{a+a\sin(c+dx)} - \frac{1}{8}d^2\cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\,\text{Si}\left(\frac{dx}{2}\right)$$

output
```
-1/2*(a+a*sin(d*x+c))^(1/2)/x^2-1/4*d*cot(1/2*c+1/4*Pi+1/2*d*x)*(a+a*sin(d
*x+c))^(1/2)/x-1/8*d^2*Ci(1/2*d*x)*csc(1/2*c+1/4*Pi+1/2*d*x)*sin(1/2*c+1/4
*Pi)*(a+a*sin(d*x+c))^(1/2)-1/8*d^2*cos(1/2*c+1/4*Pi)*csc(1/2*c+1/4*Pi+1/2
*d*x)*(a+a*sin(d*x+c))^(1/2)*Si(1/2*d*x)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx = -\frac{\sqrt{a(1+\sin(c+dx))}\left(4\cos\left(\frac{1}{2}(c+dx)\right)+2dx\cos\left(\frac{1}{2}(c+dx)\right)+d^2x^2\,\text{CosIntegral}\left(\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}\right)+\sin\right.\right.}{8x^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(\right.\right.}$$

input
```
Integrate[Sqrt[a + a*Sin[c + d*x]]/x^3,x]
```

output
```
-1/8*(Sqrt[a*(1 + Sin[c + d*x])]*(4*Cos[(c + d*x)/2] + 2*d*x*Cos[(c + d*x)
/2] + d^2*x^2*CosIntegral[(d*x)/2]*(Cos[c/2] + Sin[c/2]) + 4*Sin[(c + d*x)
/2] - 2*d*x*Sin[(c + d*x)/2] + d^2*x^2*(Cos[c/2] - Sin[c/2])*SinIntegral[(
d*x)/2]))/(x^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(c+dx)+a}}{x^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(c+dx)+a}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\int\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x^3}dx$$

$\Big\downarrow$ 3778

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\int\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x^2}dx-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{2x^2}\right)$$

$\Big\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\int\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)}{x^2}dx-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{2x^2}\right)$$

$\Big\downarrow$ 3778

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(\frac{1}{2}d\int-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}\right)-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\right.}{2x^2}$$

$\Big\downarrow$ 25

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(-\frac{1}{2}d\int\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}\right)-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\right.}{2x^2}$$

$\Big\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(-\frac{1}{2}d\int\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{x}\right)-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\right.}{2x^2}$$

$\Big\downarrow$ 3784

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(-\frac{1}{2}d\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\cos\left(\frac{dx}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\right.}{x}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(-\frac{1}{2}d\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2c+\pi)\right)\int\right.\right.\right.$$

$\Big\downarrow$ 3780

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(-\frac{1}{2}d\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\right.\right.\right.\right.$$

$$\Big\downarrow \text{3783}$$

$$\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}\left(\frac{1}{4}d\left(-\frac{1}{2}d\left(\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\right.\right.\right.\right.$$

input
```
Int[Sqrt[a + a*Sin[c + d*x]]/x^3,x]
```

output
```
Csc[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]]*(-1/2*Sin[c/2 + Pi/4 +
(d*x)/2]/x^2 + (d*(-(Cos[c/2 + Pi/4 + (d*x)/2]/x) - (d*(CosIntegral[(d*x)/
2]*Sin[(2*c + Pi)/4] + Cos[(2*c + Pi)/4]*SinIntegral[(d*x)/2]))/2))/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + a\sin(dx + c)}}{x^3} dx$$

input
```
int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

output
```
int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x^3}\, dx = \int \frac{\sqrt{a\left(\sin\left(c + dx\right) + 1\right)}}{x^3}\, dx$$

input
```
integrate((a+a*sin(d*x+c))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*(sin(c + d*x) + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x^3}\, dx = \int \frac{\sqrt{a\sin\left(dx + c\right) + a}}{x^3}\, dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a)/x^3, x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 1487, normalized size of antiderivative = 8.55

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x^3}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d^2*x^2*imag_part(cos_integ
ral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*
c)^2 + d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(-1/
2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 +
2*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/
4*d*x)^2*tan(1/4*c)^2 + 2*d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d^2*x^2*imag_pa
rt(cos_integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
)^2*tan(1/4*c) - 2*d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 2*d^2*x^2*real_part(cos_
integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan
(1/4*c) + 4*d^2*x^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin_integral(1/2*d
*x)*tan(1/4*d*x)^2*tan(1/4*c) - d^2*x^2*imag_part(cos_integral(1/2*d*x))*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d^2*x^2*imag_part(cos_
integral(-1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - d
^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*tan(1/4*d*x)^2 - d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d^2*x^2*sgn(cos(-1/4*pi + 1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx=\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{x^3}\,dx$$

input
```
int((a + a*sin(c + d*x))^(1/2)/x^3,x)
```

output
```
int((a + a*sin(c + d*x))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a + a\sin(c + dx)}}{x^3}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{\sin\left(dx + c\right) + 1} + \left(\int \frac{\sqrt{\sin(dx+c)+1}\,\cos(dx+c)}{\sin(dx+c)x^2+x^2}dx\right) d\,x^2\right)}{4x^2}$$

input
```
int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

output
```
(sqrt(a)*( - 2*sqrt(sin(c + d*x) + 1) + int((sqrt(sin(c + d*x) + 1)*cos(c
+ d*x))/(sin(c + d*x)*x**2 + x**2),x)*d*x**2))/(4*x**2)
```

3.128 $\int x^3(a+a\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 337

$$\begin{aligned}\int x^3(a+a\sin(e+fx))^{3/2}\,dx = & -\frac{1280a\sqrt{a+a\sin(e+fx)}}{9f^4} \\ & +\frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2}+\frac{640ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3} \\ & -\frac{8ax^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f} \\ & +\frac{32ax\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3} \\ & -\frac{4ax^3\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f} \\ & -\frac{64a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{27f^4} \\ & +\frac{8ax^2\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f^2}\end{aligned}$$

output
```
-1280/9*a*(a+a*sin(f*x+e))^(1/2)/f^4+16*a*x^2*(a+a*sin(f*x+e))^(1/2)/f^2+6
40/9*a*x*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f^3-8/3*a*x^3*co
t(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f+32/9*a*x*cos(1/2*e+1/4*Pi
+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f^3-4/3*a*x^3*c
os(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/
f-64/27*a*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+e))^(1/2)/f^4+8/3*a*x^2
*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+e))^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.69

$$\int x^3(a+a\sin(e+fx))^{3/2}\,dx = \frac{2a\left(-\frac{2\left((968-480fx-117f^2x^2+18f^3x^3)\cos\left(\frac{e}{2}\right)+(968+480fx-117f^2x^2-18f^3x^3)\sin\left(\frac{e}{2}\right)\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)}-\cos(fx)\left(3fx(-8+\right.\right.}{}$$

input
```
Integrate[x^3*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(2*a*((-2*((968 - 480*f*x - 117*f^2*x^2 + 18*f^3*x^3)*Cos[e/2] + (968 + 48
0*f*x - 117*f^2*x^2 - 18*f^3*x^3)*Sin[e/2]))/(Cos[e/2] + Sin[e/2]) - Cos[f
*x]*(3*f*x*(-8 + 3*f^2*x^2)*Cos[e] + 2*(8 - 9*f^2*x^2)*Sin[e]) + (2*(-8 +
9*f^2*x^2)*Cos[e] + 3*f*x*(-8 + 3*f^2*x^2)*Sin[e])*Sin[f*x] + (24*f*x*(-80
 + 3*f^2*x^2)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])))*Sqrt[a*(1 + Sin[e + f*x])])/(27*f^4)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3800, 3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3118, 3791, 3042, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3(a\sin(e+fx)+a)^{3/2}dx$$

$$\Big\downarrow \text{ 3800}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int x^3\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int x^3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{ 3792}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{3f^2}+\frac{2}{3}\int x^3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4x^{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\int x^3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4x^{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 3777}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\int x^2\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\int x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \text{ 3777}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{4\int -x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sin}{}\right.}{f}\right.\right.$$

$\downarrow$ 25

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4\int x\sin\left(\frac{e}{2}+\right.}{f}\right.}{f}\right.\right.$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4\int x\sin\left(\frac{e}{2}+\right.}{f}\right.}{f}\right.\right.$$

$\downarrow$ 3777

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4\left(\frac{2\int\cos\left(\frac{e}{2}\right.}{}\right.}{}\right.}{f}\right.\right.$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4\left(\frac{2\int\sin\left(\frac{e}{2}\right.}{}\right.}{}\right.}{f}\right.\right.$$

$\downarrow$ 3118

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{4x^2\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{2x^2}{}\right.}{}\right.\right.$$

$\downarrow$ 3791

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\left(\frac{2}{3}\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}-\frac{2x\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}\right.}{3f^2}\right.$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\left(\frac{2}{3}\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}-\frac{2x\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f}\right.}{3f^2}\right.$$

$\downarrow$ 3777

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\left(\frac{2}{3}\left(\frac{2\int\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2x\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}-\right.}{3f^2}\right.$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\left(\frac{2}{3}\left(\frac{2\int\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2x\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}-\cdots\right.}{3f^2}\right.$$

$\big\downarrow$ 3118

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{4x^2\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3f^2}+\frac{2}{3}\left(\frac{6\left(\frac{2x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{4\left(\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}\cdots\right.}{\cdots}\right.}{f}\right.\right.$$

input
```
Int[x^3*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
2*a*Csc[e/2 + Pi/4 + (f*x)/2]*((-2*x^3*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 +
 Pi/4 + (f*x)/2]^2)/(3*f) + (4*x^2*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(3*f^2) -
(8*((-2*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]^2)/(3*f) + (
4*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(9*f^2) + (2*((-2*x*Cos[e/2 + Pi/4 + (f*x)/
2])/f + (4*Sin[e/2 + Pi/4 + (f*x)/2])/f^2))/3))/(3*f^2) + (2*((-2*x^3*Cos[
e/2 + Pi/4 + (f*x)/2])/f + (6*((2*x^2*Sin[e/2 + Pi/4 + (f*x)/2])/f - (4*((
-2*x*Cos[e/2 + Pi/4 + (f*x)/2])/f + (4*Sin[e/2 + Pi/4 + (f*x)/2])/f^2))/f)
)/f))/3)*Sqrt[a + a*Sin[e + f*x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^3(a + a\sin(fx + e))^{\frac{3}{2}}\,dx$$

input
```
int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3(a + a\sin(e + fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3(a + a\sin(e + fx))^{3/2}\,dx = \int x^3(a(\sin(e + fx) + 1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x**3*(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int x^3(a + a\sin(e + fx))^{3/2}\,dx = \int (a\sin(fx + e) + a)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)*x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(259) = 518$.

Time = 0.47 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.93

$$\int x^3(a + a\sin(e + fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/216*sqrt(2)*sqrt(a)*(972*(pi^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2
*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x
 - 2*e)^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*pi*a*e*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 4*(pi - 2*f*x - 2*e)*a*e*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 4*a*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)/f^3 + 4*(9*pi^2*a
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 9*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) - 36*pi*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*(pi
- 2*f*x - 2*e)*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*a*e^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*co
s(-3/4*pi + 3/2*f*x + 3/2*e)/f^3 + 81*(pi^3*a*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - 3*pi^2*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 3*pi*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi -
2*f*x - 2*e)^3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi^2*a*e*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 12*pi*(pi - 2*f*x - 2*e)*a*e*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 6*(pi - 2*f*x - 2*e)^2*a*e*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 12*pi*a*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 12*(pi - 2
*f*x - 2*e)*a*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*a*e^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - 96*pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))...
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+a\sin(e+fx))^{3/2}\,dx=\int x^3\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

input
```
int(x^3*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(x^3*(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int x^3(a+a\sin(e+fx))^{3/2}\,dx=\sqrt{a}\,a\left(\int\sqrt{\sin(fx+e)+1}\,x^3dx+\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,x^3dx\right)$$

input
```
int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)*x**3,x) + int(sqrt(sin(e + f*x) + 1)
*sin(e + f*x)*x**3,x))
```

3.129 $\int x^2(a + a\sin(e + fx))^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 271

$$\begin{aligned}\int x^2(a+a\sin(e+fx))^{3/2}\,dx &= \frac{32ax\sqrt{a+a\sin(e+fx)}}{3f^2}\\ &+\frac{224a\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}\\ &-\frac{8ax^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}\\ &-\frac{32a\cos^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{27f^3}\\ &-\frac{4ax^2\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}\\ &+\frac{16ax\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^2}\end{aligned}$$

output
```
32/3*a*x*(a+a*sin(f*x+e))^(1/2)/f^2+224/9*a*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a
*sin(f*x+e))^(1/2)/f^3-8/3*a*x^2*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e)
)^(1/2)/f-32/27*a*cos(1/2*e+1/4*Pi+1/2*f*x)^2*cot(1/2*e+1/4*Pi+1/2*f*x)*(a
+a*sin(f*x+e))^(1/2)/f^3-4/3*a*x^2*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4
*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/f+16/9*a*x*sin(1/2*e+1/4*Pi+1/2*f*x)^2
*(a+a*sin(f*x+e))^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.70

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx = \frac{2a\left(-\frac{4\left(\left(-80-39fx+9f^2x^2\right)\cos\left(\frac{e}{2}\right)+\left(80-39fx-9f^2x^2\right)\sin\left(\frac{e}{2}\right)\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)}-\cos(fx)\left(\left(-8+9f^2x^2\right)\cos(e)-12fx\,\mathrm{s}\right.\right.}{}$$

input
```
Integrate[x^2*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(2*a*((-4*((-80 - 39*f*x + 9*f^2*x^2)*Cos[e/2] + (80 - 39*f*x - 9*f^2*x^2)
*Sin[e/2]))/(Cos[e/2] + Sin[e/2]) - Cos[f*x]*((-8 + 9*f^2*x^2)*Cos[e] - 12
*f*x*Sin[e]) + (12*f*x*Cos[e] + (-8 + 9*f^2*x^2)*Sin[e])*Sin[f*x] + (8*(-8
0 + 9*f^2*x^2)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])))*Sqrt[a*(1 + Sin[e + f*x])])/(27*f^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2(a\sin(e+fx)+a)^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int x^2\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx$$

$\big\downarrow$ 3792

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{9f^2}+\frac{2}{3}\int x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{8x\,\mathrm{si}}{}\right.$$

$\big\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{8\int\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{9f^2}+\frac{2}{3}\int x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{8x\,\mathrm{si}}{}\right.$$

$\big\downarrow$ 3113

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{16\int\left(1-\cos^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)d\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^3}+\frac{2}{3}\int x^2\sin\left(\frac{e}{2}\right.\right.$$

$\big\downarrow$ 2009

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{16\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)-\frac{1}{3}\cos^3\left(\frac{e}{2}\right.\right.}{9f^3}\right.$$

$\big\downarrow$ 3777

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{4\int x\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2x^2\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{16\left(\cos\right.}{}\right.$$

$\big\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{4\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2x^2\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{16\left(\cos\right.}{}\right.$$

$\big\downarrow$ 3777

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{4\left(\frac{2\int -\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}+\frac{2x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{f}-\frac{2x^2\cos\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.\right.$$

$\downarrow$ 25

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{4\left(\frac{2x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{2\int\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}-\frac{2x^2\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right.}{f}\right.\right.$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{4\left(\frac{2x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}-\frac{2\int\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}-\frac{2x^2\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right.}{f}\right.\right.$$

$\downarrow$ 3118

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{16\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)-\frac{1}{3}\cos^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{9f^3}+\frac{2}{3}\left(\frac{4\left(\frac{4\cos\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f^2}\right.}{}\right.\right.$$

input
```
Int[x^2*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
2*a*Csc[e/2 + Pi/4 + (f*x)/2]*((16*(Cos[e/2 + Pi/4 + (f*x)/2] - Cos[e/2 +
Pi/4 + (f*x)/2]^3/3))/(9*f^3) - (2*x^2*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 +
 Pi/4 + (f*x)/2]^2)/(3*f) + (8*x*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(9*f^2) + (2
*((-2*x^2*Cos[e/2 + Pi/4 + (f*x)/2])/f + (4*((4*Cos[e/2 + Pi/4 + (f*x)/2])
/f^2 + (2*x*Sin[e/2 + Pi/4 + (f*x)/2])/f))/f))/3)*Sqrt[a + a*Sin[e + f*x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^2(a+a\sin(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx = \int x^2(a(\sin(e+fx)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x**2*(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int x^2(a + a\sin(e + fx))^{3/2}\,dx = \int (a\sin(fx + e) + a)^{\frac{3}{2}} x^2\,dx$$

input
```
integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)*x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(205) = 410$.

Time = 0.40 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.84

$$\int x^2(a + a\sin(e + fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/108*sqrt(2)*sqrt(a)*(648*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi
 - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a*e*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)/f^2 + 24*(pi*a*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) - 2*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi
 + 3/2*f*x + 3/2*e)/f^2 + 81*(pi^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
 2*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f
*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*pi*a*e*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + 4*(pi - 2*f*x - 2*e)*a*e*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 4*a*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)/f^2 + (9*pi^2*a
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 9*(pi - 2*f*x - 2*e)^2*a*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) - 36*pi*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*(pi
- 2*f*x - 2*e)*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*a*e^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) - 32*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*si
n(-3/4*pi + 3/2*f*x + 3/2*e)/f^2)/f
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx=\int x^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

input
```
int(x^2*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(x^2*(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx=\sqrt{a}\,a\left(\int\sqrt{\sin\left(fx+e\right)+1}\,x^2dx+\int\sqrt{\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)x^2dx\right)$$

input
```
int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)*x**2,x) + int(sqrt(sin(e + f*x) + 1)
*sin(e + f*x)*x**2,x))
```

3.130 $\int x(a+a\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int x(a+a\sin(e+fx))^{3/2}\,dx=\frac{16a\sqrt{a+a\sin(e+fx)}}{3f^2}-\frac{8ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{4ax\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{8a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^2}$$

output
```
16/3*a*(a+a*sin(f*x+e))^(1/2)/f^2-8/3*a*x*cot(1/2*e+1/4*Pi+1/2*f*x)*(a+a*s
in(f*x+e))^(1/2)/f-4/3*a*x*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi+1/2*
f*x)*(a+a*sin(f*x+e))^(1/2)/f+8/9*a*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f
*x+e))^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int x(a+a\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{\left(27(-2+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2+3fx)\cos\left(\frac{3}{2}(e+fx)\right)+2(-4(7+3fx)+(-2+3fx)\cos(e+fx)\cdots\right.}{9f^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

input
```
Integrate[x*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/9*((27*(-2 + f*x)*Cos[(e + f*x)/2] + (2 + 3*f*x)*Cos[(3*(e + f*x))/2] +
 2*(-4*(7 + 3*f*x) + (-2 + 3*f*x)*Cos[e + f*x])*Sin[(e + f*x)/2])*(a*(1 +
Sin[e + f*x]))^(3/2))/(f^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3791, 3042, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x(a\sin(e+fx)+a)^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int x\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{3791}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}-\frac{2x\sin^2\left(\frac{e}{2}+\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\int x\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx+\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}-\frac{2x\sin^2\left(\frac{e}{2}+\right.}{}\right.$$

$$\Big\downarrow \text{3777}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{2\int\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f}-\frac{2x\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{4\sin^3\left(\frac{e}{2}\right.}{9}\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{2}{3}\left(\frac{2\int\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{3\pi}{4}\right)dx}{f}-\frac{2x\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)+\frac{4\sin^3\left(\frac{e}{2}\right.}{9}\right.$$

$$\Big\downarrow \text{3118}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{4\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{9f^2}+\frac{2}{3}\left(\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{2x\cos\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.\right.$$

input
```
Int[x*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
2*a*Csc[e/2 + Pi/4 + (f*x)/2]*((-2*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + P
i/4 + (f*x)/2]^2)/(3*f) + (4*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(9*f^2) + (2*((-
2*x*Cos[e/2 + Pi/4 + (f*x)/2])/f + (4*Sin[e/2 + Pi/4 + (f*x)/2])/f^2))/3)*
Sqrt[a + a*Sin[e + f*x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x(a + a\sin(fx + e))^{\frac{3}{2}} dx$$

input
```
int(x*(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(x*(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a + a\sin(e + fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x(a + a\sin(e + fx))^{3/2}\,dx = \int x(a(\sin(e + fx) + 1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x*(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int x(a + a\sin(e + fx))^{3/2}\,dx = \int (a\sin(fx + e) + a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)*x, x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.37

$$\int x(a+a\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{2}\left(\frac{108\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}+\frac{4\,a\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}+\frac{2}{}\right.}{}$$

input
```
integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/18*sqrt(2)*(108*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))/f + 4*a*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))/f + 27*(pi*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f
*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a*e*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)/f + 3*(pi*a*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*a*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) - 2*a*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*
x + 3/2*e)/f)*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int x(a+a\sin(e+fx))^{3/2}\,dx = \int x\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

input
```
int(x*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(x*(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int x(a+a\sin(e+fx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,xdx + \int \sqrt{\sin(fx+e)+1}\,xdx\right)$$

input
```
int(x*(a+a*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)*x,x) + int(sqrt(sin(e +
 f*x) + 1)*x,x))
```

3.131 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 221

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx &= \frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\\&+\frac{3}{2}a\,\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)}\\&+\frac{3}{2}a\cos\left(\frac{1}{4}(2e+\pi)\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\text{Si}\left(\frac{fx}{2}\right)\\&-\frac{1}{2}a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{3}{4}(2e-\pi)\right)\sqrt{a+a\sin(e+fx)}\text{Si}\left(\frac{3fx}{2}\right)\end{aligned}$$

output
```
-1/2*a*cos(3/2*e+1/4*Pi)*Ci(3/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*(a+a*sin(f*
x+e))^(1/2)+3/2*a*Ci(1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi)*
(a+a*sin(f*x+e))^(1/2)+3/2*a*cos(1/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*(
a+a*sin(f*x+e))^(1/2)*Si(1/2*f*x)+1/2*a*csc(1/2*e+1/4*Pi+1/2*f*x)*sin(3/2*
e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)*Si(3/2*f*x)
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx = \frac{(a(1+\sin(e+fx)))^{3/2}\left(3\,\text{CosIntegral}\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)+\text{CosIntegr}\right.}{2\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)/x,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(3*CosIntegral[(f*x)/2]*(Cos[e/2] + Sin[e/2]
) + CosIntegral[(3*f*x)/2]*(-Cos[(3*e)/2] + Sin[(3*e)/2]) + (Cos[e/2] - Si
n[e/2])*(3*SinIntegral[(f*x)/2] + (1 + 2*Sin[e])*SinIntegral[(3*f*x)/2])))
/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int \frac{\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x}dx$$

$$\Big\downarrow \text{3793}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int\left(\frac{3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{4x}+\frac{\sin\left(\frac{3e}{2}+\frac{3fx}{2}-\frac{\pi}{4}\right)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{3}{4}\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)+\frac{1}{4}\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosInte}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)/x,x]
```

output
```
2*a*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*((Cos[(3*(2*e - Pi)
)/4]*CosIntegral[(3*f*x)/2])/4 + (3*CosIntegral[(f*x)/2]*Sin[(2*e + Pi)/4]
)/4 + (3*Cos[(2*e + Pi)/4]*SinIntegral[(f*x)/2])/4 - (Sin[(3*(2*e - Pi))/4
]*SinIntegral[(3*f*x)/2])/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a + a\sin(fx + e))^{\frac{3}{2}}}{x} dx$$

input
```
int((a+a*sin(f*x+e))^(3/2)/x,x)
```

output
```
int((a+a*sin(f*x+e))^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^{3/2}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a + a\sin(e + fx))^{3/2}}{x} dx = \int \frac{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}{x} dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)/x,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx=\int \frac{(a\sin(fx+e)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx=\frac{\sqrt{2}\left(af\cos\left(\frac{3}{4}\pi-\frac{3}{2}e\right)\operatorname{Ci}\left(\frac{3}{2}fx\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3af\cos\left(\frac{1}{4}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a*f*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 3*a*f*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(3/4*pi - 3/2*e)*sin_integral(3/2*f*x) + 3*a*f*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sin(1/4*pi - 1/2*e)*sin_integral(1/2*f*x))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx=\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{x}\,dx$$

input
```
int((a + a*sin(e + f*x))^(3/2)/x,x)
```

output
```
int((a + a*sin(e + f*x))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sin(fx+e)+1}}{x}dx + \int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{x}dx\right)$$

input

```
int((a+a*sin(f*x+e))^(3/2)/x,x)
```

output

```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)/x,x) + int((sqrt(sin(e + f*x) + 1)*s
in(e + f*x))/x,x))
```

3.132 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 263

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = -\frac{3}{4}af\,\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e-\pi)\right)\sqrt{a+a\sin(e+fx)}+\frac{3}{4}af\,\text{CosIntegral}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(6e+\pi)\right)\sqrt{a+a\sin(e+fx)}-\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x}-\frac{3}{4}af\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)}\,\text{Si}\left(\frac{fx}{2}\right)+\frac{3}{4}af\cos\left(\frac{1}{4}(6e+\pi)\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\,\text{Si}\left(\frac{3fx}{2}\right)$$

output

```
3/4*a*f*Ci(1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*cos(1/2*e+1/4*Pi)*(a+a*sin(f
*x+e))^(1/2)+3/4*a*f*Ci(3/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*sin(3/2*e+1/4*P
i)*(a+a*sin(f*x+e))^(1/2)-2*a*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(a+a*sin(f*x+e))
^(1/2)/x-3/4*a*f*csc(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e+1/4*Pi)*(a+a*sin(f*x+
e))^(1/2)*Si(1/2*f*x)+3/4*a*f*cos(3/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/2*f*x)*
(a+a*sin(f*x+e))^(1/2)*Si(3/2*f*x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.86

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \frac{i\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}\left(2-6ie^{i(e+fx)}-6e^{2i(e+fx)}+2ie^{3i(e+fx)}+\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)/x^2,x]
```

output
```
((I/4)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(2 - (6*I)
*E^(I*(e + f*x)) - 6*E^((2*I)*(e + f*x)) + (2*I)*E^((3*I)*(e + f*x)) + 3*E
^(I*e + ((3*I)/2)*f*x)*f*x*ExpIntegralEi[(-1/2*I)*f*x] + (3*I)*E^((2*I)*e
+ ((3*I)/2)*f*x)*f*x*ExpIntegralEi[(I/2)*f*x] + (3*I)*E^(((3*I)/2)*f*x)*f*
x*ExpIntegralEi[((-3*I)/2)*f*x] + 3*E^(((3*I)/2)*(2*e + f*x))*f*x*ExpInteg
ralEi[((3*I)/2)*f*x]))/(Sqrt[2]*(I + E^(I*(e + f*x)))^3*x)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{x^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{x^2}dx$$
$$\Big\downarrow \text{3800}$$
$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int \frac{\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{x^2}dx$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x^2}dx$$

$\downarrow$ 3794

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{3}{2}f\int\left(\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{4x}+\frac{\sin\left(\frac{3e}{2}+\frac{3fx}{2}+\frac{\pi}{4}\right)}{4x}\right)dx-\frac{\sin^3\left(\frac{e}{2}+\right.}{x}\right.$$

$\downarrow$ 2009

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{3}{2}f\left(-\frac{1}{4}\sin\left(\frac{1}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)+\frac{1}{4}\sin\left(\frac{1}{4}(6e+\pi)\right)\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)/x^2,x]
```

output
```
2*a*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*(-(Sin[e/2 + Pi/4 +
 (f*x)/2]^3/x) + (3*f*(-1/4*(CosIntegral[(f*x)/2]*Sin[(2*e - Pi)/4]) + (Co
sIntegral[(3*f*x)/2]*Sin[(6*e + Pi)/4])/4 - (Sin[(2*e + Pi)/4]*SinIntegral
[(f*x)/2])/4 + (Cos[(6*e + Pi)/4]*SinIntegral[(3*f*x)/2])/4))/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
 - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

output
```
int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)/x**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \int \frac{(a\sin(fx+e)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(197) = 394$.

Time $= 0.39$ (sec) , antiderivative size $= 505$, normalized size of antiderivative $= 1.92$

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(3*pi*a*f^2*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sin(3/4*pi - 3/2*e) - 3*(pi - 2*f*x - 2*e)*a*f^2*cos_integral(3/2*
f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e) - 6*a*e*f^2*c
os_integral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*
e) + 3*pi*a*f^2*cos_integral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(1/4*pi - 1/2*e) - 3*(pi - 2*f*x - 2*e)*a*f^2*cos_integral(1/2*f*x)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e) - 6*a*e*f^2*cos_integ
ral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(1/4*pi - 1/2*e) - 3*p
i*a*f^2*cos(3/4*pi - 3/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integr
al(3/2*f*x) + 3*(pi - 2*f*x - 2*e)*a*f^2*cos(3/4*pi - 3/2*e)*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sin_integral(3/2*f*x) + 6*a*e*f^2*cos(3/4*pi - 3/2*
e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(3/2*f*x) - 3*pi*a*f^2*
cos(1/4*pi - 1/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(1/2*f
*x) + 3*(pi - 2*f*x - 2*e)*a*f^2*cos(1/4*pi - 1/2*e)*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sin_integral(1/2*f*x) + 6*a*e*f^2*cos(1/4*pi - 1/2*e)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(1/2*f*x) - 12*a*f^2*cos(-1/4*p
i + 1/2*f*x + 1/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*a*f^2*cos(-3/
4*pi + 3/2*f*x + 3/2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(f^2*
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx=\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{x^2}\,dx$$

input
```
int((a + a*sin(e + f*x))^(3/2)/x^2,x)
```

output
```
int((a + a*sin(e + f*x))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \frac{\sqrt{a}\,a\left(-2\sqrt{\sin(fx+e)+1}\,\sin(fx+e)+4\sqrt{\sin(fx+e)+1}+6\left(\int \frac{\sqrt{\sin}}{\sin(fx}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

output
```
(sqrt(a)*a*( - 2*sqrt(sin(e + f*x) + 1)*sin(e + f*x) + 4*sqrt(sin(e + f*x)
 + 1) + 6*int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)*x**2 + x**2),x)*x + 3*i
nt((sqrt(sin(e + f*x) + 1)*cos(e + f*x)*sin(e + f*x))/(sin(e + f*x)*x + x)
,x)*f*x + 6*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*x**2 +
 x**2),x)*x))/(2*x)
```

3.133 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 332

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx &= -\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}\right.\\
&\left.+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}-\frac{3}{16}af^2\,\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}\right.\\
&\left.+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)}\\
&-\frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{2x}\\
&-\frac{a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x^2}\\
&-\frac{3}{16}af^2\cos\left(\frac{1}{4}(2e+\pi)\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\,\text{Si}\left(\frac{fx}{2}\right)\\
&+\frac{9}{16}af^2\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{3}{4}(2e-\pi)\right)\sqrt{a+a\sin(e+fx)}\,\text{Si}\left(\frac{3fx}{2}\right)\end{aligned}$$

output
```
9/16*a*f^2*cos(3/2*e+1/4*Pi)*Ci(3/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*(a+a*si
n(f*x+e))^(1/2)-3/16*a*f^2*Ci(1/2*f*x)*csc(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*e
+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)-3/2*a*f*cos(1/2*e+1/4*Pi+1/2*f*x)*sin(1/2*
e+1/4*Pi+1/2*f*x)*(a+a*sin(f*x+e))^(1/2)/x-a*sin(1/2*e+1/4*Pi+1/2*f*x)^2*(
a+a*sin(f*x+e))^(1/2)/x^2-3/16*a*f^2*cos(1/2*e+1/4*Pi)*csc(1/2*e+1/4*Pi+1/
2*f*x)*(a+a*sin(f*x+e))^(1/2)*Si(1/2*f*x)-9/16*a*f^2*csc(1/2*e+1/4*Pi+1/2*
f*x)*sin(3/2*e+1/4*Pi)*(a+a*sin(f*x+e))^(1/2)*Si(3/2*f*x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx =$$

$$-\frac{i\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}\left(-4+12ie^{i(e+fx)}+12e^{2i(e+fx)}-4ie^{3i(e+fx)}+6ifx+6e^{i(e+fx)}fx+6i\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)/x^3,x]
```

output
```
((-1/16*I)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(-4 +
(12*I)*E^(I*(e + f*x)) + 12*E^((2*I)*(e + f*x)) - (4*I)*E^((3*I)*(e + f*x)
) + (6*I)*f*x + 6*E^(I*(e + f*x))*f*x + (6*I)*E^((2*I)*(e + f*x))*f*x + 6*
E^((3*I)*(e + f*x))*f*x + (3*I)*E^(I*e + ((3*I)/2)*f*x)*f^2*x^2*ExpIntegra
lEi[(-1/2*I)*f*x] + 3*E^((2*I)*e + ((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[(I
/2)*f*x] - 9*E^(((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[((-3*I)/2)*f*x] - (9*
I)*E^(((3*I)/2)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[((3*I)/2)*f*x]))/(Sqrt[
2]*(I + E^(I*(e + f*x)))^3*x^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.75, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 3800, 3042, 3795, 3042, 3784, 3042, 3780, 3783, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{x^3}dx$$

$$\Big\downarrow \; 3800$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int \frac{\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{x^3}dx$$

$$\Big\downarrow \; 3042$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x^3}dx$$

$$\Big\downarrow \; 3795$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{9}{8}f^2\int \frac{\sin^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{x}dx+\frac{3}{4}f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{\sin^3}{}\right.$$

$$\Big\downarrow \; 3042$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{3}{4}f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{9}{8}f^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x}dx-\frac{\sin^3\left(\right.}{}\right.$$

$$\Big\downarrow \; 3784$$

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{9}{8}f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x}dx+\frac{3}{4}f^2\left(\sin\left(\frac{1}{4}(2e+\pi)\right)\int\frac{\cos\left(\frac{fx}{2}\right.}{x}\right.\right.$$

$\downarrow$ 3042

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{9}{8}f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x}dx+\frac{3}{4}f^2\left(\sin\left(\frac{1}{4}(2e+\pi)\right)\int\frac{\sin\left(\frac{fx}{2}\right.}{x}\right.\right.$$

$\downarrow$ 3780

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{3}{4}f^2\left(\sin\left(\frac{1}{4}(2e+\pi)\right)\int\frac{\sin\left(\frac{fx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{1}{4}(2e+\pi)\right)\text{Si}\left(\frac{f}{}\right.\right.\right.$$

$\downarrow$ 3783

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{9}{8}f^2\int\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3}{x}dx+\frac{3}{4}f^2\left(\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegra}\right.\right.$$

$\downarrow$ 3793

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(-\frac{9}{8}f^2\int\left(\frac{3\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{4x}+\frac{\sin\left(\frac{3e}{2}+\frac{3fx}{2}-\frac{\pi}{4}\right)}{4x}\right)dx+\frac{3}{4}f^2\left(\text{s}\right.\right.$$

$\downarrow$ 2009

$$2a\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}\left(\frac{3}{4}f^2\left(\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)+\cos\left(\frac{1}{4}(2e+\pi)\right)\text{Si}\left(\frac{f}{2}\right.\right.\right.$$

input

```
Int[(a + a*Sin[e + f*x])^(3/2)/x^3,x]
```

output
```
2*a*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*((-3*f*Cos[e/2 + Pi
/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]^2)/(4*x) - Sin[e/2 + Pi/4 + (f*x)/
2]^3/(2*x^2) + (3*f^2*(CosIntegral[(f*x)/2]*Sin[(2*e + Pi)/4] + Cos[(2*e +
 Pi)/4]*SinIntegral[(f*x)/2]))/4 - (9*f^2*((Cos[(3*(2*e - Pi))/4]*CosInteg
ral[(3*f*x)/2])/4 + (3*CosIntegral[(f*x)/2]*Sin[(2*e + Pi)/4])/4 + (3*Cos[
(2*e + Pi)/4]*SinIntegral[(f*x)/2])/4 - (Sin[(3*(2*e - Pi))/4]*SinIntegral
[(3*f*x)/2])/4))/8)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^{\frac{3}{2}}}{x^3}dx$$

input
```
int((a+a*sin(f*x+e))^(3/2)/x^3,x)
```

output
```
int((a+a*sin(f*x+e))^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx=\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)/x**3,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx=\int \frac{(a\sin(fx+e)+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(248) = 496$.

Time $= 0.48$ (sec) , antiderivative size $= 1256$, normalized size of antiderivative $= 3.78$

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(9*pi^2*a*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*pi*(pi - 2*f*x - 2*e)*a*f^3*cos(3/4*p
i - 3/2*e)*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*(
pi - 2*f*x - 2*e)^2*a*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - 36*pi*a*e*f^3*cos(3/4*pi - 3/2*e)*cos_inte
gral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*(pi - 2*f*x - 2*e)*
a*e*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 36*a*e^2*f^3*cos(3/4*pi - 3/2*e)*cos_integral(3/2*f*x)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*pi^2*a*f^3*cos(1/4*pi - 1/2*e)*cos_inte
gral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi*(pi - 2*f*x - 2*e
)*a*f^3*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 3*(pi - 2*f*x - 2*e)^2*a*f^3*cos(1/4*pi - 1/2*e)*cos_integra
l(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 12*pi*a*e*f^3*cos(1/4*pi
- 1/2*e)*cos_integral(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*(p
i - 2*f*x - 2*e)*a*e*f^3*cos(1/4*pi - 1/2*e)*cos_integral(1/2*f*x)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*a*e^2*f^3*cos(1/4*pi - 1/2*e)*cos_integr
al(1/2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*pi^2*a*f^3*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi - 3/2*e)*sin_integral(3/2*f*x) - 18*pi
*(pi - 2*f*x - 2*e)*a*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi -
 3/2*e)*sin_integral(3/2*f*x) + 9*(pi - 2*f*x - 2*e)^2*a*f^3*sgn(cos(-1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx=\int \frac{(a+a\,\sin(e+f\,x))^{3/2}}{x^3}\,dx$$

input
```
int((a + a*sin(e + f*x))^(3/2)/x^3,x)
```

output
```
int((a + a*sin(e + f*x))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx = \frac{\sqrt{a}\,a\left(-2\sqrt{\sin(fx+e)+1}+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\cos(fx+e)}{\sin(fx+e)x^2+x^2}dx\right) f\,x^2+4\left(\int \frac{\sqrt{\sin(}{}\right.\right.}{4x^2}$$

input
```
int((a+a*sin(f*x+e))^(3/2)/x^3,x)
```

output
```
(sqrt(a)*a*( - 2*sqrt(sin(e + f*x) + 1) + int((sqrt(sin(e + f*x) + 1)*cos(
e + f*x))/(sin(e + f*x)*x**2 + x**2),x)*f*x**2 + 4*int((sqrt(sin(e + f*x)
+ 1)*sin(e + f*x))/x**3,x)*x**2))/(4*x**2)
```

3.134 $\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 417

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{4x^3\operatorname{arctanh}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} \\
&+ \frac{12ix^2\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} \\
&- \frac{12ix^2\operatorname{PolyLog}\left(2,e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} \\
&- \frac{48x\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^3\sqrt{a+a\sin(c+dx)}} \\
&+ \frac{48x\operatorname{PolyLog}\left(3,e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^3\sqrt{a+a\sin(c+dx)}} \\
&- \frac{96i\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^4\sqrt{a+a\sin(c+dx)}} \\
&+ \frac{96i\operatorname{PolyLog}\left(4,e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^4\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

output
```
-4*x^3*arctanh(exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d/(a+a
*sin(d*x+c))^(1/2)+12*I*x^2*polylog(2,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*
c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-12*I*x^2*polylog(2,exp(1/4*I*
(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-48*x
*polylog(3,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^3/(a+a*
sin(d*x+c))^(1/2)+48*x*polylog(3,exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*
Pi+1/2*d*x)/d^3/(a+a*sin(d*x+c))^(1/2)-96*I*polylog(4,-exp(1/4*I*(2*d*x+Pi
+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^4/(a+a*sin(d*x+c))^(1/2)+96*I*polylog(
4,exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^4/(a+a*sin(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sqrt[4]{-1}\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(-id^3x^3\log\left(1-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+id^3x^3\log\left(1+\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+6d^2x\right.}{}$$

input
```
Integrate[x^3/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
((-1)^(1/4)*Sqrt[2]*(I + E^(I*(c + d*x)))*((-I)*d^3*x^3*Log[1 - (-1)^(1/4)
*E^((I/2)*(c + d*x))] + I*d^3*x^3*Log[1 + (-1)^(1/4)*E^((I/2)*(c + d*x))]
+ 6*d^2*x^2*PolyLog[2, -((-1)^(1/4)*E^((I/2)*(c + d*x)))] - 6*d^2*x^2*Poly
Log[2, (-1)^(1/4)*E^((I/2)*(c + d*x))] + (24*I)*d*x*PolyLog[3, -((-1)^(1/4
)*E^((I/2)*(c + d*x)))] - (24*I)*d*x*PolyLog[3, (-1)^(1/4)*E^((I/2)*(c + d
*x))] - 48*PolyLog[4, -((-1)^(1/4)*E^((I/2)*(c + d*x)))] + 48*PolyLog[4, (
-1)^(1/4)*E^((I/2)*(c + d*x))]))/(d^4*E^((I/2)*(c + d*x))*Sqrt[((-I)*a*(I
+ E^(I*(c + d*x)))^2)/E^(I*(c + d*x))])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.61, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3800, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a\sin(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{\sqrt{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\int x^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\int x^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(-\frac{6\int x^2\log\left(1-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}+\frac{6\int x^2\log\left(1+e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}-\frac{4x^3\text{arctanh}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}\right)}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(\frac{6\left(\frac{2ix^2\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{4i\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}\right)}{d}-\frac{6\left(\frac{2ix^2\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\cdots\right)}{d}\cdots\right)}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{7163}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{4i\left(\frac{2i\int\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}-\frac{2ix\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}\right)}{d}\right)}{d}-\cdots\right.}{\sqrt{a\sin(\cdots}}$$

$\downarrow$ 2720

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{4i\left(\frac{4\int e^{-\frac{1}{4}i(2c+2dx+\pi)}\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)de^{\frac{1}{4}i(2c+2dx+\pi)}}{d^2}-\frac{2ix\operatorname{PolyLog}\cdots}{\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.}{\cdots}$$

$\downarrow$ 7143

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(-\frac{4x^3\operatorname{arctanh}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}+\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{4i\left(\frac{4\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2}-\frac{2ix\operatorname{Poly}\cdots}{\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.}{\sqrt{a\sin(c+\cdots}}$$

input
```
Int[x^3/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(((-4*x^3*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))])/d + (6*(((2*I)*x^2*PolyLo
g[2, -E^((I/4)*(2*c + Pi + 2*d*x))])/d - ((4*I)*(((-2*I)*x*PolyLog[3, -E^(
(I/4)*(2*c + Pi + 2*d*x))])/d + (4*PolyLog[4, -E^((I/4)*(2*c + Pi + 2*d*x)
)])/d^2))/d))/d - (6*(((2*I)*x^2*PolyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))])
/d - ((4*I)*(((-2*I)*x*PolyLog[3, E^((I/4)*(2*c + Pi + 2*d*x))])/d + (4*Po
lyLog[4, E^((I/4)*(2*c + Pi + 2*d*x))])/d^2))/d))/d)*Sin[c/2 + Pi/4 + (d*x
)/2])/Sqrt[a + a*Sin[c + d*x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a+a\sin(dx+c)}}dx$$

input
```
int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

input
```
integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(x**3/(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a*sin(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(x^3/(a + a*sin(c + d*x))^(1/2),x)
```

output
```
int(x^3/(a + a*sin(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(dx+c)+1}\,x^3}{\sin(dx+c)+1}dx\right)}{a}$$

input
```
int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sin(c + d*x) + 1)*x**3)/(sin(c + d*x) + 1),x))/a
```

3.135 $\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 293

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx = &-\frac{4x^2\operatorname{arctanh}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} \\ &+\frac{8ix\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} \\ &-\frac{8ix\operatorname{PolyLog}\left(2,e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} \\ &-\frac{16\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^3\sqrt{a+a\sin(c+dx)}} \\ &+\frac{16\operatorname{PolyLog}\left(3,e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^3\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

output
```
-4*x^2*arctanh(exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d/(a+a
*sin(d*x+c))^(1/2)+8*I*x*polylog(2,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1
/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-8*I*x*polylog(2,exp(1/4*I*(2*d*x
+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-16*polylog
(3,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^3/(a+a*sin(d*x+
c))^(1/2)+16*polylog(3,exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x
)/d^3/(a+a*sin(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sqrt[4]{-1}\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(4dx\,\text{PolyLog}\left(2,-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-i\left(d^2x^2\log\left(1-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-d\right.\right.}{d}$$

input
```
Integrate[x^2/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
((-1)^(1/4)*Sqrt[2]*(I + E^(I*(c + d*x)))*(4*d*x*PolyLog[2, -((-1)^(1/4)*E
^((I/2)*(c + d*x)))] - I*(d^2*x^2*Log[1 - (-1)^(1/4)*E^((I/2)*(c + d*x))]
- d^2*x^2*Log[1 + (-1)^(1/4)*E^((I/2)*(c + d*x))] - (4*I)*d*x*PolyLog[2, (
-1)^(1/4)*E^((I/2)*(c + d*x))] - 8*PolyLog[3, -((-1)^(1/4)*E^((I/2)*(c + d
*x)))] + 8*PolyLog[3, (-1)^(1/4)*E^((I/2)*(c + d*x))])))/(d^3*E^((I/2)*(c
+ d*x))*Sqrt[((-I)*a*(I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3800, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a\sin(c+dx)+a}}\,dx$$

$\downarrow$ 3042

$$\int \frac{x^2}{\sqrt{a\sin(c+dx)+a}}dx$$

$\downarrow$ 3800

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\int x^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a\sin(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\int x^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a\sin(c+dx)+a}}$$

$\downarrow$ 4671

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(-\frac{4\int x\log\left(1-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}+\frac{4\int x\log\left(1+e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}-\frac{4x^2\text{arctanh}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}\right)}{\sqrt{a\sin(c+dx)+a}}$$

$\downarrow$ 3011

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{2i\int\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}\right)}{d}-\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{2i\int \text{P}\ldots}{}\right.}{d}\right.}{\sqrt{a\sin(c+dx)+a}}$$

$\downarrow$ 2720

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{4\int e^{-\frac{1}{4}i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)de^{\frac{1}{4}i(2c+2dx+\pi)}}{d^2}\right)}{d}-\frac{4\left(\frac{2ix\,\text{Poly}\ldots}{}\right.}{}\right.}{\sqrt{a\sin(c+dx)\ldots}}$$

$\downarrow$ 7143

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(-\frac{4x^2\operatorname{arctanh}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}+\frac{4\left(\frac{2ix\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}-\frac{4\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2}\right)}{d}-\frac{4\left(\frac{2ix}{}\right.}{}\right.}{\sqrt{a\sin(c+dx)+a}}$$

input
```
Int[x^2/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(((-4*x^2*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))])/d + (4*(((2*I)*x*PolyLog[
2, -E^((I/4)*(2*c + Pi + 2*d*x))])/d - (4*PolyLog[3, -E^((I/4)*(2*c + Pi +
 2*d*x))])/d^2))/d - (4*(((2*I)*x*PolyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]
)/d - (4*PolyLog[3, E^((I/4)*(2*c + Pi + 2*d*x))])/d^2))/d)*Sin[c/2 + Pi/4
 + (d*x)/2])/Sqrt[a + a*Sin[c + d*x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a + a\sin(dx + c)}} dx$$

input
```
int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a + a\sin(c + dx)}} dx = \int \frac{x^2}{\sqrt{a\sin(dx + c) + a}} dx$$

input
```
integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a + a\sin(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a\,(\sin{(c + dx)} + 1)}}\,dx$$

input
```
integrate(x**2/(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a + a\sin(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a\sin{(dx + c)} + a}}\,dx$$

input
```
integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*sin(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + a\sin(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a\sin{(dx + c)} + a}}\,dx$$

input
```
integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(x^2/(a + a*sin(c + d*x))^(1/2),x)
```

output
```
int(x^2/(a + a*sin(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(dx+c)+1}\,x^2}{\sin(dx+c)+1}dx\right)}{a}$$

input
```
int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sin(c + d*x) + 1)*x**2)/(sin(c + d*x) + 1),x))/a
```

3.136 $\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{4x\text{arctanh}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} - \frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}}$$

output
```
-4*x*arctanh(exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d/(a+a*s
in(d*x+c))^(1/2)+4*I*polylog(2,-exp(1/4*I*(2*d*x+Pi+2*c)))*sin(1/2*c+1/4*P
i+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)-4*I*polylog(2,exp(1/4*I*(2*d*x+Pi+2*
c)))*sin(1/2*c+1/4*Pi+1/2*d*x)/d^2/(a+a*sin(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{2\left(\frac{\left(-\pi \operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{1}{4}(c+dx)\right)}{\sqrt{2}}\right)+\frac{1}{2}(2c+\pi+2dx)\left(\log\left(1-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)-\log\left(1+e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\right)+2i\left(\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+\pi+2d}\right.\right.\right.}{\sqrt{2}}\right.}{d^2\sqrt{a(1+\sin(c+}}$$

input
```
Integrate[x/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(2*(((-(Pi*ArcTanh[(-1 + Tan[(c + d*x)/4])/Sqrt[2]]) + ((2*c + Pi + 2*d*x)
*(Log[1 - E^((I/4)*(2*c + Pi + 2*d*x))] - Log[1 + E^((I/4)*(2*c + Pi + 2*d
*x))]))/2 + (2*I)*(PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*x))] - PolyLog[2,
E^((I/4)*(2*c + Pi + 2*d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/Sqr
t[2] + (c*ArcSin[Csc[(2*c + Pi + 2*d*x)/4]]*Sin[(2*c - Pi + 2*d*x)/4])/Sqr
t[(-1 + Sin[c + d*x])/(1 + Sin[c + d*x])]))/(d^2*Sqrt[a*(1 + Sin[c + d*x])
])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3800, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a\sin(c+dx)+a}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x}{\sqrt{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\int x\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\int x\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(-\frac{2\int\log\left(1-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}+\frac{2\int\log\left(1+e^{\frac{1}{4}i(2c+2dx+\pi)}\right)dx}{d}-\frac{4x\text{arctanh}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}\right)}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(\frac{4i\int e^{-\frac{1}{4}i(2c+2dx+\pi)}\log\left(1-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)de^{\frac{1}{4}i(2c+2dx+\pi)}}{d^2}-\frac{4i\int e^{-\frac{1}{4}i(2c+2dx+\pi)}\log\left(1+e^{\frac{1}{4}i(2c+2dx+\pi)}\right)de^{\frac{1}{4}i(2c+}}{d^2}\right.}{\sqrt{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\left(-\frac{4x\text{arctanh}\left(e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2}\right)}{\sqrt{a\sin(c+dx)+a}}$$

input
```
Int[x/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(((-4*x*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))])/d + ((4*I)*PolyLog[2, -E^((
I/4)*(2*c + Pi + 2*d*x))])/d^2 - ((4*I)*PolyLog[2, E^((I/4)*(2*c + Pi + 2*
d*x))])/d^2)*Sin[c/2 + Pi/4 + (d*x)/2])/Sqrt[a + a*Sin[c + d*x]]
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{a + a\sin(dx + c)}}dx$$

input
```
int(x/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
int(x/(a+a*sin(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{x}{\sqrt{a\sin(dx+c)+a}}\,dx$$

input
```
integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{x}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

input
```
integrate(x/(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(x/sqrt(a*(sin(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{x}{\sqrt{a\sin(dx+c)+a}}\,dx$$

input
```
integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*sin(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\sin(dx+c)+a}}\,dx$$

input
```
integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{x}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

input
```
int(x/(a + a*sin(c + d*x))^(1/2),x)
```

output
```
int(x/(a + a*sin(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(dx+c)+1}\,x}{\sin(dx+c)+1}dx\right)}{a}$$

input
```
int(x/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sin(c + d*x) + 1)*x)/(sin(c + d*x) + 1),x))/a
```

3.137 $\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{a+a\sin(c+dx)}},x\right)$$

output
```
Defer(Int)(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + a*Sin[c + d*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a\sin(c+dx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x\sqrt{a\sin(c+dx)+a}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x\sqrt{a\sin(c+dx)+a}}dx$$

input
```
Int[1/(x*Sqrt[a + a*Sin[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+a\sin\left(dx+c\right)}}dx$$

input
```
int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(d*x + c) + a)/(a*x*sin(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(1/x/(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*(sin(c + d*x) + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*sin(d*x + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sin(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 35.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{x\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(x*(a + a*sin(c + d*x))^(1/2)),x)
```

output
```
int(1/(x*(a + a*sin(c + d*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(dx+c)+1}}{\sin(dx+c)x+x}dx\right)}{a}$$

input
```
int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(c + d*x) + 1)/(sin(c + d*x)*x + x),x))/a
```

3.138 $\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx=\text{Int}\left(\frac{1}{x^2\sqrt{a+a\sin(c+dx)}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + a*Sin[c + d*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{x^2\sqrt{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{1}{x^2\sqrt{a\sin(c+dx)+a}}dx$$

input
```
Int[1/(x^2*Sqrt[a + a*Sin[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 \sqrt{a + a\sin(dx + c)}} dx$$

input
```
int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2 \sqrt{a + a\sin(c + dx)}} dx = \int \frac{1}{\sqrt{a\sin(dx + c) + a}x^2} dx$$

input
```
integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(d*x + c) + a)/(a*x^2*sin(d*x + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \sqrt{a + a\sin(c + dx)}} dx = \int \frac{1}{x^2 \sqrt{a(\sin(c + dx) + 1)}} dx$$

input
```
integrate(1/x**2/(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*(sin(c + d*x) + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*sin(d*x + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sin(d*x + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 36.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{1}{x^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(x^2*(a + a*sin(c + d*x))^(1/2)),x)
```

output
```
int(1/(x^2*(a + a*sin(c + d*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(dx+c)+1}}{\sin(dx+c)x^2+x^2}dx\right)}{a}$$

input
```
int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(c + d*x) + 1)/(sin(c + d*x)*x**2 + x**2),x))/a
```

3.139 $\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 691

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\operatorname{arctanh}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}} - \frac{x^3\operatorname{arctanh}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+a\sin(e+fx)}} + \frac{24i\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^4\sqrt{a+a\sin(e+fx)}} + \frac{3ix^2\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{24i\operatorname{PolyLog}\left(2,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^4\sqrt{a+a\sin(e+fx)}} - \frac{3ix^2\operatorname{PolyLog}\left(2,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{12x\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}} + \frac{12x\operatorname{PolyLog}\left(3,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}} - \frac{24i\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^4\sqrt{a+a\sin(e+fx)}} + \frac{24i\operatorname{PolyLog}\left(4,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^4\sqrt{a+a\sin(e+fx)}}$$

output
```
-3*x^2/a/f^2/(a+a*sin(f*x+e))^(1/2)-1/2*x^3*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f/
(a+a*sin(f*x+e))^(1/2)-24*x*arctanh(exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1
/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)-x^3*arctanh(exp(1/4*I*(2*f*x+P
i+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f/(a+a*sin(f*x+e))^(1/2)+24*I*polylog
(2,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+a*sin(f*
x+e))^(1/2)+3*I*x^2*polylog(2,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi
+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(1/2)-24*I*polylog(2,exp(1/4*I*(2*f*x+Pi+
2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+a*sin(f*x+e))^(1/2)-3*I*x^2*poly
log(2,exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(
f*x+e))^(1/2)-12*x*polylog(3,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+
1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)+12*x*polylog(3,exp(1/4*I*(2*f*x+Pi+2
*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)-24*I*polylog(
4,-exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^4/(a+a*sin(f*x
+e))^(1/2)+24*I*polylog(4,exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*
f*x)/a/f^4/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.52 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{(-1)^{3/4}e^{-\frac{3}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)^3\left(6(8+f^2x^2)\operatorname{PolyLog}\left(2,-\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-6(8+f^2x^2)\operatorname{PolyLog}\left(2,\sqrt[4]{}\right.\right.}{}$$
$$-\frac{x^2\left((6+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(6-fx)\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{2a^2f^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

input
```
Integrate[x^3/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/2*((-1)^(3/4)*(I + E^(I*(e + f*x)))^3*(6*(8 + f^2*x^2)*PolyLog[2, -((-1
)^(1/4)*E^((I/2)*(e + f*x)))] - 6*(8 + f^2*x^2)*PolyLog[2, (-1)^(1/4)*E^((
I/2)*(e + f*x))] - I*(24*f*x*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] + f^3
*x^3*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] - 24*f*x*Log[1 + (-1)^(1/4)*E
^((I/2)*(e + f*x))] - f^3*x^3*Log[1 + (-1)^(1/4)*E^((I/2)*(e + f*x))] - 24
*f*x*PolyLog[3, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] + 24*f*x*PolyLog[3, (-1
)^(1/4)*E^((I/2)*(e + f*x))] - (48*I)*PolyLog[4, -((-1)^(1/4)*E^((I/2)*(e
+ f*x)))] + (48*I)*PolyLog[4, (-1)^(1/4)*E^((I/2)*(e + f*x))])))/(Sqrt[2]*
E^(((3*I)/2)*(e + f*x))*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))
^(3/2)*f^4) - (x^2*((6 + f*x)*Cos[(e + f*x)/2] + (6 - f*x)*Sin[(e + f*x)/2
])*Sqrt[a*(1 + Sin[e + f*x])])/(2*a^2*f^2*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^3)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.61, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 4674, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x^3}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\int x^3\csc^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\int x^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{12\int x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{6x^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc\left(\frac{e}{2}\right.}{f}}{2a\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 3042

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{12\int x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{6x^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x^3\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc\left(\frac{e}{2}\right.}{f}}{2a\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 4671

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{12\left(-\frac{2\int\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}+\frac{2\int\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}-\frac{4x\text{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}\right)}{f^2}+\frac{1}{2}\left(-\frac{6\int x^2}{}\right.\right.}{2}$$

$\downarrow$ 2715

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{12\left(\frac{4i\int e^{-\frac{1}{4}i(2e+2fx+\pi)}\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)de^{\frac{1}{4}i(2e+2fx+\pi)}}{f^2}-\frac{4i\int e^{-\frac{1}{4}i(2e+2fx+\pi)}\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)de^{\frac{1}{4}i(2e}}{f^2}\right.}{f^2}\right.}{}$$

$\downarrow$ 2838

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6\int x^2\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}+\frac{6\int x^2\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}-\frac{4x^3\text{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}\right)+\right.}{}$$

$\downarrow$ 3011

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(\frac{6\left(\frac{2ix^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\frac{4i\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}\right)}{f}-\frac{6\left(\frac{2ix^2\,\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi}\right.}{f}\right.}{}\right.\right.$$

$\downarrow$ 7163

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(\frac{6\left(\frac{2ix^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\frac{4i\left(\frac{2i\int \text{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}-\frac{2ix\,\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}\right)}{f}\right.}{f}\right.\right.$$

$\downarrow$ 2720

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(\frac{6\left(\frac{2ix^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\frac{4i\left(\frac{4\int e^{-\frac{1}{4}i(2e+2fx+\pi)}\,\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)de^{\frac{1}{4}i(2e+2fx+\pi)}}{f^2}-\frac{2ix\,\text{P}}{}\right.}{f}\right.}{f}\right.\right.$$

$\downarrow$ 7143

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4x^3\operatorname{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}+\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\frac{4i\left(\frac{4\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f^2}-\cdots\right)}{f}\right.}{f}\right.\right.$$

input
```
Int[x^3/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(((-6*x^2*Csc[e/2 + Pi/4 + (f*x)/2])/f^2 - (x^3*Cot[e/2 + Pi/4 + (f*x)/2]*
Csc[e/2 + Pi/4 + (f*x)/2])/f + (12*((-4*x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f
*x))])/f + ((4*I)*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))])/f^2 - ((4*I)*
PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))])/f^2))/f^2 + ((-4*x^3*ArcTanh[E^(
(I/4)*(2*e + Pi + 2*f*x))])/f + (6*(((2*I)*x^2*PolyLog[2, -E^((I/4)*(2*e +
 Pi + 2*f*x))])/f - ((4*I)*(((-2*I)*x*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f
*x))])/f + (4*PolyLog[4, -E^((I/4)*(2*e + Pi + 2*f*x))])/f^2))/f))/f - (6*
(((2*I)*x^2*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))])/f - ((4*I)*(((-2*I)*
x*PolyLog[3, E^((I/4)*(2*e + Pi + 2*f*x))])/f + (4*PolyLog[4, E^((I/4)*(2*
e + Pi + 2*f*x))])/f^2))/f))/f)/2)*Sin[e/2 + Pi/4 + (f*x)/2])/(2*a*Sqrt[a
+ a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{(a + a\sin(fx + e))^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a + a\sin(e + fx))^{3/2}}\,dx = \int \frac{x^3}{(a\sin(fx + e) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e) + a)*x^3/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
 + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{x^3}{(a + a\sin(e + fx))^{3/2}}\,dx = \int \frac{x^3}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x**3/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a + a\sin(e + fx))^{3/2}}\,dx = \int \frac{x^3}{(a\sin(fx + e) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{x^3}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(x^3/(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)+1}\,x^3}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)}{a^2}$$

input
```
int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sin(e + f*x) + 1)*x**3)/(sin(e + f*x)**2 + 2*sin(e + f*
x) + 1),x))/a**2
```

3.140 $\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 435

$$\begin{aligned}
\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = &-\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} \\
&-\frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x^2\text{arctanh}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+a\sin(e+fx)}} \\
&-\frac{4\text{arctanh}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}} \\
&+\frac{2ix\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^2\sqrt{a+a\sin(e+fx)}} \\
&-\frac{2ix\,\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^2\sqrt{a+a\sin(e+fx)}} \\
&-\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}} \\
&+\frac{4\,\text{PolyLog}\left(3,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

output
```
-2*x/a/f^2/(a+a*sin(f*x+e))^(1/2)-1/2*x^2*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f/(a
+a*sin(f*x+e))^(1/2)-x^2*arctanh(exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*
Pi+1/2*f*x)/a/f/(a+a*sin(f*x+e))^(1/2)-4*arctanh(cos(1/2*e+1/4*Pi+1/2*f*x)
)*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)+2*I*x*polylog(2,-
exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e)
)^(1/2)-2*I*x*polylog(2,exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*
x)/a/f^2/(a+a*sin(f*x+e))^(1/2)-4*polylog(3,-exp(1/4*I*(2*f*x+Pi+2*e)))*si
n(1/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)+4*polylog(3,exp(1/4*I
*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^3/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{-1}e^{-\frac{3}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)^3\left(16\text{arctanh}\left(\sqrt[4]{-1}e^{\frac{1}{2}i(e+fx)}\right)-f^2x^2\log\left(1-\right.\right.}{}$$

$$-\frac{x\left((4+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(4-fx)\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{2a^2f^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

input
```
Integrate[x^2/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
((-1)^(1/4)*(I + E^(I*(e + f*x)))^3*(16*ArcTanh[(-1)^(1/4)*E^((I/2)*(e + f
*x))] - f^2*x^2*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] + f^2*x^2*Log[1 +
(-1)^(1/4)*E^((I/2)*(e + f*x))] - (4*I)*f*x*PolyLog[2, -((-1)^(1/4)*E^((I/
2)*(e + f*x)))] + (4*I)*f*x*PolyLog[2, (-1)^(1/4)*E^((I/2)*(e + f*x))] + 8
*PolyLog[3, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] - 8*PolyLog[3, (-1)^(1/4)*E
^((I/2)*(e + f*x))]))/(2*Sqrt[2]*E^(((3*I)/2)*(e + f*x))*(((-I)*a*(I + E^(
I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*f^3) - (x*((4 + f*x)*Cos[(e + f*x)
/2] + (4 - f*x)*Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])/(2*a^2*f^2*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3800, 3042, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{x^2}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \ 3800$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\int x^2\csc^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\int x^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \ 4674$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{4\int\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{4x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right.}{f}\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{4\int\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{4x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right.}{f}\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \ 4257$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\int x^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{8\text{arctanh}\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{f^3}-\frac{4x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{cs}}{f}\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 4671

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4\int x\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}+\frac{4\int x\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}-\frac{4x^2\text{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}\right)-\frac{8}{}\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 3011

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4\left(\frac{2ix\text{ PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\frac{2i\int\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}\right)}{f}-\frac{4\left(\frac{2ix\text{ PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}\right.}{}\right.\right.$$

$\downarrow$ 2720

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4\left(\frac{2ix\text{ PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\frac{4\int e^{-\frac{1}{4}i(2e+2fx+\pi)}\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)de^{\frac{1}{4}i(2e+2fx+\pi)}}{f^2}\right)}{f}-\frac{4\left(\right.}{}\right.\right.$$

$\downarrow$ 7143

$$\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(-\frac{8\text{arctanh}\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{f^3}+\frac{1}{2}\left(-\frac{4x^2\text{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}+\frac{4\left(\frac{2ix\text{ PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}-\right.}{f}\right.\right.$$

input
```
Int[x^2/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(((-8*ArcTanh[Cos[e/2 + Pi/4 + (f*x)/2]])/f^3 - (4*x*Csc[e/2 + Pi/4 + (f*x
)/2])/f^2 - (x^2*Cot[e/2 + Pi/4 + (f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2])/f +
((-4*x^2*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))])/f + (4*(((2*I)*x*PolyLog[2
, -E^((I/4)*(2*e + Pi + 2*f*x))])/f - (4*PolyLog[3, -E^((I/4)*(2*e + Pi +
2*f*x))])/f^2))/f - (4*(((2*I)*x*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))])
/f - (4*PolyLog[3, E^((I/4)*(2*e + Pi + 2*f*x))])/f^2))/f)/2)*Sin[e/2 + Pi
/4 + (f*x)/2])/(2*a*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{(a+a\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(x^2/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e) + a)*x^2/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
 + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(x^2/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(x^2/(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)+1}\,x^2}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)}{a^2}$$

input
```
int(x^2/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sin(e + f*x) + 1)*x**2)/(sin(e + f*x)**2 + 2*sin(e + f*
x) + 1),x))/a**2
```

3.141 $\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 249

$$\begin{aligned}\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}}\\ &\quad -\frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{x\operatorname{arctanh}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+a\sin(e+fx)}}\\ &\quad +\frac{i\,\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^2\sqrt{a+a\sin(e+fx)}}\\ &\quad -\frac{i\,\mathrm{PolyLog}\left(2,e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af^2\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

output

```
-1/a/f^2/(a+a*sin(f*x+e))^(1/2)-1/2*x*cot(1/2*e+1/4*Pi+1/2*f*x)/a/f/(a+a*s
in(f*x+e))^(1/2)-x*arctanh(exp(1/4*I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2
*f*x)/a/f/(a+a*sin(f*x+e))^(1/2)+I*polylog(2,-exp(1/4*I*(2*f*x+Pi+2*e)))*s
in(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(1/2)-I*polylog(2,exp(1/4*
I*(2*f*x+Pi+2*e)))*sin(1/2*e+1/4*Pi+1/2*f*x)/a/f^2/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.23 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.24

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{2fx\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-(2+fx)\left(\cos\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[x/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(2*f*x*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (2 + f*x)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + ((-(Pi*ArcTanh[(-1 + Tan[(e + f*
x)/4])/Sqrt[2]]) + ((2*e + Pi + 2*f*x)*(Log[1 - E^((I/4)*(2*e + Pi + 2*f*x
))] - Log[1 + E^((I/4)*(2*e + Pi + 2*f*x))]))/2 + (2*I)*(PolyLog[2, -E^((I
/4)*(2*e + Pi + 2*f*x))] - PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]))*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[2] + (e*ArcSin[Csc[(2*e + Pi + 2
*f*x)/4]]*(1 + Sin[e + f*x])*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[(-1 + Sin[e +
 f*x])/(1 + Sin[e + f*x])])/(2*f^2*(a*(1 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\int x\csc^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx}{2a\sqrt{a\sin(e+fx)+a}}$$

$\big\downarrow$ 3042

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\int x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a\sin(e+fx)+a}}$$

$\big\downarrow$ 4673

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\int x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{2a\sqrt{a\sin(e+fx)+a}}$$

$\big\downarrow$ 3042

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\int x\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx-\frac{2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f^2}-\frac{x\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{f}\right)}{2a\sqrt{a\sin(e+fx)+a}}$$

$\big\downarrow$ 4671

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{2\int\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}+\frac{2\int\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)dx}{f}-\frac{4x\text{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}\right)-\frac{2\csc(}{}\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

$\big\downarrow$ 2715

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4i\int e^{-\frac{1}{4}i(2e+2fx+\pi)}\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)de^{\frac{1}{4}i(2e+2fx+\pi)}}{f^2}-\frac{4i\int e^{-\frac{1}{4}i(2e+2fx+\pi)}\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)de^{\frac{1}{4}}}{f^2}\right.\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

$\big\downarrow$ 2838

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4x\text{arctanh}\left(e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{f^2}\right)-\frac{2\,\text{cs}}{}\right.}{2a\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[x/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(((-2*Csc[e/2 + Pi/4 + (f*x)/2])/f^2 - (x*Cot[e/2 + Pi/4 + (f*x)/2]*Csc[e/
2 + Pi/4 + (f*x)/2])/f + ((-4*x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))])/f +
 ((4*I)*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))])/f^2 - ((4*I)*PolyLog[2,
 E^((I/4)*(2*e + Pi + 2*f*x))])/f^2)/2)*Sin[e/2 + Pi/4 + (f*x)/2])/(2*a*Sq
rt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [F]

$$\int \frac{x}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(x/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{x}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e) + a)*x/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x +
 e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{x}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{x}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{x}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a*sin(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{x}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(x/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(x/(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)+1}\,x}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)}{a^2}$$

input
```
int(x/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sin(e + f*x) + 1)*x)/(sin(e + f*x)**2 + 2*sin(e + f*x)
+ 1),x))/a**2
```

3.142 $\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+a\sin(e+fx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 38.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + a*Sin[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x(a\sin(e+fx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x(a\sin(e+fx)+a)^{3/2}}dx$$

input
```
Int[1/(x*(a + a*Sin[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e) + a)/(a^2*x*cos(f*x + e)^2 - 2*a^2*x*sin(f*x
 + e) - 2*a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{x\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(1/(x*(a*(sin(e + f*x) + 1))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sin(f*x + e) + a)^(3/2)*x), x)
```

Giac [N/A]

Not integrable

Time = 43.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sin(f*x + e) + a)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 36.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{x\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + a*sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(x*(a + a*sin(e + f*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2x+2\sin(fx+e)x+x}dx\right)}{a^2}$$

input
```
int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2*x + 2*sin(e + f*x)*x
+ x),x))/a**2
```

3.143 $\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx = \operatorname{Int}\left(\frac{1}{x^2(a+a\sin(e+fx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 20.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + a*Sin[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/(x^2*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x^2(a\sin(e+fx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x^2(a\sin(e+fx)+a)^{3/2}}dx$$

input
```
Int[1/(x^2*(a + a*Sin[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a + a\sin(fx + e))^{\frac{3}{2}}} dx$$

input
```
int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^2(a + a\sin(e + fx))^{3/2}} dx = \int \frac{1}{(a\sin(fx + e) + a)^{\frac{3}{2}} x^2} dx$$

input
```
integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e) + a)/(a^2*x^2*cos(f*x + e)^2 - 2*a^2*x^2*sin
(f*x + e) - 2*a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a + a\sin(e + fx))^{3/2}} dx = \int \frac{1}{x^2 (a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

input
```
integrate(1/x**2/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(1/(x**2*(a*(sin(e + f*x) + 1))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{1}{(a\sin{(fx+e)}+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sin(f*x + e) + a)^(3/2)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 36.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + a*sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(x^2*(a + a*sin(e + f*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2x^2+2\sin(fx+e)x^2+x^2}dx\right)}{a^2}$$

input
```
int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2*x**2 + 2*sin(e + f*x)
*x**2 + x**2),x))/a**2
```

3.144 $\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+a\sin(c+dx)}}{x},x\right)$$

output
```
Defer(Int)((a+a*sin(d*x+c))^(1/3)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

input
```
Integrate[(a + a*Sin[c + d*x])^(1/3)/x,x]
```

output
```
Integrate[(a + a*Sin[c + d*x])^(1/3)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a\sin(c+dx)+a}}{x}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt[3]{a\sin(c+dx)+a}}{x}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{\sqrt[3]{a\sin(c+dx)+a}}{x}dx$$

input
```
Int[(a + a*Sin[c + d*x])^(1/3)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a + a\sin(dx + c))^{\frac{1}{3}}}{x} dx$$

input
```
int((a+a*sin(d*x+c))^(1/3)/x,x)
```

output
```
int((a+a*sin(d*x+c))^(1/3)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{a + a\sin(c + dx)}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{a + a\sin(c + dx)}}{x} dx = \int \frac{\sqrt[3]{a(\sin(c + dx) + 1)}}{x} dx$$

input
```
integrate((a+a*sin(d*x+c))**(1/3)/x,x)
```

output
```
Integral((a*(sin(c + d*x) + 1))**(1/3)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx = \int \frac{(a\sin(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(1/3)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx = \int \frac{(a\sin(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(1/3)/x, x)
```

Mupad [N/A]

Not integrable

Time = 35.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx = \int \frac{(a+a\,\sin(c+d\,x))^{1/3}}{x}\,dx$$

input
```
int((a + a*sin(c + d*x))^(1/3)/x,x)
```

output
```
int((a + a*sin(c + d*x))^(1/3)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a + a\sin(c + dx)}}{x}\, dx = a^{\frac{1}{3}}\left(\int \frac{(\sin(dx + c) + 1)^{\frac{1}{3}}}{x}dx\right)$$

input
```
int((a+a*sin(d*x+c))^(1/3)/x,x)
```

output
```
a**(1/3)*int((sin(c + d*x) + 1)**(1/3)/x,x)
```

3.145 $\int (c+dx)^m(a+a\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx = \text{Int}((c+dx)^m(a+a\sin(e+fx))^n,x)$$

output
```
Defer(Int)((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx = \int (c+dx)^m(a+a\sin(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\sin(e+fx)+a)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a\sin(e+fx)+a)^ndx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m(a\sin(e+fx)+a)^ndx$$

input
```
Int[(c + d*x)^m*(a + a*Sin[e + f*x])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (a+a\sin(fx+e))^n \, dx$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\sin(e+fx))^n \, dx = \int (dx+c)^m (a\sin(fx+e)+a)^n \, dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m (a+a\sin(e+fx))^n \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+a*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx = \int (dx+c)^m(a\sin(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx = \int (dx+c)^m(a\sin(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 36.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx = \int (a+a\,\sin(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sin(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((a + a*sin(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\sin(e+fx))^n\,dx=\int (dx+c)^m\left(a+a\sin\left(fx+e\right)\right)^n dx$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

output
```
int((c + d*x)**m*(sin(e + f*x)*a + a)**n,x)
```

3.146 $\int (c+dx)^m(a+a\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 449

$$\begin{aligned}\int (c+dx)^m(a+a\sin(e+fx))^3\,dx &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{8f}\\
&\quad - \frac{15a^3e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{8f}\\
&\quad + \frac{3i2^{-3-m}a^3e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}\\
&\quad - \frac{3i2^{-3-m}a^3e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}\\
&\quad + \frac{3^{-1-m}a^3e^{3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3if(c+dx)}{d}\right)}{8f}\\
&\quad + \frac{3^{-1-m}a^3e^{-3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3if(c+dx)}{d}\right)}{8f}\end{aligned}$$

output
```
5/2*a^3*(d*x+c)^(1+m)/d/(1+m)-15/8*a^3*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+
m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-15/8*a^3*(d*x+c)^m*GAMMA(1+m,I*f*
(d*x+c)/d)/exp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+3*I*2^(-3-m)*a^3*exp(2*I
*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-3
*I*2^(-3-m)*a^3*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f/d))/f/
((I*f*(d*x+c)/d)^m)+1/8*3^(-1-m)*a^3*exp(3*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+
m,-3*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)+1/8*3^(-1-m)*a^3*(d*x+c)^m*GAMM
A(1+m,3*I*f*(d*x+c)/d)/exp(3*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.84

$$\begin{aligned}\int (c+dx)^m(a+a\sin(e+fx))^3\,dx &= \frac{1}{24}a^3(c+dx)^m\left(\frac{60(c+dx)}{d(1+m)}-\frac{45e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}\right.\\&\quad-\frac{45e^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}\\&\quad+\frac{9i2^{-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}\\&\quad-\frac{9i2^{-m}e^{-2i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}\\&\quad+\frac{3^{-m}e^{3i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3if(c+dx)}{d}\right)}{f}\\&\quad\left.+\frac{3^{-m}e^{-3i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3if(c+dx)}{d}\right)}{f}\right)\end{aligned}$$

input
```
Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^3,x]
```

output
```
(a^3*(c + d*x)^m*((60*(c + d*x))/(d*(1 + m)) - (45*E^(I*(e - (c*f)/d))*Gam
ma[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (45*Gamma[
1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) +
 ((9*I)*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m
*f*(((-I)*f*(c + d*x))/d)^m) - ((9*I)*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])
/(2^m*E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (E^((3*I)*(e - (c
*f)/d))*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(3^m*f*(((-I)*f*(c + d*x))/d
)^m) + Gamma[1 + m, ((3*I)*f*(c + d*x))/d]/(3^m*E^((3*I)*(e - (c*f)/d))*f*
((I*f*(c + d*x))/d)^m)))/24
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\sin(e+fx)+a)^3 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m (a\sin(e+fx)+a)^3dx$$

$$\Big\downarrow \text{ 3799}$$

$$8a^3 \int (c+dx)^m \sin^6\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right) dx$$

$$\Big\downarrow \text{ 3042}$$

$$8a^3 \int (c+dx)^m \sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^6 dx$$

$$\Big\downarrow \text{ 3793}$$

$$8a^3 \int \left(-\frac{3}{16}\cos(2e+2fx)(c+dx)^m+\frac{15}{32}\sin(e+fx)(c+dx)^m-\frac{1}{32}\sin(3e+3fx)(c+dx)^m+\frac{5}{16}(c+dx)^m\right)$$

$$\Big\downarrow \text{ 2009}$$

$$8a^3\left(-\frac{15e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{64f}+\frac{3i2^{-m-6}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}}{f}\right.$$

input
```
Int[(c + d*x)^m*(a + a*Sin[e + f*x])^3,x]
```

output
```
8*a^3*((5*(c + d*x)^(1 + m))/(16*d*(1 + m)) - (15*E^(I*(e - (c*f)/d))*(c +
 d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(64*f*(((-I)*f*(c + d*x))/d)^m
) - (15*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(64*E^(I*(e - (c*f)/d
))*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-6 - m)*E^((2*I)*(e - (c*f)/d))*(c
 + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^
m) - ((3*I)*2^(-6 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(E
^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (3^(-1 - m)*E^((3*I)*(e
- (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(64*f*(((-I)
*f*(c + d*x))/d)^m) + (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*f*(c + d
*x))/d])/(64*E^((3*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int (dx+c)^m (a+a\sin(fx+e))^3\,dx$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.86

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx = -\frac{45\,(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+i\,de-i\,cf}{d}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,cf}{d}\right)+9\,(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)-2i\,de+2i\,cf}{d}\right)}}{}$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/24*(45*(a^3*d*m + a^3*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(
m + 1, (I*d*f*x + I*c*f)/d) + 9*(-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(-2*I*f
/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) - (a^3*d*
m + a^3*d)*e^(-(d*m*log(-3*I*f/d) - 3*I*d*e + 3*I*c*f)/d)*gamma(m + 1, -3*
(I*d*f*x + I*c*f)/d) + 45*(a^3*d*m + a^3*d)*e^(-(d*m*log(-I*f/d) - I*d*e +
 I*c*f)/d)*gamma(m + 1, (-I*d*f*x - I*c*f)/d) + 9*(I*a^3*d*m + I*a^3*d)*e^
(-(d*m*log(2*I*f/d) + 2*I*d*e - 2*I*c*f)/d)*gamma(m + 1, -2*(-I*d*f*x - I*
c*f)/d) - (a^3*d*m + a^3*d)*e^(-(d*m*log(3*I*f/d) + 3*I*d*e - 3*I*c*f)/d)*
gamma(m + 1, -3*(-I*d*f*x - I*c*f)/d) - 60*(a^3*d*f*x + a^3*c*f)*(d*x + c)
^m)/(d*f*m + d*f)
```

Sympy [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx = a^3\left(\int 3(c+dx)^m\sin(e+fx)\,dx + \int 3(c+dx)^m\sin^2(e+fx)\,dx + \int (c+dx)^m\sin^3(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*sin(f*x+e))**3,x)
```

output
```
a**3*(Integral(3*(c + d*x)**m*sin(e + f*x), x) + Integral(3*(c + d*x)**m*s
in(e + f*x)**2, x) + Integral((c + d*x)**m*sin(e + f*x)**3, x) + Integral(
(c + d*x)**m, x))
```

Maxima [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx = \int (a\sin(fx+e)+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(6*a^3*e^(m*log(d*x + c) + log(d*x
 + c)) - 6*(a^3*d*m + a^3*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) -
(a^3*d*m + a^3*d)*integrate((d*x + c)^m*sin(3*f*x + 3*e), x) + 15*(a^3*d*m
 + a^3*d)*integrate((d*x + c)^m*sin(f*x + e), x))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx = \int (a\sin(fx+e)+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx = \int (a+a\sin(e+fx))^3\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sin(e + f*x))^3*(c + d*x)^m,x)
```

output
```
int((a + a*sin(e + f*x))^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx$$
$$= \frac{a^3\left((dx+c)^m c+(dx+c)^m dx+\left(\int (dx+c)^m\sin(fx+e)^3\,dx\right)dm+\left(\int (dx+c)^m\sin(fx+e)^3\,dx\right)d\right.}{}$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

output
```
(a**3*((c + d*x)**m*c + (c + d*x)**m*d*x + int((c + d*x)**m*sin(e + f*x)**
3,x)*d*m + int((c + d*x)**m*sin(e + f*x)**3,x)*d + 3*int((c + d*x)**m*sin(
e + f*x)**2,x)*d*m + 3*int((c + d*x)**m*sin(e + f*x)**2,x)*d + 3*int((c +
d*x)**m*sin(e + f*x),x)*d*m + 3*int((c + d*x)**m*sin(e + f*x),x)*d))/(d*(m
 + 1))
```

3.147 $\int (c+dx)^m (a+a\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 299

$$\begin{aligned}\int (c+dx)^m (a+a\sin(e+fx))^2\,dx &= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} - \frac{a^2 e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}\\ &\quad - \frac{a^2 e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}\\ &\quad + \frac{i2^{-3-m}a^2 e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}\\ &\quad - \frac{i2^{-3-m}a^2 e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}\end{aligned}$$

output
```
3/2*a^2*(d*x+c)^(1+m)/d/(1+m)-a^2*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*
f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-a^2*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)
/exp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+I*2^(-3-m)*a^2*exp(2*I*(e-c*f/d))*
(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-I*2^(-3-m)*a^
2*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/
d)^m)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.87

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx$$
$$=\frac{1}{8}a^2(c+dx)^m\left(\frac{12(c+dx)}{d(1+m)}-\frac{8e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{8e^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}+\frac{i2^{-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{i2^{-m}e^{-2i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^m*((12*(c + d*x))/(d*(1 + m)) - (8*E^(I*(e - (c*f)/d))*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (8*Gamma[1
+ m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (
I*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*f*(((
-I)*f*(c + d*x))/d)^m) - (I*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m*E^((
2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)))/8
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\sin(e+fx)+a)^2\,dx$$

$\Big\downarrow$ 3042

$$\int (c+dx)^m(a\sin(e+fx)+a)^2dx$$

$\Big\downarrow$ 3799

$$4a^2\int (c+dx)^m\sin^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)dx$$

$\Big\downarrow$ 3042

$$4a^2\int (c+dx)^m\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)^4dx$$

$\Big\downarrow$ 3793

$$4a^2\int \left(-\frac{1}{8}\cos(2e+2fx)(c+dx)^m+\frac{1}{2}\sin(e+fx)(c+dx)^m+\frac{3}{8}(c+dx)^m\right)dx$$

$\Big\downarrow$ 2009

$$4a^2\left(-\frac{e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{4f}+\frac{i2^{-m-5}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(\right.}{f}\right.$$

input
```
Int[(c + d*x)^m*(a + a*Sin[e + f*x])^2,x]
```

output
```
4*a^2*((3*(c + d*x)^(1 + m))/(8*d*(1 + m)) - (E^(I*(e - (c*f)/d))*(c + d*x
)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(4*f*(((-I)*f*(c + d*x))/d)^m) - (
(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(4*E^(I*(e - (c*f)/d))*f*((I*
f*(c + d*x))/d)^m) + (I*2^(-5 - m)*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gam
ma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (I*2^(-5
 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f
)/d))*f*((I*f*(c + d*x))/d)^m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int (dx+c)^m (a+a\sin(fx+e))^2 \, dx$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.90

$$\int (c+dx)^m (a+a\sin(e+fx))^2 \, dx =$$

$$-\frac{8\,(a^2dm+a^2d)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+i\,de-i\,cf}{d}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,cf}{d}\right)-(i\,a^2dm+i\,a^2d)e^{\left(-\frac{dm\log\left(-\frac{2i\,f}{d}\right)-2i\,de+2i\,cf}{d}\right)}\Gamma\Big(\cdots}{\cdots}$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/8*(8*(a^2*d*m + a^2*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m
+ 1, (I*d*f*x + I*c*f)/d) - (I*a^2*d*m + I*a^2*d)*e^(-(d*m*log(-2*I*f/d) -
 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 8*(a^2*d*m +
 a^2*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)*gamma(m + 1, (-I*d*f*x -
I*c*f)/d) - (-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log(2*I*f/d) + 2*I*d*e - 2*I*c
*f)/d)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d) - 12*(a^2*d*f*x + a^2*c*f)*(d
*x + c)^m)/(d*f*m + d*f)
```

Sympy [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx = a^2\left(\int 2(c+dx)^m\sin(e+fx)\,dx + \int (c+dx)^m\sin^2(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*sin(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*(c + d*x)**m*sin(e + f*x), x) + Integral((c + d*x)**m*sin
(e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

Maxima [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx = \int (a\sin(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + 1/2*(a^2*e^(m*log(d*x + c) + log(d*x +
 c)) - (a^2*d*m + a^2*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + 4*(a
^2*d*m + a^2*d)*integrate((d*x + c)^m*sin(f*x + e), x))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx = \int (a\sin(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx = \int (a+a\,\sin(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sin(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + a*sin(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx$$
$$= \frac{a^2\left((dx+c)^m c+(dx+c)^m dx+\left(\int (dx+c)^m \sin(fx+e)^2\,dx\right)dm+\left(\int (dx+c)^m \sin(fx+e)^2\,dx\right)d \cdots\right.}{d\,(m+1)}$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

output
```
(a**2*((c + d*x)**m*c + (c + d*x)**m*d*x + int((c + d*x)**m*sin(e + f*x)**
2,x)*d*m + int((c + d*x)**m*sin(e + f*x)**2,x)*d + 2*int((c + d*x)**m*sin(
e + f*x),x)*d*m + 2*int((c + d*x)**m*sin(e + f*x),x)*d))/(d*(m + 1))
```

3.148 $\int (c+dx)^m (a+a\sin(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\begin{aligned}\int (c+dx)^m (a+a\sin(e+fx))\,dx &= \frac{a(c+dx)^{1+m}}{d(1+m)} - \frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f}\\ &\quad - \frac{ae^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{2f}\end{aligned}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)-1/2*a*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d
*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-1/2*a*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/e
xp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx=\frac{1}{2}a(c+dx)^m\left(\frac{2c+2dx}{d+dm}-\frac{e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{e^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + a*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^m*((2*c + 2*d*x)/(d + d*m) - (E^(I*(e - (c*f)/d))*Gamma[1 + m
, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - Gamma[1 + m, (I*f*
(c + d*x))/d]/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\sin(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a\sin(e+fx)+a)dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)^m \sin(e+fx) + a(c+dx)^m)\, dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a e^{i\left(e-\frac{cf}{d}\right)} (c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f} - \frac{a e^{-i\left(e-\frac{cf}{d}\right)} (c+dx)^m \left(\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{if(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + a*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) - (a*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gam
ma[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (a*(c +
d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c
+ d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+a\sin(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.92

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx = -\frac{(adm+ad)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+ide-icf}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+(adm+ad)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)-ide+icf}{d}\right)}\Gamma\left(m+1,\frac{-idf}{}\right.}{2\,(dfm+df)}$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-1/2*((a*d*m + a*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (
I*d*f*x + I*c*f)/d) + (a*d*m + a*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/
d)*gamma(m + 1, (-I*d*f*x - I*c*f)/d) - 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(
d*f*m + d*f)
```

Sympy [F]

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx = a\left(\int (c+dx)^m\sin(e+fx)\,dx+\int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*sin(f*x+e)),x)
```

output
```
a*(Integral((c + d*x)**m*sin(e + f*x), x) + Integral((c + d*x)**m, x))
```

Maxima [F]

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx=\int (a\sin(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
a*integrate((d*x + c)^m*sin(f*x + e), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx=\int (a\sin(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx=\int (a+a\,\sin(e+f\,x))\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sin(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + a*sin(e + f*x))*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+a\sin(e+fx))\,dx$$
$$= \frac{a\Big(-(dx+c)^m\cos(fx+e)\,dm-(dx+c)^m\cos(fx+e)\,d+(dx+c)^m\,cf+(dx+c)^m\,dfx-(dx+c)^m}{}$$

input
```
int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

output
```
(a*( - (c + d*x)**m*cos(e + f*x)*d*m - (c + d*x)**m*cos(e + f*x)*d + (c +
d*x)**m*c*f + (c + d*x)**m*d*f*x - (c + d*x)**m*d*m - (c + d*x)**m*d + 2*i
nt((c + d*x)**m/(tan((e + f*x)/2)**2*c + tan((e + f*x)/2)**2*d*x + c + d*x
),x)*d**2*m**2 + 2*int((c + d*x)**m/(tan((e + f*x)/2)**2*c + tan((e + f*x)
/2)**2*d*x + c + d*x),x)*d**2*m))/(d*f*(m + 1))
```

3.149 $\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \text{Int}\left(\frac{(c+dx)^m}{a+a\sin(e+fx)},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + a*Sin[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + a*Sin[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a\sin(e+fx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{a\sin(e+fx)+a}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{a\sin(e+fx)+a}dx$$

input
```
Int[(c + d*x)^m/(a + a*Sin[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+a\sin(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{(c+dx)^m}{\sin(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+a*sin(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(sin(e + f*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx=\int \frac{(dx+c)^m}{a\sin\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx=\int \frac{(dx+c)^m}{a\sin\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 35.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^m}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + a*sin(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + a*sin(e + f*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.70

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \frac{2(dx+c)^m\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\left(\int \frac{(dx+c)^m\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)dx+c+dx}dx\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)dm-2\left(\int \frac{(dx+c)^m\,\mathrm{ta}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+\tan}\right.}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

input
```
int((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

output
```
(2*((c + d*x)**m*tan((e + f*x)/2) - int(((c + d*x)**m*tan((e + f*x)/2))/(t
an((e + f*x)/2)*c + tan((e + f*x)/2)*d*x + c + d*x),x)*tan((e + f*x)/2)*d*
m - int(((c + d*x)**m*tan((e + f*x)/2))/(tan((e + f*x)/2)*c + tan((e + f*x
)/2)*d*x + c + d*x),x)*d*m))/(a*f*(tan((e + f*x)/2) + 1))
```

3.150 $\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \text{Int}\left(\frac{(c+dx)^m}{(a+a\sin(e+fx))^2},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + a*Sin[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + a*Sin[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\sin(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(c+dx)^m}{(a\sin(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{(c+dx)^m}{(a\sin(e+fx)+a)^2}dx$$

input
```
Int[(c + d*x)^m/(a + a*Sin[e + f*x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{(a+a\sin(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(-(d*x + c)^m/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x
)
```

Sympy [N/A]

Not integrable

Time = 10.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{(c+dx)^m}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**m/(a+a*sin(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*sin(f*x + e) + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*sin(f*x + e) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 36.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*sin(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + a*sin(e + f*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{(dx+c)^m}{\sin(fx+e)^2+2\sin(fx+e)+1}dx}{a^2}$$

input
```
int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

output
```
int((c + d*x)**m/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)/a**2
```

3.151 $\int (c+dx)^3(a+b\sin(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3} - \frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4} + \frac{3bd(c+dx)^2\sin(e+fx)}{f^2}$$

output
```
1/4*a*(d*x+c)^4/d+6*b*d^2*(d*x+c)*cos(f*x+e)/f^3-b*(d*x+c)^3*cos(f*x+e)/f-
6*b*d^3*sin(f*x+e)/f^4+3*b*d*(d*x+c)^2*sin(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\cos(e+fx)}{f^3} + \frac{3bd(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\sin(e+fx)}{f^4}$$

input
```
Integrate[(c + d*x)^3*(a + b*Sin[e + f*x]),x]
```

output
```
(a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (b*(c + d*x)*(c^2*f^
2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x])/f^3 + (3*b*d*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a+b\sin(e+fx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\sin(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (b*(c + d*x
)^3*Cos[e + f*x])/f - (6*b*d^3*Sin[e + f*x])/f^4 + (3*b*d*(c + d*x)^2*Sin[
e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{-(dx+c)fb\left((dx+c)^2f^2-6d^2\right)\cos(fx+e)+3\left((dx+c)^2f^2-2d^2\right)db\sin(fx+e)+\left(xa\left(\frac{dx}{2}+c\right)\left(\frac{1}{2}x^2d^2+cdx+c^2\right)f^3+bc^3\right.}{f^4}$
risch	$\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}-\frac{b\left(d^3f^2x^3+3c\,d^2f^2x^2+3c^2d\,f^2x+c^3f^2-6d^3x-6c\,d^2\right)\cos(f}{f^3}$
norman	$\frac{\frac{\left(c^3a\,f^3-3b\,c^2d\,f^2+6b\,d^3\right)x}{f^3}+\frac{\left(2b\,c^3f^2-12c\,d^2b\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f^3}+\frac{d^2(acf-bd)x^3}{f}+\frac{\left(c^3a\,f^3+3b\,c^2d\,f^2-6b\,d^3\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f^3}+}{}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{b\left(\frac{d^3\left(-(fx+e)^3\cos(fx+e)+3(fx+e)^2\sin(fx+e)-6\sin(fx+e)+6(fx+e)\cos(fx+e)\right)}{f^3}+\frac{3c\,d^2\left(-(fx+e)^2\cos(fx+e\right.}{}\right.}{}$
orering	$\frac{\left(d^5f^4x^6+6c\,d^4f^4x^5+15c^2d^3f^4x^4+20c^3d^2f^4x^3+14c^4d\,f^4x^2+24d^5f^2x^4+4c^5f^4x+96c\,d^4f^2x^3+156c^2d^3f^2x^2+120c^3\right.}{4f^4(dx+c)^2}$
derivativedivides	$\frac{c^3a(fx+e)-\frac{3a\,c^2de(fx+e)}{f}+\frac{3a\,c^2d(fx+e)^2}{2f}+\frac{3ac\,d^2e^2(fx+e)}{f^2}-\frac{3ac\,d^2e(fx+e)^2}{f^2}+\frac{ac\,d^2(fx+e)^3}{f^2}-\frac{a\,d^3e^3(fx+e)}{f^3}+\frac{3a\,d^3e^2(f}{2f^3}}{}$
default	$\frac{c^3a(fx+e)-\frac{3a\,c^2de(fx+e)}{f}+\frac{3a\,c^2d(fx+e)^2}{2f}+\frac{3ac\,d^2e^2(fx+e)}{f^2}-\frac{3ac\,d^2e(fx+e)^2}{f^2}+\frac{ac\,d^2(fx+e)^3}{f^2}-\frac{a\,d^3e^3(fx+e)}{f^3}+\frac{3a\,d^3e^2(f}{2f^3}}{}$

input
```
int((d*x+c)^3*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(-(d*x+c)*f*b*((d*x+c)^2*f^2-6*d^2)*cos(f*x+e)+3*((d*x+c)^2*f^2-2*d^2)*d*b
*sin(f*x+e)+(x*a*(1/2*d*x+c)*(1/2*x^2*d^2+c*d*x+c^2)*f^3+b*c^3*f^2-6*c*d^2
*b)*f)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx$$
$$= \frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 4\left(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + bc^3f^3 - 6\,bcd^2f + 3\left(bc^2df^3\right.\right.}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 - 6*b*c*d^2*f + 3*(b*c
^2*d*f^3 - 2*b*d^3*f)*x)*cos(f*x + e) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*
x + b*c^2*d*f^2 - 2*b*d^3)*sin(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.93

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx$$
$$= \begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} - \frac{bc^3\cos(e+fx)}{f} - \frac{3bc^2dx\cos(e+fx)}{f} + \frac{3bc^2d\sin(e+fx)}{f^2} - \frac{3bcd^2x^2\cos(e+fx)}{f} + \frac{6bcd}{} \\ (a+b\sin(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+b*sin(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 -
b*c**3*cos(e + f*x)/f - 3*b*c**2*d*x*cos(e + f*x)/f + 3*b*c**2*d*sin(e + f
*x)/f**2 - 3*b*c*d**2*x**2*cos(e + f*x)/f + 6*b*c*d**2*x*sin(e + f*x)/f**2
 + 6*b*c*d**2*cos(e + f*x)/f**3 - b*d**3*x**3*cos(e + f*x)/f + 3*b*d**3*x*
*2*sin(e + f*x)/f**2 + 6*b*d**3*x*cos(e + f*x)/f**3 - 6*b*d**3*sin(e + f*x
)/f**4, Ne(f, 0)), ((a + b*sin(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(88) = 176$.

Time = 0.05 (sec) , antiderivative size = 462, normalized size of antiderivative = 5.13

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx$$

$$= \frac{4(fx+e)ac^3 + \frac{(fx+e)^4ad^3}{f^3} - \frac{4(fx+e)^3ad^3e}{f^3} + \frac{6(fx+e)^2ad^3e^2}{f^3} - \frac{4(fx+e)ad^3e^3}{f^3} + \frac{4(fx+e)^3acd^2}{f^2} - \frac{12(fx+e)^2acd^2e}{f^2} + \frac{12}{}}{}$$

input
```
integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
 + 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3
*a*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2
 + 6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f - 4*b*c^3*cos(f*x +
e) + 4*b*d^3*e^3*cos(f*x + e)/f^3 - 12*b*c*d^2*e^2*cos(f*x + e)/f^2 + 12*b
*c^2*d*e*cos(f*x + e)/f - 12*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*d^3
*e^2/f^3 + 24*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*c*d^2*e/f^2 - 12*(
(f*x + e)*cos(f*x + e) - sin(f*x + e))*b*c^2*d/f + 12*(((f*x + e)^2 - 2)*c
os(f*x + e) - 2*(f*x + e)*sin(f*x + e))*b*d^3*e/f^3 - 12*(((f*x + e)^2 - 2
)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*b*c*d^2/f^2 - 4*(((f*x + e)^3 -
 6*f*x - 6*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*b*d^3/f^3)/
f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x - \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x + bc^3f^3 - 6\,bd^3fx - 6\,bcd^2f)\cos(fx+e)}{f^4} + \frac{3\,(bd^3f^2x^2 + 2\,bcd^2f^2x + bc^2df^2 - 2\,bd^3)\sin(fx+e)}{f^4}$$

input

```
integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

output

```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (b*d^3*f^3*x^3 +
 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*c^3*f^3 - 6*b*d^3*f*x - 6*b*c*d^2
*f)*cos(f*x + e)/f^4 + 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 -
2*b*d^3)*sin(f*x + e)/f^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \frac{a\,d^3\,x^4}{4} - \frac{3\,\sin(e+f\,x)\,\left(2\,b\,d^3 - b\,c^2\,d\,f^2\right)}{f^4} - \frac{\cos(e+f\,x)\,\left(b\,c^3\,f^2 - 6\,b\,c\,d^2\right)}{f^3} + a\,c^3\,x + \frac{3\,x\,\cos(e+f\,x)\,\left(2\,b\,d^3 - b\,c^2\,d\,f^2\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{b\,d^3\,x^3\,\cos(e+f\,x)}{f} + \frac{3\,b\,d^3\,x^2\,\sin(e+f\,x)}{f^2} + \frac{6\,b\,c\,d^2\,x\,\sin(e+f\,x)}{f^2} - \frac{3\,b\,c\,d^2\,x^2\,\cos(e+f\,x)}{f}$$

input

```
int((a + b*sin(e + f*x))*(c + d*x)^3,x)
```

output
```
(a*d^3*x^4)/4 - (3*sin(e + f*x)*(2*b*d^3 - b*c^2*d*f^2))/f^4 - (cos(e + f*
x)*(b*c^3*f^2 - 6*b*c*d^2))/f^3 + a*c^3*x + (3*x*cos(e + f*x)*(2*b*d^3 - b
*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (b*d^3*x^3*cos(e + f*
x))/f + (3*b*d^3*x^2*sin(e + f*x))/f^2 + (6*b*c*d^2*x*sin(e + f*x))/f^2 -
(3*b*c*d^2*x^2*cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.37

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \frac{-4\cos(fx+e)\,b\,c^3f^3-12\cos(fx+e)\,b\,c^2d\,f^3x-12\cos(fx+e)\,bc\,d^2f^3x^2+24\cos(fx+e)\,bc\,d^2f-}{}$$

input
```
int((d*x+c)^3*(a+b*sin(f*x+e)),x)
```

output
```
( - 4*cos(e + f*x)*b*c**3*f**3 - 12*cos(e + f*x)*b*c**2*d*f**3*x - 12*cos(
e + f*x)*b*c*d**2*f**3*x**2 + 24*cos(e + f*x)*b*c*d**2*f - 4*cos(e + f*x)*
b*d**3*f**3*x**3 + 24*cos(e + f*x)*b*d**3*f*x + 12*sin(e + f*x)*b*c**2*d*f
**2 + 24*sin(e + f*x)*b*c*d**2*f**2*x + 12*sin(e + f*x)*b*d**3*f**2*x**2 -
 24*sin(e + f*x)*b*d**3 + 4*a*c**3*f**4*x + 6*a*c**2*d*f**4*x**2 + 4*a*c*d
**2*f**4*x**3 + a*d**3*f**4*x**4)/(4*f**4)
```

3.152 $\int (c+dx)^2(a+b\sin(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \frac{a(c+dx)^3}{3d}+\frac{2bd^2\cos(e+fx)}{f^3}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}$$

output
```
1/3*a*(d*x+c)^3/d+2*b*d^2*cos(f*x+e)/f^3-b*(d*x+c)^2*cos(f*x+e)/f+2*b*d*(d
*x+c)*sin(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \frac{1}{3}ax(3c^2+3cdx+d^2x^2)-\frac{b(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^3}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}$$

input
```
Integrate[(c + d*x)^2*(a + b*Sin[e + f*x]),x]
```

output
```
(a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2
+ f^2*x^2))*Cos[e + f*x])/f^3 + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)^2(a+b\sin(e+fx))dx$$
$$\Big\downarrow \text{ 3798}$$
$$\int \left(a(c+dx)^2+b(c+dx)^2\sin(e+fx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a(c+dx)^3}{3d}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd^2\cos(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cos[e + f*x])/f^3 - (b*(c + d*x)^2*Cos[e
+ f*x])/f + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{-\left(\left((dx+c)^2 f^2-2d^2\right)b\cos(fx+e)+2bdf(dx+c)\sin(fx+e)+xa\left(\frac{1}{3}x^2d^2+cdx+c^2\right)f^3-b\,c^2f^2+2d^2b\right.}{f^3}$
risch	$\frac{d^2a\,x^3}{3}+adc\,x^2+a\,c^2x+\frac{a\,c^3}{3d}-\frac{b(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2)\cos(fx+e)}{f^3}+\frac{2bd(dx+c)\sin(fx+e)}{f^2}$
parts	$\frac{a(dx+c)^3}{3d}+\frac{b\left(\frac{d^2\left(-(fx+e)^2\cos(fx+e)+2\cos(fx+e)+2(fx+e)\sin(fx+e)\right)}{f^2}+\frac{2cd(\sin(fx+e)-(fx+e)\cos(fx+e))}{f}-\frac{2d^2e(\sin(f\right.}{f}$
norman	$\frac{\frac{\left(2b\,c^2f^2-4d^2b\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f^3}+\frac{c(acf-2bd)x}{f}+\frac{d(acf-bd)x^2}{f}+\frac{c(acf+2bd)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}+\frac{d(acf+bd)x^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}+\frac{d^2a}{3}}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}$
derivativedivides	$a\,c^2(fx+e)-\frac{2acde(fx+e)}{f}+\frac{acd(fx+e)^2}{f}+\frac{a\,d^2e^2(fx+e)}{f^2}-\frac{a\,d^2e(fx+e)^2}{f^2}+\frac{a\,d^2(fx+e)^3}{3f^2}-b\,c^2\cos(fx+e)+\frac{2bcde\cos(fx+e)}{f}+$
default	$a\,c^2(fx+e)-\frac{2acde(fx+e)}{f}+\frac{acd(fx+e)^2}{f}+\frac{a\,d^2e^2(fx+e)}{f^2}-\frac{a\,d^2e(fx+e)^2}{f^2}+\frac{a\,d^2(fx+e)^3}{3f^2}-b\,c^2\cos(fx+e)+\frac{2bcde\cos(fx+e)}{f}+$
orering	$\frac{(d^4f^4x^5+5c\,d^3f^4x^4+10c^2d^2f^4x^3+9c^3d\,f^4x^2+3c^4f^4x+12d^4f^2x^3+42c\,d^3f^2x^2+48c^2d^2f^2x+12c^3d\,f^2-48d^4x-12d^3}{3f^4(dx+c)^2}$

input
```
int((d*x+c)^2*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(-((d*x+c)^2*f^2-2*d^2)*b*cos(f*x+e)+2*b*d*f*(d*x+c)*sin(f*x+e)+x*a*(1/3*x
^2*d^2+c*d*x+c^2)*f^3-b*c^2*f^2+2*d^2*b)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x-3\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2-2\,bd^2\right)\cos\left(fx+e\right)+6\left(bd^2fx+bcdf\right.}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 3*(b*d^2*f^2*x^2 +
2*b*c*d*f^2*x + b*c^2*f^2 - 2*b*d^2)*cos(f*x + e) + 6*(b*d^2*f*x + b*c*d*f
)*sin(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(65) = 130.

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.22

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \begin{cases} ac^2x+acdx^2+\frac{ad^2x^3}{3}-\frac{bc^2\cos(e+fx)}{f}-\frac{2bcdx\cos(e+fx)}{f}+\frac{2bcd\sin(e+fx)}{f^2}-\frac{bd^2x^2\cos(e+fx)}{f}+\frac{2bd^2x\sin(e+fx)}{f^2}+\frac{2b}{} \\ (a+b\sin(e))\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*sin(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 - b*c**2*cos(e + f*x)/f -
 2*b*c*d*x*cos(e + f*x)/f + 2*b*c*d*sin(e + f*x)/f**2 - b*d**2*x**2*cos(e
+ f*x)/f + 2*b*d**2*x*sin(e + f*x)/f**2 + 2*b*d**2*cos(e + f*x)/f**3, Ne(f
, 0)), ((a + b*sin(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.51

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx$$
$$= \frac{3\,(fx+e)ac^2 + \frac{(fx+e)^3ad^2}{f^2} - \frac{3\,(fx+e)^2ad^2e}{f^2} + \frac{3\,(fx+e)ad^2e^2}{f^2} + \frac{3\,(fx+e)^2acd}{f} - \frac{6\,(fx+e)acde}{f} - 3\,bc^2\cos{(fx+e)} -}{}$$

input
```
integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
 + 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e
/f - 3*b*c^2*cos(f*x + e) - 3*b*d^2*e^2*cos(f*x + e)/f^2 + 6*b*c*d*e*cos(f
*x + e)/f + 6*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*d^2*e/f^2 - 6*((f*
x + e)*cos(f*x + e) - sin(f*x + e))*b*c*d/f - 3*(((f*x + e)^2 - 2)*cos(f*x
 + e) - 2*(f*x + e)*sin(f*x + e))*b*d^2/f^2)/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x - \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2)\cos{(fx+e)}}{f^3} + \frac{2\,(bd^2fx + bcdf)\sin{(fx+e)}}{f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c
^2*f^2 - 2*b*d^2)*cos(f*x + e)/f^3 + 2*(b*d^2*f*x + b*c*d*f)*sin(f*x + e)/
f^3
```

Mupad [B] (verification not implemented)

Time = 35.87 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \frac{a\,d^2\,x^3}{3}+\frac{\cos\left(e+f\,x\right)\,\left(2\,b\,d^2-b\,c^2\,f^2\right)}{f^3}+a\,c^2\,x+a\,c\,d\,x^2+\frac{2\,b\,d^2\,x\,\sin\left(e+f\,x\right)}{f^2}-\frac{b\,d^2\,x^2\,\cos\left(e+f\,x\right)}{f}+\frac{2\,b\,c\,d\,\sin\left(e+f\,x\right)}{f^2}-\frac{2\,b\,c\,d\,x\,\cos\left(e+f\,x\right)}{f}$$

input
```
int((a + b*sin(e + f*x))*(c + d*x)^2,x)
```

output
```
(a*d^2*x^3)/3 + (cos(e + f*x)*(2*b*d^2 - b*c^2*f^2))/f^3 + a*c^2*x + a*c*d
*x^2 + (2*b*d^2*x*sin(e + f*x))/f^2 - (b*d^2*x^2*cos(e + f*x))/f + (2*b*c*
d*sin(e + f*x))/f^2 - (2*b*c*d*x*cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int (c+dx)^2(a+b\sin(e+fx))\,dx = \frac{-3\cos\left(fx+e\right)b\,c^2f^2-6\cos\left(fx+e\right)bcd\,f^2x-3\cos\left(fx+e\right)b\,d^2f^2x^2+6\cos\left(fx+e\right)b\,d^2+6\sin\left(fx\right.}{3f^3}$$

input
```
int((d*x+c)^2*(a+b*sin(f*x+e)),x)
```

output
```
( - 3*cos(e + f*x)*b*c**2*f**2 - 6*cos(e + f*x)*b*c*d*f**2*x - 3*cos(e + f
*x)*b*d**2*f**2*x**2 + 6*cos(e + f*x)*b*d**2 + 6*sin(e + f*x)*b*c*d*f + 6*
sin(e + f*x)*b*d**2*f*x + 3*a*c**2*f**3*x + 3*a*c*d*f**3*x**2 + a*d**2*f**
3*x**3)/(3*f**3)
```

3.153 $\int (c+dx)(a+b\sin(e+fx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int (c+dx)(a+b\sin(e+fx))\,dx = \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2}$$

output
```
1/2*a*(d*x+c)^2/d-b*(d*x+c)*cos(f*x+e)/f+b*d*sin(f*x+e)/f^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+b\sin(e+fx))\,dx = \frac{1}{2}ax(2c+dx) - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Sin[e + f*x]),x]
```

output
```
(a*x*(2*c + d*x))/2 - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/f^
2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a+b\sin(e+fx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)+b(c+dx)\sin(e+fx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)\cos(e+fx)}{f}+\frac{bd\sin(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/
f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
risch	$\frac{a d \,x^{2}}{2}+a c x -\frac{b \left(d x +c \right) \cos \left(f x +e \right)}{f}+\frac{b d \sin \left(f x +e \right)}{f^{2}}$
parallelrisch	$\frac{-\left(d x +c \right) b f \cos \left(f x +e \right)+b d \sin \left(f x +e \right)+f \left(a x \left(\frac{d x}{2}+c \right) f -b c \right)}{f^{2}}$
parts	$a \left(\frac{1}{2} d \,x^{2}+c x \right)+\frac{b \left(\frac{d \left(\sin \left(f x +e \right)-\left(f x +e \right) \cos \left(f x +e \right)\right)}{f}-c \cos \left(f x +e \right)+\frac{d e \cos \left(f x +e \right)}{f}\right)}{f}$
derivativedivides	$\frac{a c \left(f x +e \right)-\frac{a d e \left(f x +e \right)}{f}+\frac{a d \left(f x +e \right)^{2}}{2 f}-b c \cos \left(f x +e \right)+\frac{b d e \cos \left(f x +e \right)}{f}+\frac{b d \left(\sin \left(f x +e \right)-\left(f x +e \right) \cos \left(f x +e \right)\right)}{f}}{f}$
default	$\frac{a c \left(f x +e \right)-\frac{a d e \left(f x +e \right)}{f}+\frac{a d \left(f x +e \right)^{2}}{2 f}-b c \cos \left(f x +e \right)+\frac{b d e \cos \left(f x +e \right)}{f}+\frac{b d \left(\sin \left(f x +e \right)-\left(f x +e \right) \cos \left(f x +e \right)\right)}{f}}{f}$
norman	$\frac{\frac{2 b c \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{f}+\frac{\left(a c f -b d \right) x}{f}+\frac{\left(a c f +b d \right) x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{f}+\frac{a d \,x^{2}}{2}+\frac{a d \,x^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2}+\frac{2 b d \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{f^{2}}}{1+\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}$
orering	$\frac{\left(d^{3} f^{2} x^{4}+4 c \,d^{2} f^{2} x^{3}+5 c^{2} d \,f^{2} x^{2}+2 c^{3} f^{2} x +6 d^{3} x^{2}+12 c \,d^{2} x +4 c^{2} d \right) \left(a +b \sin \left(f x +e \right)\right)}{2 \left(d x +c \right)^{2} f^{2}}-\frac{\left(2 x^{2} d^{2}+4 c d x +c^{2}\right) \left(d (a+b \sin(\ldots\right.}{(dx+\ldots}$

input
```
int((d*x+c)*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d*x^2+a*c*x-b*(d*x+c)*cos(f*x+e)/f+b*d*sin(f*x+e)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c + dx)(a + b\sin(e + fx))\, dx = \frac{adf^2x^2 + 2\,acf^2x + 2\,bd\sin\left(fx + e\right) - 2\left(bdfx + bcf\right)\cos\left(fx + e\right)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*b*d*sin(f*x + e) - 2*(b*d*f*x + b*c*f)*
cos(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int (c + dx)(a + b\sin(e + fx))\, dx = \begin{cases} acx + \frac{adx^2}{2} - \frac{bc\cos\left(e+fx\right)}{f} - \frac{bdx\cos\left(e+fx\right)}{f} + \frac{bd\sin\left(e+fx\right)}{f^2} & \text{for } f \neq 0 \\ \left(a + b\sin\left(e\right)\right)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e)),x)
```

output
```
Piecewise((a*c*x + a*d*x**2/2 - b*c*cos(e + f*x)/f - b*d*x*cos(e + f*x)/f
+ b*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a + b*sin(e))*(c*x + d*x**2/2), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(43) = 86$.

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.07

$$\int (c+dx)(a+b\sin(e+fx))\,dx = \frac{2\,(fx+e)ac + \frac{(fx+e)^2ad}{f} - \frac{2\,(fx+e)ade}{f} - 2\,bc\cos\left(fx+e\right) + \frac{2\,bde\cos(fx+e)}{f} - \frac{2\,((fx+e)\cos(fx+e)-\sin(fx+e))bd}{f}}{2\,f}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f - 2*b*c*cos
(f*x + e) + 2*b*d*e*cos(f*x + e)/f - 2*((f*x + e)*cos(f*x + e) - sin(f*x +
 e))*b*d/f)/f
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (c+dx)(a+b\sin(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{bd\sin\left(fx+e\right)}{f^2} - \frac{(bdfx+bcf)\cos\left(fx+e\right)}{f^2}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + b*d*sin(f*x + e)/f^2 - (b*d*f*x + b*c*f)*cos(f*x + e
)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (c+dx)(a+b\sin(e+fx))\,dx$$
$$=a\,c\,x-\frac{f\,\left(b\,c\,\cos\left(e+f\,x\right)+b\,d\,x\,\cos\left(e+f\,x\right)\right)-b\,d\,\sin\left(e+f\,x\right)}{f^2}+\frac{a\,d\,x^2}{2}$$

input
```
int((a + b*sin(e + f*x))*(c + d*x),x)
```

output
```
a*c*x - (f*(b*c*cos(e + f*x) + b*d*x*cos(e + f*x)) - b*d*sin(e + f*x))/f^2
 + (a*d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int (c+dx)(a+b\sin(e+fx))\,dx$$
$$=\frac{-2\cos\left(fx+e\right)bcf-2\cos\left(fx+e\right)bdfx+2\sin\left(fx+e\right)bd+2ac\,f^{2}x+ad\,f^{2}x^{2}}{2f^{2}}$$

input
```
int((d*x+c)*(a+b*sin(f*x+e)),x)
```

output
```
( - 2*cos(e + f*x)*b*c*f - 2*cos(e + f*x)*b*d*f*x + 2*sin(e + f*x)*b*d + 2
*a*c*f**2*x + a*d*f**2*x**2)/(2*f**2)
```

3.154 $\int \frac{a+b\sin(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)}{d}+\frac{b\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*ln(d*x+c)/d-b*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d+b*cos(-e+c*f/d)*Si(c*f/d+f*x
)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)+b\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

input
```
Integrate[(a + b*Sin[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x] + b*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + b*Cos[e -
(c*f)/d]*SinIntegral[f*(c/d + x)])/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(e+fx)}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{c+dx}+\frac{b\sin(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(c+dx)}{d}+\frac{b\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}$$

input
```
Int[(a + b*Sin[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x])/d + (b*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (
b*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

method	result
parts	$\frac{a\ln(dx+c)}{d}+b\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)$
derivativedivides	$\frac{\frac{fa\ln(cf-de+d(fx+e))}{d}+fb\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)}{f}$
default	$\frac{\frac{fa\ln(cf-de+d(fx+e))}{d}+fb\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)}{f}$
risch	$\frac{a\ln(dx+c)}{d}-\frac{ib\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}+\frac{ib\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$

input
```
int((a+b*sin(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+b*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*
e)/d)*sin((c*f-d*e)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx = -\frac{b\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)-b\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)-a\log\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
-(b*cos_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) - b*cos(-(d*e - c*f)
/d)*sin_integral((d*f*x + c*f)/d) - a*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx = \int \frac{a+b\sin\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*sin(e + f*x))/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.67

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx = \frac{\frac{2\,af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{\left(f\left(-i\,E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_1\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_1\left(\right.\right.}{d}}{2\,f}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(-I*exp_integral_e(1, (I*
(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d - I*
d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d
 - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/
d))*sin(-(d*e - c*f)/d))*b/d)/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 693, normalized size of antiderivative = 10.83

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2
- b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 +
2*a*log(abs(d*x + c))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 2*b*sin_integral((d*
f*x + c*f)/d)*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 2*b*real_part(cos_integral(f
*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) + 2*b*real_part(cos_integral(-f*x
 - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) - 2*b*real_part(cos_integral(f*x +
c*f/d))*tan(1/2*e)*tan(1/2*c*f/d)^2 - 2*b*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*e)*tan(1/2*c*f/d)^2 - b*imag_part(cos_integral(f*x + c*f/d))
*tan(1/2*e)^2 + b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 + 2*a
*log(abs(d*x + c))*tan(1/2*e)^2 - 2*b*sin_integral((d*f*x + c*f)/d)*tan(1/
2*e)^2 + 4*b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(1/2*c*f/d
) - 4*b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(1/2*c*f/d) +
8*b*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(1/2*c*f/d) - b*imag_part(
cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + b*imag_part(cos_integral(-f*
x - c*f/d))*tan(1/2*c*f/d)^2 + 2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2 - 2*
b*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 + 2*b*real_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*e) + 2*b*real_part(cos_integral(-f*x - c*f/d))*t
an(1/2*e) - 2*b*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*b*
real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + b*imag_part(cos_int
egral(f*x + c*f/d)) - b*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*log...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx=\int \frac{a+b\,\sin\left(e+f\,x\right)}{c+d\,x}\,dx$$

input
```
int((a + b*sin(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*sin(e + f*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a+b\sin(e+fx)}{c+dx}\,dx = \frac{\left(\int \frac{\sin(fx+e)}{dx+c}dx\right)bd+\log(dx+c)\,a}{d}$$

input
```
int((a+b*sin(f*x+e))/(d*x+c),x)
```

output
```
(int(sin(e + f*x)/(c + d*x),x)*b*d + log(c + d*x)*a)/d
```

3.155 $\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx=-\frac{a}{d(c+dx)}+\frac{bf\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)+b*f*cos(-e+c*f/d)*Ci(c*f/d+f*x)/d^2-b*sin(f*x+e)/d/(d*x+c)+b*
f*sin(-e+c*f/d)*Si(c*f/d+f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx=\frac{bf\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(a+b\sin(e+fx))}{c+dx}-bf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[(a + b*Sin[e + f*x])/(c + d*x)^2,x]
```

output
```
(b*f*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] - (d*(a + b*Sin[e + f*x]))/
(c + d*x) - b*f*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)])/d^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{ 3798}$$
$$\int \left(\frac{a}{(c+dx)^2}+\frac{b\sin(e+fx)}{(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a}{d(c+dx)}+\frac{bf\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Sin[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) + (b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
 - (b*Sin[e + f*x])/(d*(c + d*x)) - (b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f
)/d + f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{a}{d(dx+c)}+bf\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)$
derivativedivides	$\frac{-\frac{a f^2}{(cf-de+d(fx+e))d}+f^2 b\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)}{f}$
default	$\frac{-\frac{a f^2}{(cf-de+d(fx+e))d}+f^2 b\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)}{f}$
risch	$-\frac{a}{d(dx+c)}-\frac{fb\,e^{\frac{i(cf-de)}{d}}\operatorname{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^2}-\frac{bf\,e^{-\frac{i(cf-de)}{d}}\operatorname{expIntegral}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d^2}$

input
```
int((a+b*sin(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+b*f*(-sin(f*x+e)/(c*f-d*e+d*(f*x+e))/d+(Si(f*x+e+(c*f-d*e)/d)
*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx = \frac{(bdfx+bcf)\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)-bd\sin\left(fx+e\right)+(bdfx+bcf)\sin\left(-\frac{de-cf}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)-ad}{d^3x+cd^2}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b*d*f*x + b*c*f)*cos(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) - b*d
*sin(f*x + e) + (b*d*f*x + b*c*f)*sin(-(d*e - c*f)/d)*sin_integral((d*f*x
+ c*f)/d) - a*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\sin\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)**2,x)
```

output
```
Integral((a + b*sin(e + f*x))/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx = -\frac{\frac{2\,af^2}{(fx+e)d^2-d^2e+cdf}-\frac{\left(f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_2\left(\right.\right.}{(fx+e)d^2-d^2e+cdf}}{2\,f}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - (f^2*(-I*exp_integral_e(2,
 (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d
- I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(f*x
+ e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I
*c*f)/d))*sin(-(d*e - c*f)/d))*b/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(89) = 178$.

Time = 0.56 (sec) , antiderivative size = 533, normalized size of antiderivative = 6.06

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx = \frac{\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)-def^2\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{(d}{}\right.\right.}{} - \frac{a}{(dx+c)d}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c*f)/d)*cos
_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) -
 d*e*f^2*cos(-(d*e - c*f)/d)*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f) - d*e + c*f)/d) + c*f^3*cos(-(d*e - c*f)/d)*cos_integral(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + (d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*sin(-(d*e - c*f)/d)*sin_integral(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - d*e*f^2*sin
(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d*e + c*f)/d) + c*f^3*sin(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*
e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + d*f^2*sin(-(d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f)/d))*b*d^2/(((d*x + c)*d^4*(d*e/(d*x + c)
 - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + b*sin(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*sin(e + f*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx = \frac{\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)b\,c^2+\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)bcdx+ax}{c\,(dx+c)}$$

input
```
int((a+b*sin(f*x+e))/(d*x+c)^2,x)
```

output
```
(int(sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c**2 + int(sin(e + f*x
)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c*d*x + a*x)/(c*(c + d*x))
```

3.156 $\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)} - \frac{bf^2\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3} - \frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*b*f*cos(f*x+e)/d^2/(d*x+c)+1/2*b*f^2*Ci(c*f/d+f*x)*
sin(-e+c*f/d)/d^3-1/2*b*sin(f*x+e)/d/(d*x+c)^2-1/2*b*f^2*cos(-e+c*f/d)*Si(
c*f/d+f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx = -\frac{bf^2\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+\frac{d(bf(c+dx)\cos(e+fx)+d(a+b\sin(e+fx)))}{(c+dx)^2}+bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}\right.\right.}{2d^3}$$

input
```
Integrate[(a + b*Sin[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*(b*f^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + (d*(b*f*(c + d*x)*
Cos[e + f*x] + d*(a + b*Sin[e + f*x])))/(c + d*x)^2 + b*f^2*Cos[e - (c*f)/
d]*SinIntegral[f*(c/d + x)])/d^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{(c+dx)^3}+\frac{b\sin(e+fx)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2d(c+dx)^2}-\frac{bf^2\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Sin[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (b*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (b*f^2*Cos
Integral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (b*Sin[e + f*x])/(2*d*
(c + d*x)^2) - (b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

method	result
parts	$-\frac{a}{2d(dx+c)^2}+b f^2\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e\right.}{d}}{2d}\right.$
derivativedivides	$\frac{-\frac{f^3 a}{2(cf-de+d(fx+e))^2 d}+f^3 b\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\right.}{d}}{2d}\right.}{f}$
default	$\frac{-\frac{f^3 a}{2(cf-de+d(fx+e))^2 d}+f^3 b\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2 d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\right.}{d}}{2d}\right.}{f}$
risch	$-\frac{a}{2d(dx+c)^2}+\frac{i f^2 b\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{4d^3}-\frac{i f^2 b\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(-ifx-ie-\frac{icf-}{}\right.}{4d^3}$

input
```
int((a+b*sin(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2+b*f^2*(-1/2*sin(f*x+e)/(c*f-d*e+d*(f*x+e))^2/d+1/2*(-co
s(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-C
i(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx = -\frac{bd^2\sin\left(fx+e\right)+ad^2-\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)+\left(bd^2f^2x^2+2\,bcdf^2x\right.}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(b*d^2*sin(f*x + e) + a*d^2 - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*
f^2)*cos_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) + (b*d^2*f^2*x^2 +
2*b*c*d*f^2*x + b*c^2*f^2)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/
d) + (b*d^2*f*x + b*c*d*f)*cos(f*x + e))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx = \int \frac{a+b\sin\left(e+fx\right)}{\left(c+dx\right)^3}\,dx$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)**3,x)
```

output
```
Integral((a + b*sin(e + f*x))/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.15

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx =$$

$$-\frac{\frac{af^3}{(fx+e)^2d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\,(d^3e-cd^2f)(fx+e)} - \frac{\left(f^3\left(-i\,E_3\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_3\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)}{(fx+e)^2d^3+d^3e^2-2\,cd^2ef+}}{2\,f}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*
e - c*d^2*f)*(f*x + e)) - (f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d - I*d*
e + I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*c
os(-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e + I*c*f
)/d) + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(d*e -
c*f)/d))*b/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e
 - c*d^2*f)*(f*x + e)))/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 6033, normalized size of antiderivative = 49.05

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*ta
n(1/2*e)^2*tan(1/2*c*f/d)^2 - b*d^2*f^2*x^2*imag_part(cos_integral(-f*x -
c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 2*b*d^2*f^2*x^2*sin
_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 +
2*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/
2*e)^2*tan(1/2*c*f/d) + 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/
d))*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d) - 2*b*d^2*f^2*x^2*real_part
(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)*tan(1/2*c*f/d)^2 - 2
*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/
2*e)*tan(1/2*c*f/d)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))
*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 - 2*b*c*d*f^2*x*imag_part(co
s_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + 4
*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2*tan
(1/2*c*f/d)^2 - b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*f*x)^2*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))
*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/
d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*b*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)*tan(1/2*c*f/d) - 4*b*d^2*f^2*x^2*ima
g_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)*tan(1/2*c*f/d
) + 8*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx = \int \frac{a+b\,\sin\left(e+f\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a + b*sin(e + f*x))/(c + d*x)^3,x)
```

output
```
int((a + b*sin(e + f*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx$$
$$= \frac{2\left(\int \frac{\sin(fx+e)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b\,c^2d+4\left(\int \frac{\sin(fx+e)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)bc\,d^2x+2\left(\int \frac{\sin(fx+e)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b\,d}{2d\,(d^2x^2+2cdx+c^2)}$$

input
```
int((a+b*sin(f*x+e))/(d*x+c)^3,x)
```

output
```
(2*int(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b*c
**2*d + 4*int(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
,x)*b*c*d**2*x + 2*int(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)*b*d**3*x**2 - a)/(2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.157 $\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 237

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx = -\frac{3b^2d(c+dx)^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\cos(e+fx)}{f^3}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-\frac{12abd^3\sin(e+fx)}{f^4}+\frac{6abd(c+dx)^2\sin(e+fx)}{f^2}+\frac{3b^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3}-\frac{b^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f}-\frac{3b^2d^3\sin^2(e+fx)}{8f^4}+\frac{3b^2d(c+dx)^2\sin^2(e+fx)}{4f^2}$$

output
```
-3/8*b^2*d*(d*x+c)^2/f^2+1/4*a^2*(d*x+c)^4/d+1/8*b^2*(d*x+c)^4/d+12*a*b*d^
2*(d*x+c)*cos(f*x+e)/f^3-2*a*b*(d*x+c)^3*cos(f*x+e)/f-12*a*b*d^3*sin(f*x+e
)/f^4+6*a*b*d*(d*x+c)^2*sin(f*x+e)/f^2+3/4*b^2*d^2*(d*x+c)*cos(f*x+e)*sin(
f*x+e)/f^3-1/2*b^2*(d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f-3/8*b^2*d^3*sin(f*x+e
)^2/f^4+3/4*b^2*d*(d*x+c)^2*sin(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.98

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$$
$$= \frac{2(2a^2+b^2)\,f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-32abf(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\cos(e+\cdots}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*Sin[e + f*x])^2,x]
```

output
```
(2*(2*a^2 + b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 32*a*
b*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x] -
3*b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)]
+ 96*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x] - 2*b
^2*f*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e +
 f*x)])/(16*f^4)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3(a+b\sin(e+fx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\sin(e+fx)+b^2(c+dx)^3\sin^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cos(e+fx)}{f^3}+\frac{6abd(c+dx)^2\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-\frac{12abd^3\sin(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}+\frac{3b^2d(c+dx)^2\sin^2(e+fx)}{4f^2}-\frac{b^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f}-\frac{3b^2d(c+dx)^2}{8f^2}+\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\sin^2(e+fx)}{8f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Sin[e + f*x])^2,x]
```

output
```
(-3*b^2*d*(c + d*x)^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) + (b^2*(c + d*x)^
4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*a*b*(c + d*x)^3*Co
s[e + f*x])/f - (12*a*b*d^3*Sin[e + f*x])/f^4 + (6*a*b*d*(c + d*x)^2*Sin[e
 + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (
b^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*b^2*d^3*Sin[e + f*x]
^2)/(8*f^4) + (3*b^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{-4\left((dx+c)^2 f^2-\frac{3d^2}{2}\right)(dx+c)f\,b^2\sin(2fx+2e)-6d\,b^2\left((dx+c)^2 f^2-\frac{d^2}{2}\right)\cos(2fx+2e)-32(dx+c)afb\left((dx+c)^2 f^2-6d\right.}{}$
risch	$\frac{d^3a^2x^4}{4}+\frac{d^3b^2x^4}{8}+d^2a^2c\,x^3+\frac{d^2b^2c\,x^3}{2}+\frac{3d\,a^2c^2x^2}{2}+\frac{3d\,b^2c^2x^2}{4}+a^2c^3x+\frac{b^2c^3x}{2}+\frac{a^2c^4}{4d}+\frac{b^2c^4}{8d}$
parts	Expression too large to display
norman	$\frac{(\frac{1}{4}d^3a^2+\frac{1}{8}b^2d^3)x^4+(\frac{1}{2}d^3a^2+\frac{1}{4}b^2d^3)x^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{1}{4}d^3a^2+\frac{1}{8}b^2d^3)x^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{b^2d^3x^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+d^2c}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-4*((d*x+c)^2*f^2-3/2*d^2)*(d*x+c)*f*b^2*sin(2*f*x+2*e)-6*d*b^2*((d*
x+c)^2*f^2-1/2*d^2)*cos(2*f*x+2*e)-32*(d*x+c)*a*f*b*((d*x+c)^2*f^2-6*d^2)*
cos(f*x+e)+96*((d*x+c)^2*f^2-2*d^2)*a*d*b*sin(f*x+e)+16*x*(1/2*d*x+c)*(a^2
+1/2*b^2)*(1/2*x^2*d^2+c*d*x+c^2)*f^4-32*a*b*c^3*f^3+6*b^2*c^2*d*f^2+192*a
*b*c*d^2*f-3*b^2*d^3)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.61

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$$
$$=\frac{(2\,a^2+b^2)d^3f^4x^4+4\,(2\,a^2+b^2)cd^2f^4x^3+3\,(2\,(2\,a^2+b^2)c^2df^4+b^2d^3f^2)x^2-3\,(2\,b^2d^3f^2x^2+4\,b^2cd^2f^2}{}$$

input
```
integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*((2*a^2 + b^2)*d^3*f^4*x^4 + 4*(2*a^2 + b^2)*c*d^2*f^4*x^3 + 3*(2*(2*a
^2 + b^2)*c^2*d*f^4 + b^2*d^3*f^2)*x^2 - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^
2*f^2*x + 2*b^2*c^2*d*f^2 - b^2*d^3)*cos(f*x + e)^2 + 2*(2*(2*a^2 + b^2)*c
^3*f^4 + 3*b^2*c*d^2*f^2)*x - 16*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 +
a*b*c^3*f^3 - 6*a*b*c*d^2*f + 3*(a*b*c^2*d*f^3 - 2*a*b*d^3*f)*x)*cos(f*x +
 e) + 2*(24*a*b*d^3*f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 - 48*a
*b*d^3 - (2*b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 - 3*b^2*
c*d^2*f + 3*(2*b^2*c^2*d*f^3 - b^2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/f
^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(240) = 480$.

Time = 0.42 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.29

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(a+b*sin(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d*
*3*x**4/4 - 2*a*b*c**3*cos(e + f*x)/f - 6*a*b*c**2*d*x*cos(e + f*x)/f + 6*
a*b*c**2*d*sin(e + f*x)/f**2 - 6*a*b*c*d**2*x**2*cos(e + f*x)/f + 12*a*b*c
*d**2*x*sin(e + f*x)/f**2 + 12*a*b*c*d**2*cos(e + f*x)/f**3 - 2*a*b*d**3*x
**3*cos(e + f*x)/f + 6*a*b*d**3*x**2*sin(e + f*x)/f**2 + 12*a*b*d**3*x*cos
(e + f*x)/f**3 - 12*a*b*d**3*sin(e + f*x)/f**4 + b**2*c**3*x*sin(e + f*x)*
*2/2 + b**2*c**3*x*cos(e + f*x)**2/2 - b**2*c**3*sin(e + f*x)*cos(e + f*x)
/(2*f) + 3*b**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*b**2*c**2*d*x**2*cos(e +
 f*x)**2/4 - 3*b**2*c**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*b**2*c**2
*d*sin(e + f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sin(e + f*x)**2/2 + b**2*c*
d**2*x**3*cos(e + f*x)**2/2 - 3*b**2*c*d**2*x**2*sin(e + f*x)*cos(e + f*x)
/(2*f) + 3*b**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) - 3*b**2*c*d**2*x*cos(e
+ f*x)**2/(4*f**2) + 3*b**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3) + b*
*2*d**3*x**4*sin(e + f*x)**2/8 + b**2*d**3*x**4*cos(e + f*x)**2/8 - b**2*d
**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*b**2*d**3*x**2*sin(e + f*x)**
2/(8*f**2) - 3*b**2*d**3*x**2*cos(e + f*x)**2/(8*f**2) + 3*b**2*d**3*x*sin
(e + f*x)*cos(e + f*x)/(4*f**3) - 3*b**2*d**3*sin(e + f*x)**2/(8*f**4), Ne
(f, 0)), ((a + b*sin(e))**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3
*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(223) = 446$.

Time = 0.08 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.05

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*(16*(f*x + e)*a^2*c^3 + 4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^3 +
4*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*
a^2*d^3*e^2/f^3 - 16*(f*x + e)*a^2*d^3*e^3/f^3 - 4*(2*f*x + 2*e - sin(2*f*
x + 2*e))*b^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*
a^2*c*d^2*e/f^2 + 48*(f*x + e)*a^2*c*d^2*e^2/f^2 + 12*(2*f*x + 2*e - sin(2
*f*x + 2*e))*b^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 48*(f*x + e)
*a^2*c^2*d*e/f - 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2*d*e/f - 32*a*
b*c^3*cos(f*x + e) + 32*a*b*d^3*e^3*cos(f*x + e)/f^3 - 96*a*b*c*d^2*e^2*co
s(f*x + e)/f^2 + 96*a*b*c^2*d*e*cos(f*x + e)/f - 96*((f*x + e)*cos(f*x + e
) - sin(f*x + e))*a*b*d^3*e^2/f^3 + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f
*x + 2*e) - cos(2*f*x + 2*e))*b^2*d^3*e^2/f^3 + 192*((f*x + e)*cos(f*x + e
) - sin(f*x + e))*a*b*c*d^2*e/f^2 - 12*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*
f*x + 2*e) - cos(2*f*x + 2*e))*b^2*c*d^2*e/f^2 - 96*((f*x + e)*cos(f*x + e
) - sin(f*x + e))*a*b*c^2*d/f + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x +
 2*e) - cos(2*f*x + 2*e))*b^2*c^2*d/f + 96*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*a*b*d^3*e/f^3 - 2*(4*(f*x + e)^3 - 6*(f*x + e
)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*b^2*d^3*e/f^3
 - 96*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*b*c*d^
2/f^2 + 2*(4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2
 - 1)*sin(2*f*x + 2*e))*b^2*c*d^2/f^2 - 32*(((f*x + e)^3 - 6*f*x - 6*e)...
```

Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.55

$$\begin{aligned}
&\int (c+dx)^3(a+b\sin(e+fx))^2\,dx \\
&= \frac{1}{4}a^2d^3x^4 + \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 + \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{4}b^2c^2dx^2 + a^2c^3x \\
&\quad + \frac{1}{2}b^2c^3x - \frac{3\,(2\,b^2d^3f^2x^2 + 4\,b^2cd^2f^2x + 2\,b^2c^2df^2 - b^2d^3)\cos(2\,fx+2\,e)}{16\,f^4} \\
&\quad - \frac{2\,(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x + abc^3f^3 - 6\,abd^3fx - 6\,abcd^2f)\cos(fx+e)}{f^4} \\
&\quad - \frac{(2\,b^2d^3f^3x^3 + 6\,b^2cd^2f^3x^2 + 6\,b^2c^2df^3x + 2\,b^2c^3f^3 - 3\,b^2d^3fx - 3\,b^2cd^2f)\sin(2\,fx+2\,e)}{8\,f^4} \\
&\quad + \frac{6\,(abd^3f^2x^2 + 2\,abcd^2f^2x + abc^2df^2 - 2\,abd^3)\sin(fx+e)}{f^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*a^2*d^3*x^4 + 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 + 1/2*b^2*c*d^2*x^3 + 3/
2*a^2*c^2*d*x^2 + 3/4*b^2*c^2*d*x^2 + a^2*c^3*x + 1/2*b^2*c^3*x - 3/16*(2*
b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 - b^2*d^3)*cos(2*f*x
 + 2*e)/f^4 - 2*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x
 + a*b*c^3*f^3 - 6*a*b*d^3*f*x - 6*a*b*c*d^2*f)*cos(f*x + e)/f^4 - 1/8*(2*
b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 6*b^2*c^2*d*f^3*x + 2*b^2*c^3*f^3
- 3*b^2*d^3*f*x - 3*b^2*c*d^2*f)*sin(2*f*x + 2*e)/f^4 + 6*(a*b*d^3*f^2*x^2
 + 2*a*b*c*d^2*f^2*x + a*b*c^2*d*f^2 - 2*a*b*d^3)*sin(f*x + e)/f^4
```

Mupad [B] (verification not implemented)

Time = 38.10 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.10

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx = \frac{\frac{3\,b^2\,d^3\,\cos(2\,e+2\,f\,x)}{2}+8\,a^2\,c^3\,f^4\,x+4\,b^2\,c^3\,f^4\,x-96\,a\,b\,d^3\,\sin\left(e+f\,x\right)-2\,b^2\,c^3\,f^3\,\sin\left(2\,e+2\,f\,x\right)+2\,a^2\,d^3}{}$$

input
```
int((a + b*sin(e + f*x))^2*(c + d*x)^3,x)
```

output
```
((3*b^2*d^3*cos(2*e + 2*f*x))/2 + 8*a^2*c^3*f^4*x + 4*b^2*c^3*f^4*x - 96*a
*b*d^3*sin(e + f*x) - 2*b^2*c^3*f^3*sin(2*e + 2*f*x) + 2*a^2*d^3*f^4*x^4 +
 b^2*d^3*f^4*x^4 - 16*a*b*c^3*f^3*cos(e + f*x) - 3*b^2*d^3*f^2*x^2*cos(2*e
 + 2*f*x) - 2*b^2*d^3*f^3*x^3*sin(2*e + 2*f*x) + 3*b^2*c*d^2*f*sin(2*e + 2
*f*x) + 3*b^2*d^3*f*x*sin(2*e + 2*f*x) - 3*b^2*c^2*d*f^2*cos(2*e + 2*f*x)
+ 12*a^2*c^2*d*f^4*x^2 + 8*a^2*c*d^2*f^4*x^3 + 6*b^2*c^2*d*f^4*x^2 + 4*b^2
*c*d^2*f^4*x^3 + 96*a*b*c*d^2*f*cos(e + f*x) + 96*a*b*d^3*f*x*cos(e + f*x)
 - 6*b^2*c*d^2*f^2*x*cos(2*e + 2*f*x) - 6*b^2*c^2*d*f^3*x*sin(2*e + 2*f*x)
 + 48*a*b*c^2*d*f^2*sin(e + f*x) - 6*b^2*c*d^2*f^3*x^2*sin(2*e + 2*f*x) -
16*a*b*d^3*f^3*x^3*cos(e + f*x) + 48*a*b*d^3*f^2*x^2*sin(e + f*x) - 48*a*b
*c*d^2*f^3*x^2*cos(e + f*x) - 48*a*b*c^2*d*f^3*x*cos(e + f*x) + 96*a*b*c*d
^2*f^2*x*sin(e + f*x))/(8*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.35

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$$
$$= \frac{6\cos(fx+e)\sin(fx+e)\,b^2d^3fx+96\cos(fx+e)\,abc\,d^2f-16\cos(fx+e)\,ab\,d^3f^3x^3+96\cos(fx+e}{}$$

input
```
int((d*x+c)^3*(a+b*sin(f*x+e))^2,x)
```

output
```
( - 4*cos(e + f*x)*sin(e + f*x)*b**2*c**3*f**3 - 12*cos(e + f*x)*sin(e + f
*x)*b**2*c**2*d*f**3*x - 12*cos(e + f*x)*sin(e + f*x)*b**2*c*d**2*f**3*x**
2 + 6*cos(e + f*x)*sin(e + f*x)*b**2*c*d**2*f - 4*cos(e + f*x)*sin(e + f*x
)*b**2*d**3*f**3*x**3 + 6*cos(e + f*x)*sin(e + f*x)*b**2*d**3*f*x - 16*cos
(e + f*x)*a*b*c**3*f**3 - 48*cos(e + f*x)*a*b*c**2*d*f**3*x - 48*cos(e + f
*x)*a*b*c*d**2*f**3*x**2 + 96*cos(e + f*x)*a*b*c*d**2*f - 16*cos(e + f*x)*
a*b*d**3*f**3*x**3 + 96*cos(e + f*x)*a*b*d**3*f*x + 6*sin(e + f*x)**2*b**2
*c**2*d*f**2 + 12*sin(e + f*x)**2*b**2*c*d**2*f**2*x + 6*sin(e + f*x)**2*b
**2*d**3*f**2*x**2 - 3*sin(e + f*x)**2*b**2*d**3 + 48*sin(e + f*x)*a*b*c**
2*d*f**2 + 96*sin(e + f*x)*a*b*c*d**2*f**2*x + 48*sin(e + f*x)*a*b*d**3*f*
*2*x**2 - 96*sin(e + f*x)*a*b*d**3 + 8*a**2*c**3*f**4*x + 12*a**2*c**2*d*f
**4*x**2 + 8*a**2*c*d**2*f**4*x**3 + 2*a**2*d**3*f**4*x**4 + 4*b**2*c**3*f
**4*x + 6*b**2*c**2*d*f**4*x**2 - 12*b**2*c**2*d*f**2 + 4*b**2*c*d**2*f**4
*x**3 - 6*b**2*c*d**2*f**2*x + b**2*d**3*f**4*x**4 - 3*b**2*d**3*f**2*x**2
 + 6*b**2*d**3)/(8*f**4)
```

3.158 $\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 182

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx = -\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d} + \frac{4abd^2\cos(e+fx)}{f^3}-\frac{2ab(c+dx)^2\cos(e+fx)}{f} + \frac{4abd(c+dx)\sin(e+fx)}{f^2} + \frac{b^2d^2\cos(e+fx)\sin(e+fx)}{4f^3} - \frac{b^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{b^2d(c+dx)\sin^2(e+fx)}{2f^2}$$

output
```
-1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d+1/6*b^2*(d*x+c)^3/d+4*a*b*d^2*cos(f
*x+e)/f^3-2*a*b*(d*x+c)^2*cos(f*x+e)/f+4*a*b*d*(d*x+c)*sin(f*x+e)/f^2+1/4*
b^2*d^2*cos(f*x+e)*sin(f*x+e)/f^3-1/2*b^2*(d*x+c)^2*cos(f*x+e)*sin(f*x+e)/
f+1/2*b^2*d*(d*x+c)*sin(f*x+e)^2/f^2
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$$
$$= \frac{24a^2c^2f^3x+12b^2c^2f^3x+24a^2cdf^3x^2+12b^2cdf^3x^2+8a^2d^2f^3x^3+4b^2d^2f^3x^3-48ab(c^2f^2+2cdf^2x+d}{}$$

input
```
Integrate[(c + d*x)^2*(a + b*Sin[e + f*x])^2,x]
```

output
```
(24*a^2*c^2*f^3*x + 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 + 12*b^2*c*d*f^3
*x^2 + 8*a^2*d^2*f^3*x^3 + 4*b^2*d^2*f^3*x^3 - 48*a*b*(c^2*f^2 + 2*c*d*f^2
*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] - 6*b^2*d*f*(c + d*x)*Cos[2*(e + f*x
)] + 96*a*b*c*d*f*Sin[e + f*x] + 96*a*b*d^2*f*x*Sin[e + f*x] + 3*b^2*d^2*S
in[2*(e + f*x)] - 6*b^2*c^2*f^2*Sin[2*(e + f*x)] - 12*b^2*c*d*f^2*x*Sin[2*
(e + f*x)] - 6*b^2*d^2*f^2*x^2*Sin[2*(e + f*x)])/(24*f^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2(a+b\sin(e+fx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\sin(e+fx)+b^2(c+dx)^2\sin^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d}+\frac{4abd(c+dx)\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}+\frac{4abd^2\cos(e+fx)}{f^3}+\frac{b^2d(c+dx)\sin^2(e+fx)}{2f^2}-\frac{b^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{b^2(c+dx)^3}{6d}+\frac{b^2d^2\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{b^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + b*Sin[e + f*x])^2,x]
```

output
```
-1/4*(b^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) +
 (4*a*b*d^2*Cos[e + f*x])/f^3 - (2*a*b*(c + d*x)^2*Cos[e + f*x])/f + (4*a*
b*d*(c + d*x)*Sin[e + f*x])/f^2 + (b^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f
^3) - (b^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*d*(c + d*x)
*Sin[e + f*x]^2)/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{-b^2\left((dx+c)^2f^2-\frac{d^2}{2}\right)\sin(2fx+2e)-b^2df(dx+c)\cos(2fx+2e)-8\left((dx+c)^2f^2-2d^2\right)ab\cos(fx+e)+16abdf(dx+c)\sin}{4f^3}$
risch	$\frac{d^2a^2x^3}{3}+\frac{d^2b^2x^3}{6}+d\,a^2c\,x^2+\frac{d\,b^2c\,x^2}{2}+a^2c^2x+\frac{b^2c^2x}{2}+\frac{a^2c^3}{3d}+\frac{b^2c^3}{6d}-\frac{2ab(d^2x^2f^2+2cd\,f^2x+c^2f}{f^3}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{b^2\left(\frac{d^2\left((fx+e)^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx+e)\cos(fx+e)^2}{2}+\frac{\sin(fx+e)\cos(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}-\frac{(fx+}{3}\right.}{f^2}\right.}{}$
derivativedivides	$\frac{a^2c^2(fx+e)-\frac{2a^2cde(fx+e)}{f}+\frac{a^2cd(fx+e)^2}{f}+\frac{a^2d^2e^2(fx+e)}{f^2}-\frac{a^2d^2e(fx+e)^2}{f^2}+\frac{a^2d^2(fx+e)^3}{3f^2}-2ab\,c^2\cos(fx+e)+\frac{4abcde\,co}{f}}{}$
default	$\frac{a^2c^2(fx+e)-\frac{2a^2cde(fx+e)}{f}+\frac{a^2cd(fx+e)^2}{f}+\frac{a^2d^2e^2(fx+e)}{f^2}-\frac{a^2d^2e(fx+e)^2}{f^2}+\frac{a^2d^2(fx+e)^3}{3f^2}-2ab\,c^2\cos(fx+e)+\frac{4abcde\,co}{f}}{}$
norman	$\frac{(\frac{1}{3}a^2d^2+\frac{1}{6}b^2d^2)x^3+(\frac{1}{3}a^2d^2+\frac{1}{6}b^2d^2)x^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(\frac{2}{3}a^2d^2+\frac{1}{3}b^2d^2)x^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{b^2d^2x^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+dc(}{}$
orering	Expression too large to display

input
```
int((d*x+c)^2*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-b^2*((d*x+c)^2*f^2-1/2*d^2)*sin(2*f*x+2*e)-b^2*d*f*(d*x+c)*cos(2*f*x
+2*e)-8*((d*x+c)^2*f^2-2*d^2)*a*b*cos(f*x+e)+16*a*b*d*f*(d*x+c)*sin(f*x+e)
+4*x*(1/3*x^2*d^2+c*d*x+c^2)*(a^2+1/2*b^2)*f^3-8*a*b*c^2*f^2+b^2*c*d*f+16*
b*d^2*a)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.24

$$\int(c+dx)^2(a+b\sin(e+fx))^2\,dx$$

$$=\frac{2\left(2a^2+b^2\right)d^2f^3x^3+6\left(2a^2+b^2\right)cdf^3x^2-6\left(b^2d^2fx+b^2cdf\right)\cos\left(fx+e\right)^2+3\left(2\left(2a^2+b^2\right)c^2f^3+b^2d^2\right.}{}$$

input
```
integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/12*(2*(2*a^2 + b^2)*d^2*f^3*x^3 + 6*(2*a^2 + b^2)*c*d*f^3*x^2 - 6*(b^2*d
^2*f*x + b^2*c*d*f)*cos(f*x + e)^2 + 3*(2*(2*a^2 + b^2)*c^2*f^3 + b^2*d^2*
f)*x - 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2)*co
s(f*x + e) + 3*(16*a*b*d^2*f*x + 16*a*b*c*d*f - (2*b^2*d^2*f^2*x^2 + 4*b^2
*c*d*f^2*x + 2*b^2*c^2*f^2 - b^2*d^2)*cos(f*x + e))*sin(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.

Time = 0.30 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.51

$$\int (c + dx)^2 (a + b\sin(e + fx))^2\, dx = \begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} - \frac{2abc^2\cos(e+fx)}{f} - \frac{4abcdx\cos(e+fx)}{f} + \frac{4abcd\sin(e+fx)}{f^2} - \frac{2abd^2x^2\cos(e+fx)}{f} + \frac{4abd^2x\sin(e}{f^2} \\ (a + b\sin(e))^2 \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*sin(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 - 2*a*b*c**2*cos
(e + f*x)/f - 4*a*b*c*d*x*cos(e + f*x)/f + 4*a*b*c*d*sin(e + f*x)/f**2 - 2
*a*b*d**2*x**2*cos(e + f*x)/f + 4*a*b*d**2*x*sin(e + f*x)/f**2 + 4*a*b*d**
2*cos(e + f*x)/f**3 + b**2*c**2*x*sin(e + f*x)**2/2 + b**2*c**2*x*cos(e +
f*x)**2/2 - b**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + b**2*c*d*x**2*sin(
e + f*x)**2/2 + b**2*c*d*x**2*cos(e + f*x)**2/2 - b**2*c*d*x*sin(e + f*x)*
cos(e + f*x)/f + b**2*c*d*sin(e + f*x)**2/(2*f**2) + b**2*d**2*x**3*sin(e
+ f*x)**2/6 + b**2*d**2*x**3*cos(e + f*x)**2/6 - b**2*d**2*x**2*sin(e + f*
x)*cos(e + f*x)/(2*f) + b**2*d**2*x*sin(e + f*x)**2/(4*f**2) - b**2*d**2*x
*cos(e + f*x)**2/(4*f**2) + b**2*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3),
Ne(f, 0)), ((a + b*sin(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(170) = 340$.

Time = 0.05 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.76

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$$

$$= \frac{24\,(fx+e)a^2c^2 + 6\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))b^2c^2 + \frac{8\,(fx+e)^3a^2d^2}{f^2} - \frac{24\,(fx+e)^2a^2d^2e}{f^2} + \frac{24\,(fx+e)a^2d^2e^2}{f^2} + 6$$

input
```
integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/24*(24*(f*x + e)*a^2*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2 +
8*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 24*(f*x + e)*a^
2*d^2*e^2/f^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*d^2*e^2/f^2 + 24*(f
*x + e)^2*a^2*c*d/f - 48*(f*x + e)*a^2*c*d*e/f - 12*(2*f*x + 2*e - sin(2*f
*x + 2*e))*b^2*c*d*e/f - 48*a*b*c^2*cos(f*x + e) - 48*a*b*d^2*e^2*cos(f*x
+ e)/f^2 + 96*a*b*c*d*e*cos(f*x + e)/f + 96*((f*x + e)*cos(f*x + e) - sin(
f*x + e))*a*b*d^2*e/f^2 - 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e)
- cos(2*f*x + 2*e))*b^2*d^2*e/f^2 - 96*((f*x + e)*cos(f*x + e) - sin(f*x +
 e))*a*b*c*d/f + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f
*x + 2*e))*b^2*c*d/f - 48*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*si
n(f*x + e))*a*b*d^2/f^2 + (4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) -
3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*b^2*d^2/f^2)/f
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.24

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$$
$$=\frac{1}{3}a^2d^2x^3+\frac{1}{6}b^2d^2x^3+a^2cdx^2+\frac{1}{2}b^2cdx^2+a^2c^2x+\frac{1}{2}b^2c^2x-\frac{(b^2d^2fx+b^2cdf)\cos(2fx+2e)}{4f^3}-\frac{2(abd^2f^2x^2+2abcdf^2x+abc^2f^2-2abd^2)\cos(fx+e)}{f^3}-\frac{(2b^2d^2f^2x^2+4b^2cdf^2x+2b^2c^2f^2-b^2d^2)\sin(2fx+2e)}{8f^3}+\frac{4(abd^2fx+abcdf)\sin(fx+e)}{f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*a^2*d^2*x^3 + 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 + 1/2*b^2*c*d*x^2 + a^2*c^
2*x + 1/2*b^2*c^2*x - 1/4*(b^2*d^2*f*x + b^2*c*d*f)*cos(2*f*x + 2*e)/f^3 -
 2*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2)*cos(f*x +
 e)/f^3 - 1/8*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 - b^2*d
^2)*sin(2*f*x + 2*e)/f^3 + 4*(a*b*d^2*f*x + a*b*c*d*f)*sin(f*x + e)/f^3
```

Mupad [B] (verification not implemented)

Time = 36.69 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.54

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx = a^2\,c^2\,x+\frac{b^2\,c^2\,x}{2}+\frac{a^2\,d^2\,x^3}{3}+\frac{b^2\,d^2\,x^3}{6}-\frac{b^2\,c^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{b^2\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{8\,f^3}+a^2\,c\,d\,x^2+\frac{b^2\,c\,d\,x^2}{2}-\frac{2\,a\,b\,c^2\,\cos\left(e+f\,x\right)}{f}+\frac{4\,a\,b\,d^2\,\cos\left(e+f\,x\right)}{f^3}-\frac{b^2\,d^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{b^2\,c\,d\,\cos\left(2\,e+2\,f\,x\right)}{4\,f^2}-\frac{b^2\,d^2\,x\,\cos\left(2\,e+2\,f\,x\right)}{4\,f^2}+\frac{4\,a\,b\,c\,d\,\sin\left(e+f\,x\right)}{f^2}+\frac{4\,a\,b\,d^2\,x\,\sin\left(e+f\,x\right)}{f^2}-\frac{2\,a\,b\,d^2\,x^2\,\cos\left(e+f\,x\right)}{f}-\frac{b^2\,c\,d\,x\,\sin\left(2\,e+2\,f\,x\right)}{2\,f}-\frac{4\,a\,b\,c\,d\,x\,\cos\left(e+f\,x\right)}{f}$$

input
```
int((a + b*sin(e + f*x))^2*(c + d*x)^2,x)
```

output
```
a^2*c^2*x + (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 + (b^2*d^2*x^3)/6 - (b^2*c^2*s
in(2*e + 2*f*x))/(4*f) + (b^2*d^2*sin(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^2
+ (b^2*c*d*x^2)/2 - (2*a*b*c^2*cos(e + f*x))/f + (4*a*b*d^2*cos(e + f*x))/
f^3 - (b^2*d^2*x^2*sin(2*e + 2*f*x))/(4*f) - (b^2*c*d*cos(2*e + 2*f*x))/(4
*f^2) - (b^2*d^2*x*cos(2*e + 2*f*x))/(4*f^2) + (4*a*b*c*d*sin(e + f*x))/f^
2 + (4*a*b*d^2*x*sin(e + f*x))/f^2 - (2*a*b*d^2*x^2*cos(e + f*x))/f - (b^2
*c*d*x*sin(2*e + 2*f*x))/(2*f) - (4*a*b*c*d*x*cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.87

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$$
$$= \frac{-6\cos(fx+e)\sin(fx+e)\,b^2c^2f^2 - 12\cos(fx+e)\sin(fx+e)\,b^2cd\,f^2x - 6\cos(fx+e)\sin(fx+e)\,b}{}$$

input
```
int((d*x+c)^2*(a+b*sin(f*x+e))^2,x)
```

output
```
( - 6*cos(e + f*x)*sin(e + f*x)*b**2*c**2*f**2 - 12*cos(e + f*x)*sin(e + f
*x)*b**2*c*d*f**2*x - 6*cos(e + f*x)*sin(e + f*x)*b**2*d**2*f**2*x**2 + 3*
cos(e + f*x)*sin(e + f*x)*b**2*d**2 - 24*cos(e + f*x)*a*b*c**2*f**2 - 48*c
os(e + f*x)*a*b*c*d*f**2*x - 24*cos(e + f*x)*a*b*d**2*f**2*x**2 + 48*cos(e
 + f*x)*a*b*d**2 + 6*sin(e + f*x)**2*b**2*c*d*f + 6*sin(e + f*x)**2*b**2*d
**2*f*x + 48*sin(e + f*x)*a*b*c*d*f + 48*sin(e + f*x)*a*b*d**2*f*x + 12*a*
*2*c**2*f**3*x + 12*a**2*c*d*f**3*x**2 + 4*a**2*d**2*f**3*x**3 + 6*b**2*c*
*2*e*f**2 + 6*b**2*c**2*f**3*x + 6*b**2*c*d*f**3*x**2 - 12*b**2*c*d*f + 9*
b**2*d**2*e + 2*b**2*d**2*f**3*x**3 - 3*b**2*d**2*f*x)/(12*f**3)
```

3.159 $\int (c+dx)(a+b\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\begin{aligned}\int (c+dx)(a+b\sin(e+fx))^2\,dx = {} & \frac{a^2(c+dx)^2}{2d}+\frac{b^2(c+dx)^2}{4d} \\ & -\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2} \\ & -\frac{b^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f} \\ & +\frac{b^2d\sin^2(e+fx)}{4f^2}\end{aligned}$$

output
```
1/2*a^2*(d*x+c)^2/d+1/4*b^2*(d*x+c)^2/d-2*a*b*(d*x+c)*cos(f*x+e)/f+2*a*b*d
*sin(f*x+e)/f^2-1/2*b^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f+1/4*b^2*d*sin(f*x+
e)^2/f^2
```

Mathematica [A] (verified)

Time = 6.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = -\frac{2(2a^2+b^2)(e+fx)(-2cf+d(e-fx))+16abf(c+dx)\cos(e+fx)+b^2d\cos(2(e+fx))-16abd\sin(e+fx)+2b^2f(c+dx)\sin(2(e+fx))}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Sin[e + f*x])^2,x]
```

output
```
-1/8*(2*(2*a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*a*b*f*(c + d*x
)*Cos[e + f*x] + b^2*d*Cos[2*(e + f*x)] - 16*a*b*d*Sin[e + f*x] + 2*b^2*f*
(c + d*x)*Sin[2*(e + f*x)])/f^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a+b\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)+2ab(c+dx)\sin(e+fx)+b^2(c+dx)\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2}-\frac{b^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{b^2(c+dx)^2}{4d}+\frac{b^2d\sin^2(e+fx)}{4f^2}$$

input
```
Int[(c + d*x)*(a + b*Sin[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) + (b^2*(c + d*x)^2)/(4*d) - (2*a*b*(c + d*x)*Cos[e
 + f*x])/f + (2*a*b*d*Sin[e + f*x])/f^2 - (b^2*(c + d*x)*Cos[e + f*x]*Sin[
e + f*x])/(2*f) + (b^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.43 (sec), antiderivative size = 105, normalized size of antiderivative = 0.93

method	result
risch	$\frac{a^2 d x^2}{2}+a^2 c x+\frac{b^2 d x^2}{4}+\frac{b^2 c x}{2}-\frac{2 a b (d x+c) \cos(f x+e)}{f}+\frac{2 a b d \sin(f x+e)}{f^2}-\frac{b^2 d \cos(2 f x+2 e)}{8 f^2}-\frac{b^2(dx+$
parallelrisch	$\frac{-2 b^2 f (d x+c) \sin(2 f x+2 e)-b^2 d \cos(2 f x+2 e)-16 a b f (d x+c) \cos(f x+e)+16 a b d \sin(f x+e)+((2 d x^2+4 c x) f^2+d) b^2-1}{8 f^2}$
parts	$a^2\left(\frac{1}{2} d x^2+c x\right)+\frac{b^2\left(\frac{d\left((f x+e)\left(-\frac{\sin(f x+e)\cos(f x+e)}{2}+\frac{f x}{2}+\frac{e}{2}\right)-\frac{(f x+e)^2}{4}+\frac{\sin(f x+e)^2}{4}\right)}{f}+c\left(-\frac{\sin(f x+e)\cos(f x+}{2}\right.\right.}{f}$
derivativedivides	$\frac{a^2 c (f x+e)-\frac{a^2 d e (f x+e)}{f}+\frac{a^2 d (f x+e)^2}{2 f}-2 a b c \cos(f x+e)+\frac{2 a b d e \cos(f x+e)}{f}+\frac{2 a b d (\sin(f x+e)-(f x+e)\cos(f x+e))}{f}+b^2 c\left(-\mathrm{si}\right.}{}$
default	$\frac{a^2 c (f x+e)-\frac{a^2 d e (f x+e)}{f}+\frac{a^2 d (f x+e)^2}{2 f}-2 a b c \cos(f x+e)+\frac{2 a b d e \cos(f x+e)}{f}+\frac{2 a b d (\sin(f x+e)-(f x+e)\cos(f x+e))}{f}+b^2 c\left(-\mathrm{si}\right.}{}$
norman	$\frac{\left(\frac{1}{2} a^2 d+\frac{1}{4} b^2 d\right) x^2+\left(a^2 d+\frac{1}{2} b^2 d\right) x^2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^2+\left(\frac{1}{2} a^2 d+\frac{1}{4} b^2 d\right) x^2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^4+\frac{b(-b c f+4 a d)\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{f^2}+\frac{b(b c f+4}{}}{}$
orering	$\frac{(2 d^5 f^4 x^6+12 c d^4 f^4 x^5+28 c^2 d^3 f^4 x^4+32 c^3 d^2 f^4 x^3+18 c^4 d f^4 x^2+15 d^5 f^2 x^4+4 c^5 f^4 x+60 c d^4 f^2 x^3+85 c^2 d^3 f^2 x^2+50 c^3}{4 f^4 (d x+c)^4}$

input
```
int((d*x+c)*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*d*x^2+a^2*c*x+1/4*b^2*d*x^2+1/2*b^2*c*x-2*a*b*(d*x+c)*cos(f*x+e)/f
+2*a*b*d*sin(f*x+e)/f^2-1/8*b^2*d/f^2*cos(2*f*x+2*e)-1/4*b^2/f*(d*x+c)*sin
(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec), antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx$$
$$=\frac{(2a^2+b^2)df^2x^2+2(2a^2+b^2)cf^2x-b^2d\cos(fx+e)^2-8(abdfx+abcf)\cos(fx+e)+2(4abd-(b^2}{4f^2}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*d*f^2*x^2 + 2*(2*a^2 + b^2)*c*f^2*x - b^2*d*cos(f*x + e
)^2 - 8*(a*b*d*f*x + a*b*c*f)*cos(f*x + e) + 2*(4*a*b*d - (b^2*d*f*x + b^2
*c*f)*cos(f*x + e))*sin(f*x + e))/f^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(105) = 210$.

Time = 0.21 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.94

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = \begin{cases} a^2cx + \frac{a^2dx^2}{2} - \frac{2abc\cos(e+fx)}{f} - \frac{2abdx\cos(e+fx)}{f} + \frac{2abd\sin(e+fx)}{f^2} + \frac{b^2cx\sin^2(e+fx)}{2} + \frac{b^2cx\cos^2(e+fx)}{2} - \frac{b^2c\sin(e+f}{2} \\ (a+b\sin(e))^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c*x + a**2*d*x**2/2 - 2*a*b*c*cos(e + f*x)/f - 2*a*b*d*x*c
os(e + f*x)/f + 2*a*b*d*sin(e + f*x)/f**2 + b**2*c*x*sin(e + f*x)**2/2 + b
**2*c*x*cos(e + f*x)**2/2 - b**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + b**2*
d*x**2*sin(e + f*x)**2/4 + b**2*d*x**2*cos(e + f*x)**2/4 - b**2*d*x*sin(e
+ f*x)*cos(e + f*x)/(2*f) + b**2*d*sin(e + f*x)**2/(4*f**2), Ne(f, 0)), ((
a + b*sin(e))**2*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.79

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = \frac{8(fx+e)a^2c + 2(2fx+2e-\sin(2fx+2e))b^2c + \frac{4(fx+e)^2a^2d}{f} - \frac{8(fx+e)a^2de}{f} - \frac{2(2fx+2e-\sin(2fx+2e))b^2de}{f}}{}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/8*(8*(f*x + e)*a^2*c + 2*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c + 4*(f*x
 + e)^2*a^2*d/f - 8*(f*x + e)*a^2*d*e/f - 2*(2*f*x + 2*e - sin(2*f*x + 2*e
))*b^2*d*e/f - 16*a*b*c*cos(f*x + e) + 16*a*b*d*e*cos(f*x + e)/f - 16*((f*
x + e)*cos(f*x + e) - sin(f*x + e))*a*b*d/f + (2*(f*x + e)^2 - 2*(f*x + e)
*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*d/f)/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = \frac{1}{2}\,a^2dx^2 + \frac{1}{4}\,b^2dx^2 + a^2cx + \frac{1}{2}\,b^2cx - \frac{b^2d\cos\left(2\,fx+2\,e\right)}{8\,f^2} + \frac{2\,abd\sin\left(fx+e\right)}{f^2} - \frac{2\left(abdfx+abcf\right)\cos\left(fx+e\right)}{f^2} - \frac{\left(b^2dfx+b^2cf\right)\sin\left(2\,fx+2\,e\right)}{4\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d*x^2 + 1/4*b^2*d*x^2 + a^2*c*x + 1/2*b^2*c*x - 1/8*b^2*d*cos(2*f*
x + 2*e)/f^2 + 2*a*b*d*sin(f*x + e)/f^2 - 2*(a*b*d*f*x + a*b*c*f)*cos(f*x
+ e)/f^2 - 1/4*(b^2*d*f*x + b^2*c*f)*sin(2*f*x + 2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 35.98 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = \frac{a^2\,d\,x^2}{2}+\frac{b^2\,d\,x^2}{4}+a^2\,c\,x+\frac{b^2\,c\,x}{2}-\frac{b^2\,c\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{b^2\,d\sin\left(e+f\,x\right)^2}{4\,f^2}+\frac{4\,a\,b\,c\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f}-\frac{b^2\,d\,x\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{2\,a\,b\,d\,\sin\left(e+f\,x\right)}{f^2}+\frac{2\,a\,b\,d\,x\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}{f}$$

input
```
int((a + b*sin(e + f*x))^2*(c + d*x),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*d*x^2)/4 + a^2*c*x + (b^2*c*x)/2 - (b^2*c*sin(2*e + 2
*f*x))/(4*f) + (b^2*d*sin(e + f*x)^2)/(4*f^2) + (4*a*b*c*sin(e/2 + (f*x)/2
)^2)/f - (b^2*d*x*sin(2*e + 2*f*x))/(4*f) + (2*a*b*d*sin(e + f*x))/f^2 + (
2*a*b*d*x*(2*sin(e/2 + (f*x)/2)^2 - 1))/f
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.39

$$\int (c+dx)(a+b\sin(e+fx))^2\,dx = \frac{-2\cos\left(fx+e\right)\sin\left(fx+e\right)b^2cf-2\cos\left(fx+e\right)\sin\left(fx+e\right)b^2dfx-8\cos\left(fx+e\right)abcf-8\cos\left(fx+\right.}{}$$

input
```
int((d*x+c)*(a+b*sin(f*x+e))^2,x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)*b**2*c*f - 2*cos(e + f*x)*sin(e + f*x)*b**
2*d*f*x - 8*cos(e + f*x)*a*b*c*f - 8*cos(e + f*x)*a*b*d*f*x + sin(e + f*x)
**2*b**2*d + 8*sin(e + f*x)*a*b*d + 4*a**2*c*f**2*x + 2*a**2*d*f**2*x**2 -
 8*a*b*d*e + 2*b**2*c*e*f + 2*b**2*c*f**2*x + b**2*d*f**2*x**2 - 2*b**2*d)
/(4*f**2)
```

3.160 $\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx = -\frac{b^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{a^2\log(c+dx)}{d} + \frac{b^2\log(c+dx)}{2d} + \frac{2ab\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d} + \frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d} + \frac{b^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
-1/2*b^2*cos(-2*e+2*c*f/d)*Ci(2*c*f/d+2*f*x)/d+a^2*ln(d*x+c)/d+1/2*b^2*ln(
d*x+c)/d-2*a*b*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d+2*a*b*cos(-e+c*f/d)*Si(c*f/d+
f*x)/d-1/2*b^2*sin(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$$
$$= \frac{-b^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)+b^2\log(c+dx)+4ab\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right.\right.}{2d}$$

input
```
Integrate[(a + b*Sin[e + f*x])^2/(c + d*x),x]
```

output
```
(-(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x))/d]) + 2*a^2*Log[c
+ d*x] + b^2*Log[c + d*x] + 4*a*b*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d
] + 4*a*b*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + b^2*Sin[2*e - (2*c*f
)/d]*SinIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}dx$$
$$\Big\downarrow \text{ 3798}$$
$$\int \left(\frac{a^2}{c+dx}+\frac{2ab\sin(e+fx)}{c+dx}+\frac{b^2\sin^2(e+fx)}{c+dx}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}-\frac{b^2\,\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{b^2\log(c+dx)}{2d}$$

input
```
Int[(a + b*Sin[e + f*x])^2/(c + d*x),x]
```

output
```
-1/2*(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d + (a^2*Lo
g[c + d*x])/d + (b^2*Log[c + d*x])/(2*d) + (2*a*b*CosIntegral[(c*f)/d + f*
x]*Sin[e - (c*f)/d])/d + (2*a*b*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x
])/d + (b^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.30

method	result
parts	$\frac{a^2 \ln(dx+c)}{d}+\frac{b^2 \ln(cf-de+d(fx+e))}{2d}-\frac{b^2 \operatorname{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right) \sin\left(\frac{2cf-2de}{d}\right)}{2d}-\frac{b^2 \operatorname{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right) \mathrm{c}}{2d}$
derivativedivides	$\frac{\frac{a^2 f \ln(cf-de+d(fx+e))}{d}+2fab\left(\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{f b^2 \ln(cf-de+d(fx+e)}{2d}}{f}$
default	$\frac{\frac{a^2 f \ln(cf-de+d(fx+e))}{d}+2fab\left(\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{f b^2 \ln(cf-de+d(fx+e)}{2d}}{f}$
risch	$-\frac{iab\,\mathrm{e}^{\frac{i(cf-de)}{d}} \operatorname{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d}+\frac{a^2 \ln(dx+c)}{d}+\frac{b^2 \ln(dx+c)}{2d}+\frac{b^2 \mathrm{e}^{\frac{2i(cf-de)}{d}} \operatorname{expIntegral}_1\left(2i\right.}{4d}$

input
```
int((a+b*sin(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(d*x+c)/d+1/2*b^2*ln(c*f-d*e+d*(f*x+e))/d-1/2*b^2*Si(2*f*x+2*e+2*(c*
f-d*e)/d)*sin(2*(c*f-d*e)/d)/d-1/2*b^2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(
c*f-d*e)/d)/d+2*a*b*(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*
f-d*e)/d)*sin((c*f-d*e)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx =$$
$$-\frac{b^2\cos\left(-\frac{2\,(de-cf)}{d}\right)\operatorname{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+4\,ab\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)+b^2\sin\left(-\frac{2\,(de-cf)}{d}\right)\operatorname{Si}\left(\frac{2\,(dfx+cf)}{d}\right)-}{2\,d}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(b^2*cos(-2*(d*e - c*f)/d)*cos_integral(2*(d*f*x + c*f)/d) + 4*a*b*co
s_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) + b^2*sin(-2*(d*e - c*f)/d
)*sin_integral(2*(d*f*x + c*f)/d) - 4*a*b*cos(-(d*e - c*f)/d)*sin_integral
((d*f*x + c*f)/d) - (2*a^2 + b^2)*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx=\int\frac{(a+b\sin{(e+fx)})^2}{c+dx}\,dx$$

input
```
integrate((a+b*sin(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*sin(e + f*x))**2/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.15

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx = \frac{\frac{4a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{4\left(f\left(-iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E\right.\right.}{d}}{}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d + 4*(f*(-I*exp_integral_e(1,
 (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d
- I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x +
e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c
*f)/d))*sin(-(d*e - c*f)/d))*a*b/d + (f*(exp_integral_e(1, 2*(-I*(f*x + e)
*d + I*d*e - I*c*f)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*
c*f)/d))*cos(-2*(d*e - c*f)/d) + f*(-I*exp_integral_e(1, 2*(-I*(f*x + e)*d
 + I*d*e - I*c*f)/d) + I*exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*
c*f)/d))*sin(-2*(d*e - c*f)/d) + 2*f*log((f*x + e)*d - d*e + c*f))*b^2/d)/
f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 7139, normalized size of antiderivative = 45.76

$$\int \frac{(a + b\sin(e + fx))^2}{c + dx} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(4*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(
c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*log(abs(d*x + c)
)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 2*b^2*log(abs(d*x
+ c))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - b^2*real_part(
cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*
c*f/d)^2 - b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(
e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 8*a*b*sin_integral((d*f*x + c*f)/d)*t
an(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 8*a*b*real_part(cos_i
ntegral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) +
8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/
d)^2*tan(1/2*c*f/d) + 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1
/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*b^2*imag_part(cos_integra
l(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4
*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(
1/2*c*f/d)^2 - 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2
*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 2*b^2*imag_part(cos_integral(-2*f*
x - 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*b^2*si
n_integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f
/d)^2 - 8*a*b*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx=\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}{c+d\,x}\,dx$$

input
```
int((a + b*sin(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*sin(e + f*x))^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx=\frac{\left(\int \frac{\sin(fx+e)^2}{dx+c}dx\right)b^2d+2\left(\int \frac{\sin(fx+e)}{dx+c}dx\right)abd+\log(dx+c)\,a^2}{d}$$

input
```
int((a+b*sin(f*x+e))^2/(d*x+c),x)
```

output
```
(int(sin(e + f*x)**2/(c + d*x),x)*b**2*d + 2*int(sin(e + f*x)/(c + d*x),x)
*a*b*d + log(c + d*x)*a**2)/d
```

3.161 $\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx = -\frac{a^2}{d(c+dx)} + \frac{2abf\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{b^2f\,\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{2ab\sin(e+fx)}{d(c+dx)} - \frac{b^2\sin^2(e+fx)}{d(c+dx)} - \frac{2abf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{b^2f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

output

```
-a^2/d/(d*x+c)+2*a*b*f*cos(-e+c*f/d)*Ci(c*f/d+f*x)/d^2-b^2*f*Ci(2*c*f/d+2*
f*x)*sin(-2*e+2*c*f/d)/d^2-2*a*b*sin(f*x+e)/d/(d*x+c)-b^2*sin(f*x+e)^2/d/(
d*x+c)+2*a*b*f*sin(-e+c*f/d)*Si(c*f/d+f*x)/d^2+b^2*f*cos(-2*e+2*c*f/d)*Si(
2*c*f/d+2*f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{-2a^2d-b^2d+b^2d\cos(2(e+fx))+4abf(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2f(c+dx)\,\text{C}}{}$$

input
```
Integrate[(a + b*Sin[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(-2*a^2*d - b^2*d + b^2*d*Cos[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cos[e - (c*
f)/d]*CosIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CosIntegral[(2*f*(c + d
*x))/d]*Sin[2*e - (2*c*f)/d] - 4*a*b*d*Sin[e + f*x] - 4*a*b*c*f*Sin[e - (c
*f)/d]*SinIntegral[f*(c/d + x)] - 4*a*b*d*f*x*Sin[e - (c*f)/d]*SinIntegral
[f*(c/d + x)] + 2*b^2*c*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))
/d] + 2*b^2*d*f*x*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d])/(2*
d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \; 3798$$

$$\int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\sin(e+fx)}{(c+dx)^2}+\frac{b^2\sin^2(e+fx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2abf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sin(e+fx)}{d(c+dx)}+\frac{b^2f\,\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\sin^2(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Sin[e + f*x])^2/(c + d*x)^2,x]
```

output
```
-(a^2/(d*(c + d*x))) + (2*a*b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x
])/d^2 + (b^2*f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 -
 (2*a*b*Sin[e + f*x])/(d*(c + d*x)) - (b^2*Sin[e + f*x]^2)/(d*(c + d*x)) -
 (2*a*b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Cos[2*
e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.55

method	result
parts	$-\frac{a^{2}}{d(dx+c)}+\frac{b^{2}\left(-\frac{f^{2}}{2(cf-de+d(fx+e))d}-\frac{f^{2}\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2\right.}{}\right.}{d}\right.}{4}\right.}{f}$
derivativedivides	$\frac{-\frac{a^{2}f^{2}}{(cf-de+d(fx+e))d}+2f^{2}ab\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-}{}$
default	$\frac{-\frac{a^{2}f^{2}}{(cf-de+d(fx+e))d}+2f^{2}ab\left(-\frac{\sin(fx+e)}{(cf-de+d(fx+e))d}+\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-}{}$
risch	$-\frac{fab\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d^{2}}-\frac{a^{2}}{d(dx+c)}-\frac{b^{2}}{2d(dx+c)}-\frac{ib^{2}f\,\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(2ifx\right.}{2d^{2}}$

input
```
int((a+b*sin(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/(d*x+c)+b^2/f*(-1/2*f^2/(c*f-d*e+d*(f*x+e))/d-1/4*f^2*(-2*cos(2*f*x
+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*
e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d))+2*a*b*f*(-
sin(f*x+e)/(c*f-d*e+d*(f*x+e))/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d
+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx = \frac{b^2d\cos(fx+e)^2 - 2\,abd\sin(fx+e) + 2\,(abdfx+abcf)\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right) - (b^2dfx+b^2cf)\operatorname{Ci}\left(\right.}{}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
(b^2*d*cos(f*x + e)^2 - 2*a*b*d*sin(f*x + e) + 2*(a*b*d*f*x + a*b*c*f)*cos
(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*cos
_integral(2*(d*f*x + c*f)/d)*sin(-2*(d*e - c*f)/d) + (b^2*d*f*x + b^2*c*f)
*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*(a*b*d*f*x + a*
b*c*f)*sin(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) - (a^2 + b^2)*d)/
(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*sin(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*sin(e + f*x))**2/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.03

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx = -\frac{\frac{4\,a^2f^2}{(fx+e)d^2-d^2e+cdf} - \frac{4\left(f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_2 \cdots}{(fx+e)d^2-d^2e+cdf}}{\cdots}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - 4*(f^2*(-I*exp_integral_
e(2, (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e
)*d - I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
 + I*c*f)/d))*sin(-(d*e - c*f)/d))*a*b/((f*x + e)*d^2 - d^2*e + c*d*f) - (
f^2*(exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + exp_integra
l_e(2, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*cos(-2*(d*e - c*f)/d) - f^2
*(I*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - I*exp_integr
al_e(2, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*sin(-2*(d*e - c*f)/d) - 2*
f^2)*b^2/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. 2(186) = 372.

