Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.1-Sine/193-4.1.11

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**113**]. This is test number [193].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Fricas	100.00 (113)	0.00 (0)
Giac	62.83 (71)	37.17 (42)
Maxima	46.90 (53)	53.10 (60)
Sympy	23.01 (26)	76.99 (87)
Mupad	17.70 (20)	82.30 (93)
Reduce	17.70 (20)	82.30 (93)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

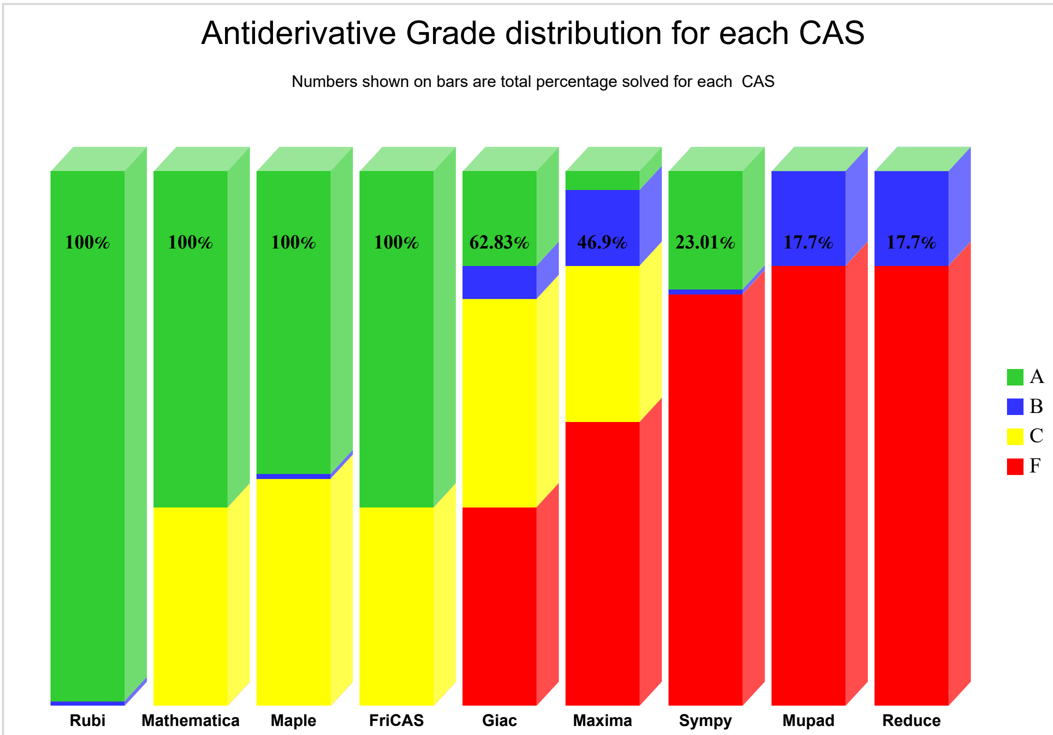
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

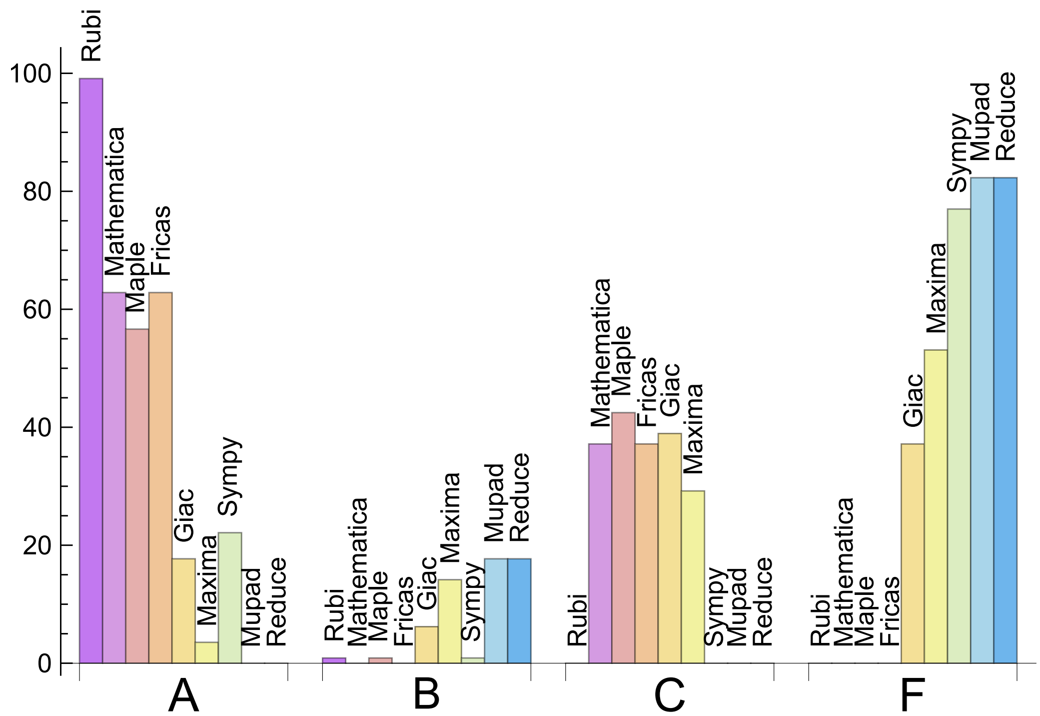
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.115	0.885	0.000	0.000
Mathematica	62.832	0.000	37.168	0.000
Fricas	62.832	0.000	37.168	0.000
Maple	56.637	0.885	42.478	0.000
Sympy	22.124	0.885	0.000	76.991
Giac	17.699	6.195	38.938	37.168
Maxima	3.540	14.159	29.204	53.097
Mupad	0.000	17.699	0.000	82.301
Reduce	0.000	17.699	0.000	82.301

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Giac	42	100.00	0.00	0.00
Maxima	60	100.00	0.00	0.00
Sympy	87	81.61	18.39	0.00
Mupad	93	0.00	100.00	0.00
Reduce	93	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Reduce	0.16
Giac	0.17
Sympy	0.81
Rubi	0.88
Maxima	1.04
Mathematica	1.16
Maple	1.30
Mupad	27.36

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	116.40	0.98	119.50	0.95
Reduce	125.55	1.03	124.00	0.99
Sympy	144.08	1.32	142.50	1.23
Mathematica	214.36	0.82	145.00	0.77
Maxima	249.96	2.85	164.00	1.80
Fricas	274.20	0.97	161.00	0.94
Maple	287.19	1.15	180.00	1.06
Rubi	321.51	1.05	181.00	1.00
Giac	2408.20	13.34	834.00	8.46

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

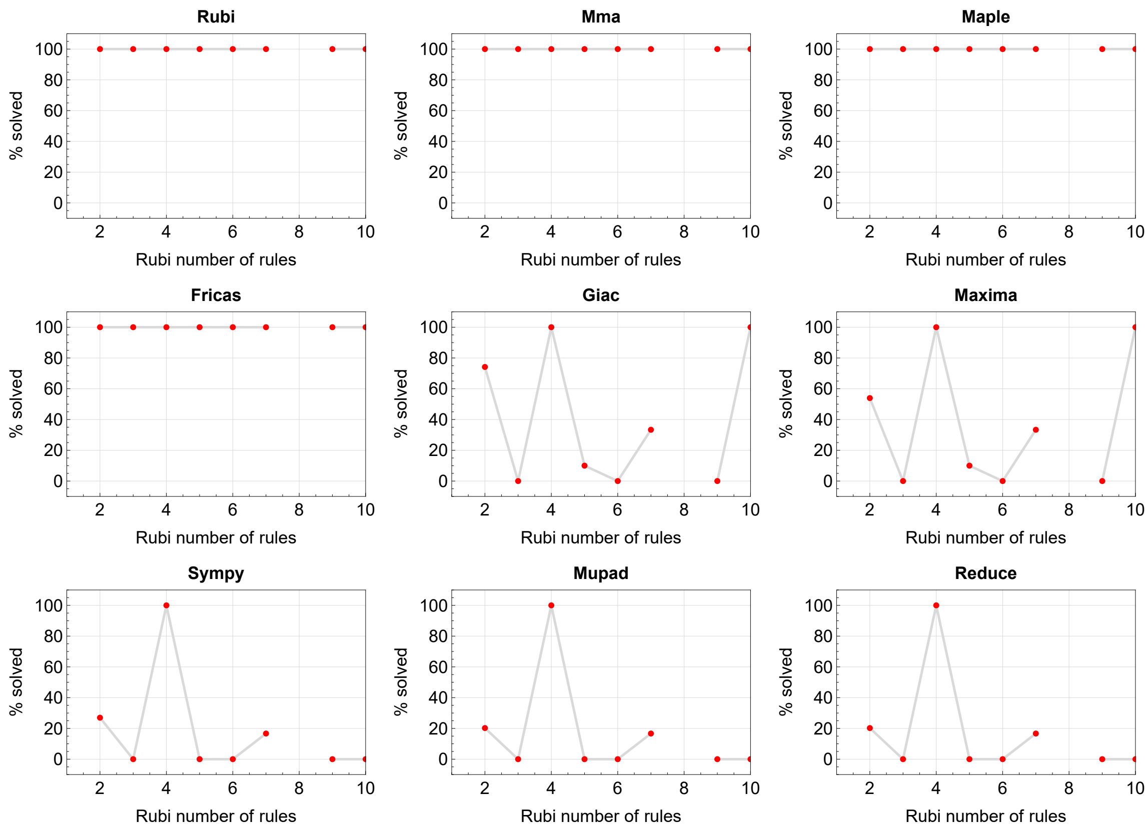


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

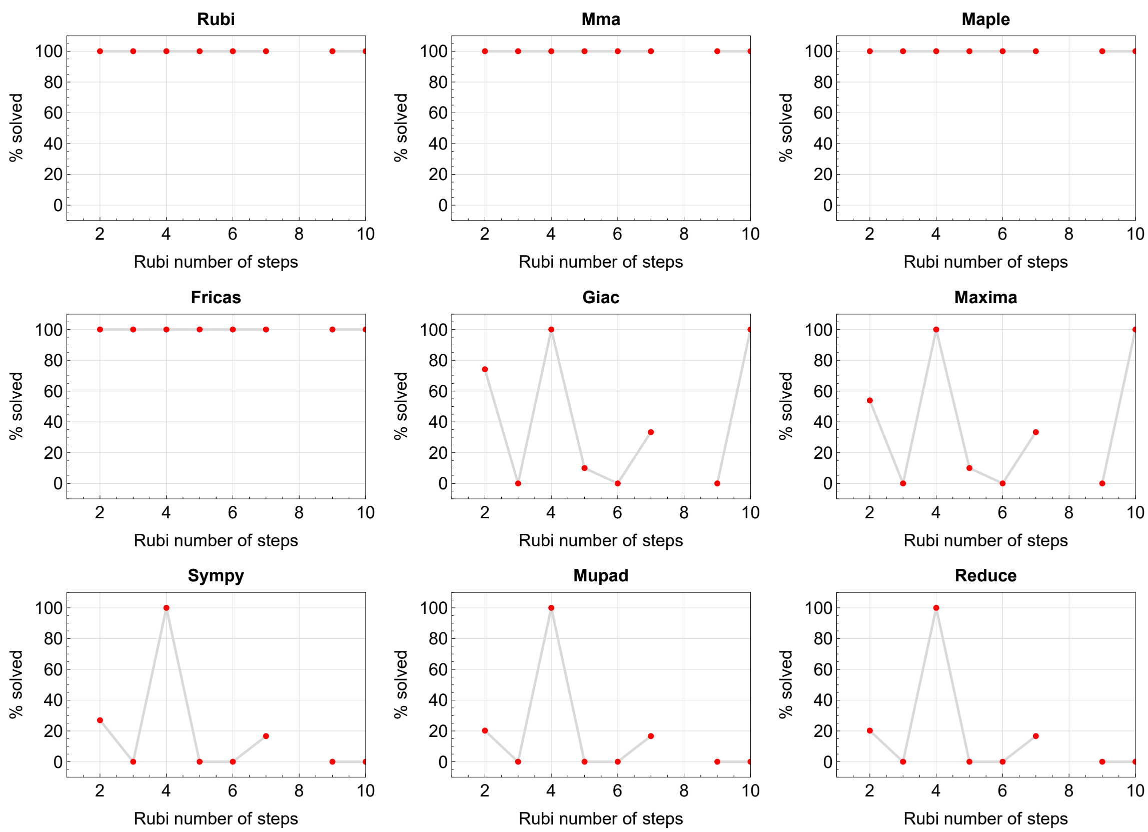


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

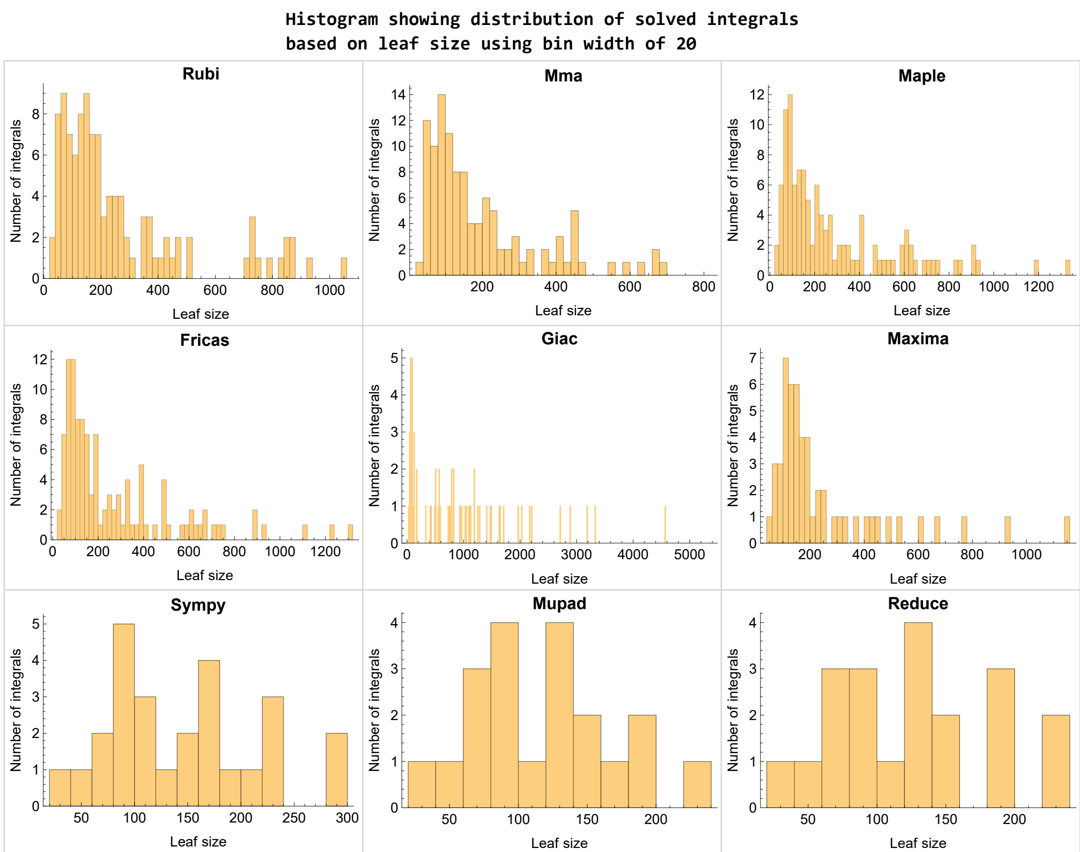


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

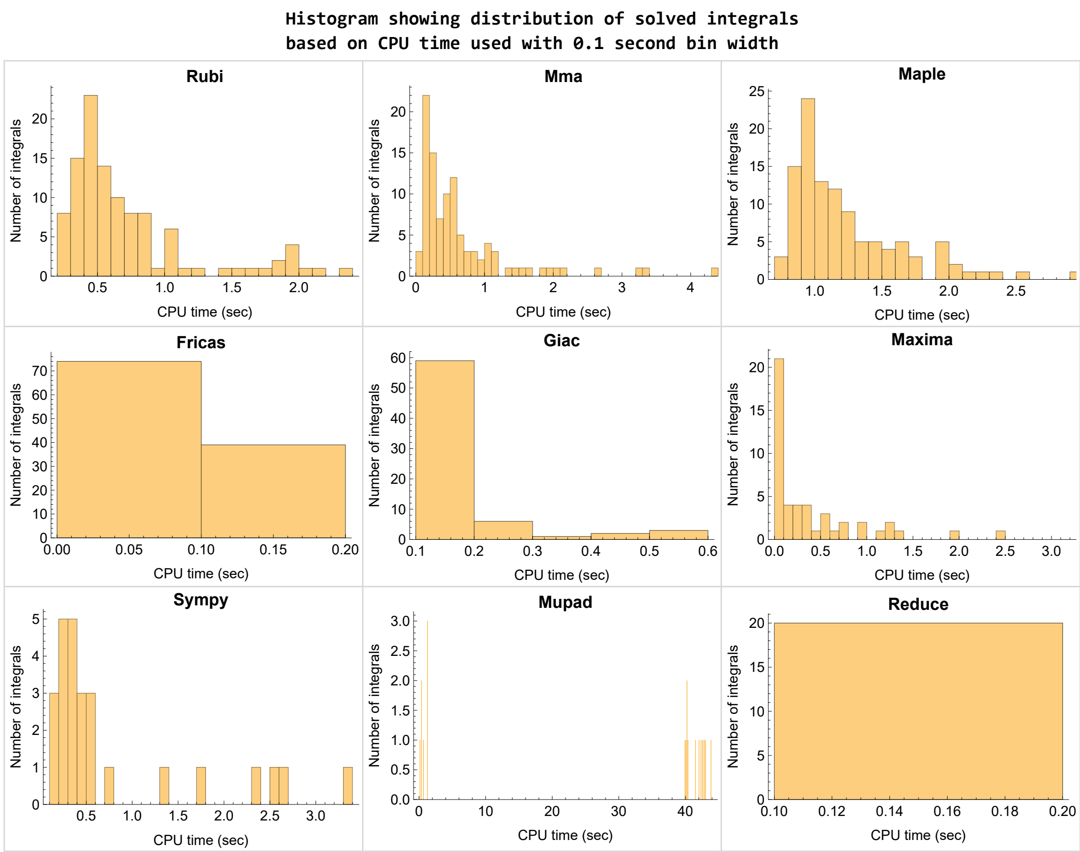


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

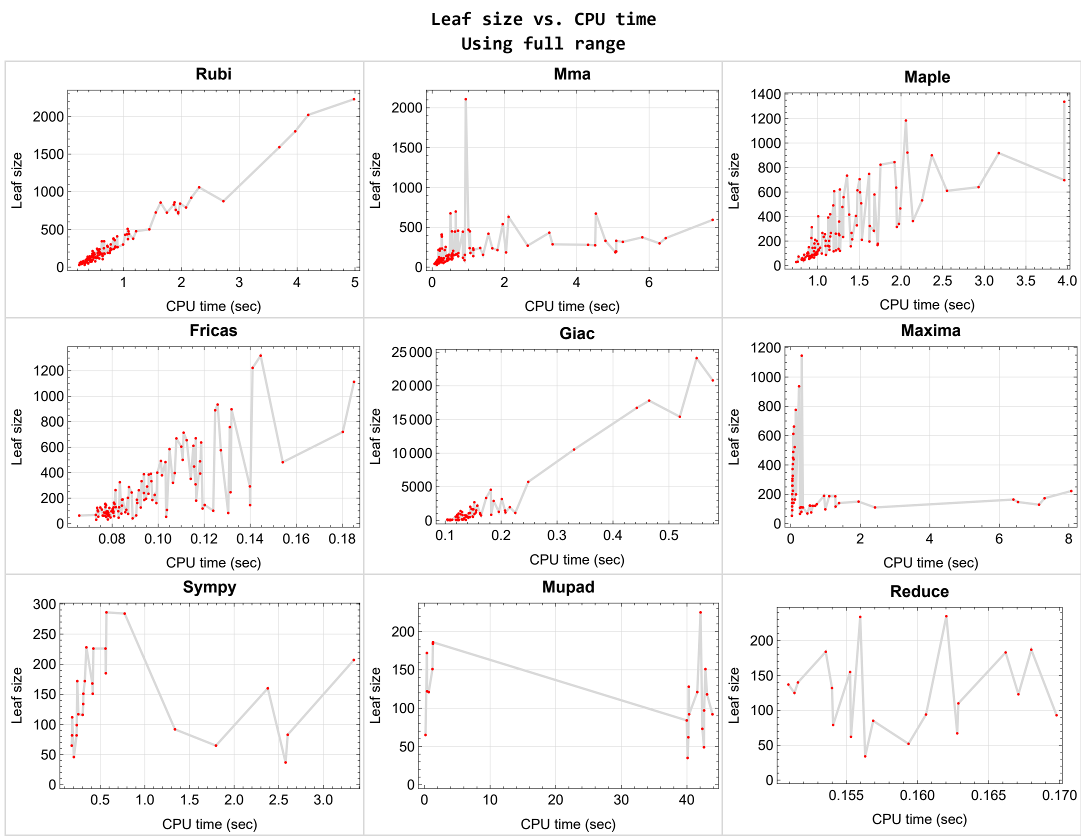


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {111}

Maple {18, 83, 90, 91}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

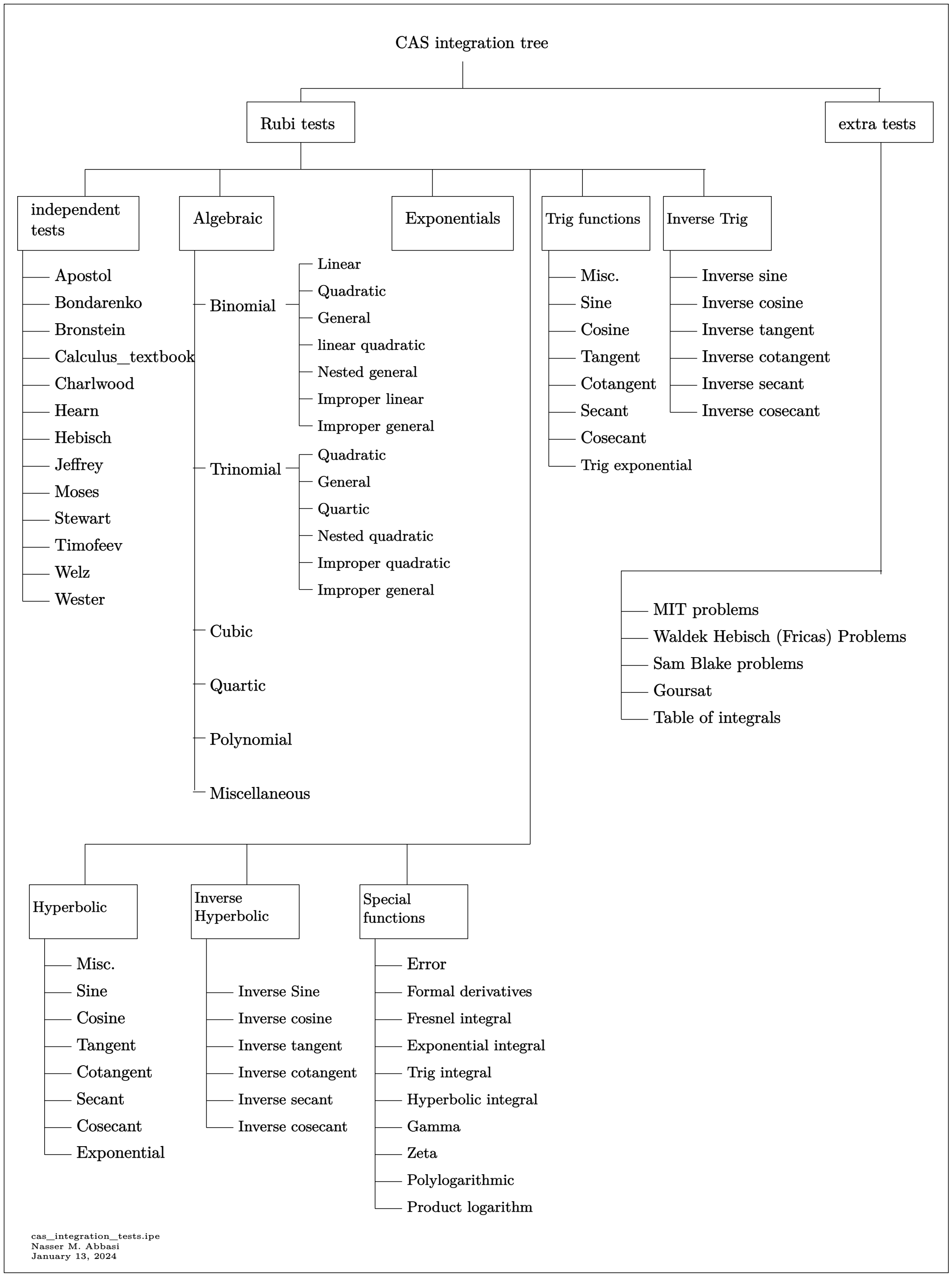
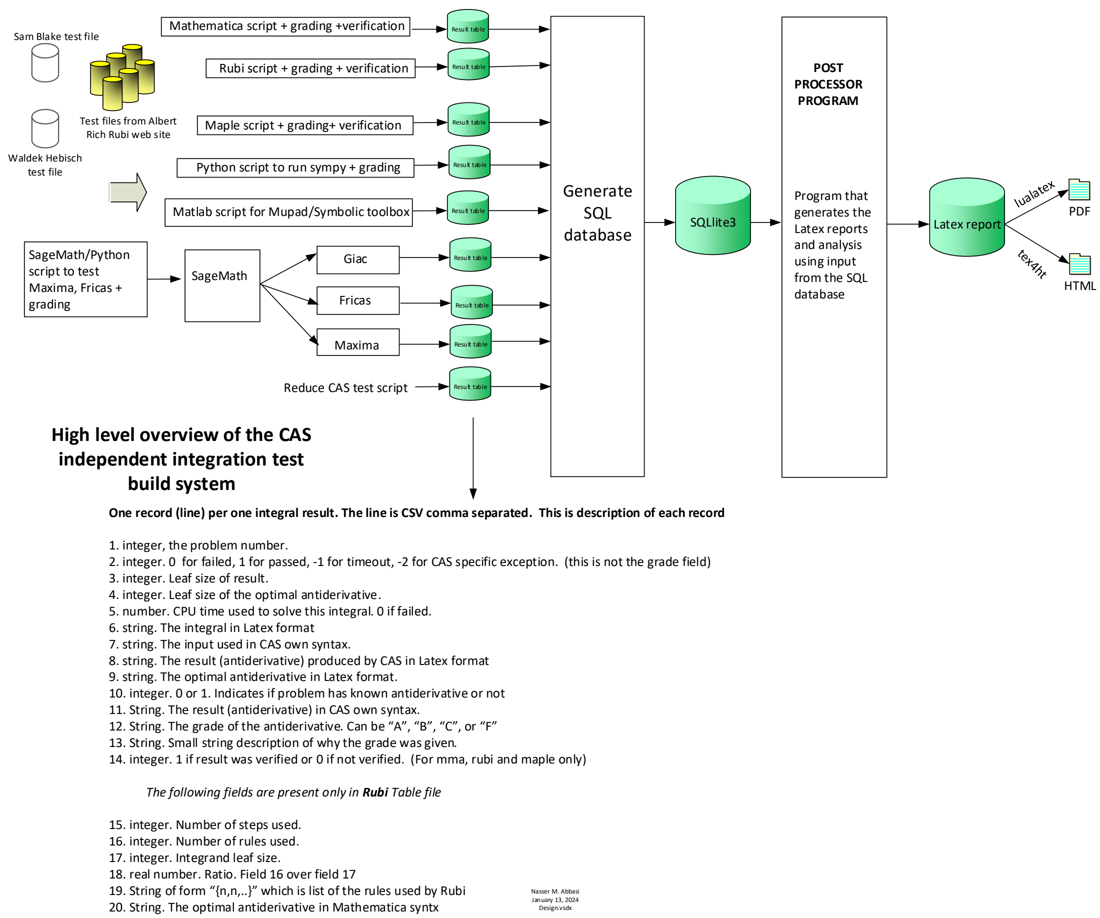


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113 }

B grade { 109 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { }

C grade { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 61, 62, 63, 64, 69, 70, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 92, 93 }

B grade { 35 }

C grade { 18, 19, 20, 21, 26, 27, 28, 29, 33, 34, 52, 57, 58, 59, 60, 65, 66, 67, 68, 71, 72, 73, 74, 77, 83, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { }

C grade { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 4, 11, 43 }

B grade { 1, 2, 10, 12, 40, 41, 42, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 21, 22, 30, 36, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F normal fail { 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

B grade { 26, 27, 28, 29, 30, 31, 32 }

C grade { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F normal fail { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 10, 11, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade { 12 }

C grade { }

F normal fail { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 76, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 113 }

F(-1) timedout fail { 72, 73, 74, 75, 77, 78, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	82	85	306	85	151	86	123	122
N.S.	1	1.00	0.65	0.67	2.43	0.67	1.20	0.68	0.98	0.97
time (sec)	N/A	0.492	0.255	0.935	0.048	0.078	0.312	0.119	0.167	0.394

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	65	67	201	67	117	68	93	92
N.S.	1	1.00	0.68	0.70	2.09	0.70	1.22	0.71	0.97	0.96
time (sec)	N/A	0.407	0.211	0.853	0.043	0.079	0.255	0.137	0.170	40.328

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	47	117	48	82	49	62	62
N.S.	1	1.00	0.69	0.72	1.80	0.74	1.26	0.75	0.95	0.95
time (sec)	N/A	0.296	0.174	0.842	0.039	0.079	0.185	0.123	0.155	40.211

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	53	30	46	31	34	35
N.S.	1	1.00	0.96	1.04	1.89	1.07	1.64	1.11	1.21	1.25
time (sec)	N/A	0.235	0.130	0.742	0.032	0.073	0.205	0.109	0.156	40.094

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	40	31	522	32	37	339	29	0
N.S.	1	1.00	1.38	1.07	18.00	1.10	1.28	11.69	1.00	0.00
time (sec)	N/A	0.327	0.123	0.760	0.113	0.078	2.577	0.134	0.165	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	56	108	54	0	569	102	0
N.S.	1	1.00	1.25	1.17	2.25	1.12	0.00	11.85	2.12	0.00
time (sec)	N/A	0.390	0.259	0.812	0.265	0.074	0.000	0.121	0.159	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	88	112	83	0	796	111	0
N.S.	1	1.00	0.85	0.99	1.26	0.93	0.00	8.94	1.25	0.00
time (sec)	N/A	0.447	0.401	0.837	0.292	0.130	0.000	0.131	0.169	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	117	111	108	0	961	110	0
N.S.	1	1.00	0.83	0.89	0.84	0.82	0.00	7.28	0.83	0.00
time (sec)	N/A	0.514	0.537	0.955	0.311	0.080	0.000	0.163	0.163	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	138	145	110	124	0	1108	111	0
N.S.	1	1.00	0.83	0.87	0.66	0.75	0.00	6.67	0.67	0.00
time (sec)	N/A	0.587	0.417	0.967	0.349	0.074	0.000	0.144	0.160	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	101	127	406	126	228	128	183	172
N.S.	1	1.00	0.54	0.68	2.18	0.68	1.23	0.69	0.98	0.92
time (sec)	N/A	0.553	0.255	1.013	0.061	0.092	0.345	0.104	0.166	0.368

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	87	94	259	95	172	95	132	128
N.S.	1	1.00	0.64	0.70	1.92	0.70	1.27	0.70	0.98	0.95
time (sec)	N/A	0.415	0.199	0.963	0.048	0.078	0.244	0.122	0.154	40.275

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	52	57	61	141	63	112	65	85	84
N.S.	1	1.04	1.14	1.22	2.82	1.26	2.24	1.30	1.70	1.68
time (sec)	N/A	0.332	0.190	0.911	0.037	0.076	0.186	0.108	0.157	39.968

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	79	80	61	92	551	58	0
N.S.	1	1.00	0.82	1.27	1.29	0.98	1.48	8.89	0.94	0.00
time (sec)	N/A	0.362	0.312	0.875	0.306	0.078	1.337	0.138	0.169	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	74	122	84	0	743	130	0
N.S.	1	1.00	0.89	1.03	1.69	1.17	0.00	10.32	1.81	0.00
time (sec)	N/A	0.426	0.257	0.909	0.564	0.081	0.000	0.164	0.168	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	95	114	189	110	0	1182	181	0
N.S.	1	1.00	0.79	0.94	1.56	0.91	0.00	9.77	1.50	0.00
time (sec)	N/A	0.526	0.455	0.996	0.957	0.076	0.000	0.155	0.172	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	154	158	187	145	0	1400	168	0
N.S.	1	1.00	0.88	0.90	1.07	0.83	0.00	8.00	0.96	0.00
time (sec)	N/A	0.672	0.535	1.135	1.106	0.077	0.000	0.128	0.161	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	204	201	186	180	0	1712	181	0
N.S.	1	1.00	0.82	0.81	0.75	0.73	0.00	6.90	0.73	0.00
time (sec)	N/A	0.773	0.582	1.136	1.282	0.117	0.000	0.147	0.161	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	158	328	0	188	0	3337	178	0
N.S.	1	1.00	0.72	1.50	0.00	0.86	0.00	15.31	0.82	0.00
time (sec)	N/A	0.768	0.660	1.470	0.000	0.084	0.000	0.173	0.157	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	117	237	0	141	0	2709	111	0
N.S.	1	1.00	0.77	1.56	0.00	0.93	0.00	17.82	0.73	0.00
time (sec)	N/A	0.552	0.553	1.151	0.000	0.084	0.000	0.152	0.163	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	99	87	180	0	108	0	2205	59	0
N.S.	1	1.11	0.98	2.02	0.00	1.21	0.00	24.78	0.66	0.00
time (sec)	N/A	0.471	0.303	0.960	0.000	0.086	0.000	0.158	0.153	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	150	776	78	0	1647	36	0
N.S.	1	1.00	0.91	2.17	11.25	1.13	0.00	23.87	0.52	0.00
time (sec)	N/A	0.371	0.163	0.894	0.144	0.084	0.000	0.151	0.165	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	73	141	63	0	597	16	0
N.S.	1	1.00	0.96	1.43	2.76	1.24	0.00	11.71	0.31	0.00
time (sec)	N/A	0.388	0.130	0.770	0.080	0.066	0.000	0.149	0.158	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	99	0	76	0	838	20	0
N.S.	1	1.00	0.86	1.36	0.00	1.04	0.00	11.48	0.27	0.00
time (sec)	N/A	0.448	0.184	0.874	0.000	0.087	0.000	0.182	0.153	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	144	0	118	0	2897	22	0
N.S.	1	1.00	0.89	1.26	0.00	1.04	0.00	25.41	0.19	0.00
time (sec)	N/A	0.581	0.415	0.938	0.000	0.119	0.000	0.187	0.164	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	202	0	186	0	4565	22	0
N.S.	1	1.00	0.93	1.07	0.00	0.98	0.00	24.15	0.12	0.00
time (sec)	N/A	0.728	0.726	1.096	0.000	0.094	0.000	0.182	0.162	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	177	734	0	287	0	1973	30	0
N.S.	1	1.00	0.76	3.15	0.00	1.23	0.00	8.47	0.13	0.00
time (sec)	N/A	0.791	1.058	1.352	0.000	0.087	0.000	0.216	0.154	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	181	153	607	0	246	0	1474	30	0
N.S.	1	1.05	0.89	3.53	0.00	1.43	0.00	8.57	0.17	0.00
time (sec)	N/A	0.665	0.904	1.198	0.000	0.131	0.000	0.207	0.167	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	117	402	0	202	0	1120	30	0
N.S.	1	1.00	0.79	2.70	0.00	1.36	0.00	7.52	0.20	0.00
time (sec)	N/A	0.635	0.852	1.006	0.000	0.087	0.000	0.225	0.157	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	96	313	0	155	0	951	28	0
N.S.	1	1.00	0.77	2.52	0.00	1.25	0.00	7.67	0.23	0.00
time (sec)	N/A	0.552	0.545	0.929	0.000	0.077	0.000	0.152	0.157	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	66	107	164	96	0	518	27	0
N.S.	1	1.06	0.92	1.49	2.28	1.33	0.00	7.19	0.38	0.00
time (sec)	N/A	0.511	0.352	0.883	0.119	0.074	0.000	0.150	0.161	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	210	0	189	0	1281	31	0
N.S.	1	1.00	0.93	1.41	0.00	1.27	0.00	8.60	0.21	0.00
time (sec)	N/A	0.638	1.138	0.969	0.000	0.085	0.000	0.196	0.159	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	184	256	0	262	0	3180	33	0
N.S.	1	1.00	0.98	1.36	0.00	1.39	0.00	16.91	0.18	0.00
time (sec)	N/A	0.761	2.044	1.087	0.000	0.092	0.000	0.201	0.157	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	235	705	0	388	0	16724	41	0
N.S.	1	1.00	0.89	2.66	0.00	1.46	0.00	63.11	0.15	0.00
time (sec)	N/A	0.898	1.134	1.503	0.000	0.094	0.000	0.443	0.164	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	154	621	0	325	0	15410	41	0
N.S.	1	1.00	0.64	2.58	0.00	1.35	0.00	63.94	0.17	0.00
time (sec)	N/A	0.831	1.407	1.266	0.000	0.083	0.000	0.519	0.154	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	157	419	0	263	0	10535	39	0
N.S.	1	1.00	0.88	2.34	0.00	1.47	0.00	58.85	0.22	0.00
time (sec)	N/A	0.625	0.651	1.150	0.000	0.081	0.000	0.330	0.157	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	105	87	145	199	164	0	5727	38	0
N.S.	1	1.01	0.84	1.39	1.91	1.58	0.00	55.07	0.37	0.00
time (sec)	N/A	0.639	0.895	1.029	0.150	0.081	0.000	0.249	0.159	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	449	359	0	391	0	17806	42	0
N.S.	1	1.00	1.72	1.38	0.00	1.50	0.00	68.22	0.16	0.00
time (sec)	N/A	0.858	1.044	1.257	0.000	0.097	0.000	0.465	0.159	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	540	405	0	500	0	20808	44	0
N.S.	1	1.00	1.81	1.35	0.00	1.67	0.00	69.59	0.15	0.00
time (sec)	N/A	0.992	1.951	1.459	0.000	0.111	0.000	0.578	0.166	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	630	466	0	585	0	24116	0	0
N.S.	1	1.00	1.67	1.24	0.00	1.55	0.00	63.97	0.00	0.00
time (sec)	N/A	1.166	2.114	1.992	0.000	0.105	0.000	0.550	0.179	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	92	89	372	95	168	97	140	121
N.S.	1	1.00	0.65	0.63	2.64	0.67	1.19	0.69	0.99	0.86
time (sec)	N/A	0.440	0.228	0.985	0.052	0.081	0.412	0.106	0.152	0.645

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	75	78	258	77	134	79	110	97
N.S.	1	1.00	0.68	0.70	2.32	0.69	1.21	0.71	0.99	0.87
time (sec)	N/A	0.370	0.134	0.903	0.046	0.078	0.310	0.108	0.163	42.612

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	57	54	165	60	99	60	79	73
N.S.	1	1.00	0.71	0.68	2.06	0.75	1.24	0.75	0.99	0.91
time (sec)	N/A	0.305	0.101	0.897	0.043	0.077	0.237	0.125	0.154	42.344

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	43	91	41	65	42	52	49
N.S.	1	1.00	0.77	0.81	1.72	0.77	1.23	0.79	0.98	0.92
time (sec)	N/A	0.244	0.066	0.829	0.043	0.089	0.181	0.105	0.159	42.591

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	54	60	66	46	65	432	41	0
N.S.	1	1.00	1.32	1.46	1.61	1.12	1.59	10.54	1.00	0.00
time (sec)	N/A	0.269	0.130	0.815	0.265	0.083	1.797	0.124	0.162	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	48	937	53	0	411	29	0
N.S.	1	1.00	1.00	1.09	21.30	1.20	0.00	9.34	0.66	0.00
time (sec)	N/A	0.282	0.094	0.807	0.239	0.103	0.000	0.141	0.152	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	73	123	62	0	766	125	0
N.S.	1	1.00	1.11	0.99	1.66	0.84	0.00	10.35	1.69	0.00
time (sec)	N/A	0.350	0.177	0.896	0.547	0.076	0.000	0.136	0.164	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	95	102	121	82	0	834	100	0
N.S.	1	1.00	0.90	0.96	1.14	0.77	0.00	7.87	0.94	0.00
time (sec)	N/A	0.423	0.159	0.993	0.635	0.077	0.000	0.122	0.158	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	131	121	101	0	1086	125	0
N.S.	1	1.00	0.84	0.88	0.81	0.68	0.00	7.29	0.84	0.00
time (sec)	N/A	0.468	0.191	1.008	0.700	0.124	0.000	0.162	0.157	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	139	160	612	154	286	162	235	186
N.S.	1	1.00	0.59	0.68	2.59	0.65	1.21	0.69	1.00	0.79
time (sec)	N/A	0.586	0.533	1.094	0.074	0.077	0.570	0.130	0.162	1.299

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	113	98	438	126	226	129	184	151
N.S.	1	1.00	0.61	0.53	2.37	0.68	1.22	0.70	0.99	0.82
time (sec)	N/A	0.492	0.254	1.049	0.081	0.075	0.424	0.132	0.154	42.814

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	86	94	292	97	172	99	137	118
N.S.	1	1.00	0.62	0.68	2.12	0.70	1.25	0.72	0.99	0.86
time (sec)	N/A	0.379	0.189	0.995	0.047	0.078	0.328	0.113	0.151	43.060

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	128	116	97	160	725	109	0
N.S.	1	1.00	0.74	1.15	1.05	0.87	1.44	6.53	0.98	0.00
time (sec)	N/A	0.388	0.428	1.141	1.289	0.083	2.377	0.138	0.159	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	156	97	96	0	1638	78	0
N.S.	1	1.00	1.00	1.61	1.00	0.99	0.00	16.89	0.80	0.00
time (sec)	N/A	0.360	0.281	1.139	0.994	0.080	0.000	0.149	0.150	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	124	150	107	0	1058	168	0
N.S.	1	1.00	0.87	1.09	1.32	0.94	0.00	9.28	1.47	0.00
time (sec)	N/A	0.419	0.486	1.219	1.951	0.104	0.000	0.142	0.158	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	114	120	140	116	0	1032	123	0
N.S.	1	1.00	0.85	0.90	1.04	0.87	0.00	7.70	0.92	0.00
time (sec)	N/A	0.456	0.448	1.267	1.389	0.080	0.000	0.124	0.169	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	157	222	129	0	1497	238	0
N.S.	1	1.00	0.69	0.89	1.25	0.73	0.00	8.46	1.34	0.00
time (sec)	N/A	0.609	0.548	1.397	8.069	0.077	0.000	0.156	0.162	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	269	323	0	240	0	0	79	0
N.S.	1	1.00	0.99	1.18	0.00	0.88	0.00	0.00	0.29	0.00
time (sec)	N/A	0.858	2.642	1.625	0.000	0.094	0.000	0.000	0.158	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	214	266	0	185	0	0	49	0
N.S.	1	1.00	1.02	1.27	0.00	0.89	0.00	0.00	0.23	0.00
time (sec)	N/A	0.570	1.806	1.406	0.000	0.097	0.000	0.000	0.165	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	216	260	0	195	0	0	38	0
N.S.	1	1.00	0.95	1.15	0.00	0.86	0.00	0.00	0.17	0.00
time (sec)	N/A	0.556	1.143	1.217	0.000	0.095	0.000	0.000	0.160	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	155	234	0	146	0	0	19	0
N.S.	1	1.00	0.88	1.32	0.00	0.82	0.00	0.00	0.11	0.00
time (sec)	N/A	0.461	0.462	1.072	0.000	0.120	0.000	0.000	0.161	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	163	225	0	187	0	0	18	0
N.S.	1	1.00	0.77	1.06	0.00	0.88	0.00	0.00	0.08	0.00
time (sec)	N/A	0.464	0.685	0.924	0.000	0.091	0.000	0.000	0.169	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	174	200	0	162	0	0	20	0
N.S.	1	1.00	0.88	1.02	0.00	0.82	0.00	0.00	0.10	0.00
time (sec)	N/A	0.590	0.561	0.990	0.000	0.091	0.000	0.000	0.148	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	238	266	0	234	0	0	22	0
N.S.	1	1.00	0.95	1.06	0.00	0.94	0.00	0.00	0.09	0.00
time (sec)	N/A	0.659	1.036	1.158	0.000	0.096	0.000	0.000	0.151	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	240	259	0	225	0	0	110	0
N.S.	1	1.00	0.89	0.96	0.00	0.83	0.00	0.00	0.41	0.00
time (sec)	N/A	0.726	1.328	1.253	0.000	0.099	0.000	0.000	0.165	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	476	298	532	0	351	0	0	32	0
N.S.	1	1.06	0.66	1.18	0.00	0.78	0.00	0.00	0.07	0.00
time (sec)	N/A	1.217	6.289	2.253	0.000	0.114	0.000	0.000	0.152	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	441	286	844	0	291	0	0	32	0
N.S.	1	1.02	0.66	1.96	0.00	0.68	0.00	0.00	0.07	0.00
time (sec)	N/A	1.094	3.331	1.922	0.000	0.095	0.000	0.000	0.186	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	429	274	748	0	333	0	0	32	0
N.S.	1	1.03	0.66	1.80	0.00	0.80	0.00	0.00	0.08	0.00
time (sec)	N/A	1.028	4.508	1.617	0.000	0.097	0.000	0.000	0.170	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	244	236	416	0	244	0	0	30	0
N.S.	1	1.02	0.99	1.74	0.00	1.02	0.00	0.00	0.13	0.00
time (sec)	N/A	0.542	1.664	1.378	0.000	0.088	0.000	0.000	0.171	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	280	491	0	333	0	0	29	0
N.S.	1	1.00	0.59	1.03	0.00	0.70	0.00	0.00	0.06	0.00
time (sec)	N/A	1.085	4.312	1.187	0.000	0.093	0.000	0.000	0.187	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	419	478	0	320	0	0	31	0
N.S.	1	1.00	0.96	1.10	0.00	0.74	0.00	0.00	0.07	0.00
time (sec)	N/A	1.060	1.556	1.296	0.000	0.106	0.000	0.000	0.171	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	330	614	0	400	0	0	33	0
N.S.	1	1.00	0.66	1.23	0.00	0.80	0.00	0.00	0.07	0.00
time (sec)	N/A	1.446	4.793	1.477	0.000	0.100	0.000	0.000	0.170	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	714	330	640	0	492	0	0	43	0
N.S.	1	1.50	0.69	1.34	0.00	1.03	0.00	0.00	0.09	0.00
time (sec)	N/A	1.944	5.092	2.931	0.000	0.101	0.000	0.000	0.180	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	759	364	900	0	604	0	0	43	0
N.S.	1	1.02	0.49	1.21	0.00	0.81	0.00	0.00	0.06	0.00
time (sec)	N/A	1.902	6.460	2.370	0.000	0.110	0.000	0.000	0.177	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	507	317	636	0	483	0	0	41	0
N.S.	1	0.99	0.62	1.24	0.00	0.94	0.00	0.00	0.08	0.00
time (sec)	N/A	1.070	5.274	1.942	0.000	0.103	0.000	0.000	0.167	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	856	374	598	0	611	0	0	40	0
N.S.	1	1.00	0.44	0.70	0.00	0.71	0.00	0.00	0.05	0.00
time (sec)	N/A	1.641	5.812	1.508	0.000	0.115	0.000	0.000	0.161	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	672	580	0	637	0	0	42	0
N.S.	1	1.00	0.92	0.79	0.00	0.87	0.00	0.00	0.06	0.00
time (sec)	N/A	1.945	4.527	1.681	0.000	0.119	0.000	0.000	0.161	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	875	875	593	922	0	714	0	0	44	0
N.S.	1	1.00	0.68	1.05	0.00	0.82	0.00	0.00	0.05	0.00
time (sec)	N/A	2.726	7.757	2.077	0.000	0.111	0.000	0.000	0.180	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	432	697	0	758	0	0	44	0
N.S.	1	1.00	0.55	0.88	0.00	0.96	0.00	0.00	0.06	0.00
time (sec)	N/A	2.080	3.239	3.960	0.000	0.131	0.000	0.000	0.170	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	101	106	449	104	185	106	155	151
N.S.	1	1.00	0.65	0.68	2.88	0.67	1.19	0.68	0.99	0.97
time (sec)	N/A	0.463	0.230	0.978	0.064	0.074	0.563	0.110	0.155	1.218

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	84	88	326	87	151	88	125	121
N.S.	1	1.00	0.67	0.70	2.59	0.69	1.20	0.70	0.99	0.96
time (sec)	N/A	0.431	0.142	0.932	0.057	0.076	0.416	0.122	0.151	41.573

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	69	224	68	116	69	94	92
N.S.	1	1.00	0.69	0.73	2.36	0.72	1.22	0.73	0.99	0.97
time (sec)	N/A	0.347	0.110	0.935	0.061	0.073	0.304	0.120	0.161	43.886

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	52	141	52	82	54	67	65
N.S.	1	1.00	0.74	0.76	2.07	0.76	1.21	0.79	0.99	0.96
time (sec)	N/A	0.263	0.088	0.832	0.055	0.077	0.236	0.110	0.163	0.157

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	98	76	58	83	510	57	0
N.S.	1	1.00	0.88	1.72	1.33	1.02	1.46	8.95	1.00	0.00
time (sec)	N/A	0.290	0.195	0.887	0.588	0.081	2.600	0.142	0.161	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	79	69	64	0	489	41	0
N.S.	1	1.00	1.00	1.41	1.23	1.14	0.00	8.73	0.73	0.00
time (sec)	N/A	0.297	0.126	0.922	0.478	0.090	0.000	0.144	0.170	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	1146	66	0	564	65	0
N.S.	1	1.00	0.94	0.93	16.37	0.94	0.00	8.06	0.93	0.00
time (sec)	N/A	0.310	0.146	0.956	0.315	0.078	0.000	0.119	0.179	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	104	87	132	89	0	796	138	0
N.S.	1	1.00	1.14	0.96	1.45	0.98	0.00	8.75	1.52	0.00
time (sec)	N/A	0.373	0.195	1.099	0.748	0.086	0.000	0.122	0.164	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	139	161	662	161	284	161	234	225
N.S.	1	1.00	0.59	0.69	2.82	0.69	1.21	0.69	1.00	0.96
time (sec)	N/A	0.579	0.391	1.125	0.092	0.099	0.774	0.133	0.156	42.069

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	112	124	489	129	226	131	187	184
N.S.	1	1.00	0.60	0.66	2.60	0.69	1.20	0.70	0.99	0.98
time (sec)	N/A	0.490	0.320	1.011	0.071	0.082	0.559	0.130	0.168	1.285

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	108	178	147	128	207	921	159	0
N.S.	1	1.00	0.67	1.11	0.91	0.80	1.29	5.72	0.99	0.00
time (sec)	N/A	0.482	0.562	1.722	6.537	0.083	3.341	0.128	0.156	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	214	129	127	0	2038	126	0
N.S.	1	1.00	1.00	1.48	0.89	0.88	0.00	14.06	0.87	0.00
time (sec)	N/A	0.462	0.418	1.414	7.142	0.081	0.000	0.145	0.163	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	138	210	110	123	0	2171	140	0
N.S.	1	1.00	0.97	1.48	0.77	0.87	0.00	15.29	0.99	0.00
time (sec)	N/A	0.414	0.443	1.530	2.423	0.079	0.000	0.157	0.171	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	135	196	173	145	0	1181	203	0
N.S.	1	1.00	0.89	1.30	1.15	0.96	0.00	7.82	1.34	0.00
time (sec)	N/A	0.472	0.721	1.621	7.302	0.140	0.000	0.208	0.169	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	148	167	164	153	0	1255	192	0
N.S.	1	1.00	0.89	1.00	0.98	0.92	0.00	7.51	1.15	0.00
time (sec)	N/A	0.500	0.595	1.718	6.405	0.081	0.000	0.130	0.171	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	231	558	0	397	0	0	49	0
N.S.	1	1.00	0.62	1.50	0.00	1.07	0.00	0.00	0.13	0.00
time (sec)	N/A	1.076	0.219	1.310	0.000	0.107	0.000	0.000	0.177	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	216	392	0	393	0	0	38	0
N.S.	1	1.00	0.61	1.10	0.00	1.10	0.00	0.00	0.11	0.00
time (sec)	N/A	0.848	0.174	1.126	0.000	0.118	0.000	0.000	0.176	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	186	266	0	292	0	0	21	0
N.S.	1	1.00	0.66	0.95	0.00	1.04	0.00	0.00	0.07	0.00
time (sec)	N/A	0.719	5.067	1.056	0.000	0.140	0.000	0.000	0.176	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	196	176	0	379	0	0	19	0
N.S.	1	1.00	0.57	0.51	0.00	1.10	0.00	0.00	0.06	0.00
time (sec)	N/A	0.624	5.091	1.005	0.000	0.102	0.000	0.000	0.170	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	196	85	0	385	0	0	18	0
N.S.	1	1.00	0.57	0.25	0.00	1.12	0.00	0.00	0.05	0.00
time (sec)	N/A	0.664	5.077	0.968	0.000	0.096	0.000	0.000	0.174	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	206	88	0	308	0	0	20	0
N.S.	1	1.00	0.68	0.29	0.00	1.02	0.00	0.00	0.07	0.00
time (sec)	N/A	0.778	0.211	0.959	0.000	0.116	0.000	0.000	0.165	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	233	116	0	448	0	0	22	0
N.S.	1	1.00	0.61	0.31	0.00	1.18	0.00	0.00	0.06	0.00
time (sec)	N/A	0.826	0.274	1.191	0.000	0.116	0.000	0.000	0.176	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	253	136	0	489	0	0	22	0
N.S.	1	1.00	0.62	0.33	0.00	1.20	0.00	0.00	0.05	0.00
time (sec)	N/A	0.890	0.373	1.250	0.000	0.118	0.000	0.000	0.210	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	725	383	1184	0	670	0	0	32	0
N.S.	1	1.02	0.54	1.66	0.00	0.94	0.00	0.00	0.04	0.00
time (sec)	N/A	1.557	0.268	2.059	0.000	0.116	0.000	0.000	0.219	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	374	214	823	0	482	0	0	32	0
N.S.	1	1.01	0.58	2.22	0.00	1.30	0.00	0.00	0.09	0.00
time (sec)	N/A	0.827	0.348	1.754	0.000	0.154	0.000	0.000	0.223	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	691	723	408	508	0	655	0	0	30	0
N.S.	1	1.05	0.59	0.74	0.00	0.95	0.00	0.00	0.04	0.00
time (sec)	N/A	1.749	0.266	1.522	0.000	0.112	0.000	0.000	0.212	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	735	830	406	248	0	669	0	0	29	0
N.S.	1	1.13	0.55	0.34	0.00	0.91	0.00	0.00	0.04	0.00
time (sec)	N/A	1.870	0.260	1.260	0.000	0.108	0.000	0.000	0.222	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	842	446	233	0	576	0	0	31	0
N.S.	1	1.22	0.64	0.34	0.00	0.83	0.00	0.00	0.04	0.00
time (sec)	N/A	1.978	0.631	1.305	0.000	0.127	0.000	0.000	0.197	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	920	445	284	0	720	0	0	33	0
N.S.	1	1.29	0.62	0.40	0.00	1.01	0.00	0.00	0.05	0.00
time (sec)	N/A	2.170	0.849	1.674	0.000	0.180	0.000	0.000	0.190	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	1059	470	315	0	898	0	0	33	0
N.S.	1	1.32	0.59	0.39	0.00	1.12	0.00	0.00	0.04	0.00
time (sec)	N/A	2.307	1.004	1.950	0.000	0.132	0.000	0.000	0.214	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	772	1592	457	1337	0	890	0	0	43	0
N.S.	1	2.06	0.59	1.73	0.00	1.15	0.00	0.00	0.06	0.00
time (sec)	N/A	3.693	0.716	3.960	0.000	0.125	0.000	0.000	0.239	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	777	860	449	918	0	935	0	0	43	0
N.S.	1	1.11	0.58	1.18	0.00	1.20	0.00	0.00	0.06	0.00
time (sec)	N/A	1.882	0.547	3.174	0.000	0.126	0.000	0.000	0.246	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1802	698	610	0	1319	0	0	41	0
N.S.	1	1.58	0.61	0.53	0.00	1.16	0.00	0.00	0.04	0.00
time (sec)	N/A	3.967	0.653	2.552	0.000	0.145	0.000	0.000	0.233	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1161	2020	675	340	0	1223	0	0	40	0
N.S.	1	1.74	0.58	0.29	0.00	1.05	0.00	0.00	0.03	0.00
time (sec)	N/A	4.192	0.506	1.975	0.000	0.141	0.000	0.000	0.237	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1163	2229	2109	363	0	1113	0	0	42	0
N.S.	1	1.92	1.81	0.31	0.00	0.96	0.00	0.00	0.04	0.00
time (sec)	N/A	4.983	0.930	2.143	0.000	0.185	0.000	0.000	0.235	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [36] had the largest ratio of [.7142859999999999976]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	2	2	1.00	15	0.133
3	A	2	2	1.00	13	0.154
4	A	4	4	1.00	12	0.333
5	A	2	2	1.00	15	0.133
6	A	2	2	1.00	15	0.133
7	A	2	2	1.00	15	0.133
8	A	2	2	1.00	15	0.133
9	A	2	2	1.00	15	0.133
10	A	2	2	1.00	17	0.118
11	A	2	2	1.00	15	0.133
12	A	7	7	1.04	14	0.500
13	A	2	2	1.00	17	0.118
14	A	2	2	1.00	17	0.118
15	A	2	2	1.00	17	0.118
16	A	2	2	1.00	17	0.118
17	A	2	2	1.00	17	0.118
18	A	2	2	1.00	17	0.118
19	A	2	2	1.00	17	0.118
20	A	2	2	1.11	17	0.118
21	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	5	1.00	14	0.357
23	A	2	2	1.00	17	0.118
24	A	2	2	1.00	17	0.118
25	A	2	2	1.00	17	0.118
26	A	2	2	1.00	17	0.118
27	A	2	2	1.05	17	0.118
28	A	2	2	1.00	17	0.118
29	A	2	2	1.00	15	0.133
30	A	7	7	1.06	14	0.500
31	A	2	2	1.00	17	0.118
32	A	2	2	1.00	17	0.118
33	A	2	2	1.00	17	0.118
34	A	2	2	1.00	17	0.118
35	A	2	2	1.00	15	0.133
36	A	10	10	1.01	14	0.714
37	A	2	2	1.00	17	0.118
38	A	2	2	1.00	17	0.118
39	A	2	2	1.00	17	0.118
40	A	2	2	1.00	17	0.118
41	A	2	2	1.00	17	0.118
42	A	2	2	1.00	15	0.133
43	A	2	2	1.00	14	0.143
44	A	2	2	1.00	17	0.118
45	A	2	2	1.00	17	0.118
46	A	2	2	1.00	17	0.118
47	A	2	2	1.00	17	0.118
48	A	2	2	1.00	17	0.118
49	A	2	2	1.00	19	0.105
50	A	2	2	1.00	17	0.118
51	A	2	2	1.00	16	0.125
52	A	2	2	1.00	19	0.105
53	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	19	0.105
55	A	2	2	1.00	19	0.105
56	A	2	2	1.00	19	0.105
57	A	2	2	1.00	19	0.105
58	A	2	2	1.00	19	0.105
59	A	2	2	1.00	19	0.105
60	A	2	2	1.00	17	0.118
61	A	2	2	1.00	16	0.125
62	A	2	2	1.00	19	0.105
63	A	2	2	1.00	19	0.105
64	A	2	2	1.00	19	0.105
65	A	5	5	1.06	19	0.263
66	A	5	5	1.02	19	0.263
67	A	5	5	1.03	19	0.263
68	A	3	3	1.02	17	0.176
69	A	2	2	1.00	16	0.125
70	A	2	2	1.00	19	0.105
71	A	2	2	1.00	19	0.105
72	A	9	9	1.50	19	0.474
73	A	6	6	1.02	19	0.316
74	A	3	3	0.99	17	0.176
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	19	0.105
77	A	2	2	1.00	19	0.105
78	A	2	2	1.00	19	0.105
79	A	2	2	1.00	17	0.118
80	A	2	2	1.00	17	0.118
81	A	2	2	1.00	15	0.133
82	A	2	2	1.00	14	0.143
83	A	2	2	1.00	17	0.118
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	17	0.118
87	A	2	2	1.00	17	0.118
88	A	2	2	1.00	16	0.125
89	A	2	2	1.00	19	0.105
90	A	2	2	1.00	19	0.105
91	A	2	2	1.00	19	0.105
92	A	2	2	1.00	19	0.105
93	A	2	2	1.00	19	0.105
94	A	2	2	1.00	19	0.105
95	A	2	2	1.00	19	0.105
96	A	2	2	1.00	19	0.105
97	A	2	2	1.00	17	0.118
98	A	2	2	1.00	16	0.125
99	A	2	2	1.00	19	0.105
100	A	2	2	1.00	19	0.105
101	A	2	2	1.00	19	0.105
102	A	5	5	1.02	19	0.263
103	A	3	3	1.01	19	0.158
104	A	5	5	1.05	17	0.294
105	A	5	5	1.13	16	0.312
106	A	5	5	1.22	19	0.263
107	A	5	5	1.29	19	0.263
108	A	5	5	1.32	19	0.263
109	B	7	7	2.06	19	0.368
110	A	6	6	1.11	19	0.316
111	A	7	7	1.58	17	0.412
112	A	7	7	1.74	16	0.438
113	A	7	7	1.92	19	0.368

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int x^3(a+bx)\sin(c+dx)\,dx = -\frac{24b\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{6a\sin(c+dx)}{d^4} - \frac{24bx\sin(c+dx)}{d^4}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}$$

output
```
-24*b*cos(d*x+c)/d^5+6*a*x*cos(d*x+c)/d^3+12*b*x^2*cos(d*x+c)/d^3-a*x^3*co
s(d*x+c)/d-b*x^4*cos(d*x+c)/d-6*a*sin(d*x+c)/d^4-24*b*x*sin(d*x+c)/d^4+3*a
*x^2*sin(d*x+c)/d^2+4*b*x^3*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int x^3(a+bx)\sin(c+dx)\,dx = \frac{-((ad^2x(-6+d^2x^2)+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+d(4bx(-6+d^2x^2)+3a(-2+d^2x^2))\sin(}{d^5}$$

input
```
Integrate[x^3*(a + b*x)*Sin[c + d*x],x]
```

output
```
(-((a*d^2*x*(-6 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x])
+ d*(4*b*x*(-6 + d^2*x^2) + 3*a*(-2 + d^2*x^2))*Sin[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)\sin(c+dx)\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(ax^3\sin(c+dx)+bx^4\sin(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{ax^3\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

input
```
Int[x^3*(a + b*x)*Sin[c + d*x],x]
```

output
```
(-24*b*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*
x])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (6*a*Sin[c + d
*x])/d^4 - (24*b*x*Sin[c + d*x])/d^4 + (3*a*x^2*Sin[c + d*x])/d^2 + (4*b*x
^3*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\left(b\,x^{4} d^{4}+a\,d^{4} x^{3}-12 x^{2} d^{2} b-6 a\,d^{2} x+24 b\right) \cos\left(d x+c\right)}{d^{5}}+\frac{\left(4 b\,d^{2} x^{3}+3 a\,d^{2} x^{2}-24 b x-6 a\right) \sin\left(d x+c\right)}{d^{4}}$
parallelrisch	$\frac{d^{2} x \left(x^{2} \left(b x+a\right) d^{2}-12 b x-6 a\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+6 d \left(x^{2} \left(\frac{4 b x}{3}+a\right) d^{2}-8 b x-2 a\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)+\left(-b\,x^{4}-a\,x^{3}\right) d^{4}+6 x \left(2 b x+a\right.}{d^{5} \left(1+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)}$
orering	$\frac{2 \left(4 b^{2} d^{4} x^{5}+7 a b\,d^{4} x^{4}+3 a^{2} d^{4} x^{3}-36 b^{2} d^{2} x^{3}-45 a b\,d^{2} x^{2}-12 x\,a^{2} d^{2}+48 b^{2} x+36 a b\right) \sin\left(d x+c\right)}{d^{6} x \left(b x+a\right)}-\frac{\left(b\,x^{4} d^{4}+a\,d^{4} x^{3}-12 x^{2}\right.}{}$
norman	$\frac{\frac{a\,x^{3} \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{b\,x^{4} \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}-\frac{48 b}{d^{5}}-\frac{12 a \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{4}}+\frac{6 a x}{d^{3}}-\frac{a\,x^{3}}{d}+\frac{12 b\,x^{2}}{d^{3}}-\frac{b\,x^{4}}{d}-\frac{6 a x \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d^{3}}+\frac{6 a\,x^{2} \tan\left(\frac{d}{2}\right.}{d^{2}}}{1+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}$
meijerg	$\frac{16 b \sin\left(c\right) \sqrt{\pi}\, \left(-\frac{x \left(d^{2}\right)^{\frac{5}{2}} \left(-\frac{5 x^{2} d^{2}}{2}+15\right) \cos\left(d x\right)}{10 \sqrt{\pi}\, d^{4}}+\frac{\left(d^{2}\right)^{\frac{5}{2}} \left(\frac{5}{8} x^{4} d^{4}-\frac{15}{2} x^{2} d^{2}+15\right) \sin\left(d x\right)}{10 \sqrt{\pi}\, d^{5}}\right)}{d^{4} \sqrt{d^{2}}}+\frac{16 b \cos\left(c\right) \sqrt{\pi}\, \left(\frac{3}{2 \sqrt{\pi}}-\frac{\left(\frac{3}{8} x^{4}\right.}{}\right.}{}$
parts	$-\frac{b\,x^{4} \cos\left(d x+c\right)}{d}-\frac{a\,x^{3} \cos\left(d x+c\right)}{d}+\frac{\frac{3 a\,c^{2} \sin\left(d x+c\right)}{d^{2}}-\frac{6 a c \left(\cos\left(d x+c\right)+\left(d x+c\right) \sin\left(d x+c\right)\right)}{d^{2}}+\frac{3 a \left(\left(d x+c\right)^{2} \sin\left(d x+c\right)-2 \sin\right.}{d}}{}$
derivativedivides	$\frac{a\,c^{3} \cos\left(d x+c\right)+3 a\,c^{2} \left(\sin\left(d x+c\right)-\cos\left(d x+c\right) \left(d x+c\right)\right)-3 a c \left(-\left(d x+c\right)^{2} \cos\left(d x+c\right)+2 \cos\left(d x+c\right)+2 \left(d x+c\right) \sin\left(d x+c\right)\right)}{}$
default	$\frac{a\,c^{3} \cos\left(d x+c\right)+3 a\,c^{2} \left(\sin\left(d x+c\right)-\cos\left(d x+c\right) \left(d x+c\right)\right)-3 a c \left(-\left(d x+c\right)^{2} \cos\left(d x+c\right)+2 \cos\left(d x+c\right)+2 \left(d x+c\right) \sin\left(d x+c\right)\right)}{}$

input
```
int(x^3*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d^4*x^4+a*d^4*x^3-12*b*d^2*x^2-6*a*d^2*x+24*b)/d^5*cos(d*x+c)+1/d^4*(4
*b*d^2*x^3+3*a*d^2*x^2-24*b*x-6*a)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int x^3(a+bx)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b)\cos(dx+c) - (4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad)\sin(dx}{d^5}$$

input
```
integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c) -
 (4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int x^3(a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \\ \left(\frac{ax^4}{4} + \frac{bx^5}{5}\right)\sin(c) \end{cases}$$

input
```
integrate(x**3*(b*x+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos
(c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**4*cos(c + d*x)/d + 4*b*x**3*
sin(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4
 - 24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*sin(c), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(126) = 252.

Time = 0.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.43

$$\int x^3(a+bx)\sin(c+dx)\,dx$$
$$=\frac{ac^3\cos(dx+c)-\frac{bc^4\cos(dx+c)}{d}-3\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac^2+\frac{4\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^3}{d}}{}$$

input
```
integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a*c^3*cos(d*x + c) - b*c^4*cos(d*x + c)/d - 3*((d*x + c)*cos(d*x + c) - s
in(d*x + c))*a*c^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d + 3
*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 6*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d - (((d*x +
c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a + 4
*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x +
 c))*b*c/d - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x +
 c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b/d)/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int x^3(a+bx)\sin(c+dx)\,dx=-\frac{(bd^4x^4+ad^4x^3-12\,bd^2x^2-6\,ad^2x+24\,b)\cos(dx+c)}{d^5}+\frac{(4\,bd^3x^3+3\,ad^3x^2-24\,bdx-6\,ad)\sin(dx+c)}{d^5}$$

input
```
integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c)/d^
5 + (4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int x^3(a+bx)\sin(c+dx)\,dx = \frac{6\,a\,x\,\cos\left(c+d\,x\right)+12\,b\,x^2\,\cos\left(c+d\,x\right)}{d^3} - \frac{6\,a\,\sin\left(c+d\,x\right)+24\,b\,x\,\sin\left(c+d\,x\right)}{d^4} - \frac{a\,x^3\,\cos\left(c+d\,x\right)+b\,x^4\,\cos\left(c+d\,x\right)}{d} + \frac{3\,a\,x^2\,\sin\left(c+d\,x\right)+4\,b\,x^3\,\sin\left(c+d\,x\right)}{d^2} - \frac{24\,b\,\cos\left(c+d\,x\right)}{d^5}$$

input
```
int(x^3*sin(c + d*x)*(a + b*x),x)
```

output
```
(6*a*x*cos(c + d*x) + 12*b*x^2*cos(c + d*x))/d^3 - (6*a*sin(c + d*x) + 24*
b*x*sin(c + d*x))/d^4 - (a*x^3*cos(c + d*x) + b*x^4*cos(c + d*x))/d + (3*a
*x^2*sin(c + d*x) + 4*b*x^3*sin(c + d*x))/d^2 - (24*b*cos(c + d*x))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)\sin(c+dx)\,dx = \frac{-\cos\left(dx+c\right)a\,d^4x^3+6\cos\left(dx+c\right)a\,d^2x-\cos\left(dx+c\right)b\,d^4x^4+12\cos\left(dx+c\right)b\,d^2x^2-24\cos\left(dx+c\right)}{d^5}$$

input
```
int(x^3*(b*x+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**4*x**3 + 6*cos(c + d*x)*a*d**2*x - cos(c + d*x)*b*d*
*4*x**4 + 12*cos(c + d*x)*b*d**2*x**2 - 24*cos(c + d*x)*b + 3*sin(c + d*x)
*a*d**3*x**2 - 6*sin(c + d*x)*a*d + 4*sin(c + d*x)*b*d**3*x**3 - 24*sin(c
+ d*x)*b*d*x)/d**5
```

3.2 $\int x^2(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int x^2(a+bx)\sin(c+dx)\,dx = \frac{2a\cos(c+dx)}{d^3}+\frac{6bx\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}$$

output
```
2*a*cos(d*x+c)/d^3+6*b*x*cos(d*x+c)/d^3-a*x^2*cos(d*x+c)/d-b*x^3*cos(d*x+c
)/d-6*b*sin(d*x+c)/d^4+2*a*x*sin(d*x+c)/d^2+3*b*x^2*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int x^2(a+bx)\sin(c+dx)\,dx = \frac{-d(bx(-6+d^2x^2)+a(-2+d^2x^2))\cos(c+dx)+(2ad^2x+3b(-2+d^2x^2))\sin(c+dx)}{d^4}$$

input
```
Integrate[x^2*(a + b*x)*Sin[c + d*x],x]
```

output
```
(-(d*(b*x*(-6 + d^2*x^2) + a*(-2 + d^2*x^2))*Cos[c + d*x]) + (2*a*d^2*x +
3*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(ax^2\sin(c+dx)+bx^3\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x)*Sin[c + d*x],x]
```

output
```
(2*a*Cos[c + d*x])/d^3 + (6*b*x*Cos[c + d*x])/d^3 - (a*x^2*Cos[c + d*x])/d
 - (b*x^3*Cos[c + d*x])/d - (6*b*Sin[c + d*x])/d^4 + (2*a*x*Sin[c + d*x])/
d^2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b\,d^2x^3+a\,d^2x^2-6bx-2a)\cos(dx+c)}{d^3}+\frac{(3x^2d^2b+2a\,d^2x-6b)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{d(x(bx+a)d^2-6b)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+((6b\,x^2+4ax)d^2-12b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-d(x^2(bx+a)d^2-6bx-4a)}{d^4\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
norman	$\frac{\frac{4a}{d^3}+\frac{a\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{b\,x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{a\,x^2}{d}-\frac{12b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6bx}{d^3}-\frac{b\,x^3}{d}+\frac{4ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{6bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}+\ldots}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
orering	$\frac{2(3b^2d^2x^4+5ab\,d^2x^3+2a^2d^2x^2-12x^2b^2-12abx-2a^2)\sin(dx+c)}{d^4x(bx+a)}-\frac{(b\,d^2x^3+a\,d^2x^2-6bx-2a)(2x(bx+a)\sin(dx+c)\ldots}{d^4x^2(bx+\ldots}$
parts	$-\frac{b\,x^3\cos(dx+c)}{d}-\frac{a\,x^2\cos(dx+c)}{d}+\frac{-\frac{2ac\sin(dx+c)}{d}+\frac{2a(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}+\frac{3b\,c^2\sin(dx+c)}{d^2}-\frac{6bc(\cos(dx\ldots}{\ldots}\ldots}{d^2\ldots}$
meijerg	$\frac{8b\sin(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2d^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{xd\left(-\frac{x^2d^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\cos(c)\sqrt{\pi}\left(\frac{xd\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-1\ldots\right.}{\ldots}\ldots\right.}{d^4}\ldots$
derivativedivides	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\ldots}{\ldots}$
default	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\ldots}{\ldots}$

input
```
int(x^2*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(b*d^2*x^3+a*d^2*x^2-6*b*x-2*a)*cos(d*x+c)+(3*b*d^2*x^2+2*a*d^2*x-6
*b)/d^4*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int x^2(a+bx)\sin(c+dx)\,dx = -\frac{(bd^3x^3+ad^3x^2-6\,bdx-2\,ad)\cos{(dx+c)}-(3\,bd^2x^2+2\,ad^2x-6\,b)\sin{(dx+c)}}{d^4}$$

input
```
integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c) - (3*b*d^2*x^2 +
2*a*d^2*x - 6*b)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int x^2(a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^2\cos{(c+dx)}}{d}+\frac{2ax\sin{(c+dx)}}{d^2}+\frac{2a\cos{(c+dx)}}{d^3}-\frac{bx^3\cos{(c+dx)}}{d}+\frac{3bx^2\sin{(c+dx)}}{d^2}+\frac{6bx\cos{(c+dx)}}{d^3}-\frac{6b\sin{(c+dx)}}{d^4} & \text{for} \\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\sin{(c)} & \text{oth} \end{cases}$$

input
```
integrate(x**2*(b*x+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c +
d*x)/d**3 - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos
(c + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/4)*
sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(96) = 192$.

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.09

$$\int x^2(a+bx)\sin(c+dx)\,dx =$$

$$-\frac{ac^2\cos(dx+c) - \frac{bc^3\cos(dx+c)}{d} - 2\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac + \frac{3\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)b}{d}}{}$$

input
```
integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*c^2*cos(d*x + c) - b*c^3*cos(d*x + c)/d - 2*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c + 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d + ((
(d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 3*(((d*x + c
)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d + (((d*x + c)^3 -
6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d)/d^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int x^2(a+bx)\sin(c+dx)\,dx = -\frac{(bd^3x^3 + ad^3x^2 - 6\,bdx - 2\,ad)\cos(dx+c)}{d^4} + \frac{(3\,bd^2x^2 + 2\,ad^2x - 6\,b)\sin(dx+c)}{d^4}$$

input
```
integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c)/d^4 + (3*b*d^2*x^2
 + 2*a*d^2*x - 6*b)*sin(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 40.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^2(a+bx)\sin(c+dx)\,dx = \frac{3\,b\,x^2\,\sin\left(c+d\,x\right)+2\,a\,x\,\sin\left(c+d\,x\right)}{d^2} + \frac{2\,a\,\cos\left(c+d\,x\right)+6\,b\,x\,\cos\left(c+d\,x\right)}{d^3} - \frac{a\,x^2\,\cos\left(c+d\,x\right)+b\,x^3\,\cos\left(c+d\,x\right)}{d} - \frac{6\,b\,\sin\left(c+d\,x\right)}{d^4}$$

input
```
int(x^2*sin(c + d*x)*(a + b*x),x)
```

output
```
(3*b*x^2*sin(c + d*x) + 2*a*x*sin(c + d*x))/d^2 + (2*a*cos(c + d*x) + 6*b*
x*cos(c + d*x))/d^3 - (a*x^2*cos(c + d*x) + b*x^3*cos(c + d*x))/d - (6*b*s
in(c + d*x))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int x^2(a+bx)\sin(c+dx)\,dx = \frac{-\cos\left(dx+c\right)a\,d^3x^2+2\cos\left(dx+c\right)ad-\cos\left(dx+c\right)b\,d^3x^3+6\cos\left(dx+c\right)bdx+2\sin\left(dx+c\right)a\,d^2x}{d^4}$$

input
```
int(x^2*(b*x+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**3*x**2 + 2*cos(c + d*x)*a*d - cos(c + d*x)*b*d**3*x*
*3 + 6*cos(c + d*x)*b*d*x + 2*sin(c + d*x)*a*d**2*x + 3*sin(c + d*x)*b*d**
2*x**2 - 6*sin(c + d*x)*b)/d**4
```

3.3 $\int x(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int x(a+bx)\sin(c+dx)\,dx=\frac{2b\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{2bx\sin(c+dx)}{d^2}$$

output
```
2*b*cos(d*x+c)/d^3-a*x*cos(d*x+c)/d-b*x^2*cos(d*x+c)/d+a*sin(d*x+c)/d^2+2*
b*x*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int x(a+bx)\sin(c+dx)\,dx=\frac{-((ad^2x+b(-2+d^2x^2))\cos(c+dx))+d(a+2bx)\sin(c+dx)}{d^3}$$

input
```
Integrate[x*(a + b*x)*Sin[c + d*x],x]
```

output
```
(-((a*d^2*x + b*(-2 + d^2*x^2))*Cos[c + d*x]) + d*(a + 2*b*x)*Sin[c + d*x]
)/d^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(ax\sin(c+dx)+bx^2\sin(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

input
```
Int[x*(a + b*x)*Sin[c + d*x],x]
```

output
```
(2*b*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (
a*Sin[c + d*x])/d^2 + (2*b*x*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\left(x^{2} d^{2} b+a\,d^{2} x-2 b\right) \cos \left(d x+c\right)}{d^{3}}+\frac{\left(2 b x+a\right) \sin \left(d x+c\right)}{d^{2}}$
parallelrisch	$\frac{\left(x \left(b x+a\right) d^{2}-4 b\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+2 d \left(2 b x+a\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-x \left(b x+a\right) d^{2}}{d^{3} \left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)}$
parts	$-\frac{b\,x^{2} \cos \left(d x+c\right)}{d}-\frac{a x \cos \left(d x+c\right)}{d}+\frac{a \sin \left(d x+c\right)-\frac{2 b c \sin \left(d x+c\right)}{d}+\frac{2 b \left(\cos \left(d x+c\right)+\left(d x+c\right) \sin \left(d x+c\right)\right)}{d}}{d^{2}}$
norman	$\frac{\frac{4 b}{d^{3}}+\frac{a x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{b\,x^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}-\frac{a x}{d}-\frac{b\,x^{2}}{d}+\frac{4 b x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}}{1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{2 \left(2 b x+a\right) \left(x^{2} d^{2} b+a\,d^{2} x-b\right) \sin \left(d x+c\right)}{d^{4} x \left(b x+a\right)}-\frac{\left(x^{2} d^{2} b+a\,d^{2} x-2 b\right) \left(\left(b x+a\right) \sin \left(d x+c\right)+x b \sin \left(d x+c\right)+x \left(b x+a\right) d \cos \left(d x+c\right.\right.}{d^{4} x \left(b x+a\right)}$
derivativedivides	$\frac{a c \cos \left(d x+c\right)+a \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)-\frac{b\,c^{2} \cos \left(d x+c\right)}{d}-\frac{2 b c \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)}{d}+\frac{b \left(-\left(d x+c\right)^{2} \cos \left(d x+c\right.\right.}{}}{d^{2}}$
default	$\frac{a c \cos \left(d x+c\right)+a \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)-\frac{b\,c^{2} \cos \left(d x+c\right)}{d}-\frac{2 b c \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)}{d}+\frac{b \left(-\left(d x+c\right)^{2} \cos \left(d x+c\right.\right.}{}}{d^{2}}$
meijerg	$\frac{4 b \sin \left(c\right) \sqrt{\pi}\, \left(\frac{x \left(d^{2}\right)^{\frac{3}{2}} \cos \left(d x\right)}{2 \sqrt{\pi}\, d^{2}}-\frac{\left(d^{2}\right)^{\frac{3}{2}} \left(-\frac{3 x^{2} d^{2}}{2}+3\right) \sin \left(d x\right)}{6 \sqrt{\pi}\, d^{3}}\right)}{d^{2} \sqrt{d^{2}}}+\frac{4 b \cos \left(c\right) \sqrt{\pi}\, \left(-\frac{1}{2 \sqrt{\pi}}+\frac{\left(-\frac{x^{2} d^{2}}{2}+1\right) \cos \left(d x\right)}{2 \sqrt{\pi}}+\frac{x d \sin \left(d x\right.}{2 \sqrt{\pi}}\right.}{d^{3}}$

input
```
int(x*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d^2*x^2+a*d^2*x-2*b)/d^3*cos(d*x+c)+1/d^2*(2*b*x+a)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int x(a+bx)\sin(c+dx)\,dx$$
$$= -\frac{(bd^2x^2+ad^2x-2\,b)\cos(dx+c)-(2\,bdx+ad)\sin(dx+c)}{d^3}$$

input
```
integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c) - (2*b*d*x + a*d)*sin(d*x + c))
/d^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int x(a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{ax\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2} + \frac{2b\cos(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**2*cos(c + d*x)
/d + 2*b*x*sin(c + d*x)/d**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x**2/
2 + b*x**3/3)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int x(a+bx)\sin(c+dx)\,dx$$

$$= \frac{ac\cos(dx+c) - \frac{bc^2\cos(dx+c)}{d} - ((dx+c)\cos(dx+c) - \sin(dx+c))a + \frac{2((dx+c)\cos(dx+c) - \sin(dx+c))bc}{d} - ((\cdots}{d^2}$$

input
```
integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a*c*cos(d*x + c) - b*c^2*cos(d*x + c)/d - ((d*x + c)*cos(d*x + c) - sin(d
*x + c))*a + 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c/d - (((d*x + c)
^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b/d)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int x(a+bx)\sin(c+dx)\,dx = -\frac{(bd^2x^2+ad^2x-2\,b)\cos(dx+c)}{d^3} + \frac{(2\,bdx+ad)\sin(dx+c)}{d^3}$$

input
```
integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c)/d^3 + (2*b*d*x + a*d)*sin(d*x +
c)/d^3
```

Mupad [B] (verification not implemented)

Time = 40.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int x(a+bx)\sin(c+dx)\,dx = \frac{a\,\sin(c+d\,x)+2\,b\,x\,\sin(c+d\,x)}{d^2} - \frac{a\,x\,\cos(c+d\,x)+b\,x^2\,\cos(c+d\,x)}{d} + \frac{2\,b\,\cos(c+d\,x)}{d^3}$$

input
```
int(x*sin(c + d*x)*(a + b*x),x)
```

output
```
(a*sin(c + d*x) + 2*b*x*sin(c + d*x))/d^2 - (a*x*cos(c + d*x) + b*x^2*cos(
c + d*x))/d + (2*b*cos(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int x(a+bx)\sin(c+dx)\,dx$$
$$=\frac{-\cos(dx+c)\,a\,d^2x-\cos(dx+c)\,b\,d^2x^2+2\cos(dx+c)\,b+\sin(dx+c)\,ad+2\sin(dx+c)\,bdx}{d^3}$$

input
```
int(x*(b*x+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**2*x - cos(c + d*x)*b*d**2*x**2 + 2*cos(c + d*x)*b +
sin(c + d*x)*a*d + 2*sin(c + d*x)*b*d*x)/d**3
```

3.4 $\int (a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (a+bx)\sin(c+dx)\,dx=-\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}$$

output
```
-(b*x+a)*cos(d*x+c)/d+b*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a+bx)\sin(c+dx)\,dx=\frac{-d(a+bx)\cos(c+dx)+b\sin(c+dx)}{d^2}$$

input
```
Integrate[(a + b*x)*Sin[c + d*x],x]
```

output
```
(-(d*(a + b*x)*Cos[c + d*x]) + b*Sin[c + d*x])/d^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+bx)\sin(c+dx)dx$$

$$\Big\downarrow \; 3777$$

$$\frac{b\int \cos(c+dx)dx}{d}-\frac{(a+bx)\cos(c+dx)}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(a+bx)\cos(c+dx)}{d}$$

$$\Big\downarrow \; 3117$$

$$\frac{b\sin(c+dx)}{d^2}-\frac{(a+bx)\cos(c+dx)}{d}$$

input
```
Int[(a + b*x)*Sin[c + d*x],x]
```

output
```
-(((a + b*x)*Cos[c + d*x])/d) + (b*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{(bx+a)\cos(dx+c)}{d}+\frac{b\sin(dx+c)}{d^2}$
parallelrisch	$\frac{-(bx+a)d\cos(dx+c)+ad+b\sin(dx+c)}{d^2}$
parts	$-\frac{bx\cos(dx+c)}{d}-\frac{a\cos(dx+c)}{d}+\frac{b\sin(dx+c)}{d^2}$
orering	$\frac{2b\sin(dx+c)}{d^2}-\frac{b\sin(dx+c)+(bx+a)d\cos(dx+c)}{d^2}$
derivativedivides	$\frac{-\cos(dx+c)a+\frac{bc\cos(dx+c)}{d}+\frac{b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}}{d}$
default	$\frac{-\cos(dx+c)a+\frac{bc\cos(dx+c)}{d}+\frac{b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}}{d}$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{bx}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
meijerg	$\frac{2b\sin(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\cos(c)\sqrt{\pi}\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sin(c)\sin(dx)}{d}+ a\cos(c)\sqrt{\pi}\cdots$

input
```
int((b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)*cos(d*x+c)/d+b*sin(d*x+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{(bdx+ad)\cos(dx+c)-b\sin(dx+c)}{d^2}$$

input
```
integrate((b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d*x + a*d)*cos(d*x + c) - b*sin(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx\cos(c+dx)}{d} + \frac{b\sin(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax+\frac{bx^2}{2}\right)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*cos(c + d*x)/d - b*x*cos(c + d*x)/d + b*sin(c + d*x)/d**2, N
e(d, 0)), ((a*x + b*x**2/2)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{a\cos(dx+c) - \frac{bc\cos(dx+c)}{d} + \frac{((dx+c)\cos(dx+c)-\sin(dx+c))b}{d}}{d}$$

input
```
integrate((b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*cos(d*x + c) - b*c*cos(d*x + c)/d + ((d*x + c)*cos(d*x + c) - sin(d*x
+ c))*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{(bdx+ad)\cos(dx+c)}{d^2} + \frac{b\sin(dx+c)}{d^2}$$

input
```
integrate((b*x+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d*x + a*d)*cos(d*x + c)/d^2 + b*sin(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 40.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (a+bx)\sin(c+dx)\,dx = \frac{b\,\sin(c+d\,x)}{d^2} - \frac{a\,\cos(c+d\,x)+b\,x\,\cos(c+d\,x)}{d}$$

input
```
int(sin(c + d*x)*(a + b*x),x)
```

output
```
(b*sin(c + d*x))/d^2 - (a*cos(c + d*x) + b*x*cos(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int (a+bx)\sin(c+dx)\,dx = \frac{-\cos(dx+c)\,ad-\cos(dx+c)\,bdx+\sin(dx+c)\,b}{d^2}$$

input
```
int((b*x+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d - cos(c + d*x)*b*d*x + sin(c + d*x)*b)/d**2
```

3.5 $\int \frac{(a+bx)\sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = -\frac{b\cos(c+dx)}{d} + a\,\mathrm{CosIntegral}(dx)\sin(c) + a\cos(c)\mathrm{Si}(dx)$$

output
```
-b*cos(d*x+c)/d+a*Ci(d*x)*sin(c)+a*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = -\frac{b\cos(c)\cos(dx)}{d} + a\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{b\sin(c)\sin(dx)}{d} + a\cos(c)\mathrm{Si}(dx)$$

input
```
Integrate[((a + b*x)*Sin[c + d*x])/x,x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c]*Sin[d*x])
/d + a*Cos[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a\sin(c+dx)}{x}+b\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\sin(c)\,\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)-\frac{b\cos(c+dx)}{d}$$

input
```
Int[((a + b*x)*Sin[c + d*x])/x,x]
```

output
```
-((b*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + a*Cos[c]*SinIntegral[d
*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

method	result
derivativedivides	$a(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))-\frac{b\cos(dx+c)}{d}$
default	$a(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))-\frac{b\cos(dx+c)}{d}$
risch	$\frac{ia\,\mathrm{e}^{ic}\,\mathrm{expIntegral}_1(-idx)}{2}-\frac{\mathrm{e}^{-ic}\pi\ \mathrm{csgn}(dx)a}{2}+\mathrm{e}^{-ic}\operatorname{Si}(dx)\,a-\frac{i\mathrm{e}^{-ic}\,\mathrm{expIntegral}_1(-idx)a}{2}-\frac{b\cos(dx+c)}{d}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d}+\frac{b\cos(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\ \operatorname{Ci}(dx)}{\sqrt{\pi}}\right)}{2}$

input
```
int((b*x+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-b*cos(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int\frac{(a+bx)\sin(c+dx)}{x}\,dx=\frac{ad\operatorname{Ci}(dx)\sin(c)+ad\cos(c)\operatorname{Si}(dx)-b\cos(dx+c)}{d}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

output
```
(a*d*cos_integral(d*x)*sin(c) + a*d*cos(c)*sin_integral(d*x) - b*cos(d*x +
 c))/d
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = -a(-\sin(c)\operatorname{Ci}(dx) - \cos(c)\operatorname{Si}(dx)) - b\left(\begin{cases} -x\sin(c) & \text{for } d=0 \\ \frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x+a)*sin(d*x+c)/x,x)
```

output
```
-a*(-sin(c)*Ci(d*x) - cos(c)*Si(d*x)) - b*Piecewise((-x*sin(c), Eq(d, 0)),
 (cos(c + d*x)/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 522, normalized size of antiderivative = 18.00

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/2*((I*exp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) +
(exp_integral_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*a + 1/2*((I
*exp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (exp_int
egral_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*b*c/d - 1/4*(2*(d*x
 + c)*(cos(c)^2 + sin(c)^2)*cos(d*x + c)^3 + 2*(d*x + c)*(cos(c)^2 + sin(c
)^2)*cos(d*x + c) - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*
x))*cos(c)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*co
s(c)*sin(c)^2 - c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x
))*sin(c)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos
(c) - (c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)
^2 + c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x)))*sin(c))
*cos(d*x + c)^2 - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x)
)*cos(c)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(
c)*sin(c)^2 - c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))
*sin(c)^3 - 2*(d*x + c)*(cos(c)^2 + sin(c)^2)*cos(d*x + c) + c*(exp_integr
al_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c) - (c*(I*exp_integral_e(
2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*exp_integral_e(2,
 I*d*x) - I*exp_integral_e(2, -I*d*x)))*sin(c))*sin(d*x + c)^2)*b/(((d*x +
 c)*(cos(c)^2 + sin(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*cos(d*x + c)^2
+ ((d*x + c)*(cos(c)^2 + sin(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*sin...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 11.69

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="giac")
```

output
```
-1/2*(a*d*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*i
mag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*sin_integ
ral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d*real_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*real_part(cos_integral(-d*x))*tan(1/2*d*
x)^2*tan(1/2*c) - a*d*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d*sin_integral(d*x)*tan(1
/2*d*x)^2 + a*d*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d*imag_part(
cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d*sin_integral(d*x)*tan(1/2*c)^2 +
2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d*real_part(cos_integral(d*x))*tan(1
/2*c) - 2*a*d*real_part(cos_integral(-d*x))*tan(1/2*c) - a*d*imag_part(cos
_integral(d*x)) + a*d*imag_part(cos_integral(-d*x)) - 2*a*d*sin_integral(d
*x) - 2*b*tan(1/2*d*x)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(1/2*c)^2
+ 2*b)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2
+ d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = a\,\mathrm{cosint}(d\,x)\,\sin(c) + a\,\mathrm{sinint}(d\,x)\,\cos(c) - \frac{b\,\cos(c+d\,x)}{d}$$

input
```
int((sin(c + d*x)*(a + b*x))/x,x)
```

output
```
a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) - (b*cos(c + d*x))/d
```

Reduce [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = \frac{-\cos(dx+c)\,b + \left(\int \frac{\sin(dx+c)}{x}dx\right)ad}{d}$$

input
```
int((b*x+a)*sin(d*x+c)/x,x)
```

output
```
( - cos(c + d*x)*b + int(sin(c + d*x)/x,x)*a*d)/d
```

3.6 $\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = ad\cos(c)\,\text{CosIntegral}(dx)+b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{x}+b\cos(c)\text{Si}(dx)-ad\sin(c)\text{Si}(dx)$$

output
```
a*d*cos(c)*Ci(d*x)+b*Ci(d*x)*sin(c)-a*sin(d*x+c)/x+b*cos(c)*Si(d*x)-a*d*si
n(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = -\frac{a\cos(dx)\sin(c)}{x}+b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\cos(c)\sin(dx)}{x}+b\cos(c)\text{Si}(dx) + ad(\cos(c)\,\text{CosIntegral}(dx)-\sin(c)\text{Si}(dx))$$

input
```
Integrate[((a + b*x)*Sin[c + d*x])/x^2,x]
```

output
```
-((a*Cos[d*x]*Sin[c])/x) + b*CosIntegral[d*x]*Sin[c] - (a*Cos[c]*Sin[d*x])
/x + b*Cos[c]*SinIntegral[d*x] + a*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*Sin
Integral[d*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a\sin(c+dx)}{x^2}+\frac{b\sin(c+dx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\cos(c)\,\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+b\sin(c)\,\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

input
```
Int[((a + b*x)*Sin[c + d*x])/x^2,x]
```

output
```
a*d*Cos[c]*CosIntegral[d*x] + b*CosIntegral[d*x]*Sin[c] - (a*Sin[c + d*x])
/x + b*Cos[c]*SinIntegral[d*x] - a*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)+\frac{b(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))}{d}\right)$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)+\frac{b(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))}{d}\right)$
risch	$\frac{ie^{ic}\operatorname{expIntegral}_1(-idx)b}{2}-\frac{a\,e^{ic}\operatorname{expIntegral}_1(-idx)d}{2}-\frac{ie^{-ic}\operatorname{expIntegral}_1(idx)b}{2}-\frac{a\,e^{-ic}\operatorname{expIntegral}_1(idx)d}{2}-$
meijerg	$\frac{b\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\operatorname{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos(c)\operatorname{Si}(dx)+\frac{a\sin(c)\sqrt{\pi}\,d^2\left(-\frac{4d^2}{x}\right.}{}$

input
```
int((b*x+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/d*b*(Si(d*x)*cos(c)
+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \frac{(adx\operatorname{Ci}(dx)+bx\operatorname{Si}(dx))\cos(c)-a\sin(dx+c)-(adx\operatorname{Si}(dx)-bx\operatorname{Ci}(dx))\sin(c)}{x}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
((a*d*x*cos_integral(d*x) + b*x*sin_integral(d*x))*cos(c) - a*sin(d*x + c)
 - (a*d*x*sin_integral(d*x) - b*x*cos_integral(d*x))*sin(c))/x
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx)\sin{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x+a)*sin(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x)*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$$
$$= \frac{((a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos{(c)}-a(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin{(c)})d^2-(b(-i\,\Gamma(-1,i\,dx)}{2\,dx}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
1/2*(((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a*(I*gamma(-1, I*
d*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 - (b*(-I*gamma(-1, I*d*x) + I*gamm
a(-1, -I*d*x))*cos(c) - b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*sin(c))*d
)*x - 2*b*cos(d*x + c))/(d*x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 569, normalized size of antiderivative = 11.85

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d
*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*x*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*imag_part(co
s_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*ta
n(1/2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*x*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*t
an(1/2*d*x)^2 - 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*
c) - 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x
*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(
-d*x))*tan(1/2*c)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + b*
x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*b*x*sin_integral(d*x)*t
an(1/2*d*x)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d*x*
imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1
/2*c) + b*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - b*x*imag_part(cos_
integral(-d*x))*tan(1/2*c)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*c)^2 - a*d*
x*real_part(cos_integral(d*x)) - a*d*x*real_part(cos_integral(-d*x)) - 2*b
*x*real_part(cos_integral(d*x))*tan(1/2*c) - 2*b*x*real_part(cos_integral(
-d*x))*tan(1/2*c) - 4*a*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*tan(1/2*d*x)*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^2}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x))/x^2,x)
```

output
```
int((sin(c + d*x)*(a + b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$$
$$= \frac{-\cos\left(dx+c\right)b+2\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2+x^2}dx\right)bx+2\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x+x}dx\right)a\,d^2x-\log(x)\,a\,d^2x-\sin\left(dx\right.}{dx}$$

input
```
int((b*x+a)*sin(d*x+c)/x^2,x)
```

output
```
( - cos(c + d*x)*b + 2*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**2 +
 x**2),x)*b*x + 2*int(1/(tan((c + d*x)/2)**2*x + x),x)*a*d**2*x - log(x)*a
*d**2*x - sin(c + d*x)*a*d + b)/(d*x)
```

3.7 $\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{ad\cos(c+dx)}{2x} + bd\cos(c)\operatorname{CosIntegral}(dx) - \frac{1}{2}ad^2\operatorname{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} - \frac{1}{2}ad^2\cos(c)\operatorname{Si}(dx) - bd\sin(c)\operatorname{Si}(dx)$$

output
```
-1/2*a*d*cos(d*x+c)/x+b*d*cos(c)*Ci(d*x)-1/2*a*d^2*Ci(d*x)*sin(c)-1/2*a*si
n(d*x+c)/x^2-b*sin(d*x+c)/x-1/2*a*d^2*cos(c)*Si(d*x)-b*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{adx\cos(c+dx) + dx^2\operatorname{CosIntegral}(dx)(-2b\cos(c) + ad\sin(c)) + a\sin(c+dx) + 2bx\sin(c+dx) + d}{2x^2}$$

input
```
Integrate[((a + b*x)*Sin[c + d*x])/x^3,x]
```

output
```
-1/2*(a*d*x*Cos[c + d*x] + d*x^2*CosIntegral[d*x]*(-2*b*Cos[c] + a*d*Sin[c
]) + a*Sin[c + d*x] + 2*b*x*Sin[c + d*x] + d*x^2*(a*d*Cos[c] + 2*b*Sin[c])
*SinIntegral[d*x])/x^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}ad^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\,\text{CosIntegral}(dx)-bd\sin(c)\text{Si}(dx)-\frac{b\sin(c+dx)}{x}$$

input
```
Int[((a + b*x)*Sin[c + d*x])/x^3,x]
```

output
```
-1/2*(a*d*Cos[c + d*x])/x + b*d*Cos[c]*CosIntegral[d*x] - (a*d^2*CosIntegr
al[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (b*Sin[c + d*x])/x - (a*d^2
*Cos[c]*SinIntegral[d*x])/2 - b*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result
derivativedivides	$d^{2}\left(a\left(-\frac{\sin(dx+c)}{2d^{2}x^{2}}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos\right.}{d}\right.$
default	$d^{2}\left(a\left(-\frac{\sin(dx+c)}{2d^{2}x^{2}}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos\right.}{d}\right.$
risch	$-\frac{\cos(c)\,\mathrm{expIntegral}_1(idx)bd}{2}+\frac{i\cos(c)\,\mathrm{expIntegral}_1(idx)a\,d^{2}}{4}-\frac{\cos(c)\,\mathrm{expIntegral}_1(-idx)bd}{2}-\frac{i\cos(c)\,\mathrm{expInteg}}{4}$
meijerg	$\frac{d^{2}b\sin(c)\sqrt{\pi}\left(-\frac{4d^{2}\cos\left(x\sqrt{d^{2}}\right)}{x\left(d^{2}\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\mathrm{Si}\left(x\sqrt{d^{2}}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^{2}}}+\frac{db\cos(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\right.}{4}$

input
```
int((b*x+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*
Ci(d*x)*sin(c))+1/d*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)\sin(c+dx)}{x^{3}}\,dx =$$
$$-\frac{adx\cos(dx+c)+\left(ad^{2}x^{2}\,\mathrm{Si}(dx)-2\,bdx^{2}\,\mathrm{Ci}(dx)\right)\cos(c)+(2\,bx+a)\sin(dx+c)+\left(ad^{2}x^{2}\,\mathrm{Ci}(dx)+\right.}{2\,x^{2}}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d*x*cos(d*x + c) + (a*d^2*x^2*sin_integral(d*x) - 2*b*d*x^2*cos_in
tegral(d*x))*cos(c) + (2*b*x + a)*sin(d*x + c) + (a*d^2*x^2*cos_integral(d
*x) + 2*b*d*x^2*sin_integral(d*x))*sin(c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx)\sin{(c+dx)}}{x^3}\,dx$$

input
```
integrate((b*x+a)*sin(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x)*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{((a(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos{(c)}-a(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin{(c)})d^3+2\,(b(\Gamma(-2,i}{2\,dx^2}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a*(gamma(-2
, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^3 + 2*(b*(gamma(-2, I*d*x) + gamma
(-2, -I*d*x))*cos(c) - b*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*sin(c)
)*d^2)*x^2 + 2*b*cos(d*x + c))/(d*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 796, normalized size of antiderivative = 8.94

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*d*x^2*real_part(cos_int
egral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*d*x^2*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2
 - 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 4*b*d*x^2*imag_part(cos_
integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*b*d*x^2*sin_integral(d*x)*tan(1/2*d
*x)^2*tan(1/2*c) + a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^2*x^2*sin_inte
gral(d*x)*tan(1/2*c)^2 + 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*
x)^2 + 2*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^2*x^
2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_inte
gral(-d*x))*tan(1/2*c) - 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)
^2 - 2*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d*x*tan(1/
2*d*x)^2*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2
*imag_part(cos_integral(-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) - 4*b*d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \int \frac{\sin(c+d\,x)\,(a+b\,x)}{x^3}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x))/x^3,x)
```

output
```
int((sin(c + d*x)*(a + b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \frac{-2\cos(dx+c)\,b-2\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2+x^2}dx\right)a\,d^2x^2-8\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^3+x^3}dx\right)b\,x^2-\sin(dx+c)\,ad-}{2d\,x^2}$$

input
```
int((b*x+a)*sin(d*x+c)/x^3,x)
```

output
```
( - 2*cos(c + d*x)*b - 2*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**2
 + x**2),x)*a*d**2*x**2 - 8*int(1/(tan((c + d*x)/2)**2*x**3 + x**3),x)*b*x
**2 - sin(c + d*x)*a*d - a*d**2*x - 2*b)/(2*d*x**2)
```

3.8 $\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 132

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = &-\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}\\ &-\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx)\\ &-\frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}\\ &-\frac{b\sin(c+dx)}{2x^2}+\frac{ad^2\sin(c+dx)}{6x}\\ &-\frac{1}{2}bd^2\cos(c)\text{Si}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)\end{aligned}$$

output
```
-1/6*a*d*cos(d*x+c)/x^2-1/2*b*d*cos(d*x+c)/x-1/6*a*d^3*cos(c)*Ci(d*x)-1/2*
b*d^2*Ci(d*x)*sin(c)-1/3*a*sin(d*x+c)/x^3-1/2*b*sin(d*x+c)/x^2+1/6*a*d^2*s
in(d*x+c)/x-1/2*b*d^2*cos(c)*Si(d*x)+1/6*a*d^3*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{adx\cos(c+dx)+3bdx^2\cos(c+dx)+d^2x^3\operatorname{CosIntegral}(dx)(ad\cos(c)+3b\sin(c))+2a\sin(c+dx)+}{6x^3}$$

input
```
Integrate[((a + b*x)*Sin[c + d*x])/x^4,x]
```

output
```
-1/6*(a*d*x*Cos[c + d*x] + 3*b*d*x^2*Cos[c + d*x] + d^2*x^3*CosIntegral[d*
x]*(a*d*Cos[c] + 3*b*Sin[c]) + 2*a*Sin[c + d*x] + 3*b*x*Sin[c + d*x] - a*d
^2*x^2*Sin[c + d*x] + d^2*x^3*(3*b*Cos[c] - a*d*Sin[c])*SinIntegral[d*x])/
x^3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{6}ad^3\cos(c)\operatorname{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\operatorname{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{2}bd^2\sin(c)\operatorname{CosIntegral}(dx)-\frac{1}{2}bd^2\cos(c)\operatorname{Si}(dx)-\frac{b\sin(c+dx)}{2x^2}-\frac{bd\cos(c+dx)}{2x}$$

input
```
Int[((a + b*x)*Sin[c + d*x])/x^4,x]
```

output
```
-1/6*(a*d*Cos[c + d*x])/x^2 - (b*d*Cos[c + d*x])/(2*x) - (a*d^3*Cos[c]*Cos
Integral[d*x])/6 - (b*d^2*CosIntegral[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(3
*x^3) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin[c + d*x])/(6*x) - (b*d^2*Cos
[c]*SinIntegral[d*x])/2 + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}\right.}{}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}\right.}{}\right.$
risch	$-\frac{i\,\mathrm{expIntegral}_1(-idx)\cos(c)b\,d^2}{4}+\frac{i\cos(c)\,\mathrm{expIntegral}_1(idx)b\,d^2}{4}+\frac{\mathrm{expIntegral}_1(-idx)\cos(c)a\,d^3}{12}+\cos(c)\,\mathrm{exp}$
meijerg	$\frac{d^2b\sin(c)\sqrt{\pi}\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6x^2d^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,x^2d^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,xd}-\frac{4\,\mathrm{Ci}(dx}{\sqrt{\pi}}\right.}{8}$

input
```
int((b*x+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+
1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d*b*(-1/2*sin(d*x+c)/d^2/x^2-1/2*
cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{(3\,bdx^2+adx)\cos{(dx+c)}+(ad^3x^3\,\mathrm{Ci}\,(dx)+3\,bd^2x^3\,\mathrm{Si}\,(dx))\cos{(c)}-(ad^2x^2-3\,bx-2\,a)\sin{(dx+}}{6\,x^3}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/6*((3*b*d*x^2 + a*d*x)*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) + 3*
b*d^2*x^3*sin_integral(d*x))*cos(c) - (a*d^2*x^2 - 3*b*x - 2*a)*sin(d*x +
c) - (a*d^3*x^3*sin_integral(d*x) - 3*b*d^2*x^3*cos_integral(d*x))*sin(c))
/x^3
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx)\sin{(c+dx)}}{x^4}\,dx$$

input
```
integrate((b*x+a)*sin(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x)*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^4-3\,(b(-i\,\Gamma(-3}{2\,dx^3}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3,
I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^4 - 3*(b*(-I*gamma(-3, I*d*x) + I*
gamma(-3, -I*d*x))*cos(c) - b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c
))*d^3)*x^3 + 2*b*cos(d*x + c))/(d*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 961, normalized size of antiderivative = 7.28

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

output
```
1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a
*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*
x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*
sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d^2*x^3*sin_integral(d*x)*tan
(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*
d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*
x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 6*b*d^2*x^3*r
eal_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d^3*x^3*real_pa
rt(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_part(cos_integral(-d*x
))*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2
+ 3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*s
in_integral(d*x)*tan(1/2*d*x)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))
*tan(1/2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d
^3*x^3*sin_integral(d*x)*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integral(d
*x))*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2
 + 6*b*d^2*x^3*sin_integral(d*x)*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_in
tegral(d*x)) - a*d^3*x^3*real_part(cos_integral(-d*x)) - 6*b*d^2*x^3*re...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = \int \frac{\sin(c+d\,x)\,(a+b\,x)}{x^4}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x))/x^4,x)
```

output
```
int((sin(c + d*x)*(a + b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$$
$$= \frac{-6\cos(dx+c)\,b+36\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^4+x^4}dx\right) b\,x^3+4\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^3+x^3}dx\right) a\,d^2x^3-2\sin(dx+c)\,ad}{6d\,x^3}$$

input
```
int((b*x+a)*sin(d*x+c)/x^4,x)
```

output
```
( - 6*cos(c + d*x)*b + 36*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**
4 + x**4),x)*b*x**3 + 4*int(1/(tan((c + d*x)/2)**2*x**3 + x**3),x)*a*d**2*
x**3 - 2*sin(c + d*x)*a*d + a*d**2*x + 6*b)/(6*d*x**3)
```

3.9 $\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 166

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = &-\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}\\&+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\,\text{CosIntegral}(dx)\\&+\frac{1}{24}ad^4\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}\\&-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{bd^2\sin(c+dx)}{6x}\\&+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{1}{6}bd^3\sin(c)\text{Si}(dx)\end{aligned}$$

output
```
-1/12*a*d*cos(d*x+c)/x^3-1/6*b*d*cos(d*x+c)/x^2+1/24*a*d^3*cos(d*x+c)/x-1/
6*b*d^3*cos(c)*Ci(d*x)+1/24*a*d^4*Ci(d*x)*sin(c)-1/4*a*sin(d*x+c)/x^4-1/3*
b*sin(d*x+c)/x^3+1/24*a*d^2*sin(d*x+c)/x^2+1/6*b*d^2*sin(d*x+c)/x+1/24*a*d
^4*cos(c)*Si(d*x)+1/6*b*d^3*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \frac{-2adx\cos(c+dx)-4bdx^2\cos(c+dx)+ad^3x^3\cos(c+dx)+d^3x^4\,\text{CosIntegral}(dx)(-4b\cos(c)+ad\sin$$

input
```
Integrate[((a + b*x)*Sin[c + d*x])/x^5,x]
```

output
```
(-2*a*d*x*Cos[c + d*x] - 4*b*d*x^2*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
 d^3*x^4*CosIntegral[d*x]*(-4*b*Cos[c] + a*d*Sin[c]) - 6*a*Sin[c + d*x] -
8*b*x*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + 4*b*d^2*x^3*Sin[c + d*x] + d
^3*x^4*(a*d*Cos[c] + 4*b*Sin[c])*SinIntegral[d*x])/(24*x^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}ad^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^3\cos(c+dx)}{24x}+\frac{ad^2\sin(c+dx)}{24x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}-\frac{1}{6}bd^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}bd^3\sin(c)\text{Si}(dx)+\frac{bd^2\sin(c+dx)}{6x}-\frac{b\sin(c+dx)}{3x^3}-\frac{bd\cos(c+dx)}{6x^2}$$

input
```
Int[((a + b*x)*Sin[c + d*x])/x^5,x]
```

output
```
-1/12*(a*d*Cos[c + d*x])/x^3 - (b*d*Cos[c + d*x])/(6*x^2) + (a*d^3*Cos[c +
 d*x])/(24*x) - (b*d^3*Cos[c]*CosIntegral[d*x])/6 + (a*d^4*CosIntegral[d*x
]*Sin[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(3*x^3) + (a*d^
2*Sin[c + d*x])/(24*x^2) + (b*d^2*Sin[c + d*x])/(6*x) + (a*d^4*Cos[c]*SinI
ntegral[d*x])/24 + (b*d^3*Sin[c]*SinIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

method	result
derivativedivides	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{b\left(-\frac{\sin(c}{3d}\right.}{}\right.$
default	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{b\left(-\frac{\sin(c}{3d}\right.}{}\right.$
risch	$\frac{\mathrm{expIntegral}_1(-idx)\cos(c)b\,d^3}{12}+\frac{\mathrm{expIntegral}_1(idx)\cos(c)b\,d^3}{12}+\frac{i\,\mathrm{expIntegral}_1(-idx)\cos(c)a\,d^4}{48}-\frac{i\,\mathrm{expIntegral}_1}{}$
meijerg	$\frac{d^4b\sin(c)\sqrt{\pi}\left(-\frac{8\left(-x^2d^2+2\right)d^2\cos\left(x\sqrt{d^2}\right)}{3x^3\left(d^2\right)^{\frac{5}{2}}\sqrt{\pi}}+\frac{8\sin\left(x\sqrt{d^2}\right)}{3x^2d^2\sqrt{\pi}}+\frac{8\,\mathrm{Si}\left(x\sqrt{d^2}\right)}{3\sqrt{\pi}}\right)}{16\sqrt{d^2}}+\frac{d^3b\cos(c)\sqrt{\pi}\left(-\frac{8}{\sqrt{\pi}\,x^2d^2}-\frac{4\left(2\gamma-\frac{11}{3}+2\right.}{3}\right.}{}$

input
```
int((b*x+a)*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*(a*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^
2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+1/d*b*(
-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(d
*x)*sin(c)-1/6*Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$
$$= \frac{(ad^3x^3 - 4\,bdx^2 - 2\,adx)\cos{(dx+c)} + (ad^4x^4\operatorname{Si}{(dx)} - 4\,bd^3x^4\operatorname{Ci}{(dx)})\cos{(c)} + (4\,bd^2x^3 + ad^2x^2 - 8\,}{24\,x^4}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
1/24*((a*d^3*x^3 - 4*b*d*x^2 - 2*a*d*x)*cos(d*x + c) + (a*d^4*x^4*sin_inte
gral(d*x) - 4*b*d^3*x^4*cos_integral(d*x))*cos(c) + (4*b*d^2*x^3 + a*d^2*x
^2 - 8*b*x - 6*a)*sin(d*x + c) + (a*d^4*x^4*cos_integral(d*x) + 4*b*d^3*x^
4*sin_integral(d*x))*sin(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx)\sin{(c+dx)}}{x^5}\,dx$$

input
```
integrate((b*x+a)*sin(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x)*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx =$$
$$-\frac{((a(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+a(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^5-4\,(b(\Gamma(-4,i\,dx}{2\,dx^4}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
-1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
 I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^5 - 4*(b*(gamma(-4, I*d*x) + gamma(
-4, -I*d*x))*cos(c) + b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*sin(c)
)*d^4)*x^4 + 2*b*cos(d*x + c))/(d*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.67

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

output
```
-1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
- a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real
_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part
(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*d^3*x^4*real_part(cos_in
tegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*
d*x)^2 - 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*b*d^3*x^4*imag_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*b*d^3*x^4*imag_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c) + a*d^4*x^4*imag_part(cos_integral(d*x))*tan
(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^4
*x^4*sin_integral(d*x)*tan(1/2*c)^2 + 4*b*d^3*x^4*real_part(cos_integral(d
*x))*tan(1/2*d*x)^2 + 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*
x)^2 - 2*a*d^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^4*x^4*r
eal_part(cos_integral(-d*x))*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_integr
al(d*x))*tan(1/2*c)^2 - 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*
c)^2 - 2*a*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_i
ntegral(d*x)) + a*d^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^5}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x))/x^5,x)
```

output
```
int((sin(c + d*x)*(a + b*x))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$
$$= \frac{-12\cos(dx+c)\,b-6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)a\,d^2x^4-96\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^5+x^5}dx\right)b\,x^4-3\sin(dx+c)\,a}{12d\,x^4}$$

input
```
int((b*x+a)*sin(d*x+c)/x^5,x)
```

output
```
( - 12*cos(c + d*x)*b - 6*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**
4 + x**4),x)*a*d**2*x**4 - 96*int(1/(tan((c + d*x)/2)**2*x**5 + x**5),x)*b
*x**4 - 3*sin(c + d*x)*a*d - a*d**2*x - 12*b)/(12*d*x**4)
```

3.10 $\int x^2(a+bx)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\begin{aligned}\int x^2(a+bx)^2 \sin(c+dx)\,dx = & -\frac{24b^2\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3} \\ & +\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3} \\ & -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d} \\ & -\frac{b^2x^4\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4} \\ & -\frac{24b^2x\sin(c+dx)}{d^4}+\frac{2a^2x\sin(c+dx)}{d^2} \\ & +\frac{6abx^2\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}\end{aligned}$$

output

```
-24*b^2*cos(d*x+c)/d^5+2*a^2*cos(d*x+c)/d^3+12*a*b*x*cos(d*x+c)/d^3+12*b^2
*x^2*cos(d*x+c)/d^3-a^2*x^2*cos(d*x+c)/d-2*a*b*x^3*cos(d*x+c)/d-b^2*x^4*co
s(d*x+c)/d-12*a*b*sin(d*x+c)/d^4-24*b^2*x*sin(d*x+c)/d^4+2*a^2*x*sin(d*x+c
)/d^2+6*a*b*x^2*sin(d*x+c)/d^2+4*b^2*x^3*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.54

$$\int x^2(a+bx)^2\sin(c+dx)\,dx$$
$$=\frac{-((2abd^2x(-6+d^2x^2)+a^2d^2(-2+d^2x^2)+b^2(24-12d^2x^2+d^4x^4))\cos(c+dx))+2d(a+2bx)\,(ad^2x}{d^5}$$

input
```
Integrate[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

output
```
(-((2*a*b*d^2*x*(-6 + d^2*x^2) + a^2*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2
*x^2 + d^4*x^4))*Cos[c + d*x]) + 2*d*(a + 2*b*x)*(a*d^2*x + b*(-6 + d^2*x^
2))*Sin[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^2\sin(c+dx)\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(a^2x^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^4\sin(c+dx)\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2a^2\cos(c+dx)}{d^3}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}-\frac{24b^2\cos(c+dx)}{d^5}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{12b^2x^2\cos(c+dx)}{d^3}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{b^2x^4\cos(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

output
```
(-24*b^2*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x])/d^3 + (12*a*b*x*Cos[c +
d*x])/d^3 + (12*b^2*x^2*Cos[c + d*x])/d^3 - (a^2*x^2*Cos[c + d*x])/d - (2*
a*b*x^3*Cos[c + d*x])/d - (b^2*x^4*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])
/d^4 - (24*b^2*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 + (6*a*b*x
^2*Sin[c + d*x])/d^2 + (4*b^2*x^3*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(b^2x^4d^4+2ab\,d^4x^3+a^2d^4x^2-12x^2d^2b^2-12ab\,d^2x-2a^2d^2+24b^2)\cos(dx+c)}{d^5}+\frac{2(2b^2d^2x^3+3ab\,d^2x^2+x\,a^2d^2-12b}{d^4}$
parallelrisch	$\frac{d^2x(x(bx+a)d^2-12b)(bx+a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4d(x(bx+a)d^2-6b)(2bx+a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-x^2(bx+a)^2d^4+4(3x^2b^2+3abx}{d^5\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$\frac{4(2b^3d^4x^5+5a\,b^2d^4x^4+4a^2b\,d^4x^3+a^3d^4x^2-18d^2x^3b^3-27a\,b^2d^2x^2-11a^2b\,d^2x-a^3d^2+24x\,b^3+12b^2a)\sin(dx+c)}{d^6x(bx+a)}-$
norman	$\frac{\frac{4a^2d^2-48b^2}{d^5}+\frac{b^2x^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{(a^2d^2-12b^2)x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}-\frac{b^2x^4}{d}-\frac{(a^2d^2-12b^2)x^2}{d^3}-\frac{24ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{8b^2x^3\tan(}{d^2}}{1+\tan\left(\frac{d}{2}\right.}$
parts	$-\frac{b^2x^4\cos(dx+c)}{d}-\frac{2ab\,x^3\cos(dx+c)}{d}-\frac{a^2x^2\cos(dx+c)}{d}+\frac{-\frac{2a^2c\sin(dx+c)}{d}+\frac{2a^2(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}+6}{}$
meijerg	$\frac{16b^2\sin(c)\sqrt{\pi}\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}x^4d^4-\frac{15}{2}x^2d^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b^2\cos(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{(3}{8}\right.}{}$
derivativedivides	$\frac{-a^2c^2\cos(dx+c)-2a^2c(\sin(dx+c)-\cos(dx+c)(dx+c))+a^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{}$
default	$\frac{-a^2c^2\cos(dx+c)-2a^2c(\sin(dx+c)-\cos(dx+c)(dx+c))+a^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{}$

input
```
int(x^2*(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^4*x^4+2*a*b*d^4*x^3+a^2*d^4*x^2-12*b^2*d^2*x^2-12*a*b*d^2*x-2*a^2*
d^2+24*b^2)/d^5*cos(d*x+c)+2/d^4*(2*b^2*d^2*x^3+3*a*b*d^2*x^2+a^2*d^2*x-12
*b^2*x-6*a*b)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = -\frac{(b^2d^4x^4+2\,abd^4x^3-12\,abd^2x-2\,a^2d^2+(a^2d^4-12\,b^2d^2)x^2+24\,b^2)\cos{(dx+c)}-2\,(2\,b^2d^3x^3+3\,ab}{d^5}$$

input
```
integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^4*x^4 + 2*a*b*d^4*x^3 - 12*a*b*d^2*x - 2*a^2*d^2 + (a^2*d^4 - 12*
b^2*d^2)*x^2 + 24*b^2)*cos(d*x + c) - 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 - 6
*a*b*d + (a^2*d^3 - 12*b^2*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.23

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = \begin{cases} -\frac{a^2x^2\cos{(c+dx)}}{d} + \frac{2a^2x\sin{(c+dx)}}{d^2} + \frac{2a^2\cos{(c+dx)}}{d^3} - \frac{2abx^3\cos{(c+dx)}}{d} + \frac{6abx^2\sin{(c+dx)}}{d^2} + \frac{12abx\cos{(c+dx)}}{d^3} - \frac{12ab\sin{(c+}}{d^4} \\ \left(\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}\right)\sin{(c)} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2
*cos(c + d*x)/d**3 - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d
**2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b**2*x**4*co
s(c + d*x)/d + 4*b**2*x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d
**3 - 24*b**2*x*sin(c + d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)),
((a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(186) = 372.