Time = 0.35 (sec) , antiderivative size = 1050, normalized size of antiderivative = 5.74

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(4*(d*x + c)*a*b*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c
*f)/d)*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d) - 4*a*b*d*e*f^2*cos(-(d*e - c*f)/d)*cos_integral(((d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + 4*a*b*c*f^3*cos(-(d*e - c
*f)/d)*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d) - 2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_int
egral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin
(-2*(d*e - c*f)/d) + 2*b^2*d*e*f^2*cos_integral(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-2*(d*e - c*f)/d) - 2*b^2*c*f^3
*cos_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f
)/d)*sin(-2*(d*e - c*f)/d) + 2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c
) + f)*f^2*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d) - 2*b^2*d*e*f^2*cos(-2*(d*e - c*f)/d)
*sin_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f
)/d) + 2*b^2*c*f^3*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + 4*(d*x + c)*a*b*(d*e/(d*x +
 c) - c*f/(d*x + c) + f)*f^2*sin(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - 4*a*b*d*e*f^2*sin(-(d
*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
 d*e + c*f)/d) + 4*a*b*c*f^3*sin(-(d*e - c*f)/d)*sin_integral(((d*x + c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + b*sin(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*sin(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{\left(\int \frac{\sin(fx+e)^2}{d^2x^2+2cdx+c^2}dx\right)b^2c^2 + \left(\int \frac{\sin(fx+e)^2}{d^2x^2+2cdx+c^2}dx\right)b^2cdx + 2\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)ab\,c^2 + 2\left(\int \frac{\sin(fx+e)}{d^2x^2+2cdx+c^2}dx\right)}{c\,(dx+c)}$$

input
```
int((a+b*sin(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(int(sin(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c**2 + int(sin(e
 + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c*d*x + 2*int(sin(e + f*x)
/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c**2 + 2*int(sin(e + f*x)/(c**2 + 2*c
*d*x + d**2*x**2),x)*a*b*c*d*x + a**2*x)/(c*(c + d*x))
```

3.162 $\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 245

$$\begin{aligned}\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = &-\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}\\&+\frac{b^2f^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{d^3}\\&-\frac{abf^2\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}\\&-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\cos(e+fx)\sin(e+fx)}{d^2(c+dx)}\\&-\frac{b^2\sin^2(e+fx)}{2d(c+dx)^2}-\frac{abf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^3}\\&-\frac{b^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{d^3}\end{aligned}$$

output

```
-1/2*a^2/d/(d*x+c)^2-a*b*f*cos(f*x+e)/d^2/(d*x+c)+b^2*f^2*cos(-2*e+2*c*f/d
)*Ci(2*c*f/d+2*f*x)/d^3+a*b*f^2*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^3-a*b*sin(f*
x+e)/d/(d*x+c)^2-b^2*f*cos(f*x+e)*sin(f*x+e)/d^2/(d*x+c)-1/2*b^2*sin(f*x+e
)^2/d/(d*x+c)^2-a*b*f^2*cos(-e+c*f/d)*Si(c*f/d+f*x)/d^3+b^2*f^2*sin(-2*e+2
*c*f/d)*Si(2*c*f/d+2*f*x)/d^3
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx =$$
$$-\frac{2a^2d^2+b^2d^2+4abcdf\cos(e+fx)+4abd^2fx\cos(e+fx)-b^2d^2\cos(2(e+fx))-4b^2f^2(c+dx)^2\cos}{}$$

input
```
Integrate[(a + b*Sin[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/4*(2*a^2*d^2 + b^2*d^2 + 4*a*b*c*d*f*Cos[e + f*x] + 4*a*b*d^2*f*x*Cos[e
 + f*x] - b^2*d^2*Cos[2*(e + f*x)] - 4*b^2*f^2*(c + d*x)^2*Cos[2*e - (2*c*
f)/d]*CosIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CosIntegral[f
*(c/d + x)]*Sin[e - (c*f)/d] + 4*a*b*d^2*Sin[e + f*x] + 2*b^2*c*d*f*Sin[2*
(e + f*x)] + 2*b^2*d^2*f*x*Sin[2*(e + f*x)] + 4*a*b*c^2*f^2*Cos[e - (c*f)/
d]*SinIntegral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cos[e - (c*f)/d]*SinIntegral
[f*(c/d + x)] + 4*a*b*d^2*f^2*x^2*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)
] + 4*b^2*c^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 8*
b^2*c*d*f^2*x*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 4*b^2*
d^2*f^2*x^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d])/(d^3*(c +
 d*x)^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\sin(e+fx)}{(c+dx)^3}+\frac{b^2\sin^2(e+fx)}{(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2}{2d(c+dx)^2}-\frac{abf^2\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{abf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{ab\sin(e+fx)}{d(c+dx)^2}+\frac{b^2f^2\,\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{b^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{b^2f\sin(e+fx)\cos(e+fx)}{d^2(c+dx)}-\frac{b^2\sin^2(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Sin[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/2*a^2/(d*(c + d*x)^2) - (a*b*f*Cos[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2
*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a*b*f^2*CosIn
tegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (a*b*Sin[e + f*x])/(d*(c + d
*x)^2) - (b^2*f*Cos[e + f*x]*Sin[e + f*x])/(d^2*(c + d*x)) - (b^2*Sin[e +
f*x]^2)/(2*d*(c + d*x)^2) - (a*b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d
+ f*x])/d^3 - (b^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x]
)/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.47

method	result
parts	$-\frac{a^2}{2d(dx+c)^2}+\frac{b^2\left(-\frac{f^3}{4(cf-de+d(fx+e))^2d}-\frac{f^3\left(-\frac{\cos(2fx+2e)}{(cf-de+d(fx+e))^2d}-\frac{-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{4\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)}{d}}{}\right.}{4}\right.}{f}$
derivativedivides	$\frac{-\frac{f^3a^2}{2(cf-de+d(fx+e))^2d}+2ab\,f^3\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+\right.}{}}{d}}{2d}\right.}{}$
default	$\frac{-\frac{f^3a^2}{2(cf-de+d(fx+e))^2d}+2ab\,f^3\left(-\frac{\sin(fx+e)}{2(cf-de+d(fx+e))^2d}+\frac{-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+\right.}{}}{d}}{2d}\right.}{}$
risch	$\frac{if^2ab\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^3}-\frac{a^2}{2d(dx+c)^2}-\frac{b^2}{4d(dx+c)^2}-\frac{b^2f^2\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}_1\left(2\right.}{2d^3}$

input
```
int((a+b*sin(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d/(d*x+c)^2+b^2/f*(-1/4*f^3/(c*f-d*e+d*(f*x+e))^2/d-1/4*f^3*(-cos
(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))^2/d-(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))
/d+2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*
(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)/d)/d))+2*a*b*f^2*(-1/2*sin(f*x+e)/(c*f-
d*e+d*(f*x+e))^2/d+1/2*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d
*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.50

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = \frac{b^2d^2\cos{(fx+e)}^2-(a^2+b^2)d^2+2\left(b^2d^2f^2x^2+2\,b^2cdf^2x+b^2c^2f^2\right)\cos\left(-\frac{2\,(de-cf)}{d}\right)\operatorname{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+2$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*cos(f*x + e)^2 - (a^2 + b^2)*d^2 + 2*(b^2*d^2*f^2*x^2 + 2*b^2
*c*d*f^2*x + b^2*c^2*f^2)*cos(-2*(d*e - c*f)/d)*cos_integral(2*(d*f*x + c*
f)/d) + 2*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*cos_integral((
d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) + 2*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x
 + b^2*c^2*f^2)*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) - 2*
(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*cos(-(d*e - c*f)/d)*sin_
integral((d*f*x + c*f)/d) - 2*(a*b*d^2*f*x + a*b*c*d*f)*cos(f*x + e) - 2*(
a*b*d^2 + (b^2*d^2*f*x + b^2*c*d*f)*cos(f*x + e))*sin(f*x + e))/(d^5*x^2 +
 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{(a+b\sin{(e+fx)})^2}{(c+dx)^3}\,dx$$

input
```
integrate((a+b*sin(f*x+e))**2/(d*x+c)**3,x)
```

output
```
Integral((a + b*sin(e + f*x))**2/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.94

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = -\frac{\frac{2a^2f^3}{(fx+e)^2d^3+d^3e^2-2cd^2ef+c^2df^2-2(d^3e-cd^2f)(fx+e)} - \frac{4\left(f^3\left(-iE_3\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_3\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-c}{d}\right.}{(fx+e)^2d^3+d^3e^2-2cd^2ef}}{}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(
d^3*e - c*d^2*f)*(f*x + e)) - 4*(f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d
- I*d*e + I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)
/d))*cos(-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e +
 I*c*f)/d) + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(
d*e - c*f)/d))*a*b/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 -
2*(d^3*e - c*d^2*f)*(f*x + e)) - (f^3*(exp_integral_e(3, 2*(-I*(f*x + e)*d
 + I*d*e - I*c*f)/d) + exp_integral_e(3, -2*(-I*(f*x + e)*d + I*d*e - I*c*
f)/d))*cos(-2*(d*e - c*f)/d) - f^3*(I*exp_integral_e(3, 2*(-I*(f*x + e)*d
+ I*d*e - I*c*f)/d) - I*exp_integral_e(3, -2*(-I*(f*x + e)*d + I*d*e - I*c
*f)/d))*sin(-2*(d*e - c*f)/d) - f^3)*b^2/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*
d^2*e*f + c^2*d*f^2 - 2*(d^3*e - c*d^2*f)*(f*x + e)))/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.20 (sec) , antiderivative size = 120406, normalized size of antiderivative = 491.45

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(a*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(f*x)^2*tan(
1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a*b*d^2*f
^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan
(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - b^2*d^2*f^2*x^2*real_pa
rt(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*t
an(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - b^2*d^2*f^2*x^2*real_part(cos_inte
gral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*ta
n(c*f/d)^2*tan(1/2*c*f/d)^2 + 2*a*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)
/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c
*f/d)^2 + 2*a*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(f*x)^
2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 2*a*b
*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(f*x)^2*tan(1/2*f*x)
^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 2*b^2*d^2*f^2*x^2*i
mag_part(cos_integral(2*f*x + 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*
e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*b^2*d^2*f^2*x^2*imag_part(co
s_integral(-2*f*x - 2*c*f/d))*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e
)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*b^2*d^2*f^2*x^2*sin_integral(2*(d*f*x
+ c*f)/d)*tan(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1
/2*c*f/d)^2 - 2*b^2*d^2*f^2*x^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(f*x)^2*tan(1/2*f*x)^2*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x))^2/(c + d*x)^3,x)
```

output
```
int((a + b*sin(e + f*x))^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = \text{too large to display}$$

input
```
int((a+b*sin(f*x+e))^2/(d*x+c)^3,x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)*a*b*c*d**2 + 2*cos(e + f*x)*sin(e + f*x)*b
**2*c**2*d*f + 2*cos(e + f*x)*sin(e + f*x)*b**2*c*d**2*f*x - 8*cos(e + f*x
)*b**2*c*d**2 - 16*int(x**2/(tan((e + f*x)/2)**4*c**3 + 3*tan((e + f*x)/2)
**4*c**2*d*x + 3*tan((e + f*x)/2)**4*c*d**2*x**2 + tan((e + f*x)/2)**4*d**
3*x**3 + 2*tan((e + f*x)/2)**2*c**3 + 6*tan((e + f*x)/2)**2*c**2*d*x + 6*t
an((e + f*x)/2)**2*c*d**2*x**2 + 2*tan((e + f*x)/2)**2*d**3*x**3 + c**3 +
3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*c**3*d**3*f**2 - 32*int(x*
*2/(tan((e + f*x)/2)**4*c**3 + 3*tan((e + f*x)/2)**4*c**2*d*x + 3*tan((e +
 f*x)/2)**4*c*d**2*x**2 + tan((e + f*x)/2)**4*d**3*x**3 + 2*tan((e + f*x)/
2)**2*c**3 + 6*tan((e + f*x)/2)**2*c**2*d*x + 6*tan((e + f*x)/2)**2*c*d**2
*x**2 + 2*tan((e + f*x)/2)**2*d**3*x**3 + c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3),x)*b**2*c**2*d**4*f**2*x - 16*int(x**2/(tan((e + f*x)/2)**4
*c**3 + 3*tan((e + f*x)/2)**4*c**2*d*x + 3*tan((e + f*x)/2)**4*c*d**2*x**2
 + tan((e + f*x)/2)**4*d**3*x**3 + 2*tan((e + f*x)/2)**2*c**3 + 6*tan((e +
 f*x)/2)**2*c**2*d*x + 6*tan((e + f*x)/2)**2*c*d**2*x**2 + 2*tan((e + f*x)
/2)**2*d**3*x**3 + c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*
c*d**5*f**2*x**2 - 16*int(tan((e + f*x)/2)/(tan((e + f*x)/2)**4*c**3 + 3*t
an((e + f*x)/2)**4*c**2*d*x + 3*tan((e + f*x)/2)**4*c*d**2*x**2 + tan((e +
 f*x)/2)**4*d**3*x**3 + 2*tan((e + f*x)/2)**2*c**3 + 6*tan((e + f*x)/2)**2
*c**2*d*x + 6*tan((e + f*x)/2)**2*c*d**2*x**2 + 2*tan((e + f*x)/2)**2*d...
```

3.163 $\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 495

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} + \frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} - \frac{6id^2(c+dx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3} + \frac{6id^2(c+dx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3} + \frac{6d^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^4} - \frac{6d^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^4}$$

output
```
-I*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/
f+I*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/f-3*d*(d*x+c)^2*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^
2)^(1/2)/f^2+3*d*(d*x+c)^2*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)
))/(a^2-b^2)^(1/2)/f^2-6*I*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^
2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^3+6*I*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*
x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^3+6*d^3*polylog(4,I*b*exp(I*(
f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^4-6*d^3*polylog(4,I*b*exp(I
*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^4
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx =$$

$$-\frac{i\left((c+dx)^3\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)+\frac{3d\left(-if^2(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + b*Sin[e + f*x]),x]
```

output
```
((-I)*((c + d*x)^3*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
 (c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (3*d*(
(-I)*f^2*(c + d*x)^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 -
b^2])] + 2*d*(f*(c + d*x)*PolyLog[3, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a
^2 - b^2])] + I*d*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])
))/f^3 + ((3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + S
qrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a
 + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])]))/f^3))/(Sqrt[a^2 - b^2]*f)
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 3804, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}dx$$

$$\Big\downarrow \text{ 3804}$$

$$2\int \frac{e^{i(e+fx)}(c+dx)^3}{2e^{i(e+fx)}a-ibe^{2i(e+fx)}+ib}dx$$

$$\Big\downarrow \text{ 2694}$$

$$2\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{2\left(a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{2\left(a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$2\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(\right.}{b}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \text{ 3011}$$

$$2\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\right.}{}\right.}{}\right.$$

$\downarrow$ 7163

$$2\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+f}}{a+\sqrt{a^2-}}\right.}{f}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 2720

$$2\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\left(\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}-\frac{i(c+dx)\operatorname{Po}\cdots}{\cdots}\right.}{f}\cdots\right.}{bf}\cdots\right.}{2\sqrt{a^2-b^2}}\cdots\right.$$

$$\Big\downarrow \text{ 7143}$$

$$2\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}\cdots\right.}{f}\cdots\right.}{bf}\cdots\right.}{2\sqrt{a^2-b^2}}\cdots\right.$$

input
```
Int[(c + d*x)^3/(a + b*Sin[e + f*x]),x]
```

output
```
2*(((-1/2*I)*b*(((c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 -
 b^2])])/(b*f) - (3*d*((I*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a
- Sqrt[a^2 - b^2])])/f - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/f + (d*PolyLog[4, (I*b*E^(I*(e + f*x)))/
(a - Sqrt[a^2 - b^2])])/f^2))/f))/(b*f)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((c
+ d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(b*f) - (3*
d*((I*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])
/f - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[
a^2 - b^2])])/f + (d*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2]
)])/f^2))/f))/(b*f)))/Sqrt[a^2 - b^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(dx+c)^3}{a+b\sin(fx+e)}dx$$

input
```
int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

output
```
int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2173 vs. $2(421) = 842$.

Time = 0.25 (sec) , antiderivative size = 2173, normalized size of antiderivative = 4.39

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(-6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(f*x + e) + a*s
in(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/
b) + 6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(f*x + e) + a*si
n(f*x + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b
) + 6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(f*x + e) + a*si
n(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b
) - 6*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(f*x + e) + a*si
n(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b
) + 3*(I*b*d^3*f^2*x^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*sqrt(-(a^2 - b
^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*
sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 3*(-I*b*d^3*f^2*x^2 - 2
*I*b*c*d^2*f^2*x - I*b*c^2*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*
x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b + 1) + 3*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*f^2*x - I*b*c^2
*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) +
 (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
3*(I*b*d^3*f^2*x^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*sqrt(-(a^2 - b^2)/
b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin
(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (b*d^3*e^3 - 3*b*c*d^2*e^2
*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\sin{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*sin(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*sin(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*sin(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+b\,\sin(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a + b*sin(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)c^3+\left(\int\frac{x^3}{\sin(fx+e)b+a}dx\right)a^2d^3f-\left(\int\frac{x^3}{\sin(fx+e)b+a}dx\right)b^2d^3f+3\left(\int\frac{x^2}{\sin(fx+e)}\right.}{f\,(a^2}$$

input
```
int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*c**3
 + int(x**3/(sin(e + f*x)*b + a),x)*a**2*d**3*f - int(x**3/(sin(e + f*x)*b
 + a),x)*b**2*d**3*f + 3*int(x**2/(sin(e + f*x)*b + a),x)*a**2*c*d**2*f -
3*int(x**2/(sin(e + f*x)*b + a),x)*b**2*c*d**2*f + 3*int(x/(sin(e + f*x)*b
 + a),x)*a**2*c**2*d*f - 3*int(x/(sin(e + f*x)*b + a),x)*b**2*c**2*d*f)/(f
*(a**2 - b**2))
```

3.164 $\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 367

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} + \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} - \frac{2id^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3} + \frac{2id^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3}$$

output
```
-I*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/
f+I*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/f-2*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)
^(1/2)/f^2+2*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(
a^2-b^2)^(1/2)/f^2-2*I*d^2*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)
))/(a^2-b^2)^(1/2)/f^3+2*I*d^2*polylog(3,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(
1/2)))/(a^2-b^2)^(1/2)/f^3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx =$$

$$-\frac{i\left((c+dx)^2\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)+\frac{2d\left(-if(c+dx)\operatorname{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+\right.}{f^2}\right.}{\sqrt{a^2-b^2}f}$$

input
```
Integrate[(c + d*x)^2/(a + b*Sin[e + f*x]),x]
```

output
```
((-I)*((c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
 (c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (2*d*(
(-I)*f*(c + d*x)*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2]
)] + d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])]))/f^2 + ((2
*I)*d*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
 + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/f^2))/(Sq
rt[a^2 - b^2]*f)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}dx$$

$$\Big\downarrow \; 3804$$

$$2\int \frac{e^{i(e+fx)}(c+dx)^2}{2e^{i(e+fx)}a-ibe^{2i(e+fx)}+ib}dx$$

$$\Big\downarrow \; 2694$$

$$2\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{2\left(a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{2\left(a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \; 2620$$

$$2\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\int(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{2d\int(c+dx)\log\left(1-\cdots\right.}{bf}\cdots\right.}{2\sqrt{a^2-b^2}}\cdots\right.$$

$$\Big\downarrow \; 3011$$

$$2\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{id\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe}{a-}\right.}{bf}\right.}{}\right.$$

$\Big\downarrow$ 2720

$$2\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c-}{}\right.}{}\right.$$

$\Big\downarrow$ 7143

$$2\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+}}{a-\sqrt{a^2}}\right.}{bf}\right.}{}\right.$$

input
```
Int[(c + d*x)^2/(a + b*Sin[e + f*x]),x]
```

output
```
2*(((-1/2*I)*b*(((c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 -
 b^2])])/(b*f) - (2*d*((I*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a -
Sqrt[a^2 - b^2])])/f - (d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 -
 b^2])])/f^2))/(b*f)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((c + d*x)^2*Log[1 - (I
*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(b*f) - (2*d*((I*(c + d*x)*Pol
yLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/f - (d*PolyLog[3, (I
*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/f^2))/(b*f)))/Sqrt[a^2 - b^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dx+c)^2}{a+b\sin(fx+e)}dx$$

input
```
int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

output
```
int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(309) = 618$.

Time = 0.24 (sec) , antiderivative size = 1543, normalized size of antiderivative = 4.20

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x + e) + a*sin(
f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b)
- 2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(f*x + e) + a*sin(f*x
 + e) - (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2
*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(f*x + e) + a*sin(f*x +
 e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*b
*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(f*x + e) + a*sin(f*x + e
) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(I*
b*d^2*f*x + I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*
sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) + 2*(-I*b*d^2*f*x - I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I
*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(-I*b*d^2*f*x - I*b*c*d*f)*sqrt(-(a^2
- b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) -
I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(I*b*d^2*f*x + I*
b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e)
- (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(
f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d
^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*...
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*sin(f*x+e)),x)
```

output
```
Integral((c + d*x)**2/(a + b*sin(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*sin(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^2/(a + b*sin(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)c^2+\left(\int \frac{x^2}{\sin(fx+e)b+a}dx\right)a^2d^2f-\left(\int \frac{x^2}{\sin(fx+e)b+a}dx\right)b^2d^2f+2\left(\int \frac{x}{\sin(fx+e}\right.}{f\left(a^2-b^2\right)}$$

input
```
int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*c**2
 + int(x**2/(sin(e + f*x)*b + a),x)*a**2*d**2*f - int(x**2/(sin(e + f*x)*b
 + a),x)*b**2*d**2*f + 2*int(x/(sin(e + f*x)*b + a),x)*a**2*c*d*f - 2*int(
x/(sin(e + f*x)*b + a),x)*b**2*c*d*f)/(f*(a**2 - b**2))
```

3.165 $\int \frac{c+dx}{a+b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 234

$$\int \frac{c+dx}{a+b\sin(e+fx)}\,dx = -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{d\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}+\frac{d\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}$$

output
```
-I*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f+
I*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f-d
*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2+d*p
olylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{a+b\sin(e+fx)}\,dx = \frac{-if(c+dx)\left(\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)-d\,\text{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+d\,\text{PolyLog}\left(\right.}{\sqrt{a^2-b^2}\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Sin[e + f*x]),x]
```

output
```
((-I)*f*(c + d*x)*(Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
 Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]) - d*PolyLog[2, ((-I
)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] + d*PolyLog[2, (I*b*E^(I*(e +
 f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a+b\sin(e+fx)}dx$$

$$\Big\downarrow \text{3804}$$

$$2\int \frac{e^{i(e+fx)}(c+dx)}{2e^{i(e+fx)}a-ibe^{2i(e+fx)}+ib}dx$$

$$\Big\downarrow \text{2694}$$

$$2\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{2\left(a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{2\left(a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{d\int\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{d\int\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \text{2715}$$

$$2\left(\frac{ib\left(\frac{id\int e^{-i(e+fx)}\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{bf^2}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{id\int e^{-i(e+fx)}\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(e+fx}}{bf^2}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \text{2838}$$

$$2\left(\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{id\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{id\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}\right.$$

input
```
Int[(c + d*x)/(a + b*Sin[e + f*x]),x]
```

output
```
2*(((-1/2*I)*b*(((c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b
^2])])/(b*f) - (I*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])
])/(b*f^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((c + d*x)*Log[1 - (I*b*E^(I*(e +
 f*x)))/(a + Sqrt[a^2 - b^2])])/(b*f) - (I*d*PolyLog[2, (I*b*E^(I*(e + f*x
)))/(a + Sqrt[a^2 - b^2])])/(b*f^2)))/Sqrt[a^2 - b^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(204) = 408.

Time = 0.69 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.10

method	result
risch	$\frac{2ic\arctan\left(\frac{2ib\,\mathrm{e}^{i(fx+e)}-2a}{2\sqrt{-a^2+b^2}}\right)}{f\sqrt{-a^2+b^2}}+\frac{d\ln\left(\frac{-ia-b\,\mathrm{e}^{i(fx+e)}+\sqrt{-a^2+b^2}}{-ia+\sqrt{-a^2+b^2}}\right)x}{f\sqrt{-a^2+b^2}}-\frac{d\ln\left(\frac{ia+b\,\mathrm{e}^{i(fx+e)}+\sqrt{-a^2+b^2}}{ia+\sqrt{-a^2+b^2}}\right)x}{f\sqrt{-a^2+b^2}}+\frac{d\ln\left(\frac{-ia-b\,\mathrm{e}^{i(fx+e}}{-ia+\sqrt{-}}\right.}{f^2\sqrt{-a^2}}$

input
```
int((d*x+c)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*I/f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^
(1/2))+1/f*d/(-a^2+b^2)^(1/2)*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/
(-I*a+(-a^2+b^2)^(1/2)))*x-1/f*d/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(f*x+e))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/f^2*d/(-a^2+b^2)^(1/2)*ln((
-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*e-1/f^2*d
/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^
2)^(1/2)))*e-I/f^2*d/(-a^2+b^2)^(1/2)*dilog((-I*a-b*exp(I*(f*x+e))+(-a^2+b
^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))+I/f^2*d/(-a^2+b^2)^(1/2)*dilog((I*a+b*
exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/f^2*d*e/(-a^2
+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(200) = 400.

Time = 0.21 (sec) , antiderivative size = 997, normalized size of antiderivative = 4.26

$$\int \frac{c+dx}{a+b\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e)
 + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
- I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) -
(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I
*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) + (b
*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*b
*d*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*c
os(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (b*d*
e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e
) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b*d*e - b*c*f)*sqrt(-(a^2 - b^2
)/b^2)*log(2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b
^2) - 2*I*a) + (b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e
) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d*e - b*
c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2
*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b
^2)*log(-(I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*
x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*f*x + b*d*e)*sqrt(-(a^2 - b^
2)/b^2)*log(-(I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) + I*b*si
n(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*f*x + b*d*e)*sqrt(-(a^2
- b^2)/b^2)*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) ...
```

Sympy [F]

$$\int \frac{c+dx}{a+b\sin(e+fx)}\,dx = \int \frac{c+dx}{a+b\sin{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*sin(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c + dx}{a + b\sin(e + fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{c + dx}{a + b\sin(e + fx)}\,dx = \int \frac{dx + c}{b\sin(fx + e) + a}\,dx$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*sin(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx}{a + b\sin(e + fx)}\,dx = \int \frac{c + d\,x}{a + b\,\sin\left(e + f\,x\right)}\,dx$$

input
```
int((c + d*x)/(a + b*sin(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{c+dx}{a+b\sin(e+fx)}\,dx$$
$$=\frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)c+\left(\int\frac{x}{\sin(fx+e)b+a}dx\right)a^2df-\left(\int\frac{x}{\sin(fx+e)b+a}dx\right)b^2df}{f\,(a^2-b^2)}$$

input

```
int((d*x+c)/(a+b*sin(f*x+e)),x)
```

output

```
(2*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*c +
int(x/(sin(e + f*x)*b + a),x)*a**2*d*f - int(x/(sin(e + f*x)*b + a),x)*b**
2*d*f)/(f*(a**2 - b**2))
```

3.166 $\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx=\text{Int}\left(\frac{1}{(c+dx)(a+b\sin(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx=\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Sin[e + f*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+b\sin(fx+e))}dx$$

input
```
int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 10.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(a+b\sin(e+fx))\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

output
```
Integral(1/((a + b*sin(e + f*x))*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*(b*sin(f*x + e) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*sin(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(a+b\,\sin(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*sin(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*sin(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.80

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \frac{-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)bc+\sin(fx+e)bdx+ac+adx}dx\right)bd+\log(dx+c)}{ad}$$

input
```
int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

output
```
( - int(sin(e + f*x)/(sin(e + f*x)*b*c + sin(e + f*x)*b*d*x + a*c + a*d*x)
,x)*b*d + log(c + d*x))/(a*d)
```

3.167 $\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\sin(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Sin[e + f*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(dx+c\right)^{2} \left(a+b\sin \left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx=\int \frac{1}{\left(dx+c\right)^{2}\left(b\sin \left(fx+e\right)+a\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 64.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx=\int \frac{1}{\left(a+b\sin \left(e+fx\right)\right)\left(c+dx\right)^{2}}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*sin(f*x+e)),x)
```

output
```
Integral(1/((a + b*sin(e + f*x))*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sin(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 36.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \int \frac{1}{(a+b\,\sin\left(e+f\,x\right))\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + b*sin(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*sin(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 8.20

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \frac{-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)b\,c^2+2\sin(fx+e)bcdx+\sin(fx+e)b\,d^2x^2+a\,c^2+2acdx+a\,d^2x^2}dx\right)b\,c^2-\left(\int \frac{\sin(fx+e)}{\sin(fx+e)b\,c^2+2\sin(fx+e)bcdx+\sin(fx+}\right.}{ac\,(dx+c)}$$

input
```
int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

output
```
( - int(sin(e + f*x)/(sin(e + f*x)*b*c**2 + 2*sin(e + f*x)*b*c*d*x + sin(e
 + f*x)*b*d**2*x**2 + a*c**2 + 2*a*c*d*x + a*d**2*x**2),x)*b*c**2 - int(si
n(e + f*x)/(sin(e + f*x)*b*c**2 + 2*sin(e + f*x)*b*c*d*x + sin(e + f*x)*b*
d**2*x**2 + a*c**2 + 2*a*c*d*x + a*d**2*x**2),x)*b*c*d*x + x)/(a*c*(c + d*
x))
```

3.168 $\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 925

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \frac{i(c+dx)^3}{(a^2-b^2)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} - \frac{3ad(c+dx)^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} + \frac{3ad(c+dx)^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} - \frac{6d^3\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^4} - \frac{6iad^2(c+dx)\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} - \frac{6d^3\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^4} + \frac{6iad^2(c+dx)\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} + \frac{6ad^3\,\mathrm{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^4} - \frac{6ad^3\,\mathrm{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^4} + \frac{b(c+dx)^3\cos(e+fx)}{(a^2-b^2)f(a+b\sin(e+fx))}$$

output
```
6*I*d^2*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2
)/f^3-3*d*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)
/f^2-6*I*a*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(
a^2-b^2)^(3/2)/f^3-3*d*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2
)))/(a^2-b^2)/f^2+I*(d*x+c)^3/(a^2-b^2)/f-I*a*(d*x+c)^3*ln(1-I*b*exp(I*(f*
x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f-3*a*d*(d*x+c)^2*polylog(2,I*b
*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+6*I*d^2*(d*x+c)*p
olylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3+3*a*d*(d*x+
c)^2*polylog(2,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2
-6*d^3*polylog(3,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4+6*I
*a*d^2*(d*x+c)*polylog(3,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)
^(3/2)/f^3-6*d^3*polylog(3,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^
2)/f^4+I*a*(d*x+c)^3*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2
)^(3/2)/f+6*a*d^3*polylog(4,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b
^2)^(3/2)/f^4-6*a*d^3*polylog(4,I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a
^2-b^2)^(3/2)/f^4+b*(d*x+c)^3*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 742, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx$$

$$= \frac{if^3(c+dx)^3 - 3df^2(c+dx)^2\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right) - 3df^2(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right) + 6id^2\Big(f(c+dx}{}$$

input
```
Integrate[(c + d*x)^3/(a + b*Sin[e + f*x])^2,x]
```

output
```
(I*f^3*(c + d*x)^3 - 3*d*f^2*(c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a
 + Sqrt[a^2 - b^2])] - 3*d*f^2*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(
a + Sqrt[a^2 - b^2])] + (6*I)*d^2*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f
*x)))/(a - Sqrt[a^2 - b^2])] + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - S
qrt[a^2 - b^2])]) + (6*I)*d^2*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x))
)/(a + Sqrt[a^2 - b^2])] + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[
a^2 - b^2])]) - (I*a*(f^3*(c + d*x)^3*Log[1 + (I*b*E^(I*(e + f*x)))/(-a +
Sqrt[a^2 - b^2])] - f^3*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqr
t[a^2 - b^2])] - (3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x
)))/(-a + Sqrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, ((-I)*b*E^(I*
(e + f*x)))/(-a + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)
))/(a - Sqrt[a^2 - b^2])]) + (3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, (I*b*E^(I
*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, (I*b*
E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e +
f*x)))/(a + Sqrt[a^2 - b^2])])))/Sqrt[a^2 - b^2] + (b*f^3*(c + d*x)^3*Cos[
e + f*x])/(a + b*Sin[e + f*x]))/((a^2 - b^2)*f^4)
```

Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 857, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3805, 3042, 3804, 2694, 27, 2620, 3011, 5030, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{(c+dx)^3}{a+b\sin(e+fx)}dx}{a^2-b^2}-\frac{3bd\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^3\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(c+dx)^3}{a+b\sin(e+fx)}dx}{a^2-b^2}-\frac{3bd\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{ 3804}$$

$$\frac{2a\int \frac{e^{i(e+fx)}(c+dx)^3}{2e^{i(e+fx)}a-ibe^{2i(e+fx)}+ib}dx}{a^2-b^2}-\frac{3bd\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2a\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{2\left(a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{2\left(a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{3bd\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2a\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^3}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{3bd\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}$$

$$\frac{3bd\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe}{a-\cdots}\right)}{bf}\cdots\right.}{\cdots}\right.}{a^2-b^2}$$

$$\frac{3bd\int\frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$\downarrow$ 5030

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe}{a-\cdots}\right)}{bf}\cdots\right.}{\cdots}\right.}{a^2-b^2}$$

$$\frac{3bd\left(\int\frac{e^{i(e+fx)}(c+dx)^2}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx+\int\frac{e^{i(e+fx)}(c+dx)^2}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx-\frac{i(c+dx)^3}{3bd}\right)}{f\left(a^2-b^2\right)}+$$

$$\frac{b(c+dx)^3\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$\downarrow$ 2620

$$2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe}{a-}\right.}{bf}\right.}{}\right.$$

$$\overline{\qquad a^2-b^2}$$

$$\frac{3bd\left(-\frac{2d\int(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bf}-\frac{2d\int(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}+\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}+\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(}}{\sqrt{a^2}}\right.}{bf}\right.}{f(a^2-b^2)}$$

$$\frac{b(c+dx)^3\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\downarrow$ 3011

$$-\frac{3bd\left(-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{id\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{id\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+}}{a+\sqrt{a^2}}\right.}{f}\right.}{bf}\right.}{f(a^2-b^2)}$$

$$2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe}{a-}\right.}{bf}\right.}{}\right.$$

$$\overline{\qquad a^2-b^2}$$

$$\frac{b(c+dx)^3\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\downarrow$ 2720

$$\frac{b\cos(e+fx)(c+dx)^3}{(a^2-b^2)f(a+b\sin(e+fx))}-$$

$$\frac{3bd\left(-\frac{i(c+dx)^3}{3bd}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{Pol}\ldots}{\ldots}\ldots\right.}{bf}\ldots\right.}{(a^2-b^2)f}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe\ldots}{a-\ldots}\right)}{bf}\ldots\right.}{\ldots}\ldots\right.}{a^2-b^2}$$

$$\Big\downarrow \; 7143$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe\ldots}{a-\ldots}\right)}{bf}\ldots\right.}{\ldots}\ldots\right.}{a^2-b^2}$$

$$\frac{3bd\left(-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f^2}\right)}{bf}\ldots\right.}{f(a^2-b^2)}$$

$$\frac{b(c+dx)^3\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \; 7163$$

$$\frac{b\cos(e+fx)(c+dx)^3}{(a^2-b^2)\,f(a+b\sin(e+fx))} -$$

$$\frac{3bd\left(-\frac{i(c+dx)^3}{3bd}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,\frac{ibe^{i(}}{a-\sqrt{}}\right.}{f^2}\right.}{bf}\right.}{(a^2-b^2)\,f}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.}{}$$

$$\downarrow 2720$$

$$\frac{b\cos(e+fx)(c+dx)^3}{(a^2-b^2)\,f(a+b\sin(e+fx))} -$$

$$\frac{3bd\left(-\frac{i(c+dx)^3}{3bd}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,\frac{ibe^{i(}}{a-\sqrt{}}\right.}{f^2}\right.}{bf}\right.}{(a^2-b^2)\,f}$$

$$2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\left(\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}-\frac{i(c+dx)\operatorname{PolyLo}}{}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 7143

$$\frac{b\cos(e+fx)(c+dx)^3}{(a^2-b^2)f(a+b\sin(e+fx))} -$$

$$\frac{3bd\left(-\frac{i(c+dx)^3}{3bd}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}+\frac{\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)(c+dx)^2}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,\frac{ibe^{i(}}{a-\sqrt{}}\right.}{f^2}\right.}{bf}\right.}{(a^2-b^2)f}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}\right.}{a^2}$$

input
```
Int[(c + d*x)^3/(a + b*Sin[e + f*x])^2,x]
```

output
```
(-3*b*d*(((-1/3*I)*(c + d*x)^3)/(b*d) + ((c + d*x)^2*Log[1 - (I*b*E^(I*(e
+ f*x)))/(a - Sqrt[a^2 - b^2])])/(b*f) + ((c + d*x)^2*Log[1 - (I*b*E^(I*(e
 + f*x)))/(a + Sqrt[a^2 - b^2])])/(b*f) - (2*d*((I*(c + d*x)*PolyLog[2, (I
*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/f - (d*PolyLog[3, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/f^2))/(b*f) - (2*d*((I*(c + d*x)*PolyLog
[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/f - (d*PolyLog[3, (I*b*E
^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/f^2))/(b*f)))/((a^2 - b^2)*f) + (2
*a*(((-1/2*I)*b*(((c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2
- b^2])])/(b*f) - (3*d*((I*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a
 - Sqrt[a^2 - b^2])])/f - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (I*b*E^(I*(
e + f*x)))/(a - Sqrt[a^2 - b^2])])/f + (d*PolyLog[4, (I*b*E^(I*(e + f*x)))
/(a - Sqrt[a^2 - b^2])])/f^2))/f))/(b*f)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((c
 + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(b*f) - (3
*d*((I*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
)/f - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])])/f + (d*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2
])])/f^2))/f))/(b*f)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (b*(c + d*x)^3*Cos[
e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(dx+c)^3}{(a+b\sin(fx+e))^2}dx$$

input
```
int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

output
```
int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5112 vs. $2(807) = 1614$.

Time = 0.40 (sec) , antiderivative size = 5112, normalized size of antiderivative = 5.53

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3/(a+b*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*sin(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(a + b*sin(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

output
```
(8*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
e + f*x)*a**5*b*c**3*f**3 - 96*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a
+ b)/sqrt(a**2 - b**2))*sin(e + f*x)*a**5*b*c*d**2*f - 48*sqrt(a**2 - b**2
)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*a**4*b**2*
c**2*d*f**2 + 384*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**
2 - b**2))*sin(e + f*x)*a**4*b**2*d**3 + 288*sqrt(a**2 - b**2)*atan((tan((
e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*a**3*b**3*c*d**2*f + 48
*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(e
+ f*x)*a**2*b**4*c**2*d*f**2 - 960*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2
)*a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*a**2*b**4*d**3 - 192*sqrt(a**2 -
b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*a*b**5
*c*d**2*f + 576*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2
- b**2))*sin(e + f*x)*b**6*d**3 + 8*sqrt(a**2 - b**2)*atan((tan((e + f*x)/
2)*a + b)/sqrt(a**2 - b**2))*a**6*c**3*f**3 - 96*sqrt(a**2 - b**2)*atan((t
an((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a**6*c*d**2*f - 48*sqrt(a**2 - b
**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a**5*b*c**2*d*f**2 +
 384*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a*
*5*b*d**3 + 288*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2
- b**2))*a**4*b**2*c*d**2*f + 48*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*
a + b)/sqrt(a**2 - b**2))*a**3*b**3*c**2*d*f**2 - 960*sqrt(a**2 - b**2)...
```

3.169 $\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 671

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \frac{i(c+dx)^2}{(a^2-b^2)f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{2id^2\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} - \frac{2ad(c+dx)\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} + \frac{2id^2\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} + \frac{2ad(c+dx)\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} - \frac{2iad^2\,\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} + \frac{2iad^2\,\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} + \frac{b(c+dx)^2\cos(e+fx)}{(a^2-b^2)f(a+b\sin(e+fx))}$$

output
```
I*(d*x+c)^2/(a^2-b^2)/f-2*d*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(
1/2)))/(a^2-b^2)/f^2-I*a*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1
/2)))/(a^2-b^2)^(3/2)/f-2*d*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(
1/2)))/(a^2-b^2)/f^2+I*a*(d*x+c)^2*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1
/2)))/(a^2-b^2)^(3/2)/f+2*I*d^2*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^
(1/2)))/(a^2-b^2)/f^3-2*a*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(a-(a^2-b
^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+2*I*d^2*polylog(2,I*b*exp(I*(f*x+e))/(a+(a
^2-b^2)^(1/2)))/(a^2-b^2)/f^3+2*a*d*(d*x+c)*polylog(2,I*b*exp(I*(f*x+e))/(
a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2-2*I*a*d^2*polylog(3,I*b*exp(I*(f*x
+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+2*I*a*d^2*polylog(3,I*b*exp(
I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+b*(d*x+c)^2*cos(f*x+e)
/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx$$

$$= \frac{if^2(c+dx)^2 - 2df(c+dx)\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right) - 2df(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right) + 2id^2\,\mathrm{PolyLog}\left(2,-\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + b*Sin[e + f*x])^2,x]
```

output
```
(I*f^2*(c + d*x)^2 - 2*d*f*(c + d*x)*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + S
qrt[a^2 - b^2])] - 2*d*f*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])] + (2*I)*d^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a
^2 - b^2])] + (2*I)*d^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b
^2])] - (I*a*(f^2*(c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2
 - b^2])] - f^2*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
b^2])] - (2*I)*d*f*(c + d*x)*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqr
t[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a +
 Sqrt[a^2 - b^2])] + 2*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2
- b^2])] - 2*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))
/Sqrt[a^2 - b^2] + (b*f^2*(c + d*x)^2*Cos[e + f*x])/(a + b*Sin[e + f*x]))/
((a^2 - b^2)*f^3)
```

Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 631, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3805, 3042, 3804, 2694, 27, 2620, 3011, 2720, 5030, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\sin(e+fx)}dx}{a^2-b^2}-\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\sin(e+fx)}dx}{a^2-b^2}-\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3804}$$

$$\frac{2a\int \frac{e^{i(e+fx)}(c+dx)^2}{2e^{i(e+fx)}a-ibe^{2i(e+fx)}+ib}dx}{a^2-b^2}-\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{2\left(a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{2\left(a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)^2}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\downarrow$ 2620

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\int (c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{2d\int (c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bf}\right.}{2\sqrt{a^2-b^2}}}{a^2-b^2}$$

$$\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\downarrow$ 3011

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{id\int \operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right.}{bf}\right.}{}}{a^2-b^2}$$

$$\frac{2bd\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\downarrow$ 2720

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\,\mathrm{lo}\cdots}{}\right.}{}\right.}{a^2-b^2\cdots}$$

$$\frac{2bd\int\frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)^2\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$\downarrow$ 5030

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\,\mathrm{lo}\cdots}{}\right.}{}\right.}{a^2-b^2\cdots}$$

$$\frac{2bd\left(\int\frac{e^{i(e+fx)}(c+dx)}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx+\int\frac{e^{i(e+fx)}(c+dx)}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx-\frac{i(c+dx)^2}{2bd}\right)}{f\left(a^2-b^2\right)}+$$
$$\frac{b(c+dx)^2\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$\downarrow$ 2620

$$2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\,\text{lo}\ldots}{\ldots}\right.}{\ldots}\right.$$

$$\overline{a^2-b^2}$$

$$\frac{2bd\left(-\frac{d\int\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bf}-\frac{d\int\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{i(c+dx\ldots}{2bd}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^2\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{2715}$$

$$2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\,\text{lo}\ldots}{\ldots}\right.}{\ldots}\right.$$

$$\overline{a^2-b^2}$$

$$\frac{2bd\left(\frac{id\int e^{-i(e+fx)}\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{bf^2}+\frac{id\int e^{-i(e+fx)}\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{bf^2}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}+\frac{(c+d\ldots}{\ldots}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^2\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-i(e+fx)}\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\ldots}{\ldots}\right.}{\ldots}\right.}{a^2-b^2}$$

$$\frac{2bd\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{id\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}-\frac{id\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}-\frac{i(c+dx)}{2bd}\ldots\right.}{f(a^2-b^2)}$$

$$\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\Big\downarrow$ 7143

$$-\frac{2bd\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{id\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}-\frac{id\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}-\frac{i(c+d}{2bd}\ldots\right.}{f(a^2-b^2)}$$

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{2d\left(\frac{i(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f}-\frac{d\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{f^2}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\ldots\right.}{\ldots}\right.}{a^2-b^2}$$

$$\frac{b(c+dx)^2\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

input
```
Int[(c + d*x)^2/(a + b*Sin[e + f*x])^2,x]
```

output
```
(-2*b*d*(((-1/2*I)*(c + d*x)^2)/(b*d) + ((c + d*x)*Log[1 - (I*b*E^(I*(e +
f*x)))/(a - Sqrt[a^2 - b^2])])/(b*f) + ((c + d*x)*Log[1 - (I*b*E^(I*(e + f
*x)))/(a + Sqrt[a^2 - b^2])])/(b*f) - (I*d*PolyLog[2, (I*b*E^(I*(e + f*x))
)/(a - Sqrt[a^2 - b^2])])/(b*f^2) - (I*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/
(a + Sqrt[a^2 - b^2])])/(b*f^2)))/((a^2 - b^2)*f) + (2*a*(((-1/2*I)*b*(((c
 + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(b*f) - (2
*d*((I*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/
f - (d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/f^2))/(b*f
)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])])/(b*f) - (2*d*((I*(c + d*x)*PolyLog[2, (I*b*E^(I*(
e + f*x)))/(a + Sqrt[a^2 - b^2])])/f - (d*PolyLog[3, (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])])/f^2))/(b*f)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (b*
(c + d*x)^2*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(a+b\sin(fx+e))^2}dx$$

input
```
int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

output
```
int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3091 vs. $2(581) = 1162$.

Time = 0.31 (sec) , antiderivative size = 3091, normalized size of antiderivative = 4.61

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2/(a+b*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*sin(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(a + b*sin(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

output

```
(6*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
e + f*x)*a**4*b*c**2*f**2 - 24*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a
+ b)/sqrt(a**2 - b**2))*sin(e + f*x)*a**4*b*d**2 - 24*sqrt(a**2 - b**2)*at
an((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*a**3*b**2*c*d*
f + 72*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*
sin(e + f*x)*a**2*b**3*d**2 + 24*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*
a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*a*b**4*c*d*f - 48*sqrt(a**2 - b**2)
*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(e + f*x)*b**5*d**2 +
 6*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a**5
*c**2*f**2 - 24*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2
- b**2))*a**5*d**2 - 24*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sq
rt(a**2 - b**2))*a**4*b*c*d*f + 72*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2
)*a + b)/sqrt(a**2 - b**2))*a**3*b**2*d**2 + 24*sqrt(a**2 - b**2)*atan((ta
n((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2*b**3*c*d*f - 48*sqrt(a**2 -
b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a*b**4*d**2 + 3*cos
(e + f*x)*a**5*b*c**2*f**2 + 12*cos(e + f*x)*a**5*b*c*d*f**2*x + 6*cos(e +
 f*x)*a**5*b*d**2*f**2*x**2 - 24*cos(e + f*x)*a**4*b**2*d**2*f*x - 3*cos(e
 + f*x)*a**3*b**3*c**2*f**2 - 24*cos(e + f*x)*a**3*b**3*c*d*f**2*x - 12*co
s(e + f*x)*a**3*b**3*d**2*f**2*x**2 + 48*cos(e + f*x)*a**2*b**4*d**2*f*x +
 12*cos(e + f*x)*a*b**5*c*d*f**2*x + 6*cos(e + f*x)*a*b**5*d**2*f**2*x*...
```

3.170 $\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 305

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} - \frac{d\log(a+b\sin(e+fx))}{(a^2-b^2)f^2} - \frac{ad\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} + \frac{ad\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} + \frac{b(c+dx)\cos(e+fx)}{(a^2-b^2)f(a+b\sin(e+fx))}$$

output
```
-I*a*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/
f+I*a*(d*x+c)*ln(1-I*b*exp(I*(f*x+e))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)
/f-d*ln(a+b*sin(f*x+e))/(a^2-b^2)/f^2-a*d*polylog(2,I*b*exp(I*(f*x+e))/(a-
(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+a*d*polylog(2,I*b*exp(I*(f*x+e))/(a+
(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+b*(d*x+c)*cos(f*x+e)/(a^2-b^2)/f/(a+
b*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \frac{-d\log(a+b\sin(e+fx)) + \frac{a\left(-if(c+dx)\left(\log\left(1+\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)\right)-d\,\text{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{-a+\sqrt{a^2-b^2}}\right)+d\,\text{PolyL}}{\sqrt{a^2-b^2}}}{(a^2-b^2)\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Sin[e + f*x])^2,x]
```

output
```
(-(d*Log[a + b*Sin[e + f*x]]) + (a*((-I)*f*(c + d*x)*(Log[1 + (I*b*E^(I*(e
 + f*x)))/(-a + Sqrt[a^2 - b^2])] - Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqr
t[a^2 - b^2])]) - d*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b
^2])] + d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/Sqrt[a
^2 - b^2] + (b*f*(c + d*x)*Cos[e + f*x])/(a + b*Sin[e + f*x]))/((a^2 - b^2
)*f^2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 3805, 3042, 3147, 16, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{c+dx}{a+b\sin(e+fx)}dx}{a^2-b^2} - \frac{bd\int \frac{\cos(e+fx)}{a+b\sin(e+fx)}dx}{f(a^2-b^2)} + \frac{b(c+dx)\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{a\int \frac{c+dx}{a+b\sin(e+fx)}dx}{a^2-b^2}-\frac{bd\int \frac{\cos(e+fx)}{a+b\sin(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$\downarrow$ 3147

$$\frac{a\int \frac{c+dx}{a+b\sin(e+fx)}dx}{a^2-b^2}-\frac{d\int \frac{1}{a+b\sin(e+fx)}d(b\sin(e+fx))}{f^2\left(a^2-b^2\right)}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$\downarrow$ 16

$$\frac{a\int \frac{c+dx}{a+b\sin(e+fx)}dx}{a^2-b^2}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\downarrow$ 3804

$$\frac{2a\int \frac{e^{i(e+fx)}(c+dx)}{2e^{i(e+fx)}a-ibe^{2i(e+fx)}+ib}dx}{a^2-b^2}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\downarrow$ 2694

$$\frac{2a\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{2\left(a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{2\left(a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\downarrow$ 27

$$\frac{2a\left(\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{a-ibe^{i(e+fx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(e+fx)}(c+dx)}{a-ibe^{i(e+fx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\downarrow$ 2620

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{d\int \log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{d\int \log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{b(c+dx)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2715

$$\frac{2a\left(\frac{ib\left(\frac{id\int e^{-i(e+fx)}\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{bf^2}+\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{id\int e^{-i(e+fx)}\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(e+fx)}}{bf^2}+\frac{(c+dx)}{}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}$$

$$\frac{b(c+dx)\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2(a^2-b^2)}$$

$\Big\downarrow$ 2838

$$\frac{2a\left(\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{bf}-\frac{id\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf}-\frac{id\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}+$$

$$\frac{b(c+dx)\cos(e+fx)}{f(a^2-b^2)(a+b\sin(e+fx))}-\frac{d\log(a+b\sin(e+fx))}{f^2(a^2-b^2)}$$

input

```
Int[(c + d*x)/(a + b*Sin[e + f*x])^2,x]
```

output

```
-((d*Log[a + b*Sin[e + f*x]])/((a^2 - b^2)*f^2)) + (2*a*(((-1/2*I)*b*(((c
 + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(b*f) - (I*d*
PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(b*f^2)))/Sqrt[a^
2 - b^2] + ((I/2)*b*(((c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/(b*f) - (I*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
b^2])])/(b*f^2)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (b*(c + d*x)*Cos[e + f*x
])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(275) = 550.

Time = 3.91 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.10

method	result
risch	$\frac{2(dx+c)\left(ib+a\,\mathrm{e}^{i(fx+e)}\right)}{f(a^2-b^2)\left(\mathrm{e}^{2i(fx+e)}b-b+2ia\,\mathrm{e}^{i(fx+e)}\right)}-\frac{2d\ln\left(\mathrm{e}^{i(fx+e)}\right)}{(-a^2+b^2)f^2}+\frac{d\ln\left(i\mathrm{e}^{2i(fx+e)}b-ib-2a\,\mathrm{e}^{i(fx+e)}\right)}{(-a^2+b^2)f^2}+\frac{2iade\arctan\left(\frac{2ib\,\mathrm{e}^{i(fx+e)}-2}{2\sqrt{-a^2+b^2}}\right.}{(-a^2+b^2)^{\frac{3}{2}}f^2}$

input
```
int((d*x+c)/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(d*x+c)*(I*b+a*exp(I*(f*x+e)))/f/(a^2-b^2)/(exp(2*I*(f*x+e))*b-b+2*I*a*e
xp(I*(f*x+e)))-2/(-a^2+b^2)/f^2*d*ln(exp(I*(f*x+e)))+1/(-a^2+b^2)/f^2*d*ln
(I*exp(2*I*(f*x+e))*b-I*b-2*a*exp(I*(f*x+e)))+2*I/(-a^2+b^2)^(3/2)/f^2*a*d
*e*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))-2*I/(-a^2+b^2)^
(3/2)/f*a*c*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))-1/(-a^
2+b^2)^(3/2)/f*a*d*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2
+b^2)^(1/2)))*x+1/(-a^2+b^2)^(3/2)/f*a*d*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^
2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/(-a^2+b^2)^(3/2)/f^2*a*d*ln((-I*a-b*
exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*e+1/(-a^2+b^2)^(
3/2)/f^2*a*d*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1
/2)))*e+I/(-a^2+b^2)^(3/2)/f^2*a*d*dilog((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)
^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))-I/(-a^2+b^2)^(3/2)/f^2*a*d*dilog((I*a+b*e
xp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1512 vs. $2(267) = 534$.

Time = 0.29 (sec) , antiderivative size = 1512, normalized size of antiderivative = 4.96

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/2*((I*a*b^2*d*sin(f*x + e) + I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*
a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*d*sin(f*x + e) - I*a^2*b*d)*sqr
t(-(a^2 - b^2)/b^2)*dilog((I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x
+ e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*d*
sin(f*x + e) - I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(f*x + e)
- a*sin(f*x + e) + (b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b
^2) - b)/b + 1) + (I*a*b^2*d*sin(f*x + e) + I*a^2*b*d)*sqrt(-(a^2 - b^2)/b
^2)*dilog((-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(
f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (a^2*b*d*f*x + a^2*b*d*e +
(a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*c
os(f*x + e) - a*sin(f*x + e) + (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(
a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*
d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(f*x + e) - a*sin(f
*x + e) - (b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/
b) - (a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sq
rt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) + (b*cos(f*x
 + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*f*x +
a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
*log(-(-I*a*cos(f*x + e) - a*sin(f*x + e) - (b*cos(f*x + e) - I*b*sin(f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx}{(a + b\sin(e + fx))^2} \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \int \frac{dx+c}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*sin(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(a + b*sin(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx$$

$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin\left(fx+e\right)abc + 2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2c + \cos\left(fx+e\right)a^2 \cdots}{\cdots}$$

input

```
int((d*x+c)/(a+b*sin(f*x+e))^2,x)
```

output

```
(2*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
e + f*x)*a*b*c + 2*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a*
*2 - b**2))*a**2*c + cos(e + f*x)*a**2*b*c - cos(e + f*x)*b**3*c + int(x/(
sin(e + f*x)**2*b**2 + 2*sin(e + f*x)*a*b + a**2),x)*sin(e + f*x)*a**4*b*d
*f - 2*int(x/(sin(e + f*x)**2*b**2 + 2*sin(e + f*x)*a*b + a**2),x)*sin(e +
 f*x)*a**2*b**3*d*f + int(x/(sin(e + f*x)**2*b**2 + 2*sin(e + f*x)*a*b + a
**2),x)*sin(e + f*x)*b**5*d*f + int(x/(sin(e + f*x)**2*b**2 + 2*sin(e + f*
x)*a*b + a**2),x)*a**5*d*f - 2*int(x/(sin(e + f*x)**2*b**2 + 2*sin(e + f*x
)*a*b + a**2),x)*a**3*b**2*d*f + int(x/(sin(e + f*x)**2*b**2 + 2*sin(e + f
*x)*a*b + a**2),x)*a*b**4*d*f)/(f*(sin(e + f*x)*a**4*b - 2*sin(e + f*x)*a*
*2*b**3 + sin(e + f*x)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.171 $\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\sin(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 43.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + b*Sin[e + f*x])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+b\sin{(fx+e)})^2}dx$$

input
```
int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sin{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2 + b^2)*d*x - (b^2*d*x + b^2*c)*cos(f*x + e)^2 + (a^2 + b^
2)*c + 2*(a*b*d*x + a*b*c)*sin(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 175.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(a+b\sin{(e+fx)})^2\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*sin(e + f*x))**2*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 7.40 (sec) , antiderivative size = 1601, normalized size of antiderivative = 80.05

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx=\int \frac{1}{(dx+c)(b\sin{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
(2*a*b*cos(2*f*x + 2*e)*cos(f*x + e) + 2*a*b*cos(f*x + e) - ((a^2*b^2 - b^
4)*d*f*x + (a^2*b^2 - b^4)*c*f + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*
c*f)*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*
cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*cos(f*x +
 e)*sin(2*f*x + 2*e) + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f)*sin(2
*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*sin(f*x +
e)^2 - 2*((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f + 2*((a^3*b - a*b^3)
*d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*((a^3*b -
 a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x + e))*integrate(-2*(a*b*d*cos
(f*x + e) + 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)^2 + 2*(a^2*d*f*x + a^2*c*
f)*sin(f*x + e)^2 + (a*b*d*cos(f*x + e) - (a*b*d*f*x + a*b*c*f)*sin(f*x +
e))*cos(2*f*x + 2*e) + (a*b*d*sin(f*x + e) + b^2*d + (a*b*d*f*x + a*b*c*f)
*cos(f*x + e))*sin(2*f*x + 2*e) + (a*b*d*f*x + a*b*c*f)*sin(f*x + e))/((a^
2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f
 + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4
)*c^2*f)*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*
b^2)*c*d*f*x + (a^4 - a^2*b^2)*c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*
d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e
)*sin(2*f*x + 2*e) + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*
x + (a^2*b^2 - b^4)*c^2*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*...
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*sin(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 35.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(a+b\,\sin(e+f\,x))^2\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*sin(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + b*sin(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 8.65

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx = \frac{-\left(\int \frac{\sin(fx+e)^2}{\sin(fx+e)^2b^2c+\sin(fx+e)^2b^2dx+2\sin(fx+e)abc+2\sin(fx+e)abdx+a^2c+a^2dx}dx\right)b^2d-2\left(\int \frac{}{\sin(fx+e)^2b^2c+\sin(fx+e)^2b^2dx\cdots}\right.}{a^2d}$$

input

```
int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

output

```
( - int(sin(e + f*x)**2/(sin(e + f*x)**2*b**2*c + sin(e + f*x)**2*b**2*d*x
 + 2*sin(e + f*x)*a*b*c + 2*sin(e + f*x)*a*b*d*x + a**2*c + a**2*d*x),x)*b
**2*d - 2*int(sin(e + f*x)/(sin(e + f*x)**2*b**2*c + sin(e + f*x)**2*b**2*
d*x + 2*sin(e + f*x)*a*b*c + 2*sin(e + f*x)*a*b*d*x + a**2*c + a**2*d*x),x
)*a*b*d + log(c + d*x))/(a**2*d)
```

3.172 $\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx=\text{Int}\left(\frac{1}{(c+dx)^2(a+b\sin(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 170.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+b\sin(fx+e))^2}dx$$

input
```
int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 5.45

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2 + b^2)*d^2*x^2 + 2*(a^2 + b^2)*c*d*x + (a^2 + b^2)*c^2 -
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*
a*b*c*d*x + a*b*c^2)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(a+b*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 20.34 (sec) , antiderivative size = 2265, normalized size of antiderivative = 113.25

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
(2*a*b*cos(2*f*x + 2*e)*cos(f*x + e) + 2*a*b*cos(f*x + e) - ((a^2*b^2 - b^
4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b
^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*c
os(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f
*x + (a^4 - a^2*b^2)*c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2
+ 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*
x + 2*e) + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b
^2 - b^4)*c^2*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^
4 - a^2*b^2)*c*d*f*x + (a^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2
 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*
((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c
^2*f)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a
^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e))*integrate(-2*
(2*a*b*d*cos(f*x + e) + 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)^2 + 2*(a^2*d*
f*x + a^2*c*f)*sin(f*x + e)^2 + (2*a*b*d*cos(f*x + e) - (a*b*d*f*x + a*b*c
*f)*sin(f*x + e))*cos(2*f*x + 2*e) + (2*a*b*d*sin(f*x + e) + 2*b^2*d + (a*
b*d*f*x + a*b*c*f)*cos(f*x + e))*sin(2*f*x + 2*e) + (a*b*d*f*x + a*b*c*f)*
sin(f*x + e))/((a^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 +
 3*(a^2*b^2 - b^4)*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f + ((a^2*b^2 - b^4)*d^
3*f*x^3 + 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)*c^2*d*f*x +...
```

Giac [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 35.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(a+b\sin(e+fx))^2(c+dx)^2}\,dx$$

input
```
int(1/((a + b*sin(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*sin(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 8816, normalized size of antiderivative = 440.80

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

output

```
(4*cos(e + f*x)*a*b**2*c*d - 2*int(cos(e + f*x)/(sin(e + f*x)**2*b**2*c**2
 + 2*sin(e + f*x)**2*b**2*c*d*x + sin(e + f*x)**2*b**2*d**2*x**2 + 2*sin(e
 + f*x)*a*b*c**2 + 4*sin(e + f*x)*a*b*c*d*x + 2*sin(e + f*x)*a*b*d**2*x**2
 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2),x)*sin(e + f*x)*a**3*b**2*c*
*3*d*f - 2*int(cos(e + f*x)/(sin(e + f*x)**2*b**2*c**2 + 2*sin(e + f*x)**2
*b**2*c*d*x + sin(e + f*x)**2*b**2*d**2*x**2 + 2*sin(e + f*x)*a*b*c**2 + 4
*sin(e + f*x)*a*b*c*d*x + 2*sin(e + f*x)*a*b*d**2*x**2 + a**2*c**2 + 2*a**
2*c*d*x + a**2*d**2*x**2),x)*sin(e + f*x)*a**3*b**2*c**2*d**2*f*x + 4*int(
cos(e + f*x)/(sin(e + f*x)**2*b**2*c**2 + 2*sin(e + f*x)**2*b**2*c*d*x + s
in(e + f*x)**2*b**2*d**2*x**2 + 2*sin(e + f*x)*a*b*c**2 + 4*sin(e + f*x)*a
*b*c*d*x + 2*sin(e + f*x)*a*b*d**2*x**2 + a**2*c**2 + 2*a**2*c*d*x + a**2*
d**2*x**2),x)*sin(e + f*x)*a**2*b**3*c**2*d**2 + 4*int(cos(e + f*x)/(sin(e
 + f*x)**2*b**2*c**2 + 2*sin(e + f*x)**2*b**2*c*d*x + sin(e + f*x)**2*b**2
*d**2*x**2 + 2*sin(e + f*x)*a*b*c**2 + 4*sin(e + f*x)*a*b*c*d*x + 2*sin(e
+ f*x)*a*b*d**2*x**2 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2),x)*sin(e
 + f*x)*a**2*b**3*c*d**3*x + 4*int(cos(e + f*x)/(sin(e + f*x)**2*b**2*c**2
 + 2*sin(e + f*x)**2*b**2*c*d*x + sin(e + f*x)**2*b**2*d**2*x**2 + 2*sin(e
 + f*x)*a*b*c**2 + 4*sin(e + f*x)*a*b*c*d*x + 2*sin(e + f*x)*a*b*d**2*x**2
 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2),x)*sin(e + f*x)*a*b**4*c**3*
d*f + 4*int(cos(e + f*x)/(sin(e + f*x)**2*b**2*c**2 + 2*sin(e + f*x)**2...
```

3.173 $\int (c+dx)^m (a+b\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m (a+b\sin(e+fx))^n\,dx = \text{Int}((c+dx)^m (a+b\sin(e+fx))^n, x)$$

output
```
Defer(Int)((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+b\sin(e+fx))^n\,dx = \int (c+dx)^m (a+b\sin(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^m(a+b\sin(e+fx))^ndx$$
$$\Big\downarrow \text{3807}$$
$$\int (c+dx)^m(a+b\sin(e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(a + b*Sin[e + f*x])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^{m} (a+b\sin (fx+e))^{n} \, dx$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+b\sin(e+fx))^n \, dx = \int (dx+c)^m (b\sin(fx+e)+a)^n \, dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m (a+b\sin(e+fx))^n \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+b*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx=\int (dx+c)^m(b\sin(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx=\int (dx+c)^m(b\sin(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 35.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx=\int (a+b\,\sin(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sin(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((a + b*sin(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx=\int (dx+c)^m\,(\sin\left(fx+e\right)b+a)^n\,dx$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

output
```
int((c + d*x)**m*(sin(e + f*x)*b + a)**n,x)
```

3.174 $\int (c+dx)^m (a+b\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 607

$$\int (c+dx)^m (a+b\sin(e+fx))^3\,dx = \frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f} - \frac{3b^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{8f} - \frac{3a^2be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{2f} - \frac{3b^3e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{8f} + \frac{3i2^{-3-m}ab^2e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f} - \frac{3i2^{-3-m}ab^2e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f} + \frac{3^{-1-m}b^3e^{3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3if(c+dx)}{d}\right)}{8f} + \frac{3^{-1-m}b^3e^{-3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3if(c+dx)}{d}\right)}{8f}$$

output
```
a^3*(d*x+c)^(1+m)/d/(1+m)+3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)-3/2*a^2*b*exp(I*
(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-3/8*
b^3*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/
d)^m)-3/2*a^2*b*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/exp(I*(e-c*f/d))/f/((I*
f*(d*x+c)/d)^m)-3/8*b^3*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/exp(I*(e-c*f/d)
)/f/((I*f*(d*x+c)/d)^m)+3*I*2^(-3-m)*a*b^2*exp(2*I*(e-c*f/d))*(d*x+c)^m*GA
MMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-3*I*2^(-3-m)*a*b^2*(d*x+c
)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+1/
8*3^(-1-m)*b^3*exp(3*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-3*I*f*(d*x+c)/d)/f/
((-I*f*(d*x+c)/d)^m)+1/8*3^(-1-m)*b^3*(d*x+c)^m*GAMMA(1+m,3*I*f*(d*x+c)/d)
/exp(3*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 11.71 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.68

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx$$
$$= \frac{i(c+dx)^m\left(-\frac{12ia(2a^2+3b^2)f(c+dx)}{d(1+m)}+9ib(4a^2+b^2)\,e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)+9ib(4a^2\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^3,x]
```

output
```
((I/24)*(c + d*x)^m*(((-12*I)*a*(2*a^2 + 3*b^2)*f*(c + d*x))/(d*(1 + m)) +
 ((9*I)*b*(4*a^2 + b^2)*E^(I*(e - (c*f)/d))*Gamma[1 + m, ((-I)*f*(c + d*x)
)/d])/(((-I)*f*(c + d*x))/d)^m + ((9*I)*b*(4*a^2 + b^2)*Gamma[1 + m, (I*f*
(c + d*x))/d])/(E^(I*(e - (c*f)/d))*((I*f*(c + d*x))/d)^m) + (9*a*b^2*E^((
2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*(((-I)*f*(c
 + d*x))/d)^m) - (9*a*b^2*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m*E^((2*
I)*(e - (c*f)/d))*((I*f*(c + d*x))/d)^m) - (I*b^3*E^((3*I)*(e - (c*f)/d))*
Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(3^m*(((-I)*f*(c + d*x))/d)^m) - (I*
b^3*Gamma[1 + m, ((3*I)*f*(c + d*x))/d])/(3^m*E^((3*I)*(e - (c*f)/d))*((I*
f*(c + d*x))/d)^m)))/f
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a+b\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^3(c+dx)^m+3a^2b(c+dx)^m\sin(e+fx)+3ab^2(c+dx)^m\sin^2(e+fx)+b^3(c+dx)^m\sin^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^{m+1}}{d(m+1)}-\frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}+\frac{3iab^22^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{3iab^22^{-m-3}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2if(c+dx)}{d}\right)}{f}+\frac{3ab^2(c+dx)^{m+1}}{2d(m+1)}-\frac{3b^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{8f}+\frac{b^33^{-m-1}e^{3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3if(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{if(c+dx)}{d}\right)}{8f}+\frac{b^33^{-m-1}e^{-3i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3if(c+dx)}{d}\right)}{8f}$$

input
```
Int[(c + d*x)^m*(a + b*Sin[e + f*x])^3,x]
```

output
```
(a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1
+ m)) - (3*a^2*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c +
 d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (3*b^3*E^(I*(e - (c*f)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m
) - (3*a^2*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*
f)/d))*f*((I*f*(c + d*x))/d)^m) - (3*b^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c
+ d*x))/d])/(8*E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3
 - m)*a*b^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c
+ d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - ((3*I)*2^(-3 - m)*a*b^2*(c + d*
x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f
*(c + d*x))/d)^m) + (3^(-1 - m)*b^3*E^((3*I)*(e - (c*f)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-3*I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) + (3^(-
1 - m)*b^3*(c + d*x)^m*Gamma[1 + m, ((3*I)*f*(c + d*x))/d])/(8*E^((3*I)*(e
 - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\sin(fx+e))^3\, dx$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.72

$$\int (c+dx)^m(a+b\sin(e+fx))^3\, dx = -\frac{9\left((4a^2b+b^3)dm+(4a^2b+b^3)d\right)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+ide-icf}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+9\left(-iab^2dm-iab^2d\right)e^{\left(-\right.}}{}$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/24*(9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*e^(-(d*m*log(I*f/d) + I
*d*e - I*c*f)/d)*gamma(m + 1, (I*d*f*x + I*c*f)/d) + 9*(-I*a*b^2*d*m - I*a
*b^2*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*
d*f*x + I*c*f)/d) - (b^3*d*m + b^3*d)*e^(-(d*m*log(-3*I*f/d) - 3*I*d*e + 3
*I*c*f)/d)*gamma(m + 1, -3*(I*d*f*x + I*c*f)/d) + 9*((4*a^2*b + b^3)*d*m +
 (4*a^2*b + b^3)*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)*gamma(m + 1,
(-I*d*f*x - I*c*f)/d) + 9*(I*a*b^2*d*m + I*a*b^2*d)*e^(-(d*m*log(2*I*f/d)
+ 2*I*d*e - 2*I*c*f)/d)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d) - (b^3*d*m +
 b^3*d)*e^(-(d*m*log(3*I*f/d) + 3*I*d*e - 3*I*c*f)/d)*gamma(m + 1, -3*(-I*
d*f*x - I*c*f)/d) - 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3 + 3*a*b^2)*c*f)*(
d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx = \int (a+b\sin{(e+fx)})^3\,(c+dx)^m\,\,dx$$

input
```
integrate((d*x+c)**m*(a+b*sin(f*x+e))**3,x)
```

output
```
Integral((a + b*sin(e + f*x))**3*(c + d*x)**m, x)
```

Maxima [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx = \int (b\sin{(fx+e)}+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(6*a*b^2*e^(m*log(d*x + c) + log(d
*x + c)) - 6*(a*b^2*d*m + a*b^2*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e),
 x) - (b^3*d*m + b^3*d)*integrate((d*x + c)^m*sin(3*f*x + 3*e), x) + 3*((4
*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*integrate((d*x + c)^m*sin(f*x + e),
 x))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx = \int (b\sin{(fx+e)}+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx=\int (a+b\,\sin\left(e+f\,x\right))^3\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + b*sin(e + f*x))^3*(c + d*x)^m,x)
```

output
```
int((a + b*sin(e + f*x))^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx=\text{too large to display}$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

output

```
( - 15*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2*tan((e + f*x)/2)**6*b**3*
c*d*f*m - 15*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2*tan((e + f*x)/2)**6
*b**3*c*d*f - 45*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2*tan((e + f*x)/2
)**4*b**3*c*d*f*m - 45*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2*tan((e +
f*x)/2)**4*b**3*c*d*f - 45*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2*tan((
e + f*x)/2)**2*b**3*c*d*f*m - 45*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2
*tan((e + f*x)/2)**2*b**3*c*d*f - 15*(c + d*x)**m*cos(e + f*x)*sin(e + f*x
)**2*b**3*c*d*f*m - 15*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)**2*b**3*c*d*
f - 108*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)**6*a*b**2*
c*d*f*m - 108*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)**6*a
*b**2*c*d*f - 324*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)*
*4*a*b**2*c*d*f*m - 324*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*
x)/2)**4*a*b**2*c*d*f - 324*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e
+ f*x)/2)**2*a*b**2*c*d*f*m - 324*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*t
an((e + f*x)/2)**2*a*b**2*c*d*f - 108*(c + d*x)**m*cos(e + f*x)*sin(e + f*
x)*a*b**2*c*d*f*m - 108*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*a*b**2*c*d*
f - 135*(c + d*x)**m*cos(e + f*x)*tan((e + f*x)/2)**6*a**2*b*c*d*f*m - 135
*(c + d*x)**m*cos(e + f*x)*tan((e + f*x)/2)**6*a**2*b*c*d*f - 30*(c + d*x)
**m*cos(e + f*x)*tan((e + f*x)/2)**6*b**3*c*d*f*m - 30*(c + d*x)**m*cos(e
+ f*x)*tan((e + f*x)/2)**6*b**3*c*d*f - 405*(c + d*x)**m*cos(e + f*x)*t...
```

3.175 $\int (c+dx)^m (a+b\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 318

$$
\begin{aligned}
\int (c+dx)^m (a+b\sin(e+fx))^2\,dx &= \frac{a^2(c+dx)^{1+m}}{d(1+m)}+\frac{b^2(c+dx)^{1+m}}{2d(1+m)} \\
&- \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f} \\
&- \frac{abe^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f} \\
&+ \frac{i2^{-3-m}b^2e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f} \\
&- \frac{i2^{-3-m}b^2e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{f}
\end{aligned}
$$

output
```
a^2*(d*x+c)^(1+m)/d/(1+m)+1/2*b^2*(d*x+c)^(1+m)/d/(1+m)-a*b*exp(I*(e-c*f/d
))*(d*x+c)^m*GAMMA(1+m,-I*f*(d*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-a*b*(d*x+c)^
m*GAMMA(1+m,I*f*(d*x+c)/d)/exp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+I*2^(-3-
m)*b^2*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/f/((-I*f*(
d*x+c)/d)^m)-I*2^(-3-m)*b^2*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)/exp(2*I*(
e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 10.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.84

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx =$$

$$-\frac{(c+dx)^m\left(-\frac{4(2a^2+b^2)f(c+dx)}{d(1+m)}+8abe^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)+8abe^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^2,x]
```

output
```
-1/8*((c + d*x)^m*((-4*(2*a^2 + b^2)*f*(c + d*x))/(d*(1 + m)) + (8*a*b*E^(
I*(e - (c*f)/d))*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(((-I)*f*(c + d*x))/d
)^m + (8*a*b*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*((I*f*(
c + d*x))/d)^m) - (I*b^2*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c
 + d*x))/d])/(2^m*(((-I)*f*(c + d*x))/d)^m) + (I*b^2*Gamma[1 + m, ((2*I)*f
*(c + d*x))/d])/(2^m*E^((2*I)*(e - (c*f)/d))*((I*f*(c + d*x))/d)^m)))/f
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a+b\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m (a+b\sin(e+fx))^2dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a^2(c+dx)^m+2ab(c+dx)^m\sin(e+fx)+b^2(c+dx)^m\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(c+dx)^{m+1}}{d(m+1)}-\frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}-\frac{abe^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{if(c+dx)}{d}\right)}{f}+\frac{ib^2 2^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{ib^2 2^{-m-3}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2if(c+dx)}{d}\right)}{f}+\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

input

```
Int[(c + d*x)^m*(a + b*Sin[e + f*x])^2,x]
```

output

```
(a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m)
) - (a*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d
])/(f*(((-I)*f*(c + d*x))/d)^m) - (a*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c +
d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*2^(-3 - m)*b^
2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d]
)/(f*(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3 - m)*b^2*(c + d*x)^m*Gamma[1 + m
, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\sin(fx+e))^2\,dx$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.87

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx =$$

$$-\frac{8\,(abdm+abd)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+i\,de-i\,cf}{d}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,cf}{d}\right)-(i\,b^2dm+i\,b^2d)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)-2i\,de+2i\,cf}{d}\right)}\Gamma\left(\right.}{}$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/8*(8*(a*b*d*m + a*b*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m
+ 1, (I*d*f*x + I*c*f)/d) - (I*b^2*d*m + I*b^2*d)*e^(-(d*m*log(-2*I*f/d) -
 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 8*(a*b*d*m +
 a*b*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)*gamma(m + 1, (-I*d*f*x -
I*c*f)/d) - (-I*b^2*d*m - I*b^2*d)*e^(-(d*m*log(2*I*f/d) + 2*I*d*e - 2*I*c
*f)/d)*gamma(m + 1, -2*(-I*d*f*x - I*c*f)/d) - 4*((2*a^2 + b^2)*d*f*x + (2
*a^2 + b^2)*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx=\int(a+b\sin(e+fx))^2\,(c+dx)^m\,dx$$

input
```
integrate((d*x+c)**m*(a+b*sin(f*x+e))**2,x)
```

output
```
Integral((a + b*sin(e + f*x))**2*(c + d*x)**m, x)
```

Maxima [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx=\int(b\sin(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + 1/2*(b^2*e^(m*log(d*x + c) + log(d*x +
 c)) - (b^2*d*m + b^2*d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + 4*(a
*b*d*m + a*b*d)*integrate((d*x + c)^m*sin(f*x + e), x))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx = \int (b\sin(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx = \int (a+b\,\sin(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sin(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + b*sin(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx = \text{too large to display}$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

output
```
( - 2*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)**4*b**2*c*d*
f*m - 2*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)**4*b**2*c*
d*f - 4*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)**2*b**2*c*
d*f*m - 4*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*tan((e + f*x)/2)**2*b**2*
c*d*f - 2*(c + d*x)**m*cos(e + f*x)*sin(e + f*x)*b**2*c*d*f*m - 2*(c + d*x
)**m*cos(e + f*x)*sin(e + f*x)*b**2*c*d*f - 6*(c + d*x)**m*cos(e + f*x)*ta
n((e + f*x)/2)**4*a*b*c*d*f*m - 6*(c + d*x)**m*cos(e + f*x)*tan((e + f*x)/
2)**4*a*b*c*d*f - 12*(c + d*x)**m*cos(e + f*x)*tan((e + f*x)/2)**2*a*b*c*d
*f*m - 12*(c + d*x)**m*cos(e + f*x)*tan((e + f*x)/2)**2*a*b*c*d*f - 6*(c +
 d*x)**m*cos(e + f*x)*a*b*c*d*f*m - 6*(c + d*x)**m*cos(e + f*x)*a*b*c*d*f
- 2*(c + d*x)**m*sin(e + f*x)*tan((e + f*x)/2)**4*b**2*c*d*f*m - 2*(c + d*
x)**m*sin(e + f*x)*tan((e + f*x)/2)**4*b**2*c*d*f - 4*(c + d*x)**m*sin(e +
 f*x)*tan((e + f*x)/2)**2*b**2*c*d*f*m - 4*(c + d*x)**m*sin(e + f*x)*tan((
e + f*x)/2)**2*b**2*c*d*f - 2*(c + d*x)**m*sin(e + f*x)*b**2*c*d*f*m - 2*(
c + d*x)**m*sin(e + f*x)*b**2*c*d*f + 3*(c + d*x)**m*tan((e + f*x)/2)**4*a
**2*c**2*f**2 + 3*(c + d*x)**m*tan((e + f*x)/2)**4*a**2*c*d*f**2*x - 6*(c
+ d*x)**m*tan((e + f*x)/2)**4*a*b*c*d*f*m - 6*(c + d*x)**m*tan((e + f*x)/2
)**4*a*b*c*d*f + 6*(c + d*x)**m*tan((e + f*x)/2)**2*a**2*c**2*f**2 + 6*(c
+ d*x)**m*tan((e + f*x)/2)**2*a**2*c*d*f**2*x - 12*(c + d*x)**m*tan((e + f
*x)/2)**2*a*b*c*d*f*m - 12*(c + d*x)**m*tan((e + f*x)/2)**2*a*b*c*d*f +...
```

3.176 $\int (c+dx)^m(a+b\sin(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx = \frac{a(c+dx)^{1+m}}{d(1+m)} - \frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f} - \frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{2f}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)-1/2*b*exp(I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-I*f*(d
*x+c)/d)/f/((-I*f*(d*x+c)/d)^m)-1/2*b*(d*x+c)^m*GAMMA(1+m,I*f*(d*x+c)/d)/e
xp(I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx=\frac{1}{2}(c+dx)^m\left(\frac{2a(c+dx)}{d(1+m)}-\frac{be^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{if(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + b*Sin[e + f*x]),x]
```

output
```
((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) - (b*E^(I*(e - (c*f)/d))*Gamma[1
 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (b*Gamma[1 + m
, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)))/2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a+b\sin(e+fx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)^m + b(c+dx)^m \sin(e+fx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^{m+1}}{d(m+1)} - \frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f} - \frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}$$

input
```
Int[(c + d*x)^m*(a + b*Sin[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) - (b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gam
ma[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (b*(c +
d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c
+ d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\sin(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.92

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx = -\frac{(bdm+bd)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+ide-icf}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+(bdm+bd)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)-ide+icf}{d}\right)}\Gamma\left(m+1,\frac{-idfx}{d}\right.}{2\,(dfm+df)}$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-1/2*((b*d*m + b*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (
I*d*f*x + I*c*f)/d) + (b*d*m + b*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/
d)*gamma(m + 1, (-I*d*f*x - I*c*f)/d) - 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(
d*f*m + d*f)
```

Sympy [F]

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx = \int (a+b\sin(e+fx))\,(c+dx)^m\,dx$$

input
```
integrate((d*x+c)**m*(a+b*sin(f*x+e)),x)
```

output
```
Integral((a + b*sin(e + f*x))*(c + d*x)**m, x)
```

Maxima [F]

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx = \int (b\sin(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
b*integrate((d*x + c)^m*sin(f*x + e), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx = \int (b\sin(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sin(e+fx))\,dx = \int (a+b\,\sin(e+f\,x))\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sin(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + b*sin(e + f*x))*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m (a+b\sin(e+fx))\,dx$$

$$= \frac{-(dx+c)^m \cos(fx+e)\,bdm - (dx+c)^m \cos(fx+e)\,bd + (dx+c)^m\,acf + (dx+c)^m\,adfx - (dx+c)}{}$$

input
```
int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

output
```
( - (c + d*x)**m*cos(e + f*x)*b*d*m - (c + d*x)**m*cos(e + f*x)*b*d + (c +
 d*x)**m*a*c*f + (c + d*x)**m*a*d*f*x - (c + d*x)**m*b*d*m - (c + d*x)**m*
b*d + 2*int((c + d*x)**m/(tan((e + f*x)/2)**2*c + tan((e + f*x)/2)**2*d*x
+ c + d*x),x)*b*d**2*m**2 + 2*int((c + d*x)**m/(tan((e + f*x)/2)**2*c + ta
n((e + f*x)/2)**2*d*x + c + d*x),x)*b*d**2*m)/(d*f*(m + 1))
```

3.177 $\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \text{Int}\left(\frac{(c+dx)^m}{a+b\sin(e+fx)},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Sin[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}dx$$

input
```
Int[(c + d*x)^m/(a + b*Sin[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+b\sin(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*sin(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(a + b*sin(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx=\int \frac{(dx+c)^m}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx=\int \frac{(dx+c)^m}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 34.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \int \frac{(c+d\,x)^m}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + b*sin(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + b*sin(e + f*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.85

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \frac{(dx+c)^m c+(dx+c)^m dx-\left(\int \frac{(dx+c)^m\sin(fx+e)}{\sin(fx+e)b+a}dx\right)bdm-\left(\int \frac{(dx+c)^m\sin(fx+e)}{\sin(fx+e)b+a}dx\right)bd}{ad\,(m+1)}$$

input
```
int((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

output
```
((c + d*x)**m*c + (c + d*x)**m*d*x - int(((c + d*x)**m*sin(e + f*x))/(sin(
e + f*x)*b + a),x)*b*d*m - int(((c + d*x)**m*sin(e + f*x))/(sin(e + f*x)*b
 + a),x)*b*d)/(a*d*(m + 1))
```

3.178 $\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \text{Int}\left(\frac{(c+dx)^m}{(a+b\sin(e+fx))^2},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Sin[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Sin[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}dx$$

input
```
Int[(c + d*x)^m/(a + b*Sin[e + f*x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{(a+b\sin(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(-(d*x + c)^m/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2
), x)
```

Sympy [N/A]

Not integrable

Time = 10.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*sin(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(a + b*sin(e + f*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*sin(f*x + e) + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*sin(f*x + e) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 35.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((c + d*x)^m/(a + b*sin(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + b*sin(e + f*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{\sin\left(fx+e\right)^2 b^2+2\sin\left(fx+e\right)ab+a^2}dx$$

input
```
int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

output
```
int((c + d*x)**m/(sin(e + f*x)**2*b**2 + 2*sin(e + f*x)*a*b + a**2),x)
```

3.179 $\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{i(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{12if^2(e+fx)\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3} - \frac{12f^3\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}$$

output
```
I*(f*x+e)^3/a/d+1/4*(f*x+e)^4/a/f+(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-
6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+12*I*f^2*(f*x+e)*polylog(2,I*ex
p(I*(d*x+c)))/a/d^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)+\frac{24f(\cos(c)+i\sin(c))\left(\frac{(e+fx)^3(\cos(c)-i\sin(c))}{3f}-\frac{(e+fx)^2\log(1+i\cos(c+dx)+\sin(c+dx))(1+i\cos(c)+\cdots}{d}\cdots\right.}{d(\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + (24*f*(Cos[c] + I*Sin[c])
*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[1 + I*Cos[c +
 d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e + f*x)*PolyL
og[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[3, (-I)*Cos[c + d*x]
 - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(d*(Cos[c] + I*(1 + Sin
[c]))) - (8*(e + f*x)^3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])))/(4*a)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin(c+dx)}{a\sin(c+dx)+a}\,dx$$
$$\Big\downarrow \text{ 5026}$$
$$\frac{\int (e+fx)^3dx}{a}-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx$$
$$\Big\downarrow \text{ 17}$$

$$\frac{(e+fx)^4}{4af}-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx$$

$\big\downarrow$ 3042

$$\frac{(e+fx)^4}{4af}-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx$$

$\big\downarrow$ 3799

$$\frac{(e+fx)^4}{4af}-\frac{\int (e+fx)^3\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$\big\downarrow$ 3042

$$\frac{(e+fx)^4}{4af}-\frac{\int (e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$\big\downarrow$ 4672

$$\frac{(e+fx)^4}{4af}-\frac{\frac{6f\int (e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\big\downarrow$ 3042

$$\frac{(e+fx)^4}{4af}-\frac{\frac{6f\int -(e+fx)^2\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\big\downarrow$ 25

$$\frac{(e+fx)^4}{4af}-\frac{-\frac{6f\int (e+fx)^2\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\big\downarrow$ 4202

$$\frac{(e+fx)^4}{4af}-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\int \frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)^2}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}$$

$\big\downarrow$ 2620

$$\frac{(e+fx)^4}{4af}-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\int (e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$\Big\downarrow$ 3011

$$\frac{\frac{(e+fx)^4}{4af}-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}\right)}{d}-\frac{i(e+fx)}{}\right.\right.}{d}}{2a}$$

$\Big\downarrow$ 2720

$$\frac{\frac{(e+fx)^4}{4af}-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi}\right.}{d^2}\right.}{d}\right.\right.}{d}}{2a}$$

$\Big\downarrow$ 7143

$$\frac{\frac{(e+fx)^4}{4af}-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log}{}\right.\right.}{d}}{2a}$$

input

```
Int[((e + f*x)^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output

```
(e + f*x)^4/(4*a*f) - ((-2*(e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/d - (6*f
*(((I/3)*(e + f*x)^3)/f - (2*I)*(((-I)*(e + f*x)^2*Log[1 + E^((I/2)*(2*c +
 3*Pi + 2*d*x))])/d + ((2*I)*f*((I*(e + f*x)*PolyLog[2, -E^((I/2)*(2*c + 3
*Pi + 2*d*x))])/d - (f*PolyLog[3, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2))/
d)))/d)/(2*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(145) = 290$.

Time = 1.05 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.70

method	result
risch	$-\frac{3f e^2 \ln(e^{2i(dx+c)}+1)}{a d^2} - \frac{12 f^3 \operatorname{polylog}(3, i e^{i(dx+c)})}{a d^4} + \frac{f^2 e x^3}{a} + \frac{3f e^2 x^2}{2a} + \frac{e^3 x}{a} - \frac{3 f^3 c^2 \ln(e^{2i(dx+c)}+1)}{a d^4} + \frac{6 f e^2 \ln(e^{i(}}{a d^2}$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)+1/a*f^2*e*x^3
+3/2/a*f*e^2*x^2+1/a*e^3*x+12*I/a/d^2*f^2*e*c*x-12*I/a/d^3*f^2*c*e*arctan(
exp(I*(d*x+c)))-3/a/d^2*f*e^2*ln(exp(2*I*(d*x+c))+1)-3/a/d^4*f^3*c^2*ln(ex
p(2*I*(d*x+c))+1)+6/a/d^2*f*e^2*ln(exp(I*(d*x+c)))+6/a/d^4*f^3*c^2*ln(exp(
I*(d*x+c)))+6/a/d^4*f^3*c^2*ln(1-I*exp(I*(d*x+c)))-6/a/d^2*f^3*ln(1-I*exp(
I*(d*x+c)))*x^2+2*I/a/d*f^3*x^3-4*I/a/d^4*f^3*c^3+12*I/a/d^3*f^3*polylog(2
,I*exp(I*(d*x+c)))*x+6*I/a/d*f^2*e*x^2+6*I/a/d^3*f^2*e*c^2+12*I/a/d^3*f^2*
e*polylog(2,I*exp(I*(d*x+c)))+6*I/a/d^4*f^3*c^2*arctan(exp(I*(d*x+c)))+6*I
/a/d^2*f*e^2*arctan(exp(I*(d*x+c)))+6/a/d^3*f^2*c*e*ln(exp(2*I*(d*x+c))+1)
-12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x-6*I/a/d^3*f^3*c^2*x-12/a/d^3*f^2*
c*e*ln(exp(I*(d*x+c)))-12/a/d^3*f^2*e*ln(1-I*exp(I*(d*x+c)))*c-12*f^3*poly
log(3,I*exp(I*(d*x+c)))/a/d^4+1/4/a*f^3*x^4+1/4/a/f*e^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(139) = 278$.

Time = 0.11 (sec) , antiderivative size = 1044, normalized size of antiderivative = 6.37

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(d^4*f^3*x^4 + 4*d^3*e^3 + 4*(d^4*e*f^2 + d^3*f^3)*x^3 + 6*(d^4*e^2*f
+ 2*d^3*e*f^2)*x^2 + 4*(d^4*e^3 + 3*d^3*e^2*f)*x + (d^4*f^3*x^4 + 4*d^3*e^
3 + 4*(d^4*e*f^2 + d^3*f^3)*x^3 + 6*(d^4*e^2*f + 2*d^3*e*f^2)*x^2 + 4*(d^4
*e^3 + 3*d^3*e^2*f)*x)*cos(d*x + c) - 24*(-I*d*f^3*x - I*d*e*f^2 + (-I*d*f
^3*x - I*d*e*f^2)*cos(d*x + c) + (-I*d*f^3*x - I*d*e*f^2)*sin(d*x + c))*di
log(I*cos(d*x + c) - sin(d*x + c)) - 24*(I*d*f^3*x + I*d*e*f^2 + (I*d*f^3*
x + I*d*e*f^2)*cos(d*x + c) + (I*d*f^3*x + I*d*e*f^2)*sin(d*x + c))*dilog(
-I*cos(d*x + c) - sin(d*x + c)) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 +
(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e*f^
2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 12*(d^
2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^2*f^3*x^2 + 2*d^2*e
*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*
x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c)
 + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^2*f^3
*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*x^2
+ 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(-I*cos(d*x + c)
 + sin(d*x + c) + 1) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 + (d^2*e^2*f
- 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3
)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) - 24*(f^3*cos(d*x
+ c) + f^3*sin(d*x + c) + f^3)*polylog(3, I*cos(d*x + c) - sin(d*x + c)...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*sin(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*si
n(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x)/(s
in(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)/(sin(c + d*x) + 1)
, x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(139) = 278$.

Time = 0.25 (sec) , antiderivative size = 1303, normalized size of antiderivative = 7.95

$$\int \frac{(e + fx)^3 \sin(c + dx)}{a + a \sin(c + dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(12*c^2*e*f^2*(1/(a*d^2 + a*d^2*sin(d*x + c)/(cos(d*x + c) + 1)) + arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e^2*f*(1/(a*d + a*d*s
in(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
(a*d)) - 6*((d*x + c)^2*cos(d*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d
*x + c)^2*sin(d*x + c) + (d*x + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d
*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(
d*x + c)^2 + 2*sin(d*x + c) + 1))*c*e*f^2/(a*d^2*cos(d*x + c)^2 + a*d^2*si
n(d*x + c)^2 + 2*a*d^2*sin(d*x + c) + a*d^2) + 4*e^3*(arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) + 3*((d
*x + c)^2*cos(d*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x + c)^2*sin(
d*x + c) + (d*x + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^2 + si
n(d*x + c)^2 + 2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2
*sin(d*x + c) + 1))*e^2*f/(a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d
*sin(d*x + c) + a*d) + 2*((d*x + c)^4*f^3 + 6*(d*x + c)^2*c^2*f^3 - 4*(d*x
 + c)*c^3*f^3 + 8*I*c^3*f^3 + 4*(d*e*f^2 - c*f^3)*(d*x + c)^3 - 24*(c^2*f^
3*cos(d*x + c) + I*c^2*f^3*sin(d*x + c) + I*c^2*f^3)*arctan2(sin(d*x + c)
+ 1, cos(d*x + c)) + 24*(I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x
+ c) + ((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(d*x + c) + (I
*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c))*sin(d*x + c))*arctan
2(cos(d*x + c), sin(d*x + c) + 1) - (I*(d*x + c)^4*f^3 - 4*(I*c^3 + 6*c...
```

Giac [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)\,(e+f\,x)^3}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - 24*int(x**2/(tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*tan((c +
 d*x)/2)*d**3*f**3 - 24*int(x**2/(tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2)
 + 1),x)*d**3*f**3 - 48*int(x/(tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) +
1),x)*tan((c + d*x)/2)*d**3*e*f**2 + 48*int(x/(tan((c + d*x)/2)**2 + 2*tan
((c + d*x)/2) + 1),x)*tan((c + d*x)/2)*d**2*f**3 - 48*int(x/(tan((c + d*x)
/2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**3*e*f**2 + 48*int(x/(tan((c + d*x)/
2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**2*f**3 + 12*log(tan((c + d*x)/2)**2
+ 1)*tan((c + d*x)/2)*d**2*e**2*f - 24*log(tan((c + d*x)/2)**2 + 1)*tan((c
 + d*x)/2)*d*e*f**2 + 24*log(tan((c + d*x)/2)**2 + 1)*tan((c + d*x)/2)*f**
3 + 12*log(tan((c + d*x)/2)**2 + 1)*d**2*e**2*f - 24*log(tan((c + d*x)/2)*
*2 + 1)*d*e*f**2 + 24*log(tan((c + d*x)/2)**2 + 1)*f**3 - 24*log(tan((c +
d*x)/2) + 1)*tan((c + d*x)/2)*d**2*e**2*f + 48*log(tan((c + d*x)/2) + 1)*t
an((c + d*x)/2)*d*e*f**2 - 48*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)*f
**3 - 24*log(tan((c + d*x)/2) + 1)*d**2*e**2*f + 48*log(tan((c + d*x)/2) +
 1)*d*e*f**2 - 48*log(tan((c + d*x)/2) + 1)*f**3 + 4*tan((c + d*x)/2)*d**4
*e**3*x + 6*tan((c + d*x)/2)*d**4*e**2*f*x**2 + 4*tan((c + d*x)/2)*d**4*e*
f**2*x**3 + tan((c + d*x)/2)*d**4*f**3*x**4 - 8*tan((c + d*x)/2)*d**3*e**3
 - 12*tan((c + d*x)/2)*d**3*e**2*f*x - 12*tan((c + d*x)/2)*d**3*e*f**2*x**
2 - 4*tan((c + d*x)/2)*d**3*f**3*x**3 + 24*tan((c + d*x)/2)*d**2*e*f**2*x
+ 12*tan((c + d*x)/2)*d**2*f**3*x**2 - 24*tan((c + d*x)/2)*d*f**3*x + 4...
```

3.180 $\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{4if^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}$$

output
```
I*(f*x+e)^2/a/d+1/3*(f*x+e)^3/a/f+(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-
4*f*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*(d*x+c)
))/a/d^3
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)+\frac{12f(\cos(c)+i\sin(c))\left(\frac{(e+fx)^2(\cos(c)-i\sin(c))}{2f}-\frac{(e+fx)\log(1+i\cos(c+dx)+\sin(c+dx))(1+i\cos(c)+\sin(c))}{d}+\frac{f\operatorname{PolyLo}}{}\right.}{d(\cos(c)+i(1+\sin(c)))}}{3a}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2) + (12*f*(Cos[c] + I*Sin[c])*(((e + f*x)^2*(
Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*
x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c +
 d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(d*(Cos[c] + I*(1 + Sin[c]))) - (6
*(e + f*x)^2*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])))/(3*a)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^2dx}{a}-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^3}{3af}-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{(e+fx)^3}{3af}-\frac{\int (e+fx)^2\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$\Big\downarrow$ 4672

$$\frac{(e+fx)^3}{3af}-\frac{\frac{4f\int (e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{(e+fx)^3}{3af}-\frac{\frac{4f\int -\left((e+fx)\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\Big\downarrow$ 25

$$\frac{(e+fx)^3}{3af}-\frac{-\frac{4f\int (e+fx)\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\Big\downarrow$ 4202

$$\frac{(e+fx)^3}{3af}-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}$$

$\Big\downarrow$ 2620

$$\frac{(e+fx)^3}{3af}-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$\Big\downarrow$ 2715

$$\frac{(e+fx)^3}{3af}-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)de^{\frac{1}{2}i(2c+2dx+3\pi)}}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+\right.}}{d}\right.\right.}{d}}{2a}$$

$\Big\downarrow$ 2838

$$\frac{\frac{(e+fx)^3}{3af}-}{\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*a*f) - ((-2*(e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/d - (4*f
*(((I/2)*(e + f*x)^2)/f - (2*I)*(((-I)*(e + f*x)*Log[1 + E^((I/2)*(2*c + 3
*Pi + 2*d*x))])/d - (f*PolyLog[2, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2)))
/d)/(2*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(113) = 226$.

Time = 0.86 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.60

method	result
risch	$\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}+\frac{2x^2f^2+4efx+2e^2}{da\left(\mathrm{e}^{i(dx+c)}+i\right)}+\frac{4fe\ln\left(\mathrm{e}^{i(dx+c)}\right)}{a\,d^2}-\frac{2fe\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a\,d^2}+\frac{2if^2x^2}{ad}-\frac{4if^2c\arctan}{a\,d}$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3+2*(f^2*x^2+2*e*f*x+e^2)/d/
a/(exp(I*(d*x+c))+I)+4/a/d^2*f*e*ln(exp(I*(d*x+c)))-2/a/d^2*f*e*ln(exp(2*I
*(d*x+c))+1)+2*I/a/d*f^2*x^2-4*I/a/d^3*f^2*c*arctan(exp(I*(d*x+c)))+2*I/a/
d^3*f^2*c^2+4*I/a/d^2*f*e*arctan(exp(I*(d*x+c)))-4/a/d^2*f^2*ln(1-I*exp(I*
(d*x+c)))*x-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+4*I*f^2*polylog(2,I*exp(I
*(d*x+c)))/a/d^3-4/a/d^3*f^2*c*ln(exp(I*(d*x+c)))+2/a/d^3*f^2*c*ln(exp(2*I
*(d*x+c))+1)+4*I/a/d^2*f^2*c*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(108) = 216$.

Time = 0.11 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.52

$$\int \frac{(e+fx)^2 \sin(c+dx)}{a+a\sin(c+dx)} \, dx = \frac{d^3f^2x^3 + 3\,d^2e^2 + 3\,(d^3ef + d^2f^2)x^2 + 3\,(d^3e^2 + 2\,d^2ef)x + (d^3f^2x^3 + 3\,d^2e^2 + 3\,(d^3ef + d^2f^2)x^2 + 3\,(d}{}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(d^3*f^2*x^3 + 3*d^2*e^2 + 3*(d^3*e*f + d^2*f^2)*x^2 + 3*(d^3*e^2 + 2*
d^2*e*f)*x + (d^3*f^2*x^3 + 3*d^2*e^2 + 3*(d^3*e*f + d^2*f^2)*x^2 + 3*(d^3
*e^2 + 2*d^2*e*f)*x)*cos(d*x + c) - 6*(-I*f^2*cos(d*x + c) - I*f^2*sin(d*x
 + c) - I*f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 6*(I*f^2*cos(d*x + c
) + I*f^2*sin(d*x + c) + I*f^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 6*
(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x +
c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 6*(d*f^2*x + c*f^2 + (d*f^2*x
 + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c
) + sin(d*x + c) + 1) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c
) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1
) - 6*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(
d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*f^2*x^3 - 3*d^2*e
^2 + 3*(d^3*e*f - d^2*f^2)*x^2 + 3*(d^3*e^2 - 2*d^2*e*f)*x)*sin(d*x + c))/
(a*d^3*cos(d*x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2 \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2 \sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*sin(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*si
n(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)/(sin(c +
 d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(108) = 216.

Time = 0.22 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.12

$$\int \frac{(e+fx)^2 \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{d^3f^2x^3 + 3\,d^3efx^2 + 3\,d^3e^2x - 6i\,d^2e^2 - 12\,(def\cos(dx+c) + i\,def\sin(dx+c) + i\,def)\arctan(\sin(dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x - 6*I*d^2*e^2 - 12*(d*e*f*cos(d
*x + c) + I*d*e*f*sin(d*x + c) + I*d*e*f)*arctan2(sin(d*x + c) + 1, cos(d*
x + c)) + 12*(d*f^2*x*cos(d*x + c) + I*d*f^2*x*sin(d*x + c) + I*d*f^2*x)*a
rctan2(cos(d*x + c), sin(d*x + c) + 1) - (I*d^3*f^2*x^3 - 3*(-I*d^3*e*f +
2*d^2*f^2)*x^2 - 3*(-I*d^3*e^2 + 4*d^2*e*f)*x)*cos(d*x + c) + 12*(f^2*cos(
d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(I*e^(I*d*x + I*c)) - 6*(d*f^2
*x + d*e*f - (I*d*f^2*x + I*d*e*f)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(d*
x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (d^3*f
^2*x^3 + 3*(d^3*e*f + 2*I*d^2*f^2)*x^2 + 3*(d^3*e^2 + 4*I*d^2*e*f)*x)*sin(
d*x + c))/(-3*I*a*d^3*cos(d*x + c) + 3*a*d^3*sin(d*x + c) + 3*a*d^3)
```

Giac [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^2\sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\sin(c+d\,x)\,(e+f\,x)^2}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{12\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)d\,f^2+12\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)d\,f^2+6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\tan\left(\frac{c}{?}\right.}{}$$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output

```
(12*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*tan((c + d*x)/2)*d*
f**2 + 12*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*d*f**2 + 6*lo
g(tan((c + d*x)/2)**2 + 1)*tan((c + d*x)/2)*e*f + 6*log(tan((c + d*x)/2)**
2 + 1)*e*f - 12*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)*e*f - 12*log(ta
n((c + d*x)/2) + 1)*e*f + 3*tan((c + d*x)/2)*d**2*e**2*x + 3*tan((c + d*x)
/2)*d**2*e*f*x**2 + tan((c + d*x)/2)*d**2*f**2*x**3 - 6*tan((c + d*x)/2)*d
*e**2 - 6*tan((c + d*x)/2)*d*e*f*x - 6*tan((c + d*x)/2)*d*f**2*x**2 + 3*d*
*2*e**2*x + 3*d**2*e*f*x**2 + d**2*f**2*x**3 + 6*d*e*f*x)/(3*a*d**2*(tan((
c + d*x)/2) + 1))
```

3.181 $\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(e+fx)^2}{2af} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}$$

output
```
1/2*(f*x+e)^2/a/f+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-2*f*ln(sin(1/2*c+1
/4*Pi+1/2*d*x))/a/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(76) = 152.

Time = 0.91 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.62

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2dfx\cos\left(c+\frac{dx}{2}\right)+\cos\left(\frac{dx}{2}\right)\left(d^2x(2e+fx)-4f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-4de\sin\left(\frac{dx}{2}\right)}{2ad^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(2*d*f*x*Cos[c + (d*x)/2] + Cos[(d*x)/2]*(d^2*x*(2*e + f*x) - 4*f*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4*d*e*Sin[(d*x)/2] - 2*d*f*x*Sin[(d*x)
/2] + 2*d^2*e*x*Sin[c + (d*x)/2] + d^2*f*x^2*Sin[c + (d*x)/2] - 4*f*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + (d*x)/2])/(2*a*d^2*(Cos[c/2] +
Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)dx}{a}-\int \frac{e+fx}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^2}{2af}-\int \frac{e+fx}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\int \frac{e+fx}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{(e+fx)^2}{2af}-\frac{\int (e+fx)\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{2f\int \cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{2f\int -\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{(e+fx)^2}{2af}-\frac{-\frac{2f\int \tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{4f\log\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)\right)}{d^2}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

input
```
Int[((e + f*x)*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*a*f) - ((-2*(e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/d + (4*f*L
og[-Cos[c/2 - Pi/4 + (d*x)/2]])/d^2)/(2*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result
risch	$\frac{f\,x^{2}}{2 a}+\frac{e x}{a}+\frac{2 i f x}{d a}+\frac{2 i f c}{d^{2} a}+\frac{2 f x +2 e}{d a \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}-\frac{2 f \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{d^{2} a}$
parallelrisch	$\frac{f \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) \ln \left(\sec \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)-2 f \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)+\left(\left(\frac{f x}{2}+e \right) \left(d x -2\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+x \left(\left(\frac{f x}{2}\right.\right.\right.}{d^{2} a \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}$
norman	$\frac{-\frac{2 e \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d a}-\frac{2 e \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d a}+\frac{\left(d e +f \right) x}{a d}+\frac{\left(d e -f \right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{a d}+\frac{\left(d e -f \right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{a d}+\frac{\left(d e +f \right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{a d}+\frac{f\,x^{2}}{2 a}+\frac{f\,x^{2}}{}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right) \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}$

input
```
int((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*f/a*x^2+1/a*e*x+2*I*f/d/a*x+2*I*f/d^2/a*c+2*(f*x+e)/d/a/(exp(I*(d*x+c)
)+I)-2*f/d^2/a*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{d^2fx^2+2\,de+2\,(d^2e+df)x+(d^2fx^2+2\,de+2\,(d^2e+df)x)\cos{(dx+c)}-2\,(f\cos{(dx+c)}+f\sin{(c}}{2\,(ad^2\cos{(dx+c)}+ad^2\sin{(dx+}}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d^2*f*x^2 + 2*d*e + 2*(d^2*e + d*f)*x + (d^2*f*x^2 + 2*d*e + 2*(d^2*e
 + d*f)*x)*cos(d*x + c) - 2*(f*cos(d*x + c) + f*sin(d*x + c) + f)*log(sin(
d*x + c) + 1) + (d^2*f*x^2 - 2*d*e + 2*(d^2*e - d*f)*x)*sin(d*x + c))/(a*d
^2*cos(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(56) = 112$.

Time = 0.67 (sec) , antiderivative size = 456, normalized size of antiderivative = 6.00

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{2d^2ex\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{2d^2ex}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{d^2fx^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{d^2fx^2}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2}+\frac{4de}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2a} \\ \frac{\left(ex+\frac{fx^2}{2}\right)\sin{(c)}}{a\sin{(c)}+a} \end{cases}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((2*d**2*e*x*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d*
*2) + 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + d**2*f*x**2*tan(
c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + d**2*f*x**2/(2*a*d**
2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*d*e/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d*
*2) - 2*d*f*x*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*
d*f*x/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*f*log(tan(c/2 + d*x/2) +
1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*f*log(tan(c
/2 + d*x/2) + 1)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*f*log(tan(c/2
+ d*x/2)**2 + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) +
 2*f*log(tan(c/2 + d*x/2)**2 + 1)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2),
Ne(d, 0)), ((e*x + f*x**2/2)*sin(c)/(a*sin(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.59

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{4\,cf\left(\frac{1}{ad+\frac{ad\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)-4\,e\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)-\frac{\left((dx+c)^2\cos(dx+c)^2+(d\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(4*c*f*(1/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*(arctan(sin(d*x + c)/(cos(d*x + c)
 + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) - ((d*x + c)^2*cos(d
*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x + c)^2*sin(d*x + c) + (d*x
 + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^2 + sin(d*x + c)^2 +
2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
 1))*f/(a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d
))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(62) = 124$.

Time = 0.27 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.63

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c) - d^2*f*x^2*tan(1/2*d*x) - d^2*f*x^
2*tan(1/2*c) + 2*d^2*e*x*tan(1/2*d*x)*tan(1/2*c) - d^2*f*x^2 - 2*d^2*e*x*t
an(1/2*d*x) - 2*d^2*e*x*tan(1/2*c) + 2*d*f*x*tan(1/2*d*x)*tan(1/2*c) - 2*d
^2*e*x + 2*d*f*x*tan(1/2*d*x) + 2*d*f*x*tan(1/2*c) + 2*d*e*tan(1/2*d*x)*ta
n(1/2*c) - 2*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 2*d*f*x + 2*d*e*tan(1/2*
d*x) + 2*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(1/2*d*x) + 2*d*e*tan(1/2*c) + 2*f*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*c) - 2*d*e + 2*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1)))/(a*d^2*tan(1/2*d*x)*tan(1/2*c) - a*d^2*tan(1/2
*d*x) - a*d^2*tan(1/2*c) - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 35.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{f\,x^2}{2\,a} - \frac{2\,f\,\ln\left(\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,d^2} + \frac{2\,(e+f\,x)}{a\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)} + \frac{x\,(d\,e+f\,2\mathrm{i})}{a\,d}$$

input
```
int((sin(c + d*x)*(e + f*x))/(a + a*sin(c + d*x)),x)
```

output
```
(f*x^2)/(2*a) - (2*f*log(exp(c*1i)*exp(d*x*1i) + 1i))/(a*d^2) + (2*(e + f*
x))/(a*d*(exp(c*1i + d*x*1i) + 1i)) + (x*(f*2i + d*e))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.42

$$\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)f+2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)f-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\tan\left(\frac{dx}{2}\right.}{}$$

input
```
int((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(2*log(tan((c + d*x)/2)**2 + 1)*tan((c + d*x)/2)*f + 2*log(tan((c + d*x)/2
)**2 + 1)*f - 4*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)*f - 4*log(tan((
c + d*x)/2) + 1)*f + 2*tan((c + d*x)/2)*d**2*e*x + tan((c + d*x)/2)*d**2*f
*x**2 - 4*tan((c + d*x)/2)*d*e - 2*tan((c + d*x)/2)*d*f*x + 2*d**2*e*x + d
**2*f*x**2 + 2*d*f*x)/(2*a*d**2*(tan((c + d*x)/2) + 1))
```

3.182 $\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{x}{a}+\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

output
```
x/a+cos(d*x+c)/d/(a+a*sin(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx \\ =\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left((c+dx)\cos\left(\frac{1}{2}(c+dx)\right)+(-2+c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad(1+\sin(c+dx))}$$

input
```
Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((c + d*x)*Cos[(c + d*x)/2] + (-2 +
 c + d*x)*Sin[(c + d*x)/2]))/(a*d*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3214}$$
$$\frac{x}{a}-\int \frac{1}{\sin(c+dx)a+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{x}{a}-\int \frac{1}{\sin(c+dx)a+a}dx$$
$$\Big\downarrow \text{3127}$$
$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}+\frac{x}{a}$$

input
```
Int[Sin[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
x/a + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{x}{a}+\frac{2}{da\left(e^{i(dx+c)}+i\right)}$	29
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}}{ad}$	37
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}}{ad}$	37
parallelrisch	$\frac{dx+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(dx-2)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	40
norman	$\frac{\frac{x}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a}+\frac{2}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	109

input
```
int(sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a+2/d/a/(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{dx+(dx+1)\cos(dx+c)+(dx-1)\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

input
```
integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x + (d*x + 1)*cos(d*x + c) + (d*x - 1)*sin(d*x + c) + 1)/(a*d*cos(d*x +
 c) + a*d*sin(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(20) = 40.

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d\neq 0 \\ \frac{x\sin(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise(((d*x*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2) + a*d) + d*x/(a*d*ta
n(c/2 + d*x/2) + a*d) + 2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x*sin(
c)/(a*sin(c) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)}{d}$$

input
```
integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(
d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{dx+c}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

input
```
integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{x}{a}+\frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

input
```
int(sin(c + d*x)/(a + a*sin(c + d*x)),x)
```

output
```
x/a + 2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)dx-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+dx}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int(sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(tan((c + d*x)/2)*d*x - 2*tan((c + d*x)/2) + d*x)/(a*d*(tan((c + d*x)/2) +
 1))
```

3.183 $\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 16.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\sin(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sin(dx+c)}{(fx+e)(a+a\sin(dx+c))}dx$$

input
```
int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

input
```
integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 373, normalized size of antiderivative = 14.35

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(2*f*cos(d*x + c) + 2*(a*d*f^3*x + a*d*e*f^2 + (a*d*f^3*x + a*d*e*f^2)*cos
(d*x + c)^2 + (a*d*f^3*x + a*d*e*f^2)*sin(d*x + c)^2 + 2*(a*d*f^3*x + a*d*
e*f^2)*sin(d*x + c))*integrate(cos(d*x + c)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a
*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*cos(d*x + c)^2 + (a*d*f^2*x
^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*sin(d*x + c)), x) + (d*f*x + (d*f*x + d*e)*cos(d*x + c)^2 + (d*
f*x + d*e)*sin(d*x + c)^2 + d*e + 2*(d*f*x + d*e)*sin(d*x + c))*log(f*x +
e))/(a*d*f^2*x + a*d*e*f + (a*d*f^2*x + a*d*e*f)*cos(d*x + c)^2 + (a*d*f^2
*x + a*d*e*f)*sin(d*x + c)^2 + 2*(a*d*f^2*x + a*d*e*f)*sin(d*x + c))
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin\left(c+d\,x\right)}{\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sin(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(sin(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(dx+c)}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(c + d*x)/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.184 $\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 12.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sin(dx+c)}{(fx+e)^2 (a+a\sin(dx+c))}dx$$

input
```
int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*
f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(sin(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2
*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 522, normalized size of antiderivative = 20.08

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(d*f*x + (d*f*x + d*e)*cos(d*x + c)^2 + (d*f*x + d*e)*sin(d*x + c)^2 + d*
e - 2*f*cos(d*x + c) - 4*(a*d*f^4*x^2 + 2*a*d*e*f^3*x + a*d*e^2*f^2 + (a*d
*f^4*x^2 + 2*a*d*e*f^3*x + a*d*e^2*f^2)*cos(d*x + c)^2 + (a*d*f^4*x^2 + 2*
a*d*e*f^3*x + a*d*e^2*f^2)*sin(d*x + c)^2 + 2*(a*d*f^4*x^2 + 2*a*d*e*f^3*x
 + a*d*e^2*f^2)*sin(d*x + c))*integrate(cos(d*x + c)/(a*d*f^3*x^3 + 3*a*d*
e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a
*d*e^2*f*x + a*d*e^3)*cos(d*x + c)^2 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*
a*d*e^2*f*x + a*d*e^3)*sin(d*x + c)^2 + 2*(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 +
 3*a*d*e^2*f*x + a*d*e^3)*sin(d*x + c)), x) + 2*(d*f*x + d*e)*sin(d*x + c)
)/(a*d*f^3*x^2 + 2*a*d*e*f^2*x + a*d*e^2*f + (a*d*f^3*x^2 + 2*a*d*e*f^2*x
+ a*d*e^2*f)*cos(d*x + c)^2 + (a*d*f^3*x^2 + 2*a*d*e*f^2*x + a*d*e^2*f)*si
n(d*x + c)^2 + 2*(a*d*f^3*x^2 + 2*a*d*e*f^2*x + a*d*e^2*f)*sin(d*x + c))
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+d\,x)}{(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(sin(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(sin(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(dx+c)}{\sin(dx+c)e^2+2\sin(dx+c)efx+\sin(dx+c)f^2x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(c + d*x)/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*x)*
f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.185 $\int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 247

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{i(e+fx)^3}{ad}-\frac{(e+fx)^4}{4af}+\frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad}-\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} - \frac{12if^2(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3} + \frac{12f^3\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4} - \frac{6f^3\sin(c+dx)}{ad^4}+\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}$$

output
```
-I*(f*x+e)^3/a/d-1/4*(f*x+e)^4/a/f+6*f^2*(f*x+e)*cos(d*x+c)/a/d^3-(f*x+e)^
3*cos(d*x+c)/a/d-(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d+6*f*(f*x+e)^2*ln(
1-I*exp(I*(d*x+c)))/a/d^2-12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d
^3+12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4-6*f^3*sin(d*x+c)/a/d^4+3*f*(f*
x+e)^2*sin(d*x+c)/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1314 vs. $2(247) = 494$.

Time = 4.64 (sec) , antiderivative size = 1314, normalized size of antiderivative = 5.32

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
((-6 + 4*I)*d^3*e^3*Cos[(c + d*x)/2] + 6*d^2*e^2*f*Cos[(c + d*x)/2] + 12*d
*e*f^2*Cos[(c + d*x)/2] - 12*f^3*Cos[(c + d*x)/2] - 4*d^4*e^3*x*Cos[(c + d
*x)/2] - (18 - 12*I)*d^3*e^2*f*x*Cos[(c + d*x)/2] + 12*d^2*e*f^2*x*Cos[(c
+ d*x)/2] + 12*d*f^3*x*Cos[(c + d*x)/2] - 6*d^4*e^2*f*x^2*Cos[(c + d*x)/2]
 - (18 - 12*I)*d^3*e*f^2*x^2*Cos[(c + d*x)/2] + 6*d^2*f^3*x^2*Cos[(c + d*x
)/2] - 4*d^4*e*f^2*x^3*Cos[(c + d*x)/2] - (6 - 4*I)*d^3*f^3*x^3*Cos[(c + d
*x)/2] - d^4*f^3*x^4*Cos[(c + d*x)/2] - 2*d^3*e^3*Cos[(3*(c + d*x))/2] - 6
*d^2*e^2*f*Cos[(3*(c + d*x))/2] + 12*d*e*f^2*Cos[(3*(c + d*x))/2] + 12*f^3
*Cos[(3*(c + d*x))/2] - 6*d^3*e^2*f*x*Cos[(3*(c + d*x))/2] - 12*d^2*e*f^2*
x*Cos[(3*(c + d*x))/2] + 12*d*f^3*x*Cos[(3*(c + d*x))/2] - 6*d^3*e*f^2*x^2
*Cos[(3*(c + d*x))/2] - 6*d^2*f^3*x^2*Cos[(3*(c + d*x))/2] - 2*d^3*f^3*x^3
*Cos[(3*(c + d*x))/2] + 24*d^2*e^2*f*Cos[(c + d*x)/2]*Log[1 + I*Cos[c + d*
x] + Sin[c + d*x]] + 48*d^2*e*f^2*x*Cos[(c + d*x)/2]*Log[1 + I*Cos[c + d*x
] + Sin[c + d*x]] + 24*d^2*f^3*x^2*Cos[(c + d*x)/2]*Log[1 + I*Cos[c + d*x]
 + Sin[c + d*x]] + (6 + 4*I)*d^3*e^3*Sin[(c + d*x)/2] + 6*d^2*e^2*f*Sin[(c
 + d*x)/2] - 12*d*e*f^2*Sin[(c + d*x)/2] - 12*f^3*Sin[(c + d*x)/2] - 4*d^4
*e^3*x*Sin[(c + d*x)/2] + (18 + 12*I)*d^3*e^2*f*x*Sin[(c + d*x)/2] + 12*d^
2*e*f^2*x*Sin[(c + d*x)/2] - 12*d*f^3*x*Sin[(c + d*x)/2] - 6*d^4*e^2*f*x^2
*Sin[(c + d*x)/2] + (18 + 12*I)*d^3*e*f^2*x^2*Sin[(c + d*x)/2] + 6*d^2*f^3
*x^2*Sin[(c + d*x)/2] - 4*d^4*e*f^2*x^3*Sin[(c + d*x)/2] + (6 + 4*I)*d^...
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.11, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.821, Rules used = {5026, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^3\sin(c+dx)dx}{a}-\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\sin(c+dx)dx}{a}-\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{3f\int (e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 3117$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 5026$$

$$\int\frac{(e+fx)^3}{\sin(c+dx)a+a}dx-\frac{\int(e+fx)^3dx}{a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 17$$

$$\int\frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \ 3799$$

$$\frac{\int (e+fx)^3\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \ 4672$$

$$\frac{\frac{6f\int (e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{6f\int -(e+fx)^2\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{6f\int(e+fx)^2\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$\downarrow$ 4202

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)^2}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$\downarrow$ 2620

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\int(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$\downarrow$ 3011

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}\right)}{d}-\frac{i(e+fx)}{}\right.\right.}{d}}{2a}+\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$\downarrow$ 2720

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{2a}$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

$\Big\downarrow$ 7143

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log\cdots}{\cdots}\cdots\right.\right.}{d}\cdots}{2a}$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{(e+fx)^4}{4af}$$

input

```
Int[((e + f*x)^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output

```
-1/4*(e + f*x)^4/(a*f) + ((-2*(e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/d - (
6*f*(((I/3)*(e + f*x)^3)/f - (2*I)*(((-I)*(e + f*x)^2*Log[1 + E^((I/2)*(2*
c + 3*Pi + 2*d*x))])/d + ((2*I)*f*((I*(e + f*x)*PolyLog[2, -E^((I/2)*(2*c
+ 3*Pi + 2*d*x))])/d - (f*PolyLog[3, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2
))/d)))/d)/(2*a) + (-(((e + f*x)^3*Cos[c + d*x])/d) + (3*f*(((e + f*x)^2*S
in[c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^
2))/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(228) = 456$.

Time = 1.75 (sec) , antiderivative size = 759, normalized size of antiderivative = 3.07

method	result
risch	$-\frac{2(f^3x^3+3e\,f^2x^2+3e^2fx+e^3)}{da(\mathrm{e}^{i(dx+c)}+i)}+\frac{12f^3\,\mathrm{polylog}(3,i\mathrm{e}^{i(dx+c)})}{a\,d^4}-\frac{f^2e\,x^3}{a}-\frac{6f\,e^2\ln(\mathrm{e}^{i(dx+c)})}{a\,d^2}-\frac{6f^3c^2\ln(\mathrm{e}^{i(dx+c)})}{a\,d^4}-6f^3c$

input
```
int((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d^3*x^3*f^3+3*I*d^2*f^3*x^2+3*e*f^2*x^2*d^3+6*I*d^2*e*f^2*x+3*e^2*f*
x*d^3+3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x-6*I*f^3-6*d*e*f^2)/a/d^4*exp(I*(d*x+
c))-2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)-1/a*f^2*e
*x^3-1/2*(d^3*x^3*f^3-3*I*d^2*f^3*x^2+3*e*f^2*x^2*d^3-6*I*d^2*e*f^2*x+3*e^
2*f*x*d^3-3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x+6*I*f^3-6*d*e*f^2)/a/d^4*exp(-I*
(d*x+c))-12/d^3/a*c*e*f^2*ln(exp(I*(d*x+c))+I)-1/4/a/f*e^4-12*I/d^3/a*e*f^
2*polylog(2,I*exp(I*(d*x+c)))+12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+12/
d^2/a*e*f^2*ln(1-I*exp(I*(d*x+c)))*x+12/d^3/a*e*f^2*ln(1-I*exp(I*(d*x+c)))
*c+6/d^2/a*f^3*ln(1-I*exp(I*(d*x+c)))*x^2-6/d^2/a*ln(exp(I*(d*x+c)))*e^2*f
+6/d^2/a*ln(exp(I*(d*x+c))+I)*e^2*f+6/d^4/a*c^2*f^3*ln(exp(I*(d*x+c))+I)-6
/d^4/a*c^2*f^3*ln(1-I*exp(I*(d*x+c)))-6/d^4/a*c^2*f^3*ln(exp(I*(d*x+c)))+4
*I/d^4/a*f^3*c^3-2*I/d/a*f^3*x^3-6*I/d/a*e*f^2*x^2+6*I/d^3/a*f^3*c^2*x-12*
I/d^3/a*f^3*polylog(2,I*exp(I*(d*x+c)))*x-1/4/a*f^3*x^4-12*I/d^2/a*e*f^2*c
*x-3/2/a*f*e^2*x^2-1/a*e^3*x+12/d^3/a*c*e*f^2*ln(exp(I*(d*x+c)))-6*I/d^3/a
*e*f^2*c^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(222) = 444$.

Time = 0.12 (sec) , antiderivative size = 1313, normalized size of antiderivative = 5.32

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*f^3*x^4 + 4*d^3*e^3 - 12*d^2*e^2*f + 4*(d^4*e*f^2 + d^3*f^3)*x^3
 + 24*f^3 + 6*(d^4*e^2*f + 2*d^3*e*f^2 - 2*d^2*f^3)*x^2 + 4*(d^3*f^3*x^3 +
 d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f^2 - 6*f^3 + 3*(d^3*e*f^2 + d^2*f^3)*x^2 +
 3*(d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c)^2 + 4*(d^4*e^3 + 3*
d^3*e^2*f - 6*d^2*e*f^2)*x + (d^4*f^3*x^4 + 8*d^3*e^3 - 24*d*e*f^2 + 4*(d^
4*e*f^2 + 2*d^3*f^3)*x^3 + 6*(d^4*e^2*f + 4*d^3*e*f^2)*x^2 + 4*(d^4*e^3 +
6*d^3*e^2*f - 6*d*f^3)*x)*cos(d*x + c) + 24*(I*d*f^3*x + I*d*e*f^2 + (I*d*
f^3*x + I*d*e*f^2)*cos(d*x + c) + (I*d*f^3*x + I*d*e*f^2)*sin(d*x + c))*di
log(I*cos(d*x + c) - sin(d*x + c)) + 24*(-I*d*f^3*x - I*d*e*f^2 + (-I*d*f^
3*x - I*d*e*f^2)*cos(d*x + c) + (-I*d*f^3*x - I*d*e*f^2)*sin(d*x + c))*dil
og(-I*cos(d*x + c) - sin(d*x + c)) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3
 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e
*f^2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 12*
(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^2*f^3*x^2 + 2*d^
2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f
^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x +
 c) + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^2*
f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*x
^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(-I*cos(d*x +
 c) + sin(d*x + c) + 1) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 + (d^2*...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3
*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d
*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)**2/(sin(c
 + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4592 vs. 2(222) = 444.

Time = 0.43 (sec) , antiderivative size = 4592, normalized size of antiderivative = 18.59

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(12*c^2*e*f^2*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 2)/(a*d^2 + a*d^2*sin(d*x + c)/(cos(d*x + c) + 1) + a*d
^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d^2*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e^2*f*
((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
2)/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1) + a*d*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/(a*d)) - 6*(((d*x + c)^2 - 1)*cos(d*x + c)^4 + ((
d*x + c)^2 - 1)*sin(d*x + c)^4 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) +
1)*cos(2*d*x + 2*c)^3 + 7*(d*x + c)*cos(d*x + c)^3 + (d*x + (d*x + c)*sin(
d*x + c) + c - cos(d*x + c))*sin(2*d*x + 2*c)^3 + (2*(d*x + c)^2 - 3)*sin(
d*x + c)^3 + (((d*x + c)^2 - 1)*cos(d*x + c)^2 + ((d*x + c)^2 - 3)*sin(d*x
 + c)^2 + (d*x + c)^2 + 6*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 - (d*x +
 c)*cos(d*x + c) - 2)*sin(d*x + c) - 1)*cos(2*d*x + 2*c)^2 + ((d*x + c)^2
- 1)*cos(d*x + c)^2 + (((d*x + c)^2 - 3)*cos(d*x + c)^2 + ((d*x + c)^2 - 1
)*sin(d*x + c)^2 + (d*x + c)^2 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) +
1)*cos(2*d*x + 2*c) + 8*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 + (d*x + c
)*cos(d*x + c) - 1)*sin(d*x + c) - 1)*sin(2*d*x + 2*c)^2 + (2*((d*x + c)^2
 - 1)*cos(d*x + c)^2 + (d*x + c)^2 + 7*(d*x + c)*cos(d*x + c) - 3)*sin(d*x
 + c)^2 + ((d*x + c)*cos(d*x + c)^3 - (2*(d*x + c)^2 - 3)*sin(d*x + c)^...
```

Giac [F]

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^3 \sin(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)^2\,(e+f\,x)^3}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)^2*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)^2*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 4*cos(c + d*x)*tan((c + d*x)/2)*d**3*e**3 - 12*cos(c + d*x)*tan((c + d
*x)/2)*d**3*e**2*f*x - 12*cos(c + d*x)*tan((c + d*x)/2)*d**3*e*f**2*x**2 -
 4*cos(c + d*x)*tan((c + d*x)/2)*d**3*f**3*x**3 + 24*cos(c + d*x)*tan((c +
 d*x)/2)*d*e*f**2 + 24*cos(c + d*x)*tan((c + d*x)/2)*d*f**3*x - 4*cos(c +
d*x)*d**3*e**3 - 12*cos(c + d*x)*d**3*e**2*f*x - 12*cos(c + d*x)*d**3*e*f*
*2*x**2 - 4*cos(c + d*x)*d**3*f**3*x**3 + 24*cos(c + d*x)*d*e*f**2 + 24*co
s(c + d*x)*d*f**3*x + 24*int(x**2/(tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2
) + 1),x)*tan((c + d*x)/2)*d**3*f**3 + 24*int(x**2/(tan((c + d*x)/2)**2 +
2*tan((c + d*x)/2) + 1),x)*d**3*f**3 + 48*int(x/(tan((c + d*x)/2)**2 + 2*t
an((c + d*x)/2) + 1),x)*tan((c + d*x)/2)*d**3*e*f**2 - 48*int(x/(tan((c +
d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*tan((c + d*x)/2)*d**2*f**3 + 48*in
t(x/(tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**3*e*f**2 - 48*int
(x/(tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**2*f**3 - 12*log(ta
n((c + d*x)/2)**2 + 1)*tan((c + d*x)/2)*d**2*e**2*f + 24*log(tan((c + d*x)
/2)**2 + 1)*tan((c + d*x)/2)*d*e*f**2 - 24*log(tan((c + d*x)/2)**2 + 1)*ta
n((c + d*x)/2)*f**3 - 12*log(tan((c + d*x)/2)**2 + 1)*d**2*e**2*f + 24*log
(tan((c + d*x)/2)**2 + 1)*d*e*f**2 - 24*log(tan((c + d*x)/2)**2 + 1)*f**3
+ 24*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)*d**2*e**2*f - 48*log(tan((
c + d*x)/2) + 1)*tan((c + d*x)/2)*d*e*f**2 + 48*log(tan((c + d*x)/2) + 1)*
tan((c + d*x)/2)*f**3 + 24*log(tan((c + d*x)/2) + 1)*d**2*e**2*f - 48*l...
```

3.186 $\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 188

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{i(e+fx)^2}{ad}-\frac{(e+fx)^3}{3af}+\frac{2f^2\cos(c+dx)}{ad^3} -\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} +\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2} -\frac{4if^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{2f(e+fx)\sin(c+dx)}{ad^2}$$

output
```
-I*(f*x+e)^2/a/d-1/3*(f*x+e)^3/a/f+2*f^2*cos(d*x+c)/a/d^3-(f*x+e)^2*cos(d*
x+c)/a/d-(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d+4*f*(f*x+e)*ln(1-I*exp(I*
(d*x+c)))/a/d^2-4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3+2*f*(f*x+e)*sin(
d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.57

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx =$$

$$-\frac{x(3e^2+3efx+f^2x^2)+\frac{3\cos(dx)\left((-2f^2+d^2(e+fx)^2)\cos(c)-2df(e+fx)\sin(c)\right)}{d^3}+\frac{12f(\cos(c)+i\sin(c))\left(\frac{(e+fx)^2(\cos(c)-i\sin(c}{2f}\right.}{}}{}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/3*(x*(3*e^2 + 3*e*f*x + f^2*x^2) + (3*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)
^2)*Cos[c] - 2*d*f*(e + f*x)*Sin[c]))/d^3 + (12*f*(Cos[c] + I*Sin[c])*(((e
 + f*x)^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] +
 Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x
] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(d*(Cos[c] + I*(1 + Sin
[c]))) - (3*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2)*Sin[c])*S
in[d*x])/d^3 - (6*(e + f*x)^2*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])))/a
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5026, 3042, 3777, 3042, 3777, 25, 3042, 3118, 5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \; 5026$$

$$\frac{\int (e+fx)^2\sin(c+dx)dx}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\sin(c+dx)dx}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)^2dx}{a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{17}$$

$$\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 3799

$$\frac{\int (e+fx)^2\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 3042

$$\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 4672

$$\frac{\frac{4f\int (e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 3042

$$\frac{\frac{4f\int -\left((e+fx)\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 25

$$\frac{-\frac{4f\int (e+fx)\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 4202

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\int \frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 2620

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)de^{\frac{1}{2}i(2c+2dx+3\pi)}}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+\right.}}{d}\right.\right.}{d}}{2a}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 2838

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\mathrm{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{(e+fx)^3}{3af}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/3*(e + f*x)^3/(a*f) + ((-2*(e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/d - (
4*f*(((I/2)*(e + f*x)^2)/f - (2*I)*(((-I)*(e + f*x)*Log[1 + E^((I/2)*(2*c
+ 3*Pi + 2*d*x))])/d - (f*PolyLog[2, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2
)))/d)/(2*a) + (-(((e + f*x)^2*Cos[c + d*x])/d) + (2*f*((f*Cos[c + d*x])/d
^2 + ((e + f*x)*Sin[c + d*x])/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(172) = 344.

Time = 2.41 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{f^{2} x^{3}}{3 a}-\frac{f e \,x^{2}}{a}-\frac{e^{2} x}{a}-\frac{e^{3}}{3 a f}-\frac{\left(d^{2} x^{2} f^{2}+2 e f x \,d^{2}+2 i d \,f^{2} x +d^{2} e^{2}+2 i d e f -2 f^{2}\right) {\mathrm e}^{i \left(d x +c \right)}}{2 a \,d^{3}}-\frac{\left(d^{2} x^{2} f^{2}+2 e f x \,d^{2}-2 i d \,f^{2} x +c\right.}{2 a \,c}$

input
```
int((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*f^2*x^3-1/a*f*e*x^2-1/a*e^2*x-1/3/a/f*e^3-1/2*(d^2*x^2*f^2+2*I*d*f^
2*x+2*e*f*x*d^2+2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(I*(d*x+c))-1/2*(d^2*x^2
*f^2-2*I*d*f^2*x+2*e*f*x*d^2-2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(-I*(d*x+c)
)-2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)+4/d^2/a*ln(exp(I*(d*x+c))
+I)*e*f-4/a/d^2*f*e*ln(exp(I*(d*x+c)))-2*I/d/a*f^2*x^2-4*I*f^2*polylog(2,I
*exp(I*(d*x+c)))/a/d^3-2*I/d^3/a*f^2*c^2+4/a/d^2*f^2*ln(1-I*exp(I*(d*x+c))
)*x+4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c-4*I/d^2/a*c*f^2*x-4/d^3/a*c*f^2*l
n(exp(I*(d*x+c))+I)+4/a/d^3*f^2*c*ln(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(167) = 334$.

Time = 0.11 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.81

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*f^2*x^3 + 3*d^2*e^2 - 6*d*e*f + 3*(d^3*e*f + d^2*f^2)*x^2 + 3*(d
^2*f^2*x^2 + d^2*e^2 + 2*d*e*f - 2*f^2 + 2*(d^2*e*f + d*f^2)*x)*cos(d*x +
c)^2 + 3*(d^3*e^2 + 2*d^2*e*f - 2*d*f^2)*x + (d^3*f^2*x^3 + 6*d^2*e^2 + 3*
(d^3*e*f + 2*d^2*f^2)*x^2 - 6*f^2 + 3*(d^3*e^2 + 4*d^2*e*f)*x)*cos(d*x + c
) + 6*(I*f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(I*cos(d*x +
c) - sin(d*x + c)) + 6*(-I*f^2*cos(d*x + c) - I*f^2*sin(d*x + c) - I*f^2)*
dilog(-I*cos(d*x + c) - sin(d*x + c)) - 6*(d*e*f - c*f^2 + (d*e*f - c*f^2)
*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x
 + c) + I) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*
x + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 6*(d*f^2
*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x +
c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 6*(d*e*f - c*f^2 + (d*e*f -
c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x + c))*log(-cos(d*x + c) + I*
sin(d*x + c) + I) + (d^3*f^2*x^3 - 3*d^2*e^2 - 6*d*e*f + 3*(d^3*e*f - d^2*
f^2)*x^2 + 3*(d^3*e^2 - 2*d^2*e*f - 2*d*f^2)*x + 3*(d^2*f^2*x^2 + d^2*e^2
- 2*d*e*f - 2*f^2 + 2*(d^2*e*f - d*f^2)*x)*cos(d*x + c))*sin(d*x + c))/(a*
d^3*cos(d*x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2
*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)**2
/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(167) = 334$.

Time = 0.31 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.20

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\,d^3f^2x^3 - 15i\,d^2e^2 - 6\,def + 3\,(2\,d^3ef - i\,d^2f^2)x^2 + 6i\,f^2 + 6\,(d^3e^2 - i\,d^2ef - df^2)x - 24\,(def\cos($$

input
```
integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output

```
-(2*d^3*f^2*x^3 - 15*I*d^2*e^2 - 6*d*e*f + 3*(2*d^3*e*f - I*d^2*f^2)*x^2 +
 6*I*f^2 + 6*(d^3*e^2 - I*d^2*e*f - d*f^2)*x - 24*(d*e*f*cos(d*x + c) + I*
d*e*f*sin(d*x + c) + I*d*e*f)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) + 24
*(d*f^2*x*cos(d*x + c) + I*d*f^2*x*sin(d*x + c) + I*d*f^2*x)*arctan2(cos(d
*x + c), sin(d*x + c) + 1) + 3*(-I*d^2*f^2*x^2 - I*d^2*e^2 + 2*d*e*f + 2*I
*f^2 + 2*(-I*d^2*e*f + d*f^2)*x)*cos(2*d*x + 2*c) - (2*I*d^3*f^2*x^3 - 3*d
^2*e^2 - 6*I*d*e*f - 3*(-2*I*d^3*e*f + 5*d^2*f^2)*x^2 + 6*f^2 - 6*(-I*d^3*
e^2 + 5*d^2*e*f + I*d*f^2)*x)*cos(d*x + c) + 24*(f^2*cos(d*x + c) + I*f^2*
sin(d*x + c) + I*f^2)*dilog(I*e^(I*d*x + I*c)) - 12*(d*f^2*x + d*e*f - (I*
d*f^2*x + I*d*e*f)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(d*x + c))*log(cos(
d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(d^2*f^2*x^2 + d^2*e
^2 + 2*I*d*e*f - 2*f^2 + 2*(d^2*e*f + I*d*f^2)*x)*sin(2*d*x + 2*c) + (2*d^
3*f^2*x^3 + 3*I*d^2*e^2 - 6*d*e*f + 3*(2*d^3*e*f + 5*I*d^2*f^2)*x^2 - 6*I*
f^2 + 6*(d^3*e^2 + 5*I*d^2*e*f - d*f^2)*x)*sin(d*x + c))/(-6*I*a*d^3*cos(d
*x + c) + 6*a*d^3*sin(d*x + c) + 6*a*d^3)
```

Giac [F]

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sin(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input

```
integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output

```
integrate((f*x + e)^2*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)^2\,(e+f\,x)^2}{a+a\,\sin(c+d\,x)}\,dx$$

input

```
int((sin(c + d*x)^2*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

output

```
int((sin(c + d*x)^2*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-12\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)d^2f^2-12\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)d^2f^2+6\cos\left(dx+c\right)f^2-6\cos\left(dx\right.}{}$$

input
```
int((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 3*cos(c + d*x)*tan((c + d*x)/2)*d**2*e**2 - 6*cos(c + d*x)*tan((c + d*
x)/2)*d**2*e*f*x - 3*cos(c + d*x)*tan((c + d*x)/2)*d**2*f**2*x**2 + 6*cos(
c + d*x)*tan((c + d*x)/2)*f**2 - 3*cos(c + d*x)*d**2*e**2 - 6*cos(c + d*x)
*d**2*e*f*x - 3*cos(c + d*x)*d**2*f**2*x**2 + 6*cos(c + d*x)*f**2 - 12*int
((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*tan((c + d*x)/2)*d**2*f**2
 - 12*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*d**2*f**2 - 6*log
(tan((c + d*x)/2)**2 + 1)*tan((c + d*x)/2)*d*e*f - 6*log(tan((c + d*x)/2)*
*2 + 1)*d*e*f + 12*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)*d*e*f + 12*l
og(tan((c + d*x)/2) + 1)*d*e*f + 6*sin(c + d*x)*tan((c + d*x)/2)*d*e*f + 6
*sin(c + d*x)*tan((c + d*x)/2)*d*f**2*x + 6*sin(c + d*x)*d*e*f + 6*sin(c +
 d*x)*d*f**2*x - 3*tan((c + d*x)/2)*d**3*e**2*x - 3*tan((c + d*x)/2)*d**3*
e*f*x**2 - tan((c + d*x)/2)*d**3*f**2*x**3 + 6*tan((c + d*x)/2)*d**2*e**2
+ 6*tan((c + d*x)/2)*d**2*e*f*x + 6*tan((c + d*x)/2)*d**2*f**2*x**2 - 3*d*
*3*e**2*x - 3*d**3*e*f*x**2 - d**3*f**2*x**3 - 6*d**2*e*f*x)/(3*a*d**3*(ta
n((c + d*x)/2) + 1))
```

3.187 $\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{(e+fx)^2}{2af}-\frac{(e+fx)\cos(c+dx)}{ad} - \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{f\sin(c+dx)}{ad^2}$$

output
```
-1/2*(f*x+e)^2/a/f-(f*x+e)*cos(d*x+c)/a/d-(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x
)/a/d+2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2+f*sin(d*x+c)/a/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(110) = 220.

Time = 7.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\left(-4de+2cde+2cf-c^2f+2d^2ex-2dfx+d^2\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(-4*d*e + 2*
c*d*e + 2*c*f - c^2*f + 2*d^2*e*x - 2*d*f*x + d^2*f*x^2 + 2*d*(e + f*x)*Co
s[c + d*x] - 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*f*Sin[c + d*
x]) + Cos[(c + d*x)/2]*(2*c*d*e + 2*c*f - c^2*f + 2*d^2*e*x + 2*d*f*x + d^
2*f*x^2 + 2*d*(e + f*x)*Cos[c + d*x] - 4*f*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] - 2*f*Sin[c + d*x])))/(a*d^2*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5026, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)\sin(c+dx)dx}{a}-\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\sin(c+dx)dx}{a}-\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\int\frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\int\frac{e+fx}{\sin(c+dx)a+a}dx-\frac{\int(e+fx)dx}{a}+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{17}$$

$$\int\frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3042}$$

$$\int\frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int(e+fx)\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int(e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2f\int\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\int-\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2f\int\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{\frac{4f\log\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)\right)}{d^2}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-\frac{(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*(e + f*x)^2/(a*f) + ((-2*(e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/d + (4*
f*Log[-Cos[c/2 - Pi/4 + (d*x)/2]])/d^2)/(2*a) + (-(((e + f*x)*Cos[c + d*x]
)/d) + (f*Sin[c + d*x])/d^2)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{-2\cos(dx+c) f\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+4\cos(dx+c) f\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-d(fx+e)\cos(2dx+2c)+\sin(2dx+2c) f-2d\left(\left(x\left(\frac{fx}{2}+\cdots\right.\right.\right.}{2d^2a\cos(dx+c)}$
risch	$-\frac{f x^2}{2a}-\frac{ex}{a}-\frac{(dxf+de+if)e^{i(dx+c)}}{2d^2a}-\frac{(dxf+de-if)e^{-i(dx+c)}}{2d^2a}-\frac{2ifx}{da}-\frac{2ifc}{d^2a}-\frac{2(fx+e)}{da\left(e^{i(dx+c)}+i\right)}+\frac{2f\ln\left(e^{i(dx\cdots\right.}{d^2a}$
default	$-\frac{\frac{4e}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\frac{2fx}{d}-\frac{2fx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{4f\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d^2}+\frac{2f\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d^2}+\frac{2e\left(\frac{2}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2\arctan\left(\cdots\right.\right.}{d}}{2a}$
norman	$\frac{\frac{-2de-2f}{d^2a}+\frac{(2de-2f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d^2a}-\frac{f x^2}{2a}-\frac{ex\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{ex\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}-\frac{f x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{f x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{f x^2\tan\left(\frac{dx}{\cdots}\right.}{a}\cdots}{}$

input
```
int((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*cos(d*x+c)*f*ln(sec(1/2*d*x+1/2*c)^2)+4*cos(d*x+c)*f*ln(tan(1/2*d*
x+1/2*c)+1)-d*(f*x+e)*cos(2*d*x+2*c)+sin(2*d*x+2*c)*f-2*d*((x*(1/2*f*x+e)*
d-2*e)*cos(d*x+c)-(f*x+e)*(sin(d*x+c)-3/2)))/d^2/a/cos(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(96) = 192$.

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{d^2fx^2+2\,(dfx+de+f)\cos\,(dx+c)^2+2\,de+2\,(d^2e+df)x+(d^2fx^2+4\,de+2\,(d^2e+2\,df)x)\cos\,(}{}$$

input
```
integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*f*x^2 + 2*(d*f*x + d*e + f)*cos(d*x + c)^2 + 2*d*e + 2*(d^2*e +
d*f)*x + (d^2*f*x^2 + 4*d*e + 2*(d^2*e + 2*d*f)*x)*cos(d*x + c) - 2*(f*cos
(d*x + c) + f*sin(d*x + c) + f)*log(sin(d*x + c) + 1) + (d^2*f*x^2 - 2*d*e
 + 2*(d^2*e - d*f)*x + 2*(d*f*x + d*e - f)*cos(d*x + c) - 2*f)*sin(d*x + c
) - 2*f)/(a*d^2*cos(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(85) = 170$.

Time = 1.25 (sec) , antiderivative size = 1867, normalized size of antiderivative = 16.97

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((-2*d**2*e*x*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 +
 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*
d**2*e*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(
c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d**2*e*x*tan(c
/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 +
 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*
x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d
**2) - d**2*f*x**2*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a
*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f
*x**2*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2
 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2*tan(c/2
 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2
*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2/(2*a*d**2*tan(c/2 + d*x
/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d*
*2) - 4*d*e*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*t
an(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*d*e*tan(c/2
 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2
*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 8*d*e/(2*a*d**2*tan(c/2 + d*x/2)**3
 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) +
4*d*f*x*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*ta...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(96) = 192$.

Time = 0.15 (sec) , antiderivative size = 1762, normalized size of antiderivative = 16.02

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*c*f*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 2)/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1) + a*d*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arct
an(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*((sin(d*x + c)/(cos(d*x +
 c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a + a*sin(d*x + c)/(c
os(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a) - (((
d*x + c)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)^2 - 1)*sin(d*x + c)^4 + ((d*x
+ c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c)^3 + 7*(d*x + c)*cos
(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*sin(2*d*x
+ 2*c)^3 + (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 + (((d*x + c)^2 - 1)*cos(d*x
 + c)^2 + ((d*x + c)^2 - 3)*sin(d*x + c)^2 + (d*x + c)^2 + 6*(d*x + c)*cos
(d*x + c) + 2*((d*x + c)^2 - (d*x + c)*cos(d*x + c) - 2)*sin(d*x + c) - 1)
*cos(2*d*x + 2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x + c)^2 + (((d*x + c)^2 - 3
)*cos(d*x + c)^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^2 + (d*x + c)^2 + ((d*x
+ c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c) + 8*(d*x + c)*cos(d
*x + c) + 2*((d*x + c)^2 + (d*x + c)*cos(d*x + c) - 1)*sin(d*x + c) - 1)*s
in(2*d*x + 2*c)^2 + (2*((d*x + c)^2 - 1)*cos(d*x + c)^2 + (d*x + c)^2 + 7*
(d*x + c)*cos(d*x + c) - 3)*sin(d*x + c)^2 + ((d*x + c)*cos(d*x + c)^3 - (
2*(d*x + c)^2 - 3)*sin(d*x + c)^3 - (4*(d*x + c)^2 - (d*x + c)*cos(d*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2952 vs. $2(96) = 192$.

Time = 0.47 (sec) , antiderivative size = 2952, normalized size of antiderivative = 26.84

$$\int \frac{(e + fx)\sin^2(c + dx)}{a + a\sin(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^2*f*x^2*tan(1/2*d*x)^3*tan
(1/2*c)^2 - d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 2*d^2*e*x*tan(1/2*d*x)
^3*tan(1/2*c)^3 + d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c) - d^2*f*x^2*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*d^2*e*x*tan(1/2*d*x)^3*tan(1/2*c)^2 + d^2*f*x^2*ta
n(1/2*d*x)*tan(1/2*c)^3 - 2*d^2*e*x*tan(1/2*d*x)^2*tan(1/2*c)^3 + 4*d*f*x*
tan(1/2*d*x)^3*tan(1/2*c)^3 - d^2*f*x^2*tan(1/2*d*x)^3 - d^2*f*x^2*tan(1/2
*d*x)^2*tan(1/2*c) + 2*d^2*e*x*tan(1/2*d*x)^3*tan(1/2*c) - d^2*f*x^2*tan(1
/2*d*x)*tan(1/2*c)^2 - 2*d^2*e*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - d^2*f*x^2*t
an(1/2*c)^3 + 2*d^2*e*x*tan(1/2*d*x)*tan(1/2*c)^3 + 4*d*e*tan(1/2*d*x)^3*t
an(1/2*c)^3 - 2*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - d^2*f*x^2*tan(1/2
*d*x)^2 - 2*d^2*e*x*tan(1/2*d*x)^3 + d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c) - 2
*d^2*e*x*tan(1/2*d*x)^2*tan(1/2*c) - d^2*f*x^2*tan(1/2*c)^2 - 2*d^2*e*x*ta
n(1/2*d*x)*tan(1/2*c)^2 - 12*d*f*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*f*log(2
*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*
x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*d*x)^3*tan(1/2*c)^2 - 2*d^2*e*x*tan(1/2*c)^3 + 2*f*log(2*(t...
```

Mupad [B] (verification not implemented)

Time = 36.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{d\,e+f\,1\mathrm{i}}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right) + e^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-d\,e+f\,1\mathrm{i}}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right) - \frac{f\,x^2}{2\,a}+\frac{2\,f\,\ln\left(e^{c\,1\mathrm{i}}\,e^{d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,d^2} - \frac{x\,(d\,e+f\,2\mathrm{i})}{a\,d}-\frac{(e+f\,x)\,2\mathrm{i}}{a\,d\,\left(-1+e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

input
```
int((sin(c + d*x)^2*(e + f*x))/(a + a*sin(c + d*x)),x)
```

output
```
exp(- c*1i - d*x*1i)*((f*1i - d*e)/(2*a*d^2) - (f*x)/(2*a*d)) - exp(c*1i +
 d*x*1i)*((f*1i + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - (f*x^2)/(2*a) + (2*f*l
og(exp(c*1i)*exp(d*x*1i) + 1i))/(a*d^2) - (x*(f*2i + d*e))/(a*d) - ((e + f
*x)*2i)/(a*d*(exp(c*1i + d*x*1i)*1i - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.48

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-2\cos(dx+c)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)de-2\cos(dx+c)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)dfx-2\cos(dx+c)de-2\cos(dx+c)dfx}{}$$

input
```
int((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*tan((c + d*x)/2)*d*e - 2*cos(c + d*x)*tan((c + d*x)/2)*
d*f*x - 2*cos(c + d*x)*d*e - 2*cos(c + d*x)*d*f*x - 2*log(tan((c + d*x)/2)
**2 + 1)*tan((c + d*x)/2)*f - 2*log(tan((c + d*x)/2)**2 + 1)*f + 4*log(tan
((c + d*x)/2) + 1)*tan((c + d*x)/2)*f + 4*log(tan((c + d*x)/2) + 1)*f + 2*
sin(c + d*x)*tan((c + d*x)/2)*f + 2*sin(c + d*x)*f - 2*tan((c + d*x)/2)*d*
*2*e*x - tan((c + d*x)/2)*d**2*f*x**2 + 4*tan((c + d*x)/2)*d*e + 2*tan((c
+ d*x)/2)*d*f*x - 2*d**2*e*x - d**2*f*x**2 - 2*d*f*x)/(2*a*d**2*(tan((c +
d*x)/2) + 1))
```

3.188 $\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)}{ad(1+\sin(c+dx))}$$

output
```
-x/a-cos(d*x+c)/a/d-cos(d*x+c)/a/d/(1+sin(d*x+c))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)(c+dx+\cos(c+dx))+(-2+c+dx+\cos(c\right.}{ad(1+\sin(c+dx))}$$

input
```
Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
-(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]*(c + d*x + Cos[
c + d*x]) + (-2 + c + d*x + Cos[c + d*x])*Sin[(c + d*x)/2]))/(a*d*(1 + Sin
[c + d*x])))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3225, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{\sin(c+dx)}{\sin(c+dx)+1}dx}{a}-\frac{\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sin(c+dx)}{\sin(c+dx)+1}dx}{a}-\frac{\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(c+dx)}{\sin(c+dx)+1}dx}{a}-\frac{\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{x-\int \frac{1}{\sin(c+dx)+1}dx}{a}-\frac{\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x-\int \frac{1}{\sin(c+dx)+1}dx}{a}-\frac{\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\cos(c+dx)}{ad}-\frac{\frac{\cos(c+dx)}{d(\sin(c+dx)+1)}+x}{a}$$

input
```
Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
-(Cos[c + d*x]/(a*d)) - (x + Cos[c + d*x]/(d*(1 + Sin[c + d*x])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-3-\cos(2dx+2c)-2dx\cos(dx+c)+2\sin(dx+c)}{2ad\cos(dx+c)}$
derivativedivides	$\frac{-\frac{2}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{ad}$
default	$\frac{-\frac{2}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{ad}$
risch	$-\frac{x}{a}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2}{da\left(e^{i(dx+c)}+i\right)}$
norman	$\frac{-\frac{2}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{x}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}-\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{a}-\frac{2\tan(\cdots}{\cdots}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

input
```
int(sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/d*(-3-cos(2*d*x+2*c)-2*d*x*cos(d*x+c)+2*sin(d*x+c))/cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{dx+(dx+2)\cos(dx+c)+\cos(dx+c)^2+(dx+\cos(dx+c)-1)\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x + (d*x + 2)*cos(d*x + c) + cos(d*x + c)^2 + (d*x + cos(d*x + c) - 1)
*sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(32) = 64$.