Time = 0.06 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.18

$$\int x^2(a+bx)^2\sin(c+dx)\,dx =$$

$$-\frac{a^2c^2\cos{(dx+c)} + \frac{b^2c^4\cos(dx+c)}{d^2} - \frac{2\,abc^3\cos(dx+c)}{d} - 2\left((dx+c)\cos{(dx+c)} - \sin{(dx+c)}\right)a^2c - \frac{4\,((dx+c)}{}}{}$$

input
```
integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a^2*c^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^2 - 2*a*b*c^3*cos(d*x + c)
/d - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 4*((d*x + c)*cos(d*
x + c) - sin(d*x + c))*b^2*c^3/d^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x +
 c))*a*b*c^2/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c
))*a^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2
*c^2/d^2 - 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
*b*c/d - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)
*sin(d*x + c))*b^2*c/d^2 + 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3
*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d + (((d*x + c)^4 - 12*(d*x + c)^2 +
24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2/d^2)/d^
3
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = -\frac{(b^2d^4x^4+2\,abd^4x^3+a^2d^4x^2-12\,b^2d^2x^2-12\,abd^2x-2\,a^2d^2+24\,b^2)\cos(dx+c)}{d^5} + \frac{2\,(2\,b^2d^3x^3+3\,abd^3x^2+a^2d^3x-12\,b^2dx-6\,abd)\sin(dx+c)}{d^5}$$

input
```
integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 12*b^2*d^2*x^2 - 12*a*b*d^2*
x - 2*a^2*d^2 + 24*b^2)*cos(d*x + c)/d^5 + 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^
2 + a^2*d^3*x - 12*b^2*d*x - 6*a*b*d)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)^2\sin(c+dx)\,dx=\frac{4\,b^2\,x^3\,\sin\left(c+d\,x\right)}{d^2}-\frac{b^2\,x^4\,\cos\left(c+d\,x\right)}{d}-\frac{2\,\cos\left(c+d\,x\right)\,\left(12\,b^2-a^2\,d^2\right)}{d^5}-\frac{12\,a\,b\,\sin\left(c+d\,x\right)}{d^4}-\frac{2\,x\,\sin\left(c+d\,x\right)\,\left(12\,b^2-a^2\,d^2\right)}{d^4}+\frac{x^2\,\cos\left(c+d\,x\right)\,\left(12\,b^2-a^2\,d^2\right)}{d^3}-\frac{2\,a\,b\,x^3\,\cos\left(c+d\,x\right)}{d}+\frac{6\,a\,b\,x^2\,\sin\left(c+d\,x\right)}{d^2}+\frac{12\,a\,b\,x\,\cos\left(c+d\,x\right)}{d^3}$$

input
```
int(x^2*sin(c + d*x)*(a + b*x)^2,x)
```

output
```
(4*b^2*x^3*sin(c + d*x))/d^2 - (b^2*x^4*cos(c + d*x))/d - (2*cos(c + d*x)*
(12*b^2 - a^2*d^2))/d^5 - (12*a*b*sin(c + d*x))/d^4 - (2*x*sin(c + d*x)*(1
2*b^2 - a^2*d^2))/d^4 + (x^2*cos(c + d*x)*(12*b^2 - a^2*d^2))/d^3 - (2*a*b
*x^3*cos(c + d*x))/d + (6*a*b*x^2*sin(c + d*x))/d^2 + (12*a*b*x*cos(c + d*
x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)^2\sin(c+dx)\,dx$$
$$=\frac{-\cos\left(dx+c\right)a^2d^4x^2+2\cos\left(dx+c\right)a^2d^2-2\cos\left(dx+c\right)ab\,d^4x^3+12\cos\left(dx+c\right)ab\,d^2x-\cos\left(dx+\right.}{}$$

input
```
int(x^2*(b*x+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**4*x**2 + 2*cos(c + d*x)*a**2*d**2 - 2*cos(c + d*x
)*a*b*d**4*x**3 + 12*cos(c + d*x)*a*b*d**2*x - cos(c + d*x)*b**2*d**4*x**4
 + 12*cos(c + d*x)*b**2*d**2*x**2 - 24*cos(c + d*x)*b**2 + 2*sin(c + d*x)*
a**2*d**3*x + 6*sin(c + d*x)*a*b*d**3*x**2 - 12*sin(c + d*x)*a*b*d + 4*sin
(c + d*x)*b**2*d**3*x**3 - 24*sin(c + d*x)*b**2*d*x)/d**5
```

3.11 $\int x(a+bx)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int x(a+bx)^2\sin(c+dx)\,dx=\frac{4ab\cos(c+dx)}{d^3}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{a^2\sin(c+dx)}{d^2}+\frac{4abx\sin(c+dx)}{d^2}+\frac{3b^2x^2\sin(c+dx)}{d^2}$$

output
```
4*a*b*cos(d*x+c)/d^3+6*b^2*x*cos(d*x+c)/d^3-a^2*x*cos(d*x+c)/d-2*a*b*x^2*c
os(d*x+c)/d-b^2*x^3*cos(d*x+c)/d-6*b^2*sin(d*x+c)/d^4+a^2*sin(d*x+c)/d^2+4
*a*b*x*sin(d*x+c)/d^2+3*b^2*x^2*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int x(a+bx)^2\sin(c+dx)\,dx=\frac{-d(a^2d^2x+b^2x(-6+d^2x^2)+2ab(-2+d^2x^2))\cos(c+dx)+(a^2d^2+4abd^2x+3b^2(-2+d^2x^2))\sin(c-}{d^4}$$

input
```
Integrate[x*(a + b*x)^2*Sin[c + d*x],x]
```

output
```
(-(d*(a^2*d^2*x + b^2*x*(-6 + d^2*x^2) + 2*a*b*(-2 + d^2*x^2))*Cos[c + d*x
]) + (a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^2\sin(c+dx)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(a^2x\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^3\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}$$

input
```
Int[x*(a + b*x)^2*Sin[c + d*x],x]
```

output
```
(4*a*b*Cos[c + d*x])/d^3 + (6*b^2*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c + d*x
])/d - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*x])/d - (6*b^2*Sin[
c + d*x])/d^4 + (a^2*Sin[c + d*x])/d^2 + (4*a*b*x*Sin[c + d*x])/d^2 + (3*b
^2*x^2*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b^2d^2x^3+2ab\,d^2x^2+x\,a^2d^2-6b^2x-4ab)\cos(dx+c)}{d^3}+\frac{(3x^2d^2b^2+4ab\,d^2x+a^2d^2-6b^2)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{d\left((bx+a)^2d^2-6b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2((3x^2b^2+4abx+a^2)d^2-6b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-d\left(x(bx+a)^2d^2-6b^2x-8ab\right)}{d^4\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$\frac{2(3b^3d^2x^4+7a\,b^2d^2x^3+5a^2b\,d^2x^2+a^3d^2x-12x^2b^3-12x\,b^2a-2a^2b)\sin(dx+c)}{d^4x(bx+a)}-\frac{(b^2d^2x^3+2ab\,d^2x^2+x\,a^2d^2-6b^2x-\cdots}{\cdots}$
parts	$-\frac{b^2x^3\cos(dx+c)}{d}-\frac{2ab\,x^2\cos(dx+c)}{d}-\frac{a^2x\cos(dx+c)}{d}+\frac{a^2\sin(dx+c)-\frac{4abc\sin(dx+c)}{d}+\frac{4ab(\cos(dx+c)+(dx+c)\,\mathrm{si}\cdots}{d}}{\cdots}$
norman	$\frac{\frac{b^2x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{\left(a^2d^2-6b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}-\frac{b^2x^3}{d}-\frac{\left(a^2d^2-6b^2\right)x}{d^3}+\frac{2\left(a^2d^2-6b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}-\frac{8ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}+\frac{6b\cdots}{\cdots}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
derivativedivides	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^2\cos(dx+c)}{d}-\frac{4abc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}+\frac{2ab\left(-(dx+c)^2\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^2\cos(dx+c)}{d}-\frac{4abc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}+\frac{2ab\left(-(dx+c)^2\cdots\right.}{\cdots}}{\cdots}$
meijerg	$\frac{8b^2\sin(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2d^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{xd\left(-\frac{x^2d^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\cos(c)\sqrt{\pi}\left(\frac{xd\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\cdots\right.}{d^4}$

input
```
int(x*(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(b^2*d^2*x^3+2*a*b*d^2*x^2+a^2*d^2*x-6*b^2*x-4*a*b)*cos(d*x+c)+(3*b
^2*d^2*x^2+4*a*b*d^2*x+a^2*d^2-6*b^2)/d^4*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x(a+bx)^2\sin(c+dx)\,dx =$$
$$-\frac{(b^2d^3x^3+2\,abd^3x^2-4\,abd+(a^2d^3-6\,b^2d)x)\cos{(dx+c)}-(3\,b^2d^2x^2+4\,abd^2x+a^2d^2-6\,b^2)\sin{(d}}{d^4}$$

input
```
integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^3*x^3 + 2*a*b*d^3*x^2 - 4*a*b*d + (a^2*d^3 - 6*b^2*d)*x)*cos(d*x
+ c) - (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.27

$$\int x(a+bx)^2\sin(c+dx)\,dx$$
$$=\begin{cases}-\frac{a^2x\cos{(c+dx)}}{d}+\frac{a^2\sin{(c+dx)}}{d^2}-\frac{2abx^2\cos{(c+dx)}}{d}+\frac{4abx\sin{(c+dx)}}{d^2}+\frac{4ab\cos{(c+dx)}}{d^3}-\frac{b^2x^3\cos{(c+dx)}}{d}+\frac{3b^2x^2\sin{(c+dx)}}{d^2} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}\right)\sin{(c)}\end{cases}$$

input
```
integrate(x*(b*x+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**2*co
s(c + d*x)/d + 4*a*b*x*sin(c + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*
x**3*cos(c + d*x)/d + 3*b**2*x**2*sin(c + d*x)/d**2 + 6*b**2*x*cos(c + d*x
)/d**3 - 6*b**2*sin(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**3/3
 + b**2*x**4/4)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.92

$$\int x(a+bx)^2\sin(c+dx)\,dx$$

$$= \frac{a^2c\cos(dx+c) + \frac{b^2c^3\cos(dx+c)}{d^2} - \frac{2abc^2\cos(dx+c)}{d} - ((dx+c)\cos(dx+c) - \sin(dx+c))a^2 - \frac{3((dx+c)\cos(dx-}{}}{}$$

input
```
integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a^2*c*cos(d*x + c) + b^2*c^3*cos(d*x + c)/d^2 - 2*a*b*c^2*cos(d*x + c)/d
- ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 3*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^2/d^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*
b*c/d + 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*
c/d^2 - 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b/
d - (((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*
x + c))*b^2/d^2)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x(a+bx)^2\sin(c+dx)\,dx$$
$$= -\frac{(b^2d^3x^3 + 2\,abd^3x^2 + a^2d^3x - 6\,b^2dx - 4\,abd)\cos(dx+c)}{d^4} + \frac{(3\,b^2d^2x^2 + 4\,abd^2x + a^2d^2 - 6\,b^2)\sin(dx+c)}{d^4}$$

input
```
integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 6*b^2*d*x - 4*a*b*d)*cos(d*x +
 c)/d^4 + (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 40.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\int x(a+bx)^2\sin(c+dx)\,dx = \frac{3\,b^2\,x^2\,\sin\left(c+d\,x\right)}{d^2}-\frac{b^2\,x^3\,\cos\left(c+d\,x\right)}{d} - \frac{\sin\left(c+d\,x\right)\,\left(6\,b^2-a^2\,d^2\right)}{d^4}+\frac{4\,a\,b\,\cos\left(c+d\,x\right)}{d^3} + \frac{x\,\cos\left(c+d\,x\right)\,\left(6\,b^2-a^2\,d^2\right)}{d^3} - \frac{2\,a\,b\,x^2\,\cos\left(c+d\,x\right)}{d}+\frac{4\,a\,b\,x\,\sin\left(c+d\,x\right)}{d^2}$$

input
```
int(x*sin(c + d*x)*(a + b*x)^2,x)
```

output
```
(3*b^2*x^2*sin(c + d*x))/d^2 - (b^2*x^3*cos(c + d*x))/d - (sin(c + d*x)*(6
*b^2 - a^2*d^2))/d^4 + (4*a*b*cos(c + d*x))/d^3 + (x*cos(c + d*x)*(6*b^2 -
 a^2*d^2))/d^3 - (2*a*b*x^2*cos(c + d*x))/d + (4*a*b*x*sin(c + d*x))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int x(a+bx)^2\sin(c+dx)\,dx = \frac{-\cos\left(dx+c\right)a^2d^3x-2\cos\left(dx+c\right)ab\,d^3x^2+4\cos\left(dx+c\right)abd-\cos\left(dx+c\right)b^2d^3x^3+6\cos\left(dx+c\right)b}{d^4}$$

input
```
int(x*(b*x+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**3*x - 2*cos(c + d*x)*a*b*d**3*x**2 + 4*cos(c + d*
x)*a*b*d - cos(c + d*x)*b**2*d**3*x**3 + 6*cos(c + d*x)*b**2*d*x + sin(c +
 d*x)*a**2*d**2 + 4*sin(c + d*x)*a*b*d**2*x + 3*sin(c + d*x)*b**2*d**2*x**
2 - 6*sin(c + d*x)*b**2)/d**4
```

3.12 $\int (a+bx)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int (a+bx)^2 \sin(c+dx)\,dx = \frac{2b^2\cos(c+dx)}{d^3} - \frac{(a+bx)^2\cos(c+dx)}{d} + \frac{2b(a+bx)\sin(c+dx)}{d^2}$$

output
```
2*b^2*cos(d*x+c)/d^3-(b*x+a)^2*cos(d*x+c)/d+2*b*(b*x+a)*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int (a+bx)^2 \sin(c+dx)\,dx = \frac{-((a^2d^2+2abd^2x+b^2(-2+d^2x^2))\cos(c+dx))+2bd(a+bx)\sin(c+dx)}{d^3}$$

input
```
Integrate[(a + b*x)^2*Sin[c + d*x],x]
```

output
```
(-((a^2*d^2 + 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*(a +
 b*x)*Sin[c + d*x])/d^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \sin(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+bx)^2 \sin(c+dx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{2b\int (a+bx)\cos(c+dx)dx}{d}-\frac{(a+bx)^2\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\int (a+bx)\sin\left(c+dx+\frac{\pi}{2}\right)dx}{d}-\frac{(a+bx)^2\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2b\left(\frac{b\int -\sin(c+dx)dx}{d}+\frac{(a+bx)\sin(c+dx)}{d}\right)}{d}-\frac{(a+bx)^2\cos(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\left(\frac{(a+bx)\sin(c+dx)}{d}-\frac{b\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(a+bx)^2\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\left(\frac{(a+bx)\sin(c+dx)}{d}-\frac{b\int \sin(c+dx)dx}{d}\right)}{d}-\frac{(a+bx)^2\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3118}$$

$$\frac{2b\left(\frac{(a+bx)\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d^2}\right)}{d}-\frac{(a+bx)^2\cos(c+dx)}{d}$$

input
```
Int[(a + b*x)^2*Sin[c + d*x],x]
```

output
```
-(((a + b*x)^2*Cos[c + d*x])/d) + (2*b*((b*Cos[c + d*x])/d^2 + ((a + b*x)*
Sin[c + d*x])/d))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(x^2d^2b^2+2ab\,d^2x+a^2d^2-2b^2)\cos(dx+c)}{d^3}+\frac{2b(bx+a)\sin(dx+c)}{d^2}$
parallelrisch	$\frac{2d^2b\left(\frac{bx}{2}+a\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4bd(bx+a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-x^2b^2-2abx-2a^2)d^2+4b^2}{d^3\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
parts	$-\frac{b^2x^2\cos(dx+c)}{d}-\frac{2abx\cos(dx+c)}{d}-\frac{a^2\cos(dx+c)}{d}+\frac{2b\left(a\sin(dx+c)-\frac{bc\sin(dx+c)}{d}+\frac{b(\cos(dx+c)+(dx+c)\sin(dx-}{d}\right.}{d^2}$
orering	$\frac{4b(x^2d^2b^2+2ab\,d^2x+a^2d^2-b^2)\sin(dx+c)}{d^4(bx+a)}-\frac{(x^2d^2b^2+2ab\,d^2x+a^2d^2-2b^2)\left(2(bx+a)\sin(dx+c)b+(bx+a)^2d\cos(dx\cdot\right.}{d^4(bx+a)^2}$
norman	$\frac{\frac{b^2x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{2a^2d^2-4b^2}{d^3}-\frac{b^2x^2}{d}+\frac{4b^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}+\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{2abx}{d}+\frac{2abx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
derivativedivides	$\frac{-\cos(dx+c)a^2+\frac{2abc\cos(dx+c)}{d}+\frac{2ab(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}-\frac{b^2c^2\cos(dx+c)}{d^2}-\frac{2b^2c(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{b}{}}{d}$
default	$\frac{-\cos(dx+c)a^2+\frac{2abc\cos(dx+c)}{d}+\frac{2ab(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}-\frac{b^2c^2\cos(dx+c)}{d^2}-\frac{2b^2c(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{b}{}}{d}$
meijerg	$\frac{4b^2\sin(c)\sqrt{\pi}\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3x^2d^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b^2\cos(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2d^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin}{2\sqrt{}}\right.}{d^3}$

input
```
int((b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2-2*b^2)/d^3*cos(d*x+c)+2*b*(b*x+a)*sin(d*
x+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int (a+bx)^2\sin(c+dx)\,dx$$
$$= -\frac{(b^2d^2x^2+2\,abd^2x+a^2d^2-2\,b^2)\cos{(dx+c)}-2\,(b^2dx+abd)\sin{(dx+c)}}{d^3}$$

input
```
integrate((b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c) - 2*(b^2*d*x
+ a*b*d)*sin(d*x + c))/d^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

$$\int (a+bx)^2 \sin(c+dx)\,dx = \begin{cases} -\frac{a^2\cos(c+dx)}{d} - \frac{2abx\cos(c+dx)}{d} + \frac{2ab\sin(c+dx)}{d^2} - \frac{b^2x^2\cos(c+dx)}{d} + \frac{2b^2x\sin(c+dx)}{d^2} + \frac{2b^2\cos(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x*cos(c + d*x)/d + 2*a*b*sin(c + d
*x)/d**2 - b**2*x**2*cos(c + d*x)/d + 2*b**2*x*sin(c + d*x)/d**2 + 2*b**2*
cos(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*sin(c), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.82

$$\int (a+bx)^2 \sin(c+dx)\,dx =$$

$$-\frac{a^2\cos(dx+c) + \frac{b^2c^2\cos(dx+c)}{d^2} - \frac{2abc\cos(dx+c)}{d} - \frac{2((dx+c)\cos(dx+c)-\sin(dx+c))b^2c}{d^2} + \frac{2((dx+c)\cos(dx+c)-\sin(dx+c))}{d}}{d}$$

input
```
integrate((b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a^2*cos(d*x + c) + b^2*c^2*cos(d*x + c)/d^2 - 2*a*b*c*cos(d*x + c)/d - 2
*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c/d^2 + 2*((d*x + c)*cos(d*x
+ c) - sin(d*x + c))*a*b/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)
*sin(d*x + c))*b^2/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (a+bx)^2\sin(c+dx)\,dx = -\frac{(b^2d^2x^2+2\,abd^2x+a^2d^2-2\,b^2)\cos{(dx+c)}}{d^3} + \frac{2\,(b^2dx+abd)\sin{(dx+c)}}{d^3}$$

input
```
integrate((b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c)/d^3 + 2*(b^2*d
*x + a*b*d)*sin(d*x + c)/d^3
```

Mupad [B] (verification not implemented)

Time = 39.97 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

$$\int (a+bx)^2\sin(c+dx)\,dx = \frac{\cos{(c+d\,x)}\,\left(2\,b^2-a^2\,d^2\right)}{d^3} - \frac{b^2\,x^2\,\cos{(c+d\,x)}}{d} + \frac{2\,a\,b\,\sin{(c+d\,x)}}{d^2} + \frac{2\,b^2\,x\,\sin{(c+d\,x)}}{d^2} - \frac{2\,a\,b\,x\,\cos{(c+d\,x)}}{d}$$

input
```
int(sin(c + d*x)*(a + b*x)^2,x)
```

output
```
(cos(c + d*x)*(2*b^2 - a^2*d^2))/d^3 - (b^2*x^2*cos(c + d*x))/d + (2*a*b*s
in(c + d*x))/d^2 + (2*b^2*x*sin(c + d*x))/d^2 - (2*a*b*x*cos(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.70

$$\int (a+bx)^2 \sin(c+dx)\,dx$$
$$= \frac{-\cos(dx+c)\,a^2d^2-2\cos(dx+c)\,ab\,d^2x-\cos(dx+c)\,b^2d^2x^2+2\cos(dx+c)\,b^2+2\sin(dx+c)\,abd-}{d^3}$$

input
```
int((b*x+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**2 - 2*cos(c + d*x)*a*b*d**2*x - cos(c + d*x)*b**2
*d**2*x**2 + 2*cos(c + d*x)*b**2 + 2*sin(c + d*x)*a*b*d + 2*sin(c + d*x)*b
**2*d*x)/d**3
```

3.13 $\int \frac{(a+bx)^2 \sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x}\,dx = -\frac{2ab\cos(c+dx)}{d} - \frac{b^2x\cos(c+dx)}{d} + a^2\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} + a^2\cos(c)\mathrm{Si}(dx)$$

output
```
-2*a*b*cos(d*x+c)/d-b^2*x*cos(d*x+c)/d+a^2*Ci(d*x)*sin(c)+b^2*sin(d*x+c)/d
^2+a^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x}\,dx = a^2\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{b(-d(2a+bx)\cos(c+dx)+b\sin(c+dx))}{d^2} + a^2\cos(c)\mathrm{Si}(dx)$$

input
```
Integrate[((a + b*x)^2*Sin[c + d*x])/x,x]
```

output
```
a^2*CosIntegral[d*x]*Sin[c] + (b*(-(d*(2*a + b*x)*Cos[c + d*x]) + b*Sin[c
+ d*x]))/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a^2 \sin(c+dx)}{x} + 2ab \sin(c+dx) + b^2 x \sin(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2 \sin(c) \text{CosIntegral}(dx) + a^2 \cos(c) \text{Si}(dx) - \frac{2ab \cos(c+dx)}{d} + \frac{b^2 \sin(c+dx)}{d^2} - \frac{b^2 x \cos(c+dx)}{d}$$

input
```
Int[((a + b*x)^2*Sin[c + d*x])/x,x]
```

output
```
(-2*a*b*Cos[c + d*x])/d - (b^2*x*Cos[c + d*x])/d + a^2*CosIntegral[d*x]*Si
n[c] + (b^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

method	result
derivativedivides	$a^2(\mathrm{Si}\,(dx)\cos\,(c)+\mathrm{Ci}\,(dx)\sin\,(c))-\frac{2ab\cos(dx+c)}{d}+\frac{2b^2c\cos(dx+c)}{d^2}+\frac{(c+1)b^2(\sin(dx+c)-\cos(dx}{d^2}$
default	$a^2(\mathrm{Si}\,(dx)\cos\,(c)+\mathrm{Ci}\,(dx)\sin\,(c))-\frac{2ab\cos(dx+c)}{d}+\frac{2b^2c\cos(dx+c)}{d^2}+\frac{(c+1)b^2(\sin(dx+c)-\cos(dx}{d^2}$
risch	$\frac{ia^2\mathrm{e}^{ic}\,\mathrm{expIntegral}_1(-idx)}{2}-\frac{i\,\mathrm{expIntegral}_1(-idx)\mathrm{e}^{-ic}a^2}{2}-\frac{\pi\,\mathrm{csgn}(dx)\mathrm{e}^{-ic}a^2}{2}+\mathrm{Si}\,(dx)\,\mathrm{e}^{-ic}a^2-\frac{b^2x\cos(c}{d}$
meijerg	$\frac{2b^2\sin(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\cos(c)\sqrt{\pi}\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2ab\sin(c)\sin(dx)}{d}+\frac{2ab\,\mathrm{co}}{}$

input
```
int((b*x+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-2*a*b*cos(d*x+c)/d+2/d^2*b^2*c*cos(d*x
+c)+(c+1)/d^2*b^2*(sin(d*x+c)-cos(d*x+c)*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx$$
$$=\frac{a^2d^2\,\mathrm{Ci}\,(dx)\sin\,(c)+a^2d^2\cos\,(c)\,\mathrm{Si}\,(dx)+b^2\sin\,(dx+c)-(b^2dx+2\,abd)\cos\,(dx+c)}{d^2}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

output
```
(a^2*d^2*cos_integral(d*x)*sin(c) + a^2*d^2*cos(c)*sin_integral(d*x) + b^2
*sin(d*x + c) - (b^2*d*x + 2*a*b*d)*cos(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx = a^2\sin(c)\operatorname{Ci}(dx) + a^2\cos(c)\operatorname{Si}(dx) + 2ab\left(\begin{cases} x\sin(c) & \text{for } d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right) + b^2x\left(\begin{cases} x\sin(c) & \text{for } d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right) - b^2\left(\begin{cases} \frac{x^2\sin(c)}{2} & \text{for } d=0 \\ -\frac{\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0 \\ x\cos(c) & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**2*sin(d*x+c)/x,x)
```

output
```
a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*Piecewise((x*sin(c), Eq(
d, 0)), (-cos(c + d*x)/d, True)) + b**2*x*Piecewise((x*sin(c), Eq(d, 0)),
(-cos(c + d*x)/d, True)) - b**2*Piecewise((x**2*sin(c)/2, Eq(d, 0)), (-Pie
cewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx$$
$$= \frac{(a^2(-i\operatorname{Ei}(i\,dx) + i\operatorname{Ei}(-i\,dx))\cos(c) + a^2(\operatorname{Ei}(i\,dx) + \operatorname{Ei}(-i\,dx))\sin(c))d^2 + 2\,b^2\sin(dx+c) - 2\,(b^2dx -}{2\,d^2}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

output
```
1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*
x))*sin(c))*d^2 + 2*b^2*sin(d*x + c) - 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))
/d^2
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 551, normalized size of antiderivative = 8.89

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

output
```
-1/2*(a^2*d^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)^2 - a^2*d^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2 + 2*a^2*d^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 -
 2*a^2*d^2*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
- 2*a^2*d^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
) - 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_
integral(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*
x))*tan(1/2*d*x + 1/2*c)^2 - 2*a^2*d^2*sin_integral(d*x)*tan(1/2*d*x + 1/2
*c)^2 + a^2*d^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^2*imag_p
art(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^2*sin_integral(d*x)*tan(1/2
*c)^2 - 4*a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2*d*x*tan(1/2*d*
x + 1/2*c)^2 - 2*a^2*d^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d
^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 2*b^2*d*x*tan(1/2*c)^2 - a^2
*d^2*imag_part(cos_integral(d*x)) + a^2*d^2*imag_part(cos_integral(-d*x))
- 2*a^2*d^2*sin_integral(d*x) - 4*a*b*d*tan(1/2*d*x + 1/2*c)^2 + 4*a*b*d*t
an(1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 2*b^2*d*x + 4*a*b*
d - 4*b^2*tan(1/2*d*x + 1/2*c))/(d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 +
 d^2*tan(1/2*d*x + 1/2*c)^2 + d^2*tan(1/2*c)^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx = b^2 \cos(c) \left(\frac{\sin(d\,x)}{d^2} - \frac{x \cos(d\,x)}{d}\right) + b^2 \sin(c) \left(\frac{\cos(d\,x)}{d^2} + \frac{x \sin(d\,x)}{d}\right) + a^2 \operatorname{cosint}(d\,x) \sin(c) + a^2 \operatorname{sinint}(d\,x) \cos(c) - \frac{2\,a\,b \cos(d\,x) \cos(c)}{d} + \frac{2\,a\,b \sin(d\,x) \sin(c)}{d}$$

input
```
int((sin(c + d*x)*(a + b*x)^2)/x,x)
```

output
```
b^2*cos(c)*(sin(d*x)/d^2 - (x*cos(d*x))/d) + b^2*sin(c)*(cos(d*x)/d^2 + (x
*sin(d*x))/d) + a^2*cosint(d*x)*sin(c) + a^2*sinint(d*x)*cos(c) - (2*a*b*c
os(d*x)*cos(c))/d + (2*a*b*sin(d*x)*sin(c))/d
```

Reduce [F]

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx = \frac{-2\cos(dx+c)\,abd - \cos(dx+c)\,b^2dx + \left(\int \frac{\sin(dx+c)}{x}dx\right) a^2d^2 + \sin(dx+c)\,b^2}{d^2}$$

input
```
int((b*x+a)^2*sin(d*x+c)/x,x)
```

output
```
( - 2*cos(c + d*x)*a*b*d - cos(c + d*x)*b**2*d*x + int(sin(c + d*x)/x,x)*a
**2*d**2 + sin(c + d*x)*b**2)/d**2
```

3.14 $\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx = &-\frac{b^2\cos(c+dx)}{d}+a^2d\cos(c)\,\text{CosIntegral}(dx)\\&+2ab\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{x}\\&+2ab\cos(c)\text{Si}(dx)-a^2d\sin(c)\text{Si}(dx)\end{aligned}$$

output
```
-b^2*cos(d*x+c)/d+a^2*d*cos(c)*Ci(d*x)+2*a*b*Ci(d*x)*sin(c)-a^2*sin(d*x+c)
/x+2*a*b*cos(c)*Si(d*x)-a^2*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx = &-\frac{b^2\cos(c+dx)}{d}+a\,\text{CosIntegral}(dx)(ad\cos(c)+2b\sin(c))\\&-\frac{a^2\sin(c+dx)}{x}-a(-2b\cos(c)+ad\sin(c))\text{Si}(dx)\end{aligned}$$

input
```
Integrate[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

output
```
-((b^2*Cos[c + d*x])/d) + a*CosIntegral[d*x]*(a*d*Cos[c] + 2*b*Sin[c]) - (
a^2*Sin[c + d*x])/x - a*(-2*b*Cos[c] + a*d*Sin[c])*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a^2 \sin(c+dx)}{x^2}+\frac{2ab \sin(c+dx)}{x}+b^2 \sin(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2 d \cos(c) \text{CosIntegral}(dx)-a^2 d \sin(c) \text{Si}(dx)-\frac{a^2 \sin(c+dx)}{x}+2ab \sin(c) \text{CosIntegral}(dx)+2ab \cos(c) \text{Si}(dx)-\frac{b^2 \cos(c+dx)}{d}$$

input
```
Int[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

output
```
-((b^2*Cos[c + d*x])/d) + a^2*d*Cos[c]*CosIntegral[d*x] + 2*a*b*CosIntegra
l[d*x]*Sin[c] - (a^2*Sin[c + d*x])/x + 2*a*b*Cos[c]*SinIntegral[d*x] - a^2
*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

method	result
derivativedivides	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx} - \mathrm{Si}(dx)\sin(c) + \mathrm{Ci}(dx)\cos(c)\right) + \frac{2ba(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d} - \frac{b^2\cos}{}\right.$
default	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx} - \mathrm{Si}(dx)\sin(c) + \mathrm{Ci}(dx)\cos(c)\right) + \frac{2ba(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d} - \frac{b^2\cos}{}\right.$
risch	$i\cos(c)\,\mathrm{expIntegral}_1(-idx)\,ab - \frac{d\cos(c)a^2\,\mathrm{expIntegral}_1(-idx)}{2} - i\cos(c)\,\mathrm{expIntegral}_1(idx)$
meijerg	$\frac{b^2\sin(c)\sin(dx)}{d} + \frac{b^2\cos(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(dx)}{\sqrt{\pi}}\right)}{d} + ab\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \right.$

input
```
int((b*x+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+2*b/d*a*(Si(d*x)*co
s(c)+Ci(d*x)*sin(c))-b^2/d^2*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx = -\frac{b^2x\cos(dx+c) + a^2d\sin(dx+c) - (a^2d^2x\,\mathrm{Ci}(dx) + 2\,abdx\,\mathrm{Si}(dx))\cos(c) + (a^2d^2x\,\mathrm{Si}(dx) - 2\,abdx}{dx}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
-(b^2*x*cos(d*x + c) + a^2*d*sin(d*x + c) - (a^2*d^2*x*cos_integral(d*x) +
 2*a*b*d*x*sin_integral(d*x))*cos(c) + (a^2*d^2*x*sin_integral(d*x) - 2*a*
b*d*x*cos_integral(d*x))*sin(c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx)^2 \sin{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x+a)**2*sin(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x)**2*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2}\,dx = \frac{\left(\left(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos{(c)}-a^2(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin{(c)}\right)d^2+2\,(ab(i\,\Gamma(-1,i}{2\,d}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
1/2*(((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a^2*(I*gamma(-1
, I*d*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 + 2*(a*b*(I*gamma(-1, I*d*x) -
 I*gamma(-1, -I*d*x))*cos(c) + a*b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*
sin(c))*d)*x - 2*(b^2*x + 2*a*b)*cos(d*x + c))/(d*x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 743, normalized size of antiderivative = 10.32

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a
^2*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^
2*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^2*x*
sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x*imag_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b*d*x*imag_part(cos_integral(
-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d*x*sin_integral(d*x)*tan(1/2*d
*x)^2*tan(1/2*c)^2 - a^2*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2
 - a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*b*d*x*real
_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x*real_part(cos_int
egral(d*x))*tan(1/2*c)^2 + a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2
*c)^2 - 2*a*b*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*a*b*d*x*
imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*b*d*x*sin_integral(d*x)
*tan(1/2*d*x)^2 + 2*a^2*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*
a^2*d^2*x*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*d^2*x*sin_integ
ral(d*x)*tan(1/2*c) + 2*a*b*d*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2
- 2*a*b*d*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 4*a*b*d*x*sin_int
egral(d*x)*tan(1/2*c)^2 + 2*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x*
real_part(cos_integral(d*x)) - a^2*d^2*x*real_part(cos_integral(-d*x)) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2}\, dx = \int \frac{\sin(c+d\,x)\,(a+b\,x)^2}{x^2}\, dx$$

input
```
int((sin(c + d*x)*(a + b*x)^2)/x^2,x)
```

output
```
int((sin(c + d*x)*(a + b*x)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2}\, dx = \frac{-2\cos(dx+c)\,ab - \cos(dx+c)\,b^2x + 4\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^2 + x^2}dx\right) abx + 2\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x + x}dx\right) a^2d^2x -}{dx}$$

input
```
int((b*x+a)^2*sin(d*x+c)/x^2,x)
```

output
```
( - 2*cos(c + d*x)*a*b - cos(c + d*x)*b**2*x + 4*int(tan((c + d*x)/2)**2/(
tan((c + d*x)/2)**2*x**2 + x**2),x)*a*b*x + 2*int(1/(tan((c + d*x)/2)**2*x
 + x),x)*a**2*d**2*x - log(x)*a**2*d**2*x - sin(c + d*x)*a**2*d + 2*a*b +
b**2*x)/(d*x)
```

3.15 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\begin{aligned}\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} dx = & -\frac{a^2 d \cos(c+dx)}{2x} + 2abd \cos(c) \operatorname{CosIntegral}(dx) \\ & + b^2 \operatorname{CosIntegral}(dx) \sin(c) - \frac{1}{2} a^2 d^2 \operatorname{CosIntegral}(dx) \sin(c) \\ & - \frac{a^2 \sin(c+dx)}{2x^2} - \frac{2ab \sin(c+dx)}{x} + b^2 \cos(c) \operatorname{Si}(dx) \\ & - \frac{1}{2} a^2 d^2 \cos(c) \operatorname{Si}(dx) - 2abd \sin(c) \operatorname{Si}(dx)\end{aligned}$$

output
```
-1/2*a^2*d*cos(d*x+c)/x+2*a*b*d*cos(c)*Ci(d*x)+b^2*Ci(d*x)*sin(c)-1/2*a^2*
d^2*Ci(d*x)*sin(c)-1/2*a^2*sin(d*x+c)/x^2-2*a*b*sin(d*x+c)/x+b^2*cos(c)*Si
(d*x)-1/2*a^2*d^2*cos(c)*Si(d*x)-2*a*b*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} \, dx = \frac{1}{2} \left(\text{CosIntegral}(dx) \left(4abd\cos(c) + \left(2b^2 - a^2d^2\right)\sin(c)\right) - \frac{a(adx\cos(c+dx) + (a+4bx)\sin(c+dx))}{x^2} + \left(\left(2b^2 - a^2d^2\right)\cos(c) - 4abd\sin(c)\right)\text{Si}(dx) \right)$$

input
```
Integrate[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

output
```
(CosIntegral[d*x]*(4*a*b*d*Cos[c] + (2*b^2 - a^2*d^2)*Sin[c]) - (a*(a*d*x*
Cos[c + d*x] + (a + 4*b*x)*Sin[c + d*x]))/x^2 + ((2*b^2 - a^2*d^2)*Cos[c]
- 4*a*b*d*Sin[c])*SinIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{a^2 \sin(c+dx)}{x^3} + \frac{2ab\sin(c+dx)}{x^2} + \frac{b^2\sin(c+dx)}{x} \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}a^2d^2\sin(c)\,\text{CosIntegral}(dx) - \frac{1}{2}a^2d^2\cos(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{2x^2} - \frac{a^2d\cos(c+dx)}{2x} + 2abd\cos(c)\,\text{CosIntegral}(dx) - 2abd\sin(c)\text{Si}(dx) - \frac{2ab\sin(c+dx)}{x} + b^2\sin(c)\,\text{CosIntegral}(dx) + b^2\cos(c)\text{Si}(dx)$$

input
```
Int[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

output
```
-1/2*(a^2*d*Cos[c + d*x])/x + 2*a*b*d*Cos[c]*CosIntegral[d*x] + b^2*CosInt
egral[d*x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (a^2*Sin[c + d*x
])/(2*x^2) - (2*a*b*Sin[c + d*x])/x + b^2*Cos[c]*SinIntegral[d*x] - (a^2*d
^2*Cos[c]*SinIntegral[d*x])/2 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

method	result
derivativedivides	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(d\right.}{d}\right.$
default	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(d\right.}{d}\right.$
risch	$-\cos(c)\,\mathrm{expIntegral}_1(-idx)\,abd-\cos(c)\,\mathrm{expIntegral}_1(idx)\,abd-\frac{i\cos(c)\,\mathrm{expIntegral}_1(-}{4}$
meijerg	$\frac{b^2\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b^2\cos(c)\,\mathrm{Si}(dx)+\frac{d^2ab\sin(c)\sqrt{\pi}\left(-\right.}{}$

input
```
int((b*x+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/
2*Ci(d*x)*sin(c))+2/d*a*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+
1/d^2*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = -\frac{a^2dx\cos(dx+c) - (4\,abdx^2\operatorname{Ci}(dx) - (a^2d^2-2\,b^2)x^2\operatorname{Si}(dx))\cos(c) + (4\,abx+a^2)\sin(dx+c) + (4}{2\,x^2}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*d*x*cos(d*x + c) - (4*a*b*d*x^2*cos_integral(d*x) - (a^2*d^2 - 2
*b^2)*x^2*sin_integral(d*x))*cos(c) + (4*a*b*x + a^2)*sin(d*x + c) + (4*a*
b*d*x^2*sin_integral(d*x) + (a^2*d^2 - 2*b^2)*x^2*cos_integral(d*x))*sin(c
))/x^2
```

Sympy [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx$$

input
```
integrate((b*x+a)**2*sin(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x)**2*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = -\frac{((a^2(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c) - a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c))d^4 + 4\,(ab(\Gamma(-}{}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(((a^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a^2*(gamm
a(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 + 4*(a*b*(gamma(-2, I*d*x) +
 gamma(-2, -I*d*x))*cos(c) - a*b*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x)
)*sin(c))*d^3 - 2*(b^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c)
- b^2*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b^2*sin(
d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1182, normalized size of antiderivative = 9.77

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
- a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^2*x^
2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2*x^2*r
eal_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d*x^2*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*
b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x
^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*
sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_in
tegral(d*x))*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(
1/2*c)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b^2*x^2*imag_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*
x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d*x^2*real_part(cos_integral(d*x))*
tan(1/2*d*x)^2 + 4*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2
- 2*a^2*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x^2*re
al_part(cos_integral(-d*x))*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx=\int \frac{\sin\left(c+d\,x\right)\,{\left(a+b\,x\right)}^2}{x^3}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x)^2)/x^3,x)
```

output
```
int((sin(c + d*x)*(a + b*x)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx$$
$$= \frac{-4\cos(dx+c)\,ab - 2\cos(dx+c)\,b^2x - 2\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2+x^2}dx\right)a^2d^2x^2 + 4\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2+x^2}dx\right)b^2}{2d\,x^2}$$

input
```
int((b*x+a)^2*sin(d*x+c)/x^3,x)
```

output
```
( - 4*cos(c + d*x)*a*b - 2*cos(c + d*x)*b**2*x - 2*int(tan((c + d*x)/2)**2
/(tan((c + d*x)/2)**2*x**2 + x**2),x)*a**2*d**2*x**2 + 4*int(tan((c + d*x)
/2)**2/(tan((c + d*x)/2)**2*x**2 + x**2),x)*b**2*x**2 - 16*int(1/(tan((c +
 d*x)/2)**2*x**3 + x**3),x)*a*b*x**2 - sin(c + d*x)*a**2*d - a**2*d**2*x -
 4*a*b + 2*b**2*x)/(2*d*x**2)
```

3.16 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\begin{aligned}\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx &= -\frac{a^2 d \cos(c+dx)}{6x^2} - \frac{abd \cos(c+dx)}{x} + b^2 d \cos(c) \text{CosIntegral}(dx) \\ &\quad - \frac{1}{6} a^2 d^3 \cos(c) \text{CosIntegral}(dx) - abd^2 \text{CosIntegral}(dx) \sin(c) \\ &\quad - \frac{a^2 \sin(c+dx)}{3x^3} - \frac{ab \sin(c+dx)}{x^2} - \frac{b^2 \sin(c+dx)}{x} + \frac{a^2 d^2 \sin(c+dx)}{6x} \\ &\quad - abd^2 \cos(c) \text{Si}(dx) - b^2 d \sin(c) \text{Si}(dx) + \frac{1}{6} a^2 d^3 \sin(c) \text{Si}(dx)\end{aligned}$$

output
```
-1/6*a^2*d*cos(d*x+c)/x^2-a*b*d*cos(d*x+c)/x+b^2*d*cos(c)*Ci(d*x)-1/6*a^2*
d^3*cos(c)*Ci(d*x)-a*b*d^2*Ci(d*x)*sin(c)-1/3*a^2*sin(d*x+c)/x^3-a*b*sin(d
*x+c)/x^2-b^2*sin(d*x+c)/x+1/6*a^2*d^2*sin(d*x+c)/x-a*b*d^2*cos(c)*Si(d*x)
-b^2*d*sin(c)*Si(d*x)+1/6*a^2*d^3*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx =$$
$$-\frac{a^2dx\cos(c+dx)+6abdx^2\cos(c+dx)+dx^3\,\text{CosIntegral}(dx)\left(\left(-6b^2+a^2d^2\right)\cos(c)+6abd\sin(c)\right)+\cdots}{\cdots}$$

input
```
Integrate[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

output
```
-1/6*(a^2*d*x*Cos[c + d*x] + 6*a*b*d*x^2*Cos[c + d*x] + d*x^3*CosIntegral[
d*x]*((-6*b^2 + a^2*d^2)*Cos[c] + 6*a*b*d*Sin[c]) + 2*a^2*Sin[c + d*x] + 6
*a*b*x*Sin[c + d*x] + 6*b^2*x^2*Sin[c + d*x] - a^2*d^2*x^2*Sin[c + d*x] +
d*x^3*(6*a*b*d*Cos[c] + 6*b^2*Sin[c] - a^2*d^2*Sin[c])*SinIntegral[d*x])/x
^3
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}-abd^2\sin(c)\,\text{CosIntegral}(dx)-abd^2\cos(c)\text{Si}(dx)-\frac{ab\sin(c+dx)}{x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\,\text{CosIntegral}(dx)-b^2d\sin(c)\text{Si}(dx)-\frac{b^2\sin(c+dx)}{x}$$

input
```
Int[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

output
```
-1/6*(a^2*d*Cos[c + d*x])/x^2 - (a*b*d*Cos[c + d*x])/x + b^2*d*Cos[c]*CosI
ntegral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - a*b*d^2*CosIntegral[d
*x]*Sin[c] - (a^2*Sin[c + d*x])/(3*x^3) - (a*b*Sin[c + d*x])/x^2 - (b^2*Si
n[c + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - a*b*d^2*Cos[c]*SinIntegral[
d*x] - b^2*d*Sin[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.90

method	result
derivativedivides	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{2d^2x^2}-\cdots\right.}{\cdots}\right.$
default	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{2d^2x^2}-\cdots\right.}{\cdots}\right.$
risch	$\frac{i\,\text{expIntegral}_1(idx)\cos(c)ab\,d^2}{2}+\frac{\text{expIntegral}_1(idx)\cos(c)a^2d^3}{12}-\frac{i\,\text{expIntegral}_1(-idx)\cos(c)ab\,d^2}{2}+\cdots$
meijerg	$\frac{d^2b^2\sin(c)\sqrt{\pi}\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\text{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{d\,b^2\cos(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{dx}{2}\cdots\right)}{\sqrt{\pi}}\cdots\right.}{4}$

input
```
int((b*x+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/
x+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d*a*b*(-1/2*sin(d*x+c)/d^2/x^2-
1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))+1/d^2*b^2*(-sin(
d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx = -\frac{(6\,abdx^2+a^2dx)\cos(dx+c)+(6\,abd^2x^3\,\mathrm{Si}(dx)+(a^2d^3-6\,b^2d)x^3\,\mathrm{Ci}(dx))\cos(c)+(6\,abx-(a^2d^2}{6\,x^3}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/6*((6*a*b*d*x^2 + a^2*d*x)*cos(d*x + c) + (6*a*b*d^2*x^3*sin_integral(d
*x) + (a^2*d^3 - 6*b^2*d)*x^3*cos_integral(d*x))*cos(c) + (6*a*b*x - (a^2*
d^2 - 6*b^2)*x^2 + 2*a^2)*sin(d*x + c) + (6*a*b*d^2*x^3*cos_integral(d*x)
- (a^2*d^3 - 6*b^2*d)*x^3*sin_integral(d*x))*sin(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x+a)**2*sin(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x)**2*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = -\frac{((a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a^2(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^5-6\,(ab(-i\,\Gamma}{}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(
-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 6*(a*b*(-I*gamma(-3, I*d*x
) + I*gamma(-3, -I*d*x))*cos(c) - a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x
))*sin(c))*d^4 - 6*(b^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + b^
2*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^3)*x^3 + 4*b^2*sin
(d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1400, normalized size of antiderivative = 8.00

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

output
```
1/12*(a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*a^2*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*
a^2*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^
2*d^3*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag
_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^2*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2 - a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(
1/2*d*x)^2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*ta
n(1/2*c) - 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*c) + a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a^2*d^3*
x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_in
tegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 + 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 - 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^2*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^3*x^3*imag_part(c
os_integral(-d*x))*tan(1/2*c) + 4*a^2*d^3*x^3*sin_integral(d*x)*tan(1/2*c)
 - 12*b^2*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = \int \frac{\sin(c+d\,x)\,(a+b\,x)^2}{x^4} \, dx$$

input
```
int((sin(c + d*x)*(a + b*x)^2)/x^4,x)
```

output
```
int((sin(c + d*x)*(a + b*x)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx$$

$$= \frac{-12\cos(dx+c)\,ab - 6\cos(dx+c)\,b^2x + 72\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)ab\,x^3 + 4\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^3+x^3}dx\right)a^2}{6d\,x^3}$$

input
```
int((b*x+a)^2*sin(d*x+c)/x^4,x)
```

output
```
( - 12*cos(c + d*x)*a*b - 6*cos(c + d*x)*b**2*x + 72*int(tan((c + d*x)/2)*
*2/(tan((c + d*x)/2)**2*x**4 + x**4),x)*a*b*x**3 + 4*int(1/(tan((c + d*x)/
2)**2*x**3 + x**3),x)*a**2*d**2*x**3 - 24*int(1/(tan((c + d*x)/2)**2*x**3
+ x**3),x)*b**2*x**3 - 2*sin(c + d*x)*a**2*d + a**2*d**2*x + 12*a*b - 6*b*
*2*x)/(6*d*x**3)
```

3.17 $\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 248

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = & -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x} \\ & +\frac{a^2d^3\cos(c+dx)}{24x}-\frac{1}{3}abd^3\cos(c)\,\text{CosIntegral}(dx) \\ & -\frac{1}{2}b^2d^2\,\text{CosIntegral}(dx)\sin(c) \\ & +\frac{1}{24}a^2d^4\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4} \\ & -\frac{2ab\sin(c+dx)}{3x^3}-\frac{b^2\sin(c+dx)}{2x^2}+\frac{a^2d^2\sin(c+dx)}{24x^2} \\ & +\frac{abd^2\sin(c+dx)}{3x}-\frac{1}{2}b^2d^2\cos(c)\text{Si}(dx) \\ & +\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{1}{3}abd^3\sin(c)\text{Si}(dx)\end{aligned}$$

output
```
-1/12*a^2*d*cos(d*x+c)/x^3-1/3*a*b*d*cos(d*x+c)/x^2-1/2*b^2*d*cos(d*x+c)/x
+1/24*a^2*d^3*cos(d*x+c)/x-1/3*a*b*d^3*cos(c)*Ci(d*x)-1/2*b^2*d^2*Ci(d*x)*
sin(c)+1/24*a^2*d^4*Ci(d*x)*sin(c)-1/4*a^2*sin(d*x+c)/x^4-2/3*a*b*sin(d*x+
c)/x^3-1/2*b^2*sin(d*x+c)/x^2+1/24*a^2*d^2*sin(d*x+c)/x^2+1/3*a*b*d^2*sin(
d*x+c)/x-1/2*b^2*d^2*cos(c)*Si(d*x)+1/24*a^2*d^4*cos(c)*Si(d*x)+1/3*a*b*d^
3*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^5} \, dx$$
$$= \frac{-2a^2dx\cos(c+dx) - 8abdx^2\cos(c+dx) - 12b^2dx^3\cos(c+dx) + a^2d^3x^3\cos(c+dx) + d^2x^4\,\text{CosIntegr}}{}$$

input
```
Integrate[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

output
```
(-2*a^2*d*x*Cos[c + d*x] - 8*a*b*d*x^2*Cos[c + d*x] - 12*b^2*d*x^3*Cos[c +
 d*x] + a^2*d^3*x^3*Cos[c + d*x] + d^2*x^4*CosIntegral[d*x]*(-8*a*b*d*Cos[
c] + (-12*b^2 + a^2*d^2)*Sin[c]) - 6*a^2*Sin[c + d*x] - 16*a*b*x*Sin[c + d
*x] - 12*b^2*x^2*Sin[c + d*x] + a^2*d^2*x^2*Sin[c + d*x] + 8*a*b*d^2*x^3*S
in[c + d*x] + d^2*x^4*(-12*b^2*Cos[c] + a^2*d^2*Cos[c] + 8*a*b*d*Sin[c])*S
inIntegral[d*x])/(24*x^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^5} \, dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(\frac{a^2 \sin(c+dx)}{x^5} + \frac{2ab\sin(c+dx)}{x^4} + \frac{b^2\sin(c+dx)}{x^3}\right) dx$$
$$\Big\downarrow \; 2009$$

$$\frac{1}{24}a^2d^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^3\cos(c+dx)}{24x}+\frac{a^2d^2\sin(c+dx)}{24x^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos(c+dx)}{12x^3}-\frac{1}{3}abd^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{3}abd^3\sin(c)\text{Si}(dx)+\frac{abd^2\sin(c+dx)}{3x}-\frac{2ab\sin(c+dx)}{3x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{1}{2}b^2d^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}b^2d^2\cos(c)\text{Si}(dx)-\frac{b^2\sin(c+dx)}{2x^2}-\frac{b^2d\cos(c+dx)}{2x}$$

input
```
Int[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

output
```
-1/12*(a^2*d*Cos[c + d*x])/x^3 - (a*b*d*Cos[c + d*x])/(3*x^2) - (b^2*d*Cos
[c + d*x])/(2*x) + (a^2*d^3*Cos[c + d*x])/(24*x) - (a*b*d^3*Cos[c]*CosInte
gral[d*x])/3 - (b^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (a^2*d^4*CosIntegral[
d*x]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (2*a*b*Sin[c + d*x])/(3*x^3
) - (b^2*Sin[c + d*x])/(2*x^2) + (a^2*d^2*Sin[c + d*x])/(24*x^2) + (a*b*d^
2*Sin[c + d*x])/(3*x) - (b^2*d^2*Cos[c]*SinIntegral[d*x])/2 + (a^2*d^4*Cos
[c]*SinIntegral[d*x])/24 + (a*b*d^3*Sin[c]*SinIntegral[d*x])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.81

method	result
derivativedivides	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{2ab\left(-\right.}{}\right.$
default	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{2ab\left(-\right.}{}\right.$
risch	$\frac{\mathrm{expIntegral}_1(-idx)\cos(c)ab\,d^3}{6}+\frac{\mathrm{expIntegral}_1(idx)\cos(c)ab\,d^3}{6}+\frac{i\,\mathrm{expIntegral}_1(-idx)\cos(c)a^2d^4}{48}-\frac{i\,\mathrm{expIntegr}}{}$
meijerg	$\frac{d^2b^2\sin(c)\sqrt{\pi}\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6x^2d^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,x^2d^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,xd}-\frac{4\,\mathrm{Ci}(}{\sqrt{\pi}}\right.}{8}$

input
```
int((b*x+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/
d^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2/d*a
*b*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*
Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d^2*b^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*
cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int\frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx=$$
$$-\frac{(8\,abdx^2+2\,a^2dx-(a^2d^3-12\,b^2d)x^3)\cos{(dx+c)}+(8\,abd^3x^4\,\mathrm{Ci}\,(dx)-(a^2d^4-12\,b^2d^2)x^4\,\mathrm{Si}\,(dx))\,\mathrm{c}}{}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
-1/24*((8*a*b*d*x^2 + 2*a^2*d*x - (a^2*d^3 - 12*b^2*d)*x^3)*cos(d*x + c) +
 (8*a*b*d^3*x^4*cos_integral(d*x) - (a^2*d^4 - 12*b^2*d^2)*x^4*sin_integra
l(d*x))*cos(c) - (8*a*b*d^2*x^3 - 16*a*b*x + (a^2*d^2 - 12*b^2)*x^2 - 6*a^
2)*sin(d*x + c) - (8*a*b*d^3*x^4*sin_integral(d*x) + (a^2*d^4 - 12*b^2*d^2
)*x^4*cos_integral(d*x))*sin(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx)^2\sin{(c+dx)}}{x^5}\,dx$$

input
```
integrate((b*x+a)**2*sin(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x)**2*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = -\frac{((a^2(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos{(c)}+a^2(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin{(c)})d^6-8\,(ab(\Gamma(-4,$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
-1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma
(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - 8*(a*b*(gamma(-4, I*d*x) +
gamma(-4, -I*d*x))*cos(c) + a*b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x)
)*sin(c))*d^5 - 12*(b^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c)
+ b^2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 6*b^2*sin(
d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1712, normalized size of antiderivative = 6.90

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

output
```
-1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*
x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4
*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*r
eal_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b*d^3*x^4*re
al_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*imag
_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2
- 16*a*b*d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) +
16*a*b*d^3*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 3
2*a*b*d^3*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^4*x^4*im
ag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(cos_integr
al(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12
*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12
*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
4*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x^
4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 8*a*b*d^3*x^4*real_part(co
s_integral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*real_part(cos_integral(d*
x))*tan(1/2*c) - 2*a^2*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx=\int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)^2}{x^5}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x)^2)/x^5,x)
```

output
```
int((sin(c + d*x)*(a + b*x)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \frac{-24\cos\left(dx+c\right)ab-12\cos\left(dx+c\right)b^2x-6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)a^2d^2x^4+72\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right.}{12d\,x^4}$$

input
```
int((b*x+a)^2*sin(d*x+c)/x^5,x)
```

output
```
( - 24*cos(c + d*x)*a*b - 12*cos(c + d*x)*b**2*x - 6*int(tan((c + d*x)/2)*
*2/(tan((c + d*x)/2)**2*x**4 + x**4),x)*a**2*d**2*x**4 + 72*int(tan((c + d
*x)/2)**2/(tan((c + d*x)/2)**2*x**4 + x**4),x)*b**2*x**4 - 192*int(1/(tan(
(c + d*x)/2)**2*x**5 + x**5),x)*a*b*x**4 - 3*sin(c + d*x)*a**2*d - a**2*d*
*2*x - 24*a*b + 12*b**2*x)/(12*d*x**4)
```

3.18 $\int \frac{x^4 \sin(c+dx)}{a+bx} dx$

Optimal result

Integrand size = 17, antiderivative size = 218

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{a+bx} dx = &-\frac{2a\cos(c+dx)}{b^2d^3} + \frac{a^3\cos(c+dx)}{b^4d} + \frac{6x\cos(c+dx)}{bd^3} \\ &- \frac{a^2x\cos(c+dx)}{b^3d} + \frac{ax^2\cos(c+dx)}{b^2d} - \frac{x^3\cos(c+dx)}{bd} \\ &+ \frac{a^4 \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} \\ &- \frac{6\sin(c+dx)}{bd^4} + \frac{a^2\sin(c+dx)}{b^3d^2} - \frac{2ax\sin(c+dx)}{b^2d^2} \\ &+ \frac{3x^2\sin(c+dx)}{bd^2} + \frac{a^4\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5}\end{aligned}$$

output
```
-2*a*cos(d*x+c)/b^2/d^3+a^3*cos(d*x+c)/b^4/d+6*x*cos(d*x+c)/b/d^3-a^2*x*co
s(d*x+c)/b^3/d+a*x^2*cos(d*x+c)/b^2/d-x^3*cos(d*x+c)/b/d-a^4*Ci(a*d/b+d*x)
*sin(-c+a*d/b)/b^5-6*sin(d*x+c)/b/d^4+a^2*sin(d*x+c)/b^3/d^2-2*a*x*sin(d*x
+c)/b^2/d^2+3*x^2*sin(d*x+c)/b/d^2+a^4*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^5
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.72

$$\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx$$
$$= \frac{a^4 d^4 \operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right) \sin\left(c-\frac{ad}{b}\right) + b(d(a^3 d^2 - a^2 b d^2 x + b^3 x(6-d^2x^2) + ab^2(-2+d^2x^2)) \cos(c}{b^5 d^4}$$

input
```
Integrate[(x^4*Sin[c + d*x])/(a + b*x),x]
```

output
```
(a^4*d^4*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a^3*d^2 - a^2*b
*d^2*x + b^3*x*(6 - d^2*x^2) + a*b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*(a^2
*d^2 - 2*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x]) + a^4*d^4*Cos[c -
 (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^5*d^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx$$
$$\Big\downarrow \ 7293$$
$$\int \left(\frac{a^4 \sin(c+dx)}{b^4(a+bx)} - \frac{a^3 \sin(c+dx)}{b^4} + \frac{a^2 x \sin(c+dx)}{b^3} - \frac{a x^2 \sin(c+dx)}{b^2} + \frac{x^3 \sin(c+dx)}{b}\right) dx$$
$$\Big\downarrow \ 2009$$

$$\frac{a^4 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^4 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3 \cos(c+dx)}{b^4 d}+\frac{a^2 \sin(c+dx)}{b^3 d^2}-\frac{a^2 x \cos(c+dx)}{b^3 d}-\frac{2a \cos(c+dx)}{b^2 d^3}-\frac{2ax \sin(c+dx)}{b^2 d^2}+\frac{ax^2 \cos(c+dx)}{b^2 d}-\frac{6 \sin(c+dx)}{bd^4}+\frac{6x \cos(c+dx)}{bd^3}+\frac{3x^2 \sin(c+dx)}{bd^2}-\frac{x^3 \cos(c+dx)}{bd}$$

input
```
Int[(x^4*Sin[c + d*x])/(a + b*x),x]
```

output
```
(-2*a*Cos[c + d*x])/(b^2*d^3) + (a^3*Cos[c + d*x])/(b^4*d) + (6*x*Cos[c +
d*x])/(b*d^3) - (a^2*x*Cos[c + d*x])/(b^3*d) + (a*x^2*Cos[c + d*x])/(b^2*d
) - (x^3*Cos[c + d*x])/(b*d) + (a^4*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*
d)/b])/b^5 - (6*Sin[c + d*x])/(b*d^4) + (a^2*Sin[c + d*x])/(b^3*d^2) - (2*
a*x*Sin[c + d*x])/(b^2*d^2) + (3*x^2*Sin[c + d*x])/(b*d^2) + (a^4*Cos[c -
(a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{i\sin\left(\frac{ad-bc}{b}\right)\pi\,\mathrm{csgn}\left(\frac{(bx+a)d}{b}\right)a^4}{2b^5}-\frac{\cos\left(\frac{ad-bc}{b}\right)\pi\,\mathrm{csgn}\left(\frac{(bx+a)d}{b}\right)a^4}{2b^5}+\frac{i\sin\left(\frac{ad-bc}{b}\right)\mathrm{Si}\left(\frac{(bx+a)d}{b}\right)a^4}{b^5}-\frac{x^3\cos}{b}$
derivativedivides	$d\,c^4\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)+\frac{4(ad-bc)d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\mathrm{Ci}(\right.}{b}$
default	$d\,c^4\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)+\frac{4(ad-bc)d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\mathrm{Ci}(\right.}{b}$

input
```
int(x^4*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^5*sin((a*d-b*c)/b)*Pi*csgn((b*x+a)*d/b)*a^4-1/2/b^5*cos((a*d-b*c)
/b)*Pi*csgn((b*x+a)*d/b)*a^4+I/b^5*sin((a*d-b*c)/b)*Si((b*x+a)*d/b)*a^4-x^
3*cos(d*x+c)/b/d+1/b^5*Ei(1,-I*(b*x+a)*d/b)*sin((a*d-b*c)/b)*a^4+1/b^5*cos
((a*d-b*c)/b)*Si((b*x+a)*d/b)*a^4+a*x^2*cos(d*x+c)/b^2/d+3*x^2*sin(d*x+c)/
b/d^2-a^2*x*cos(d*x+c)/b^3/d-2*a*x*sin(d*x+c)/b^2/d^2+a^3*cos(d*x+c)/b^4/d
+a^2*sin(d*x+c)/b^3/d^2+6*x*cos(d*x+c)/b/d^3-2*a*cos(d*x+c)/b^2/d^3-6*sin(
d*x+c)/b/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.86

$$\int \frac{x^4\sin(c+dx)}{a+bx}\,dx =$$
$$-\frac{a^4d^4\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-a^4d^4\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+\left(b^4d^3x^3-ab^3d^3x^2-a^3bd^3+2\,ab^3d+\right.}{b^5d^4}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-(a^4*d^4*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a^4*d^4*cos(
-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + (b^4*d^3*x^3 - a*b^3*d^3*x
^2 - a^3*b*d^3 + 2*a*b^3*d + (a^2*b^2*d^3 - 6*b^4*d)*x)*cos(d*x + c) - (3*
b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 - 6*b^4)*sin(d*x + c))/(b^5*d^4)
```