Time = 1.05 (sec) , antiderivative size = 422, normalized size of antiderivative = 9.38

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \begin{cases} -\frac{dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{}{ad\tan^3\left(\frac{c}{2}+\right.} \\ \frac{x\sin^2(c)}{a\sin(c)+a} \end{cases}$$

input
```
integrate(sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((-d*x*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2
 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - d*x*tan(c/2 + d*x/2)**2/(a*d*
tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d
) - d*x*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**
2 + a*d*tan(c/2 + d*x/2) + a*d) - d*x/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c
/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*
tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d
) - 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2
+ a*d*tan(c/2 + d*x/2) + a*d) - 4/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 +
 d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x*sin(c)**2/(a*sin(c
) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.87

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+2}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-2*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 2)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a)/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.71

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{\frac{dx+c}{a}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 2)/((ta
n(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 1)*
a))/d
```

Mupad [B] (verification not implemented)

Time = 35.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{x}{a}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(sin(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

output
```
- x/a - (2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^2 + 4)/(a*d*(tan(c/2
+ (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-\cos(dx+c)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos(dx+c)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)dx+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-dx}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int(sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - cos(c + d*x)*tan((c + d*x)/2) - cos(c + d*x) - tan((c + d*x)/2)*d*x +
2*tan((c + d*x)/2) - d*x)/(a*d*(tan((c + d*x)/2) + 1))
```

3.189 $\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\text{Int}\left(\frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 5048$$
$$\int \frac{\sin^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sin(dx+c)^2}{(fx+e)\,(a+a\sin(dx+c))}dx$$

input
```
int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)^2}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)),
 x)
```

Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

input
```
integrate(sin(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)
/a
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 1266, normalized size of antiderivative = 45.21

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\sin(dx+c)^2}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(d*e*(I*exp_integral_e(1, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(1,
-(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*e*(exp_integral_e(1, (I*d*f
*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f)
/f) + (d*e*(I*exp_integral_e(1, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(1,
 -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*e*(exp_integral_e(1, (I*d*
f*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f
)/f) + (d*f*(I*exp_integral_e(1, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(1
, -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*f*(exp_integral_e(1, (I*d
*f*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*
f)/f))*x)*cos(d*x + c)^2 + (d*e*(I*exp_integral_e(1, (I*d*f*x + I*d*e)/f)
- I*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*e*(ex
p_integral_e(1, (I*d*f*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I*d*e
)/f))*sin(-(d*e - c*f)/f) + (d*f*(I*exp_integral_e(1, (I*d*f*x + I*d*e)/f)
 - I*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*f*(e
xp_integral_e(1, (I*d*f*x + I*d*e)/f) + exp_integral_e(1, -(I*d*f*x + I*d*
e)/f))*sin(-(d*e - c*f)/f))*x)*sin(d*x + c)^2 + (d*f*(I*exp_integral_e(1,
(I*d*f*x + I*d*e)/f) - I*exp_integral_e(1, -(I*d*f*x + I*d*e)/f))*cos(-(d*
e - c*f)/f) + d*f*(exp_integral_e(1, (I*d*f*x + I*d*e)/f) + exp_integral_e
(1, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f)/f))*x + 4*f*cos(d*x + c) + 4*(
a*d*f^3*x + a*d*e*f^2 + (a*d*f^3*x + a*d*e*f^2)*cos(d*x + c)^2 + (a*d*f...
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)^2}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^2/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+d\,x)^2}{(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(sin(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(sin(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(dx+c)^2}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(c + d*x)**2/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.190 $\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 12.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$
$$\Big\downarrow \text{5048}$$
$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sin (dx+c)^2}{(fx+e)^2 (a+a\sin (dx+c))}dx$$

input
```
int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin (dx+c)^2}{(fx+e)^2(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2
 + 2*a*e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 13.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(sin(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x)
+ 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 1388, normalized size of antiderivative = 49.57

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)^2}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(d*e*(I*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(2,
-(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*e*(exp_integral_e(2, (I*d*f
*x + I*d*e)/f) + exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f)
/f) + (d*e*(I*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(2,
 -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*e*(exp_integral_e(2, (I*d*
f*x + I*d*e)/f) + exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f
)/f) - 2*d*e + (d*f*(I*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_inte
gral_e(2, -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*f*(exp_integral_e
(2, (I*d*f*x + I*d*e)/f) + exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin(-(
d*e - c*f)/f) - 2*d*f)*x)*cos(d*x + c)^2 + (d*e*(I*exp_integral_e(2, (I*d*
f*x + I*d*e)/f) - I*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c
*f)/f) + d*e*(exp_integral_e(2, (I*d*f*x + I*d*e)/f) + exp_integral_e(2, -
(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f)/f) - 2*d*e + (d*f*(I*exp_integral_e
(2, (I*d*f*x + I*d*e)/f) - I*exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*cos(
-(d*e - c*f)/f) + d*f*(exp_integral_e(2, (I*d*f*x + I*d*e)/f) + exp_integr
al_e(2, -(I*d*f*x + I*d*e)/f))*sin(-(d*e - c*f)/f) - 2*d*f)*x)*sin(d*x + c
)^2 - 2*d*e + (d*f*(I*exp_integral_e(2, (I*d*f*x + I*d*e)/f) - I*exp_integ
ral_e(2, -(I*d*f*x + I*d*e)/f))*cos(-(d*e - c*f)/f) + d*f*(exp_integral_e(
2, (I*d*f*x + I*d*e)/f) + exp_integral_e(2, -(I*d*f*x + I*d*e)/f))*sin(-(d
*e - c*f)/f) - 2*d*f)*x + 4*f*cos(d*x + c) + 8*(a*d*f^4*x^2 + 2*a*d*e*f...
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin\left(dx+c\right)^2}{\left(fx+e\right)^2\left(a\sin\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^2/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 35.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin\left(c+d\,x\right)^2}{\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sin(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(sin(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(dx+c)^2}{\sin(dx+c)e^2+2\sin(dx+c)efx+\sin(dx+c)f^2x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(c + d*x)**2/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*
x)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.191 $\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\begin{aligned}\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{3f(e+fx)^2}{8ad^2}+\frac{i(e+fx)^3}{ad}+\frac{3(e+fx)^4}{8af}\\
&\quad-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{(e+fx)^3\cos(c+dx)}{ad}\\
&\quad+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}\\
&\quad-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}\\
&\quad+\frac{12if^2(e+fx)\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}\\
&\quad-\frac{12f^3\,\mathrm{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}\\
&\quad+\frac{6f^3\sin(c+dx)}{ad^4}-\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}\\
&\quad+\frac{3f^2(e+fx)\cos(c+dx)\sin(c+dx)}{4ad^3}\\
&\quad-\frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2ad}\\
&\quad-\frac{3f^3\sin^2(c+dx)}{8ad^4}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4ad^2}\end{aligned}$$

output
```
-3/8*f*(f*x+e)^2/a/d^2+I*(f*x+e)^3/a/d+3/8*(f*x+e)^4/a/f-6*f^2*(f*x+e)*cos
(d*x+c)/a/d^3+(f*x+e)^3*cos(d*x+c)/a/d+(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)
/a/d-6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+12*I*f^2*(f*x+e)*polylog(2
,I*exp(I*(d*x+c)))/a/d^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+6*f^3*si
n(d*x+c)/a/d^4-3*f*(f*x+e)^2*sin(d*x+c)/a/d^2+3/4*f^2*(f*x+e)*cos(d*x+c)*s
in(d*x+c)/a/d^3-1/2*(f*x+e)^3*cos(d*x+c)*sin(d*x+c)/a/d-3/8*f^3*sin(d*x+c)
^2/a/d^4+3/4*f*(f*x+e)^2*sin(d*x+c)^2/a/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 4.14 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.46

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{48e^3x+72e^2fx^2+48ef^2x^3+12f^3x^4+\frac{192f(\cos(c)+i\sin(c))\left(\frac{(e+fx)^3(\cos(c)-i\sin(c))}{3f}-\frac{(e+fx)^2\log(1+i\cos(c+dx)+\sin(c+dx))(1+\cdots}{d}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(48*e^3*x + 72*e^2*f*x^2 + 48*e*f^2*x^3 + 12*f^3*x^4 + (192*f*(Cos[c] + I*
Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[1 + I*
Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e + f*x
)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[3, (-I)*Cos[c
 + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(d*(Cos[c] + I*(
1 + Sin[c]))) - (64*(e + f*x)^3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])) + (16*((6*I)*f^3 - 6*d*f^2*(e + f*x) -
 (3*I)*d^2*f*(e + f*x)^2 + d^3*(e + f*x)^3)*(Cos[c + d*x] - I*Sin[c + d*x]
))/d^4 + (16*((-6*I)*f^3 - 6*d*f^2*(e + f*x) + (3*I)*d^2*f*(e + f*x)^2 + d
^3*(e + f*x)^3)*(Cos[c + d*x] + I*Sin[c + d*x]))/d^4 + ((3*f^3 + (6*I)*d*f
^2*(e + f*x) - 6*d^2*f*(e + f*x)^2 - (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*
x)] - I*Sin[2*(c + d*x)]))/d^4 + ((3*f^3 - (6*I)*d*f^2*(e + f*x) - 6*d^2*f
*(e + f*x)^2 + (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x
)]))/d^4)/(32*a)
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.11, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5026, 3042, 3792, 17, 3042, 3791, 17, 5026, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^3\sin^2(c+dx)dx}{a}-\int \frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\sin(c+dx)^2dx}{a}-\int \frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{-\frac{3f^2\int (e+fx)\sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}}{a}-\int \frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{3f^2\int (e+fx)\sin^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\int \frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3f^2\int (e+fx)\sin(c+dx)^2dx}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\int \frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{-\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\int\frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 17

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\int\frac{(e+fx)^3\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 5026

$$-\frac{\int(e+fx)^3\sin(c+dx)dx}{a}+\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{\int(e+fx)^3\sin(c+dx)dx}{a}+\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\Big\downarrow$ 3777

$$\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\int(e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\Big\downarrow$ 3042

$$\int\frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\Big\downarrow$ 3777

$$\frac{\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}}{}$$

$\downarrow$ 25

$$\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\downarrow$ 3042

$$\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\downarrow$ 3777

$$\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\downarrow$ 3042

$$\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}$$

$\Big\downarrow$ 3117

$$\int \frac{(e+fx)^3\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 5026

$$-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{\int (e+fx)^3dx}{a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 17

$$-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\Big\downarrow$ 3042

$$-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\Big\downarrow$ 3799

$$-\frac{\int (e+fx)^3\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\downarrow$ 3042

$$-\frac{\int (e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\downarrow$ 4672

$$-\frac{\frac{6f\int (e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\downarrow$ 3042

$$-\frac{\frac{6f\int -(e+fx)^2\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\downarrow$ 25

$$-\frac{-\frac{6f\int (e+fx)^2\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \text{ 4202}$$

$$-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)^2}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\int (e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}+\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}\right)}{d}-\frac{i(e+\cdots}{\cdots}\cdots\right)\cdots\right)}{d}\cdots}{2a}$$

$$-\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\downarrow$ 2720

$$-\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+\cdots}\right)\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{2a}$$

$$-\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

$\downarrow$ 7143

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{a}-$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log \ldots}{\ldots}\right.\right.}{d}}{2a}$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}+\frac{(e+fx)^4}{4af}$$

input
```
Int[((e + f*x)^3*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*a*f) - ((-2*(e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/d - (6*f
*(((I/3)*(e + f*x)^3)/f - (2*I)*(((-I)*(e + f*x)^2*Log[1 + E^((I/2)*(2*c +
 3*Pi + 2*d*x))])/d + ((2*I)*f*((I*(e + f*x)*PolyLog[2, -E^((I/2)*(2*c + 3
*Pi + 2*d*x))])/d - (f*PolyLog[3, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2))/
d))/d)/(2*a) + ((e + f*x)^4/(8*f) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x
])/(2*d) + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*d^2) - (3*f^2*((e + f*x)^2/
(4*f) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x]^2)/(
4*d^2)))/(2*d^2))/a - (-(((e + f*x)^3*Cos[c + d*x])/d) + (3*f*(((e + f*x)^
2*Sin[c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])
/d^2))/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(340) = 680$.

Time = 2.20 (sec) , antiderivative size = 1054, normalized size of antiderivative = 2.86

method	result
risch	$-\frac{3f\,e^2\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a\,d^2}-\frac{12f^3\,\mathrm{polylog}\left(3,i\mathrm{e}^{i(dx+c)}\right)}{a\,d^4}+\frac{3f^2e\,x^3}{2a}+\frac{6f\,e^2\ln\left(\mathrm{e}^{i(dx+c)}\right)}{a\,d^2}+\frac{6f^3c^2\ln\left(\mathrm{e}^{i(dx+c)}\right)}{a\,d^4}+\frac{6f^3c^2\ln(1-}{a\,d}$

input
```
int((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d^3*x^3*f^3+3*I*d^2*f^3*x^2+3*e*f^2*x^2*d^3+6*I*d^2*e*f^2*x+3*e^2*f*x
*d^3+3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x-6*I*f^3-6*d*e*f^2)/a/d^4*exp(I*(d*x+c
))+1/2*(d^3*x^3*f^3-3*I*d^2*f^3*x^2+3*e*f^2*x^2*d^3-6*I*d^2*e*f^2*x+3*e^2*
f*x*d^3-3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x+6*I*f^3-6*d*e*f^2)/a/d^4*exp(-I*(d
*x+c))+2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)+3/2/a*
f^2*e*x^3+3/8/a/f*e^4+6/a/d^4*f^3*c^2*ln(exp(I*(d*x+c)))+6/a/d^2*f*e^2*ln(
exp(I*(d*x+c)))-6/a/d^2*f^3*ln(1-I*exp(I*(d*x+c)))*x^2+6/a/d^4*f^3*c^2*ln(
1-I*exp(I*(d*x+c)))+2*I/a/d*f^3*x^3-4*I/a/d^4*f^3*c^3-12*f^3*polylog(3,I*e
xp(I*(d*x+c)))/a/d^4-12*I/a/d^3*f^2*e*c*arctan(exp(I*(d*x+c)))+12*I/a/d^2*
f^2*e*c*x-12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c)))+6/a/d^3*f^2*e*c*ln(exp(2*I*(
d*x+c))+1)-12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c)))*x+6*I/a/d^2*f*e^2*arctan(
exp(I*(d*x+c)))+12*I/a/d^3*f^2*e*polylog(2,I*exp(I*(d*x+c)))-6*I/a/d^3*f^3
*c^2*x+12*I/a/d^3*f^3*polylog(2,I*exp(I*(d*x+c)))*x+6*I/a/d*f^2*e*x^2+6*I/
a/d^3*f^2*e*c^2+6*I/a/d^4*f^3*c^2*arctan(exp(I*(d*x+c)))-12/a/d^3*f^2*e*ln
(1-I*exp(I*(d*x+c)))*c+3/8/a*f^3*x^4+1/32*I*(4*d^3*x^3*f^3+6*I*d^2*f^3*x^2
+12*e*f^2*x^2*d^3+12*I*d^2*e*f^2*x+12*e^2*f*x*d^3+6*I*d^2*e^2*f+4*d^3*e^3-
6*d*f^3*x-3*I*f^3-6*d*e*f^2)/a/d^4*exp(2*I*(d*x+c))-1/32*I*(4*d^3*x^3*f^3-
6*I*d^2*f^3*x^2+12*e*f^2*x^2*d^3-12*I*d^2*e*f^2*x+12*e^2*f*x*d^3-6*I*d^2*e
^2*f+4*d^3*e^3-6*d*f^3*x+3*I*f^3-6*d*e*f^2)/a/d^4*exp(-2*I*(d*x+c))+9/4/a*
f*e^2*x^2+3/2/a*e^3*x-3/a/d^2*f*e^2*ln(exp(2*I*(d*x+c))+1)-3/a/d^4*f^3*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(334) = 668$.

Time = 0.14 (sec) , antiderivative size = 1565, normalized size of antiderivative = 4.24

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(6*d^4*f^3*x^4 + 16*d^3*e^3 - 42*d^2*e^2*f + 8*(3*d^4*e*f^2 + 2*d^3*f
^3)*x^3 + 2*(4*d^3*f^3*x^3 + 4*d^3*e^3 - 6*d^2*e^2*f - 6*d*e*f^2 + 3*f^3 +
 6*(2*d^3*e*f^2 - d^2*f^3)*x^2 + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 - d*f^3)*x)*
cos(d*x + c)^3 + 93*f^3 + 6*(6*d^4*e^2*f + 8*d^3*e*f^2 - 7*d^2*f^3)*x^2 +
2*(8*d^3*f^3*x^3 + 8*d^3*e^3 + 18*d^2*e^2*f - 48*d*e*f^2 - 45*f^3 + 6*(4*d
^3*e*f^2 + 3*d^2*f^3)*x^2 + 12*(2*d^3*e^2*f + 3*d^2*e*f^2 - 4*d*f^3)*x)*co
s(d*x + c)^2 + 12*(2*d^4*e^3 + 4*d^3*e^2*f - 7*d^2*e*f^2)*x + 3*(2*d^4*f^3
*x^4 + 8*d^3*e^3 + 2*d^2*e^2*f - 28*d*e*f^2 + 8*(d^4*e*f^2 + d^3*f^3)*x^3
- f^3 + 2*(6*d^4*e^2*f + 12*d^3*e*f^2 + d^2*f^3)*x^2 + 4*(2*d^4*e^3 + 6*d^
3*e^2*f + d^2*e*f^2 - 7*d*f^3)*x)*cos(d*x + c) - 96*(-I*d*f^3*x - I*d*e*f^
2 + (-I*d*f^3*x - I*d*e*f^2)*cos(d*x + c) + (-I*d*f^3*x - I*d*e*f^2)*sin(d
*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 96*(I*d*f^3*x + I*d*e*f^2
+ (I*d*f^3*x + I*d*e*f^2)*cos(d*x + c) + (I*d*f^3*x + I*d*e*f^2)*sin(d*x +
 c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 48*(d^2*e^2*f - 2*c*d*e*f^2 +
 c^2*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f -
 2*c*d*e*f^2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) +
I) - 48*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^2*f^3*x^
2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*x^2 + 2
*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d*x + c) + s
in(d*x + c) + 1) - 48*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*...
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sin^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3
*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d
*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)**3/(sin(c
 + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^3 \sin(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\sin\left(c+d\,x\right)^3\,\left(e+f\,x\right)^3}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((sin(c + d*x)^3*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)^3*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output

```
( - 192*cos(c + d*x)*int(x**2/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5
 + 3*tan((c + d*x)/2)**4 + 4*tan((c + d*x)/2)**3 + 3*tan((c + d*x)/2)**2 +
 2*tan((c + d*x)/2) + 1),x)*d**3*f**3 - 384*cos(c + d*x)*int(x/(tan((c + d
*x)/2)**6 + 2*tan((c + d*x)/2)**5 + 3*tan((c + d*x)/2)**4 + 4*tan((c + d*x
)/2)**3 + 3*tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**3*e*f**2 +
 128*cos(c + d*x)*int(x/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5 + 3*t
an((c + d*x)/2)**4 + 4*tan((c + d*x)/2)**3 + 3*tan((c + d*x)/2)**2 + 2*tan
((c + d*x)/2) + 1),x)*d**2*f**3 + 72*cos(c + d*x)*log(tan((c + d*x)/2)**2
+ 1)*d**2*e**2*f - 48*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*d*e*f**2 +
 16*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*f**3 - 144*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*d**2*e**2*f + 96*cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*d*e*f**2 - 32*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*f**3 + 12*cos(c +
 d*x)*sin(c + d*x)**2*d**3*e**3 + 36*cos(c + d*x)*sin(c + d*x)**2*d**3*e**
2*f*x + 36*cos(c + d*x)*sin(c + d*x)**2*d**3*e*f**2*x**2 + 12*cos(c + d*x)
*sin(c + d*x)**2*d**3*f**3*x**3 + 18*cos(c + d*x)*sin(c + d*x)**2*d**2*e**
2*f + 12*cos(c + d*x)*sin(c + d*x)**2*d**2*e*f**2*x + 6*cos(c + d*x)*sin(c
 + d*x)**2*d**2*f**3*x**2 - 6*cos(c + d*x)*sin(c + d*x)**2*d*e*f**2 + 2*co
s(c + d*x)*sin(c + d*x)**2*d*f**3*x - 7*cos(c + d*x)*sin(c + d*x)**2*f**3
- 12*cos(c + d*x)*sin(c + d*x)*d**3*e**3 - 36*cos(c + d*x)*sin(c + d*x)*d*
*3*e**2*f*x - 36*cos(c + d*x)*sin(c + d*x)*d**3*e*f**2*x**2 - 12*cos(c ...
```

3.192 $\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 278

$$\begin{aligned}\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = &-\frac{f^2x}{4ad^2}+\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{2af}-\frac{2f^2\cos(c+dx)}{ad^3}\\ &+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}\\ &-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}\\ &+\frac{4if^2\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}\\ &-\frac{2f(e+fx)\sin(c+dx)}{ad^2}+\frac{f^2\cos(c+dx)\sin(c+dx)}{4ad^3}\\ &-\frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2ad}\\ &+\frac{f(e+fx)\sin^2(c+dx)}{2ad^2}\end{aligned}$$

output

```
-1/4*f^2*x/a/d^2+I*(f*x+e)^2/a/d+1/2*(f*x+e)^3/a/f-2*f^2*cos(d*x+c)/a/d^3+
(f*x+e)^2*cos(d*x+c)/a/d+(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-4*f*(f*x+
e)*ln(1-I*exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3-
2*f*(f*x+e)*sin(d*x+c)/a/d^2+1/4*f^2*cos(d*x+c)*sin(d*x+c)/a/d^3-1/2*(f*x+
e)^2*cos(d*x+c)*sin(d*x+c)/a/d+1/2*f*(f*x+e)*sin(d*x+c)^2/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 830 vs. 2(278) = 556.

Time = 5.23 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.99

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/16*(-6*d^2*e^2*Cos[(3*(c + d*x))/2] - 14*d*e*f*Cos[(3*(c + d*x))/2] + 1
5*f^2*Cos[(3*(c + d*x))/2] - 12*d^2*e*f*x*Cos[(3*(c + d*x))/2] - 14*d*f^2*
x*Cos[(3*(c + d*x))/2] - 6*d^2*f^2*x^2*Cos[(3*(c + d*x))/2] - 2*d^2*e^2*Co
s[(5*(c + d*x))/2] + 2*d*e*f*Cos[(5*(c + d*x))/2] + f^2*Cos[(5*(c + d*x))/
2] - 4*d^2*e*f*x*Cos[(5*(c + d*x))/2] + 2*d*f^2*x*Cos[(5*(c + d*x))/2] - 2
*d^2*f^2*x^2*Cos[(5*(c + d*x))/2] - 8*Cos[(c + d*x)/2]*(-2*f^2 - 2*d*f*(e
+ f*x) + (3 - 2*I)*d^2*(e + f*x)^2 + d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 8
*d*f*(e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]) + (24 + 16*I)*d^2*e
^2*Sin[(c + d*x)/2] + 16*d*e*f*Sin[(c + d*x)/2] - 16*f^2*Sin[(c + d*x)/2]
- 24*d^3*e^2*x*Sin[(c + d*x)/2] + (48 + 32*I)*d^2*e*f*x*Sin[(c + d*x)/2] +
 16*d*f^2*x*Sin[(c + d*x)/2] - 24*d^3*e*f*x^2*Sin[(c + d*x)/2] + (24 + 16*
I)*d^2*f^2*x^2*Sin[(c + d*x)/2] - 8*d^3*f^2*x^3*Sin[(c + d*x)/2] + 64*d*e*
f*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*Sin[(c + d*x)/2] + 64*d*f^2*x*Log
[1 + I*Cos[c + d*x] + Sin[c + d*x]]*Sin[(c + d*x)/2] + (64*I)*f^2*PolyLog[
2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
 - 6*d^2*e^2*Sin[(3*(c + d*x))/2] + 14*d*e*f*Sin[(3*(c + d*x))/2] + 15*f^2
*Sin[(3*(c + d*x))/2] - 12*d^2*e*f*x*Sin[(3*(c + d*x))/2] + 14*d*f^2*x*Sin
[(3*(c + d*x))/2] - 6*d^2*f^2*x^2*Sin[(3*(c + d*x))/2] + 2*d^2*e^2*Sin[(5*
(c + d*x))/2] + 2*d*e*f*Sin[(5*(c + d*x))/2] - f^2*Sin[(5*(c + d*x))/2] +
4*d^2*e*f*x*Sin[(5*(c + d*x))/2] + 2*d*f^2*x*Sin[(5*(c + d*x))/2] + 2*d...
```

Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.964$, Rules used = {5026, 3042, 3792, 17, 3042, 3115, 24, 5026, 3042, 3777, 3042, 3777, 25, 3042, 3118, 5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^2\sin^2(c+dx)dx}{a}-\int \frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\sin(c+dx)^2dx}{a}-\int \frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}}{a}-\int \frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\int \frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int \sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\int \frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{-\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\int\frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 24

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\int\frac{(e+fx)^2\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 5026

$$-\frac{\int(e+fx)^2\sin(c+dx)dx}{a}+\int\frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{\int(e+fx)^2\sin(c+dx)dx}{a}+\int\frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$\Big\downarrow$ 3777

$$\int\frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{2f\int(e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$\Big\downarrow$ 3042

$$\int\frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$\Big\downarrow$ 3777

$$\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}$$

$$\Big\downarrow \text{3118}$$

$$\int \frac{(e+fx)^2\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{5026}$$

$$-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\int (e+fx)^2dx}{a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{17}$$

$$-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 3042

$$-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 3799

$$-\frac{\int (e+fx)^2\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 3042

$$-\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 4672

$$-\frac{\frac{4f\int (e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{4f\int -\left((e+fx)\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{4f\int (e+fx)\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$$\Big\downarrow \ 4202$$

$$-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$$\Big\downarrow \ 2620$$

$$-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 2715

$$-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)de^{\frac{1}{2}i(2c+2dx+3\pi)}}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx\cdots}\right)}{d}\cdots\right.\right.}{d}}{2a}+\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

$\Big\downarrow$ 2838

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{a}-\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}+\frac{(e+fx)^3}{3af}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*a*f) - ((-2*(e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/d - (4*f
*(((I/2)*(e + f*x)^2)/f - (2*I)*(((-I)*(e + f*x)*Log[1 + E^((I/2)*(2*c + 3
*Pi + 2*d*x))])/d - (f*PolyLog[2, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2)))
/d)/(2*a) - (-(((e + f*x)^2*Cos[c + d*x])/d) + (2*f*((f*Cos[c + d*x])/d^2
+ ((e + f*x)*Sin[c + d*x])/d))/d)/a + ((e + f*x)^3/(6*f) - ((e + f*x)^2*Co
s[c + d*x]*Sin[c + d*x])/(2*d) + (f*(e + f*x)*Sin[c + d*x]^2)/(2*d^2) - (f
^2*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(2*d^2))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(254) = 508.

Time = 2.72 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.13

method	result
risch	$\frac{f^2x^3}{2a}+\frac{3fe\,x^2}{2a}+\frac{3e^2x}{2a}+\frac{e^3}{2af}+\frac{i(2d^2x^2f^2+4efx\,d^2+2id\,f^2x+2d^2e^2+2idef-f^2)\,\mathrm{e}^{2i(dx+c)}}{16a\,d^3}+\frac{(d^2x^2f^2+2efx\,d^2+2id\,f^2x}{2a}$

input
```
int((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*f^2*x^3+3/2/a*f*e*x^2+3/2/a*e^2*x+1/2/a/f*e^3+1/16*I*(2*d^2*x^2*f^2+
2*I*d*f^2*x+4*e*f*x*d^2+2*I*d*e*f+2*d^2*e^2-f^2)/a/d^3*exp(2*I*(d*x+c))+1/
2*(d^2*x^2*f^2+2*I*d*f^2*x+2*e*f*x*d^2+2*I*d*e*f+d^2*e^2-2*f^2)/a/d^3*exp(
I*(d*x+c))+1/2*(d^2*x^2*f^2-2*I*d*f^2*x+2*e*f*x*d^2-2*I*d*e*f+d^2*e^2-2*f^
2)/a/d^3*exp(-I*(d*x+c))+2*I/a/d*f^2*x^2+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(
I*(d*x+c))+I)+4/a/d^2*f*e*ln(exp(I*(d*x+c)))-2/a/d^2*f*e*ln(exp(2*I*(d*x+c
))+1)-1/16*I*(2*d^2*x^2*f^2-2*I*d*f^2*x+4*e*f*x*d^2-2*I*d*e*f+2*d^2*e^2-f^
2)/a/d^3*exp(-2*I*(d*x+c))+2*I/a/d^3*f^2*c^2-4*I/a/d^3*f^2*c*arctan(exp(I*
(d*x+c)))+4*I/a/d^2*f^2*c*x-4/a/d^2*f^2*ln(1-I*exp(I*(d*x+c)))*x-4/a/d^3*f
^2*ln(1-I*exp(I*(d*x+c)))*c+4*I/a/d^2*f*e*arctan(exp(I*(d*x+c)))-4/a/d^3*f
^2*c*ln(exp(I*(d*x+c)))+2/a/d^3*f^2*c*ln(exp(2*I*(d*x+c))+1)+4*I*f^2*polyl
og(2,I*exp(I*(d*x+c)))/a/d^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(249) = 498$.

Time = 0.11 (sec) , antiderivative size = 846, normalized size of antiderivative = 3.04

$$\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d^3*f^2*x^3 + 4*d^2*e^2 + (2*d^2*f^2*x^2 + 2*d^2*e^2 - 2*d*e*f - f^
2 + 2*(2*d^2*e*f - d*f^2)*x)*cos(d*x + c)^3 - 7*d*e*f + 2*(3*d^3*e*f + 2*d
^2*f^2)*x^2 + 2*(2*d^2*f^2*x^2 + 2*d^2*e^2 + 3*d*e*f - 4*f^2 + (4*d^2*e*f
+ 3*d*f^2)*x)*cos(d*x + c)^2 + (6*d^3*e^2 + 8*d^2*e*f - 7*d*f^2)*x + (2*d^
3*f^2*x^3 + 6*d^2*e^2 + d*e*f + 6*(d^3*e*f + d^2*f^2)*x^2 - 7*f^2 + (6*d^3
*e^2 + 12*d^2*e*f + d*f^2)*x)*cos(d*x + c) - 8*(-I*f^2*cos(d*x + c) - I*f^
2*sin(d*x + c) - I*f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 8*(I*f^2*co
s(d*x + c) + I*f^2*sin(d*x + c) + I*f^2)*dilog(-I*cos(d*x + c) - sin(d*x +
 c)) - 8*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*s
in(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 8*(d*f^2*x + c*f^2 +
 (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*co
s(d*x + c) + sin(d*x + c) + 1) - 8*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*co
s(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x
 + c) + 1) - 8*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*
f^2)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (2*d^3*f^2*x^
3 - 4*d^2*e^2 - 7*d*e*f + 2*(3*d^3*e*f - 2*d^2*f^2)*x^2 - (2*d^2*f^2*x^2 +
 2*d^2*e^2 + 2*d*e*f - f^2 + 2*(2*d^2*e*f + d*f^2)*x)*cos(d*x + c)^2 + (6*
d^3*e^2 - 8*d^2*e*f - 7*d*f^2)*x + (2*d^2*f^2*x^2 + 2*d^2*e^2 - 8*d*e*f -
7*f^2 + 4*(d^2*e*f - 2*d*f^2)*x)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(d*
x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sin^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2
*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)**3
/(sin(c + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sin(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\sin\left(c+d\,x\right)^3\,\left(e+f\,x\right)^2}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((sin(c + d*x)^3*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)^3*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)**3*tan((c + d*x)/2)*d*f**2*x - 2*cos(c + d*x)**3*d*f**2
*x + 2*cos(c + d*x)**2*sin(c + d*x)*tan((c + d*x)/2)*d*f**2*x + 2*cos(c +
d*x)**2*sin(c + d*x)*tan((c + d*x)/2)*f**2 + 2*cos(c + d*x)**2*sin(c + d*x
)*d*f**2*x + 2*cos(c + d*x)**2*sin(c + d*x)*f**2 - 22*cos(c + d*x)**2*tan(
(c + d*x)/2)*d*f**2*x - 8*cos(c + d*x)**2*tan((c + d*x)/2)*f**2 - 22*cos(c
 + d*x)**2*d*f**2*x - 8*cos(c + d*x)**2*f**2 + 48*cos(c + d*x)*int((tan((c
 + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*tan((c + d*x)/2)*d**2*f**2 + 48*co
s(c + d*x)*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*d**2*f**2 +
24*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*tan((c + d*x)/2)*d*e*f + 24*c
os(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*d*e*f - 48*cos(c + d*x)*log(tan((
c + d*x)/2) + 1)*tan((c + d*x)/2)*d*e*f - 48*cos(c + d*x)*log(tan((c + d*x
)/2) + 1)*d*e*f + 6*cos(c + d*x)*sin(c + d*x)**2*tan((c + d*x)/2)*d**2*e**
2 + 12*cos(c + d*x)*sin(c + d*x)**2*tan((c + d*x)/2)*d**2*e*f*x + 6*cos(c
+ d*x)*sin(c + d*x)**2*tan((c + d*x)/2)*d**2*f**2*x**2 + 6*cos(c + d*x)*si
n(c + d*x)**2*tan((c + d*x)/2)*d*e*f + 4*cos(c + d*x)*sin(c + d*x)**2*tan(
(c + d*x)/2)*d*f**2*x - 3*cos(c + d*x)*sin(c + d*x)**2*tan((c + d*x)/2)*f*
*2 + 6*cos(c + d*x)*sin(c + d*x)**2*d**2*e**2 + 12*cos(c + d*x)*sin(c + d*
x)**2*d**2*e*f*x + 6*cos(c + d*x)*sin(c + d*x)**2*d**2*f**2*x**2 + 6*cos(c
 + d*x)*sin(c + d*x)**2*d*e*f + 4*cos(c + d*x)*sin(c + d*x)**2*d*f**2*x -
3*cos(c + d*x)*sin(c + d*x)**2*f**2 - 6*cos(c + d*x)*sin(c + d*x)*tan((...
```

3.193 $\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 155

$$\begin{aligned}\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{3(e+fx)^2}{4af}+\frac{(e+fx)\cos(c+dx)}{ad}\\ &\quad+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}\\ &\quad-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\sin(c+dx)}{ad^2}\\ &\quad-\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad}+\frac{f\sin^2(c+dx)}{4ad^2}\end{aligned}$$

output

```
3/4*(f*x+e)^2/a/f+(f*x+e)*cos(d*x+c)/a/d+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)
/a/d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2-f*sin(d*x+c)/a/d^2-1/2*(f*x+e
)*cos(d*x+c)*sin(d*x+c)/a/d+1/4*f*sin(d*x+c)^2/a/d^2
```

Mathematica [A] (verified)

Time = 8.75 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.92

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\left(8d(e+fx)\cos(c+dx)-f\cos(2(c+dx))+2(-8\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(8*d*(e + f*x)*Co
s[c + d*x] - f*Cos[2*(c + d*x)] + 2*(-8*d*e + 6*c*d*e + 4*c*f - 3*c^2*f +
6*d^2*e*x - 4*d*f*x + 3*d^2*f*x^2 - 8*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] - 4*f*Sin[c + d*x] - d*(e + f*x)*Sin[2*(c + d*x)])) + Cos[(c + d*x)
/2]*(8*d*(e + f*x)*Cos[c + d*x] - f*Cos[2*(c + d*x)] + 2*(6*c*d*e + 4*c*f
- 3*c^2*f + 6*d^2*e*x + 4*d*f*x + 3*d^2*f*x^2 - 8*f*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 4*f*Sin[c + d*x] - d*(e + f*x)*Sin[2*(c + d*x)]))))/(
8*a*d^2*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {5026, 3042, 3791, 17, 5026, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3799, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin^3(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \; 5026$$

$$\frac{\int (e+fx)\sin^2(c+dx)dx}{a}-\int \frac{(e+fx)\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)\sin(c+dx)^2dx}{a}-\int \frac{(e+fx)\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{1}{2}\int (e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}}{a}-\int \frac{(e+fx)\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\int \frac{(e+fx)\sin^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{5026}$$

$$-\frac{\int (e+fx)\sin(c+dx)dx}{a}+\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (e+fx)\sin(c+dx)dx}{a}+\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}$$

$$\Big\downarrow \text{3777}$$

$$\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx-\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}$$

$$\Big\downarrow \text{3117}$$

$$\int \frac{(e+fx)\sin(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 5026

$$-\int \frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\int (e+fx)dx}{a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 17

$$-\int \frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$\Big\downarrow$ 3042

$$-\int \frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$\Big\downarrow$ 3799

$$-\frac{\int (e+fx)\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$\Big\downarrow$ 3042

$$-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$\Big\downarrow$ 4672

$$-\frac{\frac{2f\int \cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2f\int -\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{2f\int \tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}+\frac{(e+fx)^2}{2af}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{a}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{\frac{4f\log\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)\right)}{d^2}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*a*f) - ((-2*(e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/d + (4*f*L
og[-Cos[c/2 - Pi/4 + (d*x)/2]])/d^2)/(2*a) - (-(((e + f*x)*Cos[c + d*x])/d
) + (f*Sin[c + d*x])/d^2)/a + ((e + f*x)^2/(4*f) - ((e + f*x)*Cos[c + d*x]
*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x]^2)/(4*d^2))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

method	result
risch	$\frac{3f\,x^2}{4a}+\frac{3ex}{2a}+\frac{(dxf+de+if)e^{i(dx+c)}}{2d^2a}+\frac{(dxf+de-if)e^{-i(dx+c)}}{2d^2a}+\frac{2ifx}{da}+\frac{2ifc}{d^2a}+\frac{2fx+2e}{da(e^{i(dx+c)}+i)}-\frac{2f\ln(e^{i(dx}}{d^2a}$
parallelrisch	$\frac{16\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)+\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)f\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-32f\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)+\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+((12x^2d^2+24d}{}$
default	$-\frac{-\frac{4e\left(\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{d}-\frac{2f\,x^2+2f\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2f\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2f\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{4fx}{d}-\frac{4fx\,\mathrm{ta}}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
norman	$\frac{\frac{de+2f}{d^2a}-\frac{2e\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{5f\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^2a}+\frac{(-3de+2f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d^2a}+\frac{(-6de+5f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d^2a}+\frac{(-5de+3f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d^2a}+\frac{(-de}{}}{}$

input
```
int((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/4*f/a*x^2+3/2/a*e*x+1/2*(d*x*f+I*f+d*e)/d^2/a*exp(I*(d*x+c))+1/2*(d*x*f-
I*f+d*e)/d^2/a*exp(-I*(d*x+c))+2*I*f/d/a*x+2*I*f/d^2/a*c+2*(f*x+e)/d/a/(ex
p(I*(d*x+c))+I)-2*f/d^2/a*ln(exp(I*(d*x+c))+I)-1/8*f/d^2/a*cos(2*d*x+2*c)-
1/4*(f*x+e)/d/a*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.61

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{6\,d^2fx^2+2\,(2\,dfx+2\,de-f)\cos\left(dx+c\right)^3+2\,(4\,dfx+4\,de+3\,f)\cos\left(dx+c\right)^2+8\,de+4\,(3\,d^2e+2\,d}{}$$

input
```
integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(6*d^2*f*x^2 + 2*(2*d*f*x + 2*d*e - f)*cos(d*x + c)^3 + 2*(4*d*f*x + 4
*d*e + 3*f)*cos(d*x + c)^2 + 8*d*e + 4*(3*d^2*e + 2*d*f)*x + (6*d^2*f*x^2
+ 12*d*e + 12*(d^2*e + d*f)*x + f)*cos(d*x + c) - 8*(f*cos(d*x + c) + f*si
n(d*x + c) + f)*log(sin(d*x + c) + 1) + (6*d^2*f*x^2 - 2*(2*d*f*x + 2*d*e
+ f)*cos(d*x + c)^2 - 8*d*e + 4*(3*d^2*e - 2*d*f)*x + 4*(d*f*x + d*e - 2*f
)*cos(d*x + c) - 7*f)*sin(d*x + c) - 7*f)/(a*d^2*cos(d*x + c) + a*d^2*sin(
d*x + c) + a*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4653 vs. $2(129) = 258$.

Time = 2.24 (sec) , antiderivative size = 4653, normalized size of antiderivative = 30.02

$$\int \frac{(e + fx)\sin^3(c + dx)}{a + a\sin(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((6*d**2*e*x*tan(c/2 + d*x/2)**5/(4*a*d**2*tan(c/2 + d*x/2)**5 +
4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan
(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*e*x*tan(
c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)*
*4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**
2*tan(c/2 + d*x/2) + 4*a*d**2) + 12*d**2*e*x*tan(c/2 + d*x/2)**3/(4*a*d**2
*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d
*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*
d**2) + 12*d**2*e*x*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*
a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c
/2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*e*x*tan(c/
2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 +
8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2*tan
(c/2 + d*x/2) + 4*a*d**2) + 6*d**2*e*x/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a
*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/
2 + d*x/2)**2 + 4*a*d**2*tan(c/2 + d*x/2) + 4*a*d**2) + 3*d**2*f*x**2*tan(
c/2 + d*x/2)**5/(4*a*d**2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)*
*4 + 8*a*d**2*tan(c/2 + d*x/2)**3 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**
2*tan(c/2 + d*x/2) + 4*a*d**2) + 3*d**2*f*x**2*tan(c/2 + d*x/2)**4/(4*a*d*
*2*tan(c/2 + d*x/2)**5 + 4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7564 vs. $2(137) = 274$.

Time = 0.91 (sec) , antiderivative size = 7564, normalized size of antiderivative = 48.80

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(6*d^2*f*x^2*tan(1/2*d*x)^5*tan(1/2*c)^5 - 6*d^2*f*x^2*tan(1/2*d*x)^5*
tan(1/2*c)^4 - 6*d^2*f*x^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 12*d^2*e*x*tan(1/
2*d*x)^5*tan(1/2*c)^5 + 12*d^2*f*x^2*tan(1/2*d*x)^5*tan(1/2*c)^3 - 6*d^2*f
*x^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 12*d^2*e*x*tan(1/2*d*x)^5*tan(1/2*c)^4
+ 12*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^5 - 12*d^2*e*x*tan(1/2*d*x)^4*tan
(1/2*c)^5 + 16*d*f*x*tan(1/2*d*x)^5*tan(1/2*c)^5 - 12*d^2*f*x^2*tan(1/2*d*
x)^5*tan(1/2*c)^2 - 12*d^2*f*x^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 24*d^2*e*x*
tan(1/2*d*x)^5*tan(1/2*c)^3 - 12*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 1
2*d^2*e*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*d*f*x*tan(1/2*d*x)^5*tan(1/2*c)^
4 - 12*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^5 + 24*d^2*e*x*tan(1/2*d*x)^3*t
an(1/2*c)^5 + 8*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^5 + 16*d*e*tan(1/2*d*x)^5*
tan(1/2*c)^5 - 8*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 + 6*d^2*f*x^2*tan(
1/2*d*x)^5*tan(1/2*c) - 12*d^2*f*x^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 24*d^2*
e*x*tan(1/2*d*x)^5*tan(1/2*c)^2 + 24*d^2*f*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3
 - 24*d^2*e*x*tan(1/2*d*x)^4*tan(1/2*c)^3 + 8*d*f*x*tan(1/2*d*x)^5*tan(1/2
*c)^3 - 12*d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - 24*d^2*e*x*tan(1/2*d*x)
^3*tan(1/2*c)^4 - 64*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*d*e*tan(1/2*...
```

Mupad [B] (verification not implemented)

Time = 41.17 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{d\,e+f\,1\mathrm{i}}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right) - \mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-d\,e+f\,1\mathrm{i}}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right) + \mathrm{e}^{-c\,2\mathrm{i}-d\,x\,2\mathrm{i}}\left(\frac{(-2\,d\,e+f\,1\mathrm{i})\,1\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) + \mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\left(\frac{(2\,d\,e+f\,1\mathrm{i})\,1\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) + \frac{3\,f\,x^2}{4\,a} - \frac{2\,f\,\ln\left(\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{a\,d^2} + \frac{2\,(e+f\,x)}{a\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)} + \frac{x\,(3\,d\,e+f\,4\mathrm{i})}{2\,a\,d}$$

input
```
int((sin(c + d*x)^3*(e + f*x))/(a + a*sin(c + d*x)),x)
```

output
```
exp(c*1i + d*x*1i)*((f*1i + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - exp(- c*1i -
 d*x*1i)*((f*1i - d*e)/(2*a*d^2) - (f*x)/(2*a*d)) + exp(- c*2i - d*x*2i)*(
((f*1i - 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) + exp(c*2i + d*x*2i)*((
(f*1i + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) + (3*f*x^2)/(4*a) - (2*f
*log(exp(c*1i)*exp(d*x*1i) + 1i))/(a*d^2) + (2*(e + f*x))/(a*d*(exp(c*1i +
 d*x*1i) + 1i)) + (x*(f*4i + 3*d*e))/(2*a*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.32

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-8f+4\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)f-8\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)f-4\cos(dx+c)}{}$$

input
```
int((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output

```
(4*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*f - 8*cos(c + d*x)*log(tan((c
 + d*x)/2) + 1)*f + 2*cos(c + d*x)*sin(c + d*x)**2*d*e + 2*cos(c + d*x)*si
n(c + d*x)**2*d*f*x + cos(c + d*x)*sin(c + d*x)**2*f - 2*cos(c + d*x)*sin(
c + d*x)*d*e - 2*cos(c + d*x)*sin(c + d*x)*d*f*x - 4*cos(c + d*x)*sin(c +
d*x)*f + 6*cos(c + d*x)*c*d*e + 4*cos(c + d*x)*c*f + 6*cos(c + d*x)*d**2*e
*x + 3*cos(c + d*x)*d**2*f*x**2 - 12*cos(c + d*x)*d*e - 8*cos(c + d*x)*d*f
*x + 8*cos(c + d*x)*f - 4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*f - 4*
log(tan((c + d*x)/2)**2 + 1)*f + 8*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*
f + 8*log(tan((c + d*x)/2) + 1)*f + 2*sin(c + d*x)**3*d*e + 2*sin(c + d*x)
**3*d*f*x - sin(c + d*x)**3*f - 4*sin(c + d*x)**2*d*e - 4*sin(c + d*x)**2*
d*f*x + 3*sin(c + d*x)**2*f - 6*sin(c + d*x)*c*d*e - 4*sin(c + d*x)*c*f -
6*sin(c + d*x)*d**2*e*x - 3*sin(c + d*x)*d**2*f*x**2 - 2*sin(c + d*x)*d*e
- 6*sin(c + d*x)*d*f*x - 4*sin(c + d*x)*f - 6*c*d*e - 4*c*f - 6*d**2*e*x -
 3*d**2*f*x**2 + 12*d*e + 8*d*f*x - 8*f)/(4*a*d**2*(cos(c + d*x) - sin(c +
 d*x) - 1))
```

3.194 $\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3x}{2a}+\frac{2\cos(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\cos(c+dx)\sin^2(c+dx)}{d(a+a\sin(c+dx))}$$

output
```
3/2*x/a+2*cos(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d+cos(d*x+c)*sin(d*x+
c)^2/d/(a+a*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.56

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)(-8+6c+6dx+4\cos(c+dx)-\sin(2(c+dx)))\right.}{4ad(1+\sin(c+dx))}$$

input
```
Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(-8 + 6*c + 6*d*x
 + 4*Cos[c + d*x] - Sin[2*(c + d*x)]) + Cos[(c + d*x)/2]*(6*c + 6*d*x + 4*
Cos[c + d*x] - Sin[2*(c + d*x)])))/(4*a*d*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3246, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^3}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3246}$$

$$\frac{\sin^2(c+dx)\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\int \sin(c+dx)(2a-3a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin^2(c+dx)\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\int \sin(c+dx)(2a-3a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 3213}$$

$$\frac{\sin^2(c+dx)\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{-\frac{2a\cos(c+dx)}{d}+\frac{3a\sin(c+dx)\cos(c+dx)}{2d}-\frac{3ax}{2}}{a^2}$$

input
```
Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x]^2)/(d*(a + a*Sin[c + d*x])) - ((-3*a*x)/2 - (2*
a*Cos[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

method	result
risch	$\frac{3x}{2a}+\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}+\frac{2}{da\left(e^{i(dx+c)}+i\right)}-\frac{\sin(2dx+2c)}{4ad}$
derivativedivides	$\frac{\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+1\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{16}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}}{ad}$
default	$\frac{\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+1\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{16}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}}{ad}$
parallelrisch	$\frac{(12dx+4)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+12dx\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+3\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)+\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)-20\sin\left(\frac{dx}{2}+\frac{c}{2}\right)+3\sin\left(3\right.}{8\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)+\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ad}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}+\frac{3x}{2a}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a}+\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}+\frac{9x\tan\left(\frac{dx}{2}+\right.}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}\left(t\right.}$

input
```
int(sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/2*x/a+1/2/a/d*exp(I*(d*x+c))+1/2/a/d*exp(-I*(d*x+c))+2/d/a/(exp(I*(d*x+c
))+I)-1/4/a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{\cos(dx+c)^3+3\,dx+3\,(dx+1)\cos(dx+c)+2\,\cos(dx+c)^2+\left(3\,dx-\cos(dx+c)^2+\cos(dx+c)-\right.}{2\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(cos(d*x + c)^3 + 3*d*x + 3*(d*x + 1)*cos(d*x + c) + 2*cos(d*x + c)^2
+ (3*d*x - cos(d*x + c)^2 + cos(d*x + c) - 2)*sin(d*x + c) + 2)/(a*d*cos(d
*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(65) = 130$.

Time = 2.01 (sec) , antiderivative size = 1127, normalized size of antiderivative = 15.03

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate(sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output

```
Piecewise((3*d*x*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*ta
n(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2
+ 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c
/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4
*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 6*d*x*tan(c/2
 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*
d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2)
 + 2*a*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*t
an(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2
 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x*tan(c/2 + d*x/2)/(2*a*d*tan(c/2
 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a
*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x/(2*a*d*ta
n(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3
+ 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 6*tan(c/2
+ d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d
*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2)
+ 2*a*d) + 6*tan(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*
a*d*tan(c/2 + d*x/2) + 2*a*d) + 10*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*
x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(71) = 142$.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.83

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+4}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
((sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 + 4)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 3*arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{3(dx+c)}{a}+\frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}+\frac{4}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{2d}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*(d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2
- tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a) + 4/(a*(tan
(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 42.79 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3\,x}{2\,a}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

input
```
int(sin(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

output
```
(3*x)/(2*a) + (tan(c/2 + (d*x)/2) + 5*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (
d*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^4 + 4)/(a*d*(tan(c/2 + (d*x)/2) + 1)*(tan
(c/2 + (d*x)/2)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.53

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\cos(dx+c)\sin(dx+c)^2-\cos(dx+c)\sin(dx+c)+3\cos(dx+c)\,dx-6\cos(dx+c)+\sin(dx+c)^3}{2ad\left(\cos(dx+c)-\sin(dx+c)-1\right)}$$

input
```
int(sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x)*sin(c + d*x) + 3*cos(c + d*x)
*d*x - 6*cos(c + d*x) + sin(c + d*x)**3 - 2*sin(c + d*x)**2 - 3*sin(c + d*
x)*d*x - sin(c + d*x) - 3*d*x + 6)/(2*a*d*(cos(c + d*x) - sin(c + d*x) - 1
))
```

3.195 $\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 11.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \ 5048$$

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sin(dx+c)^3}{(fx+e)(a+a\sin(dx+c))}dx$$

input
```
int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)^3}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*s
in(d*x + c)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sin(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(dx+c)^3}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^3/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 38.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+d\,x)^3}{(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(sin(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(sin(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(dx+c)^3}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(c + d*x)**3/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.196 $\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\text{Int}\left(\frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sin (dx+c)^3}{(fx+e)^2 (a+a\sin (dx+c))} dx$$

input
```
int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} dx = \int \frac{\sin (dx+c)^3}{(fx+e)^2(a\sin (dx+c)+a)} dx$$

input
```
integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2
 + (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin\left(dx+c\right)^3}{\left(fx+e\right)^2\left(a\sin\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^3/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 37.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin\left(c+d\,x\right)^3}{\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sin(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(sin(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sin(dx+c)^3}{\sin(dx+c)e^2+2\sin(dx+c)efx+\sin(dx+c)f^2x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sin(c + d*x)**3/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*
x)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.197 $\int \frac{(e+fx)^3 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 352

$$\begin{aligned}
\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad} \\
&\quad + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} \\
&\quad - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} \\
&\quad + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} \\
&\quad + \frac{12if^2(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3} \\
&\quad - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} \\
&\quad - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} \\
&\quad - \frac{12f^3\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4} \\
&\quad + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} \\
&\quad - \frac{6if^3\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4} + \frac{6if^3\operatorname{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4}
\end{aligned}$$

output
```
I*(f*x+e)^3/a/d-2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)^3*cot(1/2*
c+1/4*Pi+1/2*d*x)/a/d-6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+3*I*f*(f*
x+e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2+12*I*f^2*(f*x+e)*polylog(2,I*exp(I
*(d*x+c)))/a/d^3-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2-6*f^2*(f*
x+e)*polylog(3,-exp(I*(d*x+c)))/a/d^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a
/d^4+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3-6*I*f^3*polylog(4,-exp(
I*(d*x+c)))/a/d^4+6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4
```

Mathematica [A] (warning: unable to verify)

Time = 4.36 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.26

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= \frac{-2(e+fx)^3\text{arctanh}(\cos(c+dx)+i\sin(c+dx)) + \frac{3if\left(d^2(e+fx)^2\,\text{PolyLog}(2,-\cos(c+dx)-i\sin(c+dx))+2idf(e+fx)\,\text{Pol}}{}}{}$$

input
```
Integrate[((e + f*x)^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(-2*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] + ((3*I)*f*(d^2*(e
+ f*x)^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*
PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - 2*f^2*PolyLog[4, -Cos[c + d*x
] - I*Sin[c + d*x]]))/d^3 - ((3*I)*f*(d^2*(e + f*x)^2*PolyLog[2, Cos[c + d
*x] + I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*PolyLog[3, Cos[c + d*x] + I*Si
n[c + d*x]] - 2*f^2*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/d^3 + (6*f
*(Cos[c] + I*Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)
^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*
f*(d*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[
3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(Co
s[c] + I*(1 + Sin[c])) - (2*(e + f*x)^3*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(a*d)
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.654, Rules used = {5046, 3042, 3799, 3042, 4671, 3011, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\csc(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{\int (e+fx)^3\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{\int (e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\int (e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{3f\int (e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int (e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\int (e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)\cdots}{d}\cdots\right.}{d}\cdots}{a}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{\frac{6f\int (e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{6f\int -(e+fx)^2\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{6f\int (e+fx)^2\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 4202}$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right)}{d}}{a}+\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)^2}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right)}{d}}{a}+\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\int(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{\frac{3f\Big(\frac{i(e+fx)^2\operatorname{PolyLog}\big(2,-e^{i(c+dx)}\big)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\big(2,-e^{i(c+dx)}\big)dx}{d}\Big)}{d}-\frac{3f\Big(\frac{i(e+fx)^2\operatorname{PolyLog}\big(2,e^{i(c+dx)}\big)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\big(2,e^{i(c+dx)}\big)}{d}}{d}}{a}$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{2if\Big(\frac{i(e+fx)\operatorname{PolyLog}\big(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\big)}{d}-\frac{if\int\operatorname{PolyLog}\big(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\big)dx}{d}\Big)}{d}-\frac{i(e+fx)}{}}{d}$$

$$2a$$

$\downarrow$ 2720

$$\frac{\frac{3f\Big(\frac{i(e+fx)^2\operatorname{PolyLog}\big(2,-e^{i(c+dx)}\big)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\big(2,-e^{i(c+dx)}\big)dx}{d}\Big)}{d}-\frac{3f\Big(\frac{i(e+fx)^2\operatorname{PolyLog}\big(2,e^{i(c+dx)}\big)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\big(2,e^{i(c+dx)}\big)}{d}}{d}}{a}$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{2if\Big(\frac{i(e+fx)\operatorname{PolyLog}\big(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\big)}{d}-\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\operatorname{PolyLog}\big(2,-e^{\frac{1}{2}i(2c+2dx+3\pi}}{d^2}}{d}}{d}$$

$$2a$$

$\downarrow$ 7143

$$\frac{\frac{3f\Big(\frac{i(e+fx)^2\operatorname{PolyLog}\big(2,-e^{i(c+dx)}\big)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\big(2,-e^{i(c+dx)}\big)dx}{d}\Big)}{d}-\frac{3f\Big(\frac{i(e+fx)^2\operatorname{PolyLog}\big(2,e^{i(c+dx)}\big)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\big(2,e^{i(c+dx)}\big)}{d}}{d}}{a}$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{2if\Big(\frac{i(e+fx)\operatorname{PolyLog}\big(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\big)}{d}-\frac{f\operatorname{PolyLog}\big(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\big)}{d^2}\Big)}{d}-\frac{i(e+fx)^2\log}{}}{d}$$

$$2a$$

$\downarrow$ 7163

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\right.}{a}$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log}{}\right.\right.}{d}}{2a}$$

$\downarrow$ 2720

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyL}}{}\right.}{a}$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log}{}\right.\right.}{d}}{2a}$$

$\downarrow$ 7143

$$\frac{-\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f}{}}{a}$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log}{}\right.\right.}{d}}{2a}$$

input
```
Int[((e + f*x)^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((-2*(e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/d - (6*f*(((I/3)*(e + f*x
)^3)/f - (2*I)*(((-I)*(e + f*x)^2*Log[1 + E^((I/2)*(2*c + 3*Pi + 2*d*x))])
/d + ((2*I)*f*((I*(e + f*x)*PolyLog[2, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d
 - (f*PolyLog[3, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2))/d)))/d)/a + ((-2*
(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/d + (3*f*((I*(e + f*x)^2*PolyLog[2,
-E^(I*(c + d*x))])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, -E^(I*(c + d*x
))])/d + (f*PolyLog[4, -E^(I*(c + d*x))])/d^2))/d))/d - (3*f*((I*(e + f*x)
^2*PolyLog[2, E^(I*(c + d*x))])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, E
^(I*(c + d*x))])/d + (f*PolyLog[4, E^(I*(c + d*x))])/d^2))/d))/d)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(317) = 634$.

Time = 1.50 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.27

method	result	size
risch	Expression too large to display	1151

input
```
int((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4+2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e
^3)/d/a/(exp(I*(d*x+c))+I)-6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4+1/d/a*
e^3*ln(exp(I*(d*x+c))-1)-1/d/a*e^3*ln(exp(I*(d*x+c))+1)+6/a/d^4*f^3*c^2*ln
(exp(I*(d*x+c)))+6/a/d^2*f*e^2*ln(exp(I*(d*x+c)))-6/a/d^2*f^3*ln(1-I*exp(I
*(d*x+c)))*x^2+6/a/d^4*f^3*c^2*ln(1-I*exp(I*(d*x+c)))+2*I/a/d*f^3*x^3-4*I/
a/d^4*f^3*c^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+12*I/a/d^2*f^2*e*c*
x-12/a/d^3*f^2*e*c*ln(exp(I*(d*x+c)))-12/a/d^2*f^2*e*ln(1-I*exp(I*(d*x+c))
)*x+12*I/a/d^3*f^2*e*polylog(2,I*exp(I*(d*x+c)))-6*I/a/d^3*f^3*c^2*x+12*I/
a/d^3*f^3*polylog(2,I*exp(I*(d*x+c)))*x+6*I/a/d*f^2*e*x^2+6*I/a/d^3*f^2*e*
c^2-6*I/d^2/a*e*f^2*polylog(2,exp(I*(d*x+c)))*x+6*I/d^2/a*e*f^2*polylog(2,
-exp(I*(d*x+c)))*x-12/a/d^3*f^2*e*ln(1-I*exp(I*(d*x+c)))*c+6/d^3/a*e*f^2*p
olylog(3,exp(I*(d*x+c)))-6/d^3/a*e*f^2*polylog(3,-exp(I*(d*x+c)))-1/d^4/a*
c^3*f^3*ln(exp(I*(d*x+c))-1)+1/d^4/a*c^3*f^3*ln(1-exp(I*(d*x+c)))-6/d^4/a*
c^2*f^3*ln(exp(I*(d*x+c))+I)+1/d/a*f^3*ln(1-exp(I*(d*x+c)))*x^3+6/d^3/a*f^
3*polylog(3,exp(I*(d*x+c)))*x-1/d/a*f^3*ln(exp(I*(d*x+c))+1)*x^3-6/d^3/a*f
^3*polylog(3,-exp(I*(d*x+c)))*x-6/d^2/a*e^2*f*ln(exp(I*(d*x+c))+I)+3/d/a*e
^2*f*ln(1-exp(I*(d*x+c)))*x-3/d/a*e^2*f*ln(exp(I*(d*x+c))+1)*x+3/d^3/a*c^2
*e*f^2*ln(exp(I*(d*x+c))-1)+3/d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2-3/d/a*e*f
^2*ln(exp(I*(d*x+c))+1)*x^2-3/d^3/a*e*f^2*ln(1-exp(I*(d*x+c)))*c^2+3/d^2/a
*e^2*f*ln(1-exp(I*(d*x+c)))*c-3/d^2/a*c*e^2*f*ln(exp(I*(d*x+c))-1)+12/d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2924 vs. $2(305) = 610$.

Time = 0.19 (sec) , antiderivative size = 2924, normalized size of antiderivative = 8.31

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 + 2*(d^3*
f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c) - 3*(I*d
^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2*f + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^
2*x + I*d^2*e^2*f)*cos(d*x + c) + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2
*e^2*f)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*d^2*f^3
*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f + (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x -
 I*d^2*e^2*f)*cos(d*x + c) + (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2
*f)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) - 12*(-I*d*f^3*x -
I*d*e*f^2 + (-I*d*f^3*x - I*d*e*f^2)*cos(d*x + c) + (-I*d*f^3*x - I*d*e*f^
2)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 12*(I*d*f^3*x + I*
d*e*f^2 + (I*d*f^3*x + I*d*e*f^2)*cos(d*x + c) + (I*d*f^3*x + I*d*e*f^2)*s
in(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 3*(I*d^2*f^3*x^2 + 2*
I*d^2*e*f^2*x + I*d^2*e^2*f + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2
*f)*cos(d*x + c) + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2*f)*sin(d*x
 + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2
*e*f^2*x - I*d^2*e^2*f + (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f)*
cos(d*x + c) + (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f)*sin(d*x +
c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2
 + 3*d^3*e^2*f*x + d^3*e^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*
x + d^3*e^3)*cos(d*x + c) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*...
```

Sympy [F]

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*cs
c(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d*x)/(s
in(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)/(sin(c + d*x) + 1)
, x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2796 vs. $2(305) = 610$.

Time = 0.51 (sec) , antiderivative size = 2796, normalized size of antiderivative = 7.94

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(3*c*e^2*f*(2/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x +
 c)/(cos(d*x + c) + 1))/(a*d)) - e^3*(log(sin(d*x + c)/(cos(d*x + c) + 1))
/a + 2/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) + (12*I*c^2*d*e*f^2 - 4*I*
c^3*f^3 - 12*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3 - (d^2*e^2*f - 2*c*
d*e*f^2 + c^2*f^3)*cos(d*x + c) + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^
3)*sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 12*(I*(d*x + c)
^2*f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c) + ((d*x + c)^2*f^3 + 2*(d*e*f^2
 - c*f^3)*(d*x + c))*cos(d*x + c) + (I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 - I*
c*f^3)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) -
2*(-3*I*c^2*d*e*f^2 - I*(d*x + c)^3*f^3 + I*c^3*f^3 + 3*(-I*d*e*f^2 + I*c*
f^3)*(d*x + c)^2 + 3*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*(d*x + c)
- (3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x +
c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*cos(d*x + c) + (-3
*I*c^2*d*e*f^2 - I*(d*x + c)^3*f^3 + I*c^3*f^3 + 3*(-I*d*e*f^2 + I*c*f^3)*
(d*x + c)^2 + 3*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*(d*x + c))*sin(
d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) - 2*(3*I*c^2*d*e*f^2 - I
*c^3*f^3 + (3*c^2*d*e*f^2 - c^3*f^3)*cos(d*x + c) + (3*I*c^2*d*e*f^2 - I*c
^3*f^3)*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) - 1) - 2*(-I*(d*x
 + c)^3*f^3 + 3*(-I*d*e*f^2 + I*c*f^3)*(d*x + c)^2 + 3*(-I*d^2*e^2*f + 2*I
*c*d*e*f^2 - I*c^2*f^3)*(d*x + c) - ((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f...
```

Giac [F]

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^3\csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\left(\int \frac{\csc(dx+c)x^3}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)d\,f^3+\left(\int \frac{\csc(dx+c)x^3}{\sin(dx+c)+1}dx\right)d\,f^3+3\left(\int \frac{\csc(dx+c)x^2}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)de\,f^2+3}{}$$

input
```
int((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(int((csc(c + d*x)*x**3)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)*d*f**3 + i
nt((csc(c + d*x)*x**3)/(sin(c + d*x) + 1),x)*d*f**3 + 3*int((csc(c + d*x)*
x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)*d*e*f**2 + 3*int((csc(c + d*x
)*x**2)/(sin(c + d*x) + 1),x)*d*e*f**2 + 3*int((csc(c + d*x)*x)/(sin(c + d
*x) + 1),x)*tan((c + d*x)/2)*d*e**2*f + 3*int((csc(c + d*x)*x)/(sin(c + d*
x) + 1),x)*d*e**2*f + log(tan((c + d*x)/2))*tan((c + d*x)/2)*e**3 + log(ta
n((c + d*x)/2))*e**3 - 2*tan((c + d*x)/2)*e**3)/(a*d*(tan((c + d*x)/2) + 1
))
```

3.198 $\int \frac{(e+fx)^2 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{2if(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} + \frac{4if^2\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3} - \frac{2if(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} - \frac{2f^2\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} + \frac{2f^2\,\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}$$

output
```
I*(f*x+e)^2/a/d-2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)^2*cot(1/2*
c+1/4*Pi+1/2*d*x)/a/d-4*f*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d^2+2*I*f*(f*x+
e)*polylog(2,-exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/
d^3-2*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-2*f^2*polylog(3,-exp(I*(
d*x+c)))/a/d^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.33

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= \frac{(e+fx)^2 \log\left(1-e^{i(c+dx)}\right)-(e+fx)^2 \log\left(1+e^{i(c+dx)}\right)+\frac{2if\left(d(e+fx)\,\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)+if\,\mathrm{PolyLog}\left(3,-e^{i(c+dx}\right.\right.}{d^2}}{}$$

input
```
Integrate[((e + f*x)^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((e + f*x)^2*Log[1 - E^(I*(c + d*x))] - (e + f*x)^2*Log[1 + E^(I*(c + d*x)
)] + ((2*I)*f*(d*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))] + I*f*PolyLog[3, -
E^(I*(c + d*x))]))/d^2 + (2*f*((-I)*d*(e + f*x)*PolyLog[2, E^(I*(c + d*x))
] + f*PolyLog[3, E^(I*(c + d*x))]))/d^2 + (4*f*(Cos[c] + I*Sin[c])*(((e +
f*x)^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Si
n[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] -
 Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(Cos[c] + I*(1 + Sin[c]))
- (2*(e + f*x)^2*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])))/(a*d)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5046, 3042, 3799, 3042, 4671, 3011, 2720, 4672, 3042, 25, 4202, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^2 \csc(c+dx)dx}{a}-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{\int (e+fx)^2\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{2f\int (e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)}{}}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,\right.}{d^2}\right.}{d}}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{4f\int (e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,\right.}{d^2}\right.}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\frac{4f\int -\left((e+fx)\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right.}{d}}{a}$$

$\downarrow$ 25

$$-\frac{-\frac{4f\int (e+fx)\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right.}{d}}{a}$$

$\downarrow$ 4202

$$\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right.}{d}}{a}$$
$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}$$

$\downarrow$ 2620

$$\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right.}{d}}{a}$$
$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$\downarrow$ 2715

$$\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right.}{d}}{a}$$
$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)de^{\frac{1}{2}i(2c+2dx+3\pi)}}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+\right.}}{d}\right.\right.}{d}}{2a}$$

$\downarrow$ 2838

$$\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,e\right.}{d^2}\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$\downarrow$ 7143

$$\frac{-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\,\text{PolyLo}}{}\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

input

```
Int[((e + f*x)^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output

```
-1/2*((-2*(e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/d - (4*f*(((I/2)*(e + f*x
)^2)/f - (2*I)*(((-I)*(e + f*x)*Log[1 + E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d
 - (f*PolyLog[2, -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d^2)))/d)/a + ((-2*(e +
 f*x)^2*ArcTanh[E^(I*(c + d*x))])/d + (2*f*((I*(e + f*x)*PolyLog[2, -E^(I*
(c + d*x))])/d - (f*PolyLog[3, -E^(I*(c + d*x))])/d^2))/d - (2*f*((I*(e +
f*x)*PolyLog[2, E^(I*(c + d*x))])/d - (f*PolyLog[3, E^(I*(c + d*x))])/d^2)
)/d)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(223) = 446.

Time = 1.31 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{f^2 \ln(1-e^{i(dx+c)})c^2}{d^3 a} - \frac{f^2 \ln(e^{i(dx+c)}+1)x^2}{da} + \frac{f^2 \ln(1-e^{i(dx+c)})x^2}{da} + \frac{c^2 f^2 \ln(e^{i(dx+c)}-1)}{d^3 a} + \frac{2if^2c^2}{d^3 a} - \frac{4\ln(e^{i(dx+c)}+}{d^2 a}$

input
```
int((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)+1/d/a*e^2*ln(exp(I*(d*x+c))
-1)-1/d/a*e^2*ln(exp(I*(d*x+c))+1)+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d
^3-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c-4/a/d^3*f^2*c*ln(exp(I*(d*x+c)))+4
/a/d^2*f*e*ln(exp(I*(d*x+c)))-4/a/d^2*f^2*ln(1-I*exp(I*(d*x+c)))*x+2/d/a*e
*f*ln(1-exp(I*(d*x+c)))*x-2/d/a*e*f*ln(exp(I*(d*x+c))+1)*x+2/d^2/a*e*f*ln(
1-exp(I*(d*x+c)))*c-2/d^2/a*c*e*f*ln(exp(I*(d*x+c))-1)+4*I/d^2/a*f^2*c*x+2
*I/d^2/a*e*f*polylog(2,-exp(I*(d*x+c)))-2*I/d^2/a*e*f*polylog(2,exp(I*(d*x
+c)))-2*I/d^2/a*f^2*polylog(2,exp(I*(d*x+c)))*x+2*I/d^2/a*f^2*polylog(2,-e
xp(I*(d*x+c)))*x-2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+2*f^2*polylog(3,ex
p(I*(d*x+c)))/a/d^3-1/d^3/a*f^2*ln(1-exp(I*(d*x+c)))*c^2-1/d/a*f^2*ln(exp(
I*(d*x+c))+1)*x^2+1/d/a*f^2*ln(1-exp(I*(d*x+c)))*x^2+1/d^3/a*c^2*f^2*ln(ex
p(I*(d*x+c))-1)+2*I/d^3/a*f^2*c^2+2*I/d/a*f^2*x^2-4/d^2/a*ln(exp(I*(d*x+c)
)+I)*e*f+4/d^3/a*c*f^2*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1642 vs. 2(214) = 428.

Time = 0.15 (sec) , antiderivative size = 1642, normalized size of antiderivative = 6.59

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 + 2*(d^2*f^2*x^2 + 2*d^2*e*f*
x + d^2*e^2)*cos(d*x + c) - 2*(I*d*f^2*x + I*d*e*f + (I*d*f^2*x + I*d*e*f)
*cos(d*x + c) + (I*d*f^2*x + I*d*e*f)*sin(d*x + c))*dilog(cos(d*x + c) + I
*sin(d*x + c)) - 2*(-I*d*f^2*x - I*d*e*f + (-I*d*f^2*x - I*d*e*f)*cos(d*x
+ c) + (-I*d*f^2*x - I*d*e*f)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x
 + c)) - 4*(-I*f^2*cos(d*x + c) - I*f^2*sin(d*x + c) - I*f^2)*dilog(I*cos(
d*x + c) - sin(d*x + c)) - 4*(I*f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*
f^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 2*(I*d*f^2*x + I*d*e*f + (I*d
*f^2*x + I*d*e*f)*cos(d*x + c) + (I*d*f^2*x + I*d*e*f)*sin(d*x + c))*dilog
(-cos(d*x + c) + I*sin(d*x + c)) - 2*(-I*d*f^2*x - I*d*e*f + (-I*d*f^2*x -
 I*d*e*f)*cos(d*x + c) + (-I*d*f^2*x - I*d*e*f)*sin(d*x + c))*dilog(-cos(d
*x + c) - I*sin(d*x + c)) - (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 + (d^2*f^
2*x^2 + 2*d^2*e*f*x + d^2*e^2)*cos(d*x + c) + (d^2*f^2*x^2 + 2*d^2*e*f*x +
 d^2*e^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 4*(d*e*f
- c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x + c))*log
(cos(d*x + c) + I*sin(d*x + c) + I) - (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2
 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*cos(d*x + c) + (d^2*f^2*x^2 + 2*d
^2*e*f*x + d^2*e^2)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + 1) -
 4*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*s
in(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 4*(d*f^2*x + c*f^...
```

Sympy [F]

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\int \frac{e^2\csc(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{f^2x^2\csc(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{2efx\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*cs
c(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)/(sin(c +
 d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(214) = 428$.

Time = 0.21 (sec) , antiderivative size = 1418, normalized size of antiderivative = 5.69

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*c*e*f*(2/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c
)/(cos(d*x + c) + 1))/(a*d)) - e^2*(log(sin(d*x + c)/(cos(d*x + c) + 1))/a
 + 2/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) + (4*I*c^2*f^2 - 8*(-I*d*e*f
 + I*c*f^2 - (d*e*f - c*f^2)*cos(d*x + c) + (-I*d*e*f + I*c*f^2)*sin(d*x +
 c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 8*((d*x + c)*f^2*cos(d*x +
c) + I*(d*x + c)*f^2*sin(d*x + c) + I*(d*x + c)*f^2)*arctan2(cos(d*x + c),
 sin(d*x + c) + 1) - 2*(-I*(d*x + c)^2*f^2 - I*c^2*f^2 + 2*(-I*d*e*f + I*c
*f^2)*(d*x + c) - ((d*x + c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c)
)*cos(d*x + c) + (-I*(d*x + c)^2*f^2 - I*c^2*f^2 + 2*(-I*d*e*f + I*c*f^2)*
(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) - 2*(c^2*
f^2*cos(d*x + c) + I*c^2*f^2*sin(d*x + c) + I*c^2*f^2)*arctan2(sin(d*x + c
), cos(d*x + c) - 1) - 2*(-I*(d*x + c)^2*f^2 + 2*(-I*d*e*f + I*c*f^2)*(d*x
 + c) - ((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(d*x + c) + (-I
*(d*x + c)^2*f^2 + 2*(-I*d*e*f + I*c*f^2)*(d*x + c))*sin(d*x + c))*arctan2
(sin(d*x + c), -cos(d*x + c) + 1) - 4*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)
*(d*x + c))*cos(d*x + c) - 8*(f^2*cos(d*x + c) + I*f^2*sin(d*x + c) + I*f^
2)*dilog(I*e^(I*d*x + I*c)) - 4*(I*d*e*f + I*(d*x + c)*f^2 - I*c*f^2 + (d*
e*f + (d*x + c)*f^2 - c*f^2)*cos(d*x + c) + (I*d*e*f + I*(d*x + c)*f^2 - I
*c*f^2)*sin(d*x + c))*dilog(-e^(I*d*x + I*c)) - 4*(-I*d*e*f - I*(d*x + c)*
f^2 + I*c*f^2 - (d*e*f + (d*x + c)*f^2 - c*f^2)*cos(d*x + c) + (-I*d*e*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\csc(dx+c)x^2}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)d f^2+\left(\int \frac{\csc(dx+c)x^2}{\sin(dx+c)+1}dx\right)d f^2+2\left(\int \frac{\csc(dx+c)x}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)def+2(}{ad\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

input
```
int((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(int((csc(c + d*x)*x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)*d*f**2 + i
nt((csc(c + d*x)*x**2)/(sin(c + d*x) + 1),x)*d*f**2 + 2*int((csc(c + d*x)*
x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)*d*e*f + 2*int((csc(c + d*x)*x)/(
sin(c + d*x) + 1),x)*d*e*f + log(tan((c + d*x)/2))*tan((c + d*x)/2)*e**2 +
 log(tan((c + d*x)/2))*e**2 - 2*tan((c + d*x)/2)*e**2)/(a*d*(tan((c + d*x)
/2) + 1))
```

3.199 $\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} - \frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}$$

output
```
-2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a
/d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2+I*f*polylog(2,-exp(I*(d*x+c)))/
a/d^2-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 300 vs. 2(134) = 268.

Time = 7.68 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.24

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-2d(e+fx)\sin\left(\frac{1}{2}(c+dx)\right)+f(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-2*d*(e + f*x)*Sin[(c + d*x)/2] +
f*(c + d*x)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*f*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + d*e*Log[Tan
[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - c*f*Log[Tan[(c + d*
x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + f*((c + d*x)*(Log[1 - E^(I*
(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))]
- PolyLog[2, E^(I*(c + d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(a
*d^2*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5046, 3042, 3799, 3042, 4671, 2715, 2838, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\csc(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\int \frac{e+fx}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\int \frac{e+fx}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{\int (e+fx)\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4671}$$

$$\frac{-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}}{}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{2f\int \cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2f\int -\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{2f\int \tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\frac{4f\log\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)\right)}{d^2}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}$$

input
```
Int[((e + f*x)*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((-2*(e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/d + (4*f*Log[-Cos[c/2 - Pi/
4 + (d*x)/2]])/d^2)/a + ((-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + (I*f*
PolyLog[2, -E^(I*(c + d*x))])/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])/d^2)
/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(114) = 228$.

Time = 1.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.83

method	result
risch	$\frac{2fx+2e}{da\left(e^{i(dx+c)}+i\right)}+\frac{f\ln\left(1-e^{i(dx+c)}\right)x}{da}-\frac{f\ln\left(e^{i(dx+c)}+1\right)x}{da}+\frac{if\,\mathrm{polylog}\left(2,-e^{i(dx+c)}\right)}{a\,d^2}-\frac{if\,\mathrm{polylog}\left(2,e^{i(dx+c)}\right)}{a\,d^2}-\frac{2f\ln(e^{i}$

input
```
int((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f*x+e)/d/a/(exp(I*(d*x+c))+I)+1/d/a*f*ln(1-exp(I*(d*x+c)))*x-1/d/a*f*ln
(exp(I*(d*x+c))+1)*x+I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2-I*f*polylog(2,ex
p(I*(d*x+c)))/a/d^2-2*f/d^2/a*ln(exp(I*(d*x+c))+I)+2/d^2/a*f*ln(exp(I*(d*x
+c)))+1/d^2/a*f*ln(1-exp(I*(d*x+c)))*c+1/d/a*e*ln(exp(I*(d*x+c))-1)-1/d/a*
e*ln(exp(I*(d*x+c))+1)-1/d^2/a*c*f*ln(exp(I*(d*x+c))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 609, normalized size of antiderivative = 4.54

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d*f*x + 2*d*e + 2*(d*f*x + d*e)*cos(d*x + c) + (-I*f*cos(d*x + c) -
 I*f*sin(d*x + c) - I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d
*x + c) + I*f*sin(d*x + c) + I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) + (
-I*f*cos(d*x + c) - I*f*sin(d*x + c) - I*f)*dilog(-cos(d*x + c) + I*sin(d*
x + c)) + (I*f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(-cos(d*x + c)
- I*sin(d*x + c)) - (d*f*x + d*e + (d*f*x + d*e)*cos(d*x + c) + (d*f*x + d
*e)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - (d*f*x + d*e +
(d*f*x + d*e)*cos(d*x + c) + (d*f*x + d*e)*sin(d*x + c))*log(cos(d*x + c)
- I*sin(d*x + c) + 1) + (d*e - c*f + (d*e - c*f)*cos(d*x + c) + (d*e - c*f
)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (d*e -
 c*f + (d*e - c*f)*cos(d*x + c) + (d*e - c*f)*sin(d*x + c))*log(-1/2*cos(d
*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x
 + c) + (d*f*x + c*f)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1
) + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c) + (d*f*x + c*f)*sin(d*x + c)
)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(f*cos(d*x + c) + f*sin(d*x
+ c) + f)*log(sin(d*x + c) + 1) - 2*(d*f*x + d*e)*sin(d*x + c))/(a*d^2*cos
(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e\csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d*x
)/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(110) = 220$.

Time = 0.14 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.84

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(4*d*f*x*cos(d*x + c) + 4*I*d*f*x*sin(d*x + c) - 4*I*d*e - 4*(f*cos(d*x +
c) + I*f*sin(d*x + c) + I*f)*arctan2(cos(c) + sin(d*x), cos(d*x) + sin(c))
 + 2*(-I*d*f*x - I*d*e - (d*f*x + d*e)*cos(d*x + c) + (-I*d*f*x - I*d*e)*s
in(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) + 2*(d*e*cos(d*x + c)
 + I*d*e*sin(d*x + c) + I*d*e)*arctan2(sin(d*x + c), cos(d*x + c) - 1) - 2
*(d*f*x*cos(d*x + c) + I*d*f*x*sin(d*x + c) + I*d*f*x)*arctan2(sin(d*x + c
), -cos(d*x + c) + 1) + 2*(f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(
-e^(I*d*x + I*c)) - 2*(f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(e^(I
*d*x + I*c)) - (d*f*x + d*e + (-I*d*f*x - I*d*e)*cos(d*x + c) + (d*f*x + d
*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1
) + (d*f*x + d*e - (I*d*f*x + I*d*e)*cos(d*x + c) + (d*f*x + d*e)*sin(d*x
+ c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) + 2*(I*f*c
os(d*x + c) - f*sin(d*x + c) - f)*log(cos(d*x)^2 + cos(c)^2 + 2*cos(c)*sin
(d*x) + sin(d*x)^2 + 2*cos(d*x)*sin(c) + sin(c)^2))/(-2*I*a*d^2*cos(d*x +
c) + 2*a*d^2*sin(d*x + c) + 2*a*d^2)
```

Giac [F]

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)\csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Hanged}$$

input
```
int((e + f*x)/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\csc(dx+c)x}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)df+\left(\int \frac{\csc(dx+c)x}{\sin(dx+c)+1}dx\right)df+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e+\log\left(\tan\left(\right.\right.}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(int((csc(c + d*x)*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)*d*f + int((cs
c(c + d*x)*x)/(sin(c + d*x) + 1),x)*d*f + log(tan((c + d*x)/2))*tan((c + d
*x)/2)*e + log(tan((c + d*x)/2))*e - 2*tan((c + d*x)/2)*e)/(a*d*(tan((c +
d*x)/2) + 1))
```

3.200 $\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{ad} + \frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

output
```
-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/d/(a+a*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec(c+dx)\left(-1+\operatorname{arctanh}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}+\sin(c+dx)\right)}{ad}$$

input
```
Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
-((Sec[c + d*x]*(-1 + ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2] +
 Sin[c + d*x]))/(a*d))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3226, 3042, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3226}$$

$$\frac{\int \csc(c+dx)dx}{a}-\int \frac{1}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)dx}{a}-\int \frac{1}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\int \csc(c+dx)dx}{a}+\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\text{arctanh}(\cos(c+dx))}{ad}$$

input
```
Int[Csc[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
-(ArcTanh[Cos[c + d*x]]/(a*d)) + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{da}$	34
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{da}$	34
norman	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	49
parallelrisch	$\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)d}$	55
risch	$\frac{2}{da\left(e^{i(dx+c)}+i\right)}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}$	63

input
```
int(csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(ln(tan(1/2*d*x+1/2*c))+2/(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(38) = 76$.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\right.}{2\left(ad\cos(dx+c)+ad\sin(dx+c)+ad\right)}$$

input
```
integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*((cos(d*x + c) + sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos
(d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*cos(d*x + c
) + 2*sin(d*x + c) - 2)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{2}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}}{d}$$

input
```
integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2/(a + a*sin(d*x + c)/(cos(d*x +
 c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

input
```
integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
(log(abs(tan(1/2*d*x + 1/2*c)))/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 35.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} + \frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

input
```
int(1/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a*d) + 2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int(csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(log(tan((c + d*x)/2))*tan((c + d*x)/2) + log(tan((c + d*x)/2)) - 2*tan((c
 + d*x)/2))/(a*d*(tan((c + d*x)/2) + 1))
```

3.201 $\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 15.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\csc(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\csc(dx+c)}{(fx+e)(a+a\sin(dx+c))}dx$$

input
```
int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

input
```
integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 559, normalized size of antiderivative = 21.50

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc{(dx+c)}}{(fx+e)(a\sin{(dx+c)}+a)}\,dx$$

input
```
integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(2*(a*d*f^2*x + a*d*e*f + (a*d*f^2*x + a*d*e*f)*cos(d*x + c)^2 + (a*d*f^2*
x + a*d*e*f)*sin(d*x + c)^2 + 2*(a*d*f^2*x + a*d*e*f)*sin(d*x + c))*integr
ate(cos(d*x + c)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2 + 2*a
*d*e*f*x + a*d*e^2)*cos(d*x + c)^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)
*sin(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)), x
) + (a*d*f*x + a*d*e + (a*d*f*x + a*d*e)*cos(d*x + c)^2 + (a*d*f*x + a*d*e
)*sin(d*x + c)^2 + 2*(a*d*f*x + a*d*e)*sin(d*x + c))*integrate(sin(d*x + c
)/(a*f*x + (a*f*x + a*e)*cos(d*x + c)^2 + (a*f*x + a*e)*sin(d*x + c)^2 + a
*e + 2*(a*f*x + a*e)*cos(d*x + c)), x) + (a*d*f*x + a*d*e + (a*d*f*x + a*d
*e)*cos(d*x + c)^2 + (a*d*f*x + a*d*e)*sin(d*x + c)^2 + 2*(a*d*f*x + a*d*e
)*sin(d*x + c))*integrate(sin(d*x + c)/(a*f*x + (a*f*x + a*e)*cos(d*x + c)
^2 + (a*f*x + a*e)*sin(d*x + c)^2 + a*e - 2*(a*f*x + a*e)*cos(d*x + c)), x
) + 2*cos(d*x + c))/(a*d*f*x + a*d*e + (a*d*f*x + a*d*e)*cos(d*x + c)^2 +
(a*d*f*x + a*d*e)*sin(d*x + c)^2 + 2*(a*d*f*x + a*d*e)*sin(d*x + c))
```

Giac [N/A]

Not integrable

Time = 15.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc{(dx+c)}}{(fx+e)(a\sin{(dx+c)}+a)}\,dx$$

input
```
integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 36.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{1}{\sin(c+d\,x)\,(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(sin(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(sin(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc(dx+c)}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(c + d*x)/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.202 $\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\text{Int}\left(\frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 17.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\csc (dx+c)}{(fx+e)^2 (a+a\sin (dx+c))}dx$$

input
```
int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc (dx+c)}{(fx+e)^2(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*
f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc (c+dx)}{e^2\sin (c+dx)+e^2+2efx\sin (c+dx)+2efx+f^2x^2\sin (c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(csc(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2
*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 915, normalized size of antiderivative = 35.19

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(4*(a*d*f^3*x^2 + 2*a*d*e*f^2*x + a*d*e^2*f + (a*d*f^3*x^2 + 2*a*d*e*f^2*x
 + a*d*e^2*f)*cos(d*x + c)^2 + (a*d*f^3*x^2 + 2*a*d*e*f^2*x + a*d*e^2*f)*s
in(d*x + c)^2 + 2*(a*d*f^3*x^2 + 2*a*d*e*f^2*x + a*d*e^2*f)*sin(d*x + c))*
integrate(cos(d*x + c)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*
d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*cos(d*x
+ c)^2 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*sin(d*x
 + c)^2 + 2*(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*sin(
d*x + c)), x) + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2 + 2*a*
d*e*f*x + a*d*e^2)*cos(d*x + c)^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*
sin(d*x + c)^2 + 2*(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c))*int
egrate(sin(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f*
x + a*e^2)*cos(d*x + c)^2 + (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)^2
 + 2*(a*f^2*x^2 + 2*a*e*f*x + a*e^2)*cos(d*x + c)), x) + (a*d*f^2*x^2 + 2*
a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*cos(d*x + c)^2
 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)^2 + 2*(a*d*f^2*x^2 +
 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c))*integrate(sin(d*x + c)/(a*f^2*x^2 +
2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*cos(d*x + c)^2 + (a*f^
2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)^2 - 2*(a*f^2*x^2 + 2*a*e*f*x + a*e
^2)*cos(d*x + c)), x) + 2*cos(d*x + c))/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e
^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*cos(d*x + c)^2 + (a*d*f^2*x^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 36.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{1}{\sin\left(c+d\,x\right)\,\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(sin(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(sin(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc(dx+c)}{\sin(dx+c)e^2+2\sin(dx+c)efx+\sin(dx+c)f^2x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(c + d*x)/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*x)*
f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.203 $\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 463

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2i(e+fx)^3}{ad}+\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{(e+fx)^3\cot(c+dx)}{ad} + \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{3f(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{ad^2} - \frac{3if(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} - \frac{12if^2(e+fx)\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3} + \frac{3if(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} - \frac{3if^2(e+fx)\,\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3} + \frac{6f^2(e+fx)\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} + \frac{12f^3\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4} - \frac{6f^2(e+fx)\,\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} + \frac{3f^3\,\text{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2ad^4} + \frac{6if^3\,\text{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4} - \frac{6if^3\,\text{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4}$$

output
```
3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2+2*(f*x+e)^3*arctanh(exp(I*
(d*x+c)))/a/d-(f*x+e)^3*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d-(f*x+e)^3*cot(d*x+c)
/a/d+6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d^2+3*f*(f*x+e)^2*ln(1-exp(2*I
*(d*x+c)))/a/d^2-12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^3-6*I*f^
3*polylog(4,exp(I*(d*x+c)))/a/d^4-3*I*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)
))/a/d^2+6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4+6*f^2*(f*x+e)*polylog(3,
-exp(I*(d*x+c)))/a/d^3+12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4-6*f^2*(f*x
+e)*polylog(3,exp(I*(d*x+c)))/a/d^3+3/2*f^3*polylog(3,exp(2*I*(d*x+c)))/a/
d^4-2*I*(f*x+e)^3/a/d-3*I*f^2*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1052 vs. $2(463) = 926$.

Time = 10.62 (sec) , antiderivative size = 1052, normalized size of antiderivative = 2.27

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output

```
(I*d^3*e^2*(d*e - 3*f)*x - I*d^3*e^2*(d*e + 3*f)*x - ((2*I)*d^3*(e + f*x)^
3)/(-1 + E^((2*I)*c)) - 3*d^2*e*(d*e - 2*f)*f*x*Log[1 - E^((-I)*(c + d*x))
] - 3*d^2*(d*e - f)*f^2*x^2*Log[1 - E^((-I)*(c + d*x))] - d^3*f^3*x^3*Log[
1 - E^((-I)*(c + d*x))] + 3*d^2*e*f*(d*e + 2*f)*x*Log[1 + E^((-I)*(c + d*x
))] + 3*d^2*f^2*(d*e + f)*x^2*Log[1 + E^((-I)*(c + d*x))] + d^3*f^3*x^3*Lo
g[1 + E^((-I)*(c + d*x))] - d^2*e^2*(d*e - 3*f)*Log[1 - E^(I*(c + d*x))] +
 d^2*e^2*(d*e + 3*f)*Log[1 + E^(I*(c + d*x))] + (3*I)*d*e*f*(d*e + 2*f)*Po
lyLog[2, -E^((-I)*(c + d*x))] + (6*I)*d*f^2*(d*e + f)*x*PolyLog[2, -E^((-I
)*(c + d*x))] + (3*I)*d^2*f^3*x^2*PolyLog[2, -E^((-I)*(c + d*x))] - (3*I)*
d*e*(d*e - 2*f)*f*PolyLog[2, E^((-I)*(c + d*x))] - (6*I)*d*(d*e - f)*f^2*x
*PolyLog[2, E^((-I)*(c + d*x))] - (3*I)*d^2*f^3*x^2*PolyLog[2, E^((-I)*(c
+ d*x))] + 6*f^2*(d*e + f)*PolyLog[3, -E^((-I)*(c + d*x))] + 6*d*f^3*x*Pol
yLog[3, -E^((-I)*(c + d*x))] - 6*(d*e - f)*f^2*PolyLog[3, E^((-I)*(c + d*x
))] - 6*d*f^3*x*PolyLog[3, E^((-I)*(c + d*x))] - (6*I)*f^3*PolyLog[4, -E^(
(-I)*(c + d*x))] + (6*I)*f^3*PolyLog[4, E^((-I)*(c + d*x))])/(a*d^4) - (6*
f*(Cos[c] + I*Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x
)^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2
*f*(d*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog
[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(a
*d*(Cos[c] + I*(1 + Sin[c]))) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^3*Sin[(...
```

Rubi [A] (verified)

Time = 3.82 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.16, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5046, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 5046, 3042, 3799, 3042, 4671, 3011, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a \sin(c+dx)+a} \, dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^3 \csc^2(c+dx)dx}{a} - \int \frac{(e+fx)^3 \csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc(c+dx)^2dx}{a}-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{3f\int (e+fx)^2\cot(c+dx)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f\int -(e+fx)^2\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{3f\int (e+fx)^2\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{4202}$$

$$-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)^2}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\int (e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int \operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\int \frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx+$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 5046

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)^3\csc(c+dx)dx}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\int \frac{(e+fx)^3}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)^3\csc(c+dx)dx}{a}$$

$\downarrow$ 3799

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\frac{\int (e+fx)^3\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}-\frac{\int (e+fx)^3\csc(c+dx)dx}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots}{d}\cdots}{d}\cdots}{a}$$

$$\frac{\int(e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{\int(e+fx)^3\csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-\frac{3f\int(e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int(e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}+$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots}{d}\cdots}{d}\cdots}{a}$$

$$\frac{\int(e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\Bigg(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d\cdots}\right.}{d}\cdots}{d}\cdots}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots}{d}\cdots}{d}\cdots}{a}$$

$$\frac{\int(e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\Bigg(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d\cdots}\right.}{d}\cdots}{d}\cdots}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\Bigg(\frac{i(e+fx)^3}{3f}-2i\Bigg(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots}{d}\cdots}{d}\cdots}{a}$$

$$\frac{\frac{6f\int(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\downarrow$ 3042

$$\frac{-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d\cdots}}{d}\cdots\right.}{d}\cdots}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\frac{\frac{6f\int-(e+fx)^2\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\downarrow$ 25

$$\frac{-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d\cdots}}{d}\cdots\right.}{d}\cdots}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\frac{-\frac{6f\int(e+fx)^2\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\downarrow$ 4202

$$\frac{-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d\cdots}}{d}\cdots\right.}{d}\cdots}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)^2}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}$$

$\downarrow$ 2620

$$-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d}\right.}{d}\right.}{d}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\int(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$$\Big\downarrow \textbf{3011}$$

$$-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+d}\right.}{d}\right.}{d}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}\right)}{d}-\frac{i(e+fx)}{}\right.\right.}{d}}{2a}$$

$$\Big\downarrow \textbf{2720}$$

$$-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+d}{d}\right.}{d}$$

$$a$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}$$

$$a$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi}\right.}{d^2}\right.}{d}\right.\right.}{d}$$

$$2a$$

$\downarrow$ 7143

$$-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+d}{d}\right.}{d}$$

$$a$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\,\text{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi}\right.}{2d}\right.\right.}{d}$$

$$a$$

$$-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\,\text{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log}{}\right.\right.}{d}$$

$$2a$$

$\downarrow$ 7163

$$\frac{-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\cdots\right.}{\cdots}}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\cdots\right.\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log\cdots}{\cdots}\right.\right.}{d}}{2a}$$

$$\Big\downarrow \; 2720$$

$$\frac{-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{Pol}\cdots}{\cdots}\right.}{\cdots}}{a}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\cdots}\right.}{2d}\cdots\right.\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log\cdots}{\cdots}\right.\right.}{d}}{2a}$$

$$\Big\downarrow \; 7143$$

$$-\frac{-\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\cdots}{a}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\cdots}\right.}{2d}\cdots\right.\right.}{d}$$

$$\frac{\cdots}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}\right)}{d}-\frac{i(e+fx)^2\log\cdots}{\cdots}\right.\right.}{d}}{2a}$$

input
```
Int[((e + f*x)^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(-(((e + f*x)^3*Cot[c + d*x])/d) - (3*f*(((I/3)*(e + f*x)^3)/f - (2*I)*(((
-1/2*I)*(e + f*x)^2*Log[1 + E^(I*(2*c + Pi + 2*d*x))])/d + (I*f*(((I/2)*(e
 + f*x)*PolyLog[2, -E^(I*(2*c + Pi + 2*d*x))])/d - (f*PolyLog[3, -E^(I*(2*
c + Pi + 2*d*x))])/(4*d^2)))/d)))/d)/a + ((-2*(e + f*x)^3*Cot[c/2 + Pi/4 +
 (d*x)/2])/d - (6*f*(((I/3)*(e + f*x)^3)/f - (2*I)*(((-I)*(e + f*x)^2*Log[
1 + E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d + ((2*I)*f*((I*(e + f*x)*PolyLog[2,
 -E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d - (f*PolyLog[3, -E^((I/2)*(2*c + 3*Pi
 + 2*d*x))])/d^2))/d)))/d)/(2*a) - ((-2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x)
)])/d + (3*f*((I*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/d - ((2*I)*f*((
(-I)*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/d + (f*PolyLog[4, -E^(I*(c +
d*x))])/d^2))/d))/d - (3*f*((I*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/d
- ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/d + (f*PolyLog[4,
 E^(I*(c + d*x))])/d^2))/d))/d)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(419) = 838$.

Time = 1.75 (sec) , antiderivative size = 1774, normalized size of antiderivative = 3.83

method	result	size
risch	Expression too large to display	1774

input
```
int((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4-6*I*f^3*polylog(4,exp(I*(d*x+c)))
/a/d^4-2*(-2*f^3*x^3+I*exp(I*(d*x+c))*f^3*x^3-6*e*f^2*x^2+3*I*exp(I*(d*x+c
))*e*f^2*x^2-6*e^2*f*x+3*I*exp(I*(d*x+c))*e^2*f*x-2*e^3+I*exp(I*(d*x+c))*e
^3+exp(2*I*(d*x+c))*x^3*f^3+3*exp(2*I*(d*x+c))*e*f^2*x^2+3*exp(2*I*(d*x+c)
)*e^2*f*x+exp(2*I*(d*x+c))*e^3)/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/a/
d+6/d^3/a*e*f^2*polylog(3,-exp(I*(d*x+c)))-6/d^4/a*c^2*f^3*ln(1-I*exp(I*(d
*x+c)))-3/d^4/a*c^2*f^3*ln(1-exp(I*(d*x+c)))-1/d/a*f^3*ln(1-exp(I*(d*x+c))
)*x^3-6/d^3/a*f^3*polylog(3,exp(I*(d*x+c)))*x-1/d^4/a*c^3*f^3*ln(1-exp(I*(
d*x+c)))-12/d^2/a*e^2*f*ln(exp(I*(d*x+c)))-12/d^4/a*c^2*f^3*ln(exp(I*(d*x+
c)))+3/d^2/a*e^2*f*ln(exp(I*(d*x+c))-1)+3/d^4/a*c^2*f^3*ln(exp(I*(d*x+c))-
1)+3/d^4/a*c^2*f^3*ln(exp(2*I*(d*x+c))+1)+6/d^2/a*f^3*ln(1-I*exp(I*(d*x+c)
))*x^2+1/d^4/a*c^3*f^3*ln(exp(I*(d*x+c))-1)-6/d^3/a*e*f^2*polylog(3,exp(I*
(d*x+c)))+3/d^2/a*f^3*ln(1-exp(I*(d*x+c)))*x^2+3/d^2/a*f^3*ln(exp(I*(d*x+c
))+1)*x^2+12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+3/d^3/a*e*f^2*ln(1-exp(
I*(d*x+c)))*c^2-3/d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2+3/d/a*e*f^2*ln(exp(I*
(d*x+c))+1)*x^2+3/d/a*e^2*f*ln(exp(I*(d*x+c))+1)*x-3/d/a*e^2*f*ln(1-exp(I*
(d*x+c)))*x+6/d^2/a*e*f^2*ln(exp(I*(d*x+c))+1)*x+12/d^2/a*e*f^2*ln(1-I*exp
(I*(d*x+c)))*x+12*I/d^3/a*c^2*f^3*x+6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4
+6*f^3*polylog(3,exp(I*(d*x+c)))/a/d^4-1/d/a*e^3*ln(exp(I*(d*x+c))-1)+1/d/
a*e^3*ln(exp(I*(d*x+c))+1)+6/d^2/a*e*f^2*ln(1-exp(I*(d*x+c)))*x+12/d^3/...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4799 vs. $2(405) = 810$.

Time = 0.25 (sec) , antiderivative size = 4799, normalized size of antiderivative = 10.37

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3
*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d
*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)**2/(sin(c
 + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^3 \csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{2\left(\int \frac{\csc(dx+c)^2x^3}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 d f^3 + 2\left(\int \frac{\csc(dx+c)^2x^3}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right) d f^3 + 6\left(\int \frac{\csc(dx+c)^2x^2}{\sin(dx+c)+1}dx\right)\tan}{}$$

input
```
int((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(2*int((csc(c + d*x)**2*x**3)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**2*d*
f**3 + 2*int((csc(c + d*x)**2*x**3)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)
*d*f**3 + 6*int((csc(c + d*x)**2*x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)
/2)**2*d*e*f**2 + 6*int((csc(c + d*x)**2*x**2)/(sin(c + d*x) + 1),x)*tan((
c + d*x)/2)*d*e*f**2 + 6*int((csc(c + d*x)**2*x)/(sin(c + d*x) + 1),x)*tan
((c + d*x)/2)**2*d*e**2*f + 6*int((csc(c + d*x)**2*x)/(sin(c + d*x) + 1),x
)*tan((c + d*x)/2)*d*e**2*f - 2*log(tan((c + d*x)/2))*tan((c + d*x)/2)**2*
e**3 - 2*log(tan((c + d*x)/2))*tan((c + d*x)/2)*e**3 + tan((c + d*x)/2)**3
*e**3 + 6*tan((c + d*x)/2)**2*e**3 - e**3)/(2*tan((c + d*x)/2)*a*d*(tan((c
 + d*x)/2) + 1))
```

3.204 $\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 327

$$\begin{aligned}\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = &-\frac{2i(e+fx)^2}{ad}+\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad}\\&-\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{(e+fx)^2\cot(c+dx)}{ad}\\&+\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}\\&+\frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2}\\&-\frac{2if(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}\\&-\frac{4if^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}\\&+\frac{2if(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}\\&-\frac{if^2\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}\\&+\frac{2f^2\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}-\frac{2f^2\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}\end{aligned}$$

output
```
-2*I*(f*x+e)^2/a/d+2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d-(f*x+e)^2*cot(1
/2*c+1/4*Pi+1/2*d*x)/a/d-(f*x+e)^2*cot(d*x+c)/a/d+4*f*(f*x+e)*ln(1-I*exp(I
*(d*x+c)))/a/d^2+2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d^2-2*I*f*(f*x+e)*po
lylog(2,-exp(I*(d*x+c)))/a/d^2-4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3+2
*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-I*f^2*polylog(2,exp(2*I*(d*x+
c)))/a/d^3+2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3-2*f^2*polylog(3,exp(I*(d
*x+c)))/a/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 709 vs. $2(327) = 654$.

Time = 9.57 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.17

$$\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{id^2e(de-2f)x - id^2e(de+2f)x - \frac{2id^2(e+fx)^2}{-1+e^{2ic}} - 2d(de-f)fx\log\left(1-e^{-i(c+dx)}\right) - d^2f^2x^2\log\left(1-e^{-i(\cdots}\right.}{\cdots} - \frac{4f(\cos(c)+i\sin(c))\left(\frac{(e+fx)^2(\cos(c)-i\sin(c))}{2f} - \frac{(e+fx)\log(1+i\cos(c+dx)+\sin(c+dx))(1+i\cos(c)+\sin(c))}{d} + \frac{f\,\mathrm{PolyLog}(2,-\cdots}{\cdots}\right.}{ad(\cos(c)+i(1+\sin(c)))} + \frac{\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\left(e^2\sin\left(\frac{dx}{2}\right)+2efx\sin\left(\frac{dx}{2}\right)+f^2x^2\sin\left(\frac{dx}{2}\right)\right)}{2ad} + \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(e^2\sin\left(\frac{dx}{2}\right)+2efx\sin\left(\frac{dx}{2}\right)+f^2x^2\sin\left(\frac{dx}{2}\right)\right)}{2ad} + \frac{2\left(e^2\sin\left(\frac{dx}{2}\right)+2efx\sin\left(\frac{dx}{2}\right)+f^2x^2\sin\left(\frac{dx}{2}\right)\right)}{ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
Integrate[((e + f*x)^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(I*d^2*e*(d*e - 2*f)*x - I*d^2*e*(d*e + 2*f)*x - ((2*I)*d^2*(e + f*x)^2)/(
-1 + E^((2*I)*c)) - 2*d*(d*e - f)*f*x*Log[1 - E^((-I)*(c + d*x))] - d^2*f^
2*x^2*Log[1 - E^((-I)*(c + d*x))] + 2*d*f*(d*e + f)*x*Log[1 + E^((-I)*(c +
 d*x))] + d^2*f^2*x^2*Log[1 + E^((-I)*(c + d*x))] - d*e*(d*e - 2*f)*Log[1
- E^(I*(c + d*x))] + d*e*(d*e + 2*f)*Log[1 + E^(I*(c + d*x))] + (2*I)*f*(d
*e + f)*PolyLog[2, -E^((-I)*(c + d*x))] + (2*I)*d*f^2*x*PolyLog[2, -E^((-I
)*(c + d*x))] - (2*I)*(d*e - f)*f*PolyLog[2, E^((-I)*(c + d*x))] - (2*I)*d
*f^2*x*PolyLog[2, E^((-I)*(c + d*x))] + 2*f^2*PolyLog[3, -E^((-I)*(c + d*x
))] - 2*f^2*PolyLog[3, E^((-I)*(c + d*x))])/(a*d^3) - (4*f*(Cos[c] + I*Sin
[c])*(((e + f*x)^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c
 + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Co
s[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(a*d*(Cos[c] +
 I*(1 + Sin[c]))) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f
*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d) + (Sec[c/2]*Sec[c/2 + (d*
x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2
*a*d) + (2*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]
))/(a*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.15, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5046, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838, 5046, 3042, 3799, 3042, 4671, 3011, 2720, 4672, 3042, 25, 4202, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^2\csc^2(c+dx)dx}{a}-\int \frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc(c+dx)^2dx}{a}-\int \frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2f\int(e+fx)\cot(c+dx)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2f\int-\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{2f\int(e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 4202}$$

$$-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-i(2c+2dx+\pi)}\log\left(1+e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$-\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{ 5046}$$

$$\frac{\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)^2\csc(c+dx)dx}{a}+}{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e+fx)^2}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)^2\csc(c+dx)dx}{a}+}{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (e+fx)^2\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}-\frac{\int (e+fx)^2\csc(c+dx)dx}{a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{\int (e+fx)^2\csc(c+dx)dx}{a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-\frac{2f\int (e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}+\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+ \cdots}{a}$$

$$\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(\cdots\right.}{d^2 \cdots}\right.}{d}}{a}$$

$$\frac{\int (e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(\cdots\right.}{d^2 \cdots}\right.}{d}}{a}$$

$$\frac{\frac{4f\int (e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(\cdots\right.}{d^2 \cdots}\right.}{d}}{a}$$

$$\frac{\frac{4f\int -\left((e+fx)\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(\cdots}{d^2}\right.}{d}}{a}$$

$$\frac{-\frac{4f\int(e+fx)\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \; 4202$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(\cdots}{d^2}\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(\cdots}{d^2}\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(\cdots}{d^2}\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)de^{\frac{1}{2}i(2c+2dx+3\pi)}}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+\cdots}\right)}{d}\right.\right.}{d}}{2a}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\Big\downarrow$ 2838

$$-\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(\cdots}{d^2}\right.}{d}}{a}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}+$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$\Big\downarrow$ 7143

$$-\frac{-\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\operatorname{Poly}\cdots}{}\right.}{d}}{a}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}+$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

input

```
Int[((e + f*x)^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*Cot[c + d*x])/d) - (2*f*(((I/2)*(e + f*x)^2)/f - (2*I)*(((
-1/2*I)*(e + f*x)*Log[1 + E^(I*(2*c + Pi + 2*d*x))])/d - (f*PolyLog[2, -E^
(I*(2*c + Pi + 2*d*x))])/(4*d^2))))/d)/a + ((-2*(e + f*x)^2*Cot[c/2 + Pi/4
 + (d*x)/2])/d - (4*f*(((I/2)*(e + f*x)^2)/f - (2*I)*(((-I)*(e + f*x)*Log[
1 + E^((I/2)*(2*c + 3*Pi + 2*d*x))])/d - (f*PolyLog[2, -E^((I/2)*(2*c + 3*
Pi + 2*d*x))])/d^2)))/d)/(2*a) - ((-2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))]
)/d + (2*f*((I*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/d - (f*PolyLog[3, -
E^(I*(c + d*x))])/d^2))/d - (2*f*((I*(e + f*x)*PolyLog[2, E^(I*(c + d*x))]
)/d - (f*PolyLog[3, E^(I*(c + d*x))])/d^2))/d)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(296) = 592.

Time = 1.46 (sec) , antiderivative size = 984, normalized size of antiderivative = 3.01

method	result	size
risch	Expression too large to display	984

input
```
int((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*x^2*f^2+I*exp(I*(d*x+c))*f^2*x^2-4*e*f*x+2*I*exp(I*(d*x+c))*e*f*x-2
*e^2+I*exp(I*(d*x+c))*e^2+exp(2*I*(d*x+c))*x^2*f^2+2*exp(2*I*(d*x+c))*e*f*
x+exp(2*I*(d*x+c))*e^2)/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/a/d-1/d/a*
e^2*ln(exp(I*(d*x+c))-1)+1/d/a*e^2*ln(exp(I*(d*x+c))+1)-4*I*f^2*polylog(2,
I*exp(I*(d*x+c)))/a/d^3+4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+8/a/d^3*f^2*c
*ln(exp(I*(d*x+c)))-8/a/d^2*f*e*ln(exp(I*(d*x+c)))+4/a/d^2*f^2*ln(1-I*exp(
I*(d*x+c)))*x+2/d^2/a*f^2*ln(exp(I*(d*x+c))+1)*x-2/d^3/a*c*f^2*ln(exp(I*(d
*x+c))-1)-2/d^3/a*c*f^2*ln(exp(2*I*(d*x+c))+1)+2/d^2/a*e*f*ln(exp(I*(d*x+c
))-1)+2/d^2/a*e*f*ln(exp(2*I*(d*x+c))+1)+2/d^2/a*e*f*ln(exp(I*(d*x+c))+1)-
2*I/d^3/a*f^2*polylog(2,-exp(I*(d*x+c)))-4*I/d/a*f^2*x^2-4*I/d^3/a*f^2*c^2
-2/d/a*e*f*ln(1-exp(I*(d*x+c)))*x+2/d/a*e*f*ln(exp(I*(d*x+c))+1)*x-2/d^2/a
*e*f*ln(1-exp(I*(d*x+c)))*c+2/d^2/a*c*e*f*ln(exp(I*(d*x+c))-1)+2/d^2/a*f^2
*ln(1-exp(I*(d*x+c)))*x+2/d^3/a*f^2*ln(1-exp(I*(d*x+c)))*c+2*f^2*polylog(3
,-exp(I*(d*x+c)))/a/d^3-2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3+1/d^3/a*f^2*
ln(1-exp(I*(d*x+c)))*c^2+1/d/a*f^2*ln(exp(I*(d*x+c))+1)*x^2-1/d/a*f^2*ln(1
-exp(I*(d*x+c)))*x^2-1/d^3/a*c^2*f^2*ln(exp(I*(d*x+c))-1)-2*I*f^2*polylog(
2,exp(I*(d*x+c)))/a/d^3-2*I/d^2/a*e*f*polylog(2,-exp(I*(d*x+c)))+2*I/d^2/a
*e*f*polylog(2,exp(I*(d*x+c)))-4*I/d^2/a*e*f*arctan(exp(I*(d*x+c)))-2*I/d^
2/a*f^2*polylog(2,-exp(I*(d*x+c)))*x+2*I/d^2/a*f^2*polylog(2,exp(I*(d*x+c)
))*x-8*I/d^2/a*f^2*c*x+4*I/d^3/a*f^2*c*arctan(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2539 vs. $2(285) = 570$.

Time = 0.16 (sec) , antiderivative size = 2539, normalized size of antiderivative = 7.76

$$\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 - 4*(d^2*f^2*x^2 + 2*d^2*e*f
*x + d^2*e^2)*cos(d*x + c)^2 - 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*cos
(d*x + c) - 2*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f - I*f^2)*cos(d*
x + c)^2 + I*f^2 + (-I*d*f^2*x - I*d*e*f + I*f^2 + (-I*d*f^2*x - I*d*e*f +
 I*f^2)*cos(d*x + c))*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) -
 2*(I*d*f^2*x + I*d*e*f + (-I*d*f^2*x - I*d*e*f + I*f^2)*cos(d*x + c)^2 -
I*f^2 + (I*d*f^2*x + I*d*e*f - I*f^2 + (I*d*f^2*x + I*d*e*f - I*f^2)*cos(d
*x + c))*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) - 4*(-I*f^2*co
s(d*x + c)^2 + I*f^2 + (I*f^2*cos(d*x + c) + I*f^2)*sin(d*x + c))*dilog(I*
cos(d*x + c) - sin(d*x + c)) - 4*(I*f^2*cos(d*x + c)^2 - I*f^2 + (-I*f^2*c
os(d*x + c) - I*f^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) -
 2*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f + I*f^2)*cos(d*x + c)^2 -
I*f^2 + (-I*d*f^2*x - I*d*e*f - I*f^2 + (-I*d*f^2*x - I*d*e*f - I*f^2)*cos
(d*x + c))*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 2*(I*d*f^
2*x + I*d*e*f + (-I*d*f^2*x - I*d*e*f - I*f^2)*cos(d*x + c)^2 + I*f^2 + (I
*d*f^2*x + I*d*e*f + I*f^2 + (I*d*f^2*x + I*d*e*f + I*f^2)*cos(d*x + c))*s
in(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) + (d^2*f^2*x^2 + d^2*e^
2 + 2*d*e*f - (d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f + 2*(d^2*e*f + d*f^2)*x)*co
s(d*x + c)^2 + 2*(d^2*e*f + d*f^2)*x + (d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f +
2*(d^2*e*f + d*f^2)*x + (d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f + 2*(d^2*e*f +...
```

Sympy [F]

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2
*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)**2
/(sin(c + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\left(\int \frac{\csc(dx+c)^2x^2}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 d f^2 + 2\left(\int \frac{\csc(dx+c)^2x^2}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right) d f^2 + 4\left(\int \frac{\csc(dx+c)^2x}{\sin(dx+c)+1}dx\right)\tan(}{}$$

input
```
int((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(2*int((csc(c + d*x)**2*x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**2*d*
f**2 + 2*int((csc(c + d*x)**2*x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)
*d*f**2 + 4*int((csc(c + d*x)**2*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)
**2*d*e*f + 4*int((csc(c + d*x)**2*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/
2)*d*e*f - 2*log(tan((c + d*x)/2))*tan((c + d*x)/2)**2*e**2 - 2*log(tan((c
 + d*x)/2))*tan((c + d*x)/2)*e**2 + tan((c + d*x)/2)**3*e**2 + 6*tan((c +
d*x)/2)**2*e**2 - e**2)/(2*tan((c + d*x)/2)*a*d*(tan((c + d*x)/2) + 1))
```

3.205 $\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{f\log(\sin(c+dx))}{ad^2} - \frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} + \frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}$$

output
```
2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d-(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a/
d-(f*x+e)*cot(d*x+c)/a/d+2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2+f*ln(sin(
d*x+c))/a/d^2-I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2+I*f*polylog(2,exp(I*(d*
x+c)))/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 396 vs. 2(169) = 338.

Time = 8.43 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.34

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-d(e+fx)\cos\left(\frac{1}{2}(c+dx)\right)\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)+4d(e+fx)\sin\right.}{}$$

input
```
Integrate[((e + f*x)*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-(d*(e + f*x)*Cos[(c + d*x)/2]*(1
+ Cot[(c + d*x)/2])) + 4*d*(e + f*x)*Sin[(c + d*x)/2] - 2*f*(c + d*x)*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*f*Log[Sin[c + d*x]]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*d*e*Log[Tan[(c + d*x)/2]]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 2*c*f*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]) - 2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 +
E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c +
 d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + d*(e + f*x)*Sin[(c + d*x
)/2]*(1 + Tan[(c + d*x)/2])))/(2*a*d^2*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {5046, 3042, 4672, 3042, 25, 3956, 5046, 3042, 3799, 3042, 4671, 2715, 2838, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 5046}$$

$$\frac{\int (e+fx)\csc^2(c+dx)dx}{a}-\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)\csc(c+dx)^2dx}{a}-\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{\frac{f\int \cot(c+dx)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{f\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{f\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 5046}$$

$$\int \frac{e+fx}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)\csc(c+dx)dx}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{e+fx}{\sin(c+dx)a+a}dx-\frac{\int (e+fx)\csc(c+dx)dx}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{\int (e+fx)\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}-\frac{\int (e+fx)\csc(c+dx)dx}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{\int (e+fx)\csc(c+dx)dx}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}+\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}+\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2f\int \cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-\frac{-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\int -\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-\frac{-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2f\int \tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow 3956$$

$$-\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{4f\log\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)\right)}{d^2}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

input
```
Int[((e + f*x)*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
((-2*(e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/d + (4*f*Log[-Cos[c/2 - Pi/4 + (
d*x)/2]])/d^2)/(2*a) + (-(((e + f*x)*Cot[c + d*x])/d) + (f*Log[-Sin[c + d*
x]])/d^2)/a - ((-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + (I*f*PolyLog[2,
 -E^(I*(c + d*x))])/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])/d^2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(149) = 298$.

Time = 1.35 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{2\left(-2fx+i\mathrm{e}^{i(dx+c)}fx-2e+i\mathrm{e}^{i(dx+c)}e+fx\,\mathrm{e}^{2i(dx+c)}+e\,\mathrm{e}^{2i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)ad}+\frac{f\ln\left(\mathrm{e}^{i(dx+c)}+1\right)x}{da}-\frac{f\ln\left(1-\mathrm{e}^{i(dx+c)}\right)x}{da}-$ $if\,\mathrm{polylog}($

input
```
int((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*f*x+I*exp(I*(d*x+c))*f*x-2*e+I*exp(I*(d*x+c))*e+f*x*exp(2*I*(d*x+c)
)+e*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/a/d+1/d/a*f*
ln(exp(I*(d*x+c))+1)*x-1/d/a*f*ln(1-exp(I*(d*x+c)))*x-I*f*polylog(2,-exp(I
*(d*x+c)))/a/d^2+I*f*polylog(2,exp(I*(d*x+c)))/a/d^2-4/d^2/a*f*ln(exp(I*(d
*x+c)))+1/d^2/a*f*ln(exp(I*(d*x+c))-1)+1/d^2/a*f*ln(exp(2*I*(d*x+c))+1)+1/
d^2/a*f*ln(exp(I*(d*x+c))+1)+1/d^2/a*c*f*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(e
xp(I*(d*x+c))-1)+1/d/a*e*ln(exp(I*(d*x+c))+1)-1/d^2/a*f*ln(1-exp(I*(d*x+c)
))*c-2*I/d^2/a*f*arctan(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(145) = 290$.

Time = 0.13 (sec) , antiderivative size = 858, normalized size of antiderivative = 5.08

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*d*f*x - 4*(d*f*x + d*e)*cos(d*x + c)^2 + 2*d*e - 2*(d*f*x + d*e)*c
os(d*x + c) + (-I*f*cos(d*x + c)^2 + (I*f*cos(d*x + c) + I*f)*sin(d*x + c)
 + I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c)^2 + (-I*f
*cos(d*x + c) - I*f)*sin(d*x + c) - I*f)*dilog(cos(d*x + c) - I*sin(d*x +
c)) + (-I*f*cos(d*x + c)^2 + (I*f*cos(d*x + c) + I*f)*sin(d*x + c) + I*f)*
dilog(-cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c)^2 + (-I*f*cos(d*
x + c) - I*f)*sin(d*x + c) - I*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) +
(d*f*x - (d*f*x + d*e + f)*cos(d*x + c)^2 + d*e + (d*f*x + d*e + (d*f*x +
d*e + f)*cos(d*x + c) + f)*sin(d*x + c) + f)*log(cos(d*x + c) + I*sin(d*x
+ c) + 1) + (d*f*x - (d*f*x + d*e + f)*cos(d*x + c)^2 + d*e + (d*f*x + d*e
 + (d*f*x + d*e + f)*cos(d*x + c) + f)*sin(d*x + c) + f)*log(cos(d*x + c)
- I*sin(d*x + c) + 1) + ((d*e - (c + 1)*f)*cos(d*x + c)^2 - d*e + (c + 1)*
f - (d*e - (c + 1)*f + (d*e - (c + 1)*f)*cos(d*x + c))*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + ((d*e - (c + 1)*f)*cos(d*x
+ c)^2 - d*e + (c + 1)*f - (d*e - (c + 1)*f + (d*e - (c + 1)*f)*cos(d*x +
c))*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (d*f
*x - (d*f*x + c*f)*cos(d*x + c)^2 + c*f + (d*f*x + c*f + (d*f*x + c*f)*cos
(d*x + c))*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - (d*f*x
- (d*f*x + c*f)*cos(d*x + c)^2 + c*f + (d*f*x + c*f + (d*f*x + c*f)*cos(d*
x + c))*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(f*co...
```

Sympy [F]

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c +
d*x)**2/(sin(c + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)\csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{2\left(\int \frac{\csc(dx+c)^2x}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2df + 2\left(\int \frac{\csc(dx+c)^2x}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)df - 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}\right.}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ad\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

input
```
int((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(2*int((csc(c + d*x)**2*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**2*d*f +
 2*int((csc(c + d*x)**2*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)*d*f - 2*
log(tan((c + d*x)/2))*tan((c + d*x)/2)**2*e - 2*log(tan((c + d*x)/2))*tan(
(c + d*x)/2)*e + tan((c + d*x)/2)**3*e + 6*tan((c + d*x)/2)**2*e - e)/(2*t
an((c + d*x)/2)*a*d*(tan((c + d*x)/2) + 1))
```

3.206 $\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}-\frac{2\cot(c+dx)}{ad}+\frac{\cot(c+dx)}{d(a+a\sin(c+dx))}$$

output
```
arctanh(cos(d*x+c))/a/d-2*cot(d*x+c)/a/d+cot(d*x+c)/d/(a+a*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx =\frac{\sec(c+dx)\left(-1+\operatorname{arctanh}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}-\csc(c+dx)+2\sin(c+dx)\right)}{ad}$$

input
```
Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
(Sec[c + d*x]*(-1 + ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2] - C
sc[c + d*x] + 2*Sin[c + d*x]))/(a*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3247}$$

$$\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\int -\csc^2(c+dx)(2a-a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \csc^2(c+dx)(2a-a\sin(c+dx))dx}{a^2}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a-a\sin(c+dx)}{\sin(c+dx)^2}dx}{a^2}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2a\int \csc^2(c+dx)dx-a\int \csc(c+dx)dx}{a^2}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \csc(c+dx)^2dx-a\int \csc(c+dx)dx}{a^2}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{2a\int 1d\cot(c+dx)}{d}-a\int \csc(c+dx)dx}{a^2}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{-a\int \csc(c+dx)dx - \frac{2a\cot(c+dx)}{d}}{a^2} + \frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{a\text{arctanh}(\cos(c+dx))}{d} - \frac{2a\cot(c+dx)}{d}}{a^2} + \frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

input
```
Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
((a*ArcTanh[Cos[c + d*x]])/d - (2*a*Cot[c + d*x])/d)/a^2 + Cot[c + d*x]/(d
*(a + a*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{2da}$	59
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{2da}$	59
parallelrisch	$\frac{\left(-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)d}$	80
norman	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}-\frac{1}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	92
risch	$-\frac{2\left(i\mathrm{e}^{i(dx+c)}+\mathrm{e}^{2i(dx+c)}-2\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{ad}$	99

input
```
int(csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-
4/(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(51) = 102$.

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.06

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{4\cos(dx+c)^2 + \left(\cos(dx+c)^2 - (\cos(dx+c)+1)\sin(dx+c) - 1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - (\cos(d}{2\left(ad\cos(dx+c)^2\right.}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(4*cos(d*x + c)^2 + (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c)
- 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - (cos(d*x + c) + 1)*si
n(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*cos(d*x + c) + 1)*sin(
d*x + c) + 2*cos(d*x + c) - 2)/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x +
c) + a*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*sin(d*x + c)/(cos(d*x + c
) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a}}{2d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (tan(1
/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^2 +
 tan(1/2*d*x + 1/2*c))*a))/d
```

Mupad [B] (verification not implemented)

Time = 36.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(2*a*d) - log(tan(c/2 + (d*x)/2))/(a*d) - (5*tan(c/2 +
(d*x)/2) + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.00

$$\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int(csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 2*log(tan((c + d*x)/2))*tan((c + d*x)/2)**2 - 2*log(tan((c + d*x)/2))*
tan((c + d*x)/2) + tan((c + d*x)/2)**3 + 6*tan((c + d*x)/2)**2 - 1)/(2*tan
((c + d*x)/2)*a*d*(tan((c + d*x)/2) + 1))
```

3.207 $\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 35.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \text{5048}$$
$$\int \frac{\csc^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\csc(dx+c)^2}{(fx+e)(a+a\sin(dx+c))}dx$$

input
```
int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc(dx+c)^2}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)
/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 36.68 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{1}{\sin\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

input
```
int(1/(sin(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(sin(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc(dx+c)^2}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(c + d*x)**2/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.208 $\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 73.74 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\csc (dx+c)^2}{(fx+e)^2 (a+a\sin (dx+c))}dx$$

input
```
int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc (dx+c)^2}{(fx+e)^2(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*
e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc^2 (c+dx)}{e^2\sin (c+dx)+e^2+2efx\sin (c+dx)+2efx+f^2x^2\sin (c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x)
+ 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 37.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{1}{\sin\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

input
```
int(1/(sin(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(sin(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\frac{\int \frac{\csc(dx+c)^{2}}{\sin(dx+c)e^{2}+2\sin(dx+c)efx+\sin(dx+c)f^{2}x^{2}+e^{2}+2efx+f^{2}x^{2}}d x}{a}$$

input
```
int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(c + d*x)**2/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*
x)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.209 $\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 600

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{3f(e+fx)^2\csc(c+dx)}{2ad^2} - \frac{(e+fx)^3\cot(c+dx)\csc(c+dx)}{2ad} - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} - \frac{3f(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{ad^2} + \frac{3if^3\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^4} + \frac{9if(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{2ad^2} + \frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3} - \frac{3if^3\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^4} - \frac{9if(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{2ad^2} + \frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3} - \frac{9f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} - \frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4} + \frac{9f^2(e+fx)\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} - \frac{3f^3\text{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2ad^4} - \frac{9if^3\text{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4} + \frac{9if^3\text{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4}$$

output
```
9*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4-6*f^2*(f*x+e)*arctanh(exp(I*(d*x+c
)))/a/d^3-3*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)^3*cot(1/2*c+1/4*
Pi+1/2*d*x)/a/d+(f*x+e)^3*cot(d*x+c)/a/d-3/2*f*(f*x+e)^2*csc(d*x+c)/a/d^2-
1/2*(f*x+e)^3*cot(d*x+c)*csc(d*x+c)/a/d-6*f*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)
))/a/d^2-3*f*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d^2-9/2*I*f*(f*x+e)^2*poly
log(2,exp(I*(d*x+c)))/a/d^2+12*I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a
/d^3+9/2*I*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2-3*I*f^3*polylog(2,
exp(I*(d*x+c)))/a/d^4+3*I*f^3*polylog(2,-exp(I*(d*x+c)))/a/d^4-9*I*f^3*pol
ylog(4,-exp(I*(d*x+c)))/a/d^4-9*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a/d
^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+9*f^2*(f*x+e)*polylog(3,exp(I*
(d*x+c)))/a/d^3-3/2*f^3*polylog(3,exp(2*I*(d*x+c)))/a/d^4+2*I*(f*x+e)^3/a/
d+3*I*f^2*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1493 vs. $2(600) = 1200$.

Time = 21.72 (sec) , antiderivative size = 1493, normalized size of antiderivative = 2.49

$$\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(3*e^3*Log[Tan[(c + d*x)/2]])/(2*a*d) + (3*e*f^2*Log[Tan[(c + d*x)/2]])/(a
*d^3) + (9*e^2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c +
d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2, -E^(I*(c + d*x))] - Poly
Log[2, E^(I*(c + d*x))])))/(2*a*d^2) + (3*f^3*((c + d*x)*(Log[1 - E^(I*(c
+ d*x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLo
g[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^4) + (E^(I*c)
*f^3*Csc[c]*((2*d^3*x^3)/E^((2*I)*c) + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*Lo
g[1 - E^((-I)*(c + d*x))] + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*Log[1 + E^((-
I)*(c + d*x))] - 6*d*(1 - E^((-2*I)*c))*x*PolyLog[2, -E^((-I)*(c + d*x))]
- 6*d*(1 - E^((-2*I)*c))*x*PolyLog[2, E^((-I)*(c + d*x))] + (6*I)*(1 - E^(
(-2*I)*c))*PolyLog[3, -E^((-I)*(c + d*x))] + (6*I)*(1 - E^((-2*I)*c))*Poly
Log[3, E^((-I)*(c + d*x))]))/(2*a*d^4) - (9*e*f^2*(d^2*x^2*ArcTanh[Cos[c +
 d*x] + I*Sin[c + d*x]] - I*d*x*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]]
 + I*d*x*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]] + PolyLog[3, -Cos[c + d
*x] - I*Sin[c + d*x]] - PolyLog[3, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^3
) + (3*f^3*(-2*d^3*x^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] + (3*I)*d^2*
x^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] - (3*I)*d^2*x^2*PolyLog[2,
Cos[c + d*x] + I*Sin[c + d*x]] - 6*d*x*PolyLog[3, -Cos[c + d*x] - I*Sin[c
+ d*x]] + 6*d*x*PolyLog[3, Cos[c + d*x] + I*Sin[c + d*x]] - (6*I)*PolyLog[
4, -Cos[c + d*x] - I*Sin[c + d*x]] + (6*I)*PolyLog[4, Cos[c + d*x] + I*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^3\csc^3(c+dx)dx}{a}-\int \frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc(c+dx)^3dx}{a}-\int \frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{3f^2\int(e+fx)\csc(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^3\csc(c+dx)dx-\frac{3f(e+fx)^2\csc(c+dx)}{2d^2}-\frac{(e+fx)^3\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int\frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\frac{\frac{3f^2\int(e+fx)\csc(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^3\csc(c+dx)dx-\frac{3f(e+fx)^2\csc(c+dx)}{2d^2}-\frac{(e+fx)^3\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int\frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 4671

$$-\int\frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{3f^2\left(-\frac{f\int\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3f\int(e+fx)^2\log(1-e^{i(c+dx)})dx}{d}+\frac{3f\int(e+fx}{}\right.}{a}$$

$\downarrow$ 2715

$$-\int\frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{3f^2\left(\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3f\int(e+fx)^2\text{l}}{}\right.}{}$$

$\downarrow$ 2838

$$-\int\frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{1}{2}\left(-\frac{3f\int(e+fx)^2\log(1-e^{i(c+dx)})dx}{d}+\frac{3f\int(e+fx)^2\log(1+e^{i(c+dx)})dx}{d}-\frac{2(e+fx)^3\text{arctanh}(e^{i(c+dx)})}{d}\right)+\frac{3f^2\left(-\frac{2(e+fx)\text{arctanh}(}{d}\right.}{}}{a}$$

$\downarrow$ 3011

$$-\int\frac{(e+fx)^3\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+}\right.}{d}\right.}{d}\right.}{}$$

$\downarrow$ 5046

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{\int(e+fx)^3\csc^2(c+dx)dx}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{\int(e+fx)^3\csc(c+dx)^2dx}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 4672

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{\frac{3f\int(e+fx)^2\cot(c+dx)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{\frac{3f\int-(e+fx)^2\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{-\frac{3f\int(e+fx)^2\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{4202}$$

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{i(2c+2dx+\pi)}(e+fx)^2}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\int(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}+\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\right.}}{d}\right.}{d}\right.$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2\right.}}{2d}\right.\right.}{d}}{a}$$

$$\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+\cdots}}{d}\cdots\right.}{d}\cdots\right.}{\cdots}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\int\frac{(e+fx)^3\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \; 5046$$

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+\cdots}}{d}\cdots\right.}{d}\cdots\right.}{\cdots}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\int\frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{\int(e+fx)^3\csc(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+\cdots}}{d}\cdots\right.}{d}\cdots\right.}{\cdots}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\int\frac{(e+fx)^3}{\sin(c+dx)a+a}dx+\frac{\int(e+fx)^3\csc(c+dx)dx}{a}$$

$$\Big\downarrow \; 3799$$

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}$$

$$a$$

$$\frac{\int(e+fx)^3\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\int(e+fx)^3\csc(c+dx)dx}{a}$$

$\big\downarrow$ 3042

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}$$

$$a$$

$$\frac{\int(e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\int(e+fx)^3\csc(c+dx)dx}{a}$$

$\big\downarrow$ 4671

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{-\frac{3f\int(e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int(e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}$$

$$a$$

$$\frac{\int(e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$\big\downarrow$ 3011

$$\frac{\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+\ldots}}{d}\ldots}{d}\ldots\right.}{\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)\ldots}{d}\ldots}{d}\ldots}}{a}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}\ldots}{4d^2}\ldots\right.}{d}\ldots\right.\right.}{d}\ldots$$

$$\frac{\int(e+fx)^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+\ldots}}{d}\ldots}{d}\ldots\right.}{\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)\ldots}{d}\ldots}{d}\ldots}}{a}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}\ldots}{4d^2}\ldots\right.}{d}\ldots\right.\right.}{d}\ldots$$

$$\frac{\frac{6f\int(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+}{d}\right.}{d}\right.$$

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}$$

$$a$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}$$

$$a$$

$$\frac{\frac{6f\int-(e+fx)^2\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+}{d}\right.}{d}\right.$$

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}$$

$$a$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}$$

$$a$$

$$\frac{-\frac{6f\int(e+fx)^2\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}$$

$\Big\downarrow$ 4202

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+}\right.}{d}\right.}{d}\right.}{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)^2}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+}\right.}{d}\right.}{d}\right.}{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}$$

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$\frac{-\frac{2(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\int(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}}{2a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{2\operatorname{arctanh}\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right.\right.}{d}\cdots}{}$$

$$a$$

$$-\frac{\cot(c+dx)\csc(c+dx)(e+fx)^3}{2d}-\frac{3f\csc(c+dx)(e+fx)^2}{2d^2}+\frac{3f^2\left(-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right)}{d^2}\cdots\right.}{d^2}$$

$$-\frac{\cot(c+dx)(e+fx)^3}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}\cdots}{4d^2}\right.}{d}\cdots\right.\right.}{d}$$

$$a$$

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)(e+fx)^3}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}\right)}{d}-\frac{i(e+fx)\cdots}{}\right.\right.}{d}$$

$$2a$$

$\downarrow$ 2720

$$-\frac{2\operatorname{arctanh}\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right.\right.}{d}\cdots}{}$$

$$a$$

$$-\frac{\cot(c+dx)\csc(c+dx)(e+fx)^3}{2d}-\frac{3f\csc(c+dx)(e+fx)^2}{2d^2}+\frac{3f^2\left(-\frac{2(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{i(c+\cdots}\right)}{d^2}\cdots\right.}{d^2}$$

$$-\frac{\cot(c+dx)(e+fx)^3}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}\cdots}{4d^2}\right.}{d}\cdots\right.\right.}{d}$$

$$a$$

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)(e+fx)^3}{d}-\frac{6f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{2if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}-\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\operatorname{PolyLog}\left(2,-e^{\frac{1}{2}i(2c+2dx+3\cdots}\right)\cdots}{d^2}\right.}{d}\cdots\right.\right.}{d}$$

$$2a$$

input
```
Int[((e + f*x)^3*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2325 vs. 2(540) = 1080.

Time = 1.95 (sec) , antiderivative size = 2326, normalized size of antiderivative = 3.88

method	result	size
risch	Expression too large to display	2326

input
```
int((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*I*f^3*polylog(2,-exp(I*(d*x+c)))/a/d^4+9*I*f^3*polylog(4,exp(I*(d*x+c)))
/a/d^4-3*I*f^3*polylog(2,exp(I*(d*x+c)))/a/d^4-9*I*f^3*polylog(4,-exp(I*(d
*x+c)))/a/d^4+12*I/a/d*e*f^2*x^2+9/2*I/a/d^2*e^2*f*polylog(2,-exp(I*(d*x+c
)))-9/2*I/a/d^2*e^2*f*polylog(2,exp(I*(d*x+c)))-9/d^3/a*e*f^2*polylog(3,-e
xp(I*(d*x+c)))+6/d^4/a*c^2*f^3*ln(1-I*exp(I*(d*x+c)))+3/d^4/a*c^2*f^3*ln(1
-exp(I*(d*x+c)))+3/2/d/a*f^3*ln(1-exp(I*(d*x+c)))*x^3+9/d^3/a*f^3*polylog(
3,exp(I*(d*x+c)))*x+3/2/d^4/a*c^3*f^3*ln(1-exp(I*(d*x+c)))+12/d^2/a*e^2*f*
ln(exp(I*(d*x+c)))+12/d^4/a*c^2*f^3*ln(exp(I*(d*x+c)))-3/d^2/a*e^2*f*ln(ex
p(I*(d*x+c))-1)-3/d^4/a*c^2*f^3*ln(exp(I*(d*x+c))-1)-3/d^4/a*c^2*f^3*ln(ex
p(2*I*(d*x+c))+1)-6/d^2/a*f^3*ln(1-I*exp(I*(d*x+c)))*x^2-3/2/d^4/a*c^3*f^3
*ln(exp(I*(d*x+c))-1)+9/d^3/a*e*f^2*polylog(3,exp(I*(d*x+c)))-3/d^2/a*f^3*
ln(1-exp(I*(d*x+c)))*x^2-3/d^2/a*f^3*ln(exp(I*(d*x+c))+1)*x^2-12*f^3*polyl
og(3,I*exp(I*(d*x+c)))/a/d^4-9/2/d^3/a*e*f^2*ln(1-exp(I*(d*x+c)))*c^2+9/2/
d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2-9/2/d/a*e*f^2*ln(exp(I*(d*x+c))+1)*x^2-
9/2/d/a*e^2*f*ln(exp(I*(d*x+c))+1)*x+9/2/d/a*e^2*f*ln(1-exp(I*(d*x+c)))*x-
6/d^2/a*e*f^2*ln(exp(I*(d*x+c))+1)*x-12/d^2/a*e*f^2*ln(1-I*exp(I*(d*x+c)))
*x-9/2*I/a/d^2*f^3*polylog(2,exp(I*(d*x+c)))*x^2+9/2*I/a/d^2*f^3*polylog(2
,-exp(I*(d*x+c)))*x^2+6*I/a/d^3*e*f^2*polylog(2,-exp(I*(d*x+c)))+6*I/a/d^3
*e*f^2*polylog(2,exp(I*(d*x+c)))+12*I/a/d^3*e*f^2*polylog(2,I*exp(I*(d*x+c
)))+6*I/a/d^2*e^2*f*arctan(exp(I*(d*x+c)))-12*I/a/d^3*f^3*c^2*x+6*I/a/d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7842 vs. $2(522) = 1044$.

Time = 0.34 (sec) , antiderivative size = 7842, normalized size of antiderivative = 13.07

$$\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3
*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d
*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)**3/(sin(c
 + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12815 vs. $2(522) = 1044$.

Time = 10.63 (sec) , antiderivative size = 12815, normalized size of antiderivative = 21.36

$$\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(3*c*e^2*f*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)/(a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3) - (4*sin(d*x + c)/(cos(d*x + c) + 1) - s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a*d) + 12*log(sin(d*x + c)/(cos(d*x +
 c) + 1))/(a*d)) + e^3*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^
2/(cos(d*x + c) + 1)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x +
c) + 1))/a) + 8*(48*I*c^2*d*e*f^2 - 16*I*c^3*f^3 - 24*(-I*d^2*e^2*f + 2*I*
c*d*e*f^2 - I*c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(5*d*x + 5*
c) + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*cos(4*d*x + 4*c) + 2*(d^2*
e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(3*d*x + 3*c) + 2*(I*d^2*e^2*f - 2*I*c*d
*e*f^2 + I*c^2*f^3)*cos(2*d*x + 2*c) - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)
*cos(d*x + c) + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*sin(5*d*x + 5*c
) + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*sin(4*d*x + 4*c) + 2*(I*d^2*e^2*f
- 2*I*c*d*e*f^2 + I*c^2*f^3)*sin(3*d*x + 3*c) - 2*(d^2*e^2*f - 2*c*d*e*f^2
 + c^2*f^3)*sin(2*d*x + 2*c) + (-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*
sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 24*(I*(d*x + c)^2*
f^3 + 2*(I*d*e*f^2 - I*c*f^3)*(d*x + c) + ((d*x + c)^2*f^3 + 2*(d*e*f^2 -
c*f^3)*(d*x + c))*cos(5*d*x + 5*c) + (I*(d*x + c)^2*f^3 + 2*(I*d*e*f^2 ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{8\left(\int \frac{\csc(dx+c)^3x^3}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 d\,f^3+8\left(\int \frac{\csc(dx+c)^3x^3}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 d\,f^3+24\left(\int \frac{\csc(dx+c)^3x^2}{\sin(dx+c)+1}dx\right)\tan}{}$$

input
```
int((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(8*int((csc(c + d*x)**3*x**3)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**3*d*
f**3 + 8*int((csc(c + d*x)**3*x**3)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)
**2*d*f**3 + 24*int((csc(c + d*x)**3*x**2)/(sin(c + d*x) + 1),x)*tan((c +
d*x)/2)**3*d*e*f**2 + 24*int((csc(c + d*x)**3*x**2)/(sin(c + d*x) + 1),x)*
tan((c + d*x)/2)**2*d*e*f**2 + 24*int((csc(c + d*x)**3*x)/(sin(c + d*x) +
1),x)*tan((c + d*x)/2)**3*d*e**2*f + 24*int((csc(c + d*x)**3*x)/(sin(c + d
*x) + 1),x)*tan((c + d*x)/2)**2*d*e**2*f + 12*log(tan((c + d*x)/2))*tan((c
 + d*x)/2)**3*e**3 + 12*log(tan((c + d*x)/2))*tan((c + d*x)/2)**2*e**3 + t
an((c + d*x)/2)**5*e**3 - 3*tan((c + d*x)/2)**4*e**3 - 24*tan((c + d*x)/2)
**3*e**3 + 3*tan((c + d*x)/2)*e**3 - e**3)/(8*tan((c + d*x)/2)**2*a*d*(tan
((c + d*x)/2) + 1))
```

3.210 $\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 392

$$\begin{aligned}\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{2i(e+fx)^2}{ad}-\frac{3(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad}\\
&-\frac{f^2\operatorname{arctanh}(\cos(c+dx))}{ad^3}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}\\
&+\frac{(e+fx)^2\cot(c+dx)}{ad}-\frac{f(e+fx)\csc(c+dx)}{ad^2}\\
&-\frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2ad}\\
&-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}\\
&-\frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2}\\
&+\frac{3if(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}\\
&+\frac{4if^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}\\
&-\frac{3if(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}\\
&+\frac{if^2\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}\\
&-\frac{3f^2\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{3f^2\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}\end{aligned}$$

output
```
2*I*(f*x+e)^2/a/d-3*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d-f^2*arctanh(cos(
d*x+c))/a/d^3+(f*x+e)^2*cot(1/2*c+1/4*Pi+1/2*d*x)/a/d+(f*x+e)^2*cot(d*x+c)
/a/d-f*(f*x+e)*csc(d*x+c)/a/d^2-1/2*(f*x+e)^2*cot(d*x+c)*csc(d*x+c)/a/d-4*
f*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d^2-2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/
a/d^2+3*I*f*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a/d^2+4*I*f^2*polylog(2,I*e
xp(I*(d*x+c)))/a/d^3-3*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2+I*f^2*p
olylog(2,exp(2*I*(d*x+c)))/a/d^3-3*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+3*
f^2*polylog(3,exp(I*(d*x+c)))/a/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1452 vs. $2(392) = 784$.

Time = 20.64 (sec) , antiderivative size = 1452, normalized size of antiderivative = 3.70

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(3*e^2*Log[Tan[(c + d*x)/2]])/(2*a*d) + (f^2*Log[Tan[(c + d*x)/2]])/(a*d^3
) + (3*e*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]
) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2,
 E^(I*(c + d*x))])))/(a*d^2) - (3*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Si
n[c + d*x]] - I*d*x*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*Pol
yLog[2, Cos[c + d*x] + I*Sin[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[
c + d*x]] - PolyLog[3, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^3) - (2*e*f*C
sc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*
d^2*(Cos[c]^2 + Sin[c]^2)) + (4*f*(Cos[c] + I*Sin[c])*(((e + f*x)^2*(Cos[c
] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(
1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]
]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(a*d*(Cos[c] + I*(1 + Sin[c]))) + (Csc[
c]*Csc[c + d*x]^2*(-2*e*f*Cos[(d*x)/2] - 2*f^2*x*Cos[(d*x)/2] - 2*e*f*Cos[
(3*d*x)/2] - 2*f^2*x*Cos[(3*d*x)/2] - 5*d*e^2*Cos[c - (d*x)/2] - 10*d*e*f*
x*Cos[c - (d*x)/2] - 5*d*f^2*x^2*Cos[c - (d*x)/2] + d*e^2*Cos[c + (d*x)/2]
 + 2*d*e*f*x*Cos[c + (d*x)/2] + d*f^2*x^2*Cos[c + (d*x)/2] + 2*e*f*Cos[2*c
 + (d*x)/2] + 2*f^2*x*Cos[2*c + (d*x)/2] - d*e^2*Cos[c + (3*d*x)/2] - 2*d*
e*f*x*Cos[c + (3*d*x)/2] - d*f^2*x^2*Cos[c + (3*d*x)/2] + 2*e*f*Cos[2*c +
(3*d*x)/2] + 2*f^2*x*Cos[2*c + (3*d*x)/2] + 3*d*e^2*Cos[3*c + (3*d*x)/2] +
 6*d*e*f*x*Cos[3*c + (3*d*x)/2] + 3*d*f^2*x^2*Cos[3*c + (3*d*x)/2] + 4*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 5046}$$

$$\frac{\int (e+fx)^2\csc^3(c+dx)dx}{a}-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2\csc(c+dx)^3dx}{a}-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{\frac{f^2\int \csc(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc(c+dx)dx-\frac{f(e+fx)\csc(c+dx)}{d^2}-\frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\frac{\frac{f^2\int \csc(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc(c+dx)dx-\frac{f(e+fx)\csc(c+dx)}{d^2}-\frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 4257

$$\frac{\frac{1}{2}\int (e+fx)^2\csc(c+dx)dx-\frac{f^2\text{arctanh}(\cos(c+dx))}{d^3}-\frac{f(e+fx)\csc(c+dx)}{d^2}-\frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 4671

$$-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{1}{2}\left(-\frac{2f\int (e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f^2\text{arctanh}(\cos(c+dx))}{d^3}-}{a}$$

$\downarrow$ 3011

$$-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e}{}\right.}{a}$$

$\downarrow$ 2720

$$-\int \frac{(e+fx)^2\csc^2(c+dx)}{\sin(c+dx)a+a}dx+\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLo}}{}\right.}{d}\right.}{}$$

$\downarrow$ 5046

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\int(e+fx)^2\csc^2(c+dx)dx}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\int(e+fx)^2\csc(c+dx)^2dx}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 4672

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\frac{2f\int(e+fx)\cot(c+dx)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\frac{2f\int-\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{-\frac{2f\int(e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 4202

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 2620

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 2715

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-i(2c+2dx+\pi)}\log\left(1+e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}+\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx$$

$\Big\downarrow$ 2838

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{}$$

$$\int\frac{(e+fx)^2\csc(c+dx)}{\sin(c+dx)a+a}dx-\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\Big\downarrow$ 5046

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{}$$

$$\int\frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{}$$

$$\int\frac{(e+fx)^2}{\sin(c+dx)a+a}dx+\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\Big\downarrow$ 3799

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{}$$

$$\frac{\int(e+fx)^2\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{}$$

$$\frac{\int(e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 4671

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.}{-\frac{2f\int(e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int(e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}}-$$

$$a$$

$$\frac{\int(e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 3011

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.$$

$$\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)}{}$$

$$a$$

$$\frac{\int(e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 2720

$$\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e\right.}{d^2}\right.}{d}$$

$$a$$

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\int(e+fx)^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 4672

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\right.}{d^2}\right.}{d}}{a}$$

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\frac{4f\int(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\downarrow 3042$$

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\right.}{d^2}\right.}{d}}{a}$$

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}}{}\right.}{d}\right.$$

$$\frac{\frac{4f\int-\left((e+fx)\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\downarrow 25$$

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(2,\ldots}{d^2}\ldots\right)}{d}\ldots}{a}$$

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}\ldots}{\ldots}\right)}{d}\ldots\right.}{\frac{-\frac{4f\int(e+fx)\tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 4202

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}(2,\ldots}{d^2}\ldots\right)}{d}\ldots}{a}$$

$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLo}\ldots}{\ldots}\right)}{d}\ldots\right.}{\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{\frac{1}{2}i(2c+2dx+3\pi)}(e+fx)}{1+e^{\frac{1}{2}i(2c+2dx+3\pi)}}dx\right)}{d}}{2a}}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 2620

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\cdots}{d^2}\cdots\right.}{d}\cdots}{a}$$

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\cdots}{\cdots}\right.}{d}\cdots\right.$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)dx}{d}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)}{d}\right)\right)}{d}\Bigg)}{2a}-$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\cdots}{d^2}\cdots\right.}{d}\cdots}{a}$$

$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\cdots}{\cdots}\right.}{d}\cdots\right.$$

$$\frac{-\frac{2(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}-\frac{4f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-\frac{1}{2}i(2c+2dx+3\pi)}\log\left(1+e^{\frac{1}{2}i(2c+2dx+3\pi)}\right)de^{\frac{1}{2}i(2c+2dx+3\pi)}}{d^2}-\frac{i(e+fx)\log\left(1+e^{\frac{1}{2}i(2c+2dx+\cdots}\right.}{d}\cdots\right.\right.}{d}\cdots}{2a}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(359) = 718$.

Time = 1.60 (sec) , antiderivative size = 1257, normalized size of antiderivative = 3.21

method	result	size
risch	Expression too large to display	1257

input
```
int((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/2/d/a*e^2*ln(exp(I*(d*x+c))-1)-3/2/d/a*e^2*ln(exp(I*(d*x+c))+1)+4*I*f^2*
polylog(2,I*exp(I*(d*x+c)))/a/d^3-4/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c-8/a
/d^3*f^2*c*ln(exp(I*(d*x+c)))+8/a/d^2*f*e*ln(exp(I*(d*x+c)))-4/a/d^2*f^2*l
n(1-I*exp(I*(d*x+c)))*x+2*I*f^2*polylog(2,-exp(I*(d*x+c)))/a/d^3-2/d^2/a*f
^2*ln(exp(I*(d*x+c))+1)*x+2/d^3/a*c*f^2*ln(exp(I*(d*x+c))-1)+2/d^3/a*c*f^2
*ln(exp(2*I*(d*x+c))+1)-2/d^2/a*e*f*ln(exp(I*(d*x+c))-1)-2/d^2/a*e*f*ln(ex
p(2*I*(d*x+c))+1)-2/d^2/a*e*f*ln(exp(I*(d*x+c))+1)+3/d/a*e*f*ln(1-exp(I*(d
*x+c)))*x-3/d/a*e*f*ln(exp(I*(d*x+c))+1)*x+3/d^2/a*e*f*ln(1-exp(I*(d*x+c))
)*c-3/d^2/a*c*e*f*ln(exp(I*(d*x+c))-1)-2/d^2/a*f^2*ln(1-exp(I*(d*x+c)))*x-
2/d^3/a*f^2*ln(1-exp(I*(d*x+c)))*c+3*I/a/d^2*f^2*polylog(2,-exp(I*(d*x+c))
)*x+8*I/a/d^2*f^2*c*x-4*I/a/d^3*f^2*c*arctan(exp(I*(d*x+c)))-3*I/a/d^2*f^2
*polylog(2,exp(I*(d*x+c)))*x+4*I/a/d^2*e*f*arctan(exp(I*(d*x+c)))+3*I/a/d^
2*e*f*polylog(2,-exp(I*(d*x+c)))-3*I/a/d^2*e*f*polylog(2,exp(I*(d*x+c)))-3
*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+3*f^2*polylog(3,exp(I*(d*x+c)))/a/d^
3-3/2/d^3/a*f^2*ln(1-exp(I*(d*x+c)))*c^2-3/2/d/a*f^2*ln(exp(I*(d*x+c))+1)*
x^2+3/2/d/a*f^2*ln(1-exp(I*(d*x+c)))*x^2+3/2/d^3/a*c^2*f^2*ln(exp(I*(d*x+c
))-1)+4*I/a/d*f^2*x^2+4*I/a/d^3*f^2*c^2+2*I/a/d^3*f^2*polylog(2,exp(I*(d*x
+c)))+(3*d*f^2*x^2*exp(4*I*(d*x+c))+6*d*e*f*x*exp(4*I*(d*x+c))+3*d*e^2*exp
(4*I*(d*x+c))-5*d*f^2*x^2*exp(2*I*(d*x+c))-2*I*d*e*f*x*exp(I*(d*x+c))-10*d
*e*f*x*exp(2*I*(d*x+c))+2*f^2*x*exp(3*I*(d*x+c))-2*I*f^2*x*exp(4*I*(d*x...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4026 vs. $2(348) = 696$.

Time = 0.21 (sec) , antiderivative size = 4026, normalized size of antiderivative = 10.27

$$\int \frac{(e+fx)^2 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2
*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)**3
/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6160 vs. $2(348) = 696$.

Time = 2.09 (sec) , antiderivative size = 6160, normalized size of antiderivative = 15.71

$$\int \frac{(e+fx)^2 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(2*c*e*f*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1)/(a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3) - (4*sin(d*x + c)/(cos(d*x + c) + 1) - sin
(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a*d) + 12*log(sin(d*x + c)/(cos(d*x + c
) + 1))/(a*d)) + e^2*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c)
 + 1))/a) + 8*(16*I*c^2*f^2 - 16*(-I*d*e*f + I*c*f^2 - (d*e*f - c*f^2)*cos
(5*d*x + 5*c) + (-I*d*e*f + I*c*f^2)*cos(4*d*x + 4*c) + 2*(d*e*f - c*f^2)*
cos(3*d*x + 3*c) + 2*(I*d*e*f - I*c*f^2)*cos(2*d*x + 2*c) - (d*e*f - c*f^2
)*cos(d*x + c) + (-I*d*e*f + I*c*f^2)*sin(5*d*x + 5*c) + (d*e*f - c*f^2)*s
in(4*d*x + 4*c) + 2*(I*d*e*f - I*c*f^2)*sin(3*d*x + 3*c) - 2*(d*e*f - c*f^
2)*sin(2*d*x + 2*c) + (-I*d*e*f + I*c*f^2)*sin(d*x + c))*arctan2(sin(d*x +
 c) + 1, cos(d*x + c)) - 16*((d*x + c)*f^2*cos(5*d*x + 5*c) + I*(d*x + c)*
f^2*cos(4*d*x + 4*c) - 2*(d*x + c)*f^2*cos(3*d*x + 3*c) - 2*I*(d*x + c)*f^
2*cos(2*d*x + 2*c) + (d*x + c)*f^2*cos(d*x + c) + I*(d*x + c)*f^2*sin(5*d*
x + 5*c) - (d*x + c)*f^2*sin(4*d*x + 4*c) - 2*I*(d*x + c)*f^2*sin(3*d*x +
3*c) + 2*(d*x + c)*f^2*sin(2*d*x + 2*c) + I*(d*x + c)*f^2*sin(d*x + c) + I
*(d*x + c)*f^2)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*(-3*I*(d*x ...
```

Giac [F]

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^2\csc{(dx+c)}^3}{a\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csc(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{8\left(\int \frac{\csc(dx+c)^3x^2}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 d\,f^2 + 8\left(\int \frac{\csc(dx+c)^3x^2}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 d\,f^2 + 16\left(\int \frac{\csc(dx+c)^3x}{\sin(dx+c)+1}dx\right)\tan}{}$$

input
```
int((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(8*int((csc(c + d*x)**3*x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**3*d*
f**2 + 8*int((csc(c + d*x)**3*x**2)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)
**2*d*f**2 + 16*int((csc(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*tan((c + d*x
)/2)**3*d*e*f + 16*int((csc(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*tan((c +
d*x)/2)**2*d*e*f + 12*log(tan((c + d*x)/2))*tan((c + d*x)/2)**3*e**2 + 12*
log(tan((c + d*x)/2))*tan((c + d*x)/2)**2*e**2 + tan((c + d*x)/2)**5*e**2
- 3*tan((c + d*x)/2)**4*e**2 - 24*tan((c + d*x)/2)**3*e**2 + 3*tan((c + d*
x)/2)*e**2 - e**2)/(8*tan((c + d*x)/2)**2*a*d*(tan((c + d*x)/2) + 1))
```

3.211 $\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 216

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(c+dx)}{2ad^2} - \frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} - \frac{f\log(\sin(c+dx))}{ad^2} + \frac{3if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{2ad^2} - \frac{3if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{2ad^2}$$

output
```
-3*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+(f*x+e)*cot(1/2*c+1/4*Pi+1/2*d*x)/a
/d+(f*x+e)*cot(d*x+c)/a/d-1/2*f*csc(d*x+c)/a/d^2-1/2*(f*x+e)*cot(d*x+c)*cs
c(d*x+c)/a/d-2*f*ln(sin(1/2*c+1/4*Pi+1/2*d*x))/a/d^2-f*ln(sin(d*x+c))/a/d^
2+3/2*I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2-3/2*I*f*polylog(2,exp(I*(d*x+c)
))/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 484 vs. 2(216) = 432.

Time = 10.73 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.24

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-d(e+fx)\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)-16d(e+fx)\,\mathrm{si}\right.}{}$$

input
```
Integrate[((e + f*x)*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-(d*(e + f*x)*(1 + Cot[(c + d*x)/2
])*Csc[(c + d*x)/2]) - 16*d*(e + f*x)*Sin[(c + d*x)/2] + 8*f*(c + d*x)*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*(-f + 2*d*(e + f*x))*Cot[(c + d*x)/
2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 16*f*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 8*f*Log[Sin[c + d*x
]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12*d*e*Log[Tan[(c + d*x)/2]]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]) - 12*c*f*Log[Tan[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]) + 12*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))]
 - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2
, E^(I*(c + d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*(f + 2*d*(e
 + f*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Tan[(c + d*x)/2] + d*(e + f
*x)*Sec[(c + d*x)/2]*(1 + Tan[(c + d*x)/2])))/(8*a*d^2*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.32, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {5046, 3042, 4673, 3042, 4671, 2715, 2838, 5046, 3042, 4672, 3042, 25, 3956, 5046, 3042, 3799, 3042, 4671, 2715, 2838, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\csc^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \ 5046$$

$$\frac{\int (e+fx)\csc^3(c+dx)dx}{a}-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)\csc(c+dx)^3dx}{a}-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \ 4673$$

$$\frac{\frac{1}{2}\int (e+fx)\csc(c+dx)dx-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{2}\int (e+fx)\csc(c+dx)dx-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2d}}{a}-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx$$

$$\Big\downarrow \ 4671$$

$$-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx+$$
$$\frac{\frac{1}{2}\left(-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2d}}{a}$$

$$\Big\downarrow \ 2715$$

$$-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx+$$
$$\frac{\frac{1}{2}\left(\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\csc(c+dx)}{2d^2}-}{a}$$

$$\Big\downarrow \ 2838$$

$$-\int \frac{(e+fx)\csc^2(c+dx)}{\sin(c+dx)a+a}dx+$$
$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$$\Big\downarrow \ 5046$$

$$\frac{-\frac{\int (e+fx)\csc^2(c+dx)dx}{a}+\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{\int (e+fx)\csc(c+dx)^2dx}{a}+\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$\downarrow$ 4672

$$\frac{-\frac{\frac{f\int \cot(c+dx)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}+\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{f\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}+\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$\downarrow$ 25

$$\frac{-\frac{-\frac{f\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}+\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$\downarrow$ 3956

$$\frac{\int \frac{(e+fx)\csc(c+dx)}{\sin(c+dx)a+a}dx+\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$\downarrow$ 5046

$$-\int \frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\int (e+fx)\csc(c+dx)dx}{a}+$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\int \frac{e+fx}{\sin(c+dx)a+a}dx+\frac{\int (e+fx)\csc(c+dx)dx}{a}+$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3799$$

$$-\frac{\int (e+fx)\csc^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{2a}+\frac{\int (e+fx)\csc(c+dx)dx}{a}+$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\int (e+fx)\csc(c+dx)dx}{a}+$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}$$

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 4671$$

$$\frac{-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$\downarrow$ 2715

$$\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$\downarrow$ 2838

$$-\frac{\int (e+fx)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)^2dx}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$\downarrow$ 4672

$$-\frac{\frac{2f\int \cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2f\int -\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{3\pi}{4}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\;\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{2f\int \tan\left(\frac{1}{4}(2c+3\pi)+\frac{dx}{2}\right)dx}{d}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\;\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3956$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}+\frac{\frac{1}{2}\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)-\frac{f\csc(c+dx)}{2d^2}-\frac{(e+fx)\cot(c+dx)\csc(c+dx}{2d}}{a}\;\frac{\frac{4f\log\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)\right)}{d^2}-\frac{2(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{d}}{2a}-\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}$$

input
```
Int[((e + f*x)*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((-2*(e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/d + (4*f*Log[-Cos[c/2 - Pi/
4 + (d*x)/2]])/d^2)/a - (-(((e + f*x)*Cot[c + d*x])/d) + (f*Log[-Sin[c + d
*x]])/d^2)/a + ((-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + (I*f*PolyLog[2
, -E^(I*(c + d*x))])/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])/d^2)/a + (-1/
2*(f*Csc[c + d*x])/d^2 - ((e + f*x)*Cot[c + d*x]*Csc[c + d*x])/(2*d) + ((-
2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + (I*f*PolyLog[2, -E^(I*(c + d*x))
])/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])/d^2)/2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(188) = 376$.

Time = 1.43 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.25

method	result
risch	$\frac{3dfx\,\mathrm{e}^{4i(dx+c)}+3de\,\mathrm{e}^{4i(dx+c)}-5dfx\,\mathrm{e}^{2i(dx+c)}+3idfx\,\mathrm{e}^{3i(dx+c)}-5de\,\mathrm{e}^{2i(dx+c)}+f\,\mathrm{e}^{3i(dx+c)}+3ide\,\mathrm{e}^{3i(dx+c)}-if\,\mathrm{e}^{4i(dx+c)}+4dxf-i}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^{2}d^{2}\left(\mathrm{e}^{i(dx+c)}+i\right)a}$

input
```
int((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(3*d*f*x*exp(4*I*(d*x+c))+3*d*e*exp(4*I*(d*x+c))-5*d*f*x*exp(2*I*(d*x+c))+
3*I*d*f*x*exp(3*I*(d*x+c))-5*d*e*exp(2*I*(d*x+c))+f*exp(3*I*(d*x+c))+3*I*d
*e*exp(3*I*(d*x+c))-I*f*exp(4*I*(d*x+c))+4*d*x*f-I*d*f*x*exp(I*(d*x+c))+4*
d*e-exp(I*(d*x+c))*f-I*d*e*exp(I*(d*x+c))+I*f*exp(2*I*(d*x+c)))/(exp(2*I*(
d*x+c))-1)^2/d^2/(exp(I*(d*x+c))+I)/a-3/2/d/a*f*ln(exp(I*(d*x+c))+1)*x+3/2
/d/a*f*ln(1-exp(I*(d*x+c)))*x+2*I/d^2/a*f*arctan(exp(I*(d*x+c)))+4/d^2/a*f
*ln(exp(I*(d*x+c)))-3/2/d/a*e*ln(exp(I*(d*x+c))+1)+3/2/d^2/a*f*ln(1-exp(I*
(d*x+c)))*c-3/2/d^2/a*c*f*ln(exp(I*(d*x+c))-1)+3/2/d/a*e*ln(exp(I*(d*x+c))
-1)+3/2*I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2-3/2*I*f*polylog(2,exp(I*(d*x+
c)))/a/d^2-1/d^2/a*f*ln(exp(I*(d*x+c))+1)-1/d^2/a*f*ln(exp(I*(d*x+c))-1)-1
/d^2/a*f*ln(exp(2*I*(d*x+c))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(184) = 368$.

Time = 0.15 (sec) , antiderivative size = 1359, normalized size of antiderivative = 6.29

$$\int \frac{(e + fx)\csc^3(c + dx)}{a + a\sin(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(8*(d*f*x + d*e)*cos(d*x + c)^3 - 4*d*f*x + 2*(3*d*f*x + 3*d*e - f)*co
s(d*x + c)^2 - 4*d*e - 6*(d*f*x + d*e)*cos(d*x + c) - 3*(I*f*cos(d*x + c)^
3 + I*f*cos(d*x + c)^2 - I*f*cos(d*x + c) + (I*f*cos(d*x + c)^2 - I*f)*sin
(d*x + c) - I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*f*cos(d*x +
c)^3 - I*f*cos(d*x + c)^2 + I*f*cos(d*x + c) + (-I*f*cos(d*x + c)^2 + I*f)
*sin(d*x + c) + I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) - 3*(I*f*cos(d*x
 + c)^3 + I*f*cos(d*x + c)^2 - I*f*cos(d*x + c) + (I*f*cos(d*x + c)^2 - I*
f)*sin(d*x + c) - I*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 3*(-I*f*cos
(d*x + c)^3 - I*f*cos(d*x + c)^2 + I*f*cos(d*x + c) + (-I*f*cos(d*x + c)^2
 + I*f)*sin(d*x + c) + I*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) - ((3*d*
f*x + 3*d*e + 2*f)*cos(d*x + c)^3 - 3*d*f*x + (3*d*f*x + 3*d*e + 2*f)*cos(
d*x + c)^2 - 3*d*e - (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c) - (3*d*f*x - (3*
d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2 + 3*d*e + 2*f)*sin(d*x + c) - 2*f)*log
(cos(d*x + c) + I*sin(d*x + c) + 1) - ((3*d*f*x + 3*d*e + 2*f)*cos(d*x + c
)^3 - 3*d*f*x + (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2 - 3*d*e - (3*d*f*x
+ 3*d*e + 2*f)*cos(d*x + c) - (3*d*f*x - (3*d*f*x + 3*d*e + 2*f)*cos(d*x +
 c)^2 + 3*d*e + 2*f)*sin(d*x + c) - 2*f)*log(cos(d*x + c) - I*sin(d*x + c)
 + 1) + ((3*d*e - (3*c + 2)*f)*cos(d*x + c)^3 + (3*d*e - (3*c + 2)*f)*cos(
d*x + c)^2 - 3*d*e + (3*c + 2)*f - (3*d*e - (3*c + 2)*f)*cos(d*x + c) + ((
3*d*e - (3*c + 2)*f)*cos(d*x + c)^2 - 3*d*e + (3*c + 2)*f)*sin(d*x + c)...
```

Sympy [F]

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\int \frac{e\csc^3(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{fx\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c +
d*x)**3/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2080 vs. $2(184) = 368$.

Time = 0.59 (sec) , antiderivative size = 2080, normalized size of antiderivative = 9.63

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(16*d*f*x*cos(5*d*x + 5*c) + 16*I*d*f*x*sin(5*d*x + 5*c) - 16*I*d*e - 8*(f
*cos(5*d*x + 5*c) + I*f*cos(4*d*x + 4*c) - 2*f*cos(3*d*x + 3*c) - 2*I*f*co
s(2*d*x + 2*c) + f*cos(d*x + c) + I*f*sin(5*d*x + 5*c) - f*sin(4*d*x + 4*c
) - 2*I*f*sin(3*d*x + 3*c) + 2*f*sin(2*d*x + 2*c) + I*f*sin(d*x + c) + I*f
)*arctan2(cos(c) + sin(d*x), cos(d*x) + sin(c)) + 2*(-3*I*d*f*x - 3*I*d*e
- (3*d*f*x + 3*d*e + 2*f)*cos(5*d*x + 5*c) + (-3*I*d*f*x - 3*I*d*e - 2*I*f
)*cos(4*d*x + 4*c) + 2*(3*d*f*x + 3*d*e + 2*f)*cos(3*d*x + 3*c) + 2*(3*I*d
*f*x + 3*I*d*e + 2*I*f)*cos(2*d*x + 2*c) - (3*d*f*x + 3*d*e + 2*f)*cos(d*x
 + c) + (-3*I*d*f*x - 3*I*d*e - 2*I*f)*sin(5*d*x + 5*c) + (3*d*f*x + 3*d*e
 + 2*f)*sin(4*d*x + 4*c) + 2*(3*I*d*f*x + 3*I*d*e + 2*I*f)*sin(3*d*x + 3*c
) - 2*(3*d*f*x + 3*d*e + 2*f)*sin(2*d*x + 2*c) + (-3*I*d*f*x - 3*I*d*e - 2
*I*f)*sin(d*x + c) - 2*I*f)*arctan2(sin(d*x + c), cos(d*x + c) + 1) + 2*(3
*I*d*e + (3*d*e - 2*f)*cos(5*d*x + 5*c) + (3*I*d*e - 2*I*f)*cos(4*d*x + 4*
c) - 2*(3*d*e - 2*f)*cos(3*d*x + 3*c) + 2*(-3*I*d*e + 2*I*f)*cos(2*d*x + 2
*c) + (3*d*e - 2*f)*cos(d*x + c) + (3*I*d*e - 2*I*f)*sin(5*d*x + 5*c) - (3
*d*e - 2*f)*sin(4*d*x + 4*c) + 2*(-3*I*d*e + 2*I*f)*sin(3*d*x + 3*c) + 2*(
3*d*e - 2*f)*sin(2*d*x + 2*c) + (3*I*d*e - 2*I*f)*sin(d*x + c) - 2*I*f)*ar
ctan2(sin(d*x + c), cos(d*x + c) - 1) - 6*(d*f*x*cos(5*d*x + 5*c) + I*d*f*
x*cos(4*d*x + 4*c) - 2*d*f*x*cos(3*d*x + 3*c) - 2*I*d*f*x*cos(2*d*x + 2*c)
 + d*f*x*cos(d*x + c) + I*d*f*x*sin(5*d*x + 5*c) - d*f*x*sin(4*d*x + 4*...
```

Giac [F]

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)\csc(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csc(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{8\left(\int \frac{\csc(dx+c)^3 x}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 df + 8\left(\int \frac{\csc(dx+c)^3 x}{\sin(dx+c)+1}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 df + 12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{d}{2}\right.}{8\tan\left(\right.}$$

input
```
int((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(8*int((csc(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**3*d*f +
 8*int((csc(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*tan((c + d*x)/2)**2*d*f +
 12*log(tan((c + d*x)/2))*tan((c + d*x)/2)**3*e + 12*log(tan((c + d*x)/2))
*tan((c + d*x)/2)**2*e + tan((c + d*x)/2)**5*e - 3*tan((c + d*x)/2)**4*e -
 24*tan((c + d*x)/2)**3*e + 3*tan((c + d*x)/2)*e - e)/(8*tan((c + d*x)/2)*
*2*a*d*(tan((c + d*x)/2) + 1))
```

3.212 $\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3\text{arctanh}(\cos(c+dx))}{2ad}+\frac{2\cot(c+dx)}{ad}-\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))}$$

output
```
-3/2*arctanh(cos(d*x+c))/a/d+2*cot(d*x+c)/a/d-3/2*cot(d*x+c)*csc(d*x+c)/a/
d+cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{-4\csc(2(c+dx))-3\sec(c+dx)+3\text{arctanh}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)+\csc^2(c+}{2ad}$$

input
```
Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*(-4*Csc[2*(c + d*x)] - 3*Sec[c + d*x] + 3*ArcTanh[Sqrt[Cos[c + d*x]^2
]]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + Csc[c + d*x]^2*Sec[c + d*x] + 4*Tan
[c + d*x])/(a*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^3(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3247}$$

$$\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\int -\csc^3(c+dx)(3a-2a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \csc^3(c+dx)(3a-2a\sin(c+dx))dx}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-2a\sin(c+dx)}{\sin(c+dx)^3}dx}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{3a\int \csc^3(c+dx)dx-2a\int \csc^2(c+dx)dx}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \csc(c+dx)^3dx-2a\int \csc(c+dx)^2dx}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{2a\int 1d\cot(c+dx)}{d}+3a\int \csc(c+dx)^3dx}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{3a\int \csc(c+dx)^3dx+\frac{2a\cot(c+dx)}{d}}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3a\left(\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)+\frac{2a\cot(c+dx)}{d}}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)+\frac{2a\cot(c+dx)}{d}}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3a\left(-\frac{\text{arctanh}(\cos(c+dx))}{2d}-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)+\frac{2a\cot(c+dx)}{d}}{a^2}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

input
```
Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
((2*a*Cot[c + d*x])/d + 3*a*(-1/2*ArcTanh[Cos[c + d*x]]/d - (Cot[c + d*x]*
Csc[c + d*x])/(2*d)))/a^2 + (Cot[c + d*x]*Csc[c + d*x])/(d*(a + a*Sin[c +
d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$
parallelrisch	$\frac{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-24\tan\left(\frac{dx}{2}+\right.}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)ad}$
risch	$\frac{-ie^{i(dx+c)}+3ie^{3i(dx+c)}-5\,e^{2i(dx+c)}+3\,e^{4i(dx+c)}+4}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)ad}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2ad}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{2ad}$
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$

input
```
int(csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a*(1/2*tan(1/2*d*x+1/2*c)^2-2*tan(1/2*d*x+1/2*c)+8/(tan(1/2*d*x+1/2*
c)+1)-1/2/tan(1/2*d*x+1/2*c)^2+2/tan(1/2*d*x+1/2*c)+6*ln(tan(1/2*d*x+1/2*c
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(78) = 156$.

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.83

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{8\cos(dx+c)^3+6\cos(dx+c)^2-3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\right.}{}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(8*cos(d*x + c)^3 + 6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 + cos(d*x + c
)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*
sin(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(4*cos(d
*x + c)^2 + cos(d*x + c) - 2)*sin(d*x + c) - 6*cos(d*x + c) - 4)/(a*d*cos(
d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x +
c)^2 - a*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\int \frac{\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(78) = 156$.

Time = 0.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.91

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{8d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{16}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x + 1/2*c)^2 - 4*a
*tan(1/2*d*x + 1/2*c))/a^2 + 16/(a*(tan(1/2*d*x + 1/2*c) + 1)) - (18*tan(1
/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2*c)^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 36.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}+\frac{10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{2}}{d\,\left(4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

input
```
int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)^2/(8*a*d) + (3*log(tan(c/2 + (d*x)/2)))/(2*a*d) - tan(c
/2 + (d*x)/2)/(2*a*d) + ((3*tan(c/2 + (d*x)/2))/2 + 10*tan(c/2 + (d*x)/2)^
2 - 1/2)/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 4*a*tan(c/2 + (d*x)/2)^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.56

$$\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-3\tan\left(\frac{dx}{2}+\right.}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int(csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(12*log(tan((c + d*x)/2))*tan((c + d*x)/2)**3 + 12*log(tan((c + d*x)/2))*t
an((c + d*x)/2)**2 + tan((c + d*x)/2)**5 - 3*tan((c + d*x)/2)**4 - 24*tan(
(c + d*x)/2)**3 + 3*tan((c + d*x)/2) - 1)/(8*tan((c + d*x)/2)**2*a*d*(tan(
(c + d*x)/2) + 1))
```

3.213 $\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 5048$$
$$\int \frac{\csc^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+a\sin \left(dx+c\right)\right)}dx$$

input
```
int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\csc \left(dx+c\right)^{3}}{(fx+e)(a\sin \left(dx+c\right)+a)}\,dx$$

input
```
integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\frac{\int \frac{\csc^{3}\left(c+dx\right)}{e\sin \left(c+dx\right)+e+fx\sin \left(c+dx\right)+fx}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**3/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)
/a
```

Maxima [N/A]

Not integrable

Time = 11.67 (sec) , antiderivative size = 7381, normalized size of antiderivative = 263.61

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc{(dx+c)}^3}{(fx+e)(a\sin{(dx+c)}+a)}\,dx$$

input
```
integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(f*cos(4*d*x + 4*c)^2 + 2*f*cos(3*d*x + 3*c)^2 + 2*f*cos(2*d*x + 2*c)^2 +
f*cos(d*x + c)^2 + f*sin(4*d*x + 4*c)^2 + 2*f*sin(3*d*x + 3*c)^2 + 2*f*sin
(2*d*x + 2*c)^2 + f*sin(d*x + c)^2 + (4*d*f*x + 4*d*e + 3*(d*f*x + d*e)*co
s(4*d*x + 4*c) - f*cos(3*d*x + 3*c) - 5*(d*f*x + d*e)*cos(2*d*x + 2*c) + f
*cos(d*x + c) - f*sin(4*d*x + 4*c) - 3*(d*f*x + d*e)*sin(3*d*x + 3*c) + f*
sin(2*d*x + 2*c) + (d*f*x + d*e)*sin(d*x + c))*cos(5*d*x + 5*c) - (3*(d*f*
x + d*e)*cos(3*d*x + 3*c) + 3*f*cos(2*d*x + 2*c) - 2*(d*f*x + d*e)*cos(d*x
 + c) + 3*f*sin(3*d*x + 3*c) - (d*f*x + d*e)*sin(2*d*x + 2*c) - 2*f*sin(d*
x + c) - f)*cos(4*d*x + 4*c) - (5*d*f*x + 5*d*e - 4*(d*f*x + d*e)*cos(2*d*
x + 2*c) + 3*f*cos(d*x + c) + 4*f*sin(2*d*x + 2*c) - (d*f*x + d*e)*sin(d*x
 + c))*cos(3*d*x + 3*c) - (3*(d*f*x + d*e)*cos(d*x + c) + 3*f*sin(d*x + c)
 + f)*cos(2*d*x + 2*c) + 3*(d*f*x + d*e)*cos(d*x + c) + (a*d^2*f^2*x^2 + 2
*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos
(5*d*x + 5*c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(4*d*x +
4*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(3*d*x + 3*c)^2
+ 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + (a*d^
2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)^2 + (a*d^2*f^2*x^2 + 2
*a*d^2*e*f*x + a*d^2*e^2)*sin(5*d*x + 5*c)^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*
f*x + a*d^2*e^2)*sin(4*d*x + 4*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a
*d^2*e^2)*sin(3*d*x + 3*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 37.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{1}{\sin(c+d\,x)^3\,(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(sin(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(sin(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc(dx+c)^3}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(c + d*x)**3/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.214 $\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(dx+c\right)^{3}}{\left(fx+e\right)^{2} \left(a+a\sin \left(dx+c\right)\right)}dx$$

input
```
int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\csc \left(dx+c\right)^{3}}{\left(fx+e\right)^{2}\left(a\sin \left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*
e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$
$$=\frac{\int \frac{\csc^{3}\left(c+dx\right)}{e^{2}\sin \left(c+dx\right)+e^{2}+2efx\sin \left(c+dx\right)+2efx+f^{2}x^{2}\sin \left(c+dx\right)+f^{2}x^{2}}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**3/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x)
+ 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 24.48 (sec) , antiderivative size = 9726, normalized size of antiderivative = 347.36

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\csc(dx+c)^3}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(2*f*cos(4*d*x + 4*c)^2 + 4*f*cos(3*d*x + 3*c)^2 + 4*f*cos(2*d*x + 2*c)^2
+ 2*f*cos(d*x + c)^2 + 2*f*sin(4*d*x + 4*c)^2 + 4*f*sin(3*d*x + 3*c)^2 + 4
*f*sin(2*d*x + 2*c)^2 + 2*f*sin(d*x + c)^2 + (4*d*f*x + 4*d*e + 3*(d*f*x +
 d*e)*cos(4*d*x + 4*c) - 2*f*cos(3*d*x + 3*c) - 5*(d*f*x + d*e)*cos(2*d*x
+ 2*c) + 2*f*cos(d*x + c) - 2*f*sin(4*d*x + 4*c) - 3*(d*f*x + d*e)*sin(3*d
*x + 3*c) + 2*f*sin(2*d*x + 2*c) + (d*f*x + d*e)*sin(d*x + c))*cos(5*d*x +
 5*c) - (3*(d*f*x + d*e)*cos(3*d*x + 3*c) + 6*f*cos(2*d*x + 2*c) - 2*(d*f*
x + d*e)*cos(d*x + c) + 6*f*sin(3*d*x + 3*c) - (d*f*x + d*e)*sin(2*d*x + 2
*c) - 4*f*sin(d*x + c) - 2*f)*cos(4*d*x + 4*c) - (5*d*f*x + 5*d*e - 4*(d*f
*x + d*e)*cos(2*d*x + 2*c) + 6*f*cos(d*x + c) + 8*f*sin(2*d*x + 2*c) - (d*
f*x + d*e)*sin(d*x + c))*cos(3*d*x + 3*c) - (3*(d*f*x + d*e)*cos(d*x + c)
+ 6*f*sin(d*x + c) + 2*f)*cos(2*d*x + 2*c) + 3*(d*f*x + d*e)*cos(d*x + c)
+ (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^
2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(5*d*x + 5
*c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*
cos(4*d*x + 4*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*
x + a*d^2*e^3)*cos(3*d*x + 3*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c)^2 + (a*d^2*f^3*x^3 + 3*a*d^
2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + (a*d^2*f^3*x^3
 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(5*d*x + 5*c)^2 ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 38.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{1}{\sin(c+d\,x)^3\,(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(sin(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(sin(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\csc(dx+c)^3}{\sin(dx+c)e^2+2\sin(dx+c)efx+\sin(dx+c)f^2x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(csc(c + d*x)**3/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*
x)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.215 $\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 11.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\sin^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sin (dx+c)^2}{a+a\sin (dx+c)}dx$$

input
```
int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin (dx+c)^2}{a\sin (dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 13.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sin(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin\left(dx+c\right)^2}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin\left(dx+c\right)^2}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 36.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\sin\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^m}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((sin(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 3499, normalized size of antiderivative = 124.96

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output

```
(4*( - 2*(e + f*x)**m - int((e + f*x)**m/(tan((c + d*x)/2)**4*e + tan((c +
 d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*f*x + 2*
tan((c + d*x)/2)**2*e + 2*tan((c + d*x)/2)**2*f*x + 2*tan((c + d*x)/2)*e +
 2*tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x)/2)**3*d*e + 2*int((e +
 f*x)**m/(tan((c + d*x)/2)**4*e + tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x
)/2)**3*e + 2*tan((c + d*x)/2)**3*f*x + 2*tan((c + d*x)/2)**2*e + 2*tan((c
 + d*x)/2)**2*f*x + 2*tan((c + d*x)/2)*e + 2*tan((c + d*x)/2)*f*x + e + f*
x),x)*tan((c + d*x)/2)**3*f*m - int((e + f*x)**m/(tan((c + d*x)/2)**4*e +
tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*
f*x + 2*tan((c + d*x)/2)**2*e + 2*tan((c + d*x)/2)**2*f*x + 2*tan((c + d*x
)/2)*e + 2*tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x)/2)**2*d*e + 2*
int((e + f*x)**m/(tan((c + d*x)/2)**4*e + tan((c + d*x)/2)**4*f*x + 2*tan(
(c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*f*x + 2*tan((c + d*x)/2)**2*e +
2*tan((c + d*x)/2)**2*f*x + 2*tan((c + d*x)/2)*e + 2*tan((c + d*x)/2)*f*x
+ e + f*x),x)*tan((c + d*x)/2)**2*f*m - int((e + f*x)**m/(tan((c + d*x)/2)
**4*e + tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x
)/2)**3*f*x + 2*tan((c + d*x)/2)**2*e + 2*tan((c + d*x)/2)**2*f*x + 2*tan(
(c + d*x)/2)*e + 2*tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x)/2)*d*e
 + 2*int((e + f*x)**m/(tan((c + d*x)/2)**4*e + tan((c + d*x)/2)**4*f*x + 2
*tan((c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*f*x + 2*tan((c + d*x)/2)...
```

3.216 $\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \operatorname{Int}\left(\frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \ 5048$$

$$\int \frac{\sin(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sin(dx+c)}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sin(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 36.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)\,\left(e+f\,x\right)^m}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 14.50

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= \frac{(fx+e)^m \tan\left(\frac{dx}{2}+\frac{c}{2}\right) de + (fx+e)^m \tan\left(\frac{dx}{2}+\frac{c}{2}\right) dfx - 2(fx+e)^m \tan\left(\frac{dx}{2}+\frac{c}{2}\right) fm - 2(fx+e)^m \mathrm{ta}}{}$$

input
```
int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
((e + f*x)**m*tan((c + d*x)/2)*d*e + (e + f*x)**m*tan((c + d*x)/2)*d*f*x -
 2*(e + f*x)**m*tan((c + d*x)/2)*f*m - 2*(e + f*x)**m*tan((c + d*x)/2)*f +
 (e + f*x)**m*d*e + (e + f*x)**m*d*f*x + 2*int(((e + f*x)**m*tan((c + d*x)
/2))/(tan((c + d*x)/2)*e + tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x
)/2)*f**2*m**2 + 2*int(((e + f*x)**m*tan((c + d*x)/2))/(tan((c + d*x)/2)*e
 + tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x)/2)*f**2*m + 2*int(((e
+ f*x)**m*tan((c + d*x)/2))/(tan((c + d*x)/2)*e + tan((c + d*x)/2)*f*x + e
 + f*x),x)*f**2*m**2 + 2*int(((e + f*x)**m*tan((c + d*x)/2))/(tan((c + d*x
)/2)*e + tan((c + d*x)/2)*f*x + e + f*x),x)*f**2*m)/(a*d*f*(tan((c + d*x)/
2)*m + tan((c + d*x)/2) + m + 1))
```

3.217 $\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^m}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(e+fx)^m}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx=\frac{\int \frac{(e+fx)^m}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 36.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx=\int \frac{(e+f\,x)^m}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)^m/(a + a*sin(c + d*x)),x)
```

output
```
int((e + f*x)^m/(a + a*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.70

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \frac{2(fx+e)^m\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\int \frac{(fx+e)^m\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)fx+e+fx}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)fm-2\left(\int \frac{(fx+e)^m\,\mathrm{t}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e+\tan}\right.}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

output
```
(2*((e + f*x)**m*tan((c + d*x)/2) - int(((e + f*x)**m*tan((c + d*x)/2))/(t
an((c + d*x)/2)*e + tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x)/2)*f*
m - int(((e + f*x)**m*tan((c + d*x)/2))/(tan((c + d*x)/2)*e + tan((c + d*x
)/2)*f*x + e + f*x),x)*f*m))/(a*d*(tan((c + d*x)/2) + 1))
```

3.218 $\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 40.49 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)(e+fx)^m}{a\sin(c+dx)+a} \, dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\csc(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \csc (dx+c)}{a+a\sin (dx+c)}dx$$

input
```
int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc (dx+c)}{a\sin (dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 11.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \csc (c+dx)}{\sin (c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 36.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{\sin\left(c+d\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input

```
int((e + f*x)^m/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

output

```
int((e + f*x)^m/(sin(c + d*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(fx+e)^m \csc(dx+c)}{\sin(dx+c)+1}dx}{a}$$

input

```
int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

output

```
int(((e + f*x)**m*csc(c + d*x))/(sin(c + d*x) + 1),x)/a
```

3.219 $\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 45.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}\,dx$$
$$\Big\downarrow \text{5048}$$
$$\int \frac{\csc^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \csc(dx+c)^2}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 46.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m \csc\left(dx+c\right)^2}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m \csc\left(dx+c\right)^2}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 36.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{\sin\left(c+d\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(sin(c + d*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 6170, normalized size of antiderivative = 220.36

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output

```
(2*(e + f*x)**m*tan((c + d*x)/2)**3*f**2*m**2 + 2*(e + f*x)**m*tan((c + d*
x)/2)**3*f**2*m + 3*(e + f*x)**m*tan((c + d*x)/2)**2*d**2*e**2 + 3*(e + f*
x)**m*tan((c + d*x)/2)**2*d**2*e*f*x - 2*(e + f*x)**m*tan((c + d*x)/2)**2*
d*e*f*m - 2*(e + f*x)**m*tan((c + d*x)/2)**2*d*f**2*m*x - 2*(e + f*x)**m*t
an((c + d*x)/2)**2*f**2*m**2 - 2*(e + f*x)**m*tan((c + d*x)/2)**2*f**2*m +
 3*(e + f*x)**m*tan((c + d*x)/2)*d**2*e**2 + 3*(e + f*x)**m*tan((c + d*x)/
2)*d**2*e*f*x + 4*(e + f*x)**m*tan((c + d*x)/2)*d*e*f*m + 6*(e + f*x)**m*t
an((c + d*x)/2)*d*e*f - 2*(e + f*x)**m*tan((c + d*x)/2)*d*f**2*m*x - 10*(e
 + f*x)**m*tan((c + d*x)/2)*f**2*m**2 - 10*(e + f*x)**m*tan((c + d*x)/2)*f
**2*m + 10*(e + f*x)**m*f**2*m**2 + 10*(e + f*x)**m*f**2*m + 6*int((e + f*
x)**m/(tan((c + d*x)/2)**4*e + tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2
)**3*e + 2*tan((c + d*x)/2)**3*f*x + tan((c + d*x)/2)**2*e + tan((c + d*x)
/2)**2*f*x),x)*tan((c + d*x)/2)**2*d*e*f**2*m**2 + 6*int((e + f*x)**m/(tan
((c + d*x)/2)**4*e + tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2
*tan((c + d*x)/2)**3*f*x + tan((c + d*x)/2)**2*e + tan((c + d*x)/2)**2*f*x
),x)*tan((c + d*x)/2)**2*d*e*f**2*m + 6*int((e + f*x)**m/(tan((c + d*x)/2)
**4*e + tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x
)/2)**3*f*x + tan((c + d*x)/2)**2*e + tan((c + d*x)/2)**2*f*x),x)*tan((c +
 d*x)/2)*d*e*f**2*m**2 + 6*int((e + f*x)**m/(tan((c + d*x)/2)**4*e + tan((
c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*f*...
```

3.220 $\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 544

$$\begin{aligned}\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{(e+fx)^4}{4bf}+\frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} \\ &\quad-\frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} \\ &\quad+\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2} \\ &\quad-\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2} \\ &\quad+\frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3} \\ &\quad-\frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3} \\ &\quad-\frac{6af^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^4} \\ &\quad+\frac{6af^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4/b/f+I*a*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/b/(a^2-b^2)^(1/2)/d-I*a*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/b/(a^2-b^2)^(1/2)/d+3*a*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(
a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2-3*a*f*(f*x+e)^2*polylog(2,I*b*ex
p(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2+6*I*a*f^2*(f*x+e)*
polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^3-6*
I*a*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b
^2)^(1/2)/d^3-6*a*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/
(a^2-b^2)^(1/2)/d^4+6*a*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/b/(a^2-b^2)^(1/2)/d^4
```

Mathematica [A] (verified)

Time = 4.69 (sec) , antiderivative size = 956, normalized size of antiderivative = 1.76

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b} - \frac{a\left(2\sqrt{-a^2+b^2}d^3e^3\arctan\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)+3\sqrt{a^2-b^2}d^3e^2fx\log\left(1-\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)+3\sqrt{a^2-b^2}d^3ef^2\right.}{}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (a*(2*Sqrt[-a^2 +
b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*Sqrt[a^
2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2
])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a
+ Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*
x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 +
(b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^
2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b
^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - (3
*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)
*a + Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[
2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*
e*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt
[a^2 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b
^2])] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a +
Sqrt[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c +
d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (
b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^
3*PolyLog[4, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(b*Sqrt[-(
a^2 - b^2)^2]*d^4)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.89, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5026, 17, 3042, 3804, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sin(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^3dx}{b}-\frac{a\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}$$

$\big\downarrow$ 3042

$$\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}$$

$\big\downarrow$ 3804

$$\frac{(e+fx)^4}{4bf}-\frac{2a\int \frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}$$

$\big\downarrow$ 2694

$$\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}$$

$\big\downarrow$ 27

$$\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}$$

$\big\downarrow$ 2620

$$\frac{(e+fx)^4}{4bf}-$$
$$\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{b}$$

$\big\downarrow$ 3011

$$\frac{\frac{(e+fx)^4}{4bf}-2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ib}{a-\cdots}\right)}{bd}\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 7163

$$\frac{\frac{(e+fx)^4}{4bf}-2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2-b^2}}\cdots\right.}{\cdots}$$

$\downarrow$ 2720

$$\frac{(e+fx)^4}{4bf}-$$

$$2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLo}}{}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 7143

$$\frac{(e+fx)^4}{4bf}-$$

$$2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

input
```
Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x
)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*PolyLog[
4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^
2 - b^2] + ((I/2)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*
x)))/(a + Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 - b^2]))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sin(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2322 vs. $2(468) = 936$.

Time = 0.27 (sec) , antiderivative size = 2322, normalized size of antiderivative = 4.27

$$\int \frac{(e+fx)^3 \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*((a^2 - b^2)*d^4*f^3*x^4 + 4*(a^2 - b^2)*d^4*e*f^2*x^3 + 6*(a^2 - b^2)
*d^4*e^2*f*x^2 + 4*(a^2 - b^2)*d^4*e^3*x + 12*I*a*b*f^3*sqrt(-(a^2 - b^2)/
b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a*b*f^3*sqrt(-(a^2 - b^2
)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a*b*f^3*sqrt(-(a^2 - b
^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*I*a*b*f^3*sqrt(-(a^2
- b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x +
 c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(-I*a*b*d^2*f^3*x^2
 - 2*I*a*b*d^2*e*f^2*x - I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*
a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d^2*f^3*x^2 + 2*I*a*b*d^2*e*f^2*
x + I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + 6*(I*a*b*d^2*f^3*x^2 + 2*I*a*b*d^2*e*f^2*x + I*a*b*d^2*e^2*f)*
sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(-I*a*
b*d^2*f^3*x^2 - 2*I*a*b*d^2*e*f^2*x - I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b
^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^3\sin{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)\,(e+f\,x)^3}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a\,e^3+\left(\int \frac{\sin(dx+c)x^3}{\sin(dx+c)b+a}dx\right)a^2bd\,f^3-\left(\int \frac{\sin(dx+c)x^3}{\sin(dx+c)b+a}dx\right)b^3d\,f^3+3\left(\int \frac{\sin}{\sin}\right.}{}$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
*e**3 + int((sin(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*a**2*b*d*f**3 - in
t((sin(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*b**3*d*f**3 + 3*int((sin(c +
 d*x)*x**2)/(sin(c + d*x)*b + a),x)*a**2*b*d*e*f**2 - 3*int((sin(c + d*x)*
x**2)/(sin(c + d*x)*b + a),x)*b**3*d*e*f**2 + 3*int((sin(c + d*x)*x)/(sin(
c + d*x)*b + a),x)*a**2*b*d*e**2*f - 3*int((sin(c + d*x)*x)/(sin(c + d*x)*
b + a),x)*b**3*d*e**2*f + a**2*d*e**3*x - b**2*d*e**3*x)/(b*d*(a**2 - b**2
))
```

3.221 $\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 408

$$\begin{aligned}\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{(e+fx)^3}{3bf}+\frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}\\
&\quad-\frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}\\
&\quad+\frac{2af(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}\\
&\quad-\frac{2af(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}\\
&\quad+\frac{2iaf^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3}\\
&\quad-\frac{2iaf^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3/b/f+I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/b/(a^2-b^2)^(1/2)/d-I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/b/(a^2-b^2)^(1/2)/d+2*a*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-
(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2-2*a*f*(f*x+e)*polylog(2,I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2+2*I*a*f^2*polylog(3,I*
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^3-2*I*a*f^2*poly
log(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b} - \frac{ia\left(-2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)+2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{ia+\sqrt{-a^2+b^2}}\right)\right.}{}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (I*a*(-2*Sqrt[a^2 - b^2]*d*f*(e +
f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[
a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2
 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)
))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c
+ d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a +
 Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)
))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^
(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(b*Sqrt[-(a^2 - b^2)^2]*d^3)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5026, 17, 3042, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5026}$$

$$\frac{\int (e+fx)^2dx}{b}-\frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3804}$$

$$\frac{(e+fx)^3}{3bf}-\frac{2a\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{(e+fx)^3}{3bf} - \frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}} - \frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b}$$

$\downarrow$ 3011

$$\frac{(e+fx)^3}{3bf} - \frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}} - \frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{}\right)}{b}$$

$\downarrow$ 2720

$$\frac{(e+fx)^3}{3bf} - \frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}} - \frac{ib\left(\frac{(e+fx)^2\log}{}\right)}{}\right)}{b}$$

$\downarrow$ 7143

$$\frac{\frac{(e+fx)^3}{3bf}-2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\cdots\right.}{\cdots}\right.}{b}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b
*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d)
 - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))
/(b*d)))/Sqrt[a^2 - b^2]))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sin(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(348) = 696$.

Time = 0.23 (sec) , antiderivative size = 1646, normalized size of antiderivative = 4.03

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*(a^2 - b^2)*d^3*f^2*x^3 + 6*(a^2 - b^2)*d^3*e*f*x^2 + 6*(a^2 - b^2)
*d^3*e^2*x - 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c
) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) + 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c)
 + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) - 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c)
 + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) + 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c)
 + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) + 6*(-I*a*b*d*f^2*x - I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((
I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x + I*a*b*d*e*f)*sqrt(-(
a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x
 + I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*
x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
 + 1) + 6*(-I*a*b*d*f^2*x - I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*
a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*
f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2\sin{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)\,(e+f\,x)^2}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a\,e^2+\left(\int\frac{\sin(dx+c)x^2}{\sin(dx+c)b+a}dx\right)a^2bd\,f^2-\left(\int\frac{\sin(dx+c)x^2}{\sin(dx+c)b+a}dx\right)b^3d\,f^2+2\left(\int\frac{\mathrm{si}}{\mathrm{sin}}\right.}{bd\,(a^2-b^2)}$$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
*e**2 + int((sin(c + d*x)*x**2)/(sin(c + d*x)*b + a),x)*a**2*b*d*f**2 - in
t((sin(c + d*x)*x**2)/(sin(c + d*x)*b + a),x)*b**3*d*f**2 + 2*int((sin(c +
 d*x)*x)/(sin(c + d*x)*b + a),x)*a**2*b*d*e*f - 2*int((sin(c + d*x)*x)/(si
n(c + d*x)*b + a),x)*b**3*d*e*f + a**2*d*e**2*x - b**2*d*e**2*x)/(b*d*(a**
2 - b**2))
```

3.222 $\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{(e+fx)^2}{2bf} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{af\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{af\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2}$$

output
```
1/2*(f*x+e)^2/b/f+I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/b/(a^2-b^2)^(1/2)/d-I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/b/(a^2-b^2)^(1/2)/d+a*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/b/(a^2-b^2)^(1/2)/d^2-a*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/b/(a^2-b^2)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{x(2e+fx)}{2b}$$
$$-\frac{ia\left(-id\left(2\sqrt{-a^2+b^2}e\arctan\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)+\sqrt{a^2-b^2}fx\left(\log\left(1-\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)-\log\left(1+\frac{be^{i(c+dx)}}{ia+\sqrt{-a^2+b^2}}\right.\right.\right.\right.}{b\sqrt{-\left(a^2-b^2\right)^2}}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(x*(2*e + f*x))/(2*b) - (I*a*((-I)*d*(2*Sqrt[-a^2 + b^2]*e*ArcTan[(I*a + b
*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(Log[1 - (b*E^(I*
(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*
a + Sqrt[-a^2 + b^2])])) - Sqrt[a^2 - b^2]*f*PolyLog[2, (b*E^(I*(c + d*x))
)/((-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*f*PolyLog[2, -((b*E^(I*(c
 + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(b*Sqrt[-(a^2 - b^2)^2]*d^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5026, 17, 3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$\Big\downarrow \text{5026}$$
$$\frac{\int (e+fx)dx}{b}-\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b}$$
$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b}$$

$\big\downarrow$ 3042

$$\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b}$$

$\big\downarrow$ 3804

$$\frac{(e+fx)^2}{2bf}-\frac{2a\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}$$

$\big\downarrow$ 2694

$$\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}$$

$\big\downarrow$ 27

$$\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}$$

$\big\downarrow$ 2620

$$\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b}$$

$\big\downarrow$ 2715

$$\frac{(e+fx)^2}{2bf}-$$

$$2a\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\right.}{2\sqrt{a^2-b^2}}\right.$$

$$(e+fx)$$

$$b$$

$$\Big\downarrow \text{2838}$$

$$\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{b}$$

input
```
Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*
x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 -
b^2]))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(235) = 470$.

Time = 0.85 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.05

method	result
risch	$\frac{f\,x^{2}}{2b}+\frac{ex}{b}-\frac{2iae\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{db\sqrt{-a^{2}+b^{2}}}-\frac{af\ln\left(\frac{ia+\mathrm{e}^{i(dx+c)}b-\sqrt{-a^{2}+b^{2}}}{ia-\sqrt{-a^{2}+b^{2}}}\right)x}{db\sqrt{-a^{2}+b^{2}}}+\frac{af\ln\left(\frac{ia+\mathrm{e}^{i(dx+c)}b+\sqrt{-a^{2}+b^{2}}}{ia+\sqrt{-a^{2}+b^{2}}}\right)x}{db\sqrt{-a^{2}+b^{2}}}-$

input
```
int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*f*x^2+1/b*e*x-2*I/d/b*a*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(
d*x+c))-2*a)/(-a^2+b^2)^(1/2))-1/d/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d
*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d/b*a*f/(-a^2+b^2)^
(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x
-1/d^2/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(
I*a-(-a^2+b^2)^(1/2)))*c+1/d^2/b*a*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c
))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/d^2/b*a*f/(-a^2+b^2)^(1
/2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-
I/d^2/b*a*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))
/(I*a+(-a^2+b^2)^(1/2)))+2*I/d^2/b*a*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*
b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(227) = 454$.

Time = 0.21 (sec) , antiderivative size = 1052, normalized size of antiderivative = 3.94

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2 - b^2)*d^2*f*x^2 + 2*(a^2 - b^2)*d^2*e*x - I*a*b*f*sqrt(-(a^2 -
b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*a*b*f*sqrt(-(a^2 - b
^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*a*b*f*sqrt(-(a^2 - b^
2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*a*b*f*sqrt(-(a^2 - b^
2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (a*b*d*e - a*b*c*f)*sqr
t(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) + 2*I*a) - (a*b*d*e - a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log
(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*
a) + (a*b*d*e - a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (a*b*d*e - a*b*c*
f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b
*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)
/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a*b*d*f*x + a*b*c*f)*sqrt(-(a^
2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a*b*d*f*x + a*b*c*f...
```

Sympy [F]

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)\sin{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin(c+d\,x)\,(e+f\,x)}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)ae+\left(\int\frac{\sin(dx+c)x}{\sin(dx+c)b+a}dx\right)a^2bdf-\left(\int\frac{\sin(dx+c)x}{\sin(dx+c)b+a}dx\right)b^3df+a^2dex-b^2d}{bd\,(a^2-b^2)}$$

input
```
int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
*e + int((sin(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a**2*b*d*f - int((sin(c
+ d*x)*x)/(sin(c + d*x)*b + a),x)*b**3*d*f + a**2*d*e*x - b**2*d*e*x)/(b*d
*(a**2 - b**2))
```

3.223 $\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{x}{b} - \frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d}$$

output
```
x/b-2*a*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(1/2)
/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{c}{d} + x - \frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{b}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
(c/d + x - (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^
2 - b^2]*d))/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3214$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3139$$

$$\frac{x}{b}-\frac{2a\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}$$

$$\Big\downarrow \; 1083$$

$$\frac{4a\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}+\frac{x}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{b}-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}$$

input
```
Int[Sin[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
x/b - (2*a*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(b*Sq
rt[a^2 - b^2]*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}}{d}$	68
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}}{d}$	68
risch	$\frac{x}{b}-\frac{ia\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,db}+\frac{ia\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,db}$	149

input
```
int(sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*arctan(tan(1/2*d*x+1/2*c))-2/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.16

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[\frac{2\,(a^2-b^2)dx-\sqrt{-a^2+b^2}a\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right.}{2\,(a^2b-b^3)d}\right.$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)))/((a^2*b - b^3)*d), ((a^2 - b^2)*d*x + sqrt(a^2 - b^2)*a*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^2*b - b^3)
*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(44) = 88$.

Time = 11.83 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.44

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{\cos(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\sin(c)}{a+b\sin(c)} & \text{for } d=0 \\ \frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} - \frac{dx}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} + \frac{2}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} & \text{for } a=-b \\ \frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+bd} + \frac{dx}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+bd} + \frac{2}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+bd} & \text{for } a=b \\ -\frac{a\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{bd\sqrt{-a^2+b^2}} + \frac{a\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{bd\sqrt{-a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (-cos(
c + d*x)/(a*d), Eq(b, 0)), (x*sin(c)/(a + b*sin(c)), Eq(d, 0)), (d*x*tan(c
/2 + d*x/2)/(b*d*tan(c/2 + d*x/2) - b*d) - d*x/(b*d*tan(c/2 + d*x/2) - b*d
) + 2/(b*d*tan(c/2 + d*x/2) - b*d), Eq(a, -b)), (d*x*tan(c/2 + d*x/2)/(b*d
*tan(c/2 + d*x/2) + b*d) + d*x/(b*d*tan(c/2 + d*x/2) + b*d) + 2/(b*d*tan(c
/2 + d*x/2) + b*d), Eq(a, b)), (-a*log(tan(c/2 + d*x/2) + b/a - sqrt(-a**2
 + b**2)/a)/(b*d*sqrt(-a**2 + b**2)) + a*log(tan(c/2 + d*x/2) + b/a + sqrt
(-a**2 + b**2)/a)/(b*d*sqrt(-a**2 + b**2)) + x/b, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b}-\frac{dx+c}{b}}{d}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*b) - (d*x + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 36.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.44

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{x}{b} - \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b-3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^3+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{(b^2-a^2)^{3/2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

input
```
int(sin(c + d*x)/(a + b*sin(c + d*x)),x)
```

output
```
x/b - (2*a*atanh((a^4*sin(c/2 + (d*x)/2) + 2*b^4*sin(c/2 + (d*x)/2) + a*b^
3*cos(c/2 + (d*x)/2) - a^3*b*cos(c/2 + (d*x)/2) - 3*a^2*b^2*sin(c/2 + (d*x
)/2))/((b^2 - a^2)^(3/2)*(a*cos(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2))))
)/(b*d*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a+a^2dx-b^2dx}{bd\,(a^2-b^2)}$$

input
```
int(sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
 + a**2*d*x - b**2*d*x)/(b*d*(a**2 - b**2))
```

3.224 $\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 643

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} + \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} - \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^2} + \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^2} - \frac{6ia^2f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^3} + \frac{6ia^2f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^3} + \frac{6a^2f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^4} - \frac{6a^2f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^4} - \frac{6f^3\sin(c+dx)}{bd^4} + \frac{3f(e+fx)^2\sin(c+dx)}{bd^2}$$

output
```
-1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cos(d*x+c)/b/d^3-(f*x+e)^3*cos(d*x+c)
/b/d-I*a^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/(a^2
-b^2)^(1/2)/d+I*a^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/b^2/(a^2-b^2)^(1/2)/d-3*a^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^2+3*a^2*f*(f*x+e)^2*polylog(2,I*b*e
xp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^2-6*I*a^2*f^2*(f*
x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)
/d^3+6*I*a^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/b^2/(a^2-b^2)^(1/2)/d^3+6*a^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^4-6*a^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^4-6*f^3*sin(d*x+c)/b/d^4+3*f*(f
*x+e)^2*sin(d*x+c)/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1617 vs. $2(643) = 1286$.

Time = 8.40 (sec) , antiderivative size = 1617, normalized size of antiderivative = 2.51

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a^2*(2*Sqrt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2
 - b^2]] + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)
*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*
(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log
[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*
d^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt
[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b
^2])] - Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqr
t[-a^2 + b^2])] - (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^
(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*
(e + f*x)^2*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] +
6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a
^2 + b^2])] + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-
I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I
*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLo
g[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b
^2]*f^3*PolyLog[4, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I
)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 +
b^2]))]))/(b^2*Sqrt[-(a^2 - b^2)^2]*d^4) + (Cos[c + d*x]/(4*b^2*d^4) - ((I
/4)*Sin[c + d*x])/(b^2*d^4))*(-2*b*d^3*e^3 + (6*I)*b*d^2*e^2*f + 12*b*d...
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.89, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5026, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3804, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^3\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f\int (e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 25

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\Big\downarrow$ 3117

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$\downarrow$ 5026

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^3dx}{b}-\frac{a\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 17

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3804

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\int\frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}\right)}{b}$$

$\downarrow$ 2694

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}\right)}{b}$$

$\downarrow$ 27

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1- \cdots\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{b}\right.}{b}$$

$\downarrow$ 3011

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+f}{}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 7163$$

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-$$

$$a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}}{}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{}\right.$$

↓ 2720

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-$$

$$a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{}\right.$$

↓ 7143

$$\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-$$

$$a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\,\mathrm{PolyLog}\left(3,\frac{i}{a}\right.}{d}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
-((a*((e + f*x)^4/(4*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)^3*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*Po
lyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/S
qrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
 Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(
c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 - b^2]))/b))
/b) + (-(((e + f*x)^3*Cos[c + d*x])/d) + (3*f*(((e + f*x)^2*Sin[c + d*x])/
d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^2))/d))/d)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sin (dx+c)^2}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2671 vs. 2(567) = 1134.

Time = 0.31 (sec) , antiderivative size = 2671, normalized size of antiderivative = 4.15

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*((a^3 - a*b^2)*d^4*f^3*x^4 + 4*(a^3 - a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 -
 a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 - a*b^2)*d^4*e^3*x + 12*I*a^2*b*f^3*sqrt(-(
a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a^2*b*f^3*sqr
t(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a^2*b*f^3
*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) +
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*I*a^2*
b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x +
c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(I
*a^2*b*d^2*f^3*x^2 + 2*I*a^2*b*d^2*e*f^2*x + I*a^2*b*d^2*e^2*f)*sqrt(-(a^2
 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a^2*b*d^2*f^3
*x^2 - 2*I*a^2*b*d^2*e*f^2*x - I*a^2*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a^2*b*d^2*f^3*x^2 - 2*I*a^2
*b*d^2*e*f^2*x - I*a^2*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos
(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) - 6*(I*a^2*b*d^2*f^3*x^2 + 2*I*a^2*b*d^2*e*f^2*x
 + I*a^2*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3 \sin(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{6\sin(dx+c)\,b^3f^3-\cos(dx+c)\,a^2b\,d^3e^3-\cos(dx+c)\,a^2b\,d^3f^3x^3+\cos(dx+c)\,b^3d^3f^3x^3+a\,b^2d^4e^3x}{}$$

input
```
int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2
*d**3*e**3 - cos(c + d*x)*a**2*b*d**3*e**3 - 3*cos(c + d*x)*a**2*b*d**3*e*
*2*f*x - 3*cos(c + d*x)*a**2*b*d**3*e*f**2*x**2 - cos(c + d*x)*a**2*b*d**3
*f**3*x**3 + 6*cos(c + d*x)*a**2*b*d*e*f**2 + 6*cos(c + d*x)*a**2*b*d*f**3
*x + cos(c + d*x)*b**3*d**3*e**3 + 3*cos(c + d*x)*b**3*d**3*e**2*f*x + 3*c
os(c + d*x)*b**3*d**3*e*f**2*x**2 + cos(c + d*x)*b**3*d**3*f**3*x**3 - 6*c
os(c + d*x)*b**3*d*e*f**2 - 6*cos(c + d*x)*b**3*d*f**3*x - int((sin(c + d*
x)*x**3)/(sin(c + d*x)*b + a),x)*a**3*b*d**4*f**3 + int((sin(c + d*x)*x**3
)/(sin(c + d*x)*b + a),x)*a*b**3*d**4*f**3 - 3*int((sin(c + d*x)*x**2)/(si
n(c + d*x)*b + a),x)*a**3*b*d**4*e*f**2 + 3*int((sin(c + d*x)*x**2)/(sin(c
 + d*x)*b + a),x)*a*b**3*d**4*e*f**2 - 3*int((sin(c + d*x)*x)/(sin(c + d*x
)*b + a),x)*a**3*b*d**4*e**2*f + 3*int((sin(c + d*x)*x)/(sin(c + d*x)*b +
a),x)*a*b**3*d**4*e**2*f + 3*sin(c + d*x)*a**2*b*d**2*e**2*f + 6*sin(c + d
*x)*a**2*b*d**2*e*f**2*x + 3*sin(c + d*x)*a**2*b*d**2*f**3*x**2 - 6*sin(c
+ d*x)*a**2*b*f**3 - 3*sin(c + d*x)*b**3*d**2*e**2*f - 6*sin(c + d*x)*b**3
*d**2*e*f**2*x - 3*sin(c + d*x)*b**3*d**2*f**3*x**2 + 6*sin(c + d*x)*b**3*
f**3 - a**3*d**4*e**3*x + a*b**2*d**4*e**3*x)/(b**2*d**4*(a**2 - b**2))
```

3.225 $\int \frac{(e+fx)^2 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 479

$$\begin{aligned}\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cos(c+dx)}{bd^3}-\frac{(e+fx)^2\cos(c+dx)}{bd}\\
&\quad-\frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}\\
&\quad+\frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}\\
&\quad-\frac{2a^2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^2}\\
&\quad+\frac{2a^2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^2}\\
&\quad-\frac{2ia^2f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^3}\\
&\quad+\frac{2ia^2f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d^3}\\
&\quad+\frac{2f(e+fx)\sin(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/3*a*(f*x+e)^3/b^2/f+2*f^2*cos(d*x+c)/b/d^3-(f*x+e)^2*cos(d*x+c)/b/d-I*a
^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1
/2)/d+I*a^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/(a^
2-b^2)^(1/2)/d-2*a^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(
1/2)))/b^2/(a^2-b^2)^(1/2)/d^2+2*a^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c)
)/(a+(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^2-2*I*a^2*f^2*polylog(3,I*b*e
xp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^3+2*I*a^2*f^2*pol
ylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^3+2*f
*(f*x+e)*sin(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 4.26 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.11

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-ax(3e^2+3efx+f^2x^2)+\frac{3ia^2\left(-2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)+2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{ia+\sqrt{-a^2+b^2}}\right.\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + ((3*I)*a^2*(-2*Sqrt[a^2 - b^2]*d*f*(
e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*S
qrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[
-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c +
d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I
*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I
*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c +
d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((
b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(Sqrt[-(a^2 - b^2)^2]*d^3
) - (3*b*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*Cos[c] - 2*d*f*(e + f*x)*Sin
[c]))/d^3 + (3*b*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2)*Sin[
c])*Sin[d*x])/d^3)/(3*b^2)
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.90, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5026, 3042, 3777, 3042, 3777, 25, 3042, 3118, 5026, 17, 3042, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^2\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5026}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2dx}{b}-\frac{a\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3804}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\int\frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{i}{a}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}}{b}\right.}{b}$$

$\downarrow$ 3011

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1\ldots\right.}{\ldots}\right.}{\ldots}\right)}{b}\right.}{b}$$

$\downarrow$ 2720

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-$$

$$a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\right)}{b}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad b}$$

$\downarrow$ 7143

$$\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ib\ldots}{a-\ldots}\right)}{bd}\ldots\right.}{\ldots}\right)}{b}\right)}{b}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
-((a*((e + f*x)^3/(3*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]])/(b*d) - (2*f*((I*(e + f*x)*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]])/d - (f*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((
I/2)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]])
/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])]])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]])
/d^2))/(b*d)))/Sqrt[a^2 - b^2]))/b))/b) + (-(((e + f*x)^2*Cos[c + d*x])/d)
 + (2*f*((f*Cos[c + d*x])/d^2 + ((e + f*x)*Sin[c + d*x])/d))/d)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sin(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1855 vs. $2(419) = 838$.

Time = 0.27 (sec) , antiderivative size = 1855, normalized size of antiderivative = 3.87

$$\int \frac{(e+fx)^2 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*(a^3 - a*b^2)*d^3*f^2*x^3 + 6*(a^3 - a*b^2)*d^3*e*f*x^2 + 6*(a^3 -
 a*b^2)*d^3*e^2*x - 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*co
s(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) + 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*c
os(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2))/b) - 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a
*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) + 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I
*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))/b) - 6*(I*a^2*b*d*f^2*x + I*a^2*b*d*e*f)*sqrt(-(a^2 -
 b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a^2*b*d*f^2*x -
 I*a^2*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x
 + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
+ 1) - 6*(-I*a^2*b*d*f^2*x - I*a^2*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((
-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(I*a^2*b*d*f^2*x + I*a^2*b*d*e*f)*sq
rt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(a^2*b*d
^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^2\sin(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2d^2e^2-\cos(dx+c)\,a^2b\,d^2e^2-2\cos(dx+c)\,a^2b\,d^2efx-\cos(dx+c)\,c}{}$$

input
```
int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2
*d**2*e**2 - cos(c + d*x)*a**2*b*d**2*e**2 - 2*cos(c + d*x)*a**2*b*d**2*e*
f*x - cos(c + d*x)*a**2*b*d**2*f**2*x**2 + 2*cos(c + d*x)*a**2*b*f**2 + co
s(c + d*x)*b**3*d**2*e**2 + 2*cos(c + d*x)*b**3*d**2*e*f*x + cos(c + d*x)*
b**3*d**2*f**2*x**2 - 2*cos(c + d*x)*b**3*f**2 - int((sin(c + d*x)*x**2)/(
sin(c + d*x)*b + a),x)*a**3*b*d**3*f**2 + int((sin(c + d*x)*x**2)/(sin(c +
 d*x)*b + a),x)*a*b**3*d**3*f**2 - 2*int((sin(c + d*x)*x)/(sin(c + d*x)*b
+ a),x)*a**3*b*d**3*e*f + 2*int((sin(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a
*b**3*d**3*e*f + 2*sin(c + d*x)*a**2*b*d*e*f + 2*sin(c + d*x)*a**2*b*d*f**
2*x - 2*sin(c + d*x)*b**3*d*e*f - 2*sin(c + d*x)*b**3*d*f**2*x - a**3*d**3
*e**2*x + a*b**2*d**3*e**2*x)/(b**2*d**3*(a**2 - b**2))
```

3.226 $\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 309

$$\begin{aligned}\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx =& -\frac{a(e+fx)^2}{2b^2f}-\frac{(e+fx)\cos(c+dx)}{bd}\\ &-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d}\\ &+\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d}\\ &-\frac{a^2f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d^2}\\ &+\frac{a^2f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d^2}+\frac{f\sin(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/2*a*(f*x+e)^2/b^2/f-(f*x+e)*cos(d*x+c)/b/d-I*a^2*(f*x+e)*ln(1-I*b*exp(I
*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d+I*a^2*(f*x+e)*ln(1-I*
b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d-a^2*f*polylog(
2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^2+a^2*f*po
lylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/(a^2-b^2)^(1/2)/d^2+f*
sin(d*x+c)/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1973 vs. 2(309) = 618.

Time = 16.53 (sec) , antiderivative size = 1973, normalized size of antiderivative = 6.39

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
-1/2*(a*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(b^2*d^2) - ((d*e - c*f +
 f*(c + d*x))*Cos[c + d*x])/(b*d^2) + (f*Sin[c + d*x])/(b*d^2) + (((2*(d*e
 - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c +
d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*
Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a -
 b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] - (I*f*Log[1 - I*Tan[(c + d*x)/
2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^
2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + S
qrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[
-a^2 + b^2] - (I*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqr
t[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(1 + I*Tan[(c + d*
x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2
, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^
2] - (I*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b
^2]))])/Sqrt[-a^2 + b^2])*((a^2*e)/(b^2*(a + b*Sin[c + d*x])) - (a^2*c*f)/
(b^2*d*(a + b*Sin[c + d*x])) + (a^2*f*(c + d*x))/(b^2*d*(a + b*Sin[c + d*x
]))))/(d*((f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 +
b^2]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[-a^2 + b^2]*(1 - I*Tan[(c + d*x)/2]))
+ (f*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[...
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5026, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5026}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{\int (e+fx)dx}{b}-\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3804}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\int\frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b}\cdots\right)}{b}$$

$\Big\downarrow$ 2715

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+\cdots}}{bd^2}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{b}\cdots\right)}{b}$$

$\Big\downarrow$ 2838

$$\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{b}\cdots\right)}{b}$$

input
```
Int[((e + f*x)*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
-((a*((e + f*x)^2/(2*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((
e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*
f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[
a^2 - b^2]))/b))/b) + (-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^
2)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(277) = 554.

Time = 2.16 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{a f x^{2}}{2 b^{2}}-\frac{a e x}{b^{2}}-\frac{(d x f+d e+i f) \mathrm{e}^{i(d x+c)}}{2 d^{2} b}-\frac{(d x f+d e-i f) \mathrm{e}^{-i(d x+c)}}{2 d^{2} b}+\frac{2 i a^{2} e \arctan\left(\frac{2 i b\,\mathrm{e}^{i(d x+c)}-2 a}{2 \sqrt{-a^{2}+b^{2}}}\right)}{d\, b^{2} \sqrt{-a^{2}+b^{2}}}+\frac{a^{2} f \ln\left(\frac{i a+\mathrm{e}^{i(d x+}}{i a-}\right.}{d\, b^{2} \sqrt{-}}$

input
```
int((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/b^2*f*x^2-a/b^2*e*x-1/2*(d*x*f+I*f+d*e)/d^2/b*exp(I*(d*x+c))-1/2*(d
*x*f-I*f+d*e)/d^2/b*exp(-I*(d*x+c))+2*I/d/b^2*a^2*e/(-a^2+b^2)^(1/2)*arcta
n(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/d/b^2*a^2*f/(-a^2+b^2
)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
*x-1/d/b^2*a^2*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2
))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2/b^2*a^2*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(
I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/d^2/b^2*a^2*f/(
-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)
^(1/2)))*c-I/d^2/b^2*a^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b-(-
a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+I/d^2/b^2*a^2*f/(-a^2+b^2)^(1/2)*d
ilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/d
^2/b^2*a^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^
2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(269) = 538$.

Time = 0.22 (sec) , antiderivative size = 1154, normalized size of antiderivative = 3.73

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*((a^3 - a*b^2)*d^2*f*x^2 - I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*
a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) + I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I
*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) + I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((
-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog
((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(a^3 - a*b^2)*d^2*e*x - 2*(a^2*b -
 b^3)*f*sin(d*x + c) - (a^2*b*d*e - a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a
) - (a^2*b*d*e - a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) -
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (a^2*b*d*e - a^
2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (a^2*b*d*e - a^2*b*c*f)*sqrt(-(a^
2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) - 2*I*a) + (a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*lo
g(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 -
b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\sin{(dx+c)}^2}{b\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2de-\cos\left(dx+c\right)a^2bde-\cos\left(dx+c\right)a^2bdfx+\cos\left(dx+c\right)b^3de+\text{co}}{}$$

input
```
int((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2
*d*e - cos(c + d*x)*a**2*b*d*e - cos(c + d*x)*a**2*b*d*f*x + cos(c + d*x)*
b**3*d*e + cos(c + d*x)*b**3*d*f*x - int((sin(c + d*x)*x)/(sin(c + d*x)*b
+ a),x)*a**3*b*d**2*f + int((sin(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a*b**
3*d**2*f + sin(c + d*x)*a**2*b*f - sin(c + d*x)*b**3*f - a**3*d**2*e*x + a
*b**2*d**2*e*x)/(b**2*d**2*(a**2 - b**2))
```

3.227 $\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{ax}{b^2}+\frac{2a^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d}-\frac{\cos(c+dx)}{bd}$$

output
```
-a*x/b^2+2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2-b
^2)^(1/2)/d-cos(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{a(c+dx)-\frac{2a^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+b\cos(c+dx)}{b^2d}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
-((a*(c + d*x) - (2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + b*Cos[c + d*x])/(b^2*d))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3225, 25, 27, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{a\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(c+dx)}dx}{b}\right)}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(c+dx)}dx}{b}\right)}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{a\left(\frac{x}{b}-\frac{2a\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{a\left(\frac{4a\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}+\frac{x}{b}\right)}{b}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{217}$$

$$-\frac{a\left(\frac{x}{b}-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}\right)}{b}-\frac{\cos(c+dx)}{bd}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
-((a*(x/b - (2*a*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])
/(b*Sqrt[a^2 - b^2]*d)))/b) - Cos[c + d*x]/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{b}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}}{d}$
default	$\frac{-\frac{2\left(\frac{b}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}}{d}$
risch	$-\frac{ax}{b^2}-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}+\frac{a^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,d\,b^2}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,d\,b^2}$

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c)))+2/b^
2*a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.77

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}a^2\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+2\,(a^3 \cdots}{2\,(a^2b^2-b^4)d}\right.$$
$$\left.-\frac{\sqrt{a^2-b^2}a^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+(a^3-ab^2)dx+(a^2b-b^3)\cos(dx+c)}{(a^2b^2-b^4)d}\right]$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2
*(a^3 - a*b^2)*d*x + 2*(a^2*b - b^3)*cos(d*x + c))/((a^2*b^2 - b^4)*d), -(
sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) + (a^3 - a*b^2)*d*x + (a^2*b - b^3)*cos(d*x + c))/((a^2*b^2 - b^4)*
d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(61) = 122$.

Time = 110.28 (sec) , antiderivative size = 1690, normalized size of antiderivative = 22.53

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x*sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-cos(c + d*x)/(
b*d), Eq(a, 0)), (-b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 +
 b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*
x/2)) - b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*
sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*b*ta
n(c/2 + d*x/2)**2/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*ta
n(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 4*b/(b**2*d*tan(c/2
 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2
)*tan(c/2 + d*x/2)) + d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/2 +
 d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*
tan(c/2 + d*x/2)) + d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d -
b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*
sqrt(b**2)*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqr
t(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, -sqr
t(b**2))), (-b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*
d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2))
- b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b
**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*b*tan(c/2
+ d*x/2)**2/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2
+ d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 4*b/(b**2*d*tan(c/2 + ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}b^2}-\frac{(dx+c)a}{b^2}-\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(sqrt(a^2 - b^2)*b^2) - (d*x + c)*a/b^2 - 2/
((tan(1/2*d*x + 1/2*c)^2 + 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 37.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.69

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+d\,x)}{b\,d}-\frac{a\,x}{b^2}-\frac{a^2\,\mathrm{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)\,1\mathrm{i}}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{b^2-a^2}}$$

input
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

output
```
- cos(c + d*x)/(b*d) - (a*x)/b^2 - (a^2*atan(((2*b^2*sin(c/2 + (d*x)/2) -
a^2*sin(c/2 + (d*x)/2) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a
*cos(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2))))*2i)/(b^2*d*(b^2 - a^2)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2-\cos(dx+c)\,a^2b+\cos(dx+c)\,b^3-a^3dx+a\,b^2dx}{b^2d\,(a^2-b^2)}$$

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2
 - cos(c + d*x)*a**2*b + cos(c + d*x)*b**3 - a**3*d*x + a*b**2*d*x)/(b**2*
d*(a**2 - b**2))
```

3.228 $\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 789

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{3f(e+fx)^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3}+\frac{a(e+fx)^3\cos(c+dx)}{b^2d} + \frac{ia^3(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} - \frac{ia^3(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} + \frac{3a^3f(e+fx)^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^2} - \frac{3a^3f(e+fx)^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^2} + \frac{6ia^3f^2(e+fx)\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^3} - \frac{6ia^3f^2(e+fx)\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^3} - \frac{6a^3f^3\,\mathrm{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^4} + \frac{6a^3f^3\,\mathrm{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^4} + \frac{6af^3\sin(c+dx)}{b^2d^4}-\frac{3af(e+fx)^2\sin(c+dx)}{b^2d^2} + \frac{3f^2(e+fx)\cos(c+dx)\sin(c+dx)}{4bd^3} - \frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2bd} - \frac{3f^3\sin^2(c+dx)}{8bd^4}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4bd^2}$$

output
```
-3/8*f*(f*x+e)^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f+1/8*(f*x+e)^4/b/f-6*a*f^2*(
f*x+e)*cos(d*x+c)/b^2/d^3+a*(f*x+e)^3*cos(d*x+c)/b^2/d+6*I*a^3*f^2*(f*x+e)
*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d^3
+I*a^3*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-b^2
)^(1/2)/d+3*a^3*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/b^3/(a^2-b^2)^(1/2)/d^2-3*a^3*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))
/(a+(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d^2-I*a^3*(f*x+e)^3*ln(1-I*b*exp
(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d-6*I*a^3*f^2*(f*x+e)
*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d^3
-6*a^3*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)
^(1/2)/d^4+6*a^3*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3
/(a^2-b^2)^(1/2)/d^4+6*a*f^3*sin(d*x+c)/b^2/d^4-3*a*f*(f*x+e)^2*sin(d*x+c)
/b^2/d^2+3/4*f^2*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d^3-1/2*(f*x+e)^3*cos(d*x
+c)*sin(d*x+c)/b/d-3/8*f^3*sin(d*x+c)^2/b/d^4+3/4*f*(f*x+e)^2*sin(d*x+c)^2
/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1923 vs. 2(789) = 1578.

Time = 7.38 (sec) , antiderivative size = 1923, normalized size of antiderivative = 2.44

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
(16*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*e^3*x + 8*b^2*Sqrt[-(-a^2 + b^2)^2]*d^4*e
^3*x + 24*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*e^2*f*x^2 + 12*b^2*Sqrt[-(-a^2 + b^
2)^2]*d^4*e^2*f*x^2 + 16*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*e*f^2*x^3 + 8*b^2*Sq
rt[-(-a^2 + b^2)^2]*d^4*e*f^2*x^3 + 4*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*f^3*x^4
 + 2*b^2*Sqrt[-(-a^2 + b^2)^2]*d^4*f^3*x^4 - 32*a^3*Sqrt[-a^2 + b^2]*d^3*e
^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 16*a*b*Sqrt[-(a^2 -
 b^2)^2]*d^3*e^3*Cos[c + d*x] - 96*a*b*Sqrt[-(a^2 - b^2)^2]*d*e*f^2*Cos[c
+ d*x] + 48*a*b*Sqrt[-(a^2 - b^2)^2]*d^3*e^2*f*x*Cos[c + d*x] - 96*a*b*Sqr
t[-(a^2 - b^2)^2]*d*f^3*x*Cos[c + d*x] + 48*a*b*Sqrt[-(a^2 - b^2)^2]*d^3*e
*f^2*x^2*Cos[c + d*x] + 16*a*b*Sqrt[-(a^2 - b^2)^2]*d^3*f^3*x^3*Cos[c + d*
x] - 6*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*e^2*f*Cos[2*(c + d*x)] + 3*b^2*Sqrt[-(
a^2 - b^2)^2]*f^3*Cos[2*(c + d*x)] - 12*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*e*f^2
*x*Cos[2*(c + d*x)] - 6*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*f^3*x^2*Cos[2*(c + d*
x)] - 48*a^3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)
*a + Sqrt[-a^2 + b^2])] - 48*a^3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*
E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 16*a^3*Sqrt[a^2 - b^2]*d^3
*f^3*x^3*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 48*a^3
*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2
+ b^2])] + 48*a^3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x))
)/(I*a + Sqrt[-a^2 + b^2])] + 16*a^3*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 ...
```

Rubi [A] (verified)

Time = 4.27 (sec) , antiderivative size = 713, normalized size of antiderivative = 0.90, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5026, 3042, 3792, 17, 3042, 3791, 17, 5026, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3804, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5026}$$

$$\frac{\int (e+fx)^3\sin^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3 \sin(c+dx)^2dx}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{-\frac{3f^2\int (e+fx)\sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{3f^2\int (e+fx)\sin^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3f^2\int (e+fx)\sin(c+dx)^2dx}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{-\frac{3f^2\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5026}$$

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\int(e+fx)^3\sin(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\int(e+fx)^3\sin(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\int(e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 25

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 3117

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 5026

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^3dx}{b}-\frac{a\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 17

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 3804

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\int\frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 2694

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\cdots}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$\downarrow$ 27

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{\cdots}{a-\cdots}\cdots}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$\downarrow$ 2620

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int(}{}\right.}{2\sqrt{a^2-b}}\right.}{}\right.}{b}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 7163

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\right. \quad 3f$$

$$2a$$

$$a\left(\frac{(e+fx)^4}{4bf}-\right.$$

$$a\left(\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}\right.$$

$$\frac{}{b}-$$

↓ 2720

$$-\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 7143

$$\frac{-\frac{3f^2\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sin^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[((e + f*x)^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
((e + f*x)^4/(8*f) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*f*
(e + f*x)^2*Sin[c + d*x]^2)/(4*d^2) - (3*f^2*((e + f*x)^2/(4*f) - ((e + f*
x)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x]^2)/(4*d^2)))/(2*d^2)
)/b - (a*(-((a*((e + f*x)^4/(4*b*f) - (2*a*(((-1/2*I)*b*(((e + f*x)^3*Log[
1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f
*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))
/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*Po
lyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 -
b^2]))/b))/b) + (-(((e + f*x)^3*Cos[c + d*x])/d) + (3*f*(((e + f*x)^2*Sin[
c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^2))
/d))/d)/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sin(dx+c)^3}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3008 vs. $2(701) = 1402$.

Time = 0.51 (sec) , antiderivative size = 3008, normalized size of antiderivative = 3.81

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\sin(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)^3*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
( - 16*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*
a**3*d**3*e**3 + 6*cos(c + d*x)**2*a**2*b**2*d**4*e**2*f*x**2 + cos(c + d*
x)**2*a**2*b**2*d**4*f**3*x**4 - 6*cos(c + d*x)**2*a**2*b**2*d**2*e**2*f -
 3*cos(c + d*x)**2*a**2*b**2*d**2*f**3*x**2 + 3*cos(c + d*x)**2*a**2*b**2*
f**3 - 6*cos(c + d*x)**2*b**4*d**4*e**2*f*x**2 - cos(c + d*x)**2*b**4*d**4
*f**3*x**4 + 6*cos(c + d*x)**2*b**4*d**2*e**2*f + 3*cos(c + d*x)**2*b**4*d
**2*f**3*x**2 - 3*cos(c + d*x)**2*b**4*f**3 - 4*cos(c + d*x)*sin(c + d*x)*
a**2*b**2*d**3*e**3 - 12*cos(c + d*x)*sin(c + d*x)*a**2*b**2*d**3*e**2*f*x
 - 4*cos(c + d*x)*sin(c + d*x)*a**2*b**2*d**3*f**3*x**3 + 6*cos(c + d*x)*s
in(c + d*x)*a**2*b**2*d*f**3*x + 4*cos(c + d*x)*sin(c + d*x)*b**4*d**3*e**
3 + 12*cos(c + d*x)*sin(c + d*x)*b**4*d**3*e**2*f*x + 4*cos(c + d*x)*sin(c
 + d*x)*b**4*d**3*f**3*x**3 - 6*cos(c + d*x)*sin(c + d*x)*b**4*d*f**3*x +
8*cos(c + d*x)*a**3*b*d**3*e**3 + 24*cos(c + d*x)*a**3*b*d**3*e**2*f*x + 8
*cos(c + d*x)*a**3*b*d**3*f**3*x**3 - 48*cos(c + d*x)*a**3*b*d*f**3*x - 8*
cos(c + d*x)*a*b**3*d**3*e**3 - 24*cos(c + d*x)*a*b**3*d**3*e**2*f*x - 8*c
os(c + d*x)*a*b**3*d**3*f**3*x**3 + 48*cos(c + d*x)*a*b**3*d*f**3*x + 24*i
nt((sin(c + d*x)**3*x**2)/(sin(c + d*x)*b + a),x)*a**2*b**3*d**4*e*f**2 -
24*int((sin(c + d*x)**3*x**2)/(sin(c + d*x)*b + a),x)*b**5*d**4*e*f**2 + 8
*int((sin(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*a**4*b*d**4*f**3 - 8*int(
(sin(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*a**2*b**3*d**4*f**3 + 24*in...
```

3.229 $\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 592

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}+\frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3}+\frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{ia^3(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} - \frac{ia^3(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} + \frac{2a^3f(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^2} - \frac{2a^3f(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^2} + \frac{2ia^3f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^3} - \frac{2ia^3f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^3} - \frac{2af(e+fx)\sin(c+dx)}{b^2d^2}+\frac{f^2\cos(c+dx)\sin(c+dx)}{4bd^3} - \frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f(e+fx)\sin^2(c+dx)}{2bd^2}$$

output
```
-1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f+1/6*(f*x+e)^3/b/f-2*a*f^2*cos(d*x
+c)/b^2/d^3+a*(f*x+e)^2*cos(d*x+c)/b^2/d+I*a^3*(f*x+e)^2*ln(1-I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d-I*a^3*(f*x+e)^2*ln(1-I*b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d+2*a^3*f*(f*x+e)
*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d^2
-2*a^3*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/(a^
2-b^2)^(1/2)/d^2+2*I*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/
2)))/b^3/(a^2-b^2)^(1/2)/d^3-2*I*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/d^3-2*a*f*(f*x+e)*sin(d*x+c)/b^2/d^2+
1/4*f^2*cos(d*x+c)*sin(d*x+c)/b/d^3-1/2*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/b/
d+1/2*f*(f*x+e)*sin(d*x+c)^2/b/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 5.68 (sec) , antiderivative size = 1166, normalized size of antiderivative = 1.97

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
(24*a^2*Sqrt[-(a^2 - b^2)^2]*d^3*e^2*x + 12*b^2*Sqrt[-(-a^2 + b^2)^2]*d^3*
e^2*x + 24*a^2*Sqrt[-(a^2 - b^2)^2]*d^3*e*f*x^2 + 12*b^2*Sqrt[-(-a^2 + b^2
)^2]*d^3*e*f*x^2 + 8*a^2*Sqrt[-(a^2 - b^2)^2]*d^3*f^2*x^3 + 4*b^2*Sqrt[-(-
a^2 + b^2)^2]*d^3*f^2*x^3 - 48*a^3*Sqrt[-a^2 + b^2]*d^2*e^2*ArcTan[(I*a +
b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 24*a*b*Sqrt[-(a^2 - b^2)^2]*d^2*e^2*
Cos[c + d*x] - 48*a*b*Sqrt[-(a^2 - b^2)^2]*f^2*Cos[c + d*x] + 48*a*b*Sqrt[
-(a^2 - b^2)^2]*d^2*e*f*x*Cos[c + d*x] + 24*a*b*Sqrt[-(a^2 - b^2)^2]*d^2*f
^2*x^2*Cos[c + d*x] - 6*b^2*Sqrt[-(a^2 - b^2)^2]*d*e*f*Cos[2*(c + d*x)] -
6*b^2*Sqrt[-(a^2 - b^2)^2]*d*f^2*x*Cos[2*(c + d*x)] - 48*a^3*Sqrt[a^2 - b^
2]*d^2*e*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 24
*a^3*Sqrt[a^2 - b^2]*d^2*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqr
t[-a^2 + b^2])] + 48*a^3*Sqrt[a^2 - b^2]*d^2*e*f*x*Log[1 + (b*E^(I*(c + d*
x)))/(I*a + Sqrt[-a^2 + b^2])] + 24*a^3*Sqrt[a^2 - b^2]*d^2*f^2*x^2*Log[1
+ (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] + (48*I)*a^3*Sqrt[a^2 - b^
2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2]
)] - (48*I)*a^3*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*
x)))/(I*a + Sqrt[-a^2 + b^2]))] - 48*a^3*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b
*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 48*a^3*Sqrt[a^2 - b^2]*f^
2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 48*a*b*Sqr
t[-(a^2 - b^2)^2]*d*e*f*Sin[c + d*x] - 48*a*b*Sqrt[-(a^2 - b^2)^2]*d*f^...
```

Rubi [A] (verified)

Time = 3.18 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.91, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.893$, Rules used = {5026, 3042, 3792, 17, 3042, 3115, 24, 5026, 3042, 3777, 3042, 3777, 25, 3042, 3118, 5026, 17, 3042, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5026}$$

$$\frac{\int (e+fx)^2\sin^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\sin(c+dx)^2dx}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int \sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{-\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5026}$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\int (e+fx)^2\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\int(e+fx)^2\sin(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\int(e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3118$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 5026$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2dx}{b}-\frac{a\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 3804

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\int\frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 2694

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}}{\sqrt{a^2-b^2}}\right)}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 27

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{}\right.}{b}\right.}{b}$$

$\downarrow$ 3011

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\cdots\right.}{\cdots}\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{\cdots}\right)}{\cdots}\right.$$

↓ 2720

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+\ldots}}{a+\sqrt{a^2-\ldots}}\right)}{d}\ldots\right)}{\ldots}\ldots\right)}{2\sqrt{a^2-b\ldots}}\ldots\right)}{\ldots}\ldots\right)}{\ldots}\ldots\right.$$

↓ 7143

$$\frac{\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-\cdots}}\right)}{d}\cdots\right.}{\cdots}\right.}{2\sqrt{a^2-b^2}\cdots}\cdots}{\cdots}\right.}{\cdots}\right.}{b\cdots}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
((e + f*x)^3/(6*f) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*(e
 + f*x)*Sin[c + d*x]^2)/(2*d^2) - (f^2*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/(2*d^2))/b - (a*(-((a*((e + f*x)^3/(3*b*f) - (2*a*(((-1/2*I)*b*(((
e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (
2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*
d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2]))/b))/b) + (-(((e
+ f*x)^2*Cos[c + d*x])/d) + (2*f*((f*Cos[c + d*x])/d^2 + ((e + f*x)*Sin[c
+ d*x])/d))/d)/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sin(dx+c)^3}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2050 vs. $2(522) = 1044$.

Time = 0.29 (sec) , antiderivative size = 2050, normalized size of antiderivative = 3.46

$$\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(2*(2*a^4 - a^2*b^2 - b^4)*d^3*f^2*x^3 + 6*(2*a^4 - a^2*b^2 - b^4)*d^
3*e*f*x^2 - 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x +
 c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))/b) + 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x
 + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b) - 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(
d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))/b) + 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*c
os(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2))/b) - 6*((a^2*b^2 - b^4)*d*f^2*x + (a^2*b^2 - b^4)*d*e*f)*
cos(d*x + c)^2 + 12*(-I*a^3*b*d*f^2*x - I*a^3*b*d*e*f)*sqrt(-(a^2 - b^2)/b
^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*a^3*b*d*f^2*x + I*a^3*
b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) -
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
12*(I*a^3*b*d*f^2*x + I*a^3*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*co
s(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b + 1) + 12*(-I*a^3*b*d*f^2*x - I*a^3*b*d*e*f)*sqrt(-(
a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(a^3*b*d^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sin(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)^3*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\cos(dx+c)^2 a^2b^2d^2ef\,x^2 + \sin(dx+c)^2 a^2b^2d^2ef\,x^2 - 2\left(\int \frac{\sin(dx+c)^3x^2}{\sin(dx+c)b+a}dx\right) b^5d^2f^2 - 2\cos(dx+c)\sin(d}{}$$

input
```
int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**3*d*e**2 + cos(c + d*x)**2*a**2*b**2*d**2*e*f*x**2 - cos(c + d*x)**2*a**
2*b**2*e*f - cos(c + d*x)**2*b**4*d**2*e*f*x**2 + cos(c + d*x)**2*b**4*e*f
 - cos(c + d*x)*sin(c + d*x)*a**2*b**2*d*e**2 - 2*cos(c + d*x)*sin(c + d*x
)*a**2*b**2*d*e*f*x + cos(c + d*x)*sin(c + d*x)*b**4*d*e**2 + 2*cos(c + d*
x)*sin(c + d*x)*b**4*d*e*f*x + 2*cos(c + d*x)*a**3*b*d*e**2 + 4*cos(c + d*
x)*a**3*b*d*e*f*x - 2*cos(c + d*x)*a*b**3*d*e**2 - 4*cos(c + d*x)*a*b**3*d
*e*f*x + 2*int((sin(c + d*x)**3*x**2)/(sin(c + d*x)*b + a),x)*a**2*b**3*d*
*2*f**2 - 2*int((sin(c + d*x)**3*x**2)/(sin(c + d*x)*b + a),x)*b**5*d**2*f
**2 + 4*int((sin(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a**4*b*d**2*e*f - 4*i
nt((sin(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a**2*b**3*d**2*e*f + sin(c + d
*x)**2*a**2*b**2*d**2*e*f*x**2 - sin(c + d*x)**2*b**4*d**2*e*f*x**2 - 4*si
n(c + d*x)*a**3*b*e*f + 4*sin(c + d*x)*a*b**3*e*f + 2*a**4*c*d*e**2 + 2*a*
*4*d**2*e**2*x - a**2*b**2*c*d*e**2 - a**2*b**2*d**2*e**2*x - b**4*c*d*e**
2 - b**4*d**2*e**2*x)/(2*b**3*d**2*(a**2 - b**2))
```

3.230 $\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 376

$$\begin{aligned}\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{a^2(e+fx)^2}{2b^3f}+\frac{(e+fx)^2}{4bf}+\frac{a(e+fx)\cos(c+dx)}{b^2d}\\ &+\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}\\ &-\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}\\ &+\frac{a^3f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^2}\\ &-\frac{a^3f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d^2}-\frac{af\sin(c+dx)}{b^2d^2}\\ &-\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd}+\frac{f\sin^2(c+dx)}{4bd^2}\end{aligned}$$

output
```
1/2*a^2*(f*x+e)^2/b^3/f+1/4*(f*x+e)^2/b/f+a*(f*x+e)*cos(d*x+c)/b^2/d+I*a^3
*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)^(1/2)/
d-I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/(a^2-b^2)
^(1/2)/d+a^3*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/(a^2-
b^2)^(1/2)/d^2-a^3*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3
/(a^2-b^2)^(1/2)/d^2-a*f*sin(d*x+c)/b^2/d^2-1/2*(f*x+e)*cos(d*x+c)*sin(d*x
+c)/b/d+1/4*f*sin(d*x+c)^2/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2036 vs. $2(376) = 752$.

Time = 17.61 (sec) , antiderivative size = 2036, normalized size of antiderivative = 5.41

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
((2*a^2 + b^2)*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(4*b^3*d^2) + (a*(
d*e - c*f + f*(c + d*x))*Cos[c + d*x])/(b^2*d^2) - (f*Cos[2*(c + d*x)])/(8
*b*d^2) - (a*f*Sin[c + d*x])/(b^2*d^2) - ((d*e - c*f + f*(c + d*x))*Sin[2*
(c + d*x)])/(4*b*d^2) + (((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b
 - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/S
qrt[-a^2 + b^2] + (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 +
b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2
] - (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c
+ d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1
 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*
a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2, (a*(1 - I*T
an[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f
*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])/
Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + S
qrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2, (a + I*a*Tan[(c + d*
x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2])*(-((a^3*e)/(b^3
*(a + b*Sin[c + d*x]))) + (a^3*c*f)/(b^3*d*(a + b*Sin[c + d*x])) - (a^3*f*
(c + d*x))/(b^3*d*(a + b*Sin[c + d*x]))))/(d*((f*Log[1 - (a*(1 - I*Tan[(c
+ d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt...
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.97, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {5026, 3042, 3791, 17, 5026, 3042, 3777, 3042, 3117, 5026, 17, 3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5026}$$

$$\frac{\int (e+fx)\sin^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)\sin(c+dx)^2dx}{b}-\frac{a\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{1}{2}\int (e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5026$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\int (e+fx)\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\int (e+fx)\sin(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3117$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5026}$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)dx}{b}-\frac{a\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3804}$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\int\frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 27

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 2620

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\cdots\right)}{2\sqrt{\cdots}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$\Big\downarrow$ 2715

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{\cdots}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\cdots\right.}{2\cdots}\right.}{b}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
((e + f*x)^2/(4*f) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[
c + d*x]^2)/(4*d^2))/b - (a*(-((a*((e + f*x)^2/(2*b*f) - (2*a*(((-1/2*I)*b
*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) -
 (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/S
qrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/b))/b) + (-(((e + f*x)*Cos[c + d
*x])/d) + (f*Sin[c + d*x])/d^2)/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5026
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sin[c +
 d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sin[c + d*x]^(n - 1)/(a
 + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(338) = 676.

Time = 2.77 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.82

method	result
risch	$\frac{a^2 f x^2}{2b^3}+\frac{f x^2}{4b}+\frac{a^2 e x}{b^3}+\frac{e x}{2b}+\frac{a(dxf+de+if)\mathrm{e}^{i(dx+c)}}{2b^2d^2}+\frac{a(dxf+de-if)\mathrm{e}^{-i(dx+c)}}{2b^2d^2}+\frac{2ia^3 fc\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{d^2b^3\sqrt{-a^2+b^2}}-$

input
```
int((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3*a^2*f*x^2+1/4/b*f*x^2+1/b^3*a^2*e*x+1/2/b*e*x+1/2*a*(d*x*f+I*f+d*e
)/b^2/d^2*exp(I*(d*x+c))+1/2*a*(d*x*f-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c))+2*I
/d^2/b^3*a^3*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-
a^2+b^2)^(1/2))-1/d/b^3*a^3*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-
a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d/b^3*a^3*f/(-a^2+b^2)^(1/2)*l
n((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/d^2/
b^3*a^3*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))*c+1/d^2/b^3*a^3*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+
c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I/d/b^3*a^3*e/(-a^2+b^
2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2/b^3
*a^3*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))-I/d^2/b^3*a^3*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x
+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-1/8*f/d^2/b*cos(2*d*x+2*c
)-1/4*(f*x+e)/d/b*sin(2*d*x+2*c)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(330) = 660$.

Time = 0.25 (sec) , antiderivative size = 1247, normalized size of antiderivative = 3.32

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d
*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b + 1) - 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) - 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) -
a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) + 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c
) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b + 1) - (2*a^4 - a^2*b^2 - b^4)*d^2*f*x^2 - 2*(2*a^4 - a^2*b^2
 - b^4)*d^2*e*x + (a^2*b^2 - b^4)*f*cos(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*
c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(a^3*b*d*e - a^3*b*c*f)*sqrt(-(a^2 -
 b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) - 2*I*a) - 2*(a^3*b*d*e - a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2
*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)
 - 2*(a^3*b*d*e - a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(a^3*b*d*f*
x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) +
 2*(a^3*b*d*f*x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\sin(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)^3*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**5*d*e - cos(c + d*x)*sin(c + d*x)*a**4*b**2*d*e + cos(c + d*x)*sin(c + d
*x)*a**3*b**3*f + cos(c + d*x)*sin(c + d*x)*a**2*b**4*d*e - 4*cos(c + d*x)
*sin(c + d*x)*a**2*b**4*d*f*x - cos(c + d*x)*sin(c + d*x)*a*b**5*f + 4*cos
(c + d*x)*sin(c + d*x)*b**6*d*f*x + 2*cos(c + d*x)*a**5*b*d*e - 2*cos(c +
d*x)*a**3*b**3*d*e - 4*cos(c + d*x)*a**3*b**3*d*f*x - 16*cos(c + d*x)*a**2
*b**4*f + 4*cos(c + d*x)*a*b**5*d*f*x + 16*cos(c + d*x)*b**6*f - 16*int((t
an((c + d*x)/2)*x)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*ta
n((c + d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a +
2*tan((c + d*x)/2)*b + a),x)*a**4*b**3*d**2*f + 80*int((tan((c + d*x)/2)*x
)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c + d*x)/2)**4
*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2
)*b + a),x)*a**2*b**5*d**2*f - 64*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/
2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c + d*x)/2)**4*a + 4*tan((c + d
*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**7*
d**2*f + 32*int(x/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan
((c + d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2
*tan((c + d*x)/2)*b + a),x)*a**3*b**4*d**2*f - 32*int(x/(tan((c + d*x)/2)*
*6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c + d*x)/2)**4*a + 4*tan((c + d*x)
/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b**...
```

3.231 $\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx=\frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}+\frac{a\cos(c+dx)}{b^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
1/2*(2*a^2+b^2)*x/b^3-2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/b^3/(a^2-b^2)^(1/2)/d+a*cos(d*x+c)/b^2/d-1/2*cos(d*x+c)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx=\frac{2(2a^2+b^2)(c+dx)-\frac{8a^3\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+4ab\cos(c+dx)-b^2\sin(2(c+dx))}{4b^3d}$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
(2*(2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*b*Cos[c + d*x] - b^2*Sin[2*(c + d*x)])/(
4*b^3*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3272, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{-2a\sin^2(c+dx)+b\sin(c+dx)+a}{a+b\sin(c+dx)}dx}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-2a\sin(c+dx)^2+b\sin(c+dx)+a}{a+b\sin(c+dx)}dx}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{2a^3\int\frac{1}{a+b\sin(c+dx)}dx}{b}}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{2a^3\int\frac{1}{a+b\sin(c+dx)}dx}{b}}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 3139

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\int\frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 1083

$$\frac{\frac{\frac{8a^3\int\frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}+\frac{x(2a^2+b^2)}{b}}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}}{b}+\frac{2a\cos(c+dx)}{bd}}{2b}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
((((2*a^2 + b^2)*x)/b - (4*a^3*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt
[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d))/b + (2*a*Cos[c + d*x])/(b*d))/(2*b)
- (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b^{2}}{2}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}ab-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^{2}}{2}+ab\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+\left(2a^{2}+b^{2}\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^{3}}-\frac{2a^{3}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\cdots}{2\sqrt{a^{2}-b^{2}}}\right)}{b^{3}\sqrt{a^{2}-b^{2}}}}{d}$
default	$\frac{\frac{\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b^{2}}{2}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}ab-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^{2}}{2}+ab\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+\left(2a^{2}+b^{2}\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^{3}}-\frac{2a^{3}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\cdots}{2\sqrt{a^{2}-b^{2}}}\right)}{b^{3}\sqrt{a^{2}-b^{2}}}}{d}$
risch	$\frac{x\,a^{2}}{b^{3}}+\frac{x}{2b}+\frac{a\,e^{i(dx+c)}}{2b^{2}d}+\frac{a\,e^{-i(dx+c)}}{2b^{2}d}+\frac{i a^{3}\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^{2}-b^{2}}\,a-a^{2}+b^{2}\right)}{\sqrt{a^{2}-b^{2}}\,b}\right)}{\sqrt{a^{2}-b^{2}}\,d\,b^{3}}-\frac{i a^{3}\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^{2}-\cdots}\right.}{\sqrt{a\cdots}}\right.}{\sqrt{a^{2}-b^{2}}\,d\,b^{3}\cdots}$

input
```
int(sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^3*((1/2*tan(1/2*d*x+1/2*c)^3*b^2+tan(1/2*d*x+1/2*c)^2*a*b-1/2*tan
(1/2*d*x+1/2*c)*b^2+a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2+b^2)*arctan
(tan(1/2*d*x+1/2*c)))-2*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.36

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}a^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-(2a}{2(a^2b^3-b^5)d}\right.$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*a^3*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*s
qrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
(2*a^4 - a^2*b^2 - b^4)*d*x + (a^2*b^2 - b^4)*cos(d*x + c)*sin(d*x + c) -
2*(a^3*b - a*b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d), 1/2*(2*sqrt(a^2 - b^2
)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (2*a^
4 - a^2*b^2 - b^4)*d*x - (a^2*b^2 - b^4)*cos(d*x + c)*sin(d*x + c) + 2*(a^
3*b - a*b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}b^3}-\frac{(2a^2+b^2)(dx+c)}{b^3}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
 1/2*c) + b)/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2 - b^2)*b^3) - (2*a^2 + b^2)*(
d*x + c)/b^3 - 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 -
b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 37.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} - \frac{\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d} + \frac{2\,a^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d} + \frac{a\,\cos\left(c+d\,x\right)}{b^2\,d} + \frac{a^3\operatorname{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2\right)1\mathrm{i}}{\sqrt{b^2-a^2}\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)2\mathrm{i}}{b^3\,d\,\sqrt{b^2-a^2}}$$

input
```
int(sin(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

output
```
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(b*d) - sin(2*c + 2*d*x)/(4*b*
d) + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) + (a*cos(
c + d*x))/(b^2*d) + (a^3*atan(((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (
d*x)/2) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x
)/2) + 2*b*sin(c/2 + (d*x)/2))))*2i)/(b^3*d*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.56

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^3-\cos\left(dx+c\right)\sin\left(dx+c\right)a^2b^2+\cos\left(dx+c\right)\sin\left(dx+c\right)b^4+2\cos}{2b^3d\left(a^2-b^2\right)}$$

input
```
int(sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**3 - cos(c + d*x)*sin(c + d*x)*a**2*b**2 + cos(c + d*x)*sin(c + d*x)*b**4
 + 2*cos(c + d*x)*a**3*b - 2*cos(c + d*x)*a*b**3 + 2*a**4*c + 2*a**4*d*x -
 a**2*b**2*c - a**2*b**2*d*x - b**4*c - b**4*d*x)/(2*b**3*d*(a**2 - b**2))
```

3.232 $\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 732

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d^2} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d^2} - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} + \frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d^3} - \frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d^3} - \frac{6if^3\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4} + \frac{6if^3\operatorname{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4} - \frac{6bf^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d^4} + \frac{6bf^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d^4}$$

output
```
-2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+I*b*(f*x+e)^3*ln(1-I*b*exp(I*(d*x
+c))/(a-(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d-I*b*(f*x+e)^3*ln(1-I*b*exp(I
*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d+3*I*f*(f*x+e)^2*polylog
(2,-exp(I*(d*x+c)))/a/d^2-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2+
3*b*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/(a^2-b
^2)^(1/2)/d^2-3*b*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1
/2)))/a/(a^2-b^2)^(1/2)/d^2-6*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a/d^3
+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3+6*I*b*f^2*(f*x+e)*polylog(3
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d^3-6*I*b*f^2*(
f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)
/d^3-6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4+6*I*f^3*polylog(4,exp(I*(d*x
+c)))/a/d^4-6*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/(a
^2-b^2)^(1/2)/d^4+6*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/a/(a^2-b^2)^(1/2)/d^4
```

Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.22

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{-2d^3(e+fx)^3\text{arctanh}(\cos(c+dx)+i\sin(c+dx)) + \frac{b\left(3d^2f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)+i\left(2id^3e^3\arctan\left(\frac{ia+}{}\right.\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)^3*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*d^3*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] + (b*(3*d^2*f*(
e + f*x)^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + I
*((2*I)*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*d^3*
e^2*f*x*Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + 3*d^3*e*f^
2*x^2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + d^3*f^3*x^3*
Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - 3*d^3*e^2*f*x*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - 3*d^3*e*f^2*x^2*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - d^3*f^3*x^3*Log[1 - (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (3*I)*d^2*f*(e + f*x)^2*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 6*d*f^2*(e + f*x)*PolyLog
[3, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - 6*d*e*f^2*PolyLog[3
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - 6*d*f^3*x*PolyLog[3, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (6*I)*f^3*PolyLog[4, ((-I)*b*E^
(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - (6*I)*f^3*PolyLog[4, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])))/Sqrt[a^2 - b^2] + (3*I)*f*(d^2*(e + f*
x)^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*Poly
Log[3, -Cos[c + d*x] - I*Sin[c + d*x]] - 2*f^2*PolyLog[4, -Cos[c + d*x] -
I*Sin[c + d*x]]) - (3*I)*f*(d^2*(e + f*x)^2*PolyLog[2, Cos[c + d*x] + I*Si
n[c + d*x]] + (2*I)*d*f*(e + f*x)*PolyLog[3, Cos[c + d*x] + I*Sin[c + d*x]
] - 2*f^2*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^4)
```

Rubi [A] (verified)

Time = 2.62 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5046, 3042, 3804, 2694, 27, 2620, 3011, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3804

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\int \frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a}$$

$\Big\downarrow$ 2694

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a}$$

$\Big\downarrow$ 27

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a}$$

$\Big\downarrow$ 2620

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{a}$$

$\Big\downarrow$ 3011

$$\frac{\int (e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ib}{a-}\right.}{bd}\right.}{}\right.}{a}$$

$\Big\downarrow$ 4671

$$\frac{-\frac{3f\int (e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int (e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-$$

$$\frac{2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ib}{a-}\right.}{bd}\right.}{}\right.}{a}$$

$\downarrow$ 3011

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\frac{2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ib}{a-}\right.}{bd}\right.}{}\right.}{a}$$

$\downarrow$ 7163

$$\frac{\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int \operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\cdots\right.}{\cdots}}{a}$$

$$2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int \operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2-b^2}}\cdots\right.$$

$\downarrow$ 2720

$$\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyL}}{}\right.}{}$$

$$a$$

$$2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLo}}{}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

↓ 7143

$$-\frac{2(e+fx)^3\text{arctanh}(e^{i(c+dx)})}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f}{}$$

$$a$$

$$2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

input
```
Int[((e + f*x)^3*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/d + (3*f*((I*(e + f*x)^2*PolyLo
g[2, -E^(I*(c + d*x))])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, -E^(I*(c
+ d*x))])/d + (f*PolyLog[4, -E^(I*(c + d*x))])/d^2))/d))/d - (3*f*((I*(e +
 f*x)^2*PolyLog[2, E^(I*(c + d*x))])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog
[3, E^(I*(c + d*x))])/d + (f*PolyLog[4, E^(I*(c + d*x))])/d^2))/d))/d)/a -
 (2*b*(((-1/2*I)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(
((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) -
 (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 - b^2]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \csc(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3584 vs. $2(634) = 1268$.

Time = 0.40 (sec) , antiderivative size = 3584, normalized size of antiderivative = 4.90

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3\csc\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)b\,e^3+\left(\int \frac{\csc(dx+c)x^3}{\sin(dx+c)b+a}dx\right)a^3d\,f^3-\left(\int \frac{\csc(dx+c)x^3}{\sin(dx+c)b+a}dx\right)a\,b^2d\,f^3+3\left(\int \frac{\mathrm{cs}}{\mathrm{sin}}\right.}{}$$

input
```
int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*b
*e**3 + int((csc(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*a**3*d*f**3 - int(
(csc(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*a*b**2*d*f**3 + 3*int((csc(c +
 d*x)*x**2)/(sin(c + d*x)*b + a),x)*a**3*d*e*f**2 - 3*int((csc(c + d*x)*x*
*2)/(sin(c + d*x)*b + a),x)*a*b**2*d*e*f**2 + 3*int((csc(c + d*x)*x)/(sin(
c + d*x)*b + a),x)*a**3*d*e**2*f - 3*int((csc(c + d*x)*x)/(sin(c + d*x)*b
+ a),x)*a*b**2*d*e**2*f + log(tan((c + d*x)/2))*a**2*e**3 - log(tan((c + d
*x)/2))*b**2*e**3)/(a*d*(a**2 - b**2))
```

3.233 $\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 528

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} + \frac{2if(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} - \frac{2if(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} + \frac{2bf(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d^2} - \frac{2bf(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d^2} - \frac{2f^2\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} + \frac{2f^2\,\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} + \frac{2ibf^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d^3} - \frac{2ibf^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d^3}$$

output

```
-2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d+I*b*(f*x+e)^2*ln(1-I*b*exp(I*(d*x
+c))/(a-(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d-I*b*(f*x+e)^2*ln(1-I*b*exp(I
*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d+2*I*f*(f*x+e)*polylog(2
,-exp(I*(d*x+c)))/a/d^2-2*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2+2*b*
f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1
/2)/d^2-2*b*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/
(a^2-b^2)^(1/2)/d^2-2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+2*f^2*polylog(3
,exp(I*(d*x+c)))/a/d^3+2*I*b*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)
^(1/2)))/a/(a^2-b^2)^(1/2)/d^3-2*I*b*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{d^2(e+fx)^2 \log\left(1-e^{i(c+dx)}\right) - d^2(e+fx)^2 \log\left(1+e^{i(c+dx)}\right) + 2idf(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right) - 2i}{}$$

input
```
Integrate[((e + f*x)^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(d^2*(e + f*x)^2*Log[1 - E^(I*(c + d*x))] - d^2*(e + f*x)^2*Log[1 + E^(I*(
c + d*x))] + (2*I)*d*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))] - (2*I)*d*f*
(e + f*x)*PolyLog[2, E^(I*(c + d*x))] - 2*f^2*PolyLog[3, -E^(I*(c + d*x))]
 + 2*f^2*PolyLog[3, E^(I*(c + d*x))] + (b*(2*d*f*(e + f*x)*PolyLog[2, ((-I
)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + I*((2*I)*d^2*e^2*ArcTan[(I*
a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 2*d^2*e*f*x*Log[1 + (I*b*E^(I*(c
 + d*x)))/(-a + Sqrt[a^2 - b^2])] + d^2*f^2*x^2*Log[1 + (I*b*E^(I*(c + d*x
)))/(-a + Sqrt[a^2 - b^2])] - 2*d^2*e*f*x*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])] - d^2*f^2*x^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])] + (2*I)*d*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])] + 2*f^2*PolyLog[3, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a
^2 - b^2])] - 2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])))/Sqrt[a^2 - b^2])/(a*d^3)
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5046, 3042, 3804, 2694, 27, 2620, 3011, 2720, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3804}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-$$

$$\frac{2b\Bigg(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\Bigg(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}}{2\sqrt{a^2-b^2}}}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right.}{}\right.}{a}$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\,\text{lo}}{}\right.}{}\right.}{a}$$

$\downarrow$ 4671

$$\frac{-\frac{2f\int (e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\,\text{lo}}{}\right.}{}\right.}{a}$$

$\downarrow$ 3011

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)\cdots}{\cdots}}{a}$$

$$\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\operatorname{lo}\cdots}{\cdots}\right.}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 2720

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e\cdots\right)}{d^2}\cdots\right)}{d}\cdots}{a}$$

$$\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\operatorname{lo}\cdots}{\cdots}\right.}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 7143

$$\frac{-\frac{2(e+fx)^2\operatorname{arctanh}(e^{i(c+dx)})}{d}+\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLo}}{}\right.}{d}}{a}$$

$$\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/d + (2*f*((I*(e + f*x)*PolyLog[
2, -E^(I*(c + d*x))])/d - (f*PolyLog[3, -E^(I*(c + d*x))])/d^2))/d - (2*f*
((I*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/d - (f*PolyLog[3, E^(I*(c + d*x
))])/d^2))/d)/a - (2*b*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((
e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (
2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])
/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*
d)))/Sqrt[a^2 - b^2]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \csc(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2412 vs. $2(454) = 908$.

Time = 0.31 (sec) , antiderivative size = 2412, normalized size of antiderivative = 4.57

$$\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*s
in(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) - 2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) + 2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) - 2*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) - 2*(a^2 - b^2)*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) - 2*(a^2 -
 b^2)*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 2*(a^2 - b^2)*f^2*po
lylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 2*(a^2 - b^2)*f^2*polylog(3, -c
os(d*x + c) - I*sin(d*x + c)) - 2*(-I*b^2*d*f^2*x - I*b^2*d*e*f)*sqrt(-(a^
2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b^2*d*f^2*x +
 I*b^2*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
 c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
1) - 2*(I*b^2*d*f^2*x + I*b^2*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*co
s(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*b^2*d*f^2*x - I*b^2*d*e*f)*sqrt(-(a^2 -
 b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) ...
```

Sympy [F]

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2\csc\,(c+dx)}{a+b\sin\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)b\,e^2+\left(\int\frac{\csc(dx+c)x^2}{\sin(dx+c)b+a}dx\right)a^3d\,f^2-\left(\int\frac{\csc(dx+c)x^2}{\sin(dx+c)b+a}dx\right)a\,b^2d\,f^2+2\left(\int\frac{c}{\sin}\right.}{ad\,(a^2-}$$

input
```
int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*b
*e**2 + int((csc(c + d*x)*x**2)/(sin(c + d*x)*b + a),x)*a**3*d*f**2 - int(
(csc(c + d*x)*x**2)/(sin(c + d*x)*b + a),x)*a*b**2*d*f**2 + 2*int((csc(c +
 d*x)*x)/(sin(c + d*x)*b + a),x)*a**3*d*e*f - 2*int((csc(c + d*x)*x)/(sin(
c + d*x)*b + a),x)*a*b**2*d*e*f + log(tan((c + d*x)/2))*a**2*e**2 - log(ta
n((c + d*x)/2))*b**2*e**2)/(a*d*(a**2 - b**2))
```

3.234 $\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 325

$$\begin{aligned}\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = &-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad}\\ &+\frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d}\\ &-\frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d}\\ &+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}\\ &+\frac{bf\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d^2}-\frac{bf\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d^2}\end{aligned}$$

output

```
-2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d+I*b*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))
/(a-(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d-I*b*(f*x+e)*ln(1-I*b*exp(I*(d*x+
c))/(a+(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d+I*f*polylog(2,-exp(I*(d*x+c))
)/a/d^2-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2+b*f*polylog(2,I*b*exp(I*(d*x+c
))/(a-(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d^2-b*f*polylog(2,I*b*exp(I*(d*x
+c))/(a+(a^2-b^2)^(1/2)))/a/(a^2-b^2)^(1/2)/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2016 vs. 2(325) = 650.

Time = 16.59 (sec) , antiderivative size = 2016, normalized size of antiderivative = 6.20

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(e*Log[Tan[(c + d*x)/2]])/(a*d) - (c*f*Log[Tan[(c + d*x)/2]])/(a*d^2) + (f
*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(Pol
yLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^2) + (((2*
(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] - (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(
c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1
- I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I
*a - b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] - (I*f*Log[1 - I*Tan[(c + d
*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt
[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b
 + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/S
qrt[-a^2 + b^2] - (I*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b +
 Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(1 + I*Tan[(c
+ d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyL
og[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2
+ b^2] - (I*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2
 + b^2]))])/Sqrt[-a^2 + b^2])*(-((b*e)/(a*(a + b*Sin[c + d*x]))) + (b*c*f)
/(a*d*(a + b*Sin[c + d*x])) - (b*f*(c + d*x))/(a*d*(a + b*Sin[c + d*x]))))
/(d*((f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2])
)]*Sec[(c + d*x)/2]^2)/(2*Sqrt[-a^2 + b^2]*(1 - I*Tan[(c + d*x)/2])) + ...
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5046, 3042, 3804, 2694, 27, 2620, 2715, 2838, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3804}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \ 2715$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\,\mathrm{l}}{}\right)}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \ 2838$$

$$\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \ 4671$$

$$\frac{-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \ 2715$$

$$\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a}$$

$$\Big\downarrow \textbf{2838}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a}$$

input
```
Int[((e + f*x)*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + (I*f*PolyLog[2, -E^(I*(c + d*
x))])/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])/d^2)/a - (2*b*(((-1/2*I)*b*(
((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (
I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqr
t[a^2 - b^2] + ((I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(287) = 574$.

Time = 1.12 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.00

method	result
risch	$\frac{e\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}-\frac{fb\ln\left(\frac{-ia-\mathrm{e}^{i(dx+c)}b+\sqrt{-a^2+b^2}}{-ia+\sqrt{-a^2+b^2}}\right)x}{da\sqrt{-a^2+b^2}}-\frac{2ieb\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx+c)}-2a}{2\sqrt{-a^2+b^2}}\right)}{da\sqrt{-a^2+b^2}}+\frac{if\,\mathrm{dilog}\left(\mathrm{e}^{i(dx+c)}\right)}{d^2a}-\frac{cf\ln\left(\mathrm{e}^{i(dx\cdots\right.}}{d^2a}$

input
```
int((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*e*ln(exp(I*(d*x+c))-1)-1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*
x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x-2*I/d*e*b/a/(-a^2+b^2
)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2*f/a*
dilog(exp(I*(d*x+c)))-1/d^2/a*c*f*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d
*x+c))+1)+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+1/d*f*b/a/(-a^2+b^2)^(1/2)*ln(
(I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*f*
b/a/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2
+b^2)^(1/2)))*c-1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a
^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*c-1/d/a*f*ln(exp(I*(d*x+c))+1)*x+I
/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))
/(-I*a+(-a^2+b^2)^(1/2)))-I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d
*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/d^2*c*f*b/a/(-a^2+b
^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1428 vs. $2(275) = 550$.

Time = 0.32 (sec) , antiderivative size = 1428, normalized size of antiderivative = 4.39

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
1) + I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) - I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x +
c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) - I*(a^2 - b^2)*f*dilog(cos(d*x + c) + I*sin(d*x + c)) + I*(a^2 - b^2)*f
*dilog(cos(d*x + c) - I*sin(d*x + c)) - I*(a^2 - b^2)*f*dilog(-cos(d*x + c
) + I*sin(d*x + c)) + I*(a^2 - b^2)*f*dilog(-cos(d*x + c) - I*sin(d*x + c)
) - (b^2*d*e - b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*
b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b^2*d*e - b^2*c*f)
*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) + (b^2*d*e - b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)
*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) +
 2*I*a) + (b^2*d*e - b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c)
 - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b^2*d*f*x +
 b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) +
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (...
```

Sympy [F]

$$\int \frac{(e + fx)\csc(c + dx)}{a + b\sin(c + dx)}\,dx = \int \frac{(e + fx)\csc{(c + dx)}}{a + b\sin{(c + dx)}}\,dx$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)be+\left(\int\frac{\csc(dx+c)x}{\sin(dx+c)b+a}dx\right)a^3df-\left(\int\frac{\csc(dx+c)x}{\sin(dx+c)b+a}dx\right)a\,b^2df+\log\left(\tan\left(\frac{dx}{2}\right.\right.}{ad\left(a^2-b^2\right)}$$

input
```
int((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*b
*e + int((csc(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a**3*d*f - int((csc(c +
d*x)*x)/(sin(c + d*x)*b + a),x)*a*b**2*d*f + log(tan((c + d*x)/2))*a**2*e
- log(tan((c + d*x)/2))*b**2*e)/(a*d*(a**2 - b**2))
```

3.235 $\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2b\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{ad}$$

output
```
-2*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(1/2)/d-
arctanh(cos(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
 Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3226, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3226}$$

$$\frac{\int \csc(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\int \csc(c+dx)dx}{a}-\frac{2b\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4b\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{\int \csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \csc(c+dx)dx}{a}-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}$$

$$\Big\downarrow \textbf{4257}$$

$$-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}$$

input
```
Int[Csc[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*b*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(a*Sqrt[a^
2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{ib\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,da}-\frac{ib\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}+\frac{\ln\left(e^{i(dx+c}\right.}{ad}$

input
```
int(csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 297, normalized size of antiderivative = 4.43

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}b\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+(a^2-}{2\,(a^3-ab^2)d}\right.$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*b*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + (a
^2 - b^2)*log(1/2*cos(d*x + c) + 1/2) - (a^2 - b^2)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^3 - a*b^2)*d), 1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^2 - b^2)*log(1/2*cos(d*x + c)
+ 1/2) + (a^2 - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a) - log(abs(tan(1/2*d*x + 1
/2*c)))/a)/d
```

Mupad [B] (verification not implemented)

Time = 39.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.58

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{b^2-a^2}\,\left(-1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)}{1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+3\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2-4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

input
```
int(1/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) + (2*b*atanh(((b^2 - a^2)
^(1/2)*(b^2*sin(c/2 + (d*x)/2)*4i - a^2*sin(c/2 + (d*x)/2)*1i + a*b*cos(c/
2 + (d*x)/2)*2i))/(a^3*cos(c/2 + (d*x)/2)*1i - b^3*sin(c/2 + (d*x)/2)*4i -
 a*b^2*cos(c/2 + (d*x)/2)*2i + a^2*b*sin(c/2 + (d*x)/2)*3i)))/(a*d*(b^2 -
a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{ad\,(a^2-b^2)}$$

input
```
int(csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*b
 + log(tan((c + d*x)/2))*a**2 - log(tan((c + d*x)/2))*b**2)/(a*d*(a**2 - b
**2))
```

3.236 $\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 882

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^3}{ad}+\frac{2b(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{3f(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{ad^2} - \frac{3ibf(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2} + \frac{3ibf(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2} - \frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^2} + \frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^2} - \frac{3if^2(e+fx)\,\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3} + \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{a^2d^3} - \frac{6bf^2(e+fx)\,\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{a^2d^3} - \frac{6ib^2f^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^3} + \frac{6ib^2f^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^3} + \frac{3f^3\,\text{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2ad^4} + \frac{6ibf^3\,\text{PolyLog}\left(4,-e^{i(c+dx)}\right)}{a^2d^4} - \frac{6ibf^3\,\text{PolyLog}\left(4,e^{i(c+dx)}\right)}{a^2d^4} + \frac{6b^2f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^4} - \frac{6b^2f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^4}$$

output
```
-3*I*b*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+2*b*(f*x+e)^3*arctan
h(exp(I*(d*x+c)))/a^2/d-(f*x+e)^3*cot(d*x+c)/a/d+6*I*b*f^3*polylog(4,-exp(
I*(d*x+c)))/a^2/d^4+6*I*b^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a
^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^3+3*f*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)
))/a/d^2+3*I*b*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a^2/d^2+I*b^2*(f*x+e)
^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d-3*b^
2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/(a^2-b
^2)^(1/2)/d^2+3*b^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/a^2/(a^2-b^2)^(1/2)/d^2-6*I*b*f^3*polylog(4,exp(I*(d*x+c)))/a^2/d^
4+6*b*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c)))/a^2/d^3-6*b*f^2*(f*x+e)*polyl
og(3,exp(I*(d*x+c)))/a^2/d^3-I*b^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d-6*I*b^2*f^2*(f*x+e)*polylog(3,I*b*ex
p(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^3+3/2*f^3*polylog(
3,exp(2*I*(d*x+c)))/a/d^4-I*(f*x+e)^3/a/d-3*I*f^2*(f*x+e)*polylog(2,exp(2*
I*(d*x+c)))/a/d^3+6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/a^2/(a^2-b^2)^(1/2)/d^4-6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2
-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^4
```

Mathematica [A] (warning: unable to verify)

Time = 11.29 (sec) , antiderivative size = 1735, normalized size of antiderivative = 1.97

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(I*d^3*e^2*(b*d*e - 3*a*f)*x - I*d^3*e^2*(b*d*e + 3*a*f)*x - ((2*I)*a*d^3*
(e + f*x)^3)/(-1 + E^((2*I)*c)) - 3*d^2*e*f*(b*d*e - 2*a*f)*x*Log[1 - E^((
-I)*(c + d*x))] - 3*d^2*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)*(c + d*x))]
- b*d^3*f^3*x^3*Log[1 - E^((-I)*(c + d*x))] + 3*d^2*e*f*(b*d*e + 2*a*f)*x*
Log[1 + E^((-I)*(c + d*x))] + 3*d^2*f^2*(b*d*e + a*f)*x^2*Log[1 + E^((-I)*
(c + d*x))] + b*d^3*f^3*x^3*Log[1 + E^((-I)*(c + d*x))] - d^2*e^2*(b*d*e -
 3*a*f)*Log[1 - E^(I*(c + d*x))] + d^2*e^2*(b*d*e + 3*a*f)*Log[1 + E^(I*(c
 + d*x))] + (3*I)*d*e*f*(b*d*e + 2*a*f)*PolyLog[2, -E^((-I)*(c + d*x))] +
(6*I)*d*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^((-I)*(c + d*x))] + (3*I)*b*d^2*
f^3*x^2*PolyLog[2, -E^((-I)*(c + d*x))] - (3*I)*d*e*f*(b*d*e - 2*a*f)*Poly
Log[2, E^((-I)*(c + d*x))] - (6*I)*d*f^2*(b*d*e - a*f)*x*PolyLog[2, E^((-I
)*(c + d*x))] - (3*I)*b*d^2*f^3*x^2*PolyLog[2, E^((-I)*(c + d*x))] + 6*f^2
*(b*d*e + a*f)*PolyLog[3, -E^((-I)*(c + d*x))] + 6*b*d*f^3*x*PolyLog[3, -E
^((-I)*(c + d*x))] + 6*f^2*(-(b*d*e) + a*f)*PolyLog[3, E^((-I)*(c + d*x))]
 - 6*b*d*f^3*x*PolyLog[3, E^((-I)*(c + d*x))] - (6*I)*b*f^3*PolyLog[4, -E^
((-I)*(c + d*x))] + (6*I)*b*f^3*PolyLog[4, E^((-I)*(c + d*x))])/(a^2*d^4)
+ (b^2*(2*Sqrt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a
^2 - b^2]] + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-
I)*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(
I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^...
```

Rubi [A] (verified)

Time = 4.56 (sec) , antiderivative size = 836, normalized size of antiderivative = 0.95, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5046, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 5046, 3042, 3804, 2694, 27, 2620, 3011, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5046}$$

$$\frac{\int (e+fx)^3\csc^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3\csc(c+dx)^2dx}{a}-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4672

$$\frac{\frac{3f\int (e+fx)^2\cot(c+dx)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{3f\int -(e+fx)^2\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 25

$$\frac{-\frac{3f\int (e+fx)^2\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4202

$$-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)^2}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}}{a}$$

$\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\int (e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int \operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2\ldots}\right.}{2d}\right.\right.}{d}}{a}$$

$\downarrow$ 2720

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{-\frac{b\int\frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\Big\downarrow \text{5046}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{-\frac{b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{-\frac{b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\Big\downarrow \text{3804}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}$$

$$\frac{b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\int\frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}}{d}}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a}\right)}{a}$$

$\downarrow$ 27

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}}{d}}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}}{d}}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int}{}\right.}{2\sqrt{a^2-}}}{a}}{a}$$

$\downarrow$ 3011

$$-\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$b\left(\frac{\int(e+fx)^3\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad a}$$

$\downarrow$ 4671

$$-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$b\left(\frac{-\frac{3f\int(e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int(e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\right.\right.}{}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right.}{d}\right.\right.}{d}}{a}$$

$$b\left(\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\right.}{d}}{a}\right.$$

$\downarrow$ 7143

$$-\frac{-\frac{(e+fx)^3\cot(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\,\text{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right.\right.}{d}}{a}$$

$$b\left(\frac{\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right.}{d}\right.}{d}}{a}\right.$$

$\downarrow$ 7163

$$-\frac{\cot(c+dx)(e+fx)^3}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi}\right.}{2d}\right.\right.}{d}$$
$$a$$

$$b\left(\frac{-\frac{2\operatorname{arctanh}\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-}{a}\right.$$

↓ 2720

$$\frac{-\frac{\cot(c+dx)(e+fx)^3}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\,\mathrm{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi}\right.}{2d}\right.\right.}{d}}{a}$$

$$b\left(\frac{-\frac{2\,\mathrm{arctanh}\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{3f\left(\frac{i(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\,\mathrm{PolyLog}\left(3,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\,\mathrm{PolyLog}\left(3,-\right.}{d}\right.}{d}\right.}{d}}{a}\right.$$

↓ 7143

$$\frac{-\frac{\cot(c+dx)(e+fx)^3}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\,\mathrm{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi}\right.}{2d}\right.\right.}{d}}{a}$$

$$b\left(\frac{-\frac{2\,\mathrm{arctanh}\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{3f\left(\frac{i(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(4,-e^{i(c+dx)}\right)}{d^2}-\frac{i(e+fx)\,\mathrm{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i}{\cdots}\right.}{\cdots}}{a}\right.$$

input
```
Int[((e + f*x)^3*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-(((e + f*x)^3*Cot[c + d*x])/d) - (3*f*(((I/3)*(e + f*x)^3)/f - (2*I)*(((
-1/2*I)*(e + f*x)^2*Log[1 + E^(I*(2*c + Pi + 2*d*x))])/d + (I*f*(((I/2)*(e
 + f*x)*PolyLog[2, -E^(I*(2*c + Pi + 2*d*x))])/d - (f*PolyLog[3, -E^(I*(2*
c + Pi + 2*d*x))])/(4*d^2)))/d)))/d)/a - (b*(((-2*(e + f*x)^3*ArcTanh[E^(I
*(c + d*x))])/d + (3*f*((I*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/d - (
(2*I)*f*(((-I)*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/d + (f*PolyLog[4, -
E^(I*(c + d*x))])/d^2))/d))/d - (3*f*((I*(e + f*x)^2*PolyLog[2, E^(I*(c +
d*x))])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/d + (f*
PolyLog[4, E^(I*(c + d*x))])/d^2))/d))/d)/a - (2*b*(((-1/2*I)*b*(((e + f*x
)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I
*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d -
((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d
^2))/d))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e +
 f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d + (f*Poly
Log[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqr
t[a^2 - b^2]))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \csc(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4562 vs. $2(771) = 1542$.

Time = 0.45 (sec) , antiderivative size = 4562, normalized size of antiderivative = 5.17

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3 \csc^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,b^2e^3-\cos(dx+c)\,a^3e^3+\cos(dx+c)\,a\,b^2e^3+\left(\int\frac{\csc(dx+c)^2}{\sin(dx+c)b}\right.}{}$$

input
```
int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*b**2*e**3 - cos(c + d*x)*a**3*e**3 + cos(c + d*x)*a*b**2*e**3 + i
nt((csc(c + d*x)**2*x**3)/(sin(c + d*x)*b + a),x)*sin(c + d*x)*a**4*d*f**3
 - int((csc(c + d*x)**2*x**3)/(sin(c + d*x)*b + a),x)*sin(c + d*x)*a**2*b*
*2*d*f**3 + 3*int((csc(c + d*x)**2*x**2)/(sin(c + d*x)*b + a),x)*sin(c + d
*x)*a**4*d*e*f**2 - 3*int((csc(c + d*x)**2*x**2)/(sin(c + d*x)*b + a),x)*s
in(c + d*x)*a**2*b**2*d*e*f**2 + 3*int((csc(c + d*x)**2*x)/(sin(c + d*x)*b
 + a),x)*sin(c + d*x)*a**4*d*e**2*f - 3*int((csc(c + d*x)**2*x)/(sin(c + d
*x)*b + a),x)*sin(c + d*x)*a**2*b**2*d*e**2*f - log(tan((c + d*x)/2))*sin(
c + d*x)*a**2*b*e**3 + log(tan((c + d*x)/2))*sin(c + d*x)*b**3*e**3)/(sin(
c + d*x)*a**2*d*(a**2 - b**2))
```

3.237 $\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 639

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^2}{ad}+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2} - \frac{2ibf(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2} + \frac{2ibf(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2} - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^2} + \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^2} - \frac{if^2\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3} + \frac{2bf^2\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{a^2d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{a^2d^3} - \frac{2ib^2f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^3} + \frac{2ib^2f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d^3}$$

output
```
-I*(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a^2/d-(f*x+e)^2*cot
(d*x+c)/a/d-I*b^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a
^2/(a^2-b^2)^(1/2)/d+I*b^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/a^2/(a^2-b^2)^(1/2)/d+2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d^2-2*I
*b*f*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+2*I*b*f*(f*x+e)*polylog(2,
exp(I*(d*x+c)))/a^2/d^2-2*b^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^2+2*b^2*f*(f*x+e)*polylog(2,I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^2-I*f^2*polylog(2,ex
p(2*I*(d*x+c)))/a/d^3+2*b*f^2*polylog(3,-exp(I*(d*x+c)))/a^2/d^3-2*b*f^2*p
olylog(3,exp(I*(d*x+c)))/a^2/d^3-2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^3+2*I*b^2*f^2*polylog(3,I*b*exp
(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 10.90 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.46

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(I*d^2*e*(b*d*e - 2*a*f)*x - I*d^2*e*(b*d*e + 2*a*f)*x - ((2*I)*a*d^2*(e +
 f*x)^2)/(-1 + E^((2*I)*c)) - 2*d*f*(b*d*e - a*f)*x*Log[1 - E^((-I)*(c + d
*x))] - b*d^2*f^2*x^2*Log[1 - E^((-I)*(c + d*x))] + 2*d*f*(b*d*e + a*f)*x*
Log[1 + E^((-I)*(c + d*x))] + b*d^2*f^2*x^2*Log[1 + E^((-I)*(c + d*x))] -
d*e*(b*d*e - 2*a*f)*Log[1 - E^(I*(c + d*x))] + d*e*(b*d*e + 2*a*f)*Log[1 +
 E^(I*(c + d*x))] + (2*I)*f*(b*d*e + a*f)*PolyLog[2, -E^((-I)*(c + d*x))]
+ (2*I)*b*d*f^2*x*PolyLog[2, -E^((-I)*(c + d*x))] + (2*I)*f*(-(b*d*e) + a*
f)*PolyLog[2, E^((-I)*(c + d*x))] - (2*I)*b*d*f^2*x*PolyLog[2, E^((-I)*(c
+ d*x))] + 2*b*f^2*PolyLog[3, -E^((-I)*(c + d*x))] - 2*b*f^2*PolyLog[3, E^
((-I)*(c + d*x))])/(a^2*d^3) + (I*b^2*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*Po
lyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b
^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]
))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt
[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x))
)/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-
a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I
)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c +
 d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(a^2*Sqrt[-(a^2 - b^2)^2]*d^3) + (Cs
c[c/2]*Csc[c/2 + (d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x
^2*Sin[(d*x)/2]))/(2*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(e^2*Sin[(d*x)...
```

Rubi [A] (verified)

Time = 3.22 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5046, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838, 5046, 3042, 3804, 2694, 27, 2620, 3011, 2720, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)^2\csc^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc(c+dx)^2dx}{a}-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4672

$$\frac{\frac{2f\int (e+fx)\cot(c+dx)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2f\int -\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{-\frac{2f\int (e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)^2\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4202

$$-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int \log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\Big\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-i(2c+2dx+\pi)}\log\left(1+e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$\Big\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}$$

$$\Big\downarrow \text{5046}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$\Big\downarrow \text{3804}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\int\frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{2694}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(\cdots}{\cdots}\right.}{2\sqrt{a^2-b^2}}\cdots}{a}\cdots\right)}{a}$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib}{\cdots}\right)}{a}\cdots\right)}{a}$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\mathrm{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)d\ldots}{d^2}\ldots\right.}{bd}\ldots\right.}{2\sqrt{a^2-b^2}}\ldots\right.}{\ldots}\ldots\right.}{a}$$

$\downarrow$ 4671

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{2f\int(e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int(e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-$$

$$b\left(\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)^2\mathcal{E}}{}\cdots}{a}\right.$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-$$

$$b\left(\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\right.}{d^2}\right.}{d}}{a}\right.$$

$\downarrow$ 7143

$$\frac{-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\mathrm{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{2f\left(\frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\,\mathrm{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\mathrm{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\,\mathrm{PolyLog}\left(3\right.}{d}\right.}{d}}{a}\right.$$

input
```
Int[((e + f*x)^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*Cot[c + d*x])/d) - (2*f*(((I/2)*(e + f*x)^2)/f - (2*I)*(((
-1/2*I)*(e + f*x)*Log[1 + E^(I*(2*c + Pi + 2*d*x))])/d - (f*PolyLog[2, -E^
(I*(2*c + Pi + 2*d*x))])/(4*d^2))))/d)/a - (b*(((-2*(e + f*x)^2*ArcTanh[E^
(I*(c + d*x))])/d + (2*f*((I*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/d - (
f*PolyLog[3, -E^(I*(c + d*x))])/d^2))/d - (2*f*((I*(e + f*x)*PolyLog[2, E^
(I*(c + d*x))])/d - (f*PolyLog[3, E^(I*(c + d*x))])/d^2))/d)/a - (2*b*(((-
1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2]))/a))/
a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \csc(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2972 vs. 2(556) = 1112.

Time = 0.34 (sec) , antiderivative size = 2972, normalized size of antiderivative = 4.65

$$\int \frac{(e+fx)^2 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*si
n(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
)*sin(d*x + c) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x
 + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b)*sin(d*x + c) + 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -
(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)
*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 2*(a^2*b - b^3)*f^2
*polylog(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 2*(a^2*b - b^3)*
f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*(a^2*b - b^
3)*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*(a^2*b
- b^3)*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*(I*
b^3*d*f^2*x + I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c)
- a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b + 1)*sin(d*x + c) + 2*(-I*b^3*d*f^2*x - I*b^3*d*e*f)*sqrt(-(a^2
 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(-I*
b^3*d*f^2*x - I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c)
 - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^...
```

Sympy [F]

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^2\csc^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin\left(dx+c\right)b^2e^2-\cos\left(dx+c\right)a^3e^2+\cos\left(dx+c\right)a\,b^2e^2+\left(\int \frac{\csc(dx+c)^2}{\sin(dx+c)b}\right.}{}$$

input
```
int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*b**2*e**2 - cos(c + d*x)*a**3*e**2 + cos(c + d*x)*a*b**2*e**2 + i
nt((csc(c + d*x)**2*x**2)/(sin(c + d*x)*b + a),x)*sin(c + d*x)*a**4*d*f**2
 - int((csc(c + d*x)**2*x**2)/(sin(c + d*x)*b + a),x)*sin(c + d*x)*a**2*b*
*2*d*f**2 + 2*int((csc(c + d*x)**2*x)/(sin(c + d*x)*b + a),x)*sin(c + d*x)
*a**4*d*e*f - 2*int((csc(c + d*x)**2*x)/(sin(c + d*x)*b + a),x)*sin(c + d*
x)*a**2*b**2*d*e*f - log(tan((c + d*x)/2))*sin(c + d*x)*a**2*b*e**2 + log(
tan((c + d*x)/2))*sin(c + d*x)*b**3*e**2)/(sin(c + d*x)*a**2*d*(a**2 - b**
2))
```

3.238 $\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 370

$$\begin{aligned}\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{2b(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{a^2d}-\frac{(e+fx)\cot(c+dx)}{ad}\\&\quad-\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}d}\\&\quad+\frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}d}\\&\quad+\frac{f\log(\sin(c+dx))}{ad^2}-\frac{ibf\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}\\&\quad+\frac{ibf\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}-\frac{b^2f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}d^2}\\&\quad+\frac{b^2f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}d^2}\end{aligned}$$

output
```
2*b*(f*x+e)*arctanh(exp(I*(d*x+c)))/a^2/d-(f*x+e)*cot(d*x+c)/a/d-I*b^2*(f*
x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d+I*
b^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/
2)/d+f*ln(sin(d*x+c))/a/d^2-I*b*f*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+I*b*f
*polylog(2,exp(I*(d*x+c)))/a^2/d^2-b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^2+b^2*f*polylog(2,I*b*exp(I*(d*x+c)
)/(a+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)^(1/2)/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2171 vs. $2(370) = 740$.

Time = 17.04 (sec) , antiderivative size = 2171, normalized size of antiderivative = 5.87

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d*x)*Cos[(c + d*
x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) + (f*Log[Sin[c + d*x]])/(a*d^2) - (b*e*
Log[Tan[(c + d*x)/2]])/(a^2*d) + (b*c*f*Log[Tan[(c + d*x)/2]])/(a^2*d^2) -
 (b*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I
*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a^2*d^2)
+ (Sec[(c + d*x)/2]*(d*e*Sin[(c + d*x)/2] - c*f*Sin[(c + d*x)/2] + f*(c +
d*x)*Sin[(c + d*x)/2]))/(2*a*d^2) + (((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*Log[1 + I*Tan[(c + d*x
)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2
 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b -
Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))])/Sqr
t[-a^2 + b^2] - (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2]
 + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2]
+ (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2,
 (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 +
 b^2] + (I*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2
 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/
(I*a - b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2, (a + I*a
*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2])*...
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.96, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {5046, 3042, 4672, 3042, 25, 3956, 5046, 3042, 3804, 2694, 27, 2620, 2715, 2838, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5046}$$

$$\frac{\int (e+fx)\csc^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc(c+dx)^2dx}{a}-\frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4672

$$\frac{\frac{f\int \cot(c+dx)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{-\frac{f\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3956

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 5046

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$\Big\downarrow$ 3804

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\csc(c+dx)dx}{a}-\frac{2b\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a}\right)}{a}$$

$\Big\downarrow$ 2694

$$\frac{\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-b\left(\frac{\int(e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a}\right)}{a}$$

$\downarrow$ 27

$$\frac{\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-b\left(\frac{\int(e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-b\left(\frac{\int(e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int\log\left(1-\cdots\right.}{bd}\cdots}{2\sqrt{a^2-b^2}}\cdots}{a}\cdots}{a}$$

$\downarrow$ 2715

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c\cdots}}{a-\sqrt{\cdots}}\right)\cdots}{bd^2}\cdots\right)}{\cdots}\right)}{a}\cdots\right)}{a}$$

$\downarrow$ 2838

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\csc(c+dx)dx}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ib\cdots}{a-\cdots}\right)}{bd^2}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{a}\cdots\right)}{a}$$

$\downarrow$ 4671

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-\frac{b\left(\frac{-\frac{f\int\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+d\cdots}}{a+\sqrt{a^2-\cdots}}\right)}{bd^2}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 2715

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}-\frac{2b\left(ib\left(\frac{(e+fx)\log\left(1-\frac{ib}{\sqrt{}}\right.}{bd}\right.\right.}{}\right.}{a}$$

$\downarrow$ 2838

$$\frac{\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}}{a}-\frac{2b\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c}}{a+\sqrt{a}}\right.}{bd^2}\right.}{2\sqrt{a^2-b^2}}\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-(((e + f*x)*Cot[c + d*x])/d) + (f*Log[-Sin[c + d*x]])/d^2)/a - (b*(((-2*
(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + (I*f*PolyLog[2, -E^(I*(c + d*x))])
/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])/d^2)/a - (2*b*(((-1/2*I)*b*(((e +
 f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*f*P
olyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2
 - b^2] + ((I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5046
```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csc[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csc[c + d*x]^(n - 1)/(a + b*S
in[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ
[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(332) = 664.

Time = 1.24 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{i b^{2} f \,\mathrm{dilog}\left(\frac{i a+e^{i(dx+c)} b-\sqrt{-a^{2}+b^{2}}}{i a-\sqrt{-a^{2}+b^{2}}}\right)}{d^{2} a^{2} \sqrt{-a^{2}+b^{2}}}+\frac{c b f \ln\left(e^{i(dx+c)}-1\right)}{d^{2} a^{2}}-\frac{i b f \,\mathrm{dilog}\left(e^{i(dx+c)}\right)}{d^{2} a^{2}}+\frac{2 i b^{2} e \arctan\left(\frac{2 i b\, e^{i(dx+c)}-2 a}{2 \sqrt{-a^{2}+b^{2}}}\right)}{d\, a^{2} \sqrt{-a^{2}+b^{2}}}+ b^{2}$

input
```
int((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(
1/2))/(I*a-(-a^2+b^2)^(1/2)))+1/d^2/a^2*c*b*f*ln(exp(I*(d*x+c))-1)-I/d^2/a
^2*b*f*dilog(exp(I*(d*x+c)))+2*I/d/a^2*b^2*e/(-a^2+b^2)^(1/2)*arctan(1/2*(
2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/d^2/a^2*b^2*f/(-a^2+b^2)^(1/
2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/
d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/
(I*a+(-a^2+b^2)^(1/2)))*c+1/d/a^2*b*f*ln(exp(I*(d*x+c))+1)*x-2*I*(f*x+e)/d
/a/(exp(2*I*(d*x+c))-1)+I/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*
(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-I/d^2/a^2*b*f*dilog(e
xp(I*(d*x+c))+1)-2*I/d^2/a^2*c*b^2*f/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*ex
p(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-1/d/a^2*b*e*ln(exp(I*(d*x+c))-1)+1/d/a
^2*b*e*ln(exp(I*(d*x+c))+1)+1/d/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(
d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d/a^2*b^2*f/(-a^2+
b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2
)))*x+1/d^2/a*f*ln(exp(I*(d*x+c))-1)-2/d^2/a*f*ln(exp(I*(d*x+c)))+1/d^2/a*
f*ln(exp(I*(d*x+c))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(320) = 640$.

Time = 0.32 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.56

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
)*sin(d*x + c) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) -
a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1)*sin(d*x + c) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*co
s(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*d
ilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^2*b - b^3)*f*d
ilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - I*(a^2*b - b^3)*f*dilog
(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + I*(a^2*b - b^3)*f*dilog(-co
s(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - I*(a^2*b - b^3)*f*dilog(-cos(d
*x + c) - I*sin(d*x + c))*sin(d*x + c) + (b^3*d*e - b^3*c*f)*sqrt(-(a^2 -
b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a)*sin(d*x + c) + (b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*
log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2
*I*a)*sin(d*x + c) - (b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*c
os(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin
(d*x + c) - (b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x +...
```

Sympy [F]

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)\csc^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,b^2e-\cos(dx+c)\,a^3e+\cos(dx+c)\,a\,b^2e+\left(\int\frac{\csc(dx+c)^2x}{\sin(dx+c)b+a}d\right.}{}$$

input
```
int((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*b**2*e - cos(c + d*x)*a**3*e + cos(c + d*x)*a*b**2*e + int((csc(c
 + d*x)**2*x)/(sin(c + d*x)*b + a),x)*sin(c + d*x)*a**4*d*f - int((csc(c +
 d*x)**2*x)/(sin(c + d*x)*b + a),x)*sin(c + d*x)*a**2*b**2*d*f - log(tan((
c + d*x)/2))*sin(c + d*x)*a**2*b*e + log(tan((c + d*x)/2))*sin(c + d*x)*b*
*3*e)/(sin(c + d*x)*a**2*d*(a**2 - b**2))
```

3.239 $\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}d} + \frac{b\,\text{arctanh}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

output
```
2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(1/2)
/d+b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{4b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a\cot\left(\frac{1}{2}(c+dx)\right) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + a\tan}{2a^2d}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
((4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- a*Cot[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2
]] + a*Tan[(c + d*x)/2])/(2*a^2*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3281, 25, 27, 3042, 3226, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{b\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{1}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{3226}$$

$$-\frac{b\left(\frac{\int \csc(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\sin(c+dx)}dx}{a}\right)}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(\frac{\int \csc(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\sin(c+dx)}dx}{a}\right)}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \; 3139$$

$$-\frac{b\left(\frac{\int \csc(c+dx)dx}{a}-\frac{2b\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \; 1083$$

$$-\frac{b\left(\frac{4b\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{\int \csc(c+dx)dx}{a}\right)}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \; 217$$

$$-\frac{b\left(\frac{\int \csc(c+dx)dx}{a}-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}\right)}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \; 4257$$

$$-\frac{b\left(-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{\text{arctanh}(\cos(c+dx))}{ad}\right)}{a}-\frac{\cot(c+dx)}{ad}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
-((b*((-2*b*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(a*S
qrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a*d)))/a) - Cot[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{b^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,d\,a^2}-\frac{b^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a-}{\sqrt{-a^2+b}}\right)}{\sqrt{-a^2+b^2}\,d\,a^2}$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tan(1/2*d*x+1/2*c)+2*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln
(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(78) = 156.

Time = 0.18 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.82

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}b^2\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin(dx+\right.$$
$$-\frac{2\sqrt{a^2-b^2}b^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\sin(dx+c)-(a^2b-b^3)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)}{2(a^4-a^2b^2)d\sin(dx+c}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*b^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin
(d*x + c) - (a^2*b - b^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (a^2*
b - b^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(a^3 - a*b^2)*cos(d
*x + c))/((a^4 - a^2*b^2)*d*sin(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*b^2*arc
tan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - (
a^2*b - b^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (a^2*b - b^3)*log(
-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^
4 - a^2*b^2)*d*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.57

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}a^2} - \frac{2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} + \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^2/(sqrt(a^2 - b^2)*a^2) - 2*b*log(abs(tan(
1/2*d*x + 1/2*c)))/a^2 + tan(1/2*d*x + 1/2*c)/a + (2*b*tan(1/2*d*x + 1/2*c
) - a)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 37.97 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.67

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\,b^2-a^3}{a^4\,d\tan\left(c+d\,x\right)-a^2\,b^2\,d\tan\left(c+d\,x\right)} + \frac{b^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+b^2\,\mathrm{atan}\left(\frac{-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a\,b}{-a^3-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b+2\,a\,b^2+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^3}\right.}{a^4\,d-a^2\,b^2\,d}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
(a*b^2 - a^3)/(a^4*d*tan(c + d*x) - a^2*b^2*d*tan(c + d*x)) + (b^3*log(tan
(c/2 + (d*x)/2)) + b^2*atan((b^2*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i -
 a^2*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + a*b*(b^2 - a^2)^(1/2)*2i)/(
2*a*b^2 - a^3 + 4*b^3*tan(c/2 + (d*x)/2) - 3*a^2*b*tan(c/2 + (d*x)/2)))*(b
^2 - a^2)^(1/2)*2i - a^2*b*log(tan(c/2 + (d*x)/2)))/(a^4*d - a^2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin\left(dx+c\right)b^2-\cos\left(dx+c\right)a^3+\cos\left(dx+c\right)a\,b^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\,\mathrm{s}}{\sin\left(dx+c\right)a^2d\left(a^2-b^2\right)}$$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*b**2 - cos(c + d*x)*a**3 + cos(c + d*x)*a*b**2 - log(tan((c + d*x
)/2))*sin(c + d*x)*a**2*b + log(tan((c + d*x)/2))*sin(c + d*x)*b**3)/(sin(
c + d*x)*a**2*d*(a**2 - b**2))
```

3.240 $\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx = \text{Int}\left(\frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}, x\right)$$

output
```
Defer(Int)((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 18.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

input
```
Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\sin^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sin(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 16.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sin(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\sin\left(dx+c\right)^2}{b\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\sin\left(dx+c\right)^2}{b\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{{\sin\left(c+d\,x\right)}^2\,{\left(e+f\,x\right)}^m}{a+b\,\sin\left(c+d\,x\right)} \,dx$$

input
```
int((sin(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{\left(f x +e \right)^{m} \sin \left(d x +c \right)^{2}}{\sin \left(d x +c \right) b +a}d x$$

input
```
int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*sin(c + d*x)**2)/(sin(c + d*x)*b + a),x)
```

3.241 $\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)} \, dx = \operatorname{Int}\left(\frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)} \, dx$$

input
```
Integrate[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\sin(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sin(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(e+f\,x\right)^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((sin(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
```

output
```
int((sin(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sin\left(dx+c\right)}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*sin(c + d*x))/(sin(c + d*x)*b + a),x)
```

3.242 $\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\text{Int}\left(\frac{(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)} dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)} dx$$

input
```
Int[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)^m/(a + b*sin(c + d*x)),x)
```

output
```
int((e + f*x)^m/(a + b*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.85

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \frac{(fx+e)^m e+(fx+e)^m fx-\left(\int \frac{(fx+e)^m \sin(dx+c)}{\sin(dx+c)b+a}dx\right) bfm-\left(\int \frac{(fx+e)^m \sin(dx+c)}{\sin(dx+c)b+a}dx\right) bf}{af\,(m+1)}$$

input
```
int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

output
```
((e + f*x)**m*e + (e + f*x)**m*f*x - int(((e + f*x)**m*sin(c + d*x))/(sin(
c + d*x)*b + a),x)*b*f*m - int(((e + f*x)**m*sin(c + d*x))/(sin(c + d*x)*b
 + a),x)*b*f)/(a*f*(m + 1))
```

3.243 $\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \operatorname{Int}\left(\frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 37.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\csc(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \csc(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 11.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{\sin\left(c+d\,x\right)\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(sin(c + d*x)*(a + b*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc\left(dx+c\right)}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*csc(c + d*x))/(sin(c + d*x)*b + a),x)
```

3.244 $\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 46.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 5048$$

$$\int \frac{\csc^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \csc (dx+c)^2}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m \csc (dx+c)^2}{b\sin (dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 50.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^m \csc^2 (c+dx)}{a+b\sin (c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 10.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{\sin\left(c+d\,x\right)^2\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(sin(c + d*x)^2*(a + b*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \csc\left(dx+c\right)^2}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*csc(c + d*x)**2)/(sin(c + d*x)*b + a),x)
```

3.245 $\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 574

$$\begin{aligned}
\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} \\
&- \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} \\
&- \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} \\
&+ \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{af\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d^2} \\
&+ \frac{a^2f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2} - \frac{f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} \\
&- \frac{a^2f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2} + \frac{f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} \\
&- \frac{a(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))}
\end{aligned}$$

output
```
I*a^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/
2)/d-I*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1
/2)/d-I*a^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^
2)^(3/2)/d+I*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b
^2)^(1/2)/d+a*f*ln(a+b*sin(d*x+c))/b/(a^2-b^2)/d^2+a^2*f*polylog(2,I*b*exp
(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-f*polylog(2,I*b*exp
(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2-a^2*f*polylog(2,I*b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+f*polylog(2,I*b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2-a*(f*x+e)*cos(d
*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2286 vs. 2(574) = 1148.

Time = 17.04 (sec) , antiderivative size = 2286, normalized size of antiderivative = 3.98

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(-(a*d*e*Cos[c + d*x]) + a*c*f*Cos[c + d*x] - a*f*(c + d*x)*Cos[c + d*x])/
((a - b)*(a + b)*d^2*(a + b*Sin[c + d*x])) + (((-2*b*d*e*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*b*c*f*ArcTan[(b + a*
Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (a*f*Log[Sec[(c + d*
x)/2]^2])/b + (a*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])])/b + (I*b*
f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/
2])/(I*a + b - Sqrt[-a^2 + b^2])]])/Sqrt[-a^2 + b^2] - (I*b*f*Log[1 - I*Tan
[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b
+ Sqrt[-a^2 + b^2]))]])/Sqrt[-a^2 + b^2] + (I*b*f*Log[1 - I*Tan[(c + d*x)/2
]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2
 + b^2])]])/Sqrt[-a^2 + b^2] - (I*b*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b +
Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]])/Sqrt
[-a^2 + b^2] + (I*b*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b +
Sqrt[-a^2 + b^2]))]])/Sqrt[-a^2 + b^2] - (I*b*f*PolyLog[2, (a*(1 + I*Tan[(c
 + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]])/Sqrt[-a^2 + b^2] - (I*b*f*Po
lyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]])/Sqrt[-a
^2 + b^2] + (I*b*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt
[-a^2 + b^2]))]])/Sqrt[-a^2 + b^2])*(-((b*e)/((a^2 - b^2)*(a + b*Sin[c + d*
x]))) + (b*c*f)/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (b*f*(c + d*x))/((a
^2 - b^2)*d*(a + b*Sin[c + d*x])) + (a*f*Cos[c + d*x])/((a^2 - b^2)*d*(...
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e+fx}{b(a+b\sin(c+dx))}-\frac{a(e+fx)}{b(a+b\sin(c+dx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 f \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2 (a^2-b^2)^{3/2}} - \frac{a^2 f \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2 (a^2-b^2)^{3/2}} - \frac{f \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}} + \frac{f \operatorname{PolyLog}\left(2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}} + \frac{af\log(a+b\sin(c+dx))}{bd^2(a^2-b^2)} + \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd(a^2-b^2)^{3/2}} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd(a^2-b^2)^{3/2}} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}} + \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}} - \frac{a(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}$$

input
```
Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(3/2)*d) - (I*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e + f*x
)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]
*d) + (a*f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) + (a^2*f*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2)
 - (f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2
 - b^2]*d^2) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(b*(a^2 - b^2)^(3/2)*d^2) + (f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (a*(e + f*x)*Cos[c + d*x])/((a
^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.31

method	result
risch	$\frac{2ia(fx+e)\left(b-ia\,\mathrm{e}^{i(dx+c)}\right)}{b(-a^2+b^2)d\left(2ia\,\mathrm{e}^{i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}-b\right)}+\frac{af\ln\left(ib\,\mathrm{e}^{2i(dx+c)}-2a\,\mathrm{e}^{i(dx+c)}-ib\right)}{b\,d^2(a^2-b^2)}-\frac{2af\ln\left(\mathrm{e}^{i(dx+c)}\right)}{b\,d^2(a^2-b^2)}+\frac{2ibcf\arctan\left(\frac{2ib\,\mathrm{e}^{i(dx-}}{2\sqrt{-a}}\right.}{d^2(a^2-b^2)\sqrt{-a^2-}}$

input
```
int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*a*(f*x+e)*(b-I*a*exp(I*(d*x+c)))/b/(-a^2+b^2)/d/(2*I*a*exp(I*(d*x+c))+
b*exp(2*I*(d*x+c))-b)+1/b/d^2/(a^2-b^2)*a*f*ln(I*b*exp(2*I*(d*x+c))-2*a*ex
p(I*(d*x+c))-I*b)-2/b/d^2/(a^2-b^2)*a*f*ln(exp(I*(d*x+c)))+2*I*b/d^2/(a^2-
b^2)*c*f/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)
^(1/2))-b/d/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^
2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+b/d/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I
*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-b/d^2/(a^2
-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(
-a^2+b^2)^(1/2)))*c+b/d^2/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+
c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I*b/d^2/(a^2-b^2)*f/(-a^
2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)
^(1/2)))-I*b/d^2/(a^2-b^2)*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b+
(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I*b/d/(a^2-b^2)*e/(-a^2+b^2)^(
1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1506 vs. 2(503) = 1006.

Time = 0.37 (sec) , antiderivative size = 1506, normalized size of antiderivative = 2.62

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((-I*b^4*f*sin(d*x + c) - I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) - b)/b + 1) + (I*b^4*f*sin(d*x + c) + I*a*b^3*f)*sqrt(-(
a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*b^4*f*sin(d*x
 + c) + I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b + 1) + (-I*b^4*f*sin(d*x + c) - I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilo
g((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f
*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c)
- a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b) - (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f*x + b^4*c*f)*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a*b^3*d*f*
x + a*b^3*c*f + (b^4*d*f*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f
*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x + c)
 - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{(fx+e)\sin(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,b^2e-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)abe-\cos(dx+c)\,a}{}$$

input
```
int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*s
in(c + d*x)*b**2*e - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqr
t(a**2 - b**2))*a*b*e - cos(c + d*x)*a**3*e + cos(c + d*x)*a*b**2*e + int(
(sin(c + d*x)*x)/(sin(c + d*x)**2*b**2 + 2*sin(c + d*x)*a*b + a**2),x)*sin
(c + d*x)*a**4*b*d*f - 2*int((sin(c + d*x)*x)/(sin(c + d*x)**2*b**2 + 2*si
n(c + d*x)*a*b + a**2),x)*sin(c + d*x)*a**2*b**3*d*f + int((sin(c + d*x)*x
)/(sin(c + d*x)**2*b**2 + 2*sin(c + d*x)*a*b + a**2),x)*sin(c + d*x)*b**5*
d*f + int((sin(c + d*x)*x)/(sin(c + d*x)**2*b**2 + 2*sin(c + d*x)*a*b + a*
*2),x)*a**5*d*f - 2*int((sin(c + d*x)*x)/(sin(c + d*x)**2*b**2 + 2*sin(c +
 d*x)*a*b + a**2),x)*a**3*b**2*d*f + int((sin(c + d*x)*x)/(sin(c + d*x)**2
*b**2 + 2*sin(c + d*x)*a*b + a**2),x)*a*b**4*d*f)/(d*(sin(c + d*x)*a**4*b
- 2*sin(c + d*x)*a**2*b**3 + sin(c + d*x)*b**5 + a**5 - 2*a**3*b**2 + a*b*
*4))
```

3.246 $\int \frac{(e+fx)^2 \sin(c+dx)}{(a+b\sin(c+dx))^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 1106

$$\int \frac{(e+fx)^2 \sin(c+dx)}{(a+b\sin(c+dx))^2} \, dx = \text{Too large to display}$$

output
```
-I*a^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^
(3/2)/d+2*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-
b^2)/d^2-2*I*a*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^
2-b^2)/d^3+2*I*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^
2-b^2)^(1/2)/d^3+2*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/b/(a^2-b^2)/d^2-2*I*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/b/(a^2-b^2)^(1/2)/d^3-2*I*a^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3+I*a^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d+2*a^2*f*(f*x+e)*polylog(2,I*b*exp
(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-2*f*(f*x+e)*polylog
(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2+I*(f*x+e)
^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d-2*a^2*
f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3
/2)/d^2+2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a
^2-b^2)^(1/2)/d^2-I*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/b/(a^2-b^2)^(1/2)/d+2*I*a^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)
^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-I*a*(f*x+e)^2/b/(a^2-b^2)/d-2*I*a*f^2*polyl
og(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3-a*(f*x+e)^2*c
os(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3595 vs. $2(1106) = 2212$.

Time = 25.60 (sec) , antiderivative size = 3595, normalized size of antiderivative = 3.25

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(2*b*e*f*((Pi*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
 b^2] + (2*(-c + Pi/2 - d*x)*ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sq
rt[-a^2 + b^2]] - 2*(-c + ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(-c + Pi/2
 - d*x)/2])/Sqrt[-a^2 + b^2]] + (ArcCos[-(a/b)] - (2*I)*(ArcTanh[((a + b)*
Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - ArcTanh[((-a + b)*Tan[(-c +
Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[Sqrt[-a^2 + b^2]/(Sqrt[2]*Sqrt[b]*
E^((I/2)*(-c + Pi/2 - d*x))*Sqrt[a + b*Sin[c + d*x]])] + (ArcCos[-(a/b)] +
 (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - Arc
Tanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-a^
2 + b^2]*E^((I/2)*(-c + Pi/2 - d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Sin[c +
d*x]])] - (ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/
2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^
2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c +
 Pi/2 - d*x)/2]))] + (-ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c +
Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a + I*Sqrt[-a^2 + b^2])*(a +
b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^
2]*Tan[(-c + Pi/2 - d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*(
a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2
+ b^2]*Tan[(-c + Pi/2 - d*x)/2]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*
(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-...
```

Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{(e+fx)^2}{b(a+b\sin(c+dx))}-\frac{a(e+fx)^2}{b(a+b\sin(c+dx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^{3/2}d}-\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^{3/2}d}+\frac{2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^{3/2}d^2}-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^{3/2}d^2}+\frac{2if^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^{3/2}d^3}-\frac{2if^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^{3/2}d^3}-\frac{i(e+fx)^2a}{b(a^2-b^2)d}+\frac{2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)d^2}+\frac{2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)d^2}-\frac{2if^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)d^3}-\frac{2if^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)d^3}-\frac{(e+fx)^2\cos(c+dx)a}{(a^2-b^2)d(a+b\sin(c+dx))}-\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d}+\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d}-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2}+\frac{2f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2}-\frac{2if^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^3}+\frac{2if^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^3}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
((-I)*a*(e + f*x)^2)/(b*(a^2 - b^2)*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*
x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^
(3/2)*d) - (I*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^2*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) +
(I*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sq
rt[a^2 - b^2]*d) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt
[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (2*f*(e
 + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a
^2 - b^2]*d^2) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) + (2*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2
 - b^2]*d^2) + ((2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - ((2*I)*f^2*PolyLog[3, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^2*f^
2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(a+b\sin(dx+c))^2}dx$$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3122 vs. $2(958) = 1916$.

Time = 0.34 (sec) , antiderivative size = 3122, normalized size of antiderivative = 2.82

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{(fx+e)^2\sin(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{too large to display}$$

input

```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output

```
(12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin
(c + d*x)*a**6*b*f**2 - 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/
sqrt(a**2 - b**2))*sin(c + d*x)*a**4*b**3*d**2*e**2 + 12*sqrt(a**2 - b**2)
*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**4*b**3*f
**2 + 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2)
)*sin(c + d*x)*a**3*b**4*d*e*f - 96*sqrt(a**2 - b**2)*atan((tan((c + d*x)/
2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**2*b**5*f**2 - 24*sqrt(a**2 -
b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a*b**6
*d*e*f + 72*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b*
*2))*sin(c + d*x)*b**7*f**2 + 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a + b)/sqrt(a**2 - b**2))*a**7*f**2 - 6*sqrt(a**2 - b**2)*atan((tan((c + d
*x)/2)*a + b)/sqrt(a**2 - b**2))*a**5*b**2*d**2*e**2 + 12*sqrt(a**2 - b**2
)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**5*b**2*f**2 + 24*sqr
t(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**4*b**3*
d*e*f - 96*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**
2))*a**3*b**4*f**2 - 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sq
rt(a**2 - b**2))*a**2*b**5*d*e*f + 72*sqrt(a**2 - b**2)*atan((tan((c + d*x
)/2)*a + b)/sqrt(a**2 - b**2))*a*b**6*f**2 - 3*cos(c + d*x)*a**7*b*d**2*e*
*2 - 6*cos(c + d*x)*a**7*b*d**2*e*f*x - 3*cos(c + d*x)*a**7*b*d**2*f**2*x*
*2 + 6*cos(c + d*x)*a**6*b**2*d*f**2*x + 3*cos(c + d*x)*a**5*b**3*d**2*...
```

3.247 $\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 1512

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

output
```
-I*a^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^
(3/2)/d+3*a*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^
2-b^2)/d^2-6*I*a*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/
2)))/b/(a^2-b^2)/d^3-I*a*(f*x+e)^3/b/(a^2-b^2)/d+3*a*f*(f*x+e)^2*ln(1-I*b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^2-6*I*f^2*(f*x+e)*polylo
g(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^3+6*I*f^2*
(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2
)/d^3-6*I*a^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/b/(a^2-b^2)^(3/2)/d^3+3*a^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-
(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3*f*(f*x+e)^2*polylog(2,I*b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^2+I*(f*x+e)^3*ln(1-I*b*e
xp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d-3*a^2*f*(f*x+e)^2*p
olylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+3*f
*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(
1/2)/d^2+6*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-
b^2)/d^4-I*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b
^2)^(1/2)/d+6*I*a^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^
(1/2)))/b/(a^2-b^2)^(3/2)/d^3+6*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2
-b^2)^(1/2)))/b/(a^2-b^2)/d^4-6*I*a*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3+I*a^2*(f*x+e)^3*ln(1-I*b*exp(I*...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4970 vs. $2(1512) = 3024$.

Time = 24.86 (sec) , antiderivative size = 4970, normalized size of antiderivative = 3.29

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(3*b*e^2*f*((Pi*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2
 - b^2] + (2*(-c + Pi/2 - d*x)*ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/
Sqrt[-a^2 + b^2]] - 2*(-c + ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(-c + Pi
/2 - d*x)/2])/Sqrt[-a^2 + b^2]] + (ArcCos[-(a/b)] - (2*I)*(ArcTanh[((a + b
)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - ArcTanh[((-a + b)*Tan[(-c
+ Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[Sqrt[-a^2 + b^2]/(Sqrt[2]*Sqrt[b
]*E^((I/2)*(-c + Pi/2 - d*x))*Sqrt[a + b*Sin[c + d*x]])] + (ArcCos[-(a/b)]
 + (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - A
rcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-
a^2 + b^2]*E^((I/2)*(-c + Pi/2 - d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Sin[c
+ d*x]])] - (ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x
)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-
a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c
 + Pi/2 - d*x)/2]))] + (-ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c
+ Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a + I*Sqrt[-a^2 + b^2])*(a
+ b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 +
b^2]*Tan[(-c + Pi/2 - d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])
*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^
2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2]
)*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt...
```

Rubi [A] (verified)

Time = 3.61 (sec) , antiderivative size = 1512, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{(e+fx)^3}{b(a+b\sin(c+dx))}-\frac{a(e+fx)^3}{b(a+b\sin(c+dx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6a\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{b\,(a^2-b^2)\,d^4}+\frac{6a\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^3}{b\,(a^2-b^2)\,d^4}+\frac{6\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{b\sqrt{a^2-b^2}d^4}-\frac{6a^2\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{b\,(a^2-b^2)^{3/2}\,d^4}-\frac{6\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^3}{b\sqrt{a^2-b^2}d^4}+\frac{6a^2\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^3}{b\,(a^2-b^2)^{3/2}\,d^4}-\frac{6ia(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^2}{b\,(a^2-b^2)\,d^3}-\frac{6ia(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^2}{b\,(a^2-b^2)\,d^3}-\frac{6i(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^2}{b\sqrt{a^2-b^2}d^3}+\frac{6ia^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^2}{b\,(a^2-b^2)^{3/2}\,d^3}+\frac{6i(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^2}{b\sqrt{a^2-b^2}d^3}-\frac{6ia^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^2}{b\,(a^2-b^2)^{3/2}\,d^3}+\frac{3a(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f}{b\,(a^2-b^2)\,d^2}+\frac{3a(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f}{b\,(a^2-b^2)\,d^2}-\frac{3(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f}{b\sqrt{a^2-b^2}d^2}+\frac{3a^2(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f}{b\,(a^2-b^2)^{3/2}\,d^2}+\frac{3(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f}{b\sqrt{a^2-b^2}d^2}-\frac{3a^2(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f}{b\,(a^2-b^2)^{3/2}\,d^2}-\frac{ia(e+fx)^3}{b\,(a^2-b^2)\,d}-\frac{i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d}+\frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d}+\frac{i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d}-\frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}\,d}-\frac{a(e+fx)^3\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))}$$

input
```
Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
((-I)*a*(e + f*x)^3)/(b*(a^2 - b^2)*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e +
f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)^(3/2)*d) - (I*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
) + (I*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b*Sqrt[a^2 - b^2]*d) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^2*f*(e + f*x)^2*Po
lyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2
)*d^2) - (3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^2*f*(e
 + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2
 - b^2)^(3/2)*d^2) + (3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + (6*a*f^3*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((6*I)*a^2*f^
2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)^(3/2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(a+b\sin(dx+c))^2}dx$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5184 vs. $2(1320) = 2640$.

Time = 0.53 (sec) , antiderivative size = 5184, normalized size of antiderivative = 3.43

$$\int \frac{(e + fx)^3 \sin(c + dx)}{(a + b\sin(c + dx))^2} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e + fx)^3 \sin(c + dx)}{(a + b\sin(c + dx))^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{(fx+e)^3 \sin(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
(48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin
(c + d*x)*a**7*b*d*e*f**2 - 144*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a
 + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**6*b**2*f**3 - 8*sqrt(a**2 - b**2)
*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**5*b**3*d
**3*e**3 + 48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 -
b**2))*sin(c + d*x)*a**5*b**3*d*e*f**2 + 48*sqrt(a**2 - b**2)*atan((tan((c
 + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**4*b**4*d**2*e**2*f -
240*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin
(c + d*x)*a**4*b**4*f**3 - 384*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a
+ b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**3*b**5*d*e*f**2 - 48*sqrt(a**2 - b
**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**2*b*
*6*d**2*e**2*f + 1248*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt
(a**2 - b**2))*sin(c + d*x)*a**2*b**6*f**3 + 288*sqrt(a**2 - b**2)*atan((t
an((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a*b**7*d*e*f**2 - 8
64*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*b**8*f**3 + 48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sq
rt(a**2 - b**2))*a**8*d*e*f**2 - 144*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a + b)/sqrt(a**2 - b**2))*a**7*b*f**3 - 8*sqrt(a**2 - b**2)*atan((tan(
(c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**6*b**2*d**3*e**3 + 48*sqrt(a**2
- b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**6*b**2*d*e*...
```

3.248 $\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 751

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d} - \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{5/2}d} + \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d} + \frac{3a^2f\log(a+b\sin(c+dx))}{2b(a^2-b^2)^2d^2} - \frac{f\log(a+b\sin(c+dx))}{b(a^2-b^2)d^2} + \frac{3a^3f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{5/2}d^2} - \frac{3af\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d^2} - \frac{3a^3f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{5/2}d^2} + \frac{3af\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d^2} - \frac{a(e+fx)\cos(c+dx)}{2(a^2-b^2)d(a+b\sin(c+dx))^2} - \frac{af}{2b(a^2-b^2)d^2(a+b\sin(c+dx))} - \frac{3a^2(e+fx)\cos(c+dx)}{2(a^2-b^2)^2d(a+b\sin(c+dx))} + \frac{(e+fx)\cos(c+dx)}{(a^2-b^2)d(a+b\sin(c+dx))}$$

output
```
3/2*I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)
^(5/2)/d-3/2*I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a
^2-b^2)^(3/2)/d-3/2*I*a^3*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/
2)))/b/(a^2-b^2)^(5/2)/d+3/2*I*a*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d+3/2*a^2*f*ln(a+b*sin(d*x+c))/b/(a^2-b^2)^2
/d^2-f*ln(a+b*sin(d*x+c))/b/(a^2-b^2)/d^2+3/2*a^3*f*polylog(2,I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2-3/2*a*f*polylog(2,I*b*ex
p(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3/2*a^3*f*polylog(
2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2+3/2*a*f*po
lylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-1/2*
a*(f*x+e)*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-1/2*a*f/b/(a^2-b^2)/d^
2/(a+b*sin(d*x+c))-3/2*a^2*(f*x+e)*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c
))+(f*x+e)*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2666 vs. $2(751) = 1502$.

Time = 19.80 (sec) , antiderivative size = 2666, normalized size of antiderivative = 3.55

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
(-(a*d*e*Cos[c + d*x]) + a*c*f*Cos[c + d*x] - a*f*(c + d*x)*Cos[c + d*x])/
(2*(a - b)*(a + b)*d^2*(a + b*Sin[c + d*x])^2) + (-(a^3*f) + a*b^2*f - a^2
*b*d*e*Cos[c + d*x] - 2*b^3*d*e*Cos[c + d*x] + a^2*b*c*f*Cos[c + d*x] + 2*
b^3*c*f*Cos[c + d*x] - a^2*b*f*(c + d*x)*Cos[c + d*x] - 2*b^3*f*(c + d*x)*
Cos[c + d*x])/(2*(a - b)^2*b*(a + b)^2*d^2*(a + b*Sin[c + d*x])) + (((-3*a
*b*d*e*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (3*a*b*c*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] - ((a^2 + 2*b^2)*f*Log[Sec[(c + d*x)/2]^2])/(2*b) + (a^2*f*Log[Sec[(c
+ d*x)/2]^2*(a + b*Sin[c + d*x])])/(2*b) + b*f*Log[Sec[(c + d*x)/2]^2*(a +
 b*Sin[c + d*x])] + (((3*I)/2)*a*b*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b -
Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/Sqrt
[-a^2 + b^2] - (((3*I)/2)*a*b*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqr
t[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))])/Sqrt[-
a^2 + b^2] + (((3*I)/2)*a*b*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a
^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^
2 + b^2] - (((3*I)/2)*a*b*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b
^2] + (((3*I)/2)*a*b*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b +
 Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] - (((3*I)/2)*a*b*f*PolyLog[2, (a*(1
 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^...
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e+fx}{b(a+b\sin(c+dx))^2}-\frac{a(e+fx)}{b(a+b\sin(c+dx))^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3af\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}+\frac{3af\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}-\frac{af}{2bd^2\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)}+\frac{3a^2f\log(a+b\sin(c+dx))}{2bd^2\left(a^2-b^2\right)^2}-\frac{f\log(a+b\sin(c+dx))}{bd^2\left(a^2-b^2\right)}-\frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd\left(a^2-b^2\right)^{3/2}}+\frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd\left(a^2-b^2\right)^{3/2}}-\frac{3a^2(e+fx)\cos(c+dx)}{2d\left(a^2-b^2\right)^2\left(a+b\sin(c+dx)\right)}-\frac{a(e+fx)\cos(c+dx)}{2d\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)^2}+\frac{(e+fx)\cos(c+dx)}{d\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)}+\frac{3a^3f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{3a^3f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}+\frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd\left(a^2-b^2\right)^{5/2}}-\frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd\left(a^2-b^2\right)^{5/2}}$$

input
```
Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
(((3*I)/2)*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (((3*I)/2)*a^3
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 -
 b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
 Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*Log[a + b*Sin[c + d
*x]])/(2*b*(a^2 - b^2)^2*d^2) - (f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)
*d^2) + (3*a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(2*b*(a^2 - b^2)^(5/2)*d^2) - (3*a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) - (3*a^3*f*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d^2) + (3
*a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 -
b^2)^(3/2)*d^2) - (a*(e + f*x)*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c
 + d*x])^2) - (a*f)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e
 + f*x)*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x
)*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 6.60 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.44

method	result	size
risch	Expression too large to display	1084

input
```
int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
I*(5*I*b^3*a*d*e*exp(I*(d*x+c))-2*I*b^2*f*exp(2*I*(d*x+c))*a^2-3*I*b^3*a*d
*f*x*exp(3*I*(d*x+c))+4*I*b*a^3*d*f*x*exp(I*(d*x+c))+2*a^4*d*f*x*exp(2*I*(
d*x+c))+5*exp(2*I*(d*x+c))*a^2*b^2*d*f*x+2*b^4*d*f*x*exp(2*I*(d*x+c))-3*I*
b^3*a*d*e*exp(3*I*(d*x+c))+2*I*a^4*f*exp(2*I*(d*x+c))+5*I*b^3*a*d*f*x*exp(
I*(d*x+c))+4*I*b*a^3*d*e*exp(I*(d*x+c))+2*a^4*d*e*exp(2*I*(d*x+c))+exp(3*I
*(d*x+c))*a^3*b*f+5*exp(2*I*(d*x+c))*a^2*b^2*d*e-b^3*a*f*exp(3*I*(d*x+c))+
2*b^4*d*e*exp(2*I*(d*x+c))-a^2*b^2*d*f*x-2*b^4*d*f*x-b*a^3*f*exp(I*(d*x+c)
)-a^2*b^2*d*e+b^3*a*f*exp(I*(d*x+c))-2*b^4*d*e)/(-I*b*exp(2*I*(d*x+c))+2*a
*exp(I*(d*x+c))+I*b)^2/(a^2-b^2)^2/d^2/b-1/b/d^2/(-a^2+b^2)^2*a^2*f*ln(exp
(I*(d*x+c)))+1/2/b/d^2/(-a^2+b^2)^2*a^2*f*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(
I*(d*x+c))-I*b)-2*b/d^2/(-a^2+b^2)^2*f*ln(exp(I*(d*x+c)))+b/d^2/(-a^2+b^2)
^2*f*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-3*I*b/d/(-a^2+b^2)^(5
/2)*a*e*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+3*I*b/d^2/
(-a^2+b^2)^(5/2)*a*f*c*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1
/2))-3/2*b/d/(-a^2+b^2)^(5/2)*a*f*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*x+3/2*b/d/(-a^2+b^2)^(5/2)*a*f*ln((I*a+exp(I*(d
*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-3/2*b/d^2/(-a^2+b^2)^
(5/2)*a*f*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)
))*c+3/2*b/d^2/(-a^2+b^2)^(5/2)*a*f*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1
/2))/(I*a+(-a^2+b^2)^(1/2)))*c+3/2*I*b/d^2/(-a^2+b^2)^(5/2)*a*f*dilog((...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2429 vs. $2(651) = 1302$.