Sympy [F]

$$\int \frac{x^4 \sin(c + dx)}{a + bx} \, dx = \int \frac{x^4 \sin(c + dx)}{a + bx} \, dx$$

input
```
integrate(x**4*sin(d*x+c)/(b*x+a),x)
```

output
```
Integral(x**4*sin(c + d*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c + dx)}{a + bx} \, dx = \int \frac{x^4 \sin(dx + c)}{bx + a} \, dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(((6*a*b^2*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral
_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 6*a*b^2*(I*exp_integral_e(2, (I*b*
d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a
^3*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*
d*x + I*a*d)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b)
 + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d^2 - 4*(a^2*b*(ex
p_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d
)/b))*cos(c)^2 + a^2*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integ
ral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) - (6*a*b^
2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x +
I*a*d)/b))*cos(c)^2 + 6*a*b^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + ex
p_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral_e(2,
(I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
 a^3*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x
 + I*a*d)/b))*sin(c)^2)*d^2 - 4*(a^2*b*(-I*exp_integral_e(2, (I*b*d*x + I*
a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*b*(-I*
exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((6*a*b^2*(I*
exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I
*a*d)/b))*cos(c)^2 + 6*a*b^2*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 3337, normalized size of antiderivative = 15.31

$$\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx = \text{Too large to display}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
2*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4
*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^4*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b
^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^4*d^4*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) +
 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b) + 2*b^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a
*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^4*d^3*x
^3*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*c)^2 - a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*c)^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^4*d^4*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^4*im...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{a+bx}\,dx = \int \frac{x^4\,\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

input
```
int((x^4*sin(c + d*x))/(a + b*x),x)
```

output
```
int((x^4*sin(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x^4 \sin(c + dx)}{a + bx} \, dx$$

$$= \frac{\cos(dx + c)\, a^3 d^3 - \cos(dx + c)\, a^2 b\, d^3 x + \cos(dx + c)\, a\, b^2 d^3 x^2 - 2\cos(dx + c)\, a\, b^2 d - \cos(dx + c)\, b^3 d^3}{}$$

input
```
int(x^4*sin(d*x+c)/(b*x+a),x)
```

output
```
(cos(c + d*x)*a**3*d**3 - cos(c + d*x)*a**2*b*d**3*x + cos(c + d*x)*a*b**2
*d**3*x**2 - 2*cos(c + d*x)*a*b**2*d - cos(c + d*x)*b**3*d**3*x**3 + 6*cos
(c + d*x)*b**3*d*x + int(sin(c + d*x)/(a + b*x),x)*a**4*d**4 + sin(c + d*x
)*a**2*b*d**2 - 2*sin(c + d*x)*a*b**2*d**2*x + 3*sin(c + d*x)*b**3*d**2*x*
*2 - 6*sin(c + d*x)*b**3)/(b**4*d**4)
```

3.19 $\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 152

$$\begin{aligned}\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = {} & \frac{2\cos(c+dx)}{bd^3} - \frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} \\ & - \frac{x^2\cos(c+dx)}{bd} - \frac{a^3 \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} \\ & - \frac{a\sin(c+dx)}{b^2d^2} + \frac{2x\sin(c+dx)}{bd^2} - \frac{a^3\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^4}\end{aligned}$$

output
```
2*cos(d*x+c)/b/d^3-a^2*cos(d*x+c)/b^3/d+a*x*cos(d*x+c)/b^2/d-x^2*cos(d*x+c
)/b/d+a^3*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4-a*sin(d*x+c)/b^2/d^2+2*x*sin(d*x
+c)/b/d^2-a^3*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = -\frac{a^3d^3 \operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + b((a^2d^2-abd^2x+b^2(-2+d^2x^2))\cos(c+dx)+bd(a-2b}{b^4d^3}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x),x]
```

output
```
-((a^3*d^3*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*((a^2*d^2 - a*b*d
^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*d*(a - 2*b*x)*Sin[c + d*x]) +
a^3*d^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^4*d^3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{a^3 \sin(c+dx)}{b^3(a+bx)}+\frac{a^2 \sin(c+dx)}{b^3}-\frac{ax \sin(c+dx)}{b^2}+\frac{x^2 \sin(c+dx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^3 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \cos(c+dx)}{b^3 d}-\frac{a \sin(c+dx)}{b^2 d^2}+\frac{ax \cos(c+dx)}{b^2 d}+\frac{2 \cos(c+dx)}{bd^3}+\frac{2x \sin(c+dx)}{bd^2}-\frac{x^2 \cos(c+dx)}{bd}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x),x]
```

output
```
(2*Cos[c + d*x])/(b*d^3) - (a^2*Cos[c + d*x])/(b^3*d) + (a*x*Cos[c + d*x])
/(b^2*d) - (x^2*Cos[c + d*x])/(b*d) - (a^3*CosIntegral[(a*d)/b + d*x]*Sin[
c - (a*d)/b])/b^4 - (a*Sin[c + d*x])/(b^2*d^2) + (2*x*Sin[c + d*x])/(b*d^2
) - (a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.56

method	result
risch	$\frac{i\,\mathrm{expIntegral}_1\left(\frac{i(bx+a)d}{b}\right)\cos\left(\frac{ad-bc}{b}\right)a^3}{2b^4}-\frac{i\,\mathrm{expIntegral}_1\left(-\frac{i(bx+a)d}{b}\right)\cos\left(\frac{ad-bc}{b}\right)a^3}{2b^4}-\frac{x^2\cos(dx+c)}{bd}-\mathrm{expInt}$
derivativedivides	$-d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)-\frac{3(ad-bc)d\,c^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\mathrm{C}\right.}{b}$
default	$-d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)-\frac{3(ad-bc)d\,c^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\mathrm{C}\right.}{b}$

input
```
int(x^3*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^4*Ei(1,I*(b*x+a)*d/b)*cos((a*d-b*c)/b)*a^3-1/2*I/b^4*Ei(1,-I*(b*x+
a)*d/b)*cos((a*d-b*c)/b)*a^3-x^2*cos(d*x+c)/b/d-1/2/b^4*Ei(1,I*(b*x+a)*d/b
)*sin((a*d-b*c)/b)*a^3-1/2/b^4*Ei(1,-I*(b*x+a)*d/b)*sin((a*d-b*c)/b)*a^3+a
*x*cos(d*x+c)/b^2/d+2*x*sin(d*x+c)/b/d^2-a^2*cos(d*x+c)/b^3/d-a*sin(d*x+c)
/b^2/d^2+2*cos(d*x+c)/b/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{x^3\sin(c+dx)}{a+bx}\,dx = \frac{a^3d^3\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right) - a^3d^3\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right) - \left(b^3d^2x^2 - ab^2d^2x + a^2bd^2 - 2\,b^3\right)\cos\left(dx\right.}{b^4d^3}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
(a^3*d^3*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a^3*d^3*cos(-
(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - (b^3*d^2*x^2 - a*b^2*d^2*x
+ a^2*b*d^2 - 2*b^3)*cos(d*x + c) + (2*b^3*d*x - a*b^2*d)*sin(d*x + c))/(b
^4*d^3)
```

Sympy [F]

$$\int \frac{x^3\sin(c+dx)}{a+bx}\,dx = \int \frac{x^3\sin\left(c+dx\right)}{a+bx}\,dx$$

input
```
integrate(x**3*sin(d*x+c)/(b*x+a),x)
```

output
```
Integral(x**3*sin(c + d*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{x^3\sin(c+dx)}{a+bx}\,dx = \int \frac{x^3\sin\left(dx+c\right)}{bx+a}\,dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(((2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -
(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d
)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(-I*exp_in
tegral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/
b))*cos(c)^2 + a^2*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_inte
gral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a*b
*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x
 + I*a*d)/b))*cos(c)^2 + 2*a*b*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) -
 I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(exp_integral_
e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c
)^2 + a^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((2*
a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x
+ I*a*d)/b))*cos(c)^2 + 2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + ex
p_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(-I*exp_integral_e(
2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c
)^2 + a^2*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2,
 -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a*b*(I*exp_i
ntegral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)
/b))*cos(c)^2 + 2*a*b*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 2709, normalized size of antiderivative = 17.82

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^3*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 + 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2 + a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*c)^2 - a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*c)^2 - 4*a^3*d^3*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d^3*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)
 - 8*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)*tan(1/2*a*d/b) + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = \int \frac{x^3 \sin(c+d\,x)}{a+b\,x} \, dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x),x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx}\,dx$$
$$= \frac{-\cos(dx+c)\,a^2d^2 + \cos(dx+c)\,ab\,d^2x - \cos(dx+c)\,b^2d^2x^2 + 2\cos(dx+c)\,b^2 - \left(\int \frac{\sin(dx+c)}{bx+a}dx\right)a^3d^3}{b^3d^3}$$

input
```
int(x^3*sin(d*x+c)/(b*x+a),x)
```

output
```
( - cos(c + d*x)*a**2*d**2 + cos(c + d*x)*a*b*d**2*x - cos(c + d*x)*b**2*d
**2*x**2 + 2*cos(c + d*x)*b**2 - int(sin(c + d*x)/(a + b*x),x)*a**3*d**3 -
 sin(c + d*x)*a*b*d + 2*sin(c + d*x)*b**2*d*x)/(b**3*d**3)
```

3.20 $\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{x^2\sin(c+dx)}{a+bx}\,dx=\frac{(a-bx)\cos(c+dx)}{b^2d}+\frac{a^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}+\frac{\sin(c+dx)}{bd^2}+\frac{a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

output
```
(-b*x+a)*cos(d*x+c)/b^2/d-a^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3+sin(d*x+c)/b
/d^2+a^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{x^2\sin(c+dx)}{a+bx}\,dx=\frac{a^2d^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+b(d(a-bx)\cos(c+dx)+b\sin(c+dx))+a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{S}}{b^3d^2}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x),x]
```

output
```
(a^2*d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a - b*x)*Cos[c
+ d*x] + b*Sin[c + d*x]) + a^2*d^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x
)])/(b^3*d^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{a^2 \sin(c+dx)}{b^2(a+bx)} - \frac{a \sin(c+dx)}{b^2} + \frac{x \sin(c+dx)}{b}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{a^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{a \cos(c+dx)}{b^2 d} + \frac{\sin(c+dx)}{bd^2} - \frac{x \cos(c+dx)}{bd}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x),x]
```

output
```
(a*Cos[c + d*x])/(b^2*d) - (x*Cos[c + d*x])/(b*d) + (a^2*CosIntegral[(a*d)
/b + d*x]*Sin[c - (a*d)/b])/b^3 + Sin[c + d*x]/(b*d^2) + (a^2*Cos[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{(dxb-ad)\cos(dx+c)}{d^2b^2}+\frac{\sin(dx+c)}{b\,d^2}-\frac{ia^2\cos\left(\frac{ad-bc}{b}\right)\text{expIntegral}_1\left(\frac{i(bx+a)d}{b}\right)}{2b^3}+\frac{ia^2\cos\left(\frac{ad-bc}{b}\right)\text{expIntegral}_1 \cdots}{2b^3}$
derivativedivides	$\frac{d\,c^2\left(\frac{\text{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)+\frac{2(ad-bc)dc\left(\frac{\text{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\text{Ci}\left(d \cdots\right)\right)}{b}\cdots}{\cdots}$
default	$\frac{d\,c^2\left(\frac{\text{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)+\frac{2(ad-bc)dc\left(\frac{\text{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\text{Ci}\left(d \cdots\right)\right)}{b}\cdots}{\cdots}$

input
```
int(x^2*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(b*d*x-a*d)/b^2*cos(d*x+c)+sin(d*x+c)/b/d^2-1/2*I*a^2/b^3*cos((a*d-
b*c)/b)*Ei(1,I*(b*x+a)*d/b)+1/2*I*a^2/b^3*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)
*d/b)+1/2*a^2/b^3*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)+1/2*a^2/b^3*sin((a*
d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int \frac{x^2 \sin(c + dx)}{a + bx} \, dx = -\frac{a^2 d^2 \operatorname{Ci}\left(\frac{bdx+ad}{b}\right) \sin\left(-\frac{bc-ad}{b}\right) - a^2 d^2 \cos\left(-\frac{bc-ad}{b}\right) \operatorname{Si}\left(\frac{bdx+ad}{b}\right) - b^2 \sin(dx + c) + (b^2 dx - abd) \cos(dx}{b^3 d^2}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-(a^2*d^2*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a^2*d^2*cos(
-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - b^2*sin(d*x + c) + (b^2*d*
x - a*b*d)*cos(d*x + c))/(b^3*d^2)
```

Sympy [F]

$$\int \frac{x^2 \sin(c + dx)}{a + bx} \, dx = \int \frac{x^2 \sin(c + dx)}{a + bx} \, dx$$

input
```
integrate(x**2*sin(d*x+c)/(b*x+a),x)
```

output
```
Integral(x**2*sin(c + d*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c + dx)}{a + bx} \, dx = \int \frac{x^2 \sin(dx + c)}{bx + a} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^2*cos(d*x + c) - ((a*(I*exp_integral_e
(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(
c)^2 + a*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -
(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(2
, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2
 + a*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x
 + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - (b*cos(c)^2
+ b*sin(c)^2)*x*sin(d*x + c) - ((a*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/
b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_integr
al_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(2, (I*b*d*x + I*a*d)/b)
 + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(exp_integral_e(2
, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2
)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + ((b*d*x^2*cos(c) + b*x*sin(c))*cos
(d*x + c)^2 + (b*d*x^2*cos(c) + b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c)
 - 2*(((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 + a^2*b*s
in(c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x +
(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(1/2*x*cos
(d*x + c)/(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2), x) - 2*(((a*b^2*cos(c)^2
+ a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*cos(d*...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 2205, normalized size of antiderivative = 24.78

$$\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*d^2*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 - 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/
2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d
*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2*d^2*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) -
 4*a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)*tan(1/2*a*d/b) + 8*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx = \int \frac{x^2\,\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x),x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx$$
$$= \frac{\cos(dx+c)\,ad - \cos(dx+c)\,bdx + \left(\int \frac{\sin(dx+c)}{bx+a}dx\right)a^2d^2 + \sin(dx+c)\,b}{b^2d^2}$$

input
```
int(x^2*sin(d*x+c)/(b*x+a),x)
```

output
```
(cos(c + d*x)*a*d - cos(c + d*x)*b*d*x + int(sin(c + d*x)/(a + b*x),x)*a**
2*d**2 + sin(c + d*x)*b)/(b**2*d**2)
```

3.21 $\int \frac{x \sin(c+dx)}{a+bx} dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x \sin(c+dx)}{a+bx} dx = -\frac{\cos(c+dx)}{bd} - \frac{a \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2} - \frac{a \cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^2}$$

output
```
-cos(d*x+c)/b/d+a*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^2-a*cos(-c+a*d/b)*Si(a*d/b
+d*x)/b^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x \sin(c+dx)}{a+bx} dx = -\frac{b\cos(c+dx) + ad \operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + ad\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2 d}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x),x]
```

output
```
-((b*Cos[c + d*x] + a*d*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + a*d*Co
s[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^2*d))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(c+dx)}{a+bx} \, dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{\sin(c+dx)}{b} - \frac{a \sin(c+dx)}{b(a+bx)}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{\cos(c+dx)}{bd}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x),x]
```

output
```
-(Cos[c + d*x]/(b*d)) - (a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^
2 - (a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{\cos(dx+c)}{bd}+\frac{ia\cos\left(\frac{ad-bc}{b}\right)\text{expIntegral}_1\left(\frac{i(bx+a)d}{b}\right)}{2b^2}-\frac{ia\cos\left(\frac{ad-bc}{b}\right)\text{expIntegral}_1\left(-\frac{i(bx+a)d}{b}\right)}{2b^2}-\frac{a\sin\left(\frac{ad-}{b}\right.}{} \cdots$
derivativedivides	$\frac{-\frac{(ad-bc)d\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{b}-\frac{d\cos(dx+c)}{b}-dc\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-}{b}\right.}{b}\right. \cdots}{d^2}$
default	$\frac{-\frac{(ad-bc)d\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{b}-\frac{d\cos(dx+c)}{b}-dc\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-}{b}\right.}{b}\right. \cdots}{d^2}$

input
```
int(x*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-cos(d*x+c)/b/d+1/2*I*a/b^2*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)-1/2*I*a/b
^2*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)-1/2*a/b^2*sin((a*d-b*c)/b)*Ei(1,I
*(b*x+a)*d/b)-1/2*a/b^2*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \frac{ad\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-ad\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)-b\cos(dx+c)}{b^2d}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
(a*d*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a*d*cos(-(b*c - a
*d)/b)*sin_integral((b*d*x + a*d)/b) - b*cos(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \frac{x \sin(c + dx)}{a + bx} \, dx = \int \frac{x \sin{(c + dx)}}{a + bx} \, dx$$

input
```
integrate(x*sin(d*x+c)/(b*x+a),x)
```

output
```
Integral(x*sin(c + d*x)/(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 776, normalized size of antiderivative = 11.25

$$\int \frac{x \sin(c + dx)}{a + bx} \, dx = \text{Too large to display}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*((d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_
integral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d
*(exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1,
 -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))*c/b + ((d*x + c
)*b*d*cos(d*x + c)^3 + (d*x + c)*b*d*cos(d*x + c) - ((b*c*d*(exp_integral_
e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*
b - I*b*c + I*a*d)/b)) - a*d^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c +
 I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-
(b*c - a*d)/b) - (a*d^2*(I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*
d)/b) - I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(
-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e
(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*sin(-(b*c - a*d)/b))*cos(d*x + c
)^2 + ((d*x + c)*b*d*cos(d*x + c) - (b*c*d*(exp_integral_e(2, (I*(d*x + c)
*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d
)/b)) - a*d^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_
integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a*d)/b) +
(a*d^2*(I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I*exp_int
egral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_integral_e
(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*x + c)
*b - I*b*c + I*a*d)/b)))*sin(-(b*c - a*d)/b))*sin(d*x + c)^2)/(((d*x + ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1647, normalized size of antiderivative = 23.87

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2
*a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 2*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 4*a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b) - 4*a*d*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*d*sin_integral((b*d*x + a*d)/b
)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - a*d*imag_part(cos_integral(d*
x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a*d*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*sin_integral((b*d*x + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x),x)
```

output
```
int((x*sin(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \frac{-\cos\left(dx+c\right)-\left(\int \frac{\sin(dx+c)}{bx+a}dx\right)ad}{bd}$$

input
```
int(x*sin(d*x+c)/(b*x+a),x)
```

output
```
( - (cos(c + d*x) + int(sin(c + d*x)/(a + b*x),x)*a*d))/(b*d)
```

3.22 $\int \frac{\sin(c+dx)}{a+bx} dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sin(c+dx)}{a+bx}\,dx=\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

output
```
-Ci(a*d/b+d*x)*sin(-c+a*d/b)/b+cos(-c+a*d/b)*Si(a*d/b+d*x)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{a+bx}\,dx=\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)+\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x),x]
```

output
```
(CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b] + Cos[c - (a*d)/b]*SinIntegra
l[(a*d)/b + d*x])/b
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+bx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)}{a+bx}dx$$

$$\Big\downarrow \text{3784}$$

$$\sin\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(xd+\frac{ad}{b}\right)}{a+bx}dx+\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}\right)}{a+bx}dx$$

$$\Big\downarrow \text{3042}$$

$$\sin\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}\right)}{a+bx}dx$$

$$\Big\downarrow \text{3780}$$

$$\sin\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

input
```
Int[Sin[c + d*x]/(a + b*x),x]
```

output
```
(CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b + (Cos[c - (a*d)/b]*SinInt
egral[(a*d)/b + d*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

method	result	size
derivativedivides	$\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}$	73
default	$\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}$	73
risch	$\frac{i e^{-\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2b}-\frac{i e^{\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(idx+ic+\frac{i(ad-bc)}{b}\right)}{2b}$	98

input
```
int(sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*
c)/b)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = -\frac{\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)}{b}$$

input
```
integrate(sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-(cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - cos(-(b*c - a*d)/b)*
sin_integral((b*d*x + a*d)/b))/b
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = \int \frac{\sin(c+dx)}{a+bx}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x+a),x)
```

output
```
Integral(sin(c + d*x)/(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int \frac{\sin(c+dx)}{a+bx}\,dx$$
$$= \frac{d\left(-i\,E_1\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_1\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d\left(E_1\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_1\left(-\frac{i}{\ }\right.\right.}{2\,bd}$$

input
```
integrate(sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_in
tegral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(
exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -
(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 597, normalized size of antiderivative = 11.71

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*c)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2
*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*sin_integral((b*d*x + a*d)/b)*tan(1/2*c
)*tan(1/2*a*d/b) - imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 +
 imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*sin_integral((
b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*c) + 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*a*d/b) + imag_part(cos_integral(d*x + a*d/b)) - imag
_part(cos_integral(-d*x - a*d/b)) + 2*sin_integral((b*d*x + a*d)/b))/(b*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + b*tan(1/2*c)^2 + b*tan(1/2*a*d/b)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a+bx}\,dx=\int \frac{\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x),x)
```

output
```
int(sin(c + d*x)/(a + b*x), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{a+bx}\,dx=\int \frac{\sin\left(dx+c\right)}{bx+a}dx$$

input
```
int(sin(d*x+c)/(b*x+a),x)
```

output
```
int(sin(c + d*x)/(a + b*x),x)
```

3.23 $\int \frac{\sin(c+dx)}{x(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx=\frac{\text{CosIntegral}(dx)\sin(c)}{a}-\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a}$$

output
```
Ci(d*x)*sin(c)/a+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a+cos(c)*Si(d*x)/a-cos(-c+a*d
/b)*Si(a*d/b+d*x)/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx=\frac{\text{CosIntegral}(dx)\sin(c)-\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+\cos(c)\text{Si}(dx)-\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{a}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x)),x]
```

output
```
(CosIntegral[d*x]*Sin[c] - CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + Cos
[c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/a
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{\sin(c+dx)}{ax}-\frac{b\sin(c+dx)}{a(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\sin(c)\,\text{CosIntegral}(dx)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x)),x]
```

output
```
(CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b]
)/a + (Cos[c]*SinIntegral[d*x])/a - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b
+ d*x])/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result
derivativedivides	$-\frac{b\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{a}+\frac{\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c)}{a}$
default	$-\frac{b\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{a}+\frac{\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c)}{a}$
risch	$\frac{i\mathrm{e}^{ic}\operatorname{expIntegral}_1(-idx)}{2a}-\frac{i\mathrm{e}^{-\frac{i(ad-bc)}{b}}\operatorname{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2a}-\frac{\mathrm{e}^{-ic}\pi\operatorname{csgn}(dx)}{2a}+\frac{\mathrm{e}^{-ic}\operatorname{Si}(dx)}{a}-$

input
```
int(sin(d*x+c)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-b/a*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((
a*d-b*c)/b)/b)+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int\frac{\sin(c+dx)}{x(a+bx)}\,dx$$
$$=\frac{\operatorname{Ci}(dx)\sin(c)+\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)+\cos(c)\operatorname{Si}(dx)-\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)}{a}$$

input
```
integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

output
```
(cos_integral(d*x)*sin(c) + cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)
/b) + cos(c)*sin_integral(d*x) - cos(-(b*c - a*d)/b)*sin_integral((b*d*x +
 a*d)/b))/a
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \frac{\sin{(c+dx)}}{x\,(a+bx)}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x+a),x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \frac{\sin{(dx+c)}}{(bx+a)x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)*x), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 838, normalized size of antiderivative = 11.48

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_in
tegral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_integral(d*x)*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*
a*d/b) + 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/
b) - 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 -
2*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_i
ntegral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*c)^2 + imag_part(cos_integral(d*x))*tan(1/2*c)^2 + imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - imag_part(cos_integral(-d*x)
)*tan(1/2*c)^2 + 2*sin_integral(d*x)*tan(1/2*c)^2 - 2*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b) - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1
/2*a*d/b) + 8*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - imag_part(cos_integr
al(d*x))*tan(1/2*a*d/b)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
a*d/b)^2 + imag_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*sin_integ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x\,\left(a+b\,x\right)}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x)),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \frac{\sin (dx+c)}{b\,x^2+ax}dx$$

input
```
int(sin(d*x+c)/x/(b*x+a),x)
```

output
```
int(sin(c + d*x)/(a*x + b*x**2),x)
```

3.24 $\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx &= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a}-\frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2}\\&\quad+\frac{b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}-\frac{\sin(c+dx)}{ax}\\&\quad-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{d\sin(c)\text{Si}(dx)}{a}+\frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}\end{aligned}$$

output
```
d*cos(c)*Ci(d*x)/a-b*Ci(d*x)*sin(c)/a^2-b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^2-
sin(d*x+c)/a/x-b*cos(c)*Si(d*x)/a^2-d*sin(c)*Si(d*x)/a+b*cos(-c+a*d/b)*Si(
a*d/b+d*x)/a^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \frac{x\,\text{CosIntegral}(dx)(ad\cos(c)-b\sin(c))+bx\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)-a\sin(c+dx)-bx\,\text{co}}{a^2x}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

output
```
(x*CosIntegral[d*x]*(a*d*Cos[c] - b*Sin[c]) + b*x*CosIntegral[d*(a/b + x)]
*Sin[c - (a*d)/b] - a*Sin[c + d*x] - b*x*Cos[c]*SinIntegral[d*x] - a*d*x*S
in[c]*SinIntegral[d*x] + b*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a
^2*x)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{b^2\sin(c+dx)}{a^2(a+bx)}-\frac{b\sin(c+dx)}{a^2x}+\frac{\sin(c+dx)}{ax^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\sin(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}+\frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a}-\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sin(c+dx)}{ax}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

output
```
(d*Cos[c]*CosIntegral[d*x])/a - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*CosIn
tegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 - Sin[c + d*x]/(a*x) - (b*Cos[
c]*SinIntegral[d*x])/a^2 - (d*Sin[c]*SinIntegral[d*x])/a + (b*Cos[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

method	result
derivativedivides	$d\left(-\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^2 d}+\frac{b^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{a^2 d}+\frac{-\sin\ldots}{}\right.$
default	$d\left(-\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^2 d}+\frac{b^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{a^2 d}+\frac{-\sin\ldots}{}\right.$
risch	$-\frac{ib\,e^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a^2}+\frac{ib\,e^{-\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2a^2}-\frac{d\,e^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a}-d\,e\ldots$

input
```
int(sin(d*x+c)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
d*(-b/a^2/d*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+b^2/a^2/d*(Si(d*x+c+(a*d-b*c)/
b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+1/a*(-sin(
d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx =$$
$$-\frac{bx\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-bx\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)-\left(adx\operatorname{Ci}\left(dx\right)-bx\operatorname{Si}\left(dx\right)\right)\cos\left(c\right)+a\sin\left(c\right.}{a^2x}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - b*x*cos(-(b*c -
a*d)/b)*sin_integral((b*d*x + a*d)/b) - (a*d*x*cos_integral(d*x) - b*x*sin
_integral(d*x))*cos(c) + a*sin(d*x + c) + (a*d*x*sin_integral(d*x) + b*x*c
os_integral(d*x))*sin(c))/(a^2*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin\left(c+dx\right)}{x^2\left(a+bx\right)}\,dx$$

input
```
integrate(sin(d*x+c)/x**2/(b*x+a),x)
```

output
```
Integral(sin(c + d*x)/(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx+a)x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)*x^2), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 2897, normalized size of antiderivative = 25.41

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*
x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
- 2*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*x*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b) - 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b) - a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*t
an(1/2*a*d/b)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^2\,\left(a+b\,x\right)}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x)),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin\left(dx+c\right)}{b\,x^3+a\,x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x+a),x)
```

output
```
int(sin(c + d*x)/(a*x**2 + b*x**3),x)
```

3.25 $\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 189

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = & -\frac{d\cos(c+dx)}{2ax}-\frac{bd\cos(c)\,\text{CosIntegral}(dx)}{a^2}\\ & +\frac{b^2\,\text{CosIntegral}(dx)\sin(c)}{a^3}-\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a}\\ & -\frac{b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}-\frac{\sin(c+dx)}{2ax^2}\\ & +\frac{b\sin(c+dx)}{a^2x}+\frac{b^2\cos(c)\text{Si}(dx)}{a^3}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}\\ & +\frac{bd\sin(c)\text{Si}(dx)}{a^2}-\frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}\end{aligned}$$

output
```
-1/2*d*cos(d*x+c)/a/x-b*d*cos(c)*Ci(d*x)/a^2+b^2*Ci(d*x)*sin(c)/a^3-1/2*d^
2*Ci(d*x)*sin(c)/a+b^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3-1/2*sin(d*x+c)/a/x^
2+b*sin(d*x+c)/a^2/x+b^2*cos(c)*Si(d*x)/a^3-1/2*d^2*cos(c)*Si(d*x)/a+b*d*s
in(c)*Si(d*x)/a^2-b^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = -\frac{a^2dx\cos(c+dx)+x^2\,\text{CosIntegral}(dx)\,(2abd\cos(c)+(-2b^2+a^2d^2)\sin(c))+2b^2x^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

output
```
-1/2*(a^2*d*x*Cos[c + d*x] + x^2*CosIntegral[d*x]*(2*a*b*d*Cos[c] + (-2*b^
2 + a^2*d^2)*Sin[c]) + 2*b^2*x^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b]
 + a^2*Sin[c + d*x] - 2*a*b*x*Sin[c + d*x] - 2*b^2*x^2*Cos[c]*SinIntegral[
d*x] + a^2*d^2*x^2*Cos[c]*SinIntegral[d*x] - 2*a*b*d*x^2*Sin[c]*SinIntegra
l[d*x] + 2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a^3*x^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{b^3\sin(c+dx)}{a^3(a+bx)}+\frac{b^2\sin(c+dx)}{a^3x}-\frac{b\sin(c+dx)}{a^2x^2}+\frac{\sin(c+dx)}{ax^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\sin(c)\,\text{CosIntegral}(dx)}{a^3}-\frac{b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{b^2\cos(c)\text{Si}(dx)}{a^3}-\frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{bd\cos(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{bd\sin(c)\text{Si}(dx)}{a^2}+\frac{b\sin(c+dx)}{a^2x}-\frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

input
```
Int[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

output
```
-1/2*(d*Cos[c + d*x])/(a*x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (b^2*Cos
Integral[d*x]*Sin[c])/a^3 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^2*Cos
Integral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(2*a*x^2) + (
b*Sin[c + d*x])/(a^2*x) + (b^2*Cos[c]*SinIntegral[d*x])/a^3 - (d^2*Cos[c]*
SinIntegral[d*x])/(2*a) + (b*d*Sin[c]*SinIntegral[d*x])/a^2 - (b^2*Cos[c -
 (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07

method	result
derivativedivides	$d^{2}\left(-\frac{b^{3}\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{d^{2}a^{3}}+\frac{b^{2}(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^{3}d^{2}}+\frac{-}{}\right.$
default	$d^{2}\left(-\frac{b^{3}\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{d^{2}a^{3}}+\frac{b^{2}(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^{3}d^{2}}+\frac{-}{}\right.$
risch	$\frac{db\,\mathrm{e}^{ic}\,\mathrm{expIntegral}_{1}(-idx)}{2a^{2}}+\frac{ib^{2}\mathrm{e}^{ic}\,\mathrm{expIntegral}_{1}(-idx)}{2a^{3}}-\frac{ib^{2}\mathrm{e}^{-\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_{1}\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2a^{3}}-\frac{id^{2}}{}$

input
```
int(sin(d*x+c)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
d^2*(-1/d^2*b^3/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*
d-b*c)/b)*sin((a*d-b*c)/b)/b)+b^2/a^3/d^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+
1/a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(
d*x)*sin(c))-b/a^2/d*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int\frac{\sin(c+dx)}{x^{3}(a+bx)}\,dx$$
$$=\frac{2\,b^{2}x^{2}\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-2\,b^{2}x^{2}\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)-a^{2}dx\cos\left(dx+c\right)-\left(2\,abdx^{2}\,\mathrm{Ci}\left(dx\right)+\right.}{}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - 2*b^2*x
^2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - a^2*d*x*cos(d*x + c
) - (2*a*b*d*x^2*cos_integral(d*x) + (a^2*d^2 - 2*b^2)*x^2*sin_integral(d*
x))*cos(c) + (2*a*b*x - a^2)*sin(d*x + c) + (2*a*b*d*x^2*sin_integral(d*x)
 - (a^2*d^2 - 2*b^2)*x^2*cos_integral(d*x))*sin(c))/(a^3*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\sin\left(c+dx\right)}{x^3\left(a+bx\right)}\,dx$$

input
```
integrate(sin(d*x+c)/x**3/(b*x+a),x)
```

output
```
Integral(sin(c + d*x)/(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx+a)x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)*x^3), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 4565, normalized size of antiderivative = 24.15

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

output
```
1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*
d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 + 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
 4*a*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 4*a*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(
1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(a+b\,x\right)}\,dx$$

input
```
int(sin(c + d*x)/(x^3*(a + b*x)),x)
```

output
```
int(sin(c + d*x)/(x^3*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\sin\left(dx+c\right)}{b\,x^4+a\,x^3}dx$$

input
```
int(sin(d*x+c)/x^3/(b*x+a),x)
```

output
```
int(sin(c + d*x)/(a*x**3 + b*x**4),x)
```

3.26 $\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 233

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx = {} & \frac{2\cos(c+dx)}{b^2d^3} - \frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} \\ & - \frac{x^2\cos(c+dx)}{b^2d} + \frac{a^4d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^6} \\ & - \frac{4a^3\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} \\ & - \frac{2a\sin(c+dx)}{b^3d^2} + \frac{2x\sin(c+dx)}{b^2d^2} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} \\ & - \frac{4a^3\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{a^4d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^6}\end{aligned}$$

output

```
2*cos(d*x+c)/b^2/d^3-3*a^2*cos(d*x+c)/b^4/d+2*a*x*cos(d*x+c)/b^3/d-x^2*cos
(d*x+c)/b^2/d+a^4*d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^6+4*a^3*Ci(a*d/b+d*x)*si
n(-c+a*d/b)/b^5-2*a*sin(d*x+c)/b^3/d^2+2*x*sin(d*x+c)/b^2/d^2-a^4*sin(d*x+
c)/b^5/(b*x+a)-4*a^3*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^5+a^4*d*sin(-c+a*d/b)*S
i(a*d/b+d*x)/b^6
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx = \frac{a^3 \operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-4b\sin\left(c-\frac{ad}{b}\right)\right)-\frac{b(b(a+bx)(3a^2d^2-2abd^2x+b^2(-2+d^2x^2))\cos(c+dx)}{d^3(a+bx)}}{b^6}$$

input
```
Integrate[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
(a^3*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 4*b*Sin[c - (a*d)/b]
) - (b*(b*(a + b*x)*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c +
 d*x] + d*(2*a^2*b^2 + a^4*d^2 - 2*b^4*x^2)*Sin[c + d*x]))/(d^3*(a + b*x))
 - a^3*(4*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b +
x)])/b^6
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{a^4 \sin(c+dx)}{b^4(a+bx)^2}-\frac{4a^3 \sin(c+dx)}{b^4(a+bx)}+\frac{3a^2 \sin(c+dx)}{b^4}-\frac{2ax \sin(c+dx)}{b^3}+\frac{x^2 \sin(c+dx)}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$
\frac{a^4 d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^6}-\frac{a^4 d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^6}-\frac{a^4\sin(c+dx)}{b^5(a+bx)}-\frac{4a^3\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{4a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{3a^2\cos(c+dx)}{b^4 d}-\frac{2a\sin(c+dx)}{b^3 d^2}+\frac{2ax\cos(c+dx)}{b^3 d}+\frac{2\cos(c+dx)}{b^2 d^3}+\frac{2x\sin(c+dx)}{b^2 d^2}-\frac{x^2\cos(c+dx)}{b^2 d}
$$

input
```
Int[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
(2*Cos[c + d*x])/(b^2*d^3) - (3*a^2*Cos[c + d*x])/(b^4*d) + (2*a*x*Cos[c +
 d*x])/(b^3*d) - (x^2*Cos[c + d*x])/(b^2*d) + (a^4*d*Cos[c - (a*d)/b]*CosI
ntegral[(a*d)/b + d*x])/b^6 - (4*a^3*CosIntegral[(a*d)/b + d*x]*Sin[c - (a
*d)/b])/b^5 - (2*a*Sin[c + d*x])/(b^3*d^2) + (2*x*Sin[c + d*x])/(b^2*d^2)
- (a^4*Sin[c + d*x])/(b^5*(a + b*x)) - (4*a^3*Cos[c - (a*d)/b]*SinIntegral
[(a*d)/b + d*x])/b^5 - (a^4*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])
/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 734, normalized size of antiderivative = 3.15

method	result
risch	$-\frac{i\left(-2ib^6d^5x^5+4ia\,b^5d^5x^4-6ib^6c\,d^4x^4-8ia^3b^3d^5x^2+10ia^4b^2d^5x-24ia^3b^3c\,d^4x+4ib^6d^3x^3+12ia^5b\,d^5-18ia^4b^2c\,d^4\right.}{2d^4b^5(bx+a)(-dxb-ad)(-dxb+2ad-3b}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^4*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d^4/b^5*(-2*I*b^6*d^5*x^5+4*I*b^6*d^3*x^3+12*I*a^5*b*d^5-8*I*a^3*b^
3*d^3+24*I*a*b^5*c*d^2*x-24*I*a^3*b^3*c*d^4*x-8*I*a^3*b^3*d^5*x^2+12*I*b^6
*c*d^2*x^2+10*I*a^4*b^2*d^5*x+12*I*a^2*b^4*c*d^2-18*I*a^4*b^2*c*d^4-12*I*a
^2*b^4*d^3*x+4*I*a*b^5*d^5*x^4-6*I*b^6*c*d^4*x^4)/(b*x+a)/(-b*d*x-a*d)/(-b
*d*x+2*a*d-3*b*c)*cos(d*x+c)+1/2/d^4/b^5*(-2*a^4*b^2*d^6*x^2+4*b^6*d^4*x^4
+2*a^5*b*d^6*x-6*a^4*b^2*c*d^5*x-4*a*b^5*d^4*x^3+12*b^6*c*d^3*x^3+4*a^6*d^
6-6*a^5*b*c*d^5-12*a^2*b^4*d^4*x^2+12*a*b^5*c*d^3*x^2+4*a^3*b^3*d^4*x-12*a
^2*b^4*c*d^3*x+8*a^4*b^2*d^4-12*a^3*b^3*c*d^3)/(b*x+a)/(-b*d*x-a*d)/(-b*d*
x+2*a*d-3*b*c)*sin(d*x+c)-2*I/b^5*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^
3+2*I/b^5*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^3-1/2*d/b^6*cos((a*d-b*c)
/b)*Ei(1,-I*(b*x+a)*d/b)*a^4-1/2*d/b^6*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b
)*a^4-2/b^5*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^3-2/b^5*sin((a*d-b*c)/
b)*Ei(1,I*(b*x+a)*d/b)*a^3+1/2*I*d/b^6*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/
b)*a^4-1/2*I*d/b^6*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.23

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2}\,dx = -\frac{(b^5d^2x^3 - ab^4d^2x^2 + 3\,a^3b^2d^2 - 2\,ab^4 + (a^2b^3d^2 - 2\,b^5)x)\cos(dx+c) - \left((a^4bd^4x + a^5d^4)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right) \right.}{}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 - 2*a*b^4 + (a^2*b^3*d^2 -
2*b^5)*x)*cos(d*x + c) - ((a^4*b*d^4*x + a^5*d^4)*cos_integral((b*d*x + a*
d)/b) - 4*(a^3*b^2*d^3*x + a^4*b*d^3)*sin_integral((b*d*x + a*d)/b))*cos(-
(b*c - a*d)/b) + (a^4*b*d^3 - 2*b^5*d*x^2 + 2*a^2*b^3*d)*sin(d*x + c) - (4
*(a^3*b^2*d^3*x + a^4*b*d^3)*cos_integral((b*d*x + a*d)/b) + (a^4*b*d^4*x
+ a^5*d^4)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^7*d^3*x
+ a*b^6*d^3)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^4 \sin{(c+dx)}}{(a+bx)^2} \, dx$$

input
```
integrate(x**4*sin(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x**4*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^4 \sin{(dx+c)}}{(bx+a)^2} \, dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*((2*a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(
3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(exp_integral_e(3, (I*b*d*x +
 I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(-I*
exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3, -(I*b*d*x + I
*a*d)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*ex
p_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) +
(2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -
(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I
*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp
_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)
/b))*cos(c)^2 + a^3*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral
_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c
)^2 + 2*((2*a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e
(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(exp_integral_e(3, (I*b*d*x
+ I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(-I
*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3, -(I*b*d*x +
I*a*d)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*e
xp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) +
 (2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3,
-(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(I*exp_integral_e(3, (I*b*d*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1973 vs. $2(236) = 472$.

Time = 0.22 (sec) , antiderivative size = 1973, normalized size of antiderivative = 8.47

$$\int \frac{x^4 \sin(c + dx)}{(a + bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output

```
((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*cos(-(b*c - a*d)/b)
*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/
b) - a^4*b*c*d^4*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^5*d^5*cos(-(b*c - a*d)/b)*cos_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (
b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*sin(-(b*c - a*d)/b)*s
in_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 - a^4*b*c*d^4*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b) + a^5*d^5*sin(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 4*(
b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos_integral(((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/
b) - 4*a^3*b^2*c*d^3*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 4*a^4*b*d^4*cos_integral(((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d
)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*c
 - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b
*c + a*d)/b) + 4*a^3*b^2*c*d^3*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - 4*a^4*b*d^4*cos(-(b
*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^4 \sin(c+d x)}{(a+b x)^2} \, dx$$

input

```
int((x^4*sin(c + d*x))/(a + b*x)^2,x)
```

output

```
int((x^4*sin(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin(dx+c)\,x^4}{b^2x^2+2abx+a^2}dx$$

input

```
int(x^4*sin(d*x+c)/(b*x+a)^2,x)
```

output

```
int((sin(c + d*x)*x**4)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.27 $\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 172

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx = \frac{(2a-bx)\cos(c+dx)}{b^3d} - \frac{a^3d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sin(c+dx)}{b^2d^2} + \frac{a^3\sin(c+dx)}{b^4(a+bx)} + \frac{3a^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{a^3d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5}$$

output

```
(-b*x+2*a)*cos(d*x+c)/b^3/d-a^3*d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^5-3*a^2*Ci
(a*d/b+d*x)*sin(-c+a*d/b)/b^4+sin(d*x+c)/b^2/d^2+a^3*sin(d*x+c)/b^4/(b*x+a
)+3*a^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4-a^3*d*sin(-c+a*d/b)*Si(a*d/b+d*x)/
b^5
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx = \frac{-a^2 \operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-3b\sin\left(c-\frac{ad}{b}\right)\right)+\frac{b(bd(2a^2+abx-b^2x^2)\cos(c+dx)+(ab^2+a^3d^2+b^3x)}{d^2(a+bx)}}{b^5}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
(-(a^2*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 3*b*Sin[c - (a*d)/
b])) + (b*(b*d*(2*a^2 + a*b*x - b^2*x^2)*Cos[c + d*x] + (a*b^2 + a^3*d^2 +
 b^3*x)*Sin[c + d*x]))/(d^2*(a + b*x)) + a^2*(3*b*Cos[c - (a*d)/b] + a*d*S
in[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{a^3 \sin(c+dx)}{b^3(a+bx)^2}+\frac{3a^2 \sin(c+dx)}{b^3(a+bx)}-\frac{2a \sin(c+dx)}{b^3}+\frac{x \sin(c+dx)}{b^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3 \sin(c+dx)}{b^4(a+bx)}+\frac{3a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{3a^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{2a \cos(c+dx)}{b^3 d}+\frac{\sin(c+dx)}{b^2 d^2}-\frac{x \cos(c+dx)}{b^2 d}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
(2*a*Cos[c + d*x])/(b^3*d) - (x*Cos[c + d*x])/(b^2*d) - (a^3*d*Cos[c - (a*
d)/b]*CosIntegral[(a*d)/b + d*x])/b^5 + (3*a^2*CosIntegral[(a*d)/b + d*x]*
Sin[c - (a*d)/b])/b^4 + Sin[c + d*x]/(b^2*d^2) + (a^3*Sin[c + d*x])/(b^4*(
a + b*x)) + (3*a^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4 + (a^3
*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 607, normalized size of antiderivative = 3.53

method	result
risch	$-\frac{i\left(-2ib^4d^3x^4+4iab^3d^3x^3-6ib^4c\,d^2x^3+6ia^2b^2d^3x^2-8ia^3b\,d^3x+18ia^2b^2c\,d^2x-8ia^4d^3+12ia^3bc\,d^2\right)\cos(dx+c)}{2d^2b^3(bx+a)(-dxb+2ad-3bc)(-dxb-ad)}+$
derivativedivides	$\frac{-d^2c^3\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)+3d^2c^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right.}{b}\right.}{}$
default	$\frac{-d^2c^3\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)+3d^2c^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right.}{b}\right.}{}$

input
```
int(x^3*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d^2/b^3*(-2*I*b^4*d^3*x^4+18*I*a^2*b^2*c*d^2*x+4*I*a*b^3*d^3*x^3-6*
I*b^4*c*d^2*x^3+12*I*a^3*b*c*d^2+6*I*a^2*b^2*d^3*x^2-8*I*a^3*b*d^3*x-8*I*a
^4*d^3)/(b*x+a)/(-b*d*x+2*a*d-3*b*c)/(-b*d*x-a*d)*cos(d*x+c)+1/2/d^2/b^4*(
2*a^3*b^2*d^4*x^2-2*a^4*b*d^4*x+6*a^3*b^2*c*d^3*x+2*b^5*d^2*x^3-4*a^5*d^4+
6*a^4*b*c*d^3+6*b^5*c*d*x^2-6*a^2*b^3*d^2*x+12*a*b^4*c*d*x-4*a^3*b^2*d^2+6
*a^2*b^3*c*d)/(b*x+a)/(-b*d*x+2*a*d-3*b*c)/(-b*d*x-a*d)*sin(d*x+c)+1/2*d/b
^5*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^3+1/2*d/b^5*cos((a*d-b*c)/b)*Ei(
1,-I*(b*x+a)*d/b)*a^3-3/2*I/b^4*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^2+3
/2*I/b^4*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^2+1/2*I*d/b^5*sin((a*d-b*
c)/b)*Ei(1,I*(b*x+a)*d/b)*a^3-1/2*I*d/b^5*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)
*d/b)*a^3+3/2/b^4*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^2+3/2/b^4*sin((a*
d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.43

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx = -\frac{(b^4dx^2 - ab^3dx - 2a^2b^2d)\cos(dx+c) + \left((a^3bd^3x + a^4d^3)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right) - 3(a^2b^2d^2x + a^3bd^2)\,\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\right.}{}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*cos(d*x + c) + ((a^3*b*d^3*x + a^4
*d^3)*cos_integral((b*d*x + a*d)/b) - 3*(a^2*b^2*d^2*x + a^3*b*d^2)*sin_in
tegral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (a^3*b*d^2 + b^4*x + a*b^3)
*sin(d*x + c) + (3*(a^2*b^2*d^2*x + a^3*b*d^2)*cos_integral((b*d*x + a*d)/
b) + (a^3*b*d^3*x + a^4*d^3)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*
d)/b))/(b^6*d^2*x + a*b^5*d^2)
```

Sympy [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx$$

input
```
integrate(x**3*sin(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x**3*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx+a)^2}\,dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^3*cos(d*x + c) - 2*((a^2*(I*exp_in
tegral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/
b))*cos(c)^2 + a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integ
ral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a^2*(exp_
integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/
b))*cos(c)^2 + a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_
e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2
- 2*((a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3,
-(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*
d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a
*d)/b) - (a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3,
-(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)
/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/
b))*sin(d*x + c)^2 + ((b^2*d*x^3*cos(c) + b^2*x^2*sin(c) + 2*a*b*x*sin(c))
*cos(d*x + c)^2 + (b^2*d*x^3*cos(c) + b^2*x^2*sin(c) + 2*a*b*x*sin(c))*sin
(d*x + c)^2)*cos(d*x + 2*c) + 2*(((a^2*b^4*cos(c)^2 + a^2*b^4*sin(c)^2)*d^
3*x^2 + 2*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (a^4*b^2*cos(c)^2
+ a^4*b^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a^2*b^4*cos(c)^2 + a^2*b^4*sin
(c)^2)*d^3*x^2 + 2*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (a^4*b^2*
cos(c)^2 + a^4*b^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(x*cos(d*x +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. $2(177) = 354$.

Time = 0.21 (sec) , antiderivative size = 1474, normalized size of antiderivative = 8.57

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*c - a*d)/b
)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)
/b) - a^3*b*c*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^4*d^4*cos(-(b*c - a*d)/b)*cos_
integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) +
(b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*sin(-(b*c - a*d)/b)*
sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
) - a^3*b*c*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^4*d^4*sin(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 3*
(b*x + a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos_integral(((b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)
/b) - 3*a^2*b^2*c*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 3*a^3*b*d^3*cos_integral(((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*
d)/b) - 3*(b*x + a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*
c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b) + 3*a^2*b^2*c*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - 3*a^3*b*d^3*cos(-(
b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3\,\sin\left(c+d\,x\right)}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{\sin(dx+c)\, x^3}{b^2x^2+2abx+a^2} dx$$

input
```
int(x^3*sin(d*x+c)/(b*x+a)^2,x)
```

output
```
int((sin(c + d*x)*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.28 $\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} dx = -\frac{\cos(c+dx)}{b^2 d} + \frac{a^2 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2a \text{CosIntegral}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2 \sin(c+dx)}{b^3(a+bx)} - \frac{2a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a^2 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}$$

output
```
-cos(d*x+c)/b^2/d+a^2*d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^4+2*a*Ci(a*d/b+d*x)*
sin(-c+a*d/b)/b^3-a^2*sin(d*x+c)/b^3/(b*x+a)-2*a*cos(-c+a*d/b)*Si(a*d/b+d*
x)/b^3+a^2*d*sin(-c+a*d/b)*Si(a*d/b+d*x)/b^4
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} dx = \frac{a \text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-2b\sin\left(c-\frac{ad}{b}\right)\right)+b\left(-\frac{b\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{a+bx}\right)-a\left(2b\cos\left(c\right.\right.}{b^4}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
(a*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 2*b*Sin[c - (a*d)/b])
+ b*(-((b*Cos[c + d*x])/d) - (a^2*Sin[c + d*x])/(a + b*x)) - a*(2*b*Cos[c
- (a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^4
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a^2 \sin(c+dx)}{b^2(a+bx)^2} - \frac{2a \sin(c+dx)}{b^2(a+bx)} + \frac{\sin(c+dx)}{b^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^2 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^2 \sin(c+dx)}{b^3(a+bx)} - \frac{2a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{2a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{\cos(c+dx)}{b^2 d}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
-(Cos[c + d*x]/(b^2*d)) + (a^2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*
x])/b^4 - (2*a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3 - (a^2*Sin
[c + d*x])/(b^3*(a + b*x)) - (2*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d
*x])/b^3 - (a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.70

method	result
risch	$-\frac{i(2ib^3d^2x^2+4ia\,b^2d^2x+2ia^2b\,d^2)\cos(dx+c)}{2d^2b^3(bx+a)(-dxb-ad)}+\frac{(2a^2b\,d^3x+2a^3d^3)\sin(dx+c)}{2d^2b^3(bx+a)(-dxb-ad)}-\frac{i\cos\left(\frac{ad-bc}{b}\right)\text{expIntegral}_1\left(-\frac{i(}{}\right.}{b^3}$
derivativedivides	$\frac{d^2c^2\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\text{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)+\frac{2d^2(ad-bc)c\left(-\frac{\sin(dx}{(ad-bc+b(}\right.}{}}{}$
default	$\frac{d^2c^2\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\text{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)+\frac{2d^2(ad-bc)c\left(-\frac{\sin(dx}{(ad-bc+b(}\right.}{}}{}$

input
```
int(x^2*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d^2/b^3*(4*I*a*b^2*d^2*x+2*I*a^2*b*d^2+2*I*b^3*d^2*x^2)/(b*x+a)/(-b
*d*x-a*d)*cos(d*x+c)+1/2/d^2/b^3*(2*a^2*b*d^3*x+2*a^3*d^3)/(b*x+a)/(-b*d*x
-a*d)*sin(d*x+c)-I/b^3*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a+I/b^3*cos((
a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a-1/2*d/b^4*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+
a)*d/b)*a^2-1/2*d/b^4*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^2-1/b^3*sin((
a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a-1/b^3*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d
/b)*a+1/2*I*d/b^4*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^2-1/2*I*d/b^4*si
n((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = -\frac{a^2bd\sin(dx+c)+(b^3x+ab^2)\cos(dx+c)-\left((a^2bd^2x+a^3d^2)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)-2\left(ab^2dx+a^2bd\right)\operatorname{Si}\left(\frac{bdx-}{b}\right.\right.}{b^5dx+ab}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(a^2*b*d*sin(d*x + c) + (b^3*x + a*b^2)*cos(d*x + c) - ((a^2*b*d^2*x + a^
3*d^2)*cos_integral((b*d*x + a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*sin_integra
l((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (2*(a*b^2*d*x + a^2*b*d)*cos_int
egral((b*d*x + a*d)/b) + (a^2*b*d^2*x + a^3*d^2)*sin_integral((b*d*x + a*d
)/b))*sin(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)
```

Sympy [F]

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx$$

input
```
integrate(x**2*sin(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x**2*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^2\sin(dx+c)}{(bx+a)^2}\,dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x^2*cos(d*x + c) + (x^2*cos(d*x + c)^2*cos(c)
+ x^2*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b^2*cos(c)^2 + a*b^2*
sin(c)^2)*d*x^2 + 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2
+ a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x
^2 + 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^
2)*d)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b^3*d*x^3 + 3*a*b^2*d*x^2
+ 3*a^2*b*d*x + a^3*d), x) - 2*(((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 +
 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d
)*cos(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 + 2*(a^2*b*cos
(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c
)^2)*integrate(x*cos(d*x + c)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x +
a^3*d)*cos(d*x + c)^2 + (b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d)*
sin(d*x + c)^2), x) + (x^2*cos(d*x + c)^2*sin(c) + x^2*sin(d*x + c)^2*sin(
c))*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^2 + 2*(a*b*cos(c)^
2 + a*b*sin(c)^2)*d*x + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 +
((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^2 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x
 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(152) = 304$.

Time = 0.23 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.52

$$\int \frac{x^2 \sin(c + dx)}{(a + bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)
*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/
b) - a^2*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^3*d^3*cos(-(b*c - a*d)/b)*cos_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (
b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*s
in_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 - a^2*b*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b) + a^3*d^3*sin(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*(
b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) -
 2*a*b^2*c*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
 b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 2*a^2*b*d^2*cos_integral(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2
*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos(-(b*c - a*d)/b)*s
in_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 + 2*a*b^2*c*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b) - 2*a^2*b*d^2*cos(-(b*c - a*d)/b)*sin
_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^2 \sin(c+d\,x)}{{(a+b\,x)}^2} \,dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{\sin (dx+c) x^2}{b^2x^2+2abx+a^2} dx$$

input
```
int(x^2*sin(d*x+c)/(b*x+a)^2,x)
```

output
```
int((sin(c + d*x)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.29 $\int \frac{x \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x \sin(c+dx)}{(a+bx)^2} \, dx = -\frac{ad \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{b^2} + \frac{a \sin(c+dx)}{b^2(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{ad \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

output
```
-a*d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^3-Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^2+a*sin
(d*x+c)/b^2/(b*x+a)+cos(-c+a*d/b)*Si(a*d/b+d*x)/b^2-a*d*sin(-c+a*d/b)*Si(a
*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x \sin(c+dx)}{(a+bx)^2} \, dx = \frac{\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(-ad \cos\left(c-\frac{ad}{b}\right)+b \sin\left(c-\frac{ad}{b}\right)\right)+\frac{ab \sin(c+dx)}{a+bx}+\left(b \cos\left(c-\frac{ad}{b}\right)+ad \sin\left(c-\right.\right.}{b^3}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
(CosIntegral[d*(a/b + x)]*(-(a*d*Cos[c - (a*d)/b]) + b*Sin[c - (a*d)/b]) +
 (a*b*Sin[c + d*x])/(a + b*x) + (b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b]
)*SinIntegral[d*(a/b + x)])/b^3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{\sin(c+dx)}{b(a+bx)}-\frac{a\sin(c+dx)}{b(a+bx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

output
```
-((a*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^3) + (CosIntegral[(a
*d)/b + d*x]*Sin[c - (a*d)/b])/b^2 + (a*Sin[c + d*x])/(b^2*(a + b*x)) + (C
os[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2 + (a*d*Sin[c - (a*d)/b]*Si
nIntegral[(a*d)/b + d*x])/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.52

method	result
risch	$\frac{(-2abdx-2a^2d)\sin(dx+c)}{2b^2(bx+a)(-dxb-ad)}+\frac{\cos\left(\frac{ad-bc}{b}\right)\operatorname{expIntegral}_1\left(\frac{i(bx+a)d}{b}\right)ad}{2b^3}+\frac{\cos\left(\frac{ad-bc}{b}\right)\operatorname{expIntegral}_1\left(-\frac{i(bx+a)d}{b}\right)ad}{2b^3}$
derivativedivides	$\frac{\frac{d^2\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{b}-\frac{d^2(ad-bc)\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)}{b}\right.}{b}}{}$
default	$\frac{\frac{d^2\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{b}-\frac{d^2(ad-bc)\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)}{b}\right.}{b}}{}$

input
```
int(x*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(-2*a*b*d*x-2*a^2*d)/(b*x+a)/(-b*d*x-a*d)*sin(d*x+c)+1/2/b^3*cos((
a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a*d+1/2/b^3*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+
a)*d/b)*a*d-1/2*I/b^2*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)+1/2*I/b^2*cos((
a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)+1/2*I/b^3*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)
*d/b)*a*d-1/2*I/b^3*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a*d+1/2/b^2*sin(
(a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)+1/2/b^2*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*
d/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.25

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{ab\sin\left(dx+c\right) - \left(\left(abdx + a^2d\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) - \left(b^2x+ab\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right) - \left(\left(b^2x+ab\right)\operatorname{Ci}\left(\frac{bdx+}{b}\right.\right.}{b^4x+ab^3}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(a*b*sin(d*x + c) - ((a*b*d*x + a^2*d)*cos_integral((b*d*x + a*d)/b) - (b^
2*x + a*b)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - ((b^2*x +
a*b)*cos_integral((b*d*x + a*d)/b) + (a*b*d*x + a^2*d)*sin_integral((b*d*x
 + a*d)/b))*sin(-(b*c - a*d)/b))/(b^4*x + a*b^3)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\sin\left(dx+c\right)}{\left(bx+a\right)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((b*cos(c)^2 + b*sin(c)^2)*x*cos(d*x + c) + ((a*(exp_integral_e(3, (I
*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a
*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(3, (I*b*d*x
 + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*
exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((a*(exp_integra
l_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos
(c)^2 + a*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(3,
 (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^
2 + a*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + (b*x*co
s(d*x + c)^2*cos(c) + b*x*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^4
*cos(c)^2 + b^4*sin(c)^2)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x
+ (a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^4*cos(c)^2
 + b^4*sin(c)^2)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^
2*cos(c)^2 + a^2*b^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x
+ c)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d), x) + 2*(((b^4*cos(
c)^2 + b^4*sin(c)^2)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(130) = 260$.