Time = 0.35 (sec) , antiderivative size = 2429, normalized size of antiderivative = 3.23

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(3*(I*a*b^5*f*cos(d*x + c)^2 - 2*I*a^2*b^4*f*sin(d*x + c) - I*(a^3*b^
3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
1) + 3*(-I*a*b^5*f*cos(d*x + c)^2 + 2*I*a^2*b^4*f*sin(d*x + c) + I*(a^3*b^
3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x +
 c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
1) + 3*(-I*a*b^5*f*cos(d*x + c)^2 + 2*I*a^2*b^4*f*sin(d*x + c) + I*(a^3*b^
3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x
+ c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
 1) + 3*(I*a*b^5*f*cos(d*x + c)^2 - 2*I*a^2*b^4*f*sin(d*x + c) - I*(a^3*b^
3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x
+ c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b +
 1) + 3*((a^3*b^3 + a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c*f - (a*b^5*d*f*x +
a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*c*f)*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - 3*((a^3*b^3 +
a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c*f - (a*b^5*d*f*x + a*b^5*c*f)*cos(d*x +
 c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 3*((a^3*b^3 + a*b^5)*d*f*x + (a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{(fx+e)\sin(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
( - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*
sin(c + d*x)**2*a**2*b**3*e - 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**3*b**2*e - 12*sqrt(a**2 - b**2)*
atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**4*b*e - 2*cos(c + d*x)
*sin(c + d*x)*a**5*b*e - 2*cos(c + d*x)*sin(c + d*x)*a**3*b**3*e + 4*cos(c
 + d*x)*sin(c + d*x)*a*b**5*e - 4*cos(c + d*x)*a**6*e + 2*cos(c + d*x)*a**
4*b**2*e + 2*cos(c + d*x)*a**2*b**4*e + 4*int((sin(c + d*x)*x)/(sin(c + d*
x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a**3),x)*s
in(c + d*x)**2*a**7*b**2*d*f - 12*int((sin(c + d*x)*x)/(sin(c + d*x)**3*b*
*3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a**3),x)*sin(c + d
*x)**2*a**5*b**4*d*f + 12*int((sin(c + d*x)*x)/(sin(c + d*x)**3*b**3 + 3*s
in(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a**3),x)*sin(c + d*x)**2*a
**3*b**6*d*f - 4*int((sin(c + d*x)*x)/(sin(c + d*x)**3*b**3 + 3*sin(c + d*
x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a**3),x)*sin(c + d*x)**2*a*b**8*d*f
 + 8*int((sin(c + d*x)*x)/(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2
 + 3*sin(c + d*x)*a**2*b + a**3),x)*sin(c + d*x)*a**8*b*d*f - 24*int((sin(
c + d*x)*x)/(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d
*x)*a**2*b + a**3),x)*sin(c + d*x)*a**6*b**3*d*f + 24*int((sin(c + d*x)*x)
/(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b
+ a**3),x)*sin(c + d*x)*a**4*b**5*d*f - 8*int((sin(c + d*x)*x)/(sin(c +...
```

3.249 $\int \frac{(e+fx)^2 \sin(c+dx)}{(a+b\sin(c+dx))^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 1584

$$\int \frac{(e+fx)^2 \sin(c+dx)}{(a+b\sin(c+dx))^3} \, dx = \text{Too large to display}$$

output

```
-1/2*a*(f*x+e)^2*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-3/2*a^2*(f*x+e)
^2*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-3/2*I*a^2*(f*x+e)^2/b/(a^2-b^
2)^2/d-a*f*(f*x+e)/b/(a^2-b^2)/d^2/(a+b*sin(d*x+c))+3/2*I*a^3*(f*x+e)^2*ln
(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d+3*I*a*f^2*p
olylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3+3*I
*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/
2)/d^3+3*a^2*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2
-b^2)^2/d^2+3*a^2*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b
/(a^2-b^2)^2/d^2-3*a^3*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)
^(1/2)))/b/(a^2-b^2)^(5/2)/d^2+3*a*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+3*a^3*f*(f*x+e)*polylog(2,I*b*e
xp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^2-3*a*f*(f*x+e)*pol
ylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3/2*I
*a^3*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5
/2)/d-3/2*I*a*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^
2-b^2)^(3/2)/d-3*I*a^3*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)
))/b/(a^2-b^2)^(5/2)/d^3-3*I*a*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-3*I*a^2*f^2*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3-3*I*a^2*f^2*polylog(2,I*b*exp(I*(d*
x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3+3/2*I*a*(f*x+e)^2*ln(1-I*b...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 13567 vs. $2(1584) = 3168$.

Time = 23.90 (sec) , antiderivative size = 13567, normalized size of antiderivative = 8.57

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output

```
Result too large to show
```

Rubi [A] (verified)

Time = 6.00 (sec) , antiderivative size = 1584, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{(e+fx)^2}{b(a+b\sin(c+dx))^2}-\frac{a(e+fx)^2}{b(a+b\sin(c+dx))^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{2b\left(a^2-b^2\right)^{5/2}d}-\frac{3i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{2b\left(a^2-b^2\right)^{5/2}d}+\frac{3f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{b\left(a^2-b^2\right)^{5/2}d^2}-\frac{3f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{b\left(a^2-b^2\right)^{5/2}d^2}+\frac{3if^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{b\left(a^2-b^2\right)^{5/2}d^3}-\frac{3if^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{b\left(a^2-b^2\right)^{5/2}d^3}-\frac{3i(e+fx)^2a^2}{2b\left(a^2-b^2\right)^2d}+\frac{3f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^2d^2}+\frac{3f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^2d^2}-\frac{3if^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^2d^3}-\frac{3if^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^2d^3}-\frac{3(e+fx)^2\cos(c+dx)a^2}{2\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}+\frac{2f^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)a}{b\left(a^2-b^2\right)^{3/2}d^3}-\frac{3i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{2b\left(a^2-b^2\right)^{3/2}d}+\frac{3i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{2b\left(a^2-b^2\right)^{3/2}d}-\frac{3f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b\left(a^2-b^2\right)^{3/2}d^2}+\frac{3f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{3if^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3if^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b\left(a^2-b^2\right)^{3/2}d^3}-\frac{f(e+fx)a}{b\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}-\frac{(e+fx)^2\cos(c+dx)a}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}-\frac{2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2}-\frac{2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2}+\frac{2if^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3}+\frac{2if^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3}+\frac{(e+fx)^2\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))}$$

input
```
Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
(((-3*I)/2)*a^2*(e + f*x)^2)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^2)/(b*(a^2
 - b^2)*d) + (2*a*f^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b
*(a^2 - b^2)^(3/2)*d^3) + (3*a^2*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (((3*I)/
2)*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*(e + f*x)
*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^
2) - (2*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)*d^2) - (((3*I)/2)*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*
x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f
*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)
^(3/2)*d) - ((3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(b*(a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^3*f*(e + f*x)*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d
^2) - (3*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*(a^2 - b^2)^(3/2)*d^2) - ((3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^3) + ((2*I)*f^2*Poly...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(a+b\sin(dx+c))^3}dx$$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5755 vs. $2(1385) = 2770$.

Time = 0.44 (sec) , antiderivative size = 5755, normalized size of antiderivative = 3.63

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{(fx+e)^2\sin(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2 \sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
(80*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin
(c + d*x)**2*a**10*b**2*f**2 - 48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)
*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**8*b**4*d**2*e**2 - 120*sqrt(
a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)
**2*a**8*b**4*f**2 + 96*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sq
rt(a**2 - b**2))*sin(c + d*x)**2*a**7*b**5*d*e*f + 240*sqrt(a**2 - b**2)*a
tan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**6*b**6*
d**2*e**2 - 440*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2
- b**2))*sin(c + d*x)**2*a**6*b**6*f**2 - 528*sqrt(a**2 - b**2)*atan((tan(
(c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**5*b**7*d*e*f - 3
00*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)**2*a**4*b**8*d**2*e**2 + 432*sqrt(a**2 - b**2)*atan((tan((c + d*x
)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**4*b**8*f**2 + 912*sqrt(a
**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*
*2*a**3*b**9*d*e*f + 368*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/s
qrt(a**2 - b**2))*sin(c + d*x)**2*a**2*b**10*f**2 - 480*sqrt(a**2 - b**2)*
atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a*b**11*d
*e*f - 320*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**
2))*sin(c + d*x)**2*b**12*f**2 + 160*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**11*b*f**2 - 96*sqrt(a**2 ...
```

3.250 $\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 2348

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3} \, dx = \text{Too large to display}$$

output
```
9*a*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2
)/d^4+9*a^2*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b
^2)^2/d^4+9*a^2*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a
^2-b^2)^2/d^4+3*a*f^3*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/
(a^2-b^2)^(3/2)/d^4-3*a*f^3*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/b/(a^2-b^2)^(3/2)/d^4-3*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^
2)^(1/2)))/b/(a^2-b^2)/d^2-3*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b
^2)^(1/2)))/b/(a^2-b^2)/d^2+9*a^3*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2
-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^4-9*a*f^3*polylog(4,I*b*exp(I*(d*x+c))/(
a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^4-9*a^3*f^3*polylog(4,I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(5/2)/d^4+3*I*a*f^2*(f*x+e)*ln(1-I
*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-3*I*a*f^2*(f*
x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-9*
I*a^3*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2
-b^2)^(5/2)/d^3-9*I*a*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2
)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-9*I*a^2*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d
*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3-9*I*a^2*f^2*(f*x+e)*polylog(
2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^2/d^3+9*I*a*f^2*(f*x
+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^
3+9*I*a^3*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 29732 vs. 2(2348) = 4696.

Time = 28.73 (sec) , antiderivative size = 29732, normalized size of antiderivative = 12.66

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 7.83 (sec) , antiderivative size = 2348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{(e+fx)^3}{b(a+b\sin(c+dx))^2}-\frac{a(e+fx)^3}{b(a+b\sin(c+dx))^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{2b(a^2-b^2)^{5/2}d}-\frac{3i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{2b(a^2-b^2)^{5/2}d}+\frac{9f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{2b(a^2-b^2)^{5/2}d^2}-\frac{9f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{2b(a^2-b^2)^{5/2}d^2}+\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{b(a^2-b^2)^{5/2}d^3}-\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{b(a^2-b^2)^{5/2}d^3}-\frac{9f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{b(a^2-b^2)^{5/2}d^4}+\frac{9f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{b(a^2-b^2)^{5/2}d^4}-\frac{3i(e+fx)^3a^2}{2b(a^2-b^2)^2d}+\frac{9f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{2b(a^2-b^2)^2d^2}+\frac{9f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{2b(a^2-b^2)^2d^2}-\frac{9if^2(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^2d^3}-\frac{9if^2(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^2d^3}+\frac{9f^3\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^2d^4}+\frac{9f^3\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b(a^2-b^2)^2d^4}-\frac{3(e+fx)^3\cos(c+dx)a^2}{2(a^2-b^2)^2d(a+b\sin(c+dx))}-\frac{3i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{2b(a^2-b^2)^{3/2}d}-\frac{3if^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^3}+\frac{3i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{2b(a^2-b^2)^{3/2}d}+\frac{3if^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^3}-\frac{3f^3\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^4}-\frac{9f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{2b(a^2-b^2)^{3/2}d^2}+\frac{3f^3\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^4}+\frac{9f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{2b(a^2-b^2)^{3/2}d^2}-\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^3}+\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^3}+\frac{9f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^4}-\frac{9f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a}{b(a^2-b^2)^{3/2}d^4}-\frac{3f(e+fx)^2a}{2b(a^2-b^2)d^2(a+b\sin(c+dx))}-\frac{(e+fx)^3\cos(c+dx)a}{2(a^2-b^2)d(a+b\sin(c+dx))^2}+\frac{i(e+fx)^3}{b(a^2-b^2)d}-\frac{3f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^2}-\frac{3f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^2}+\frac{6if^2(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^3}+\frac{6if^2(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^3}-\frac{6f^3\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^4}-\frac{6f^3\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^4}+\frac{(e+fx)^3\cos(c+dx)}{(a^2-b^2)d(a+b\sin(c+dx))}$$

input
```
Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
(((-3*I)/2)*a^2*(e + f*x)^3)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^3)/(b*(a^2
 - b^2)*d) - ((3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*
f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^
2 - b^2)*d^2) + (((3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^3*
Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)
*d) + ((3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e +
f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)*d^2) - (((3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^3*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (
3*a*f^3*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 -
 b^2)^(3/2)*d^4) - ((9*I)*a^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*Po
lyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3)
 + (9*a^3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(a+b\sin(dx+c))^3} dx$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10614 vs. 2(2046) = 4092.

Time = 0.88 (sec) , antiderivative size = 10614, normalized size of antiderivative = 4.52

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3} dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{(fx+e)^3\sin(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Hanged}$$

input
```
int((sin(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
(480*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*si
n(c + d*x)**2*a**12*b**2*d*e*f**2 - 624*sqrt(a**2 - b**2)*atan((tan((c + d
*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**11*b**3*f**3 - 96*sqrt
(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x
)**2*a**10*b**4*d**3*e**3 - 1920*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**10*b**4*d*e*f**2 + 288*sqrt(a
**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*
*2*a**9*b**5*d**2*e**2*f - 72*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a +
 b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**9*b**5*f**3 + 720*sqrt(a**2 - b*
*2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**8*
b**6*d**3*e**3 - 840*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(
a**2 - b**2))*sin(c + d*x)**2*a**8*b**6*d*e*f**2 - 2304*sqrt(a**2 - b**2)*
atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**7*b**7
*d**2*e**2*f + 336*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a*
*2 - b**2))*sin(c + d*x)**2*a**7*b**7*f**3 - 1800*sqrt(a**2 - b**2)*atan((
tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**6*b**8*d**3*
e**3 + 9192*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b*
*2))*sin(c + d*x)**2*a**6*b**8*d*e*f**2 + 6696*sqrt(a**2 - b**2)*atan((tan
((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**5*b**9*d**2*e**
2*f + 24840*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 -...
```

3.251 $\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 151

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{i(e+fx)^4}{4af}+\frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{6if(e+fx)^2\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2} + \frac{12f^2(e+fx)\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3} + \frac{12if^3\,\text{PolyLog}\left(4,ie^{i(c+dx)}\right)}{ad^4}$$

output
```
-1/4*I*(f*x+e)^4/a/f+2*(f*x+e)^3*ln(1-I*exp(I*(d*x+c)))/a/d-6*I*f*(f*x+e)^
2*polylog(2,I*exp(I*(d*x+c)))/a/d^2+12*f^2*(f*x+e)*polylog(3,I*exp(I*(d*x+
c)))/a/d^3+12*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4
```

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.83

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)}{4a\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)} - \frac{2(\cos(c)+i\sin(c))\left(\frac{(e+fx)^4(\cos(c)-i\sin(c))}{4f}+\frac{3f\left(d^2(e+fx)^2\operatorname{PolyLog}(2,-i\cos(c+dx)-\sin(c+dx))-2idf(e+fx)\operatorname{PolyLog}(3,-i\right.}{}\right.}{a}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*(Cos[c/2] - Sin[c/2]))/(4*a
*(Cos[c/2] + Sin[c/2])) - (2*(Cos[c] + I*Sin[c])*(((e + f*x)^4*(Cos[c] - I
*Sin[c]))/(4*f) + (3*f*(d^2*(e + f*x)^2*PolyLog[2, (-I)*Cos[c + d*x] - Sin
[c + d*x]] - (2*I)*d*f*(e + f*x)*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*
x]] - 2*f^2*PolyLog[4, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*Sin[
c])*(1 - I*Cos[c] + Sin[c]))/d^4 - ((e + f*x)^3*Log[1 + I*Cos[c + d*x] + S
in[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d))/(a*(Cos[c] + I*(1 + Sin[c])))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5028, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \; 5028$$

$$2\int \frac{e^{i(c+dx)}(e+fx)^3}{a-iae^{i(c+dx)}}dx-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \; 2620$$

$$2\left(\frac{(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{3f\int(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)dx}{ad}\right)-\frac{i(e+fx)^4}{4af}$$

$\Big\downarrow$ 3011

$$2\left(\frac{(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right)}{ad}\right)-\frac{i(e+fx)^4}{4af}$$

$\Big\downarrow$ 7163

$$2\left(\frac{(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int\text{PolyLog}\left(3,ie^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\text{PolyLog}\left(3,ie^{i(c+\cdots}}{d}\right.}{d}\right.}{ad}\right.$$

$$\frac{i(e+fx)^4}{4af}$$

$\Big\downarrow$ 2720

$$2\left(\frac{(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(3,ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\cdots}{}\right.}{d}\right.}{ad}\right.$$

$$\frac{i(e+fx)^4}{4af}$$

$\Big\downarrow$ 7143

$$2\left(\frac{(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,ie^{i(c+dx)}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{d}\right)}{d}\right)}{ad}\right.$$

$$\frac{i(e+fx)^4}{4af}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((-1/4*I)*(e + f*x)^4)/(a*f) + 2*(((e + f*x)^3*Log[1 - I*E^(I*(c + d*x))])
/(a*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/d - ((2*I)*f*
(((-I)*(e + f*x)*PolyLog[3, I*E^(I*(c + d*x))])/d + (f*PolyLog[4, I*E^(I*(
c + d*x))])/d^2))/d))/(a*d))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5028
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + Simp[2   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - I*b*E^(I*(c + d*x)
))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2,
 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(134) = 268.

Time = 1.30 (sec) , antiderivative size = 691, normalized size of antiderivative = 4.58

method	result
risch	$\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)e^{3}}{da}-\frac{if^{3}x^{4}}{4a}+\frac{ie^{3}x}{a}+\frac{ie^{4}}{4fa}+\frac{2f^{3}\ln\left(1-i\,\mathrm{e}^{i(dx+c)}\right)x^{3}}{da}-\frac{2c^{3}f^{3}\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d^{4}a}+\frac{2c^{3}f^{3}\ln\left(\mathrm{e}^{i(dx+c)}\right)}{d^{4}a}+$ 12

input
```
int((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*ln(exp(I*(d*x+c)))*e^3+2/d/a*ln(exp(I*(d*x+c))+I)*e^3-1/4*I*f^3/a*x
^4+2/d/a*f^3*ln(1-I*exp(I*(d*x+c)))*x^3-2/d^4/a*c^3*f^3*ln(exp(I*(d*x+c))+
I)+2/d^4/a*c^3*f^3*ln(exp(I*(d*x+c)))+12/d^3/a*f^3*polylog(3,I*exp(I*(d*x+
c)))*x+12/d^3/a*f^2*e*polylog(3,I*exp(I*(d*x+c)))+2/d^4/a*c^3*f^3*ln(1-I*e
xp(I*(d*x+c)))-3/2*I/d^4/a*f^3*c^4+I/a*e^3*x+1/4*I/f/a*e^4-I*f^2/a*e*x^3+1
2*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-6*I/d/a*e^2*f*c*x+6*I/d^2/a*f^2*
e*c^2*x-12*I/d^2/a*f^2*e*polylog(2,I*exp(I*(d*x+c)))*x+6/d/a*f^2*e*ln(1-I*
exp(I*(d*x+c)))*x^2+6/d^3/a*c^2*e*f^2*ln(exp(I*(d*x+c))+I)-6/d^3/a*c^2*e*f
^2*ln(exp(I*(d*x+c)))-6/d^2/a*c*e^2*f*ln(exp(I*(d*x+c))+I)+6/d^2/a*c*e^2*f
*ln(exp(I*(d*x+c)))+6/d/a*e^2*f*ln(1-I*exp(I*(d*x+c)))*x+6/d^2/a*e^2*f*ln(
1-I*exp(I*(d*x+c)))*c-6/d^3/a*c^2*e*f^2*ln(1-I*exp(I*(d*x+c)))-6*I/d^2/a*f
^3*polylog(2,I*exp(I*(d*x+c)))*x^2+4*I/d^3/a*f^2*e*c^3-3*I/d^2/a*e^2*f*c^2
-2*I/d^3/a*f^3*c^3*x-6*I/d^2/a*e^2*f*polylog(2,I*exp(I*(d*x+c)))-3/2*I*f/a
*e^2*x^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(126) = 252.

Time = 0.11 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.26

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{6i\,f^3\text{polylog}(4,i\,\cos{(dx+c)}-\sin{(dx+c)})-6i\,f^3\text{polylog}(4,-i\,\cos{(dx+c)}-\sin{(dx+c)})-3\,(i\,d^2f}{}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(6*I*f^3*polylog(4, I*cos(d*x + c) - sin(d*x + c)) - 6*I*f^3*polylog(4, -I
*cos(d*x + c) - sin(d*x + c)) - 3*(I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2
*e^2*f)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2
*e*f^2*x - I*d^2*e^2*f)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (d^3*e^3 -
 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cos(d*x + c) + I*sin(d*x + c
) + I) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f -
3*c^2*d*e*f^2 + c^3*f^3)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*f^3
*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c
^3*f^3)*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*e^3 - 3*c*d^2*e^2*f
 + 3*c^2*d*e*f^2 - c^3*f^3)*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 6*(d
*f^3*x + d*e*f^2)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 6*(d*f^3*x +
 d*e*f^2)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)))/(a*d^4)
```

Sympy [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*co
s(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*cos(c + d*x)/(s
in(c + d*x) + 1), x) + Integral(3*e**2*f*x*cos(c + d*x)/(sin(c + d*x) + 1)
, x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(126) = 252$.

Time = 0.13 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.44

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx =$$
$$-\frac{\frac{12\,ce^2f\log(ad\sin(dx+c)+ad)}{ad}-\frac{4\,e^3\log(a\sin(dx+c)+a)}{a}-\frac{-i\,(dx+c)^4f^3-4\,(i\,def^2-i\,cf^3)\,(dx+c)^3+48i\,f^3\text{Li}_4(i\,e^{(i\,dx+i\,c)})-6\,(i\,d^2e^2f}{}}{}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(12*c*e^2*f*log(a*d*sin(d*x + c) + a*d)/(a*d) - 4*e^3*log(a*sin(d*x +
 c) + a)/a - (-I*(d*x + c)^4*f^3 - 4*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^3 + 4
8*I*f^3*polylog(4, I*e^(I*d*x + I*c)) - 6*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I
*c^2*f^3)*(d*x + c)^2 - 4*(3*I*c^2*d*e*f^2 - I*c^3*f^3)*(d*x + c) - 8*(-3*
I*c^2*d*e*f^2 + I*c^3*f^3)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 8*(I*
(d*x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c)^2 + 3*(I*d^2*e^2*f - 2
*I*c*d*e*f^2 + I*c^2*f^3)*(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) +
1) - 24*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*(d*x + c)^2*f^3 + I*c^2*f^3 + 2*(
I*d*e*f^2 - I*c*f^3)*(d*x + c))*dilog(I*e^(I*d*x + I*c)) + 4*(3*c^2*d*e*f^
2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e
^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)
^2 + 2*sin(d*x + c) + 1) + 48*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*polylog(3,
 I*e^(I*d*x + I*c)))/(a*d^3))/d
```

Giac [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)\,{\left(e+f\,x\right)}^3}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x)^3)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{-8\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x^3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)d\,f^3-24\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)de\,f^2-24\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)d\,e^2f-4\log\left(\tan\left(\right.\right.}{4ad}$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - 8*int((tan((c + d*x)/2)*x**3)/(tan((c + d*x)/2) + 1),x)*d*f**3 - 24*in
t((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2) + 1),x)*d*e*f**2 - 24*int((tan
((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*d*e**2*f - 4*log(tan((c + d*x)/
2)**2 + 1)*e**3 + 8*log(tan((c + d*x)/2) + 1)*e**3 + 6*d*e**2*f*x**2 + 4*d
*e*f**2*x**3 + d*f**3*x**4)/(4*a*d)
```

3.252 $\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{4if(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2} + \frac{4f^2\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}$$

output
```
-1/3*I*(f*x+e)^3/a/f+2*(f*x+e)^2*ln(1-I*exp(I*(d*x+c)))/a/d-4*I*f*(f*x+e)*
polylog(2,I*exp(I*(d*x+c)))/a/d^2+4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)}{3a\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)} - \frac{2(\cos(c)+i\sin(c))\left(\frac{(e+fx)^3(\cos(c)-i\sin(c))}{3f}-\frac{(e+fx)^2\log(1+i\cos(c+dx)+\sin(c+dx))(1+i\cos(c)+\sin(c))}{d}+\frac{2f(d(e+fx)\operatorname{Pol}}{}\right.}{a(\cos(c)+i(1+\sin(c)))}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*(Cos[c/2] - Sin[c/2]))/(3*a*(Cos[c/2] + Sin
[c/2])) - (2*(Cos[c] + I*Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f)
- ((e + f*x)^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[
c]))/d + (2*f*(d*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] -
I*f*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])
))/d^3))/(a*(Cos[c] + I*(1 + Sin[c])))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5028, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5028}$$

$$2\int \frac{e^{i(c+dx)}(e+fx)^2}{a-iae^{i(c+dx)}}dx-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2f\int (e+fx)\log\left(1-ie^{i(c+dx)}\right)dx}{ad}\right)-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}(2,ie^{i(c+dx)})}{d}-\frac{if\int \text{PolyLog}(2,ie^{i(c+dx)})dx}{d}\right)}{ad}\right)-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2720}$$

$$2\left(\frac{(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{ad}\right)-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{7143}$$

$$2\left(\frac{(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{d^2}\right)}{ad}\right)-\frac{i(e+fx)^3}{3af}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((-1/3*I)*(e + f*x)^3)/(a*f) + 2*(((e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])
/(a*d) - (2*f*((I*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/d - (f*PolyLog[
3, I*E^(I*(c + d*x))])/d^2))/(a*d))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5028
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + Simp[2   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - I*b*E^(I*(c + d*x)
))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2,
 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(101) = 202$.

Time = 1.09 (sec), antiderivative size = 433, normalized size of antiderivative = 3.80

method	result
risch	$\frac{ie^2x}{a}+\frac{4if^2c^3}{3d^3a}-\frac{4ief\,\mathrm{polylog}\left(2,ie^{i(dx+c)}\right)}{d^2a}+\frac{2if^2c^2x}{d^2a}-\frac{2\ln\left(e^{i(dx+c)}\right)e^2}{da}-\frac{2c^2f^2\ln\left(e^{i(dx+c)}\right)}{d^3a}+\frac{2\ln\left(e^{i(dx+c)}+i\right)e^2}{da}+$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I/a*e^2*x+4/3*I/d^3/a*f^2*c^3-4*I/d^2/a*e*f*polylog(2,I*exp(I*(d*x+c)))+2*
I/d^2/a*f^2*c^2*x-2/d/a*ln(exp(I*(d*x+c)))*e^2-2/d^3/a*c^2*f^2*ln(exp(I*(d
*x+c)))+2/d/a*ln(exp(I*(d*x+c))+I)*e^2+4*f^2*polylog(3,I*exp(I*(d*x+c)))/a
/d^3+2/d/a*f^2*ln(1-I*exp(I*(d*x+c)))*x^2+4/d/a*e*f*ln(1-I*exp(I*(d*x+c)))
*x-1/3*I*f^2/a*x^3-I*f/a*e*x^2+2/d^3/a*c^2*f^2*ln(exp(I*(d*x+c))+I)-2/d^3/
a*c^2*f^2*ln(1-I*exp(I*(d*x+c)))-2*I/d^2/a*e*f*c^2-4*I/d^2/a*f^2*polylog(2
,I*exp(I*(d*x+c)))*x+1/3*I/f/a*e^3-4*I/d/a*e*f*c*x+4/d^2/a*e*f*ln(1-I*exp(
I*(d*x+c)))*c-4/d^2/a*c*e*f*ln(exp(I*(d*x+c))+I)+4/d^2/a*c*e*f*ln(exp(I*(d
*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(95) = 190$.

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.67

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{2\,f^2\mathrm{polylog}(3, i\,\cos{(dx+c)} - \sin{(dx+c)}) + 2\,f^2\mathrm{polylog}(3, -i\,\cos{(dx+c)} - \sin{(dx+c)}) - 2\,(i\,df^2x}{}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(2*f^2*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 2*f^2*polylog(3, -I*cos
(d*x + c) - sin(d*x + c)) - 2*(I*d*f^2*x + I*d*e*f)*dilog(I*cos(d*x + c) -
 sin(d*x + c)) - 2*(-I*d*f^2*x - I*d*e*f)*dilog(-I*cos(d*x + c) - sin(d*x
+ c)) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(cos(d*x + c) + I*sin(d*x + c)
+ I) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(I*cos(d*x + c
) + sin(d*x + c) + 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*
log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*
log(-cos(d*x + c) + I*sin(d*x + c) + I))/(a*d^3)
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*co
s(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*cos(c + d*x)/(sin(c +
 d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.61

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx =$$
$$-\frac{\frac{6\,cef\log(ad\sin(dx+c)+ad)}{ad} - \frac{3\,e^2\log(a\sin(dx+c)+a)}{a} - \frac{-i\,(dx+c)^3f^2 - 3i\,(dx+c)c^2f^2 + 6i\,c^2f^2\arctan(\sin(dx+c)+1,\cos(dx+c)) - 3\,(i\,c}{}}{}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(6*c*e*f*log(a*d*sin(d*x + c) + a*d)/(a*d) - 3*e^2*log(a*sin(d*x + c)
 + a)/a - (-I*(d*x + c)^3*f^2 - 3*I*(d*x + c)*c^2*f^2 + 6*I*c^2*f^2*arctan
2(sin(d*x + c) + 1, cos(d*x + c)) - 3*(I*d*e*f - I*c*f^2)*(d*x + c)^2 + 12
*f^2*polylog(3, I*e^(I*d*x + I*c)) - 6*(I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I
*c*f^2)*(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 12*(I*d*e*f +
 I*(d*x + c)*f^2 - I*c*f^2)*dilog(I*e^(I*d*x + I*c)) + 3*((d*x + c)^2*f^2
+ c^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)
^2 + 2*sin(d*x + c) + 1))/(a*d^2))/d
```

Giac [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^2\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\cos(c+d\,x)\,(e+f\,x)^2}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x)^2)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{-6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)d\,f^2-12\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)def-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)e^2+6\log\left(\tan\left(\frac{dx}{2}\right.\right.}{3ad}$$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - 6*int((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2) + 1),x)*d*f**2 - 12*in
t((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*d*e*f - 3*log(tan((c + d*
x)/2)**2 + 1)*e**2 + 6*log(tan((c + d*x)/2) + 1)*e**2 + 3*d*e*f*x**2 + d*f
**2*x**3)/(3*a*d)
```

3.253 $\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}$$

output
```
-1/2*I*(f*x+e)^2/a/f+2*(f*x+e)*ln(1-I*exp(I*(d*x+c)))/a/d-2*I*f*polylog(2,
I*exp(I*(d*x+c)))/a/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 246 vs. 2(79) = 158.

Time = 6.55 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.11

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-ic^2f+icf\pi-2icdfx+idf\pi x-id^2fx^2+4f\pi\log\left(1+e^{-i(c+dx)}\right)+4cf\log\left(1-ie^{i(c+dx)}\right)+2f\pi\log\left(\right.}{}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((-I)*c^2*f + I*c*f*Pi - (2*I)*c*d*f*x + I*d*f*Pi*x - I*d^2*f*x^2 + 4*f*Pi
*Log[1 + E^((-I)*(c + d*x))] + 4*c*f*Log[1 - I*E^(I*(c + d*x))] + 2*f*Pi*L
og[1 - I*E^(I*(c + d*x))] + 4*d*f*x*Log[1 - I*E^(I*(c + d*x))] - 4*f*Pi*Lo
g[Cos[(c + d*x)/2]] + 4*d*e*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*c
*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*f*Pi*Log[Sin[(2*c + Pi + 2
*d*x)/4]] - (4*I)*f*PolyLog[2, I*E^(I*(c + d*x))])/(2*a*d^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5028, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{ 5028}$$

$$2\int \frac{e^{i(c+dx)}(e+fx)}{a-iae^{i(c+dx)}}dx-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{ 2620}$$

$$2\left(\frac{(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{f\int \log\left(1-ie^{i(c+dx)}\right)dx}{ad}\right)-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{ 2715}$$

$$2\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{ad^2}+\frac{(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}\right)-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{ 2838}$$

$$2\left(\frac{(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}\right)-\frac{i(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((-1/2*I)*(e + f*x)^2)/(a*f) + 2*(((e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(
a*d) - (I*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5028
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + Simp[2   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - I*b*E^(I*(c + d*x)
))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2,
 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(69) = 138.

Time = 1.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.57

method	result
risch	$-\frac{if\,x^2}{2a}+\frac{iex}{a}-\frac{2\ln\left(e^{i(dx+c)}\right)e}{da}+\frac{2\ln\left(e^{i(dx+c)}+i\right)e}{da}-\frac{2ifcx}{da}-\frac{if\,c^2}{d^2a}+\frac{2f\ln\left(1-ie^{i(dx+c)}\right)x}{da}+\frac{2f\ln\left(1-ie^{i(dx+c)}\right)c}{d^2a}-$

input
```
int((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/a*f*x^2+I/a*e*x-2/d/a*ln(exp(I*(d*x+c)))*e+2/d/a*ln(exp(I*(d*x+c))+
I)*e-2*I/d/a*f*c*x-I/d^2/a*f*c^2+2/d/a*f*ln(1-I*exp(I*(d*x+c)))*x+2/d^2/a*
f*ln(1-I*exp(I*(d*x+c)))*c-2*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-2/d^2/a
*c*f*ln(exp(I*(d*x+c))+I)+2/d^2/a*c*f*ln(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(64) = 128.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.97

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{-i\,f\mathrm{Li}_2(i\cos(dx+c)-\sin(dx+c))+i\,f\mathrm{Li}_2(-i\cos(dx+c)-\sin(dx+c))+(de-cf)\log(\cos(dx-$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*f*dilog(I*cos(d*x + c) - sin(d*x + c)) + I*f*dilog(-I*cos(d*x + c) - s
in(d*x + c)) + (d*e - c*f)*log(cos(d*x + c) + I*sin(d*x + c) + I) + (d*f*x
 + c*f)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d*f*x + c*f)*log(-I*cos(
d*x + c) + sin(d*x + c) + 1) + (d*e - c*f)*log(-cos(d*x + c) + I*sin(d*x +
 c) + I))/(a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*cos(c + d*x
)/(sin(c + d*x) + 1), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-i\,d^2fx^2 - 2i\,d^2ex - 4i\,dfx\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right) + 4i\,de\arctan\left(\sin\left(dx+c\right)+1,\cos\left(\right.\right.}{2\,ad^2}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(-I*d^2*f*x^2 - 2*I*d^2*e*x - 4*I*d*f*x*arctan2(cos(d*x + c), sin(d*x
+ c) + 1) + 4*I*d*e*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 4*I*f*dilog(
I*e^(I*d*x + I*c)) + 2*(d*f*x + d*e)*log(cos(d*x + c)^2 + sin(d*x + c)^2 +
 2*sin(d*x + c) + 1))/(a*d^2)
```

Giac [F]

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e+f\,x)}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x))/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x))/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-4\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right)df-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)e+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)e+df\,x^2}{2ad}$$

input
```
int((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - 4*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2) + 1),x)*d*f - 2*log(tan((
c + d*x)/2)**2 + 1)*e + 4*log(tan((c + d*x)/2) + 1)*e + d*f*x**2)/(2*a*d)
```

3.254 $\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log(1+\sin(c+dx))}{ad}$$

output
```
ln(1+sin(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log(1+\sin(c+dx))}{ad}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
Log[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3146}$$
$$\frac{\int \frac{1}{\sin(c+dx)a+a}d(a\sin(c+dx))}{ad}$$
$$\Big\downarrow \text{16}$$
$$\frac{\log(a\sin(c+dx)+a)}{ad}$$

input
```
Int[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
Log[a + a*Sin[c + d*x]]/(a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\ln(a+a\sin(dx+c))}{da}$	19
default	$\frac{\ln(a+a\sin(dx+c))}{da}$	19
parallelrisch	$\frac{-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	37
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\ln(e^{i(dx+c)}+i)}{ad}$	40
norman	$\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{ad}$	44

input
```
int(cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*ln(a+a*sin(d*x+c))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

input
```
integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
log(sin(d*x + c) + 1)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{\log(\sin(c+dx)+1)}{ad} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((log(sin(c + d*x) + 1)/(a*d), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log(a\sin(dx+c)+a)}{ad}$$

input
```
integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
log(a*sin(d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log(|\sin(dx+c)+1|)}{ad}$$

input
```
integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(sin(d*x + c) + 1))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a\,d}$$

input
```
int(cos(c + d*x)/(a + a*sin(c + d*x)),x)
```

output
```
log(sin(c + d*x) + 1)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\log(\sin\left(dx+c\right)+1)}{ad}$$

input
```
int(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
log(sin(c + d*x) + 1)/(a*d)
```

3.255 $\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{ 5048}$$

$$\int \frac{\cos(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cos (dx+c)}{(fx+e)\,(a+a\sin (dx+c))}dx$$

input
```
int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\cos (dx+c)}{(fx+e)(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos (c+dx)}{e\sin (c+dx)+e+fx\sin (c+dx)+fx}\,dx}{a}$$

input
```
integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\cos\left(dx+c\right)}{(fx+e)(a\sin\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\cos\left(dx+c\right)}{(fx+e)(a\sin\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 37.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{(e+f\,x)\,\left(a+a\,\sin(c+d\,x)\right)}\,dx$$

input
```
int(cos(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{2\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)fx+e+fx}dx\right)f-\log(fx+e)}{af}$$

input
```
int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
(2*int(1/(tan((c + d*x)/2)*e + tan((c + d*x)/2)*f*x + e + f*x),x)*f - log(
e + f*x))/(a*f)
```

3.256 $\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 7.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 5048$$
$$\int \frac{\cos(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cos (dx+c)}{(fx+e)^2 (a+a\sin (dx+c))}dx$$

input
```
int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos (dx+c)}{(fx+e)^2(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*
f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos (c+dx)}{e^2\sin (c+dx)+e^2+2efx\sin (c+dx)+2efx+f^2x^2\sin (c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(cos(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2
*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 38.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(e+f\,x\right)}^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cos(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 5.92

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$
$$= \frac{2\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e^2+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)efx+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)f^2x^2+e^2+2efx+f^2x^2}dx\right)e^2+2\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e^2+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)efx+\tan\left(\frac{d}{}\right.}\right.}{ae\left(fx+e\right)}$$

input
```
int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
(2*int(1/(tan((c + d*x)/2)*e**2 + 2*tan((c + d*x)/2)*e*f*x + tan((c + d*x)
/2)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)*e**2 + 2*int(1/(tan((c + d*
x)/2)*e**2 + 2*tan((c + d*x)/2)*e*f*x + tan((c + d*x)/2)*f**2*x**2 + e**2
+ 2*e*f*x + f**2*x**2),x)*e*f*x - x)/(a*e*(e + f*x))
```

3.257 $\int \frac{(e+fx)^3 \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 99

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(e+fx)^4}{4af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{6f^3\sin(c+dx)}{ad^4} - \frac{3f(e+fx)^2\sin(c+dx)}{ad^2}$$

output
```
1/4*(f*x+e)^4/a/f-6*f^2*(f*x+e)*cos(d*x+c)/a/d^3+(f*x+e)^3*cos(d*x+c)/a/d+
6*f^3*sin(d*x+c)/a/d^4-3*f*(f*x+e)^2*sin(d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{d^4x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)+4d(e+fx)\left(-6f^2+d^2(e+fx)^2\right)\cos(c+dx)-12f(-2f^2+d^2(e}{4ad^4}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*d*(e + f*x)*(-6*f^2
 + d^2*(e + f*x)^2)*Cos[c + d*x] - 12*f*(-2*f^2 + d^2*(e + f*x)^2)*Sin[c +
 d*x])/(4*a*d^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5034, 17, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{ 5034}$$

$$\frac{\int (e+fx)^3dx}{a}-\frac{\int (e+fx)^3\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(e+fx)^4}{4af}-\frac{\int (e+fx)^3\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{\int (e+fx)^3\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\int (e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*a*f) - (-(((e + f*x)^3*Cos[c + d*x])/d) + (3*f*(((e + f*x)^
2*Sin[c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])
/d^2))/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{\left((fx+e)^2d^2-6f^2\right)(fx+e)d\cos(dx+c)-3\left((fx+e)^2d^2-2f^2\right)f\sin(dx+c)+\left(x\left(\frac{fx}{2}+e\right)\left(\frac{1}{2}x^2f^2+efx+e^2\right)d^3+d^2e^3-6e\right.}{a\,d^4}$
risch	$\frac{f^3x^4}{4a}+\frac{f^2e\,x^3}{a}+\frac{3f\,e^2x^2}{2a}+\frac{e^3x}{a}+\frac{e^4}{4af}+\frac{(d^2x^3f^3+3d^2e\,f^2x^2+3d^2e^2fx+d^2e^3-6f^3x-6e\,f^2)\cos(dx+c)}{d^3a}-$
derivativedivides	$\frac{-\cos(dx+c)c^3f^3+3\cos(dx+c)c^2de\,f^2-3c^2f^3(\sin(dx+c)-\cos(dx+c)(dx+c))-3\cos(dx+c)c\,d^2e^2f+6cde\,f^2(\sin(dx+}{}$
default	$\frac{-\cos(dx+c)c^3f^3+3\cos(dx+c)c^2de\,f^2-3c^2f^3(\sin(dx+c)-\cos(dx+c)(dx+c))-3\cos(dx+c)c\,d^2e^2f+6cde\,f^2(\sin(dx+}{}$
norman	$\frac{2e\left(d^2e^2-6f^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^2a}+\frac{2e\left(d^2e^2-6f^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d^2a}+\frac{3ef(de+2f)x^2}{2da}+\frac{3f\left(d^2e^2+2def-4f^2\right)x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d^2a}+\frac{3f\left(d\right.}{}$

input
```
int((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(((f*x+e)^2*d^2-6*f^2)*(f*x+e)*d*cos(d*x+c)-3*((f*x+e)^2*d^2-2*f^2)*f*sin(
d*x+c)+(x*(1/2*f*x+e)*(1/2*x^2*f^2+e*f*x+e^2)*d^3+d^2*e^3-6*e*f^2)*d)/a/d^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{d^4f^3x^4+4\,d^4ef^2x^3+6\,d^4e^2fx^2+4\,d^4e^3x+4\,(d^3f^3x^3+3\,d^3ef^2x^2+d^3e^3-6\,def^2+3\,(d^3e^2f-2\,df^3)x}{4\,ad^4}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 4*(d^
3*f^3*x^3 + 3*d^3*e*f^2*x^2 + d^3*e^3 - 6*d*e*f^2 + 3*(d^3*e^2*f - 2*d*f^3
)*x)*cos(d*x + c) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 2*f^3)*s
in(d*x + c))/(a*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(88) = 176$.

Time = 2.31 (sec) , antiderivative size = 984, normalized size of antiderivative = 9.94

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((4*d**4*e**3*x*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2
 + 4*a*d**4) + 4*d**4*e**3*x/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 6
*d**4*e**2*f*x**2*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*
d**4) + 6*d**4*e**2*f*x**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 4*d
**4*e*f**2*x**3*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d*
*4) + 4*d**4*e*f**2*x**3/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + d**4*
f**3*x**4*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) +
d**4*f**3*x**4/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 8*d**3*e**3/(4*
a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 12*d**3*e**2*f*x*tan(c/2 + d*x/2)
**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 12*d**3*e**2*f*x/(4*a*d**4
*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 12*d**3*e*f**2*x**2*tan(c/2 + d*x/2)**2
/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 12*d**3*e*f**2*x**2/(4*a*d**4
*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 4*d**3*f**3*x**3*tan(c/2 + d*x/2)**2/(4
*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 4*d**3*f**3*x**3/(4*a*d**4*tan(c
/2 + d*x/2)**2 + 4*a*d**4) - 24*d**2*e**2*f*tan(c/2 + d*x/2)/(4*a*d**4*tan
(c/2 + d*x/2)**2 + 4*a*d**4) - 48*d**2*e*f**2*x*tan(c/2 + d*x/2)/(4*a*d**4
*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 24*d**2*f**3*x**2*tan(c/2 + d*x/2)/(4*a
*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 48*d*e*f**2/(4*a*d**4*tan(c/2 + d*
x/2)**2 + 4*a*d**4) + 24*d*f**3*x*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 +
d*x/2)**2 + 4*a*d**4) - 24*d*f**3*x/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(97) = 194$.

Time = 0.15 (sec) , antiderivative size = 534, normalized size of antiderivative = 5.39

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(8*c^3*f^3*(1/(a*d^3 + a*d^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^3)) - 24*c^2*e*f^2*(1/(a*d^2 +
 a*d^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/(a*d^2)) + 24*c*e^2*f*(1/(a*d + a*d*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 8*e^3*(arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2)) - 6*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*
e^2*f/(a*d) + 12*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))
*c*e*f^2/(a*d^2) - 6*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x +
 c))*c^2*f^3/(a*d^3) - 4*((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d*x + c) -
 6*(d*x + c)*sin(d*x + c))*e*f^2/(a*d^2) + 4*((d*x + c)^3 + 3*((d*x + c)^2
 - 2)*cos(d*x + c) - 6*(d*x + c)*sin(d*x + c))*c*f^3/(a*d^3) - ((d*x + c)^
4 + 4*((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 12*((d*x + c)^2 - 2)*sin(
d*x + c))*f^3/(a*d^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(97) = 194$.

Time = 0.16 (sec) , antiderivative size = 1077, normalized size of antiderivative = 10.88

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(d^4*f^3*x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*d^4*e*f^2*x^3*tan(1/2*d*x
)^2*tan(1/2*c)^2 + d^4*f^3*x^4*tan(1/2*d*x)^2 + d^4*f^3*x^4*tan(1/2*c)^2 +
 6*d^4*e^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*d^3*f^3*x^3*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 4*d^4*e*f^2*x^3*tan(1/2*d*x)^2 + 4*d^4*e*f^2*x^3*tan(1/2*
c)^2 + 4*d^4*e^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*d^3*e*f^2*x^2*tan(1/2*
d*x)^2*tan(1/2*c)^2 + d^4*f^3*x^4 + 6*d^4*e^2*f*x^2*tan(1/2*d*x)^2 - 4*d^3
*f^3*x^3*tan(1/2*d*x)^2 - 16*d^3*f^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 6*d^4*e
^2*f*x^2*tan(1/2*c)^2 - 4*d^3*f^3*x^3*tan(1/2*c)^2 + 12*d^3*e^2*f*x*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 4*d^4*e*f^2*x^3 + 4*d^4*e^3*x*tan(1/2*d*x)^2 - 12*
d^3*e*f^2*x^2*tan(1/2*d*x)^2 - 48*d^3*e*f^2*x^2*tan(1/2*d*x)*tan(1/2*c) +
24*d^2*f^3*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*d^4*e^3*x*tan(1/2*c)^2 - 12*d
^3*e*f^2*x^2*tan(1/2*c)^2 + 24*d^2*f^3*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*d
^3*e^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*d^4*e^2*f*x^2 + 4*d^3*f^3*x^3 - 12*
d^3*e^2*f*x*tan(1/2*d*x)^2 - 48*d^3*e^2*f*x*tan(1/2*d*x)*tan(1/2*c) + 48*d
^2*e*f^2*x*tan(1/2*d*x)^2*tan(1/2*c) - 12*d^3*e^2*f*x*tan(1/2*c)^2 + 48*d^
2*e*f^2*x*tan(1/2*d*x)*tan(1/2*c)^2 - 24*d*f^3*x*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 4*d^4*e^3*x + 12*d^3*e*f^2*x^2 - 24*d^2*f^3*x^2*tan(1/2*d*x) - 4*d^3*
e^3*tan(1/2*d*x)^2 - 24*d^2*f^3*x^2*tan(1/2*c) - 16*d^3*e^3*tan(1/2*d*x)*t
an(1/2*c) + 24*d^2*e^2*f*tan(1/2*d*x)^2*tan(1/2*c) - 4*d^3*e^3*tan(1/2*c)^
2 + 24*d^2*e^2*f*tan(1/2*d*x)*tan(1/2*c)^2 - 24*d*e*f^2*tan(1/2*d*x)^2*...
```

Mupad [B] (verification not implemented)

Time = 38.57 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.86

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{e^3\,x+\frac{3\,e^2\,f\,x^2}{2}+e\,f^2\,x^3+\frac{f^3\,x^4}{4}}{a} - \frac{d\,\left(6\,x\,\cos\left(c+d\,x\right)\,f^3+6\,e\,\cos\left(c+d\,x\right)\,f^2\right)+d^2\,\left(3\,f^3\,x^2\,\sin\left(c+d\,x\right)+3\,e^2\,f\,\sin\left(c+d\,x\right)+6\,e\,f^2\,x\right.}{}$$

input
```
int((cos(c + d*x)^2*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

output
```
(e^3*x + (f^3*x^4)/4 + (3*e^2*f*x^2)/2 + e*f^2*x^3)/a - (d*(6*e*f^2*cos(c
+ d*x) + 6*f^3*x*cos(c + d*x)) + d^2*(3*f^3*x^2*sin(c + d*x) + 3*e^2*f*sin
(c + d*x) + 6*e*f^2*x*sin(c + d*x)) - d^3*(e^3*cos(c + d*x) + f^3*x^3*cos(
c + d*x) + 3*e^2*f*x*cos(c + d*x) + 3*e*f^2*x^2*cos(c + d*x)) - 6*f^3*sin(
c + d*x))/(a*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.19

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{4\cos(dx+c)\,d^3e^3+12\cos(dx+c)\,d^3e^2fx+12\cos(dx+c)\,d^3e\,f^2x^2+4\cos(dx+c)\,d^3f^3x^3-24\cos(d}{}$$

input
```
int((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(4*cos(c + d*x)*d**3*e**3 + 12*cos(c + d*x)*d**3*e**2*f*x + 12*cos(c + d*x
)*d**3*e*f**2*x**2 + 4*cos(c + d*x)*d**3*f**3*x**3 - 24*cos(c + d*x)*d*e*f
**2 - 24*cos(c + d*x)*d*f**3*x - 12*sin(c + d*x)*d**2*e**2*f - 24*sin(c +
d*x)*d**2*e*f**2*x - 12*sin(c + d*x)*d**2*f**3*x**2 + 24*sin(c + d*x)*f**3
 + 4*d**4*e**3*x + 6*d**4*e**2*f*x**2 + 4*d**4*e*f**2*x**3 + d**4*f**3*x**
4 - 4*d**3*e**3 + 24*d*e*f**2)/(4*a*d**4)
```

3.258 $\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 75

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(e+fx)^3}{3af}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}$$

output
```
1/3*(f*x+e)^3/a/f-2*f^2*cos(d*x+c)/a/d^3+(f*x+e)^2*cos(d*x+c)/a/d-2*f*(f*x
+e)*sin(d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{d^3x(3e^2+3efx+f^2x^2)+3(-2f^2+d^2(e+fx)^2)\cos(c+dx)-6df(e+fx)\sin(c+dx)}{3ad^3}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c +
d*x] - 6*d*f*(e + f*x)*Sin[c + d*x])/(3*a*d^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5034, 17, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int (e+fx)^2dx}{a}-\frac{\int (e+fx)^2\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^3}{3af}-\frac{\int (e+fx)^2\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{\int (e+fx)^2\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*a*f) - (-(((e + f*x)^2*Cos[c + d*x])/d) + (2*f*((f*Cos[c +
d*x])/d^2 + ((e + f*x)*Sin[c + d*x])/d))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{\left(3(fx+e)^2d^2-6f^2\right)\cos(dx+c)-6df(fx+e)\sin(dx+c)+\left(f^2x^3+3x^2ef+3x\,e^2\right)d^3+3d^2e^2-6f^2}{3a\,d^3}$
risch	$\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}+\frac{\left(d^2x^2f^2+2efx\,d^2+d^2e^2-2f^2\right)\cos(dx+c)}{a\,d^3}-\frac{2f(fx+e)\sin(dx+c)}{a\,d^2}$
derivativedivides	$-\frac{-\cos(dx+c)c^2f^2+2\cos(dx+c)cdef-2c\,f^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\cos(dx+c)d^2e^2+2def(\sin(dx+c)-\cos(}{}$
default	$-\frac{-\cos(dx+c)c^2f^2+2\cos(dx+c)cdef-2c\,f^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\cos(dx+c)d^2e^2+2def(\sin(dx+c)-\cos(}{}$
norman	$\frac{\frac{2d^2e^2+4def-4f^2}{a\,d^3}+\frac{4ef\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d^2a}+\frac{\left(2d^2e^2-4f^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\,d^3}+\frac{e(de+2f)x}{da}+\frac{f(de+f)x^2}{ad}+\frac{\left(d^2e^2-2def-4f^2\right)x\tan\left(\frac{dx}{2}+\right.}{d^2a}}{}$

input
```
int((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*((3*(f*x+e)^2*d^2-6*f^2)*cos(d*x+c)-6*d*f*(f*x+e)*sin(d*x+c)+(f^2*x^3+
3*e*f*x^2+3*e^2*x)*d^3+3*d^2*e^2-6*f^2)/a/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+3\,(d^2f^2x^2+2\,d^2efx+d^2e^2-2\,f^2)\cos(dx+c)-6\,(df^2x+def)\sin(dx}{3\,ad^3}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*(d^2*f^2*x^2 + 2*d^2*e*
f*x + d^2*e^2 - 2*f^2)*cos(d*x + c) - 6*(d*f^2*x + d*e*f)*sin(d*x + c))/(a
*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(65) = 130.

Time = 1.74 (sec) , antiderivative size = 605, normalized size of antiderivative = 8.07

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{3d^3e^2x\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3} + \frac{3d^3e^2x}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3} + \frac{3d^3efx^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3} + \frac{3d^3efx^2}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad^3} + \frac{d^3f^2x^3\tan^2\left(\frac{c}{2}\right.}{3ad^3\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right.} \\ \frac{\left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\cos^2(c)}{a\sin(c)+a} \end{cases}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((3*d**3*e**2*x*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2
 + 3*a*d**3) + 3*d**3*e**2*x/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 3
*d**3*e*f*x**2*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**
3) + 3*d**3*e*f*x**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + d**3*f**2
*x**3*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + d**3
*f**2*x**3/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 6*d**2*e**2/(3*a*d*
*3*tan(c/2 + d*x/2)**2 + 3*a*d**3) - 6*d**2*e*f*x*tan(c/2 + d*x/2)**2/(3*a
*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 6*d**2*e*f*x/(3*a*d**3*tan(c/2 + d
*x/2)**2 + 3*a*d**3) - 3*d**2*f**2*x**2*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(
c/2 + d*x/2)**2 + 3*a*d**3) + 3*d**2*f**2*x**2/(3*a*d**3*tan(c/2 + d*x/2)*
*2 + 3*a*d**3) - 12*d*e*f*tan(c/2 + d*x/2)/(3*a*d**3*tan(c/2 + d*x/2)**2 +
 3*a*d**3) - 12*d*f**2*x*tan(c/2 + d*x/2)/(3*a*d**3*tan(c/2 + d*x/2)**2 +
3*a*d**3) - 12*f**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3), Ne(d, 0)),
((e**2*x + e*f*x**2 + f**2*x**3/3)*cos(c)**2/(a*sin(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.12

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{6\,c^2f^2\left(\frac{1}{ad^2+\frac{ad^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad^2}\right)-12\,cef\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)+6\,e^2\left(\frac{\arctan}{}\right.}{}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*(6*c^2*f^2*(1/(a*d^2 + a*d^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e*f*(1/(a*d + a*d*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/(a*d)) + 6*e^2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2)) + 3*((d*x + c)^2 + 2*(d*x + c)*cos(d*x
 + c) - 2*sin(d*x + c))*e*f/(a*d) - 3*((d*x + c)^2 + 2*(d*x + c)*cos(d*x +
 c) - 2*sin(d*x + c))*c*f^2/(a*d^2) + ((d*x + c)^3 + 3*((d*x + c)^2 - 2)*c
os(d*x + c) - 6*(d*x + c)*sin(d*x + c))*f^2/(a*d^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.75

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/3*(d^3*f^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d^3*e*f*x^2*tan(1/2*d*x)^
2*tan(1/2*c)^2 + d^3*f^2*x^3*tan(1/2*d*x)^2 + d^3*f^2*x^3*tan(1/2*c)^2 + 3
*d^3*e^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d^2*f^2*x^2*tan(1/2*d*x)^2*tan(
1/2*c)^2 + 3*d^3*e*f*x^2*tan(1/2*d*x)^2 + 3*d^3*e*f*x^2*tan(1/2*c)^2 + 6*d
^2*e*f*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*f^2*x^3 + 3*d^3*e^2*x*tan(1/2*d
*x)^2 - 3*d^2*f^2*x^2*tan(1/2*d*x)^2 - 12*d^2*f^2*x^2*tan(1/2*d*x)*tan(1/2
*c) + 3*d^3*e^2*x*tan(1/2*c)^2 - 3*d^2*f^2*x^2*tan(1/2*c)^2 + 3*d^2*e^2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 3*d^3*e*f*x^2 - 6*d^2*e*f*x*tan(1/2*d*x)^2 - 2
4*d^2*e*f*x*tan(1/2*d*x)*tan(1/2*c) + 12*d*f^2*x*tan(1/2*d*x)^2*tan(1/2*c)
 - 6*d^2*e*f*x*tan(1/2*c)^2 + 12*d*f^2*x*tan(1/2*d*x)*tan(1/2*c)^2 + 3*d^3
*e^2*x + 3*d^2*f^2*x^2 - 3*d^2*e^2*tan(1/2*d*x)^2 - 12*d^2*e^2*tan(1/2*d*x
)*tan(1/2*c) + 12*d*e*f*tan(1/2*d*x)^2*tan(1/2*c) - 3*d^2*e^2*tan(1/2*c)^2
 + 12*d*e*f*tan(1/2*d*x)*tan(1/2*c)^2 - 6*f^2*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 6*d^2*e*f*x - 12*d*f^2*x*tan(1/2*d*x) - 12*d*f^2*x*tan(1/2*c) + 3*d^2*e^
2 - 12*d*e*f*tan(1/2*d*x) + 6*f^2*tan(1/2*d*x)^2 - 12*d*e*f*tan(1/2*c) + 2
4*f^2*tan(1/2*d*x)*tan(1/2*c) + 6*f^2*tan(1/2*c)^2 - 6*f^2)/(a*d^3*tan(1/2
*d*x)^2*tan(1/2*c)^2 + a*d^3*tan(1/2*d*x)^2 + a*d^3*tan(1/2*c)^2 + a*d^3)
```

Mupad [B] (verification not implemented)

Time = 39.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{e^2\,x+e\,f\,x^2+\frac{f^2\,x^3}{3}}{a} - \frac{2\,f^2\,\cos\left(c+d\,x\right)-d^2\,\left(e^2\,\cos\left(c+d\,x\right)+f^2\,x^2\,\cos\left(c+d\,x\right)+2\,e\,f\,x\,\cos\left(c+d\,x\right)\right)+d\,\left(2\,x\,\sin\left(c+d\right.\right.}{a\,d^3}$$

input
```
int((cos(c + d*x)^2*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

output
```
(e^2*x + (f^2*x^3)/3 + e*f*x^2)/a - (2*f^2*cos(c + d*x) - d^2*(e^2*cos(c +
 d*x) + f^2*x^2*cos(c + d*x) + 2*e*f*x*cos(c + d*x)) + d*(2*f^2*x*sin(c +
d*x) + 2*e*f*sin(c + d*x)))/(a*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.75

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{3\cos(dx+c)\,d^2e^2+6\cos(dx+c)\,d^2efx+3\cos(dx+c)\,d^2f^2x^2-6\cos(dx+c)\,f^2-6\sin(dx+c)\,def}{3a\,d^3}$$

input
```
int((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(3*cos(c + d*x)*d**2*e**2 + 6*cos(c + d*x)*d**2*e*f*x + 3*cos(c + d*x)*d**
2*f**2*x**2 - 6*cos(c + d*x)*f**2 - 6*sin(c + d*x)*d*e*f - 6*sin(c + d*x)*
d*f**2*x + 3*d**3*e**2*x + 3*d**3*e*f*x**2 + d**3*f**2*x**3 - 3*d**2*e**2
+ 6*f**2)/(3*a*d**3)
```

3.259 $\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{(e+fx)^2}{2af}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\sin(c+dx)}{ad^2}$$

output
```
1/2*(f*x+e)^2/a/f+(f*x+e)*cos(d*x+c)/a/d-f*sin(d*x+c)/a/d^2
```

Mathematica [A] (verified)

Time = 2.80 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx =-\frac{(c+dx)(-2de+cf-dfx)-2d(e+fx)\cos(c+dx)+2f\sin(c+dx)}{2ad^2}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((c + d*x)*(-2*d*e + c*f - d*f*x) - 2*d*(e + f*x)*Cos[c + d*x] + 2*f*
Sin[c + d*x])/(a*d^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5034, 17, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int (e+fx)dx}{a}-\frac{\int (e+fx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^2}{2af}-\frac{\int (e+fx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{\int (e+fx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

input
```
Int[((e + f*x)*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*a*f) - (-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^
2)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{(fx+e)d\cos(dx+c)-f\sin(dx+c)+d\left(x\left(\frac{fx}{2}+e\right)d+e\right)}{d^{2}a}$
risch	$\frac{f\,x^{2}}{2a}+\frac{ex}{a}+\frac{(fx+e)\cos(dx+c)}{ad}-\frac{f\sin(dx+c)}{a\,d^{2}}$
derivativedivides	$\frac{-\cos(dx+c)cf+\cos(dx+c)de-f(\sin(dx+c)-\cos(dx+c)(dx+c))-fc(dx+c)+ed(dx+c)+\frac{f(dx+c)^{2}}{2}}{d^{2}a}$
default	$\frac{-\cos(dx+c)cf+\cos(dx+c)de-f(\sin(dx+c)-\cos(dx+c)(dx+c))-fc(dx+c)+ed(dx+c)+\frac{f(dx+c)^{2}}{2}}{d^{2}a}$
norman	$\frac{\frac{2e}{da}+\frac{f\,x^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{f\,x^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a}+\frac{(de+f)x}{ad}-\frac{2f\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{d^{2}a}+\frac{(2de-2f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^{2}a}+\frac{(de-f)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}+\cdots}{}$

input
```
int((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((f*x+e)*d*cos(d*x+c)-f*sin(d*x+c)+d*(x*(1/2*f*x+e)*d+e))/d^2/a
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{d^2fx^2+2\,d^2ex+2\,(dfx+de)\cos{(dx+c)}-2\,f\sin{(dx+c)}}{2\,ad^2}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d^2*f*x^2 + 2*d^2*e*x + 2*(d*f*x + d*e)*cos(d*x + c) - 2*f*sin(d*x +
c))/(a*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(39) = 78$.

Time = 1.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 6.39

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{2d^2ex\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{2d^2ex}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{d^2fx^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{d^2fx^2}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{4de}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{\left(ex+\frac{fx^2}{2}\right)\cos^2(c)}{a\sin(c)+a} \end{cases}$$

input
```
integrate((f*x+e)*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((2*d**2*e*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 +
2*a*d**2) + 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + d**2*f*
x**2*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + d**2*
f*x**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + 4*d*e/(2*a*d**2*tan(c/2
 + d*x/2)**2 + 2*a*d**2) - 2*d*f*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 +
 d*x/2)**2 + 2*a*d**2) + 2*d*f*x/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2)
 - 4*f*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2), Ne(d, 0
)), ((e*x + f*x**2/2)*cos(c)**2/(a*sin(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(49) = 98$.

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.96

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{4\,cf\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}} + \frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right) - 4\,e\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right) - \frac{\left((dx+c)^2+2\,(dx+c\right.}{}}{2\,d}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(4*c*f*(1/(a*d + a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*(arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)) - ((d*x + c)
^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*f/(a*d))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 6.31

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{d^2fx^2\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,d^2ex\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + d^2fx^2\tan\left(\frac{1}{2}\,dx\right)^2 + d^2fx^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,d}{}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d^2*f*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*d^2*e*x*tan(1/2*d*x)^2*tan(
1/2*c)^2 + d^2*f*x^2*tan(1/2*d*x)^2 + d^2*f*x^2*tan(1/2*c)^2 + 2*d*f*x*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*d^2*e*x*tan(1/2*d*x)^2 + 2*d^2*e*x*tan(1/2*c)
^2 + 2*d*e*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*f*x^2 - 2*d*f*x*tan(1/2*d*x)^
2 - 8*d*f*x*tan(1/2*d*x)*tan(1/2*c) - 2*d*f*x*tan(1/2*c)^2 + 2*d^2*e*x - 2
*d*e*tan(1/2*d*x)^2 - 8*d*e*tan(1/2*d*x)*tan(1/2*c) + 4*f*tan(1/2*d*x)^2*t
an(1/2*c) - 2*d*e*tan(1/2*c)^2 + 4*f*tan(1/2*d*x)*tan(1/2*c)^2 + 2*d*f*x +
 2*d*e - 4*f*tan(1/2*d*x) - 4*f*tan(1/2*c))/(a*d^2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + a*d^2*tan(1/2*d*x)^2 + a*d^2*tan(1/2*c)^2 + a*d^2)
```

Mupad [B] (verification not implemented)

Time = 37.88 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{f\,x^2}{2}+e\,x}{a} - \frac{f\,\sin\left(c+d\,x\right)-d\,\left(e\,\cos\left(c+d\,x\right)+f\,x\,\cos\left(c+d\,x\right)\right)}{a\,d^2}$$

input
```
int((cos(c + d*x)^2*(e + f*x))/(a + a*sin(c + d*x)),x)
```

output
```
(e*x + (f*x^2)/2)/a - (f*sin(c + d*x) - d*(e*cos(c + d*x) + f*x*cos(c + d*
x)))/(a*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\cos(dx+c)\,de+2\cos(dx+c)\,dfx-2\sin(dx+c)\,f+2cde+2cf+2d^2ex+d^2f\,x^2-2de}{2a\,d^2}$$

input
```
int((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(2*cos(c + d*x)*d*e + 2*cos(c + d*x)*d*f*x - 2*sin(c + d*x)*f + 2*c*d*e +
2*c*f + 2*d**2*e*x + d**2*f*x**2 - 2*d*e)/(2*a*d**2)
```

3.260 $\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{x}{a}+\frac{\cos(c+dx)}{ad}$$

output
```
x/a+cos(d*x+c)/a/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(19) = 38.

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.11

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\cos^3(c+dx)\left(2\arcsin\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+(-1+\sin(c+dx))\sqrt{1+\sin(c+dx)}\right)}{ad(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^3*(2*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
 + d*x]] + (-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]]))/(a*d*(-1 + Sin[c +
 d*x])^2*(1 + Sin[c + d*x])^(3/2)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(c+dx)^2}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3161}$$
$$\frac{\int 1dx}{a}+\frac{\cos(c+dx)}{ad}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
x/a + Cos[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{dx+\cos(dx+c)-1}{ad}$
risch	$\frac{x}{a}+\frac{\cos(dx+c)}{ad}$
derivativedivides	$\frac{\frac{2}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$
default	$\frac{\frac{2}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$
norman	$\frac{\frac{x}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{a}+\frac{2}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a}+\frac{2\tan\left(\frac{dx}{2}+\right.}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

input
```
int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*(d*x+cos(d*x+c)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{dx+\cos(dx+c)}{ad}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x + cos(d*x + c))/(a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 1.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.63

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + d*x/(
a*d*tan(c/2 + d*x/2)**2 + a*d) + 2/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d,
0)), (x*cos(c)**2/(a*sin(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{dx+c}{a} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 37.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{x}{a} + \frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

output
```
x/a + 2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\cos(dx+c)+dx-1}{ad}$$

input
```
int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(cos(c + d*x) + d*x - 1)/(a*d)
```

3.261 $\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\log(e+fx)}{af} - \frac{\text{CosIntegral}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}$$

output
```
ln(f*x+e)/a/f-Ci(d*e/f+d*x)*sin(c-d*e/f)/a/f-cos(c-d*e/f)*Si(d*e/f+d*x)/a/
f
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\log(e+fx) - \text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right)\sin\left(c-\frac{de}{f}\right) - \cos\left(c-\frac{de}{f}\right)\text{Si}\left(d\left(\frac{e}{f}+x\right)\right)}{af}$$

input
```
Integrate[Cos[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
(Log[e + f*x] - CosIntegral[d*(e/f + x)]*Sin[c - (d*e)/f] - Cos[c - (d*e)/
f]*SinIntegral[d*(e/f + x)])/(a*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5034, 16, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int \frac{1}{e+fx}dx}{a}-\frac{\int \frac{\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(e+fx)}{af}-\frac{\int \frac{\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(e+fx)}{af}-\frac{\int \frac{\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\log(e+fx)}{af}-\frac{\sin\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(e+fx)}{af}-\frac{\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx+\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\log(e+fx)}{af}-\frac{\sin\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}$$

$$\Big\downarrow \; 3783$$

$$\frac{\log(e+fx)}{af}-\frac{\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}$$

input
```
Int[Cos[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Log[e + f*x]/(a*f) - ((CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/f + (C
os[c - (d*e)/f]*SinIntegral[(d*e)/f + d*x])/f)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

method	result	si
derivativedivides	$-\frac{-\frac{\mathrm{Si}\left(-dx-c-\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}-\frac{\mathrm{Ci}\left(dx+c+\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}-\frac{\ln(-cf+de+f(dx+c))}{f}}{a}$	1
default	$-\frac{-\frac{\mathrm{Si}\left(-dx-c-\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}-\frac{\mathrm{Ci}\left(dx+c+\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}-\frac{\ln(-cf+de+f(dx+c))}{f}}{a}$	1
risch	$\frac{\ln(fx+e)}{af}-\frac{i e^{\frac{i(cf-de)}{f}}\operatorname{expIntegral}_1\left(-idx-ic-\frac{-icf+ide}{f}\right)}{2af}+\frac{i e^{-\frac{i(cf-de)}{f}}\operatorname{expIntegral}_1\left(idx+ic-\frac{i(cf-de)}{f}\right)}{2af}$	1

input
```
int(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-Si(-d*x-c-(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*
sin((-c*f+d*e)/f)/f-ln(-c*f+d*e+f*(d*x+c))/f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = -\frac{\mathrm{Ci}\left(\frac{dfx+de}{f}\right)\sin\left(-\frac{de-cf}{f}\right)+\cos\left(-\frac{de-cf}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)-\log\left(fx+e\right)}{af}$$

input
```
integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-(cos_integral((d*f*x + d*e)/f)*sin(-(d*e - c*f)/f) + cos(-(d*e - c*f)/f)*
sin_integral((d*f*x + d*e)/f) - log(f*x + e))/(a*f)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)
/a
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.26

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{d\left(i\,E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)-i\,E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+d\left(E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_1\left(-\frac{i}{}\right.\right.}{2\,adf}$$

input
```
integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(d*(I*exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - I*exp_int
egral_e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) + d*(e
xp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integral_e(1, -(
I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) + 2*d*log(d*e + (d*
x + c)*f - c*f))/(a*d*f)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 670, normalized size of antiderivative = 9.31

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 -
 imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 2*l
og(abs(f*x + e))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*sin_integral((d*f*x + d
*e)/f)*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*real_part(cos_integral(d*x + d*e/
f))*tan(1/2*c)^2*tan(1/2*d*e/f) + 2*real_part(cos_integral(-d*x - d*e/f))*
tan(1/2*c)^2*tan(1/2*d*e/f) - 2*real_part(cos_integral(d*x + d*e/f))*tan(1
/2*c)*tan(1/2*d*e/f)^2 - 2*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c
)*tan(1/2*d*e/f)^2 - imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2 + i
mag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2 - 2*log(abs(f*x + e))*ta
n(1/2*c)^2 - 2*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^2 + 4*imag_part(co
s_integral(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) - 4*imag_part(cos_integ
ral(-d*x - d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) + 8*sin_integral((d*f*x + d*e
)/f)*tan(1/2*c)*tan(1/2*d*e/f) - imag_part(cos_integral(d*x + d*e/f))*tan(
1/2*d*e/f)^2 + imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f)^2 - 2*
log(abs(f*x + e))*tan(1/2*d*e/f)^2 - 2*sin_integral((d*f*x + d*e)/f)*tan(1
/2*d*e/f)^2 + 2*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c) + 2*real_p
art(cos_integral(-d*x - d*e/f))*tan(1/2*c) - 2*real_part(cos_integral(d*x
+ d*e/f))*tan(1/2*d*e/f) - 2*real_part(cos_integral(-d*x - d*e/f))*tan(1/2
*d*e/f) + imag_part(cos_integral(d*x + d*e/f)) - imag_part(cos_integral(-d
*x - d*e/f)) - 2*log(abs(f*x + e)) + 2*sin_integral((d*f*x + d*e)/f))/(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cos(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(cos(c + d*x)^2/((e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{-\left(\int \frac{\sin(dx+c)}{fx+e}dx\right)f+\log(fx+e)}{af}$$

input
```
int(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
( - int(sin(c + d*x)/(e + f*x),x)*f + log(e + f*x))/(a*f)
```

3.262 $\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = -\frac{1}{af(e+fx)} - \frac{d\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2} + \frac{\sin(c+dx)}{af(e+fx)} + \frac{d\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af^2}$$

output
```
-1/a/f/(f*x+e)-d*cos(c-d*e/f)*Ci(d*e/f+d*x)/a/f^2+sin(d*x+c)/a/f/(f*x+e)+d
*sin(c-d*e/f)*Si(d*e/f+d*x)/a/f^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{-d(e+fx)\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right)+f(-1+\sin(c+dx))+d(e+fx)\sin\left(c-\frac{de}{f}\right)\text{Si}\left(d\right.}{af^2(e+fx)}$$

input
```
Integrate[Cos[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
(-(d*(e + f*x)*Cos[c - (d*e)/f]*CosIntegral[d*(e/f + x)]) + f*(-1 + Sin[c
+ d*x]) + d*(e + f*x)*Sin[c - (d*e)/f]*SinIntegral[d*(e/f + x)])/(a*f^2*(e
 + f*x))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5034, 17, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int \frac{1}{(e+fx)^2}dx}{a}-\frac{\int \frac{\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\int \frac{\sin(c+dx)}{(e+fx)^2}dx}{a}-\frac{1}{af(e+fx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(c+dx)}{(e+fx)^2}dx}{a}-\frac{1}{af(e+fx)}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{\frac{d\int \frac{\cos(c+dx)}{e+fx}dx}{f}-\frac{\sin(c+dx)}{f(e+fx)}}{a}-\frac{1}{af(e+fx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{d\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{e+fx}dx}{f}-\frac{\sin(c+dx)}{f(e+fx)}}{a}-\frac{1}{af(e+fx)}$$

$\Big\downarrow$ 3784

$$-\frac{\frac{d\left(\cos\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}dx-\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{\sin(c+dx)}{f(e+fx)}}{a}-\frac{1}{af(e+fx)}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{d\left(\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx-\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{\sin(c+dx)}{f(e+fx)}}{a}-\frac{1}{af(e+fx)}$$

$\Big\downarrow$ 3780

$$-\frac{\frac{d\left(\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\sin(c+dx)}{f(e+fx)}}{a}-\frac{1}{af(e+fx)}$$

$\Big\downarrow$ 3783

$$-\frac{\frac{d\left(\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\sin(c+dx)}{f(e+fx)}}{a}-\frac{1}{af(e+fx)}$$

input
```
Int[Cos[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
-(1/(a*f*(e + f*x))) - (-(Sin[c + d*x]/(f*(e + f*x))) + (d*((Cos[c - (d*e)
/f]*CosIntegral[(d*e)/f + d*x])/f - (Sin[c - (d*e)/f]*SinIntegral[(d*e)/f
+ d*x])/f))/f)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{d\left(\frac{\sin(dx+c)}{(-cf+de+f(dx+c))f}-\frac{-\frac{\mathrm{Si}\left(-dx-c-\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}+\frac{\mathrm{Ci}\left(dx+c+\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}}{f}-\frac{1}{(-cf+de+f(dx+c))f}\right)}{a}$
default	$\frac{d\left(\frac{\sin(dx+c)}{(-cf+de+f(dx+c))f}-\frac{-\frac{\mathrm{Si}\left(-dx-c-\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}+\frac{\mathrm{Ci}\left(dx+c+\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)}{f}}{f}-\frac{1}{(-cf+de+f(dx+c))f}\right)}{a}$
risch	$-\frac{1}{af(fx+e)}+\frac{d\,\mathrm{e}^{\frac{i(cf-de)}{f}}\,\mathrm{expIntegral}_1\left(-idx-ic-\frac{-icf+ide}{f}\right)}{2a\,f^2}+\frac{d\,\mathrm{e}^{-\frac{i(cf-de)}{f}}\,\mathrm{expIntegral}_1\left(idx+ic-\frac{i(cf-de)}{f}\right)}{2a\,f^2}$

input
```
int(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
d/a*(sin(d*x+c)/(-c*f+d*e+f*(d*x+c))/f-(-Si(-d*x-c-(-c*f+d*e)/f)*sin((-c*f
+d*e)/f)/f+Ci(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f)/f-1/(-c*f+d*e+f*(d*
x+c))/f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = -\frac{(dfx+de)\cos\left(-\frac{de-cf}{f}\right)\mathrm{Ci}\left(\frac{dfx+de}{f}\right)-(dfx+de)\sin\left(-\frac{de-cf}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)-f\sin(dx+c)+f}{af^3x+aef^2}$$

input
```
integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-((d*f*x + d*e)*cos(-(d*e - c*f)/f)*cos_integral((d*f*x + d*e)/f) - (d*f*x
 + d*e)*sin(-(d*e - c*f)/f)*sin_integral((d*f*x + d*e)/f) - f*sin(d*x + c)
 + f)/(a*f^3*x + a*e*f^2)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x)
+ 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.81

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{d^2\left(i\,E_2\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)-i\,E_2\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+d^2\left(E_2\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_2\left(-\right.\right.}{2\,(adef+(dx+c)af^2-acf^2)d}$$

input
```
integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(d^2*(I*exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - I*exp_i
ntegral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) + d^
2*(exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integral_e(2
, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - 2*d^2)/((a*d*
e*f + (d*x + c)*a*f^2 - a*c*f^2)*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 3192, normalized size of antiderivative = 33.60

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*d*e/f)^2 + d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(d*x
+ d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) + 2*d*f*x*imag_part(c
os_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) - 4*
d*f*x*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*
e/f) + 2*d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(-d*x - d*e/f))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 + 4*d*f*x*sin_integral((d*f*x + d*e)
/f)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*d*e/f)^2 + d*e*real_part(cos_integra
l(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + d*e*real_pa
rt(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f)^
2 - d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 4*d*f*x*real_part(cos_integral(d*x + d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*d*e/f) + 4*d*f*x*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*d*e/f) - 2*d*e*imag_part(cos_integral(d*x + d*e/f))
*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) + 2*d*e*imag_part(cos_integral
(-d*x - d*e/f))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) - 4*d*e*sin_int
egral((d*f*x + d*e)/f)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*d*e/f) - d*f...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cos(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(cos(c + d*x)^2/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{-\left(\int \frac{\cos(dx+c)}{fx+e}dx\right)d\,e^2-\left(\int \frac{\cos(dx+c)}{fx+e}dx\right)defx+\sin\left(dx+c\right)e+fx}{aef\left(fx+e\right)}$$

input
```
int(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - int(cos(c + d*x)/(e + f*x),x)*d*e**2 - int(cos(c + d*x)/(e + f*x),x)*d
*e*f*x + sin(c + d*x)*e + f*x)/(a*e*f*(e + f*x))
```

3.263 $\int \frac{(e+fx)^3 \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 219

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3f^3x}{8ad^3}+\frac{(e+fx)^3}{4ad}-\frac{6f^3\cos(c+dx)}{ad^4} + \frac{3f(e+fx)^2\cos(c+dx)}{ad^2}-\frac{6f^2(e+fx)\sin(c+dx)}{ad^3} + \frac{(e+fx)^3\sin(c+dx)}{ad}+\frac{3f^3\cos(c+dx)\sin(c+dx)}{8ad^4} - \frac{3f(e+fx)^2\cos(c+dx)\sin(c+dx)}{4ad^2} + \frac{3f^2(e+fx)\sin^2(c+dx)}{4ad^3}-\frac{(e+fx)^3\sin^2(c+dx)}{2ad}$$

output
```
-3/8*f^3*x/a/d^3+1/4*(f*x+e)^3/a/d-6*f^3*cos(d*x+c)/a/d^4+3*f*(f*x+e)^2*co
s(d*x+c)/a/d^2-6*f^2*(f*x+e)*sin(d*x+c)/a/d^3+(f*x+e)^3*sin(d*x+c)/a/d+3/8
*f^3*cos(d*x+c)*sin(d*x+c)/a/d^4-3/4*f*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/a/d
^2+3/4*f^2*(f*x+e)*sin(d*x+c)^2/a/d^3-1/2*(f*x+e)^3*sin(d*x+c)^2/a/d
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.60

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{96f(-2f^2+d^2(e+fx)^2)\cos(c+dx)+4d(e+fx)\left(-3f^2+2d^2(e+fx)^2\right)\cos(2(c+dx))+4(8d(e+f}{32ad^4}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(96*f*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + 4*d*(e + f*x)*(-3*f^2 + 2*
d^2*(e + f*x)^2)*Cos[2*(c + d*x)] + 4*(8*d*(e + f*x)*(-6*f^2 + d^2*(e + f*
x)^2) - 3*f*(-f^2 + 2*d^2*(e + f*x)^2)*Cos[c + d*x])*Sin[c + d*x])/(32*a*d
^4)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5034, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4904, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int (e+fx)^3\cos(c+dx)dx}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{3f\int -(e+fx)^2\sin(c+dx)dx}{d}+\frac{(e+fx)^3\sin(c+dx)}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$\Big\downarrow$ 3118

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}$$

$\Big\downarrow$ 4904

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sin^2(c+dx)dx}{2d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sin(c+dx)^2dx}{2d}}{a}$$

$\Big\downarrow$ 3792

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{a}$$

$\Big\downarrow$ 17

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int\sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{a}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{a}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{a}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(((e + f*x)^3*Sin[c + d*x])/d - (3*f*(-(((e + f*x)^2*Cos[c + d*x])/d) + (2
*f*((f*Cos[c + d*x])/d^2 + ((e + f*x)*Sin[c + d*x])/d))/d))/d)/a - (((e +
f*x)^3*Sin[c + d*x]^2)/(2*d) - (3*f*((e + f*x)^3/(6*f) - ((e + f*x)^2*Cos[
c + d*x]*Sin[c + d*x])/(2*d) + (f*(e + f*x)*Sin[c + d*x]^2)/(2*d^2) - (f^2
*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(2*d^2)))/(2*d))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{2(fx+e)\left(\left(fx+e\right)^2d^2-\frac{3f^2}{2}\right)d\cos(2dx+2c)-3\left(\left(fx+e\right)^2d^2-\frac{f^2}{2}\right)f\sin(2dx+2c)+8\left(\left(fx+e\right)^2d^2-6f^2\right)(fx+e)d\sin(dx\ldots}{8d^4a}$
risch	$\frac{3f\left(d^2x^2f^2+2efx\,d^2+d^2e^2-2f^2\right)\cos(dx+c)}{d^4a}+\frac{\left(d^2x^3f^3+3d^2e\,f^2x^2+3d^2e^2fx+d^2e^3-6f^3x-6e\,f^2\right)\sin(dx+c)}{d^3a}+\ldots$
derivativedivides	$\frac{-\frac{c^3f^3\cos(dx+c)^2}{2}+\frac{3c^2de\,f^2\cos(dx+c)^2}{2}-3c^2f^3\left(-\frac{(dx+c)\cos(dx+c)^2}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)-\frac{3c\,d^2e^2f\cos(dx+c)\ldots}{2}\ldots}{\ldots}$
default	$\frac{-\frac{c^3f^3\cos(dx+c)^2}{2}+\frac{3c^2de\,f^2\cos(dx+c)^2}{2}-3c^2f^3\left(-\frac{(dx+c)\cos(dx+c)^2}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)-\frac{3c\,d^2e^2f\cos(dx+c)\ldots}{2}\ldots}{\ldots}$

input
```
int((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*(f*x+e)*((f*x+e)^2*d^2-3/2*f^2)*d*cos(2*d*x+2*c)-3*((f*x+e)^2*d^2-1
/2*f^2)*f*sin(2*d*x+2*c)+8*((f*x+e)^2*d^2-6*f^2)*(f*x+e)*d*sin(d*x+c)+24*(
(f*x+e)^2*d^2-2*f^2)*f*cos(d*x+c)-2*d^3*e^3+24*d^2*e^2*f+3*d*e*f^2-48*f^3)
/d^4/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\,d^3f^3x^3+6\,d^3ef^2x^2-2\,(2\,d^3f^3x^3+6\,d^3ef^2x^2+2\,d^3e^3-3\,def^2+3\,(2\,d^3e^2f-df^3)x)\cos{(dx+c)}^2}{}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 - 2*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2
 + 2*d^3*e^3 - 3*d*e*f^2 + 3*(2*d^3*e^2*f - d*f^3)*x)*cos(d*x + c)^2 + 3*(
2*d^3*e^2*f - d*f^3)*x - 24*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 2*f
^3)*cos(d*x + c) - (8*d^3*f^3*x^3 + 24*d^3*e*f^2*x^2 + 8*d^3*e^3 - 48*d*e*
f^2 + 24*(d^3*e^2*f - 2*d*f^3)*x - 3*(2*d^2*f^3*x^2 + 4*d^2*e*f^2*x + 2*d^
2*e^2*f - f^3)*cos(d*x + c))*sin(d*x + c))/(a*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2725 vs. 2(204) = 408.

Time = 4.61 (sec) , antiderivative size = 2725, normalized size of antiderivative = 12.44

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((16*d**3*e**3*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 + d*x/2)**4
+ 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 16*d**3*e**3*tan(c/2 + d*x/2
)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d
**4) + 16*d**3*e**3*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*
d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e**2*f*x*tan(c/2 + d*x/2)**4
/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4)
 + 48*d**3*e**2*f*x*tan(c/2 + d*x/2)**3/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16
*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 36*d**3*e**2*f*x*tan(c/2 + d*x/2
)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d
**4) + 48*d**3*e**2*f*x*tan(c/2 + d*x/2)/(8*a*d**4*tan(c/2 + d*x/2)**4 + 1
6*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e**2*f*x/(8*a*d**4*tan(c
/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 6*d**3*e*f**2
*x**2*tan(c/2 + d*x/2)**4/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/
2 + d*x/2)**2 + 8*a*d**4) + 48*d**3*e*f**2*x**2*tan(c/2 + d*x/2)**3/(8*a*d
**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) - 36*d
**3*e*f**2*x**2*tan(c/2 + d*x/2)**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d
**4*tan(c/2 + d*x/2)**2 + 8*a*d**4) + 48*d**3*e*f**2*x**2*tan(c/2 + d*x/2)
/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 + d*x/2)**2 + 8*a*d**4)
 + 6*d**3*e*f**2*x**2/(8*a*d**4*tan(c/2 + d*x/2)**4 + 16*a*d**4*tan(c/2 +
d*x/2)**2 + 8*a*d**4) + 2*d**3*f**3*x**3*tan(c/2 + d*x/2)**4/(8*a*d**4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(207) = 414$.

Time = 0.07 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.61

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(8*(sin(d*x + c)^2 - 2*sin(d*x + c))*e^3/a - 24*(sin(d*x + c)^2 - 2*
sin(d*x + c))*c*e^2*f/(a*d) + 24*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^2*e*f
^2/(a*d^2) - 8*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^3*f^3/(a*d^3) - 6*(2*(d
*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin
(2*d*x + 2*c))*e^2*f/(a*d) + 12*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c
)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*c*e*f^2/(a*d^2) - 6*(2
*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) -
sin(2*d*x + 2*c))*c^2*f^3/(a*d^3) - 6*((2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c
) + 16*(d*x + c)*cos(d*x + c) - 2*(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x + c
)^2 - 2)*sin(d*x + c))*e*f^2/(a*d^2) + 6*((2*(d*x + c)^2 - 1)*cos(2*d*x +
2*c) + 16*(d*x + c)*cos(d*x + c) - 2*(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x
+ c)^2 - 2)*sin(d*x + c))*c*f^3/(a*d^3) - (2*(2*(d*x + c)^3 - 3*d*x - 3*c)
*cos(2*d*x + 2*c) + 48*((d*x + c)^2 - 2)*cos(d*x + c) - 3*(2*(d*x + c)^2 -
 1)*sin(2*d*x + 2*c) + 16*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*f^3/(a
*d^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3893 vs. $2(207) = 414$.

Time = 0.30 (sec) , antiderivative size = 3893, normalized size of antiderivative = 17.78

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(2*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 16*d^3*f^3*x^3*tan(1/2*d*
x)^4*tan(1/2*c)^3 - 16*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(1/2*c)^4 + 6*d^3*e*f
^2*x^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 12*d^3*f^3*x^3*tan(1/2*d*x)^4*tan(1/2
*c)^2 - 32*d^3*f^3*x^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 48*d^3*e*f^2*x^2*tan(
1/2*d*x)^4*tan(1/2*c)^3 - 12*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 48*
d^3*e*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4 + 6*d^3*e^2*f*x*tan(1/2*d*x)^4*t
an(1/2*c)^4 + 24*d^2*f^3*x^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 16*d^3*f^3*x^3*
tan(1/2*d*x)^4*tan(1/2*c) - 36*d^3*e*f^2*x^2*tan(1/2*d*x)^4*tan(1/2*c)^2 -
 96*d^3*e*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 48*d^3*e^2*f*x*tan(1/2*d*x
)^4*tan(1/2*c)^3 + 12*d^2*f^3*x^2*tan(1/2*d*x)^4*tan(1/2*c)^3 - 16*d^3*f^3
*x^3*tan(1/2*d*x)*tan(1/2*c)^4 - 36*d^3*e*f^2*x^2*tan(1/2*d*x)^2*tan(1/2*c
)^4 - 48*d^3*e^2*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 12*d^2*f^3*x^2*tan(1/2*
d*x)^3*tan(1/2*c)^4 + 2*d^3*e^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 48*d^2*e*f^2
*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2*d^3*f^3*x^3*tan(1/2*d*x)^4 + 32*d^3*f^3
*x^3*tan(1/2*d*x)^3*tan(1/2*c) - 48*d^3*e*f^2*x^2*tan(1/2*d*x)^4*tan(1/2*c
) + 72*d^3*f^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 36*d^3*e^2*f*x*tan(1/2*d*
x)^4*tan(1/2*c)^2 + 32*d^3*f^3*x^3*tan(1/2*d*x)*tan(1/2*c)^3 - 96*d^3*e^2*
f*x*tan(1/2*d*x)^3*tan(1/2*c)^3 - 96*d^2*f^3*x^2*tan(1/2*d*x)^3*tan(1/2*c)
^3 - 16*d^3*e^3*tan(1/2*d*x)^4*tan(1/2*c)^3 + 24*d^2*e*f^2*x*tan(1/2*d*x)^
4*tan(1/2*c)^3 + 2*d^3*f^3*x^3*tan(1/2*c)^4 - 48*d^3*e*f^2*x^2*tan(1/2*...
```

Mupad [B] (verification not implemented)

Time = 38.76 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= \frac{\frac{3 f^3 \sin(2 c+2 d x)}{2} - 48 f^3 \cos(c+d x) + 8 d^3 e^3 \sin(c+d x) + 2 d^3 e^3 \cos(2 c+2 d x) - 3 d^2 e^2 f \sin(2 c+}{}$$

input
```
int((cos(c + d*x)^3*(e + f*x)^3)/(a + a*sin(c + d*x)),x)
```

output
```
((3*f^3*sin(2*c + 2*d*x))/2 - 48*f^3*cos(c + d*x) + 8*d^3*e^3*sin(c + d*x)
 + 2*d^3*e^3*cos(2*c + 2*d*x) - 3*d^2*e^2*f*sin(2*c + 2*d*x) + 24*d^2*f^3*
x^2*cos(c + d*x) + 8*d^3*f^3*x^3*sin(c + d*x) - 48*d*e*f^2*sin(c + d*x) -
48*d*f^3*x*sin(c + d*x) + 2*d^3*f^3*x^3*cos(2*c + 2*d*x) - 3*d^2*f^3*x^2*s
in(2*c + 2*d*x) - 3*d*e*f^2*cos(2*c + 2*d*x) + 24*d^2*e^2*f*cos(c + d*x) -
 3*d*f^3*x*cos(2*c + 2*d*x) + 48*d^2*e*f^2*x*cos(c + d*x) + 24*d^3*e^2*f*x
*sin(c + d*x) + 6*d^3*e^2*f*x*cos(2*c + 2*d*x) - 6*d^2*e*f^2*x*sin(2*c + 2
*d*x) + 24*d^3*e*f^2*x^2*sin(c + d*x) + 6*d^3*e*f^2*x^2*cos(2*c + 2*d*x))/
(8*a*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.83

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-12\cos(dx+c)\sin(dx+c)\,d^2ef^2x+8d^3e^3+2d^3f^3x^3+3\cos(dx+c)\sin(dx+c)\,f^3-4\sin(dx+c)^2}{}$$

input
```
int((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)*d**2*e**2*f - 12*cos(c + d*x)*sin(c + d*x)
*d**2*e*f**2*x - 6*cos(c + d*x)*sin(c + d*x)*d**2*f**3*x**2 + 3*cos(c + d*
x)*sin(c + d*x)*f**3 + 24*cos(c + d*x)*d**2*e**2*f + 48*cos(c + d*x)*d**2*
e*f**2*x + 24*cos(c + d*x)*d**2*f**3*x**2 - 48*cos(c + d*x)*f**3 - 4*sin(c
 + d*x)**2*d**3*e**3 - 12*sin(c + d*x)**2*d**3*e**2*f*x - 12*sin(c + d*x)*
*2*d**3*e*f**2*x**2 - 4*sin(c + d*x)**2*d**3*f**3*x**3 + 6*sin(c + d*x)**2
*d*e*f**2 + 6*sin(c + d*x)**2*d*f**3*x + 8*sin(c + d*x)*d**3*e**3 + 24*sin
(c + d*x)*d**3*e**2*f*x + 24*sin(c + d*x)*d**3*e*f**2*x**2 + 8*sin(c + d*x
)*d**3*f**3*x**3 - 48*sin(c + d*x)*d*e*f**2 - 48*sin(c + d*x)*d*f**3*x + 8
*d**3*e**3 + 6*d**3*e**2*f*x + 6*d**3*e*f**2*x**2 + 2*d**3*f**3*x**3 - 12*
d*e*f**2 - 3*d*f**3*x)/(8*a*d**4)
```

3.264 $\int \frac{(e+fx)^2 \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(e+fx)^2}{4ad}+\frac{2f(e+fx)\cos(c+dx)}{ad^2} - \frac{2f^2\sin(c+dx)}{ad^3}+\frac{(e+fx)^2\sin(c+dx)}{ad} - \frac{f(e+fx)\cos(c+dx)\sin(c+dx)}{2ad^2} + \frac{f^2\sin^2(c+dx)}{4ad^3}-\frac{(e+fx)^2\sin^2(c+dx)}{2ad}$$

output
```
1/4*(f*x+e)^2/a/d+2*f*(f*x+e)*cos(d*x+c)/a/d^2-2*f^2*sin(d*x+c)/a/d^3+(f*x
+e)^2*sin(d*x+c)/a/d-1/2*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)/a/d^2+1/4*f^2*sin
(d*x+c)^2/a/d^3-1/2*(f*x+e)^2*sin(d*x+c)^2/a/d
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{16df(e+fx)\cos(c+dx)+(-f^2+2d^2(e+fx)^2)\cos(2(c+dx))-4(-2(-2f^2+d^2(e+fx)^2)+df(e+}{8ad^3}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(16*d*f*(e + f*x)*Cos[c + d*x] + (-f^2 + 2*d^2*(e + f*x)^2)*Cos[2*(c + d*x
)] - 4*(-2*(-2*f^2 + d^2*(e + f*x)^2) + d*f*(e + f*x)*Cos[c + d*x])*Sin[c
+ d*x])/(8*a*d^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5034, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4904, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int (e+fx)^2\cos(c+dx)dx}{a}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}}{a}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}}{a}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}}{a}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{4904}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\int(e+fx)\sin^2(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\int(e+fx)\sin(c+dx)^2dx}{d}}{a}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{d}}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}}{a}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(((e + f*x)^2*Sin[c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*S
in[c + d*x])/d^2))/d)/a - (((e + f*x)^2*Sin[c + d*x]^2)/(2*d) - (f*((e + f
*x)^2/(4*f) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x
]^2)/(4*d^2)))/d)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{\left(2(fx+e)^2d^2-f^2\right)\cos(2dx+2c)-2df(fx+e)\sin(2dx+2c)+8\left((fx+e)^2d^2-2f^2\right)\sin(dx+c)+16df(fx+e)\cos(dx+c)-}{8a\,d^3}$
risch	$\frac{2f(fx+e)\cos(dx+c)}{a\,d^2}+\frac{(d^2x^2f^2+2efx\,d^2+d^2e^2-2f^2)\sin(dx+c)}{a\,d^3}+\frac{(2d^2x^2f^2+4efx\,d^2+2d^2e^2-f^2)\cos(2dx+2c)}{8a\,d^3}$
derivativedivides	$-\frac{-\frac{c^2f^2\cos(dx+c)^2}{2}+cdef\cos(dx+c)^2-2c\,f^2\left(-\frac{(dx+c)\cos(dx+c)^2}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)-\frac{d^2e^2\cos(dx+c)^2}{2}+2}{}$
default	$-\frac{-\frac{c^2f^2\cos(dx+c)^2}{2}+cdef\cos(dx+c)^2-2c\,f^2\left(-\frac{(dx+c)\cos(dx+c)^2}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)-\frac{d^2e^2\cos(dx+c)^2}{2}+2}{}$
norman	$\frac{\frac{4ef}{d^2a}+\frac{\left(5def-3f^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a\,d^3}+\frac{\left(7def-3f^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a\,d^3}+\frac{\left(2d^2e^2+def-4f^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a\,d^3}+\frac{\left(2d^2e^2+3def-4f^2\right)\tan\left(\frac{d}{}\right.}{a\,d^3}}{}$

input
```
int((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(f*x+e)^2*d^2-f^2)*cos(2*d*x+2*c)-2*d*f*(f*x+e)*sin(2*d*x+2*c)+8*(
(f*x+e)^2*d^2-2*f^2)*sin(d*x+c)+16*d*f*(f*x+e)*cos(d*x+c)-2*d^2*e^2+16*d*e
*f+f^2)/a/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{d^2f^2x^2+2\,d^2efx-(2\,d^2f^2x^2+4\,d^2efx+2\,d^2e^2-f^2)\cos(dx+c)^2-8\,(df^2x+def)\cos(dx+c)-}{4\,ad^3}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^2*f^2*x^2 + 2*d^2*e*f*x - (2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2
 - f^2)*cos(d*x + c)^2 - 8*(d*f^2*x + d*e*f)*cos(d*x + c) - 2*(2*d^2*f^2*x
^2 + 4*d^2*e*f*x + 2*d^2*e^2 - 4*f^2 - (d*f^2*x + d*e*f)*cos(d*x + c))*sin
(d*x + c))/(a*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. 2(131) = 262.

Time = 3.44 (sec) , antiderivative size = 1528, normalized size of antiderivative = 10.26

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((8*d**2*e**2*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 +
 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 8*d**2*e**2*tan(c/2 + d*x/2)**
2/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3)
 + 8*d**2*e**2*tan(c/2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*t
an(c/2 + d*x/2)**2 + 4*a*d**3) + 2*d**2*e*f*x*tan(c/2 + d*x/2)**4/(4*a*d**
3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 16*d**2
*e*f*x*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/
2 + d*x/2)**2 + 4*a*d**3) - 12*d**2*e*f*x*tan(c/2 + d*x/2)**2/(4*a*d**3*ta
n(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 16*d**2*e*f
*x*tan(c/2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x
/2)**2 + 4*a*d**3) + 2*d**2*e*f*x/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3
*tan(c/2 + d*x/2)**2 + 4*a*d**3) + d**2*f**2*x**2*tan(c/2 + d*x/2)**4/(4*a
*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 8*d
**2*f**2*x**2*tan(c/2 + d*x/2)**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3
*tan(c/2 + d*x/2)**2 + 4*a*d**3) - 6*d**2*f**2*x**2*tan(c/2 + d*x/2)**2/(4
*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 8
*d**2*f**2*x**2*tan(c/2 + d*x/2)/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*
tan(c/2 + d*x/2)**2 + 4*a*d**3) + d**2*f**2*x**2/(4*a*d**3*tan(c/2 + d*x/2
)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*d**3) + 4*d*e*f*tan(c/2 + d*x/2)
**3/(4*a*d**3*tan(c/2 + d*x/2)**4 + 8*a*d**3*tan(c/2 + d*x/2)**2 + 4*a*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(141) = 282$.

Time = 0.05 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.94

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)e^2}{a} - \frac{8\left(\sin(dx+c)^2-2\sin(dx+c)\right)cef}{ad} + \frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)c^2f^2}{ad^2} - \frac{2(2(dx+c)\cos(2dx+2c)-}{}}{}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(4*(sin(d*x + c)^2 - 2*sin(d*x + c))*e^2/a - 8*(sin(d*x + c)^2 - 2*si
n(d*x + c))*c*e*f/(a*d) + 4*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^2*f^2/(a*d
^2) - 2*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d
*x + c) - sin(2*d*x + 2*c))*e*f/(a*d) + 2*(2*(d*x + c)*cos(2*d*x + 2*c) +
8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*c*f^2/(a*d^2
) - ((2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*(d*x + c)*cos(d*x + c) - 2*
(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)*sin(d*x + c))*f^2/(a*d^2)
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2190 vs. $2(141) = 282$.

Time = 0.23 (sec) , antiderivative size = 2190, normalized size of antiderivative = 14.70

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(2*d^2*f^2*x^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 16*d^2*f^2*x^2*tan(1/2*d*
x)^4*tan(1/2*c)^3 - 16*d^2*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^4 + 4*d^2*e*f
*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 12*d^2*f^2*x^2*tan(1/2*d*x)^4*tan(1/2*c)^
2 - 32*d^2*f^2*x^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 32*d^2*e*f*x*tan(1/2*d*x)
^4*tan(1/2*c)^3 - 12*d^2*f^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - 32*d^2*e*f*
x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 2*d^2*e^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16
*d*f^2*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 16*d^2*f^2*x^2*tan(1/2*d*x)^4*tan(1
/2*c) - 24*d^2*e*f*x*tan(1/2*d*x)^4*tan(1/2*c)^2 - 64*d^2*e*f*x*tan(1/2*d*
x)^3*tan(1/2*c)^3 - 16*d^2*e^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 8*d*f^2*x*tan
(1/2*d*x)^4*tan(1/2*c)^3 - 16*d^2*f^2*x^2*tan(1/2*d*x)*tan(1/2*c)^4 - 24*d
^2*e*f*x*tan(1/2*d*x)^2*tan(1/2*c)^4 - 16*d^2*e^2*tan(1/2*d*x)^3*tan(1/2*c
)^4 + 8*d*f^2*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 16*d*e*f*tan(1/2*d*x)^4*tan(
1/2*c)^4 + 2*d^2*f^2*x^2*tan(1/2*d*x)^4 + 32*d^2*f^2*x^2*tan(1/2*d*x)^3*ta
n(1/2*c) - 32*d^2*e*f*x*tan(1/2*d*x)^4*tan(1/2*c) + 72*d^2*f^2*x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 12*d^2*e^2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 32*d^2*f^2
*x^2*tan(1/2*d*x)*tan(1/2*c)^3 - 32*d^2*e^2*tan(1/2*d*x)^3*tan(1/2*c)^3 -
64*d*f^2*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 8*d*e*f*tan(1/2*d*x)^4*tan(1/2*c)
^3 + 2*d^2*f^2*x^2*tan(1/2*c)^4 - 32*d^2*e*f*x*tan(1/2*d*x)*tan(1/2*c)^4 -
 12*d^2*e^2*tan(1/2*d*x)^2*tan(1/2*c)^4 + 8*d*e*f*tan(1/2*d*x)^3*tan(1/2*c
)^4 - f^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16*d^2*f^2*x^2*tan(1/2*d*x)^3 +...
```

Mupad [B] (verification not implemented)

Time = 38.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{8\,d^2\,e^2\,\sin(c+d\,x)-f^2\,\cos(2\,c+2\,d\,x)-16\,f^2\,\sin(c+d\,x)+2\,d^2\,e^2\,\cos(2\,c+2\,d\,x)+8\,d^2\,f^2\,x^2\,\sin(\ldots}{\ldots}$$

input
```
int((cos(c + d*x)^3*(e + f*x)^2)/(a + a*sin(c + d*x)),x)
```

output
```
(8*d^2*e^2*sin(c + d*x) - f^2*cos(2*c + 2*d*x) - 16*f^2*sin(c + d*x) + 2*d
^2*e^2*cos(2*c + 2*d*x) + 8*d^2*f^2*x^2*sin(c + d*x) - 2*d*e*f*sin(2*c + 2
*d*x) + 16*d*f^2*x*cos(c + d*x) + 2*d^2*f^2*x^2*cos(2*c + 2*d*x) - 2*d*f^2
*x*sin(2*c + 2*d*x) + 16*d*e*f*cos(c + d*x) + 4*d^2*e*f*x*cos(2*c + 2*d*x)
 + 16*d^2*e*f*x*sin(c + d*x))/(8*a*d^3)
```

Reduce [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-2\cos(dx+c)\sin(dx+c)\,def-2\cos(dx+c)\sin(dx+c)\,d\,f^2x+8\cos(dx+c)\,def+8\cos(dx+c)\,c}{}$$

input
```
int((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)*d*e*f - 2*cos(c + d*x)*sin(c + d*x)*d*f**2
*x + 8*cos(c + d*x)*d*e*f + 8*cos(c + d*x)*d*f**2*x - 2*sin(c + d*x)**2*d*
*2*e**2 - 4*sin(c + d*x)**2*d**2*e*f*x - 2*sin(c + d*x)**2*d**2*f**2*x**2
+ sin(c + d*x)**2*f**2 + 4*sin(c + d*x)*d**2*e**2 + 8*sin(c + d*x)*d**2*e*
f*x + 4*sin(c + d*x)*d**2*f**2*x**2 - 8*sin(c + d*x)*f**2 + 4*d**2*e**2 +
2*d**2*e*f*x + d**2*f**2*x**2 - 2*f**2)/(4*a*d**3)
```

3.265 $\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{fx}{4ad}+\frac{f\cos(c+dx)}{ad^2}+\frac{(e+fx)\sin(c+dx)}{ad} - \frac{f\cos(c+dx)\sin(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+dx)}{2ad}$$

output
```
1/4*f*x/a/d+f*cos(d*x+c)/a/d^2+(f*x+e)*sin(d*x+c)/a/d-1/4*f*cos(d*x+c)*sin
(d*x+c)/a/d^2-1/2*(f*x+e)*sin(d*x+c)^2/a/d
```

Mathematica [A] (verified)

Time = 7.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-f\cos(c+dx)(-4+\sin(c+dx))+d(e+fx)(\cos(2(c+dx))+4\sin(c+dx))}{4ad^2}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(-(f*Cos[c + d*x]*(-4 + Sin[c + d*x])) + d*(e + f*x)*(Cos[2*(c + d*x)] + 4
*Sin[c + d*x]))/(4*a*d^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5034, 3042, 3777, 25, 3042, 3118, 4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int (e+fx)\cos(c+dx)dx}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{4904}$$

$$\frac{\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int\sin^2(c+dx)dx}{2d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int\sin(c+dx)^2dx}{2d}}{a}$$

$\Big\downarrow$ 3115

$$\frac{\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{a}$$

$\Big\downarrow$ 24

$$\frac{\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{a}$$

input
```
Int[((e + f*x)*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
((f*Cos[c + d*x])/d^2 + ((e + f*x)*Sin[c + d*x])/d)/a - (((e + f*x)*Sin[c
+ d*x]^2)/(2*d) - (f*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(2*d))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{2d(fx+e)\cos(2dx+2c)-\sin(2dx+2c)f+8d(fx+e)\sin(dx+c)-2de+8\cos(dx+c)f-8f}{8d^2a}$
risch	$\frac{f\cos(dx+c)}{a\,d^2}+\frac{(fx+e)\sin(dx+c)}{ad}+\frac{(fx+e)\cos(2dx+2c)}{4ad}-\frac{f\sin(2dx+2c)}{8d^2a}$
default	$\frac{\frac{4e\left(-\frac{\sin(dx+c)^2}{2}+\sin(dx+c)\right)}{d}+\frac{4f\left(\frac{(dx+c)\cos(dx+c)^2}{2}-\frac{\cos(dx+c)\sin(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}-\frac{c\cos(dx+c)^2}{2}+\cos(dx+c)+(dx+c)\sin(dx+c)-c \cdots\right.}{d^2}}{4a}$
norman	$\frac{\frac{2f}{d^2a}+\frac{(2de+2f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d^2a}+\frac{(2de+4f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d^2a}+\frac{fx}{4ad}+\frac{5f\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d^2a}+\frac{7f\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d^2a}+\frac{(4de+f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2d^2a}+\frac{(4de+3 \cdots}{\cdots}}{\left(1+\tan\left(\cdots\right.\right.}$

input
```
int((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*d*(f*x+e)*cos(2*d*x+2*c)-sin(2*d*x+2*c)*f+8*d*(f*x+e)*sin(d*x+c)-2*
d*e+8*cos(d*x+c)*f-8*f)/d^2/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{dfx-2\,(dfx+de)\cos{(dx+c)}^2-4\,f\cos{(dx+c)}-(4\,dfx+4\,de-f\cos{(dx+c)})\sin{(dx+c)}}{4\,ad^2}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d*f*x - 2*(d*f*x + d*e)*cos(d*x + c)^2 - 4*f*cos(d*x + c) - (4*d*f*x
 + 4*d*e - f*cos(d*x + c))*sin(d*x + c))/(a*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(78) = 156.

Time = 2.65 (sec) , antiderivative size = 724, normalized size of antiderivative = 7.96

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((8*d*e*tan(c/2 + d*x/2)**3/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d
**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) - 8*d*e*tan(c/2 + d*x/2)**2/(4*a*d**2*
tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*d*e*tan
(c/2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2
 + 4*a*d**2) + d*f*x*tan(c/2 + d*x/2)**4/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8
*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*d*f*x*tan(c/2 + d*x/2)**3/(4*a
*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) - 6*d
*f*x*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2
+ d*x/2)**2 + 4*a*d**2) + 8*d*f*x*tan(c/2 + d*x/2)/(4*a*d**2*tan(c/2 + d*x
/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + d*f*x/(4*a*d**2*tan(c/
2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 2*f*tan(c/2 + d
*x/2)**3/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*
a*d**2) + 8*f*tan(c/2 + d*x/2)**2/(4*a*d**2*tan(c/2 + d*x/2)**4 + 8*a*d**2
*tan(c/2 + d*x/2)**2 + 4*a*d**2) - 2*f*tan(c/2 + d*x/2)/(4*a*d**2*tan(c/2
+ d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2) + 8*f/(4*a*d**2*tan
(c/2 + d*x/2)**4 + 8*a*d**2*tan(c/2 + d*x/2)**2 + 4*a*d**2), Ne(d, 0)), ((
e*x + f*x**2/2)*cos(c)**3/(a*sin(c) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)e}{a} - \frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)cf}{ad} - \frac{(2(dx+c)\cos(2dx+2c)+8(dx+c)\sin(dx+c)+8\cos(dx+c)-\sin(2d}{ad}}{8d}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(4*(sin(d*x + c)^2 - 2*sin(d*x + c))*e/a - 4*(sin(d*x + c)^2 - 2*sin(
d*x + c))*c*f/(a*d) - (2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x
+ c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*f/(a*d))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(85) = 170$.

Time = 0.17 (sec) , antiderivative size = 947, normalized size of antiderivative = 10.41

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)
^3 - 8*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + d*e*tan(1/2*d*x)^4*tan(1/2*c)^4
 - 6*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^2 - 16*d*f*x*tan(1/2*d*x)^3*tan(1/2*c
)^3 - 8*d*e*tan(1/2*d*x)^4*tan(1/2*c)^3 - 6*d*f*x*tan(1/2*d*x)^2*tan(1/2*c
)^4 - 8*d*e*tan(1/2*d*x)^3*tan(1/2*c)^4 + 4*f*tan(1/2*d*x)^4*tan(1/2*c)^4
- 8*d*f*x*tan(1/2*d*x)^4*tan(1/2*c) - 6*d*e*tan(1/2*d*x)^4*tan(1/2*c)^2 -
16*d*e*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*f*tan(1/2*d*x)^4*tan(1/2*c)^3 - 8*d
*f*x*tan(1/2*d*x)*tan(1/2*c)^4 - 6*d*e*tan(1/2*d*x)^2*tan(1/2*c)^4 + 2*f*t
an(1/2*d*x)^3*tan(1/2*c)^4 + d*f*x*tan(1/2*d*x)^4 + 16*d*f*x*tan(1/2*d*x)^
3*tan(1/2*c) - 8*d*e*tan(1/2*d*x)^4*tan(1/2*c) + 36*d*f*x*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 16*d*f*x*tan(1/2*d*x)*tan(1/2*c)^3 - 16*f*tan(1/2*d*x)^3*tan
(1/2*c)^3 + d*f*x*tan(1/2*c)^4 - 8*d*e*tan(1/2*d*x)*tan(1/2*c)^4 + 8*d*f*x
*tan(1/2*d*x)^3 + d*e*tan(1/2*d*x)^4 + 16*d*e*tan(1/2*d*x)^3*tan(1/2*c) -
2*f*tan(1/2*d*x)^4*tan(1/2*c) + 36*d*e*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*f*
tan(1/2*d*x)^3*tan(1/2*c)^2 + 8*d*f*x*tan(1/2*c)^3 + 16*d*e*tan(1/2*d*x)*t
an(1/2*c)^3 - 12*f*tan(1/2*d*x)^2*tan(1/2*c)^3 + d*e*tan(1/2*c)^4 - 2*f*ta
n(1/2*d*x)*tan(1/2*c)^4 - 6*d*f*x*tan(1/2*d*x)^2 + 8*d*e*tan(1/2*d*x)^3 -
4*f*tan(1/2*d*x)^4 - 16*d*f*x*tan(1/2*d*x)*tan(1/2*c) - 16*f*tan(1/2*d*x)^
3*tan(1/2*c) - 6*d*f*x*tan(1/2*c)^2 + 8*d*e*tan(1/2*c)^3 - 16*f*tan(1/2*d*
x)*tan(1/2*c)^3 - 4*f*tan(1/2*c)^4 + 8*d*f*x*tan(1/2*d*x) - 6*d*e*tan(1...
```

Mupad [B] (verification not implemented)

Time = 38.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{f\,\sin(2\,c+2\,d\,x)}{2}+8\,f\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,d\,e\,\sin\left(c+d\,x\right)+2\,d\,e\,\sin\left(c+d\,x\right)^2-4\,d\,f\,x\,\sin\left(c+d\,x\right)+d\,f\,x}{4\,a\,d^2}$$

input
```
int((cos(c + d*x)^3*(e + f*x))/(a + a*sin(c + d*x)),x)
```

output
```
-((f*sin(2*c + 2*d*x))/2 + 8*f*sin(c/2 + (d*x)/2)^2 - 4*d*e*sin(c + d*x) +
 2*d*e*sin(c + d*x)^2 - 4*d*f*x*sin(c + d*x) + d*f*x*(2*sin(c + d*x)^2 - 1
))/(4*a*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-\cos(dx+c)\sin(dx+c)\,f+4\cos(dx+c)\,f-2\sin(dx+c)^2\,de-2\sin(dx+c)^2\,dfx+4\sin(dx+c)}{4a\,d^2}$$

input
```
int((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - cos(c + d*x)*sin(c + d*x)*f + 4*cos(c + d*x)*f - 2*sin(c + d*x)**2*d*e
 - 2*sin(c + d*x)**2*d*f*x + 4*sin(c + d*x)*d*e + 4*sin(c + d*x)*d*f*x + c
*f + 4*d*e + d*f*x)/(4*a*d**2)
```

3.266 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{(a-a\sin(c+dx))^2}{2a^3d}$$

output
```
-1/2*(a-a*sin(d*x+c))^2/a^3/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{(-2+\sin(c+dx))\sin(c+dx)}{2ad}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((-2 + Sin[c + d*x])*Sin[c + d*x])/(a*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(c+dx)^3}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3146}$$
$$\frac{\int (a-a\sin(c+dx))d(a\sin(c+dx))}{a^3d}$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{(a-a\sin(c+dx))^2}{2a^3d}$$

input
```
Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*(a - a*Sin[c + d*x])^2/(a^3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{-\frac{\sin(dx+c)^2}{2}+\sin(dx+c)}{ad}$	25
default	$\frac{-\frac{\sin(dx+c)^2}{2}+\sin(dx+c)}{ad}$	25
parallelrisch	$\frac{4\sin(dx+c)-1+\cos(2dx+2c)}{4ad}$	28
risch	$\frac{\sin(dx+c)}{ad}+\frac{\cos(2dx+2c)}{4ad}$	32
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	105

input
```
int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*(-1/2*sin(d*x+c)^2+sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\cos\left(dx+c\right)^2+2\,\sin\left(dx+c\right)}{2\,ad}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(19) = 38$.

Time = 1.96 (sec) , antiderivative size = 158, normalized size of antiderivative = 6.87

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \begin{cases} \frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{fo} \\ \frac{x\cos^3(c)}{a\sin(c)+a} & \text{ot} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2
+ d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*
d*tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4
 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2ad}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2\,ad}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 37.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sin(c+d\,x)\,(\sin(c+d\,x)-2)}{2\,a\,d}$$

input
```
int(cos(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

output
```
-(sin(c + d*x)*(sin(c + d*x) - 2))/(2*a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\sin(dx+c)\,(-\sin(dx+c)+2)}{2ad}$$

input
```
int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(sin(c + d*x)*( - sin(c + d*x) + 2))/(2*a*d)
```

3.267 $\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 128

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af} - \frac{\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)\sin\left(2c-\frac{2de}{f}\right)}{2af} - \frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af} - \frac{\cos\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{2af}$$

output
```
cos(c-d*e/f)*Ci(d*e/f+d*x)/a/f-1/2*Ci(2*d*e/f+2*d*x)*sin(2*c-2*d*e/f)/a/f-
sin(c-d*e/f)*Si(d*e/f+d*x)/a/f-1/2*cos(2*c-2*d*e/f)*Si(2*d*e/f+2*d*x)/a/f
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.82

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = -\frac{-2\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right)+\text{CosIntegral}\left(\frac{2d(e+fx)}{f}\right)\sin\left(2c-\frac{2de}{f}\right)+2\sin\left(c-\frac{de}{f}\right)\text{Si}}{2af}$$

input
```
Integrate[Cos[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
-1/2*(-2*Cos[c - (d*e)/f]*CosIntegral[d*(e/f + x)] + CosIntegral[(2*d*(e +
 f*x))/f]*Sin[2*c - (2*d*e)/f] + 2*Sin[c - (d*e)/f]*SinIntegral[d*(e/f + x
)] + Cos[2*c - (2*d*e)/f]*SinIntegral[(2*d*(e + f*x))/f])/(a*f)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {5034, 3042, 3784, 3042, 3780, 3783, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int \frac{\cos(c+dx)}{e+fx}dx}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\cos\left(c-\frac{de}{f}\right)\int\frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}dx-\sin\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}-\frac{\int\frac{\cos(c+dx)\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx-\sin\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}-\frac{\int\frac{\cos(c+dx)\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\cos\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int\frac{\cos(c+dx)\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int\frac{\cos(c+dx)\sin(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int\frac{\sin(2c+2dx)}{2(e+fx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int\frac{\sin(2c+2dx)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int\frac{\sin(2c+2dx)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\sin\left(2c-\frac{2de}{f}\right)\int\frac{\cos\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+\cos\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx\right)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\sin\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx+\frac{\pi}{2}\right)}{e+fx}dx+\cos\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx\right)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\sin\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{\cos\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{f}}{2a}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\frac{\sin\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{f}+\frac{\cos\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{f}}{2a}$$

input
```
Int[Cos[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
((Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/f - (Sin[c - (d*e)/f]*SinIn
tegral[(d*e)/f + d*x])/f)/a - ((CosIntegral[(2*d*e)/f + 2*d*x]*Sin[2*c - (
2*d*e)/f])/f + (Cos[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f + 2*d*x])/f)/(2
*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.27

method	result
derivativedivides	$-\frac{\frac{-\operatorname{Si}\left(-2dx-2c+\frac{2cf-2de}{f}\right)\cos\left(\frac{2cf-2de}{f}\right)+\operatorname{Ci}\left(2dx+2c-\frac{2(cf-de)}{f}\right)\sin\left(\frac{2cf-2de}{f}\right)}{2f}+\frac{\operatorname{Si}\left(-dx-c-\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}-\cdots}{a}$
default	$-\frac{\frac{-\operatorname{Si}\left(-2dx-2c+\frac{2cf-2de}{f}\right)\cos\left(\frac{2cf-2de}{f}\right)+\operatorname{Ci}\left(2dx+2c-\frac{2(cf-de)}{f}\right)\sin\left(\frac{2cf-2de}{f}\right)}{2f}+\frac{\operatorname{Si}\left(-dx-c-\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)}{f}-\cdots}{a}$
risch	$-\frac{e^{-\frac{i(cf-de)}{f}}\operatorname{expIntegral}_1\left(idx+ic-\frac{i(cf-de)}{f}\right)}{2af}-\frac{e^{\frac{i(cf-de)}{f}}\operatorname{expIntegral}_1\left(-idx-ic-\frac{-icf+ide}{f}\right)}{2af}-\frac{ie^{\frac{2i(cf-de)}{f}}\cdots}{\cdots}$

input
```
int(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(1/2/f*(-Si(-2*d*x-2*c+2*(c*f-d*e)/f)*cos(2*(c*f-d*e)/f)+Ci(2*d*x+2*c
-2*(c*f-d*e)/f)*sin(2*(c*f-d*e)/f))+Si(-d*x-c-(-c*f+d*e)/f)*sin((-c*f+d*e)
/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{2\cos\left(-\frac{de-cf}{f}\right)\operatorname{Ci}\left(\frac{dfx+de}{f}\right)-\operatorname{Ci}\left(\frac{2\,(dfx+de)}{f}\right)\sin\left(-\frac{2\,(de-cf)}{f}\right)-\cos\left(-\frac{2\,(de-cf)}{f}\right)\operatorname{Si}\left(\frac{2\,(dfx+de)}{f}\right)-2\sin\left(\right.}{2\,af}$$

input
```
integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*cos(-(d*e - c*f)/f)*cos_integral((d*f*x + d*e)/f) - cos_integral(2*
(d*f*x + d*e)/f)*sin(-2*(d*e - c*f)/f) - cos(-2*(d*e - c*f)/f)*sin_integra
l(2*(d*f*x + d*e)/f) - 2*sin(-(d*e - c*f)/f)*sin_integral((d*f*x + d*e)/f)
)/(a*f)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.20

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = -\frac{2\,d\left(E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-d\left(-i\,E_1\left(\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)-\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(2*d*(exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_inte
gral_e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) - d*(-I
*exp_integral_e(1, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + I*exp_integral_
e(1, -2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f))*cos(-2*(d*e - c*f)/f) + 2*d*(
-I*exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + I*exp_integral_e
(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - d*(exp_inte
gral_e(1, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + exp_integral_e(1, -2*(-I
*d*e - I*(d*x + c)*f + I*c*f)/f))*sin(-2*(d*e - c*f)/f))/(a*d*f)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 4510, normalized size of antiderivative = 35.23

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(3*pi + 3*pi*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*imag_part
(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2
 + 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*t
an(1/2*d*e/f)^2 - 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(
d*e/f)^2*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(-d*x - d*e/f))*tan(1/
2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*sin_integral(2*(d*f*x + d*e)/f)*t
an(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral(d*x +
 d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) - 8*imag_part(cos_integr
al(-d*x - d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) + 16*sin_integr
al((d*f*x + d*e)/f)*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) - 4*real_part
(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 -
 4*real_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)*tan(1
/2*d*e/f)^2 - 8*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^3*tan(d*e/
f)^2*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)
^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*real_part(cos_integral(2*d*x + 2*d*e/
f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*real_part(cos_integral(
-2*d*x - 2*d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 16*sin_int
egral((d*f*x + d*e)/f)*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 3*pi*t
an(1/2*c)^4*tan(d*e/f)^2 - 2*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(
1/2*c)^4*tan(d*e/f)^2 + 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cos(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(cos(c + d*x)^3/((e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos(dx+c)}{fx+e}dx - \left(\int \frac{\cos(dx+c)\sin(dx+c)}{fx+e}dx\right)}{a}$$

input
```
int(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
(int(cos(c + d*x)/(e + f*x),x) - int((cos(c + d*x)*sin(c + d*x))/(e + f*x)
,x))/a
```

3.268 $\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{af^2} - \frac{d\,\text{CosIntegral}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2} + \frac{\sin(2c+2dx)}{2af(e+fx)} - \frac{d\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af^2} + \frac{d\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{af^2}$$

output
```
-cos(d*x+c)/a/f/(f*x+e)-d*cos(2*c-2*d*e/f)*Ci(2*d*e/f+2*d*x)/a/f^2-d*Ci(d*
e/f+d*x)*sin(c-d*e/f)/a/f^2+1/2*sin(2*d*x+2*c)/a/f/(f*x+e)-d*cos(c-d*e/f)*
Si(d*e/f+d*x)/a/f^2+d*sin(2*c-2*d*e/f)*Si(2*d*e/f+2*d*x)/a/f^2
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$
$$= \frac{-2f\cos(c+dx)-2d(e+fx)\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2d(e+fx)}{f}\right)-2d(e+fx)\text{CosIntegral}\left(d\left(\frac{e}{f}+\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
(-2*f*Cos[c + d*x] - 2*d*(e + f*x)*Cos[2*c - (2*d*e)/f]*CosIntegral[(2*d*(
e + f*x))/f] - 2*d*(e + f*x)*CosIntegral[d*(e/f + x)]*Sin[c - (d*e)/f] + f
*Sin[2*(c + d*x)] - 2*d*e*Cos[c - (d*e)/f]*SinIntegral[d*(e/f + x)] - 2*d*
f*x*Cos[c - (d*e)/f]*SinIntegral[d*(e/f + x)] + 2*d*e*Sin[2*c - (2*d*e)/f]
*SinIntegral[(2*d*(e + f*x))/f] + 2*d*f*x*Sin[2*c - (2*d*e)/f]*SinIntegral
[(2*d*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5034, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 5034$$
$$\frac{\int \frac{\cos(c+dx)}{(e+fx)^2}dx}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$
$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{(e+fx)^2}dx}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{\frac{d\int -\frac{\sin(c+dx)}{e+fx}dx}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{d\int \frac{\sin(c+dx)}{e+fx}dx}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{d\int \frac{\sin(c+dx)}{e+fx}dx}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{-\frac{d\left(\sin\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{d\left(\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx+\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{-\frac{d\left(\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3783}$$

$$\frac{-\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{-\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int \frac{\sin(2c+2dx)}{2(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int\frac{\sin(2c+2dx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\int\frac{\sin(2c+2dx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\frac{2d\int\frac{\cos(2c+2dx)}{e+fx}dx}{f}-\frac{\sin(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\frac{2d\int\frac{\sin\left(2c+2dx+\frac{\pi}{2}\right)}{e+fx}dx}{f}-\frac{\sin(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\frac{2d\left(\cos\left(2c-\frac{2de}{f}\right)\int\frac{\cos\left(\frac{2de}{f}+2dx\right)}{e+fx}dx-\sin\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx\right)}{e+fx}dx\right)}{f}-\frac{\sin(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\frac{2d\left(\cos\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx+\frac{\pi}{2}\right)}{e+fx}dx-\sin\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx\right)}{e+fx}dx\right)}{f}-\frac{\sin(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\frac{2d\left(\cos\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2de}{f}+2dx+\frac{\pi}{2}\right)}{e+fx}dx-\frac{\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{f}\right)}{f}-\frac{\sin(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \; 3783$$

$$\frac{-\frac{d\left(\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{f}+\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{\cos(c+dx)}{f(e+fx)}}{a}-\frac{\frac{2d\left(\frac{\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{f}-\frac{\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{f}\right)}{f}-\frac{\sin(2c+2dx)}{f(e+fx)}}{2a}$$

input
```
Int[Cos[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
(-(Cos[c + d*x]/(f*(e + f*x))) - (d*((CosIntegral[(d*e)/f + d*x]*Sin[c - (
d*e)/f])/f + (Cos[c - (d*e)/f]*SinIntegral[(d*e)/f + d*x])/f))/f)/a - (-(S
in[2*c + 2*d*x]/(f*(e + f*x))) + (2*d*((Cos[2*c - (2*d*e)/f]*CosIntegral[(
2*d*e)/f + 2*d*x])/f - (Sin[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f + 2*d*x
])/f))/f)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{d\left(-\frac{\sin(2dx+2c)}{2f(cf-de-f(dx+c))}-\frac{\mathrm{Si}\left(-2dx-2c+\frac{2cf-2de}{f}\right)\sin\left(\frac{2cf-2de}{f}\right)+\mathrm{Ci}\left(2dx+2c-\frac{2(cf-de)}{f}\right)\cos\left(\frac{2cf-2de}{f}\right)}{f^2}-\frac{\cos(dx+c)}{(-cf+de+f(dx}\right.}{a}$
default	$\frac{d\left(-\frac{\sin(2dx+2c)}{2f(cf-de-f(dx+c))}-\frac{\mathrm{Si}\left(-2dx-2c+\frac{2cf-2de}{f}\right)\sin\left(\frac{2cf-2de}{f}\right)+\mathrm{Ci}\left(2dx+2c-\frac{2(cf-de)}{f}\right)\cos\left(\frac{2cf-2de}{f}\right)}{f^2}-\frac{\cos(dx+c)}{(-cf+de+f(dx}\right.}{a}$
risch	$\frac{id\,\mathrm{e}^{-\frac{i(cf-de)}{f}}\mathrm{expIntegral}_1\left(idx+ic-\frac{i(cf-de)}{f}\right)}{2a\,f^2}-\frac{id\,\mathrm{e}^{\frac{i(cf-de)}{f}}\mathrm{expIntegral}_1\left(-idx-ic-\frac{-icf+ide}{f}\right)}{2a\,f^2}+\frac{d\,\mathrm{e}^{\frac{2i(cf-de)}{f}}}{}$

input
```
int(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
d/a*(-1/2*sin(2*d*x+2*c)/f/(c*f-d*e-f*(d*x+c))-1/f^2*(Si(-2*d*x-2*c+2*(c*f
-d*e)/f)*sin(2*(c*f-d*e)/f)+Ci(2*d*x+2*c-2*(c*f-d*e)/f)*cos(2*(c*f-d*e)/f)
)-cos(d*x+c)/(-c*f+d*e+f*(d*x+c))/f-(-Si(-d*x-c-(-c*f+d*e)/f)*cos((-c*f+d*
e)/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*sin((-c*f+d*e)/f)/f)/f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07

$$\int\frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx =$$
$$-\frac{(dfx+de)\cos\left(-\frac{2\,(de-cf)}{f}\right)\mathrm{Ci}\left(\frac{2\,(dfx+de)}{f}\right)-f\cos(dx+c)\sin(dx+c)+(dfx+de)\,\mathrm{Ci}\left(\frac{dfx+de}{f}\right)\sin\left(\right.}{af}$$

input
```
integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-((d*f*x + d*e)*cos(-2*(d*e - c*f)/f)*cos_integral(2*(d*f*x + d*e)/f) - f*
cos(d*x + c)*sin(d*x + c) + (d*f*x + d*e)*cos_integral((d*f*x + d*e)/f)*si
n(-(d*e - c*f)/f) - (d*f*x + d*e)*sin(-2*(d*e - c*f)/f)*sin_integral(2*(d*
f*x + d*e)/f) + (d*f*x + d*e)*cos(-(d*e - c*f)/f)*sin_integral((d*f*x + d*
e)/f) + f*cos(d*x + c))/(a*f^3*x + a*e*f^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.77

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = -\frac{2\,d^2\left(E_2\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_2\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-d^2\left(-i\,E_2\left(\frac{2\,(-i\,de-i\,(dx+c)f+i\,cf)}{f}\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(2*d^2*(exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_in
tegral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) - d^2
*(-I*exp_integral_e(2, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + I*exp_integ
ral_e(2, -2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f))*cos(-2*(d*e - c*f)/f) + 2
*d^2*(-I*exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + I*exp_inte
gral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - d^2*(
exp_integral_e(2, 2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f) + exp_integral_e(2
, -2*(-I*d*e - I*(d*x + c)*f + I*c*f)/f))*sin(-2*(d*e - c*f)/f))/((a*d*e*f
 + (d*x + c)*a*f^2 - a*c*f^2)*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.12 (sec) , antiderivative size = 46878, normalized size of antiderivative = 267.87

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(d*f*x*imag_part(cos_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*f*x*imag_part(cos_integral
(-d*x - d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/
2*d*e/f)^2 - d*f*x*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - d*f*x*real_part(c
os_integral(-2*d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(
d*e/f)^2*tan(1/2*d*e/f)^2 + 2*d*f*x*sin_integral((d*f*x + d*e)/f)*tan(d*x)
^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 2*d*f*x*rea
l_part(cos_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*t
an(d*e/f)^2*tan(1/2*d*e/f) + 2*d*f*x*real_part(cos_integral(-d*x - d*e/f))
*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) + 2*d*
f*x*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan
(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 - 2*d*f*x*imag_part(cos_integral(-2*
d*x - 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*
d*e/f)^2 + 4*d*f*x*sin_integral(2*(d*f*x + d*e)/f)*tan(d*x)^2*tan(1/2*d*x)
^2*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 - 4*d*f*x*imag_part(cos_integr
al(2*d*x + 2*d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*t
an(1/2*d*e/f)^2 + 4*d*f*x*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*
x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*d*f*x*r
eal_part(cos_integral(d*x + d*e/f))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cos(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(cos(c + d*x)^3/((e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos(dx+c)^3}{\sin(dx+c)e^2+2\sin(dx+c)efx+\sin(dx+c)f^2x^2+e^2+2efx+f^2x^2}dx}{a}$$

input
```
int(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(cos(c + d*x)**3/(sin(c + d*x)*e**2 + 2*sin(c + d*x)*e*f*x + sin(c + d*
x)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x)/a
```

3.269 $\int \frac{(e+fx)^3 \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 502

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\arctan\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\arctan\left(e^{i(c+dx)}\right)}{ad} + \frac{3f^2(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{ad^3} + \frac{3if^3\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^4} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2} - \frac{3if^3\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^4} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^2} - \frac{3if^3\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2ad^4} - \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3} - \frac{3if^3\operatorname{PolyLog}\left(4,-ie^{i(c+dx)}\right)}{ad^4} + \frac{3if^3\operatorname{PolyLog}\left(4,ie^{i(c+dx)}\right)}{ad^4} - \frac{3f(e+fx)^2\sec(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^2(c+dx)}{2ad} + \frac{3f(e+fx)^2\tan(c+dx)}{2ad^2} + \frac{(e+fx)^3\sec(c+dx)\tan(c+dx)}{2ad}$$

output
```
3*I*f^3*polylog(2,-I*exp(I*(d*x+c)))/a/d^4-3/2*I*f^3*polylog(2,-exp(2*I*(d
*x+c)))/a/d^4-3/2*I*f*(f*x+e)^2/a/d^2+3*f^2*(f*x+e)*ln(1+exp(2*I*(d*x+c)))
/a/d^3-I*(f*x+e)^3*arctan(exp(I*(d*x+c)))/a/d-3/2*I*f*(f*x+e)^2*polylog(2,
I*exp(I*(d*x+c)))/a/d^2+3*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-3*I*f^3*
polylog(4,-I*exp(I*(d*x+c)))/a/d^4-6*I*f^2*(f*x+e)*arctan(exp(I*(d*x+c)))/
a/d^3-3*f^2*(f*x+e)*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+3*f^2*(f*x+e)*polyl
og(3,I*exp(I*(d*x+c)))/a/d^3-3*I*f^3*polylog(2,I*exp(I*(d*x+c)))/a/d^4+3/2
*I*f*(f*x+e)^2*polylog(2,-I*exp(I*(d*x+c)))/a/d^2-3/2*f*(f*x+e)^2*sec(d*x+
c)/a/d^2-1/2*(f*x+e)^3*sec(d*x+c)^2/a/d+3/2*f*(f*x+e)^2*tan(d*x+c)/a/d^2+1
/2*(f*x+e)^3*sec(d*x+c)*tan(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1025 vs. 2(502) = 1004.

Time = 10.31 (sec) , antiderivative size = 1025, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(8*a*(Cos[c/2] - Sin[c/2])
*(Cos[c/2] + Sin[c/2])) - ((Cos[c] + I*Sin[c])*(((e + f*x)^3*Log[1 - I*Cos
[c + d*x] - Sin[c + d*x]]*(1 - I*Cos[c] - Sin[c]))/d + ((e + f*x)^4*(Cos[c
] - I*Sin[c]))/(4*f) + (3*f*(d^2*(e + f*x)^2*PolyLog[2, I*Cos[c + d*x] + S
in[c + d*x]] - (2*I)*d*f*(e + f*x)*PolyLog[3, I*Cos[c + d*x] + Sin[c + d*x
]] - 2*f^2*PolyLog[4, I*Cos[c + d*x] + Sin[c + d*x]])*(Cos[c] + I*(-1 + Si
n[c]))*(I*Cos[c] + Sin[c]))/d^4))/(2*a*(Cos[c] + I*(-1 + Sin[c]))) - ((Cos
[c] + I*Sin[c])*(((12*f^2 + d^2*(e + f*x)^2)^2*(Cos[c] - I*Sin[c]))/(4*d^2
*f) + (3*f*(d^2*e^2 + 4*f^2)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*
(Cos[c] - I*Sin[c])*(1 - I*Cos[c] + Sin[c]))/d^2 + 6*e*f^2*x*PolyLog[2, (-
I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(1 - I*Cos[c] + Sin[c]
) + 3*f^3*x^2*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1
+ Sin[c])) - (6*f^3*PolyLog[4, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] -
 I*(1 + Sin[c])))/d^2 - (3*f*(d^2*e^2 + 4*f^2)*x*Log[1 + I*Cos[c + d*x] +
Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d - 3*d*e*f^2
*x^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] +
I*(1 + Sin[c])) - d*f^3*x^3*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c]
 - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) - (e*(d^2*e^2 + 12*f^2)*Log[Cos[c +
 d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])
))/d - (6*e*f^2*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - ...
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.99, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {5042, 3042, 4674, 3042, 4669, 2715, 2838, 3011, 4909, 3042, 4672, 25, 3042, 4202, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sec(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)^3\sec^3(c+dx)dx}{a}-\frac{\int (e+fx)^3\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3 \csc\left(c+dx+\frac{\pi}{2}\right)^3 dx}{a}-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \; 4674$$

$$\frac{\frac{3f^2\int (e+fx)\sec(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\sec(c+dx)dx-\frac{3f(e+fx)^2\sec(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f^2\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{3f(e+fx)^2\sec(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}+$$
$$\frac{\frac{3f^2\left(-\frac{f\int \log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3f\int (e+fx)^2\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{3f\int (e+fx)^{2}\ldots}{\ldots}\right.}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}+$$
$$\frac{\frac{3f^2\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3f\int (e+fx)^2\log\ldots}{\ldots}\right.}{\ldots}$$

$$\Big\downarrow \; 2838$$

$$-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}+$$
$$\frac{\frac{1}{2}\left(-\frac{3f\int (e+fx)^2\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{3f\int (e+fx)^2\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)^3\arctan\left(e^{i(c+dx)}\right)}{d}\right)+\frac{3f^2\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+\ldots}\right.}{d}\right.}{\ldots}}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{\int (e+fx)^3 \sec^2(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{4909}$$

$$-\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sec^2(c+dx)dx}{2d}}{a}+\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{2d}}{a}+\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{2f\int -((e+fx)\tan(c+dx))dx}{d}+\frac{(e+fx)^2\tan(c+dx)}{d}\right)}{2d}}{a}+\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\int (e+fx)\tan(c+dx)dx}{d}\right)}{2d}}{a}+\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\int(e+fx)\tan(c+dx)dx}{d}\right)}{2d}}{a}+$$

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,ie^{\cdots}\right.}{d}\right.}{d}\right.}{}$$

$\downarrow$ 4202

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,ie^{\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{2i(c+dx)}(e+fx)}{1+e^{2i(c+dx)}}dx\right)}{d}\right)}{2d}}{a}$$

$\downarrow$ 2620

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,ie^{\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{2i(c+dx)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)}{2d}}{a}$$

$\downarrow$ 2715

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,ie^{\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-2i(c+dx)}\log\left(1+e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)}{2d}}{a}$$

$\downarrow$ 2838

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,ie^{i\cdots}\right.}{d}\right.}{d}\right.}{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)}{2d}}$$

a

$\downarrow$ 7163

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c\cdots}\right.}{d}\right.}{}\right.}{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)}{2d}}$$

a

$\downarrow$ 2720

$$\frac{\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)\cdots}{}\right.}{}\right.}{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)}{2d}}$$

a

$\downarrow$ 7143

$$\frac{\frac{3f^2\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)}{d^2}+\frac{1}{2}\left(-\frac{2i(e+fx)^3\arctan(e^{i(c+dx)})}{d}+\frac{3f\left(\frac{i(e+fx)^2\,\text{I}}{}\right.}{}\right.}{}$$

$$\frac{\frac{(e+fx)^3\sec^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)}{2d}}{a}$$

input
```
Int[((e + f*x)^3*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((3*f^2*(((-2*I)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/d + (I*f*PolyLog[2, (-
I)*E^(I*(c + d*x))])/d^2 - (I*f*PolyLog[2, I*E^(I*(c + d*x))])/d^2))/d^2 +
 (((-2*I)*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/d + (3*f*((I*(e + f*x)^2*Po
lyLog[2, (-I)*E^(I*(c + d*x))])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (
-I)*E^(I*(c + d*x))])/d + (f*PolyLog[4, (-I)*E^(I*(c + d*x))])/d^2))/d))/d
 - (3*f*((I*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/d - ((2*I)*f*(((-I)
*(e + f*x)*PolyLog[3, I*E^(I*(c + d*x))])/d + (f*PolyLog[4, I*E^(I*(c + d*
x))])/d^2))/d))/d)/2 - (3*f*(e + f*x)^2*Sec[c + d*x])/(2*d^2) + ((e + f*x)
^3*Sec[c + d*x]*Tan[c + d*x])/(2*d))/a - (((e + f*x)^3*Sec[c + d*x]^2)/(2*
d) - (3*f*((-2*f*(((I/2)*(e + f*x)^2)/f - (2*I)*(((-1/2*I)*(e + f*x)*Log[1
 + E^((2*I)*(c + d*x))])/d - (f*PolyLog[2, -E^((2*I)*(c + d*x))])/(4*d^2))
))/d + ((e + f*x)^2*Tan[c + d*x])/d))/(2*d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(444) = 888$.

Time = 2.51 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.38

method	result	size
risch	Expression too large to display	1196

input
```
int((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-3*I*f^3*polylog(4,-I*exp(I*(d*x+
c)))/a/d^4-3*I/a/d^2*e*f^2*polylog(2,I*exp(I*(d*x+c)))*x+3*I/a/d^2*e*f^2*p
olylog(2,-I*exp(I*(d*x+c)))*x+3*I/a/d^2*e^2*f*c*arctan(exp(I*(d*x+c)))-3*I
/a/d^3*e*f^2*c^2*arctan(exp(I*(d*x+c)))-I*(d*exp(I*(d*x+c))*f^3*x^3+3*d*ex
p(I*(d*x+c))*e*f^2*x^2+3*d*exp(I*(d*x+c))*e^2*f*x+d*exp(I*(d*x+c))*e^3+3*f
^3*x^2-3*I*f^3*x^2*exp(I*(d*x+c))+6*e*f^2*x-6*I*e*f^2*x*exp(I*(d*x+c))+3*e
^2*f-3*I*e^2*f*exp(I*(d*x+c)))/d^2/(exp(I*(d*x+c))+I)^2/a+3/2*I/a/d^2*f^3*
polylog(2,-I*exp(I*(d*x+c)))*x^2-3/2*I/a/d^2*e^2*f*polylog(2,I*exp(I*(d*x+
c)))+3/2*I/a/d^2*e^2*f*polylog(2,-I*exp(I*(d*x+c)))-3/2*I/a/d^2*f^3*polylo
g(2,I*exp(I*(d*x+c)))*x^2+6*I/a/d^4*f^3*c*arctan(exp(I*(d*x+c)))-6*I/a/d^3
*e*f^2*arctan(exp(I*(d*x+c)))+I/a/d^4*f^3*c^3*arctan(exp(I*(d*x+c)))+3/2/a
/d^3*e*f^2*ln(1+I*exp(I*(d*x+c)))*c^2+3/2/a/d*e^2*f*ln(1-I*exp(I*(d*x+c)))
*x+3/2/a/d^2*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-3/2/a/d*e^2*f*ln(1+I*exp(I*(d*
x+c)))*x-3/2/a/d^2*e^2*f*ln(1+I*exp(I*(d*x+c)))*c+3/2/a/d*e*f^2*ln(1-I*exp
(I*(d*x+c)))*x^2-3/2/a/d*e*f^2*ln(1+I*exp(I*(d*x+c)))*x^2-3/2/a/d^3*e*f^2*
ln(1-I*exp(I*(d*x+c)))*c^2-6*I/a/d^3*f^3*c*x-6/a/d^3*e*f^2*ln(exp(I*(d*x+c
)))+3/a/d^3*e*f^2*ln(exp(2*I*(d*x+c))+1)+3/a/d^3*e*f^2*polylog(3,I*exp(I*(
d*x+c)))-3/a/d^3*e*f^2*polylog(3,-I*exp(I*(d*x+c)))+1/2/a/d^4*c^3*f^3*ln(1
-I*exp(I*(d*x+c)))-1/2/a/d^4*c^3*f^3*ln(1+I*exp(I*(d*x+c)))+6/a/d^3*f^3*ln
(1-I*exp(I*(d*x+c)))*x+6/a/d^4*f^3*ln(1-I*exp(I*(d*x+c)))*c+6/a/d^4*f^3...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1884 vs. $2(421) = 842$.

Time = 0.25 (sec) , antiderivative size = 1884, normalized size of antiderivative = 3.75

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 + 6*(d^2
*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*cos(d*x + c) + 3*(I*d^2*f^3*x^2 + 2*
I*d^2*e*f^2*x + I*d^2*e^2*f + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2
*f)*sin(d*x + c))*dilog(I*cos(d*x + c) + sin(d*x + c)) + 3*(I*d^2*f^3*x^2
+ 2*I*d^2*e*f^2*x + I*d^2*e^2*f + 4*I*f^3 + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2
*x + I*d^2*e^2*f + 4*I*f^3)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x +
 c)) + 3*(-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f + (-I*d^2*f^3*x^2
 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f)*sin(d*x + c))*dilog(-I*cos(d*x + c) + si
n(d*x + c)) + 3*(-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f - 4*I*f^3
+ (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f - 4*I*f^3)*sin(d*x + c))
*dilog(-I*cos(d*x + c) - sin(d*x + c)) - (d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2
 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2 + 4)*
d*e*f^2 - (c^3 + 12*c)*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c
) + I) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*e^3 - 3
*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sin(d*x + c))*log(cos(d*x + c) - I
*sin(d*x + c) + I) - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^
2*d*e*f^2 + (c^3 + 12*c)*f^3 + 3*(d^3*e^2*f + 4*d*f^3)*x + (d^3*f^3*x^3 +
3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 + 12*c)*f^3 + 3*(d^
3*e^2*f + 4*d*f^3)*x)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1)
 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*se
c(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d*x)/(s
in(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)/(sin(c + d*x) + 1)
, x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3854 vs. $2(421) = 842$.

Time = 0.86 (sec) , antiderivative size = 3854, normalized size of antiderivative = 7.68

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(3*c*e^2*f*(2/(a*d*sin(d*x + c) + a*d) - log(sin(d*x + c) + 1)/(a*d) +
 log(sin(d*x + c) - 1)/(a*d)) + e^3*(log(sin(d*x + c) + 1)/a - log(sin(d*x
 + c) - 1)/a - 2/(a*sin(d*x + c) + a)) - 4*(12*d^2*e^2*f - 24*c*d*e*f^2 +
12*c^2*f^3 + 2*(3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 - (3*(c^2 + 4)*d*e*
f^2 - (c^3 + 12*c)*f^3)*cos(2*d*x + 2*c) + 2*(3*(-I*c^2 - 4*I)*d*e*f^2 + (
I*c^3 + 12*I*c)*f^3)*cos(d*x + c) + (3*(-I*c^2 - 4*I)*d*e*f^2 + (I*c^3 + 1
2*I*c)*f^3)*sin(2*d*x + 2*c) + 2*(3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3)*
sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 2*(3*c^2*d*e*f^2 -
 c^3*f^3 - (3*c^2*d*e*f^2 - c^3*f^3)*cos(2*d*x + 2*c) - 2*(3*I*c^2*d*e*f^2
 - I*c^3*f^3)*cos(d*x + c) - (3*I*c^2*d*e*f^2 - I*c^3*f^3)*sin(2*d*x + 2*c
) + 2*(3*c^2*d*e*f^2 - c^3*f^3)*sin(d*x + c))*arctan2(sin(d*x + c) - 1, co
s(d*x + c)) - 2*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^
2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c) - ((d*x + c)^3*f^3 + 3*(d
*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*
(d*x + c))*cos(2*d*x + 2*c) - 2*(I*(d*x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^
3)*(d*x + c)^2 + 3*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + (I*c^2 + 4*I)*f^3)*(d*x
+ c))*cos(d*x + c) - (I*(d*x + c)^3*f^3 + 3*(I*d*e*f^2 - I*c*f^3)*(d*x + c
)^2 + 3*(I*d^2*e^2*f - 2*I*c*d*e*f^2 + (I*c^2 + 4*I)*f^3)*(d*x + c))*sin(2
*d*x + 2*c) + 2*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^
2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c))*sin(d*x + c))*arctan2...
```

Giac [F]

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^3\sec\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Hanged}$$

input
```
int((e + f*x)^3/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output

```
( - 12*cos(c + d*x)*d**3*e**2*f*x - 12*cos(c + d*x)*d**3*e*f**2*x**2 - 4*c
os(c + d*x)*d**3*f**3*x**3 - 12*cos(c + d*x)*d**2*e**2*f + 48*cos(c + d*x)
*d**2*e*f**2*x + 24*cos(c + d*x)*d**2*f**3*x**2 + 24*cos(c + d*x)*d*e*f**2
 - 120*cos(c + d*x)*d*f**3*x - 48*cos(c + d*x)*f**3 - 48*int(x**2/(tan((c
+ d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*sin(c +
d*x)*d**3*f**3 - 48*int(x**2/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3
- 2*tan((c + d*x)/2) - 1),x)*d**3*f**3 - 16*int((tan((c + d*x)/2)*x**3)/(t
an((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*si
n(c + d*x)*d**4*f**3 - 16*int((tan((c + d*x)/2)*x**3)/(tan((c + d*x)/2)**4
 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*d**4*f**3 - 48*int((
tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*ta
n((c + d*x)/2) - 1),x)*sin(c + d*x)*d**4*e*f**2 + 48*int((tan((c + d*x)/2)
*x**2)/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) -
 1),x)*sin(c + d*x)*d**3*f**3 - 48*int((tan((c + d*x)/2)*x**2)/(tan((c + d
*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*d**4*e*f**2
 + 48*int((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2
)**3 - 2*tan((c + d*x)/2) - 1),x)*d**3*f**3 - 48*int((tan((c + d*x)/2)*x)/
(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*
sin(c + d*x)*d**4*e**2*f + 96*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**
4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)*d**...
```

3.270 $\int \frac{(e+fx)^2 \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 278

$$\begin{aligned}\int \frac{(e+fx)^2 \sec(c+dx)}{a+a\sin(c+dx)}\,dx = & -\frac{i(e+fx)^2 \arctan\left(e^{i(c+dx)}\right)}{ad} \\ & + \frac{f^2 \operatorname{arctanh}(\sin(c+dx))}{ad^3} + \frac{f^2 \log(\cos(c+dx))}{ad^3} \\ & + \frac{if(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^2} \\ & - \frac{if(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2} \\ & - \frac{f^2 \operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3} \\ & + \frac{f^2 \operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3} - \frac{f(e+fx)\sec(c+dx)}{ad^2} \\ & - \frac{(e+fx)^2 \sec^2(c+dx)}{2ad} + \frac{f(e+fx)\tan(c+dx)}{ad^2} \\ & + \frac{(e+fx)^2 \sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

output
```
-I*(f*x+e)^2*arctan(exp(I*(d*x+c)))/a/d+f^2*arctanh(sin(d*x+c))/a/d^3+f^2*
ln(cos(d*x+c))/a/d^3+I*f*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/a/d^2-I*f*(f
*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^2-f^2*polylog(3,-I*exp(I*(d*x+c)))/a
/d^3+f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3-f*(f*x+e)*sec(d*x+c)/a/d^2-1/2*
(f*x+e)^2*sec(d*x+c)^2/a/d+f*(f*x+e)*tan(d*x+c)/a/d^2+1/2*(f*x+e)^2*sec(d*
x+c)*tan(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 725 vs. $2(278) = 556$.

Time = 9.08 (sec) , antiderivative size = 725, normalized size of antiderivative = 2.61

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{\frac{(e+fx)^3}{(-i+e^{ic})f}+\frac{3(e+fx)^2\log\left(1-ie^{-i(c+dx)}\right)}{d}+\frac{6f\left(id(e+fx)\operatorname{PolyLog}\left(2,ie^{-i(c+dx)}\right)+f\operatorname{PolyLog}\left(3,ie^{-i(c+dx)}\right)\right)}{d^3}}{6a}$$
$$+\frac{x(3e^2+3efx+f^2x^2)}{6a\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$
$$-\frac{(\cos(c)+i\sin(c))\left(d^2e^2x\cos(c)+4f^2x\cos(c)+d^2efx^2\cos(c)+\frac{1}{3}d^2f^2x^3(\cos(c)-i\sin(c))-id^2e^2x\,\mathrm{s}\right.}{}$$
$$-\frac{(e+fx)^2}{2ad\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$+\frac{2\left(ef\sin\left(\frac{dx}{2}\right)+f^2x\sin\left(\frac{dx}{2}\right)\right)}{ad^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
Integrate[((e + f*x)^2*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
-1/6*((e + f*x)^3/((-I + E^(I*c))*f) + (3*(e + f*x)^2*Log[1 - I/E^(I*(c +
d*x))])/d + (6*f*(I*d*(e + f*x)*PolyLog[2, I/E^(I*(c + d*x))] + f*PolyLog[
3, I/E^(I*(c + d*x))]))/d^3)/a + (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(6*a*(Cos
[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])) - ((Cos[c] + I*Sin[c])*(d^2*e^2*x
*Cos[c] + 4*f^2*x*Cos[c] + d^2*e*f*x^2*Cos[c] + (d^2*f^2*x^3*(Cos[c] - I*S
in[c]))/3 - I*d^2*e^2*x*Sin[c] - (4*I)*f^2*x*Sin[c] - I*d^2*e*f*x^2*Sin[c]
 + 2*e*f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin
[c])) + 2*f^2*x*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(
1 + Sin[c])) - 2*d*e*f*x*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] -
I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) - d*f^2*x^2*Log[1 + I*Cos[c + d*x] + S
in[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) - ((d^2*e^2 + 4
*f^2)*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I*Sin[c])*(Cos[c]
 + I*(1 + Sin[c])))/d - (2*f^2*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]
]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d + (d^2*e^2 + 4*f^2)*x*(
I*Cos[c] + Sin[c])*(Cos[c] + I*(1 + Sin[c]))))/(2*a*d^2*(Cos[c] + I*(1 + S
in[c]))) - (e + f*x)^2/(2*a*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2)
 + (2*(e*f*Sin[(d*x)/2] + f^2*x*Sin[(d*x)/2]))/(a*d^2*(Cos[c/2] + Sin[c/2]
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5042, 3042, 4674, 3042, 4257, 4669, 3011, 2720, 4909, 3042, 4672, 25, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sec(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{ 5042}$$

$$\frac{\int (e+fx)^2\sec^3(c+dx)dx}{a}-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{f^2\int \sec(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\sec(c+dx)dx-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{1}{2}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{f^2\text{arctanh}(\sin(c+dx))}{d^3}-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}+$$
$$\frac{\frac{1}{2}\left(-\frac{2f\int (e+fx)\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}\right)+\frac{f^2\text{arctanh}(\sin(c+dx))}{d^3}-f}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}+$$
$$\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right)}{d}-2\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)dx}{a}+$$
$$\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{Poly}}{}\right.}{d}\right.$$

$$\Big\downarrow \text{4909}$$

$$-\frac{\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\int(e+fx)\sec^2(c+dx)dx}{d}}{a}+\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{Poly}\right.}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\int(e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{d}}{a}+\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{Poly}\right.}{d}\right.$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\left(\frac{f\int-\tan(c+dx)dx}{d}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d}}{a}+\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{Poly}\right.}{d}\right.$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int\tan(c+dx)dx}{d}\right)}{d}}{a}+\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{Poly}\right.}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int\tan(c+dx)dx}{d}\right)}{d}}{a}+\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{Poly}\right.}{d}\right.$$

$$\Big\downarrow \ 3956$$

$$\frac{-\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d}}{a}+\frac{\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{Poly}}{}\right.}{d}\right.}{}$$

$$\Big\downarrow \ 7143$$

$$\frac{-\frac{(e+fx)^2\sec^2(c+dx)}{2d}-\frac{f\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d}}{a}+\frac{\frac{1}{2}\left(-\frac{2i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\operatorname{Po}}{}\right.}{d}\right.}{a}$$

input
```
Int[((e + f*x)^2*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
((f^2*ArcTanh[Sin[c + d*x]])/d^3 + (((-2*I)*(e + f*x)^2*ArcTan[E^(I*(c + d
*x))])/d + (2*f*((I*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/d - (f*Pol
yLog[3, (-I)*E^(I*(c + d*x))])/d^2))/d - (2*f*((I*(e + f*x)*PolyLog[2, I*E
^(I*(c + d*x))])/d - (f*PolyLog[3, I*E^(I*(c + d*x))])/d^2))/d)/2 - (f*(e
+ f*x)*Sec[c + d*x])/d^2 + ((e + f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*d))/
a - (((e + f*x)^2*Sec[c + d*x]^2)/(2*d) - (f*((f*Log[Cos[c + d*x]])/d^2 +
((e + f*x)*Tan[c + d*x])/d))/d)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(257) = 514$.

Time = 1.42 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.22

method	result
risch	$-\frac{i f^2 c^2 \arctan\left(\mathrm{e}^{i(dx+c)}\right)}{a\, d^3}-\frac{f^2 \operatorname{polylog}\left(3,-i \mathrm{e}^{i(dx+c)}\right)}{a\, d^3}-\frac{c^2 f^2 \ln\left(1-i \mathrm{e}^{i(dx+c)}\right)}{2 d^3 a}+\frac{f^2 \ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a\, d^3}+\frac{e f \ln\left(1-i \mathrm{e}^{i(dx+c)}\right) x}{d a}$

input
```
int((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/a/d^3*f^2*c^2*arctan(exp(I*(d*x+c)))-f^2*polylog(3,-I*exp(I*(d*x+c)))/a
/d^3-1/2/d^3/a*c^2*f^2*ln(1-I*exp(I*(d*x+c)))+1/a/d^3*f^2*ln(exp(2*I*(d*x+
c))+1)+1/d/a*e*f*ln(1-I*exp(I*(d*x+c)))*x+f^2*polylog(3,I*exp(I*(d*x+c)))/
a/d^3-1/a/d^2*ln(1+I*exp(I*(d*x+c)))*c*e*f+2*I/a/d^2*e*f*c*arctan(exp(I*(d
*x+c)))-2/a/d^3*f^2*ln(exp(I*(d*x+c)))-I/a/d^2*e*f*polylog(2,I*exp(I*(d*x+
c)))+1/d^2/a*e*f*ln(1-I*exp(I*(d*x+c)))*c-I*(d*exp(I*(d*x+c))*f^2*x^2+2*d*
exp(I*(d*x+c))*e*f*x+d*exp(I*(d*x+c))*e^2+2*f^2*x-2*I*f^2*x*exp(I*(d*x+c))
+2*e*f-2*I*e*f*exp(I*(d*x+c)))/d^2/(exp(I*(d*x+c))+I)^2/a+I/a/d^2*e*f*poly
log(2,-I*exp(I*(d*x+c)))-I/a/d*e^2*arctan(exp(I*(d*x+c)))+I/a/d^2*f^2*poly
log(2,-I*exp(I*(d*x+c)))*x-1/a/d*ln(1+I*exp(I*(d*x+c)))*e*f*x+1/2/d/a*f^2*
ln(1-I*exp(I*(d*x+c)))*x^2+1/2/a/d^3*f^2*ln(1+I*exp(I*(d*x+c)))*c^2-2*I/a/
d^3*f^2*arctan(exp(I*(d*x+c)))-I/a/d^2*f^2*polylog(2,I*exp(I*(d*x+c)))*x-1
/2/a/d*ln(1+I*exp(I*(d*x+c)))*f^2*x^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(248) = 496$.

Time = 0.13 (sec) , antiderivative size = 1064, normalized size of antiderivative = 3.83

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 + 4*(d*f^2*x + d*e*f)*cos(d*
x + c) + 2*(I*d*f^2*x + I*d*e*f + (I*d*f^2*x + I*d*e*f)*sin(d*x + c))*dilo
g(I*cos(d*x + c) + sin(d*x + c)) + 2*(I*d*f^2*x + I*d*e*f + (I*d*f^2*x + I
*d*e*f)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + 2*(-I*d*f^2*x
 - I*d*e*f + (-I*d*f^2*x - I*d*e*f)*sin(d*x + c))*dilog(-I*cos(d*x + c) +
sin(d*x + c)) + 2*(-I*d*f^2*x - I*d*e*f + (-I*d*f^2*x - I*d*e*f)*sin(d*x +
 c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - (d^2*e^2 - 2*c*d*e*f + (c^2 +
 4)*f^2 + (d^2*e^2 - 2*c*d*e*f + (c^2 + 4)*f^2)*sin(d*x + c))*log(cos(d*x
+ c) + I*sin(d*x + c) + I) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2 + (d^2*e^2 - 2
*c*d*e*f + c^2*f^2)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + I) -
 (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2 + (d^2*f^2*x^2 + 2*d^2*e
*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c
) + 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2 + (d^2*f^2*x^2 +
 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(I*cos(d*x + c) - sin
(d*x + c) + 1) - (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2 + (d^2*f
^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(-I*cos(d*x +
 c) + sin(d*x + c) + 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2
 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(-I*
cos(d*x + c) - sin(d*x + c) + 1) - (d^2*e^2 - 2*c*d*e*f + (c^2 + 4)*f^2 +
(d^2*e^2 - 2*c*d*e*f + (c^2 + 4)*f^2)*sin(d*x + c))*log(-cos(d*x + c) +...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\int \frac{e^2\sec(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{f^2x^2\sec(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{2efx\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*se
c(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)/(sin(c +
 d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1932 vs. $2(248) = 496$.

Time = 0.29 (sec) , antiderivative size = 1932, normalized size of antiderivative = 6.95

$$\int \frac{(e+fx)^2 \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(2*c*e*f*(2/(a*d*sin(d*x + c) + a*d) - log(sin(d*x + c) + 1)/(a*d) + l
og(sin(d*x + c) - 1)/(a*d)) + e^2*(log(sin(d*x + c) + 1)/a - log(sin(d*x +
 c) - 1)/a - 2/(a*sin(d*x + c) + a)) - 4*(8*(d*x + c)*f^2*cos(2*d*x + 2*c)
 + 8*I*(d*x + c)*f^2*sin(2*d*x + 2*c) + 8*d*e*f - 8*c*f^2 - 2*((c^2 + 4)*f
^2*cos(2*d*x + 2*c) - 2*(-I*c^2 - 4*I)*f^2*cos(d*x + c) - (-I*c^2 - 4*I)*f
^2*sin(2*d*x + 2*c) - 2*(c^2 + 4)*f^2*sin(d*x + c) - (c^2 + 4)*f^2)*arctan
2(sin(d*x + c) + 1, cos(d*x + c)) + 2*(c^2*f^2*cos(2*d*x + 2*c) + 2*I*c^2*
f^2*cos(d*x + c) + I*c^2*f^2*sin(2*d*x + 2*c) - 2*c^2*f^2*sin(d*x + c) - c
^2*f^2)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) - 2*((d*x + c)^2*f^2 + 2*(
d*e*f - c*f^2)*(d*x + c) - ((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))
*cos(2*d*x + 2*c) - 2*(I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c)
)*cos(d*x + c) - (I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*sin
(2*d*x + 2*c) + 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x
+ c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*((d*x + c)^2*f^2 + 2*(d*
e*f - c*f^2)*(d*x + c) - ((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*c
os(2*d*x + 2*c) - 2*(I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*
cos(d*x + c) - (I*(d*x + c)^2*f^2 + 2*(I*d*e*f - I*c*f^2)*(d*x + c))*sin(2
*d*x + 2*c) + 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x +
c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 4*((d*x + c)^2*f^2 - 2*I*d*
e*f + (c^2 + 2*I*c)*f^2 + 2*(d*e*f - (c - I)*f^2)*(d*x + c))*cos(d*x + ...
```

Giac [F]

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sec(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - 12*cos(c + d*x)*d**2*e*f*x - 6*cos(c + d*x)*d**2*f**2*x**2 - 12*cos(c
+ d*x)*d*e*f + 24*cos(c + d*x)*d*f**2*x + 12*cos(c + d*x)*f**2 - 24*int((t
an((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan
((c + d*x)/2) - 1),x)*sin(c + d*x)*d**3*f**2 - 24*int((tan((c + d*x)/2)*x*
*2)/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1)
,x)*d**3*f**2 - 48*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**4 + 2*tan((
c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)*d**3*e*f + 48*int
((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan
((c + d*x)/2) - 1),x)*sin(c + d*x)*d**2*f**2 - 48*int((tan((c + d*x)/2)*x)
/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)
*d**3*e*f + 48*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**4 + 2*tan((c +
d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*d**2*f**2 - 48*int(x/(tan((c + d*x
)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)*
d**2*f**2 - 48*int(x/(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**3 - 2*tan(
(c + d*x)/2) - 1),x)*d**2*f**2 - 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d
*x)*d*e*f + 24*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*f**2 - 12*log(tan
((c + d*x)/2)**2 + 1)*d*e*f + 24*log(tan((c + d*x)/2)**2 + 1)*f**2 - 3*log
(tan((c + d*x)/2) - 1)*sin(c + d*x)*d**2*e**2 - 3*log(tan((c + d*x)/2) - 1
)*d**2*e**2 + 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*d**2*e**2 + 24*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)*d*e*f - 48*log(tan((c + d*x)/2) + 1)...
```

3.271 $\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = & -\frac{i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{ad}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2} \\ & -\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^2}-\frac{f\sec(c+dx)}{2ad^2} \\ & -\frac{(e+fx)\sec^2(c+dx)}{2ad}+\frac{f\tan(c+dx)}{2ad^2} \\ & +\frac{(e+fx)\sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

output
```
-I*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d+1/2*I*f*polylog(2,-I*exp(I*(d*x+c)))
/a/d^2-1/2*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-1/2*f*sec(d*x+c)/a/d^2-1/
2*(f*x+e)*sec(d*x+c)^2/a/d+1/2*f*tan(d*x+c)/a/d^2+1/2*(f*x+e)*sec(d*x+c)*t
an(d*x+c)/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 655 vs. 2(172) = 344.

Time = 9.59 (sec) , antiderivative size = 655, normalized size of antiderivative = 3.81

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
-1/4*(2*d*(e + f*x) - 4*f*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]) + (c + d*x)*(c*f - d*(2*e + f*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2 + d*e*(c + d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 - c*f*(c + d*x + 2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + d*e*(c + d*
x - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 - c*f*(c + d*x - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (f*((-1)^(3/4)*(c + d*x)^2 + ((
-3*I)*Pi*(c + d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] + 2*(-2*c + Pi - 2*d
*x)*Log[1 + I*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] - 2*Pi*Log[Sin
[(2*c - Pi + 2*d*x)/4]] + (4*I)*PolyLog[2, (-I)*E^(I*(c + d*x))])/Sqrt[2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/Sqrt[2] + (f*((-1)^(1/4)*(c + d*
x)^2 + ((-I)*Pi*(c + d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(2*c + Pi
 + 2*d*x)*Log[1 - I*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] + 2*Pi*L
og[Sin[(2*c + Pi + 2*d*x)/4]] + (4*I)*PolyLog[2, I*E^(I*(c + d*x))])/Sqrt[
2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/Sqrt[2])/(a*d^2*(1 + Sin[c +
d*x]))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5042, 3042, 4673, 3042, 4669, 2715, 2838, 4909, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sec(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)\sec^3(c+dx)dx}{a}-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{1}{2}\int (e+fx)\sec(c+dx)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{1}{2}\left(-\frac{f\int \log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{1}{2}\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}+}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{4909}$$

$$\frac{-\frac{\frac{(e+fx)\sec^2(c+dx)}{2d}-\frac{f\int \sec^2(c+dx)dx}{2d}}{a}+\frac{\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)\sec^2(c+dx)}{2d}-\frac{f\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{2d}}{a}+\frac{\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{f\int 1d(-\tan(c+dx))}{2d^2}+\frac{(e+fx)\sec^2(c+dx)}{2d}}{a}+\frac{\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{(e+fx)\sec^2(c+dx)}{2d}-\frac{f\tan(c+dx)}{2d^2}}{a}+\frac{\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}}{a}$$

input
```
Int[((e + f*x)*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
-((((e + f*x)*Sec[c + d*x]^2)/(2*d) - (f*Tan[c + d*x])/(2*d^2))/a) + ((((-
2*I)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/d + (I*f*PolyLog[2, (-I)*E^(I*(c +
 d*x))])/d^2 - (I*f*PolyLog[2, I*E^(I*(c + d*x))])/d^2)/2 - (f*Sec[c + d*x
])/(2*d^2) + ((e + f*x)*Sec[c + d*x]*Tan[c + d*x])/(2*d))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{i\left(dfx\,\mathrm{e}^{i(dx+c)}+de\,\mathrm{e}^{i(dx+c)}+f-if\,\mathrm{e}^{i(dx+c)}\right)}{d^{2}\left(\mathrm{e}^{i(dx+c)}+i\right)^{2}a}-\frac{ie\arctan\left(\mathrm{e}^{i(dx+c)}\right)}{da}+\frac{f\ln\left(1-i\mathrm{e}^{i(dx+c)}\right)x}{2da}+\frac{f\ln\left(1-i\mathrm{e}^{i(dx+c)}\right)c}{2d^{2}a}-\frac{if\,\mathrm{polyl}}{}$

input
```
int((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I*(d*f*x*exp(I*(d*x+c))+d*e*exp(I*(d*x+c))+f-I*f*exp(I*(d*x+c)))/d^2/(exp
(I*(d*x+c))+I)^2/a-I/d/a*e*arctan(exp(I*(d*x+c)))+1/2/d/a*f*ln(1-I*exp(I*(
d*x+c)))*x+1/2/d^2/a*f*ln(1-I*exp(I*(d*x+c)))*c-1/2*I*f*polylog(2,I*exp(I*
(d*x+c)))/a/d^2-1/2/d/a*f*ln(1+I*exp(I*(d*x+c)))*x-1/2/d^2/a*f*ln(1+I*exp(
I*(d*x+c)))*c+1/2*I*f*polylog(2,-I*exp(I*(d*x+c)))/a/d^2+I/d^2/a*f*c*arcta
n(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(144) = 288.

Time = 0.11 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.95

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\,dfx+2\,de+2\,f\cos\left(dx+c\right)-\left(-i\,f\sin\left(dx+c\right)-i\,f\right)\mathrm{Li}_2(i\,\cos\left(dx+c\right)+\sin\left(dx+c\right))-\left(-i\,f\sin\right.}{}$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(2*d*f*x + 2*d*e + 2*f*cos(d*x + c) - (-I*f*sin(d*x + c) - I*f)*dilog
(I*cos(d*x + c) + sin(d*x + c)) - (-I*f*sin(d*x + c) - I*f)*dilog(I*cos(d*
x + c) - sin(d*x + c)) - (I*f*sin(d*x + c) + I*f)*dilog(-I*cos(d*x + c) +
sin(d*x + c)) - (I*f*sin(d*x + c) + I*f)*dilog(-I*cos(d*x + c) - sin(d*x +
 c)) - (d*e - c*f + (d*e - c*f)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x
 + c) + I) + (d*e - c*f + (d*e - c*f)*sin(d*x + c))*log(cos(d*x + c) - I*s
in(d*x + c) + I) - (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*log(I*cos(d*
x + c) + sin(d*x + c) + 1) + (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*lo
g(I*cos(d*x + c) - sin(d*x + c) + 1) - (d*f*x + c*f + (d*f*x + c*f)*sin(d*
x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d*f*x + c*f + (d*f*x +
c*f)*sin(d*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - (d*e - c*f +
(d*e - c*f)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (d*e -
 c*f + (d*e - c*f)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I))/
(a*d^2*sin(d*x + c) + a*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e\sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d*x
)/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(144) = 288$.

Time = 0.17 (sec) , antiderivative size = 725, normalized size of antiderivative = 4.22

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(2*(d*e*cos(2*d*x + 2*c) + 2*I*d*e*cos(d*x + c) + I*d*e*sin(2*d*x + 2*c) -
 2*d*e*sin(d*x + c) - d*e)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 2*(d*
e*cos(2*d*x + 2*c) + 2*I*d*e*cos(d*x + c) + I*d*e*sin(2*d*x + 2*c) - 2*d*e
*sin(d*x + c) - d*e)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) - 2*(d*f*x*co
s(2*d*x + 2*c) + 2*I*d*f*x*cos(d*x + c) + I*d*f*x*sin(2*d*x + 2*c) - 2*d*f
*x*sin(d*x + c) - d*f*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*(d*f*
x*cos(2*d*x + 2*c) + 2*I*d*f*x*cos(d*x + c) + I*d*f*x*sin(2*d*x + 2*c) - 2
*d*f*x*sin(d*x + c) - d*f*x)*arctan2(cos(d*x + c), -sin(d*x + c) + 1) - 4*
(d*f*x + d*e - I*f)*cos(d*x + c) - 2*(f*cos(2*d*x + 2*c) + 2*I*f*cos(d*x +
 c) + I*f*sin(2*d*x + 2*c) - 2*f*sin(d*x + c) - f)*dilog(I*e^(I*d*x + I*c)
) + 2*(f*cos(2*d*x + 2*c) + 2*I*f*cos(d*x + c) + I*f*sin(2*d*x + 2*c) - 2*
f*sin(d*x + c) - f)*dilog(-I*e^(I*d*x + I*c)) + (I*d*f*x + I*d*e + (-I*d*f
*x - I*d*e)*cos(2*d*x + 2*c) + 2*(d*f*x + d*e)*cos(d*x + c) + (d*f*x + d*e
)*sin(2*d*x + 2*c) - 2*(-I*d*f*x - I*d*e)*sin(d*x + c))*log(cos(d*x + c)^2
 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (-I*d*f*x - I*d*e + (I*d*f*x + I
*d*e)*cos(2*d*x + 2*c) - 2*(d*f*x + d*e)*cos(d*x + c) - (d*f*x + d*e)*sin(
2*d*x + 2*c) - 2*(I*d*f*x + I*d*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(
d*x + c)^2 - 2*sin(d*x + c) + 1) - 4*(I*d*f*x + I*d*e + f)*sin(d*x + c) -
4*f)/(-4*I*a*d^2*cos(2*d*x + 2*c) + 8*a*d^2*cos(d*x + c) + 4*a*d^2*sin(2*d
*x + 2*c) + 8*I*a*d^2*sin(d*x + c) + 4*I*a*d^2)
```

Giac [F]

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)\sec\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Hanged}$$

input
```
int((e + f*x)/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{-2\cos\left(dx+c\right)dfx-2\cos\left(dx+c\right)f+2\left(\int \frac{\sin(dx+c)x}{\sin(dx+c)+1}dx\right)\sin\left(dx+c\right)d^2f+2\left(\int \frac{\sin(dx+c)x}{\sin(dx+c)+1}dx\right)d^2f+}{}$$

input
```
int((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*d*f*x - 2*cos(c + d*x)*f + 2*int((sin(c + d*x)*x)/(sin(
c + d*x) + 1),x)*sin(c + d*x)*d**2*f + 2*int((sin(c + d*x)*x)/(sin(c + d*x
) + 1),x)*d**2*f + 2*int((sin(c + d*x)*x)/(cos(c + d*x)*sin(c + d*x) + cos
(c + d*x)),x)*sin(c + d*x)*d**2*f + 2*int((sin(c + d*x)*x)/(cos(c + d*x)*s
in(c + d*x) + cos(c + d*x)),x)*d**2*f - 2*log(tan((c + d*x)/2)**2 + 1)*sin
(c + d*x)*f - 2*log(tan((c + d*x)/2)**2 + 1)*f - log(tan((c + d*x)/2) - 1)
*sin(c + d*x)*d*e - log(tan((c + d*x)/2) - 1)*d*e + log(tan((c + d*x)/2) +
 1)*sin(c + d*x)*d*e + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*f + log(ta
n((c + d*x)/2) + 1)*d*e + 4*log(tan((c + d*x)/2) + 1)*f - sin(c + d*x)*d**
2*f*x**2 - d**2*f*x**2 - d*e - 2*d*f*x)/(2*a*d**2*(sin(c + d*x) + 1))
```

3.272 $\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{1}{2d(a+a\sin(c+dx))}$$

output
```
1/2*arctanh(sin(d*x+c))/a/d-1/2/d/(a+a*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{1+\sin(c+dx)}}{2ad}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
(ArcTanh[Sin[c + d*x]] - (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3146}$$

$$\frac{a\int \frac{1}{(a-a\sin(c+dx))(\sin(c+dx)a+a)^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{a\int \left(\frac{1}{2(a^2-a^2\sin^2(c+dx))a}+\frac{1}{2(\sin(c+dx)a+a)^2a}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{2a^2}-\frac{1}{2a(a\sin(c+dx)+a)}\right)}{d}$$

input
```
Int[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
(a*(ArcTanh[Sin[c + d*x]]/(2*a^2) - 1/(2*a*(a + a*Sin[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}-\frac{1}{2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4}}{da}$	43
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}-\frac{1}{2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4}}{da}$	43
parallelrisch	$\frac{(-1-\sin(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\sin(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\sin(dx+c)}{2a(1+\sin(dx+c))d}$	70
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$	71
risch	$-\frac{i\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{i(dx+c)}+i\right)^{2}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$	76

input
```
int(sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/4*ln(sin(d*x+c)-1)-1/2/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(\sin(dx+c)+1)\log(-\sin(dx+c)+1)-2}{4\,(ad\sin(dx+c)+ad)}$$

input
```
integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-si
n(d*x + c) + 1) - 2)/(a*d*sin(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} - \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a - 2/(a*sin(d*x + c)
 + a))/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{4\,ad} - \frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{4\,ad} - \frac{1}{2\,ad(\sin\left(dx+c\right)+1)}$$

input
```
integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*log(abs(sin(d*x + c) + 1))/(a*d) - 1/4*log(abs(sin(d*x + c) - 1))/(a*d
) - 1/2/(a*d*(sin(d*x + c) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\mathrm{atanh}(\sin\left(c+d\,x\right))}{2\,a\,d} - \frac{1}{2\,d\,\left(a+a\,\sin\left(c+d\,x\right)\right)}$$

input
```
int(1/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
atanh(sin(c + d*x))/(2*a*d) - 1/(2*d*(a + a*sin(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.30

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sin\left(dx+c\right)+\log}{2ad\left(\sin\left(dx+c\right)+1\right)}$$

input
```
int(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*sin(c + d*x) - log(tan((c + d*x)/2) - 1) + l
og(tan((c + d*x)/2) + 1)*sin(c + d*x) + log(tan((c + d*x)/2) + 1) - 1)/(2*
a*d*(sin(c + d*x) + 1))
```

3.273 $\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 17.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \ 5048$$
$$\int \frac{\sec(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sec (dx+c)}{(fx+e)(a+a\sin (dx+c))}dx$$

input
```
int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\sec (dx+c)}{(fx+e)(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\frac{\int \frac{\sec (c+dx)}{e\sin (c+dx)+e+fx\sin (c+dx)+fx}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 1504, normalized size of antiderivative = 57.85

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*(d*f*x + d*e)*cos(d*x + c)^2 + 2*(d*f*x + d*e)*sin(d*x + c)^2 - (f*cos
(d*x + c) + (d*f*x + d*e)*sin(d*x + c))*cos(2*d*x + 2*c) - f*cos(d*x + c)
 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*
f*x + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a
*d^2*e^2)*cos(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*c
os(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)
*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*
x + c)^2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + 2*(a*d^2*f^2*x^2
 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^
2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*integrate(1/2*(d^2*f^2*x^
2 + 2*d^2*e*f*x + d^2*e^2 + 4*f^2)*cos(d*x + c)/(a*d^2*f^3*x^3 + 3*a*d^2*e
*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^
2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2
*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 + 2*(a*d^2*f^3*x^
3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)), x) - (
a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
 + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^
2*e^2)*cos(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(
d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*si
n(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*...
```

Giac [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec\left(dx+c\right)}{(fx+e)(a\sin\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 37.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cos(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)*(e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 5.08

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\left(\int \frac{\sin(dx+c)}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx\right)f+\left(\int \frac{1}{\cos(dx+c)\sin(dx+c)e+\cos(dx+c)\sin(dx+c)fx+\cos(dx+c)e+\cos(dx+c)fx}dx\right)f+}{af}$$

input
```
int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
(int(sin(c + d*x)/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)*f + int
(1/(cos(c + d*x)*sin(c + d*x)*e + cos(c + d*x)*sin(c + d*x)*f*x + cos(c +
d*x)*e + cos(c + d*x)*f*x),x)*f + int(1/(sin(c + d*x)*e + sin(c + d*x)*f*x
 + e + f*x),x)*f - log(e + f*x))/(a*f)
```

3.274 $\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \operatorname{Int}\left(\frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 24.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)} \, dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sec (dx+c)}{(fx+e)^2 (a+a\sin (dx+c))}dx$$

input
```
int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec (dx+c)}{(fx+e)^2(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*
f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 8.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sec (c+dx)}{e^2\sin (c+dx)+e^2+2efx\sin (c+dx)+2efx+f^2x^2\sin (c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2
*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 2018, normalized size of antiderivative = 77.62

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*(d*f*x + d*e)*cos(d*x + c)^2 + 2*(d*f*x + d*e)*sin(d*x + c)^2 - (2*f*c
os(d*x + c) + (d*f*x + d*e)*sin(d*x + c))*cos(2*d*x + 2*c) - 2*f*cos(d*x +
 c) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (
a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x
 + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2
*e^3)*cos(d*x + c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*
f*x + a*d^2*e^3)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*
e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3
*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2
*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d
^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)
)*cos(2*d*x + 2*c) + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*
x + a*d^2*e^3)*sin(d*x + c))*integrate(1/2*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^
2*e^2 + 12*f^2)*cos(d*x + c)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*
e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3
*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*cos(d*x + c)^2 +
 (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*
x + a*d^2*e^4)*sin(d*x + c)^2 + 2*(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a
*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*sin(d*x + c)), x) - (a*d^2
*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3...
```

Giac [N/A]

Not integrable

Time = 11.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 38.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)\,(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(cos(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23269, normalized size of antiderivative = 894.96

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output

```
( - 18*cos(c + d*x)*int(tan((c + d*x)/2)**3/(tan((c + d*x)/2)**5*e**2 + 2*
tan((c + d*x)/2)**5*e*f*x + tan((c + d*x)/2)**5*f**2*x**2 + tan((c + d*x)/
2)**4*e**2 + 2*tan((c + d*x)/2)**4*e*f*x + tan((c + d*x)/2)**4*f**2*x**2 -
 2*tan((c + d*x)/2)**3*e**2 - 4*tan((c + d*x)/2)**3*e*f*x - 2*tan((c + d*x
)/2)**3*f**2*x**2 - 2*tan((c + d*x)/2)**2*e**2 - 4*tan((c + d*x)/2)**2*e*f
*x - 2*tan((c + d*x)/2)**2*f**2*x**2 + tan((c + d*x)/2)*e**2 + 2*tan((c +
d*x)/2)*e*f*x + tan((c + d*x)/2)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x**2),x
)*sin(c + d*x)*e**2*f**2 - 18*cos(c + d*x)*int(tan((c + d*x)/2)**3/(tan((c
 + d*x)/2)**5*e**2 + 2*tan((c + d*x)/2)**5*e*f*x + tan((c + d*x)/2)**5*f**
2*x**2 + tan((c + d*x)/2)**4*e**2 + 2*tan((c + d*x)/2)**4*e*f*x + tan((c +
 d*x)/2)**4*f**2*x**2 - 2*tan((c + d*x)/2)**3*e**2 - 4*tan((c + d*x)/2)**3
*e*f*x - 2*tan((c + d*x)/2)**3*f**2*x**2 - 2*tan((c + d*x)/2)**2*e**2 - 4*
tan((c + d*x)/2)**2*e*f*x - 2*tan((c + d*x)/2)**2*f**2*x**2 + tan((c + d*x
)/2)*e**2 + 2*tan((c + d*x)/2)*e*f*x + tan((c + d*x)/2)*f**2*x**2 + e**2 +
 2*e*f*x + f**2*x**2),x)*sin(c + d*x)*e*f**3*x - 18*cos(c + d*x)*int(tan((
c + d*x)/2)**3/(tan((c + d*x)/2)**5*e**2 + 2*tan((c + d*x)/2)**5*e*f*x + t
an((c + d*x)/2)**5*f**2*x**2 + tan((c + d*x)/2)**4*e**2 + 2*tan((c + d*x)/
2)**4*e*f*x + tan((c + d*x)/2)**4*f**2*x**2 - 2*tan((c + d*x)/2)**3*e**2 -
 4*tan((c + d*x)/2)**3*e*f*x - 2*tan((c + d*x)/2)**3*f**2*x**2 - 2*tan((c
+ d*x)/2)**2*e**2 - 4*tan((c + d*x)/2)**2*e*f*x - 2*tan((c + d*x)/2)**2...
```

3.275 $\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 475

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2i(e+fx)^3}{3ad}-\frac{if(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{ad^2}+\frac{f^3\text{arctanh}(\sin(c+dx))}{ad^4}+\frac{2f(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{ad^2}+\frac{f^3\log(\cos(c+dx))}{ad^4}+\frac{if^2(e+fx)\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2(e+fx)\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2if^2(e+fx)\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{ad^3}-\frac{f^3\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^4}+\frac{f^3\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{f^3\,\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)}{ad^4}-\frac{f^2(e+fx)\sec(c+dx)}{ad^3}-\frac{f(e+fx)^2\sec^2(c+dx)}{2ad^2}-\frac{(e+fx)^3\sec^3(c+dx)}{3ad}+\frac{f^2(e+fx)\tan(c+dx)}{ad^3}+\frac{2(e+fx)^3\tan(c+dx)}{3ad}+\frac{f(e+fx)^2\sec(c+dx)\tan(c+dx)}{2ad^2}+\frac{(e+fx)^3\sec^2(c+dx)\tan(c+dx)}{3ad}$$

output

```
-I*f*(f*x+e)^2*arctan(exp(I*(d*x+c)))/a/d^2+I*f^2*(f*x+e)*polylog(2,-I*exp
(I*(d*x+c)))/a/d^3+f^3*arctanh(sin(d*x+c))/a/d^4+2*f*(f*x+e)^2*ln(1+exp(2*
I*(d*x+c)))/a/d^2+f^3*ln(cos(d*x+c))/a/d^4-2*I*f^2*(f*x+e)*polylog(2,-exp(
2*I*(d*x+c)))/a/d^3-I*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/a/d^3-2/3*I*
(f*x+e)^3/a/d-f^3*polylog(3,-I*exp(I*(d*x+c)))/a/d^4+f^3*polylog(3,I*exp(I
*(d*x+c)))/a/d^4+f^3*polylog(3,-exp(2*I*(d*x+c)))/a/d^4-f^2*(f*x+e)*sec(d*
x+c)/a/d^3-1/2*f*(f*x+e)^2*sec(d*x+c)^2/a/d^2-1/3*(f*x+e)^3*sec(d*x+c)^3/a
/d+f^2*(f*x+e)*tan(d*x+c)/a/d^3+2/3*(f*x+e)^3*tan(d*x+c)/a/d+1/2*f*(f*x+e)
^2*sec(d*x+c)*tan(d*x+c)/a/d^2+1/3*(f*x+e)^3*sec(d*x+c)^2*tan(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1173 vs. $2(475) = 950$.

Time = 10.16 (sec) , antiderivative size = 1173, normalized size of antiderivative = 2.47

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
((d^3*(e + f*x)^3)/(-I + E^(I*c)) + 3*d^2*f*(e + f*x)^2*Log[1 - I/E^(I*(c
+ d*x))] + 6*f^2*(I*d*(e + f*x)*PolyLog[2, I/E^(I*(c + d*x))] + f*PolyLog[
3, I/E^(I*(c + d*x))]))/(2*a*d^4) - (f*(Cos[c] + I*Sin[c])*(5*d^2*e^2*x*Co
s[c] + 4*f^2*x*Cos[c] + 5*d^2*e*f*x^2*Cos[c] + (5*d^2*f^2*x^3*(Cos[c] - I*
Sin[c]))/3 - (5*I)*d^2*e^2*x*Sin[c] - (4*I)*f^2*x*Sin[c] - (5*I)*d^2*e*f*x
^2*Sin[c] + 10*e*f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] -
I*(1 + Sin[c])) + 10*f^2*x*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(C
os[c] - I*(1 + Sin[c])) - 10*d*e*f*x*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]
]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) - 5*d*f^2*x^2*Log[1 + I*Co
s[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) -
 ((5*d^2*e^2 + 4*f^2)*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])]*(Cos[c] - I
*Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d - (10*f^2*PolyLog[3, (-I)*Cos[c + d*
x] - Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d + (5*d
^2*e^2 + 4*f^2)*x*(I*Cos[c] + Sin[c])*(Cos[c] + I*(1 + Sin[c]))))/(2*a*d^3
*(Cos[c] + I*(1 + Sin[c]))) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] +
 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/(2*a*d*(Cos[c/2] - Sin[c
/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (e^3*Sin[(d*x)/2] + 3*e^
2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/(3*a
*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (-
(d*e^3*Cos[c/2]) - 3*e^2*f*Cos[c/2] - 3*d*e^2*f*x*Cos[c/2] - 6*e*f^2*x*...
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5042, 3042, 4674, 3042, 4672, 25, 3042, 3956, 4202, 2620, 3011, 2720, 4909, 3042, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)^3\sec^4(c+dx)dx}{a}-\frac{\int (e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{a}-\frac{\int (e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{f^2\int (e+fx)\sec^2(c+dx)dx}{d^2}+\frac{2}{3}\int (e+fx)^3\sec^2(c+dx)dx-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{d^2}+\frac{2}{3}\int (e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{f^2\left(\frac{f\int -\tan(c+dx)dx}{d}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{3f\int -(e+fx)^2\tan(c+dx)dx}{d}+\frac{(e+fx)^3\tan(c+dx)}{d}\right)-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}+(e+fx)^3}{a}-\frac{\int (e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow 25$$

$$\frac{\frac{f^2\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int\tan(c+dx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\int(e+fx)^2\tan(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(}{}}{a}$$

$$\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{f^2\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int\tan(c+dx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\int(e+fx)^2\tan(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(}{}}{a}$$

$$\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow 3956$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\int(e+fx)^2\tan(c+dx)dx}{d}\right)+\frac{f^2\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d^2}-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(}{}}{a}$$

$$\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow 4202$$

$$-\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{2i(c+dx)}(e+fx)^2}{1+e^{2i(c+dx)}}dx\right)}{d}\right)+\frac{f^2\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d^2}-\frac{f(e+fx)^2\sec^2(c+dx)}{2d^2}}{a}$$

$$\Big\downarrow 2620$$

$$-\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\int(e+fx)\log\left(1+e^{2i(c+dx)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)+\frac{f^2\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx}{}\right)}{d^2}}{a}$$

$$\Big\downarrow 3011$$

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{if\int\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right.$$

$$-\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}+$$

$$\overline{\qquad\qquad a}$$

$\downarrow$ 2720

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2}{\cdots}\right.\right.}{d}\right.$$

$$-\frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)dx}{a}+$$

$$\overline{\qquad\qquad a}$$

$\downarrow$ 4909

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2}{\cdots}\right.\right.}{d}\right.$$

$$-\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\int(e+fx)^2\sec^3(c+dx)dx}{d}}{a}+$$

$$\overline{\qquad\qquad a}$$

$\downarrow$ 3042

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2}{\cdots}\right.\right.}{d}\right.$$

$$-\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\int(e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{d}}{a}+$$

$$\overline{\qquad\qquad a}$$

$$\Big\downarrow \; 4674$$

$$-\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{f^2\int \sec(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\sec(c+dx)dx-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}\right)}{d}}{a}+\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\cdots}{\cdots}\cdots\right)\cdots\right)}{d}\cdots\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{f^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}\right)}{d}}{a}+\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\cdots}{\cdots}\cdots\right)\cdots\right)}{d}\cdots\right)}{a}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{f^2\operatorname{arctanh}(\sin(c+dx))}{d^3}-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}\right)}{d}}{a}+\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\cdots}{\cdots}\cdots\right)\cdots\right)}{d}\cdots\right)}{a}$$

$$\Big\downarrow \; 4669$$

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2}{\cdots}\cdots\right.\right.}{d}\cdots\right.$$

$$\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\left(-\frac{2f\int(e+fx)\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{2f\int(e+fx)\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}\right)+\frac{f^2\operatorname{arctanh}(\sin(\cdots}{d^3}\cdots\right.}{d}\cdots}{a}$$

$\Big\downarrow$ 3011

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2}{\cdots}\cdots\right.\right.}{d}\cdots\right.$$

$$\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{if\int\operatorname{PolyL\cdots}}{\cdots}\right.}{d}\cdots\right.\right.}{\cdots}\cdots}{a}$$

$\Big\downarrow$ 2720

$$\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\int e^{-2i(c+dx)}\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2}{\cdots}\cdots\right.\right.}{d}\cdots\right.$$

$$\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+\cdots}\right.}{d}\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}\cdots}{a}$$

$\Big\downarrow$ 7143

$$\frac{\frac{f^2\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tan(c+dx)}{d}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(3,-e^{2i}\right.}{4d^2}\right.}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{\frac{(e+fx)^3\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\left(-\frac{2i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLo}}{}\right.}{}\right.\right.}{d}}{a}$$

input
```
Int[((e + f*x)^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(-1/2*(f*(e + f*x)^2*Sec[c + d*x]^2)/d^2 + ((e + f*x)^3*Sec[c + d*x]^2*Tan
[c + d*x])/(3*d) + (f^2*((f*Log[Cos[c + d*x]])/d^2 + ((e + f*x)*Tan[c + d*
x])/d))/d^2 + (2*((-3*f*(((I/3)*(e + f*x)^3)/f - (2*I)*(((-1/2*I)*(e + f*x
)^2*Log[1 + E^((2*I)*(c + d*x))])/d + (I*f*(((I/2)*(e + f*x)*PolyLog[2, -E
^((2*I)*(c + d*x))])/d - (f*PolyLog[3, -E^((2*I)*(c + d*x))])/(4*d^2)))/d)
))/d + ((e + f*x)^3*Tan[c + d*x])/d))/3)/a - (((e + f*x)^3*Sec[c + d*x]^3)
/(3*d) - (f*((f^2*ArcTanh[Sin[c + d*x]])/d^3 + (((-2*I)*(e + f*x)^2*ArcTan
[E^(I*(c + d*x))])/d + (2*f*((I*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))]
)/d - (f*PolyLog[3, (-I)*E^(I*(c + d*x))])/d^2))/d - (2*f*((I*(e + f*x)*Po
lyLog[2, I*E^(I*(c + d*x))])/d - (f*PolyLog[3, I*E^(I*(c + d*x))])/d^2))/d
)/2 - (f*(e + f*x)*Sec[c + d*x])/d^2 + ((e + f*x)^2*Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/d)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(438) = 876$.

Time = 2.73 (sec) , antiderivative size = 1135, normalized size of antiderivative = 2.39

method	result	size
risch	Expression too large to display	1135

input
```
int((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/a/d^2*e*f^2*ln(1+I*exp(I*(d*x+c)))*x+3/a/d^3*e*f^2*ln(1+I*exp(I*(d*x+c))
)*c+4*I/a/d^3*c^2*f^3*x-4*I/a/d*e*f^2*x^2-I/a/d^2*e^2*f*arctan(exp(I*(d*x+
c)))-4*I/a/d^3*c^2*e*f^2-I/a/d^4*f^3*c^2*arctan(exp(I*(d*x+c)))-5*I/a/d^3*
e*f^2*polylog(2,I*exp(I*(d*x+c)))-3*I/a/d^3*e*f^2*polylog(2,-I*exp(I*(d*x+
c)))-5*I/a/d^3*f^3*polylog(2,I*exp(I*(d*x+c)))*x-3*I/a/d^3*f^3*polylog(2,-
I*exp(I*(d*x+c)))*x-3/2/a/d^4*c^2*f^3*ln(1+I*exp(I*(d*x+c)))-8*I/a/d^2*e*f
^2*c*x+2*I/a/d^3*e*f^2*c*arctan(exp(I*(d*x+c)))-4/3*I/a/d*x^3*f^3-5/2/d^4/
a*c^2*f^3*ln(1-I*exp(I*(d*x+c)))-4/d^2/a*e^2*f*ln(exp(I*(d*x+c)))-4/d^4/a*
c^2*f^3*ln(exp(I*(d*x+c)))+2/d^4/a*c^2*f^3*ln(exp(2*I*(d*x+c))+1)+5/2/d^2/
a*f^3*ln(1-I*exp(I*(d*x+c)))*x^2+5*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+8
/3*I/a/d^4*c^3*f^3-2*I/a/d^4*f^3*arctan(exp(I*(d*x+c)))+5/d^2/a*e*f^2*ln(1
-I*exp(I*(d*x+c)))*x+3*f^3*polylog(3,-I*exp(I*(d*x+c)))/a/d^4-2/a/d^4*f^3*
ln(exp(I*(d*x+c)))+1/a/d^4*f^3*ln(exp(2*I*(d*x+c))+1)+5/d^3/a*e*f^2*ln(1-I
*exp(I*(d*x+c)))*c-4/d^3/a*c*e*f^2*ln(exp(2*I*(d*x+c))+1)+8/d^3/a*c*f^2*e*
ln(exp(I*(d*x+c)))+2/d^2/a*e^2*f*ln(exp(2*I*(d*x+c))+1)-1/3*(8*d^2*f^3*x^3
*exp(I*(d*x+c))+6*I*f^3*x*exp(2*I*(d*x+c))+12*I*d^2*e^2*f*x+3*I*d*f^3*x^2*
exp(3*I*(d*x+c))+6*f^3*x*exp(3*I*(d*x+c))+24*d^2*e*f^2*x^2*exp(I*(d*x+c))+
24*d^2*e^2*f*x*exp(I*(d*x+c))+3*I*d*e^2*f*exp(I*(d*x+c))+12*I*d^2*e*f^2*x^
2+6*I*d*e*f^2*x*exp(3*I*(d*x+c))+6*I*d*e*f^2*x*exp(I*(d*x+c))+6*I*f^3*x+8*
d^2*e^3*exp(I*(d*x+c))+3*I*d*f^3*x^2*exp(I*(d*x+c))+4*I*d^2*e^3+3*I*d*e...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1531 vs. $2(426) = 852$.

Time = 0.16 (sec) , antiderivative size = 1531, normalized size of antiderivative = 3.22

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*d^3*f^3*x^3 + 12*d^3*e*f^2*x^2 + 12*d^3*e^2*f*x + 4*d^3*e^3 - 4*(2
*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 2*d^3*e^3 + 3*d*e*f^2 + 3*(2*d^3*e^2*f +
d*f^3)*x)*cos(d*x + c)^2 - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*cos
(d*x + c) - 18*((-I*d*f^3*x - I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (-I*d
*f^3*x - I*d*e*f^2)*cos(d*x + c))*dilog(I*cos(d*x + c) + sin(d*x + c)) - 3
0*((I*d*f^3*x + I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (I*d*f^3*x + I*d*e*
f^2)*cos(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 18*((I*d*f^3*x +
 I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (I*d*f^3*x + I*d*e*f^2)*cos(d*x +
c))*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 30*((-I*d*f^3*x - I*d*e*f^2)*c
os(d*x + c)*sin(d*x + c) + (-I*d*f^3*x - I*d*e*f^2)*cos(d*x + c))*dilog(-I
*cos(d*x + c) - sin(d*x + c)) + 3*((5*d^2*e^2*f - 10*c*d*e*f^2 + (5*c^2 +
4)*f^3)*cos(d*x + c)*sin(d*x + c) + (5*d^2*e^2*f - 10*c*d*e*f^2 + (5*c^2 +
 4)*f^3)*cos(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + 9*((d^2*e^
2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)*sin(d*x + c) + (d^2*e^2*f - 2*c*
d*e*f^2 + c^2*f^3)*cos(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + I) +
15*((d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)*sin
(d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x
+ c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) + 9*((d^2*f^3*x^2 + 2*d^2*e*f
^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)*sin(d*x + c) + (d^2*f^3*x^2 + 2
*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c))*log(I*cos(d*x + c) ...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3
*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d
*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)**2/(sin(c
 + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5130 vs. $2(426) = 852$.

Time = 0.99 (sec) , antiderivative size = 5130, normalized size of antiderivative = 10.80

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(24*c^2*e*f^2*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d^2 + 2*
a*d^2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*d^2*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - a*d^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 6*(4*(8*(d*x + c)
*cos(d*x + c) - sin(3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*
d*x + 4*(d*x + c)*sin(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*
cos(3*d*x + 3*c)^2 + 8*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d
*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2
- 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c)
 + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c)
 + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d
*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3
*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x
 + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos
(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4
*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*
c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x
 + c)^2 - 2*sin(d*x + c) + 1) + 4*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c
+ cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(
d*x + c) - 4*sin(d*x + c) - 1)*sin(3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 ...
```

Giac [F]

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^3\sec\left(dx+c\right)^2}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output

```
(576*cos(c + d*x)*int(x**2/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5 -
tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((c + d*x)/2)**2 + 2*tan(
(c + d*x)/2) + 1),x)*sin(c + d*x)*d**3*f**3 + 576*cos(c + d*x)*int(x**2/(t
an((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5 - tan((c + d*x)/2)**4 - 4*tan((
c + d*x)/2)**3 - tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**3*f**
3 - 1296*cos(c + d*x)*int((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**6 + 2
*tan((c + d*x)/2)**5 - tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((
c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*sin(c + d*x)*d**3*f**3 - 1296*
cos(c + d*x)*int((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**6 + 2*tan((c +
 d*x)/2)**5 - tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((c + d*x)/
2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**3*f**3 - 2592*cos(c + d*x)*int((tan(
(c + d*x)/2)*x)/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5 - tan((c + d*
x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2
) + 1),x)*sin(c + d*x)*d**3*e*f**2 + 8208*cos(c + d*x)*int((tan((c + d*x)/
2)*x)/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5 - tan((c + d*x)/2)**4 -
 4*tan((c + d*x)/2)**3 - tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*
sin(c + d*x)*d**2*f**3 - 2592*cos(c + d*x)*int((tan((c + d*x)/2)*x)/(tan((
c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5 - tan((c + d*x)/2)**4 - 4*tan((c +
d*x)/2)**3 - tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*d**3*e*f**2
+ 8208*cos(c + d*x)*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**6 + 2*t...
```

3.276 $\int \frac{(e+fx)^2 \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 343

$$\begin{aligned}\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx =& -\frac{2i(e+fx)^2}{3ad}-\frac{2if(e+fx)\arctan\left(e^{i(c+dx)}\right)}{3ad^2}\\ &+\frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}\\ &+\frac{if^2\,\mathrm{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{3ad^3}-\frac{if^2\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{3ad^3}\\ &-\frac{2if^2\,\mathrm{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{3ad^3}-\frac{f^2\sec(c+dx)}{3ad^3}\\ &-\frac{f(e+fx)\sec^2(c+dx)}{3ad^2}-\frac{(e+fx)^2\sec^3(c+dx)}{3ad}\\ &+\frac{f^2\tan(c+dx)}{3ad^3}+\frac{2(e+fx)^2\tan(c+dx)}{3ad}\\ &+\frac{f(e+fx)\sec(c+dx)\tan(c+dx)}{3ad^2}\\ &+\frac{(e+fx)^2\sec^2(c+dx)\tan(c+dx)}{3ad}\end{aligned}$$

output
```
-2/3*I*(f*x+e)^2/a/d-2/3*I*f*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d^2+4/3*f*(f
*x+e)*ln(1+exp(2*I*(d*x+c)))/a/d^2+1/3*I*f^2*polylog(2,-I*exp(I*(d*x+c)))/
a/d^3-1/3*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3-2/3*I*f^2*polylog(2,-exp
(2*I*(d*x+c)))/a/d^3-1/3*f^2*sec(d*x+c)/a/d^3-1/3*f*(f*x+e)*sec(d*x+c)^2/a
/d^2-1/3*(f*x+e)^2*sec(d*x+c)^3/a/d+1/3*f^2*tan(d*x+c)/a/d^3+2/3*(f*x+e)^2
*tan(d*x+c)/a/d+1/3*f*(f*x+e)*sec(d*x+c)*tan(d*x+c)/a/d^2+1/3*(f*x+e)^2*se
c(d*x+c)^2*tan(d*x+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.26 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.86

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{12d^2f\left(\frac{f\,\mathrm{PolyLog}(2,i\cos(c+dx)+\sin(c+dx))(\cos(c)-i(-1+\sin(c)))}{d^2}+\frac{(e+fx)\log(1-i\cos(c+dx)-\sin(c+dx))(1-i\cos(c)-\sin(c))}{d}+\frac{(e+fx)^2(\cos(c)-i\sin(c))}{2f}\right)(\cdots}{\cos(c)+i(-1+\sin(c))}\cdots}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
((12*d^2*f*((f*PolyLog[2, I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*(-1 +
 Sin[c])))/d^2 + ((e + f*x)*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(1 - I*
Cos[c] - Sin[c]))/d + ((e + f*x)^2*(Cos[c] - I*Sin[c]))/(2*f))*(Cos[c] + I
*Sin[c]))/(Cos[c] + I*(-1 + Sin[c])) - (20*d^2*f*(Cos[c] + I*Sin[c])*(((e
+ f*x)^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] +
Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x]
 - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(Cos[c] + I*(1 + Sin[c])
) + (-2*f^2*Cos[c] - 2*d*f*(e + f*x)*Cos[d*x] + 2*d^2*e^2*Cos[c + d*x] + 4
*f^2*Cos[c + d*x] + 4*d^2*e*f*x*Cos[c + d*x] + 2*d^2*f^2*x^2*Cos[c + d*x]
- 2*d*e*f*Cos[2*c + d*x] - 2*d*f^2*x*Cos[2*c + d*x] - 4*d^2*e^2*Cos[c + 2*
d*x] - 2*f^2*Cos[c + 2*d*x] - 8*d^2*e*f*x*Cos[c + 2*d*x] - 4*d^2*f^2*x^2*C
os[c + 2*d*x] + 8*d^2*e^2*Sin[d*x] + 2*f^2*Sin[d*x] + 16*d^2*e*f*x*Sin[d*x
] + 8*d^2*f^2*x^2*Sin[d*x] + d^2*e^2*Sin[2*(c + d*x)] + 2*f^2*Sin[2*(c + d
*x)] + 2*d^2*e*f*x*Sin[2*(c + d*x)] + d^2*f^2*x^2*Sin[2*(c + d*x)] - 2*f^2
*Sin[2*c + d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(12*a*
d^3)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5042, 3042, 4674, 3042, 4254, 24, 4672, 25, 3042, 4202, 2620, 2715, 2838, 4909, 3042, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)^2\sec^4(c+dx)dx}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{f^2\int \sec^2(c+dx)dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\sec^2(c+dx)dx-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{f^2\int 1d(-\tan(c+dx))}{3d^3}+\frac{2}{3}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{2}{3}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 4672

$$\frac{\frac{2}{3}\left(\frac{2f\int -((e+fx)\tan(c+dx))dx}{d}+\frac{(e+fx)^2\tan(c+dx)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 25

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\int (e+fx)\tan(c+dx)dx}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\int (e+fx)\tan(c+dx)dx}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 4202

$$-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{2i(c+dx)}(e+fx)}{1+e^{2i(c+dx)}}dx\right)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{a}$$

$\downarrow$ 2620

$$-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{2i(c+dx)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+}{3d^2}}{a}$$

$\downarrow$ 2715

$$-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-2i(c+dx)}\log\left(1+e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-}{a}$$

$\downarrow$ 2838

$$-\frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(}{3d^2}}{a}$$

$\downarrow$ 4909

$$-\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\int (e+fx)\sec^3(c+dx)dx}{3d}}{a}+$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(}{3d^2}}{a}$$

$\downarrow$ 3042

$$-\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}}{a}+$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(}{3d^2}}{a}$$

$\downarrow$ 4673

$$-\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\left(\frac{1}{2}\int (e+fx)\sec(c+dx)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)}{3d}}{a}+$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(}{3d^2}}{a}$$

$\downarrow$ 3042

$$-\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\left(\frac{1}{2}\int(e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)}{3d}}{a}+$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(c}{3d^2}}{a}$$

$$\Big\downarrow \; 4669$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(c}{3d^2}}{a}$$

$$\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\left(\frac{1}{2}\left(-\frac{f\int\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{f\int\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\,\text{se}}{2d}}{3d}}{a}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(c}{3d^2}}{a}$$

$$\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\left(\frac{1}{2}\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)}{3d}}{a}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)-\frac{f(e+fx)\sec^2(c}{3d^2}}{a}$$

$$\frac{\frac{(e+fx)^2\sec^3(c+dx)}{3d}-\frac{2f\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx}{2d}}{3d}}{a}$$

input
```
Int[((e + f*x)^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(-1/3*(f*(e + f*x)*Sec[c + d*x]^2)/d^2 + (f^2*Tan[c + d*x])/(3*d^3) + ((e
+ f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (2*((-2*f*(((I/2)*(e + f*x)^
2)/f - (2*I)*(((-1/2*I)*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/d - (f*Pol
yLog[2, -E^((2*I)*(c + d*x))])/(4*d^2))))/d + ((e + f*x)^2*Tan[c + d*x])/d
))/3)/a - (((e + f*x)^2*Sec[c + d*x]^3)/(3*d) - (2*f*((((-2*I)*(e + f*x)*A
rcTan[E^(I*(c + d*x))])/d + (I*f*PolyLog[2, (-I)*E^(I*(c + d*x))])/d^2 - (
I*f*PolyLog[2, I*E^(I*(c + d*x))])/d^2)/2 - (f*Sec[c + d*x])/(2*d^2) + ((e
 + f*x)*Sec[c + d*x]*Tan[c + d*x])/(2*d)))/(3*d))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{2\left(8\,\mathrm{e}^{i(dx+c)}d^2efx+idf^2x\,\mathrm{e}^{3i(dx+c)}+idef\,\mathrm{e}^{3i(dx+c)}+idf^2x\,\mathrm{e}^{i(dx+c)}+idef\,\mathrm{e}^{i(dx+c)}+4id^2efx+4\,\mathrm{e}^{i(dx+c)}d^2f^2x^2+2id^2x^2f^2+if}{3\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^3d^3a}$

input
```
int((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(8*exp(I*(d*x+c))*d^2*e*f*x+I*d*f^2*x*exp(3*I*(d*x+c))+I*d*e*f*exp(3*
I*(d*x+c))+I*d*f^2*x*exp(I*(d*x+c))+I*d*e*f*exp(I*(d*x+c))+4*I*d^2*e*f*x+4
*exp(I*(d*x+c))*d^2*f^2*x^2+2*I*d^2*x^2*f^2+I*f^2*exp(2*I*(d*x+c))+f^2*exp
(I*(d*x+c))+f^2*exp(3*I*(d*x+c))+4*exp(I*(d*x+c))*d^2*e^2+I*f^2+2*I*d^2*e^
2)/(exp(I*(d*x+c))-I)/(exp(I*(d*x+c))+I)^3/d^3/a+8/3/a/d^3*f^2*c*ln(exp(I*
(d*x+c)))-8/3/a/d^2*f*e*ln(exp(I*(d*x+c)))+2/3*I/d^3/a*f^2*c*arctan(exp(I*
(d*x+c)))-4/3/d^3/a*f^2*c*ln(exp(2*I*(d*x+c))+1)+5/3/a/d^2*f^2*ln(1-I*exp(
I*(d*x+c)))*x-8/3*I/d^2/a*c*f^2*x-I/d^3/a*f^2*polylog(2,-I*exp(I*(d*x+c)))
-4/3*I/d^3/a*f^2*c^2+5/3/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c+1/d^2/a*f^2*ln
(1+I*exp(I*(d*x+c)))*x-2/3*I/d^2/a*e*f*arctan(exp(I*(d*x+c)))+1/d^3/a*f^2*
ln(1+I*exp(I*(d*x+c)))*c+4/3/d^2/a*e*f*ln(exp(2*I*(d*x+c))+1)-5/3*I/d^3/a*
f^2*polylog(2,I*exp(I*(d*x+c)))-4/3*I/d/a*f^2*x^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(290) = 580$.

Time = 0.12 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.50

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 - 2*(2*d^2*f^2*x^2 + 4*d^2*e*
f*x + 2*d^2*e^2 + f^2)*cos(d*x + c)^2 - 2*(d*f^2*x + d*e*f)*cos(d*x + c) -
 3*(-I*f^2*cos(d*x + c)*sin(d*x + c) - I*f^2*cos(d*x + c))*dilog(I*cos(d*x
 + c) + sin(d*x + c)) - 5*(I*f^2*cos(d*x + c)*sin(d*x + c) + I*f^2*cos(d*x
 + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - 3*(I*f^2*cos(d*x + c)*sin(d*
x + c) + I*f^2*cos(d*x + c))*dilog(-I*cos(d*x + c) + sin(d*x + c)) - 5*(-I
*f^2*cos(d*x + c)*sin(d*x + c) - I*f^2*cos(d*x + c))*dilog(-I*cos(d*x + c)
 - sin(d*x + c)) + 5*((d*e*f - c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*e*f -
 c*f^2)*cos(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + 3*((d*e*f -
 c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*e*f - c*f^2)*cos(d*x + c))*log(cos(
d*x + c) - I*sin(d*x + c) + I) + 5*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x
 + c) + (d*f^2*x + c*f^2)*cos(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c)
+ 1) + 3*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*f^2*x + c*f^2)*
cos(d*x + c))*log(I*cos(d*x + c) - sin(d*x + c) + 1) + 5*((d*f^2*x + c*f^2
)*cos(d*x + c)*sin(d*x + c) + (d*f^2*x + c*f^2)*cos(d*x + c))*log(-I*cos(d
*x + c) + sin(d*x + c) + 1) + 3*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x +
c) + (d*f^2*x + c*f^2)*cos(d*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) +
1) + 5*((d*e*f - c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*e*f - c*f^2)*cos(d*
x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 3*((d*e*f - c*f^2)*cos(d
*x + c)*sin(d*x + c) + (d*e*f - c*f^2)*cos(d*x + c))*log(-cos(d*x + c) ...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2
*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)**2
/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(290) = 580$.

Time = 0.40 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.87

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(8*d^2*e^2 + 4*f^2*cos(2*d*x + 2*c) + 4*I*f^2*sin(2*d*x + 2*c) + 4*f^2 -
10*(d*e*f*cos(4*d*x + 4*c) + 2*I*d*e*f*cos(3*d*x + 3*c) + 2*I*d*e*f*cos(d*
x + c) + I*d*e*f*sin(4*d*x + 4*c) - 2*d*e*f*sin(3*d*x + 3*c) - 2*d*e*f*sin
(d*x + c) - d*e*f)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 6*(d*e*f*cos(
4*d*x + 4*c) + 2*I*d*e*f*cos(3*d*x + 3*c) + 2*I*d*e*f*cos(d*x + c) + I*d*e
*f*sin(4*d*x + 4*c) - 2*d*e*f*sin(3*d*x + 3*c) - 2*d*e*f*sin(d*x + c) - d*
e*f)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) + 10*(d*f^2*x*cos(4*d*x + 4*c
) + 2*I*d*f^2*x*cos(3*d*x + 3*c) + 2*I*d*f^2*x*cos(d*x + c) + I*d*f^2*x*si
n(4*d*x + 4*c) - 2*d*f^2*x*sin(3*d*x + 3*c) - 2*d*f^2*x*sin(d*x + c) - d*f
^2*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 6*(d*f^2*x*cos(4*d*x + 4*c
) + 2*I*d*f^2*x*cos(3*d*x + 3*c) + 2*I*d*f^2*x*cos(d*x + c) + I*d*f^2*x*si
n(4*d*x + 4*c) - 2*d*f^2*x*sin(3*d*x + 3*c) - 2*d*f^2*x*sin(d*x + c) - d*f
^2*x)*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 8*(d^2*f^2*x^2 + 2*d^2*e*
f*x)*cos(4*d*x + 4*c) + 4*(4*I*d^2*f^2*x^2 + d*e*f - I*f^2 + (8*I*d^2*e*f
+ d*f^2)*x)*cos(3*d*x + 3*c) + 4*(-4*I*d^2*e^2 + d*f^2*x + d*e*f - I*f^2)*
cos(d*x + c) + 10*(f^2*cos(4*d*x + 4*c) + 2*I*f^2*cos(3*d*x + 3*c) + 2*I*f
^2*cos(d*x + c) + I*f^2*sin(4*d*x + 4*c) - 2*f^2*sin(3*d*x + 3*c) - 2*f^2*
sin(d*x + c) - f^2)*dilog(I*e^(I*d*x + I*c)) + 6*(f^2*cos(4*d*x + 4*c) + 2
*I*f^2*cos(3*d*x + 3*c) + 2*I*f^2*cos(d*x + c) + I*f^2*sin(4*d*x + 4*c) -
2*f^2*sin(3*d*x + 3*c) - 2*f^2*sin(d*x + c) - f^2)*dilog(-I*e^(I*d*x + ...
```

Giac [F]

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^2\sec(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Hanged}$$

input
```
int((e + f*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 432*cos(c + d*x)*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**6 + 2*tan
((c + d*x)/2)**5 - tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((c +
d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*sin(c + d*x)*d**2*f**2 - 432*cos(c
 + d*x)*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)
**5 - tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((c + d*x)/2)**2 +
2*tan((c + d*x)/2) + 1),x)*d**2*f**2 + 192*cos(c + d*x)*int(x/(tan((c + d*
x)/2)**6 + 2*tan((c + d*x)/2)**5 - tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2
)**3 - tan((c + d*x)/2)**2 + 2*tan((c + d*x)/2) + 1),x)*sin(c + d*x)*d**2*
f**2 + 192*cos(c + d*x)*int(x/(tan((c + d*x)/2)**6 + 2*tan((c + d*x)/2)**5
 - tan((c + d*x)/2)**4 - 4*tan((c + d*x)/2)**3 - tan((c + d*x)/2)**2 + 2*t
an((c + d*x)/2) + 1),x)*d**2*f**2 - 24*cos(c + d*x)*log(tan((c + d*x)/2)**
2 + 1)*sin(c + d*x)*d*e*f + 124*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*
sin(c + d*x)*f**2 - 24*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*d*e*f + 1
24*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*f**2 + 18*cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)*d*e*f + 15*cos(c + d*x)*log(tan((c + d*x)
/2) - 1)*sin(c + d*x)*f**2 + 18*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*d*e
*f + 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*f**2 + 30*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)*d*e*f - 263*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)*f**2 + 30*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*
d*e*f - 263*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*f**2 + 12*cos(c + d*...
```

3.277 $\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{f\operatorname{arctanh}(\sin(c+dx))}{6ad^2}+\frac{2f\log(\cos(c+dx))}{3ad^2}-\frac{f\sec^2(c+dx)}{6ad^2}-\frac{(e+fx)\sec^3(c+dx)}{3ad}+\frac{2(e+fx)\tan(c+dx)}{3ad}+\frac{f\sec(c+dx)\tan(c+dx)}{6ad^2}+\frac{(e+fx)\sec^2(c+dx)\tan(c+dx)}{3ad}$$

output

```
1/6*f*arctanh(sin(d*x+c))/a/d^2+2/3*f*ln(cos(d*x+c))/a/d^2-1/6*f*sec(d*x+c
)^2/a/d^2-1/3*(f*x+e)*sec(d*x+c)^3/a/d+2/3*(f*x+e)*tan(d*x+c)/a/d+1/6*f*se
c(d*x+c)*tan(d*x+c)/a/d^2+1/3*(f*x+e)*sec(d*x+c)^2*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 7.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.52

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{-2d(e+fx)(\cos(2(c+dx))-2\sin(c+dx))+\cos(c+dx)\left(de-f-cf+3f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\mathrm{si}\right.\right.}{6ad^2\left(\cos\left(\right.\right.}$$

input
```
Integrate[((e + f*x)*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(-2*d*(e + f*x)*(Cos[2*(c + d*x)] - 2*Sin[c + d*x]) + Cos[c + d*x]*(d*e -
f - c*f + 3*f*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*f*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + (d*e - c*f + 3*f*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 5*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]
))/(6*a*d^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5042, 3042, 4673, 3042, 4672, 25, 3042, 3956, 4909, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sec^2(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \; 5042$$

$$\frac{\int (e+fx)\sec^4(c+dx)dx}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int (e+fx)\sec^2(c+dx)dx-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2}{3}\left(\frac{f\int -\tan(c+dx)dx}{d}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int \tan(c+dx)dx}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int \tan(c+dx)dx}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4909}$$

$$\frac{\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\frac{(e+fx)\sec^3(c+dx)}{3d}-\frac{f\int \sec^3(c+dx)dx}{3d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\frac{(e+fx)\sec^3(c+dx)}{3d}-\frac{f\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}}{a}$$

$$\Big\downarrow \; 4255$$

$$\frac{\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\frac{(e+fx)\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{3d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\frac{(e+fx)\sec^3(c+dx)}{3d}-\frac{f\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{3d}}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}}{a}-\frac{\frac{(e+fx)\sec^3(c+dx)}{3d}-\frac{f\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{3d}}{a}$$

input
```
Int[((e + f*x)*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(-1/6*(f*Sec[c + d*x]^2)/d^2 + ((e + f*x)*Sec[c + d*x]^2*Tan[c + d*x])/(3*
d) + (2*((f*Log[Cos[c + d*x]])/d^2 + ((e + f*x)*Tan[c + d*x])/d))/3)/a - (
((e + f*x)*Sec[c + d*x]^3)/(3*d) - (f*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[
c + d*x]*Tan[c + d*x])/(2*d)))/(3*d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4ifx}{3da}-\frac{4ifc}{3d^2a}-\frac{i\left(f\,\mathrm{e}^{3i(dx+c)}+4dxf-8idfx\,\mathrm{e}^{i(dx+c)}+4de+\mathrm{e}^{i(dx+c)}f-8ide\,\mathrm{e}^{i(dx+c)}\right)}{3\left(\mathrm{e}^{i(dx+c)}+i\right)^3d^2\left(\mathrm{e}^{i(dx+c)}-i\right)a}+\frac{f\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d^2a}+\frac{5f\ln(\mathrm{e}^{i}}{6}$
parallelrisch	$\frac{-4f(\sin(2dx+2c)+2\cos(dx+c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+3f(\sin(2dx+2c)+2\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5f(\sin(2dx+2c)-}{6d^2a(\sin(2dx+2c)}$
default	$\frac{e\left(-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}\right)}{d}+\frac{f\left(-\frac{dx+c}{3\cos(dx+c)^3}+\frac{\sec(dx+c)\tan(dx+c)}{6}\right.}{}$
norman	$\frac{\frac{2e}{3da}-\frac{2e\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}+\frac{fx}{3ad}+\frac{(-6de+f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d^2a}-\frac{(2de+f)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d^2a}-\frac{4fx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{2fx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{ad}-\frac{4fx\tan\left(\frac{dx}{2}+\right.}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

input
```
int((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*I*f/d/a*x-4/3*I*f/d^2/a*c-1/3*I*(f*exp(3*I*(d*x+c))+4*d*x*f-8*I*d*f*x
*exp(I*(d*x+c))+4*d*e+exp(I*(d*x+c))*f-8*I*d*e*exp(I*(d*x+c)))/(exp(I*(d*x
+c))+I)^3/d^2/(exp(I*(d*x+c))-I)/a+1/2*f/d^2/a*ln(exp(I*(d*x+c))-I)+5/6*f/
d^2/a*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{4\,dfx-8\,(dfx+de)\cos{(dx+c)}^2+4\,de-2\,f\cos{(dx+c)}+5\,(f\cos{(dx+c)}\sin{(dx+c)}+f\cos{(dx+}}{12\,(ad^2\cos{(dx}}$$

input
```
integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*d*f*x - 8*(d*f*x + d*e)*cos(d*x + c)^2 + 4*d*e - 2*f*cos(d*x + c)
+ 5*(f*cos(d*x + c)*sin(d*x + c) + f*cos(d*x + c))*log(sin(d*x + c) + 1) +
 3*(f*cos(d*x + c)*sin(d*x + c) + f*cos(d*x + c))*log(-sin(d*x + c) + 1) +
 8*(d*f*x + d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c)*sin(d*x + c) + a*d^2*co
s(d*x + c))
```

Sympy [F]

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e\sec^2{(c+dx)}}{\sin{(c+dx)}+1}\,dx+\int \frac{fx\sec^2{(c+dx)}}{\sin{(c+dx)}+1}\,dx}{a}$$

input
```
integrate((f*x+e)*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c +
d*x)**2/(sin(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(138) = 276$.

Time = 0.07 (sec) , antiderivative size = 1115, normalized size of antiderivative = 7.34

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(8*c*f*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d + 2*a*d*sin(
d*x + c)/(cos(d*x + c) + 1) - 2*a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -
a*d*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (4*(8*(d*x + c)*cos(d*x + c) -
sin(3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x + 4*(d*x + c
)*sin(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*d*x + 3*c)
^2 + 8*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos
(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x +
3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*x + c))
*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(3*d*x
+ 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1)*log
(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3*d*x
 + 3*c) + 2*sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*co
s(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*
(cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 -
4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x
 + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(
d*x + c) + 1) + 4*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*
c) + cos(d*x + c))*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin
(d*x + c) - 1)*sin(3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5555 vs. $2(138) = 276$.

Time = 1.29 (sec) , antiderivative size = 5555, normalized size of antiderivative = 36.55

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(4*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16*d*f*x*tan(1/2*d*x)^4*tan(1
/2*c)^3 + 16*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 4*d*e*tan(1/2*d*x)^4*tan(
1/2*c)^4 - 3*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*f*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^4*tan(1/2*c)^4 - 24*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^2 - 64*d*f*x*tan
(1/2*d*x)^3*tan(1/2*c)^3 + 16*d*e*tan(1/2*d*x)^4*tan(1/2*c)^3 + 6*f*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x
)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^4*tan(1/2*c)^3 + 10*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^3
 - 24*d*f*x*tan(1/2*d*x)^2*tan(1/2*c)^4 + 16*d*e*tan(1/2*d*x)^3*tan(1/2*c)
^4 + 6*f*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*...
```

Mupad [B] (verification not implemented)

Time = 44.83 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.58

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\,(d\,e+d\,f\,x)}{3\,a\,d^2\,\left(3\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}-\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,3\mathrm{i}-\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}+1\mathrm{i}\right)} - \frac{3\,d\,e+3\,d\,f\,x+f\,2\mathrm{i}}{6\,a\,d^2\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)} + \frac{e+f\,x}{2\,a\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}-\mathrm{i}\right)} - \frac{(24\,d\,e+24\,d\,f\,x-f\,8\mathrm{i})\,1\mathrm{i}}{24\,a\,d^2\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}-1+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,2\mathrm{i}\right)} - \frac{f\,x\,4\mathrm{i}}{3\,a\,d} + \frac{f\,\ln\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}-\mathrm{i}\right)}{2\,a\,d^2} + \frac{5\,f\,\ln\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{6\,a\,d^2}$$

input
```
int((e + f*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
(2*(d*e + d*f*x))/(3*a*d^2*(3*exp(c*1i + d*x*1i) - exp(c*2i + d*x*2i)*3i -
 exp(c*3i + d*x*3i) + 1i)) - (f*2i + 3*d*e + 3*d*f*x)/(6*a*d^2*(exp(c*1i +
 d*x*1i) + 1i)) + (e + f*x)/(2*a*d*(exp(c*1i + d*x*1i) - 1i)) - ((24*d*e -
 f*8i + 24*d*f*x)*1i)/(24*a*d^2*(exp(c*1i + d*x*1i)*2i + exp(c*2i + d*x*2i
) - 1)) - (f*x*4i)/(3*a*d) + (f*log(exp(c*1i + d*x*1i) - 1i))/(2*a*d^2) +
(5*f*log(exp(c*1i + d*x*1i) + 1i))/(6*a*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.74

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= \frac{-4\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,f-4\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)f+3\cos}{}$$

input
```
int((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*f - 4*cos(c +
 d*x)*log(tan((c + d*x)/2)**2 + 1)*f + 3*cos(c + d*x)*log(tan((c + d*x)/2)
 - 1)*sin(c + d*x)*f + 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*f + 5*cos(
c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*f + 5*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*f + 4*cos(c + d*x)*sin(c + d*x)*d*e + 4*cos(c + d*x)*d*
e - cos(c + d*x)*f + 4*sin(c + d*x)**2*d*e + 4*sin(c + d*x)**2*d*f*x + 4*s
in(c + d*x)*d*e + 4*sin(c + d*x)*d*f*x - 2*d*e - 2*d*f*x)/(6*cos(c + d*x)*
a*d**2*(sin(c + d*x) + 1))
```

3.278 $\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\tan(c+dx)}{3ad}$$

output
```
-1/3*sec(d*x+c)/d/(a+a*sin(d*x+c))+2/3*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-\cos(2(c+dx))\sec(c+dx)+2\tan(c+dx)}{3ad(1+\sin(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
(-(Cos[2*(c + d*x)]*Sec[c + d*x]) + 2*Tan[c + d*x])/(3*a*d*(1 + Sin[c + d*
x]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^2(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{2\int \sec^2(c+dx)dx}{3a}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{3a}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{2\int 1d(-\tan(c+dx))}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
-1/3*Sec[c + d*x]/(d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{4\left(2\,{\mathrm e}^{i\left(d x+c\right)}+i\right)}{3\left({\mathrm e}^{i\left(d x+c\right)}+i\right)^{3}\left({\mathrm e}^{i\left(d x+c\right)}-i\right)a d}$	51
derivativedivides	$\frac{-\frac{2}{3\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}}{a d}$	70
default	$\frac{-\frac{2}{3\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}}{a d}$	70
parallelrisch	$\frac{2-6\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}-6\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-2\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{3 d a\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}$	74
norman	$\frac{-\frac{2\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{a d}+\frac{2}{3 a d}-\frac{2\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{a d}-\frac{2\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{3 a d}}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}$	92

input
```
int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*exp(I*(d*x+c))+I)/(exp(I*(d*x+c))+I)^3/(exp(I*(d*x+c))-I)/a/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\cos(dx+c)^2-2\sin(dx+c)-1}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(2*cos(d*x + c)^2 - 2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)*sin(d*x + c
) + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.07

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
2/3*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/((a + 2*a*sin(d*x + c)/(co
s(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1/2*c)^2 + 12*ta
n(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 37.75 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^3}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
-(2*(tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^3
- 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\cos(dx+c)\sin(dx+c)+2\cos(dx+c)+2\sin(dx+c)^2+2\sin(dx+c)-1}{3\cos(dx+c)\,ad\,(\sin(dx+c)+1)}$$

input
```
int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
(2*cos(c + d*x)*sin(c + d*x) + 2*cos(c + d*x) + 2*sin(c + d*x)**2 + 2*sin(
c + d*x) - 1)/(3*cos(c + d*x)*a*d*(sin(c + d*x) + 1))
```

3.279 $\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 21.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \text{ 5048}$$
$$\int \frac{\sec^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sec (dx+c)^2}{(fx+e)\,(a+a\sin (dx+c))}dx$$

input
```
int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec (dx+c)^2}{(fx+e)(a\sin (dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sec^2(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)
/a
```

Maxima [N/A]

Not integrable

Time = 10.23 (sec) , antiderivative size = 3760, normalized size of antiderivative = 134.29

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec{(dx+c)}^2}{(fx+e)(a\sin{(dx+c)}+a)}\,dx$$

input
```
integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(4*f^2*cos(2*d*x + 2*c)*cos(d*x + c) - 2*(d*f^2*x + d*e*f)*cos(3*d*x
+ 3*c)^2 + 2*f^2*cos(d*x + c) - 2*(d*f^2*x + d*e*f)*cos(d*x + c)^2 - 2*(d*
f^2*x + d*e*f)*sin(3*d*x + 3*c)^2 - 2*(d*f^2*x + d*e*f)*sin(d*x + c)^2 + (
2*f^2*cos(3*d*x + 3*c) - 2*f^2*sin(2*d*x + 2*c) + 2*(4*d^2*f^2*x^2 + 8*d^2
*e*f*x + 4*d^2*e^2 + f^2)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(3*d*x + 3*c
) + (d*f^2*x + d*e*f)*sin(d*x + c))*cos(4*d*x + 4*c) + 2*(4*d^2*f^2*x^2 +
8*d^2*e*f*x + 4*d^2*e^2 + 2*f^2*cos(2*d*x + 2*c) + f^2 - 2*(d*f^2*x + d*e*
f)*cos(d*x + c) + 8*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))*co
s(3*d*x + 3*c) + 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x +
a*d^3*e^3 + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e
^3)*cos(4*d*x + 4*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^
2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x
^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c)*cos(d*x + c) + 4*(a*d^3
*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c)^2
 + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(4
*d*x + 4*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a
*d^3*e^3)*sin(3*d*x + 3*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*
d^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c)^2 - 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2
*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2
+ 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(3*d*x + 3*c) + 2*(a*d^3*f^3*x^3 + 3*...
```

Giac [N/A]

Not integrable

Time = 30.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)^2}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 38.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,(e+f\,x)\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^2*(e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sec(dx+c)^2}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(c + d*x)**2/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.280 $\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 27.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 5048$$
$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sec(dx+c)^2}{(fx+e)^2(a+a\sin(dx+c))}dx$$

input
```
int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)^2}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*
e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 8.92 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sec^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x)
+ 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 21.45 (sec) , antiderivative size = 4597, normalized size of antiderivative = 164.18

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)^2}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(12*f^2*cos(2*d*x + 2*c)*cos(d*x + c) - 4*(d*f^2*x + d*e*f)*cos(3*d*x
 + 3*c)^2 + 6*f^2*cos(d*x + c) - 4*(d*f^2*x + d*e*f)*cos(d*x + c)^2 - 4*(d
*f^2*x + d*e*f)*sin(3*d*x + 3*c)^2 - 4*(d*f^2*x + d*e*f)*sin(d*x + c)^2 +
2*(3*f^2*cos(3*d*x + 3*c) - 3*f^2*sin(2*d*x + 2*c) + (4*d^2*f^2*x^2 + 8*d^
2*e*f*x + 4*d^2*e^2 + 3*f^2)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(3*d*x +
3*c) + (d*f^2*x + d*e*f)*sin(d*x + c))*cos(4*d*x + 4*c) + 2*(4*d^2*f^2*x^2
 + 8*d^2*e*f*x + 4*d^2*e^2 + 6*f^2*cos(2*d*x + 2*c) + 3*f^2 - 4*(d*f^2*x +
 d*e*f)*cos(d*x + c) + 8*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c
))*cos(3*d*x + 3*c) + 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f
^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3
+ 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*cos(4*d*x + 4*c)^2 +
4*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f
*x + a*d^3*e^4)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3
+ 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*cos(3*d*x + 3*c)*cos(
d*x + c) + 4*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*
a*d^3*e^3*f*x + a*d^3*e^4)*cos(d*x + c)^2 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3
*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(4*d*x + 4*c)
^2 + 4*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*
e^3*f*x + a*d^3*e^4)*sin(3*d*x + 3*c)^2 + 4*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3
*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x + c)^...
```

Giac [N/A]

Not integrable

Time = 69.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(dx+c)^2}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

input
```
integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 38.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^2*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 57348, normalized size of antiderivative = 2048.14

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output

```
(120*cos(c + d*x)*int(tan((c + d*x)/2)**5/(tan((c + d*x)/2)**7*e**2 + 2*ta
n((c + d*x)/2)**7*e*f*x + tan((c + d*x)/2)**7*f**2*x**2 + tan((c + d*x)/2)
**6*e**2 + 2*tan((c + d*x)/2)**6*e*f*x + tan((c + d*x)/2)**6*f**2*x**2 - 3
*tan((c + d*x)/2)**5*e**2 - 6*tan((c + d*x)/2)**5*e*f*x - 3*tan((c + d*x)/
2)**5*f**2*x**2 - 3*tan((c + d*x)/2)**4*e**2 - 6*tan((c + d*x)/2)**4*e*f*x
 - 3*tan((c + d*x)/2)**4*f**2*x**2 + 3*tan((c + d*x)/2)**3*e**2 + 6*tan((c
 + d*x)/2)**3*e*f*x + 3*tan((c + d*x)/2)**3*f**2*x**2 + 3*tan((c + d*x)/2)
**2*e**2 + 6*tan((c + d*x)/2)**2*e*f*x + 3*tan((c + d*x)/2)**2*f**2*x**2 -
 tan((c + d*x)/2)*e**2 - 2*tan((c + d*x)/2)*e*f*x - tan((c + d*x)/2)*f**2*
x**2 - e**2 - 2*e*f*x - f**2*x**2),x)*sin(c + d*x)**2*e**2*f**2 + 120*cos(
c + d*x)*int(tan((c + d*x)/2)**5/(tan((c + d*x)/2)**7*e**2 + 2*tan((c + d*
x)/2)**7*e*f*x + tan((c + d*x)/2)**7*f**2*x**2 + tan((c + d*x)/2)**6*e**2
+ 2*tan((c + d*x)/2)**6*e*f*x + tan((c + d*x)/2)**6*f**2*x**2 - 3*tan((c +
 d*x)/2)**5*e**2 - 6*tan((c + d*x)/2)**5*e*f*x - 3*tan((c + d*x)/2)**5*f**
2*x**2 - 3*tan((c + d*x)/2)**4*e**2 - 6*tan((c + d*x)/2)**4*e*f*x - 3*tan(
(c + d*x)/2)**4*f**2*x**2 + 3*tan((c + d*x)/2)**3*e**2 + 6*tan((c + d*x)/2
)**3*e*f*x + 3*tan((c + d*x)/2)**3*f**2*x**2 + 3*tan((c + d*x)/2)**2*e**2
+ 6*tan((c + d*x)/2)**2*e*f*x + 3*tan((c + d*x)/2)**2*f**2*x**2 - tan((c +
 d*x)/2)*e**2 - 2*tan((c + d*x)/2)*e*f*x - tan((c + d*x)/2)*f**2*x**2 - e*
*2 - 2*e*f*x - f**2*x**2),x)*sin(c + d*x)**2*e*f**3*x - 120*cos(c + d*x...
```

3.281 $\int \frac{(e+fx)^3 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 698

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\arctan\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\arctan\left(e^{i(c+dx)}\right)}{4ad} + \frac{f^2(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{ad^3} + \frac{5if^3\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^4} + \frac{9if(e+fx)^2\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{8ad^2} - \frac{5if^3\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^4} - \frac{9if(e+fx)^2\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{8ad^2} - \frac{if^3\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2ad^4} - \frac{9f^2(e+fx)\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3} + \frac{9f^2(e+fx)\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{4ad^3} - \frac{9if^3\,\text{PolyLog}\left(4,-ie^{i(c+dx)}\right)}{4ad^4} + \frac{9if^3\,\text{PolyLog}\left(4,ie^{i(c+dx)}\right)}{4ad^4} - \frac{f^3\sec(c+dx)}{4ad^4} - \frac{9f(e+fx)^2\sec(c+dx)}{8ad^2} - \frac{f^2(e+fx)\sec^2(c+dx)}{4ad^3} - \frac{f(e+fx)^2\sec^3(c+dx)}{4ad^2} - \frac{(e+fx)^3\sec^4(c+dx)}{4ad} + \frac{f^3\tan(c+dx)}{4ad^4} + \frac{f(e+fx)^2\tan(c+dx)}{2ad^2} + \frac{f^2(e+fx)\sec(c+dx)\tan(c+dx)}{4ad^3} + \frac{3(e+fx)^3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{f(e+fx)^2\sec^2(c+dx)\tan(c+dx)}{4ad^2} + \frac{(e+fx)^3\sec^3(c+dx)\tan(c+dx)}{4ad}$$

output
```
5/2*I*f^3*polylog(2,-I*exp(I*(d*x+c)))/a/d^4-1/2*I*f^3*polylog(2,-exp(2*I*
(d*x+c)))/a/d^4-1/2*I*f*(f*x+e)^2/a/d^2+f^2*(f*x+e)*ln(1+exp(2*I*(d*x+c)))
/a/d^3-3/4*I*(f*x+e)^3*arctan(exp(I*(d*x+c)))/a/d-9/8*I*f*(f*x+e)^2*polylo
g(2,I*exp(I*(d*x+c)))/a/d^2+9/4*I*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4-9/
4*I*f^3*polylog(4,-I*exp(I*(d*x+c)))/a/d^4-5*I*f^2*(f*x+e)*arctan(exp(I*(d
*x+c)))/a/d^3-9/4*f^2*(f*x+e)*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+9/4*f^2*(
f*x+e)*polylog(3,I*exp(I*(d*x+c)))/a/d^3-5/2*I*f^3*polylog(2,I*exp(I*(d*x+
c)))/a/d^4+9/8*I*f*(f*x+e)^2*polylog(2,-I*exp(I*(d*x+c)))/a/d^2-1/4*f^3*se
c(d*x+c)/a/d^4-9/8*f*(f*x+e)^2*sec(d*x+c)/a/d^2-1/4*f^2*(f*x+e)*sec(d*x+c)
^2/a/d^3-1/4*f*(f*x+e)^2*sec(d*x+c)^3/a/d^2-1/4*(f*x+e)^3*sec(d*x+c)^4/a/d
+1/4*f^3*tan(d*x+c)/a/d^4+1/2*f*(f*x+e)^2*tan(d*x+c)/a/d^2+1/4*f^2*(f*x+e)
*sec(d*x+c)*tan(d*x+c)/a/d^3+3/8*(f*x+e)^3*sec(d*x+c)*tan(d*x+c)/a/d+1/4*f
*(f*x+e)^2*sec(d*x+c)^2*tan(d*x+c)/a/d^2+1/4*(f*x+e)^3*sec(d*x+c)^3*tan(d*
x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2278 vs. 2(698) = 1396.

Time = 12.73 (sec) , antiderivative size = 2278, normalized size of antiderivative = 3.26

$$\int \frac{(e+fx)^3 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(-3*(6*d^4*e^2*f*x^2 + 8*d^2*f^3*x^2 + 4*d^4*e*f^2*x^3 + d^4*f^3*x^4 - (4*
I)*d^4*e^3*x*Cos[c] - (16*I)*d^2*e*f^2*x*Cos[c] - (4*I)*d^3*e^3*Log[-Cos[c
 + d*x] - I*(-1 + Sin[c + d*x])] - (16*I)*d*e*f^2*Log[-Cos[c + d*x] - I*(-
1 + Sin[c + d*x])] - (12*I)*d^3*e^2*f*x*Log[1 - I*Cos[c + d*x] - Sin[c + d
*x]] - (16*I)*d*f^3*x*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]] - (12*I)*d^3*
e*f^2*x^2*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]] - (4*I)*d^3*f^3*x^3*Log[1
 - I*Cos[c + d*x] - Sin[c + d*x]] - 24*f^3*PolyLog[4, I*Cos[c + d*x] + Sin
[c + d*x]] + 24*d*f^2*(e + f*x)*PolyLog[3, I*Cos[c + d*x] + Sin[c + d*x]]*
(Cos[c] + I*(-1 + Sin[c])) + 4*f*(4*f^2 + 3*d^2*(e + f*x)^2)*PolyLog[2, I*
Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] - Sin[c]) + 4*d^3*e^3*Log[-Cos[
c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] + I*Sin[c]) + 16*d*e*f^2*Log[-Co
s[c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] + I*Sin[c]) + 12*d^3*e^2*f*x*L
og[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*Sin[c]) + 16*d*f^3*x*Log
[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*Sin[c]) + 12*d^3*e*f^2*x^2
*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*Sin[c]) + 4*d^3*f^3*x^
3*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*Sin[c]) + 4*d^4*e^3*x
*Sin[c] + 16*d^2*e*f^2*x*Sin[c] + 24*f^3*PolyLog[4, I*Cos[c + d*x] + Sin[c
 + d*x]]*((-I)*Cos[c] + Sin[c])))/(32*a*d^4*(Cos[c] + I*(-1 + Sin[c]))) -
((Cos[c] + I*Sin[c])*(((28*f^2 + 3*d^2*(e + f*x)^2)^2*(Cos[c] - I*Sin[c]))
/(12*d^2*f) + (f*(9*d^2*e^2 + 28*f^2)*PolyLog[2, (-I)*Cos[c + d*x] - Si...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sec^3(c+dx)}{a \sin(c+dx)+a} \, dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)^3 \sec^5(c+dx)dx}{a}-\frac{\int (e+fx)^3 \sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3 \csc\left(c+dx+\frac{\pi}{2}\right)^5 dx}{a}-\frac{\int (e+fx)^3 \sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{f^2\int(e+fx)\sec^3(c+dx)dx}{2d^2}+\frac{3}{4}\int(e+fx)^3\sec^3(c+dx)dx-\frac{f(e+fx)^2\sec^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int(e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{f^2\int(e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{2d^2}+\frac{3}{4}\int(e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f(e+fx)^2\sec^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int(e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 4673

$$\frac{\frac{f^2\left(\frac{1}{2}\int(e+fx)\sec(c+dx)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int(e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f(e+fx)^2\sec^3(c+dx)}{4d^2}}{a}\ \frac{\int(e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{f^2\left(\frac{1}{2}\int(e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int(e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f(e+fx)^2\sec^3(c+}{4d^2}}{a}\ \frac{\int(e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\downarrow$ 4669

$$-\frac{\int(e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{f^2\left(\frac{1}{2}\left(-\frac{f\int\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{f\int\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int(e+}{a}$$

$\downarrow$ 2715

$$-\frac{\int(e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{f^2\left(\frac{1}{2}\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan}{}}{2d^2}}{a}$$

$\downarrow$ 2838

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{3}{4}\int (e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{f^2\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c}{2d}}{2d^2}}{a}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{3}{4}\left(\frac{3f^2\int (e+fx)\sec(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\sec(c+dx)dx-\frac{3f(e+fx)^2\sec(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{f^2\left(\frac{1}{2}\left(\right.\right.}{}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{3}{4}\left(\frac{3f^2\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{3f(e+fx)^2\sec(c+dx)}{2d^2}+\frac{(e+fx)^3\tan(c+dx)\sec(c+dx)}{2d}\right)}{}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{3}{4}\left(\frac{3f^2\left(-\frac{f\int \log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3f\int (e+fx)^2\log(1-ie^{i(c+dx)})dx}{d}+\frac{3f\int (e+}{}\right.\right.}{}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{\frac{3}{4}\left(\frac{3f^2\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3f\int (e+fx)}{}\right.\right.}{}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{3f\int (e+fx)^2\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{3f\int (e+fx)^2\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)^3\arctan\left(e^{i(c+dx)}\right)}{d}\right)+\frac{3f^2\left(-\frac{2i(e+fx)\arctan\left(e\right.}{d}\right.}{}\right.$$

$$\Big\downarrow \textbf{3011}$$

$$-\frac{\int (e+fx)^3\sec^4(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(\right.}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \textbf{4909}$$

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\sec^4(c+dx)dx}{4d}}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(\right.}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{4d}}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(\right.}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \textbf{4674}$$

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int \sec^2(c+dx)dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\sec^2(c+dx)dx-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{4d}}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(\right.}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{3d^2}+\frac{2}{3}\int(e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{4d}}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$\downarrow$ 4254

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(-\frac{f^2\int 1d(-\tan(c+dx))}{3d^3}+\frac{2}{3}\int(e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{4d}}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$\downarrow$ 24

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\int(e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{4d}}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$\downarrow$ 4672

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\left(\frac{2f\int-((e+fx)\tan(c+dx))dx}{d}+\frac{(e+fx)^2\tan(c+dx)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+}{3d}\right.}{4d}}{a}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$\downarrow$ 25

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\int(e+fx)\tan(c+dx)dx}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{4d}}{a}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$$\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\int(e+fx)\tan(c+dx)dx}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^2(c+dx)}{3d}}{4d}}{a}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$$\downarrow \text{4202}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$$\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int\frac{e^{2i(c+dx)}(e+fx)}{1+e^{2i(c+dx)}}dx\right)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-\frac{f(e+fx)\sec^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tan(c}{}}{4d}}{a}$$

$$\downarrow \text{2620}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.$$

$$\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int\log\left(1+e^{2i(c+dx)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)+\frac{f^2\tan(c+dx)}{3d^3}-}{4d}}{a}$$

$$\downarrow \text{2715}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-2i(c+dx)}\log\left(1+e^{2i(c+dx)}\right)de^{2i(c+dx)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right.\right.}{4d}}$$

$$a$$

$\downarrow$ 2838

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)\right.}{4d}}$$

$$a$$

$\downarrow$ 7163

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}(2,}{d}\right.}{}\right.\right.}{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\tan(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{2d}\right)\right)}{d}\right)\right.}{4d}}$$

$$a$$

$\downarrow$ 2720

$$-\frac{f(e+fx)^2\sec^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{f^2\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{i(\right.}{d^2}\right.\right.}{2d^2}$$

$$\frac{\frac{(e+fx)^3\sec^4(c+dx)}{4d}-\frac{3f\left(\frac{\tan(c+dx)f^2}{3d^3}-\frac{(e+fx)\sec^2(c+dx)f}{3d^2}+\frac{(e+fx)^2\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{2}{3}\left(\frac{(e+fx)^2\tan(c+dx)}{d}-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{i(e+fx}{}\right.\right.}{}\right.\right.}{4d}}{a}$$

input
```
Int[((e + f*x)^3*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2040 vs. 2(612) = 1224.

Time = 3.19 (sec) , antiderivative size = 2041, normalized size of antiderivative = 2.92

method	result	size
risch	Expression too large to display	2041

input
```
int((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-9/4*I*f^3*polylog(4,-I*exp(I*(d*x+c)))/a/d^4-3/2/a/d^3*f^3*ln(1+I*exp(I*(
d*x+c)))*x-3/2/a/d^4*f^3*ln(1+I*exp(I*(d*x+c)))*c+5*I/a/d^4*f^3*c*arctan(e
xp(I*(d*x+c)))-5*I/a/d^3*e*f^2*arctan(exp(I*(d*x+c)))+9/8*I/a/d^2*e^2*f*po
lylog(2,-I*exp(I*(d*x+c)))-2*I/a/d^3*f^3*c*x-9/8*I/a/d^2*f^3*polylog(2,I*e
xp(I*(d*x+c)))*x^2+9/8*I/a/d^2*f^3*polylog(2,-I*exp(I*(d*x+c)))*x^2-9/8*I/
a/d^2*e^2*f*polylog(2,I*exp(I*(d*x+c)))+3/4*I/a/d^4*f^3*c^3*arctan(exp(I*(
d*x+c)))-1/4*I*(2*f^3+2*d^3*f^3*x^3*exp(3*I*(d*x+c))+4*d^2*f^3*x^2+8*d^2*e
*f^2*x+6*I*d^3*e^3*exp(4*I*(d*x+c))+9*d^3*e*f^2*x^2*exp(5*I*(d*x+c))+9*d^3
*e^2*f*x*exp(5*I*(d*x+c))+44*d^2*e*f^2*x*exp(2*I*(d*x+c))+4*f^3*exp(2*I*(d
*x+c))+18*d^2*e^2*f*exp(4*I*(d*x+c))+4*d*f^3*x*exp(3*I*(d*x+c))+4*d*e*f^2*
exp(3*I*(d*x+c))+22*d^2*f^3*x^2*exp(2*I*(d*x+c))+22*d^2*e^2*f*exp(2*I*(d*x
+c))-6*I*d^3*e^3*exp(2*I*(d*x+c))+2*d*f^3*x*exp(5*I*(d*x+c))+2*d*e*f^2*exp
(5*I*(d*x+c))+3*d^3*f^3*x^3*exp(5*I*(d*x+c))+18*d^2*f^3*x^2*exp(4*I*(d*x+c
))+2*d*f^3*x*exp(I*(d*x+c))+2*d*e*f^2*exp(I*(d*x+c))+3*d^3*f^3*x^3*exp(I*(
d*x+c))+2*I*d^2*e*f^2*x*exp(I*(d*x+c))+2*f^3*exp(4*I*(d*x+c))+4*d^2*e^2*f-
4*I*f^3*exp(3*I*(d*x+c))+3*d^3*e^3*exp(5*I*(d*x+c))+3*d^3*e^3*exp(I*(d*x+c
))-2*I*f^3*exp(I*(d*x+c))-18*I*d^3*e*f^2*x^2*exp(2*I*(d*x+c))-18*I*d^3*e^2
*f*x*exp(2*I*(d*x+c))-16*I*d^2*e*f^2*x*exp(3*I*(d*x+c))+6*d^3*e*f^2*x^2*ex
p(3*I*(d*x+c))+6*d^3*e^2*f*x*exp(3*I*(d*x+c))-6*I*d^3*f^3*x^3*exp(2*I*(d*x
+c))+I*d^2*f^3*x^2*exp(I*(d*x+c))+I*d^2*e^2*f*exp(I*(d*x+c))+9*d^3*e^2*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2572 vs. $2(589) = 1178$.

Time = 0.25 (sec) , antiderivative size = 2572, normalized size of antiderivative = 3.68

$$\int \frac{(e+fx)^3 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 - 4*(2*d
^2*f^3*x^2 + 4*d^2*e*f^2*x + 2*d^2*e^2*f + f^3)*cos(d*x + c)^3 - 2*(3*d^3*
f^3*x^3 + 9*d^3*e*f^2*x^2 + 3*d^3*e^3 + 2*d*e*f^2 + (9*d^3*e^2*f + 2*d*f^3
)*x)*cos(d*x + c)^2 - 14*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*cos(d*x
 + c) - 3*((3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f + 4*I*f^3)*c
os(d*x + c)^2*sin(d*x + c) + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*
e^2*f + 4*I*f^3)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) + sin(d*x + c)) + ((
-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f - 28*I*f^3)*cos(d*x +
c)^2*sin(d*x + c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f -
 28*I*f^3)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 3*((-3*I
*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f - 4*I*f^3)*cos(d*x + c)^2*s
in(d*x + c) + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f - 4*I*f^
3)*cos(d*x + c)^2)*dilog(-I*cos(d*x + c) + sin(d*x + c)) + ((9*I*d^2*f^3*x
^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f + 28*I*f^3)*cos(d*x + c)^2*sin(d*x +
 c) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f + 28*I*f^3)*cos(
d*x + c)^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + ((3*d^3*e^3 - 9*c*d^2*
e^2*f + (9*c^2 + 28)*d*e*f^2 - (3*c^3 + 28*c)*f^3)*cos(d*x + c)^2*sin(d*x
+ c) + (3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c^2 + 28)*d*e*f^2 - (3*c^3 + 28*c)*
f^3)*cos(d*x + c)^2)*log(cos(d*x + c) + I*sin(d*x + c) + I) - 3*((d^3*e^3
- 3*c*d^2*e^2*f + (3*c^2 + 4)*d*e*f^2 - (c^3 + 4*c)*f^3)*cos(d*x + c)^2...
```

Sympy [F]

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^3\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**3*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**3*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3
*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d
*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)**3/(sin(c
 + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\sec(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - 87204*cos(c + d*x)*sin(c + d*x)**2*d**2*e**2*f - 174408*cos(c + d*x)*s
in(c + d*x)**2*d**2*e*f**2*x - 87204*cos(c + d*x)*sin(c + d*x)**2*d**2*f**
3*x**2 - 148824*cos(c + d*x)*sin(c + d*x)**2*d*e*f**2 + 1013896*cos(c + d*
x)*sin(c + d*x)**2*d*f**3*x + 940012*cos(c + d*x)*sin(c + d*x)**2*f**3 + 3
6504*cos(c + d*x)*sin(c + d*x)*d**3*e**2*f*x + 36504*cos(c + d*x)*sin(c +
d*x)*d**3*e*f**2*x**2 + 12168*cos(c + d*x)*sin(c + d*x)*d**3*f**3*x**3 + 2
2308*cos(c + d*x)*sin(c + d*x)*d**2*e**2*f + 112008*cos(c + d*x)*sin(c + d
*x)*d**2*e*f**2*x + 56004*cos(c + d*x)*sin(c + d*x)*d**2*f**3*x**2 - 19656
*cos(c + d*x)*sin(c + d*x)*d*e*f**2 + 182848*cos(c + d*x)*sin(c + d*x)*d*f
**3*x + 40252*cos(c + d*x)*sin(c + d*x)*f**3 + 73008*cos(c + d*x)*d**3*e**
2*f*x + 73008*cos(c + d*x)*d**3*e*f**2*x**2 + 24336*cos(c + d*x)*d**3*f**3
*x**3 + 125736*cos(c + d*x)*d**2*e**2*f + 386256*cos(c + d*x)*d**2*e*f**2*
x + 193128*cos(c + d*x)*d**2*f**3*x**2 + 134784*cos(c + d*x)*d*e*f**2 - 13
20560*cos(c + d*x)*d*f**3*x - 936176*cos(c + d*x)*f**3 - 194688*int((tan((
c + d*x)/2)*x**3)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7 - 2*tan((c
+ d*x)/2)**6 - 6*tan((c + d*x)/2)**5 + 6*tan((c + d*x)/2)**3 + 2*tan((c +
d*x)/2)**2 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)**3*d**4*f**3 - 194688
*int((tan((c + d*x)/2)*x**3)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7
- 2*tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5 + 6*tan((c + d*x)/2)**3 +
2*tan((c + d*x)/2)**2 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)**2*d**4...
```

3.282 $\int \frac{(e+fx)^2 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 431

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\text{arctanh}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+dx))}{3ad^3} + \frac{3if(e+fx)\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{4ad^2} - \frac{3if(e+fx)\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{4ad^2} - \frac{3f^2\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3} + \frac{3f^2\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{4ad^3} - \frac{3f(e+fx)\sec(c+dx)}{4ad^2} - \frac{f^2\sec^2(c+dx)}{12ad^3} - \frac{f(e+fx)\sec^3(c+dx)}{6ad^2} - \frac{(e+fx)^2\sec^4(c+dx)}{4ad} + \frac{f(e+fx)\tan(c+dx)}{3ad^2} + \frac{f^2\sec(c+dx)\tan(c+dx)}{12ad^3} + \frac{3(e+fx)^2\sec(c+dx)\tan(c+dx)}{8ad} + \frac{f(e+fx)\sec^2(c+dx)\tan(c+dx)}{6ad^2} + \frac{(e+fx)^2\sec^3(c+dx)\tan(c+dx)}{4ad}$$

output
```
-3/4*I*(f*x+e)^2*arctan(exp(I*(d*x+c)))/a/d+5/6*f^2*arctanh(sin(d*x+c))/a/
d^3+1/3*f^2*ln(cos(d*x+c))/a/d^3-3/4*I*f*(f*x+e)*polylog(2,I*exp(I*(d*x+c)
))/a/d^2+3/4*I*f*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/a/d^2-3/4*f^2*polylo
g(3,-I*exp(I*(d*x+c)))/a/d^3+3/4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/d^3-3/4
*f*(f*x+e)*sec(d*x+c)/a/d^2-1/12*f^2*sec(d*x+c)^2/a/d^3-1/6*f*(f*x+e)*sec(
d*x+c)^3/a/d^2-1/4*(f*x+e)^2*sec(d*x+c)^4/a/d+1/3*f*(f*x+e)*tan(d*x+c)/a/d
^2+1/12*f^2*sec(d*x+c)*tan(d*x+c)/a/d^3+3/8*(f*x+e)^2*sec(d*x+c)*tan(d*x+c
)/a/d+1/6*f*(f*x+e)*sec(d*x+c)^2*tan(d*x+c)/a/d^2+1/4*(f*x+e)^2*sec(d*x+c)
^3*tan(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1579 vs. 2(431) = 862.

Time = 10.01 (sec) , antiderivative size = 1579, normalized size of antiderivative = 3.66

$$\int \frac{(e+fx)^2 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
-1/8*((Cos[c] + I*Sin[c])*(3*d^2*e^2*x*Cos[c] + 4*f^2*x*Cos[c] + 3*d^2*e*f
*x^2*Cos[c] + 6*e*f*PolyLog[2, I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*
(-1 + Sin[c])) + 6*f^2*x*PolyLog[2, I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c]
 - I*(-1 + Sin[c])) + d^2*f^2*x^3*(Cos[c] - I*Sin[c]) + ((3*d^2*e^2 + 4*f^
2)*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])]*(Cos[c] + I*(-1 + Sin[c]))*(
Cos[c] - I*Sin[c]))/d + 6*d*e*f*x*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(
Cos[c] + I*(-1 + Sin[c]))*(Cos[c] - I*Sin[c]) + 3*d*f^2*x^2*Log[1 - I*Cos[
c + d*x] - Sin[c + d*x]]*(Cos[c] + I*(-1 + Sin[c]))*(Cos[c] - I*Sin[c]) +
(6*f^2*PolyLog[3, I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] + I*(-1 + Sin[c])
)*(Cos[c] - I*Sin[c]))/d - (3*I)*d^2*e^2*x*Sin[c] - (4*I)*f^2*x*Sin[c] - (
3*I)*d^2*e*f*x^2*Sin[c] + (3*d^2*e^2 + 4*f^2)*x*(Cos[c] - I*Sin[c])*(-1 -
I*Cos[c] + Sin[c])))/(a*d^2*(Cos[c] + I*(-1 + Sin[c]))) - ((Cos[c] + I*Sin
[c])*(9*d^2*e^2*x*Cos[c] + 28*f^2*x*Cos[c] + 9*d^2*e*f*x^2*Cos[c] + 3*d^2*
f^2*x^3*Cos[c] - (9*I)*d^2*e^2*x*Sin[c] - (28*I)*f^2*x*Sin[c] - (9*I)*d^2*
e*f*x^2*Sin[c] - (3*I)*d^2*f^2*x^3*Sin[c] + 18*e*f*PolyLog[2, (-I)*Cos[c +
 d*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])) + 18*f^2*x*PolyLog[2, (-I)
*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])) - 18*d*e*f*x*Log[1
 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin
[c])) - 9*d*f^2*x^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin
[c])*(Cos[c] + I*(1 + Sin[c])) - ((9*d^2*e^2 + 28*f^2)*Log[Cos[c + d*x]...
```

Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.98, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5042, 3042, 4674, 3042, 4255, 3042, 4257, 4674, 3042, 4257, 4669, 3011, 2720, 4909, 3042, 4673, 3042, 4672, 25, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)^2\sec^5(c+dx)dx}{a}-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{a}-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{f^2\int \sec^3(c+dx)dx}{6d^2}+\frac{3}{4}\int (e+fx)^2\sec^3(c+dx)dx-\frac{f(e+fx)\sec^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f(e+fx)\sec^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{f^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f(e+fx)\sec^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tan(c+dx)\sec^3}{4d}}{a}-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\left(\frac{1}{2}\int \csc(c+dx+\frac{\pi}{2})dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f(e+fx)\sec^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tan(c+dx)}{4d}}{a}$$

$$\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\Big\downarrow$ 4257

$$\frac{\frac{3}{4}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{f^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{6d^2}-\frac{f(e+fx)\sec^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tan(c+dx}{4d}}{a}$$

$$\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\Big\downarrow$ 4674

$$\frac{\frac{3}{4}\left(\frac{f^2\int\sec(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\sec(c+dx)dx-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{f^2\left(\frac{\text{arctanh}(\sin(}{2d}\right.}{}}{a}$$

$$\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3}{4}\left(\frac{f^2\int\csc(c+dx+\frac{\pi}{2})dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{f^2\left(\text{arc}\right.}{}}{a}$$

$$\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\Big\downarrow$ 4257

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)^2\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{f^2\text{arctanh}(\sin(c+dx))}{d^3}-\frac{f(e+fx)\sec(c+dx)}{d^2}+\frac{(e+fx)^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{f^2\left(\right.}{}}{a}$$

$$\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\Big\downarrow$ 4669

$$-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2f\int (e+fx)\log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}\right)+\frac{f^2\text{arctanh}(\sin(c+dx))}{d^3}-\right.$$

$$\Big\downarrow \; 3011$$

$$-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)dx}{d}\right)}{d}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$-\frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)dx}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 4909$$

$$-\frac{\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\int (e+fx)\sec^4(c+dx)dx}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 4673$$

$$-\frac{\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\int (e+fx)\sec^2(c+dx)dx-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\int(e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 4672$$

$$-\frac{-\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\left(\frac{f\int-\tan(c+dx)dx}{d}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int\tan(c+dx)dx}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tan(c+dx)}{d}-\frac{f\int\tan(c+dx)dx}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 3956$$

$$-\frac{\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}}{}\right.}{d}\right.\right.}{}$$

$$\Big\downarrow \textbf{7143}$$

$$-\frac{\frac{(e+fx)^2\sec^4(c+dx)}{4d}-\frac{f\left(\frac{2}{3}\left(\frac{f\log(\cos(c+dx))}{d^2}+\frac{(e+fx)\tan(c+dx)}{d}\right)-\frac{f\sec^2(c+dx)}{6d^2}+\frac{(e+fx)\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{2d}}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2i(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d}-\right.}{d}\right.\right.}{}$$

input
```
Int[((e + f*x)^2*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(-1/6*(f*(e + f*x)*Sec[c + d*x]^3)/d^2 + ((e + f*x)^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (f^2*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c +
 d*x])/(2*d)))/(6*d^2) + (3*((f^2*ArcTanh[Sin[c + d*x]])/d^3 + (((-2*I)*(e
 + f*x)^2*ArcTan[E^(I*(c + d*x))])/d + (2*f*((I*(e + f*x)*PolyLog[2, (-I)*
E^(I*(c + d*x))])/d - (f*PolyLog[3, (-I)*E^(I*(c + d*x))])/d^2))/d - (2*f*
((I*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/d - (f*PolyLog[3, I*E^(I*(c +
 d*x))])/d^2))/d)/2 - (f*(e + f*x)*Sec[c + d*x])/d^2 + ((e + f*x)^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)))/4)/a - (((e + f*x)^2*Sec[c + d*x]^4)/(4*d) -
(f*(-1/6*(f*Sec[c + d*x]^2)/d^2 + ((e + f*x)*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d) + (2*((f*Log[Cos[c + d*x]])/d^2 + ((e + f*x)*Tan[c + d*x])/d))/3))/(
2*d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(382) = 764.

Time = 2.02 (sec) , antiderivative size = 1035, normalized size of antiderivative = 2.40

method	result	size
risch	Expression too large to display	1035

input
```
int((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*I*(9*d^2*f^2*x^2*exp(5*I*(d*x+c))+9*exp(I*(d*x+c))*d^2*f^2*x^2+6*exp
(3*I*(d*x+c))*d^2*f^2*x^2+36*exp(4*I*(d*x+c))*d*f^2*x+44*exp(2*I*(d*x+c))*
d*f^2*x+36*exp(4*I*(d*x+c))*d*e*f+44*exp(2*I*(d*x+c))*d*e*f+9*d^2*e^2*exp(
5*I*(d*x+c))+4*f^2*exp(3*I*(d*x+c))+2*f^2*exp(I*(d*x+c))-18*I*d*f^2*x*exp(
5*I*(d*x+c))-18*I*d*e*f*exp(5*I*(d*x+c))-18*I*d^2*f^2*x^2*exp(2*I*(d*x+c))
-36*I*d^2*e*f*x*exp(2*I*(d*x+c))+36*I*d^2*e*f*x*exp(4*I*(d*x+c))+8*d*f^2*x
+2*I*d*f^2*x*exp(I*(d*x+c))+2*I*d*e*f*exp(I*(d*x+c))-16*I*d*f^2*x*exp(3*I*
(d*x+c))-16*I*d*e*f*exp(3*I*(d*x+c))+2*f^2*exp(5*I*(d*x+c))+9*exp(I*(d*x+c
))*d^2*e^2+6*exp(3*I*(d*x+c))*d^2*e^2+18*exp(I*(d*x+c))*d^2*e*f*x+12*exp(3
*I*(d*x+c))*d^2*e*f*x-18*I*d^2*e^2*exp(2*I*(d*x+c))+18*I*d^2*e^2*exp(4*I*(
d*x+c))+8*d*e*f+18*I*d^2*f^2*x^2*exp(4*I*(d*x+c))+18*d^2*e*f*x*exp(5*I*(d*
x+c)))/(exp(I*(d*x+c))+I)^4/d^3/(exp(I*(d*x+c))-I)^2/a+1/3/a/d^3*f^2*ln(ex
p(2*I*(d*x+c))+1)+3/8/d/a*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-5/3*I/a/d^3*f^2*a
rctan(exp(I*(d*x+c)))-3/4/a/d*ln(1+I*exp(I*(d*x+c)))*e*f*x-3/4*I/a/d^2*f^2
*polylog(2,I*exp(I*(d*x+c)))*x+3/4/d/a*e*f*ln(1-I*exp(I*(d*x+c)))*x+3/4*I/
a/d^2*f^2*polylog(2,-I*exp(I*(d*x+c)))*x-3/4*I/a/d^2*e*f*polylog(2,I*exp(I
*(d*x+c)))+3/4*I/a/d^2*e*f*polylog(2,-I*exp(I*(d*x+c)))-3/4*I/a/d*e^2*arct
an(exp(I*(d*x+c)))-3/4*I/a/d^3*f^2*c^2*arctan(exp(I*(d*x+c)))+3/2*I/a/d^2*
e*f*c*arctan(exp(I*(d*x+c)))+3/4/d^2/a*e*f*ln(1-I*exp(I*(d*x+c)))*c-2/3/a/
d^3*f^2*ln(exp(I*(d*x+c)))-3/4*f^2*polylog(3,-I*exp(I*(d*x+c)))/a/d^3-3...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(373) = 746$.

Time = 0.19 (sec) , antiderivative size = 1517, normalized size of antiderivative = 3.52

$$\int \frac{(e+fx)^2 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(6*d^2*f^2*x^2 + 12*d^2*e*f*x + 6*d^2*e^2 - 16*(d*f^2*x + d*e*f)*cos(
d*x + c)^3 - 2*(9*d^2*f^2*x^2 + 18*d^2*e*f*x + 9*d^2*e^2 + 2*f^2)*cos(d*x
+ c)^2 - 28*(d*f^2*x + d*e*f)*cos(d*x + c) - 18*((I*d*f^2*x + I*d*e*f)*cos
(d*x + c)^2*sin(d*x + c) + (I*d*f^2*x + I*d*e*f)*cos(d*x + c)^2)*dilog(I*c
os(d*x + c) + sin(d*x + c)) - 18*((I*d*f^2*x + I*d*e*f)*cos(d*x + c)^2*sin
(d*x + c) + (I*d*f^2*x + I*d*e*f)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - s
in(d*x + c)) - 18*((-I*d*f^2*x - I*d*e*f)*cos(d*x + c)^2*sin(d*x + c) + (-
I*d*f^2*x - I*d*e*f)*cos(d*x + c)^2)*dilog(-I*cos(d*x + c) + sin(d*x + c))
 - 18*((-I*d*f^2*x - I*d*e*f)*cos(d*x + c)^2*sin(d*x + c) + (-I*d*f^2*x -
I*d*e*f)*cos(d*x + c)^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + ((9*d^2*e
^2 - 18*c*d*e*f + (9*c^2 + 28)*f^2)*cos(d*x + c)^2*sin(d*x + c) + (9*d^2*e
^2 - 18*c*d*e*f + (9*c^2 + 28)*f^2)*cos(d*x + c)^2)*log(cos(d*x + c) + I*s
in(d*x + c) + I) - 3*((3*d^2*e^2 - 6*c*d*e*f + (3*c^2 + 4)*f^2)*cos(d*x +
c)^2*sin(d*x + c) + (3*d^2*e^2 - 6*c*d*e*f + (3*c^2 + 4)*f^2)*cos(d*x + c)
^2)*log(cos(d*x + c) - I*sin(d*x + c) + I) + 9*((d^2*f^2*x^2 + 2*d^2*e*f*x
 + 2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2*sin(d*x + c) + (d^2*f^2*x^2 + 2*d^2
*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2)*log(I*cos(d*x + c) + sin(d*x
 + c) + 1) - 9*((d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x
+ c)^2*sin(d*x + c) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*co
s(d*x + c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) + 9*((d^2*f^2*x^2 ...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e^2\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**2*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e**2*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2
*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)**3
/(sin(c + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sec(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Hanged}$$

input
```
int((e + f*x)^2/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - 4472*cos(c + d*x)*sin(c + d*x)**2*d*e*f - 4472*cos(c + d*x)*sin(c + d*
x)**2*d*f**2*x - 3816*cos(c + d*x)*sin(c + d*x)**2*f**2 + 1872*cos(c + d*x
)*sin(c + d*x)*d**2*e*f*x + 936*cos(c + d*x)*sin(c + d*x)*d**2*f**2*x**2 +
 1144*cos(c + d*x)*sin(c + d*x)*d*e*f + 2872*cos(c + d*x)*sin(c + d*x)*d*f
**2*x - 504*cos(c + d*x)*sin(c + d*x)*f**2 + 3744*cos(c + d*x)*d**2*e*f*x
+ 1872*cos(c + d*x)*d**2*f**2*x**2 + 6448*cos(c + d*x)*d*e*f + 9904*cos(c
+ d*x)*d*f**2*x + 3456*cos(c + d*x)*f**2 - 14976*int((tan((c + d*x)/2)*x**
2)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7 - 2*tan((c + d*x)/2)**6 -
6*tan((c + d*x)/2)**5 + 6*tan((c + d*x)/2)**3 + 2*tan((c + d*x)/2)**2 - 2*
tan((c + d*x)/2) - 1),x)*sin(c + d*x)**3*d**3*f**2 - 14976*int((tan((c + d
*x)/2)*x**2)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7 - 2*tan((c + d*x
)/2)**6 - 6*tan((c + d*x)/2)**5 + 6*tan((c + d*x)/2)**3 + 2*tan((c + d*x)/
2)**2 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)**2*d**3*f**2 + 14976*int((
tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7 - 2*ta
n((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5 + 6*tan((c + d*x)/2)**3 + 2*tan(
(c + d*x)/2)**2 - 2*tan((c + d*x)/2) - 1),x)*sin(c + d*x)*d**3*f**2 + 1497
6*int((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7
 - 2*tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5 + 6*tan((c + d*x)/2)**3 +
 2*tan((c + d*x)/2)**2 - 2*tan((c + d*x)/2) - 1),x)*d**3*f**2 - 29952*int(
(tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**8 + 2*tan((c + d*x)/2)**7 - 2*t...
```

3.283 $\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 241

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{4ad} + \frac{3if\,\mathrm{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{8ad^2} - \frac{3if\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{8ad^2} - \frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \frac{f\tan(c+dx)}{4ad^2} + \frac{3(e+fx)\sec(c+dx)\tan(c+dx)}{8ad} + \frac{(e+fx)\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{f\tan^3(c+dx)}{12ad^2}$$

output
```
-3/4*I*(f*x+e)*arctan(exp(I*(d*x+c)))/a/d+3/8*I*f*polylog(2,-I*exp(I*(d*x+
c)))/a/d^2-3/8*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-3/8*f*sec(d*x+c)/a/d^
2-1/12*f*sec(d*x+c)^3/a/d^2-1/4*(f*x+e)*sec(d*x+c)^4/a/d+1/4*f*tan(d*x+c)/
a/d^2+3/8*(f*x+e)*sec(d*x+c)*tan(d*x+c)/a/d+1/4*(f*x+e)*sec(d*x+c)^3*tan(d
*x+c)/a/d+1/12*f*tan(d*x+c)^3/a/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1171 vs. 2(241) = 482.

Time = 13.46 (sec) , antiderivative size = 1171, normalized size of antiderivative = 4.86

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(-6*d*e - f + 6*c*f - 6*f*(c + d*x))/(24*d^2*(a + a*Sin[c + d*x])) + (-(d*
e) + c*f - f*(c + d*x))/(8*d^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a
+ a*Sin[c + d*x])) + (f*Sin[(c + d*x)/2])/(12*d^2*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])*(a + a*Sin[c + d*x])) + (7*f*Sin[(c + d*x)/2]*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]))/(12*d^2*(a + a*Sin[c + d*x])) + (3*(c + d*x)*(2*d
*e - 2*c*f + f*(c + d*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(16*d^2
*(a + a*Sin[c + d*x])) + (3*e*((-c - d*x)/2 - Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(8*d*(a + a*Sin[c +
 d*x])) - (3*c*f*((-c - d*x)/2 - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(8*d^2*(a + a*Sin[c + d*x])) - (
3*e*((c + d*x)/2 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^2)/(8*d*(a + a*Sin[c + d*x])) + (3*c*f*((c + d*x)
/2 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2)/(8*d^2*(a + a*Sin[c + d*x])) - (3*f*((c + d*x)^2/(4*E^((I/4)
*Pi)) - (((-3*I)/4)*Pi*(c + d*x) - Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(-1/
4*Pi + (c + d*x)/2)*Log[1 - E^((2*I)*(-1/4*Pi + (c + d*x)/2))] + Pi*Log[Co
s[(c + d*x)/2]] - (Pi*Log[-Sin[Pi/4 + (-c - d*x)/2]])/2 + I*PolyLog[2, E^(
(2*I)*(-1/4*Pi + (c + d*x)/2))])/Sqrt[2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2)/(4*Sqrt[2]*d^2*(a + a*Sin[c + d*x])) - (3*f*((E^((I/4)*Pi)*(c + d
*x)^2)/4 + ((-1/4*I)*Pi*(c + d*x) - Pi*Log[1 + E^((-I)*(c + d*x))] - 2*...
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5042, 3042, 4673, 3042, 4673, 3042, 4669, 2715, 2838, 4909, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sec^3(c+dx)}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{5042}$$

$$\frac{\int (e+fx)\sec^5(c+dx)dx}{a}-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{a}-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{3}{4}\int (e+fx)\sec^3(c+dx)dx-\frac{f\sec^3(c+dx)}{12d^2}+\frac{(e+fx)\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{f\sec^3(c+dx)}{12d^2}+\frac{(e+fx)\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)\sec(c+dx)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{f\sec^3(c+dx)}{12d^2}+\frac{(e+fx)\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{f\sec^3(c+dx)}{12d^2}+\frac{(e+fx)\tan(c+dx)\sec^3(c+}{4d}}{a}$$

$$\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}$$

$\big\downarrow$ 4669

$$-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{f\int \log\left(1-ie^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+ie^{i(c+dx)}\right)dx}{d}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right.}{a}$$

$\big\downarrow$ 2715

$$-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{if\int e^{-i(c+dx)}\log\left(1-ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+ie^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}\right)-\frac{f\sec(c+dx)}{2d^2}\right.}{a}$$

$\big\downarrow$ 2838

$$-\frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)dx}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+}{2d}\right.}{a}$$

$\big\downarrow$ 4909

$$-\frac{\frac{(e+fx)\sec^4(c+dx)}{4d}-\frac{f\int \sec^4(c+dx)dx}{4d}}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+}{2d}\right.}{a}$$

$\big\downarrow$ 3042

$$-\frac{\frac{(e+fx)\sec^4(c+dx)}{4d}-\frac{f\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{4d}}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+}{2d}\right.}{a}$$

$\big\downarrow$ 4254

$$-\frac{\frac{f\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{4d^2}+\frac{(e+fx)\sec^4(c+dx)}{4d}}{a}+$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+}{2d}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{f\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{4d^2}+\frac{(e+fx)\sec^4(c+dx)}{4d}}{a}+\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2i(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}\right)-\frac{f\sec(c+dx)}{2d^2}+\frac{(e+fx)\tan(c+dx)\sec(c+dx)}{2d}\right.}{a}$$

input
```
Int[((e + f*x)*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(-1/12*(f*Sec[c + d*x]^3)/d^2 + ((e + f*x)*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d) + (3*(((((-2*I)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/d + (I*f*PolyLog[2,
(-I)*E^(I*(c + d*x))])/d^2 - (I*f*PolyLog[2, I*E^(I*(c + d*x))])/d^2)/2 -
(f*Sec[c + d*x])/(2*d^2) + ((e + f*x)*Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4
)/a - (((e + f*x)*Sec[c + d*x]^4)/(4*d) + (f*(-Tan[c + d*x] - Tan[c + d*x]
^3/3))/(4*d^2))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5042
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sec[c +
 d*x]^(n + 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a
^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(210) = 420$.

Time = 2.60 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{i\left(-18 i d f x\, e^{2 i (d x+c)}+9 d f x\, e^{5 i (d x+c)}-8 i f\, e^{3 i (d x+c)}+18 i d f x\, e^{4 i (d x+c)}+9 d e\, e^{5 i (d x+c)}-18 i d e\, e^{2 i (d x+c)}+6 d f x\, e^{3 i (d x+c)}+i e^{i (d x+c}\right.}{12\left(e^{i (d x+c)}+i\right)^{4} d^{2}\left(e^{i (d\right.}}$

input
```
int((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*I*(-18*I*d*f*x*exp(2*I*(d*x+c))+9*d*f*x*exp(5*I*(d*x+c))-8*I*f*exp(3
*I*(d*x+c))+18*I*d*f*x*exp(4*I*(d*x+c))+9*d*e*exp(5*I*(d*x+c))-18*I*d*e*ex
p(2*I*(d*x+c))+6*d*f*x*exp(3*I*(d*x+c))+I*exp(I*(d*x+c))*f-9*I*f*exp(5*I*(
d*x+c))+6*d*e*exp(3*I*(d*x+c))+18*f*exp(4*I*(d*x+c))+9*d*f*x*exp(I*(d*x+c)
)+18*I*d*e*exp(4*I*(d*x+c))+9*d*e*exp(I*(d*x+c))+22*f*exp(2*I*(d*x+c))+4*f
)/(exp(I*(d*x+c))+I)^4/d^2/(exp(I*(d*x+c))-I)^2/a-3/4*I/d/a*e*arctan(exp(I
*(d*x+c)))+3/8/d/a*f*ln(1-I*exp(I*(d*x+c)))*x+3/8/d^2/a*f*ln(1-I*exp(I*(d*
x+c)))*c-3/8*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2-3/8/d/a*f*ln(1+I*exp(I*
(d*x+c)))*x-3/8/d^2/a*f*ln(1+I*exp(I*(d*x+c)))*c+3/8*I*f*polylog(2,-I*exp(
I*(d*x+c)))/a/d^2+3/4*I/d^2/a*f*c*arctan(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(206) = 412$.

Time = 0.18 (sec) , antiderivative size = 792, normalized size of antiderivative = 3.29

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/48*(8*f*cos(d*x + c)^3 - 6*d*f*x + 18*(d*f*x + d*e)*cos(d*x + c)^2 - 6*
d*e + 14*f*cos(d*x + c) + 9*(I*f*cos(d*x + c)^2*sin(d*x + c) + I*f*cos(d*x
 + c)^2)*dilog(I*cos(d*x + c) + sin(d*x + c)) + 9*(I*f*cos(d*x + c)^2*sin(
d*x + c) + I*f*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) + 9*(-
I*f*cos(d*x + c)^2*sin(d*x + c) - I*f*cos(d*x + c)^2)*dilog(-I*cos(d*x + c
) + sin(d*x + c)) + 9*(-I*f*cos(d*x + c)^2*sin(d*x + c) - I*f*cos(d*x + c)
^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 9*((d*e - c*f)*cos(d*x + c)^2*
sin(d*x + c) + (d*e - c*f)*cos(d*x + c)^2)*log(cos(d*x + c) + I*sin(d*x +
c) + I) + 9*((d*e - c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*e - c*f)*cos(d*x
 + c)^2)*log(cos(d*x + c) - I*sin(d*x + c) + I) - 9*((d*f*x + c*f)*cos(d*x
 + c)^2*sin(d*x + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(I*cos(d*x + c) +
sin(d*x + c) + 1) + 9*((d*f*x + c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*f*x
+ c*f)*cos(d*x + c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) - 9*((d*f*x
+ c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(-I*
cos(d*x + c) + sin(d*x + c) + 1) + 9*((d*f*x + c*f)*cos(d*x + c)^2*sin(d*x
 + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(-I*cos(d*x + c) - sin(d*x + c) +
 1) - 9*((d*e - c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*e - c*f)*cos(d*x + c
)^2)*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 9*((d*e - c*f)*cos(d*x + c)
^2*sin(d*x + c) + (d*e - c*f)*cos(d*x + c)^2)*log(-cos(d*x + c) - I*sin(d*
x + c) + I) - 2*(9*d*f*x + 9*d*e - 5*f*cos(d*x + c))*sin(d*x + c))/(a*d...
```

Sympy [F]

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{e\sec^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
(Integral(e*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c +
d*x)**3/(sin(c + d*x) + 1), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)\sec(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{8\left(\int \frac{\sec(dx+c)^3x}{\sin(dx+c)+1}dx\right)\sin(dx+c)^3 df + 8\left(\int \frac{\sec(dx+c)^3x}{\sin(dx+c)+1}dx\right)\sin(dx+c)^2 df - 8\left(\int \frac{\sec(dx+c)^3x}{\sin(dx+c)+1}dx\right)\sin(dx+c$$

input
```
int((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
(8*int((sec(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*sin(c + d*x)**3*d*f + 8*i
nt((sec(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*sin(c + d*x)**2*d*f - 8*int((
sec(c + d*x)**3*x)/(sin(c + d*x) + 1),x)*sin(c + d*x)*d*f - 8*int((sec(c +
 d*x)**3*x)/(sin(c + d*x) + 1),x)*d*f - 3*log(tan((c + d*x)/2) - 1)*sin(c
+ d*x)**3*e - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*e + 3*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)*e + 3*log(tan((c + d*x)/2) - 1)*e + 3*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**3*e + 3*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x)**2*e - 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*e - 3*log(tan((c + d
*x)/2) + 1)*e - 3*sin(c + d*x)**3*e - 6*sin(c + d*x)**2*e + 5*e)/(8*a*d*(s
in(c + d*x)**3 + sin(c + d*x)**2 - sin(c + d*x) - 1))
```

3.284 $\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3\operatorname{arctanh}(\sin(c+dx))}{8ad} + \frac{1}{8d(a-a\sin(c+dx))} - \frac{1}{4d(a+a\sin(c+dx))} - \frac{a^3}{8d\left(a^2+a^2\sin(c+dx)\right)^2}$$

output
```
3/8*arctanh(sin(d*x+c))/a/d+1/8/d/(a-a*sin(d*x+c))-1/4/d/(a+a*sin(d*x+c))-
1/8*a^3/d/(a^2+a^2*sin(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec^2(c+dx)\left(2-3\sin(c+dx)-3\sin^2(c+dx)+3\operatorname{arctanh}(\sin(c+dx))(-1+\sin(c+dx))(1+\sin(c\right.}{8ad(1+\sin(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
-1/8*(Sec[c + d*x]^2*(2 - 3*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 3*ArcTanh[Si
n[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2))/(a*d*(1 + Sin[c + d
*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^3(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3146}$$

$$\frac{a^3\int \frac{1}{(a-a\sin(c+dx))^2(\sin(c+dx)a+a)^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^3\int \left(\frac{1}{8a^3(a-a\sin(c+dx))^2}+\frac{1}{4a^3(\sin(c+dx)a+a)^2}+\frac{1}{4a^2(\sin(c+dx)a+a)^3}+\frac{3}{8a^3(a^2-a^2\sin^2(c+dx))}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{3\text{arctanh}(\sin(c+dx))}{8a^4}+\frac{1}{8a^3(a-a\sin(c+dx))}-\frac{1}{4a^3(a\sin(c+dx)+a)}-\frac{1}{8a^2(a\sin(c+dx)+a)^2}\right)}{d}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
(a^3*((3*ArcTanh[Sin[c + d*x]])/(8*a^4) + 1/(8*a^3*(a - a*Sin[c + d*x])) -
 1/(8*a^2*(a + a*Sin[c + d*x])^2) - 1/(4*a^3*(a + a*Sin[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{1}{8(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{16}}{da}$
default	$\frac{-\frac{1}{8(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{16}}{da}$
risch	$-\frac{i\left(6i\,e^{4i(dx+c)}-6i\,e^{2i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{5i(dx+c)}+3\,e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}+i\right)^4\left(e^{i(dx+c)}-i\right)^2 ad}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{8ad}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{8ad}$
parallelrisch	$\frac{(-6\cos(2dx+2c)-3\sin(dx+c)-3\sin(3dx+3c)-6)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(6\cos(2dx+2c)+3\sin(dx+c)+3\sin(3dx+3c)}{8ad(\sin(3dx+3c)+\sin(dx+c)+2\cos(2dx+2c)}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8ad}+\frac{3\ln\left(\tan\left(\right.\right.}{8a}$

input
```
int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/8/(sin(d*x+c)-1)-3/16*ln(sin(d*x+c)-1)-1/8/(1+sin(d*x+c))^2-1/4/
(1+sin(d*x+c))+3/16*ln(1+sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.51

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{6\cos(dx+c)^2 - 3\left(\cos(dx+c)^2\sin(dx+c)+\cos(dx+c)^2\right)\log(\sin(dx+c)+1) + 3\left(\cos(dx+c)\right.}{16\left(ad\cos(dx+c)^2\sin(dx+c)+ad\,\mathrm{co}\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/16*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)
*log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*
log(-sin(d*x + c) + 1) - 6*sin(d*x + c) - 2)/(a*d*cos(d*x + c)^2*sin(d*x +
 c) + a*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{2\left(3\sin(dx+c)^2+3\sin(dx+c)-2\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{3\log(\sin(dx+c)+1)}{a}+\frac{3\log(\sin(dx+c)-1)}{a}}{16\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin
(d*x + c)^2 - a*sin(d*x + c) - a) - 3*log(sin(d*x + c) + 1)/a + 3*log(sin(
d*x + c) - 1)/a)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3\log(|\sin(dx+c)+1|)}{16\,ad}-\frac{3\log(|\sin(dx+c)-1|)}{16\,ad}-\frac{3\sin(dx+c)^2+3\sin(dx+c)-2}{8\,ad(\sin(dx+c)+1)^2(\sin(dx+c)-1)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
3/16*log(abs(sin(d*x + c) + 1))/(a*d) - 3/16*log(abs(sin(d*x + c) - 1))/(a
*d) - 1/8*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*d*(sin(d*x + c) + 1)^
2*(sin(d*x + c) - 1))
```

Mupad [B] (verification not implemented)

Time = 37.63 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3\,\mathrm{atanh}(\sin(c+d\,x))}{8\,a\,d} + \frac{\frac{3\sin(c+d\,x)^2}{8}+\frac{3\,\sin(c+d\,x)}{8}-\frac{1}{4}}{d\,\left(-a\sin(c+d\,x)^3-a\sin(c+d\,x)^2+a\,\sin(c+d\,x)+a\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

output
```
(3*atanh(sin(c + d*x)))/(8*a*d) + ((3*sin(c + d*x))/8 + (3*sin(c + d*x)^2)
/8 - 1/4)/(d*(a + a*sin(c + d*x) - a*sin(c + d*x)^2 - a*sin(c + d*x)^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.58

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^3-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{}$$

input
```
int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3 - 3*log(tan((c + d*x)/2) -
 1)*sin(c + d*x)**2 + 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x) + 3*log(tan
((c + d*x)/2) - 1) + 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3 + 3*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2 - 3*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x) - 3*log(tan((c + d*x)/2) + 1) - 3*sin(c + d*x)**3 - 6*sin(c + d*x)**
2 + 5)/(8*a*d*(sin(c + d*x)**3 + sin(c + d*x)**2 - sin(c + d*x) - 1))
```

3.285 $\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 39.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \text{ 5048}$$
$$\int \frac{\sec^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)}dx$$

input
```
Int[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sec \left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+a\sin \left(dx+c\right)\right)}dx$$

input
```
int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\sec \left(dx+c\right)^{3}}{(fx+e)(a\sin \left(dx+c\right)+a)}\,dx$$

input
```
integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\frac{\int \frac{\sec^3\left(c+dx\right)}{e\sin\left(c+dx\right)+e+fx\sin\left(c+dx\right)+fx}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)
/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 38.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^3\,\left(e+f\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^3*(e + f*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\sec(dx+c)^3}{\sin(dx+c)e+\sin(dx+c)fx+e+fx}dx}{a}$$

input
```
int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(c + d*x)**3/(sin(c + d*x)*e + sin(c + d*x)*f*x + e + f*x),x)/a
```

3.286 $\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Int}\left(\frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 51.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

input
```
Integrate[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
Integrate[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 5048$$
$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)}dx$$

input
```
Int[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sec (dx + c)^3}{(fx + e)^2 (a + a \sin (dx + c))} dx$$

input
```
int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{\sec^3(c + dx)}{(e + fx)^2(a + a\sin(c + dx))} dx = \int \frac{\sec (dx + c)^3}{(fx + e)^2(a \sin (dx + c) + a)} dx$$

input
```
integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*
e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 9.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{\sec^3(c + dx)}{(e + fx)^2(a + a\sin(c + dx))} dx = \frac{\int \frac{\sec^3 (c+dx)}{e^2 \sin (c+dx)+e^2+2efx \sin (c+dx)+2efx+f^2x^2 \sin (c+dx)+f^2x^2} dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x)
+ 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 38.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(e+f\,x)^2\,(a+a\,\sin(c+d\,x))}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^3*(e + f*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 97475, normalized size of antiderivative = 3481.25

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

output
```
( - 1050*cos(c + d*x)*int(tan((c + d*x)/2)**7/(tan((c + d*x)/2)**9*e**2 +
2*tan((c + d*x)/2)**9*e*f*x + tan((c + d*x)/2)**9*f**2*x**2 + tan((c + d*x
)/2)**8*e**2 + 2*tan((c + d*x)/2)**8*e*f*x + tan((c + d*x)/2)**8*f**2*x**2
 - 4*tan((c + d*x)/2)**7*e**2 - 8*tan((c + d*x)/2)**7*e*f*x - 4*tan((c + d
*x)/2)**7*f**2*x**2 - 4*tan((c + d*x)/2)**6*e**2 - 8*tan((c + d*x)/2)**6*e
*f*x - 4*tan((c + d*x)/2)**6*f**2*x**2 + 6*tan((c + d*x)/2)**5*e**2 + 12*t
an((c + d*x)/2)**5*e*f*x + 6*tan((c + d*x)/2)**5*f**2*x**2 + 6*tan((c + d*
x)/2)**4*e**2 + 12*tan((c + d*x)/2)**4*e*f*x + 6*tan((c + d*x)/2)**4*f**2*
x**2 - 4*tan((c + d*x)/2)**3*e**2 - 8*tan((c + d*x)/2)**3*e*f*x - 4*tan((c
 + d*x)/2)**3*f**2*x**2 - 4*tan((c + d*x)/2)**2*e**2 - 8*tan((c + d*x)/2)*
*2*e*f*x - 4*tan((c + d*x)/2)**2*f**2*x**2 + tan((c + d*x)/2)*e**2 + 2*tan
((c + d*x)/2)*e*f*x + tan((c + d*x)/2)*f**2*x**2 + e**2 + 2*e*f*x + f**2*x
**2),x)*sin(c + d*x)**3*e**2*f**2 - 1050*cos(c + d*x)*int(tan((c + d*x)/2)
**7/(tan((c + d*x)/2)**9*e**2 + 2*tan((c + d*x)/2)**9*e*f*x + tan((c + d*x
)/2)**9*f**2*x**2 + tan((c + d*x)/2)**8*e**2 + 2*tan((c + d*x)/2)**8*e*f*x
 + tan((c + d*x)/2)**8*f**2*x**2 - 4*tan((c + d*x)/2)**7*e**2 - 8*tan((c +
 d*x)/2)**7*e*f*x - 4*tan((c + d*x)/2)**7*f**2*x**2 - 4*tan((c + d*x)/2)**
6*e**2 - 8*tan((c + d*x)/2)**6*e*f*x - 4*tan((c + d*x)/2)**6*f**2*x**2 + 6
*tan((c + d*x)/2)**5*e**2 + 12*tan((c + d*x)/2)**5*e*f*x + 6*tan((c + d*x)
/2)**5*f**2*x**2 + 6*tan((c + d*x)/2)**4*e**2 + 12*tan((c + d*x)/2)**4*...
```

3.287 $\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 449

$$
\begin{aligned}
\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx
&= \frac{(e+fx)^{1+m}}{2af(1+m)} + \frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{8ad} \\
&+ \frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{id(e+fx)}{f}\right)}{8ad} \\
&- \frac{i2^{-3-m}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{2id(e+fx)}{f}\right)}{ad} \\
&+ \frac{i2^{-3-m}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{2id(e+fx)}{f}\right)}{ad} \\
&+ \frac{3^{-1-m}e^{3i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{3id(e+fx)}{f}\right)}{8ad} \\
&+ \frac{3^{-1-m}e^{-3i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{3id(e+fx)}{f}\right)}{8ad}
\end{aligned}
$$

output
```
1/2*(f*x+e)^(1+m)/a/f/(1+m)+1/8*exp(I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-I*d*
(f*x+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+1/8*(f*x+e)^m*GAMMA(1+m,I*d*(f*x+e)/f)
/a/d/exp(I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)-I*2^(-3-m)*exp(2*I*(c-d*e/f))*(f
*x+e)^m*GAMMA(1+m,-2*I*d*(f*x+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+I*2^(-3-m)*(f
*x+e)^m*GAMMA(1+m,2*I*d*(f*x+e)/f)/a/d/exp(2*I*(c-d*e/f))/((I*d*(f*x+e)/f)
^m)+1/8*3^(-1-m)*exp(3*I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-3*I*d*(f*x+e)/f)/
a/d/((-I*d*(f*x+e)/f)^m)+1/8*3^(-1-m)*(f*x+e)^m*GAMMA(1+m,3*I*d*(f*x+e)/f)
/a/d/exp(3*I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)
```

Mathematica [A] (verified)

Time = 12.20 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.90

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{i(e+fx)^m\left(-\frac{12id(e+fx)}{f(1+m)} - 3ie^{i\left(c-\frac{de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right) - 3ie^{-i\left(c-\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(\right.\right.}{}$$

input
```
Integrate[((e + f*x)^m*Cos[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

output
```
((I/24)*(e + f*x)^m*(((-12*I)*d*(e + f*x))/(f*(1 + m)) - ((3*I)*E^(I*(c -
(d*e)/f))*Gamma[1 + m, ((-I)*d*(e + f*x))/f])/(((-I)*d*(e + f*x))/f)^m - (
(3*I)*Gamma[1 + m, (I*d*(e + f*x))/f])/(E^(I*(c - (d*e)/f))*((I*d*(e + f*x
))/f)^m) - (3*E^((2*I)*(c - (d*e)/f))*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f]
)/(2^m*(((-I)*d*(e + f*x))/f)^m) + (3*Gamma[1 + m, ((2*I)*d*(e + f*x))/f])
/(2^m*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m) - (I*E^((3*I)*(c - (d
*e)/f))*Gamma[1 + m, ((-3*I)*d*(e + f*x))/f])/(3^m*(((-I)*d*(e + f*x))/f)^
m) - (I*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(3^m*E^((3*I)*(c - (d*e)/f))*
((I*d*(e + f*x))/f)^m))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(a*d*(1 +
 Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5034, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5034}$$

$$\frac{\int (e+fx)^m\cos^2(c+dx)dx}{a}-\frac{\int (e+fx)^m\cos^2(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^m\sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a}-\frac{\int (e+fx)^m\cos^2(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{1}{2}\cos(2c+2dx)(e+fx)^m+\frac{1}{2}(e+fx)^m\right)dx}{a}-\frac{\int (e+fx)^m\cos^2(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int (e+fx)^m\cos^2(c+dx)\sin(c+dx)dx}{a}+$$
$$\frac{-\frac{i2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{2id(e+fx)}{f}\right)}{d}+\frac{i2^{-m-3}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{2id(e+fx)}{f}\right)}{d}+ (}{a}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{1}{4}\sin(c+dx)(e+fx)^m+\frac{1}{4}\sin(3c+3dx)(e+fx)^m\right)dx}{a}+$$
$$\frac{-\frac{i2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{2id(e+fx)}{f}\right)}{d}+\frac{i2^{-m-3}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{2id(e+fx)}{f}\right)}{d}+ (}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{i2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{2id(e+fx)}{f}\right)}{d}+\frac{i2^{-m-3}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{2id(e+fx)}{f}\right)}{d}+\cdots}{a}$$

$$\frac{-\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{8d}-\frac{3^{-m-1}e^{3i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{3id(e+fx)}{f}\right)}{8d}-\frac{e^{-i\left(c-\frac{de}{f}\right)}\cdots}{}}{a}$$

input
```
Int[((e + f*x)^m*Cos[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

output
```
((e + f*x)^(1 + m)/(2*f*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*(c - (d*e)/f))*(
e + f*x)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f])/(d*(((-I)*d*(e + f*x))/f)
^m) + (I*2^(-3 - m)*(e + f*x)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f])/(d*E^
((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m))/a - (-1/8*(E^(I*(c - (d*e)/f
))*(e + f*x)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f])/(d*(((-I)*d*(e + f*x))/
f)^m) - ((e + f*x)^m*Gamma[1 + m, (I*d*(e + f*x))/f])/(8*d*E^(I*(c - (d*e)
/f))*((I*d*(e + f*x))/f)^m) - (3^(-1 - m)*E^((3*I)*(c - (d*e)/f))*(e + f*x
)^m*Gamma[1 + m, ((-3*I)*d*(e + f*x))/f])/(8*d*(((-I)*d*(e + f*x))/f)^m) -
 (3^(-1 - m)*(e + f*x)^m*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(8*d*E^((3*I
)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [F]

$$\int \frac{(fx+e)^m \cos(dx+c)^4}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3\,(fm+f)e^{\left(-\frac{fm\log\left(\frac{i\,d}{f}\right)-i\,de+i\,cf}{f}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,de}{f}\right)-3\,(i\,fm+i\,f)e^{\left(-\frac{fm\log\left(-\frac{2i\,d}{f}\right)+2i\,de-2i\,cf}{f}\right)}\Gamma\left(m+1,-\right.}{}$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(3*(f*m + f)*e^(-(f*m*log(I*d/f) - I*d*e + I*c*f)/f)*gamma(m + 1, (I*
d*f*x + I*d*e)/f) - 3*(I*f*m + I*f)*e^(-(f*m*log(-2*I*d/f) + 2*I*d*e - 2*I
*c*f)/f)*gamma(m + 1, -2*(I*d*f*x + I*d*e)/f) + (f*m + f)*e^(-(f*m*log(-3*
I*d/f) + 3*I*d*e - 3*I*c*f)/f)*gamma(m + 1, -3*(I*d*f*x + I*d*e)/f) + 3*(f
*m + f)*e^(-(f*m*log(-I*d/f) + I*d*e - I*c*f)/f)*gamma(m + 1, (-I*d*f*x -
I*d*e)/f) - 3*(-I*f*m - I*f)*e^(-(f*m*log(2*I*d/f) - 2*I*d*e + 2*I*c*f)/f)
*gamma(m + 1, -2*(-I*d*f*x - I*d*e)/f) + (f*m + f)*e^(-(f*m*log(3*I*d/f) -
 3*I*d*e + 3*I*c*f)/f)*gamma(m + 1, -3*(-I*d*f*x - I*d*e)/f) + 12*(d*f*x +
 d*e)*(f*x + e)^m)/(a*d*f*m + a*d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f*x+e)**m*cos(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)^4\,(e+f\,x)^m}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^4*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)^4*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(fx+e)^m \cos(dx+c)^4}{\sin(dx+c)+1}dx}{a}$$

input
```
int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*cos(c + d*x)**4)/(sin(c + d*x) + 1),x)/a
```

3.288 $\int \frac{(e+fx)^m \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 277

$$\begin{aligned}\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad}\\
&\quad+\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{id(e+fx)}{f}\right)}{2ad}\\
&\quad+\frac{2^{-3-m}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{2id(e+fx)}{f}\right)}{ad}\\
&\quad+\frac{2^{-3-m}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{2id(e+fx)}{f}\right)}{ad}\end{aligned}$$

output
```
-1/2*I*exp(I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-I*d*(f*x+e)/f)/a/d/((-I*d*(f*
x+e)/f)^m)+1/2*I*(f*x+e)^m*GAMMA(1+m,I*d*(f*x+e)/f)/a/d/exp(I*(c-d*e/f))/(
(I*d*(f*x+e)/f)^m)+2^(-3-m)*exp(2*I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-2*I*d*
(f*x+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+2^(-3-m)*(f*x+e)^m*GAMMA(1+m,2*I*d*(f*
x+e)/f)/a/d/exp(2*I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)
```

Mathematica [A] (verified)

Time = 9.89 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91

$$\int \frac{(e+fx)^m \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{2^{-3-m}e^{-\frac{2i(de+cf)}{f}}(e+fx)^m\left(\frac{d^2(e+fx)^2}{f^2}\right)^{-m}\left(-i2^{2+m}e^{i\left(3c+\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^m\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)+i2^{2+m}e^{i(}\right.}{}$$

input
```
Integrate[((e + f*x)^m*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(2^(-3 - m)*(e + f*x)^m*((-I)*2^(2 + m)*E^(I*(3*c + (d*e)/f))*((I*d*(e + f
*x))/f)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f] + I*2^(2 + m)*E^(I*(c + (3*d*
e)/f))*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, (I*d*(e + f*x))/f] + E^((4*I)
*c)*((I*d*(e + f*x))/f)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f] + E^(((4*I)
*d*e)/f)*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f]))/(a
*d*E^(((2*I)*(d*e + c*f))/f)*((d^2*(e + f*x)^2)/f^2)^m)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5034, 3042, 3788, 26, 2612, 4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(e+fx)^m}{a\sin(c+dx)+a}\,dx$$
$$\Big\downarrow \text{5034}$$
$$\frac{\int (e+fx)^m\cos(c+dx)dx}{a}-\frac{\int (e+fx)^m\cos(c+dx)\sin(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int (e+fx)^m\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\int (e+fx)^m\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{\int (e+fx)^m \cos(c+dx)\sin(c+dx)dx}{a}+\frac{\frac{1}{2}i\int -ie^{-i(c+dx)}(e+fx)^m dx-\frac{1}{2}i\int ie^{i(c+dx)}(e+fx)^m dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\int (e+fx)^m \cos(c+dx)\sin(c+dx)dx}{a}+\frac{\frac{1}{2}\int e^{-i(c+dx)}(e+fx)^m dx+\frac{1}{2}\int e^{i(c+dx)}(e+fx)^m dx}{a}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{\int (e+fx)^m \cos(c+dx)\sin(c+dx)dx}{a}+\frac{\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}}{a}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \frac{1}{2}(e+fx)^m \sin(2c+2dx)dx}{a}+\frac{\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int (e+fx)^m \sin(2c+2dx)dx}{2a}+\frac{\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (e+fx)^m \sin(2c+2dx)dx}{2a}+\frac{\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}}{a}$$

$$\Big\downarrow \text{3789}$$

$$\frac{\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}}{a}-\frac{\frac{1}{2}i\int e^{-2i(c+dx)}(e+fx)^m dx-\frac{1}{2}i\int e^{2i(c+dx)}(e+fx)^m dx}{2a}$$

$$\Big\downarrow \ 2612$$

$$\frac{\frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}}{a}-\frac{-\frac{2^{-m-2}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{2id(e+fx)}{f}\right)}{d}-\frac{2^{-m-2}e^{-2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{2id(e+fx)}{f}\right)}{d}}{2a}$$

input
```
Int[((e + f*x)^m*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

output
```
(((-1/2*I)*E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-I)*d*(e + f*x))
/f])/(d*(((-I)*d*(e + f*x))/f)^m) + ((I/2)*(e + f*x)^m*Gamma[1 + m, (I*d*(
e + f*x))/f])/(d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m))/a - (-((2^(-2
 - m)*E^((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x)
)/f])/(d*(((-I)*d*(e + f*x))/f)^m)) - (2^(-2 - m)*(e + f*x)^m*Gamma[1 + m,
 ((2*I)*d*(e + f*x))/f])/(d*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [F]

$$\int \frac{(fx+e)^m \cos(dx+c)^3}{a+a\sin(dx+c)}\,dx$$

input
```
int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{4i\,e^{\left(-\frac{fm\log\left(\frac{i\,d}{f}\right)-i\,de+i\,cf}{f}\right)}\Gamma\left(m+1,\frac{i\,dfx+i\,de}{f}\right)+e^{\left(-\frac{fm\log\left(-\frac{2i\,d}{f}\right)+2i\,de-2i\,cf}{f}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,de)}{f}\right)-4i\,e^{\left(-\frac{f\cdots}{}\right.}\cdots}{8\,ad}$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*I*e^(-(f*m*log(I*d/f) - I*d*e + I*c*f)/f)*gamma(m + 1, (I*d*f*x + I
*d*e)/f) + e^(-(f*m*log(-2*I*d/f) + 2*I*d*e - 2*I*c*f)/f)*gamma(m + 1, -2*
(I*d*f*x + I*d*e)/f) - 4*I*e^(-(f*m*log(-I*d/f) + I*d*e - I*c*f)/f)*gamma(
m + 1, (-I*d*f*x - I*d*e)/f) + e^(-(f*m*log(2*I*d/f) - 2*I*d*e + 2*I*c*f)/
f)*gamma(m + 1, -2*(-I*d*f*x - I*d*e)/f))/(a*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f*x+e)**m*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m\cos(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m\cos(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)^3\,(e+f\,x)^m}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^3*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)^3*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^m \cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(fx+e)^m \cos(dx+c)^3}{\sin(dx+c)+1}dx}{a}$$

input
```
int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*cos(c + d*x)**3)/(sin(c + d*x) + 1),x)/a
```

3.289 $\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(e+fx)^{1+m}}{af(1+m)}+\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,\frac{id(e+fx)}{f}\right)}{2ad}$$

output
```
(f*x+e)^(1+m)/a/f/(1+m)+1/2*exp(I*(c-d*e/f))*(f*x+e)^m*GAMMA(1+m,-I*d*(f*x
+e)/f)/a/d/((-I*d*(f*x+e)/f)^m)+1/2*(f*x+e)^m*GAMMA(1+m,I*d*(f*x+e)/f)/a/d
/exp(I*(c-d*e/f))/((I*d*(f*x+e)/f)^m)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m \left(\frac{d^2(e+fx)^2}{f^2}\right)^{-m} \left(2de^{-i\left(c-\frac{de}{f}\right)}(e+fx)\left(\frac{d^2(e+fx)^2}{f^2}\right)^m + f(1+m)\left(\frac{id(e+fx)}{f}\right)^m \Gamma\left(1+m,\right.\right.}{2adf(1+m)(}$$

input
```
Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(E^(I*(c - (d*e)/f))*(e + f*x)^m*((2*d*(e + f*x)*((d^2*(e + f*x)^2)/f^2)^m
)/E^(I*(c - (d*e)/f)) + f*(1 + m)*((I*d*(e + f*x))/f)^m*Gamma[1 + m, ((-I)
*d*(e + f*x))/f] + (f*(1 + m)*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, (I*d*(
e + f*x))/f])/E^((2*I)*(c - (d*e)/f)))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^2)/(2*a*d*f*(1 + m)*((d^2*(e + f*x)^2)/f^2)^m*(1 + Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5034, 17, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}\,dx$$
$$\Big\downarrow \; 5034$$
$$\frac{\int (e+fx)^m dx}{a} - \frac{\int (e+fx)^m \sin(c+dx)dx}{a}$$
$$\Big\downarrow \; 17$$
$$\frac{(e+fx)^{m+1}}{af(m+1)} - \frac{\int (e+fx)^m \sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^{m+1}}{af(m+1)}-\frac{\int (e+fx)^m\sin(c+dx)dx}{a}$$

$$\Big\downarrow \text{3789}$$

$$\frac{(e+fx)^{m+1}}{af(m+1)}-\frac{\frac{1}{2}i\int e^{-i(c+dx)}(e+fx)^m dx-\frac{1}{2}i\int e^{i(c+dx)}(e+fx)^m dx}{a}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(e+fx)^{m+1}}{af(m+1)}-\frac{-\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,-\frac{id(e+fx)}{f}\right)}{2d}-\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(m+1,\frac{id(e+fx)}{f}\right)}{2d}}{a}$$

input
```
Int[((e + f*x)^m*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
(e + f*x)^(1 + m)/(a*f*(1 + m)) - (-1/2*(E^(I*(c - (d*e)/f))*(e + f*x)^m*G
amma[1 + m, ((-I)*d*(e + f*x))/f])/(d*(((-I)*d*(e + f*x))/f)^m) - ((e + f*
x)^m*Gamma[1 + m, (I*d*(e + f*x))/f])/(2*d*E^(I*(c - (d*e)/f))*((I*d*(e +
f*x))/f)^m))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5034
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cos[c +
d*x]^(n - 2), x], x] - Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[
c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^
2 - b^2, 0]
```

Maple [F]

$$\int \frac{(fx+e)^m \cos(dx+c)^2}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(fm+f)e^{\left(-\frac{fm\log\left(\frac{id}{f}\right)-ide+icf}{f}\right)}\Gamma\left(m+1,\frac{idfx+ide}{f}\right)+(fm+f)e^{\left(-\frac{fm\log\left(-\frac{id}{f}\right)+ide-icf}{f}\right)}\Gamma\left(m+1,\frac{-idfx-ide}{f}\right.}{2\,(adfm+adf)}$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((f*m + f)*e^(-(f*m*log(I*d/f) - I*d*e + I*c*f)/f)*gamma(m + 1, (I*d*f
*x + I*d*e)/f) + (f*m + f)*e^(-(f*m*log(-I*d/f) + I*d*e - I*c*f)/f)*gamma(
m + 1, (-I*d*f*x - I*d*e)/f) + 2*(d*f*x + d*e)*(f*x + e)^m)/(a*d*f*m + a*d
*f)
```

Sympy [F]

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m\cos^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*cos(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m\cos(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\cos(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\cos(c+d\,x)^2\,(e+f\,x)^m}{a+a\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)^2*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{(fx+e)^m\cos(dx+c)\,fm+(fx+e)^m\cos(dx+c)\,f+(fx+e)^m\,de+(fx+e)^m\,dfx-\left(\int \frac{(fx+e)^m\cos(\ldots}{fx+e}\right.}{adf\,(m+1)}$$

input
```
int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
((e + f*x)**m*cos(c + d*x)*f*m + (e + f*x)**m*cos(c + d*x)*f + (e + f*x)**
m*d*e + (e + f*x)**m*d*f*x - int(((e + f*x)**m*cos(c + d*x))/(e + f*x),x)*
f**2*m**2 - int(((e + f*x)**m*cos(c + d*x))/(e + f*x),x)*f**2*m)/(a*d*f*(m
 + 1))
```

3.290 $\int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)} \, dx = \mathrm{Int}\left(\frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)}, x\right)$$

output
```
Defer(Int)((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 12.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)} \, dx$$

input
```
Integrate[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(e+fx)^m}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 5048}$$

$$\int \frac{\cos(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \cos(dx+c)}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*cos(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\cos\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\cos\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)\,{\left(e+f\,x\right)}^m}{a+a\,\sin\left(c+d\,x\right)} \,dx$$

input
```
int((cos(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x)^m)/(a + a*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-(fx+e)^m e-(fx+e)^m fx+2\left(\int \frac{(fx+e)^m}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right) fm+2\left(\int \frac{(fx+e)^m}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}dx\right) f}{af\,(m+1)}$$

input
```
int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
( - (e + f*x)**m*e - (e + f*x)**m*f*x + 2*int((e + f*x)**m/(tan((c + d*x)/
2) + 1),x)*f*m + 2*int((e + f*x)**m/(tan((c + d*x)/2) + 1),x)*f)/(a*f*(m +
 1))
```

3.291 $\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^m}{a\sin(c+dx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e+fx)^m}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m/(sin(c + d*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx=\int \frac{(e+f\,x)^m}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)^m/(a + a*sin(c + d*x)),x)
```

output
```
int((e + f*x)^m/(a + a*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.70

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$
$$=\frac{2(fx+e)^m\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\int \frac{(fx+e)^m\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)fx+e+fx}dx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)fm-2\left(\int \frac{(fx+e)^m\,\mathrm{t}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e+\mathrm{tan}}\right.}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

output
```
(2*((e + f*x)**m*tan((c + d*x)/2) - int(((e + f*x)**m*tan((c + d*x)/2))/(t
an((c + d*x)/2)*e + tan((c + d*x)/2)*f*x + e + f*x),x)*tan((c + d*x)/2)*f*
m - int(((e + f*x)**m*tan((c + d*x)/2))/(tan((c + d*x)/2)*e + tan((c + d*x
)/2)*f*x + e + f*x),x)*f*m))/(a*d*(tan((c + d*x)/2) + 1))
```

3.292 $\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \operatorname{Int}\left(\frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 141.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\sec(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input

```
Int[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 5048

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sec(dx+c)}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 25.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 37.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{\cos\left(c+d\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(cos(c + d*x)*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(fx+e)^m \sec(dx+c)}{\sin(dx+c)+1}dx}{a}$$

input
```
int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*sec(c + d*x))/(sin(c + d*x) + 1),x)/a
```

3.293 $\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 25.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}\,dx$$
$$\Big\downarrow \; 5048$$
$$\int \frac{\sec^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a}dx$$

input
```
Int[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sec(dx+c)^2}{a+a\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 104.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{(e+fx)^m \sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate((f*x+e)**m*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 38.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(e+f\,x)^m}{\cos\left(c+d\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(cos(c + d*x)^2*(a + a*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 6074, normalized size of antiderivative = 216.93

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

output

```
(3*(e + f*x)**m*cos(c + d*x)*sin(c + d*x)*d*e + 3*(e + f*x)**m*cos(c + d*x
)*sin(c + d*x)*d*f*x + (e + f*x)**m*cos(c + d*x)*sin(c + d*x)*f*m + (e + f
*x)**m*cos(c + d*x)*sin(c + d*x)*f + 3*(e + f*x)**m*cos(c + d*x)*d*e + 3*(
e + f*x)**m*cos(c + d*x)*d*f*x + 7*(e + f*x)**m*cos(c + d*x)*f*m + 7*(e +
f*x)**m*cos(c + d*x)*f - 12*cos(c + d*x)*int((e + f*x)**m/(tan((c + d*x)/2
)**5*e + tan((c + d*x)/2)**5*f*x + 3*tan((c + d*x)/2)**4*e + 3*tan((c + d*
x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*f*x - 2*tan
((c + d*x)/2)**2*e - 2*tan((c + d*x)/2)**2*f*x - 3*tan((c + d*x)/2)*e - 3*
tan((c + d*x)/2)*f*x - e - f*x),x)*sin(c + d*x)*d*e*f*m - 12*cos(c + d*x)*
int((e + f*x)**m/(tan((c + d*x)/2)**5*e + tan((c + d*x)/2)**5*f*x + 3*tan(
(c + d*x)/2)**4*e + 3*tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e +
2*tan((c + d*x)/2)**3*f*x - 2*tan((c + d*x)/2)**2*e - 2*tan((c + d*x)/2)**
2*f*x - 3*tan((c + d*x)/2)*e - 3*tan((c + d*x)/2)*f*x - e - f*x),x)*sin(c
+ d*x)*d*e*f + 14*cos(c + d*x)*int((e + f*x)**m/(tan((c + d*x)/2)**5*e + t
an((c + d*x)/2)**5*f*x + 3*tan((c + d*x)/2)**4*e + 3*tan((c + d*x)/2)**4*f
*x + 2*tan((c + d*x)/2)**3*e + 2*tan((c + d*x)/2)**3*f*x - 2*tan((c + d*x)
/2)**2*e - 2*tan((c + d*x)/2)**2*f*x - 3*tan((c + d*x)/2)*e - 3*tan((c + d
*x)/2)*f*x - e - f*x),x)*sin(c + d*x)*f**2*m**2 + 14*cos(c + d*x)*int((e +
 f*x)**m/(tan((c + d*x)/2)**5*e + tan((c + d*x)/2)**5*f*x + 3*tan((c + d*x
)/2)**4*e + 3*tan((c + d*x)/2)**4*f*x + 2*tan((c + d*x)/2)**3*e + 2*tan...
```

3.294 $\int \frac{(e+fx)^3 \cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 432

$$\begin{aligned}\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{i(e+fx)^4}{4bf}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} \\ &\quad+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} \\ &\quad-\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} \\ &\quad-\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} \\ &\quad+\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} \\ &\quad+\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^3} \\ &\quad+\frac{6if^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4} \\ &\quad+\frac{6if^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}\end{aligned}$$

output
```
-1/4*I*(f*x+e)^4/b/f+(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/b/d+(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d-3*I*f*(f*
x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2-3*I*f*(f*x+
e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2+6*f^2*(f*x+e)
*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3+6*f^2*(f*x+e)*pol
ylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3+6*I*f^3*polylog(4,I*b
*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^4+6*I*f^3*polylog(4,I*b*exp(I*(d*
x+c))/(a+(a^2-b^2)^(1/2)))/b/d^4
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{-\frac{i(e+fx)^4}{f} + \frac{4(e+fx)^3\log\left(1+\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)}{d} + \frac{4(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} + 12f\left(-id^2(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)+2f\left(d(e\right.\right.}{}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(((-I)*(e + f*x)^4)/f + (4*(e + f*x)^3*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
 Sqrt[a^2 - b^2])])/d + (4*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/d + (12*f*((-I)*d^2*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])] + 2*f*(d*(e + f*x)*PolyLog[3, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])] + I*f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])))/d^4 + (12*f*((-I)*d^2*(e + f*x)^2*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 2*f*(d*(e + f*x)*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + I*f*PolyLog[4, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])))/d^4)/(4*b)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5030, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5030}$$

$$\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^4}{4bf}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-1/4*I)*(e + f*x)^4)/(b*f) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog
[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d) - (3*f*((I*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - ((2*I)*f
*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])
/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/d))
/(b*d)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1777 vs. $2(370) = 740$.

Time = 0.23 (sec) , antiderivative size = 1777, normalized size of antiderivative = 4.11

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-6*I*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*f^3*polylog(4, -
(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))/b) + 6*I*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin
(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 + 6*I*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 3*(I*d^2*f^3*x^2 + 2*I
*d^2*e*f^2*x + I*d^2*e^2*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(I
*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2*f)*dilog((I*a*cos(d*x + c) - a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f)*dilog((-I
*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*
d^2*e^2*f)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (d^3*e^3 - 3*c*d^2*e
^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
+ 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d
*e*f^2 - c^3*f^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a
^2 - b^2)/b^2) - 2*I*a) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e+f\,x)^3}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{8\left(\int \frac{x^3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a}dx\right)abd\,f^3+24\left(\int \frac{x^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a}dx\right)abde\,f^2+8\left(\int \frac{}{\tan\left(\frac{dx}{2}\right.}\right.}{}$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(8*int(x**3/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b*d*f*
*3 + 24*int(x**2/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b
*d*e*f**2 + 8*int((tan((c + d*x)/2)*x**3)/(tan((c + d*x)/2)**2*a + 2*tan((
c + d*x)/2)*b + a),x)*b**2*d*f**3 + 24*int((tan((c + d*x)/2)*x**2)/(tan((c
 + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**2*d*e*f**2 + 24*int((tan
((c + d*x)/2)*x)/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**
2*d*e**2*f + 24*int(x/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x
)*a*b*d*e**2*f - 4*log(tan((c + d*x)/2)**2 + 1)*a*e**3 + 4*log(tan((c + d*
x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*a*e**3 - 6*b*d*e**2*f*x**2 - 4*b*d*
e*f**2*x**3 - b*d*f**3*x**4)/(4*a*b*d)
```

3.295 $\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 320

$$\begin{aligned}\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = &-\frac{i(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\\
&+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd}\\
&-\frac{2if(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\\
&-\frac{2if(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\\
&+\frac{2f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}\\
&+\frac{2f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}\end{aligned}$$

output
```
-1/3*I*(f*x+e)^3/b/f+(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/b/d+(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d-2*I*f*(f*
x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2-2*I*f*(f*x+e)
*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2+2*f^2*polylog(3,I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^3+2*f^2*polylog(3,I*b*exp(I*(d*
x+c))/(a+(a^2-b^2)^(1/2)))/b/d^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$=\frac{-\frac{i(e+fx)^3}{f}+\frac{3(e+fx)^2\log\left(1+\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)}{d}+\frac{3(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}+\frac{6f\left(-id(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)+f\operatorname{PolyLog}\right.}{d^3}}{3b}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(((-I)*(e + f*x)^3)/f + (3*(e + f*x)^2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
 Sqrt[a^2 - b^2])])/d + (3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/d + (6*f*((-I)*d*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])] + f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt
[a^2 - b^2])]))/d^3 + (6*f*((-I)*d*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])] + f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])]))/d^3)/(3*b)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5030, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5030}$$

$$\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^3}{3bf}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-1/3*I)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(274) = 548$.

Time = 0.22 (sec) , antiderivative size = 1235, normalized size of antiderivative = 3.86

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*f^2*polylog(3, -(I*a*
cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2))/b) + 2*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*f^
2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(I*d*f^2*x + I*d*e*f)*dilog((
I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*d*f^2*x + I*d*e*f)*dilog((I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) - 2*(-I*d*f^2*x - I*d*e*f)*dilog((-I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1) - 2*(-I*d*f^2*x - I*d*e*f)*dilog((-I*a*cos(d*x + c) - a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 - b)/b + 1) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cos(d*x + c) + 2*I*
b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d^2*e^2 - 2*c*d*e*
f + c^2*f^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
b^2)/b^2) - 2*I*a) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(-2*b*cos(d*x + c)
 + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d^2*e^2 - 2
*c*d*e*f + c^2*f^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2\cos{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e+f\,x)^2}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{6\left(\int \frac{x^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a}dx\right)abd\,f^2+6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)x^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a}dx\right)b^2d\,f^2+12\left(\int \frac{}{\tan\left(\frac{dx}{2}+\right.}\right.}{}$$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(6*int(x**2/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b*d*f*
*2 + 6*int((tan((c + d*x)/2)*x**2)/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x
)/2)*b + a),x)*b**2*d*f**2 + 12*int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)
**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**2*d*e*f + 12*int(x/(tan((c + d*x)/
2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b*d*e*f - 3*log(tan((c + d*x)/2)*
*2 + 1)*a*e**2 + 3*log(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*a
*e**2 - 3*b*d*e*f*x**2 - b*d*f**2*x**3)/(3*a*b*d)
```

3.296 $\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} - \frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}$$

output
```
-1/2*I*(f*x+e)^2/b/f+(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/
b/d+(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/d-I*f*polylog(2
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/d^2-I*f*polylog(2,I*b*exp(I*(d*
x+c))/(a+(a^2-b^2)^(1/2)))/b/d^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i\left(d(e+fx)\left(de+dfx+2if\log\left(1+\frac{ibe^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)+2if\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)\right)+2f^2\operatorname{PolyLog}\left(2,-\frac{ib}{-a}\right.\right.}{2bd^2f}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-1/2*I)*(d*(e + f*x)*(d*e + d*f*x + (2*I)*f*Log[1 + (I*b*E^(I*(c + d*x))
)/(-a + Sqrt[a^2 - b^2])] + (2*I)*f*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])]) + 2*f^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2
 - b^2])] + 2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])
)/(b*d^2*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5030, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \ 5030$$

$$\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^2}{2bf}$$

$$\Big\downarrow \ 2620$$

$$-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2715}$$

$$\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}$$

input
```
Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-1/2*I)*(e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*d^2)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(187) = 374$.

Time = 0.97 (sec) , antiderivative size = 1006, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{if\,x^2}{2b}-\frac{ibf\,\mathrm{dilog}\left(\frac{ia+\mathrm{e}^{i(dx+c)}b-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right)}{d^2(-a^2+b^2)}+\frac{bf\ln\left(\frac{ia+\mathrm{e}^{i(dx+c)}b-\sqrt{-a^2+b^2}}{ia-\sqrt{-a^2+b^2}}\right)c}{d^2(-a^2+b^2)}-\frac{f\ln\left(\frac{ia+\mathrm{e}^{i(dx+c)}b+\sqrt{-a^2+b^2}}{ia+\sqrt{-a^2+b^2}}\right)a^2c}{d^2b(-a^2+b^2)}-$

input
```
int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*f*x^2-I/d^2*b*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)
^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+1/d^2*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c)
)*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/d^2/b*f/(-a^2+b^2)*ln((I
*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c-1/d^2/
b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^
(1/2)))*a^2*c-2/d/b*e*ln(exp(I*(d*x+c)))-2*I/d/b*f*c*x+I/d^2/b*f/(-a^2+b^2
)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^
2+1/d*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2
+b^2)^(1/2)))*x+1/d/b*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+1/
d*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2
)^(1/2)))*x-1/d/b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/
(I*a-(-a^2+b^2)^(1/2)))*a^2*x-1/d/b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+
(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x-1/d^2/b*c*f*ln(I*b*exp(2*I
*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+I/d^2/b*f/(-a^2+b^2)*dilog((I*a+exp(I*(d
*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2-I/d^2/b*f*c^2-I/d^2
*b*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b
^2)^(1/2)))+I/b*e*x+1/d^2*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^
2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+2/d^2/b*c*f*ln(exp(I*(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(178) = 356$.