Time = 0.15 (sec) , antiderivative size = 951, normalized size of antiderivative = 7.67

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*
cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
) - a*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*cos(-(b*c - a*d)/b)*cos_inte
gral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x
 + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*
b*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*sin(-(b*c - a*d)/b)*sin_integral((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)*
b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - b^2*c*d*co
s_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*
sin(-(b*c - a*d)/b) + a*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*
x + a) - a*d/(b*x + a) + d)*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*
(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + b^2*c*d*cos(-(b*c -
a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
+ a*d)/b) - a*b*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a*b*d^2*sin(-(b*x + a)*(b*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x*sin(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin(dx+c)\,x}{b^2x^2+2abx+a^2}dx$$

input
```
int(x*sin(d*x+c)/(b*x+a)^2,x)
```

output
```
int((sin(c + d*x)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.30 $\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{\sin(c+dx)}{b(a+bx)} - \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}$$

output
```
d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^2-sin(d*x+c)/b/(b*x+a)+d*sin(-c+a*d/b)*Si(
a*d/b+d*x)/b^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right) - \frac{b\sin(c+dx)}{a+bx} - d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x)^2,x]
```

output
```
(d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b + x)] - (b*Sin[c + d*x])/(a + b*x)
- d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b^2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{d\int \frac{\cos(c+dx)}{a+bx}dx}{b}-\frac{\sin(c+dx)}{b(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a+bx}dx}{b}-\frac{\sin(c+dx)}{b(a+bx)}$$

$$\Big\downarrow \text{3784}$$

$$\frac{d\left(\cos\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(xd+\frac{ad}{b}\right)}{a+bx}dx-\sin\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}-\frac{\sin(c+dx)}{b(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\left(\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}+\frac{\pi}{2}\right)}{a+bx}dx-\sin\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}-\frac{\sin(c+dx)}{b(a+bx)}$$

$$\Big\downarrow \text{3780}$$

$$\frac{d\left(\cos\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(xd+\frac{ad}{b}+\frac{\pi}{2}\right)}{a+bx}dx-\frac{\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}-\frac{\sin(c+dx)}{b(a+bx)}$$

$$\Big\downarrow \text{3783}$$

$$\frac{d\left(\frac{\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b}-\frac{\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}-\frac{\sin(c+dx)}{b(a+bx)}$$

input
```
Int[Sin[c + d*x]/(a + b*x)^2,x]
```

output
```
-(Sin[c + d*x]/(b*(a + b*x))) + (d*((Cos[c - (a*d)/b]*CosIntegral[(a*d)/b
+ d*x])/b - (Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)$
default	$d\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)$
risch	$-\frac{d\,\mathrm{e}^{-\frac{i(ad-bc)}{b}}\operatorname{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2b^2}-\frac{d\,\mathrm{e}^{\frac{i(ad-bc)}{b}}\operatorname{expIntegral}_1\left(idx+ic+\frac{i(ad-bc)}{b}\right)}{2b^2}-\frac{(-2dxb-2ad}{2b(bx+a)(-}$

input
```
int(sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*(-sin(d*x+c)/(a*d-b*c+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/
b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{(bdx+ad)\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+(bdx+ad)\sin\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)-b\sin\left(dx+c\right)}{b^3x+ab^2}$$

input
```
integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*d*x + a*d)*cos(-(b*c - a*d)/b)*cos_integral((b*d*x + a*d)/b) + (b*d*x
+ a*d)*sin(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - b*sin(d*x + c))
/(b^3*x + a*b^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin{(c+dx)}}{(a+bx)^2}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d^2\left(-i\,E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_2\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^2\left(E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_2\left(-\right.\right.}{2\left((dx+c)b^2-b^2c+abd\right)d}$$

input
```
integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(d^2*(-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_
integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d
^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(
2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)*b
^2 - b^2*c + a*b*d)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(73) = 146$.

Time = 0.15 (sec) , antiderivative size = 518, normalized size of antiderivative = 7.19

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)-bcd^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{(bx\cdots}{}\right.\right.}{}$$

input
```
integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*cos
_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) -
 b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b) + a*d^3*cos(-(b*c - a*d)/b)*cos_integral(((
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral(((
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - b*c*d^2*sin
(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d) - b*c + a*d)/b) + a*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + b*d^2*sin(-(b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b^5*c + a*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin(c+d\,x)}{(a+b\,x)^2}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x)^2,x)
```

output
```
int(sin(c + d*x)/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin(dx+c)}{b^2x^2+2abx+a^2}dx$$

input
```
int(sin(d*x+c)/(b*x+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.31 $\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\begin{aligned}\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = &-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{ab}+\frac{\text{CosIntegral}(dx)\sin(c)}{a^2}\\&-\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}+\frac{\sin(c+dx)}{a(a+bx)}+\frac{\cos(c)\text{Si}(dx)}{a^2}\\&-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{ab}\end{aligned}$$

output
```
-d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/a/b+Ci(d*x)*sin(c)/a^2+Ci(a*d/b+d*x)*sin(-c
+a*d/b)/a^2+sin(d*x+c)/a/(b*x+a)+cos(c)*Si(d*x)/a^2-cos(-c+a*d/b)*Si(a*d/b
+d*x)/a^2-d*sin(-c+a*d/b)*Si(a*d/b+d*x)/a/b
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$

$$= \frac{\frac{a\cos(dx)\sin(c)}{a+bx}+\text{CosIntegral}(dx)\sin(c)-\frac{\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)+b\sin\left(c-\frac{ad}{b}\right)\right)}{b}+\frac{a\cos(c)\sin(dx)}{a+bx}+\cos(c)\text{S}\ldots}{a^2}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

output
```
((a*Cos[d*x]*Sin[c])/(a + b*x) + CosIntegral[d*x]*Sin[c] - (CosIntegral[d*
(a/b + x)]*(a*d*Cos[c - (a*d)/b] + b*Sin[c - (a*d)/b]))/b + (a*Cos[c]*Sin[
d*x])/(a + b*x) + Cos[c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d
*(a/b + x)] + (a*d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b)/a^2
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{b\sin(c+dx)}{a^2(a+bx)}+\frac{\sin(c+dx)}{a^2x}-\frac{b\sin(c+dx)}{a(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{ab}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{ab}+\frac{\sin(c+dx)}{a(a+bx)}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

output
```
-((d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/(a*b)) + (CosIntegral[d*
x]*Sin[c])/a^2 - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 + Sin[c
 + d*x]/(a*(a + b*x)) + (Cos[c]*SinIntegral[d*x])/a^2 - (Cos[c - (a*d)/b]*
SinIntegral[(a*d)/b + d*x])/a^2 + (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b
+ d*x])/(a*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c)}{a^{2}}-\frac{db\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\cdots\right)}{a}$
default	$\frac{\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c)}{a^{2}}-\frac{db\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\cdots\right)}{a}$
risch	$\frac{i e^{ic}\operatorname{expIntegral}_{1}(-idx)}{2a^{2}}-\frac{i e^{-\frac{i(ad-bc)}{b}}\operatorname{expIntegral}_{1}\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2a^{2}}+\frac{d\,e^{-\frac{i(ad-bc)}{b}}\operatorname{expIntegral}_{1}\left(-idx-ic-\cdots\right)}{2ab}$

input
```
int(sin(d*x+c)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-d*b/a*(-sin(d*x+c)/(a*d-b*c+b*(d*x+c
))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((
a*d-b*c)/b)/b)/b)-b/a^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c
+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$
$$= \frac{ab\sin(dx+c)+(b^2x+ab)\operatorname{Ci}(dx)\sin(c)+(b^2x+ab)\cos(c)\operatorname{Si}(dx)-\left((abdx+a^2d)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+(b^2\ldots\right.}{a^2b^2x+\ldots}$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(a*b*sin(d*x + c) + (b^2*x + a*b)*cos_integral(d*x)*sin(c) + (b^2*x + a*b)
*cos(c)*sin_integral(d*x) - ((a*b*d*x + a^2*d)*cos_integral((b*d*x + a*d)/
b) + (b^2*x + a*b)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) + ((
b^2*x + a*b)*cos_integral((b*d*x + a*d)/b) - (a*b*d*x + a^2*d)*sin_integra
l((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin(dx+c)}{(bx+a)^2x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)^2*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(153) = 306$.

Time = 0.20 (sec) , antiderivative size = 1281, normalized size of antiderivative = 8.60

$$\int \frac{\sin(c + dx)}{x(a + bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
-((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*
cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
) - a*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*cos(-(b*c - a*d)/b)*cos_inte
gral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x
 + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*
b*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*sin(-(b*c - a*d)/b)*sin_integral((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - (b*x + a)*
b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos_integral((b*x + a)*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)/b - c)*sin(c) + b^2*c*d*cos_integral((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - a*b*d^2*cos_integral((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - (b*x + a)*b*(b*
c/(b*x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + b^2*c*d*cos_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-
(b*c - a*d)/b) - a*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + (b*x + a)*b*(b*c/(b*x + a
) - a*d/(b*x + a) + d)*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin(c+d\,x)}{x\,(a+b\,x)^2}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x)^2),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin(dx+c)}{b^2x^3+2ab\,x^2+a^2x}dx$$

input
```
int(sin(d*x+c)/x/(b*x+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)
```

3.32 $\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 188

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx &= \frac{d\cos(c)\,\mathrm{CosIntegral}(dx)}{a^2} + \frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2} \\ &\quad - \frac{2b\,\mathrm{CosIntegral}(dx)\sin(c)}{a^3} + \frac{2b\,\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\ &\quad - \frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} - \frac{2b\cos(c)\mathrm{Si}(dx)}{a^3} - \frac{d\sin(c)\mathrm{Si}(dx)}{a^2} \\ &\quad + \frac{2b\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{a^2}\end{aligned}$$

output

```
d*cos(c)*Ci(d*x)/a^2+d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/a^2-2*b*Ci(d*x)*sin(c)/
a^3-2*b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3-sin(d*x+c)/a^2/x-b*sin(d*x+c)/a^2/
(b*x+a)-2*b*cos(c)*Si(d*x)/a^3-d*sin(c)*Si(d*x)/a^2+2*b*cos(-c+a*d/b)*Si(a
*d/b+d*x)/a^3+d*sin(-c+a*d/b)*Si(a*d/b+d*x)/a^2
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx =$$
$$-\frac{-ad\cos(c)\,\text{CosIntegral}(dx) - ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right) + \frac{a(a+2bx)\cos(dx)\sin(c)}{x(a+bx)} + 2b\,\text{CosIn}}{}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

output
```
-((-(a*d*Cos[c]*CosIntegral[d*x]) - a*d*Cos[c - (a*d)/b]*CosIntegral[d*(a/
b + x)] + (a*(a + 2*b*x)*Cos[d*x]*Sin[c])/(x*(a + b*x)) + 2*b*CosIntegral[
d*x]*Sin[c] - 2*b*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (a*(a + 2*b*
x)*Cos[c]*Sin[d*x])/(x*(a + b*x)) + 2*b*Cos[c]*SinIntegral[d*x] + a*d*Sin[
c]*SinIntegral[d*x] - 2*b*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a*d*
Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/a^3)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(\frac{2b^2\sin(c+dx)}{a^3(a+bx)} - \frac{2b\sin(c+dx)}{a^3x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^2} + \frac{\sin(c+dx)}{a^2x^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{2b\sin(c)\,\text{CosIntegral}(dx)}{a^3}+\frac{2b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\cos(c)\text{Si}(dx)}{a^3}+\frac{2b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\sin(c+dx)}{a^2(a+bx)}+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2}-\frac{d\sin(c)\text{Si}(dx)}{a^2}-\frac{\sin(c+dx)}{a^2x}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

output
```
(d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b
+ d*x])/a^2 - (2*b*CosIntegral[d*x]*Sin[c])/a^3 + (2*b*CosIntegral[(a*d)/b
 + d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(a^2*x) - (b*Sin[c + d*x])/(a
^2*(a + b*x)) - (2*b*Cos[c]*SinIntegral[d*x])/a^3 - (d*Sin[c]*SinIntegral[
d*x])/a^2 + (2*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^3 - (d*Sin
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.36

method	result
derivativedivides	$d\left(\frac{b^2\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)}{a^2}+\frac{2b^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-b}{b}\right.}{b}\right.}{}\right.$
default	$d\left(\frac{b^2\left(-\frac{\sin(dx+c)}{(ad-bc+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}}{b}\right)}{a^2}+\frac{2b^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{ad-b}{b}\right.}{b}\right.}{}\right.$
risch	$-\frac{d\,e^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a^2}-\frac{ib\,e^{ic}\,\mathrm{expIntegral}_1(-idx)}{a^3}+\frac{ib\,e^{-\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{a^3}-\frac{d\,e}{}$

input
```
int(sin(d*x+c)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*(b^2/a^2*(-sin(d*x+c)/(a*d-b*c+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((
a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+2/d*b^2/a^3*(Si
(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)
/b)/b)+1/a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-2/d/a^3*b*(Si
(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.39

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$$
$$=\frac{\left((abdx^2+a^2dx)\,\mathrm{Ci}\,(dx)-2\,(b^2x^2+abx)\,\mathrm{Si}\,(dx)\right)\cos\,(c)+\left((abdx^2+a^2dx)\,\mathrm{Ci}\,\left(\frac{bdx+ad}{b}\right)+2\,(b^2x^2+abx)\right.}{}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(((a*b*d*x^2 + a^2*d*x)*cos_integral(d*x) - 2*(b^2*x^2 + a*b*x)*sin_integr
al(d*x))*cos(c) + ((a*b*d*x^2 + a^2*d*x)*cos_integral((b*d*x + a*d)/b) + 2
*(b^2*x^2 + a*b*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (2
*a*b*x + a^2)*sin(d*x + c) - (2*(b^2*x^2 + a*b*x)*cos_integral(d*x) + (a*b
*d*x^2 + a^2*d*x)*sin_integral(d*x))*sin(c) - (2*(b^2*x^2 + a*b*x)*cos_int
egral((b*d*x + a*d)/b) - (a*b*d*x^2 + a^2*d*x)*sin_integral((b*d*x + a*d)/
b))*sin(-(b*c - a*d)/b))/(a^3*b*x^2 + a^4*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\sin{(c+dx)}}{x^2{(a+bx)}^2}\,dx$$

input
```
integrate(sin(d*x+c)/x**2/(b*x+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(x**2*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\sin{(dx+c)}}{{(bx+a)}^2x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3180 vs. $2(191) = 382$.

Time = 0.20 (sec) , antiderivative size = 3180, normalized size of antiderivative = 16.91

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*cos(c)*cos_integr
al((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)/b - 2*(b*x + a)*a*
(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*cos(c)*cos_integral((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + a*b*c^2*d^2*cos(c)*cos_integral
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + (b*x + a)*a^2*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(c)*cos_integral((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d)/b - c)/b - a^2*c*d^3*cos(c)*cos_integral((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + (b*x + a)^2*a*(b*c/(b*x
+ a) - a*d/(b*x + a) + d)^2*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)/b - 2*(b*x + a)*a*(b
*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*cos(-(b*c - a*d)/b)*cos_integral((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a*b*c^2*d^
2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d) - b*c + a*d)/b) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d
)*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*
x + a) + d) - b*c + a*d)/b)/b - a^2*c*d^3*cos(-(b*c - a*d)/b)*cos_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a
)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*sin(c)*sin_integral(-(b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)/b - 2*(b*x + a)*a*(b*c/(b*
x + a) - a*d/(b*x + a) + d)*c*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x)^2),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\sin\left(dx+c\right)}{b^2x^4+2ab\,x^3+a^2x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)
```

3.33 $\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 265

$$\begin{aligned}\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx =& -\frac{\cos(c+dx)}{b^3 d}+\frac{a^3 d\cos(c+dx)}{2b^5(a+bx)} \\ &+\frac{3a^2 d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} \\ &-\frac{3a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} \\ &+\frac{a^3 d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^6}+\frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} \\ &-\frac{3a^2\sin(c+dx)}{b^4(a+bx)}-\frac{3a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} \\ &+\frac{a^3 d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^6}-\frac{3a^2 d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5}\end{aligned}$$

output
```
-cos(d*x+c)/b^3/d+1/2*a^3*d*cos(d*x+c)/b^5/(b*x+a)+3*a^2*d*cos(-c+a*d/b)*C
i(a*d/b+d*x)/b^5+3*a*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4-1/2*a^3*d^2*Ci(a*d/b+
d*x)*sin(-c+a*d/b)/b^6+1/2*a^3*sin(d*x+c)/b^4/(b*x+a)^2-3*a^2*sin(d*x+c)/b
^4/(b*x+a)-3*a*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4+1/2*a^3*d^2*cos(-c+a*d/b)*S
i(a*d/b+d*x)/b^6+3*a^2*d*sin(-c+a*d/b)*Si(a*d/b+d*x)/b^5
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx =$$
$$-\frac{b\cos(dx)\left(-((a+bx)\left(-2ab^2+a^3d^2-2b^3x\right)\cos(c))+a^2bd(5a+6bx)\sin(c)\right)+b(a^2bd(5a+6bx)\cos(\cdots}{\cdots}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

output
```
-1/2*(b*Cos[d*x]*(-((a + b*x)*(-2*a*b^2 + a^3*d^2 - 2*b^3*x)*Cos[c]) + a^2
*b*d*(5*a + 6*b*x)*Sin[c]) + b*(a^2*b*d*(5*a + 6*b*x)*Cos[c] + (a + b*x)*(
-2*a*b^2 + a^3*d^2 - 2*b^3*x)*Sin[c])*Sin[d*x] - a*d*(a + b*x)^2*(CosInteg
ral[d*(a/b + x)]*(6*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (a
*d)/b]) + ((-6*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 6*a*b*d*Sin[c - (a*d)/b])
*SinIntegral[d*(a/b + x)]))/(b^6*d*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(-\frac{a^3\sin(c+dx)}{b^3(a+bx)^3}+\frac{3a^2\sin(c+dx)}{b^3(a+bx)^2}-\frac{3a\sin(c+dx)}{b^3(a+bx)}+\frac{\sin(c+dx)}{b^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^3d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{a^3d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{a^3d\cos(c+dx)}{2b^5(a+bx)}+\frac{a^3\sin(c+dx)}{2b^4(a+bx)^2}+\frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{3a^2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{3a^2\sin(c+dx)}{b^4(a+bx)}-\frac{3a\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{3a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{\cos(c+dx)}{b^3d}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

output
```
-(Cos[c + d*x]/(b^3*d)) + (a^3*d*Cos[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*
d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^5 - (3*a*CosIntegral[(a*d
)/b + d*x]*Sin[c - (a*d)/b])/b^4 + (a^3*d^2*CosIntegral[(a*d)/b + d*x]*Sin
[c - (a*d)/b])/(2*b^6) + (a^3*Sin[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*S
in[c + d*x])/(b^4*(a + b*x)) - (3*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b +
 d*x])/b^4 + (a^3*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^6)
 - (3*a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.66

method	result
risch	$\frac{i(2ia^3b^3d^6x^3+6ia^4b^2d^6x^2-4ib^6d^4x^4+6ia^5b\,d^6x-16ia\,b^5d^4x^3+2ia^6d^6-24ia^2b^4d^4x^2-16ia^3b^3d^4x-4ia^4b^2d^4)\cos(dx}{4b^5d^3(bx+a)^2(-x^2d^2b^2-2ab\,d^2x-a^2d^2)}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^3*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/b^5/d^3*(-4*I*b^6*d^4*x^4-4*I*a^4*b^2*d^4+2*I*a^3*b^3*d^6*x^3+6*I*a^
4*b^2*d^6*x^2+6*I*a^5*b*d^6*x-24*I*a^2*b^4*d^4*x^2-16*I*a^3*b^3*d^4*x+2*I*
a^6*d^6-16*I*a*b^5*d^4*x^3)/(b*x+a)^2/(-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*c
os(d*x+c)+1/4/b^5/d^3*(12*a^2*b^4*d^5*x^3+34*a^3*b^3*d^5*x^2+32*a^4*b^2*d^
5*x+10*a^5*b*d^5)/(b*x+a)^2/(-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*sin(d*x+c)-
3/2*d/b^5*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^2-3/2*d/b^5*cos((a*d-b*c
)/b)*Ei(1,I*(b*x+a)*d/b)*a^2+1/4*I*d^2/b^6*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a
)*d/b)*a^3-1/4*I*d^2/b^6*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^3-3/2*I/b^
4*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a+3/2*I/b^4*cos((a*d-b*c)/b)*Ei(1,
I*(b*x+a)*d/b)*a+3/2*I*d/b^5*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a^2-3/2
*I*d/b^5*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a^2+1/4*d^2/b^6*sin((a*d-b*c
)/b)*Ei(1,-I*(b*x+a)*d/b)*a^3+1/4*d^2/b^6*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*
d/b)*a^3-3/2/b^4*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)*a-3/2/b^4*sin((a*d-
b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.46

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx$$
$$= \frac{(a^4bd^2-2\,b^5x^2-2\,a^2b^3+(a^3b^2d^2-4\,ab^4)x)\cos\left(dx+c\right)+\left(6\,(a^2b^3d^2x^2+2\,a^3b^2d^2x+a^4bd^2)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right.\right.}{}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*((a^4*b*d^2 - 2*b^5*x^2 - 2*a^2*b^3 + (a^3*b^2*d^2 - 4*a*b^4)*x)*cos(d
*x + c) + (6*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*cos_integral(
(b*d*x + a*d)/b) + (a^5*d^3 - 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a*b^4*d)*x^2
+ 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c
- a*d)/b) - (6*a^2*b^3*d*x + 5*a^3*b^2*d)*sin(d*x + c) - ((a^5*d^3 - 6*a^3
*b^2*d + (a^3*b^2*d^3 - 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*co
s_integral((b*d*x + a*d)/b) - 6*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b
*d^2)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a
*b^7*d*x + a^2*b^6*d)
```

Sympy [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^3\sin\left(c+dx\right)}{\left(a+bx\right)^3}\,dx$$

input
```
integrate(x**3*sin(d*x+c)/(b*x+a)**3,x)
```

output
```
Integral(x**3*sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^3\sin\left(dx+c\right)}{\left(bx+a\right)^3}\,dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^3*cos(d*x + c) + 3*((a^2*(-I*exp_i
ntegral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I*b*d*x + I*a*d)
/b))*cos(c)^2 + a^2*(-I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_int
egral_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a^2*(ex
p_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d
)/b))*cos(c)^2 + a^2*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integra
l_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^
2 - 3*(a*b*cos(c)^2 + a*b*sin(c)^2)*x*sin(d*x + c) + 3*((a^2*(-I*exp_integ
ral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))
*cos(c)^2 + a^2*(-I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integra
l_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a^2*(exp_in
tegral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b)
)*cos(c)^2 + a^2*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(
4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 +
((b^2*d*x^3*cos(c) + 3*a*b*x*sin(c))*cos(d*x + c)^2 + (b^2*d*x^3*cos(c) +
3*a*b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 6*(((a^2*b^5*cos(c)^2 + a
^2*b^5*sin(c)^2)*d^3*x^3 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2
 + 3*(a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5
*b^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2
)*d^3*x^3 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 16724, normalized size of antiderivative = 63.11

$$\int \frac{x^3 \sin(c + dx)}{(a + bx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^3*x^
2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b^2*d^3*x^
2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 - 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^4*b*d^3*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^4
*b*d^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 - a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c)^2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*sin_integral((b*d*x + a*d
)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^3*b^2*d^3*x^2*imag_part(cos_inte...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^3 \sin(c+d\,x)}{(a+b\,x)^3} \, dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x)^3,x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3} \, dx = \int \frac{\sin(dx+c)\, x^3}{b^3x^3 + 3a\, b^2x^2 + 3a^2bx + a^3} dx$$

input
```
int(x^3*sin(d*x+c)/(b*x+a)^3,x)
```

output
```
int((sin(c + d*x)*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
```

3.34 $\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 241

$$\begin{aligned}\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} dx = & -\frac{a^2 d \cos(c+dx)}{2b^4(a+bx)} - \frac{2ad \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4} \\ & + \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{b^3} \\ & - \frac{a^2 d^2 \text{CosIntegral}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{2b^5} - \frac{a^2 \sin(c+dx)}{2b^3(a+bx)^2} \\ & + \frac{2a \sin(c+dx)}{b^3(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^3} \\ & - \frac{a^2 d^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{2b^5} + \frac{2ad \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}\end{aligned}$$

output
```
-1/2*a^2*d*cos(d*x+c)/b^4/(b*x+a)-2*a*d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^4-Ci
(a*d/b+d*x)*sin(-c+a*d/b)/b^3+1/2*a^2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5-
1/2*a^2*sin(d*x+c)/b^3/(b*x+a)^2+2*a*sin(d*x+c)/b^3/(b*x+a)+cos(-c+a*d/b)*
Si(a*d/b+d*x)/b^3-1/2*a^2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^5-2*a*d*sin(-c
+a*d/b)*Si(a*d/b+d*x)/b^4
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.64

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{-\operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(-4abd\cos\left(c-\frac{ad}{b}\right)+\left(2b^2-a^2d^2\right)\sin\left(c-\frac{ad}{b}\right)\right)+\frac{ab(ad(a+bx)\cos(c+dx)-b(3a+}{(a+bx)^2}}{2b^5}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

output
```
-1/2*(-(CosIntegral[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (2*b^2 - a^2
*d^2)*Sin[c - (a*d)/b])) + (a*b*(a*d*(a + b*x)*Cos[c + d*x] - b*(3*a + 4*b
*x)*Sin[c + d*x]))/(a + b*x)^2 + ((-2*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 4*
a*b*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{a^2\sin(c+dx)}{b^2(a+bx)^3}-\frac{2a\sin(c+dx)}{b^2(a+bx)^2}+\frac{\sin(c+dx)}{b^2(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}-\frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{2ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{2a\sin(c+dx)}{b^3(a+bx)}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

output
```
-1/2*(a^2*d*Cos[c + d*x])/(b^4*(a + b*x)) - (2*a*d*Cos[c - (a*d)/b]*CosInt
egral[(a*d)/b + d*x])/b^4 + (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/
b^3 - (a^2*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^5) - (a^2
*Sin[c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Sin[c + d*x])/(b^3*(a + b*x)) +
(Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3 - (a^2*d^2*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/(2*b^5) + (2*a*d*Sin[c - (a*d)/b]*SinInteg
ral[(a*d)/b + d*x])/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{i(2ia^2b^3d^4x^3+6ia^3b^2d^4x^2+6ia^4b\,d^4x+2ia^5d^4)\cos(dx+c)}{4b^4d(bx+a)^2(-x^2d^2b^2-2ab\,d^2x-a^2d^2)}-\frac{(8a\,b^3d^3x^3+22a^2b^2d^3x^2+20a^3b\,d^3x+6a^4d^3)\sin(da}{4b^3d(bx+a)^2(-x^2d^2b^2-2ab\,d^2x-a^2d^2)}$
derivativedivides	$\frac{d^3c^2\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}}{b}}{2b}\right)+-}{}$
default	$\frac{d^3c^2\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}}{b}}{2b}\right)+-}{}$

input

```
int(x^2*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/4*I/b^4/d*(6*I*a^3*b^2*d^4*x^2+6*I*a^4*b*d^4*x+2*I*a^2*b^3*d^4*x^3+2*I*
a^5*d^4)/(b*x+a)^2/(-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*cos(d*x+c)-1/4/b^3/d
*(8*a*b^3*d^3*x^3+22*a^2*b^2*d^3*x^2+20*a^3*b*d^3*x+6*a^4*d^3)/(b*x+a)^2/(
-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*sin(d*x+c)-1/4*I/b^5*cos((a*d-b*c)/b)*Ei
(1,-I*(b*x+a)*d/b)*a^2*d^2+1/4*I/b^5*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*
a^2*d^2+1/2*I/b^3*cos((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)+1/b^4*cos((a*d-b*c
)/b)*Ei(1,-I*(b*x+a)*d/b)*a*d-1/2*I/b^3*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/
b)+1/b^4*cos((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a*d-1/4/b^5*sin((a*d-b*c)/b)
*Ei(1,-I*(b*x+a)*d/b)*a^2*d^2-1/4/b^5*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)
*a^2*d^2+1/2/b^3*sin((a*d-b*c)/b)*Ei(1,-I*(b*x+a)*d/b)-I/b^4*sin((a*d-b*c)
/b)*Ei(1,-I*(b*x+a)*d/b)*a*d+1/2/b^3*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)+
I/b^4*sin((a*d-b*c)/b)*Ei(1,I*(b*x+a)*d/b)*a*d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.35

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{(a^2b^2dx+a^3bd)\cos(dx+c)+\left(4\left(ab^3dx^2+2\,a^2b^2dx+a^3bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^4d^2-2\,a^2b^2+\left(a^2b^2d^2-\right.\right.\right.}{}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2*b^2*d*x + a^3*b*d)*cos(d*x + c) + (4*(a*b^3*d*x^2 + 2*a^2*b^2*d
*x + a^3*b*d)*cos_integral((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2 + (a^2*
b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a*
d)/b))*cos(-(b*c - a*d)/b) - (4*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) - ((a^4*
d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*c
os_integral((b*d*x + a*d)/b) - 4*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*s
in_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x +
a^2*b^5)
```

Sympy [F]

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx$$

input
```
integrate(x**2*sin(d*x+c)/(b*x+a)**3,x)
```

output
```
Integral(x**2*sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^2 \sin{(dx+c)}}{(bx+a)^3}\,dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^2*cos(d*x + c) + ((a*(I*exp_integral_e
(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(
c)^2 + a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -
(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(4
, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2
 + a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x
 + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + (b*cos(c)^2
+ b*sin(c)^2)*x*sin(d*x + c) + ((a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/
b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_integr
al_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*
sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b)
 + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(exp_integral_e(4
, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2
)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + ((b*d*x^2*cos(c) - b*x*sin(c))*cos
(d*x + c)^2 + (b*d*x^2*cos(c) - b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c)
 - 6*(((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a
^2*b^3*sin(c)^2)*d^3*x^2 + 3*(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^3*x +
 (a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2
+ a*b^4*sin(c)^2)*d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^
2 + 3*(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 15410, normalized size of antiderivative = 63.94

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x
^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x
^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*
x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*
x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^2 \sin(c+d\,x)}{(a+b\,x)^3} \,dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x)^3,x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} \, dx = \int \frac{\sin(dx+c)\, x^2}{b^3x^3 + 3a\, b^2x^2 + 3a^2bx + a^3} dx$$

input
```
int(x^2*sin(d*x+c)/(b*x+a)^3,x)
```

output
```
int((sin(c + d*x)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
```

3.35 $\int \frac{x \sin(c+dx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 179

$$\begin{aligned}\int \frac{x \sin(c+dx)}{(a+bx)^3} dx = &\frac{ad \cos(c+dx)}{2b^3(a+bx)} + \frac{d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3} \\ &+ \frac{ad^2 \text{CosIntegral}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{2b^4} \\ &+ \frac{a \sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} \\ &+ \frac{ad^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{2b^4} - \frac{d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}\end{aligned}$$

output
```
1/2*a*d*cos(d*x+c)/b^3/(b*x+a)+d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/b^3-1/2*a*d^2
*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4+1/2*a*sin(d*x+c)/b^2/(b*x+a)^2-sin(d*x+c)
/b^2/(b*x+a)+1/2*a*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4+d*sin(-c+a*d/b)*Si(
a*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx$$
$$= \frac{b\cos(dx)(ad(a+bx)\cos(c)-b(a+2bx)\sin(c))-b(b(a+2bx)\cos(c)+ad(a+bx)\sin(c))\sin(dx)+d(c}{}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

output
```
(b*Cos[d*x]*(a*d*(a + b*x)*Cos[c] - b*(a + 2*b*x)*Sin[c]) - b*(b*(a + 2*b*
x)*Cos[c] + a*d*(a + b*x)*Sin[c])*Sin[d*x] + d*(a + b*x)^2*(CosIntegral[d*
(a/b + x)]*(2*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b]) + (a*d*Cos[c - (a
*d)/b] - 2*b*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)]))/(2*b^4*(a + b*x)
^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(\frac{\sin(c+dx)}{b(a+bx)^2}-\frac{a\sin(c+dx)}{b(a+bx)^3}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{ad^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{ad^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\cos(c+dx)}{2b^3(a+bx)}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

output
```
(a*d*Cos[c + d*x])/(2*b^3*(a + b*x)) + (d*Cos[c - (a*d)/b]*CosIntegral[(a*
d)/b + d*x])/b^3 + (a*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*
b^4) + (a*Sin[c + d*x])/(2*b^2*(a + b*x)^2) - Sin[c + d*x]/(b^2*(a + b*x))
 + (a*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^4) - (d*Sin[c
- (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(174) = 348$.

Time = 1.15 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.34

method	result
derivativedivides	$\frac{-\frac{d^3(ad-bc)\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2 b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}}{b}}{2b}\cdots\right.}{b}\cdots}{\cdots}$
default	$\frac{-\frac{d^3(ad-bc)\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2 b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}}{b}}{2b}\cdots\right.}{b}\cdots}{\cdots}$
risch	$\frac{i(2iab^3d^3x^3+6ia^2b^2d^3x^2+6ia^3bd^3x+2ia^4d^3)\cos(dx+c)}{4b^3(bx+a)^2(-x^2d^2b^2-2abd^2x-a^2d^2)}+\frac{(4b^4d^2x^3+10ab^3d^2x^2+8a^2b^2d^2x+2a^3bd^2)\sin(dx+c)}{4b^3(bx+a)^2(-x^2d^2b^2-2abd^2x-a^2d^2)}\cdots$

input
```
int(x*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-d^3*(a*d-b*c)/b*(-1/2*sin(d*x+c)/(a*d-b*c+b*(d*x+c))^2/b+1/2*(-cos
(d*x+c)/(a*d-b*c+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci
(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+d^3/b*(-sin(d*x+c)/(a*d-b*c+
b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/
b)*cos((a*d-b*c)/b)/b)/b)-d^3*c*(-1/2*sin(d*x+c)/(a*d-b*c+b*(d*x+c))^2/b+1
/2*(-cos(d*x+c)/(a*d-b*c+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)
/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.47

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx$$
$$= \frac{(ab^2dx+a^2bd)\cos(dx+c)+\left(2\left(b^3dx^2+2ab^2dx+a^2bd\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(ab^2d^2x^2+2a^2bd^2x+a^3d^2\right)\mathrm{Si}\left(\cdots\right.\right.}{\cdots}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*((a*b^2*d*x + a^2*b*d)*cos(d*x + c) + (2*(b^3*d*x^2 + 2*a*b^2*d*x + a^
2*b*d)*cos_integral((b*d*x + a*d)/b) + (a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^
3*d^2)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (2*b^3*x + a*b
^2)*sin(d*x + c) - ((a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)*cos_integral
((b*d*x + a*d)/b) - 2*(b^3*d*x^2 + 2*a*b^2*d*x + a^2*b*d)*sin_integral((b*
d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\sin{(c+dx)}}{(a+bx)^3}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)**3,x)
```

output
```
Integral(x*sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\sin{(dx+c)}}{(bx+a)^3}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((b*cos(c)^2 + b*sin(c)^2)*x*cos(d*x + c) + ((a*(exp_integral_e(4, (I
*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a
*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(4, (I*b*d*x
 + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*
exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((a*(exp_integra
l_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos
(c)^2 + a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(4,
 (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^
2 + a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + (b*x*co
s(d*x + c)^2*cos(c) + b*x*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 4*(((b^5
*cos(c)^2 + b^5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d*x^
2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*cos(c)^2 + a^3*
b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)*d*x^3 + 3
*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*s
in(c)^2)*d*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)*sin(d*x + c)^2)*in
tegrate(1/2*x*cos(d*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 10535, normalized size of antiderivative = 58.85

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2*x^2*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2
*a*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b) + 2*a*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b^2*d^2*x^2*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*
a*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d^2*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*
b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b*d^2*x*sin_integral((b*d
*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^2*d
^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 4*a*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^3}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x)^3,x)
```

output
```
int((x*sin(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{\sin(dx+c)\,x}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx$$

input
```
int(x*sin(d*x+c)/(b*x+a)^3,x)
```

output
```
int((sin(c + d*x)*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
```

3.36 $\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{d\cos(c+dx)}{2b^2(a+bx)} - \frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^3} - \frac{\sin(c+dx)}{2b(a+bx)^2} - \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

output
```
-1/2*d*cos(d*x+c)/b^2/(b*x+a)+1/2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3-1/2*
sin(d*x+c)/b/(b*x+a)^2-1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{d^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + \frac{b(d(a+bx)\cos(c+dx)+b\sin(c+dx))}{(a+bx)^2} + d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^3}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x)^3,x]
```

output
```
-1/2*(d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (b*(d*(a + b*x)*Cos[
c + d*x] + b*Sin[c + d*x]))/(a + b*x)^2 + d^2*Cos[c - (a*d)/b]*SinIntegral
[d*(a/b + x)])/b^3
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+bx)^3} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{d\int \frac{\cos(c+dx)}{(a+bx)^2}dx}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{(a+bx)^2}dx}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{d\left(\frac{d\int -\frac{\sin(c+dx)}{a+bx}dx}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(-\frac{d\int \frac{\sin(c+dx)}{a+bx}dx}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\left(-\frac{d\int\frac{\sin(c+dx)}{a+bx}dx}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$\Big\downarrow$ 3784

$$\frac{d\left(-\frac{d\left(\sin\left(c-\frac{ad}{b}\right)\int\frac{\cos\left(xd+\frac{ad}{b}\right)}{a+bx}dx+\cos\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$\Big\downarrow$ 3042

$$\frac{d\left(-\frac{d\left(\sin\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(xd+\frac{ad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+\cos\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$\Big\downarrow$ 3780

$$\frac{d\left(-\frac{d\left(\sin\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(xd+\frac{ad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

$\Big\downarrow$ 3783

$$\frac{d\left(-\frac{d\left(\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}-\frac{\cos(c+dx)}{b(a+bx)}\right)}{2b}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

input
```
Int[Sin[c + d*x]/(a + b*x)^3,x]
```

output
```
-1/2*Sin[c + d*x]/(b*(a + b*x)^2) + (d*(-(Cos[c + d*x]/(b*(a + b*x))) - (d
*((CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b + (Cos[c - (a*d)/b]*SinI
ntegral[(a*d)/b + d*x])/b))/b))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.39

method	result
derivativedivides	$d^2\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}}{b}}{2b}\right.$
default	$d^2\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}}{b}}{2b}\right.$
risch	$-\frac{id^2e^{-\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{4b^3}+\frac{id^2e^{\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(idx+ic+\frac{i(ad-bc)}{b}\right)}{4b^3}+\frac{i(-2ib^3d^3}{}$

input
```
int(sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(-1/2*sin(d*x+c)/(a*d-b*c+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(a*d-b*c+b*(
d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*
sin((a*d-b*c)/b)/b)/b)/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.58

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx =$$
$$-\frac{b^2\sin(dx+c)-(b^2d^2x^2+2\,abd^2x+a^2d^2)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)+(b^2d^2x^2+2\,abd^2x+a^2d^2)\cos\left(\right.}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^2*sin(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos_integra
l((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*
d^2)*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + (b^2*d*x + a*b*d)
*cos(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{\sin(c+dx)}{(a+bx)^3}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x+a)**3,x)
```

output
```
Integral(sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \frac{d^3\left(-i\,E_3\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_3\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^3\left(E_3\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_3\left(-\right.\right.}{2\left((dx+c)^2b^3+b^3c^2-2\,ab^2cd+a^2bd^2-2\,(b^3c-ab^2d)(dx+c)\right)d}$$

input
```
integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(d^3*(-I*exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_
integral_e(3, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d
^3*(exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(
3, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)^2
*b^3 + b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - 2*(b^3*c - a*b^2*d)*(d*x + c))*
d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 5727, normalized size of antiderivative = 55.07

$$\int \frac{\sin(c+dx)}{(a+bx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*
d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) + 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*d^2*x^2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*
d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 4*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b) - 4*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*d^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^3}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x)^3,x)
```

output
```
int(sin(c + d*x)/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx$$

input
```
int(sin(d*x+c)/(b*x+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
```

3.37 $\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 261

$$\begin{aligned}\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx &= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b} \\ &+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\ &+ \frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2ab^2} + \frac{\sin(c+dx)}{2a(a+bx)^2} \\ &+ \frac{\sin(c+dx)}{a^2(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^3} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} \\ &+ \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2ab^2} + \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2b}\end{aligned}$$

output
```
1/2*d*cos(d*x+c)/a/b/(b*x+a)-d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/a^2/b+Ci(d*x)*s
in(c)/a^3+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3-1/2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d
/b)/a/b^2+1/2*sin(d*x+c)/a/(b*x+a)^2+sin(d*x+c)/a^2/(b*x+a)+cos(c)*Si(d*x)
/a^3-cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3+1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a
/b^2-d*sin(-c+a*d/b)*Si(a*d/b+d*x)/a^2/b
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.72

$$\int \frac{\sin(c+dx)}{x(a+bx)^3} dx$$
$$= \frac{a^3bd\cos(c+dx) + a^2b^2dx\cos(c+dx) + 2b^2(a+bx)^2 \text{CosIntegral}(dx)\sin(c) + (a+bx)^2 \text{CosIntegral}\left(d\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

output
```
(a^3*b*d*Cos[c + d*x] + a^2*b^2*d*x*Cos[c + d*x] + 2*b^2*(a + b*x)^2*CosIn
tegral[d*x]*Sin[c] + (a + b*x)^2*CosIntegral[d*(a/b + x)]*(-2*a*b*d*Cos[c
- (a*d)/b] + (-2*b^2 + a^2*d^2)*Sin[c - (a*d)/b]) + 3*a^2*b^2*Sin[c + d*x]
 + 2*a*b^3*x*Sin[c + d*x] + 2*a^2*b^2*Cos[c]*SinIntegral[d*x] + 4*a*b^3*x*
Cos[c]*SinIntegral[d*x] + 2*b^4*x^2*Cos[c]*SinIntegral[d*x] - 2*a^2*b^2*Co
s[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a^4*d^2*Cos[c - (a*d)/b]*SinInte
gral[d*(a/b + x)] - 4*a*b^3*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] +
2*a^3*b*d^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 2*b^4*x^2*Cos[c
- (a*d)/b]*SinIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^2*Cos[c - (a*d)/b]*Sin
Integral[d*(a/b + x)] + 2*a^3*b*d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)
] + 4*a^2*b^2*d*x*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a*b^3*d*x^
2*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(2*a^3*b^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x(a+bx)^3} dx$$
$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{b\sin(c+dx)}{a^3(a+bx)}+\frac{\sin(c+dx)}{a^3x}-\frac{b\sin(c+dx)}{a^2(a+bx)^2}-\frac{b\sin(c+dx)}{a(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^3}+\frac{\cos(c)\text{Si}(dx)}{a^3}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2b}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2b}+\frac{\sin(c+dx)}{a^2(a+bx)}+\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2ab^2}+\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2ab^2}+\frac{\sin(c+dx)}{2a(a+bx)^2}+\frac{d\cos(c+dx)}{2ab(a+bx)}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

output
```
(d*Cos[c + d*x])/(2*a*b*(a + b*x)) - (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)
/b + d*x])/(a^2*b) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a*d)/b
+ d*x]*Sin[c - (a*d)/b])/a^3 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*
d)/b])/(2*a*b^2) + Sin[c + d*x]/(2*a*(a + b*x)^2) + Sin[c + d*x]/(a^2*(a +
 b*x)) + (Cos[c]*SinIntegral[d*x])/a^3 - (Cos[c - (a*d)/b]*SinIntegral[(a*
d)/b + d*x])/a^3 + (d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a*
b^2) + (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(a^2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{d^2 b\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2 b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}(\cdots}{b}}{2b}\right.}{a}$
default	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{d^2 b\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2 b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\mathrm{Ci}(\cdots}{b}}{2b}\right.}{a}$
risch	$\frac{i e^{\frac{i(ad-bc)}{b}} \mathrm{expIntegral}_1\left(idx+ic+\frac{i(ad-bc)}{b}\right)}{2a^3} - \frac{i e^{-\frac{i(ad-bc)}{b}} \mathrm{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2a^3} + \frac{id^2 e^{-\frac{i(ad-bc)}{b}} \mathrm{ex}\cdots}{}$

input

```
int(sin(d*x+c)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-d^2*b/a*(-1/2*sin(d*x+c)/(a*d-b*c+b*
(d*x+c))^2/b+1/2*(-cos(d*x+c)/(a*d-b*c+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)
*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-b/a^3*
(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b
*c)/b)/b)-d*b/a^2*(-sin(d*x+c)/(a*d-b*c+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b
)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.50

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \frac{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)\mathrm{Ci}\left(dx\right)\sin\left(c\right)+2\left(b^4x^2+2\,ab^3x+a^2b^2\right)\cos\left(c\right)\mathrm{Si}\left(dx\right)+\left(a^2b^2dx+a^3bd\right)\cos\left(d\cdots\right.}{}$$

input

```
integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*cos_integral(d*x)*sin(c) + 2*(b^4*x
^2 + 2*a*b^3*x + a^2*b^2)*cos(c)*sin_integral(d*x) + (a^2*b^2*d*x + a^3*b*
d)*cos(d*x + c) - (2*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*cos_integral(
(b*d*x + a*d)/b) - (a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a
^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b)
+ (2*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) - ((a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*
d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_integral((b*d*x + a*d)/b
) + 2*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_integral((b*d*x + a*d)/b
))*sin(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\sin(c+dx)}{x\,(a+bx)^3}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)**3,x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\sin(dx+c)}{(bx+a)^3x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)^3*x), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 17806, normalized size of antiderivative = 68.22

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\, dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^
2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^
2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 - 2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^2*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x
*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x\,{\left(a+b\,x\right)}^3}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x)^3),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{b^3x^4+3a\,b^2x^3+3a^2b\,x^2+a^3x}dx$$

input
```
int(sin(d*x+c)/x/(b*x+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)
```

3.38 $\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 299

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = &-\frac{d\cos(c+dx)}{2a^2(a+bx)}+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^3}\\
&+\frac{2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}\\
&-\frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4}+\frac{3b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4}\\
&-\frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2a^2b}-\frac{\sin(c+dx)}{a^3x}\\
&-\frac{b\sin(c+dx)}{2a^2(a+bx)^2}-\frac{2b\sin(c+dx)}{a^3(a+bx)}-\frac{3b\cos(c)\text{Si}(dx)}{a^4}\\
&-\frac{d\sin(c)\text{Si}(dx)}{a^3}+\frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4}\\
&-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2a^2b}-\frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}\end{aligned}$$

output
```
-1/2*d*cos(d*x+c)/a^2/(b*x+a)+d*cos(c)*Ci(d*x)/a^3+2*d*cos(-c+a*d/b)*Ci(a*
d/b+d*x)/a^3-3*b*Ci(d*x)*sin(c)/a^4-3*b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^4+1/
2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^2/b-sin(d*x+c)/a^3/x-1/2*b*sin(d*x+c)/
a^2/(b*x+a)^2-2*b*sin(d*x+c)/a^3/(b*x+a)-3*b*cos(c)*Si(d*x)/a^4-d*sin(c)*S
i(d*x)/a^3+3*b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^4-1/2*d^2*cos(-c+a*d/b)*Si(a*
d/b+d*x)/a^2/b+2*d*sin(-c+a*d/b)*Si(a*d/b+d*x)/a^3
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.81

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{a^3bdx\cos(c+dx)+a^2b^2dx^2\cos(c+dx)+2bx(a+bx)^2\,\mathrm{CosIntegral}(dx)(-ad\cos(c)+3b\sin(c))+x(}{}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

output
```
-1/2*(a^3*b*d*x*Cos[c + d*x] + a^2*b^2*d*x^2*Cos[c + d*x] + 2*b*x*(a + b*x
)^2*CosIntegral[d*x]*(-(a*d*Cos[c]) + 3*b*Sin[c]) + x*(a + b*x)^2*CosInteg
ral[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (
a*d)/b]) + 2*a^3*b*Sin[c + d*x] + 9*a^2*b^2*x*Sin[c + d*x] + 6*a*b^3*x^2*S
in[c + d*x] + 6*a^2*b^2*x*Cos[c]*SinIntegral[d*x] + 12*a*b^3*x^2*Cos[c]*Si
nIntegral[d*x] + 6*b^4*x^3*Cos[c]*SinIntegral[d*x] + 2*a^3*b*d*x*Sin[c]*Si
nIntegral[d*x] + 4*a^2*b^2*d*x^2*Sin[c]*SinIntegral[d*x] + 2*a*b^3*d*x^3*S
in[c]*SinIntegral[d*x] - 6*a^2*b^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b +
 x)] + a^4*d^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*a*b^3*x^2*
Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^2*Cos[c - (a*d)/
b]*SinIntegral[d*(a/b + x)] - 6*b^4*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/
b + x)] + a^2*b^2*d^2*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 4*a^
3*b*d*x*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 8*a^2*b^2*d*x^2*Sin[c
- (a*d)/b]*SinIntegral[d*(a/b + x)] + 4*a*b^3*d*x^3*Sin[c - (a*d)/b]*SinIn
tegral[d*(a/b + x)])/(a^4*b*x*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3} \, dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{3b^2\sin(c+dx)}{a^4(a+bx)} - \frac{3b\sin(c+dx)}{a^4x} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)^2} + \frac{\sin(c+dx)}{a^3x^2} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^3}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3b\sin(c)\,\text{CosIntegral}(dx)}{a^4} + \frac{3b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^4} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} + \frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^4} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{2b\sin(c+dx)}{a^3(a+bx)} + \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^3} - \frac{d\sin(c)\text{Si}(dx)}{a^3} - \frac{\sin(c+dx)}{a^3x} - \frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2a^2b} - \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2a^2b} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{d\cos(c+dx)}{2a^2(a+bx)}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

output
```
-1/2*(d*Cos[c + d*x])/(a^2*(a + b*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 +
(2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/a^3 - (3*b*CosIntegral[d
*x]*Sin[c])/a^4 + (3*b*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^4 -
(d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^2*b) - Sin[c + d*x]
/(a^3*x) - (b*Sin[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Sin[c + d*x])/(a^3*
(a + b*x)) - (3*b*Cos[c]*SinIntegral[d*x])/a^4 - (d*Sin[c]*SinIntegral[d*x
])/a^3 + (3*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4 - (d^2*Cos[
c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^2*b) - (2*d*Sin[c - (a*d)/b]
*SinIntegral[(a*d)/b + d*x])/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.35

method	result
derivativedivides	$d\left(-\frac{3b(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))}{d\,a^4}+\frac{3b^2\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{d\,a^4}+-\right.$
default	$d\left(-\frac{3b(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))}{d\,a^4}+\frac{3b^2\left(\frac{\operatorname{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-bc}{b}\right)}{b}-\frac{\operatorname{Ci}\left(dx+c+\frac{ad-bc}{b}\right)\sin\left(\frac{ad-bc}{b}\right)}{b}\right)}{d\,a^4}+-\right.$
risch	$-\frac{d\,e^{ic}\operatorname{expIntegral}_1(-idx)}{2a^3}-\frac{3ib\,e^{ic}\operatorname{expIntegral}_1(-idx)}{2a^4}+\frac{3ib\,e^{-\frac{i(ad-bc)}{b}}\operatorname{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{2a^4}-$

input
```
int(sin(d*x+c)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d*(-3/d/a^4*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+3/d*b^2/a^4*(Si(d*x+c+(a*d-b
*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+d*b^2/
a^2*(-1/2*sin(d*x+c)/(a*d-b*c+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(a*d-b*c+b*(
d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*
sin((a*d-b*c)/b)/b)/b)/b)+2*b^2/a^3*(-sin(d*x+c)/(a*d-b*c+b*(d*x+c))/b+(Si
(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)
/b)/b)/b)+1/a^3*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.67

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{(a^2b^2dx^2+a^3bdx)\cos(dx+c)-2\left((ab^3dx^3+2\,a^2b^2dx^2+a^3bdx)\operatorname{Ci}(dx)-3\,(b^4x^3+2\,ab^3x^2+a^2b^2x\right.}{}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2*b^2*d*x^2 + a^3*b*d*x)*cos(d*x + c) - 2*((a*b^3*d*x^3 + 2*a^2*b
^2*d*x^2 + a^3*b*d*x)*cos_integral(d*x) - 3*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b
^2*x)*sin_integral(d*x))*cos(c) - (4*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*
b*d*x)*cos_integral((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3
*b*d^2 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*sin_integral((b*d*x + a*d
)/b))*cos(-(b*c - a*d)/b) + (6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*sin(d*x
+ c) + 2*(3*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*cos_integral(d*x) + (a*b^3
*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*sin_integral(d*x))*sin(c) - (((a^2*b
^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*
x)*cos_integral((b*d*x + a*d)/b) + 4*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*
b*d*x)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^4*b^3*x^3 +
2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin{(c+dx)}}{x^2\,(a+bx)^3}\,dx$$

input
```
integrate(sin(d*x+c)/x**2/(b*x+a)**3,x)
```

output
```
Integral(sin(c + d*x)/(x**2*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin(dx+c)}{(bx+a)^3x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)^3*x^2), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 20808, normalized size of antiderivative = 69.59

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x
^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x
^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^
2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^3*real_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 + 2*a*b^3*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*b^2*d^2*x^3*im...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(a+b\,x\right)}^3}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x)^3),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin(dx+c)}{b^3x^5+3a\,b^2x^4+3a^2b\,x^3+a^3x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3*x**2 + 3*a**2*b*x**3 + 3*a*b**2*x**4 + b**3*x**5),x
)
```

3.39 $\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 377

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx &= -\frac{d\cos(c+dx)}{2a^3x}+\frac{bd\cos(c+dx)}{2a^3(a+bx)}-\frac{3bd\cos(c)\operatorname{CosIntegral}(dx)}{a^4}\\
&-\frac{3bd\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4}\\
&+\frac{6b^2\operatorname{CosIntegral}(dx)\sin(c)}{a^5}-\frac{d^2\operatorname{CosIntegral}(dx)\sin(c)}{2a^3}\\
&-\frac{6b^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5}\\
&+\frac{d^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2a^3}\\
&-\frac{\sin(c+dx)}{2a^3x^2}+\frac{3b\sin(c+dx)}{a^4x}+\frac{b^2\sin(c+dx)}{2a^3(a+bx)^2}\\
&+\frac{3b^2\sin(c+dx)}{a^4(a+bx)}+\frac{6b^2\cos(c)\operatorname{Si}(dx)}{a^5}-\frac{d^2\cos(c)\operatorname{Si}(dx)}{2a^3}\\
&+\frac{3bd\sin(c)\operatorname{Si}(dx)}{a^4}-\frac{6b^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{a^5}\\
&+\frac{d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{2a^3}+\frac{3bd\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{a^4}\end{aligned}$$

output
```
-1/2*d*cos(d*x+c)/a^3/x+1/2*b*d*cos(d*x+c)/a^3/(b*x+a)-3*b*d*cos(c)*Ci(d*x
)/a^4-3*b*d*cos(-c+a*d/b)*Ci(a*d/b+d*x)/a^4+6*b^2*Ci(d*x)*sin(c)/a^5-1/2*d
^2*Ci(d*x)*sin(c)/a^3+6*b^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^5-1/2*d^2*Ci(a*d
/b+d*x)*sin(-c+a*d/b)/a^3-1/2*sin(d*x+c)/a^3/x^2+3*b*sin(d*x+c)/a^4/x+1/2*
b^2*sin(d*x+c)/a^3/(b*x+a)^2+3*b^2*sin(d*x+c)/a^4/(b*x+a)+6*b^2*cos(c)*Si(
d*x)/a^5-1/2*d^2*cos(c)*Si(d*x)/a^3+3*b*d*sin(c)*Si(d*x)/a^4-6*b^2*cos(-c+
a*d/b)*Si(a*d/b+d*x)/a^5+1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3-3*b*d*sin
(-c+a*d/b)*Si(a*d/b+d*x)/a^4
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.67

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \frac{-a^4dx\cos(c+dx)-a^3bdx^2\cos(c+dx)-x^2(a+bx)^2\,\text{CosIntegral}(dx)\,(6abd\cos(c)+(-12b^2+a^2d^2)\,\text{si}}{}$$

input
```
Integrate[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

output
```
(-(a^4*d*x*Cos[c + d*x]) - a^3*b*d*x^2*Cos[c + d*x] - x^2*(a + b*x)^2*CosI
ntegral[d*x]*(6*a*b*d*Cos[c] + (-12*b^2 + a^2*d^2)*Sin[c]) + x^2*(a + b*x)
^2*CosIntegral[d*(a/b + x)]*(-6*a*b*d*Cos[c - (a*d)/b] + (-12*b^2 + a^2*d^
2)*Sin[c - (a*d)/b]) - a^4*Sin[c + d*x] + 4*a^3*b*x*Sin[c + d*x] + 18*a^2*
b^2*x^2*Sin[c + d*x] + 12*a*b^3*x^3*Sin[c + d*x] + 12*a^2*b^2*x^2*Cos[c]*S
inIntegral[d*x] - a^4*d^2*x^2*Cos[c]*SinIntegral[d*x] + 24*a*b^3*x^3*Cos[c
]*SinIntegral[d*x] - 2*a^3*b*d^2*x^3*Cos[c]*SinIntegral[d*x] + 12*b^4*x^4*
Cos[c]*SinIntegral[d*x] - a^2*b^2*d^2*x^4*Cos[c]*SinIntegral[d*x] + 6*a^3*
b*d*x^2*Sin[c]*SinIntegral[d*x] + 12*a^2*b^2*d*x^3*Sin[c]*SinIntegral[d*x]
 + 6*a*b^3*d*x^4*Sin[c]*SinIntegral[d*x] - 12*a^2*b^2*x^2*Cos[c - (a*d)/b]
*SinIntegral[d*(a/b + x)] + a^4*d^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/
b + x)] - 24*a*b^3*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b
*d^2*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*b^4*x^4*Cos[c - (a
*d)/b]*SinIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^4*Cos[c - (a*d)/b]*SinInte
gral[d*(a/b + x)] + 6*a^3*b*d*x^2*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)
] + 12*a^2*b^2*d*x^3*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 6*a*b^3*d
*x^4*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(2*a^5*x^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{6b^3\sin(c+dx)}{a^5(a+bx)}+\frac{6b^2\sin(c+dx)}{a^5x}-\frac{3b^3\sin(c+dx)}{a^4(a+bx)^2}-\frac{3b\sin(c+dx)}{a^4x^2}-\frac{b^3\sin(c+dx)}{a^3(a+bx)^3}+\frac{\sin(c+dx)}{a^3x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6b^2\sin(c)\text{CosIntegral}(dx)}{a^5}-\frac{6b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{6b^2\cos(c)\text{Si}(dx)}{a^5}-\frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{3b^2\sin(c+dx)}{a^4(a+bx)}-\frac{3bd\cos(c)\text{CosIntegral}(dx)}{a^4}-\frac{3bd\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{3bd\sin(c)\text{Si}(dx)}{a^4}+\frac{3bd\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{3b\sin(c+dx)}{a^4x}+\frac{b^2\sin(c+dx)}{2a^3(a+bx)^2}+\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2a^3}+\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2a^3}+\frac{bd\cos(c+dx)}{2a^3(a+bx)}-\frac{d^2\sin(c)\text{CosIntegral}(dx)}{2a^3}-\frac{d^2\cos(c)\text{Si}(dx)}{2a^3}-\frac{\sin(c+dx)}{2a^3x^2}-\frac{d\cos(c+dx)}{2a^3x}$$

input
```
Int[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

output
```
-1/2*(d*Cos[c + d*x])/(a^3*x) + (b*d*Cos[c + d*x])/(2*a^3*(a + b*x)) - (3*
b*d*Cos[c]*CosIntegral[d*x])/a^4 - (3*b*d*Cos[c - (a*d)/b]*CosIntegral[(a*
d)/b + d*x])/a^4 + (6*b^2*CosIntegral[d*x]*Sin[c])/a^5 - (d^2*CosIntegral[
d*x]*Sin[c])/(2*a^3) - (6*b^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])
/a^5 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^3) - Sin[c +
 d*x]/(2*a^3*x^2) + (3*b*Sin[c + d*x])/(a^4*x) + (b^2*Sin[c + d*x])/(2*a^3
*(a + b*x)^2) + (3*b^2*Sin[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Cos[c]*SinIn
tegral[d*x])/a^5 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) + (3*b*d*Sin[c]*S
inIntegral[d*x])/a^4 - (6*b^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])
/a^5 + (d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^3) + (3*b*d*
Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.24

method	result
derivativedivides	$d^2\left(\frac{6b^2(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2a^5}-\frac{b^3\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-}{b}\right.}{b}}{2b}\right.}{a^3}\right.$
default	$d^2\left(\frac{6b^2(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2a^5}-\frac{b^3\left(-\frac{\sin(dx+c)}{2(ad-bc+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(ad-bc+b(dx+c))b}-\frac{\mathrm{Si}\left(dx+c+\frac{ad-bc}{b}\right)\cos\left(\frac{ad-}{b}\right.}{b}}{2b}\right.}{a^3}\right.$
risch	$\frac{3db\,\mathrm{e}^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a^4}+\frac{3ib^2\mathrm{e}^{ic}\,\mathrm{expIntegral}_1(-idx)}{a^5}-\frac{3ib^2\mathrm{e}^{-\frac{i(ad-bc)}{b}}\,\mathrm{expIntegral}_1\left(-idx-ic-\frac{iad-ibc}{b}\right)}{a^5}+$

input
```
int(sin(d*x+c)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(6/d^2/a^5*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-b^3/a^3*(-1/2*sin(d*x+c
)/(a*d-b*c+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(a*d-b*c+b*(d*x+c))/b-(Si(d*x+c
+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
/b)/b)-6/d^2*b^3/a^5*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a
*d-b*c)/b)*sin((a*d-b*c)/b)/b)+1/a^3*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+
c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))-3/d*b^3/a^4*(-sin(d*x+c)/(a*
d-b*c+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d
-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-3/d/a^4*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+
Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.55