Time = 0.20 (sec) , antiderivative size = 773, normalized size of antiderivative = 3.65

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-I*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*f*dilog((I*a*cos(d*x
 + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b + 1) + I*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*f
*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (d*e - c*f)*log(2*b*cos(d*x +
c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d*e - c*f
)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) + (d*e - c*f)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) + 2*I*a) + (d*e - c*f)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (d*f*x + c*f)*log(-(I*
a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b) + (d*f*x + c*f)*log(-(I*a*cos(d*x + c) - a*sin(
d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b) + (d*f*x + c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (d*f*x + c*f)*lo
g(-(-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/(b*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)\cos{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e+f\,x)}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\left(\int \frac{\cos(dx+c)x}{\sin(dx+c)b+a}dx\right)bdf + \log(\sin(dx+c)\,b+a)\,e}{bd}$$

input
```
int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(int((cos(c + d*x)*x)/(sin(c + d*x)*b + a),x)*b*d*f + log(sin(c + d*x)*b +
 a)*e)/(b*d)
```

3.297 $\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log(a+b\sin(c+dx))}{bd}$$

output
```
ln(a+b*sin(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log(a+b\sin(c+dx))}{bd}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
Log[a + b*Sin[c + d*x]]/(b*d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3147}$$
$$\frac{\int \frac{1}{a+b\sin(c+dx)}d(b\sin(c+dx))}{bd}$$
$$\Big\downarrow \text{16}$$
$$\frac{\log(a+b\sin(c+dx))}{bd}$$

input
```
Int[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
Log[a + b*Sin[c + d*x]]/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\sin(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\sin(dx+c))}{bd}$	19
parallelrisch	$\frac{-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a\right)}{bd}$	50
risch	$-\frac{ix}{b}-\frac{2ic}{bd}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{bd}$	54
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}{bd}-\frac{\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{bd}$	59

input
```
int(cos(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sin(d*x+c))/b/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
log(b*sin(d*x + c) + a)/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \begin{cases} \frac{x\cos(c)}{a} & \text{for } b=0\wedge d=0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cos(c)}{a+b\sin(c)} & \text{for } d=0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)
), (x*cos(c)/(a + b*sin(c)), Eq(d, 0)), (log(a/b + sin(c + d*x))/(b*d), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log(b\sin(dx+c)+a)}{bd}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*sin(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log\left(|b\sin\left(dx+c\right)+a|\right)}{bd}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*sin(d*x + c) + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b\,d}$$

input
```
int(cos(c + d*x)/(a + b*sin(c + d*x)),x)
```

output
```
log(a + b*sin(c + d*x))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log(\sin\left(dx+c\right)b+a)}{bd}$$

input
```
int(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
log(sin(c + d*x)*b + a)/(b*d)
```

3.298 $\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 618

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d} + \frac{3\sqrt{a^2-b^2}f(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2} - \frac{3\sqrt{a^2-b^2}f(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2} + \frac{6i\sqrt{a^2-b^2}f^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3} - \frac{6i\sqrt{a^2-b^2}f^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3} - \frac{6\sqrt{a^2-b^2}f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4} + \frac{6\sqrt{a^2-b^2}f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^4} + \frac{6f^3\sin(c+dx)}{bd^4} - \frac{3f(e+fx)^2\sin(c+dx)}{bd^2}$$

output
```
1/4*a*(f*x+e)^4/b^2/f-6*f^2*(f*x+e)*cos(d*x+c)/b/d^3+(f*x+e)^3*cos(d*x+c)/
b/d+I*(a^2-b^2)^(1/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/b^2/d-I*(a^2-b^2)^(1/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/b^2/d+3*(a^2-b^2)^(1/2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(
a-(a^2-b^2)^(1/2)))/b^2/d^2-3*(a^2-b^2)^(1/2)*f*(f*x+e)^2*polylog(2,I*b*ex
p(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^2+6*I*(a^2-b^2)^(1/2)*f^2*(f*x+e)*
polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^3-6*I*(a^2-b^2)^(1
/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^3-
6*(a^2-b^2)^(1/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^
2/d^4+6*(a^2-b^2)^(1/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/b^2/d^4+6*f^3*sin(d*x+c)/b/d^4-3*f*(f*x+e)^2*sin(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 4.21 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.66

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{ad^4x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)+4bd(e+fx)\left(-6f^2+d^2(e+fx)^2\right)\cos(c+dx)+\frac{4(-a^2+b^2)\left(2\sqrt{-a^2}\right.}{}}{}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*b*d*(e + f*x)*(-6
*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + (4*(-a^2 + b^2)*(2*Sqrt[-a^2 + b^2]
*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*Sqrt[a^2 -
b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])]
 + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sq
rt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*x)))
/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E
^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^
2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b^2]*
d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - (3*I)*
Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a +
 Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*e*f^
2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt[a^2
 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])
] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt
[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)
))/(I*a + Sqrt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^
(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^3*Po
lyLog[4, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/Sqrt[-(a^2 ...
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.93, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {5036, 17, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3804, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \cos^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \ 5036$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^3dx}{b^2}-\frac{\int (e+fx)^3\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 17$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^3\sin(c+dx)dx}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 3042

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^3\sin(c+dx)dx}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 3777

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\int (e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 3042

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 3777

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 25

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 3042

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$\Big\downarrow$ 3777

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{3804}$$

$$-\frac{2\left(a^2-b^2\right)\int\frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$\downarrow$ 2620

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1 \cdots\right.}{b\cdots}\right.}{2\sqrt{a^2-b^2}}\cdots\right.}{b^2}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{b^2}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$\downarrow$ 7163

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3\right.}{d}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\cdots\right)}{d}\right)}{bd}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{\cdots}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

$\downarrow$ 7143

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\cdots}\right)}{d}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{\cdots}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a*(e + f*x)^4)/(4*b^2*f) - (2*(a^2 - b^2)*(((-1/2*I)*b*(((e + f*x)^3*Log[
1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f
*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))
/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*Po
lyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 -
b^2]))/b^2 - (-(((e + f*x)^3*Cos[c + d*x])/d) + (3*f*(((e + f*x)^2*Sin[c +
 d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^2))/d)
)/d)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2331 vs. $2(542) = 1084$.

Time = 0.38 (sec) , antiderivative size = 2331, normalized size of antiderivative = 3.77

$$\int \frac{(e+fx)^3 \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
 + 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin
(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin
(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*si
n(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) + 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(-I*a*cos(d*x + c) + a*s
in(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) - 6*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*e*f^2*x + I*b*d^2*e^2*f)*sqrt(-(a^2 -
b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*b*d^2*f^3*x^2 -
2*I*b*d^2*e*f^2*x - I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d
*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - 6*(-I*b*d^2*f^3*x^2 - 2*I*b*d^2*e*f^2*x - I*b*d^
2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) -
 6*(I*b*d^2*f^3*x^2 + 2*I*b*d^2*e*f^2*x + I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)
/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(b*d^3*e^3 - 3*b*c*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
( - 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**3*d**3*e**3 + 4*cos(c + d*x)*a**3*b*d**3*e**3 - 24*cos(c + d*x)*a**2*b**
2*d**2*e**2*f - 48*cos(c + d*x)*a**2*b**2*d**2*e*f**2*x - 24*cos(c + d*x)*
a**2*b**2*d**2*f**3*x**2 + 48*cos(c + d*x)*a**2*b**2*f**3 + 12*cos(c + d*x
)*a*b**3*d**3*e**2*f*x + 12*cos(c + d*x)*a*b**3*d**3*e*f**2*x**2 + 4*cos(c
 + d*x)*a*b**3*d**3*f**3*x**3 - 24*cos(c + d*x)*a*b**3*d*e*f**2 - 24*cos(c
 + d*x)*a*b**3*d*f**3*x + 24*cos(c + d*x)*b**4*d**2*e**2*f + 48*cos(c + d*
x)*b**4*d**2*e*f**2*x + 24*cos(c + d*x)*b**4*d**2*f**3*x**2 - 48*cos(c + d
*x)*b**4*f**3 + 16*int(x**3/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3
*b + 2*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**3*b**2*d**4
*f**3 - 16*int(x**3/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*t
an((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b**4*d**4*f**3 + 48*
int(x**2/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x
)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**3*b**2*d**4*e*f**2 - 48*int(x*
*2/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**
2*a + 2*tan((c + d*x)/2)*b + a),x)*a*b**4*d**4*e*f**2 + 32*int((tan((c + d
*x)/2)*x**3)/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c +
 d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**2*b**3*d**4*f**3 - 32*int(
(tan((c + d*x)/2)*x**3)/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b +
 2*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**5*d**4*f**3 ...
```

3.299 $\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 460

$$\begin{aligned}\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{a(e+fx)^3}{3b^2f}-\frac{2f^2\cos(c+dx)}{bd^3}+\frac{(e+fx)^2\cos(c+dx)}{bd}\\
&+\frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}\\
&-\frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}\\
&+\frac{2\sqrt{a^2-b^2}f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}\\
&-\frac{2\sqrt{a^2-b^2}f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}\\
&+\frac{2i\sqrt{a^2-b^2}f^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}\\
&-\frac{2i\sqrt{a^2-b^2}f^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}\\
&-\frac{2f(e+fx)\sin(c+dx)}{bd^2}\end{aligned}$$

output
```
1/3*a*(f*x+e)^3/b^2/f-2*f^2*cos(d*x+c)/b/d^3+(f*x+e)^2*cos(d*x+c)/b/d+I*(a
^2-b^2)^(1/2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d
-I*(a^2-b^2)^(1/2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
b^2/d+2*(a^2-b^2)^(1/2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2
)^(1/2)))/b^2/d^2-2*(a^2-b^2)^(1/2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))
/(a+(a^2-b^2)^(1/2)))/b^2/d^2+2*I*(a^2-b^2)^(1/2)*f^2*polylog(3,I*b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^3-2*I*(a^2-b^2)^(1/2)*f^2*polylog(3,I*
b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^3-2*f*(f*x+e)*sin(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 3.70 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.17

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{ax(3e^2+3efx+f^2x^2) + \frac{3i(-a^2+b^2)\left(-2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)+2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{i(c}}{ia+\sqrt{-}}\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a*x*(3*e^2 + 3*e*f*x + f^2*x^2) + ((3*I)*(-a^2 + b^2)*(-2*Sqrt[a^2 - b^2]
*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])]
 + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a +
 Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I
*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (
b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x
)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I
*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[
3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(Sqrt[-(a^2 - b^2)^
2]*d^3) + (3*b*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*Cos[c] - 2*d*f*(e + f*
x)*Sin[c]))/d^3 - (3*b*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2
)*Sin[c])*Sin[d*x])/d^3)/(3*b^2)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5036, 17, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5036}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^2dx}{b^2}-\frac{\int (e+fx)^2\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^2\sin(c+dx)dx}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^2\sin(c+dx)dx}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 3804}$$

$$-\frac{2\left(a^2-b^2\right)\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\cdots\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\cdots\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$\big\downarrow$ 3011

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$\big\downarrow$ 2720

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\cdots\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

$\big\downarrow$ 7143

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ib}{a-}\right.}{bd}\right.}{}\right.}{b^2}$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a*(e + f*x)^3)/(3*b^2*f) - (2*(a^2 - b^2)*(((-1/2*I)*b*(((e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*
x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog
[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 -
 b^2] + ((I/2)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2]))/b^2 - (-(((e + f*x)^2*Cos[c + d
*x])/d) + (2*f*((f*Cos[c + d*x])/d^2 + ((e + f*x)*Sin[c + d*x])/d))/d)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(400) = 800$.

Time = 0.27 (sec) , antiderivative size = 1636, normalized size of antiderivative = 3.56

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*f^2*x^3 + 6*a*d^3*e*f*x^2 + 6*a*d^3*e^2*x - 6*b*f^2*sqrt(-(a^
2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 -
 b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*sqrt(-(a^2 - b^
2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
+ I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 - b^2)
/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(I*b*d*f^2*x + I*b*d*e*f)
*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(
d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*b*
d*f^2*x - I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) - 6*(-I*b*d*f^2*x - I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*
a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(I*b*d*f^2*x + I*b*d*e*f)*sqrt(-(a^2 -
b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(b*d^2*e^2 - 2*b*c*
d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin
(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(b*d^2*e^2 - 2*b*c*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input

```
int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output

```
( - 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**3*d**2*e**2 + 3*cos(c + d*x)*a**3*b*d**2*e**2 - 12*cos(c + d*x)*a**2*b**
2*d*e*f - 12*cos(c + d*x)*a**2*b**2*d*f**2*x + 6*cos(c + d*x)*a*b**3*d**2*
e*f*x + 3*cos(c + d*x)*a*b**3*d**2*f**2*x**2 - 6*cos(c + d*x)*a*b**3*f**2
+ 12*cos(c + d*x)*b**4*d*e*f + 12*cos(c + d*x)*b**4*d*f**2*x + 12*int(x**2
/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*
a + 2*tan((c + d*x)/2)*b + a),x)*a**3*b**2*d**3*f**2 - 12*int(x**2/(tan((c
 + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*a + 2*ta
n((c + d*x)/2)*b + a),x)*a*b**4*d**3*f**2 + 24*int((tan((c + d*x)/2)*x**2)
/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*
a + 2*tan((c + d*x)/2)*b + a),x)*a**2*b**3*d**3*f**2 - 24*int((tan((c + d*
x)/2)*x**2)/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c +
d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**5*d**3*f**2 + 48*int((tan((
c + d*x)/2)*x)/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c
 + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**2*b**3*d**3*e*f - 48*int
((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2
*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**5*d**3*e*f + 24*i
nt(x/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)
**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**3*b**2*d**3*e*f - 24*int(x/(tan((c
 + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*a + 2...
```

3.300 $\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a(e+fx)^2}{2b^2f}+\frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d} + \frac{\sqrt{a^2-b^2}f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2} - \frac{\sqrt{a^2-b^2}f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2} - \frac{f\sin(c+dx)}{bd^2}$$

output
```
1/2*a*(f*x+e)^2/b^2/f+(f*x+e)*cos(d*x+c)/b/d+I*(a^2-b^2)^(1/2)*(f*x+e)*ln(
1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d-I*(a^2-b^2)^(1/2)*(f*x+e)*
ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d+(a^2-b^2)^(1/2)*f*polyl
og(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^2/d^2-(a^2-b^2)^(1/2)*f*pol
ylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^2/d^2-f*sin(d*x+c)/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1991 vs. 2(297) = 594.

Time = 16.39 (sec) , antiderivative size = 1991, normalized size of antiderivative = 6.70

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(2*b^2*d^2) + ((d*e - c*f + f*
(c + d*x))*Cos[c + d*x])/(b*d^2) - (f*Sin[c + d*x])/(b*d^2) + (((2*(d*e -
c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x
)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*Tan
[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b
+ Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] - (I*f*Log[1 - I*Tan[(c + d*x)/2]]
*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 +
 b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt
[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^
2 + b^2] - (I*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-
a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/
2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (
a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2]
- (I*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]
))])/Sqrt[-a^2 + b^2])*(((-a^2 + b^2)*e)/(b^2*(a + b*Sin[c + d*x])) - ((-a
^2 + b^2)*c*f)/(b^2*d*(a + b*Sin[c + d*x])) + ((-a^2 + b^2)*f*(c + d*x))/(
b^2*d*(a + b*Sin[c + d*x]))))/(d*((f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/
(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[-a^2 + b^2]*(1
 - I*Tan[(c + d*x)/2])) + (f*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d...
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5036, 17, 3042, 3777, 3042, 3117, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5036}$$

$$-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)dx}{b^2}-\frac{\int (e+fx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)\sin(c+dx)dx}{b}+\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)\sin(c+dx)dx}{b}+\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{3804}$$

$$-\frac{2(a^2-b^2)\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 2694$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 2620$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}$$
$$\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 2715$$

$$-\frac{2\left(a^2-b^2\right)\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}\right.}{2\sqrt{a^2-b^2}}\right.}{b^2}$$
$$\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 2838$$

$$-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}$$

$$\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}$$

input
```
Int[((e + f*x)*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(a*(e + f*x)^2)/(2*b^2*f) - (2*(a^2 - b^2)*(((-1/2*I)*b*(((e + f*x)*Log[1
 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((
I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^
2)))/Sqrt[a^2 - b^2]))/b^2 - (-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c + d
*x])/d^2)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(265) = 530$.

Time = 2.25 (sec) , antiderivative size = 1105, normalized size of antiderivative = 3.72

method	result	size
risch	Expression too large to display	1105

input
```
int((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/b^2*f*x^2+a/b^2*e*x+1/2*(d*x*f+I*f+d*e)/d^2/b*exp(I*(d*x+c))+1/2*(d*
x*f-I*f+d*e)/d^2/b*exp(-I*(d*x+c))+2*I/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*
I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-2*I/d/b^2*a^2*e/(-a^2+b^2)^(1/2)
*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/d*f/(-a^2+b^2)^
(1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))
*x-1/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*
a+(-a^2+b^2)^(1/2)))*c+1/d/b^2*a^2*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c
))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*f/(-a^2+b^2)^(1/2)*
ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*c-I/d
^2*f/(-a^2+b^2)^(1/2)*dilog((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a
+(-a^2+b^2)^(1/2)))-I/d^2/b^2*a^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x
+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/d^2/b^2*f*a^2*c/(-a^2
+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2/
b^2*a^2*f/(-a^2+b^2)^(1/2)*dilog((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/
(-I*a+(-a^2+b^2)^(1/2)))-1/d^2/b^2*a^2*f/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(
d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*c+1/d^2/b^2*a^2*f/(-a
^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(
1/2)))*c-1/d/b^2*a^2*f/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^
2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x-1/d*f/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*
(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-2*I/d^2*f*c/(-a^...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(257) = 514$.

Time = 0.22 (sec) , antiderivative size = 1034, normalized size of antiderivative = 3.48

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*f*x^2 + 2*a*d^2*e*x - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*c
os(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*
x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b + 1) - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x
+ c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b + 1) - 2*b*f*sin(d*x + c) - (b*d*e - b*c*f)*sqrt(-(a^2 - b
^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) + 2*I*a) - (b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*d*e - b
*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^
2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) + (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x + c
) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) - b)/b) - (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x
 + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^3de+2\cos\left(dx+c\right)a^3bde-4\cos\left(dx+c\right)a^2b^2f+2\cos\left(dx+c\right)a\,b^3}{}$$

input
```
int((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**3*d*e + 2*cos(c + d*x)*a**3*b*d*e - 4*cos(c + d*x)*a**2*b**2*f + 2*cos(c
 + d*x)*a*b**3*d*f*x + 4*cos(c + d*x)*b**4*f + 16*int((tan((c + d*x)/2)*x)
/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*
a + 2*tan((c + d*x)/2)*b + a),x)*a**2*b**3*d**2*f - 16*int((tan((c + d*x)/
2)*x)/(tan((c + d*x)/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2
)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b**5*d**2*f + 8*int(x/(tan((c + d*x)
/2)**4*a + 2*tan((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*a + 2*tan((c +
d*x)/2)*b + a),x)*a**3*b**2*d**2*f - 8*int(x/(tan((c + d*x)/2)**4*a + 2*ta
n((c + d*x)/2)**3*b + 2*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),
x)*a*b**4*d**2*f - 4*sin(c + d*x)*a**2*b**2*d*f*x - 2*sin(c + d*x)*a*b**3*
f + 4*sin(c + d*x)*b**4*d*f*x + 2*a**4*d**2*e*x - 2*a**3*b*d*e - a**2*b**2
*d**2*f*x**2 + 4*a**2*b**2*f + 2*b**4*d**2*f*x**2 - 4*b**4*f)/(2*a**3*b**2
*d**2)
```

3.301 $\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{ax}{b^2} - \frac{2\sqrt{a^2-b^2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d} + \frac{\cos(c+dx)}{bd}$$

output
```
a*x/b^2-2*(a^2-b^2)^(1/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/b^2/d+cos(d*x+c)/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 361 vs. 2(70) = 140.

Time = 1.42 (sec) , antiderivative size = 361, normalized size of antiderivative = 5.16

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\cos(c+dx)\left(2(a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{a+b}\left(-2\sqrt{a-b}\text{arctanh}\left(\cdots\right.\right.\right.}{\sqrt{a-b}b\sqrt{a+b}d\cdots}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
(Cos[c + d*x]*(2*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x])
)/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sqrt[
1 - Sin[c + d*x]] + Sqrt[a + b]*(-2*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 + Sin[c
 + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]]*Sqrt[1 - Sin
[c + d*x]] + Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*(2*Sqrt[b]*ArcSinh[(
Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])] +
Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]))
))/(Sqrt[a - b]*b*Sqrt[a + b]*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin
[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3174, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}+\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}dx}{b}+\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin(c+dx)}dx}{b}}{b}+\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin(c+dx)}dx}{b}}{b}+\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \ 3139$$

$$\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}+\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{4(a^2-b^2)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}+\frac{ax}{b}}{b}+\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{ax}{b}-\frac{2\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{bd}}{b}+\frac{\cos(c+dx)}{bd}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
((a*x)/b - (2*Sqrt[a^2 - b^2]*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[
a^2 - b^2])])/(b*d))/b + Cos[c + d*x]/(b*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}+\frac{\frac{2b}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$
default	$\frac{\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}+\frac{\frac{2b}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$
risch	$\frac{ax}{b^2}+\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia-\sqrt{-a^2+b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia+\sqrt{-a^2+b^2}}{b}\right.}{d\,b^2}$

input
```
int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-a^2+b^2)/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))+2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*
x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.06

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[\frac{2\,adx + 2\,b\cos\left(dx+c\right) + \sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(d}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right.}{2\,b^2d}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*a*d*x + 2*b*cos(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2)))/(b^2*d), (a*d*x + b*cos(d*x + c) + sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/(b^2*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. 2(58) = 116.

Time = 113.18 (sec) , antiderivative size = 1923, normalized size of antiderivative = 27.47

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x*cos(c)**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(
tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(d*tan(c/2 + d*x/2)**2 + d) + log(ta
n(c/2 + d*x/2))/(d*tan(c/2 + d*x/2)**2 + d) + 2/(d*tan(c/2 + d*x/2)**2 + d
))/b, Eq(a, 0)), (b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 +
b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x
/2)) + b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*s
qrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*b/(b*
*2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b
*d*sqrt(b**2)*tan(c/2 + d*x/2)) - d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2
*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d
*sqrt(b**2)*tan(c/2 + d*x/2)) - d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2
 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 +
d*x/2)) - 2*sqrt(b**2)*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2
*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2))
, Eq(a, -sqrt(b**2))), (b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)
**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2
 + d*x/2)) + b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d +
 b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2
*b/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)*
*3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + d*x*sqrt(b**2)*tan(c/2 + d*x/2)...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{(dx+c)a}{b^2} - \frac{2\left(\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)*a/b^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 + 2/((tan(
1/2*d*x + 1/2*c)^2 + 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 40.40 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.54

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2-\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4-64\,a^2\,b^2}\right)}{b^2\,d}$$

$$+ \frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{b^2-a^2}}{64\,a^2\,b-\frac{64\,a^4}{b}-128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{64\,a^2-\frac{64\,a^4}{b^2}-\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{\cdots}{64\,a^4\cdots}\cdots}{b^2\,d}$$

input
```
int(cos(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

output
```
2/(b*d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2*a*atan((64*a^2*tan(c/2 + (d*x)/2))
/(64*a^2 - (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 - 64*a^2*b^
2)))/(b^2*d) + (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(64*a^2*b - (64*a^4)/b
- 128*a^3*tan(c/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(64*a^2 - (64*a^4)/b^2 - (128*a^3*tan(c/
2 + (d*x)/2))/b + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2))/(64*a^4 - 64*a^2*b^2 - 128*a*b^3*tan(c/2 + (d*x)/2) +
128*a^3*b*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)+\cos\left(dx+c\right)b+ac+adx-b}{b^2d}$$

input
```
int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2)) +
 cos(c + d*x)*b + a*c + a*d*x - b)/(b**2*d)
```

3.302 $\int \frac{(e+fx)^3 \cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 737

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}+\frac{i(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2} - \frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} - \frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2} + \frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2} - \frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3} - \frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3} - \frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^4} - \frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^4} - \frac{6af^2(e+fx)\sin(c+dx)}{b^2d^3}+\frac{a(e+fx)^3\sin(c+dx)}{b^2d} + \frac{3f^3\cos(c+dx)\sin(c+dx)}{8bd^4} - \frac{3f(e+fx)^2\cos(c+dx)\sin(c+dx)}{4bd^2} + \frac{3f^2(e+fx)\sin^2(c+dx)}{4bd^3}-\frac{(e+fx)^3\sin^2(c+dx)}{2bd}$$

output
```
-3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(
d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^4-6*a*f^3*cos(d*x+c)/b^2/d^4+3*a*f*(f*x
+e)^2*cos(d*x+c)/b^2/d^2-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/b^3/d-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-
b^2)^(1/2)))/b^3/d-6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/b^3/d^4+3*I*(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c)
)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I
*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^3-6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I
*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^3+1/4*I*(a^2-b^2)*(f*x+e)^4/b
^3/f+3*I*(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(
1/2)))/b^3/d^2-6*a*f^2*(f*x+e)*sin(d*x+c)/b^2/d^3+a*(f*x+e)^3*sin(d*x+c)/b
^2/d+3/8*f^3*cos(d*x+c)*sin(d*x+c)/b/d^4-3/4*f*(f*x+e)^2*cos(d*x+c)*sin(d*
x+c)/b/d^2+3/4*f^2*(f*x+e)*sin(d*x+c)^2/b/d^3-1/2*(f*x+e)^3*sin(d*x+c)^2/b
/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2452 vs. $2(737) = 1474$.

Time = 10.42 (sec) , antiderivative size = 2452, normalized size of antiderivative = 3.33

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
(-32*(a^2 - b^2)*e^3*x*Cot[c] - 48*(a^2 - b^2)*e^2*f*x^2*Cot[c] - 32*(a^2
- b^2)*e*f^2*x^3*Cot[c] - 8*(a^2 - b^2)*f^3*x^4*Cot[c] + (16*(a^2 - b^2)*(
(4*I)*d^4*e^3*E^((2*I)*c)*x + (6*I)*d^4*e^2*E^((2*I)*c)*f*x^2 + (4*I)*d^4*
e*E^((2*I)*c)*f^2*x^3 + I*d^4*E^((2*I)*c)*f^3*x^4 + (2*I)*d^3*e^3*ArcTan[(
2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - (2*I)*d^3*e^3*E^((2
*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + d^3*
e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - d^
3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c +
d*x)))^2] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqr
t[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*
(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^
2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] - 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a
*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(
I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^
((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2
+ b^2)*E^((2*I)*c)])] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[
1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a...
```

Rubi [A] (verified)

Time = 3.65 (sec) , antiderivative size = 667, normalized size of antiderivative = 0.91, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5036, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4904, 3042, 3792, 17, 3042, 3115, 24, 5030, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 5036$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^3\cos(c+dx)dx}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{3f\int -(e+fx)^2\sin(c+dx)dx}{d}+\frac{(e+fx)^3\sin(c+dx)}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 4904$$

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sin^2(c+dx)dx}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sin(c+dx)^2dx}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3792$$

$$-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 17$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int \sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3115$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$\downarrow$ 5030

$$-\frac{\left(a^2-b^2\right)\left(\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^4}{4bf}\right)}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$\downarrow$ 2620

$$-\frac{\left(a^2-b^2\right)\left(-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3 \cdots}{\cdots}\right.}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$\downarrow$ 3011

$$-\frac{(a^2-b^2)\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\ldots}{}\right)}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-$$

$$\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$$\Big\downarrow \; 7163$$

$$-\frac{(a^2-b^2)\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+\ldots}{}\right.}{}\right.}{}$$

$$\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-$$

$$\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{(a^2-b^2)\left(-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\cdots\right)}{bd}\cdots\right.}{\cdots}$$

$$\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-$$

$$\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$$\Big\downarrow \; 7143$$

$$-\frac{(a^2-b^2)\left(-\frac{3f\left(\frac{i(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-$$

$$\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
-(((a^2 - b^2)*(((-1/4*I)*(e + f*x)^4)/(b*f) + ((e + f*x)^3*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^3*Log[1 - (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(
e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*P
olyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d) -
(3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/d^2))/d))/(b*d)))/b^2) + (a*(((e + f*x)^3*Sin[c + d*x])/d - (3*f*(-
(((e + f*x)^2*Cos[c + d*x])/d) + (2*f*((f*Cos[c + d*x])/d^2 + ((e + f*x)*S
in[c + d*x])/d))/d))/d))/b^2 - (((e + f*x)^3*Sin[c + d*x]^2)/(2*d) - (3*f*
((e + f*x)^3/(6*f) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*(e
 + f*x)*Sin[c + d*x]^2)/(2*d^2) - (f^2*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/(2*d^2)))/(2*d))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos (dx+c)^3}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2684 vs. $2(673) = 1346$.

Time = 0.30 (sec) , antiderivative size = 2684, normalized size of antiderivative = 3.64

$$\int \frac{(e+fx)^3 \cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(2*b^2*d^3*f^3*x^3 + 6*b^2*d^3*e*f^2*x^2 - 24*I*(a^2 - b^2)*f^3*polyl
og(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*I*(a^2 - b^2)*f^3*polylog(4, -(I*a*c
os(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2))/b) + 24*I*(a^2 - b^2)*f^3*polylog(4, -(-I*a*cos(d*x + c)
+ a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b) + 24*I*(a^2 - b^2)*f^3*polylog(4, -(-I*a*cos(d*x + c) + a*sin(d*x
+ c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*
(2*b^2*d^3*f^3*x^3 + 6*b^2*d^3*e*f^2*x^2 + 2*b^2*d^3*e^3 - 3*b^2*d*e*f^2 +
 3*(2*b^2*d^3*e^2*f - b^2*d*f^3)*x)*cos(d*x + c)^2 + 3*(2*b^2*d^3*e^2*f -
b^2*d*f^3)*x - 24*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a*b*d^2*e^2*f - 2
*a*b*f^3)*cos(d*x + c) + 12*(-I*(a^2 - b^2)*d^2*f^3*x^2 - 2*I*(a^2 - b^2)*
d^2*e*f^2*x - I*(a^2 - b^2)*d^2*e^2*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x
 + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
+ 1) + 12*(-I*(a^2 - b^2)*d^2*f^3*x^2 - 2*I*(a^2 - b^2)*d^2*e*f^2*x - I*(a
^2 - b^2)*d^2*e^2*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*(a^2
- b^2)*d^2*f^3*x^2 + 2*I*(a^2 - b^2)*d^2*e*f^2*x + I*(a^2 - b^2)*d^2*e^2*f
)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 12*(I*(a^2 - b^2)*d^2*f^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^3\cos(dx+c)^3}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

3.303 $\int \frac{(e+fx)^2 \cos^3(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 536

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{(e+fx)^2}{4bd}+\frac{i(a^2-b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cos(c+dx)}{b^2d^2}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{2i(a^2-b^2)f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2i(a^2-b^2)f(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}-\frac{2(a^2-b^2)f^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2(a^2-b^2)f^2\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2af^2\sin(c+dx)}{b^2d^3}+\frac{a(e+fx)^2\sin(c+dx)}{b^2d}-\frac{f(e+fx)\cos(c+dx)\sin(c+dx)}{2bd^2}+\frac{f^2\sin^2(c+dx)}{4bd^3}-\frac{(e+fx)^2\sin^2(c+dx)}{2bd}$$

output

```
1/4*(f*x+e)^2/b/d+1/3*I*(a^2-b^2)*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*cos(d*x+c)
/b^2/d^2-(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/
b^3/d-(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3
/d+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)
))/b^3/d^2+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^
2)^(1/2)))/b^3/d^2-2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/b^3/d^3-2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^
2)^(1/2)))/b^3/d^3-2*a*f^2*sin(d*x+c)/b^2/d^3+a*(f*x+e)^2*sin(d*x+c)/b^2/d
-1/2*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d^2+1/4*f^2*sin(d*x+c)^2/b/d^3-1/2*
(f*x+e)^2*sin(d*x+c)^2/b/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2283 vs. 2(536) = 1072.

Time = 5.69 (sec) , antiderivative size = 2283, normalized size of antiderivative = 4.26

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
((48*I)*a^2*d^3*e^2*E^((2*I)*c)*x - (48*I)*b^2*d^3*e^2*E^((2*I)*c)*x + (48
*I)*a^2*d^3*e*E^((2*I)*c)*f*x^2 - (48*I)*b^2*d^3*e*E^((2*I)*c)*f*x^2 + (16
*I)*a^2*d^3*E^((2*I)*c)*f^2*x^3 - (16*I)*b^2*d^3*E^((2*I)*c)*f^2*x^3 + (24
*I)*a*b*d^2*e^2*E^(I*c)*Cos[d*x] - (24*I)*a*b*d^2*e^2*E^((3*I)*c)*Cos[d*x]
 + 48*a*b*d*e*E^(I*c)*f*Cos[d*x] + 48*a*b*d*e*E^((3*I)*c)*f*Cos[d*x] - (48
*I)*a*b*E^(I*c)*f^2*Cos[d*x] + (48*I)*a*b*E^((3*I)*c)*f^2*Cos[d*x] + (48*I
)*a*b*d^2*e*E^(I*c)*f*x*Cos[d*x] - (48*I)*a*b*d^2*e*E^((3*I)*c)*f*x*Cos[d*
x] + 48*a*b*d*E^(I*c)*f^2*x*Cos[d*x] + 48*a*b*d*E^((3*I)*c)*f^2*x*Cos[d*x]
 + (24*I)*a*b*d^2*E^(I*c)*f^2*x^2*Cos[d*x] - (24*I)*a*b*d^2*E^((3*I)*c)*f^
2*x^2*Cos[d*x] + 6*b^2*d^2*e^2*Cos[2*d*x] + 6*b^2*d^2*e^2*E^((4*I)*c)*Cos[
2*d*x] - (6*I)*b^2*d*e*f*Cos[2*d*x] + (6*I)*b^2*d*e*E^((4*I)*c)*f*Cos[2*d*
x] - 3*b^2*f^2*Cos[2*d*x] - 3*b^2*E^((4*I)*c)*f^2*Cos[2*d*x] + 12*b^2*d^2*
e*f*x*Cos[2*d*x] + 12*b^2*d^2*e*E^((4*I)*c)*f*x*Cos[2*d*x] - (6*I)*b^2*d*f
^2*x*Cos[2*d*x] + (6*I)*b^2*d*E^((4*I)*c)*f^2*x*Cos[2*d*x] + 6*b^2*d^2*f^2
*x^2*Cos[2*d*x] + 6*b^2*d^2*E^((4*I)*c)*f^2*x^2*Cos[2*d*x] - 48*a^2*d^2*e^
2*E^((2*I)*c)*Log[b - (2*I)*a*E^(I*(c + d*x)) - b*E^((2*I)*(c + d*x))] + 4
8*b^2*d^2*e^2*E^((2*I)*c)*Log[b - (2*I)*a*E^(I*(c + d*x)) - b*E^((2*I)*(c
+ d*x))] - 96*a^2*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a
*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 96*b^2*d^2*e*E^((2*I)*c)*f*x
*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I...
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.90, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {5036, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4904, 3042, 3791, 17, 5030, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5036}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^2\cos(c+dx)dx}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{4904}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\int (e+fx)\sin^2(c+dx)dx}{d}}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\int (e+fx)\sin(c+dx)^2dx}{d}}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3791$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{d}}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 17$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{b}$$

$$\Big\downarrow \; 5030$$

$$-\frac{(a^2-b^2)\left(\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^3}{3bf}\right)}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{(a^2-b^2)\left(-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log}{}\right.}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{(a^2-b^2)\Bigg(-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{2f\Big(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int \text{PolyLog}(\cdots}{\cdots}}{bd}\cdots}{b^2\cdots}$$

$$-\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{b}$$

$\downarrow$ 2720

$$-\frac{(a^2-b^2)\Bigg(-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}-\frac{2f\Big(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\cdots}{\cdots}\cdots}{b^2\cdots}$$

$$-\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{b}$$

$\downarrow$ 7143

$$-\frac{(a^2-b^2)\Bigg(-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}-\frac{2f\Big(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\big(3,\frac{ibe^{i(\cdots}}{a+\sqrt{\cdots}}}{d^2}\cdots}{bd}\cdots}{b^2\cdots}$$

$$-\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}}{b}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
-(((a^2 - b^2)*(((-1/3*I)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*
E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d) - (2*f*((I*(e + f*x)*
PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3,
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/b^2) + (a*(((
e + f*x)^2*Sin[c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[
c + d*x])/d^2))/d))/b^2 - (((e + f*x)^2*Sin[c + d*x]^2)/(2*d) - (f*((e + f
*x)^2/(4*f) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x
]^2)/(4*d^2)))/d)/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)^3}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1779 vs. 2(488) = 976.

Time = 0.25 (sec) , antiderivative size = 1779, normalized size of antiderivative = 3.32

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 4*(a^2 - b^2)*f^2*polylog(3, -(I
*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))/b) + 4*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c)
+ a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b) + 4*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c
) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(a^
2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - (2*b^2*d^2*f^2*x^2 +
 4*b^2*d^2*e*f*x + 2*b^2*d^2*e^2 - b^2*f^2)*cos(d*x + c)^2 - 8*(a*b*d*f^2*
x + a*b*d*e*f)*cos(d*x + c) + 4*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*
e*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 4*(-I*(a^2 - b^2)*d*f^2*x -
 I*(a^2 - b^2)*d*e*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 4*(I*(a^2
- b^2)*d*f^2*x + I*(a^2 - b^2)*d*e*f)*dilog((-I*a*cos(d*x + c) - a*sin(d*x
 + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
+ 1) + 4*(I*(a^2 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*e*f)*dilog((-I*a*cos(d*x
 + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b + 1) + 2*((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (
a^2 - b^2)*c^2*f^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{too large to display}$$

input

```
int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output

```
( - 36*cos(c + d*x)*sin(c + d*x)*a**4*b**3*d**2*e*f*x - 18*cos(c + d*x)*si
n(c + d*x)*a**4*b**3*d**2*f**2*x**2 + 9*cos(c + d*x)*sin(c + d*x)*a**4*b**
3*f**2 - 18*cos(c + d*x)*sin(c + d*x)*a**3*b**4*d*e*f - 18*cos(c + d*x)*si
n(c + d*x)*a**3*b**4*d*f**2*x + 84*cos(c + d*x)*sin(c + d*x)*a**2*b**5*d**
2*e*f*x + 42*cos(c + d*x)*sin(c + d*x)*a**2*b**5*d**2*f**2*x**2 - 21*cos(c
 + d*x)*sin(c + d*x)*a**2*b**5*f**2 + 12*cos(c + d*x)*sin(c + d*x)*a*b**6*
d*e*f + 12*cos(c + d*x)*sin(c + d*x)*a*b**6*d*f**2*x - 48*cos(c + d*x)*sin
(c + d*x)*b**7*d**2*e*f*x - 24*cos(c + d*x)*sin(c + d*x)*b**7*d**2*f**2*x*
*2 + 12*cos(c + d*x)*sin(c + d*x)*b**7*f**2 - 72*cos(c + d*x)*a**4*b**3*d*
e*f - 72*cos(c + d*x)*a**4*b**3*d*f**2*x + 48*cos(c + d*x)*a**3*b**4*d**2*
e*f*x + 24*cos(c + d*x)*a**3*b**4*d**2*f**2*x**2 - 48*cos(c + d*x)*a**3*b*
*4*f**2 + 288*cos(c + d*x)*a**2*b**5*d*e*f + 288*cos(c + d*x)*a**2*b**5*d*
f**2*x - 48*cos(c + d*x)*a*b**6*d**2*e*f*x - 24*cos(c + d*x)*a*b**6*d**2*f
**2*x**2 + 48*cos(c + d*x)*a*b**6*f**2 - 192*cos(c + d*x)*b**7*d*e*f - 192
*cos(c + d*x)*b**7*d*f**2*x + 96*int(x**2/(tan((c + d*x)/2)**6*a + 2*tan((
c + d*x)/2)**5*b + 3*tan((c + d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*t
an((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**5*b**3*d**3*f**2 -
288*int(x**2/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c +
 d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan(
(c + d*x)/2)*b + a),x)*a**3*b**5*d**3*f**2 + 192*int(x**2/(tan((c + d*x...
```

3.304 $\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 351

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{fx}{4bd}+\frac{i(a^2-b^2)(e+fx)^2}{2b^3f}+\frac{af\cos(c+dx)}{b^2d}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{i(a^2-b^2)f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{i(a^2-b^2)f\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{a(e+fx)\sin(c+dx)}{b^2d}-\frac{f\cos(c+dx)\sin(c+dx)}{4bd^2}-\frac{(e+fx)\sin^2(c+dx)}{2bd}$$

output
```
1/4*f*x/b/d+1/2*I*(a^2-b^2)*(f*x+e)^2/b^3/f+a*f*cos(d*x+c)/b^2/d^2-(a^2-b^
2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d-(a^2-b^2)*(f
*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d+I*(a^2-b^2)*f*pol
ylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b^3/d^2+I*(a^2-b^2)*f*polyl
og(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b^3/d^2+a*(f*x+e)*sin(d*x+c)/
b^2/d-1/4*f*cos(d*x+c)*sin(d*x+c)/b/d^2-1/2*(f*x+e)*sin(d*x+c)^2/b/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 816 vs. 2(351) = 702.

Time = 4.07 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.32

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{8abf\cos(c+dx)+2b^2d(e+fx)\cos(2(c+dx))-8a^2de\log\left(1+\frac{b\sin(c+dx)}{a}\right)+8b^2de\log\left(1+\frac{b\sin(c+dx)}{a}\right)}{}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output
```
(8*a*b*f*Cos[c + d*x] + 2*b^2*d*(e + f*x)*Cos[2*(c + d*x)] - 8*a^2*d*e*Log
[1 + (b*Sin[c + d*x])/a] + 8*b^2*d*e*Log[1 + (b*Sin[c + d*x])/a] + 8*a^2*c
*f*Log[1 + (b*Sin[c + d*x])/a] - 8*b^2*c*f*Log[1 + (b*Sin[c + d*x])/a] - a
^2*f*(I*(-2*c + Pi - 2*d*x)^2 - (32*I)*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]]*Arc
Tan[((a - b)*Cot[(2*c + Pi + 2*d*x)/4])/Sqrt[a^2 - b^2]] - 4*(-2*c + Pi -
2*d*x + 4*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 - (I*(-a + Sqrt[a^2 - b^2
]))/(b*E^(I*(c + d*x)))] - 4*(-2*c + Pi - 2*d*x - 4*ArcSin[Sqrt[(a + b)/b]
/Sqrt[2]])*Log[1 + (I*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))] + 4*(-2*
c + Pi - 2*d*x)*Log[a + b*Sin[c + d*x]] + 8*(c + d*x)*Log[a + b*Sin[c + d*
x]] + (8*I)*(PolyLog[2, (I*(-a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))] +
PolyLog[2, ((-I)*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))])) + b^2*f*(I*
(-2*c + Pi - 2*d*x)^2 - (32*I)*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]]*ArcTan[((a
- b)*Cot[(2*c + Pi + 2*d*x)/4])/Sqrt[a^2 - b^2]] - 4*(-2*c + Pi - 2*d*x +
4*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 - (I*(-a + Sqrt[a^2 - b^2]))/(b*E
^(I*(c + d*x)))] - 4*(-2*c + Pi - 2*d*x - 4*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]
])*Log[1 + (I*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))] + 4*(-2*c + Pi -
 2*d*x)*Log[a + b*Sin[c + d*x]] + 8*(c + d*x)*Log[a + b*Sin[c + d*x]] + (8
*I)*(PolyLog[2, (I*(-a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))] + PolyLog[
2, ((-I)*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))])) + 8*a*b*d*(e + f*x)
*Sin[c + d*x] - b^2*f*Sin[2*(c + d*x)])/(8*b^3*d^2)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5036, 3042, 3777, 25, 3042, 3118, 4904, 3042, 3115, 24, 5030, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5036}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)\cos(c+dx)dx}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{4904}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int \sin^2(c+dx)dx}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2-b^2\right)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int \sin(c+dx)^2dx}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{5030}$$

$$-\frac{(a^2-b^2)\left(\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^2}{2bf}\right)}{b^2}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(a^2-b^2)\left(-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(a^2-b^2)\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{(a^2-b^2)\left(-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}$$

input

```
Int[((e + f*x)*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

output

```
-(((a^2 - b^2)*(((-1/2*I)*(e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c
+ d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/b^2) + (a*((f*Cos[c + d*x])/d^2 +
 ((e + f*x)*Sin[c + d*x])/d))/b^2 - (((e + f*x)*Sin[c + d*x]^2)/(2*d) - (f
*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(2*d))/b
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1749 vs. $2(320) = 640$.

Time = 3.61 (sec) , antiderivative size = 1750, normalized size of antiderivative = 4.99

method	result	size
risch	Expression too large to display	1750

input
```
int((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/b^3*a^2*f*c*x-I/d^2/b^3*a^4*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b
-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+1/16*(2*d*x*f+I*f+2*d*e)/d^2/b*
exp(2*I*(d*x+c))+1/16*(2*d*x*f-I*f+2*d*e)/d^2/b*exp(-2*I*(d*x+c))+1/d^2*b*
f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1
/2)))*c+1/d*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))*x+1/d*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^
2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2*b*f/(-a^2+b^2)*ln((I*a+exp(I*(d*
x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-2/d^2/b^3*c*f*a^2*ln(e
xp(I*(d*x+c)))+1/d^2/b^3*c*f*a^2*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c)
)-I*b)+I/d^2/b^3*a^2*f*c^2-I/d^2*b*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*
b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-I/d^2*b*f/(-a^2+b^2)*dilog((I*
a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I/d/b*f*c*x
-I/b^3*a^2*e*x+1/d^2/b^3*a^4*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b
^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2/d^2/b*a^2*f/(-a^2+b^2)*ln((I*a+exp(
I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+1/d/b^3*a^4*f/(-a
^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
*x-2/d/b*a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-
(-a^2+b^2)^(1/2)))*x+1/d/b^3*a^4*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a
^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-2/d/b*a^2*f/(-a^2+b^2)*ln((I*a+ex
p(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/d^2/b^3*a^...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(315) = 630$.

Time = 0.23 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.95

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(b^2*d*f*x - 4*a*b*f*cos(d*x + c) - 2*(b^2*d*f*x + b^2*d*e)*cos(d*x +
 c)^2 - 2*I*(a^2 - b^2)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*I*(a
^2 - b^2)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*(a^2 - b^2)*f*di
log((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*(a^2 - b^2)*f*dilog((-I*a*cos
(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) + 2*((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*
cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) +
2*((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*((a^2 - b^2)*d*e - (a^2 -
b^2)*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) + 2*I*a) + 2*((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x
+ c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*((a^2
- b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) +
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*((a
^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c)
 - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + 2*
((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos{(dx+c)}^3}{b\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)*a**4*b**3*d*f*x - 3*cos(c + d*x)*sin(c + d
*x)*a**3*b**4*f + 14*cos(c + d*x)*sin(c + d*x)*a**2*b**5*d*f*x + 2*cos(c +
 d*x)*sin(c + d*x)*a*b**6*f - 8*cos(c + d*x)*sin(c + d*x)*b**7*d*f*x - 12*
cos(c + d*x)*a**4*b**3*f + 8*cos(c + d*x)*a**3*b**4*d*f*x + 48*cos(c + d*x
)*a**2*b**5*f - 8*cos(c + d*x)*a*b**6*d*f*x - 32*cos(c + d*x)*b**7*f + 96*
int((tan((c + d*x)/2)*x)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b
 + 3*tan((c + d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**
2*a + 2*tan((c + d*x)/2)*b + a),x)*a**4*b**4*d**2*f - 224*int((tan((c + d*
x)/2)*x)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c + d*x
)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan((c +
 d*x)/2)*b + a),x)*a**2*b**6*d**2*f + 128*int((tan((c + d*x)/2)*x)/(tan((c
 + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c + d*x)/2)**4*a + 4*ta
n((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),
x)*b**8*d**2*f + 32*int(x/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b
 + 3*tan((c + d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)*
*2*a + 2*tan((c + d*x)/2)*b + a),x)*a**5*b**3*d**2*f - 96*int(x/(tan((c +
d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b + 3*tan((c + d*x)/2)**4*a + 4*tan((
c + d*x)/2)**3*b + 3*tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*
a**3*b**5*d**2*f + 64*int(x/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5
*b + 3*tan((c + d*x)/2)**4*a + 4*tan((c + d*x)/2)**3*b + 3*tan((c + d*x...
```

3.305 $\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)} \, dx = -\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d} + \frac{a\sin(c+dx)}{b^2d} - \frac{\sin^2(c+dx)}{2bd}$$

output
```
-(a^2-b^2)*ln(a+b*sin(d*x+c))/b^3/d+a*sin(d*x+c)/b^2/d-1/2*sin(d*x+c)^2/b/
d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)} \, dx = \frac{-((a^2-b^2)\log(a+b\sin(c+dx))) + ab\sin(c+dx) - \frac{1}{2}b^2\sin^2(c+dx)}{b^3d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
(-((a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + a*b*Sin[c + d*x] - (b^2*Sin[c +
d*x]^2)/2)/(b^3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^3}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{\int \frac{b^2-b^2\sin^2(c+dx)}{a+b\sin(c+dx)}d(b\sin(c+dx))}{b^3d}$$

$$\Big\downarrow \text{ 476}$$

$$\frac{\int \left(a-b\sin(c+dx)+\frac{b^2-a^2}{a+b\sin(c+dx)}\right)d(b\sin(c+dx))}{b^3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\left(a^2-b^2\right)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)}{b^3d}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
(-((a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + a*b*Sin[c + d*x] - (b^2*Sin[c +
d*x]^2)/2)/(b^3*d)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{-\frac{b\sin(dx+c)^2}{2}+a\sin(dx+c)}{b^2}+\frac{\left(-a^2+b^2\right)\ln(a+b\sin(dx+c))}{b^3}}{d}$
default	$\frac{\frac{-\frac{b\sin(dx+c)^2}{2}+a\sin(dx+c)}{b^2}+\frac{\left(-a^2+b^2\right)\ln(a+b\sin(dx+c))}{b^3}}{d}$
parallelrisch	$\frac{4a^2\left(\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right)\right)-4b^2\left(\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+\sec\left(\right.\right.\right.}{4b^3d}$
risch	$\frac{ix\,a^2}{b^3}-\frac{ix}{b}+\frac{e^{2i(dx+c)}}{8bd}-\frac{ia\,e^{i(dx+c)}}{2b^2d}+\frac{ia\,e^{-i(dx+c)}}{2b^2d}+\frac{e^{-2i(dx+c)}}{8bd}+\frac{2ia^2c}{b^3d}-\frac{2ic}{bd}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(}}{b}\right.}{b^3d}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{bd}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{bd}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{b^2d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{(a^2-b^2)\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^3d}$

input
```
int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*(-1/2*b*sin(d*x+c)^2+a*sin(d*x+c))+(-a^2+b^2)/b^3*ln(a+b*sin(d*
x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^2\cos\left(dx+c\right)^2+2\,ab\sin\left(dx+c\right)-2\,(a^2-b^2)\log\left(b\sin\left(dx+c\right)+a\right)}{2\,b^3d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - 2*(a^2 - b^2)*log(b*sin(d*x
 + c) + a))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\,(a^2-b^2)\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(b*sin(
d*x + c) + a)/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{b^3d} - \frac{bd\sin(dx+c)^2-2\,ad\sin(dx+c)}{2\,b^2d^2}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/(b^3*d) - 1/2*(b*d*sin(d*x + c)^
2 - 2*a*d*sin(d*x + c))/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{\sin(c+d\,x)^2}{2\,b}+\frac{\ln(a+b\,\sin(c+d\,x))\,(a^2-b^2)}{b^3}-\frac{a\,\sin(c+d\,x)}{b^2}}{d}$$

input
```
int(cos(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

output
```
-(sin(c + d*x)^2/(2*b) + (log(a + b*sin(c + d*x))*(a^2 - b^2))/b^3 - (a*si
n(c + d*x))/b^2)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.31

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^2-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)b^2-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+\right.}{2b^3d}$$

input
```
int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

output
```
(2*log(tan((c + d*x)/2)**2 + 1)*a**2 - 2*log(tan((c + d*x)/2)**2 + 1)*b**2
 - 2*log(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*a**2 + 2*log(ta
n((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*b**2 - sin(c + d*x)**2*b**
2 + 2*sin(c + d*x)*a*b + 2*b**2)/(2*b**3*d)
```

3.306 $\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 937

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

output
```
-6*I*a*f^3*polylog(4,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^4-b*(f*x+e)^3*ln(1-I*b
*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d-b*(f*x+e)^3*ln(1-I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d+b*(f*x+e)^3*ln(1+exp(2*I*(d*x+
c)))/(a^2-b^2)/d-3/2*I*b*f*(f*x+e)^2*polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2
)/d^2+6*I*a*f^3*polylog(4,I*exp(I*(d*x+c)))/(a^2-b^2)/d^4+3*I*b*f*(f*x+e)^
2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2+3*I*a*f*
(f*x+e)^2*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^2+3/4*I*b*f^3*polylog(4
,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^4-6*a*f^2*(f*x+e)*polylog(3,-I*exp(I*(d*x+
c)))/(a^2-b^2)/d^3+6*a*f^2*(f*x+e)*polylog(3,I*exp(I*(d*x+c)))/(a^2-b^2)/d
^3-6*b*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-
b^2)/d^3-6*b*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/(a^2-b^2)/d^3+3/2*b*f^2*(f*x+e)*polylog(3,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^
3-6*I*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^
4-6*I*b*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^
4-3*I*a*f*(f*x+e)^2*polylog(2,I*exp(I*(d*x+c)))/(a^2-b^2)/d^2+3*I*b*f*(f*x
+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2-2*I*
a*(f*x+e)^3*arctan(exp(I*(d*x+c)))/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2506 vs. 2(937) = 1874.

Time = 11.80 (sec) , antiderivative size = 2506, normalized size of antiderivative = 2.67

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((I*b*(e + f*x)^4)/f - (2*(a - b)*(1 + E^((2*I)*c))*(e + f*x)^3*Log[1 - I/
E^(I*(c + d*x))])/d + (2*(a + b)*(1 + E^((2*I)*c))*(e + f*x)^3*Log[1 + I/E
^(I*(c + d*x))])/d + (6*(a + b)*(1 + E^((2*I)*c))*f*(I*d^2*(e + f*x)^2*Pol
yLog[2, (-I)/E^(I*(c + d*x))] + 2*f*(d*(e + f*x)*PolyLog[3, (-I)/E^(I*(c +
 d*x))] - I*f*PolyLog[4, (-I)/E^(I*(c + d*x))])))/d^4 - ((6*I)*(a - b)*(1
+ E^((2*I)*c))*f*(d^2*(e + f*x)^2*PolyLog[2, I/E^(I*(c + d*x))] - (2*I)*d*
f*(e + f*x)*PolyLog[3, I/E^(I*(c + d*x))] - 2*f^2*PolyLog[4, I/E^(I*(c + d
*x))]))/d^4)/(2*(a^2 - b^2)*(1 + E^((2*I)*c))) + (b*((-4*I)*d^4*e^3*E^((2*
I)*c)*x - (6*I)*d^4*e^2*E^((2*I)*c)*f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^
3 - I*d^4*E^((2*I)*c)*f^3*x^4 - (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))
/(b*(-1 + E^((2*I)*(c + d*x))))] + (2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E
^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - d^3*e^3*Log[4*a^2*E^((2*
I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] + d^3*e^3*E^((2*I)*c)*Lo
g[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 6*d^3*e^
2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(
I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*
E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^
2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(...
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 814, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5044, 5030, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5044}$$

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 5030}$$

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\left(\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^4}{4bf}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-$$
$$\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1-\frac{ibe}{\sqrt{a}}\right.}{bd}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-$$
$$\frac{b^2\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\operatorname{Poly}}{}\right.}{bd}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-$$

$$b^2\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}(\ldots}{d}\right.}{\ldots}\right.$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-$$

$$b^2\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\ldots\right.}{\ldots}\right.$$

$\downarrow$ 7143

$$\frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-$$

$$b^2\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ib\ldots}{a+\ldots}\right.}{d}\right.}{\ldots}\right.$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^3\sec(c+dx)-b(e+fx)^3\tan(c+dx)\right)dx}{a^2-b^2} - $$

$$b^2\left(-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ib}{a+}\right.}{d}\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{-\frac{ib(e+fx)^4}{4f}-\frac{2ia\arctan\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{b\log\left(1+e^{2i(c+dx)}\right)(e+fx)^3}{d}+\frac{3iaf\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)(e+fx)^2}{d^2}-\frac{3iaf\operatorname{PolyLog}\left(2,ie^{i(\right.}}{d^2}}{}$$

$$b^2\left(-\frac{i(e+fx)^4}{4bf}+\frac{\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^3}{bd}+\frac{\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)(e+fx)^3}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4\right.}{}\right.}{}\right.}{bd}\right.$$

input
```
Int[((e + f*x)^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-((b^2*(((-1/4*I)*(e + f*x)^4)/(b*f) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)
*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*PolyLog[4
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d) - (3*f*((I
*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d -
((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d
^2))/d))/(b*d)))/(a^2 - b^2)) + (((-1/4*I)*b*(e + f*x)^4)/f - ((2*I)*a*(e
+ f*x)^3*ArcTan[E^(I*(c + d*x))])/d + (b*(e + f*x)^3*Log[1 + E^((2*I)*(c +
 d*x))])/d + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/d^2
- ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/d^2 - (((3*I)/2)*b
*f*(e + f*x)^2*PolyLog[2, -E^((2*I)*(c + d*x))])/d^2 - (6*a*f^2*(e + f*x)*
PolyLog[3, (-I)*E^(I*(c + d*x))])/d^3 + (6*a*f^2*(e + f*x)*PolyLog[3, I*E^
(I*(c + d*x))])/d^3 + (3*b*f^2*(e + f*x)*PolyLog[3, -E^((2*I)*(c + d*x))])
/(2*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/d^4 + ((6*I)*a*f
^3*PolyLog[4, I*E^(I*(c + d*x))])/d^4 + (((3*I)/4)*b*f^3*PolyLog[4, -E^((2
*I)*(c + d*x))])/d^4)/(a^2 - b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5044
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-b^2/(a^2 - b^2)   Int[(e + f
*x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Simp[1/(a^2 - b
^2)   Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sec (dx+c)}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3069 vs. $2(822) = 1644$.

Time = 0.38 (sec) , antiderivative size = 3069, normalized size of antiderivative = 3.28

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3 \sec (c+dx)}{a+b\sin (c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\sec(dx+c)x^3}{\sin(dx+c)b+a}dx\right)a^2d\,f^3 - \left(\int \frac{\sec(dx+c)x^3}{\sin(dx+c)b+a}dx\right)b^2d\,f^3 + 3\left(\int \frac{\sec(dx+c)x^2}{\sin(dx+c)b+a}dx\right)a^2de\,f^2 - 3\left(\int \frac{\sec(dx+c)x^2}{\sin(dx+c)b+a}dx\right.}{}$$

input
```
int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(int((sec(c + d*x)*x**3)/(sin(c + d*x)*b + a),x)*a**2*d*f**3 - int((sec(c
+ d*x)*x**3)/(sin(c + d*x)*b + a),x)*b**2*d*f**3 + 3*int((sec(c + d*x)*x**
2)/(sin(c + d*x)*b + a),x)*a**2*d*e*f**2 - 3*int((sec(c + d*x)*x**2)/(sin(
c + d*x)*b + a),x)*b**2*d*e*f**2 + 3*int((sec(c + d*x)*x)/(sin(c + d*x)*b
+ a),x)*a**2*d*e**2*f - 3*int((sec(c + d*x)*x)/(sin(c + d*x)*b + a),x)*b**
2*d*e**2*f - log(tan((c + d*x)/2) - 1)*a*e**3 + log(tan((c + d*x)/2) - 1)*
b*e**3 + log(tan((c + d*x)/2) + 1)*a*e**3 + log(tan((c + d*x)/2) + 1)*b*e*
*3 - log(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*b*e**3)/(d*(a**
2 - b**2))
```

3.307 $\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 667

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2ia(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} + \frac{b(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{(a^2-b^2)\,d} + \frac{2iaf(e+fx)\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} - \frac{2iaf(e+fx)\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{2ibf(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d^2} + \frac{2ibf(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d^2} - \frac{ibf(e+fx)\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{(a^2-b^2)\,d^2} - \frac{2af^2\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^3} + \frac{2af^2\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^3} - \frac{2bf^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d^3} - \frac{2bf^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d^3} + \frac{bf^2\,\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)}{2\,(a^2-b^2)\,d^3}$$

output
```
-2*I*a*(f*x+e)^2*arctan(exp(I*(d*x+c)))/(a^2-b^2)/d-b*(f*x+e)^2*ln(1-I*b*e
xp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d-b*(f*x+e)^2*ln(1-I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d+b*(f*x+e)^2*ln(1+exp(2*I*(d*x+c)
))/(a^2-b^2)/d+2*I*a*f*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^2-
2*I*a*f*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/(a^2-b^2)/d^2+2*I*b*f*(f*x+e)*
polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2+2*I*b*f*(f
*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^2-I*b*
f*(f*x+e)*polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^2-2*a*f^2*polylog(3,-I*
exp(I*(d*x+c)))/(a^2-b^2)/d^3+2*a*f^2*polylog(3,I*exp(I*(d*x+c)))/(a^2-b^2
)/d^3-2*b*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/
d^3-2*b*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^
3+1/2*b*f^2*polylog(3,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1461 vs. 2(667) = 1334.

Time = 6.91 (sec) , antiderivative size = 1461, normalized size of antiderivative = 2.19

$$\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((2*(((2*I)*b*(e + f*x)^3)/f - (3*(a - b)*(1 + E^((2*I)*c))*(e + f*x)^2*Lo
g[1 - I/E^(I*(c + d*x))])/d + (3*(a + b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log
[1 + I/E^(I*(c + d*x))])/d + (6*(a + b)*(1 + E^((2*I)*c))*f*(I*d*(e + f*x)
*PolyLog[2, (-I)/E^(I*(c + d*x))] + f*PolyLog[3, (-I)/E^(I*(c + d*x))]))/d
^3 - ((6*I)*(a - b)*(1 + E^((2*I)*c))*f*(d*(e + f*x)*PolyLog[2, I/E^(I*(c
+ d*x))] - I*f*PolyLog[3, I/E^(I*(c + d*x))]))/d^3))/((a^2 - b^2)*(1 + E^(
(2*I)*c))) + (2*b*((-6*I)*d^3*e^2*E^((2*I)*c)*x - (6*I)*d^3*e*E^((2*I)*c)*
f*x^2 - (2*I)*d^3*E^((2*I)*c)*f^2*x^3 - 3*d^2*e^2*Log[b - (2*I)*a*E^(I*(c
+ d*x)) - b*E^((2*I)*(c + d*x))] + 3*d^2*e^2*E^((2*I)*c)*Log[b - (2*I)*a*E
^(I*(c + d*x)) - b*E^((2*I)*(c + d*x))] - 6*d^2*e*f*x*Log[1 + (b*E^(I*(2*c
 + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*e*E^((2*
I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E
^((2*I)*c)])] - 3*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) -
 Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 3*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E
^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2*
e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] + 6*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(
I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 3*d^2*f^2*x^2*Log[1 + (b*E^(I*(2
*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 3*d^2*E^((2*
I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +...
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5044, 5030, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \ 5044$$

$$\frac{\int (e+fx)^2 \sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 5030$$

$$\frac{\int (e+fx)^2 \sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} - \frac{b^2\left(\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx + \int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx - \frac{i(e+fx)^3}{3bf}\right)}{a^2-b^2}$$

$\downarrow$ 2620

$$\frac{\int (e+fx)^2 \sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} - \frac{b^2\left(-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd} - \frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c}}{\sqrt{a^2-}}\right.}{bd}\right.}{a^2-b^2}$$

$\downarrow$ 3011

$$\frac{\int (e+fx)^2 \sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} - \frac{b^2\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d} - \frac{if\int \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd} - \frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} - \frac{if\int \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{d}\right.}{bd}\right.}{a^2-b^2}$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^2 \sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} - \frac{b^2\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d} - \frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd} - \frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} - \frac{f\int e^{-i(c}}{}\right.}{bd}\right.}{a^2-b^2}$$

$\downarrow$ 7143

$$\frac{\int (e+fx)^2 \sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} - \frac{b^2\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d} - \frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd} - \frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} - \frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd} + \right.}{a^2-b^2}$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^2\sec(c+dx)-b(e+fx)^2\tan(c+dx)\right)dx}{a^2-b^2}-$$

$$\frac{b^2\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}+\right.}{a^2-b^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2ia(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{d}-\frac{2af^2\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{d^3}+\frac{2af^2\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)}{d^3}+\frac{2iaf(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{2ia}{}}{a^2}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}+\right.}{a^2-b^2}$$

input
```
Int[((e + f*x)^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-((b^2*(((-1/3*I)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c
+ d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d) - (2*f*((I*(e + f*x)*PolyLog[
2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/(a^2 - b^2)) + (((-1/
3*I)*b*(e + f*x)^3)/f - ((2*I)*a*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/d +
(b*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/d + ((2*I)*a*f*(e + f*x)*Poly
Log[2, (-I)*E^(I*(c + d*x))])/d^2 - ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(I
*(c + d*x))])/d^2 - (I*b*f*(e + f*x)*PolyLog[2, -E^((2*I)*(c + d*x))])/d^2
 - (2*a*f^2*PolyLog[3, (-I)*E^(I*(c + d*x))])/d^3 + (2*a*f^2*PolyLog[3, I*
E^(I*(c + d*x))])/d^3 + (b*f^2*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*d^3))/
(a^2 - b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5044
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-b^2/(a^2 - b^2)   Int[(e + f
*x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Simp[1/(a^2 - b
^2)   Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sec(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(588) = 1176$.

Time = 0.32 (sec) , antiderivative size = 2035, normalized size of antiderivative = 3.05

$$\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*b*f^2*polylog(3, -
(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))/b) + 2*b*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin
(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 + 2*b*f^2*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a - b)*f^2*polylog(
3, I*cos(d*x + c) + sin(d*x + c)) - 2*(a + b)*f^2*polylog(3, I*cos(d*x + c
) - sin(d*x + c)) + 2*(a - b)*f^2*polylog(3, -I*cos(d*x + c) + sin(d*x + c
)) - 2*(a + b)*f^2*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) - 2*(I*b*d*f
^2*x + I*b*d*e*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b*d*f^2*x
 + I*b*d*e*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*b*d*f^2*x -
I*b*d*e*f)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*b*d*f^2*x - I*
b*d*e*f)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*(a - b)*d*f^2*x
- I*(a - b)*d*e*f)*dilog(I*cos(d*x + c) + sin(d*x + c)) - 2*(-I*(a + b)*d*
f^2*x - I*(a + b)*d*e*f)*dilog(I*cos(d*x + c) - sin(d*x + c)) - 2*(I*(a...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\sec(dx+c)x^2}{\sin(dx+c)b+a}dx\right)a^2d\,f^2 - \left(\int \frac{\sec(dx+c)x^2}{\sin(dx+c)b+a}dx\right)b^2d\,f^2 + 2\left(\int \frac{\sec(dx+c)x}{\sin(dx+c)b+a}dx\right)a^2def - 2\left(\int \frac{\sec(dx+c)x}{\sin(dx+c)b+a}dx\right)}{}$$

input
```
int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(int((sec(c + d*x)*x**2)/(sin(c + d*x)*b + a),x)*a**2*d*f**2 - int((sec(c
+ d*x)*x**2)/(sin(c + d*x)*b + a),x)*b**2*d*f**2 + 2*int((sec(c + d*x)*x)/
(sin(c + d*x)*b + a),x)*a**2*d*e*f - 2*int((sec(c + d*x)*x)/(sin(c + d*x)*
b + a),x)*b**2*d*e*f - log(tan((c + d*x)/2) - 1)*a*e**2 + log(tan((c + d*x
)/2) - 1)*b*e**2 + log(tan((c + d*x)/2) + 1)*a*e**2 + log(tan((c + d*x)/2)
 + 1)*b*e**2 - log(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*b*e**
2)/(d*(a**2 - b**2))
```

3.308 $\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx =& -\frac{2ia(e+fx)\arctan\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d} \\ &-\frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} \\ &-\frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d} \\ &+\frac{b(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{(a^2-b^2)\,d} \\ &+\frac{iaf\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} \\ &-\frac{iaf\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^2}+\frac{ibf\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d^2} \\ &+\frac{ibf\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,d^2}-\frac{ibf\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2\,(a^2-b^2)\,d^2}\end{aligned}$$

output
```
-2*I*a*(f*x+e)*arctan(exp(I*(d*x+c)))/(a^2-b^2)/d-b*(f*x+e)*ln(1-I*b*exp(I
*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d-b*(f*x+e)*ln(1-I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d+b*(f*x+e)*ln(1+exp(2*I*(d*x+c)))/(a^2-
b^2)/d+I*a*f*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^2-I*a*f*polylog(2,I*
exp(I*(d*x+c)))/(a^2-b^2)/d^2+I*b*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b
^2)^(1/2)))/(a^2-b^2)/d^2+I*b*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/(a^2-b^2)/d^2-1/2*I*b*f*polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1185 vs. 2(413) = 826.

Time = 10.97 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.87

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-((b*e*Log[1 + (b*Sin[c + d*x])/a])/((a^2 - b^2)*d)) + (b*c*f*Log[1 + (b*S
in[c + d*x])/a])/((a^2 - b^2)*d^2) - (b^2*f*(((c + d*x)*Log[a + b*Sin[c +
d*x]])/b - ((-1/2*I)*(-c + Pi/2 - d*x)^2 + (4*I)*ArcSin[Sqrt[(a + b)/b]/Sq
rt[2]]*ArcTan[((a - b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[a^2 - b^2]] + (-c +
Pi/2 - d*x + 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 + ((a - Sqrt[a^2 - b
^2])*E^(I*(-c + Pi/2 - d*x)))/b] + (-c + Pi/2 - d*x - 2*ArcSin[Sqrt[(a + b
)/b]/Sqrt[2]])*Log[1 + ((a + Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b]
- (-c + Pi/2 - d*x)*Log[a + b*Sin[c + d*x]] - I*(PolyLog[2, ((-a - Sqrt[a^
2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] + PolyLog[2, ((-a + Sqrt[a^2 - b^2])
*E^(I*(-c + Pi/2 - d*x)))/b]))/b))/((a^2 - b^2)*d^2) + ((d*e + d*f*x)*(((-
I)*b*(d*e + d*f*x)^2)/f + 2*(a - b)*(d*e - c*f)*Log[1 - Tan[(c + d*x)/2]]
- 4*b*(d*e + d*f*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*(a + b)*(d*e -
 c*f)*Log[1 + Tan[(c + d*x)/2]] - (4*I)*b*f*PolyLog[2, -Cos[c + d*x] + I*S
in[c + d*x]] + (2*I)*(a + b)*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(1/2 - I/2
)*(1 + Tan[(c + d*x)/2])] + PolyLog[2, ((1 + I) - (1 - I)*Tan[(c + d*x)/2]
)/2]) - (2*I)*(a + b)*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(1/2 + I/2)*(1 +
Tan[(c + d*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(c + d*x)/2])]) + (2
*I)*(a - b)*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(c +
 d*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/2]) - (2*I)*(
a - b)*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(c + d...
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5044, 5030, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5044}$$

$$\frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{5030}$$

$$\frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\left(\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^2}{2bf}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\left(-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)}{2bf}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)}{}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\left(-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+f}{2bf}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int (a(e+fx)\sec(c+dx)-b(e+fx)\tan(c+dx))dx}{a^2-b^2}-\frac{b^2\left(-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+f}{2bf}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2ia(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d}+\frac{iaf\,\mathrm{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2}-\frac{iaf\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2}-\frac{ibf\,\mathrm{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{2d^2}+\frac{b(e+fx)\log(1+e}{d}}{a^2-b^2}$$

$$\frac{b^2\left(-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx}{2bf}\right.}{a^2-b^2}$$

input
```
Int[((e + f*x)*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-((b^2*(((-1/2*I)*(e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*
x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*d^2)))/(a^2 - b^2)) + (((-1/2*I)*b*(e + f*x)^2)/
f - ((2*I)*a*(e + f*x)*ArcTan[E^(I*(c + d*x))])/d + (b*(e + f*x)*Log[1 + E
^((2*I)*(c + d*x))])/d + (I*a*f*PolyLog[2, (-I)*E^(I*(c + d*x))])/d^2 - (I
*a*f*PolyLog[2, I*E^(I*(c + d*x))])/d^2 - ((I/2)*b*f*PolyLog[2, -E^((2*I)*
(c + d*x))])/d^2)/(a^2 - b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5044
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-b^2/(a^2 - b^2)   Int[(e + f
*x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Simp[1/(a^2 - b
^2)   Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(373) = 746$.

Time = 1.40 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{eb\ln\left(ib\,\mathrm{e}^{2i(dx+c)}-2a\,\mathrm{e}^{i(dx+c)}-ib\right)}{d(a-b)(a+b)}-\frac{4e\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d(4a+4b)}+\frac{4e\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d(4a-4b)}-\frac{fb\ln\left(\frac{-ib\,\mathrm{e}^{i(dx+c)}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{d(a-b)(a+b)}-\frac{fb\ln}{}$

input
```
int((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*b/(a-b)/(a+b)*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-4/d*e
/(4*a+4*b)*ln(exp(I*(d*x+c))-I)+4/d*e/(4*a-4*b)*ln(exp(I*(d*x+c))+I)-1/d*f
*b/(a-b)/(a+b)*ln((-I*exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/
2)))*x-1/d^2*f*b/(a-b)/(a+b)*ln((-I*exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)+a)/(a
+(a^2-b^2)^(1/2)))*c-1/d*f*b/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^
(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x-1/d^2*f*b/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+
c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-4*I/d^2*f/(4*a+4*b)*ln(-I*e
xp(I*(d*x+c)))*ln(-I*(-exp(I*(d*x+c))+I))+I/d^2*f*b/(a-b)/(a+b)*dilog((-I*
exp(I*(d*x+c))*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-4/d*f/(4*a+4*b)*l
n(-I*(-exp(I*(d*x+c))+I))*x-4/d^2*f/(4*a+4*b)*ln(-I*(-exp(I*(d*x+c))+I))*c
+I/d^2*f*b/(a-b)/(a+b)*dilog((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a
^2-b^2)^(1/2)))-4*I/d^2*f/(4*a+4*b)*dilog(-I*exp(I*(d*x+c)))+4/d*f/(4*a-4*
b)*ln(-I*(exp(I*(d*x+c))+I))*x+4/d^2*f/(4*a-4*b)*ln(-I*(exp(I*(d*x+c))+I))
*c-4*I/d^2*f/(4*a-4*b)*dilog(-I*(exp(I*(d*x+c))+I))+1/d^2*c*f*b/(a-b)/(a+b
)*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+4/d^2*c*f/(4*a+4*b)*ln(e
xp(I*(d*x+c))-I)-4/d^2*c*f/(4*a-4*b)*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(356) = 712$.

Time = 0.29 (sec) , antiderivative size = 1181, normalized size of antiderivative = 2.86

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(-I*b*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*b*f*dilog((I*a*co
s(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b + 1) + I*b*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x +
c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + I*b*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*(a - b)*f*dilog(I*c
os(d*x + c) + sin(d*x + c)) + I*(a + b)*f*dilog(I*cos(d*x + c) - sin(d*x +
 c)) - I*(a - b)*f*dilog(-I*cos(d*x + c) + sin(d*x + c)) - I*(a + b)*f*dil
og(-I*cos(d*x + c) - sin(d*x + c)) + (b*d*e - b*c*f)*log(2*b*cos(d*x + c)
+ 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d*e - b*c*
f)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a) + (b*d*e - b*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d*e - b*c*f)*log(-2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*d*f*x + b*
c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f)*log(-(I*a*cos(
d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f)*log(-(-I*a*cos(d*x + c) - a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - ...
```

Sympy [F]

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)\sec{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\sec(dx+c)x}{\sin(dx+c)b+a}dx\right)a^2df - \left(\int \frac{\sec(dx+c)x}{\sin(dx+c)b+a}dx\right)b^2df - \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)ae + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)be}{d\left(a^2\right.}$$

input
```
int((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(int((sec(c + d*x)*x)/(sin(c + d*x)*b + a),x)*a**2*d*f - int((sec(c + d*x)
*x)/(sin(c + d*x)*b + a),x)*b**2*d*f - log(tan((c + d*x)/2) - 1)*a*e + log
(tan((c + d*x)/2) - 1)*b*e + log(tan((c + d*x)/2) + 1)*a*e + log(tan((c +
d*x)/2) + 1)*b*e - log(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a)*b
*e)/(d*(a**2 - b**2))
```

3.309 $\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

output
```
-1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-b*ln(a+b*sin(d*
x+c))/(a^2-b^2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{(-a+b)\log(1-\sin(c+dx))+(a+b)\log(1+\sin(c+dx))-2b\log(a+b\sin(c+dx))}{2(a-b)(a+b)d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*Log[
a + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{b\int \frac{1}{(a+b\sin(c+dx))(b^2-b^2\sin^2(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 477}$$

$$\frac{\int \left(-\frac{b^2}{(a^2-b^2)(a+b\sin(c+dx))}+\frac{b}{2(a+b)(b-b\sin(c+dx))}+\frac{b}{2(a-b)(\sin(c+dx)b+b)}\right)d(b\sin(c+dx))}{bd}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^2\log(a+b\sin(c+dx))}{a^2-b^2}-\frac{b\log(b-b\sin(c+dx))}{2(a+b)}+\frac{b\log(b\sin(c+dx)+b)}{2(a-b)}}{bd}$$

input
```
Int[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
(-1/2*(b*Log[b - b*Sin[c + d*x]])/(a + b) - (b^2*Log[a + b*Sin[c + d*x]])/
(a^2 - b^2) + (b*Log[b + b*Sin[c + d*x]])/(2*(a - b)))/(b*d)
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}-\frac{b\ln(a+b\sin(dx+c))}{(a-b)(a+b)}}{d}$
default	$\frac{\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}-\frac{b\ln(a+b\sin(dx+c))}{(a-b)(a+b)}}{d}$
parallelrisch	$\frac{-b\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a\right)+(-a+b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)(a+b)}{d(a^{2}-b^{2})}$
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d(a+b)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}{d(a^{2}-b^{2})}$
risch	$-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{ix}{a+b}+\frac{ic}{d(a+b)}+\frac{2ibx}{a^{2}-b^{2}}+\frac{2ibc}{d(a^{2}-b^{2})}+\frac{\ln(e^{i(dx+c)}+i)}{d(a-b)}-\frac{\ln(e^{i(dx+c)}-i)}{d(a+b)}-\frac{b\ln\left(e^{2i(}\right.}{}$

input
```
int(sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*a-2*b)*ln(1+sin(d*x+c))-1/(2*a+2*b)*ln(sin(d*x+c)-1)-b/(a-b)/(a+
b)*ln(a+b*sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)+(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) + (a - b
)*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{2\,b\log(b\sin(dx+c)+a)}{a^2-b^2} - \frac{\log(\sin(dx+c)+1)}{a-b} + \frac{\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*b*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
 b) + log(sin(d*x + c) - 1)/(a + b))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{b^2\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^2bd-b^3d} + \frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{2\left(ad-bd\right)} - \frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{2\left(ad+bd\right)}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*sin(d*x + c) + a))/(a^2*b*d - b^3*d) + 1/2*log(abs(sin(d*x
+ c) + 1))/(a*d - b*d) - 1/2*log(abs(sin(d*x + c) - 1))/(a*d + b*d)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)} - \frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)} - \frac{b\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^2-b^2\right)}$$

input
```
int(1/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
log(sin(c + d*x) + 1)/(2*d*(a - b)) - log(sin(c + d*x) - 1)/(2*d*(a + b))
- (b*log(a + b*sin(c + d*x)))/(d*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,(a^2-b^2)}$$

input
```
int(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) - 1)*b + log(tan((c
 + d*x)/2) + 1)*a + log(tan((c + d*x)/2) + 1)*b - log(tan((c + d*x)/2)**2*
a + 2*tan((c + d*x)/2)*b + a)*b)/(d*(a**2 - b**2))
```

3.310 $\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 923

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{ia(e+fx)^3}{(a^2-b^2)\,d}-\frac{6ibf(e+fx)^2\arctan\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2}+\frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}+\frac{3af(e+fx)^2\log\left(1+e^{2i(c+dx)}\right)}{(a^2-b^2)\,d^2}+\frac{6ibf^2(e+fx)\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^3}-\frac{6ibf^2(e+fx)\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^3}+\frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2}-\frac{3b^2f(e+fx)^2\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2}-\frac{3iaf^2(e+fx)\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{(a^2-b^2)\,d^3}-\frac{6bf^3\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^4}+\frac{6bf^3\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^4}+\frac{6ib^2f^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^3}-\frac{6ib^2f^2(e+fx)\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^3}+\frac{3af^3\,\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)}{2\,(a^2-b^2)\,d^4}-\frac{6b^2f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^4}+\frac{6b^2f^3\,\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^4}-\frac{b(e+fx)^3\sec(c+dx)}{(a^2-b^2)\,d}+\frac{a(e+fx)^3\tan(c+dx)}{(a^2-b^2)\,d}$$

output
```
6*I*b*f^2*(f*x+e)*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^3+6*I*b^2*f^2*(
f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d
^3-I*a*(f*x+e)^3/(a^2-b^2)/d-6*I*b*f*(f*x+e)^2*arctan(exp(I*(d*x+c)))/(a^2
-b^2)/d^2+3*a*f*(f*x+e)^2*ln(1+exp(2*I*(d*x+c)))/(a^2-b^2)/d^2+I*b^2*(f*x+
e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-6*I*b^
2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^
(3/2)/d^3+3*b^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2
)))/(a^2-b^2)^(3/2)/d^2-3*b^2*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+
(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-I*b^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+
c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-6*b*f^3*polylog(3,-I*exp(I*(d*x
+c)))/(a^2-b^2)/d^4+6*b*f^3*polylog(3,I*exp(I*(d*x+c)))/(a^2-b^2)/d^4-6*I*
b*f^2*(f*x+e)*polylog(2,I*exp(I*(d*x+c)))/(a^2-b^2)/d^3-3*I*a*f^2*(f*x+e)*
polylog(2,-exp(2*I*(d*x+c)))/(a^2-b^2)/d^3+3/2*a*f^3*polylog(3,-exp(2*I*(d
*x+c)))/(a^2-b^2)/d^4-6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^
(1/2)))/(a^2-b^2)^(3/2)/d^4+6*b^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2
-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^4-b*(f*x+e)^3*sec(d*x+c)/(a^2-b^2)/d+a*(f*
x+e)^3*tan(d*x+c)/(a^2-b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 10.31 (sec) , antiderivative size = 1438, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(f*(((2*I)*a*(e + f*x)^3)/f + (3*(a - b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log
[1 - I/E^(I*(c + d*x))])/d + (3*(a + b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log[
1 + I/E^(I*(c + d*x))])/d + (6*(a + b)*(1 + E^((2*I)*c))*f*(I*d*(e + f*x)*
PolyLog[2, (-I)/E^(I*(c + d*x))] + f*PolyLog[3, (-I)/E^(I*(c + d*x))]))/d^
3 + (6*(a - b)*(1 + E^((2*I)*c))*f*(I*d*(e + f*x)*PolyLog[2, I/E^(I*(c + d
*x))] + f*PolyLog[3, I/E^(I*(c + d*x))]))/d^3))/((a^2 - b^2)*d*(1 + E^((2*
I)*c))) + (b^2*(2*Sqrt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x))
)/Sqrt[a^2 - b^2]] + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*
x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1
- (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f
^3*x^3*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a
^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])
] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqr
t[-a^2 + b^2])] - Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/
(I*a + Sqrt[-a^2 + b^2])] - (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLo
g[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b
^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 +
b^2]))] + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a
 + Sqrt[-a^2 + b^2])] + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c +
d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog...
```

Rubi [A] (verified)

Time = 3.44 (sec) , antiderivative size = 774, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5044, 3042, 3804, 2694, 27, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5044}$$

$$\frac{\int (e+fx)^3\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$\Big\downarrow$ 3804

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\int \frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a^2-b^2}$$

$\Big\downarrow$ 2694

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$\Big\downarrow$ 27

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$\Big\downarrow$ 2620

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}$$

$\Big\downarrow$ 3011

$$\frac{\int (e+fx)^3\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\cdots\right)}{bd}\cdots\right.}{\cdots}\right)}{a^2-b^2}$$

$\downarrow$ 7163

$$\frac{\int (e+fx)^3\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{if\int \operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2-b^2}}\cdots\right.}{\cdots}$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} -$$

$$2b^2 \left(\frac{ib \left(\frac{(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd} - \frac{3f \left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} - \frac{2if \left(\frac{f \int e^{-i(c+dx)} \operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right) de^{i(c+dx)}}{d^2} - \frac{i(e+fx) \operatorname{PolyL}}{}\right.}{d} \right.}{bd} \right.}{2\sqrt{a^2-b^2}} \right.$$

$\downarrow$ 7143

$$\frac{\int (e+fx)^3 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2} -$$

$$2b^2 \left(\frac{ib \left(\frac{(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd} - \frac{3f \left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} - \frac{2if \left(\frac{f \operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2} - \frac{i(e+fx) \operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d} \right)}{d} \right.}{bd} \right.}{2\sqrt{a^2-b^2}} \right.$$

a

$$\Big\downarrow \; 7293$$

$$\frac{\int \left(a(e+fx)^3\sec^2(c+dx)-b(e+fx)^3\sec(c+dx)\tan(c+dx)\right)dx}{a^2-b^2}-$$

$$2b^2\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right.}{bd}}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3af^3\operatorname{PolyLog}(3,-e^{2i(c+dx)})}{2d^4}-\frac{3iaf^2(e+fx)\operatorname{PolyLog}(2,-e^{2i(c+dx)})}{d^3}+\frac{3af(e+fx)^2\log(1+e^{2i(c+dx)})}{d^2}+\frac{a(e+fx)^3\tan(c+dx)}{d}-\frac{ia(e+fx)}{d}}{}$$

$$2b^2\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{d}\right)}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

a

input
```
Int[((e + f*x)^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*b^2*(((-1/2*I)*b*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt
[a^2 - b^2])])/(b*d) - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b
*(((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d)
 - (3*f*((I*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/d - ((2*I)*f*(((-I)*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a +
 Sqrt[a^2 - b^2])])/d + (f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/d^2))/d))/(b*d)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (((-I)*a*(e +
f*x)^3)/d - ((6*I)*b*f*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/d^2 + (3*a*f*(
e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/d^2 + ((6*I)*b*f^2*(e + f*x)*Poly
Log[2, (-I)*E^(I*(c + d*x))])/d^3 - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^
(I*(c + d*x))])/d^3 - ((3*I)*a*f^2*(e + f*x)*PolyLog[2, -E^((2*I)*(c + d*x
))])/d^3 - (6*b*f^3*PolyLog[3, (-I)*E^(I*(c + d*x))])/d^4 + (6*b*f^3*PolyL
og[3, I*E^(I*(c + d*x))])/d^4 + (3*a*f^3*PolyLog[3, -E^((2*I)*(c + d*x))])
/(2*d^4) - (b*(e + f*x)^3*Sec[c + d*x])/d + (a*(e + f*x)^3*Tan[c + d*x])/d
)/(a^2 - b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5044
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-b^2/(a^2 - b^2)   Int[(e + f
*x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Simp[1/(a^2 - b
^2)   Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^3 \sec(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4116 vs. 2(810) = 1620.

Time = 0.45 (sec) , antiderivative size = 4116, normalized size of antiderivative = 4.46

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3 \sec^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3 \sec(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output

```
( - 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*c
os(c + d*x)*a**3*b**2*d**3*e**3 + 24*cos(c + d*x)*int(x**2/(tan((c + d*x)/
2)**6*a + 2*tan((c + d*x)/2)**5*b - tan((c + d*x)/2)**4*a - 4*tan((c + d*x
)/2)**3*b - tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**6*b*d*
*3*f**3 - 48*cos(c + d*x)*int(x**2/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x
)/2)**5*b - tan((c + d*x)/2)**4*a - 4*tan((c + d*x)/2)**3*b - tan((c + d*x
)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a**4*b**3*d**3*f**3 + 24*cos(c +
d*x)*int(x**2/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b - tan((c +
d*x)/2)**4*a - 4*tan((c + d*x)/2)**3*b - tan((c + d*x)/2)**2*a + 2*tan((c
+ d*x)/2)*b + a),x)*a**2*b**5*d**3*f**3 - 32*cos(c + d*x)*int((tan((c + d*
x)/2)**3*x**3)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b - tan((c +
 d*x)/2)**4*a - 4*tan((c + d*x)/2)**3*b - tan((c + d*x)/2)**2*a + 2*tan((c
 + d*x)/2)*b + a),x)*a**4*b**3*d**4*f**3 + 64*cos(c + d*x)*int((tan((c + d
*x)/2)**3*x**3)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b - tan((c
+ d*x)/2)**4*a - 4*tan((c + d*x)/2)**3*b - tan((c + d*x)/2)**2*a + 2*tan((
c + d*x)/2)*b + a),x)*a**2*b**5*d**4*f**3 - 32*cos(c + d*x)*int((tan((c +
d*x)/2)**3*x**3)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b - tan((c
 + d*x)/2)**4*a - 4*tan((c + d*x)/2)**3*b - tan((c + d*x)/2)**2*a + 2*tan(
(c + d*x)/2)*b + a),x)*b**7*d**4*f**3 - 48*cos(c + d*x)*int((tan((c + d*x)
/2)**3*x**2)/(tan((c + d*x)/2)**6*a + 2*tan((c + d*x)/2)**5*b - tan((c ...
```

3.311 $\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 659

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{ia(e+fx)^2}{(a^2-b^2)\,d} - \frac{4ibf(e+fx)\arctan\left(e^{i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{2af(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{(a^2-b^2)\,d^2} + \frac{2ibf^2\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^3} - \frac{2ibf^2\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{(a^2-b^2)\,d^3} + \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} - \frac{iaf^2\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{(a^2-b^2)\,d^3} + \frac{2ib^2f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^3} - \frac{2ib^2f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^3} - \frac{b(e+fx)^2\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a(e+fx)^2\tan(c+dx)}{(a^2-b^2)\,d}$$

output
```
-I*a*(f*x+e)^2/(a^2-b^2)/d-4*I*b*f*(f*x+e)*arctan(exp(I*(d*x+c)))/(a^2-b^2
)/d^2+I*b^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^
2)^(3/2)/d-I*b^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a
^2-b^2)^(3/2)/d+2*a*f*(f*x+e)*ln(1+exp(2*I*(d*x+c)))/(a^2-b^2)/d^2+2*I*b*f
^2*polylog(2,-I*exp(I*(d*x+c)))/(a^2-b^2)/d^3-2*I*b*f^2*polylog(2,I*exp(I*
(d*x+c)))/(a^2-b^2)/d^3+2*b^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-2*b^2*f*(f*x+e)*polylog(2,I*b*exp(I*(d
*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-I*a*f^2*polylog(2,-exp(2*I
*(d*x+c)))/(a^2-b^2)/d^3+2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3-2*I*b^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3-b*(f*x+e)^2*sec(d*x+c)/(a^2-b^2)/
d+a*(f*x+e)^2*tan(d*x+c)/(a^2-b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.91 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.70

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(I*b^2*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((
-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -(
(b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2
]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*
f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])]
- Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b
^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sq
rt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]
))])))/(Sqrt[-(a^2 - b^2)^2]*(-a^2 + b^2)*d^3) + (b*(e + f*x)^2*Sec[c])/((
-a^2 + b^2)*d) + (2*a*e*f*Sec[c]*(Cos[c]*Log[Cos[c]*Cos[d*x] - Sin[c]*Sin[
d*x]] + d*x*Sin[c]))/((a^2 - b^2)*d^2*(Cos[c]^2 + Sin[c]^2)) + ((4*I)*b*e*
f*ArcTan[((-I)*Sin[c] - I*Cos[c]*Tan[(d*x)/2])/Sqrt[Cos[c]^2 + Sin[c]^2]])
/((a^2 - b^2)*d^2*Sqrt[Cos[c]^2 + Sin[c]^2]) + (a*f^2*Csc[c]*((d^2*x^2)/E^
(I*ArcTan[Cot[c]]) - (Cot[c]*(I*d*x*(-Pi - 2*ArcTan[Cot[c]]) - Pi*Log[1 +
E^((-2*I)*d*x)] - 2*(d*x - ArcTan[Cot[c]])*Log[1 - E^((2*I)*(d*x - ArcTan[
Cot[c]]))] + Pi*Log[Cos[d*x]] - 2*ArcTan[Cot[c]]*Log[Sin[d*x - ArcTan[Cot[
c]]]] + I*PolyLog[2, E^((2*I)*(d*x - ArcTan[Cot[c]]))]))/Sqrt[1 + Cot[c]^2
])*Sec[c])/((a^2 - b^2)*d^3*Sqrt[Csc[c]^2*(Cos[c]^2 + Sin[c]^2)]) + (2*b*f
^2*(-((Csc[c]*((d*x - ArcTan[Cot[c]])*(Log[1 - E^(I*(d*x - ArcTan[Cot[c]])
)] - Log[1 + E^(I*(d*x - ArcTan[Cot[c]]))]) + I*(PolyLog[2, -E^(I*(d*x ...
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5044, 3042, 3804, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sec^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5044}$$

$$\frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3804}$$

$$\frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a^2-b^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)^2 \sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int (e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)d}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\int (e+fx)^2\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int \operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-b}}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{a^2-b^2}$$

$\downarrow$ 2720

$$\frac{\int (e+fx)^2\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{a^2-b^2}$$

$\downarrow$ 7143

$$\frac{\int (e+fx)^2\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\cdots\right.}{\cdots}\cdots\right)}{a^2-b^2}$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^2\sec^2(c+dx)-b(e+fx)^2\sec(c+dx)\tan(c+dx)\right)dx}{a^2-b^2}-$$

$$\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\right.}{}\right.}{a^2-b^2}$$

$\downarrow$ 2009

$$\frac{-\frac{iaf^2\operatorname{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{d^3}+\frac{2af(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{d^2}+\frac{a(e+fx)^2\tan(c+dx)}{d}-\frac{ia(e+fx)^2}{d}-\frac{4ibf(e+fx)\arctan\left(e^{i(c+dx)}\right)}{d^2}+2}{a^2-b^2}$$

$$\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\right.}{}\right.}{a^2-b^2}$$

input
```
Int[((e + f*x)^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*b^2*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt
[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt
[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*
x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog
[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 -
 b^2]))/(a^2 - b^2) + (((-I)*a*(e + f*x)^2)/d - ((4*I)*b*f*(e + f*x)*ArcTa
n[E^(I*(c + d*x))])/d^2 + (2*a*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/d
^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/d^3 - ((2*I)*b*f^2*Pol
yLog[2, I*E^(I*(c + d*x))])/d^3 - (I*a*f^2*PolyLog[2, -E^((2*I)*(c + d*x))
])/d^3 - (b*(e + f*x)^2*Sec[c + d*x])/d + (a*(e + f*x)^2*Tan[c + d*x])/d)/
(a^2 - b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5044
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-b^2/(a^2 - b^2)   Int[(e + f
*x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Simp[1/(a^2 - b
^2)   Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{(fx+e)^2 \sec (dx+c)^2}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2659 vs. $2(574) = 1148$.

Time = 0.36 (sec) , antiderivative size = 2659, normalized size of antiderivative = 4.03

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, -(I*a*cos(d
*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3,
 -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b) + 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c
)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*b^3*f^2*sqrt(-(a^2 - b^2)/b^2
)*cos(d*x + c)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2*b - b^3)*d^
2*f^2*x^2 + 4*(a^2*b - b^3)*d^2*e*f*x + 2*(a^2*b - b^3)*d^2*e^2 - 2*I*(a^3
 - a^2*b - a*b^2 + b^3)*f^2*cos(d*x + c)*dilog(I*cos(d*x + c) + sin(d*x +
c)) + 2*I*(a^3 + a^2*b - a*b^2 - b^3)*f^2*cos(d*x + c)*dilog(I*cos(d*x + c
) - sin(d*x + c)) + 2*I*(a^3 - a^2*b - a*b^2 + b^3)*f^2*cos(d*x + c)*dilog
(-I*cos(d*x + c) + sin(d*x + c)) - 2*I*(a^3 + a^2*b - a*b^2 - b^3)*f^2*cos
(d*x + c)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 2*(I*b^3*d*f^2*x + I*b^3
*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(d*x + c) - a*si
n(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + 2*(-I*b^3*d*f^2*x - I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*
x + c)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(-I*b^3*d*f^2*x - I*...
```

Sympy [F]

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^2\sec^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\sec(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\cos\left(dx+c\right)b^2e^2+\cos\left(dx+c\right)\left(\int\frac{\sec(dx+c)^2x^2}{\sin(dx+c)b+a}dx\right)a^4d\,f^2-2\cos\left(dx+\right.}{}$$

input
```
int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*c
os(c + d*x)*b**2*e**2 + cos(c + d*x)*int((sec(c + d*x)**2*x**2)/(sin(c + d
*x)*b + a),x)*a**4*d*f**2 - 2*cos(c + d*x)*int((sec(c + d*x)**2*x**2)/(sin
(c + d*x)*b + a),x)*a**2*b**2*d*f**2 + cos(c + d*x)*int((sec(c + d*x)**2*x
**2)/(sin(c + d*x)*b + a),x)*b**4*d*f**2 + 2*cos(c + d*x)*int((sec(c + d*x
)**2*x)/(sin(c + d*x)*b + a),x)*a**4*d*e*f - 4*cos(c + d*x)*int((sec(c + d
*x)**2*x)/(sin(c + d*x)*b + a),x)*a**2*b**2*d*e*f + 2*cos(c + d*x)*int((se
c(c + d*x)**2*x)/(sin(c + d*x)*b + a),x)*b**4*d*e*f + cos(c + d*x)*a**2*b*
e**2 - cos(c + d*x)*b**3*e**2 + sin(c + d*x)*a**3*e**2 - sin(c + d*x)*a*b*
*2*e**2 - a**2*b*e**2 + b**3*e**2)/(cos(c + d*x)*d*(a**4 - 2*a**2*b**2 + b
**4))
```

3.312 $\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 349

$$\begin{aligned}\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{bf\operatorname{arctanh}(\sin(c+dx))}{(a^2-b^2)\,d^2} + \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\ &\quad - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\ &\quad + \frac{af\log(\cos(c+dx))}{(a^2-b^2)\,d^2} + \frac{b^2f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} \\ &\quad - \frac{b^2f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} \\ &\quad - \frac{b(e+fx)\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a(e+fx)\tan(c+dx)}{(a^2-b^2)\,d}\end{aligned}$$

output
```
b*f*arctanh(sin(d*x+c))/(a^2-b^2)/d^2+I*b^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c)
)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-I*b^2*(f*x+e)*ln(1-I*b*exp(I*(d*x
+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+a*f*ln(cos(d*x+c))/(a^2-b^2)/d
^2+b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)
/d^2-b^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/
2)/d^2-b*(f*x+e)*sec(d*x+c)/(a^2-b^2)/d+a*(f*x+e)*tan(d*x+c)/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2180 vs. $2(349) = 698$.

Time = 16.12 (sec) , antiderivative size = 2180, normalized size of antiderivative = 6.25

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(b*(d*e - c*f + f*(c + d*x)))/((-a^2 + b^2)*d^2) + (f*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/((a + b)*d^2) + (f*Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]])/((a - b)*d^2) + (d*e*Sin[(c + d*x)/2] - c*f*Sin[(c + d*x)/2] + f*
(c + d*x)*Sin[(c + d*x)/2])/((a + b)*d^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])) + (d*e*Sin[(c + d*x)/2] - c*f*Sin[(c + d*x)/2] + f*(c + d*x)*Sin[(c
+ d*x)/2])/((a - b)*d^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (((2*(d*e
 - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c +
d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*
Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a -
 b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] - (I*f*Log[1 - I*Tan[(c + d*x)/
2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^
2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + S
qrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[
-a^2 + b^2] - (I*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqr
t[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(1 + I*Tan[(c + d*
x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2
, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^
2] - (I*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b
^2]))])/Sqrt[-a^2 + b^2])*(-((b^2*e)/((a^2 - b^2)*(a + b*Sin[c + d*x]))...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5044, 3042, 3804, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5044}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{b^2\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3804}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a^2-b^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx \cdots}{}\right.}{2\sqrt{a^2-b^2}}\cdots}{a^2-b^2}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(a(e+fx)\sec^2(c+dx)-b(e+fx)\sec(c+dx)\tan(c+dx)\right)dx}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{af\log(\cos(c+dx))}{d^2}+\frac{a(e+fx)\tan(c+dx)}{d}+\frac{bf\text{arctanh}(\sin(c+dx))}{d^2}-\frac{b(e+fx)\sec(c+dx)}{d}}{a^2-b^2}-\frac{2b^2\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

input
```
Int[((e + f*x)*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*b^2*(((-1/2*I)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/(a^2 - b^2)
+ ((b*f*ArcTanh[Sin[c + d*x]])/d^2 + (a*f*Log[Cos[c + d*x]])/d^2 - (b*(e +
 f*x)*Sec[c + d*x])/d + (a*(e + f*x)*Tan[c + d*x])/d)/(a^2 - b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5044
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-b^2/(a^2 - b^2)   Int[(e + f
*x)^m*(Sec[c + d*x]^(n - 2)/(a + b*Sin[c + d*x])), x], x] + Simp[1/(a^2 - b
^2)   Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2064 vs. 2(319) = 638.

Time = 1.77 (sec) , antiderivative size = 2065, normalized size of antiderivative = 5.92

method	result	size
risch	Expression too large to display	2065

input
```
int((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2-b^2)^(3/2)/d^2*b^4*f/(a-b)/(a+b)*dilog(-(I*b*exp(I*(d*x+c))-a-(a^2-
b^2)^(1/2))/(a+(a^2-b^2)^(1/2)))-1/(a^2-b^2)^(3/2)/d^2*b^4*f/(a-b)/(a+b)*d
ilog((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/(a^2-b
^2)/d^2*b^4*f/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c)
)-2*a)/(-a^2+b^2)^(1/2))-2/(a^2-b^2)/d^2*a^4*f/(a+b)/(a-b)/(-a^2+b^2)^(1/2
)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-2/(a^2-b^2)/d^2*
a^2*f/(a-b)/(a+b)*(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(
-a^2+b^2)^(1/2))+1/(a^2-b^2)/d^2*b^2*f/(a-b)/(a+b)*(-a^2+b^2)^(1/2)*arctan
(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/(a^2-b^2)/d*e*b^2/(a-b
)/(a+b)*(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^
(1/2))+I/(a^2-b^2)^(3/2)/d^2*b^4*f/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-a-(
a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2)))*c+I/(a^2-b^2)^(3/2)/d*b^4*f/(a-b)/(a+
b)*ln(-(I*b*exp(I*(d*x+c))-a-(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2)))*x-I/(a^
2-b^2)/d^2*f*c*b^2/(a-b)/(a+b)*(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d
*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/(a^2-b^2)/d^2*f*c*b^4/(a-b)/(a+b)/(-a^2+b^
2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/(a^2-b^
2)/d*e*b^2/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2
*a)/(-a^2+b^2)^(1/2))*a^2-I/(a^2-b^2)^(3/2)/d*b^2*f/(a-b)/(a+b)*ln(-(I*b*e
xp(I*(d*x+c))-a-(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2)))*a^2*x-I/(a^2-b^2)^(3
/2)/d^2*b^2*f/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-a-(a^2-b^2)^(1/2))/(a...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(311) = 622$.

Time = 0.27 (sec) , antiderivative size = 1267, normalized size of antiderivative = 3.63

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*cos(d*x + c)
- a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) - b)/b + 1) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog((I*a*c
os(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) - I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*
dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*
cos(d*x + c)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*(a^2*b - b^3)*d*
f*x - (a^3 + a^2*b - a*b^2 - b^3)*f*cos(d*x + c)*log(sin(d*x + c) + 1) - (
a^3 - a^2*b - a*b^2 + b^3)*f*cos(d*x + c)*log(-sin(d*x + c) + 1) + (b^3*d*
e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) + 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b^3*d*e - b^3*c*
f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b^3*d*e - b^3*c*f)*sqrt(-(
a^2 - b^2)/b^2)*cos(d*x + c)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2
)/b^2)*cos(d*x + c)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(
-(a^2 - b^2)/b^2) - 2*I*a) - (b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*
cos(d*x + c)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) ...
```

Sympy [F]

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)\sec^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\sec{(dx+c)}^2}{b\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,b^2e+\cos(dx+c)\left(\int\frac{\sec(dx+c)^2x}{\sin(dx+c)b+a}dx\right)a^4df-2\cos(dx+c)}{}$$

input
```
int((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*c
os(c + d*x)*b**2*e + cos(c + d*x)*int((sec(c + d*x)**2*x)/(sin(c + d*x)*b
+ a),x)*a**4*d*f - 2*cos(c + d*x)*int((sec(c + d*x)**2*x)/(sin(c + d*x)*b
+ a),x)*a**2*b**2*d*f + cos(c + d*x)*int((sec(c + d*x)**2*x)/(sin(c + d*x)
*b + a),x)*b**4*d*f + cos(c + d*x)*a**2*b*e - cos(c + d*x)*b**3*e + sin(c
+ d*x)*a**3*e - sin(c + d*x)*a*b**2*e - a**2*b*e + b**3*e)/(cos(c + d*x)*d
*(a**4 - 2*a**2*b**2 + b**4))
```

3.313 $\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx=-\frac{2b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}$$

output
```
-2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-
sec(d*x+c)*(b-a*sin(d*x+c))/(a^2-b^2)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.81

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}(b-b\cos(c+dx)-a\sin(c+dx))}{(-a+b)(a+b)\sqrt{a^2-b^2}d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
(2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(b - b*Cos[c + d*x] - a*Sin[c + d*x]))/((-a + b)*(a + b)*Sqrt
[a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3175, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3175}$$

$$-\frac{\int \frac{b^2}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{2b^2\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\,(a^2-b^2)}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4b^2\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2-b^2\right)}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

$\Big\downarrow$ 217

$$-\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*b^2*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/((a^2 -
b^2)^(3/2)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

method	result	si
derivativedivides	$\frac{-\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$	11
default	$\frac{-\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$	11
risch	$\frac{-2ia+2\,e^{i(dx+c)}b}{d(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}-\frac{b^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}$	20

input
```
int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-2/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)-2/(2*a-2*b)/(tan
(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.63

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-a^2+b^2}b^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right.}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)} \right.$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(
d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b
^2 + b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b
^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c)
 - b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 39.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)} - \frac{2\,b^2\,\mathrm{atan}\left(\frac{\frac{b^2\,\left(2\,a^2\,b-2\,b^3\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}}{2\,b^2}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (
d*x)/2)^2 - 1)) - (2*b^2*atan(((b^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a -
 b)^(3/2)) + (2*a*b^2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a -
b)^(3/2)))/(2*b^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,b^2+\cos(dx+c)\,a^2b-\cos(dx+c)\,b^3+\sin(dx+c)\,a^3-}{\cos(dx+c)\,d\,(a^4-2a^2b^2+b^4)}$$

input
```
int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*c
os(c + d*x)*b**2 + cos(c + d*x)*a**2*b - cos(c + d*x)*b**3 + sin(c + d*x)*
a**3 - sin(c + d*x)*a*b**2 - a**2*b + b**3)/(cos(c + d*x)*d*(a**4 - 2*a**2
*b**2 + b**4))
```

3.314 $\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx=\operatorname{Int}\left(\frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 14.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5048}$$

$$\int \frac{\cos^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \cos(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 16.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*cos(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\cos\left(dx+c\right)^2}{b\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m\cos\left(dx+c\right)^2}{b\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 38.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(e+f\,x\right)^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((cos(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)^2*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos\left(dx+c\right)^2}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*cos(c + d*x)**2)/(sin(c + d*x)*b + a),x)
```

3.315 $\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)} \, dx = \text{Int}\left(\frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)} \, dx$$

input
```
Integrate[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 5048}$$

$$\int \frac{\cos(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \cos(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e+f\,x)^m}{a+b\,\sin(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e + f*x)^m)/(a + b*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 8.69

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-(fx+e)^m e-(fx+e)^m fx+2\left(\int \frac{(fx+e)^m}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a}dx\right)afm+2\left(\int \frac{(fx+e)^m}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{af(m+}$$

input
```
int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - (e + f*x)**m*e - (e + f*x)**m*f*x + 2*int((e + f*x)**m/(tan((c + d*x)/
2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*f*m + 2*int((e + f*x)**m/(tan((c
+ d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*a*f + 2*int(((e + f*x)**m*ta
n((c + d*x)/2))/(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*b + a),x)*b*f*
m + 2*int(((e + f*x)**m*tan((c + d*x)/2))/(tan((c + d*x)/2)**2*a + 2*tan((
c + d*x)/2)*b + a),x)*b*f)/(a*f*(m + 1))
```

3.316 $\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\text{Int}\left(\frac{(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{b\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m}{b\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx=\int \frac{(e+f\,x)^m}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)^m/(a + b*sin(c + d*x)),x)
```

output
```
int((e + f*x)^m/(a + b*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.85

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \frac{(fx+e)^m e+(fx+e)^m fx-\left(\int \frac{(fx+e)^m \sin(dx+c)}{\sin(dx+c)b+a}dx\right)bfm-\left(\int \frac{(fx+e)^m \sin(dx+c)}{\sin(dx+c)b+a}dx\right)bf}{af\,(m+1)}$$

input
```
int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

output
```
((e + f*x)**m*e + (e + f*x)**m*f*x - int(((e + f*x)**m*sin(c + d*x))/(sin(
c + d*x)*b + a),x)*b*f*m - int(((e + f*x)**m*sin(c + d*x))/(sin(c + d*x)*b
 + a),x)*b*f)/(a*f*(m + 1))
```

3.317 $\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \operatorname{Int}\left(\frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 161.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5048\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 5048}$$

$$\int \frac{\sec(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sec(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 26.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 38.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+f\,x)^m}{\cos\left(c+d\,x\right)\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(cos(c + d*x)*(a + b*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec\left(dx+c\right)}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*sec(c + d*x))/(sin(c + d*x)*b + a),x)
```

3.318 $\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Int}\left(\frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)},x\right)$$

output
```
Defer(Int)((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 26.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5048}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}\,dx$$
$$\Big\downarrow \text{ 5048}$$
$$\int \frac{\sec^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)}dx$$

input
```
Int[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 5048
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c + d
*x]^n)/(a + b*Sin[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && Tr
igQ[F]
```

Maple [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^m \sec(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 106.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**m*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**m*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 18.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^m \sec(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+f\,x)^m}{\cos\left(c+d\,x\right)^2\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^m/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

output
```
int((e + f*x)^m/(cos(c + d*x)^2*(a + b*sin(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^m \sec\left(dx+c\right)^2}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int(((e + f*x)**m*sec(c + d*x)**2)/(sin(c + d*x)*b + a),x)
```

3.319 $\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{2f\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{e+fx}{bd(a+b\sin(c+dx))}$$

output
```
2*f*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(1/2)/d^2
-(f*x+e)/b/d/(a+b*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{2f\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{d(e+fx)}{a+b\sin(c+dx)}}{bd^2}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
((2*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
(d*(e + f*x))/(a + b*Sin[c + d*x]))/(b*d^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4922, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4922}$$

$$\frac{f\int \frac{1}{a+b\sin(c+dx)}dx}{bd}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{f\int \frac{1}{a+b\sin(c+dx)}dx}{bd}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{2f\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd^2}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{4f\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2f\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

input
```
Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(2*f*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(b*Sqrt[a^2
 - b^2]*d^2) - (e + f*x)/(b*d*(a + b*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.52

method	result	size
risch	$-\frac{2 i \left(f x +e \right) {\mathrm e}^{i \left(d x +c \right)}}{b d \left(2 i a \,{\mathrm e}^{i \left(d x +c \right)}+b \,{\mathrm e}^{2 i \left(d x +c \right)}-b \right)}-\frac{f \ln \left({\mathrm e}^{i \left(d x +c \right)}+\frac{i \sqrt{-a^{2}+b^{2}}\, a -a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\, b}\right)}{\sqrt{-a^{2}+b^{2}}\, d^{2} b}+\frac{f \ln \left({\mathrm e}^{i \left(d x +c \right)}+\frac{i \sqrt{-a^{2}+b^{2}}\, a +a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\, b}\right)}{\sqrt{-a^{2}+b^{2}}\, d^{2} b}$	194

input
```
int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(f*x+e)*exp(I*(d*x+c))/b/d/(2*I*a*exp(I*(d*x+c))+b*exp(2*I*(d*x+c))-b
)-1/(-a^2+b^2)^(1/2)*f/d^2/b*ln(exp(I*(d*x+c))+(I*(-a^2+b^2)^(1/2)*a-a^2+b
^2)/(-a^2+b^2)^(1/2)/b)+1/(-a^2+b^2)^(1/2)*f/d^2/b*ln(exp(I*(d*x+c))+(I*(-
a^2+b^2)^(1/2)*a+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.40

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \left[-\frac{2\,(a^2-b^2)dfx+2\,(a^2-b^2)de+(bf\sin{(dx+c)}+af)\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-\cdots}{b^2\cos(dx+\cdots}\right.}{2\,((a^2b^2-b^4)d^2\sin{(dx+c)}+(a^3b-ab^3)d^2)}\right.$$
$$\left.-\frac{(a^2-b^2)dfx+(a^2-b^2)de+(bf\sin{(dx+c)}+af)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)}{(a^2b^2-b^4)d^2\sin{(dx+c)}+(a^3b-ab^3)d^2}\right]$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^2 - b^2)*d*f*x + 2*(a^2 - b^2)*d*e + (b*f*sin(d*x + c) + a*f)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/((a^2*b^2 - b
^4)*d^2*sin(d*x + c) + (a^3*b - a*b^3)*d^2), -((a^2 - b^2)*d*f*x + (a^2 -
b^2)*d*e + (b*f*sin(d*x + c) + a*f)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^2*b^2 - b^4)*d^2*sin(d*x + c)
+ (a^3*b - a*b^3)*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{(fx+e)\cos(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,bf+2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)af-a^2de-a^2dfx+b}{b\,d^2\left(\sin(dx+c)\,a^2b-\sin(dx+c)\,b^3+a^3-a\,b^2\right)}$$

input
```
int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*b*f + 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2
 - b**2))*a*f - a**2*d*e - a**2*d*f*x + b**2*d*e + b**2*d*f*x)/(b*d**2*(si
n(c + d*x)*a**2*b - sin(c + d*x)*b**3 + a**3 - a*b**2))
```

3.320 $\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 280

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{2f^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^3} + \frac{2f^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^3} - \frac{(e+fx)^2}{bd(a+b\sin(c+dx))}$$

output
```
-2*I*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1
/2)/d^2+2*I*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-
b^2)^(1/2)/d^2-2*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(
a^2-b^2)^(1/2)/d^3+2*f^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/b/(a^2-b^2)^(1/2)/d^3-(f*x+e)^2/b/d/(a+b*sin(d*x+c))
```

Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.11

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{2if\left(-id\left(2\sqrt{-a^2+b^2}e\arctan\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)+\sqrt{a^2-b^2}fx\left(\log\left(1-\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)-\log\left(1+\frac{be^{i(c+dx)}}{ia+\sqrt{-a^2+b^2}}\right.\right.\right.\right.}{b\sqrt{-(a^2-b^2)^2}d}$$
$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
((2*I)*f*((-I)*d*(2*Sqrt[-a^2 + b^2]*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sq
rt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a
+ Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]
)])) - Sqrt[a^2 - b^2]*f*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^
2 + b^2])] + Sqrt[a^2 - b^2]*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqr
t[-a^2 + b^2]))]))/(b*Sqrt[-(a^2 - b^2)^2]*d^3) - (e + f*x)^2/(b*d*(a + b*
Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4922, 3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$\Big\downarrow \; 4922$$
$$\frac{2f\int \frac{e+fx}{a+b\sin(c+dx)}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{2f\int \frac{e+fx}{a+b\sin(c+dx)}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}$$

$\Big\downarrow$ 3804

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}+\frac{4f\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{bd}$$

$\Big\downarrow$ 2694

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}+\frac{4f\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{bd}$$

$\Big\downarrow$ 27

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}+\frac{4f\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{bd}$$

$\Big\downarrow$ 2620

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}+\frac{4f\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{bd}$$

$\Big\downarrow$ 2715

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}+\frac{4f\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)}{}\right.}{2\sqrt{a^2-b^2}}}{bd}$$

$\Big\downarrow$ 2838

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))}+\frac{4f\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{bd}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(4*f*(((-1/2*I)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(
c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/(b*d) - (e + f*
x)^2/(b*d*(a + b*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(250) = 500.

Time = 4.18 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{2i\left(x^{2} f^{2}+2efx+e^{2}\right){\mathrm e}^{i(dx+c)}}{bd\left(2ia\,{\mathrm e}^{i(dx+c)}+b\,{\mathrm e}^{2i(dx+c)}-b\right)}+\frac{4ife\arctan\left(\frac{2ib\,{\mathrm e}^{i(dx+c)}-2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{d^{2} b\sqrt{-a^{2}+b^{2}}}+\frac{2f^{2}\ln\left(\frac{ia+{\mathrm e}^{i(dx+c)}b-\sqrt{-a^{2}+b^{2}}}{ia-\sqrt{-a^{2}+b^{2}}}\right)x}{d^{2} b\sqrt{-a^{2}+b^{2}}}-\frac{2f^{2}\ln\left(\frac{ia+{\mathrm e}^{i(dx}}{ia+}\right.}{d^{2} b\sqrt{}}$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(f^2*x^2+2*e*f*x+e^2)*exp(I*(d*x+c))/b/d/(2*I*a*exp(I*(d*x+c))+b*exp(
2*I*(d*x+c))-b)+4*I/d^2/b*f*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*
x+c))-2*a)/(-a^2+b^2)^(1/2))+2/d^2/b*f^2/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d
*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-2/d^2/b*f^2/(-a^2+b^2
)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
*x+2/d^3/b*f^2/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))
/(I*a-(-a^2+b^2)^(1/2)))*c-2/d^3/b*f^2/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x
+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I/d^3/b*f^2/(-a^2+b^2
)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2
)))+2*I/d^3/b*f^2/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^
(1/2))/(I*a+(-a^2+b^2)^(1/2)))-4*I/d^3/b*f^2*c/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1393 vs. $2(242) = 484$.

Time = 0.23 (sec) , antiderivative size = 1393, normalized size of antiderivative = 4.98

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + (a^2 - b^2)*d^2*e^2
+ (-I*b^2*f^2*sin(d*x + c) - I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) + (I*b^2*f^2*sin(d*x + c) + I*a*b*f^2)*sqrt(-
(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*b^2*f^2*sin(
d*x + c) + I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*
sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) + (-I*b^2*f^2*sin(d*x + c) - I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)
*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (a*b*d*e*f - a*b*c*f^2 + (b^2*
d*e*f - b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (a*b*d*e*f
 - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) + (a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) + 2*I*a) + (a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c
*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (a*b*d*f^2*x + a*b*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cos(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
(4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*a**3*b*d*e*f - 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/
sqrt(a**2 - b**2))*sin(c + d*x)*a**2*b**2*f**2 + 8*sqrt(a**2 - b**2)*atan(
(tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*b**4*f**2 + 4*sqr
t(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**4*d*e*f
 - 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a*
*3*b*f**2 + 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 -
b**2))*a*b**3*f**2 + 2*cos(c + d*x)*a**4*b*d*f**2*x - 6*cos(c + d*x)*a**2*
b**3*d*f**2*x + 4*cos(c + d*x)*b**5*d*f**2*x + 8*int(x/(tan((c + d*x)/2)**
4*a**2 + 4*tan((c + d*x)/2)**3*a*b + 2*tan((c + d*x)/2)**2*a**2 + 4*tan((c
 + d*x)/2)**2*b**2 + 4*tan((c + d*x)/2)*a*b + a**2),x)*sin(c + d*x)*a**4*b
**3*d**2*f**2 - 16*int(x/(tan((c + d*x)/2)**4*a**2 + 4*tan((c + d*x)/2)**3
*a*b + 2*tan((c + d*x)/2)**2*a**2 + 4*tan((c + d*x)/2)**2*b**2 + 4*tan((c
+ d*x)/2)*a*b + a**2),x)*sin(c + d*x)*a**2*b**5*d**2*f**2 + 8*int(x/(tan((
c + d*x)/2)**4*a**2 + 4*tan((c + d*x)/2)**3*a*b + 2*tan((c + d*x)/2)**2*a*
*2 + 4*tan((c + d*x)/2)**2*b**2 + 4*tan((c + d*x)/2)*a*b + a**2),x)*sin(c
+ d*x)*b**7*d**2*f**2 + 8*int(x/(tan((c + d*x)/2)**4*a**2 + 4*tan((c + d*x
)/2)**3*a*b + 2*tan((c + d*x)/2)**2*a**2 + 4*tan((c + d*x)/2)**2*b**2 + 4*
tan((c + d*x)/2)*a*b + a**2),x)*a**5*b**2*d**2*f**2 - 16*int(x/(tan((c + d
*x)/2)**4*a**2 + 4*tan((c + d*x)/2)**3*a*b + 2*tan((c + d*x)/2)**2*a**2...
```

3.321 $\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 418

$$\begin{aligned}\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = &-\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}\\&+\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}\\&-\frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3}\\&+\frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^3}\\&-\frac{6if^3\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^4}\\&+\frac{6if^3\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^4}-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}\end{aligned}$$

output
```
-3*I*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^
(1/2)/d^2+3*I*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(
a^2-b^2)^(1/2)/d^2-6*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)
^(1/2)))/b/(a^2-b^2)^(1/2)/d^3+6*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^3-6*I*f^3*polylog(3,I*b*exp(I*(d*
x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^4+6*I*f^3*polylog(3,I*b*exp
(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(1/2)/d^4-(f*x+e)^3/b/d/(a+b*
sin(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 3.15 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{3if\left(-2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)+2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{ia+\sqrt{-a^2+b^2}}\right.\right.}{}$$

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
((3*I)*f*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/
((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2,
-((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b
^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2
]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])
] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 -
 b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*
Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^
2]))])))/(b*Sqrt[-(a^2 - b^2)^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sin[c + d*
x]))
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4922, 3042, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{4922}$$

$$\frac{3f\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3f\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3804}$$

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{bd}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{bd}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{bd}$$

$\big\downarrow$ 3011

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\cdots\right)}{bd}\cdots\right)}{\cdots}\cdots\right)}{bd}$$

$\big\downarrow$ 2720

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{bd}$$

$\big\downarrow$ 7143

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))}+\frac{6f\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\cdots\right)}{\cdots}\right)}{bd}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

output
```
(6*f*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^2*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*
PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3,
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^
2]))/(b*d) - (e + f*x)^3/(b*d*(a + b*Sin[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos(dx+c)}{(a+b\sin(dx+c))^2}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2284 vs. $2(360) = 720$.

Time = 0.28 (sec) , antiderivative size = 2284, normalized size of antiderivative = 5.46

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 - b^2)*d^3*f^3*x^3 + 6*(a^2 - b^2)*d^3*e*f^2*x^2 + 6*(a^2 - b
^2)*d^3*e^2*f*x + 2*(a^2 - b^2)*d^3*e^3 - 6*(I*a*b*d*f^3*x + I*a*b*d*e*f^2
 + (I*b^2*d*f^3*x + I*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*di
log((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a*b*d*f^3*x - I*a*b*d*e*f^2
+ (-I*b^2*d*f^3*x - I*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*di
log((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(-I*a*b*d*f^3*x - I*a*b*d*e*f^2
+ (-I*b^2*d*f^3*x - I*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*di
log((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 6*(I*a*b*d*f^3*x + I*a*b*d*e*f^2
+ (I*b^2*d*f^3*x + I*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*dil
og((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2
 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a
*b*c^2*f^3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) + 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cos(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

output
```
(6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*a**4*b*d**2*e**2*f - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**4*b*f**3 - 24*sqrt(a**2 - b**2)*
atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**3*b**2*d*
e*f**2 + 72*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b*
*2))*sin(c + d*x)*a**2*b**3*f**3 + 24*sqrt(a**2 - b**2)*atan((tan((c + d*x
)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a*b**4*d*e*f**2 - 72*sqrt(a**2
 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*b**
5*f**3 + 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**
2))*a**5*d**2*e**2*f - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/
sqrt(a**2 - b**2))*a**5*f**3 - 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)
*a + b)/sqrt(a**2 - b**2))*a**4*b*d*e*f**2 + 72*sqrt(a**2 - b**2)*atan((ta
n((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**3*b**2*f**3 + 24*sqrt(a**2 - b
**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2*b**3*d*e*f**2 -
 72*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a*b
**4*f**3 + 6*cos(c + d*x)*a**5*b*d**2*e*f**2*x + 3*cos(c + d*x)*a**5*b*d**
2*f**3*x**2 - 18*cos(c + d*x)*a**4*b**2*d*f**3*x - 18*cos(c + d*x)*a**3*b*
*3*d**2*e*f**2*x - 9*cos(c + d*x)*a**3*b**3*d**2*f**3*x**2 + 54*cos(c + d*
x)*a**2*b**4*d*f**3*x + 12*cos(c + d*x)*a*b**5*d**2*e*f**2*x + 6*cos(c + d
*x)*a*b**5*d**2*f**3*x**2 - 36*cos(c + d*x)*b**6*d*f**3*x + 12*int(x**2...
```

3.322 $\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{af\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2} - \frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}$$

output
```
a*f*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(3/2)/d^2
-1/2*(f*x+e)/b/d/(a+b*sin(d*x+c))^2+1/2*f*cos(d*x+c)/(a^2-b^2)/d^2/(a+b*si
n(d*x+c))
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{2af\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}} + \frac{-\frac{d(e+fx)}{b}+\frac{f\cos(c+dx)(a+b\sin(c+dx))}{(a-b)(a+b)}}{(a+b\sin(c+dx))^2}}{2d^2}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
((2*a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(
3/2)) + (-((d*(e + f*x))/b) + (f*Cos[c + d*x]*(a + b*Sin[c + d*x]))/((a -
b)*(a + b)))/(a + b*Sin[c + d*x])^2)/(2*d^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4922, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{4922}$$

$$\frac{f\int \frac{1}{(a+b\sin(c+dx))^2}dx}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{f\int \frac{1}{(a+b\sin(c+dx))^2}dx}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3143}$$

$$\frac{f\left(\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}-\frac{\int -\frac{a}{a+b\sin(c+dx)}dx}{a^2-b^2}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{a}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{a\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{f\left(\frac{a\int\frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{f\left(\frac{2a\int\frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}+\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f\left(\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}-\frac{4a\int\frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2-b^2)}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{f\left(\frac{2a\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}}+\frac{b\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

input
```
Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)/(b*d*(a + b*Sin[c + d*x])^2) + (f*((2*a*ArcTan[(2*b + 2*a*T
an[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) + (b*Cos[c +
d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))))/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.70 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.01

method	result
risch	$\frac{2a^2dfx\,e^{2i(dx+c)}-2b^2dfx\,e^{2i(dx+c)}+2ia^2f\,e^{2i(dx+c)}+ib^2f\,e^{2i(dx+c)}+2a^2de\,e^{2i(dx+c)}+baf\,e^{3i(dx+c)}-2b^2de\,e^{2i(dx+c)}-ib^2f-3abf}{\left(2ia\,e^{i(dx+c)}+b\,e^{2i(dx+c)}-b\right)^2d^2(a^2-b^2)b}$

input
```
int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(2*a^2*d*f*x*exp(2*I*(d*x+c))-2*b^2*d*f*x*exp(2*I*(d*x+c))+2*I*a^2*f*exp(2
*I*(d*x+c))+I*b^2*f*exp(2*I*(d*x+c))+2*a^2*d*e*exp(2*I*(d*x+c))+b*a*f*exp(
3*I*(d*x+c))-2*b^2*d*e*exp(2*I*(d*x+c))-I*b^2*f-3*a*b*f*exp(I*(d*x+c)))/(2
*I*a*exp(I*(d*x+c))+b*exp(2*I*(d*x+c))-b)^2/d^2/(a^2-b^2)/b-1/2/(-a^2+b^2)
^(1/2)*f*a/(a+b)/(a-b)/d^2/b*ln(exp(I*(d*x+c))+(I*(-a^2+b^2)^(1/2)*a-a^2+b
^2)/(-a^2+b^2)^(1/2)/b)+1/2/(-a^2+b^2)^(1/2)*f*a/(a+b)/(a-b)/d^2/b*ln(exp(
I*(d*x+c))+(I*(-a^2+b^2)^(1/2)*a+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(107) = 214.

Time = 0.15 (sec) , antiderivative size = 625, normalized size of antiderivative = 5.39

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \left[\frac{2\,(a^4-2\,a^2b^2+b^4)dfx-2\,(a^2b^2-b^4)f\cos{(dx+c)}\sin{(dx+c)}+2\,(a^4-2\,a^2b^2+b^4)de-2\,(a^3b-ab}{4\,((a^4b^3-2\,a^2b^5+b^7)}\right.$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^4 - 2*a^2*b^2 + b^4)*d*f*x - 2*(a^2*b^2 - b^4)*f*cos(d*x + c)*s
in(d*x + c) + 2*(a^4 - 2*a^2*b^2 + b^4)*d*e - 2*(a^3*b - a*b^3)*f*cos(d*x
+ c) + (a*b^2*f*cos(d*x + c)^2 - 2*a^2*b*f*sin(d*x + c) - (a^3 + a*b^2)*f)
*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
 b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/((a^4*b^3 -
 2*a^2*b^5 + b^7)*d^2*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^2
*sin(d*x + c) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^2), 1/2*((a^4 - 2*a^2*
b^2 + b^4)*d*f*x - (a^2*b^2 - b^4)*f*cos(d*x + c)*sin(d*x + c) + (a^4 - 2*
a^2*b^2 + b^4)*d*e - (a^3*b - a*b^3)*f*cos(d*x + c) - (a*b^2*f*cos(d*x + c
)^2 - 2*a^2*b*f*sin(d*x + c) - (a^3 + a*b^2)*f)*sqrt(a^2 - b^2)*arctan(-(a
*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^4*b^3 - 2*a^2*b^5
+ b^7)*d^2*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^2*sin(d*x +
c) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e + fx)\cos(c + dx)}{(a + b\sin(c + dx))^3} dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e + fx)\cos(c + dx)}{(a + b\sin(c + dx))^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{(fx+e)\cos(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*(e + f*x))/(a + b*sin(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 455, normalized size of antiderivative = 3.92

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)^2\,a^2b^2f + 8\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,a^3bf +}{}$$

input
```
int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
(4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)**2*a**2*b**2*f + 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b
)/sqrt(a**2 - b**2))*sin(c + d*x)*a**3*b*f + 4*sqrt(a**2 - b**2)*atan((tan
((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**4*f + 2*cos(c + d*x)*sin(c + d*
x)*a**3*b**2*f - 2*cos(c + d*x)*sin(c + d*x)*a*b**4*f + 2*cos(c + d*x)*a**
4*b*f - 2*cos(c + d*x)*a**2*b**3*f + sin(c + d*x)**2*a**2*b**3*f - sin(c +
 d*x)**2*b**5*f + 2*sin(c + d*x)*a**3*b**2*f - 2*sin(c + d*x)*a*b**4*f - 2
*a**5*d*e - 2*a**5*d*f*x + a**4*b*f + 4*a**3*b**2*d*e + 4*a**3*b**2*d*f*x
- a**2*b**3*f - 2*a*b**4*d*e - 2*a*b**4*d*f*x)/(4*a*b*d**2*(sin(c + d*x)**
2*a**4*b**2 - 2*sin(c + d*x)**2*a**2*b**4 + sin(c + d*x)**2*b**6 + 2*sin(c
 + d*x)*a**5*b - 4*sin(c + d*x)*a**3*b**3 + 2*sin(c + d*x)*a*b**5 + a**6 -
 2*a**4*b**2 + a**2*b**4))
```

3.323 $\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 357

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^2} + \frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^2} - \frac{f^2\log(a+b\sin(c+dx))}{b(a^2-b^2)d^3} - \frac{af^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{af^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} - \frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2} + \frac{f(e+fx)\cos(c+dx)}{(a^2-b^2)d^2(a+b\sin(c+dx))}$$

output
```
-I*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3
/2)/d^2+I*a*f*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-
b^2)^(3/2)/d^2-f^2*ln(a+b*sin(d*x+c))/b/(a^2-b^2)/d^3-a*f^2*polylog(2,I*b*
exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3+a*f^2*polylog(2,
I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^3-1/2*(f*x+e)^
2/b/d/(a+b*sin(d*x+c))^2+f*(f*x+e)*cos(d*x+c)/(a^2-b^2)/d^2/(a+b*sin(d*x+c
))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1017 vs. $2(357) = 714$.

Time = 15.87 (sec) , antiderivative size = 1017, normalized size of antiderivative = 2.85

$$\int \frac{(e+fx)^2 \cos(c+dx)}{(a+b\sin(c+dx))^3} \, dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
(f^2*x*Cot[c])/(b*(-a^2 + b^2)*d^2) - ((2*I)*E^(I*c)*f*(E^(I*c)*f*x + (I*a
*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*E^(
I*c)) - (I*a*e*E^(I*c)*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] - ((I/2)*f*Log[b - (2*I)*a*E^(I*(c + d*x)) - b*E^((2*I)*(c
 + d*x))])/(d*E^(I*c)) + ((I/2)*E^(I*c)*f*Log[b - (2*I)*a*E^(I*(c + d*x))
- b*E^((2*I)*(c + d*x))])/d + ((I/2)*a*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)
*c)] - ((I/2)*a*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c)
 - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - ((I/
2)*a*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + ((I/2)*a*E^((2*I)*c)*f*x*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (a*(-1 + E^((2*I)*c))*f*PolyLog[2, (I*
b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(2*d
*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (a*(-1 + E^((2*I)*c))*f*PolyLog[2, -((b
*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/(2*d
*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])))/(b*(-a^2 + b^2)*d^2*(-1 + E^((2*I)*c)))
 - (f^2*x*Csc[c/2]*Sec[c/2]*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))/(
2*b*(-a + b)*(a + b)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sin[c + d*x])^2) + (
Csc[c/2]*Sec[c/2]*(-(a*e*f*Cos[c]) - a*f^2*x*Cos[c] - b*e*f*Sin[d*x] - ...
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {4922, 3042, 3805, 3042, 3147, 16, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{4922}$$

$$\frac{f\int \frac{e+fx}{(a+b\sin(c+dx))^2}dx}{bd}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{f\int \frac{e+fx}{(a+b\sin(c+dx))^2}dx}{bd}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3805}$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{bf\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{bf\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3147}$$

$$\frac{f\left(-\frac{f\int \frac{1}{a+b\sin(c+dx)}d(b\sin(c+dx))}{d^2(a^2-b^2)}+\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{f\log(a+b\sin(c+dx))}{d^2(a^2-b^2)}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3804

$$-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\left(\frac{2a\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a^2-b^2}-\frac{f\log(a+b\sin(c+dx))}{d^2(a^2-b^2)}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}$$

$\downarrow$ 2694

$$-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\left(\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+b\sin(c+dx))}{d^2(a^2-b^2)}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}$$

$\downarrow$ 27

$$-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\left(\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+b\sin(c+dx))}{d^2(a^2-b^2)}+\frac{b(e+fx)\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{bd}$$

$\downarrow$ 2620

$$-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+\cdots}{d^2(\cdots}\right.}{bd}$$

$\downarrow$ 2715

$$-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\left(\frac{2a\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\ldots}{\ldots}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}\ldots\right)}{bd}$$

$\downarrow$ 2838

$$-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a\ldots}{d^2\ldots}\ldots\right)}{bd}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^2/(b*d*(a + b*Sin[c + d*x])^2) + (f*(-((f*Log[a + b*Sin[c +
 d*x]])/((a^2 - b^2)*d^2)) + (2*a*(((-1/2*I)*b*(((e + f*x)*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*((
(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I
*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt
[a^2 - b^2]))/(a^2 - b^2) + (b*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d*(a +
 b*Sin[c + d*x]))))/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(325) = 650$.

Time = 6.58 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.65

method	result
risch	$\frac{2a^2d f^2x^2e^{2i(dx+c)}-2b^2d f^2x^2e^{2i(dx+c)}+4ia^2f^2x e^{2i(dx+c)}-2ib^2f^2x+4a^2defx e^{2i(dx+c)}+2ba f^2x e^{3i(dx+c)}-4b^2defx e^{2i(dx+c)}-}{(2ia e^{i(dx}}$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(a^2*d*f^2*x^2*exp(2*I*(d*x+c))-b^2*d*f^2*x^2*exp(2*I*(d*x+c))+2*I*a^2*f
^2*x*exp(2*I*(d*x+c))-I*b^2*f^2*x+2*a^2*d*e*f*x*exp(2*I*(d*x+c))+b*a*f^2*x
*exp(3*I*(d*x+c))-2*b^2*d*e*f*x*exp(2*I*(d*x+c))+I*b^2*e*f*exp(2*I*(d*x+c)
)+I*b^2*f^2*x*exp(2*I*(d*x+c))+a^2*d*e^2*exp(2*I*(d*x+c))+b*a*e*f*exp(3*I*
(d*x+c))-b^2*d*e^2*exp(2*I*(d*x+c))+2*I*a^2*e*f*exp(2*I*(d*x+c))-3*a*b*f^2
*x*exp(I*(d*x+c))-I*b^2*e*f-3*a*b*e*f*exp(I*(d*x+c)))/(2*I*a*exp(I*(d*x+c)
)+b*exp(2*I*(d*x+c))-b)^2/d^2/(a^2-b^2)/b+1/b/(-a^2+b^2)/d^3*f^2*ln(I*b*ex
p(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/b/(-a^2+b^2)/d^3*f^2*ln(exp(I*(d*
x+c)))+2*I/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*c*arctan(1/2*(2*I*b*exp(I*(d*x+c))
-2*a)/(-a^2+b^2)^(1/2))-2*I/b/(-a^2+b^2)^(3/2)/d^2*f*a*e*arctan(1/2*(2*I*b
*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-1/b/(-a^2+b^2)^(3/2)/d^2*f^2*a*ln((
I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/b/(-a^2
+b^2)^(3/2)/d^2*f^2*a*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^
2+b^2)^(1/2)))*x-1/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*ln((I*a+exp(I*(d*x+c))*b-(
-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*
ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/b/(
-a^2+b^2)^(3/2)/d^3*f^2*a*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I
*a-(-a^2+b^2)^(1/2)))-I/b/(-a^2+b^2)^(3/2)/d^3*f^2*a*dilog((I*a+exp(I*(d*x
+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2375 vs. $2(317) = 634$.

Time $= 0.27$ (sec) , antiderivative size $= 2375$, normalized size of antiderivative $= 6.65$

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d^2*e
*f*x + (a^4 - 2*a^2*b^2 + b^4)*d^2*e^2 - 2*((a^2*b^2 - b^4)*d*f^2*x + (a^2
*b^2 - b^4)*d*e*f)*cos(d*x + c)*sin(d*x + c) - (-I*a*b^3*f^2*cos(d*x + c)^
2 + 2*I*a^2*b^2*f^2*sin(d*x + c) + I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^2
)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (I*a*b^3*f^2*cos(d*x + c)
^2 - 2*I*a^2*b^2*f^2*sin(d*x + c) - I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^
2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (I*a*b^3*f^2*cos(d*x + c
)^2 - 2*I*a^2*b^2*f^2*sin(d*x + c) - I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b
^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (-I*a*b^3*f^2*cos(d*x
+ c)^2 + 2*I*a^2*b^2*f^2*sin(d*x + c) + I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2
- b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + ((a^3*b + a*b^3)*d*
f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cos(d*x + c)
^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - ((a^3*b + a*b^3)*d*f^2*x + (a^3*b
 + a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cos(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
(4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)**2*a**2*b**2*e*f + 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a +
 b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**3*b*e*f + 4*sqrt(a**2 - b**2)*atan(
(tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a**4*e*f + 2*cos(c + d*x)*sin(
c + d*x)*a**3*b**2*e*f - 2*cos(c + d*x)*sin(c + d*x)*a*b**4*e*f + 2*cos(c
+ d*x)*a**4*b*e*f - 2*cos(c + d*x)*a**2*b**3*e*f + 2*int((cos(c + d*x)*x**
2)/(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*
b + a**3),x)*sin(c + d*x)**2*a**5*b**3*d**2*f**2 - 4*int((cos(c + d*x)*x**
2)/(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*
b + a**3),x)*sin(c + d*x)**2*a**3*b**5*d**2*f**2 + 2*int((cos(c + d*x)*x**
2)/(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*
b + a**3),x)*sin(c + d*x)**2*a*b**7*d**2*f**2 + 4*int((cos(c + d*x)*x**2)/
(sin(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b +
 a**3),x)*sin(c + d*x)*a**6*b**2*d**2*f**2 - 8*int((cos(c + d*x)*x**2)/(si
n(c + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a*
*3),x)*sin(c + d*x)*a**4*b**4*d**2*f**2 + 4*int((cos(c + d*x)*x**2)/(sin(c
 + d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a**3)
,x)*sin(c + d*x)*a**2*b**6*d**2*f**2 + 2*int((cos(c + d*x)*x**2)/(sin(c +
d*x)**3*b**3 + 3*sin(c + d*x)**2*a*b**2 + 3*sin(c + d*x)*a**2*b + a**3),x)
*a**7*b*d**2*f**2 - 4*int((cos(c + d*x)*x**2)/(sin(c + d*x)**3*b**3 + 3...
```

3.324 $\int \frac{(e+fx)^3 \cos(c+dx)}{(a+b\sin(c+dx))^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 753

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{3if(e+fx)^2}{2b(a^2-b^2)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^3} - \frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^3} + \frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{2b(a^2-b^2)^{3/2}d^2} + \frac{3if^3\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^4} - \frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} + \frac{3if^3\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)d^4} + \frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^3} - \frac{3iaf^3\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^4} + \frac{3iaf^3\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}d^4} - \frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{3f(e+fx)^2\cos(c+dx)}{2(a^2-b^2)d^2(a+b\sin(c+dx))}$$

output
```
3/2*I*f*(f*x+e)^2/b/(a^2-b^2)/d^2-3*f^2*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a
-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3-3/2*I*a*f*(f*x+e)^2*ln(1-I*b*exp(I*(d*x
+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2-3*f^2*(f*x+e)*ln(1-I*b*exp
(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^3+3/2*I*a*f*(f*x+e)^2*ln(1-
I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^2+3*I*f^3*poly
log(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^4-3*a*f^2*(f*x
+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/b/(a^2-b^2)^(3/2)/d^
3+3*I*f^3*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2-b^2)/d^
4+3*a*f^2*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/b/(a^2
-b^2)^(3/2)/d^3-3*I*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2))
)/b/(a^2-b^2)^(3/2)/d^4+3*I*a*f^3*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2
)^(1/2)))/b/(a^2-b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sin(d*x+c))^2+3/2*f
*(f*x+e)^2*cos(d*x+c)/(a^2-b^2)/d^2/(a+b*sin(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 8825 vs. 2(753) = 1506.

Time = 20.27 (sec) , antiderivative size = 8825, normalized size of antiderivative = 11.72

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.09 (sec) , antiderivative size = 672, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {4922, 3042, 3805, 3042, 3804, 2694, 27, 2620, 3011, 2720, 5030, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$\Big\downarrow$ 4922

$$\frac{3f\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}dx}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{3f\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}dx}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3805

$$\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{2bf\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)^2\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{2bf\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)^2\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3804

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+\frac{3f\left(\frac{2a\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{a^2-b^2}-\frac{2bf\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)^2\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}$$

$\Big\downarrow$ 2694

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+\frac{3f\left(\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{2bf\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)^2\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}$$

$\Big\downarrow$ 27

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+\frac{3f\left(\frac{2a\left(\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{2bf\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{d(a^2-b^2)}+\frac{b(e+fx)^2\cos(c+dx)}{d(a^2-b^2)(a+b\sin(c+dx))}\right)}{2bd}$$

$\downarrow$ 2620

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+\frac{3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}\right.}{2bd}$$

$\downarrow$ 3011

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\cdots\right.}{\cdots}\right)}{a^2-b^2}\cdots\right)$$

$$\overline{\qquad 2bd\cdots}$$

$\downarrow$ 2720

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\cdots}{\cdots}\right.}{\cdots}\right)}{a^2-b^2}\cdots\right)$$

$\downarrow$ 5030

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2}{\cdots}\right.}{\cdots}\right.}{a^2-b^2}\right.$$

$\downarrow$ 2620

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2}{\ldots}\right.}{\ldots}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2}{\ldots}\right.}{\ldots}\right.}{a^2-b^2}\right.$$

$\downarrow$ 2838

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(\frac{2a\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2}{\cdots}\right.}{\cdots}\right)}{a^2-b^2}\cdots\right)$$

$\downarrow$ 7143

$$-\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2}+$$

$$3f\left(-\frac{2bf\left(-\frac{if\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}\right)}{d(a^2-b^2)}+\right.$$

input
```
Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^3/(b*d*(a + b*Sin[c + d*x])^2) + (3*f*((-2*b*f*(((-1/2*I)*(
e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(b*d^2)))/((a^2 - b^2)*d) + (2*a*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*Pol
yLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2]
+ ((I/2)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/d^2))/(b*d)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (b*(e + f*x)^2*Cos[c +
d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))))/(2*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos (dx+c)}{(a+b\sin (dx+c))^3}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4917 vs. $2(653) = 1306$.

Time = 0.43 (sec) , antiderivative size = 4917, normalized size of antiderivative = 6.53

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cos(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx=\text{Hanged}$$

input
```
int((cos(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

output
```
(48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin
(c + d*x)**2*a**8*b**2*d**2*e**2*f - 312*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**8*b**2*f**3 - 96*sqrt
(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x
)**2*a**7*b**3*d*e*f**2 - 240*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a +
 b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**6*b**4*d**2*e**2*f + 1020*sqrt(a
**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*
*2*a**6*b**4*f**3 + 528*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sq
rt(a**2 - b**2))*sin(c + d*x)**2*a**5*b**5*d*e*f**2 + 300*sqrt(a**2 - b**2
)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**4*b*
*6*d**2*e**2*f - 240*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(
a**2 - b**2))*sin(c + d*x)**2*a**4*b**6*f**3 - 912*sqrt(a**2 - b**2)*atan(
(tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**3*b**7*d*e*
f**2 - 912*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**
2))*sin(c + d*x)**2*a**2*b**8*f**3 + 480*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a*b**9*d*e*f**2 + 480*sq
rt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d
*x)**2*b**10*f**3 + 96*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqr
t(a**2 - b**2))*sin(c + d*x)*a**9*b*d**2*e**2*f - 624*sqrt(a**2 - b**2)*at
an((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(c + d*x)*a**9*b*f**3...
```

3.325 $\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 765

$$
\begin{aligned}
\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx
&= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd} \\
&\quad + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} \\
&\quad - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} - \frac{3\sqrt{a^2-b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2} \\
&\quad + \frac{3\sqrt{a^2-b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd^2} - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} \\
&\quad + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} - \frac{6i\sqrt{a^2-b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^3} \\
&\quad + \frac{6i\sqrt{a^2-b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd^3} \\
&\quad - \frac{6if^3\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)}{ad^4} + \frac{6if^3\operatorname{PolyLog}\left(4,e^{i(c+dx)}\right)}{ad^4} \\
&\quad + \frac{6\sqrt{a^2-b^2}f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^4} - \frac{6\sqrt{a^2-b^2}f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd^4}
\end{aligned}
$$

output
```
-1/4*(f*x+e)^4/b/f-2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d-3*I*f*(f*x+e)^2
*polylog(2,exp(I*(d*x+c)))/a/d^2-I*(a^2-b^2)^(1/2)*(f*x+e)^3*ln(1-I*b*exp(
I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b/d+6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/
d^4+6*I*(a^2-b^2)^(1/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/a/b/d^3-3*(a^2-b^2)^(1/2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x
+c))/(a-(a^2-b^2)^(1/2)))/a/b/d^2+3*(a^2-b^2)^(1/2)*f*(f*x+e)^2*polylog(2,
I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d^2-6*f^2*(f*x+e)*polylog(3,-e
xp(I*(d*x+c)))/a/d^3+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3+3*I*f*(
f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2+I*(a^2-b^2)^(1/2)*(f*x+e)^3*ln(1
-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d-6*I*f^3*polylog(4,-exp(I*(d
*x+c)))/a/d^4-6*I*(a^2-b^2)^(1/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))
/(a-(a^2-b^2)^(1/2)))/a/b/d^3+6*(a^2-b^2)^(1/2)*f^3*polylog(4,I*b*exp(I*(d
*x+c))/(a-(a^2-b^2)^(1/2)))/a/b/d^4-6*(a^2-b^2)^(1/2)*f^3*polylog(4,I*b*ex
p(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d^4
```

Mathematica [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-1/4*(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/b + ((a^2 - b^2)*(2*S
qrt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]]
+ 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt
[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)
))/((-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E
^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f
*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^
2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - S
qrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 +
b^2])] - (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d
*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)
^2*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^
2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2]
)] + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sq
rt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x
)))/(I*a + Sqrt[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b
*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*P
olyLog[4, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^
2 - b^2]*f^3*PolyLog[4, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))
/(a*b*Sqrt[-(a^2 - b^2)^2]*d^4) + (I*((2*I)*(e + f*x)^3*ArcTanh[Cos[c +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 5054$$

$$\frac{\int (e+fx)^3\cos(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\int (e+fx)^3\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\int (e+fx)^3\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{3f\int (e+fx)^2\cos(c+dx)dx}{d}+\int (e+fx)^3\csc(c+dx)dx+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{3f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\int (e+fx)^3\csc(c+dx)dx+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{3f\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}+\int (e+fx)^3\csc(c+dx)dx+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}+\int (e+fx)^3\csc(c+dx)dx+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{d}+\int (e+fx)^3\csc(c+dx)dx+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4671$$

$$-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{3f\int(e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int(e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{d}-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(\ldots}{\ldots}\right.}{\ldots}\right.}{d}}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)\ldots}{d}\right.}{d}}{a}$$

$$\Big\downarrow \; 5036$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^3dx}{b^2}-\frac{\int (e+fx)^3\sin(c+dx)dx}{b}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 17

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^3\sin(c+dx)dx}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^3\sin(c+dx)dx}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\int (e+fx)^2\cos(c+dx)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{2f\int-((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$\big\downarrow$ 3117

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}\right)}{a}+\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$\big\downarrow$ 3804

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\int\frac{e^{i(c+dx)}(e+fx)^3}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+d}{d}}{b}\right.}{a}$$

$$\Big\downarrow \; 2694$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}\right.}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^3}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-(e+f\right.}{d}\right.}{d}}{b}\right.}{a}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(1-\frac{ibe}{a-}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{b^2}\right.$$

$$\Big\downarrow \; 3011$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{a}$$

$$b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{2if\int(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^3}{}\right.}{}\right.}{b^2}\right.$$

$$\Big\downarrow \; 7163$$

$$-\frac{2\operatorname{arctanh}\left(e^{i(c+dx)}\right)(e+fx)^3}{d}+\frac{\cos(c+dx)(e+fx)^3}{d}-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyL}}{}\right.}{}$$

$$b\left(\frac{a(e+fx)^4}{4b^2f}-\frac{\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{(e+fx)^3\cos(c+dx)}{d}}{b}-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos (dx+c) \cot (dx+c)}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3089 vs. 2(665) = 1330.

Time = 0.38 (sec) , antiderivative size = 3089, normalized size of antiderivative = 4.04

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3 \cos{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*cot(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)\cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.326 $\int \frac{(e+fx)^2 \cos(c+dx) \cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 557

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{(e+fx)^3}{3bf}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd} + \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd} + \frac{2if(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} - \frac{2if(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2} - \frac{2\sqrt{a^2-b^2}f(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2} + \frac{2\sqrt{a^2-b^2}f(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd^2} - \frac{2f^2\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} + \frac{2f^2\,\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3} - \frac{2i\sqrt{a^2-b^2}f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^3} + \frac{2i\sqrt{a^2-b^2}f^2\,\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd^3}$$

output
```
-1/3*(f*x+e)^3/b/f-2*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d-I*(a^2-b^2)^(1/
2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b/d+I*(a^2-b^2
)^(1/2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d+2*I*f
*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a/d^2-2*I*f*(f*x+e)*polylog(2,exp(I*(d
*x+c)))/a/d^2-2*(a^2-b^2)^(1/2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-
(a^2-b^2)^(1/2)))/a/b/d^2+2*(a^2-b^2)^(1/2)*f*(f*x+e)*polylog(2,I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d^2-2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d
^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3-2*I*(a^2-b^2)^(1/2)*f^2*polylog(3
,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b/d^3+2*I*(a^2-b^2)^(1/2)*f^2*p
olylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d^3
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{x(3e^2+3efx+f^2x^2)}{3b}$$
$$+\frac{(e+fx)^2\log\left(1-e^{i(c+dx)}\right)-(e+fx)^2\log\left(1+e^{i(c+dx)}\right)+\frac{2if\left(d(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)+if\operatorname{PolyLog}\left(3,-e^{i(c\cdots}\right.\right.}{d^2}}{ad}$$
$$+\frac{i(a^2-b^2)\left(-2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-ia+\sqrt{-a^2+b^2}}\right)+2\sqrt{a^2-b^2}df(e+fx)\operatorname{PolyLog}\left(2,-\frac{\cdots}{ia\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-1/3*(x*(3*e^2 + 3*e*f*x + f^2*x^2))/b + ((e + f*x)^2*Log[1 - E^(I*(c + d*
x))] - (e + f*x)^2*Log[1 + E^(I*(c + d*x))] + ((2*I)*f*(d*(e + f*x)*PolyLo
g[2, -E^(I*(c + d*x))] + I*f*PolyLog[3, -E^(I*(c + d*x))]))/d^2 + (2*f*((-
I)*d*(e + f*x)*PolyLog[2, E^(I*(c + d*x))] + f*PolyLog[3, E^(I*(c + d*x))]
))/d^2)/(a*d) + (I*(a^2 - b^2)*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2
, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f
*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I
*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 -
b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)
*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b
^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + S
qrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))
/(I*a + Sqrt[-a^2 + b^2]))])))/(a*b*Sqrt[-(a^2 - b^2)^2]*d^3)
```

Rubi [A] (verified)

Time = 4.11 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.13, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5054, 4908, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4671, 3011, 2720, 5036, 17, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$\Big\downarrow$ 5054

$$\frac{\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4908

$$\frac{\int (e+fx)^2\csc(c+dx)dx-\int (e+fx)^2\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\int (e+fx)^2\csc(c+dx)dx-\int (e+fx)^2\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{2f\int (e+fx)\cos(c+dx)dx}{d}+\int (e+fx)^2\csc(c+dx)dx+\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\int (e+fx)^2\csc(c+dx)dx+\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}+\int (e+fx)^2\csc(c+dx)dx+\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{-\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}+\int (e+fx)^2\csc(c+dx)dx+\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}+\int(e+fx)^2\csc(c+dx)dx+\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3118$$

$$\frac{\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}+\frac{(e+fx)^2\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4671$$

$$-\frac{b\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{2f\int(e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int(e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}-\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)}{}}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}(2,}{d^2}\right.}{d}}{a}$$

$$\Big\downarrow \; 5036$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^2dx}{b^2}-\frac{\int(e+fx)^2\sin(c+dx)dx}{b}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}(2,}{d^2}\right.}{d}}{a}$$

$$\Big\downarrow \; 17$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^2\sin(c+dx)dx}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\cdots\right)}{d^2}\cdots\right)}{d}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^2\sin(c+dx)dx}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\cdots\right)}{d^2}\cdots\right)}{d}}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\int (e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\cdots\right)}{d^2}\cdots\right)}{d}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\cdots\right)}{d^2}\cdots\right)}{d}}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\cdots\right)}{d^2}\cdots\right)}{d}}{a}$$

$\downarrow$ 25

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e\cdots\right.}{d^2}\cdots\right.}{d}\cdots}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e\cdots\right.}{d^2}\cdots\right.}{d}\cdots}{a}$$

$\downarrow$ 3118

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}\right)}{a}+\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e\cdots\right.}{d^2}\cdots\right.}{d}\cdots}{a}$$

$\downarrow$ 3804

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e\cdots\right.}{d^2}\cdots\right.}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\int \frac{e^{i(c+dx)}(e+fx)^2}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}}{b}\right)}{a}$$

$\downarrow$ 2694

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\cdots}{d^2}\cdots\right)}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\cdots}{b}\cdots\right)}{a}$$

$\downarrow$ 27

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\cdots}{d^2}\cdots\right)}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^3}{3b^2f}-\frac{\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cdots}{\cdots}}{b}\cdots\right)}{a}$$

$\downarrow$ 2620

$$\frac{\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\cdots}{d^2}\cdots\right)}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{\cdots}}\cdots\right)\cdots}{bd}\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{b^2}\cdots\right)}{a}$$

$\downarrow$ 3011

$$\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\ldots\right.}{d^2}\right.}{d}$$

$$a$$

$$b\left(-\frac{2\left(a^2-b^2\right)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{i\ldots}{a\ldots}\right.}{bd}\right.}{\ldots}\right.}{b^2}\right.$$

↓ 2720

$$\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,e\right.}{d^2}\right.}{d}$$

$$a$$

$$b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\right.}{}\right.}{b^2}\right.$$

↓ 7143

$$-\frac{2(e+fx)^2\operatorname{arctanh}(e^{i(c+dx)})}{d}+\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\operatorname{PolyLo}}{}\right.}{d}$$

$$a$$

$$b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c-}}{a-\sqrt{a}}\right.}{bd}\right.}{}\right.}{b^2}\right.$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/d + ((e + f*x)^2*Cos[c + d*x])/
d + (2*f*((I*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/d - (f*PolyLog[3, -E^
(I*(c + d*x))])/d^2))/d - (2*f*((I*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/
d - (f*PolyLog[3, E^(I*(c + d*x))])/d^2))/d - (2*f*((f*Cos[c + d*x])/d^2 +
 ((e + f*x)*Sin[c + d*x])/d))/d)/a - (b*((a*(e + f*x)^3)/(3*b^2*f) - (2*(a
^2 - b^2)*(((-1/2*I)*b*(((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)^2*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e +
f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyL
og[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/Sqrt[a^2
 - b^2]))/b^2 - (-(((e + f*x)^2*Cos[c + d*x])/d) + (2*f*((f*Cos[c + d*x])/
d^2 + ((e + f*x)*Sin[c + d*x])/d))/d)/b))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)\cot(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2109 vs. $2(481) = 962$.

Time = 0.32 (sec) , antiderivative size = 2109, normalized size of antiderivative = 3.79

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/6*(2*a*d^3*f^2*x^3 + 6*a*d^3*e*f*x^2 + 6*a*d^3*e^2*x - 6*b*f^2*sqrt(-(a
^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2
- b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*sqrt(-(a^2 - b
^2)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(a^2 - b^2
)/b^2)*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*polylog(3, cos(d*x
 + c) + I*sin(d*x + c)) - 6*b*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c)
) + 6*b*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 6*b*f^2*polylog(3
, -cos(d*x + c) - I*sin(d*x + c)) + 6*(-I*b*d*f^2*x - I*b*d*e*f)*sqrt(-(a^
2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 6*(I*b*d*f^2*x + I
*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c)
- (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 6*(I*b*d*f^2*x + I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x +
c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) - b)/b + 1) + 6*(-I*b*d*f^2*x - I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*d
ilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*cot(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2 \cos(c+dx) \cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2 \cos(dx+c) \cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.327 $\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 350

$$\begin{aligned}\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx =& -\frac{(e+fx)^2}{2bf}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad}\\&-\frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd}\\&+\frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd}\\&+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}\\&-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}\\&-\frac{\sqrt{a^2-b^2}f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}\\&+\frac{\sqrt{a^2-b^2}f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{abd^2}\end{aligned}$$

output
```
-1/2*(f*x+e)^2/b/f-2*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d-I*(a^2-b^2)^(1/2)
*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b/d+I*(a^2-b^2)^(1
/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d+I*f*polylog
(2,-exp(I*(d*x+c)))/a/d^2-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2-(a^2-b^2)^(1
/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b/d^2+(a^2-b^2)^
(1/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2079 vs. $2(350) = 700$.

Time = 16.12 (sec) , antiderivative size = 2079, normalized size of antiderivative = 5.94

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-1/2*((c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(b*d^2) + (e*Log[Tan[(c + d
*x)/2]])/(a*d) - (c*f*Log[Tan[(c + d*x)/2]])/(a*d^2) + (f*((c + d*x)*(Log[
1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c
+ d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^2) + (((2*(d*e - c*f)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*Log[1
+ I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a
 + b - Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*Tan[(c + d*x)
/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^
2 + b^2]))])/Sqrt[-a^2 + b^2] - (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b +
Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])])/S
qrt[-a^2 + b^2] + (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^
2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] -
 (I*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]
))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a -
I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(I + Tan
[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*Pol
yLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])/Sqrt[
-a^2 + b^2])*(((a^2 - b^2)*e)/(a*b*(a + b*Sin[c + d*x])) - ((a^2 - b^2)*c*
f)/(a*b*d*(a + b*Sin[c + d*x])) + ((a^2 - b^2)*f*(c + d*x))/(a*b*d*(a + b*
Sin[c + d*x]))))/(d*((f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b ...
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.16, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5054, 4908, 3042, 3777, 3042, 3117, 4671, 2715, 2838, 5036, 17, 3042, 3777, 3042, 3117, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 5054$$

$$\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{\int (e+fx)\csc(c+dx)dx-\int (e+fx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)\csc(c+dx)dx-\int (e+fx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\int (e+fx)\csc(c+dx)dx-\frac{f\int \cos(c+dx)dx}{d}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)\csc(c+dx)dx-\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\int (e+fx)\csc(c+dx)dx-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}-\frac{f\sin(c+dx)}{d^2}+(e+}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 5036}$$

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)dx}{b^2}-\frac{\int(e+fx)\sin(c+dx)dx}{b}\right)}{a}+-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 17

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\sin(c+dx)dx}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{a}+-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\sin(c+dx)dx}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{a}+-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{a}+-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{a}+-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 3117

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{a}+-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}$$

$\Big\downarrow$ 3804

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\left(-\frac{2(a^2-b^2)\int\frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{a}$$

$\Big\downarrow$ 2694

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{a}$$

$\Big\downarrow$ 27

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{a}$$

$\Big\downarrow$ 2620

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}+\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}\right.}{2\sqrt{a^2-b^2}}}{b^2}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\left(-\frac{2(a^2-b^2)\left(\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}+\right.}{a}$$

input
```
Int[((e + f*x)*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/d + ((e + f*x)*Cos[c + d*x])/d +
(I*f*PolyLog[2, -E^(I*(c + d*x))])/d^2 - (I*f*PolyLog[2, E^(I*(c + d*x))])
/d^2 - (f*Sin[c + d*x])/d^2)/a - (b*((a*(e + f*x)^2)/(2*b^2*f) - (2*(a^2 -
 b^2)*(((-1/2*I)*b*(((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*(((e + f*x)*Log[1 - (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/b^2 - (-(((e +
 f*x)*Cos[c + d*x])/d) + (f*Sin[c + d*x])/d^2)/b))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1188 vs. 2(310) = 620.

Time = 1.67 (sec) , antiderivative size = 1189, normalized size of antiderivative = 3.40

method	result	size
risch	Expression too large to display	1189

input
```
int((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
-1/2/b*f*x^2-1/b*e*x-I*b/d^2*f/a/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c)
)*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+1/d/a*e*ln(exp(I*(d*x+c))-1)
-1/d/a*e*ln(exp(I*(d*x+c))+1)-1/b/d*f*a/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*
x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/b/d*f*a/(-a^2+b^2)^(
1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*
x-1/b/d^2*f*a/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/
(I*a+(-a^2+b^2)^(1/2)))*c+1/b/d^2*f*a/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*x
+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*c-I/b/d^2*f*a/(-a^2+b^2)
^(1/2)*dilog((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/
2)))-b/d^2*f/a/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2)
)/(-I*a+(-a^2+b^2)^(1/2)))*c+b/d^2*f/a/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x
+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I*b/d*e/a/(-a^2+b^2)^
(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2*f/a*di
log(exp(I*(d*x+c)))+2*I*b/d^2*f*c/a/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp
(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+I/b/d
^2*f*a/(-a^2+b^2)^(1/2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a
+(-a^2+b^2)^(1/2)))-b/d*f/a/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a
^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x+b/d*f/a/(-a^2+b^2)^(1/2)*ln((I*a
+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-2*I/b/d^2*a*
f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(298) = 596$.

Time = 0.29 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.64

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(a*d^2*f*x^2 + 2*a*d^2*e*x - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos
(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b + 1) + I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d
*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x
 + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b + 1) + I*b*f*dilog(cos(d*x + c) + I*sin(d*x + c)) - I*b*f
*dilog(cos(d*x + c) - I*sin(d*x + c)) + I*b*f*dilog(-cos(d*x + c) + I*sin(
d*x + c)) - I*b*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) - (b*d*e - b*c*f)*
sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) + 2*I*a) - (b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
) + (b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d*e - b*c*f)*sqrt(-
(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a
^2 - b^2)/b^2) - 2*I*a) + (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-(I
*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) - (b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*...
```

Sympy [F]

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)\cos{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*cot(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)\cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.328 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd} - \frac{\operatorname{arctanh}(\cos(c+dx))}{ad}$$

output
```
-x/b+2*(a^2-b^2)^(1/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/
b/d-arctanh(cos(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{ac+adx-2\sqrt{a^2-b^2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

input
```
Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-((a*c + a*d*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]] + b*Log[Cos[(c + d*x)/2]] - b*Log[Sin[(c + d*x)/2]])/(a*b*d))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3368, 3042, 3537, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{\sin(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3368}$$

$$\int \frac{\left(1-\sin^2(c+dx)\right)\csc(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin(c+dx)^2}{\sin(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3537}$$

$$\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}dx+\frac{\int \csc(c+dx)dx}{a}-\frac{x}{b}$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}dx+\frac{\int \csc(c+dx)dx}{a}-\frac{x}{b}$$

$$\Big\downarrow \text{3139}$$

$$\frac{2\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}+\frac{\int \csc(c+dx)dx}{a}-\frac{x}{b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\int \csc(c+dx)dx}{a}-\frac{x}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \csc(c+dx)dx}{a}+\frac{2\left(\frac{a}{b}-\frac{b}{a}\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}-\frac{x}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2\left(\frac{a}{b}-\frac{b}{a}\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}-\frac{\text{arctanh}(\cos(c+dx))}{ad}-\frac{x}{b}$$

input
```
Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-(x/b) + (2*(a/b - b/a)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 -
b^2])])/(Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3537
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[C*(x
/(b*d)), x] + (Simp[(A*b^2 + a^2*C)/(b*(b*c - a*d))   Int[1/(a + b*Sin[e +
f*x]), x], x] - Simp[(c^2*C + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e +
 f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{\left(2a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ab\sqrt{a^2-b^2}}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{\left(2a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ab\sqrt{a^2-b^2}}}{d}$
risch	$-\frac{x}{b}+\frac{i\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}+\frac{i\left(a+\sqrt{a^2-b^2}\right)}{b}\right)}{dba}-\frac{i\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}-\frac{i\left(-a+\sqrt{a^2-b^2}\right)}{b}\right)}{dba}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}$

input
```
int(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tan(1/2*d*x+1/2*c))-2/b*arctan(tan(1/2*d*x+1/2*c))+(2*a^2-2*b^
2)/a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.49

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[-\frac{2\,adx + b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) - b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) - \sqrt{-a^2+b^2}\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)}{}\right.}{2\,abd}\right.$$
$$-\frac{2\,adx + b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) - b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) + 2\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+}\right.}{2\,abd}$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
 1/2) - sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b
*d), -1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x +
c) + 1/2) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))))/(a*b*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos\left(c+dx\right)\cot\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{dx+c}{b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2))
)*sqrt(a^2 - b^2)/(a*b))/d
```

Mupad [B] (verification not implemented)

Time = 40.77 (sec) , antiderivative size = 896, normalized size of antiderivative = 11.95

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)*cot(c + d*x))/(a + b*sin(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a*d) + (2*atan((64*a^3)/(64*a^2*b - 64*b^3 + 64*a
^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) - (64*a*b^2)/(64*a^2*
b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) + (6
4*b^3*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) -
 64*a*b^2*tan(c/2 + (d*x)/2)) - (64*a^2*b*tan(c/2 + (d*x)/2))/(64*a^2*b -
64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2))))/(b*d)
- (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2
 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*ta
n(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3) - (512*b^2*(b^2 -
 a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)
/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (10
24*b^6*tan(c/2 + (d*x)/2))/a^3) + (512*b^4*(b^2 - a^2)^(1/2))/(256*a^4*b +
 512*b^5 - 768*a^2*b^3 - 64*a^5*tan(c/2 + (d*x)/2) - 1792*a*b^4*tan(c/2 +
(d*x)/2) + 832*a^3*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/
a) - (1280*b^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(256*a^3*b - 768*a*b^
3 + (512*b^5)/a - 64*a^4*tan(c/2 + (d*x)/2) - 1792*b^4*tan(c/2 + (d*x)/2)
+ 832*a^2*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a^2) + (1
024*b^5*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(512*a*b^5 + 256*a^5*b - 768
*a^3*b^3 - 64*a^6*tan(c/2 + (d*x)/2) + 1024*b^6*tan(c/2 + (d*x)/2) - 1792*
a^2*b^4*tan(c/2 + (d*x)/2) + 832*a^4*b^2*tan(c/2 + (d*x)/2)) + (320*a*b...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-adx}{abd}$$

input
```
int(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2)) + lo
g(tan((c + d*x)/2))*b - a*d*x)/(a*b*d)
```

3.329 $\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 763

$$
\begin{aligned}
&\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{i(e+fx)^4}{4af}-\frac{i(a^2-b^2)(e+fx)^4}{4ab^2f}+\frac{6f^3\cos(c+dx)}{bd^4}-\frac{3f(e+fx)^2\cos(c+dx)}{bd^2} \\
&\quad+\frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d} \\
&\quad+\frac{(e+fx)^3\log\left(1-e^{2i(c+dx)}\right)}{ad}-\frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2} \\
&\quad-\frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d^2} \\
&\quad-\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2ad^2}+\frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^3} \\
&\quad+\frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d^3}+\frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2ad^3} \\
&\quad+\frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^4}+\frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d^4} \\
&\quad+\frac{3if^3\operatorname{PolyLog}\left(4,e^{2i(c+dx)}\right)}{4ad^4}+\frac{6f^2(e+fx)\sin(c+dx)}{bd^3}-\frac{(e+fx)^3\sin(c+dx)}{bd}
\end{aligned}
$$

output
```
6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/
d^4+6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/
b^2/d^4+6*f^3*cos(d*x+c)/b/d^4-3*f*(f*x+e)^2*cos(d*x+c)/b/d^2+(a^2-b^2)*(f
*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d+(a^2-b^2)*(f*
x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d+(f*x+e)^3*ln(1
-exp(2*I*(d*x+c)))/a/d-1/4*I*(a^2-b^2)*(f*x+e)^4/a/b^2/f-3*I*(a^2-b^2)*f*(
f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^2-3*I*(
a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b
^2/d^2+6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(
1/2)))/a/b^2/d^3+6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(
a^2-b^2)^(1/2)))/a/b^2/d^3+3/2*f^2*(f*x+e)*polylog(3,exp(2*I*(d*x+c)))/a/d
^3+3/4*I*f^3*polylog(4,exp(2*I*(d*x+c)))/a/d^4-3/2*I*f*(f*x+e)^2*polylog(2
,exp(2*I*(d*x+c)))/a/d^2-1/4*I*(f*x+e)^4/a/f+6*f^2*(f*x+e)*sin(d*x+c)/b/d^
3-(f*x+e)^3*sin(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4052 vs. $2(763) = 1526$.

Time = 11.40 (sec) , antiderivative size = 4052, normalized size of antiderivative = 5.31

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x
]
```

output
```
-1/2*(e*E^(I*c)*f^2*Csc[c]*((2*d^3*x^3)/E^((2*I)*c) + (3*I)*d^2*(1 - E^((-
2*I)*c))*x^2*Log[1 - E^((-I)*(c + d*x))] + (3*I)*d^2*(1 - E^((-2*I)*c))*x^
2*Log[1 + E^((-I)*(c + d*x))] - 6*d*(1 - E^((-2*I)*c))*x*PolyLog[2, -E^((-
I)*(c + d*x))] - 6*d*(1 - E^((-2*I)*c))*x*PolyLog[2, E^((-I)*(c + d*x))] +
 (6*I)*(1 - E^((-2*I)*c))*PolyLog[3, -E^((-I)*(c + d*x))] + (6*I)*(1 - E^(
(-2*I)*c))*PolyLog[3, E^((-I)*(c + d*x))]))/(a*d^3) - (E^(I*c)*f^3*Csc[c]*
((d^4*x^4)/E^((2*I)*c) + (2*I)*d^3*(1 - E^((-2*I)*c))*x^3*Log[1 - E^((-I)*
(c + d*x))] + (2*I)*d^3*(1 - E^((-2*I)*c))*x^3*Log[1 + E^((-I)*(c + d*x))]
 - 6*d^2*(1 - E^((-2*I)*c))*x^2*PolyLog[2, -E^((-I)*(c + d*x))] - 6*d^2*(1
 - E^((-2*I)*c))*x^2*PolyLog[2, E^((-I)*(c + d*x))] + (12*I)*d*(1 - E^((-2
*I)*c))*x*PolyLog[3, -E^((-I)*(c + d*x))] + (12*I)*d*(1 - E^((-2*I)*c))*x*
PolyLog[3, E^((-I)*(c + d*x))] + 12*(1 - E^((-2*I)*c))*PolyLog[4, -E^((-I)
*(c + d*x))] + 12*(1 - E^((-2*I)*c))*PolyLog[4, E^((-I)*(c + d*x))]))/(4*a
*d^4) + ((a^2 - b^2)*((-4*I)*d^4*e^3*E^((2*I)*c)*x - (6*I)*d^4*e^2*E^((2*I
)*c)*f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 - I*d^4*E^((2*I)*c)*f^3*x^4 -
 (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x)))
] + (2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*
I)*(c + d*x))))] - d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2
*I)*(c + d*x)))^2] + d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b
^2*(-1 + E^((2*I)*(c + d*x)))^2] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)^3\cos^2(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)^3\cot(c+dx)dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -(e+fx)^3\tan\left(c+dx+\frac{\pi}{2}\right)dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 25

$$\frac{-\int (e+fx)^3\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4202

$$-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)^3}{1+e^{i(2c+2dx+\pi)}}dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^4}{4f}}{a}$$

$\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\int (e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)}{4f}}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int (e+fx)}{a}$$

$\downarrow$ 4904

$$-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\int (e+fx)^2}{2}}{a}$$

$\downarrow$ 3042

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\int(e+fx)^2\cdots}{\cdots}}{a}$$

$\downarrow$ 3792

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\cdots\right)}{2d}\cdots\right.}{a}$$

$\downarrow$ 17

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+\cdots}{\cdots}\right)}{2d}\cdots\right.}{a}$$

$\downarrow$ 3042

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+\cdots}{\cdots}\right)}{2d}\cdots\right.}{a}$$

$\downarrow$ 3115

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\left(\frac{\int1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(2c+2d\cdots}\right.}{2d}\cdots\right.}{\cdots}\right.}{a}$$

$\downarrow$ 24

$$-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$
$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{}\right.}{}}{a}$$

$\downarrow$ 5036

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^3\cos(c+dx)dx}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$
$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{}\right.}{}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$
$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{}\right.}{}}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{3f\int-(e+fx)^2\sin(c+dx)dx}{d}+\frac{(e+fx)^3\sin(c+dx)}{d}\right)}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$
$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{}\right.}{}}{a}$$

$\downarrow$ 25

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int(e+fx)^2\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int(e+fx)^2\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int(e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int(e+fx)^3\cos(c+dx)\,\mathrm{si}}{b}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int(e+fx)^3\cos(c+d\cdots}{b}\cdots}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)\cdots}{2\cdots}\cdots\right.}{\cdots}\cdots}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int(e+\cdots}{\cdots}\cdots}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)\cdots}{2\cdots}\cdots\right.}{\cdots}\cdots}{a}$$

$\downarrow$ 25

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int(e+f}{}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int(e+f}{}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

$\downarrow$ 3118

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}\cdots\right.}{d}\cdots\right.}{b^2}\cdots\right.}{a}\cdots$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\cdots\right.}{}\cdots}{a}\cdots$$

$\downarrow$ 4904

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sin^2(c+dx)dx}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\cdots}{}\right.}{d}\cdots\right.}{}\cdots\right.}{b^2}\cdots\right.}{a}\cdots$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\cdots\right.}{}\cdots}{a}\cdots$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sin(c+dx)^2dx}{2d}}{b}+\frac{a\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)}{}\right.}{d}\right.}{}\right.}{b^2}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{}\right.}{}}{a}$$

$\downarrow$ 3792

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos}{2d}\right.}{2d}}{b}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{}\right.}{}}{a}$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int \sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx}{6f}\right.}{2d}}{b}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^3\sin^2(c+dx)}{2d}-\frac{3f\left(-\frac{f^2\int \sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx}{6f}\right.}{2d}}{b}\right.}{a}$$

$$\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{3f\left(\frac{f(e+fx)}{2}\right.}{}}{a}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos(dx+c)^2 \cot(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3449 vs. 2(688) = 1376.

Time = 0.44 (sec) , antiderivative size = 3449, normalized size of antiderivative = 4.52

$$\int \frac{(e+fx)^3 \cos^2(c+dx) \cot(c+dx)}{a+b\sin(c+dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3\cos^2{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos{(dx+c)}^2\cot{(dx+c)}}{b\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)^2*cot(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)^2\cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.330 $\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 566

$$\begin{aligned}
\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx
&= -\frac{i(e+fx)^3}{3af}-\frac{i(a^2-b^2)(e+fx)^3}{3ab^2f}-\frac{2f(e+fx)\cos(c+dx)}{bd^2} \\
&\quad+\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d} \\
&\quad+\frac{(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{ad}-\frac{2i(a^2-b^2)f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2} \\
&\quad-\frac{2i(a^2-b^2)f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{if(e+fx)\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^2} \\
&\quad+\frac{2(a^2-b^2)f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^3}+\frac{2(a^2-b^2)f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d^3} \\
&\quad+\frac{f^2\operatorname{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2ad^3}+\frac{2f^2\sin(c+dx)}{bd^3}-\frac{(e+fx)^2\sin(c+dx)}{bd}
\end{aligned}$$

output
```
-1/3*I*(f*x+e)^3/a/f-1/3*I*(a^2-b^2)*(f*x+e)^3/a/b^2/f-2*f*(f*x+e)*cos(d*x
+c)/b/d^2+(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))
/a/b^2/d+(a^2-b^2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/
a/b^2/d+(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d-2*I*(a^2-b^2)*f*(f*x+e)*polyl
og(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^2-2*I*(a^2-b^2)*f*(f*
x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d^2-I*f*(f*x+
e)*polylog(2,exp(2*I*(d*x+c)))/a/d^2+2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(
d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d^3+2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I
*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d^3+1/2*f^2*polylog(3,exp(2*I*(d*x+c)
))/a/d^3+2*f^2*sin(d*x+c)/b/d^3-(f*x+e)^2*sin(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1741 vs. 2(566) = 1132.

Time = 9.94 (sec) , antiderivative size = 1741, normalized size of antiderivative = 3.08

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x
]
```

output
```
-1/6*(E^(I*c)*f^2*Csc[c]*((2*d^3*x^3)/E^((2*I)*c) + (3*I)*d^2*(1 - E^((-2*
I)*c))*x^2*Log[1 - E^((-I)*(c + d*x))] + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*
Log[1 + E^((-I)*(c + d*x))] - 6*d*(1 - E^((-2*I)*c))*x*PolyLog[2, -E^((-I)
*(c + d*x))] - 6*d*(1 - E^((-2*I)*c))*x*PolyLog[2, E^((-I)*(c + d*x))] + (
6*I)*(1 - E^((-2*I)*c))*PolyLog[3, -E^((-I)*(c + d*x))] + (6*I)*(1 - E^((-
2*I)*c))*PolyLog[3, E^((-I)*(c + d*x))]))/(a*d^3) + ((a^2 - b^2)*((-6*I)*d
^3*e^2*E^((2*I)*c)*x - (6*I)*d^3*e*E^((2*I)*c)*f*x^2 - (2*I)*d^3*E^((2*I)*
c)*f^2*x^3 - 3*d^2*e^2*Log[b - (2*I)*a*E^(I*(c + d*x)) - b*E^((2*I)*(c + d
*x))] + 3*d^2*e^2*E^((2*I)*c)*Log[b - (2*I)*a*E^(I*(c + d*x)) - b*E^((2*I)
*(c + d*x))] - 6*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 3*d^2*f^2*x
^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] + 3*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I
*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2*e*f*x*Log[1 + (b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*e*E^((2*I
)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)])] - 3*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 3*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^
(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I...
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.24, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.853, Rules used = {5054, 4908, 3042, 25, 4202, 2620, 3011, 2720, 4904, 3042, 3791, 17, 5036, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4904, 3042, 3791, 17, 5030, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5054}$$

$$\frac{\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4908}$$

$$\frac{\int (e+fx)^2\cot(c+dx)dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\int -(e+fx)^2\tan\left(c+dx+\frac{\pi}{2}\right)dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{-\int (e+fx)^2\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4202

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)^2}{1+e^{i(2c+2dx+\pi)}}dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int (e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^3}{3f}}{a}$$

$\Big\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int \text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int (e+fx)^2\cos(c}{a}$$

$\Big\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 4904

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 3042

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 3791

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 17

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 5036

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^2\cos(c+dx)dx}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\big\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\big\downarrow$ 25

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\big\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\Big\downarrow$ 3777

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\Big\downarrow$ 3117

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}\right)}{a}$$

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 4904

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\int(e+fx)\sin^2(c+dx)dx}{d}}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+\cdots}{d}\right.}{d}\right.}{b^2}\right.}{a}$$

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\int(e+fx)\sin(c+dx)^2dx}{d}}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+\cdots}{d}\right.}{d}\right.}{b^2}\right.}{a}$$

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 3791

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{d}}{b}+\frac{a\left(\frac{(e+fx)^2\sin(c+}{d}\right.}{}\right.}{a}$$

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{b^2}-\frac{\frac{(e+fx)^2\sin^2(c+dx)}{2d}-\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}\right.}{}}{}\right.}{a}$$

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$\downarrow$ 5030

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{a}$$

$$\frac{b\left(-\frac{(a^2-b^2)\left(\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)^2}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^3}{3bf}\right)}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx}{d}\right.}{d}\right.}{b^2}\right.}{a}$$

$\downarrow$ 2620

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{b\left(-\frac{\left(a^2-b^2\right)\left(-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}}{b^2}\right.}$$

a

$\downarrow$ 3011

$$\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)}{b\left(-\frac{\left(a^2-b^2\right)\left(-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{if\int\text{PolyLog}\left(2,\frac{ibe}{a+}\right.}{d}\right.}{bd}}{b^2}\right.}$$

a

$\downarrow$ 2720

$$2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)$$

$$a$$

$$b\left(-\frac{\left(a^2-b^2\right)\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\ldots\right.}{\ldots}\right.}{b^2}\right.$$

$$\Big\downarrow \ 7143$$

$$2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(3,-e^{i(2c+2dx+\pi)}\right)}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)+\frac{f\left(\frac{f\sin^2(c+dx)}{4d^2}-\frac{(e+fx)\sin\ldots}{\ldots}\right.}{\ldots}$$

$$a$$

$$b\left(-\frac{\left(a^2-b^2\right)\left(-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\ldots\right.}{bd}\right.}{b^2}\right.$$

input

```
Int[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((-1/2*I)*(e + f*x)^2*Log[1 + E^(I*(2*c
 + Pi + 2*d*x))])/d + (I*f*(((I/2)*(e + f*x)*PolyLog[2, -E^(I*(2*c + Pi +
2*d*x))])/d - (f*PolyLog[3, -E^(I*(2*c + Pi + 2*d*x))])/(4*d^2)))/d) - ((e
 + f*x)^2*Sin[c + d*x]^2)/(2*d) + (f*((e + f*x)^2/(4*f) - ((e + f*x)*Cos[c
 + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x]^2)/(4*d^2)))/d)/a - (b*(-(((
a^2 - b^2)*(((-1/3*I)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*f*((I*(e + f*x)*PolyLog[2,
 (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/d^2))/(b*d) - (2*f*((I*(e + f*x)*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d - (f*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/d^2))/(b*d)))/b^2) + (a*(((e +
f*x)^2*Sin[c + d*x])/d - (2*f*(-(((e + f*x)*Cos[c + d*x])/d) + (f*Sin[c +
d*x])/d^2))/d))/b^2 - (((e + f*x)^2*Sin[c + d*x]^2)/(2*d) - (f*((e + f*x)^
2/(4*f) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (f*Sin[c + d*x]^2)
/(4*d^2)))/d)/b))/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)^2 \cot(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2260 vs. 2(511) = 1022.

Time = 0.40 (sec) , antiderivative size = 2260, normalized size of antiderivative = 3.99

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*(2*b^2*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 2*b^2*f^2*polyl
og(3, cos(d*x + c) - I*sin(d*x + c)) + 2*b^2*f^2*polylog(3, -cos(d*x + c)
+ I*sin(d*x + c)) + 2*b^2*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) +
 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f
^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*polylog(3, -(
-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x +
c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b) - 4*(a*b*d*f^2*x + a*b*d*e*f)*cos(d*x + c) - 2*(I*(a^2 - b^2)*d
*f^2*x + I*(a^2 - b^2)*d*e*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*
(I*(a^2 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*e*f)*dilog((I*a*cos(d*x + c) - a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*e*f)*dilog((-I*a
*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*
d*e*f)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*b^2*d*f^2*x + I*...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2\cos^2{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)^2\cot(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)^2*cot(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2 \cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2 \cos(dx+c)^2 \cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.331 $\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 379

$$\begin{aligned}
&\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{i(e+fx)^2}{2af}-\frac{i(a^2-b^2)(e+fx)^2}{2ab^2f}-\frac{f\cos(c+dx)}{bd^2} \\
&\quad+\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d} \\
&\quad+\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d}+\frac{(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad} \\
&\quad-\frac{i(a^2-b^2)f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{i(a^2-b^2)f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^2d^2} \\
&\quad-\frac{if\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2ad^2}-\frac{(e+fx)\sin(c+dx)}{bd}
\end{aligned}$$

output
```
-1/2*I*(f*x+e)^2/a/f-1/2*I*(a^2-b^2)*(f*x+e)^2/a/b^2/f-f*cos(d*x+c)/b/d^2+
(a^2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^2/d+(a^
2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^2/d+(f*x+e
)*ln(1-exp(2*I*(d*x+c)))/a/d-I*(a^2-b^2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a
-(a^2-b^2)^(1/2)))/a/b^2/d^2-I*(a^2-b^2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a
+(a^2-b^2)^(1/2)))/a/b^2/d^2-1/2*I*f*polylog(2,exp(2*I*(d*x+c)))/a/d^2-(f*
x+e)*sin(d*x+c)/b/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 868 vs. $2(379) = 758$.

Time = 5.71 (sec) , antiderivative size = 868, normalized size of antiderivative = 2.29

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(-(a*b*f*Cos[c + d*x]) + b^2*d*e*Log[Sin[c + d*x]] - b^2*c*f*Log[Sin[c + d
*x]] + a^2*d*e*Log[1 + (b*Sin[c + d*x])/a] - b^2*d*e*Log[1 + (b*Sin[c + d*
x])/a] - a^2*c*f*Log[1 + (b*Sin[c + d*x])/a] + b^2*c*f*Log[1 + (b*Sin[c +
d*x])/a] + (a^2*f*(I*(-2*c + Pi - 2*d*x)^2 - (32*I)*ArcSin[Sqrt[(a + b)/b]
/Sqrt[2]]*ArcTan[((a - b)*Cot[(2*c + Pi + 2*d*x)/4])/Sqrt[a^2 - b^2]] - 4*
(-2*c + Pi - 2*d*x + 4*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 - (I*(-a + S
qrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))] - 4*(-2*c + Pi - 2*d*x - 4*ArcSin[Sq
rt[(a + b)/b]/Sqrt[2]])*Log[1 + (I*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x
)))] + 4*(-2*c + Pi - 2*d*x)*Log[a + b*Sin[c + d*x]] + 8*(c + d*x)*Log[a +
 b*Sin[c + d*x]] + (8*I)*(PolyLog[2, (I*(-a + Sqrt[a^2 - b^2]))/(b*E^(I*(c
 + d*x)))] + PolyLog[2, ((-I)*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))])
))/8 - (b^2*f*(I*(-2*c + Pi - 2*d*x)^2 - (32*I)*ArcSin[Sqrt[(a + b)/b]/Sqr
t[2]]*ArcTan[((a - b)*Cot[(2*c + Pi + 2*d*x)/4])/Sqrt[a^2 - b^2]] - 4*(-2*
c + Pi - 2*d*x + 4*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 - (I*(-a + Sqrt[
a^2 - b^2]))/(b*E^(I*(c + d*x)))] - 4*(-2*c + Pi - 2*d*x - 4*ArcSin[Sqrt[(
a + b)/b]/Sqrt[2]])*Log[1 + (I*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))]
 + 4*(-2*c + Pi - 2*d*x)*Log[a + b*Sin[c + d*x]] + 8*(c + d*x)*Log[a + b*S
in[c + d*x]] + (8*I)*(PolyLog[2, (I*(-a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d
*x)))] + PolyLog[2, ((-I)*(a + Sqrt[a^2 - b^2]))/(b*E^(I*(c + d*x)))])))/8
 + b^2*f*((c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c + d*x)^2 +...
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.22, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5054, 4908, 3042, 25, 4202, 2620, 2715, 2838, 4904, 3042, 3115, 24, 5036, 3042, 3777, 25, 3042, 3118, 4904, 3042, 3115, 24, 5030, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)\cos^2(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)\cot(c+dx)dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\int (e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4202}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int \log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int (e+fx)\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{f\int e^{-i(2c+2dx+\pi)}\log\left(1+e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int (e+fx)\cos(c+dx)\sin(c+dx)d}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\int (e+fx)\cos(c+dx)\sin(c+dx)dx+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 4904}$$

$$\frac{-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{f\int \sin^2(c+dx)dx}{2d}+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{i(e+fx)^2}{2f}}{a}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{f\int \sin(c+dx)^2dx}{2d}+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{5036}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)\cos(c+dx)dx}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 4904}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int\sin^2(c+dx)dx}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int \sin(c+dx)^2dx}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}(2,-e^{i(2c+2dx+\pi)})}{4d^2}-\frac{i(e+fx)\log(1+e^{i(2c+2dx+\pi)})}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}(2,-e^{i(2c+2dx+\pi)})}{4d^2}-\frac{i(e+fx)\log(1+e^{i(2c+2dx+\pi)})}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}(2,-e^{i(2c+2dx+\pi)})}{4d^2}-\frac{i(e+fx)\log(1+e^{i(2c+2dx+\pi)})}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{5030}$$

$$\frac{2i\left(-\frac{f\,\text{PolyLog}(2,-e^{i(2c+2dx+\pi)})}{4d^2}-\frac{i(e+fx)\log(1+e^{i(2c+2dx+\pi)})}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}-\frac{b\left(-\frac{(a^2-b^2)\left(\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}}dx+\int \frac{e^{i(c+dx)}(e+fx)}{a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}}dx-\frac{i(e+fx)^2}{2bf}\right)}{b^2}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx}{2d}}{}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{f\int\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)dx}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}\right)}{b^2}+\cdots\right)}{a}$$

$\downarrow$ 2715

$$\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}-\frac{b\left(-\frac{(a^2-b^2)\left(\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{if\int e^{-i(c+dx)}\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\cdots}{\cdots}\right)}{b^2}\cdots\right)}{a}$$

$\downarrow$ 2838

$$\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}-\frac{i(e+fx)^2}{2f}}{a}-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e+fx)^2}{2bf}\right)}{b^2}\cdots\right)}{a}$$

input
```
Int[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((-1/2*I)*(e + f*x)*Log[1 + E^(I*(2*c +
 Pi + 2*d*x))])/d - (f*PolyLog[2, -E^(I*(2*c + Pi + 2*d*x))])/(4*d^2)) - (
(e + f*x)*Sin[c + d*x]^2)/(2*d) + (f*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2
*d)))/(2*d))/a - (b*(-(((a^2 - b^2)*(((-1/2*I)*(e + f*x)^2)/(b*f) + ((e +
f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) + ((e + f
*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*f*Pol
yLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - (I*f*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2)))/b^2) + (a*((
f*Cos[c + d*x])/d^2 + ((e + f*x)*Sin[c + d*x])/d))/b^2 - (((e + f*x)*Sin[c
 + d*x]^2)/(2*d) - (f*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(2*d))/b)
)/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(344) = 688$.

Time = 7.58 (sec) , antiderivative size = 1694, normalized size of antiderivative = 4.47

method	result	size
risch	Expression too large to display	1694

input
```
int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d/a*e*ln(exp(I*(d*x+c))+1)+1/d/a*e*ln(exp(I*(d*x+c))-1)+1/d/a*f*ln(exp(I
*(d*x+c))+1)*x-1/d^2/a*c*f*ln(exp(I*(d*x+c))-1)-1/2*I*(d*x*f-I*f+d*e)/d^2/
b*exp(-I*(d*x+c))+1/d^2*f*c/a*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I
*b)+1/d/b^2*a*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/d/b^2*a*
e*ln(exp(I*(d*x+c)))-I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+2/d^2/b^2*c*f*a*ln(
exp(I*(d*x+c)))-1/d^2/b^2*a*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))
-I*b)+2/d*a*f/(-a^2+b^2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a
+(-a^2+b^2)^(1/2)))*x+2/d*a*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^
2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+2/d^2*a*f/(-a^2+b^2)*ln((-I*a-exp(I*(d
*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*c+2/d^2*a*f/(-a^2+b^2)
*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I/d^
2/b^2*a*f*c^2-2*I/d^2*a*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2
)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/d^2*a*f/(-a^2+b^2)*dilog((-I*a-exp(I*
(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))+I/d^2*f/a*dilog(exp(
I*(d*x+c)))+I/d^2/b^2*a^3*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b
^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-1/d/b^2*a^3*f/(-a^2+b^2)*ln((-I*a-exp(I
*(d*x+c))*b+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x-1/d/b^2*a^3*f/(-a
^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
*x-1/d^2/b^2*a^3*f/(-a^2+b^2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/
(-I*a+(-a^2+b^2)^(1/2)))*c-1/d^2/b^2*a^3*f/(-a^2+b^2)*ln((I*a+exp(I*(d*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(338) = 676$.

Time = 0.31 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.41

$$\int \frac{(e + fx)\cos^2(c + dx)\cot(c + dx)}{a + b\sin(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*(2*a*b*f*cos(d*x + c) + I*b^2*f*dilog(cos(d*x + c) + I*sin(d*x + c))
- I*b^2*f*dilog(cos(d*x + c) - I*sin(d*x + c)) - I*b^2*f*dilog(-cos(d*x +
c) + I*sin(d*x + c)) + I*b^2*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) + I*(
a^2 - b^2)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + I*(a^2 - b^2)*f*dil
og((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x
 + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) - b)/b + 1) - I*(a^2 - b^2)*f*dilog((-I*a*cos(d*x + c) - a*sin(d
*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b + 1) - ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d*x + c) + 2*I*b*
sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - ((a^2 - b^2)*d*e - (a
^2 - b^2)*c*f)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) - 2*I*a) - ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*
x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - ((a^2
- b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +
 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*
f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c
*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin...
```

Sympy [F]

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)\cos^2{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)^2\cot(dx+c)}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((f*x + e)*cos(d*x + c)^2*cot(d*x + c)/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)^2\cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.332 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log(\sin(c+dx))}{ad} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d} - \frac{\sin(c+dx)}{bd}$$

output
```
ln(sin(d*x+c))/a/d+(a^2-b^2)*ln(a+b*sin(d*x+c))/a/b^2/d-sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^2\log(\sin(c+dx)) + (a^2-b^2)\log(a+b\sin(c+dx)) - ab\sin(c+dx)}{ab^2d}$$

input
```
Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(b^2*Log[Sin[c + d*x]] + (a^2 - b^2)*Log[a + b*Sin[c + d*x]] - a*b*Sin[c +
 d*x])/(a*b^2*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{\sin(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\csc(c+dx)(b^2-b^2\sin^2(c+dx))}{a+b\sin(c+dx)}d(b\sin(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\csc(c+dx)(b^2-b^2\sin^2(c+dx))}{b(a+b\sin(c+dx))}d(b\sin(c+dx))}{b^2d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a^2-b^2}{a(a+b\sin(c+dx))}+\frac{b\csc(c+dx)}{a}-1\right)d(b\sin(c+dx))}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a}+\frac{b^2\log(b\sin(c+dx))}{a}-b\sin(c+dx)}{b^2d}$$

input
```
Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
((b^2*Log[b*Sin[c + d*x]])/a + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/a - b
*Sin[c + d*x])/(b^2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b}+\frac{(a^2-b^2)\ln(a+b\sin(dx+c))}{b^2 a}+\frac{\ln(\sin(dx+c))}{a}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{b}+\frac{(a^2-b^2)\ln(a+b\sin(dx+c))}{b^2 a}+\frac{\ln(\sin(dx+c))}{a}}{d}$
risch	$-\frac{iax}{b^2}+\frac{i e^{i(dx+c)}}{2bd}-\frac{i e^{-i(dx+c)}}{2bd}-\frac{2iac}{b^2 d}+\frac{a\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{b^2 d}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{ad}$

input
```
int(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*sin(d*x+c)+1/b^2*(a^2-b^2)/a*ln(a+b*sin(d*x+c))+1/a*ln(sin(d*x+c
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^2\log\left(-\frac{1}{2}\sin(dx+c)\right)-ab\sin(dx+c)+(a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2d}$$

input
```
integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(b^2*log(-1/2*sin(d*x + c)) - a*b*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x
 + c) + a))/(a*b^2*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{\log(\sin(dx+c))}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2}}{d}$$

input
```
integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c))/a - sin(d*x + c)/b + (a^2 - b^2)*log(b*sin(d*x + c) + a
)/(a*b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log(|\sin(dx+c)|)}{ad} - \frac{\sin(dx+c)}{bd} + \frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{ab^2d}$$

input
```
integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(sin(d*x + c)))/(a*d) - sin(d*x + c)/(b*d) + (a^2 - b^2)*log(abs(b*
sin(d*x + c) + a))/(a*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 40.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{\sin(c+d\,x)}{b\,d} + \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(\frac{a}{b^2}-\frac{1}{a}\right)}{d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b^2\,d}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x))/(a + b*sin(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a*d) - sin(c + d*x)/(b*d) + (log(a + 2*b*tan(c/2
+ (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d - (a*log(tan(c/2 + (
d*x)/2)^2 + 1))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.02

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)a^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\right.}{a\,b^2d}$$

input
```
int(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2)**2 + 1)*a**2 + log(tan((c + d*x)/2)**2*a + 2*tan(
(c + d*x)/2)*b + a)*a**2 - log(tan((c + d*x)/2)**2*a + 2*tan((c + d*x)/2)*
b + a)*b**2 + log(tan((c + d*x)/2))*b**2 - sin(c + d*x)*a*b)/(a*b**2*d)
```

3.333 $\int \frac{(e+fx)^3 \cos^3(c+dx) \cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1125

$$\int \frac{(e+fx)^3 \cos^3(c+dx) \cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

output
```
6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/
b^3/d^4-6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1
/2)))/a/b^3/d^4+3*I*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a/d^2+3/8*f*(f*
x+e)^2/b/d^2-2*(f*x+e)^3*arctanh(exp(I*(d*x+c)))/a/d+6*f^3*sin(d*x+c)/a/d^
4-3/4*f*(f*x+e)^2*cos(d*x+c)^2/b/d^2-6*f^2*(f*x+e)*polylog(3,-exp(I*(d*x+c
)))/a/d^3+6*f^2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a/d^3-6*I*f^3*polylog(4,
-exp(I*(d*x+c)))/a/d^4-1/8*(f*x+e)^4/b/f+1/4*(a^2-b^2)*(f*x+e)^4/b^3/f+3/8
*f^3*cos(d*x+c)^2/b/d^4+I*(a^2-b^2)^(3/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c)
)/(a-(a^2-b^2)^(1/2)))/a/b^3/d-3*(a^2-b^2)^(3/2)*f*(f*x+e)^2*polylog(2,I*b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^2+3*(a^2-b^2)^(3/2)*f*(f*x+e)
^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^2-I*(a^2-b^2)
^(3/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d-6*I*
(a^2-b^2)^(3/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2
)))/a/b^3/d^3+6*I*(a^2-b^2)^(3/2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))
/(a-(a^2-b^2)^(1/2)))/a/b^3/d^3+6*(a^2-b^2)*f^3*sin(d*x+c)/a/b^2/d^4+(f*x+
e)^3*cos(d*x+c)/a/d+3/4*f^2*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d^3+(a^2-b^2)*
(f*x+e)^3*cos(d*x+c)/a/b^2/d+6*I*f^3*polylog(4,exp(I*(d*x+c)))/a/d^4-3*(a^
2-b^2)*f*(f*x+e)^2*sin(d*x+c)/a/b^2/d^2-6*(a^2-b^2)*f^2*(f*x+e)*cos(d*x+c)
/a/b^2/d^3-3*I*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a/d^2-6*f^2*(f*x+e)*c
os(d*x+c)/a/d^3-3*f*(f*x+e)^2*sin(d*x+c)/a/d^2-1/2*(f*x+e)^3*cos(d*x+c)...
```

Mathematica [A] (verified)

Time = 7.98 (sec) , antiderivative size = 1181, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x
]
```

output
```
(8*a*(2*a^2 - 3*b^2)*d^4*e^3*x + 12*a*(2*a^2 - 3*b^2)*d^4*e^2*f*x^2 + 8*a*
(2*a^2 - 3*b^2)*d^4*e*f^2*x^3 + 2*a*(2*a^2 - 3*b^2)*d^4*f^3*x^4 - 32*b^3*d
^3*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 96*a^2*b*d*f^2*(e
+ f*x)*Cos[c + d*x] + 16*a^2*b*d^3*(e + f*x)^3*Cos[c + d*x] + 3*a*b^2*f^3*
Cos[2*(c + d*x)] - 6*a*b^2*d^2*f*(e + f*x)^2*Cos[2*(c + d*x)] + 48*(a^2 -
b^2)^(3/2)*d^2*f*(e + f*x)^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqr
t[a^2 - b^2])] + (16*I)*(a^2 - b^2)^(3/2)*((2*I)*d^3*e^3*ArcTan[(I*a + b*E
^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*d^3*e^2*f*x*Log[1 + (I*b*E^(I*(c + d*
x)))/(-a + Sqrt[a^2 - b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (I*b*E^(I*(c + d*x)
))/(-a + Sqrt[a^2 - b^2])] + d^3*f^3*x^3*Log[1 + (I*b*E^(I*(c + d*x)))/(-a
 + Sqrt[a^2 - b^2])] - 3*d^3*e^2*f*x*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])] - 3*d^3*e*f^2*x^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])] - d^3*f^3*x^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])] + (3*I)*d^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])] + 6*d*f^2*(e + f*x)*PolyLog[3, ((-I)*b*E^(I*(c + d*x)))/(-a
+ Sqrt[a^2 - b^2])] - 6*d*e*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])] - 6*d*f^3*x*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])] + (6*I)*f^3*PolyLog[4, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b
^2])] - (6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])
 + (48*I)*b^3*f*(d^2*(e + f*x)^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)^3\cos^3(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)^3\cos(c+dx)\cot(c+dx)dx-\int (e+fx)^3\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4905$$

$$\frac{-\frac{f\int (e+fx)^2\cos^3(c+dx)dx}{d}+\int (e+fx)^3\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^3\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{d}+\int (e+fx)^3\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^3\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3792$$

$$\frac{-\frac{f\left(-\frac{2f^2\int \cos^3(c+dx)dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\cos(c+dx)dx+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int (e+fx)^3\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^3\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\left(-\frac{2f^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int (e+fx)^3\cos(c+\cdots}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3113$$

$$\frac{-\frac{f\left(\frac{2f^2\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{9d^3}+\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int (e+fx)^3\cdots}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{f\left(\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int (e+fx)^3\cos(c}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\big\downarrow$ 3777

$$\frac{-\frac{f\left(\frac{2}{3}\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\big\downarrow$ 25

$$\frac{-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int (}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\big\downarrow$ 3042

$$\frac{-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int (e+fx)\sin(c+dx)dx}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int (}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\big\downarrow$ 3777

$$\frac{-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos}{3d}}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\big\downarrow$ 3042

$$-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+d}{3d}\right.}{d}$$
$$a$$
$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3117

$$\int(e+fx)^3\cos(c+dx)\cot(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx}{d^2}\right.}{}\right.\right.}{d}$$
$$a$$
$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4908

$$-\int(e+fx)^3\sin(c+dx)dx+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2 s}{}\right.\right.}{}$$
$$a$$
$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$-\int(e+fx)^3\sin(c+dx)dx+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2 s}{}\right.\right.}{}$$
$$a$$
$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3777

$$-\frac{3f\int(e+fx)^2\cos(c+dx)dx}{d}+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+c}{d}\right.\right.}{d}$$
$$a$$
$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$-\frac{3f\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sin}{d}\right.\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$-\frac{3f\left(\frac{2f\int-((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{d}+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c}{9d^2}\right.}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx}{9d^2}\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{d}+\int(e+fx)^3\csc(c+dx)dx-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx}{9d^2}\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$\int(e+fx)^3\csc(c+dx)dx-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+2f(e\right.}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\right.}{}}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3117

$$\frac{\int (e+fx)^3\csc(c+dx)dx-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)}{d}-\frac{f\left(\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx}{}\right.}{}}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4671

$$-\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{-\frac{3f\int(e+fx)^2\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{3f\int(e+fx)^2\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^3\text{arctanh}\left(e^{i(c+dx)}\right)}{d}-\frac{3f\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\sin(}{}\right.}{}\right.}{d}}{}$$

$\downarrow$ 3011

$$-\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{}$$

$\downarrow$ 5036

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^3\cos^2(c+dx)dx}{b^2}-\frac{\int(e+fx)^3\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$

$$\frac{\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}}{}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}-\frac{\int (e+fx)^3\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}$$

$\downarrow$ 3792

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(-\frac{3f^2\int (e+fx)\cos^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx+\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}\right)}{b^2}\right.}{a}$$

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(-\frac{3f^2\int (e+fx)\cos^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{b^2}-\int\right.}{a}$$

$$\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{2if\int (e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}\right.}{d}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos (dx+c)^3 \cot (dx+c)}{a+b \sin (dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4205 vs. $2(1013) = 2026$.

Time = 0.74 (sec) , antiderivative size = 4205, normalized size of antiderivative = 3.74

$$\int \frac{(e+fx)^3 \cos^3(c+dx) \cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3\cos^3{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos\left(dx+c\right)^3\cot\left(dx+c\right)}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.334 $\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 825

$$
\begin{aligned}
&\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx \\
&= \frac{f^2x}{4bd^2}-\frac{(e+fx)^3}{6bf}+\frac{(a^2-b^2)(e+fx)^3}{3b^3f}-\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad} \\
&\quad-\frac{2f^2\cos(c+dx)}{ad^3}-\frac{2(a^2-b^2)f^2\cos(c+dx)}{ab^2d^3} \\
&\quad+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(a^2-b^2)(e+fx)^2\cos(c+dx)}{ab^2d} \\
&\quad-\frac{f(e+fx)\cos^2(c+dx)}{2bd^2}+\frac{i(a^2-b^2)^{3/2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d} \\
&\quad-\frac{i(a^2-b^2)^{3/2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{2if(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2} \\
&\quad-\frac{2if(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{2(a^2-b^2)^{3/2}f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d^2} \\
&\quad-\frac{2(a^2-b^2)^{3/2}f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^3d^2}-\frac{2f^2\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3} \\
&\quad+\frac{2f^2\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{2i(a^2-b^2)^{3/2}f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d^3} \\
&\quad-\frac{2i(a^2-b^2)^{3/2}f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^3d^3} \\
&\quad-\frac{2f(e+fx)\sin(c+dx)}{ad^2}-\frac{2(a^2-b^2)f(e+fx)\sin(c+dx)}{ab^2d^2} \\
&\quad+\frac{f^2\cos(c+dx)\sin(c+dx)}{4bd^3}-\frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2bd}
\end{aligned}
$$

output
```
1/4*f^2*x/b/d^2-1/6*(f*x+e)^3/b/f+1/3*(a^2-b^2)*(f*x+e)^3/b^3/f-2*(f*x+e)^
2*arctanh(exp(I*(d*x+c)))/a/d-2*f^2*cos(d*x+c)/a/d^3-2*(a^2-b^2)*f^2*cos(d
*x+c)/a/b^2/d^3+(f*x+e)^2*cos(d*x+c)/a/d+(a^2-b^2)*(f*x+e)^2*cos(d*x+c)/a/
b^2/d-1/2*f*(f*x+e)*cos(d*x+c)^2/b/d^2+I*(a^2-b^2)^(3/2)*(f*x+e)^2*ln(1-I*
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d+2*I*f*(f*x+e)*polylog(2,-exp
(I*(d*x+c)))/a/d^2+2*I*(a^2-b^2)^(3/2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a
-(a^2-b^2)^(1/2)))/a/b^3/d^3-2*I*(a^2-b^2)^(3/2)*f^2*polylog(3,I*b*exp(I*(
d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^3+2*(a^2-b^2)^(3/2)*f*(f*x+e)*polylog
(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^2-2*(a^2-b^2)^(3/2)*f*(
f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^2-2*f^2*p
olylog(3,-exp(I*(d*x+c)))/a/d^3+2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3-I*(a
^2-b^2)^(3/2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3
/d-2*I*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a/d^2-2*f*(f*x+e)*sin(d*x+c)/a/
d^2-2*(a^2-b^2)*f*(f*x+e)*sin(d*x+c)/a/b^2/d^2+1/4*f^2*cos(d*x+c)*sin(d*x+
c)/b/d^3-1/2*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.52

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x
]
```

output

```
-1/24*(-24*a^3*d^3*e^2*x + 36*a*b^2*d^3*e^2*x - 24*a^3*d^3*e*f*x^2 + 36*a*
b^2*d^3*e*f*x^2 - 8*a^3*d^3*f^2*x^3 + 12*a*b^2*d^3*f^2*x^3 + 48*(a^2 - b^2
)^(3/2)*d^2*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] - 24*a^2
*b*d^2*e^2*Cos[c + d*x] + 48*a^2*b*f^2*Cos[c + d*x] - 48*a^2*b*d^2*e*f*x*C
os[c + d*x] - 24*a^2*b*d^2*f^2*x^2*Cos[c + d*x] + 6*a*b^2*d*e*f*Cos[2*(c +
 d*x)] + 6*a*b^2*d*f^2*x*Cos[2*(c + d*x)] - 24*b^3*d^2*e^2*Log[1 - E^(I*(c
 + d*x))] - 48*b^3*d^2*e*f*x*Log[1 - E^(I*(c + d*x))] - 24*b^3*d^2*f^2*x^2
*Log[1 - E^(I*(c + d*x))] + 24*b^3*d^2*e^2*Log[1 + E^(I*(c + d*x))] + 48*b
^3*d^2*e*f*x*Log[1 + E^(I*(c + d*x))] + 24*b^3*d^2*f^2*x^2*Log[1 + E^(I*(c
 + d*x))] - (48*I)*(a^2 - b^2)^(3/2)*d^2*e*f*x*Log[1 + (I*b*E^(I*(c + d*x)
))/(-a + Sqrt[a^2 - b^2])] - (24*I)*(a^2 - b^2)^(3/2)*d^2*f^2*x^2*Log[1 +
(I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + (48*I)*(a^2 - b^2)^(3/2)*d
^2*e*f*x*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (24*I)*(a^
2 - b^2)^(3/2)*d^2*f^2*x^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])] - (48*I)*b^3*d*e*f*PolyLog[2, -E^(I*(c + d*x))] - (48*I)*b^3*d*f^2*x
*PolyLog[2, -E^(I*(c + d*x))] + (48*I)*b^3*d*e*f*PolyLog[2, E^(I*(c + d*x)
)] + (48*I)*b^3*d*f^2*x*PolyLog[2, E^(I*(c + d*x))] - 48*(a^2 - b^2)^(3/2)
*d*e*f*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - 48*(a
^2 - b^2)^(3/2)*d*f^2*x*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2
 - b^2])] + 48*(a^2 - b^2)^(3/2)*d*e*f*PolyLog[2, (I*b*E^(I*(c + d*x)))...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)^2\cos^3(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx-\int (e+fx)^2\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4905$$

$$\frac{-\frac{2f\int (e+fx)\cos^3(c+dx)dx}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3791$$

$$\frac{-\frac{2f\left(\frac{2}{3}\int (e+fx)\cos(c+dx)dx+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos}{3d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2f\left(\frac{2}{3}\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2}{}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{2f\left(\frac{2}{3}\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx+}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx+}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3118

$$\frac{\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\frac{(e+f}{}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4908

$$\frac{-\int(e+fx)^2\sin(c+dx)dx+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+d}{3c}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{-\int(e+fx)^2\sin(c+dx)dx+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+d}{3c}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3777

$$\frac{-\frac{2f\int(e+fx)\cos(c+dx)dx}{d}+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(}{3d}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\,c}{3d}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{2f\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx}{}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{-\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\,s}{}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}+\int(e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\,s}{}\right.}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3118

$$\frac{\int (e+fx)^2\csc(c+dx)dx - \frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d} - \frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+\cdots}{}\right.}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4671

$$-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{-\frac{2f\int(e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int(e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\cdots\right.\right.}{}}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)\cdots}{}}{}$$

$\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{\cdots}\right.}{d^2}\right.}{d}}{}$$

$\downarrow$ 5036

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^2\cos^2(c+dx)dx}{b^2}-\frac{\int(e+fx)^2\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$

$$\frac{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,e^{\cdots}\right.}{d^2}\right.}{d}}{}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}-\frac{\int (e+fx)^2\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+$$

$$\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,e^{i(c+dx)}\right.}{d^2}\right.}{d}$$

$$\downarrow \text{3792}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(-\frac{f^2\int\cos^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}\right)}{b^2}-\frac{\int (e\cdots}{}\right.}{a}$$

$$\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,e^{i(c+dx)}\right.}{d^2}\right.}{d}$$

$$\downarrow \text{17}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(-\frac{f^2\int\cos^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{b^2}-\frac{\int (e+fx)^2\cdots}{}\right.}{a}$$

$$\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,e^{i(c+dx)}\right.}{d^2}\right.}{d}$$

$$\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(-\frac{f^2\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{2d^2}+\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{b^2}-\frac{\int (e+f\cdots}{}\right.}{a}$$

$$\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,e^{i(c+dx)}\right.}{d^2}\right.}{d}$$

$$\downarrow \text{3115}$$

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(-\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{b^2}\right.}{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}(2,}{d^2}\right.}{d}}$$

a

$\downarrow$ 24

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)^2\cos^2(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{f(e+fx)\cos^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+f}{}\right.}{b^2}\right.}{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}(2,}{d^2}\right.}{d}}$$

a

$\downarrow$ 4905

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\int(e+fx)\cos^3(c+dx)dx}{3d}-\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{b}+\frac{a\left(\frac{f(e+fx)\cos^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right.}{2d^2}\right.}{b^2}\right.}{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}(2,}{d^2}\right.}{d}}$$

a

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}-\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{b}+\frac{a\left(\frac{f(e+fx)\cos^2(c+dx)}{2d^2}-\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx}{2d}\right.}{2d^2}\right.}{}\right.}{\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}(2,}{d^2}\right.}{d}}$$

a

input
```
Int[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)^3 \cot(dx+c)}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2787 vs. $2(739) = 1478$.

Time = 0.41 (sec) , antiderivative size = 2787, normalized size of antiderivative = 3.38

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/12*(2*(2*a^3 - 3*a*b^2)*d^3*f^2*x^3 + 6*(2*a^3 - 3*a*b^2)*d^3*e*f*x^2 +
12*b^3*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 12*b^3*f^2*polylog(
3, cos(d*x + c) - I*sin(d*x + c)) - 12*b^3*f^2*polylog(3, -cos(d*x + c) +
I*sin(d*x + c)) - 12*b^3*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) -
12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c)
+ a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b) + 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos
(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) - 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
 -(-I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)
*polylog(3, -(-I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(a*b^2*d*f^2*x + a*b^2*d*e*f)*
cos(d*x + c)^2 - 12*(I*(a^2*b - b^3)*d*f^2*x + I*(a^2*b - b^3)*d*e*f)*sqrt
(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x +
 c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(-I*(a^2*b
 - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a
*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) - b)/b + 1) - 12*(-I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b -
b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)^2\cos^3{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)^3\cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.335 $\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 512

$$\begin{aligned}\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{(e+fx)^2}{4bf}+\frac{(a^2-b^2)(e+fx)^2}{2b^3f}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{ad}\\
&+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(a^2-b^2)(e+fx)\cos(c+dx)}{ab^2d}\\
&-\frac{f\cos^2(c+dx)}{4bd^2}+\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d}\\
&-\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}\\
&-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{(a^2-b^2)^{3/2}f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d^2}\\
&-\frac{(a^2-b^2)^{3/2}f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{ab^3d^2}-\frac{f\sin(c+dx)}{ad^2}\\
&-\frac{(a^2-b^2)f\sin(c+dx)}{ab^2d^2}-\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd}\end{aligned}$$

output
```
-1/4*(f*x+e)^2/b/f+1/2*(a^2-b^2)*(f*x+e)^2/b^3/f-2*(f*x+e)*arctanh(exp(I*(
d*x+c)))/a/d+(f*x+e)*cos(d*x+c)/a/d+(a^2-b^2)*(f*x+e)*cos(d*x+c)/a/b^2/d-1
/4*f*cos(d*x+c)^2/b/d^2+I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/a/b^3/d-I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(d*x
+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d+I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2-I*f
*polylog(2,exp(I*(d*x+c)))/a/d^2+(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*
x+c))/(a-(a^2-b^2)^(1/2)))/a/b^3/d^2-(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I
*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a/b^3/d^2-f*sin(d*x+c)/a/d^2-(a^2-b^2)*f*si
n(d*x+c)/a/b^2/d^2-1/2*(f*x+e)*cos(d*x+c)*sin(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2194 vs. $2(512) = 1024$.

Time = 17.19 (sec) , antiderivative size = 2194, normalized size of antiderivative = 4.29

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-1/4*((-2*a^2 + 3*b^2)*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(b^3*d^2)
+ (a*(d*e - c*f + f*(c + d*x))*Cos[c + d*x])/(b^2*d^2) - (f*Cos[2*(c + d*x
)])/(8*b*d^2) + (e*Log[Tan[(c + d*x)/2]])/(a*d) - (c*f*Log[Tan[(c + d*x)/2
]])/(a*d^2) + (f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c +
d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))
/(a*d^2) - (a*f*Sin[c + d*x])/(b^2*d^2) - ((d*e - c*f + f*(c + d*x))*Sin[2
*(c + d*x)])/(4*b*d^2) + (((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(
b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])])/
Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 +
 b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^
2] - (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Log[
1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I
*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2, (a*(1 - I*
Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (I*
f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))])
/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b +
Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2, (a + I*a*Tan[(c + d
*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2])*(-(((a^2 - ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5054}$$

$$\frac{\int (e+fx)\cos^3(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4908}$$

$$\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx-\int (e+fx)\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 4905$$

$$\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx - \frac{f\int \cos^3(c+dx)dx}{3d} + \frac{(e+fx)\cos^3(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx - \frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d} + \frac{(e+fx)\cos^3(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3113$$

$$\frac{\frac{f\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{3d^2} + \int (e+fx)\cos(c+dx)\cot(c+dx)dx + \frac{(e+fx)\cos^3(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 2009$$

$$\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} + \frac{(e+fx)\cos^3(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 4908$$

$$\frac{-\int (e+fx)\sin(c+dx)dx + \int (e+fx)\csc(c+dx)dx + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} + \frac{(e+fx)\cos^3(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\int (e+fx)\sin(c+dx)dx + \int (e+fx)\csc(c+dx)dx + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} + \frac{(e+fx)\cos^3(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{\int (e+fx)\csc(c+dx)dx - \frac{f\int \cos(c+dx)dx}{d} + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} + \frac{(e+fx)\cos^3(c+dx)}{3d} + \frac{(e+fx)\cos(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{\int (e+fx)\csc(c+dx)dx - \frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d} + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} + \frac{(e+fx)\cos^3(c+dx)}{3d} + \frac{(e+fx)\cos(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3117

$$\frac{\int (e+fx)\csc(c+dx)dx + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} - \frac{f\sin(c+dx)}{d^2} + \frac{(e+fx)\cos^3(c+dx)}{3d} + \frac{(e+fx)\cos(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4671

$$-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a} + \frac{-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d} + \frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d} - \frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d} + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} - \frac{f\sin(c+dx)}{d^2} + \frac{(e+}{}}{a}$$

$\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a} + \frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2} - \frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2} - \frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d} + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin\right.}{3d^2}}{a}$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a} + \frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d} + \frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2} - \frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2} + \frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2} - \frac{f\sin(c+dx)}{d^2} + \frac{(}{}}{a}$$

$\downarrow$ 5036

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)\cos^2(c+dx)dx}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx))}{3d^2}-\frac{f\sin(c+dx)}{d^2}+(}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)\sin(c+dx+\frac{\pi}{2})^2dx}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx))}{3d^2}-\frac{f\sin(c+dx)}{d^2}+(}{a}$$

$\downarrow$ 3791

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{1}{2}\int(e+fx)dx+\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}\right.}{a}$$
$$\frac{-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx))}{3d^2}-\frac{f\sin(c+dx)}{d^2}+(}{a}$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}\right)}{a}+\frac{-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx))}{3d^2}-\frac{f\sin(c+dx)}{d^2}+(}{a}$$

$\downarrow$ 4905

$$-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int\cos^3(c+dx)dx}{3d}-\frac{(e+fx)\cos^3(c+dx)}{3d}}{b}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}\right)}{a}+\frac{-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx))}{3d^2}-\frac{f\sin(c+dx)}{d^2}+(}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}-\frac{(e+fx)\cos^3(c+dx)}{3d}}{b}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}\cdots\right)}{a}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$\downarrow$ 3113

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{-\frac{f\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{3d^2}-\frac{(e+fx)\cos^3(c+dx)}{3d}}{b}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\cdots\right)}{b^2}\cdots\right)}{a}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$\downarrow$ 2009

$$-\frac{b\left(-\frac{(a^2-b^2)\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}-\frac{-\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{(e+fx)\cos^3\cdots}{3d}}{b}\cdots\right)}{a}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$\downarrow$ 5036

$$-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)dx}{b^2}-\frac{\int(e+fx)\sin(c+dx)dx}{b}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}\cdots\right)}{a}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$\downarrow$ 17

$$-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{(a^2-b^2)\int\frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\sin(c+dx)dx}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}-\cdots\right)}{a}$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int (e+fx)\sin(c+dx)dx}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}-\cdots}{a}-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\cdots\right)}{b^2}\cdots}{a}-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}}{b}+\frac{a(e+fx)^2}{2b^2f}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c-\cdots}{2d}\cdots\right)}{b^2}\cdots}{a}-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{-\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{(a^2-b^2)\int \frac{e+fx}{a+b\sin(c+dx)}dx}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\cdots\right)}{b^2}\cdots}{a}-\frac{2(e+fx)\text{arctanh}(e^{i(c+dx)})}{d}+\frac{if\,\text{PolyLog}(2,-e^{i(c+dx)})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{i(c+dx)})}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\cdots}{a}$$

$$\Big\downarrow \; 3804$$

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(\cdots}{}}{a}$$

$$\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{2\left(a^2-b^2\right)\int\frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a-ibe^{2i(c+dx)}+ib}dx}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{b^2}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos\cdots}{2d}\right.}{b^2}\right.}{a}$$

$\downarrow$ 2694

$$\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(\cdots}{}}{a}$$

$$\frac{b\left(-\frac{(a^2-b^2)\left(-\frac{2\left(a^2-b^2\right)\left(\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i(c+dx)}(e+fx)}{2\left(a-ibe^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a(e+fx)^2}{2b^2f}-\frac{\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}}{b}\right)}{b^2}\cdots\right.}{a}$$

input
```
Int[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1873 vs. $2(466) = 932$.

Time = 8.29 (sec) , antiderivative size = 1874, normalized size of antiderivative = 3.66

method	result	size
risch	Expression too large to display	1874

input
```
int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
2*I*b/d^2*f*c/a/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a
^2+b^2)^(1/2))-1/d/a*e*ln(exp(I*(d*x+c))+1)+1/d/a*e*ln(exp(I*(d*x+c))-1)-4
*I/d^2/b*a*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^
2+b^2)^(1/2))+2*I/d^2/b^3*a^3*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I
*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-1/d/a*f*ln(exp(I*(d*x+c))+1)*x-1/d^2/a*c*
f*ln(exp(I*(d*x+c))-1)-1/16*I*(2*d*x*f-I*f+2*d*e)/d^2/b*exp(-2*I*(d*x+c))+
1/2*a*(d*x*f-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c))+1/d^2/b^3*f*a^3/(-a^2+b^2)^(
1/2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+
I/d^2/b^3*f*a^3/(-a^2+b^2)^(1/2)*dilog((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(
1/2))/(-I*a+(-a^2+b^2)^(1/2)))-I/d^2/b^3*f*a^3/(-a^2+b^2)^(1/2)*dilog((I*a
+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/d/b^3*a^3*
e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
+4*I/d/b*a*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+
b^2)^(1/2))-2*I/d^2/b*f*a/(-a^2+b^2)^(1/2)*dilog((-I*a-exp(I*(d*x+c))*b+(-
a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))-3/4/b*f*x^2-3/2/b*e*x+I/d^2*f/a*d
ilog(exp(I*(d*x+c)))+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+1/2*a*(d*x*f+I*f+d*
e)/b^2/d^2*exp(I*(d*x+c))+b/d*f/a/(-a^2+b^2)^(1/2)*ln((I*a+exp(I*(d*x+c))*
b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-I*b/d^2*f/a/(-a^2+b^2)^(1/2)
*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2/b
/d^2*f*a/(-a^2+b^2)^(1/2)*ln((-I*a-exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(454) = 908$.

Time = 0.49 (sec) , antiderivative size = 1611, normalized size of antiderivative = 3.15

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/4*((2*a^3 - 3*a*b^2)*d^2*f*x^2 - a*b^2*f*cos(d*x + c)^2 + 2*(2*a^3 - 3*a
*b^2)*d^2*e*x - 2*I*b^3*f*dilog(cos(d*x + c) + I*sin(d*x + c)) + 2*I*b^3*f
*dilog(cos(d*x + c) - I*sin(d*x + c)) - 2*I*b^3*f*dilog(-cos(d*x + c) + I*
sin(d*x + c)) + 2*I*b^3*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 2*I*(a^2
*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c
) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)
 + 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*
sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) + 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d
*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog
((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)
*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*
c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*c
*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*c
*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +...
```

Sympy [F]

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)\cos^3{(c+dx)}\cot{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)^3\cot(dx+c)}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

3.336 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{2(a^2-b^2)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
1/2*(2*a^2-3*b^2)*x/b^3-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/
(a^2-b^2)^(1/2))/a/b^3/d-arctanh(cos(d*x+c))/a/d+a*cos(d*x+c)/b^2/d-1/2*co
s(d*x+c)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{-4a^3c+6ab^2c-4a^3dx+6ab^2dx+8(a^2-b^2)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-4a^2b\cos(c+dx)+4b^3\,\mathrm{lo}}{4ab^3d}$$

input
```
Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
-1/4*(-4*a^3*c + 6*a*b^2*c - 4*a^3*d*x + 6*a*b^2*d*x + 8*(a^2 - b^2)^(3/2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a^2*b*Cos[c + d*x] +
 4*b^3*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Sin[(c + d*x)/2]] + a*b^2*Sin[2*(
c + d*x)])/(a*b^3*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 3374, 25, 3042, 3536, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{\sin(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3374}$$

$$-\frac{\int -\frac{\csc(c+dx)\left(2b^2+a\sin(c+dx)b+\left(2a^2-3b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc(c+dx)\left(2b^2+a\sin(c+dx)b+\left(2a^2-3b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2b^2+a\sin(c+dx)b+\left(2a^2-3b^2\right)\sin(c+dx)^2}{\sin(c+dx)(a+b\sin(c+dx))}dx}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3536}$$

$$\frac{-\frac{2(a^2-b^2)^2\int\frac{1}{a+b\sin(c+dx)}dx}{ab}+\frac{2b^2\int\csc(c+dx)dx}{a}+\frac{x(2a^2-3b^2)}{b}}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2(a^2-b^2)^2\int\frac{1}{a+b\sin(c+dx)}dx}{ab}+\frac{2b^2\int\csc(c+dx)dx}{a}+\frac{x(2a^2-3b^2)}{b}}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 3139

$$\frac{-\frac{4(a^2-b^2)^2\int\frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{abd}+\frac{2b^2\int\csc(c+dx)dx}{a}+\frac{x(2a^2-3b^2)}{b}}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 1083

$$\frac{\frac{8(a^2-b^2)^2\int\frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd}+\frac{2b^2\int\csc(c+dx)dx}{a}+\frac{x(2a^2-3b^2)}{b}}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 217

$$\frac{\frac{2b^2\int\csc(c+dx)dx}{a}-\frac{4(a^2-b^2)^{3/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{abd}+\frac{x(2a^2-3b^2)}{b}}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$\Big\downarrow$ 4257

$$\frac{-\frac{4(a^2-b^2)^{3/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{abd}+\frac{x(2a^2-3b^2)}{b}-\frac{2b^2\text{arctanh}(\cos(c+dx))}{ad}}{2b^2}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

input
```
Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

output
```
(((2*a^2 - 3*b^2)*x)/b - (4*(a^2 - b^2)^(3/2)*ArcTan[(2*b + 2*a*Tan[(c + d
*x)/2])/(2*Sqrt[a^2 - b^2])])/(a*b*d) - (2*b^2*ArcTanh[Cos[c + d*x]])/(a*d
))/(2*b^2) + (a*Cos[c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3374
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a +
b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x] - Simp[1/(b^2*(m + n + 3
)*(m + n + 4))   Int[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n
+ 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n +
 2)*(n + 3) - b^2*(m + n + 3)*(m + n + 5))*Sin[e + f*x]^2, x], x], x]) /; F
reeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Integ
ersQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
 n + 4, 0]
```

rule 3536
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d))   Int[1/(a + b*Sin[e + f*x]), x], x] - Simp[(c^2*C - B*c*d
 + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a
, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{\frac{\left(-2a^4+4a^2b^2-2b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\,b^3\sqrt{a^2-b^2}}+\frac{\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3b^2}{2}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2ab-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{2}+ab\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(2a^2-3b^2\right)}{b^3}}{d}$
default	$\frac{\frac{\left(-2a^4+4a^2b^2-2b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\,b^3\sqrt{a^2-b^2}}+\frac{\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3b^2}{2}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2ab-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{2}+ab\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(2a^2-3b^2\right)}{b^3}}{d}$
risch	$\frac{x\,a^2}{b^3}-\frac{3x}{2b}+\frac{a\,e^{i(dx+c)}}{2b^2d}+\frac{a\,e^{-i(dx+c)}}{2b^2d}+\frac{i\sqrt{a^2-b^2}\,a\ln\left(e^{i(dx+c)}-\frac{i\left(-a+\sqrt{a^2-b^2}\right)}{b}\right)}{d\,b^3}-\frac{i\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}-\right.}{dba}$

input
```
int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-2*a^4+4*a^2*b^2-2*b^4)/a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/b^3*((1/2*tan(1/2*d*x+1/2*c)^3*b^2+ta
n(1/2*d*x+1/2*c)^2*a*b-1/2*tan(1/2*d*x+1/2*c)*b^2+a*b)/(1+tan(1/2*d*x+1/2*
c)^2)^2+1/2*(2*a^2-3*b^2)*arctan(tan(1/2*d*x+1/2*c)))+1/a*ln(tan(1/2*d*x+1
/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.82

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \left[-\frac{ab^2\cos(dx+c)\sin(dx+c)-2\,a^2b\cos(dx+c)+b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b^3\log\left(-\frac{1}{2}\cos(dx+\right.}{}\right.$$

$$-\frac{ab^2\cos(dx+c)\sin(dx+c)-2\,a^2b\cos(dx+c)+b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b^3\log\left(-\frac{1}{2}\cos(dx+c\right.}{2\,ab^3d}$$

input
```
integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log(1/
2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*b^
2)*d*x - (-a^2 + b^2)^(3/2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*s
qrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(
a*b^3*d), -1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b
^3*log(1/2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3
 - 3*a*b^2)*d*x - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a
^2 - b^2)*cos(d*x + c))))/(a*b^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{(2\,a^2-3\,b^2)(dx+c)}{b^3} - \frac{4\,(a^4-2\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}ab^3} + \frac{2\left(b\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{2\,d}$$

input
```
integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a + (2*a^2 - 3*b^2)*(d*x + c)/b^3 -
4*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arcta
n((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b^3) +
 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
+ 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 42.72 (sec) , antiderivative size = 1320, normalized size of antiderivative = 10.65

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x)),x)
```

output
```
log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - sin(2*c + 2*d*x)/(4*b*d
) - (3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2
*cos(c/2 + (d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2)
+ 3*a*b^2*sin(c/2 + (d*x)/2))))/(b*d) + (a*cos(c + d*x))/(b^2*d) + (2*a^2*
atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*cos(c/
2 + (d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3*a*b
^2*sin(c/2 + (d*x)/2))))/(b^3*d) + (atan((b^6*sin(c/2 + (d*x)/2)*(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*64i - a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6
- 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*16i - a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3
*a^2*b^4 + 3*a^4*b^2)^(3/2)*16i - a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 -
3*a^2*b^4 + 3*a^4*b^2)^(3/2)*42i + a^3*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*66i - a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6
- 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*176i + a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*178i - a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 -
a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*81i - a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 -
 a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*116i + a^4*b^2*sin(c/2 + (d*x)/2)*(b^6
 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*72i + a^2*b^10*sin(c/2 + (d*x)/2)*(b
^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*148i - a^4*b^8*sin(c/2 + (d*x)/2)*
(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*460i + a^6*b^6*sin(c/2 + (d*x)/2
)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*577i - a^8*b^4*sin(c/2 + (d...
```

Reduce [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2+4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)b^2-\cos(dx+c)\sin(dx+c)\,a\,b^2}{2a\,b^3d}$$

input
```
int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*a
**2 + 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))
*b**2 - cos(c + d*x)*sin(c + d*x)*a*b**2 + 2*cos(c + d*x)*a**2*b + 2*log(t
an((c + d*x)/2))*b**3 + 2*a**3*c + 2*a**3*d*x - 3*a*b**2*c - 3*a*b**2*d*x)
/(2*a*b**3*d)
```

3.337 $\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 852

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i(a^2-b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx)^3\csc(c+dx)}{ad} - \frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd} - \frac{(a^2-b^2)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd} - \frac{b(e+fx)^3\log\left(1-e^{2i(c+dx)}\right)}{a^2d} + \frac{6if^2(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^3} - \frac{6if^2(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^3} + \frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2} + \frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd^2} + \frac{3ibf(e+fx)^2\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2} - \frac{6f^3\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^4} + \frac{6f^3\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^4} - \frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^3} - \frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd^3} - \frac{3bf^2(e+fx)\operatorname{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2a^2d^3} - \frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^4} - \frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd^4} - \frac{3ibf^3\operatorname{PolyLog}\left(4,e^{2i(c+dx)}\right)}{4a^2d^4}$$

output
```
6*I*f^2*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a/d^3+3/2*I*b*f*(f*x+e)^2*polyl
og(2,exp(2*I*(d*x+c)))/a^2/d^2-6*f*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d^2
-(f*x+e)^3*csc(d*x+c)/a/d-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/a^2/b/d-(a^2-b^2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a
^2-b^2)^(1/2)))/a^2/b/d-b*(f*x+e)^3*ln(1-exp(2*I*(d*x+c)))/a^2/d+3*I*(a^2-
b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d
^2+1/4*I*(a^2-b^2)*(f*x+e)^4/a^2/b/f-3/4*I*b*f^3*polylog(4,exp(2*I*(d*x+c)
))/a^2/d^4+3*I*(a^2-b^2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/a^2/b/d^2-6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/a^2/b/d^4-6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4+6*f^3*po
lylog(3,exp(I*(d*x+c)))/a/d^4-6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,I*b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d^3-6*(a^2-b^2)*f^2*(f*x+e)*polylog(3,
I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^3-3/2*b*f^2*(f*x+e)*polylo
g(3,exp(2*I*(d*x+c)))/a^2/d^3-6*I*(a^2-b^2)*f^3*polylog(4,I*b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^4+1/4*I*b*(f*x+e)^4/a^2/f-6*I*f^2*(f*x+e)*
polylog(2,exp(I*(d*x+c)))/a/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3039 vs. $2(852) = 1704$.

Time = 14.87 (sec) , antiderivative size = 3039, normalized size of antiderivative = 3.57

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x
]
```

output
```
-1/2*(((-2*I)*e^2*(b*d*e - 3*a*f)*x)/d - ((2*I)*e^2*(b*d*e + 3*a*f)*x)/d -
 (I*b*(e + f*x)^4)/((-1 + E^((2*I)*c))*f) + (6*e*f*(b*d*e - 2*a*f)*x*Log[1
 - E^((-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)*(c
+ d*x))])/d^2 + (2*b*f^3*x^3*Log[1 - E^((-I)*(c + d*x))])/d + (6*e*f*(b*d*
e + 2*a*f)*x*Log[1 + E^((-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e + a*f)*x^2*L
og[1 + E^((-I)*(c + d*x))])/d^2 + (2*b*f^3*x^3*Log[1 + E^((-I)*(c + d*x))]
)/d + (2*e^2*(b*d*e - 3*a*f)*Log[1 - E^(I*(c + d*x))])/d^2 + (2*e^2*(b*d*e
 + 3*a*f)*Log[1 + E^(I*(c + d*x))])/d^2 + ((6*I)*e*f*(b*d*e + 2*a*f)*PolyL
og[2, -E^((-I)*(c + d*x))])/d^3 + ((12*I)*f^2*(b*d*e + a*f)*x*PolyLog[2, -
E^((-I)*(c + d*x))])/d^3 + ((6*I)*b*f^3*x^2*PolyLog[2, -E^((-I)*(c + d*x))
])/d^2 + ((6*I)*e*f*(b*d*e - 2*a*f)*PolyLog[2, E^((-I)*(c + d*x))])/d^3 +
((12*I)*f^2*(b*d*e - a*f)*x*PolyLog[2, E^((-I)*(c + d*x))])/d^3 + ((6*I)*b
*f^3*x^2*PolyLog[2, E^((-I)*(c + d*x))])/d^2 + (12*f^2*(b*d*e + a*f)*PolyL
og[3, -E^((-I)*(c + d*x))])/d^4 + (12*b*f^3*x*PolyLog[3, -E^((-I)*(c + d*x
))])/d^3 + (12*f^2*(b*d*e - a*f)*PolyLog[3, E^((-I)*(c + d*x))])/d^4 + (12
*b*f^3*x*PolyLog[3, E^((-I)*(c + d*x))])/d^3 - ((12*I)*b*f^3*PolyLog[4, -E
^((-I)*(c + d*x))])/d^4 - ((12*I)*b*f^3*PolyLog[4, E^((-I)*(c + d*x))])/d^
4)/a^2 + ((a^2 - b^2)*((4*I)*d^4*e^3*E^((2*I)*c)*x + (6*I)*d^4*e^2*E^((2*I
)*c)*f*x^2 + (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 + I*d^4*E^((2*I)*c)*f^3*x^4 +
 (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x)...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 5054$$

$$\frac{\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{\int (e+fx)^3\cot(c+dx)\csc(c+dx)dx-\int (e+fx)^3\cos(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)^3\cot(c+dx)\csc(c+dx)dx - \int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{3f\int -(e+fx)^2\sin(c+dx)dx}{d} + \int (e+fx)^3\cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^3\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d} + \int (e+fx)^3\cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^3\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d} + \int (e+fx)^3\cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^3\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{3f\left(\frac{2f\int (e+fx)\cos(c+dx)dx}{d} - \frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d} + \int (e+fx)^3\cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^3\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\left(\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d} - \frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d} + \int (e+fx)^3\cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^3\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{3f\left(\frac{2f\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx-\frac{(e+fx)^3\sin(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx-\frac{(e+fx)^3\sin(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx-\frac{(e+fx)^3\sin(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3118

$$\frac{\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\sin(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4910

$$\frac{\frac{3f\int(e+fx)^2\csc(c+dx)dx}{d}+\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{(e+fx)^3\csc(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\int (e+fx)^2\csc(c+dx)dx}{d}+\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{(e+fx)^3\csc(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4671

$$-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{2f\int (e+fx)\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{2f\int (e+fx)\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)^2\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d}+\frac{3f\left(\frac{2f\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}\cdots\right.}{d}\cdots}{a}$$

$\downarrow$ 3011

$$-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{if\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)dx}{d}\right)}{d}-\frac{2(e+fx)^2\cdots}{}\right.}{d}\cdots}{a}$$

$\downarrow$ 2720

$$-\frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2\cdots\right.}{d^2}\right.}{d}\cdots\right.}{d}\cdots$$

$\downarrow$ 5054

$$-\frac{b\left(\frac{\int (e+fx)^3\cos^2(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\text{PolyLog}\left(2\cdots\right.}{d^2}\right.}{d}\cdots\right.}{d}\cdots$$

$\downarrow$ 4908

$$-\frac{b\left(\frac{\int (e+fx)^3\cot(c+dx)dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{\int -(e+fx)^3\tan\left(c+dx+\frac{\pi}{2}\right)dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{-\int (e+fx)^3\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$\downarrow$ 4202

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\,\text{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)^3}{1+e^{i(2c+2dx+\pi)}}dx-\int (e+fx)^3\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^4}{4f}}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\int(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int(e+fx)^3\cos(c+dx)\sin(c+dx)dx}{a}\right.}{a}$$

$\downarrow$ 3011

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c\right.}}{2d}\right.}{a}\right.}{a}$$

$\downarrow$ 4904

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c\right.}}{2d}\right.}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{d}\right)}{2d}-\frac{i(e+fx)^3\log\left(1+e^{i(2c}\right.}{2d}\right.}{a}\right.}{a}$$

$\downarrow$ 3792

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2}{}\right.}{}\right.}{}\right.}{a}$$

$\downarrow$ 17

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sin^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyL}}{}\right.}{}\right.}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$$b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sin(c+dx)^2dx}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+2i\left(\frac{3if\left(\frac{i(e+fx)^2\operatorname{PolyL}\right.}{}\right.}{a}\right.$$

$$\Big\downarrow \; 3115$$

$$\frac{3f\left(\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}\right)}{d}-\frac{2f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{d}-\frac{f\int e^{-i(c+dx)}\operatorname{PolyLog}\left(2\right.}{d^2}\right.}{d}\right.}{d}$$

$$b\left(-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sin^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+2i\left(\frac{3if\left(\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos(dx+c) \cot(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3903 vs. $2(761) = 1522$.

Time = 0.42 (sec) , antiderivative size = 3903, normalized size of antiderivative = 4.58

$$\int \frac{(e+fx)^3 \cos(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \cos(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3 \cos{(c+dx)} \cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \cos(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^3 \cos(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3 \cos{(dx+c)} \cot{(dx+c)}^2}{b\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((f*x + e)^3*cos(d*x + c)*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*cot(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.338 $\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 616

$$\begin{aligned}
&\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx\\
&=\frac{ib(e+fx)^3}{3a^2f}+\frac{i(a^2-b^2)(e+fx)^3}{3a^2bf}-\frac{4f(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{ad^2}\\
&-\frac{(e+fx)^2\csc(c+dx)}{ad}-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd}\\
&-\frac{(a^2-b^2)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd}-\frac{b(e+fx)^2\log\left(1-e^{2i(c+dx)}\right)}{a^2d}\\
&+\frac{2if^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^3}-\frac{2if^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^3}\\
&+\frac{2i(a^2-b^2)f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}\\
&+\frac{2i(a^2-b^2)f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd^2}\\
&+\frac{ibf(e+fx)\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{a^2d^2}-\frac{2(a^2-b^2)f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^3}\\
&-\frac{2(a^2-b^2)f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd^3}-\frac{bf^2\operatorname{PolyLog}\left(3,e^{2i(c+dx)}\right)}{2a^2d^3}
\end{aligned}$$

output
```
1/3*I*b*(f*x+e)^3/a^2/f+1/3*I*(a^2-b^2)*(f*x+e)^3/a^2/b/f-4*f*(f*x+e)*arct
anh(exp(I*(d*x+c)))/a/d^2-(f*x+e)^2*csc(d*x+c)/a/d-(a^2-b^2)*(f*x+e)^2*ln(
1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d-(a^2-b^2)*(f*x+e)^2*ln(1
-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d-b*(f*x+e)^2*ln(1-exp(2*I*
(d*x+c)))/a^2/d+2*I*f^2*polylog(2,-exp(I*(d*x+c)))/a/d^3-2*I*f^2*polylog(2
,exp(I*(d*x+c)))/a/d^3+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c)
)/(a-(a^2-b^2)^(1/2)))/a^2/b/d^2+2*I*(a^2-b^2)*f*(f*x+e)*polylog(2,I*b*exp
(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^2+I*b*f*(f*x+e)*polylog(2,exp(2*I
*(d*x+c)))/a^2/d^2-2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^
2)^(1/2)))/a^2/b/d^3-2*(a^2-b^2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-
b^2)^(1/2)))/a^2/b/d^3-1/2*b*f^2*polylog(3,exp(2*I*(d*x+c)))/a^2/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1752 vs. $2(616) = 1232$.

Time = 12.27 (sec) , antiderivative size = 1752, normalized size of antiderivative = 2.84

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x
]
```

output
```
-1/3*(((-3*I)*e*(b*d*e - 2*a*f)*x)/d - ((3*I)*e*(b*d*e + 2*a*f)*x)/d - ((2
*I)*b*(e + f*x)^3)/((-1 + E^((2*I)*c))*f) + (6*f*(b*d*e - a*f)*x*Log[1 - E
^((-I)*(c + d*x))])/d^2 + (3*b*f^2*x^2*Log[1 - E^((-I)*(c + d*x))])/d + (6
*f*(b*d*e + a*f)*x*Log[1 + E^((-I)*(c + d*x))])/d^2 + (3*b*f^2*x^2*Log[1 +
 E^((-I)*(c + d*x))])/d + (3*e*(b*d*e - 2*a*f)*Log[1 - E^(I*(c + d*x))])/d
^2 + (3*e*(b*d*e + 2*a*f)*Log[1 + E^(I*(c + d*x))])/d^2 + ((6*I)*f*(b*d*e
+ a*f)*PolyLog[2, -E^((-I)*(c + d*x))])/d^3 + ((6*I)*b*f^2*x*PolyLog[2, -E
^((-I)*(c + d*x))])/d^2 + ((6*I)*f*(b*d*e - a*f)*PolyLog[2, E^((-I)*(c + d
*x))])/d^3 + ((6*I)*b*f^2*x*PolyLog[2, E^((-I)*(c + d*x))])/d^2 + (6*b*f^2
*PolyLog[3, -E^((-I)*(c + d*x))])/d^3 + (6*b*f^2*PolyLog[3, E^((-I)*(c + d
*x))])/d^3)/a^2 + ((a^2 - b^2)*((6*I)*d^3*e^2*E^((2*I)*c)*x + (6*I)*d^3*e*
E^((2*I)*c)*f*x^2 + (2*I)*d^3*E^((2*I)*c)*f^2*x^3 + 3*d^2*e^2*Log[b - (2*I
)*a*E^(I*(c + d*x)) - b*E^((2*I)*(c + d*x))] - 3*d^2*e^2*E^((2*I)*c)*Log[b
 - (2*I)*a*E^(I*(c + d*x)) - b*E^((2*I)*(c + d*x))] + 6*d^2*e*f*x*Log[1 +
(b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*
d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-
a^2 + b^2)*E^((2*I)*c)])] + 3*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I
*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 3*d^2*E^((2*I)*c)*f^2*x^2*
Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)
])] + 6*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx-\int (e+fx)^2\cos(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)^2 \cot(c+dx)\csc(c+dx)dx - \int (e+fx)^2 \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{-\frac{2f\int -((e+fx)\sin(c+dx))dx}{d} + \int (e+fx)^2 \cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^2\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2f\int (e+fx)\sin(c+dx)dx}{d} + \int (e+fx)^2 \cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^2\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2f\int (e+fx)\sin(c+dx)dx}{d} + \int (e+fx)^2 \cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^2\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d} - \frac{(e+fx)\cos(c+dx)}{d}\right)}{d} + \int (e+fx)^2 \cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^2\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2f\left(\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d} - \frac{(e+fx)\cos(c+dx)}{d}\right)}{d} + \int (e+fx)^2 \cot(c+dx)\csc(c+dx)dx - \frac{(e+fx)^2\sin(c+dx)}{d}}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3117$$

$$\frac{\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4910}$$

$$\frac{\frac{2f\int (e+fx)\csc(c+dx)dx}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{(e+fx)^2\csc(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f\int (e+fx)\csc(c+dx)dx}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{(e+fx)^2\csc(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{2f\left(-\frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int \log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c+}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{2f\left(\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+f}{}\right.}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$$\Big\downarrow \text{5054}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$\downarrow$ 4908

$$-\frac{b\left(\frac{\int (e+fx)^2\cot(c+dx)dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{\int -(e+fx)^2\tan\left(c+dx+\frac{\pi}{2}\right)dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{-\int (e+fx)^2\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$\downarrow$ 4202

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)^2}{1+e^{i(2c+2dx+\pi)}}dx-\int (e+fx)^2\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^3}{3f}}{a}\right)}{a}$$

$\downarrow$ 2620

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c \cdots}{d}}{a} \cdots$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx- \cdots}{a}\right.}{a}$$

$\downarrow$ 3011

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c \cdots}{d}}{a} \cdots$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right) \cdots}{2d}\right.}{a}\right.}{a}$$

$\downarrow$ 2720

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c \cdots}{d}}{a} \cdots$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(\cdots}{\cdots}\right.}{a}\right.}{a}$$

$\downarrow$ 4904

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c\cdots}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c\cdots}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{a}$$

$\downarrow$ 3791

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c\cdots}{d}\cdots}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 17

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$$b\left(-\frac{b\int\frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(}{}\right.}{}\right.$$

$$a$$

$\downarrow$ 5036

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^2\cos(c+dx)dx}{b^2}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2dx+}\right.}{2d}\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)\,\text{PolyLog}\left(2,-e^{i(2c+2}\right.}{2d}\right.}{}\right.}{}\right.$$

$\downarrow$ 3777

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{2f\int-((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)}{b^2}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e-}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\sin(c}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)^2\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(\frac{if\left(\frac{i(e+fx)}{}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos (dx+c) \cot (dx+c)^2}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2529 vs. $2(549) = 1098$.

Time = 0.32 (sec) , antiderivative size = 2529, normalized size of antiderivative = 4.11

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*e*f*x + 2*a*b*d^2*e^2 + 2*b^2*f^2*poly
log(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3,
cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, -cos(d*
x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, -cos(d*x + c)
 - I*sin(d*x + c))*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d
*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x
+ c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x +
c) + a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(-I*a*cos(d*x + c)
 + a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b)*sin(d*x + c) + 2*(-I*(a^2 - b^2)*d*f^2*x - I*(a^2 - b^2)*d*e*f)*d
ilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(-I*(a^2 - b^2)*d*
f^2*x - I*(a^2 - b^2)*d*e*f)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d
*x + c) + 2*(I*(a^2 - b^2)*d*f^2*x + I*(a^2 - b^2)*d*e*f)*dilog((-I*a*cos(
d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*(I*(a^2 - b^2)*d*f^2*x + I*(a...
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2\cos{(c+dx)}\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos{(dx+c)}\cot{(dx+c)}^2}{b\sin{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*cot(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.339 $\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 386

$$\begin{aligned}\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{ib(e+fx)^2}{2a^2f}+\frac{i(a^2-b^2)(e+fx)^2}{2a^2bf} \\ &\quad -\frac{f\operatorname{arctanh}(\cos(c+dx))}{ad^2} \\ &\quad -\frac{(e+fx)\csc(c+dx)}{ad} \\ &\quad -\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd} \\ &\quad -\frac{(a^2-b^2)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd} \\ &\quad -\frac{b(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{a^2d} \\ &\quad +\frac{i(a^2-b^2)f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2} \\ &\quad +\frac{i(a^2-b^2)f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2bd^2} \\ &\quad +\frac{ibf\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}\end{aligned}$$

output
```
1/2*I*b*(f*x+e)^2/a^2/f+1/2*I*(a^2-b^2)*(f*x+e)^2/a^2/b/f-f*arctanh(cos(d*
x+c))/a/d^2-(f*x+e)*csc(d*x+c)/a/d-(a^2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c
))/(a-(a^2-b^2)^(1/2)))/a^2/b/d-(a^2-b^2)*(f*x+e)*ln(1-I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/a^2/b/d-b*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a^2/d+I*(a^2
-b^2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b/d^2+I*(a^2
-b^2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b/d^2+1/2*I*
b*f*polylog(2,exp(2*I*(d*x+c)))/a^2/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1111 vs. $2(386) = 772$.

Time = 9.01 (sec) , antiderivative size = 1111, normalized size of antiderivative = 2.88

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d*x)*Cos[(c + d*
x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) - (b*e*Log[Sin[c + d*x]])/(a^2*d) + (b*
c*f*Log[Sin[c + d*x]])/(a^2*d^2) - (e*Log[1 + (b*Sin[c + d*x])/a])/(b*d) +
 (b*e*Log[1 + (b*Sin[c + d*x])/a])/(a^2*d) + (c*f*Log[1 + (b*Sin[c + d*x])
/a])/(b*d^2) - (b*c*f*Log[1 + (b*Sin[c + d*x])/a])/(a^2*d^2) + (f*Log[Tan[
(c + d*x)/2]])/(a*d^2) - (f*(((c + d*x)*Log[a + b*Sin[c + d*x]])/b - ((-1/
2*I)*(-c + Pi/2 - d*x)^2 + (4*I)*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]]*ArcTan[((
a - b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[a^2 - b^2]] + (-c + Pi/2 - d*x + 2*A
rcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 + ((a - Sqrt[a^2 - b^2])*E^(I*(-c +
Pi/2 - d*x)))/b] + (-c + Pi/2 - d*x - 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*L
og[1 + ((a + Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] - (-c + Pi/2 - d
*x)*Log[a + b*Sin[c + d*x]] - I*(PolyLog[2, ((-a - Sqrt[a^2 - b^2])*E^(I*(
-c + Pi/2 - d*x)))/b] + PolyLog[2, ((-a + Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2
 - d*x)))/b]))/b))/d^2 + (b^2*f*(((c + d*x)*Log[a + b*Sin[c + d*x]])/b - (
(-1/2*I)*(-c + Pi/2 - d*x)^2 + (4*I)*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]]*ArcTa
n[((a - b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[a^2 - b^2]] + (-c + Pi/2 - d*x +
 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 + ((a - Sqrt[a^2 - b^2])*E^(I*(-
c + Pi/2 - d*x)))/b] + (-c + Pi/2 - d*x - 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]
])*Log[1 + ((a + Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] - (-c + Pi/2
 - d*x)*Log[a + b*Sin[c + d*x]] - I*(PolyLog[2, ((-a - Sqrt[a^2 - b^2])...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx-\int (e+fx)\cos(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx-\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3777$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx-\frac{f\int -\sin(c+dx)dx}{d}-\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx+\frac{f\int \sin(c+dx)dx}{d}-\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx+\frac{f\int \sin(c+dx)dx}{d}-\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3118$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 4910$$

$$\frac{\frac{f\int \csc(c+dx)dx}{d}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{f\int \csc(c+dx)dx}{d}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 4257$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \ 5054$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 4908$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\cot(c+dx)dx-\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(\frac{\int-\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(\frac{-\int(e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 4202$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int\frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx-\int(e+fx)\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \ 2620$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int(e+fx)\cos(c+dx)\sin(c+dx)dx-\frac{i(e+fx)^2}{2f}}{a}\right.}{a}$$

$\downarrow$ 2715

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(\frac{f\int e^{-i(2c+2dx+\pi)}\log\left(1+e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\int(e+fx)\cos(c+dx)}{a}\right.}{a}$$

$\downarrow$ 2838

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\int(e+fx)\cos(c+dx)\sin(c+dx)dx+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{i(e+}{2}}{a}\right.}{a}$$

$\downarrow$ 4904

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{f\int\sin^2(c+dx)dx}{2d}+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{i(e+}{2}}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{f\int\sin(c+dx)^2dx}{2d}+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{i(e+}{2}}{a}\right.}{a}$$

$\downarrow$ 3115

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d}+2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin\cdots}{2\cdots}}{a}\cdots\right.}{a}$$

$\downarrow$ 24

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)-\frac{(e+fx)\sin^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+\cdots}{2d}\right.}{2d}\cdots}{a}\cdots\right.}{a}$$

$\downarrow$ 5036

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)\cos(c+dx)dx}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx\cdots}{\cdots}\right.\cdots}{\cdots}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e\cdots}{\cdots}\right.\cdots}{\cdots}\right.}{a}$$

$\downarrow$ 3777

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c\cdots}\right.}{4d^2}\cdots\right.}{\cdots}\right.}{a}$$

$\downarrow$ 25

$$\frac{-\frac{f\operatorname{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2}\right.}{4d^2}\right.}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{f\operatorname{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2}\right.}{4d^2}\right.}{}\right.}{a}$$

$\downarrow$ 3118

$$\frac{-\frac{f\operatorname{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\cos(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+}\right.}{4d^2}\right.}{}\right.}{a}$$

$\downarrow$ 4904

$$\frac{-\frac{f\operatorname{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int\sin^2(c+dx)dx}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c}\right.}{4d^2}\right.}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{\frac{\left(a^2-b^2\right)\int\frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{(e+fx)\sin^2(c+dx)}{2d}-\frac{f\int\sin(c+dx)^2dx}{2d}}{b}+\frac{a\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{b^2}\right)}{a}+\frac{2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c}\right.}{4d^2}\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1731 vs. $2(351) = 702$.

Time = 2.03 (sec) , antiderivative size = 1732, normalized size of antiderivative = 4.49

method	result	size
risch	Expression too large to display	1732

input
```
int((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d^2/a*f*ln(exp(I*(d*x+c))-1)-1/d^2/a*f*ln(exp(I*(d*x+c))+1)-2/b/d^2*f*c*
ln(exp(I*(d*x+c)))+I/b/d^2*f*c^2+b/d/a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp
(I*(d*x+c))-I*b)-b/d/a^2*e*ln(exp(I*(d*x+c))-1)-b/d/a^2*e*ln(exp(I*(d*x+c)
)+1)-I/b*e*x-1/b/d*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2/b/d
*e*ln(exp(I*(d*x+c)))+1/2*I/b*f*x^2+b^3/d/a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(
d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+b^3/d/a^2*f/(-a^2+b^
2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/
b/d*a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2
+b^2)^(1/2)))*x+1/b/d*a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/b/d^2*a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(
d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/b/d^2*a^2*f/(-a^2+
b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+
b^3/d^2/a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(
-a^2+b^2)^(1/2)))*c+b^3/d^2/a^2*f/(-a^2+b^2)*ln((I*a+exp(I*(d*x+c))*b-(-a^
2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-I*b^3/d^2/a^2*f/(-a^2+b^2)*dilog((
I*a+exp(I*(d*x+c))*b-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I*b^3/d^2/a
^2*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b
^2)^(1/2)))-I/b/d^2*a^2*f/(-a^2+b^2)*dilog((I*a+exp(I*(d*x+c))*b-(-a^2+b^2
)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/b/d^2*a^2*f/(-a^2+b^2)*dilog((I*a+exp(I
*(d*x+c))*b+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I*(f*x+e)*exp(I...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(343) = 686$.

Time = 0.30 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.68

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*(2*a*b*d*f*x - I*b^2*f*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
 c) + I*b^2*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + I*b^2*f*
dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - I*b^2*f*dilog(-cos(d*
x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*a*b*d*e - I*(a^2 - b^2)*f*dilog(
(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - I*(a^2 - b^2)*f*dilog((I*
a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^2 - b^2)*f*dilog((-I*a*
cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + I*(a^2 - b^2)*f*dilog((-I*a*co
s(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + ((a^2 - b^2)*d*e - (a^2 - b^2)*c
*f)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 + 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d
*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x
 + c) + ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*
sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + ((a^2 -
b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d*f*x + (a^
2 - b^2)*c*f)*log(-(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c)...
```

Sympy [F]

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)\cos{(c+dx)}\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)\cot(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((f*x + e)*cos(d*x + c)*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)*cot(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.340 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\csc(c+dx)}{ad} - \frac{b\log(\sin(c+dx))}{a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

output
```
-csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d-(1-b^2/a^2)*ln(a+b*sin(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-ab\csc(c+dx) - b^2\log(\sin(c+dx)) + (-a^2+b^2)\log(a+b\sin(c+dx))}{a^2bd}$$

input
```
Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-(a*b*Csc[c + d*x]) - b^2*Log[Sin[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sin[
c + d*x]])/(a^2*b*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{\sin(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\csc^2(c+dx)(b^2-b^2\sin^2(c+dx))}{a+b\sin(c+dx)}d(b\sin(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\csc^2(c+dx)(b^2-b^2\sin^2(c+dx))}{b^2(a+b\sin(c+dx))}d(b\sin(c+dx))}{bd}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\csc^2(c+dx)}{a}-\frac{b\csc(c+dx)}{a^2}+\frac{b^2-a^2}{a^2(a+b\sin(c+dx))}\right)d(b\sin(c+dx))}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2\log(b\sin(c+dx))}{a^2}-\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))-\frac{b\csc(c+dx)}{a}}{bd}$$

input
```
Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-((b*Csc[c + d*x])/a) - (b^2*Log[b*Sin[c + d*x]])/a^2 - (1 - b^2/a^2)*Log
[a + b*Sin[c + d*x]])/(b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\left(-a^2+b^2\right)\ln(a+b\sin(dx+c))}{a^2b}-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}}{d}$
default	$\frac{\frac{\left(-a^2+b^2\right)\ln(a+b\sin(dx+c))}{a^2b}-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}}{d}$
risch	$\frac{ix}{b}+\frac{2ic}{bd}-\frac{2ie^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{bd}+\frac{b\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(}}{}\right.}{a^2d}$

input
```
int(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-a^2+b^2)/a^2/b*ln(a+b*sin(d*x+c))-1/a/sin(d*x+c)-b/a^2*ln(sin(d*x+c
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{b^2\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)+ab}{a^2bd\sin\left(dx+c\right)}$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-(b^2*log(1/2*sin(d*x + c))*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x + c)
+ a)*sin(d*x + c) + a*b)/(a^2*b*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{b\log(\sin(dx+c))}{a^2} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{a^2b} + \frac{1}{a\sin(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*log(sin(d*x + c))/a^2 + (a^2 - b^2)*log(b*sin(d*x + c) + a)/(a^2*b) +
1/(a*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{b\log(|\sin(dx+c)|)}{a^2d} - \frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{a^2bd} - \frac{1}{ad\sin(dx+c)}$$

input
```
integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-b*log(abs(sin(d*x + c)))/(a^2*d) - (a^2 - b^2)*log(abs(b*sin(d*x + c) + a
))/(a^2*b*d) - 1/(a*d*sin(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 39.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,\left(\frac{b}{a^2}-\frac{1}{b}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{b\,d} - \frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}$$

input
```
int((cos(c + d*x)*cot(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
(log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(b/a^2 - 1/b))/d
 - tan(c/2 + (d*x)/2)/(2*a*d) - cot(c/2 + (d*x)/2)/(2*a*d) + log(tan(c/2 +
 (d*x)/2)^2 + 1)/(b*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.42

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,a^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)\sin(dx+c)\,a^2+\mathrm{lo}}{\sin(dx+c)}$$

input
```
int(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**2 - log(tan((c + d*x)/2)**2*
a + 2*tan((c + d*x)/2)*b + a)*sin(c + d*x)*a**2 + log(tan((c + d*x)/2)**2*
a + 2*tan((c + d*x)/2)*b + a)*sin(c + d*x)*b**2 - log(tan((c + d*x)/2))*si
n(c + d*x)*b**2 - a*b)/(sin(c + d*x)*a**2*b*d)
```

3.341 $\int \frac{(e+fx)^3 \cos^2(c+dx) \cot^2(c+dx)}{a+b \sin(c+dx)} dx$

Optimal result

Integrand size = 36, antiderivative size = 1144

$$\int \frac{(e+fx)^3 \cos^2(c+dx) \cot^2(c+dx)}{a+b \sin(c+dx)} dx = \text{Too large to display}$$

output
```
-6*(a^2-b^2)*f^3*sin(d*x+c)/a^2/b/d^4+3*b*f*(f*x+e)^2*sin(d*x+c)/a^2/d^2+6
*b*f^2*(f*x+e)*cos(d*x+c)/a^2/d^3-6*(a^2-b^2)^(3/2)*f^3*polylog(4,I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d^4+6*(a^2-b^2)^(3/2)*f^3*polylog(
4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^4+6*b*f^2*(f*x+e)*poly
log(3,-exp(I*(d*x+c)))/a^2/d^3+3/2*f^3*polylog(3,exp(2*I*(d*x+c)))/a/d^4-I
*(f*x+e)^3/a/d+3*(a^2-b^2)^(3/2)*f*(f*x+e)^2*polylog(2,I*b*exp(I*(d*x+c))/
(a+(a^2-b^2)^(1/2)))/a^2/b^2/d^2-3*(a^2-b^2)^(3/2)*f*(f*x+e)^2*polylog(2,I
*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^2-I*(a^2-b^2)^(3/2)*(f*x+
e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d+I*(a^2-b^2)^(3
/2)*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d-b*(f*
x+e)^3*cos(d*x+c)/a^2/d-(a^2-b^2)*(f*x+e)^3*cos(d*x+c)/a^2/b/d+3*(a^2-b^2)
*f*(f*x+e)^2*sin(d*x+c)/a^2/b/d^2+6*(a^2-b^2)*f^2*(f*x+e)*cos(d*x+c)/a^2/b
/d^3-3*I*b*f*(f*x+e)^2*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+2*b*(f*x+e)^3*ar
ctanh(exp(I*(d*x+c)))/a^2/d-6*I*(a^2-b^2)^(3/2)*f^2*(f*x+e)*polylog(3,I*b*
exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^3+6*I*(a^2-b^2)^(3/2)*f^2*(f
*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d^3-6*b*f^
2*(f*x+e)*polylog(3,exp(I*(d*x+c)))/a^2/d^3-6*I*b*f^3*polylog(4,exp(I*(d*x
+c)))/a^2/d^4-(f*x+e)^3*cot(d*x+c)/a/d+6*I*b*f^3*polylog(4,-exp(I*(d*x+c))
)/a^2/d^4+3*I*b*f*(f*x+e)^2*polylog(2,exp(I*(d*x+c)))/a^2/d^2-1/4*(f*x+e)^
4/a/f+3*f*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a/d^2-3*I*f^2*(f*x+e)*polylo...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3915 vs. 2(1144) = 2288.

Time = 9.84 (sec) , antiderivative size = 3915, normalized size of antiderivative = 3.42

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])
,x]
```

output
```
(I*d^3*e^2*(b*d*e - 3*a*f)*x - I*d^3*e^2*(b*d*e + 3*a*f)*x - ((2*I)*a*d^3*
(e + f*x)^3)/(-1 + E^((2*I)*c)) - 3*d^2*e*f*(b*d*e - 2*a*f)*x*Log[1 - E^((
-I)*(c + d*x))] - 3*d^2*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)*(c + d*x))]
- b*d^3*f^3*x^3*Log[1 - E^((-I)*(c + d*x))] + 3*d^2*e*f*(b*d*e + 2*a*f)*x*
Log[1 + E^((-I)*(c + d*x))] + 3*d^2*f^2*(b*d*e + a*f)*x^2*Log[1 + E^((-I)*
(c + d*x))] + b*d^3*f^3*x^3*Log[1 + E^((-I)*(c + d*x))] - d^2*e^2*(b*d*e -
 3*a*f)*Log[1 - E^(I*(c + d*x))] + d^2*e^2*(b*d*e + 3*a*f)*Log[1 + E^(I*(c
 + d*x))] + (3*I)*d*e*f*(b*d*e + 2*a*f)*PolyLog[2, -E^((-I)*(c + d*x))] +
(6*I)*d*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^((-I)*(c + d*x))] + (3*I)*b*d^2*
f^3*x^2*PolyLog[2, -E^((-I)*(c + d*x))] - (3*I)*d*e*f*(b*d*e - 2*a*f)*Poly
Log[2, E^((-I)*(c + d*x))] - (6*I)*d*f^2*(b*d*e - a*f)*x*PolyLog[2, E^((-I
)*(c + d*x))] - (3*I)*b*d^2*f^3*x^2*PolyLog[2, E^((-I)*(c + d*x))] + 6*f^2
*(b*d*e + a*f)*PolyLog[3, -E^((-I)*(c + d*x))] + 6*b*d*f^3*x*PolyLog[3, -E
^((-I)*(c + d*x))] + 6*f^2*(-(b*d*e) + a*f)*PolyLog[3, E^((-I)*(c + d*x))]
 - 6*b*d*f^3*x*PolyLog[3, E^((-I)*(c + d*x))] - (6*I)*b*f^3*PolyLog[4, -E^
((-I)*(c + d*x))] + (6*I)*b*f^3*PolyLog[4, E^((-I)*(c + d*x))])/(a^2*d^4)
+ (Sqrt[-(a^2 - b^2)^2]*(-2*Sqrt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*
(c + d*x)))/Sqrt[a^2 - b^2]] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^
(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*
x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 5054}$$

$$\frac{\int (e+fx)^3\cos^2(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4908}$$

$$\frac{\int (e+fx)^3\cot^2(c+dx)dx-\int (e+fx)^3\cos^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^3 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \int (e+fx)^3 \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3792

$$\frac{\frac{3f^2\int (e+fx)\cos^2(c+dx)dx}{2d^2} + \int (e+fx)^3 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{1}{2}\int (e+fx)^3 dx - \frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2} - \frac{(e+fx)^3\sin(c+}{2}}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 17

$$\frac{\frac{3f^2\int (e+fx)\cos^2(c+dx)dx}{2d^2} + \int (e+fx)^3 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2} - \frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{(e+f}{8}}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{3f^2\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^2 dx}{2d^2} + \int (e+fx)^3 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2} - \frac{(e+fx)^3\sin(c+dx)\cos(c+dx)}{2d} - (}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3791

$$\frac{\frac{3f^2\left(\frac{1}{2}\int (e+fx)dx + \frac{f\cos^2(c+dx)}{4d^2} + \frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2} + \int (e+fx)^3 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2} - (e+}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 17

$$\frac{\int (e+fx)^3 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx + \frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2} + \frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d} + \frac{(e+fx)^2}{4f}\right)}{2d^2} - \frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2} - \frac{(e+fx)^3}{}}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4203$$

$$\frac{-\frac{3f\int -(e+fx)^2\cot(c+dx)dx}{d}-\int (e+fx)^3dx+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}-}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{3f\int -(e+fx)^2\cot(c+dx)dx}{d}+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}-\frac{(e+fx)^3\cot(c+dx)}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3f\int (e+fx)^2\cot(c+dx)dx}{d}+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}-\frac{(e+fx)^3\cot(c+dx)}{d}-}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\int -(e+fx)^2\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}-\frac{(e+fx)^3\cot(c+d}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{3f\int (e+fx)^2\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}-\frac{(e+fx)^3\cot(}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4202$$

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$
$$\frac{-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\int\frac{e^{i(2c+2dx+\pi)}(e+fx)^2}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cos^2(c+dx)}{4d^2}-\frac{(e+}{}}{a}$$

$\downarrow$ 2620

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$
$$\frac{-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\int(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}+\frac{3f^2\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(}{}\right.}{2d^2}}{a}$$

$\downarrow$ 3011

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$
$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{if\int\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)dx}{2d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}+\frac{3f^2\left(\frac{f}{}\right.}{}$$

$\downarrow$ 2720

$$-\frac{b\int\frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$
$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c}\right.}{2d}\right.\right.}{d}$$

$\downarrow$ 5054

$$-\frac{b\left(\frac{\int(e+fx)^3\cos^3(c+dx)\cot(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$
$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c}\right.}{2d}\right.\right.}{d}$$

$\downarrow$ 4908

$$-\frac{b\left(\frac{\int(e+fx)^3\cos(c+dx)\cot(c+dx)dx-\int(e+fx)^3\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\right.\right.}{d}$$

$\downarrow$ 4905

$$-\frac{b\left(\frac{-\frac{f\int(e+fx)^2\cos^3(c+dx)dx}{d}+\int(e+fx)^3\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^3\cos^3(c+dx)}{3d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\right.\right.}{d}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{-\frac{f\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{d}+\int(e+fx)^3\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^3\cos^3(c+dx)}{3d}}{a}-\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+$$

$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\right.\right.}{d}$$

$\downarrow$ 3792

$$-\frac{b\left(\frac{-\frac{f\left(-\frac{2f^2\int\cos^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cos(c+dx)dx+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}+\int(e+fx)^3\cos(c+dx)\cot(c+dx)dx\cdots}{a}\right.}{a}$$

$$-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\right.\right.}{d}$$

$\downarrow$ 3042

$$-\frac{-\frac{b\left(\frac{-\frac{f\left(-\frac{2f^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}}{a}+\int (e+fx)^3\cos(c+dx)\cot(\cdots\right)}{a}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\cdots\right)\right)}{d}}{\cdots}$$

$\downarrow$ 3113

$$-\frac{-\frac{b\left(\frac{-\frac{f\left(\frac{2f^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{9d^3}+\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}}{a}+\int (e+fx)^3\cos(c\cdots\right)}{a}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\cdots\right)\right)}{d}}{\cdots}$$

$\downarrow$ 2009

$$-\frac{-\frac{b\left(\frac{-\frac{f\left(\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}}{a}+\int (e+fx)^3\cos(c+dx)\cdots\right)}{a}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\cdots}\right)}{2d}\cdots\right)\right)}{d}}{\cdots}$$

$\downarrow$ 3777

$$-\frac{b\left(-\frac{\frac{f\left(\frac{2}{3}\left(\frac{2f\int -((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}}{a}\ldots\right)}{a}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\ldots}\right)}{2d}\ldots\right)\right)}{d}$$

$\downarrow$ 25

$$-\frac{b\left(-\frac{\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}}{a}+\int\ldots\right)}{a}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\ldots}\right)}{2d}\ldots\right)\right)}{d}$$

$\downarrow$ 3042

$$-\frac{b\left(-\frac{\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{d}}{a}+\int\ldots\right)}{a}-\frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c\ldots}\right)}{2d}\ldots\right)\right)}{d}$$

$\downarrow$ 3777

$$-\frac{b\left(\frac{-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sin(c+dx}{3d}}{d}}{a}}{} - \frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c}\right.}{2d}\right.}{d}\;a$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(\frac{-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)+\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}+\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+(e+fx)^2\sin(}{d}}{a}}{} - \frac{3f\left(\frac{i(e+fx)^3}{3f}-2i\left(\frac{if\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{2d}-\frac{f\int e^{-i(2c+2dx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}\right)}{d}-\frac{i(e+fx)^2\log\left(1+e^{i(2c}\right.}{2d}\right.}{d}\;a$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos (dx+c)^2 \cot (dx+c)^2}{a+b\sin (dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4706 vs. $2(1029) = 2058$.

Time = 0.55 (sec) , antiderivative size = 4706, normalized size of antiderivative = 4.11

$$\int \frac{(e+fx)^3 \cos^2(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^3\cos^2{(c+dx)}\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)^2\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.342 $\int \frac{(e+fx)^2 \cos^2(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 840

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= -\frac{i(e+fx)^2}{ad}-\frac{(e+fx)^3}{3af}-\frac{(a^2-b^2)\,(e+fx)^3}{3ab^2f}$$
$$+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{a^2d}+\frac{2bf^2\cos(c+dx)}{a^2d^3}+\frac{2(a^2-b^2)\,f^2\cos(c+dx)}{a^2bd^3}$$
$$-\frac{b(e+fx)^2\cos(c+dx)}{a^2d}-\frac{(a^2-b^2)\,(e+fx)^2\cos(c+dx)}{a^2bd}$$
$$-\frac{(e+fx)^2\cot(c+dx)}{ad}-\frac{i(a^2-b^2)^{3/2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d}$$
$$+\frac{i(a^2-b^2)^{3/2}\,(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{2f(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{ad^2}$$
$$-\frac{2ibf(e+fx)\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{2ibf(e+fx)\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}$$
$$-\frac{2(a^2-b^2)^{3/2}\,f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d^2}$$
$$+\frac{2(a^2-b^2)^{3/2}\,f(e+fx)\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^2d^2}-\frac{if^2\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}$$
$$+\frac{2bf^2\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)}{a^2d^3}-\frac{2bf^2\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)}{a^2d^3}$$
$$-\frac{2i(a^2-b^2)^{3/2}\,f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d^3}+\frac{2i(a^2-b^2)^{3/2}\,f^2\operatorname{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^2d^3}$$
$$+\frac{2bf(e+fx)\sin(c+dx)}{a^2d^2}+\frac{2(a^2-b^2)\,f(e+fx)\sin(c+dx)}{a^2bd^2}$$

output
```
-I*f^2*polylog(2,exp(2*I*(d*x+c)))/a/d^3-1/3*(f*x+e)^3/a/f-1/3*(a^2-b^2)*(
f*x+e)^3/a/b^2/f+2*b*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a^2/d+2*b*f^2*cos(d
*x+c)/a^2/d^3+2*(a^2-b^2)*f^2*cos(d*x+c)/a^2/b/d^3-b*(f*x+e)^2*cos(d*x+c)/
a^2/d-(a^2-b^2)*(f*x+e)^2*cos(d*x+c)/a^2/b/d-(f*x+e)^2*cot(d*x+c)/a/d-I*(a
^2-b^2)^(3/2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b
^2/d-2*I*(a^2-b^2)^(3/2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/
2)))/a^2/b^2/d^3+2*f*(f*x+e)*ln(1-exp(2*I*(d*x+c)))/a/d^2-I*(f*x+e)^2/a/d-
2*I*b*f*(f*x+e)*polylog(2,-exp(I*(d*x+c)))/a^2/d^2-2*(a^2-b^2)^(3/2)*f*(f*
x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d^2+2*(a^2-
b^2)^(3/2)*f*(f*x+e)*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2
/b^2/d^2+2*I*b*f*(f*x+e)*polylog(2,exp(I*(d*x+c)))/a^2/d^2+2*b*f^2*polylog
(3,-exp(I*(d*x+c)))/a^2/d^3-2*b*f^2*polylog(3,exp(I*(d*x+c)))/a^2/d^3+I*(a
^2-b^2)^(3/2)*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b
^2/d+2*I*(a^2-b^2)^(3/2)*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/
2)))/a^2/b^2/d^3+2*b*f*(f*x+e)*sin(d*x+c)/a^2/d^2+2*(a^2-b^2)*f*(f*x+e)*si
n(d*x+c)/a^2/b/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 9.11 (sec) , antiderivative size = 973, normalized size of antiderivative = 1.16

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{12\left(id^2e(bde-2af)x-id^2e(bde+2af)x-\frac{2iad^2(e+fx)^2}{-1+e^{2ic}}-2df(bde-af)x\log\left(1-e^{-i(c+dx)}\right)-bd^2f^2x^2\right.}{}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])
,x]
```

output
```
(12*(I*d^2*e*(b*d*e - 2*a*f)*x - I*d^2*e*(b*d*e + 2*a*f)*x - ((2*I)*a*d^2*
(e + f*x)^2)/(-1 + E^((2*I)*c)) - 2*d*f*(b*d*e - a*f)*x*Log[1 - E^((-I)*(c
 + d*x))] - b*d^2*f^2*x^2*Log[1 - E^((-I)*(c + d*x))] + 2*d*f*(b*d*e + a*f
)*x*Log[1 + E^((-I)*(c + d*x))] + b*d^2*f^2*x^2*Log[1 + E^((-I)*(c + d*x))
] - d*e*(b*d*e - 2*a*f)*Log[1 - E^(I*(c + d*x))] + d*e*(b*d*e + 2*a*f)*Log
[1 + E^(I*(c + d*x))] + (2*I)*f*(b*d*e + a*f)*PolyLog[2, -E^((-I)*(c + d*x
))] + (2*I)*b*d*f^2*x*PolyLog[2, -E^((-I)*(c + d*x))] + (2*I)*f*(-(b*d*e)
+ a*f)*PolyLog[2, E^((-I)*(c + d*x))] - (2*I)*b*d*f^2*x*PolyLog[2, E^((-I)
*(c + d*x))] + 2*b*f^2*PolyLog[3, -E^((-I)*(c + d*x))] - 2*b*f^2*PolyLog[3
, E^((-I)*(c + d*x))]) - ((12*I)*Sqrt[-(a^2 - b^2)^2]*(-2*Sqrt[a^2 - b^2]*
d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])]
+ 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a +
Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*
(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b
*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)
))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*
(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3
, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/b^2 + (a*Csc[c]*Csc[
c + d*x]*(-2*a^2*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Cos[d*x] + 2*a^2*d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2)*Cos[2*c + d*x] + 3*b*(-(a*(-2*f^2 + d^2*(e ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5054}$$

$$\frac{\int (e+fx)^2\cos^2(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4908}$$

$$\frac{\int (e+fx)^2\cot^2(c+dx)dx-\int (e+fx)^2\cos^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e+fx)^2 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \int (e+fx)^2 \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{f^2\int \cos^2(c+dx)dx}{2d^2} + \int (e+fx)^2 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{1}{2}\int (e+fx)^2 dx - \frac{f(e+fx)\cos^2(c+dx)}{2d^2} - \frac{(e+fx)^2\sin(c+dx)\cos(c+}{2d}}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{f^2\int \cos^2(c+dx)dx}{2d^2} + \int (e+fx)^2 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{f(e+fx)\cos^2(c+dx)}{2d^2} - \frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{(e+fx)^3}{6f}}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{2d^2} + \int (e+fx)^2 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{f(e+fx)\cos^2(c+dx)}{2d^2} - \frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{(e+fx)^3}{6f}}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{2d^2} + \int (e+fx)^2 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{f(e+fx)\cos^2(c+dx)}{2d^2} - \frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{(e+}{}}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{\int (e+fx)^2 \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{f(e+fx)\cos^2(c+dx)}{2d^2} + \frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2} - \frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{(e+fx}{6f}}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4203$$

$$\frac{-\frac{2f\int -((e+fx)\cot(c+dx))dx}{d}-\int (e+fx)^2dx-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{(e+f}{}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{2f\int -((e+fx)\cot(c+dx))dx}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{(e+fx)^2\sin(c+dx)\cos(c+}{2d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2f\int (e+fx)\cot(c+dx)dx}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{(e+fx)^2\sin(c+dx)\cos(c+dx)}{2d}-}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2f\int -\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{(e+fx)^2\sin(c+dx)\cos(}{2d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{2f\int (e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{(e+fx)^2\sin(c+dx)\cos}{2d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4202$$

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-\frac{(e+fx)^2\cot(c+dx)}{d}-\frac{(e+fx)^2\sin}{}}{a}$$

$$\Big\downarrow \textbf{2620}$$

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{if\int \log\left(1+e^{i(2c+2dx+\pi)}\right)dx}{2d}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}-}{a}$$

$$\Big\downarrow \textbf{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(\frac{f\int e^{-i(2c+2dx+\pi)}\log\left(1+e^{i(2c+2dx+\pi)}\right)de^{i(2c+2dx+\pi)}}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\text{s}\right.}{}}{a}$$

$$\Big\downarrow \textbf{2838}$$

$$-\frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$$\Big\downarrow \textbf{5054}$$

$$-\frac{b\left(\frac{\int(e+fx)^2\cos^3(c+dx)\cot(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$$\Big\downarrow \textbf{4908}$$

$$-\frac{b\left(\frac{\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx-\int(e+fx)^2\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$$\Big\downarrow \textbf{4905}$$

$$-\frac{b\left(\frac{-\frac{2f\int (e+fx)\cos^3(c+dx)dx}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\cdots}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{-\frac{2f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\cdots}{a}$$

$\downarrow$ 3791

$$-\frac{b\left(\frac{-\frac{2f\left(\frac{2}{3}\int (e+fx)\cos(c+dx)dx+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{a}-b\int \cdots\right)}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\cdots}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{-\frac{2f\left(\frac{2}{3}\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int (e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3(c+dx)}{3d}}{a}-b\cdots\right)}{a}+\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\,\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\cdots}{a}$$

$\downarrow$ 3777

$$-\frac{b\left(\frac{-\frac{2f\left(\frac{2}{3}\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\text{co}}{3d}}{a}}{a}$$
$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3}{3d}}{a}}{a}$$
$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$\downarrow$ 3042

$$-\frac{b\left(\frac{-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^3}{3d}}{a}}{a}$$
$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$\downarrow$ 3118

$$-\frac{b\left(\frac{\int(e+fx)^2\cos(c+dx)\cot(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\frac{(e+fx)^2\cos^3(c+dx}{3d}}{a}}{a}$$
$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\text{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}}{a}$$

$\downarrow$ 4908

$$-\frac{b\left(\frac{-\int (e+fx)^2\sin(c+dx)dx+\int (e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\ldots}{a}\right.}{a}$$

$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\ldots}{a}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(\frac{-\int (e+fx)^2\sin(c+dx)dx+\int (e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\ldots}{a}\right.}{a}$$

$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\ldots}{a}$$

$\Big\downarrow$ 3777

$$-\frac{b\left(\frac{-\frac{2f\int (e+fx)\cos(c+dx)dx}{d}+\int (e+fx)^2\csc(c+dx)dx-\frac{2f\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d}+\ldots}{a}\right.}{a}$$

$$\frac{-\frac{2f\left(\frac{i(e+fx)^2}{2f}-2i\left(-\frac{f\operatorname{PolyLog}\left(2,-e^{i(2c+2dx+\pi)}\right)}{4d^2}-\frac{i(e+fx)\log\left(1+e^{i(2c+2dx+\pi)}\right)}{2d}\right)\right)}{d}-\frac{f(e+fx)\cos^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}\ldots}{a}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)^2 \cot(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3075 vs. $2(753) = 1506$.

Time = 0.44 (sec) , antiderivative size = 3075, normalized size of antiderivative = 3.66

$$\int \frac{(e+fx)^2 \cos^2(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2\cos^2{(c+dx)}\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)^2\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.343 $\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 508

$$\begin{aligned}
&\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{(e+fx)^2}{2af}-\frac{(a^2-b^2)(e+fx)^2}{2ab^2f}+\frac{2b(e+fx)\operatorname{arctanh}\left(e^{i(c+dx)}\right)}{a^2d} \\
&\quad -\frac{b(e+fx)\cos(c+dx)}{a^2d}-\frac{(a^2-b^2)(e+fx)\cos(c+dx)}{a^2bd} \\
&\quad -\frac{(e+fx)\cot(c+dx)}{ad}-\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d} \\
&\quad +\frac{i(a^2-b^2)^{3/2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{f\log(\sin(c+dx))}{ad^2} \\
&\quad -\frac{ibf\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2} \\
&\quad -\frac{(a^2-b^2)^{3/2}f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d^2}+\frac{(a^2-b^2)^{3/2}f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^2d^2} \\
&\quad +\frac{bf\sin(c+dx)}{a^2d^2}+\frac{(a^2-b^2)f\sin(c+dx)}{a^2bd^2}
\end{aligned}$$

output
```
-1/2*(f*x+e)^2/a/f-1/2*(a^2-b^2)*(f*x+e)^2/a/b^2/f+2*b*(f*x+e)*arctanh(exp
(I*(d*x+c)))/a^2/d-b*(f*x+e)*cos(d*x+c)/a^2/d-(a^2-b^2)*(f*x+e)*cos(d*x+c)
/a^2/b/d-(f*x+e)*cot(d*x+c)/a/d-I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*exp(I*(
d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^2/d+I*(a^2-b^2)^(3/2)*(f*x+e)*ln(1-I*b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^2/d+f*ln(sin(d*x+c))/a/d^2-I*b*f
*polylog(2,-exp(I*(d*x+c)))/a^2/d^2+I*b*f*polylog(2,exp(I*(d*x+c)))/a^2/d^
2-(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/
b^2/d^2+(a^2-b^2)^(3/2)*f*polylog(2,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2))
)/a^2/b^2/d^2+b*f*sin(d*x+c)/a^2/d^2+(a^2-b^2)*f*sin(d*x+c)/a^2/b/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2279 vs. $2(508) = 1016$.

Time = 18.02 (sec) , antiderivative size = 2279, normalized size of antiderivative = 4.49

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x
]
```

output

```
-1/2*(a*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(b^2*d^2) - ((d*e - c*f +
 f*(c + d*x))*Cos[c + d*x])/(b*d^2) + ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[
(c + d*x)/2] - f*(c + d*x)*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) +
 (f*Log[Sin[c + d*x]])/(a*d^2) - (b*e*Log[Tan[(c + d*x)/2]])/(a^2*d) + (b*
c*f*Log[Tan[(c + d*x)/2]])/(a^2*d^2) - (b*f*((c + d*x)*(Log[1 - E^(I*(c +
d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - Pol
yLog[2, E^(I*(c + d*x))])))/(a^2*d^2) + (Sec[(c + d*x)/2]*(d*e*Sin[(c + d*
x)/2] - c*f*Sin[(c + d*x)/2] + f*(c + d*x)*Sin[(c + d*x)/2]))/(2*a*d^2) +
(f*Sin[c + d*x])/(b*d^2) + (((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*Log[1 + I*Tan[(c + d*x)/2]]*Log
[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]
)/Sqrt[-a^2 + b^2] + (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 +
b^2] - (I*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[
(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] + (I*f*Lo
g[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/
(I*a + b + Sqrt[-a^2 + b^2])])/Sqrt[-a^2 + b^2] - (I*f*PolyLog[2, (a*(1 -
I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))])/Sqrt[-a^2 + b^2] + (
I*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))
])/Sqrt[-a^2 + b^2] + (I*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a -...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)\cos^2(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)\cot^2(c+dx)dx-\int (e+fx)\cos^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3791

$$\frac{\int (e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{1}{2}\int (e+fx)dx-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 17

$$\frac{\int (e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 4203

$$\frac{-\frac{f\int -\cot(c+dx)dx}{d}-\int (e+fx)dx-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 17

$$\frac{-\frac{f\int -\cot(c+dx)dx}{d}-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{\frac{f\int \cot(c+dx)dx}{d}-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{f\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{d}-\frac{f\cos^2(c+dx)}{4d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3956$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5054$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int (e+fx)\cos^3(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx-\int (e+fx)\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4905$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx-\frac{f\int \cos^3(c+dx)dx}{3d}+\frac{(e+fx)\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int (e+fx)\cos(c+dx)\cot(c+dx)dx-\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d}+\frac{(e+fx)\cos^3(c+dx)}{3d}}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3113}$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\frac{f\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{3d^2}+\int(e+fx)\cos(c+dx)\cot(c+dx)dx+\frac{(e+fx)\cos^3(c+dx)}{3d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int(e+fx)\cos(c+dx)\cot(c+dx)dx+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}+\frac{(e+fx)\cos^3(c+dx)}{3d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{-\int(e+fx)\sin(c+dx)dx+\int(e+fx)\csc(c+dx)dx+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}+\frac{(e+fx)\cos^3(c+dx)}{3d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{-\int(e+fx)\sin(c+dx)dx+\int(e+fx)\csc(c+dx)dx+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}+\frac{(e+fx)\cos^3(c+dx)}{3d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int(e+fx)\csc(c+dx)dx-\frac{f\int\cos(c+dx)dx}{d}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}+\frac{(e+fx)\cos^3(c+dx)}{3d}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int(e+fx)\csc(c+dx)dx-\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}+\frac{(e+fx)\cos^3(c+dx)}{3d}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}}{}$$

$\downarrow$ 3117

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(\frac{\int(e+fx)\csc(c+dx)dx+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-\frac{f\sin(c+dx)}{d^2}+\frac{(e+fx)\cos^3(c+dx)}{3d}+\frac{(e+fx)\cos(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$\downarrow$ 4671

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{f\int\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}-f}{a}\right.}{a}$$

$\downarrow$ 2715

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}}{a}\right.}{a}$$

$\downarrow$ 2838

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}+\frac{f\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{3d^2}}{a}\right.}{a}$$

$\downarrow$ 5036

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-$$

$$b\left(-\frac{b\left(-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)\cos^2(c+dx)dx}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ Poly}\ldots}{\ldots}}{\ldots}\right.$$

$$a$$

$\downarrow$ 3042

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-$$

$$b\left(-\frac{b\left(-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\ldots}{\ldots}}{\ldots}\right.$$

$$a$$

$\downarrow$ 3791

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-$$

$$b\left(-\frac{b\left(-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}+\frac{a\left(\frac{1}{2}\int(e+fx)dx+\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}\right)}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}\right)}{a}+\frac{-\frac{2\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 17

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-$$

$$b\left(-\frac{b\left(-\frac{\left(a^2-b^2\right)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\int(e+fx)\cos^2(c+dx)\sin(c+dx)dx}{b}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}\right)}{a}+\frac{-\frac{2(e+\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 4905

$$\frac{-\frac{f\cos^2(c+dx)}{4d^2}+\frac{f\log(-\sin(c+dx))}{d^2}-\frac{(e+fx)\cot(c+dx)}{d}-\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3(e+fx)^2}{4f}}{a}-$$

$$b\left(-\frac{b\left(-\frac{(a^2-b^2)\int\frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}dx}{b^2}-\frac{\frac{f\int\cos^3(c+dx)dx}{3d}-\frac{(e+fx)\cos^3(c+dx)}{3d}}{b}+\frac{a\left(\frac{f\cos^2(c+dx)}{4d^2}+\frac{(e+fx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{b^2}\right)}{a}+\cdots\right.$$

input
```
Int[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5036
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[a/b^2   Int[(e + f*x)^m*Cos[c
+ d*x]^(n - 2), x], x] + (-Simp[1/b   Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*
Sin[c + d*x], x], x] - Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Cos[c + d*x]
^(n - 2)/(a + b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5309 vs. $2(466) = 932$.

Time = 8.19 (sec) , antiderivative size = 5310, normalized size of antiderivative = 10.45

method	result	size
risch	Expression too large to display	5310

input
```
int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(454) = 908$.

Time = 0.33 (sec) , antiderivative size = 1751, normalized size of antiderivative = 3.45

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/2*(2*a^2*b*f*cos(d*x + c)^2 - I*b^3*f*dilog(cos(d*x + c) + I*sin(d*x +
c))*sin(d*x + c) + I*b^3*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x +
c) - I*b^3*f*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + I*b^3*f*
dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - I*(a^2*b - b^3)*f*sqr
t(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) +
 I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x + c) - a*sin(
d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b + 1)*sin(d*x + c) + I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*
a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - I*(a^2*b - b^3)*f*sqrt(-(a^2
 - b^2)/b^2)*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 2*a^2
*b*f - ((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a
)*sin(d*x + c) - ((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)
/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) - 2*I*a)*sin(d*x + c) + ((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(
a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + ((a^2*b - b^3)*d*e - (a^2*b - b^3...
```

Sympy [F]

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)^2\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.344 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 104

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{ax}{b^2} + \frac{2(a^2-b^2)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d} + \frac{b\,\text{arctanh}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad}$$

output
```
-a*x/b^2+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a^2/b^2/d+b*arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/b/d-cot(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2a^3c + 2a^3dx - 4(a^2-b^2)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + 2a^2b\cos(c+dx) + ab^2\cot\left(\frac{1}{2}(c+dx)\right) - 2b^3}{2a^2b^2d}$$

input
```
Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
-1/2*(2*a^3*c + 2*a^3*d*x - 4*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]] + 2*a^2*b*Cos[c + d*x] + a*b^2*Cot[(c + d*x)/2] - 2
*b^3*Log[Cos[(c + d*x)/2]] + 2*b^3*Log[Sin[(c + d*x)/2]] - a*b^2*Tan[(c +
d*x)/2])/(a^2*b^2*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = $\{3042, 3373, 3042, 3536, 3042, 3139, 1083, 217, 4257\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{\sin(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3373}$$

$$-\frac{\int \frac{\csc(c+dx)\left(b^2+2a\sin(c+dx)b+a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{b^2+2a\sin(c+dx)b+a^2\sin(c+dx)^2}{\sin(c+dx)(a+b\sin(c+dx))}dx}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3536}$$

$$-\frac{-\frac{\left(a^2-b^2\right)^2\int \frac{1}{a+b\sin(c+dx)}dx}{ab}+\frac{b^2\int \csc(c+dx)dx}{a}+\frac{a^2x}{b}}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\left(a^2-b^2\right)^2\int \frac{1}{a+b\sin(c+dx)}dx}{ab}+\frac{b^2\int \csc(c+dx)dx}{a}+\frac{a^2x}{b}}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{-\frac{2(a^2-b^2)^2\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{abd}+\frac{b^2\int \csc(c+dx)dx}{a}+\frac{a^2x}{b}}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{4(a^2-b^2)^2\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd}+\frac{b^2\int \csc(c+dx)dx}{a}+\frac{a^2x}{b}}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{b^2\int \csc(c+dx)dx}{a}-\frac{2(a^2-b^2)^{3/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{abd}+\frac{a^2x}{b}}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{2(a^2-b^2)^{3/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{abd}+\frac{a^2x}{b}-\frac{b^2\text{arctanh}(\cos(c+dx))}{ad}}{ab}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

input
```
Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
-(((a^2*x)/b - (2*(a^2 - b^2)^(3/2)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2
*Sqrt[a^2 - b^2])])/(a*b*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a*d))/(a*b)) -
Cos[c + d*x]/(b*d) - Cot[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3373
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 2)/(b*d
^2*f*(m + n + 4))), x] + Simp[1/(a*b*d*(n + 1)*(m + n + 4))   Int[(a + b*Si
n[e + f*x])^m*(d*Sin[e + f*x])^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m +
n + 2)*(m + n + 4) + a*b*(m + 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*
(m + n + 3)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, d, e,
f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !
m < -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

rule 3536
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d))   Int[1/(a + b*Sin[e + f*x]), x], x] - Simp[(c^2*C - B*c*d
 + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a
, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{\left(4a^4-8a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2b^2\sqrt{a^2-b^2}}-\frac{2\left(\frac{b}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}-\frac{1}{2a\tan\left(\frac{dx}{2}+\right.}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{\left(4a^4-8a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2b^2\sqrt{a^2-b^2}}-\frac{2\left(\frac{b}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}-\frac{1}{2a\tan\left(\frac{dx}{2}+\right.}}{d}$
risch	$-\frac{ax}{b^2}-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\sqrt{-a^2+b^2}\ln\left(\right.}{}$

input
```
int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tan(1/2*d*x+1/2*c)+1/2*(4*a^4-8*a^2*b^2+4*b^4)/a^2/b^2/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/b^2*(b/
(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c)))-1/2/a/tan(1/2*d*x+1
/2*c)-1/a^2*b*ln(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.81

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[\frac{b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2ab^2\cos(dx+c)}{}\right.$$

input
```
integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
[1/2*(b^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 2*a*b^2*cos(d*x + c) - (a^2 - b^2)*sqrt(-a^2 +
b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*(a^3*d*x
+ a^2*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c)), 1/2*(b^3*log
(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x + c) + 1/2)*s
in(d*x + c) - 2*a*b^2*cos(d*x + c) - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*
x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*(a^3*d*x + a^
2*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos^2{(c+dx)}\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(99) = 198$.

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.12

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{6\,(dx+c)a}{b^2} + \frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{12\,(a^4-2\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^2b^2}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(6*(d*x + c)*a/b^2 + 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1
/2*d*x + 1/2*c)/a - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqr
t(a^2 - b^2)*a^2*b^2) - (2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x
+ 1/2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 2*b^2*tan(1/2*d*x + 1/2*c) - 3*
a*b)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^2*b))/d
```

Mupad [B] (verification not implemented)

Time = 41.42 (sec) , antiderivative size = 1167, normalized size of antiderivative = 11.22

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
(atan((16*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)
 - 4*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4
*a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 12*a^3
*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^5*b^
7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a^7*b^5
*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 6*a^9*b^3*
cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 29*a^2*b^4*
sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 18*a^4*b^2*
sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^2*b^10*si
n(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a^4*b^8*sin
(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 22*a^6*b^6*sin
(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 32*a^8*b^4*sin
(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 18*a^10*b^2*si
n(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a*b^5*cos(c
/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 5*a^5*b*cos(c/2
+ (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^11*b*cos(c/2 +
(d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/
2)*16i + a*b^14*cos(c/2 + (d*x)/2)*8i - a^14*b*sin(c/2 + (d*x)/2)*3i - a^3
*b^12*cos(c/2 + (d*x)/2)*48i + a^5*b^10*cos(c/2 + (d*x)/2)*123i - a^7*b^8*
cos(c/2 + (d*x)/2)*167i + a^9*b^6*cos(c/2 + (d*x)/2)*126i - a^11*b^4*co...
```

Reduce [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.94

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,a^2-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)\,b^2-\cos(d}{}$$

input
```
int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**2 - b**2))*sin(
c + d*x)*a**2 - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a + b)/sqrt(a**
2 - b**2))*sin(c + d*x)*b**2 - cos(c + d*x)*sin(c + d*x)*a**2*b - cos(c +
d*x)*a*b**2 - log(tan((c + d*x)/2))*sin(c + d*x)*b**3 - sin(c + d*x)*a**3*
c - sin(c + d*x)*a**3*d*x + sin(c + d*x)*a**2*b)/(sin(c + d*x)*a**2*b**2*d
)
```

3.345 $\int \frac{(e+fx)^3 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 1432

$$\int \frac{(e+fx)^3 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} \, dx = \text{Too large to display}$$

output
```
-3/2*b*f^2*(f*x+e)*polylog(3,exp(2*I*(d*x+c)))/a^2/d^3-6*I*f^2*(f*x+e)*pol
ylog(2,exp(I*(d*x+c)))/a/d^3-3/4*I*b*f^3*polylog(4,exp(2*I*(d*x+c)))/a^2/d
^4-3/4*b*f^2*(f*x+e)*sin(d*x+c)^2/a^2/d^3+1/2*(a^2-b^2)*(f*x+e)^3*sin(d*x+
c)^2/a^2/b/d-3/8*b*f^3*cos(d*x+c)*sin(d*x+c)/a^2/d^4+6*(a^2-b^2)*f^3*cos(d
*x+c)/a/b^2/d^4+3/8*(a^2-b^2)*f^3*x/a^2/b/d^3-1/4*I*(a^2-b^2)^2*(f*x+e)^4/
a^2/b^3/f+(a^2-b^2)^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)
))/a^2/b^3/d+(a^2-b^2)^2*(f*x+e)^3*ln(1-I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1
/2)))/a^2/b^3/d-3*f*(f*x+e)^2*cos(d*x+c)/a/d^2+6*f^2*(f*x+e)*sin(d*x+c)/a/
d^3-6*f*(f*x+e)^2*arctanh(exp(I*(d*x+c)))/a/d^2+6*I*f^2*(f*x+e)*polylog(2,
-exp(I*(d*x+c)))/a/d^3+3/2*I*b*f*(f*x+e)^2*polylog(2,exp(2*I*(d*x+c)))/a^2
/d^2+1/4*I*b*(f*x+e)^4/a^2/f-1/4*b*(f*x+e)^3/a^2/d+6*(a^2-b^2)^2*f^2*(f*x+
e)*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^3/d^3+6*(a^2-b^
2)^2*f^2*(f*x+e)*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^3
/d^3+6*I*(a^2-b^2)^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))
/a^2/b^3/d^4+6*I*(a^2-b^2)^2*f^3*polylog(4,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)
^(1/2)))/a^2/b^3/d^4+6*f^3*cos(d*x+c)/a/d^4-6*f^3*polylog(3,-exp(I*(d*x+c)
))/a/d^4+6*f^3*polylog(3,exp(I*(d*x+c)))/a/d^4+6*(a^2-b^2)*f^2*(f*x+e)*sin
(d*x+c)/a/b^2/d^3-(f*x+e)^3*csc(d*x+c)/a/d-3/4*(a^2-b^2)*f^2*(f*x+e)*sin(d
*x+c)^2/a^2/b/d^3-3/8*(a^2-b^2)*f^3*cos(d*x+c)*sin(d*x+c)/a^2/b/d^4+3/4*b*
f*(f*x+e)^2*cos(d*x+c)*sin(d*x+c)/a^2/d^2-3*(a^2-b^2)*f*(f*x+e)^2*cos(d...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4009 vs. 2(1432) = 2864.

Time = 13.60 (sec) , antiderivative size = 4009, normalized size of antiderivative = 2.80

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])
,x]
```

output
```
((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x^3)*Csc[c + d*x])/(a*d) - (((-2*I)
*e^2*(b*d*e - 3*a*f)*x)/d - ((2*I)*e^2*(b*d*e + 3*a*f)*x)/d - (I*b*(e + f*
x)^4)/((-1 + E^((2*I)*c))*f) + (6*e*f*(b*d*e - 2*a*f)*x*Log[1 - E^((-I)*(c
 + d*x))])/d^2 + (6*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)*(c + d*x))])/d^2
 + (2*b*f^3*x^3*Log[1 - E^((-I)*(c + d*x))])/d + (6*e*f*(b*d*e + 2*a*f)*x*
Log[1 + E^((-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e + a*f)*x^2*Log[1 + E^((-I
)*(c + d*x))])/d^2 + (2*b*f^3*x^3*Log[1 + E^((-I)*(c + d*x))])/d + (2*e^2*
(b*d*e - 3*a*f)*Log[1 - E^(I*(c + d*x))])/d^2 + (2*e^2*(b*d*e + 3*a*f)*Log
[1 + E^(I*(c + d*x))])/d^2 + ((6*I)*e*f*(b*d*e + 2*a*f)*PolyLog[2, -E^((-I
)*(c + d*x))])/d^3 + ((12*I)*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^((-I)*(c +
d*x))])/d^3 + ((6*I)*b*f^3*x^2*PolyLog[2, -E^((-I)*(c + d*x))])/d^2 + ((6*
I)*e*f*(b*d*e - 2*a*f)*PolyLog[2, E^((-I)*(c + d*x))])/d^3 + ((12*I)*f^2*(
b*d*e - a*f)*x*PolyLog[2, E^((-I)*(c + d*x))])/d^3 + ((6*I)*b*f^3*x^2*Poly
Log[2, E^((-I)*(c + d*x))])/d^2 + (12*f^2*(b*d*e + a*f)*PolyLog[3, -E^((-I
)*(c + d*x))])/d^4 + (12*b*f^3*x*PolyLog[3, -E^((-I)*(c + d*x))])/d^3 + (1
2*f^2*(b*d*e - a*f)*PolyLog[3, E^((-I)*(c + d*x))])/d^4 + (12*b*f^3*x*Poly
Log[3, E^((-I)*(c + d*x))])/d^3 - ((12*I)*b*f^3*PolyLog[4, -E^((-I)*(c + d
*x))])/d^4 - ((12*I)*b*f^3*PolyLog[4, E^((-I)*(c + d*x))])/d^4)/(2*a^2) +
((a^2 - b^2)^2*((-4*I)*d^4*e^3*E^((2*I)*c)*x - (6*I)*d^4*e^2*E^((2*I)*c)*f
*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 - I*d^4*E^((2*I)*c)*f^3*x^4 - (2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{5054}$$

$$\frac{\int (e+fx)^3\cos^3(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{4908}$$

$$\frac{\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\int (e+fx)^3\cos^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\frac{2f^2\int (e+fx)\cos^3(c+dx)dx}{3d^2}-\frac{2}{3}\int (e+fx)^3\cos(c+dx)dx+\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\frac{f(e+fx)^2\cos^3(c+d}{3d^2}}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\frac{f(e+fx)}{}}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{3f\int -(e+fx)^2\sin(c+dx)dx}{d}+\frac{(e+fx)^3\sin(c+dx)}{d}\right)+\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}\right)+\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f^2\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}\right)+\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{2f^2\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int(e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)+\int(e+fx)^3\cos(c+}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2f^2\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)+\int(e+fx)^3\cos(c}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{2f^2\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)+\int(e+}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{2f^2\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)+\int(e+}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2f^2\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}\right)+\int(e+}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{2f^2\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{3d^2}+\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\frac{f(e+fx)^2\cos^3(c+dx)}{3d^2}-\frac{2}{3}\left(\frac{(e+fx)^3\sin(c+dx)}{d}-\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\int(e+fx)\cos(c+dx)dx+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d^2}+\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\frac{f(e+fx)^2\cos}{3d}}{a}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d^2}+\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-\frac{f(e+fx}{}}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2f^2\left(\frac{2}{3}\left(\frac{f\int -\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d^2}+\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 25

$$\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d^2}+\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d^2}+\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx-$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3118

$$\int(e+fx)^3\cos(c+dx)\cot^2(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4908

$$-\int (e+fx)^3\cos(c+dx)dx+\int (e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}\right.}{3d^2}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$-\int (e+fx)^3\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int (e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos\right.}{3d^2}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3777

$$-\frac{3f\int -(e+fx)^2\sin(c+dx)dx}{d}+\int (e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\right.}{3d^2}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 25

$$\frac{3f\int (e+fx)^2\sin(c+dx)dx}{d}+\int (e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx}{}\right.}{3d^2}$$

$$\frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{3f\int(e+fx)^2\sin(c+dx)dx}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)+\frac{f\cos^3(c+dx)}{9d^2}+\frac{(e+fx}{}\right.}{3d^2}}{\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f\left(\frac{2f\int(e+fx)\cos(c+dx)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)\right.}{}}{\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\left(\frac{2f\int(e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+d}{d}\right.\right.}{}}{\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}}$$

$\Big\downarrow$ 3777

$$\frac{\frac{3f\left(\frac{2f\left(\frac{f\int-\sin(c+dx)dx}{d}+\frac{(e+fx)\sin(c+dx)}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\right.\right.}{}}{\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}}$$

$\Big\downarrow$ 25

$$\frac{\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\right.\right.}{}}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3f\left(\frac{2f\left(\frac{(e+fx)\sin(c+dx)}{d}-\frac{f\int\sin(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\cos(c+dx)}{d}\right)}{d}+\int(e+fx)^3\cot(c+dx)\csc(c+dx)dx+\frac{2f^2\left(\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\right.\right.}{}}{}$$

$$\frac{b\int\frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

input
```
Int[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^3 \cos(dx+c)^3 \cot(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4916 vs. 2(1307) = 2614.

Time = 0.67 (sec) , antiderivative size = 4916, normalized size of antiderivative = 3.43

$$\int \frac{(e+fx)^3 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorit
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^3 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} dx = \int \frac{(e+fx)^3 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} dx$$

input
```
integrate((f*x+e)**3*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cos(c + d*x)**3*cot(c + d*x)**2/(a + b*sin(c + d*x))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)^2*(e + f*x)^3)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^3\cos(dx+c)^3\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.346 $\int \frac{(e+fx)^2 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 1014

$$\int \frac{(e+fx)^2 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)} \, dx = \text{Too large to display}$$

output
```
2*(a^2-b^2)*f^2*sin(d*x+c)/a/b^2/d^3-1/4*(a^2-b^2)*f^2*sin(d*x+c)^2/a^2/b/
d^3+1/2*(a^2-b^2)*(f*x+e)^2*sin(d*x+c)^2/a^2/b/d-1/3*I*(a^2-b^2)^2*(f*x+e)
^3/a^2/b^3/f+2*(a^2-b^2)^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a+(a^2-b^2)^(
1/2)))/a^2/b^3/d^3+2*(a^2-b^2)^2*f^2*polylog(3,I*b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/a^2/b^3/d^3+(a^2-b^2)^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/(a+
(a^2-b^2)^(1/2)))/a^2/b^3/d+(a^2-b^2)^2*(f*x+e)^2*ln(1-I*b*exp(I*(d*x+c))/
(a-(a^2-b^2)^(1/2)))/a^2/b^3/d-2*f*(f*x+e)*cos(d*x+c)/a/d^2+1/3*I*b*(f*x+e
)^3/a^2/f+I*b*f*(f*x+e)*polylog(2,exp(2*I*(d*x+c)))/a^2/d^2-1/4*b*(f*x+e)^
2/a^2/d+2*f^2*sin(d*x+c)/a/d^3-2*(a^2-b^2)*f*(f*x+e)*cos(d*x+c)/a/b^2/d^2+
1/2*b*f*(f*x+e)*cos(d*x+c)*sin(d*x+c)/a^2/d^2+1/2*(a^2-b^2)*f*(f*x+e)*cos(
d*x+c)*sin(d*x+c)/a^2/b/d^2-2*I*(a^2-b^2)^2*f*(f*x+e)*polylog(2,I*b*exp(I*
(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^3/d^2-2*I*(a^2-b^2)^2*f*(f*x+e)*polylo
g(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^3/d^2+2*I*f^2*polylog(2,
-exp(I*(d*x+c)))/a/d^3-b*(f*x+e)^2*ln(1-exp(2*I*(d*x+c)))/a^2/d-(f*x+e)^2*
csc(d*x+c)/a/d-(a^2-b^2)*(f*x+e)^2*sin(d*x+c)/a/b^2/d-1/4*(a^2-b^2)*(f*x+e
)^2/a^2/b/d-1/4*b*f^2*sin(d*x+c)^2/a^2/d^3+1/2*b*(f*x+e)^2*sin(d*x+c)^2/a^
2/d-4*f*(f*x+e)*arctanh(exp(I*(d*x+c)))/a/d^2-1/2*b*f^2*polylog(3,exp(2*I*
(d*x+c)))/a^2/d^3-2*I*f^2*polylog(2,exp(I*(d*x+c)))/a/d^3-(f*x+e)^2*sin(d*
x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5075 vs. $2(1014) = 2028$.

Time = 12.22 (sec) , antiderivative size = 5075, normalized size of antiderivative = 5.00

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])
,x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 5054$$

$$\frac{\int (e+fx)^2\cos^3(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\int (e+fx)^2\cos^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{2f^2\int \cos^3(c+dx)dx}{9d^2}-\frac{2}{3}\int (e+fx)^2\cos(c+dx)dx+\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}-(\ldots}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2f^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{9d^2}-\frac{2}{3}\int (e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f(e+fx)\cos^3(\ldots}{9d^2}}{a}-\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3113$$

$$\frac{-\frac{2f^2\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{9d^3}-\frac{2}{3}\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-}{a}$$
$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{2}{3}\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+}{}}{a}$$
$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{2}{3}\left(\frac{2f\int-((e+fx)\sin(c+dx))dx}{d}+\frac{(e+fx)^2\sin(c+dx)}{d}\right)+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+\right.}{9d^3}}{a}$$
$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 25

$$\frac{-\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right.}{9d^3}}{a}$$
$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\int(e+fx)\sin(c+dx)dx}{d}\right)+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right.}{9d^3}}{a}$$
$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\Big\downarrow$ 3777

$$\frac{-\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+\right.}{9}}{a}$$
$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{-\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2f\left(\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}\right)+\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin\right.}{}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3117

$$\frac{\int(e+fx)^2\cos(c+dx)\cot^2(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}-\frac{2}{3}\left(\frac{(e+fx)^2\sin(c+dx)}{d}-\frac{2}{}\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4908

$$\frac{-\int(e+fx)^2\cos(c+dx)dx+\int(e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c}{9d^2}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{-\int(e+fx)^2\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int(e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)}{9}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3777

$$\frac{-\frac{2f\int-((e+fx)\sin(c+dx))dx}{d}+\int(e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx}{9d^2}}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2f\int (e+fx)\sin(c+dx)dx}{d}+\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}-\frac{2}{3}\cdots}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2f\int (e+fx)\sin(c+dx)dx}{d}+\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}-\frac{2}{3}\cdots}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{2f\left(\frac{f\int \cos(c+dx)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}+\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos\cdots}{9d^2}\cdots}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2f\left(\frac{f\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}+\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx\cdots}{\cdots}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{\int (e+fx)^2\cot(c+dx)\csc(c+dx)dx-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+\cdots}{d}\right)}{d}\cdots}{a}$$

$$\frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4910$$

$$\frac{\frac{2f\int(e+fx)\csc(c+dx)dx}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{2}{3}\left(\frac{(e+fx)^2}{}\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{2f\int(e+fx)\csc(c+dx)dx}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+dx)}{9d^2}+\frac{2f\left(\frac{f\sin(c+dx)}{d^2}-\frac{(e+fx)\cos(c+dx)}{d}\right)}{d}-\frac{2}{3}\left(\frac{(e+fx)^2}{}\right.}{a}$$

$$\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4671

$$-\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{\frac{2f\left(-\frac{f\int\log\left(1-e^{i(c+dx)}\right)dx}{d}+\frac{f\int\log\left(1+e^{i(c+dx)}\right)dx}{d}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c+d}{9d^2}}{}$$

$\downarrow$ 2715

$$-\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{\frac{2f\left(\frac{if\int e^{-i(c+dx)}\log\left(1-e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{if\int e^{-i(c+dx)}\log\left(1+e^{i(c+dx)}\right)de^{i(c+dx)}}{d^2}-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\text{si}\right.}{9d^3}}{}$$

$\downarrow$ 2838

$$-\frac{b\int\frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}+$$

$$\frac{\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c-}{9d^2}}{}$$

$\downarrow$ 5054

$$-\frac{b\left(\frac{\int (e+fx)^2\cos^4(c+dx)\cot(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c-}{9d^2}$$

$\big\downarrow$ 4908

$$-\frac{b\left(\frac{\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)dx-\int (e+fx)^2\cos^3(c+dx)\sin(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c-}{9d^2}$$

$\big\downarrow$ 4905

$$-\frac{b\left(\frac{-\frac{f\int (e+fx)\cos^4(c+dx)dx}{2d}+\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^4(c+dx)}{4d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c-}{9d^2}$$

$\big\downarrow$ 3042

$$-\frac{b\left(\frac{-\frac{f\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{2d}+\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)dx+\frac{(e+fx)^2\cos^4(c+dx)}{4d}}{a}-\frac{b\int \frac{(e+fx)^2\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}+\frac{2f\left(-\frac{2(e+fx)\text{arctanh}\left(e^{i(c+dx)}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{i(c+dx)}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{i(c+dx)}\right)}{d^2}\right)}{d}-\frac{2f^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cos^3(c-}{9d^2}$$

input
```
Int[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{(fx+e)^2 \cos(dx+c)^3 \cot(dx+c)^2}{a+b\sin(dx+c)}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3131 vs. $2(925) = 1850$.

Time = 0.41 (sec) , antiderivative size = 3131, normalized size of antiderivative = 3.09

$$\int \frac{(e+fx)^2 \cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(e+fx)^2 \cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^2 \cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cos(c + d*x)**3*cot(c + d*x)**2/(a + b*sin(c + d*x))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)^2\cos(dx+c)^3\cot(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((f*x + e)^2*cos(d*x + c)^3*cot(d*x + c)^2/(b*sin(d*x + c) + a),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)^2*(e + f*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)^2\cos(dx+c)^3\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.347 $\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 641

$$\begin{aligned}
&\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{bfx}{4a^2d} - \frac{(a^2-b^2)fx}{4a^2bd} + \frac{ib(e+fx)^2}{2a^2f} - \frac{i(a^2-b^2)^2(e+fx)^2}{2a^2b^3f} \\
&\quad - \frac{f\operatorname{arctanh}(\cos(c+dx))}{ad^2} - \frac{f\cos(c+dx)}{ad^2} - \frac{(a^2-b^2)f\cos(c+dx)}{ab^2d^2} \\
&\quad - \frac{(e+fx)\csc(c+dx)}{ad} + \frac{(a^2-b^2)^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^3d} \\
&\quad + \frac{(a^2-b^2)^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^3d} - \frac{b(e+fx)\log\left(1-e^{2i(c+dx)}\right)}{a^2d} \\
&\quad - \frac{i(a^2-b^2)^2f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^3d^2} - \frac{i(a^2-b^2)^2f\operatorname{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{a^2b^3d^2} \\
&\quad + \frac{ibf\operatorname{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2} - \frac{(e+fx)\sin(c+dx)}{ad} - \frac{(a^2-b^2)(e+fx)\sin(c+dx)}{ab^2d} \\
&\quad + \frac{bf\cos(c+dx)\sin(c+dx)}{4a^2d^2} + \frac{(a^2-b^2)f\cos(c+dx)\sin(c+dx)}{4a^2bd^2} \\
&\quad + \frac{b(e+fx)\sin^2(c+dx)}{2a^2d} + \frac{(a^2-b^2)(e+fx)\sin^2(c+dx)}{2a^2bd}
\end{aligned}$$

output
```
-1/4*b*f*x/a^2/d-1/4*(a^2-b^2)*f*x/a^2/b/d-1/2*I*(a^2-b^2)^2*(f*x+e)^2/a^2
/b^3/f-I*(a^2-b^2)^2*f*polylog(2,I*b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a
^2/b^3/d^2-f*arctanh(cos(d*x+c))/a/d^2-f*cos(d*x+c)/a/d^2-(a^2-b^2)*f*cos(
d*x+c)/a/b^2/d^2-(f*x+e)*csc(d*x+c)/a/d+(a^2-b^2)^2*(f*x+e)*ln(1-I*b*exp(I
*(d*x+c))/(a-(a^2-b^2)^(1/2)))/a^2/b^3/d+(a^2-b^2)^2*(f*x+e)*ln(1-I*b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/a^2/b^3/d-b*(f*x+e)*ln(1-exp(2*I*(d*x+c)))
/a^2/d+1/2*I*b*(f*x+e)^2/a^2/f-I*(a^2-b^2)^2*f*polylog(2,I*b*exp(I*(d*x+c)
)/(a+(a^2-b^2)^(1/2)))/a^2/b^3/d^2+1/2*I*b*f*polylog(2,exp(2*I*(d*x+c)))/a
^2/d^2-(f*x+e)*sin(d*x+c)/a/d-(a^2-b^2)*(f*x+e)*sin(d*x+c)/a/b^2/d+1/4*b*f
*cos(d*x+c)*sin(d*x+c)/a^2/d^2+1/4*(a^2-b^2)*f*cos(d*x+c)*sin(d*x+c)/a^2/b
/d^2+1/2*b*(f*x+e)*sin(d*x+c)^2/a^2/d+1/2*(a^2-b^2)*(f*x+e)*sin(d*x+c)^2/a
^2/b/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1650 vs. 2(641) = 1282.

Time = 10.32 (sec) , antiderivative size = 1650, normalized size of antiderivative = 2.57

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x
]
```

output
```
-((a*f*Cos[c + d*x])/(b^2*d^2)) - ((d*e - c*f + f*(c + d*x))*Cos[2*(c + d*
x)])/(4*b*d^2) + ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c +
 d*x)*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) - (b*e*Log[Sin[c + d*x
]])/(a^2*d) + (b*c*f*Log[Sin[c + d*x]])/(a^2*d^2) + (a^2*e*Log[1 + (b*Sin[
c + d*x])/a])/(b^3*d) - (2*e*Log[1 + (b*Sin[c + d*x])/a])/(b*d) + (b*e*Log
[1 + (b*Sin[c + d*x])/a])/(a^2*d) - (a^2*c*f*Log[1 + (b*Sin[c + d*x])/a])/
(b^3*d^2) + (2*c*f*Log[1 + (b*Sin[c + d*x])/a])/(b*d^2) - (b*c*f*Log[1 + (
b*Sin[c + d*x])/a])/(a^2*d^2) + (f*Log[Tan[(c + d*x)/2]])/(a*d^2) - (2*f*(
((c + d*x)*Log[a + b*Sin[c + d*x]])/b - ((-1/2*I)*(-c + Pi/2 - d*x)^2 + (4
*I)*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]]*ArcTan[((a - b)*Tan[(-c + Pi/2 - d*x)/
2])/Sqrt[a^2 - b^2]] + (-c + Pi/2 - d*x + 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]
])*Log[1 + ((a - Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] + (-c + Pi/2
 - d*x - 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 + ((a + Sqrt[a^2 - b^2])
*E^(I*(-c + Pi/2 - d*x)))/b] - (-c + Pi/2 - d*x)*Log[a + b*Sin[c + d*x]] -
 I*(PolyLog[2, ((-a - Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] + PolyL
og[2, ((-a + Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b]))/b))/d^2 + (a^2
*f*(((c + d*x)*Log[a + b*Sin[c + d*x]])/b - ((-1/2*I)*(-c + Pi/2 - d*x)^2
+ (4*I)*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]]*ArcTan[((a - b)*Tan[(-c + Pi/2 - d
*x)/2])/Sqrt[a^2 - b^2]] + (-c + Pi/2 - d*x + 2*ArcSin[Sqrt[(a + b)/b]/Sqr
t[2]])*Log[1 + ((a - Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] + (-c...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 5054}$$

$$\frac{\int (e+fx)\cos^3(c+dx)\cot^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 4908}$$

$$\frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)dx-\int (e+fx)\cos^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)^3 dx}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3791}$$

$$\frac{-\frac{2}{3}\int (e+fx)\cos(c+dx)dx + \int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2}{3}\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx + \int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin(c+dx)\cos^2(c+d}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{2}{3}\left(\frac{f\int -\sin(c+dx)dx}{d} + \frac{(e+fx)\sin(c+dx)}{d}\right) + \int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin(c+dx)\cos}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d} - \frac{f\int \sin(c+dx)dx}{d}\right) + \int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin(c+dx)\cos^2(}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2}{3}\left(\frac{(e+fx)\sin(c+dx)}{d} - \frac{f\int \sin(c+dx)dx}{d}\right) + \int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin(c+dx)\cos^2(}{3d}}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3118$$

$$\frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)dx - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin(c+dx)\cos^2(c+dx)}{3d}}{a}$$
$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{-\int (e+fx)\cos(c+dx)dx + \int (e+fx)\cot(c+dx)\csc(c+dx)dx - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+d}{9d^2}}{a}$$
$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\int (e+fx)\sin\left(c+dx+\frac{\pi}{2}\right)dx + \int (e+fx)\cot(c+dx)\csc(c+dx)dx - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - f\,co}{a}$$
$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx - \frac{f\int -\sin(c+dx)dx}{d} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - (e+fx)\,s}{a}$$
$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx + \frac{f\int \sin(c+dx)dx}{d} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin}{d}}{a}$$
$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx + \frac{f\int \sin(c+dx)dx}{d} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{(e+fx)\sin}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3118

$$\frac{\int (e+fx)\cot(c+dx)\csc(c+dx)dx - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{f\cos(c+dx)}{d^2} - \frac{(e+fx)\sin(c+}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4910

$$\frac{\frac{f\int \csc(c+dx)dx}{d} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{f\cos(c+dx)}{d^2} - \frac{(e+fx)\sin(c+dx)}{d} - \frac{(e+fx)\csc(c+dx)}{d} -}{a}$$

$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 3042

$$\frac{\frac{f\int \csc(c+dx)dx}{d} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{f\cos(c+dx)}{d^2} - \frac{(e+fx)\sin(c+dx)}{d} - \frac{(e+fx)\csc(c+dx)}{d} -}{a}$$

$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 4257

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{f\cos(c+dx)}{d^2} - \frac{(e+fx)\sin(c+dx)}{d} - \frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}dx}{a}$$

$\downarrow$ 5054

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2} - \frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right) - \frac{f\cos^3(c+dx)}{9d^2} - \frac{f\cos(c+dx)}{d^2} - \frac{(e+fx)\sin(c+dx)}{d} - \frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\cos^4(c+dx)\cot(c+dx)dx}{a} - \frac{b\int \frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx-\int(e+fx)\cos^3(c+dx)\sin(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4905$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx-\frac{f\int\cos^4(c+dx)dx}{4d}+\frac{(e+fx)\cos^4(c+dx)}{4d}}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx-\frac{f\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{4d}+\frac{(e+fx)\cos^4(c+dx)}{4d}}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx-\frac{f\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{4d}+\frac{(e+fx)\cos^4(c+dx)}{4d}}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c\cdots}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx-\frac{f\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{4d}+\frac{(e+fx)\cos^4(c+dx)}{4d}}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c\cdots}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx-\frac{f\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{4d}+\frac{(e+fx)\cos^4(c+dx)}{4d}}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx\cdots}{a}\cdots}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c\cdots}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cos^2(c+dx)\cot(c+dx)dx+\frac{(e+fx)\cos^4(c+dx)}{4d}-\frac{f\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{a}-\frac{b\int\frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4908$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c\cdots}{d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\cot(c+dx)dx-\int(e+fx)\cos(c+dx)\sin(c+dx)dx+\frac{(e+fx)\cos^4(c+dx)}{4d}-\frac{f\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{a}-b\int\cdots\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{\int -\left((e+fx)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx+\frac{(e+fx)\cos^4(c+dx)}{4d}-\frac{f\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{a}\right.}{a}$$

$\downarrow$ 25

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(\frac{-\int (e+fx)\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx+\frac{(e+fx)\cos^4(c+dx)}{4d}-\frac{f\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right.}{4d}}{a}\right.}{a}$$

$\downarrow$ 4202

$$\frac{-\frac{f\text{arctanh}(\cos(c+dx))}{d^2}-\frac{2}{3}\left(\frac{f\cos(c+dx)}{d^2}+\frac{(e+fx)\sin(c+dx)}{d}\right)-\frac{f\cos^3(c+dx)}{9d^2}-\frac{f\cos(c+dx)}{d^2}-\frac{(e+fx)\sin(c+dx)}{d}-\frac{(e+fx)\csc(c}{d}}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)\cos^5(c+dx)}{a+b\sin(c+dx)}dx}{a}+\frac{2i\int \frac{e^{i(2c+2dx+\pi)}(e+fx)}{1+e^{i(2c+2dx+\pi)}}dx-\int (e+fx)\cos(c+dx)\sin(c+dx)dx+\frac{(e+fx)\cos^4(c+dx)}{4d}-\frac{f\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\right.}{4}}{a}\right.}{a}$$

input

```
Int[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5054
```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[1/a   Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Simp[b/a   I
nt[(e + f*x)^m*Cos[c + d*x]^(p + 1)*(Cot[c + d*x]^(n - 1)/(a + b*Sin[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5209 vs. $2(594) = 1188$.

Time = 17.61 (sec) , antiderivative size = 5210, normalized size of antiderivative = 8.13

method	result	size
risch	Expression too large to display	5210

input
```
int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1707 vs. $2(582) = 1164$.

Time = 0.34 (sec) , antiderivative size = 1707, normalized size of antiderivative = 2.66

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/4*(a^2*b^2*f*cos(d*x + c)^3 - 2*I*b^4*f*dilog(cos(d*x + c) + I*sin(d*x
+ c))*sin(d*x + c) + 2*I*b^4*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*
x + c) + 2*I*b^4*f*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 2*
I*b^4*f*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - a^2*b^2*f*cos
(d*x + c) + 4*(a^3*b + a*b^3)*d*f*x + 2*I*(a^4 - 2*a^2*b^2 + b^4)*f*dilog(
(I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) + 2*I*(a^4 - 2*a^2*b^2 + b^
4)*f*dilog((I*a*cos(d*x + c) - a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 2*I*(a^4 - 2*a
^2*b^2 + b^4)*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) + (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d*x + c) - 2*
I*(a^4 - 2*a^2*b^2 + b^4)*f*dilog((-I*a*cos(d*x + c) - a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1)*sin(d
*x + c) + 4*(a^3*b + a*b^3)*d*e - 4*(a^3*b*d*f*x + a^3*b*d*e)*cos(d*x + c)
^2 - 2*((a^4 - 2*a^2*b^2 + b^4)*d*e - (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(2*b
*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*s
in(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4)*d*e - (a^4 - 2*a^2*b^2 + b^4)*c*f
)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4)*d*e - (a^4 - 2*a^2*b^2 +
 b^4)*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 ...
```

Sympy [F]

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(e+fx)\cos^3{(c+dx)}\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral((e + f*x)*cos(c + d*x)**3*cot(c + d*x)**2/(a + b*sin(c + d*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(fx+e)\cos(dx+c)^3\cot(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((f*x + e)*cos(d*x + c)^3*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Hanged}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)^2*(e + f*x))/(a + b*sin(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{(fx+e)\cos\left(dx+c\right)^3\cot\left(dx+c\right)^2}{\sin\left(dx+c\right)b+a}dx$$

input
```
int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

3.348 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 96

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

output
```
-csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^2/
b^3/d-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-\frac{2\csc(c+dx)}{a}-\frac{2b\log(\sin(c+dx))}{a^2}+\frac{2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3}-\frac{2a\sin(c+dx)}{b^2}+\frac{\sin^2(c+dx)}{b}}{2d}$$

input
```
Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
((-2*Csc[c + d*x])/a - (2*b*Log[Sin[c + d*x]])/a^2 + (2*(a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3) - (2*a*Sin[c + d*x])/b^2 + Sin[c + d*x]^2/b
)/(2*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^5}{\sin(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\csc^2(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)^2}{a+b\sin(c+dx)}d(b\sin(c+dx))}{b^5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\csc^2(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^2(a+b\sin(c+dx))}d(b\sin(c+dx))}{b^3d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(-\frac{\csc(c+dx)b^3}{a^2}+\frac{\csc^2(c+dx)b^2}{a}+\sin(c+dx)b-a+\frac{\left(a^2-b^2\right)^2}{a^2(a+b\sin(c+dx))}\right)d(b\sin(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^4\log(b\sin(c+dx))}{a^2}+\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2}-\frac{b^3\csc(c+dx)}{a}-ab\sin(c+dx)+\frac{1}{2}b^2\sin^2(c+dx)}{b^3d}$$

input
```
Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

output
```
(-((b^3*Csc[c + d*x])/a) - (b^4*Log[b*Sin[c + d*x]])/a^2 + ((a^2 - b^2)^2*
Log[a + b*Sin[c + d*x]])/a^2 - a*b*Sin[c + d*x] + (b^2*Sin[c + d*x]^2)/2)/
(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{-\frac{b\sin(dx+c)^2}{2}+a\sin(dx+c)}{b^2}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{b^3a^2}-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}}{d}$
default	$\frac{-\frac{-\frac{b\sin(dx+c)^2}{2}+a\sin(dx+c)}{b^2}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{b^3a^2}-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}}{d}$
risch	$-\frac{ix\,a^2}{b^3}+\frac{2ix}{b}-\frac{e^{2i(dx+c)}}{8bd}+\frac{ia\,e^{i(dx+c)}}{2b^2d}-\frac{ia\,e^{-i(dx+c)}}{2b^2d}-\frac{e^{-2i(dx+c)}}{8bd}-\frac{2ia^2c}{b^3d}+\frac{4ic}{bd}-\frac{2ie^{i(dx+c)}}{da(e^{2i(dx+c)}-1)}$

input
```
int(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*b*sin(d*x+c)^2+a*sin(d*x+c))+1/b^3*(a^4-2*a^2*b^2+b^4)/a
^2*ln(a+b*sin(d*x+c))-1/a/sin(d*x+c)-b/a^2*ln(sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.39

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$=\frac{4\,a^3b\cos{(dx+c)}^2-4\,b^4\log\left(\frac{1}{2}\,\sin{(dx+c)}\right)\sin{(dx+c)}-4\,a^3b-4\,ab^3+4\,(a^4-2\,a^2b^2+b^4)\log{(b\sin}}{4\,a^2b^3d\sin{(dx+c)}}$$

input
```
integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/4*(4*a^3*b*cos(d*x + c)^2 - 4*b^4*log(1/2*sin(d*x + c))*sin(d*x + c) - 4
*a^3*b - 4*a*b^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)*sin(d
*x + c) - (2*a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))/(a^2*b^3*d*si
n(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{2\,b\log(\sin(dx+c))}{a^2} - \frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2} + \frac{2}{a\sin(dx+c)} - \frac{2\,(a^4-2\,a^2b^2+b^4)\log(b\sin(dx+c)+a)}{a^2b^3}}{2\,d}$$

input
```
integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
-1/2*(2*b*log(sin(d*x + c))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^
2 + 2/(a*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)
/(a^2*b^3))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{b\log(|\sin(dx+c)|)}{a^2d} + \frac{bd\sin(dx+c)^2 - 2\,ad\sin(dx+c)}{2\,b^2d^2} - \frac{1}{ad\sin(dx+c)} + \frac{(a^4-2\,a^2b^2+b^4)\log(|b\sin(dx+c)+a|)}{a^2b^3d}$$

input
```
integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-b*log(abs(sin(d*x + c)))/(a^2*d) + 1/2*(b*d*sin(d*x + c)^2 - 2*a*d*sin(d*
x + c))/(b^2*d^2) - 1/(a*d*sin(d*x + c)) + (a^4 - 2*a^2*b^2 + b^4)*log(abs
(b*sin(d*x + c) + a))/(a^2*b^3*d)
```

Mupad [B] (verification not implemented)

Time = 42.90 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.43

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-b^2\right)^2}{a^2\,b^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^2-2\,b^2\right)}{b^3\,d} - \frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+b^2\right)}{b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(4\,a^2+b^2\right)}{b^2}-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{b}+1}{d\,\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input
```
int((cos(c + d*x)^3*cot(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

output
```
(log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^2)/(
a^2*b^3*d) - tan(c/2 + (d*x)/2)/(2*a*d) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(
a^2 - 2*b^2))/(b^3*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d) - ((2*tan(c/2
+ (d*x)/2)^2*(2*a^2 + b^2))/b^2 + (tan(c/2 + (d*x)/2)^4*(4*a^2 + b^2))/b^2
 - (4*a*tan(c/2 + (d*x)/2)^3)/b + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*tan(
c/2 + (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^5))
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cos(dx+c)^3\cot(dx+c)^2}{\sin(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

output
```
int(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file