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx =$$

$$-\frac{(a^3bdx^2+a^4dx)\cos(dx+c)+(6\,(ab^3dx^4+2\,a^2b^2dx^3+a^3bdx^2)\,\mathrm{Ci}\,(dx)+((a^2b^2d^2-12\,b^4)x^4+2\,(a^3$$

input
```
integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^3*b*d*x^2 + a^4*d*x)*cos(d*x + c) + (6*(a*b^3*d*x^4 + 2*a^2*b^2*d
*x^3 + a^3*b*d*x^2)*cos_integral(d*x) + ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a
^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral(d*x))*
cos(c) + (6*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*cos_integral((b*
d*x + a*d)/b) - ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3
 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c -
a*d)/b) - (12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*sin(d*x + c) +
 (((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 -
12*a^2*b^2)*x^2)*cos_integral(d*x) - 6*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^
3*b*d*x^2)*sin_integral(d*x))*sin(c) + (((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a
^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*cos_integral((b*d*x
 + a*d)/b) + 6*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*sin_integral(
(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^
2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\sin{(c+dx)}}{x^3\,(a+bx)^3}\,dx$$

input
```
integrate(sin(d*x+c)/x**3/(b*x+a)**3,x)
```

output
```
Integral(sin(c + d*x)/(x**3*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\sin{(dx+c)}}{(bx+a)^3x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x + a)^3*x^3), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 24116, normalized size of antiderivative = 63.97

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
- a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
 + 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*
a^3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*imag_part(co...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\sin(c+d\,x)}{x^3\,(a+b\,x)^3}\,dx$$

input
```
int(sin(c + d*x)/(x^3*(a + b*x)^3),x)
```

output
```
int(sin(c + d*x)/(x^3*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \text{too large to display}$$

input
```
int(sin(d*x+c)/x^3/(b*x+a)^3,x)
```

output
```
( - 12*cos(c + d*x)*a*b**2*x - 12*cos(c + d*x)*b**3*x**2 - 24*int(x/(tan((
c + d*x)/2)**2*a**2 + 2*tan((c + d*x)/2)**2*a*b*x + tan((c + d*x)/2)**2*b*
*2*x**2 + a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**3*d**2*x**2 - 48*int(x/(t
an((c + d*x)/2)**2*a**2 + 2*tan((c + d*x)/2)**2*a*b*x + tan((c + d*x)/2)**
2*b**2*x**2 + a**2 + 2*a*b*x + b**2*x**2),x)*a*b**4*d**2*x**3 - 24*int(x/(
tan((c + d*x)/2)**2*a**2 + 2*tan((c + d*x)/2)**2*a*b*x + tan((c + d*x)/2)*
*2*b**2*x**2 + a**2 + 2*a*b*x + b**2*x**2),x)*b**5*d**2*x**4 + 2*int(1/(ta
n((c + d*x)/2)**2*a**2*x**2 + 2*tan((c + d*x)/2)**2*a*b*x**3 + tan((c + d*
x)/2)**2*b**2*x**4 + a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*a**5*d**2*x**2
 + 4*int(1/(tan((c + d*x)/2)**2*a**2*x**2 + 2*tan((c + d*x)/2)**2*a*b*x**3
 + tan((c + d*x)/2)**2*b**2*x**4 + a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*
a**4*b*d**2*x**3 + 2*int(1/(tan((c + d*x)/2)**2*a**2*x**2 + 2*tan((c + d*x
)/2)**2*a*b*x**3 + tan((c + d*x)/2)**2*b**2*x**4 + a**2*x**2 + 2*a*b*x**3
+ b**2*x**4),x)*a**3*b**2*d**2*x**4 - 24*int(1/(tan((c + d*x)/2)**2*a**2*x
**2 + 2*tan((c + d*x)/2)**2*a*b*x**3 + tan((c + d*x)/2)**2*b**2*x**4 + a**
2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*a**3*b**2*x**2 - 48*int(1/(tan((c + d*
x)/2)**2*a**2*x**2 + 2*tan((c + d*x)/2)**2*a*b*x**3 + tan((c + d*x)/2)**2*
b**2*x**4 + a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*a**2*b**3*x**3 - 24*int
(1/(tan((c + d*x)/2)**2*a**2*x**2 + 2*tan((c + d*x)/2)**2*a*b*x**3 + tan((
c + d*x)/2)**2*b**2*x**4 + a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*a*b**...
```

3.40 $\int x^3(a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = -\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}-\frac{6a\sin(c+dx)}{d^4}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}$$

output

```
-120*b*x*cos(d*x+c)/d^5+6*a*x*cos(d*x+c)/d^3+20*b*x^3*cos(d*x+c)/d^3-a*x^3
*cos(d*x+c)/d-b*x^5*cos(d*x+c)/d+120*b*sin(d*x+c)/d^6-6*a*sin(d*x+c)/d^4-6
0*b*x^2*sin(d*x+c)/d^4+3*a*x^2*sin(d*x+c)/d^2+5*b*x^4*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int x^3(a+bx^2)\sin(c+dx)\,dx$$
$$=\frac{-dx(ad^2(-6+d^2x^2)+b(120-20d^2x^2+d^4x^4))\cos(c+dx)+(3ad^2(-2+d^2x^2)+5b(24-12d^2x^2+d}{d^6}$$

input
```
Integrate[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(-(d*x*(a*d^2*(-6 + d^2*x^2) + b*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x
]) + (3*a*d^2*(-2 + d^2*x^2) + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*
x])/d^6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^2)\sin(c+dx)\,dx$$
$$\Big\downarrow \; 3820$$
$$\int (ax^3\sin(c+dx)+bx^5\sin(c+dx))\,dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{ax^3\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}-\frac{120bx\cos(c+dx)}{d^5}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{20bx^3\cos(c+dx)}{d^3}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{bx^5\cos(c+dx)}{d}$$

input
```
Int[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(-120*b*x*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (20*b*x^3*Cos[c +
 d*x])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[
c + d*x])/d^6 - (6*a*Sin[c + d*x])/d^4 - (60*b*x^2*Sin[c + d*x])/d^4 + (3*
a*x^2*Sin[c + d*x])/d^2 + (5*b*x^4*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{-d\left(x^2\left(b\,x^2+a\right)d^4+\left(-20b\,x^2-6a\right)d^2+120b\right)x\cos(dx+c)+3\sin(dx+c)\left(x^2\left(\frac{5b\,x^2}{3}+a\right)d^4+\left(-20b\,x^2-2a\right)d^2+40b\right)}{d^6}$
risch	$-\frac{x\left(b\,x^4d^4+a\,d^4x^2-20x^2d^2b-6d^2a+120b\right)\cos(dx+c)}{d^5}+\frac{\left(5b\,x^4d^4+3a\,d^4x^2-60x^2d^2b-6d^2a+120b\right)\sin(dx+c)}{d^6}$
orering	$\frac{2\left(5b^2d^4x^6+8ab\,d^4x^4+3a^2d^4x^2-80b^2d^2x^4-78ab\,d^2x^2-12a^2d^2+360x^2b^2+240ab\right)\sin(dx+c)}{d^6\left(b\,x^2+a\right)}-\frac{\left(b\,x^4d^4+a\,d^4x^2-20\cdots\right.}{\cdots}$
norman	$\frac{\frac{b\,x^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{\left(d^2a-20b\right)x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}-\frac{b\,x^5}{d}+\frac{6\left(d^2a-20b\right)x}{d^5}-\frac{\left(d^2a-20b\right)x^3}{d^3}-\frac{12\left(d^2a-20b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^6}+\frac{10b\,x^4\,\mathrm{ta}\cdots}{\cdots}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
meijerg	$\frac{32b\sin(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}x^4d^4-\frac{45}{2}x^2d^2+45\right)\cos(dx)}{12\sqrt{\pi}}+\frac{xd\left(\frac{3}{8}x^4d^4-\frac{15}{2}x^2d^2+45\right)\sin(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32b\cos(c)\sqrt{\pi}\left(-\frac{xd\left(\frac{7}{8}x^4\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{b\,x^5\cos(dx+c)}{d}-\frac{a\,x^3\cos(dx+c)}{d}+\frac{\frac{3a\,c^2\sin(dx+c)}{d^2}-\frac{6ac(\cos(dx+c)+(dx+c)\sin(dx+c))}{d^2}+\frac{3a\left((dx+c)^2\sin(dx+c)-2\sin\cdots\right.}{d\cdots}}{\cdots}$
derivativedivides	$\frac{a\,c^3\cos(dx+c)+3a\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))-3ac\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)\cdots}{\cdots}$
default	$\frac{a\,c^3\cos(dx+c)+3a\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))-3ac\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)\cdots}{\cdots}$

input
```
int(x^3*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(-d*(x^2*(b*x^2+a)*d^4+(-20*b*x^2-6*a)*d^2+120*b)*x*cos(d*x+c)+3*sin(d*x+c
)*(x^2*(5/3*b*x^2+a)*d^4+(-20*b*x^2-2*a)*d^2+40*b))/d^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^5x^5+(ad^5-20\,bd^3)x^3-6\,(ad^3-20\,bd)x)\cos(dx+c)-(5\,bd^4x^4-6\,ad^2+3\,(ad^4-20\,bd^2)x^2+}{d^6}$$

input
```
integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^5*x^5 + (a*d^5 - 20*b*d^3)*x^3 - 6*(a*d^3 - 20*b*d)*x)*cos(d*x + c)
 - (5*b*d^4*x^4 - 6*a*d^2 + 3*(a*d^4 - 20*b*d^2)*x^2 + 120*b)*sin(d*x + c)
)/d^6
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.19

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^3\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{6ax\cos(c+dx)}{d^3}-\frac{6a\sin(c+dx)}{d^4}-\frac{bx^5\cos(c+dx)}{d}+\frac{5bx^4\sin(c+dx)}{d^2}+\frac{20bx^3\cos(c+dx)}{d^3}- \\ \left(\frac{ax^4}{4}+\frac{bx^6}{6}\right)\sin(c) \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos
(c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**5*cos(c + d*x)/d + 5*b*x**4*
sin(c + d*x)/d**2 + 20*b*x**3*cos(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d
**4 - 120*b*x*cos(c + d*x)/d**5 + 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*
x**4/4 + b*x**6/6)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(141) = 282.

Time = 0.05 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.64

$$\int x^3(a+bx^2)\sin(c+dx)\,dx$$

$$= \frac{ac^3\cos(dx+c) + \frac{bc^5\cos(dx+c)}{d^2} - 3\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac^2 - \frac{5\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^4}{d^2}}{}$$

input
```
integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a*c^3*cos(d*x + c) + b*c^5*cos(d*x + c)/d^2 - 3*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c^2 - 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^2
 + 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c + 10*
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^2 - ((
(d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c)
)*a - 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*s
in(d*x + c))*b*c^2/d^2 + 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x +
c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^2 - (((d*x + c)^5 -
 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x
 + c)^2 + 24)*sin(d*x + c))*b/d^2)/d^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^5x^5+ad^5x^3-20\,bd^3x^3-6\,ad^3x+120\,bdx)\cos(dx+c)}{d^6} + \frac{(5\,bd^4x^4+3\,ad^4x^2-60\,bd^2x^2-6\,ad^2+120\,b)\sin(dx+c)}{d^6}$$

input
```
integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^5*x^5 + a*d^5*x^3 - 20*b*d^3*x^3 - 6*a*d^3*x + 120*b*d*x)*cos(d*x +
c)/d^6 + (5*b*d^4*x^4 + 3*a*d^4*x^2 - 60*b*d^2*x^2 - 6*a*d^2 + 120*b)*sin(
d*x + c)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = \frac{6\,\sin(c+d\,x)\,(20\,b-a\,d^2)}{d^6} + \frac{x^3\,\cos(c+d\,x)\,(20\,b-a\,d^2)}{d^3} - \frac{3\,x^2\,\sin(c+d\,x)\,(20\,b-a\,d^2)}{d^4} - \frac{6\,x\,\cos(c+d\,x)\,(20\,b-a\,d^2)}{d^5} - \frac{b\,x^5\,\cos(c+d\,x)}{d} + \frac{5\,b\,x^4\,\sin(c+d\,x)}{d^2}$$

input
```
int(x^3*sin(c + d*x)*(a + b*x^2),x)
```

output
```
(6*sin(c + d*x)*(20*b - a*d^2))/d^6 + (x^3*cos(c + d*x)*(20*b - a*d^2))/d^
3 - (3*x^2*sin(c + d*x)*(20*b - a*d^2))/d^4 - (6*x*cos(c + d*x)*(20*b - a*
d^2))/d^5 - (b*x^5*cos(c + d*x))/d + (5*b*x^4*sin(c + d*x))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^2)\sin(c+dx)\,dx$$
$$= \frac{-\cos(dx+c)\,a\,d^5x^3+6\cos(dx+c)\,a\,d^3x-\cos(dx+c)\,b\,d^5x^5+20\cos(dx+c)\,b\,d^3x^3-120\cos(dx+$$

input
```
int(x^3*(b*x^2+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**5*x**3 + 6*cos(c + d*x)*a*d**3*x - cos(c + d*x)*b*d*
*5*x**5 + 20*cos(c + d*x)*b*d**3*x**3 - 120*cos(c + d*x)*b*d*x + 3*sin(c +
 d*x)*a*d**4*x**2 - 6*sin(c + d*x)*a*d**2 + 5*sin(c + d*x)*b*d**4*x**4 - 6
0*sin(c + d*x)*b*d**2*x**2 + 120*sin(c + d*x)*b)/d**6
```

3.41 $\int x^2(a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{24b\cos(c+dx)}{d^5}+\frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{24bx\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}$$

output
```
-24*b*cos(d*x+c)/d^5+2*a*cos(d*x+c)/d^3+12*b*x^2*cos(d*x+c)/d^3-a*x^2*cos(
d*x+c)/d-b*x^4*cos(d*x+c)/d-24*b*x*sin(d*x+c)/d^4+2*a*x*sin(d*x+c)/d^2+4*b
*x^3*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \frac{-((ad^2(-2+d^2x^2)+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+2dx(ad^2+2b(-6+d^2x^2))\sin(c+dx)}{d^5}$$

input
```
Integrate[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(-((a*d^2*(-2 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) +
2*d*x*(a*d^2 + 2*b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^2) \sin(c + dx) \, dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(ax^2 \sin(c + dx) + bx^4 \sin(c + dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\cos(c+dx)}{d^3} + \frac{2ax\sin(c+dx)}{d^2} - \frac{ax^2\cos(c+dx)}{d} - \frac{24b\cos(c+dx)}{d^5} - \frac{24bx\sin(c+dx)}{d^4} + \frac{12bx^2\cos(c+dx)}{d^3} + \frac{4bx^3\sin(c+dx)}{d^2} - \frac{bx^4\cos(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(-24*b*Cos[c + d*x])/d^5 + (2*a*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x]
)/d^3 - (a*x^2*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (24*b*x*Sin[c +
d*x])/d^4 + (2*a*x*Sin[c + d*x])/d^2 + (4*b*x^3*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b x^4 d^4+a d^4 x^2-12x^2 d^2 b-2d^2 a+24b)\cos(dx+c)}{d^5}+\frac{2x(2x^2d^2b+d^2a-12b)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{d^2x^2((bx^2+a)d^2-12b)\tan(\frac{dx}{2}+\frac{c}{2})^2+4dx((2bx^2+a)d^2-12b)\tan(\frac{dx}{2}+\frac{c}{2})+(-bx^4-ax^2)d^4+4(3bx^2+a)d^2-48b}{d^5(1+\tan(\frac{dx}{2}+\frac{c}{2})^2)}$
norman	$\frac{\frac{4d^2a-48b}{d^5}+\frac{bx^4\tan(\frac{dx}{2}+\frac{c}{2})^2}{d}+\frac{(d^2a-12b)x^2\tan(\frac{dx}{2}+\frac{c}{2})^2}{d^3}-\frac{bx^4}{d}-\frac{(d^2a-12b)x^2}{d^3}+\frac{8bx^3\tan(\frac{dx}{2}+\frac{c}{2})}{d^2}+\frac{4(d^2a-12b)x\tan(\frac{d}{2}}{d^4}}{1+\tan(\frac{dx}{2}+\frac{c}{2})^2}$
orering	$\frac{4(2b^2d^4x^6+3abd^4x^4+a^2d^4x^2-18b^2d^2x^4-14abd^2x^2-a^2d^2+24x^2b^2+12ab)\sin(dx+c)}{d^6x(bx^2+a)}-\frac{(bx^4d^4+ad^4x^2-12x^2d^2b}{}$
parts	$-\frac{bx^4\cos(dx+c)}{d}-\frac{ax^2\cos(dx+c)}{d}+\frac{-\frac{2ac\sin(dx+c)}{d}+\frac{2a(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}-\frac{4bc^3\sin(dx+c)}{d^3}+\frac{12bc^2(\cos(}{}}{}$
meijerg	$\frac{16b\sin(c)\sqrt{\pi}\left(-\frac{x(d^2)^{\frac{5}{2}}(-\frac{5x^2d^2}{2}+15)\cos(dx)}{10\sqrt{\pi}d^4}+\frac{(d^2)^{\frac{5}{2}}(\frac{5}{8}x^4d^4-\frac{15}{2}x^2d^2+15)\sin(dx)}{10\sqrt{\pi}d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\cos(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{(\frac{3}{8}x^4}{}\right.}{}$
derivativedivides	$\frac{-ac^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c))-}{}$
default	$\frac{-ac^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c))-}{}$

input
```
int(x^2*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d^4*x^4+a*d^4*x^2-12*b*d^2*x^2-2*a*d^2+24*b)/d^5*cos(d*x+c)+2/d^4*x*(2
*b*d^2*x^2+a*d^2-12*b)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^4x^4-2\,ad^2+(ad^4-12\,bd^2)x^2+24\,b)\cos(dx+c)-2\,(2\,bd^3x^3+(ad^3-12\,bd)x)\sin(dx+c)}{d^5}$$

input
```
integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^4*x^4 - 2*a*d^2 + (a*d^4 - 12*b*d^2)*x^2 + 24*b)*cos(d*x + c) - 2*(
2*b*d^3*x^3 + (a*d^3 - 12*b*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^2\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}- \\ \left(\frac{ax^3}{3}+\frac{bx^5}{5}\right)\sin(c) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c +
d*x)/d**3 - b*x**4*cos(c + d*x)/d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2
*cos(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne
(d, 0)), ((a*x**3/3 + b*x**5/5)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(111) = 222$.

Time = 0.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.32

$$\int x^2(a+bx^2)\sin(c+dx)\,dx =$$

$$-\frac{ac^2\cos(dx+c) + \frac{bc^4\cos(dx+c)}{d^2} - 2\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac - \frac{4\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)b}{d^2}}{}$$

input
```
integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*c^2*cos(d*x + c) + b*c^4*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*a*c - 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^2
+ (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a + 6*(((d*x
 + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^2 - 4*(((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*
c/d^2 + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3
 - 6*d*x - 6*c)*sin(d*x + c))*b/d^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x^2 - 12\,bd^2x^2 - 2\,ad^2 + 24\,b)\cos(dx+c)}{d^5} + \frac{2\,(2\,bd^3x^3 + ad^3x - 12\,bdx)\sin(dx+c)}{d^5}$$

input
```
integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^4*x^4 + a*d^4*x^2 - 12*b*d^2*x^2 - 2*a*d^2 + 24*b)*cos(d*x + c)/d^5
+ 2*(2*b*d^3*x^3 + a*d^3*x - 12*b*d*x)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 42.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \frac{x^2\cos(c+d\,x)\,(12\,b-a\,d^2)}{d^3} - \frac{2\cos(c+d\,x)\,(12\,b-a\,d^2)}{d^5} - \frac{2\,x\sin(c+d\,x)\,(12\,b-a\,d^2)}{d^4} - \frac{b\,x^4\cos(c+d\,x)}{d} + \frac{4\,b\,x^3\sin(c+d\,x)}{d^2}$$

input
```
int(x^2*sin(c + d*x)*(a + b*x^2),x)
```

output
```
(x^2*cos(c + d*x)*(12*b - a*d^2))/d^3 - (2*cos(c + d*x)*(12*b - a*d^2))/d^
5 - (2*x*sin(c + d*x)*(12*b - a*d^2))/d^4 - (b*x^4*cos(c + d*x))/d + (4*b*
x^3*sin(c + d*x))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \frac{-\cos(dx+c)\,a\,d^4x^2+2\cos(dx+c)\,a\,d^2-\cos(dx+c)\,b\,d^4x^4+12\cos(dx+c)\,b\,d^2x^2-24\cos(dx+c)}{d^5}$$

input
```
int(x^2*(b*x^2+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**4*x**2 + 2*cos(c + d*x)*a*d**2 - cos(c + d*x)*b*d**4
*x**4 + 12*cos(c + d*x)*b*d**2*x**2 - 24*cos(c + d*x)*b + 2*sin(c + d*x)*a
*d**3*x + 4*sin(c + d*x)*b*d**3*x**3 - 24*sin(c + d*x)*b*d*x)/d**5
```

3.42 $\int x(a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{6bx\cos(c+dx)}{d^3} - \frac{ax\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4} + \frac{a\sin(c+dx)}{d^2} + \frac{3bx^2\sin(c+dx)}{d^2}$$

output
```
6*b*x*cos(d*x+c)/d^3-a*x*cos(d*x+c)/d-b*x^3*cos(d*x+c)/d-6*b*sin(d*x+c)/d^
4+a*sin(d*x+c)/d^2+3*b*x^2*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{-dx(ad^2+b(-6+d^2x^2))\cos(c+dx)+(ad^2+3b(-2+d^2x^2))\sin(c+dx)}{d^4}$$

input
```
Integrate[x*(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(-(d*x*(a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x]) + (a*d^2 + 3*b*(-2 + d^2*x
^2))*Sin[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)\sin(c+dx)\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(ax\sin(c+dx)+bx^3\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

input
```
Int[x*(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(6*b*x*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^3*Cos[c + d*x])/d -
 (6*b*Sin[c + d*x])/d^4 + (a*Sin[c + d*x])/d^2 + (3*b*x^2*Sin[c + d*x])/d^
2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{-\left(\left(b\,x^{2}+a\right)d^{2}-6b\right)dx\cos\left(dx+c\right)+\sin\left(dx+c\right)\left(\left(3b\,x^{2}+a\right)d^{2}-6b\right)}{d^{4}}$
risch	$-\frac{x\left(x^{2}d^{2}b+d^{2}a-6b\right)\cos\left(dx+c\right)}{d^{3}}+\frac{\left(3x^{2}d^{2}b+d^{2}a-6b\right)\sin\left(dx+c\right)}{d^{4}}$
parts	$-\frac{b\,x^{3}\cos\left(dx+c\right)}{d}-\frac{ax\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)+\frac{3b\,c^{2}\sin\left(dx+c\right)}{d^{2}}-\frac{6bc\left(\cos\left(dx+c\right)+\left(dx+c\right)\sin\left(dx+c\right)\right)}{d^{2}}+\frac{3b\left(\left(dx+c\right)^{2}\sin(\cdots\right)}{\cdots}}{d^{2}}$
norman	$\frac{\frac{b\,x^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{\left(d^{2}a-6b\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d^{3}}-\frac{b\,x^{3}}{d}-\frac{\left(d^{2}a-6b\right)x}{d^{3}}+\frac{2\left(d^{2}a-6b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^{4}}+\frac{6b\,x^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^{2}}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{2\left(3b^{2}d^{2}x^{4}+4ab\,d^{2}x^{2}+a^{2}d^{2}-12x^{2}b^{2}-6ab\right)\sin\left(dx+c\right)}{d^{4}\left(b\,x^{2}+a\right)}-\frac{\left(x^{2}d^{2}b+d^{2}a-6b\right)\left(\left(b\,x^{2}+a\right)\sin\left(dx+c\right)+2x^{2}b\sin\left(dx+c\right)+x(\cdots\right)}{d^{4}\left(b\,x^{2}+a\right)}$
meijerg	$\frac{8b\sin\left(c\right)\sqrt{\pi}\,\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^{2}d^{2}}{2}+3\right)\cos\left(dx\right)}{4\sqrt{\pi}}-\frac{xd\left(-\frac{x^{2}d^{2}}{2}+3\right)\sin\left(dx\right)}{4\sqrt{\pi}}\right)}{d^{4}}+\frac{8b\cos\left(c\right)\sqrt{\pi}\,\left(\frac{xd\left(-\frac{5x^{2}d^{2}}{2}+15\right)\cos\left(dx\right)}{20\sqrt{\pi}}-\frac{\left(-\cdots\right.}{\cdots}\right.}{d^{4}}$
derivativedivides	$\frac{ac\cos\left(dx+c\right)+a\left(\sin\left(dx+c\right)-\cos\left(dx+c\right)\left(dx+c\right)\right)+\frac{b\,c^{3}\cos\left(dx+c\right)}{d^{2}}+\frac{3b\,c^{2}\left(\sin\left(dx+c\right)-\cos\left(dx+c\right)\left(dx+c\right)\right)}{d^{2}}-\frac{3bc\left(-\left(dx+c\right)^{2}\cos(\cdots\right)}{\cdots}}{d^{2}}$
default	$\frac{ac\cos\left(dx+c\right)+a\left(\sin\left(dx+c\right)-\cos\left(dx+c\right)\left(dx+c\right)\right)+\frac{b\,c^{3}\cos\left(dx+c\right)}{d^{2}}+\frac{3b\,c^{2}\left(\sin\left(dx+c\right)-\cos\left(dx+c\right)\left(dx+c\right)\right)}{d^{2}}-\frac{3bc\left(-\left(dx+c\right)^{2}\cos(\cdots\right)}{\cdots}}{d^{2}}$

input
```
int(x*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(-((b*x^2+a)*d^2-6*b)*d*x*cos(d*x+c)+sin(d*x+c)*((3*b*x^2+a)*d^2-6*b))/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x(a+bx^2)\sin(c+dx)\,dx$$
$$= -\frac{(bd^3x^3+(ad^3-6\,bd)x)\cos(dx+c)-(3\,bd^2x^2+ad^2-6\,b)\sin(dx+c)}{d^4}$$

input
```
integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^3*x^3 + (a*d^3 - 6*b*d)*x)*cos(d*x + c) - (3*b*d^2*x^2 + a*d^2 - 6*
b)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int x(a+bx^2)\sin(c+dx)\,dx$$
$$= \begin{cases} -\frac{ax\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}+\frac{6bx\cos(c+dx)}{d^3}-\frac{6b\sin(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{ax^2}{2}+\frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**3*cos(c + d*x)
/d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*
x)/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.06

$$\int x(a+bx^2)\sin(c+dx)\,dx$$
$$= \frac{ac\cos(dx+c) + \frac{bc^3\cos(dx+c)}{d^2} - ((dx+c)\cos(dx+c) - \sin(dx+c))a - \frac{3((dx+c)\cos(dx+c)-\sin(dx+c))bc^2}{d^2} + \frac{3}{}}{d^2}$$

input
```
integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a*c*cos(d*x + c) + b*c^3*cos(d*x + c)/d^2 - ((d*x + c)*cos(d*x + c) - sin
(d*x + c))*a - 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d^2 + 3*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d^2 - (((d*x
+ c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d
^2)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^3x^3 + ad^3x - 6\,bdx)\cos(dx+c)}{d^4} + \frac{(3\,bd^2x^2 + ad^2 - 6\,b)\sin(dx+c)}{d^4}$$

input
```
integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^3*x^3 + a*d^3*x - 6*b*d*x)*cos(d*x + c)/d^4 + (3*b*d^2*x^2 + a*d^2 -
 6*b)*sin(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 42.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{x\,\cos(c+d\,x)\,(6\,b-a\,d^2)}{d^3} - \frac{\sin(c+d\,x)\,(6\,b-a\,d^2)}{d^4} - \frac{b\,x^3\,\cos(c+d\,x)}{d} + \frac{3\,b\,x^2\,\sin(c+d\,x)}{d^2}$$

input
```
int(x*sin(c + d*x)*(a + b*x^2),x)
```

output
```
(x*cos(c + d*x)*(6*b - a*d^2))/d^3 - (sin(c + d*x)*(6*b - a*d^2))/d^4 - (b
*x^3*cos(c + d*x))/d + (3*b*x^2*sin(c + d*x))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{-\cos(dx+c)\,a\,d^3x-\cos(dx+c)\,b\,d^3x^3+6\cos(dx+c)\,bdx+\sin(dx+c)\,a\,d^2+3\sin(dx+c)\,b\,d^2x^2 -}{d^4}$$

input
```
int(x*(b*x^2+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**3*x - cos(c + d*x)*b*d**3*x**3 + 6*cos(c + d*x)*b*d*
x + sin(c + d*x)*a*d**2 + 3*sin(c + d*x)*b*d**2*x**2 - 6*sin(c + d*x)*b)/d
**4
```

3.43 $\int (a+bx^2) \sin(c+dx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int (a+bx^2) \sin(c+dx) \, dx = \frac{2b\cos(c+dx)}{d^3} - \frac{a\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2}$$

output
```
2*b*cos(d*x+c)/d^3-a*cos(d*x+c)/d-b*x^2*cos(d*x+c)/d+2*b*x*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int (a+bx^2) \sin(c+dx) \, dx = \frac{-((ad^2+b(-2+d^2x^2))\cos(c+dx)) + 2bdx\sin(c+dx)}{d^3}$$

input
```
Integrate[(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(-((a*d^2 + b*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*x*Sin[c + d*x])/d^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\sin(c+dx)\,dx$$

$$\Big\downarrow \text{3810}$$

$$\int \left(a\sin(c+dx)+bx^2\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cos(c+dx)}{d}+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

input
```
Int[(a + b*x^2)*Sin[c + d*x],x]
```

output
```
(2*b*Cos[c + d*x])/d^3 - (a*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (2*
b*x*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3810
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(x^{2} d^{2} b+d^{2} a-2 b\right) \cos \left(d x+c\right)}{d^{3}}+\frac{2 b x \sin \left(d x+c\right)}{d^{2}}$
parallelrisch	$\frac{\left(\left(-b \,x^{2}-a \right) d^{2}+2 b \right) \cos \left(d x+c \right)+2 \sin \left(d x+c \right) b d x+d^{2} a-2 b}{d^{3}}$
parts	$-\frac{b \,x^{2} \cos \left(d x+c \right)}{d}-\frac{a \cos \left(d x+c \right)}{d}+\frac{2 b \left(\cos \left(d x+c \right)+\left(d x+c \right) \sin \left(d x+c \right)-c \sin \left(d x+c \right)\right)}{d^{3}}$
norman	$\frac{\frac{b \,x^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}-\frac{2 d^{2} a-4 b}{d^{3}}-\frac{b \,x^{2}}{d}+\frac{4 b x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}}{1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{4 b x \left(x^{2} d^{2} b+d^{2} a-b \right) \sin \left(d x+c \right)}{d^{4} \left(b \,x^{2}+a \right)}-\frac{\left(x^{2} d^{2} b+d^{2} a-2 b \right) \left(2 x b \sin \left(d x+c \right)+\left(b \,x^{2}+a \right) d \cos \left(d x+c \right)\right)}{d^{4} \left(b \,x^{2}+a \right)}$
derivativedivides	$\frac{-\cos \left(d x+c \right) a-\frac{b \,c^{2} \cos \left(d x+c \right)}{d^{2}}-\frac{2 b c \left(\sin \left(d x+c \right)-\cos \left(d x+c \right) \left(d x+c \right)\right)}{d^{2}}+\frac{b \left(-\left(d x+c \right)^{2} \cos \left(d x+c \right)+2 \cos \left(d x+c \right)+2 \left(d x+c \right) \sin \left(d x+c \right)\right)}{d^{2}}}{d}$
default	$\frac{-\cos \left(d x+c \right) a-\frac{b \,c^{2} \cos \left(d x+c \right)}{d^{2}}-\frac{2 b c \left(\sin \left(d x+c \right)-\cos \left(d x+c \right) \left(d x+c \right)\right)}{d^{2}}+\frac{b \left(-\left(d x+c \right)^{2} \cos \left(d x+c \right)+2 \cos \left(d x+c \right)+2 \left(d x+c \right) \sin \left(d x+c \right)\right)}{d^{2}}}{d}$
meijerg	$\frac{4 b \sin \left(c \right) \sqrt{\pi}\, \left(\frac{x \left(d^{2}\right)^{\frac{3}{2}} \cos \left(d x \right)}{2 \sqrt{\pi}\, d^{2}}-\frac{\left(d^{2}\right)^{\frac{3}{2}} \left(-\frac{3 x^{2} d^{2}}{2}+3\right) \sin \left(d x \right)}{6 \sqrt{\pi}\, d^{3}}\right)}{d^{2} \sqrt{d^{2}}}+\frac{4 b \cos \left(c \right) \sqrt{\pi}\, \left(-\frac{1}{2 \sqrt{\pi}}+\frac{\left(-\frac{x^{2} d^{2}}{2}+1\right) \cos \left(d x \right)}{2 \sqrt{\pi}}+\frac{x d \sin(dx}{2 \sqrt{\pi}}\right.}{d^{3}}$

input
```
int((b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d^2*x^2+a*d^2-2*b)/d^3*cos(d*x+c)+2*b*x*sin(d*x+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)\sin(c+dx)\,dx=\frac{2\,bdx\sin\left(dx+c\right)-\left(bd^2x^2+ad^2-2\,b\right)\cos\left(dx+c\right)}{d^3}$$

input
```
integrate((b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
(2*b*d*x*sin(d*x + c) - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int (a+bx^2)\sin(c+dx)\,dx = \begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2} + \frac{2b\cos(c+dx)}{d^3} & \text{for } d\neq 0 \\ \left(ax+\frac{bx^3}{3}\right)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*cos(c + d*x)/d - b*x**2*cos(c + d*x)/d + 2*b*x*sin(c + d*x)/
d**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int (a+bx^2)\sin(c+dx)\,dx = -\frac{a\cos(dx+c) + \frac{bc^2\cos(dx+c)}{d^2} - \frac{2((dx+c)\cos(dx+c)-\sin(dx+c))bc}{d^2} + \frac{\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)b}{d^2}}{d}$$

input
```
integrate((b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*cos(d*x + c) + b*c^2*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) - si
n(d*x + c))*b*c/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*
x + c))*b/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int (a+bx^2)\sin(c+dx)\,dx = \frac{2\,bx\sin(dx+c)}{d^2} - \frac{(bd^2x^2+ad^2-2\,b)\cos(dx+c)}{d^3}$$

input
```
integrate((b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
2*b*x*sin(d*x + c)/d^2 - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c)/d^3
```

Mupad [B] (verification not implemented)

Time = 42.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int (a+bx^2)\sin(c+dx)\,dx = \frac{\cos(c+d\,x)\,\left(2\,b-a\,d^2\right)}{d^3} + \frac{2\,b\,x\,\sin(c+d\,x)}{d^2} - \frac{b\,x^2\,\cos(c+d\,x)}{d}$$

input
```
int(sin(c + d*x)*(a + b*x^2),x)
```

output
```
(cos(c + d*x)*(2*b - a*d^2))/d^3 + (2*b*x*sin(c + d*x))/d^2 - (b*x^2*cos(c
 + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int (a+bx^2)\sin(c+dx)\,dx = \frac{-\cos(dx+c)\,a\,d^2-\cos(dx+c)\,b\,d^2x^2+2\cos(dx+c)\,b+2\sin(dx+c)\,bdx}{d^3}$$

input
```
int((b*x^2+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**2 - cos(c + d*x)*b*d**2*x**2 + 2*cos(c + d*x)*b + 2*
sin(c + d*x)*b*d*x)/d**3
```

3.44 $\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = -\frac{bx\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{b\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

output
```
-b*x*cos(d*x+c)/d+a*Ci(d*x)*sin(c)+b*sin(d*x+c)/d^2+a*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = -\frac{b\cos(dx)(dx\cos(c)-\sin(c))}{d^2} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{b(\cos(c)+dx\sin(c))\sin(dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

input
```
Integrate[((a + b*x^2)*Sin[c + d*x])/x,x]
```

output
```
-((b*Cos[d*x]*(d*x*Cos[c] - Sin[c]))/d^2) + a*CosIntegral[d*x]*Sin[c] + (b
*(Cos[c] + d*x*Sin[c])*Sin[d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x} \, dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(\frac{a\sin(c+dx)}{x} + bx\sin(c+dx)\right) dx$$
$$\Big\downarrow \text{2009}$$
$$a\sin(c)\operatorname{CosIntegral}(dx) + a\cos(c)\operatorname{Si}(dx) + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^2)*Sin[c + d*x])/x,x]
```

output
```
-((b*x*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c + d*x])/d^2
 + a*Cos[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

method	result
derivativedivides	$a(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))+\frac{2cb\cos(dx+c)}{d^2}+\frac{(c+1)b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}$
default	$a(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))+\frac{2cb\cos(dx+c)}{d^2}+\frac{(c+1)b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}$
risch	$-\frac{e^{-ic}\pi\ \operatorname{csgn}(dx)a}{2}+\frac{ia\,e^{ic}\operatorname{expIntegral}_1(-idx)}{2}-\frac{ie^{-ic}\operatorname{expIntegral}_1(-idx)a}{2}+e^{-ic}\operatorname{Si}(dx)\,a-\frac{bx\cos(dx\ldots}{d}$
meijerg	$\frac{2b\sin(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\cos(c)\sqrt{\pi}\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln(d^2\ldots}{\sqrt{\pi}}\ldots\right.}{\ldots}$

input
```
int((b*x^2+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+2/d^2*c*b*cos(d*x+c)+(c+1)/d^2*b*(sin(d*
x+c)-cos(d*x+c)*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = \frac{ad^2\operatorname{Ci}(dx)\sin(c)+ad^2\cos(c)\operatorname{Si}(dx)-bdx\cos(dx+c)+b\sin(dx+c)}{d^2}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

output
```
(a*d^2*cos_integral(d*x)*sin(c) + a*d^2*cos(c)*sin_integral(d*x) - b*d*x*c
os(d*x + c) + b*sin(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = a\sin(c)\operatorname{Ci}(dx) + a\cos(c)\operatorname{Si}(dx) + bx\left(\begin{cases} x\sin(c) & \text{for } d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} \frac{x^2\sin(c)}{2} & \text{for } d=0 \\ -\frac{\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d\neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)*sin(d*x+c)/x,x)
```

output
```
a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x*Piecewise((x*sin(c), Eq(d, 0)),
(-cos(c + d*x)/d, True)) - b*Piecewise((x**2*sin(c)/2, Eq(d, 0)), (-Piecew
ise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = -\frac{2\,bdx\cos(dx+c) - (a(-i\operatorname{Ei}(i\,dx) + i\operatorname{Ei}(-i\,dx))\cos(c) + a(\operatorname{Ei}(i\,dx) + \operatorname{Ei}(-i\,dx))\sin(c))d^2 - 2\,b\sin}{2\,d^2}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/2*(2*b*d*x*cos(d*x + c) - (a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(
Ei(I*d*x) + Ei(-I*d*x))*sin(c))*d^2 - 2*b*sin(d*x + c))/d^2
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 432, normalized size of antiderivative = 10.54

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="giac")
```

output
```
-1/2*(a*d^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d
^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*sin
_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*real_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2*tan(1/2*c) + 2*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*im
ag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d^2*imag_part(cos_integral(-
d*x))*tan(1/2*d*x)^2 - 2*a*d^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*im
ag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*imag_part(cos_integral(-d*
x))*tan(1/2*c)^2 + 2*a*d^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*d*x*tan(1/
2*d*x)^2 - 2*a*d^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^2*real_
part(cos_integral(-d*x))*tan(1/2*c) - 8*b*d*x*tan(1/2*d*x)*tan(1/2*c) - 2*
b*d*x*tan(1/2*c)^2 - a*d^2*imag_part(cos_integral(d*x)) + a*d^2*imag_part(
cos_integral(-d*x)) - 2*a*d^2*sin_integral(d*x) + 4*b*tan(1/2*d*x)^2*tan(1
/2*c) + 4*b*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b*d*x - 4*b*tan(1/2*d*x) - 4*b*t
an(1/2*c))/(d^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*tan(1/2*d*x)^2 + d^2*tan
(1/2*c)^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = a\,\mathrm{cosint}(d\,x)\,\sin(c) + a\,\mathrm{sinint}(d\,x)\,\cos(c) + \frac{b\,(\sin(c+d\,x) - d\,x\,\cos(c+d\,x))}{d^2}$$

input
```
int((sin(c + d*x)*(a + b*x^2))/x,x)
```

output
```
a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) + (b*(sin(c + d*x) - d*x*cos(c
 + d*x)))/d^2
```

Reduce [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = \frac{-\cos(dx+c)\,bdx+\left(\int \frac{\sin(dx+c)}{x}dx\right)a\,d^2+\sin(dx+c)\,b}{d^2}$$

input
```
int((b*x^2+a)*sin(d*x+c)/x,x)
```

output
```
( - cos(c + d*x)*b*d*x + int(sin(c + d*x)/x,x)*a*d**2 + sin(c + d*x)*b)/d*
*2
```

3.45 $\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = -\frac{b\cos(c+dx)}{d} + ad\cos(c)\,\mathrm{CosIntegral}(dx) - \frac{a\sin(c+dx)}{x} - ad\sin(c)\mathrm{Si}(dx)$$

output
```
-b*cos(d*x+c)/d+a*d*cos(c)*Ci(d*x)-a*sin(d*x+c)/x-a*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = -\frac{b\cos(c+dx)}{d} + ad\cos(c)\,\mathrm{CosIntegral}(dx) - \frac{a\sin(c+dx)}{x} - ad\sin(c)\mathrm{Si}(dx)$$

input
```
Integrate[((a + b*x^2)*Sin[c + d*x])/x^2,x]
```

output
```
-((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
 a*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a\sin(c+dx)}{x^2}+b\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\cos(c)\,\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^2)*Sin[c + d*x])/x^2,x]
```

output
```
-((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
 a*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)-\frac{b\cos(dx+c)}{d^2}\right)$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)-\frac{b\cos(dx+c)}{d^2}\right)$
risch	$-\frac{d\cos(c)a\ \mathrm{expIntegral}_1(-idx)}{2}-\frac{d\cos(c)a\ \mathrm{expIntegral}_1(idx)}{2}-\frac{id\sin(c)a\ \mathrm{expIntegral}_1(-idx)}{2}+\frac{id\sin(c)a\ \mathrm{exp}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d}+\frac{b\cos(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\sin(c)\sqrt{\pi}\,d^2\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\operatorname{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{a\cos(c)\sqrt{\pi}\,d\,(}{}$

input
```
int((b*x^2+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-1/d^2*b*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \frac{ad^2x\cos(c)\operatorname{Ci}(dx)-ad^2x\sin(c)\operatorname{Si}(dx)-bx\cos(dx+c)-ad\sin(dx+c)}{dx}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
(a*d^2*x*cos(c)*cos_integral(d*x) - a*d^2*x*sin(c)*sin_integral(d*x) - b*x
*cos(d*x + c) - a*d*sin(d*x + c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx=\int \frac{(a+bx^2)\sin{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x**2+a)*sin(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**2)*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 937, normalized size of antiderivative = 21.30

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(((I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^3
 + (I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)*sin(c
)^2 + (exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*sin(c)^3 + (I
*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c) + ((exp_in
tegral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^2 + exp_integral_e(
2, I*d*x) + exp_integral_e(2, -I*d*x))*sin(c))*b*c^2/((d*x + c)*(cos(c)^2
+ sin(c)^2)*d^2 - (c*cos(c)^2 + c*sin(c)^2)*d^2) - ((I*exp_integral_e(2, I
*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^3 + (I*exp_integral_e(2, I*d*x
) - I*exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 + (exp_integral_e(2, I*d*
x) + exp_integral_e(2, -I*d*x))*sin(c)^3 + (I*exp_integral_e(2, I*d*x) - I
*exp_integral_e(2, -I*d*x))*cos(c) + ((exp_integral_e(2, I*d*x) + exp_inte
gral_e(2, -I*d*x))*cos(c)^2 + exp_integral_e(2, I*d*x) + exp_integral_e(2,
 -I*d*x))*sin(c))*a/(c*cos(c)^2 + c*sin(c)^2 - (d*x + c)*(cos(c)^2 + sin(c
)^2)) + 2*(((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*
sin(c)^2)*(d*x + c))*cos(d*x + c)^3 + (b*c^2*(exp_integral_e(3, I*d*x) + e
xp_integral_e(3, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp
_integral_e(3, -I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*
x) + I*exp_integral_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d*
x) + exp_integral_e(3, -I*d*x))*cos(c) + (b*c^2*(-I*exp_integral_e(3, I*d*
x) + I*exp_integral_e(3, -I*d*x))*cos(c)^2 + b*c^2*(-I*exp_integral_e(3...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 411, normalized size of antiderivative = 9.34

$$\int \frac{(a + bx^2)\sin(c + dx)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a
*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2
*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^2*x*sin_integr
al(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x*real_part(cos_integral(d*x))*t
an(1/2*d*x)^2 - a*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + a*d
^2*x*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^2*x*real_part(cos_int
egral(-d*x))*tan(1/2*c)^2 + 2*a*d^2*x*imag_part(cos_integral(d*x))*tan(1/2
*c) - 2*a*d^2*x*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^2*x*sin_i
ntegral(d*x)*tan(1/2*c) + 2*b*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x*real
_part(cos_integral(d*x)) - a*d^2*x*real_part(cos_integral(-d*x)) - 4*a*d*t
an(1/2*d*x)^2*tan(1/2*c) - 4*a*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*b*x*tan(1/2
*d*x)^2 - 8*b*x*tan(1/2*d*x)*tan(1/2*c) - 2*b*x*tan(1/2*c)^2 + 4*a*d*tan(1
/2*d*x) + 4*a*d*tan(1/2*c) + 2*b*x)/(d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*x
*tan(1/2*d*x)^2 + d*x*tan(1/2*c)^2 + d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^2}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2))/x^2,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \frac{-\cos\left(dx+c\right)b+\left(\int \frac{\sin(dx+c)}{x^2}dx\right)ad}{d}$$

input
```
int((b*x^2+a)*sin(d*x+c)/x^2,x)
```

output
```
( - cos(c + d*x)*b + int(sin(c + d*x)/x**2,x)*a*d)/d
```

3.46 $\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{ad\cos(c+dx)}{2x} + b\,\text{CosIntegral}(dx)\sin(c) - \frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} + b\cos(c)\text{Si}(dx) - \frac{1}{2}ad^2\cos(c)\text{Si}(dx)$$

output
```
-1/2*a*d*cos(d*x+c)/x+b*Ci(d*x)*sin(c)-1/2*a*d^2*Ci(d*x)*sin(c)-1/2*a*sin(
d*x+c)/x^2+b*cos(c)*Si(d*x)-1/2*a*d^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\cos(dx)(dx\cos(c)+\sin(c))}{2x^2} + \frac{a(-\cos(c)+dx\sin(c))\sin(dx)}{2x^2} + b\cos(c)\text{Si}(dx) - \frac{1}{2}ad^2(\text{CosIntegral}(dx)\sin(c)+\cos(c)\text{Si}(dx))$$

input
```
Integrate[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

output
```
b*CosIntegral[d*x]*Sin[c] - (a*Cos[d*x]*(d*x*Cos[c] + Sin[c]))/(2*x^2) + (
a*(-Cos[c] + d*x*Sin[c])*Sin[d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (
a*d^2*(CosIntegral[d*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}ad^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+b\sin(c)\,\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

input
```
Int[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

output
```
-1/2*(a*d*Cos[c + d*x])/x + b*CosIntegral[d*x]*Sin[c] - (a*d^2*CosIntegral
[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (
a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}\right)$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}\right)$
risch	$-\frac{i\,\mathrm{expIntegral}_1(-idx)\cos(c)a\,d^2}{4}+\frac{i\,\mathrm{expIntegral}_1(idx)\cos(c)a\,d^2}{4}+\frac{i\,\mathrm{expIntegral}_1(-idx)\cos(c)b}{2}-\frac{i\,\mathrm{expIntegra}}{}$
meijerg	$\frac{b\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos\left(c\right)\mathrm{Si}\left(dx\right)+\frac{a\sin(c)\sqrt{\pi}\,d^2\left(-\frac{}{\sqrt{\pi}}\right.}{}$

input
```
int((b*x^2+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*
Ci(d*x)*sin(c))+b/d^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{(ad^2-2\,b)x^2\,\mathrm{Ci}\left(dx\right)\sin\left(c\right)+(ad^2-2\,b)x^2\cos\left(c\right)\mathrm{Si}\left(dx\right)+adx\cos\left(dx+c\right)+a\sin\left(dx+c\right)}{2\,x^2}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/2*((a*d^2 - 2*b)*x^2*cos_integral(d*x)*sin(c) + (a*d^2 - 2*b)*x^2*cos(c
)*sin_integral(d*x) + a*d*x*cos(d*x + c) + a*sin(d*x + c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx^2)\sin{(c+dx)}}{x^3}\,dx$$

input
```
integrate((b*x**2+a)*sin(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**2)*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{2\,bdx\cos{(dx+c)} + ((a(-i\,\Gamma(-2,i\,dx) + i\,\Gamma(-2,-i\,dx))\cos{(c)} - a(\Gamma(-2,i\,dx) + \Gamma(-2,-i\,dx))\sin{(}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*d*x*cos(d*x + c) + ((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x
))*cos(c) - a*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 - 2*(b*(-
I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - b*(gamma(-2, I*d*x) + g
amma(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b*sin(d*x + c))/(d^2*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 766, normalized size of antiderivative = 10.35

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x^2*imag_part(cos_int
egral(d*x))*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1
/2*d*x)^2 - 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*x^2*imag_
part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(-d
*x))*tan(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b*x^2*imag_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x^2*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part(cos_integral(d*x))
*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 4*b*x
^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x^2*real_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_pa
rt(cos_integral(-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) + 2*b*x^2*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(-d*x))
*tan(1/2*d*x)^2 + 4*b*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 2*b*x^2*imag_
part(cos_integral(d*x))*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_integral(-...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^3}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2))/x^3,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$$
$$= \frac{-2\cos\left(dx+c\right)bx-2\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2+x^2}dx\right)a\,d^2x^2+4\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2+x^2}dx\right)b\,x^2-\sin\left(dx+c\right)ad -}{2d\,x^2}$$

input
```
int((b*x^2+a)*sin(d*x+c)/x^3,x)
```

output
```
( - 2*cos(c + d*x)*b*x - 2*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x*
*2 + x**2),x)*a*d**2*x**2 + 4*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2
*x**2 + x**2),x)*b*x**2 - sin(c + d*x)*a*d - a*d**2*x + 2*b*x)/(2*d*x**2)
```

3.47 $\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\begin{aligned}\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = &-\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\,\text{CosIntegral}(dx)\\&-\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx)-\frac{a\sin(c+dx)}{3x^3}\\&-\frac{b\sin(c+dx)}{x}+\frac{ad^2\sin(c+dx)}{6x}\\&-bd\sin(c)\text{Si}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)\end{aligned}$$

output
```
-1/6*a*d*cos(d*x+c)/x^2+b*d*cos(c)*Ci(d*x)-1/6*a*d^3*cos(c)*Ci(d*x)-1/3*a*
sin(d*x+c)/x^3-b*sin(d*x+c)/x+1/6*a*d^2*sin(d*x+c)/x-b*d*sin(c)*Si(d*x)+1/
6*a*d^3*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$$
$$= \frac{-adx\cos(c+dx)+d(6b-ad^2)\,x^3\cos(c)\operatorname{CosIntegral}(dx)-2a\sin(c+dx)-6bx^2\sin(c+dx)+ad^2x^2\,\mathrm{s}}{6x^3}$$

input
```
Integrate[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

output
```
(-(a*d*x*Cos[c + d*x]) + d*(6*b - a*d^2)*x^3*Cos[c]*CosIntegral[d*x] - 2*a
*Sin[c + d*x] - 6*b*x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d*(-6*b +
a*d^2)*x^3*Sin[c]*SinIntegral[d*x])/(6*x^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{6}ad^3\cos(c)\operatorname{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\operatorname{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\operatorname{CosIntegral}(dx)-bd\sin(c)\operatorname{Si}(dx)-\frac{b\sin(c+dx)}{x}$$

input
```
Int[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

output
```
-1/6*(a*d*Cos[c + d*x])/x^2 + b*d*Cos[c]*CosIntegral[d*x] - (a*d^3*Cos[c]*
CosIntegral[d*x])/6 - (a*Sin[c + d*x])/(3*x^3) - (b*Sin[c + d*x])/x + (a*d
^2*Sin[c + d*x])/(6*x) - b*d*Sin[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinIn
tegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\,\mathrm{si}\cdots\right.}{d^2}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\,\mathrm{si}\cdots\right.}{d^2}\right.$
risch	$\frac{\mathrm{expIntegral}_1(-idx)\cos(c)a\,d^3}{12}+\frac{\mathrm{expIntegral}_1(idx)\cos(c)a\,d^3}{12}-\frac{\mathrm{expIntegral}_1(-idx)\cos(c)bd}{2}-\frac{\mathrm{expIntegral}_1(id\cdots}{2}$
meijerg	$\frac{d^2b\sin(c)\sqrt{\pi}\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\mathrm{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{db\cos(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\cdots\right.}{4}$

input
```
int((b*x^2+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+
1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+b/d^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(
c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = -\frac{(ad^3-6\,bd)x^3\cos(c)\,\mathrm{Ci}\,(dx)-(ad^3-6\,bd)x^3\sin(c)\,\mathrm{Si}\,(dx)+adx\cos(dx+c)-((ad^2-6\,b)x^2-2\,a}{6\,x^3}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/6*((a*d^3 - 6*b*d)*x^3*cos(c)*cos_integral(d*x) - (a*d^3 - 6*b*d)*x^3*s
in(c)*sin_integral(d*x) + a*d*x*cos(d*x + c) - ((a*d^2 - 6*b)*x^2 - 2*a)*s
in(d*x + c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x**2+a)*sin(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**2)*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = -\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c))d^5-6\,(b(\Gamma(-3,i}{}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3,
I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 6*(b*(gamma(-3, I*d*x) + gamma
(-3, -I*d*x))*cos(c) + b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c
))*d^3)*x^3 + 2*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(d^2*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 834, normalized size of antiderivative = 7.87

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

output
```
1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a
*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*
x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*
sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/
2*d*x)^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3
*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(
-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*
x))*tan(1/2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*
a*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 12*b*d*x^3*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*d*x^3*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*
tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(
cos_integral(-d*x)) + 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^
2 + 6*b*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*d^2*x^2*t
an(1/2*d*x)^2*tan(1/2*c) - 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*
c)^2 - 6*b*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a*d^2*x^2*
tan(1/2*d*x)*tan(1/2*c)^2 - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx=\int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)}{x^4}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2))/x^4,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$$
$$= \frac{-6\cos(dx+c)\,bx + 4\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^3+x^3}dx\right) a\,d^2x^3 - 24\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^3+x^3}dx\right) b\,x^3 - 2\sin(dx+c)\,a}{6d\,x^3}$$

input
```
int((b*x^2+a)*sin(d*x+c)/x^4,x)
```

output
```
( - 6*cos(c + d*x)*b*x + 4*int(1/(tan((c + d*x)/2)**2*x**3 + x**3),x)*a*d*
*2*x**3 - 24*int(1/(tan((c + d*x)/2)**2*x**3 + x**3),x)*b*x**3 - 2*sin(c +
 d*x)*a*d + a*d**2*x - 6*b*x)/(6*d*x**3)
```

3.48 $\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\begin{aligned}\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = &-\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}\\&+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c)\\&+\frac{1}{24}ad^4\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}\\&-\frac{b\sin(c+dx)}{2x^2}+\frac{ad^2\sin(c+dx)}{24x^2}\\&-\frac{1}{2}bd^2\cos(c)\text{Si}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)\end{aligned}$$

output
```
-1/12*a*d*cos(d*x+c)/x^3-1/2*b*d*cos(d*x+c)/x+1/24*a*d^3*cos(d*x+c)/x-1/2*
b*d^2*Ci(d*x)*sin(c)+1/24*a*d^4*Ci(d*x)*sin(c)-1/4*a*sin(d*x+c)/x^4-1/2*b*
sin(d*x+c)/x^2+1/24*a*d^2*sin(d*x+c)/x^2-1/2*b*d^2*cos(c)*Si(d*x)+1/24*a*d
^4*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$$
$$= \frac{-2adx\cos(c+dx)-12bdx^3\cos(c+dx)+ad^3x^3\cos(c+dx)+d^2(-12b+ad^2)\,x^4\,\text{CosIntegral}(dx)\sin(c}{24x^4}$$

input
```
Integrate[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

output
```
(-2*a*d*x*Cos[c + d*x] - 12*b*d*x^3*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x]
+ d^2*(-12*b + a*d^2)*x^4*CosIntegral[d*x]*Sin[c] - 6*a*Sin[c + d*x] - 12*
b*x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d^2*(-12*b + a*d^2)*x^4*Cos[
c]*SinIntegral[d*x])/(24*x^4)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{24}ad^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^3\cos(c+dx)}{24x}+\frac{ad^2\sin(c+dx)}{24x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}-\frac{1}{2}bd^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}bd^2\cos(c)\text{Si}(dx)-\frac{b\sin(c+dx)}{2x^2}-\frac{bd\cos(c+dx)}{2x}$$

input
```
Int[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

output
```
-1/12*(a*d*Cos[c + d*x])/x^3 - (b*d*Cos[c + d*x])/(2*x) + (a*d^3*Cos[c + d
*x])/(24*x) - (b*d^2*CosIntegral[d*x]*Sin[c])/2 + (a*d^4*CosIntegral[d*x]*
Sin[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*
Sin[c + d*x])/(24*x^2) - (b*d^2*Cos[c]*SinIntegral[d*x])/2 + (a*d^4*Cos[c]
*SinIntegral[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

method	result
derivativedivides	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\ldots\right.$
default	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\ldots\right.$
risch	$-\frac{i\,\mathrm{expIntegral}_1(idx)\cos(c)a\,d^4}{48}+\frac{i\cos(c)\,\mathrm{expIntegral}_1(-idx)a\,d^4}{48}+\frac{i\,\mathrm{expIntegral}_1(idx)\cos(c)b\,d^2}{4}-\ldots$
meijerg	$\frac{d^2b\sin(c)\sqrt{\pi}\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6x^2d^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,x^2d^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,xd}-\ldots\right.}{8}$

input
```
int((b*x^2+a)*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*(a*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^
2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+1/d^2*b
*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x
)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$$
$$= \frac{(ad^4-12\,bd^2)x^4\operatorname{Ci}(dx)\sin(c)+(ad^4-12\,bd^2)x^4\cos(c)\operatorname{Si}(dx)+((ad^3-12\,bd)x^3-2\,adx)\cos(dx+c}{24\,x^4}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
1/24*((a*d^4 - 12*b*d^2)*x^4*cos_integral(d*x)*sin(c) + (a*d^4 - 12*b*d^2)
*x^4*cos(c)*sin_integral(d*x) + ((a*d^3 - 12*b*d)*x^3 - 2*a*d*x)*cos(d*x +
 c) + ((a*d^2 - 12*b)*x^2 - 6*a)*sin(d*x + c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$$

input
```
integrate((b*x**2+a)*sin(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x**2)*sin(c + dx)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = -\frac{((a(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+a(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^6-12\,(b(i\,\Gamma(-4,i$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
-1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
 I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - 12*(b*(I*gamma(-4, I*d*x) - I*g
amma(-4, -I*d*x))*cos(c) + b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c)
)*d^4)*x^4 + 2*b*d*x*cos(d*x + c) + 6*b*sin(d*x + c))/(d^2*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1086, normalized size of antiderivative = 7.29

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

output
```
-1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
- a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real
_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part
(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^4*x^4*imag_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 - 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^4*x^4*ima
g_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(
-d*x))*tan(1/2*c)^2 + 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b*d^
2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*
x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b*d^2*x
^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_part(c
os_integral(d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_integral(-d*x))*t
an(1/2*c) + 24*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1
/2*c) + 24*b*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*
c) - 2*a*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_int
egral(d*x)) + a*d^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_in
tegral(d*x) + 12*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 1
2*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*b*d^2*x^4*si
n_integral(d*x)*tan(1/2*d*x)^2 - 12*b*d^2*x^4*imag_part(cos_integral(d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^5}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2))/x^5,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$$
$$= \frac{-12\cos(dx+c)\,bx-6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)a\,d^2x^4+72\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)b\,x^4-3\sin(dx+c)}{12d\,x^4}$$

input
```
int((b*x^2+a)*sin(d*x+c)/x^5,x)
```

output
```
( - 12*cos(c + d*x)*b*x - 6*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x
**4 + x**4),x)*a*d**2*x**4 + 72*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)*
*2*x**4 + x**4),x)*b*x**4 - 3*sin(c + d*x)*a*d - a*d**2*x + 12*b*x)/(12*d*
x**4)
```

3.49 $\int x^2(a+bx^2)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 236

$$\int x^2\left(a+bx^2\right)^2\sin(c+dx)\,dx=\frac{720b^2\cos(c+dx)}{d^7}-\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{720b^2x\sin(c+dx)}{d^6}-\frac{48abx\sin(c+dx)}{d^4}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{6b^2x^5\sin(c+dx)}{d^2}$$

output

```
720*b^2*cos(d*x+c)/d^7-48*a*b*cos(d*x+c)/d^5+2*a^2*cos(d*x+c)/d^3-360*b^2*
x^2*cos(d*x+c)/d^5+24*a*b*x^2*cos(d*x+c)/d^3-a^2*x^2*cos(d*x+c)/d+30*b^2*x
^4*cos(d*x+c)/d^3-2*a*b*x^4*cos(d*x+c)/d-b^2*x^6*cos(d*x+c)/d+720*b^2*x*si
n(d*x+c)/d^6-48*a*b*x*sin(d*x+c)/d^4+2*a^2*x*sin(d*x+c)/d^2-120*b^2*x^3*si
n(d*x+c)/d^4+8*a*b*x^3*sin(d*x+c)/d^2+6*b^2*x^5*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int x^2 \left(a+bx^2\right)^2 \sin(c+dx)\,dx$$
$$= \frac{-((a^2d^4(-2+d^2x^2)+2abd^2(24-12d^2x^2+d^4x^4)+b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx)}{d^7}$$

input
```
Integrate[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

output
```
(-((a^2*d^4*(-2 + d^2*x^2) + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*(
-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + 2*d*x*(a^2*d^4
 + 4*a*b*d^2*(-6 + d^2*x^2) + 3*b^2*(120 - 20*d^2*x^2 + d^4*x^4))*Sin[c +
d*x])/d^7
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^2 \sin(c+dx)\,dx$$
$$\Big\downarrow \text{ 3820}$$
$$\int \left(a^2x^2\sin(c+dx)+2abx^4\sin(c+dx)+b^2x^6\sin(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^2\cos(c+dx)}{d^3}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{48ab\cos(c+dx)}{d^5}-\frac{48abx\sin(c+dx)}{d^4}+\frac{24abx^2\cos(c+dx)}{d^3}+\frac{8abx^3\sin(c+dx)}{d^2}-\frac{2abx^4\cos(c+dx)}{d}+\frac{720b^2\cos(c+dx)}{d^7}+\frac{720b^2x\sin(c+dx)}{d^6}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{30b^2x^4\cos(c+dx)}{d^3}+\frac{6b^2x^5\sin(c+dx)}{d^2}-\frac{b^2x^6\cos(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

output
```
(720*b^2*Cos[c + d*x])/d^7 - (48*a*b*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*
x])/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 + (24*a*b*x^2*Cos[c + d*x])/d^3 -
 (a^2*x^2*Cos[c + d*x])/d + (30*b^2*x^4*Cos[c + d*x])/d^3 - (2*a*b*x^4*Cos
[c + d*x])/d - (b^2*x^6*Cos[c + d*x])/d + (720*b^2*x*Sin[c + d*x])/d^6 - (
48*a*b*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 - (120*b^2*x^3*Sin
[c + d*x])/d^4 + (8*a*b*x^3*Sin[c + d*x])/d^2 + (6*b^2*x^5*Sin[c + d*x])/d
^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(b^2x^6d^6+2ab\,d^6x^4+a^2d^6x^2-30b^2x^4d^4-24ab\,d^4x^2-2a^2d^4+360x^2d^2b^2+48b\,d^2a-720b^2)\cos(dx+c)}{d^7}+\frac{2x(3b^2x^4d\cdots}{\cdots}$
parallelrisch	$\frac{\left(x^2(b\,x^2+a)^2d^6+(-30b^2x^4-24ab\,x^2-4a^2)d^4+(360x^2b^2+96ab)d^2-1440b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4dx\left((3b\,x^2+a)(b\,x^2+a\cdots\right.}{d^7\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cdots\right.}$
norman	$\frac{\frac{b^2x^6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{(a^2d^4-24b\,d^2a+360b^2)x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^5}-\frac{b^2x^6}{d}-\frac{(a^2d^4-24b\,d^2a+360b^2)x^2}{d^5}-\frac{(4a^2d^4-96b\,d^2a+1440b^2)\cdots}{d^7}\cdots}{\cdots}$
orering	$\frac{4(3b^3d^6x^8+7a\,b^2d^6x^6+5a^2b\,d^6x^4-75b^3d^4x^6+a^3d^6x^2-93a\,b^2d^4x^4-27a^2b\,d^4x^2+720b^3d^2x^4-a^3d^4+432a\,b^2d^2x^2+24\cdots}{d^8x(b\,x^2+a)}$
meijerg	$\frac{64b^2\sin(c)\sqrt{\pi}\left(\frac{x(d^2)^{\frac{7}{2}}\left(\frac{21}{8}x^4d^4-\frac{105}{2}x^2d^2+315\right)\cos(dx)}{28\sqrt{\pi}\,d^6}-\frac{(d^2)^{\frac{7}{2}}\left(-\frac{7}{16}x^6d^6+\frac{105}{8}x^4d^4-\frac{315}{2}x^2d^2+315\right)\sin(dx)}{28\sqrt{\pi}\,d^7}\right)}{d^6\sqrt{d^2}}+\frac{64b^{\cdots}}{\cdots}$
parts	$-\frac{b^2x^6\cos(dx+c)}{d}-\frac{2ab\,x^4\cos(dx+c)}{d}-\frac{a^2x^2\cos(dx+c)}{d}+\frac{-\frac{2a^2c\sin(dx+c)}{d}+\frac{2a^2(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}-\cdots}{\cdots}$
derivativedivides	$\frac{-a^2c^2\cos(dx+c)-2a^2c(\sin(dx+c)-\cos(dx+c)(dx+c))+a^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)\cdots}{\cdots}$
default	$\frac{-a^2c^2\cos(dx+c)-2a^2c(\sin(dx+c)-\cos(dx+c)(dx+c))+a^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)\cdots}{\cdots}$

input
```
int(x^2*(b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^6*x^6+2*a*b*d^6*x^4+a^2*d^6*x^2-30*b^2*d^4*x^4-24*a*b*d^4*x^2-2*a^
2*d^4+360*b^2*d^2*x^2+48*a*b*d^2-720*b^2)/d^7*cos(d*x+c)+2/d^6*x*(3*b^2*d^
4*x^4+4*a*b*d^4*x^2+a^2*d^4-60*b^2*d^2*x^2-24*a*b*d^2+360*b^2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.65

$$\int x^2\left(a+bx^2\right)^2\sin(c+dx)\,dx = -\frac{(b^2d^6x^6-2\,a^2d^4+2\,(abd^6-15\,b^2d^4)x^4+48\,abd^2+(a^2d^6-24\,abd^4+360\,b^2d^2)x^2-720\,b^2)\cos\left(dx+\right.}{d^7}$$

input
```
integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^6*x^6 - 2*a^2*d^4 + 2*(a*b*d^6 - 15*b^2*d^4)*x^4 + 48*a*b*d^2 + (
a^2*d^6 - 24*a*b*d^4 + 360*b^2*d^2)*x^2 - 720*b^2)*cos(d*x + c) - 2*(3*b^2
*d^5*x^5 + 4*(a*b*d^5 - 15*b^2*d^3)*x^3 + (a^2*d^5 - 24*a*b*d^3 + 360*b^2*
d)*x)*sin(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.21

$$\int x^2\left(a+bx^2\right)^2\sin(c+dx)\,dx = \begin{cases} -\frac{a^2x^2\cos(c+dx)}{d}+\frac{2a^2x\sin(c+dx)}{d^2}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{2abx^4\cos(c+dx)}{d}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{48abx\sin(}{d^4} \\ \left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right)\sin(c) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2
*cos(c + d*x)/d**3 - 2*a*b*x**4*cos(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d
**2 + 24*a*b*x**2*cos(c + d*x)/d**3 - 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*
cos(c + d*x)/d**5 - b**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d*
*2 + 30*b**2*x**4*cos(c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 36
0*b**2*x**2*cos(c + d*x)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*co
s(c + d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7)*si
n(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(236) = 472$.

Time = 0.07 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.59

$$\int x^2 (a + bx^2)^2 \sin(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a^2*c^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^4 + 2*a*b*c^4*cos(d*x + c)
/d^2 - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 6*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*b^2*c^5/d^4 - 8*((d*x + c)*cos(d*x + c) - sin(d*x
 + c))*a*b*c^3/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x
 + c))*a^2 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c)
)*b^2*c^4/d^4 + 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x +
 c))*a*b*c^2/d^2 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x
+ c)^2 - 2)*sin(d*x + c))*b^2*c^3/d^4 - 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos
(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^2 + 15*(((d*x + c)^4
 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d
*x + c))*b^2*c^2/d^4 + 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c)
 - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^2 - 6*(((d*x + c)^5 -
 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x
 + c)^2 + 24)*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^6 - 30*(d*x + c)^4 + 3
60*(d*x + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120
*d*x + 120*c)*sin(d*x + c))*b^2/d^4)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.69

$$\int x^2 (a + bx^2)^2 \sin(c + dx)\, dx = - \frac{(b^2d^6x^6 + 2\,abd^6x^4 + a^2d^6x^2 - 30\,b^2d^4x^4 - 24\,abd^4x^2 - 2\,a^2d^4 + 360\,b^2d^2x^2 + 48\,abd^2 - 720\,b^2)\cos (d}{d^7} + \frac{2\,(3\,b^2d^5x^5 + 4\,abd^5x^3 + a^2d^5x - 60\,b^2d^3x^3 - 24\,abd^3x + 360\,b^2dx)\sin (dx + c)}{d^7}$$

input
```
integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + a^2*d^6*x^2 - 30*b^2*d^4*x^4 - 24*a*b*d^4*
x^2 - 2*a^2*d^4 + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2)*cos(d*x + c)/d^7
 + 2*(3*b^2*d^5*x^5 + 4*a*b*d^5*x^3 + a^2*d^5*x - 60*b^2*d^3*x^3 - 24*a*b*
d^3*x + 360*b^2*d*x)*sin(d*x + c)/d^7
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.79

$$\int x^2 (a+bx^2)^2 \sin(c+dx)\,dx = \frac{2\,\cos(c+d\,x)\,\left(a^2\,d^4-24\,a\,b\,d^2+360\,b^2\right)}{d^7} - \frac{b^2\,x^6\,\cos(c+d\,x)}{d} + \frac{6\,b^2\,x^5\,\sin(c+d\,x)}{d^2} + \frac{2\,x\,\sin(c+d\,x)\,\left(a^2\,d^4-24\,a\,b\,d^2+360\,b^2\right)}{d^6} - \frac{x^2\,\cos(c+d\,x)\,\left(a^2\,d^4-24\,a\,b\,d^2+360\,b^2\right)}{d^5} + \frac{2\,x^4\,\cos(c+d\,x)\,\left(15\,b^2-a\,b\,d^2\right)}{d^3} - \frac{8\,x^3\,\sin(c+d\,x)\,\left(15\,b^2-a\,b\,d^2\right)}{d^4}$$

input
```
int(x^2*sin(c + d*x)*(a + b*x^2)^2,x)
```

output
```
(2*cos(c + d*x)*(360*b^2 + a^2*d^4 - 24*a*b*d^2))/d^7 - (b^2*x^6*cos(c + d
*x))/d + (6*b^2*x^5*sin(c + d*x))/d^2 + (2*x*sin(c + d*x)*(360*b^2 + a^2*d
^4 - 24*a*b*d^2))/d^6 - (x^2*cos(c + d*x)*(360*b^2 + a^2*d^4 - 24*a*b*d^2)
)/d^5 + (2*x^4*cos(c + d*x)*(15*b^2 - a*b*d^2))/d^3 - (8*x^3*sin(c + d*x)*
(15*b^2 - a*b*d^2))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00

$$\int x^2 (a + bx^2)^2 \sin(c + dx)\, dx$$
$$= \frac{-\cos(dx+c)\,a^2d^6x^2 + 2\cos(dx+c)\,a^2d^4 - 2\cos(dx+c)\,ab\,d^6x^4 + 24\cos(dx+c)\,ab\,d^4x^2 - 48\cos(dx$$

input
```
int(x^2*(b*x^2+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**6*x**2 + 2*cos(c + d*x)*a**2*d**4 - 2*cos(c + d*x
)*a*b*d**6*x**4 + 24*cos(c + d*x)*a*b*d**4*x**2 - 48*cos(c + d*x)*a*b*d**2
 - cos(c + d*x)*b**2*d**6*x**6 + 30*cos(c + d*x)*b**2*d**4*x**4 - 360*cos(
c + d*x)*b**2*d**2*x**2 + 720*cos(c + d*x)*b**2 + 2*sin(c + d*x)*a**2*d**5
*x + 8*sin(c + d*x)*a*b*d**5*x**3 - 48*sin(c + d*x)*a*b*d**3*x + 6*sin(c +
 d*x)*b**2*d**5*x**5 - 120*sin(c + d*x)*b**2*d**3*x**3 + 720*sin(c + d*x)*
b**2*d*x)/d**7
```

3.50 $\int x(a+bx^2)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\begin{aligned}\int x(a+bx^2)^2\sin(c+dx)\,dx = &-\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}\\&-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}\\&-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}\\&+\frac{120b^2\sin(c+dx)}{d^6}-\frac{12ab\sin(c+dx)}{d^4}\\&+\frac{a^2\sin(c+dx)}{d^2}-\frac{60b^2x^2\sin(c+dx)}{d^4}\\&+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{5b^2x^4\sin(c+dx)}{d^2}\end{aligned}$$

output
```
-120*b^2*x*cos(d*x+c)/d^5+12*a*b*x*cos(d*x+c)/d^3-a^2*x*cos(d*x+c)/d+20*b^
2*x^3*cos(d*x+c)/d^3-2*a*b*x^3*cos(d*x+c)/d-b^2*x^5*cos(d*x+c)/d+120*b^2*s
in(d*x+c)/d^6-12*a*b*sin(d*x+c)/d^4+a^2*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+
c)/d^4+6*a*b*x^2*sin(d*x+c)/d^2+5*b^2*x^4*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int x\left(a+bx^2\right)^2\sin(c+dx)\,dx$$
$$=\frac{-dx(a^2d^4+2abd^2(-6+d^2x^2)+b^2(120-20d^2x^2+d^4x^4))\cos(c+dx)+(a^2d^4+6abd^2(-2+d^2x^2)+5}{d^6}$$

input
```
Integrate[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

output
```
(-(d*x*(a^2*d^4 + 2*a*b*d^2*(-6 + d^2*x^2) + b^2*(120 - 20*d^2*x^2 + d^4*x
^4))*Cos[c + d*x]) + (a^2*d^4 + 6*a*b*d^2*(-2 + d^2*x^2) + 5*b^2*(24 - 12*
d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^2\sin(c+dx)\,dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(a^2x\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^5\sin(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}+\frac{120b^2\sin(c+dx)}{d^6}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{60b^2x^2\sin(c+dx)}{d^4}+\frac{20b^2x^3\cos(c+dx)}{d^3}+\frac{5b^2x^4\sin(c+dx)}{d^2}-\frac{b^2x^5\cos(c+dx)}{d}$$

input
```
Int[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

output
```
(-120*b^2*x*Cos[c + d*x])/d^5 + (12*a*b*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c
 + d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (2*a*b*x^3*Cos[c + d*x])/d -
(b^2*x^5*Cos[c + d*x])/d + (120*b^2*Sin[c + d*x])/d^6 - (12*a*b*Sin[c + d*
x])/d^4 + (a^2*Sin[c + d*x])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (6*a*b*
x^2*Sin[c + d*x])/d^2 + (5*b^2*x^4*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{-\left(\left(b\,x^{2}+a\right)^{2} d^{4}+\left(-20 x^{2} b^{2}-12 a b\right) d^{2}+120 b^{2}\right) d x \cos \left(d x+c\right)+\left(\left(5 b\,x^{2}+a\right) \left(b\,x^{2}+a\right) d^{4}-12 b \left(5 b\,x^{2}+a\right) d^{2}+120 b^{2}\right) \sin \left(d x\right.}{d^{6}}$
risch	$-\frac{x \left(b^{2} x^{4} d^{4}+2 a b\,d^{4} x^{2}+a^{2} d^{4}-20 x^{2} d^{2} b^{2}-12 b\,d^{2} a+120 b^{2}\right) \cos \left(d x+c\right)}{d^{5}}+\frac{\left(5 b^{2} x^{4} d^{4}+6 a b\,d^{4} x^{2}+a^{2} d^{4}-60 x^{2} d^{2} b^{2}-12 b\right.}{d^{6}}$
orering	$\frac{2 \left(5 b^{3} d^{4} x^{6}+11 a\,b^{2} d^{4} x^{4}+7 a^{2} b\,d^{4} x^{2}-80 b^{3} d^{2} x^{4}+a^{3} d^{4}-76 a\,b^{2} d^{2} x^{2}-12 a^{2} b\,d^{2}+360 b^{3} x^{2}+120 b^{2} a\right) \sin \left(d x+c\right)}{d^{6} \left(b\,x^{2}+a\right)}-\frac{\left(b^{2} x^{4}\right.}{}$
norman	$\frac{\frac{b^{2} x^{5} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{\left(a^{2} d^{4}-12 b\,d^{2} a+120 b^{2}\right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d^{5}}-\frac{b^{2} x^{5}}{d}-\frac{\left(a^{2} d^{4}-12 b\,d^{2} a+120 b^{2}\right) x}{d^{5}}+\frac{2 \left(a^{2} d^{4}-12 b\,d^{2} a+120 b^{2}\right) \tan}{d^{6}}}{1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right.}$
meijerg	$\frac{32 b^{2} \sin \left(c\right) \sqrt{\pi}\, \left(-\frac{15}{4 \sqrt{\pi}}+\frac{\left(\frac{15}{8} x^{4} d^{4}-\frac{45}{2} x^{2} d^{2}+45\right) \cos \left(d x\right)}{12 \sqrt{\pi}}+\frac{x d \left(\frac{3}{8} x^{4} d^{4}-\frac{15}{2} x^{2} d^{2}+45\right) \sin \left(d x\right)}{12 \sqrt{\pi}}\right)}{d^{6}}+\frac{32 b^{2} \cos \left(c\right) \sqrt{\pi}\, \left(-\frac{x d \left(\frac{7}{8}\right.}{}\right.}{}$
parts	$-\frac{b^{2} x^{5} \cos \left(d x+c\right)}{d}-\frac{2 a b\,x^{3} \cos \left(d x+c\right)}{d}-\frac{a^{2} x \cos \left(d x+c\right)}{d}+\frac{a^{2} \sin \left(d x+c\right)+\frac{6 a b\,c^{2} \sin \left(d x+c\right)}{d^{2}}-\frac{12 a b c \left(\cos \left(d x+c\right)+\left(d x+\right.\right.}{d^{2}}}{}$
derivativedivides	$\frac{a^{2} c \cos \left(d x+c\right)+a^{2} \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)+\frac{2 a b\,c^{3} \cos \left(d x+c\right)}{d^{2}}+\frac{6 a b\,c^{2} \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)}{d^{2}}-\frac{6 a b c \left(-\left(d x+c\right.\right.}{}}{}$
default	$\frac{a^{2} c \cos \left(d x+c\right)+a^{2} \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)+\frac{2 a b\,c^{3} \cos \left(d x+c\right)}{d^{2}}+\frac{6 a b\,c^{2} \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)}{d^{2}}-\frac{6 a b c \left(-\left(d x+c\right.\right.}{}}{}$

input
```
int(x*(b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(-((b*x^2+a)^2*d^4+(-20*b^2*x^2-12*a*b)*d^2+120*b^2)*d*x*cos(d*x+c)+((5*b*
x^2+a)*(b*x^2+a)*d^4-12*b*(5*b*x^2+a)*d^2+120*b^2)*sin(d*x+c))/d^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int x\left(a+bx^2\right)^2\sin(c+dx)\,dx = -\frac{(b^2d^5x^5+2\,(abd^5-10\,b^2d^3)x^3+(a^2d^5-12\,abd^3+120\,b^2d)x)\cos\left(dx+c\right)-(5\,b^2d^4x^4+a^2d^4-12\,ab}{d^6}$$

input
```
integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^5*x^5 + 2*(a*b*d^5 - 10*b^2*d^3)*x^3 + (a^2*d^5 - 12*a*b*d^3 + 12
0*b^2*d)*x)*cos(d*x + c) - (5*b^2*d^4*x^4 + a^2*d^4 - 12*a*b*d^2 + 6*(a*b*
d^4 - 10*b^2*d^2)*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int x(a+bx^2)^2\sin(c+dx)\,dx$$
$$=\begin{cases} -\frac{a^2x\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{12abx\cos(c+dx)}{d^3}-\frac{12ab\sin(c+dx)}{d^4}-\frac{b^2x^5\cos(c+dx)}{d} \\ \left(\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}\right)\sin(c) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**3*co
s(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 -
 12*a*b*sin(c + d*x)/d**4 - b**2*x**5*cos(c + d*x)/d + 5*b**2*x**4*sin(c +
 d*x)/d**2 + 20*b**2*x**3*cos(c + d*x)/d**3 - 60*b**2*x**2*sin(c + d*x)/d*
*4 - 120*b**2*x*cos(c + d*x)/d**5 + 120*b**2*sin(c + d*x)/d**6, Ne(d, 0)),
 ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(185) = 370.

Time = 0.08 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.37

$$\int x(a+bx^2)^2\sin(c+dx)\,dx$$
$$=\frac{a^2c\cos(dx+c)+\frac{b^2c^5\cos(dx+c)}{d^4}+\frac{2\,abc^3\cos(dx+c)}{d^2}-((dx+c)\cos(dx+c)-\sin(dx+c))a^2-\frac{5\,((dx+c)\cos(dx+c)}{}}{}$$

input
```
integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a^2*c*cos(d*x + c) + b^2*c^5*cos(d*x + c)/d^4 + 2*a*b*c^3*cos(d*x + c)/d^
2 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 5*((d*x + c)*cos(d*x + c
) - sin(d*x + c))*b^2*c^4/d^4 - 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*
a*b*c^2/d^2 + 10*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c
))*b^2*c^3/d^4 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x +
 c))*a*b*c/d^2 - 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x +
c)^2 - 2)*sin(d*x + c))*b^2*c^2/d^4 - 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d
*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^2 + 5*(((d*x + c)^4 - 12
*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x +
c))*b^2*c/d^4 - (((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x
+ c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x + c))*b^2/d^4)/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int x(a+bx^2)^2\sin(c+dx)\,dx = -\frac{(b^2d^5x^5+2\,abd^5x^3+a^2d^5x-20\,b^2d^3x^3-12\,abd^3x+120\,b^2dx)\cos(dx+c)}{d^6} + \frac{(5\,b^2d^4x^4+6\,abd^4x^2+a^2d^4-60\,b^2d^2x^2-12\,abd^2+120\,b^2)\sin(dx+c)}{d^6}$$

input
```
integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^5*x^5 + 2*a*b*d^5*x^3 + a^2*d^5*x - 20*b^2*d^3*x^3 - 12*a*b*d^3*x
+ 120*b^2*d*x)*cos(d*x + c)/d^6 + (5*b^2*d^4*x^4 + 6*a*b*d^4*x^2 + a^2*d^4
 - 60*b^2*d^2*x^2 - 12*a*b*d^2 + 120*b^2)*sin(d*x + c)/d^6
```

Mupad [B] (verification not implemented)

Time = 42.81 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)^2\sin(c+dx)\,dx = \frac{\sin(c+d\,x)\,\left(a^2\,d^4-12\,a\,b\,d^2+120\,b^2\right)}{d^6} - \frac{b^2\,x^5\,\cos(c+d\,x)}{d} + \frac{5\,b^2\,x^4\,\sin(c+d\,x)}{d^2} - \frac{x\,\cos(c+d\,x)\,\left(a^2\,d^4-12\,a\,b\,d^2+120\,b^2\right)}{d^5} + \frac{2\,x^3\,\cos(c+d\,x)\,\left(10\,b^2-a\,b\,d^2\right)}{d^3} - \frac{6\,x^2\,\sin(c+d\,x)\,\left(10\,b^2-a\,b\,d^2\right)}{d^4}$$

input
```
int(x*sin(c + d*x)*(a + b*x^2)^2,x)
```

output
```
(sin(c + d*x)*(120*b^2 + a^2*d^4 - 12*a*b*d^2))/d^6 - (b^2*x^5*cos(c + d*x
))/d + (5*b^2*x^4*sin(c + d*x))/d^2 - (x*cos(c + d*x)*(120*b^2 + a^2*d^4 -
 12*a*b*d^2))/d^5 + (2*x^3*cos(c + d*x)*(10*b^2 - a*b*d^2))/d^3 - (6*x^2*s
in(c + d*x)*(10*b^2 - a*b*d^2))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int x(a+bx^2)^2\sin(c+dx)\,dx = \frac{-\cos(dx+c)\,a^2d^5x - 2\cos(dx+c)\,ab\,d^5x^3 + 12\cos(dx+c)\,ab\,d^3x - \cos(dx+c)\,b^2d^5x^5 + 20\cos(dx+c)}{}$$

input
```
int(x*(b*x^2+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**5*x - 2*cos(c + d*x)*a*b*d**5*x**3 + 12*cos(c + d
*x)*a*b*d**3*x - cos(c + d*x)*b**2*d**5*x**5 + 20*cos(c + d*x)*b**2*d**3*x
**3 - 120*cos(c + d*x)*b**2*d*x + sin(c + d*x)*a**2*d**4 + 6*sin(c + d*x)*
a*b*d**4*x**2 - 12*sin(c + d*x)*a*b*d**2 + 5*sin(c + d*x)*b**2*d**4*x**4 -
 60*sin(c + d*x)*b**2*d**2*x**2 + 120*sin(c + d*x)*b**2)/d**6
```

3.51 $\int (a+bx^2)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{24b^2\cos(c+dx)}{d^5}+\frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d} +\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d} -\frac{b^2x^4\cos(c+dx)}{d}-\frac{24b^2x\sin(c+dx)}{d^4} +\frac{4abx\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}$$

output
```
-24*b^2*cos(d*x+c)/d^5+4*a*b*cos(d*x+c)/d^3-a^2*cos(d*x+c)/d+12*b^2*x^2*co
s(d*x+c)/d^3-2*a*b*x^2*cos(d*x+c)/d-b^2*x^4*cos(d*x+c)/d-24*b^2*x*sin(d*x+
c)/d^4+4*a*b*x*sin(d*x+c)/d^2+4*b^2*x^3*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx$$
$$=\frac{-((a^2d^4+2abd^2(-2+d^2x^2)+b^2(24-12d^2x^2+d^4x^4))\cos(c+dx))+4bdx(ad^2+b(-6+d^2x^2))\sin(c}{d^5}$$

input
```
Integrate[(a + b*x^2)^2*Sin[c + d*x],x]
```

output
```
(-((a^2*d^4 + 2*a*b*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*x^2 + d^4*x^4))*
Cos[c + d*x]) + 4*b*d*x*(a*d^2 + b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx$$
$$\Big\downarrow \text{3810}$$
$$\int \left(a^2\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^4\sin(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2\cos(c+dx)}{d}+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}-\frac{24b^2\cos(c+dx)}{d^5}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{12b^2x^2\cos(c+dx)}{d^3}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{b^2x^4\cos(c+dx)}{d}$$

input
```
Int[(a + b*x^2)^2*Sin[c + d*x],x]
```

output

```
(-24*b^2*Cos[c + d*x])/d^5 + (4*a*b*Cos[c + d*x])/d^3 - (a^2*Cos[c + d*x])
/d + (12*b^2*x^2*Cos[c + d*x])/d^3 - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^4
*Cos[c + d*x])/d - (24*b^2*x*Sin[c + d*x])/d^4 + (4*a*b*x*Sin[c + d*x])/d^
2 + (4*b^2*x^3*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3810

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\left(b^{2} x^{4} d^{4}+2 a b\, d^{4} x^{2}+a^{2} d^{4}-12 x^{2} d^{2} b^{2}-4 b\, d^{2} a+24 b^{2}\right) \cos \left(d x+c\right)}{d^{5}}+\frac{4 b x \left(x^{2} d^{2} b+d^{2} a-6 b\right) \sin \left(d x+c\right)}{d^{4}}$
parallelrisch	$\frac{2\left(\left(\frac{b\,x^{2}}{2}+a\right) d^{2}-6 b\right) d^{2} b\,x^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+8\left(\left(b\,x^{2}+a\right) d^{2}-6 b\right) d b x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+\left(-b^{2} x^{4}-2 a b\,x^{2}-2 a^{2}\right) d^{4}+\left(12 x^{2} b^{2}+8 \cdots\right.}{d^{5}\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)}$
norman	$\frac{\frac{b^{2} x^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}-\frac{2 a^{2} d^{4}-8 b\,d^{2} a+48 b^{2}}{d^{5}}-\frac{b^{2} x^{4}}{d}+\frac{8 b^{2} x^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}-\frac{2 b\left(d^{2} a-6 b\right) x^{2}}{d^{3}}+\frac{8 b\left(d^{2} a-6 b\right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{4}}+\frac{2 b\left(d^{2} a-\cdots\right.}{\cdots}}{1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{8 b x \left(b^{2} x^{4} d^{4}+2 a b\, d^{4} x^{2}+a^{2} d^{4}-9 x^{2} d^{2} b^{2}-5 b\, d^{2} a+12 b^{2}\right) \sin \left(d x+c\right)}{d^{6} \left(b\,x^{2}+a\right)}-\frac{\left(b^{2} x^{4} d^{4}+2 a b\, d^{4} x^{2}+a^{2} d^{4}-12 x^{2} d^{2} b^{2}-4 b\, d^{2} a+2 \cdots\right.}{d^{6} \left(\cdots\right.}$
parts	$-\frac{b^{2} x^{4} \cos \left(d x+c\right)}{d}-\frac{2 a b\,x^{2} \cos \left(d x+c\right)}{d}-\frac{a^{2} \cos \left(d x+c\right)}{d}+\frac{4 b\left(-a c \sin \left(d x+c\right)+a\left(\cos \left(d x+c\right)+\left(d x+c\right) \sin \left(d x+c\right)\right)-\cdots\right.}{\cdots}$
meijerg	$\frac{16 b^{2} \sin \left(c\right) \sqrt{\pi}\,\left(-\frac{x \left(d^{2}\right)^{\frac{5}{2}} \left(-\frac{5 x^{2} d^{2}}{2}+15\right) \cos \left(d x\right)}{10 \sqrt{\pi}\, d^{4}}+\frac{\left(d^{2}\right)^{\frac{5}{2}} \left(\frac{5}{8} x^{4} d^{4}-\frac{15}{2} x^{2} d^{2}+15\right) \sin \left(d x\right)}{10 \sqrt{\pi}\, d^{5}}\right)}{d^{4} \sqrt{d^{2}}}+\frac{16 b^{2} \cos \left(c\right) \sqrt{\pi}\,\left(\frac{3}{2 \sqrt{\pi}}-\frac{\left(\frac{3}{8} \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\cos \left(d x+c\right) a^{2}-\frac{2 a b\,c^{2} \cos \left(d x+c\right)}{d^{2}}-\frac{4 a b c \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)}{d^{2}}+\frac{2 a b\left(-\left(d x+c\right)^{2} \cos \left(d x+c\right)+2 \cos \left(d x+c\right)+2 \left(d x+c\right) \sin \left(d \cdots\right.\right.}{d^{2}}}{\cdots}$
default	$\frac{-\cos \left(d x+c\right) a^{2}-\frac{2 a b\,c^{2} \cos \left(d x+c\right)}{d^{2}}-\frac{4 a b c \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)}{d^{2}}+\frac{2 a b\left(-\left(d x+c\right)^{2} \cos \left(d x+c\right)+2 \cos \left(d x+c\right)+2 \left(d x+c\right) \sin \left(d \cdots\right.\right.}{d^{2}}}{\cdots}$

input
```
int((b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4-12*b^2*d^2*x^2-4*a*b*d^2+24*b^2)/d^5*c
os(d*x+c)+4*b*x/d^4*(b*d^2*x^2+a*d^2-6*b)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx = -\frac{(b^2d^4x^4 + a^2d^4 - 4\,abd^2 + 2\,(abd^4 - 6\,b^2d^2)x^2 + 24\,b^2)\cos\left(dx+c\right) - 4\,(b^2d^3x^3 + (abd^3 - 6\,b^2d)x)\sin}{d^5}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^4*x^4 + a^2*d^4 - 4*a*b*d^2 + 2*(a*b*d^4 - 6*b^2*d^2)*x^2 + 24*b^
2)*cos(d*x + c) - 4*(b^2*d^3*x^3 + (a*b*d^3 - 6*b^2*d)*x)*sin(d*x + c))/d^
5
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25

$$\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx = \begin{cases} -\frac{a^2\cos(c+dx)}{d} - \frac{2abx^2\cos(c+dx)}{d} + \frac{4abx\sin(c+dx)}{d^2} + \frac{4ab\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + \frac{4b^2x^3\sin(c+dx)}{d^2} + \frac{12b^2x^2\cos(c+d}{d^3} \\ \left(a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}\right)\sin(c) \end{cases}$$

input
```
integrate((b*x**2+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**2*cos(c + d*x)/d + 4*a*b*x*sin(
c + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x**4*cos(c + d*x)/d + 4*b**
2*x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3 - 24*b**2*x*sin(
c + d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b*x**
3/3 + b**2*x**5/5)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(138) = 276.

Time = 0.05 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.12

$$\int (a+bx^2)^2 \sin(c+dx)\,dx =$$

$$-\frac{a^2\cos(dx+c)+\frac{b^2c^4\cos(dx+c)}{d^4}+\frac{2\,abc^2\cos(dx+c)}{d^2}-\frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^3}{d^4}-\frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c}{d^2}}{}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^4 + 2*a*b*c^2*cos(d*x + c)/d^2
 - 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^3/d^4 - 4*((d*x + c)*co
s(d*x + c) - sin(d*x + c))*a*b*c/d^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) -
 2*(d*x + c)*sin(d*x + c))*b^2*c^2/d^4 + 2*(((d*x + c)^2 - 2)*cos(d*x + c)
 - 2*(d*x + c)*sin(d*x + c))*a*b/d^2 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(
d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^4 - 1
2*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x +
 c))*b^2/d^4)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{(b^2d^4x^4+2\,abd^4x^2+a^2d^4-12\,b^2d^2x^2-4\,abd^2+24\,b^2)\cos(dx+c)}{d^5} + \frac{4\,(b^2d^3x^3+abd^3x-6\,b^2dx)\sin(dx+c)}{d^5}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + a^2*d^4 - 12*b^2*d^2*x^2 - 4*a*b*d^2 + 24*
b^2)*cos(d*x + c)/d^5 + 4*(b^2*d^3*x^3 + a*b*d^3*x - 6*b^2*d*x)*sin(d*x +
c)/d^5
```

Mupad [B] (verification not implemented)

Time = 43.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx = \frac{4\,b^2\,x^3\,\sin\left(c+d\,x\right)}{d^2} - \frac{b^2\,x^4\,\cos\left(c+d\,x\right)}{d} - \frac{\cos\left(c+d\,x\right)\,\left(a^2\,d^4-4\,a\,b\,d^2+24\,b^2\right)}{d^5} - \frac{4\,x\,\sin\left(c+d\,x\right)\,\left(6\,b^2-a\,b\,d^2\right)}{d^4} + \frac{2\,x^2\,\cos\left(c+d\,x\right)\,\left(6\,b^2-a\,b\,d^2\right)}{d^3}$$

input
```
int(sin(c + d*x)*(a + b*x^2)^2,x)
```

output
```
(4*b^2*x^3*sin(c + d*x))/d^2 - (b^2*x^4*cos(c + d*x))/d - (cos(c + d*x)*(2
4*b^2 + a^2*d^4 - 4*a*b*d^2))/d^5 - (4*x*sin(c + d*x)*(6*b^2 - a*b*d^2))/d
^4 + (2*x^2*cos(c + d*x)*(6*b^2 - a*b*d^2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx = \frac{-\cos\left(dx+c\right)a^2d^4 - 2\cos\left(dx+c\right)ab\,d^4x^2 + 4\cos\left(dx+c\right)ab\,d^2 - \cos\left(dx+c\right)b^2d^4x^4 + 12\cos\left(dx+c\right)}{d^5}$$

input
```
int((b*x^2+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**4 - 2*cos(c + d*x)*a*b*d**4*x**2 + 4*cos(c + d*x)
*a*b*d**2 - cos(c + d*x)*b**2*d**4*x**4 + 12*cos(c + d*x)*b**2*d**2*x**2 -
 24*cos(c + d*x)*b**2 + 4*sin(c + d*x)*a*b*d**3*x + 4*sin(c + d*x)*b**2*d*
*3*x**3 - 24*sin(c + d*x)*b**2*d*x)/d**5
```

3.52 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx = \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2 \operatorname{CosIntegral}(dx)\sin(c) - \frac{6b^2\sin(c+dx)}{d^4} + \frac{2ab\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2} + a^2\cos(c)\operatorname{Si}(dx)$$

output
```
6*b^2*x*cos(d*x+c)/d^3-2*a*b*x*cos(d*x+c)/d-b^2*x^3*cos(d*x+c)/d+a^2*Ci(d*
x)*sin(c)-6*b^2*sin(d*x+c)/d^4+2*a*b*sin(d*x+c)/d^2+3*b^2*x^2*sin(d*x+c)/d
^2+a^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx = -\frac{bx(2ad^2+b(-6+d^2x^2))\cos(c+dx)}{d^3} + a^2 \operatorname{CosIntegral}(dx)\sin(c) + \frac{b(2ad^2+3b(-2+d^2x^2))\sin(c+dx)}{d^4} + a^2\cos(c)\operatorname{Si}(dx)$$

input
```
Integrate[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

output
```
-((b*x*(2*a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x])/d^3) + a^2*CosIntegral[d
*x]*Sin[c] + (b*(2*a*d^2 + 3*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4 + a^2*Cos
[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a^2\sin(c+dx)}{x}+2abx\sin(c+dx)+b^2x^3\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2\sin(c)\operatorname{CosIntegral}(dx)+a^2\cos(c)\operatorname{Si}(dx)+\frac{2ab\sin(c+dx)}{d^2}-\frac{2abx\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

output
```
(6*b^2*x*Cos[c + d*x])/d^3 - (2*a*b*x*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d
*x])/d + a^2*CosIntegral[d*x]*Sin[c] - (6*b^2*Sin[c + d*x])/d^4 + (2*a*b*S
in[c + d*x])/d^2 + (3*b^2*x^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d
*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{b^2 x^3 \cos(dx+c)}{d}-\frac{i a^2 \mathrm{e}^{-ic} \operatorname{expIntegral}_1(idx)}{2}+\frac{i a^2 \mathrm{e}^{ic} \operatorname{expIntegral}_1(-idx)}{2}+\frac{3 b^2 x^2 \sin(dx+c)}{d^2}-\frac{2abx \cos(dx$
derivativedivides	$a^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))+\frac{4abc\cos(dx+c)}{d^2}+\frac{2(c+1)ab(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+$
default	$a^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))+\frac{4abc\cos(dx+c)}{d^2}+\frac{2(c+1)ab(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+$
meijerg	$\frac{8b^2\sin(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2d^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{xd\left(-\frac{x^2d^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\cos(c)\sqrt{\pi}\left(\frac{xd\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\right.}{d^4}$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-b^2*x^3*cos(d*x+c)/d-1/2*I*a^2*exp(-I*c)*Ei(1,I*d*x)+1/2*I*a^2*exp(I*c)*E
i(1,-I*d*x)+3*b^2*x^2*sin(d*x+c)/d^2-2*a*b*x*cos(d*x+c)/d+2*a*b*sin(d*x+c)
/d^2+6*b^2*x*cos(d*x+c)/d^3-6*b^2*sin(d*x+c)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x}\,dx$$
$$= \frac{a^2d^4\operatorname{Ci}(dx)\sin(c) + a^2d^4\cos(c)\operatorname{Si}(dx) - (b^2d^3x^3 + 2(abd^3 - 3b^2d)x)\cos(dx+c) + (3b^2d^2x^2 + 2abd^2}{d^4}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

output
```
(a^2*d^4*cos_integral(d*x)*sin(c) + a^2*d^4*cos(c)*sin_integral(d*x) - (b^
2*d^3*x^3 + 2*(a*b*d^3 - 3*b^2*d)*x)*cos(d*x + c) + (3*b^2*d^2*x^2 + 2*a*b
*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x}\,dx$$
$$= a^2\sin(c)\operatorname{Ci}(dx) + a^2\cos(c)\operatorname{Si}(dx) + 2abx\left(\begin{cases} x\sin(c) & \text{for}\ d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)$$
$$- 2ab\left(\begin{cases} \frac{x^2\sin(c)}{2} & \text{for}\ d=0 \\ -\frac{\begin{cases}\frac{\sin(c+dx)}{d} & \text{for}\ d\neq 0 \\ x\cos(c) & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right) + b^2x^3\left(\begin{cases} x\sin(c) & \text{for}\ d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)$$
$$- 3b^2\left(\begin{cases} \frac{x^4\sin(c)}{4} & \text{for}\ d=0 \\ -\frac{\begin{cases}\frac{x^2\sin(c+dx)}{d} + \frac{2x\cos(c+dx)}{d^2} - \frac{2\sin(c+dx)}{d^3} & \text{for}\ d\neq 0 \\ \frac{x^3\cos(c)}{3} & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*sin(d*x+c)/x,x)
```

output
```
a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x*Piecewise((x*sin(c), E
q(d, 0)), (-cos(c + d*x)/d, True)) - 2*a*b*Piecewise((x**2*sin(c)/2, Eq(d,
 0)), (-Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
+ b**2*x**3*Piecewise((x*sin(c), Eq(d, 0)), (-cos(c + d*x)/d, True)) - 3*b
**2*Piecewise((x**4*sin(c)/4, Eq(d, 0)), (-Piecewise((x**2*sin(c + d*x)/d
+ 2*x*cos(c + d*x)/d**2 - 2*sin(c + d*x)/d**3, Ne(d, 0)), (x**3*cos(c)/3,
True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx$$
$$= \frac{(a^2(-i \operatorname{Ei}(i\,dx) + i \operatorname{Ei}(-i\,dx)) \cos{(c)} + a^2(\operatorname{Ei}(i\,dx) + \operatorname{Ei}(-i\,dx)) \sin{(c)})d^4 - 2\,(b^2d^3x^3 + 2\,(abd^3 - 3\,b^2d)}{2\,d^4}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

output
```
1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*
x))*sin(c))*d^4 - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 - 3*b^2*d)*x)*cos(d*x + c) +
 2*(3*b^2*d^2*x^2 + 2*a*b*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 725, normalized size of antiderivative = 6.53

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(2*b^2*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^4*imag_part
(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_par
t(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^4*sin_
integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*b^2*d^3*x^3*tan(1/2*
d*x + 1/2*c)^2 + 2*a^2*d^4*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c) + 2*a^2*d^4*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/
2*c)^2*tan(1/2*c) - 2*b^2*d^3*x^3*tan(1/2*c)^2 + 4*a*b*d^3*x*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(d*x))*tan(1/2*d*x
+ 1/2*c)^2 - a^2*d^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2
+ 2*a^2*d^4*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2 - a^2*d^4*imag_part(c
os_integral(d*x))*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 - 2*a^2*d^4*sin_integral(d*x)*tan(1/2*c)^2 + 12*b^2*d^2*x^2*tan(
1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 2*b^2*d^3*x^3 + 4*a*b*d^3*x*tan(1/2*d*x +
1/2*c)^2 + 2*a^2*d^4*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^2*d^4*r
eal_part(cos_integral(-d*x))*tan(1/2*c) - 4*a*b*d^3*x*tan(1/2*c)^2 - 12*b^
2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral
(d*x)) - a^2*d^4*imag_part(cos_integral(-d*x)) + 2*a^2*d^4*sin_integral(d*
x) + 12*b^2*d^2*x^2*tan(1/2*d*x + 1/2*c) + 8*a*b*d^2*tan(1/2*d*x + 1/2*c)*
tan(1/2*c)^2 - 4*a*b*d^3*x - 12*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*d*
x*tan(1/2*c)^2 + 8*a*b*d^2*tan(1/2*d*x + 1/2*c) - 24*b^2*tan(1/2*d*x + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)^2}{x}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2)^2)/x,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x}\,dx$$

$$= \frac{-2\cos(dx+c)\,ab\,d^3x - \cos(dx+c)\,b^2d^3x^3 + 6\cos(dx+c)\,b^2dx + \left(\int \frac{\sin(dx+c)}{x}dx\right)a^2d^4 + 2\sin(dx+c)}{d^4}$$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x,x)
```

output
```
( - 2*cos(c + d*x)*a*b*d**3*x - cos(c + d*x)*b**2*d**3*x**3 + 6*cos(c + d*
x)*b**2*d*x + int(sin(c + d*x)/x,x)*a**2*d**4 + 2*sin(c + d*x)*a*b*d**2 +
3*sin(c + d*x)*b**2*d**2*x**2 - 6*sin(c + d*x)*b**2)/d**4
```

3.53 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} dx = \frac{2b^2\cos(c+dx)}{d^3} - \frac{2ab\cos(c+dx)}{d} - \frac{b^2x^2\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) - \frac{a^2\sin(c+dx)}{x} + \frac{2b^2x\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)$$

output
```
2*b^2*cos(d*x+c)/d^3-2*a*b*cos(d*x+c)/d-b^2*x^2*cos(d*x+c)/d+a^2*d*cos(c)*
Ci(d*x)-a^2*sin(d*x+c)/x+2*b^2*x*sin(d*x+c)/d^2-a^2*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} dx = \frac{2b^2\cos(c+dx)}{d^3} - \frac{2ab\cos(c+dx)}{d} - \frac{b^2x^2\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) - \frac{a^2\sin(c+dx)}{x} + \frac{2b^2x\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)$$

input
```
Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

output
```
(2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])
/d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c
 + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(\frac{a^2 \sin(c+dx)}{x^2} + 2ab\sin(c+dx) + b^2x^2\sin(c+dx)\right) dx$$
$$\Big\downarrow \text{2009}$$
$$a^2 d\cos(c)\,\text{CosIntegral}(dx) - a^2 d\sin(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{x} - \frac{2ab\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d^3} + \frac{2b^2x\sin(c+dx)}{d^2} - \frac{b^2x^2\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

output
```
(2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])
/d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c
 + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

method	result
derivativedivides	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)-\frac{2ab\cos(dx+c)}{d^2}-\frac{6b^2c^2\cos(dx+c)}{d^4}-\frac{4c}{}\right.$
default	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)-\frac{2ab\cos(dx+c)}{d^2}-\frac{6b^2c^2\cos(dx+c)}{d^4}-\frac{4c}{}\right.$
risch	$-\frac{-\pi\ \mathrm{csgn}(dx)\sin(c)a^2d^4x+2\ \mathrm{Si}(dx)\sin(c)a^2d^4x-i\pi\ \mathrm{csgn}(dx)\cos(c)a^2d^4x+2i\,\mathrm{Si}(dx)\cos(c)a^2d^4x+2\ \mathrm{expIntegral}_1(}{2d}$
meijerg	$\frac{4b^2\sin(c)\sqrt{\pi}\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3x^2d^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b^2\cos(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2d^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin}{2\sqrt{}}\right.}{d^3}$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-2/d^2*a*b*cos(d*x+c
)-6/d^4*b^2*c^2*cos(d*x+c)-4*c*b^2*(2*c+1)/d^4*(sin(d*x+c)-cos(d*x+c)*(d*x
+c))+(3*c^2+2*c+1)/d^4*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*s
in(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$$
$$= \frac{a^2d^4x\cos(c)\,\mathrm{Ci}(dx) - a^2d^4x\sin(c)\,\mathrm{Si}(dx) - (b^2d^2x^3 + 2\,(abd^2 - b^2)x)\cos(dx+c) - (a^2d^3 - 2\,b^2dx^2)\,\mathrm{s}}{d^3x}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
(a^2*d^4*x*cos(c)*cos_integral(d*x) - a^2*d^4*x*sin(c)*sin_integral(d*x) -
 (b^2*d^2*x^3 + 2*(a*b*d^2 - b^2)*x)*cos(d*x + c) - (a^2*d^3 - 2*b^2*d*x^2
)*sin(d*x + c))/(d^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx = \int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$$

input
```
integrate((b*x**2+a)**2*sin(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**2)**2*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$$
$$= \frac{(a^2(\Gamma(-1, i\,dx) + \Gamma(-1, -i\,dx))\cos(c) + a^2(-i\,\Gamma(-1, i\,dx) + i\,\Gamma(-1, -i\,dx))\sin(c))d^4 + 4\,b^2dx\sin(dx}{2\,d^3}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1
, I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^4 + 4*b^2*d*x*sin(d*x + c) - 2*(
b^2*d^2*x^2 + 2*a*b*d^2 - 2*b^2)*cos(d*x + c))/d^3
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1638, normalized size of antiderivative = 16.89

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4*x*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x*i
mag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2
*c) + 4*a^2*d^4*x*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*
tan(1/2*c) - 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x)^2 - a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x)^2 + a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x
)^2*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 2*a^2
*d^4*x*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*
a^2*d^4*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
+ 4*a^2*d^4*x*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*
d^4*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^4*x*sin
_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - 2*b^2*d^2*x^3*tan(1/2*d*x + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^2}\,dx = \int \frac{\sin(c+d\,x)\,(b\,x^2+a)^2}{x^2}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^2,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^2}\,dx = \frac{-2\cos(dx+c)\,ab\,d^2-\cos(dx+c)\,b^2d^2x^2+2\cos(dx+c)\,b^2+\left(\int\frac{\sin(dx+c)}{x^2}dx\right)a^2d^3+2\sin(dx+c)\,b^2d}{d^3}$$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^2,x)
```

output
```
( - 2*cos(c + d*x)*a*b*d**2 - cos(c + d*x)*b**2*d**2*x**2 + 2*cos(c + d*x)
*b**2 + int(sin(c + d*x)/x**2,x)*a**2*d**3 + 2*sin(c + d*x)*b**2*d*x)/d**3
```

3.54 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} \, dx = -\frac{a^2 d\cos(c+dx)}{2x} - \frac{b^2 x\cos(c+dx)}{d} + 2ab \,\text{CosIntegral}(dx)\sin(c) - \frac{1}{2}a^2 d^2 \,\text{CosIntegral}(dx)\sin(c) + \frac{b^2 \sin(c+dx)}{d^2} - \frac{a^2 \sin(c+dx)}{2x^2} + 2ab\cos(c)\text{Si}(dx) - \frac{1}{2}a^2 d^2 \cos(c)\text{Si}(dx)$$

output
```
-1/2*a^2*d*cos(d*x+c)/x-b^2*x*cos(d*x+c)/d+2*a*b*Ci(d*x)*sin(c)-1/2*a^2*d^
2*Ci(d*x)*sin(c)+b^2*sin(d*x+c)/d^2-1/2*a^2*sin(d*x+c)/x^2+2*a*b*cos(c)*Si
(d*x)-1/2*a^2*d^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3}\,dx = \frac{1}{2}\left(-\frac{a^2 d\cos(c+dx)}{x} - \frac{2b^2 x\cos(c+dx)}{d} + a(4b-ad^2)\,\text{CosIntegral}(dx)\sin(c) + \frac{2b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x^2} + a(4b-ad^2)\cos(c)\text{Si}(dx)\right)$$

input
```
Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

output
```
(-((a^2*d*Cos[c + d*x])/x) - (2*b^2*x*Cos[c + d*x])/d + a*(4*b - a*d^2)*Co
sIntegral[d*x]*Sin[c] + (2*b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x^2
+ a*(4*b - a*d^2)*Cos[c]*SinIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a^2\sin(c+dx)}{x^3} + \frac{2ab\sin(c+dx)}{x} + b^2 x\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a^2d^2\sin(c)\,\text{CosIntegral}(dx) - \frac{1}{2}a^2d^2\cos(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{2x^2} - \frac{a^2 d\cos(c+dx)}{2x} + 2ab\sin(c)\,\text{CosIntegral}(dx) + 2ab\cos(c)\text{Si}(dx) + \frac{b^2\sin(c+dx)}{d^2} - \frac{b^2 x\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

output
```
-1/2*(a^2*d*Cos[c + d*x])/x - (b^2*x*Cos[c + d*x])/d + 2*a*b*CosIntegral[d
*x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (b^2*Sin[c + d*x])/d^2
- (a^2*Sin[c + d*x])/(2*x^2) + 2*a*b*Cos[c]*SinIntegral[d*x] - (a^2*d^2*Co
s[c]*SinIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

method	result
derivativedivides	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ab(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}+\right.$
default	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ab(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}+\right.$
risch	$-\frac{-\cos(c)\pi\ \mathrm{csgn}(dx)a^2d^4x^2+2\cos(c)\,\mathrm{Si}(dx)a^2d^4x^2+i\sin(c)\pi\ \mathrm{csgn}(dx)a^2d^4x^2+4\cos(c)\pi\ \mathrm{csgn}(dx)ab\,d^2x^2-2i\sin(c)}{}$
meijerg	$\frac{2b^2\sin(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\cos(c)\sqrt{\pi}\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+ab\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln}{}\right.$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/
2*Ci(d*x)*sin(c))+2/d^2*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+4/d^4*b^2*c*co
s(d*x+c)+(3*c+1)/d^4*b^2*(sin(d*x+c)-cos(d*x+c)*(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx = -\frac{(a^2d^4 - 4\,abd^2)x^2 \operatorname{Ci}(dx) \sin(c) + (a^2d^4 - 4\,abd^2)x^2 \cos(c) \operatorname{Si}(dx) + (a^2d^3x + 2\,b^2dx^3) \cos(dx+c) +}{2\,d^2x^2}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2*d^4 - 4*a*b*d^2)*x^2*cos_integral(d*x)*sin(c) + (a^2*d^4 - 4*a*
b*d^2)*x^2*cos(c)*sin_integral(d*x) + (a^2*d^3*x + 2*b^2*d*x^3)*cos(d*x +
c) + (a^2*d^2 - 2*b^2*x^2)*sin(d*x + c))/(d^2*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx = \int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx$$

input
```
integrate((b*x**2+a)**2*sin(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**2)**2*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx = \frac{((a^2(i\,\Gamma(-2,i\,dx) - i\,\Gamma(-2,-i\,dx)) \cos(c) + a^2(\Gamma(-2,i\,dx) + \Gamma(-2,-i\,dx)) \sin(c))d^4 - 4\,(ab(i\,\Gamma(-2,i}{}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/2*(((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(
-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 - 4*(a*b*(I*gamma(-2, I*d*x) -
 I*gamma(-2, -I*d*x))*cos(c) + a*b*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*
sin(c))*d^2)*x^2 - 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(d*x + c) + 2*(b^2*x^2 - 2
*a*b)*sin(d*x + c))/(d^2*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1058, normalized size of antiderivative = 9.28

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a^2*d^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
- a^2*d^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^
2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^2*r
eal_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^2*imag_
part(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 +
 a^2*d^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^2*imag_
part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^2*sin_integral(d*x)*ta
n(1/2*c)^2 - 4*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 8*a*b*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*a^2*d^4*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^2*real
_part(cos_integral(-d*x))*tan(1/2*c) + 8*a*b*d^2*x^2*real_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d^2*x^2*real_part(cos_integral(-
d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2
- 4*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^2*imag_part(cos_inte
gral(d*x)) + a^2*d^4*x^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^4*x^2*sin
_integral(d*x) + 4*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2
 - 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 8*a*b*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^3}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)^2}{x^3}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^3,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a + bx^2)^2 \sin(c + dx)}{x^3}\, dx$$
$$= \frac{-4\cos(dx+c)\,abdx - 2\cos(dx+c)\,b^2 d\,x^3 - 2\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^2 + x^2} dx\right) a^2 d^3 x^2 + 8\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^2 + x^2} dx\right.}{2d^2x^2}$$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^3,x)
```

output
```
( - 4*cos(c + d*x)*a*b*d*x - 2*cos(c + d*x)*b**2*d*x**3 - 2*int(tan((c + d
*x)/2)**2/(tan((c + d*x)/2)**2*x**2 + x**2),x)*a**2*d**3*x**2 + 8*int(tan(
(c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**2 + x**2),x)*a*b*d*x**2 - sin(c +
d*x)*a**2*d**2 + 2*sin(c + d*x)*b**2*x**2 - a**2*d**3*x + 4*a*b*d*x)/(2*d*
*2*x**2)
```

3.55 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\begin{aligned}\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} dx = &-\frac{b^2\cos(c+dx)}{d} - \frac{a^2 d\cos(c+dx)}{6x^2} \\ &+ 2abd\cos(c)\,\text{CosIntegral}(dx) \\ &- \frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx) - \frac{a^2\sin(c+dx)}{3x^3} \\ &- \frac{2ab\sin(c+dx)}{x} + \frac{a^2d^2\sin(c+dx)}{6x} \\ &- 2abd\sin(c)\text{Si}(dx) + \frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)\end{aligned}$$

output
```
-b^2*cos(d*x+c)/d-1/6*a^2*d*cos(d*x+c)/x^2+2*a*b*d*cos(c)*Ci(d*x)-1/6*a^2*
d^3*cos(c)*Ci(d*x)-1/3*a^2*sin(d*x+c)/x^3-2*a*b*sin(d*x+c)/x+1/6*a^2*d^2*s
in(d*x+c)/x-2*a*b*d*sin(c)*Si(d*x)+1/6*a^2*d^3*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^4}\,dx = \frac{1}{6}\left(-\frac{6b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{x^2}-ad\left(-12b+ad^2\right)\cos(c)\operatorname{CosIntegral}(dx)-\frac{2a^2\sin(c+dx)}{x^3}-\frac{12ab\sin(c+dx)}{x}+\frac{a^2d^2\sin(c+dx)}{x}+ad\left(-12b+ad^2\right)\sin(c)\operatorname{Si}(dx)\right)$$

input
```
Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

output
```
((-6*b^2*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x^2 - a*d*(-12*b + a*d^2)*
Cos[c]*CosIntegral[d*x] - (2*a^2*Sin[c + d*x])/x^3 - (12*a*b*Sin[c + d*x])
/x + (a^2*d^2*Sin[c + d*x])/x + a*d*(-12*b + a*d^2)*Sin[c]*SinIntegral[d*x
])/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \text{ 3820}$$

$$\int \left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^2}+b^2\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\,\text{CosIntegral}(dx)-2abd\sin(c)\text{Si}(dx)-\frac{2ab\sin(c+dx)}{x}-\frac{b^2\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

output
```
-((b^2*Cos[c + d*x])/d) - (a^2*d*Cos[c + d*x])/(6*x^2) + 2*a*b*d*Cos[c]*Co
sIntegral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - (a^2*Sin[c + d*x])/
(3*x^3) - (2*a*b*Sin[c + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - 2*a*b*d*
Sin[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

method	result
derivativedivides	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{2ba\left(-\frac{\sin(dx+c)}{dx}-\text{Si}(d\right.}{}\right.$
default	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{2ba\left(-\frac{\sin(dx+c)}{dx}-\text{Si}(d\right.}{}\right.$
risch	$\frac{d^3\cos(c)a^2\,\text{expIntegral}_1(-idx)}{12}+\frac{d^3\cos(c)a^2\,\text{expIntegral}_1(idx)}{12}-d\cos(c)\,\text{expIntegral}_1(-idx)\,ab-$
meijerg	$\frac{b^2\sin(c)\sin(dx)}{d}+\frac{b^2\cos(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{d^2ab\sin(c)\sqrt{\pi}\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\text{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{2\sqrt{d^2}}+\frac{dab\cos(c)\sqrt{}}{}$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/
x+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d^2*b*a*(-sin(d*x+c)/d/x-Si(d*x
)*sin(c)+Ci(d*x)*cos(c))-1/d^4*b^2*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^4}\,dx = -\frac{(a^2d^4-12\,abd^2)x^3\cos{(c)}\operatorname{Ci}{(dx)}-(a^2d^4-12\,abd^2)x^3\sin{(c)}\operatorname{Si}{(dx)}+(a^2d^2x+6\,b^2x^3)\cos{(dx+c)}-}{6\,dx^3}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/6*((a^2*d^4 - 12*a*b*d^2)*x^3*cos(c)*cos_integral(d*x) - (a^2*d^4 - 12*
a*b*d^2)*x^3*sin(c)*sin_integral(d*x) + (a^2*d^2*x + 6*b^2*x^3)*cos(d*x +
c) + (2*a^2*d - (a^2*d^3 - 12*a*b*d)*x^2)*sin(d*x + c))/(d*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)^2\sin{(c+dx)}}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**2*sin(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**2)**2*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} dx =$$
$$-\frac{((a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c))d^5-12\,(ab(\Gamma(-$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(
-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 12*(a*b*(gamma(-3, I*d*x)
+ gamma(-3, -I*d*x))*cos(c) + a*b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*
x))*sin(c))*d^3)*x^3 + 8*a*b*sin(d*x + c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(
d*x + c))/(d^2*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1032, normalized size of antiderivative = 7.70

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

output
```
1/12*(a^2*d^4*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + a^2*d^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*a^2*d^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*
a^2*d^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^
2*d^4*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^3*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^4*x^3*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^3*real_part(cos_integral(d*x))*tan(1/2*
c)^2 + a^2*d^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 12*a*b*d^2
*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b*d^2
*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4
*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^3*imag_part(cos
_integral(-d*x))*tan(1/2*c) + 4*a^2*d^4*x^3*sin_integral(d*x)*tan(1/2*c) -
 24*a*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2
4*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 48
*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^3*rea
l_part(cos_integral(d*x)) - a^2*d^4*x^3*real_part(cos_integral(-d*x)) + 12
*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 12*a*b*d^2*x^3*
real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a^2*d^3*x^2*tan(1/2*d*x)^
2*tan(1/2*c) - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 -
12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a^2*d^3*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^4}\,dx=\int \frac{\sin\left(c+d\,x\right)\,{\left(b\,x^2+a\right)}^2}{x^4}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^4,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4}\,dx$$

$$= \frac{-12\cos(dx+c)\,abx - 6\cos(dx+c)\,b^2x^3 + 4\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^3+x^3}dx\right)a^2d^2x^3 - 48\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^3+x^3}dx\right.}{6d\,x^3}$$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^4,x)
```

output
```
( - 12*cos(c + d*x)*a*b*x - 6*cos(c + d*x)*b**2*x**3 + 4*int(1/(tan((c + d
*x)/2)**2*x**3 + x**3),x)*a**2*d**2*x**3 - 48*int(1/(tan((c + d*x)/2)**2*x
**3 + x**3),x)*a*b*x**3 - 2*sin(c + d*x)*a**2*d + a**2*d**2*x - 12*a*b*x)/
(6*d*x**3)
```

3.56 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\begin{aligned}\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} dx = &-\frac{a^2 d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + \frac{a^2d^3\cos(c+dx)}{24x} \\ &+ b^2 \operatorname{CosIntegral}(dx)\sin(c) - abd^2 \operatorname{CosIntegral}(dx)\sin(c) \\ &+ \frac{1}{24}a^2d^4 \operatorname{CosIntegral}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{4x^4} \\ &- \frac{ab\sin(c+dx)}{x^2} + \frac{a^2d^2\sin(c+dx)}{24x^2} + b^2\cos(c)\operatorname{Si}(dx) \\ &- abd^2\cos(c)\operatorname{Si}(dx) + \frac{1}{24}a^2d^4\cos(c)\operatorname{Si}(dx)\end{aligned}$$

output

```
-1/12*a^2*d*cos(d*x+c)/x^3-a*b*d*cos(d*x+c)/x+1/24*a^2*d^3*cos(d*x+c)/x+b^
2*Ci(d*x)*sin(c)-a*b*d^2*Ci(d*x)*sin(c)+1/24*a^2*d^4*Ci(d*x)*sin(c)-1/4*a^
2*sin(d*x+c)/x^4-a*b*sin(d*x+c)/x^2+1/24*a^2*d^2*sin(d*x+c)/x^2+b^2*cos(c)
*Si(d*x)-a*b*d^2*cos(c)*Si(d*x)+1/24*a^2*d^4*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+bx^2\right)^2 \sin(c+dx)}{x^5}\,dx$$
$$= \frac{adx(-24bx^2+a(-2+d^2x^2))\cos(c+dx)+(24b^2-24abd^2+a^2d^4)\,x^4\,\text{CosIntegral}(dx)\sin(c)+a(-24b}{24x^4}$$

input
```
Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

output
```
(a*d*x*(-24*b*x^2 + a*(-2 + d^2*x^2))*Cos[c + d*x] + (24*b^2 - 24*a*b*d^2
+ a^2*d^4)*x^4*CosIntegral[d*x]*Sin[c] + a*(-24*b*x^2 + a*(-6 + d^2*x^2))*
Sin[c + d*x] + (24*b^2 - 24*a*b*d^2 + a^2*d^4)*x^4*Cos[c]*SinIntegral[d*x]
)/(24*x^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2 \sin(c+dx)}{x^5}\,dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(\frac{a^2 \sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{24}a^2d^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^3\cos(c+dx)}{24x}+\frac{a^2d^2\sin(c+dx)}{24x^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos(c+dx)}{12x^3}-abd^2\sin(c)\,\text{CosIntegral}(dx)-abd^2\cos(c)\text{Si}(dx)-\frac{ab\sin(c+dx)}{x^2}-\frac{abd\cos(c+dx)}{x}+b^2\sin(c)\,\text{CosIntegral}(dx)+b^2\cos(c)\text{Si}(dx)$$

input
```
Int[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

output
```
-1/12*(a^2*d*Cos[c + d*x])/x^3 - (a*b*d*Cos[c + d*x])/x + (a^2*d^3*Cos[c +
 d*x])/(24*x) + b^2*CosIntegral[d*x]*Sin[c] - a*b*d^2*CosIntegral[d*x]*Sin
[c] + (a^2*d^4*CosIntegral[d*x]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) -
(a*b*Sin[c + d*x])/x^2 + (a^2*d^2*Sin[c + d*x])/(24*x^2) + b^2*Cos[c]*SinI
ntegral[d*x] - a*b*d^2*Cos[c]*SinIntegral[d*x] + (a^2*d^4*Cos[c]*SinIntegr
al[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.89

method	result
derivativedivides	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{2ab\left(-\right.}{}\right.$
default	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{2ab\left(-\right.}{}\right.$
risch	$-\frac{i\cos(c)\,\mathrm{expIntegral}_1(idx)a^2d^4}{48}+\frac{i\cos(c)\,\mathrm{expIntegral}_1(-idx)a^2d^4}{48}+\frac{i\cos(c)\,\mathrm{expIntegral}_1(idx)ab\,d^2}{2}-\frac{i\cos(c)}{}$
meijerg	$\frac{b^2\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b^2\cos(c)\,\mathrm{Si}(dx)+\frac{d^2ab\sin(c)\sqrt{\pi}\left(-\right.}{}$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/
d^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2/d^2
*a*b*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci
(d*x)*sin(c))+1/d^4*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx$$
$$= \frac{(a^2d^4 - 24\,abd^2 + 24\,b^2)x^4\,\mathrm{Ci}\,(dx)\sin{(c)} + (a^2d^4 - 24\,abd^2 + 24\,b^2)x^4\cos{(c)}\,\mathrm{Si}\,(dx) - (2\,a^2dx - (a^2d^3 -}{24\,x^4}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
1/24*((a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos_integral(d*x)*sin(c) + (a^2*
d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos(c)*sin_integral(d*x) - (2*a^2*d*x - (a^
2*d^3 - 24*a*b*d)*x^3)*cos(d*x + c) + ((a^2*d^2 - 24*a*b)*x^2 - 6*a^2)*sin
(d*x + c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)^2\sin{(c+dx)}}{x^5}\,dx$$

input
```
integrate((b*x**2+a)**2*sin(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x**2)**2*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx =$$
$$-\frac{((a^2(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos{(c)}+a^2(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin{(c)})d^8-24\,(ab(i\,\Gamma(-$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
-1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma
(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^8 - 24*(a*b*(I*gamma(-4, I*d*x)
 - I*gamma(-4, -I*d*x))*cos(c) + a*b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x)
)*sin(c))*d^6 - 24*(b^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c)
 - b^2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 2*(b^2*d^
3*x^3 + 2*(a*b*d^3 - b^2*d)*x)*cos(d*x + c) + 2*(b^2*d^2*x^2 + 6*a*b*d^2 -
 6*b^2)*sin(d*x + c))/(d^4*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1497, normalized size of antiderivative = 8.46

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

output
```
-1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*
x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4
*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^4*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2
 + a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*ima
g_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*
tan(1/2*c)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*a^2*d^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^4
*real_part(cos_integral(-d*x))*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_int
egral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^2*d^4*x^4*imag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_par
t(cos_integral(-d*x)) - 2*a^2*d^4*x^4*sin_integral(d*x) + 24*a*b*d^2*x^4*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 24*a*b*d^2*x^4*imag_part(cos_
integral(-d*x))*tan(1/2*d*x)^2 + 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)^2}{x^5}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^5,x)
```

output
```
int((sin(c + d*x)*(a + b*x^2)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx$$

$$= \frac{-24\cos(dx+c)\,ab\,d^2x - 12\cos(dx+c)\,b^2d^2x^3 + 24\cos(dx+c)\,b^2x - 6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)a^2d^4x^4}{}$$

input
```
int((b*x^2+a)^2*sin(d*x+c)/x^5,x)
```

output
```
( - 24*cos(c + d*x)*a*b*d**2*x - 12*cos(c + d*x)*b**2*d**2*x**3 + 24*cos(c
 + d*x)*b**2*x - 6*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**4 + x**
4),x)*a**2*d**4*x**4 + 144*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x*
*4 + x**4),x)*a*b*d**2*x**4 - 144*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2
)**2*x**4 + x**4),x)*b**2*x**4 - 3*sin(c + d*x)*a**2*d**3 - 12*sin(c + d*x
)*b**2*d*x**2 - a**2*d**4*x + 24*a*b*d**2*x - 24*b**2*x)/(12*d**3*x**4)
```

3.57 $\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 273

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx = {} & \frac{2\cos(c+dx)}{bd^3} + \frac{a\cos(c+dx)}{b^2 d} - \frac{x^2\cos(c+dx)}{bd} \\ & - \frac{(-a)^{3/2}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} \\ & + \frac{(-a)^{3/2}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} \\ & + \frac{2x\sin(c+dx)}{bd^2} - \frac{(-a)^{3/2}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} \\ & - \frac{(-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}\end{aligned}$$

output

```
2*cos(d*x+c)/b/d^3+a*cos(d*x+c)/b^2/d-x^2*cos(d*x+c)/b/d-1/2*(-a)^(3/2)*Ci
((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/b^(5/2)+1/2*(-a)^(3
/2)*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/b^(5/2)+2*x*s
in(d*x+c)/b/d^2+1/2*(-a)^(3/2)*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*
d/b^(1/2)+d*x)/b^(5/2)-1/2*(-a)^(3/2)*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^
(1/2)*d/b^(1/2)+d*x)/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.99

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx$$
$$= \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(a^{3/2}d^3e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-a^{3/2}d^3\,\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+e^{ic}\left(a^{3/2}d^3e\right.\right.}{}$$

input
```
Integrate[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
(E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(a^(3/2)*d^3*E^((2*Sqrt[a]*d)/Sqrt[b])*E
xpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - a^(3/2)*d^3*ExpIntegralEi[(
Sqrt[a]*d)/Sqrt[b] - I*d*x] + E^(I*c)*(a^(3/2)*d^3*E^((2*Sqrt[a]*d)/Sqrt[b
])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*(Cos[c] + I*Sin[c]) - a^(
3/2)*d^3*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*(Cos[c] + I*Sin[c]) -
4*Sqrt[b]*E^((Sqrt[a]*d)/Sqrt[b])*((-2*b - a*d^2 + b*d^2*x^2)*Cos[c + d*x]
 - 2*b*d*x*Sin[c + d*x]))))/(4*b^(5/2)*d^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx$$
$$\Big\downarrow \; 3826$$
$$\int \left(\frac{a^2\sin(c+dx)}{b^2(a+bx^2)}-\frac{a\sin(c+dx)}{b^2}+\frac{x^2\sin(c+dx)}{b}\right)dx$$
$$\Big\downarrow \; 2009$$

$$
-\frac{(-a)^{3/2}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{(-a)^{3/2}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{a\cos(c+dx)}{b^2d}+\frac{2\cos(c+dx)}{bd^3}+\frac{2x\sin(c+dx)}{bd^2}-\frac{x^2\cos(c+dx)}{bd}
$$

input
```
Int[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
(2*Cos[c + d*x])/(b*d^3) + (a*Cos[c + d*x])/(b^2*d) - (x^2*Cos[c + d*x])/(
b*d) - ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]])/(2*b^(5/2)) + ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(5/2)) + (2*x*Sin[c + d*x])/(b
*d^2) - ((-a)^(3/2)*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*S
inIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\sqrt{ab}\,\text{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}a}{4b^3}-\frac{\sqrt{ab}\,\text{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}a}{4b^3}+\frac{\sqrt{a}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^4*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^3*(a*b)^(1/2)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*b*c
-d*(a*b)^(1/2))/b)*a-1/4/b^3*(a*b)^(1/2)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*
x+I*c))/b)*exp((I*b*c+d*(a*b)^(1/2))/b)*a+1/4/b^3*(a*b)^(1/2)*exp(-(I*b*c+
d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*a-1/4/b^3*(
a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d
*x+I*c))/b)*a-(b*d^2*x^2-a*d^2-2*b)/b^2/d^3*cos(d*x+c)-2/d^3/b*(d^2*x^2+3*
c*d*x)/(-d*x-3*c)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.88

$$\int \frac{x^4\sin(c+dx)}{a+bx^2}\,dx$$
$$= \frac{\sqrt{\frac{ad^2}{b}}ad^2\text{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}ad^2\text{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}ad^2\text{Ei}\left(-i\,dx-\right.}{}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a*d^2/b)*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b))
 - sqrt(a*d^2/b)*a*d^2*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) +
 sqrt(a*d^2/b)*a*d^2*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) -
 sqrt(a*d^2/b)*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) +
 8*b*d*x*sin(d*x + c) - 4*(b*d^2*x^2 - a*d^2 - 2*b)*cos(d*x + c))/(b^2*d^3
)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \sin{(c+dx)}}{a+bx^2}\,dx$$

input
```
integrate(x**4*sin(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x**4*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \sin{(dx+c)}}{bx^2+a}\,dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(((b*d^2*x^4*cos(c) + 2*b*d*x^3*sin(c) - 2*b*x^2*cos(c) + 2*a*d*x*sin
(c))*cos(d*x + c)^2 + (b*d^2*x^4*cos(c) + 2*b*d*x^3*sin(c) - 2*b*x^2*cos(c
) + 2*a*d*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((b*cos(c)^2 + b*sin(
c)^2)*d^2*x^4 - 2*(b*cos(c)^2 + b*sin(c)^2)*x^2)*cos(d*x + c) + 2*(((b^2*c
os(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^3)*cos(d
*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*s
in(c)^2)*d^3)*sin(d*x + c)^2)*integrate(((a^2*d^2 + 2*a*b)*x*cos(d*x + c)
+ (a*b*d*x^2 + a^2*d)*sin(d*x + c))/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b
*d^3), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*
b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 +
 (a*b*cos(c)^2 + a*b*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(((a^2*d^2 +
2*a*b)*x*cos(d*x + c) + (a*b*d*x^2 + a^2*d)*sin(d*x + c))/((b^3*d^3*x^4 +
2*a*b^2*d^3*x^2 + a^2*b*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^4 + 2*a*b^2*d^3*x
^2 + a^2*b*d^3)*sin(d*x + c)^2), x) + ((b*d^2*x^4*sin(c) - 2*b*d*x^3*cos(c
) - 2*a*d*x*cos(c) - 2*b*x^2*sin(c))*cos(d*x + c)^2 + (b*d^2*x^4*sin(c) -
2*b*d*x^3*cos(c) - 2*a*d*x*cos(c) - 2*b*x^2*sin(c))*sin(d*x + c)^2)*sin(d*
x + 2*c) - 2*((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*
d*x)*sin(d*x + c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2
 + a*b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*
x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^4 \sin{(dx+c)}}{bx^2+a} \, dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^4*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^4\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

input
```
int((x^4*sin(c + d*x))/(a + b*x^2),x)
```

output
```
int((x^4*sin(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx = \frac{\cos\left(dx+c\right)a\,d^2-\cos\left(dx+c\right)b\,d^2x^2+2\cos\left(dx+c\right)b+\left(\int \frac{\sin(dx+c)}{b\,x^2+a}dx\right)a^2d^3+2\sin\left(dx+c\right)bdx}{b^2d^3}$$

input
```
int(x^4*sin(d*x+c)/(b*x^2+a),x)
```

output
```
(cos(c + d*x)*a*d**2 - cos(c + d*x)*b*d**2*x**2 + 2*cos(c + d*x)*b + int(s
in(c + d*x)/(a + b*x**2),x)*a**2*d**3 + 2*sin(c + d*x)*b*d*x)/(b**2*d**3)
```

3.58 $\int \frac{x^3\sin(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\int \frac{x^3\sin(c+dx)}{a+bx^2}\,dx = -\frac{x\cos(c+dx)}{bd} - \frac{a\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} - \frac{a\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{bd^2} + \frac{a\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}$$

output
```
-x*cos(d*x+c)/b/d-1/2*a*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^
(1/2))/b^2-1/2*a*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/
b^2+sin(d*x+c)/b/d^2-1/2*a*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^
(1/2)+d*x)/b^2-1/2*a*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d
*x)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = \frac{-iae^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+iae^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{Exp}\cdots\right.}{\cdots}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
((-I)*a*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInt
egralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b
] - I*d*x]) + I*a*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])
*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)
/Sqrt[b] + I*d*x]) - (4*b*Cos[d*x]*(d*x*Cos[c] - Sin[c]))/d^2 + (4*b*(Cos[
c] + d*x*Sin[c])*Sin[d*x])/d^2)/(4*b^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{3826, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(\frac{x\sin(c+dx)}{b}-\frac{ax\sin(c+dx)}{b\left(a+bx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{a\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}+\frac{\sin(c+dx)}{bd^2}-\frac{x\cos(c+dx)}{bd}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
-((x*Cos[c + d*x])/(b*d)) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(b*d^2) + (a*
Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*
b^2) - (a*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{i\,\text{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)e^{\frac{ibc+d\sqrt{ab}}{b}}a}{4b^2}-\frac{i\,\text{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)e^{\frac{ibc-d\sqrt{ab}}{b}}a}{4b^2}+\frac{i\,\text{expInt}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^3*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/b^2*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*b*c+d*(a*b)^
(1/2))/b)*a-1/4*I/b^2*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*b
*c-d*(a*b)^(1/2))/b)*a+1/4*I/b^2*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))
/b)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*a+1/4*I/b^2*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b
*(I*d*x+I*c))/b)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*a-x*cos(d*x+c)/b/d+sin(d*x+
c)/b/d^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx = \frac{i\,ad^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,ad^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-i\,ad^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\right.}}{4\,b^2d^2}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(I*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*a*d^2*E
i(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x - sqr
t(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*
e^(-I*c - sqrt(a*d^2/b)) - 4*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(b^2*d
^2)
```

Sympy [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx$$

input
```
integrate(x**3*sin(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x**3*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^3 \sin(dx+c)}{bx^2+a} \, dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^3*cos(d*x + c) - (cos(c)^2 + sin(c)^2)*x^2
*sin(d*x + c) + ((d*x^3*cos(c) + x^2*sin(c))*cos(d*x + c)^2 + (d*x^3*cos(c
) + x^2*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c
)^2)*d^2*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^
2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*i
ntegrate(-(a*d*x^2*cos(d*x + c) - a*x*sin(d*x + c))/(b^2*d^2*x^4 + 2*a*b*d
^2*x^2 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c)^
2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 +
 (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(a*d*x^2*cos(d*
x + c) - a*x*sin(d*x + c))/((b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2)*cos(d*
x + c)^2 + (b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2)*sin(d*x + c)^2), x) + (
(d*x^3*sin(c) - x^2*cos(c))*cos(d*x + c)^2 + (d*x^3*sin(c) - x^2*cos(c))*s
in(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*co
s(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*
x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^3 \sin(dx+c)}{bx^2+a} \, dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^3*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^3 \sin(c+d\,x)}{b\,x^2+a} \, dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x^2),x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx = \frac{-\cos(dx+c)\,dx - \left(\int \frac{\sin(dx+c)x}{b\,x^2+a} dx\right) a\,d^2 + \sin(dx+c)}{b\,d^2}$$

input
```
int(x^3*sin(d*x+c)/(b*x^2+a),x)
```

output
```
( - cos(c + d*x)*d*x - int((sin(c + d*x)*x)/(a + b*x**2),x)*a*d**2 + sin(c
 + d*x))/(b*d**2)
```

3.59 $\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx = -\frac{\cos(c+dx)}{bd} - \frac{\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}$$

output
```
-cos(d*x+c)/b/d-1/2*(-a)^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/
2)*d/b^(1/2))/b^(3/2)+1/2*(-a)^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-
a)^(1/2)*d/b^(1/2))/b^(3/2)+1/2*(-a)^(1/2)*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(
-(-a)^(1/2)*d/b^(1/2)+d*x)/b^(3/2)-1/2*(-a)^(1/2)*cos(c-(-a)^(1/2)*d/b^(1/
2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} dx$$
$$= -\frac{\cos(c)\cos(dx)}{bd} + \frac{\sqrt{a}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{4b^{3/2}} + \frac{\sqrt{a}e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)\right)}{4b^{3/2}} + \frac{\sin(c)\sin(dx)}{bd}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
-((Cos[c]*Cos[d*x])/(b*d)) + (Sqrt[a]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-(
E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) +
 ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/(4*b^(3/2)) + (Sqrt[a]*E^(I*
c - (Sqrt[a]*d)/Sqrt[b])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqr
t[a]*d)/Sqrt[b]) + I*d*x]) + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/
(4*b^(3/2)) + (Sin[c]*Sin[d*x])/(b*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} dx$$

$$\downarrow \text{3826}$$

$$\int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b(a+bx^2)}\right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\cos(c+dx)}{bd}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
-(Cos[c + d*x]/(b*d)) - (Sqrt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*
Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) + (Sqrt[-a]*CosIntegral[(Sqrt[-
a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a
]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b^(3/2)) - (Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]
*d)/Sqrt[b] + d*x])/(2*b^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.15

method	result
risch	$\frac{\sqrt{ab}\,e^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4b^2}-\frac{e^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(-\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)\sqrt{ab}}{4b^2}-$
derivativedivides	$d^2c^2\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos\left(\right.}{}\right.$
default	$d^2c^2\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos\left(\right.}{}\right.$

input
```
int(x^2*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*(a*b)^(1/2)*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)
-b*(I*d*x+I*c))/b)-1/4/b^2*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*c+d*(a
*b)^(1/2)+b*(I*d*x+I*c))/b)*(a*b)^(1/2)-1/4/b^2*(a*b)^(1/2)*exp(-(I*b*c+d*
(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/4/b^2*(a*b)
^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I
*c))/b)-cos(d*x+c)/b/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int \frac{x^2\sin(c+dx)}{a+bx^2}\,dx =$$
$$-\frac{\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right.}{4\,bd}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sq
rt(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2
/b)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei
(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*cos(d*x + c))/(b*d)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} dx = \int \frac{x^2 \sin{(c+dx)}}{a+bx^2} dx$$

input
```
integrate(x**2*sin(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x**2*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} dx = \int \frac{x^2 \sin{(dx+c)}}{bx^2+a} dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x^2*cos(d*x + c) + (x^2*cos(d*x + c)^2*cos(c)
+ x^2*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b*cos(c)^2 + a*b*sin(
c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b*cos(
c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)
^2)*integrate(x*cos(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 2*(((
a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(
d*x + c)^2 + ((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*si
n(c)^2)*d)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/((b^2*d*x^4 + 2*a*b*d*
x^2 + a^2*d)*cos(d*x + c)^2 + (b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*sin(d*x +
c)^2), x) + (x^2*cos(d*x + c)^2*sin(c) + x^2*sin(d*x + c)^2*sin(c))*sin(d*
x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)
*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c
)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^2 \sin(dx+c)}{bx^2+a} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^2*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^2 \sin(c+d\,x)}{b\,x^2+a} \, dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x^2),x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx = \frac{-\cos(dx+c) - \left(\int \frac{\sin(dx+c)}{b\,x^2+a} dx\right) ad}{bd}$$

input
```
int(x^2*sin(d*x+c)/(b*x^2+a),x)
```

output
```
( - (cos(c + d*x) + int(sin(c + d*x)/(a + b*x**2),x)*a*d))/(b*d)
```

3.60 $\int \frac{x \sin(c+dx)}{a+bx^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 177

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}$$

output
```
1/2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/b+1/2*Ci((-a)
^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/b+1/2*cos(c+(-a)^(1/2)*d
/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/b+1/2*cos(c-(-a)^(1/2)*d/b^(1/2))*
Si((-a)^(1/2)*d/b^(1/2)+d*x)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-e^{2ic}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralE}\right.\right.}{4b}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
((I/4)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInte
gralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b]
 - I*d*x] - E^((2*I)*c)*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a
]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x^2),x]
```

output
```
(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2
*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b
]])/(2*b) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x])/(2*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] + d*x])/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.32

method	result
risch	$\frac{i\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}}{4b}+\frac{i\,\mathrm{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}}{4b}-\frac{i\mathrm{e}^{-\frac{ibc+d\sqrt{ab}}{b}}}{}$
derivativedivides	$\frac{-\frac{d^2(d\sqrt{-ab}+bc)\left(\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)\right)}{2b^2\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}+\frac{d^2(d\sqrt{-ab}-bc)\left(\mathrm{Si}\left(\right.\right.}{}}{}$
default	$\frac{-\frac{d^2(d\sqrt{-ab}+bc)\left(\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)\right)}{2b^2\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}+\frac{d^2(d\sqrt{-ab}-bc)\left(\mathrm{Si}\left(\right.\right.}{}}{}$

input
```
int(x*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/b*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*b*c+d*(a*b)^(1/
2))/b)+1/4*I/b*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*b*c-d*(a
*b)^(1/2))/b)-1/4*I/b*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(
1/2)-b*(I*d*x+I*c))/b)-1/4*I/b*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c-
d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{-i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}}{4\,b}$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(-I*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*Ei(I*d*x + s
qrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I
*c + sqrt(a*d^2/b)) + I*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)
))/b
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\sin(c+dx)}{a+bx^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\sin(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*
cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d*x^
2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^
2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(b*x
^2 - a)*cos(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + 2*(((b*cos(c)
^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b
*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^
2)*integrate(1/2*(b*x^2 - a)*cos(d*x + c)/((b^2*d*x^4 + 2*a*b*d*x^2 + a^2*
d)*cos(d*x + c)^2 + (b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*sin(d*x + c)^2), x)
+ (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b
*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^
2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*
x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\sin(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\,\sin(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x^2),x)
```

output
```
int((x*sin(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin(dx+c)\,x}{b\,x^2+a}dx$$

input
```
int(x*sin(d*x+c)/(b*x^2+a),x)
```

output
```
int((sin(c + d*x)*x)/(a + b*x**2),x)
```

3.61 $\int \frac{\sin(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = -\frac{\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} + \frac{\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}$$

output
```
-1/2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b
^(1/2)+1/2*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(
1/2)/b^(1/2)+1/2*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)
/(-a)^(1/2)/b^(1/2)-1/2*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2
)+d*x)/(-a)^(1/2)/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+e^{2ic}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\right.\right.}{4\sqrt{a}\sqrt{b}}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x^2),x]
```

output
```
(E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi
[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d
*x] + E^((2*I)*c)*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/S
qrt[b]) + I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(4*Sqrt[a
]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{3814}$$

$$\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

input
```
Int[Sin[c + d*x]/(a + b*x^2),x]
```

output
```
-1/2*(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]
])/(Sqrt[-a]*Sqrt[b]) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (
Sqrt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.06

method	result
derivativedivides	$d\left(-\frac{\text{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\text{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\text{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\text{co}}{}\right.$
default	$d\left(-\frac{\text{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\text{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\text{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\text{co}}{}\right.$
risch	$\frac{\sqrt{ab}\,e^{-\frac{ibc+d\sqrt{ab}}{b}}\text{expIntegral}_1\left(-\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4ab}-\frac{\sqrt{ab}\,e^{-\frac{ibc-d\sqrt{ab}}{b}}\text{expIntegral}_1\left(-\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4ab}-$

input
```
int(sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*(-1/2/b/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*co
s((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(
1/2)+b*c)/b))-1/2/b/((d*(-a*b)^(1/2)-b*c)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-b
*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d
*(-a*b)^(1/2)-b*c)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \frac{\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e}{4\,ad}$$

input
```
integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqr
t(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/
b)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(
-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/(a*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin(c+dx)}{a+bx^2}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x^2),x)
```

output
```
int(sin(c + d*x)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin\left(dx+c\right)}{b\,x^2+a}dx$$

input

```
int(sin(d*x+c)/(b*x^2+a),x)
```

output

```
int(sin(c + d*x)/(a + b*x**2),x)
```

3.62 $\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} + \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a}$$

output
```
Ci(d*x)*sin(c)/a-1/2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/
2))/a-1/2*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/a+cos(c
)*Si(d*x)/a-1/2*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/
a-1/2*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)}\,dx$$
$$= \frac{-ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-e^{2ic}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegral}\right.\right.}{4a}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

output
```
((-I)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInteg
ralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b]
- I*d*x] - E^((2*I)*c)*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]
*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) + 4*C
osIntegral[d*x]*Sin[c] + 4*Cos[c]*SinIntegral[d*x])/(4*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)}\,dx$$
$$\Big\downarrow \text{3826}$$
$$\int \left(\frac{\sin(c+dx)}{ax}-\frac{bx\sin(c+dx)}{a\,(a+bx^2)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\sin(c)\operatorname{CosIntegral}(dx)}{a}+\frac{\cos(c)\operatorname{Si}(dx)}{a}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

output
```
(CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a) + (Cos[c]*SinIntegral[d*x])/a + (C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a
) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(2*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\operatorname{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\operatorname{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2a}-\operatorname{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos\left(\cdots\right.$
default	$-\frac{\operatorname{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\operatorname{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2a}-\operatorname{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos\left(\cdots\right.$
risch	$\frac{i e^{ic}\operatorname{expIntegral}_1(-idx)}{2a}-\frac{i e^{\frac{ibc+d\sqrt{ab}}{b}}\operatorname{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a}-\frac{i e^{\frac{ibc-d\sqrt{ab}}{b}}\operatorname{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-\cdots}{b}\right.}{4a}$

input
```
int(sin(d*x+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*(Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*
x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(1/2)+b*c)/b))-1/2/a*(Si(d*x+c+(
d*(-a*b)^(1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)-Ci(d*x+c+(d*(-a*b)^(1/2
)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b))+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.82

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx$$
$$= \frac{i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-}{4\,a}$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(I*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*Ei(I*d*x + sq
rt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*
c + sqrt(a*d^2/b)) - I*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b))
 + 4*cos_integral(d*x)*sin(c) + 4*cos(c)*sin_integral(d*x))/a
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x**2+a),x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(c+d\,x)}{x\,(b\,x^2+a)}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x^2)),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{b\,x^3+ax}dx$$

input
```
int(sin(d*x+c)/x/(b*x^2+a),x)
```

output
```
int(sin(c + d*x)/(a*x + b*x**3),x)
```

3.63 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 250

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx &= \frac{d\cos(c)\operatorname{CosIntegral}(dx)}{a} \\
&\quad - \frac{\sqrt{b}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} \\
&\quad + \frac{\sqrt{b}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sin(c+dx)}{ax} \\
&\quad - \frac{d\sin(c)\operatorname{Si}(dx)}{a} - \frac{\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} \\
&\quad - \frac{\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}
\end{aligned}
$$

output
```
d*cos(c)*Ci(d*x)/a-1/2*b^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/
2)*d/b^(1/2))/(-a)^(3/2)+1/2*b^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-
a)^(1/2)*d/b^(1/2))/(-a)^(3/2)-sin(d*x+c)/a/x-d*sin(c)*Si(d*x)/a+1/2*b^(1/
2)*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)-1/
2*b^(1/2)*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \frac{\sqrt{b}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{4a^{3/2}} + \frac{\sqrt{b}e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)\right)}{4a^{3/2}} - \frac{\cos(dx)\sin(c)}{ax} - \frac{\cos(c)\sin(dx)}{ax} + \frac{d(\cos(c)\,\text{CosIntegral}(dx)-\sin(c)\text{Si}(dx))}{a}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

output
```
(Sqrt[b]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*Exp
IntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + ExpIntegralEi[(Sqrt[a]*d)/Sq
rt[b] - I*d*x]))/(4*a^(3/2)) + (Sqrt[b]*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(-(E
^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) +
ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/(4*a^(3/2)) - (Cos[d*x]*Sin[c
])/(a*x) - (Cos[c]*Sin[d*x])/(a*x) + (d*(Cos[c]*CosIntegral[d*x] - Sin[c]*
SinIntegral[d*x]))/a
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$$

$$\Big\downarrow \; 3826$$

$$\int \left(\frac{\sin(c+dx)}{ax^2}-\frac{b\sin(c+dx)}{a\left(a+bx^2\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}}+\frac{\sqrt{b}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}}+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a}-\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sin(c+dx)}{ax}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

output
```
(d*Cos[c]*CosIntegral[d*x])/a - (Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) + (Sqrt[b]*CosIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)
) - Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - (Sqrt[b]*Cos[c +
(Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/
2)) - (Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*(-a)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06

method	result
derivativedivides	$d\left(-\frac{b\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos(\cdots}{\cdots}\right)}{a}\cdots\right.$
default	$d\left(-\frac{b\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos(\cdots}{\cdots}\right)}{a}\cdots\right.$
risch	$-\frac{d\,\mathrm{expIntegral}_1(-idx)\mathrm{e}^{ic}}{2a}+\frac{\sqrt{ab}\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^2}-\frac{\sqrt{ab}\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1(\cdots}{4a^2}$

input
```
int(sin(d*x+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*(-b/a*(-1/2/b/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+b*c)
/b)*cos((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-
a*b)^(1/2)+b*c)/b))-1/2/b/((d*(-a*b)^(1/2)-b*c)/b+c)*(Si(d*x+c+(d*(-a*b)^(
1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*
sin((d*(-a*b)^(1/2)-b*c)/b)))+1/a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*
cos(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)}\,dx$$
$$=\frac{4\,ad^2x\cos(c)\,\mathrm{Ci}(dx)-4\,ad^2x\sin(c)\,\mathrm{Si}(dx)-\sqrt{\frac{ad^2}{b}}bx\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}bx\mathrm{Ei}\left(i\,dx\cdots\right.}{\cdots}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(4*a*d^2*x*cos(c)*cos_integral(d*x) - 4*a*d^2*x*sin(c)*sin_integral(d*
x) - sqrt(a*d^2/b)*b*x*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) +
 sqrt(a*d^2/b)*b*x*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - sqr
t(a*d^2/b)*b*x*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + sqrt(
a*d^2/b)*b*x*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*d*s
in(d*x + c))/(a^2*d*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$$

input
```
integrate(sin(d*x+c)/x**2/(b*x**2+a),x)
```

output
```
Integral(sin(c + d*x)/(x**2*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)}\,dx=\int \frac{\sin\left(dx+c\right)}{(bx^2+a)x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^2\,\left(b\,x^2+a\right)}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x^2)),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)}\,dx=\int \frac{\sin\left(dx+c\right)}{b\,x^4+a\,x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x^2+a),x)
```

output
```
int(sin(c + d*x)/(a*x**2 + b*x**4),x)
```

3.64 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx &= -\frac{d\cos(c+dx)}{2ax}-\frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2}\\&\quad-\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a}\\&\quad+\frac{b\,\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2}\\&\quad+\frac{b\,\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2}\\&\quad-\frac{\sin(c+dx)}{2ax^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}\\&\quad-\frac{b\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2a^2}\\&\quad+\frac{b\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{2a^2}\end{aligned}$$

output
```
-1/2*d*cos(d*x+c)/a/x-b*Ci(d*x)*sin(c)/a^2-1/2*d^2*Ci(d*x)*sin(c)/a+1/2*b*
Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/a^2+1/2*b*Ci((-a)
^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/a^2-1/2*sin(d*x+c)/a/x^2
-b*cos(c)*Si(d*x)/a^2-1/2*d^2*cos(c)*Si(d*x)/a+1/2*b*cos(c+(-a)^(1/2)*d/b^
(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^2+1/2*b*cos(c-(-a)^(1/2)*d/b^(1/2))
*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \frac{ibe^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\operatorname{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)-ibe^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIn}\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

output
```
(I*b*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegr
alEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] -
 I*d*x]) - I*b*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*Ex
pIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sq
rt[b] + I*d*x]) - (2*a*Cos[d*x]*(d*x*Cos[c] + Sin[c]))/x^2 + (2*a*(-Cos[c]
 + d*x*Sin[c])*Sin[d*x])/x^2 - 2*(2*b + a*d^2)*(CosIntegral[d*x]*Sin[c] +
Cos[c]*SinIntegral[d*x]))/(4*a^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$$

$$\Big\downarrow \ \text{3826}$$

$$\int \left(\frac{b^2x\sin(c+dx)}{a^2(a+bx^2)}-\frac{b\sin(c+dx)}{a^2x}+\frac{\sin(c+dx)}{ax^3}\right)dx$$

$$\Big\downarrow \ \text{2009}$$

$$-\frac{b\sin(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{b\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{b\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}-\frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

input
```
Int[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*(d*Cos[c + d*x])/(a*x) - (b*CosIntegral[d*x]*Sin[c])/a^2 - (d^2*CosIn
tegral[d*x]*Sin[c])/(2*a) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) - Sin[c + d*x]/(2*a*x^2) - (
b*Cos[c]*SinIntegral[d*x])/a^2 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - (b*
Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*
a^2) + (b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.96

method	result
derivativedivides	$d^2\left(-\frac{\sin(dx+c)}{2a\,d^2x^2}-\frac{\cos(dx+c)}{2adx}-\frac{(d^2a+2b)(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{2a^2d^2}+\frac{b\left(\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-a}}{b}\right.\right.}{}\right.$
default	$d^2\left(-\frac{\sin(dx+c)}{2a\,d^2x^2}-\frac{\cos(dx+c)}{2adx}-\frac{(d^2a+2b)(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{2a^2d^2}+\frac{b\left(\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-a}}{b}\right.\right.}{}\right.$
risch	$-\frac{id^2\mathrm{e}^{ic}\,\mathrm{expIntegral}_1(-idx)}{4a}-\frac{i\mathrm{e}^{ic}\,\mathrm{expIntegral}_1(-idx)b}{2a^2}+\frac{ib\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^2}+$

input
```
int(sin(d*x+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d^2*(-1/2*sin(d*x+c)/a/d^2/x^2-1/2*cos(d*x+c)/a/d/x-1/2/a^2*(a*d^2+2*b)/d^
2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/2*b/a^2/d^2*(Si(d*x+c-(d*(-a*b)^(1/2)+
b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((
d*(-a*b)^(1/2)+b*c)/b))+1/2*b/a^2/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*co
s((d*(-a*b)^(1/2)-b*c)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d*(-a*b)^(
1/2)-b*c)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$$
$$=\frac{-i\,bx^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,bx^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,bx^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(\right.}}{}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(-I*b*x^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*b*x^2*
Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x - sq
rt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x + sqrt(a*d^2/b))
*e^(-I*c - sqrt(a*d^2/b)) - 2*(a*d^2 + 2*b)*x^2*cos_integral(d*x)*sin(c) -
 2*(a*d^2 + 2*b)*x^2*cos(c)*sin_integral(d*x) - 2*a*d*x*cos(d*x + c) - 2*a
*sin(d*x + c))/(a^2*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3\,(a+bx^2)}\,dx = \int \frac{\sin\left(c+dx\right)}{x^3\left(a+bx^2\right)}\,dx$$

input
```
integrate(sin(d*x+c)/x**3/(b*x**2+a),x)
```

output
```
Integral(sin(c + d*x)/(x**3*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3\,(a+bx^2)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx^2+a)x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\sin(c+d\,x)}{x^3\,(b\,x^2+a)}\,dx$$

input
```
int(sin(c + d*x)/(x^3*(a + b*x^2)),x)
```

output
```
int(sin(c + d*x)/(x^3*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \frac{-\cos(dx+c)\,dx - \left(\int \frac{\sin(dx+c)}{b\,x^3+ax}dx\right) a\,d^2x^2 - 2\left(\int \frac{\sin(dx+c)}{b\,x^3+ax}dx\right) b\,x^2 - \left(\int \frac{\sin(dx+c)x}{b\,x^2+a}dx\right) b\,d^2x^2 - \sin(dx+}{2a\,x^2}$$

input
```
int(sin(d*x+c)/x^3/(b*x^2+a),x)
```

output
```
( - cos(c + d*x)*d*x - int(sin(c + d*x)/(a*x + b*x**3),x)*a*d**2*x**2 - 2*
int(sin(c + d*x)/(a*x + b*x**3),x)*b*x**2 - int((sin(c + d*x)*x)/(a + b*x*
*2),x)*b*d**2*x**2 - sin(c + d*x))/(2*a*x**2)
```

3.65 $\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 450

$$\begin{aligned}\int \frac{x^4\sin(c+dx)}{(a+bx^2)^2}\,dx =& -\frac{\cos(c+dx)}{b^2d}-\frac{ad\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}\\&-\frac{ad\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3}\\&-\frac{3\sqrt{-a}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}\\&+\frac{3\sqrt{-a}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}\\&+\frac{x\sin(c+dx)}{2b^2}-\frac{x^3\sin(c+dx)}{2b(a+bx^2)}\\&-\frac{3\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}\\&-\frac{ad\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}\\&-\frac{3\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}\\&+\frac{ad\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3}\end{aligned}$$

output
```
-cos(d*x+c)/b^2/d-1/4*a*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1
/2)-d*x)/b^3-1/4*a*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d
*x)/b^3-3/4*(-a)^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(
1/2))/b^(5/2)+3/4*(-a)^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)
*d/b^(1/2))/b^(5/2)+1/2*x*sin(d*x+c)/b^2-1/2*x^3*sin(d*x+c)/b/(b*x^2+a)+3/
4*(-a)^(1/2)*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^(
5/2)+1/4*a*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^3
-3/4*(-a)^(1/2)*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/b
^(5/2)+1/4*a*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/b^
3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.66

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx =$$
$$-\frac{\sqrt{a}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(-3\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{}$$

input
```
Integrate[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/8*(Sqrt[a]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^
((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (-
3*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + Sqrt[
a]*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)
/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (-3*Sqrt[b] + Sq
rt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - 4*b*Cos[d*x]*((-2*C
os[c])/d + (a*x*Sin[c])/(a + b*x^2)) - 4*b*((a*x*Cos[c])/(a + b*x^2) + (2*
Sin[c])/d)*Sin[d*x])/b^3
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 3824}$$

$$\frac{3\int \frac{x^2 \sin(c+dx)}{bx^2+a}dx}{2b} + \frac{d\int \frac{x^3 \cos(c+dx)}{bx^2+a}dx}{2b} - \frac{x^3 \sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 3826}$$

$$\frac{3\int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b(bx^2+a)}\right)dx}{2b} + \frac{d\int \frac{x^3 \cos(c+dx)}{bx^2+a}dx}{2b} - \frac{x^3 \sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\int \frac{x^3 \cos(c+dx)}{bx^2+a}dx}{2b} + \frac{3\left(-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}\right.}{2b}$$

$$\frac{x^3 \sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 3827}$$

$$\frac{d\int \left(\frac{x\cos(c+dx)}{b} - \frac{ax\cos(c+dx)}{b(bx^2+a)}\right)dx}{2b} + \frac{3\left(-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}\right.}{2b}$$

$$\frac{x^3 \sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\left(-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}\cdots\right.}{2b}$$

$$\frac{d\left(-\frac{a\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{a\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\sin\left(c-\cdots\right.}{}\right.}{2b}$$

$$\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*Sin[c + d*x])/(b*(a + b*x^2)) + (3*(-(Cos[c + d*x]/(b*d)) - (Sqr
t[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]
])/(2*b^(3/2)) + (Sqrt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c +
 (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a]*Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cos[c
 - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/
2))))/(2*b) + (d*(Cos[c + d*x]/(b*d^2) - (a*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cos[c - (Sqrt[-a]*d)
/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2) + (x*Sin[c + d*
x])/(b*d) - (a*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(2*b^2) + (a*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(2*b^2)))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.18

method	result
risch	$\frac{e^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)ad}{8b^3}+\frac{e^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)ad}{8b^3}+\frac{3\sqrt{ab}\,e^{\frac{ibc}{}}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^4*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/b^3*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*
c))/b)*a*d+1/8/b^3*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-
b*(I*d*x+I*c))/b)*a*d+3/8/b^3*(a*b)^(1/2)*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(
1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-3/8/b^3*(a*b)^(1/2)*exp((I*b*c-d*
(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/8/b^3*Ei(1,-
(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*a*d+1
/8/b^3*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*b*c-d*(a*b)^(1
/2))/b)*a*d-3/8/b^3*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/
2)*exp(-(I*b*c+d*(a*b)^(1/2))/b)+3/8/b^3*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d
*x+I*c))/b)*(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)-cos(d*x+c)/b^2/d+1/2
*d^2*a*x/b^2/(b*d^2*x^2+a*d^2)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.78

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx$$

$$= \frac{4\,abdx\sin\left(dx+c\right) - \left(abd^2x^2 + a^2d^2 + 3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} - \left(abd^2x^2 + \right.}{}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*s
qrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2
*x^2 + a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b)
)*e^(I*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sqr
t(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2
*x^2 + a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b
))*e^(-I*c - sqrt(a*d^2/b)) - 8*(b^2*x^2 + a*b)*cos(d*x + c))/(b^4*d*x^2 +
 a*b^3*d)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^4 \sin\left(c+dx\right)}{(a+bx^2)^2} \, dx$$

input
```
integrate(x**4*sin(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x**4*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^4\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4\sin(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^4*cos(d*x + c) - 4*(a*cos(c)^2 + a*sin
(c)^2)*x*sin(d*x + c) + ((b*d*x^4*cos(c) + 4*a*x*sin(c))*cos(d*x + c)^2 +
(b*d*x^4*cos(c) + 4*a*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((b^3*
cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2
*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c
)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2
+ (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-2*(a^2
*d*x*cos(d*x + c) - (3*a*b*x^2 - a^2)*sin(d*x + c))/(b^4*d^2*x^6 + 3*a*b^3
*d^2*x^4 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 2*(((b^3*cos(c)^2 + b^3*si
n(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos
(c)^2 + a^2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^
2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2
 + a^2*b*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-2*(a^2*d*x*cos(d*x + c)
 - (3*a*b*x^2 - a^2)*sin(d*x + c))/((b^4*d^2*x^6 + 3*a*b^3*d^2*x^4 + 3*a^2
*b^2*d^2*x^2 + a^3*b*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^6 + 3*a*b^3*d^2*x^4
+ 3*a^2*b^2*d^2*x^2 + a^3*b*d^2)*sin(d*x + c)^2), x) + ((b*d*x^4*sin(c) -
4*a*x*cos(c))*cos(d*x + c)^2 + (b*d*x^4*sin(c) - 4*a*x*cos(c))*sin(d*x + c
)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*co
s(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*
cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(...
```

Giac [F]

$$\int \frac{x^4\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4\sin(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^4*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((x^4*sin(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x^4*sin(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\sin\left(dx+c\right)x^4}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(x^4*sin(d*x+c)/(b*x^2+a)^2,x)
```

output
```
int((sin(c + d*x)*x**4)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.66 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 431

$$\begin{aligned}\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}\,dx &= \frac{\sqrt{-a}d \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ &\quad - \frac{\sqrt{-a}d \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} \\ &\quad + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} \\ &\quad + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{2b^2} \\ &\quad - \frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} \\ &\quad + \frac{\sqrt{-a}d \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ &\quad + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} \\ &\quad + \frac{\sqrt{-a}d \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}\end{aligned}$$

output
```
1/4*(-a)^(1/2)*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/
b^(5/2)-1/4*(-a)^(1/2)*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/
2)+d*x)/b^(5/2)+1/2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2
))/b^2+1/2*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/b^2+1/
2*sin(d*x+c)/b^2-1/2*x^2*sin(d*x+c)/b/(b*x^2+a)+1/2*cos(c+(-a)^(1/2)*d/b^(
1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^2-1/4*(-a)^(1/2)*d*sin(c+(-a)^(1/2)*
d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)+1/2*cos(c-(-a)^(1/2)*d/b^
(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/b^2+1/4*(-a)^(1/2)*d*sin(c-(-a)^(1/2)*
d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.66

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} dx = \frac{ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(2\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(2\sqrt{b}-\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-id\right.}{}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
(I*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((2*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]
*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (2*Sqrt[b] -
Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) - I*E^(I*c - (Sqrt[
a]*d)/Sqrt[b])*((2*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInteg
ralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (2*Sqrt[b] - Sqrt[a]*d)*ExpIntegra
lEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (4*a*Sqrt[b]*Cos[d*x]*Sin[c])/(a + b*x
^2) + (4*a*Sqrt[b]*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \; 3824$$

$$\frac{\int \frac{x\sin(c+dx)}{bx^2+a}dx}{b}+\frac{d\int \frac{x^2\cos(c+dx)}{bx^2+a}dx}{2b}-\frac{x^2\sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \; 3826$$

$$\frac{d\int \frac{x^2\cos(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \left(\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{b}-\frac{x^2\sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\int \frac{x^2\cos(c+dx)}{bx^2+a}dx}{2b}+\frac{\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(x\right.}{2b}}{b}$$
$$\frac{x^2\sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \; 3827$$

$$\frac{d\int \left(\frac{\cos(c+dx)}{b}-\frac{a\cos(c+dx)}{b(bx^2+a)}\right)dx}{2b}+\frac{\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(x\right.}{2b}}{b}$$
$$\frac{x^2\sin(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\left(\frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}+\cdots}{2b}$$

$$\frac{\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\cdots\right)}{2b}\cdots}{b}$$

$$\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^2*Sin[c + d*x])/(b*(a + b*x^2)) + ((CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b) + (CosIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b) - (Cos[c + (Sqrt[-a]*
d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Cos[c - (Sqr
t[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b))/b + (d*(
(Sqrt[-a]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sin[c + d*x]/(b*d) + (Sqrt[-a]*S
in[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b
^(3/2)) + (Sqrt[-a]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(2*b^(3/2))))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.96

method	result
risch	$\frac{i\sqrt{ab}\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8b^3}-\frac{i\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(-\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)\sqrt{ab}\,d}{8b^3}+$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^3*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/b^3*(a*b)^(1/2)*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/
2)-b*(I*d*x+I*c))/b)*d-1/8*I/b^3*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*
c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)*(a*b)^(1/2)*d+1/4*I/b^2*Ei(1,(I*b*c+d*(a
*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*b*c+d*(a*b)^(1/2))/b)+1/4*I/b^2*exp((I*
b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/8*I/
b^3*(a*b)^(1/2)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b
*(I*d*x+I*c))/b)*d-1/8*I/b^3*(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(
1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)*d-1/4*I/b^2*exp(-(I*b*c+d*(a*b)^
(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/4*I/b^2*exp(-(I*b
*c-d*(a*b)^(1/2))/b)*Ei(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/d^4*(1
/2/(2*I*(I*d*x+I*c)*b*c-(I*d*x+I*c)^2*b+d^2*a+b*c^2)/b^2/a*(3*I*(I*d*x+I*c
)*a*b*c*d^2-I*(I*d*x+I*c)*b^2*c^3+a^2*d^4-b^2*c^4)*d^2+1/2*c^3*d^3*x/(-2*I
*(I*d*x+I*c)*b*c+(I*d*x+I*c)^2*b-d^2*a-b*c^2)/a-3/2*c^2*d^2*(I*(I*d*x+I*c)
*b*c+d^2*a+b*c^2)/a/b/(2*I*(I*d*x+I*c)*b*c-(I*d*x+I*c)^2*b+d^2*a+b*c^2)-3/
2*I*c*d^2*(-I*a*c*d^2-I*b*c^3-(I*d*x+I*c)*a*d^2+b*(I*d*x+I*c)*c^2)/a/b/(-2
*I*(I*d*x+I*c)*b*c+(I*d*x+I*c)^2*b-d^2*a-b*c^2))*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.68

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\left(2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}+\right.}{}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 2*I*a)*Ei(I*d*x - sqrt
(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (2*I*b*x^2 - (I*b*x^2 + I*a)*sqrt(a*d
^2/b) + 2*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-2*I*b
*x^2 - (I*b*x^2 + I*a)*sqrt(a*d^2/b) - 2*I*a)*Ei(-I*d*x - sqrt(a*d^2/b))*e
^(-I*c + sqrt(a*d^2/b)) + (-2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 2
*I*a)*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x +
c))/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{x^3 \sin(c + dx)}{(a + bx^2)^2} dx = \int \frac{x^3 \sin(c + dx)}{(a + bx^2)^2} dx$$

input
```
integrate(x**3*sin(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x**3*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c + dx)}{(a + bx^2)^2} dx = \int \frac{x^3 \sin(dx + c)}{(bx^2 + a)^2} dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - d*x^2*
sin(c) - 2*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - d*x^2*sin(c) - 2*x
*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*x^3 -
 2*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^
2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2
*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 +
2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^
3)*sin(d*x + c)^2)*integrate((2*a*d*x*sin(d*x + c) + ((2*a*d^2 + 3*b)*x^2
- a)*cos(d*x + c))/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*
d^3), x) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a
*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 +
 ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*
d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate((2*
a*d*x*sin(d*x + c) + ((2*a*d^2 + 3*b)*x^2 - a)*cos(d*x + c))/((b^3*d^3*x^6
 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3)*cos(d*x + c)^2 + (b^3*d^3*
x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3)*sin(d*x + c)^2), x) + (
(d^2*x^3*sin(c) + d*x^2*cos(c) - 2*x*sin(c))*cos(d*x + c)^2 + (d^2*x^3*sin
(c) + d*x^2*cos(c) - 2*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^2*co
s(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 +
 (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b...
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^3 \sin(dx+c)}{(bx^2+a)^2} \, dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^3 \sin(c+dx)}{(bx^2+a)^2} \, dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{\sin(dx+c)\, x^3}{b^2x^4+2ab\,x^2+a^2} dx$$

input
```
int(x^3*sin(d*x+c)/(b*x^2+a)^2,x)
```

output
```
int((sin(c + d*x)*x**3)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.67 $\int \frac{x^2\sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}\int \frac{x^2\sin(c+dx)}{(a+bx^2)^2}\,dx =\ & \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} \\ & +\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2} \\ & -\frac{\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} \\ & +\frac{\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} \\ & -\frac{x\sin(c+dx)}{2b(a+bx^2)}-\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} \\ & +\frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} \\ & -\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} \\ & -\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2}\end{aligned}$$

output
```
1/4*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/b^2+1/4*d*c
os(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/b^2-1/4*Ci((-a)^(1
/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(3/2)+1/4*Ci((
-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(3/2)-1/
2*x*sin(d*x+c)/b/(b*x^2+a)+1/4*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*
d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)-1/4*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-
a)^(1/2)*d/b^(1/2)+d*x)/b^2-1/4*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*
d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)-1/4*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a
)^(1/2)*d/b^(1/2)+d*x)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.66

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2} \, dx = \frac{\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(-\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{a}}+\frac{e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{Ex}\right.}{}}{8b^2}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
((E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)
/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (-Sqrt[b] + Sqrt
[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[a] + (E^(I*c - (S
qrt[a]*d)/Sqrt[b])*((Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInt
egralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (-Sqrt[b] + Sqrt[a]*d)*ExpIntegr
alEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[a] - (4*b*x*Cos[d*x]*Sin[c])/(a +
 b*x^2) - (4*b*x*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*b^2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3814, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \; 3824$$

$$\frac{\int \frac{\sin(c+dx)}{bx^2+a}dx}{2b}+\frac{d\int \frac{x\cos(c+dx)}{bx^2+a}dx}{2b}-\frac{x\sin(c+dx)}{2b\,(a+bx^2)}$$

$$\Big\downarrow \; 3814$$

$$\frac{d\int \frac{x\cos(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{x\sin(c+dx)}{2b\,(a+bx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\int \frac{x\cos(c+dx)}{bx^2+a}dx}{2b}+\frac{-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}}{2\sqrt{-a}}}{2b}$$
$$\frac{x\sin(c+dx)}{2b\,(a+bx^2)}$$

$$\Big\downarrow \; 3827$$

$$\frac{d\int \left(\frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{2b}+\frac{-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}}{2\sqrt{-a}}}{2b}$$
$$\frac{x\sin(c+dx)}{2b\,(a+bx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}}{2\sqrt{-a}}}{2b}$$

$$\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{S}}{2}\right.}{2b}$$

$$\frac{x\sin(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*Sin[c + d*x])/(b*(a + b*x^2)) + (-1/2*(CosIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(Sqrt[-a]*Sqrt[b]) + (CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]
*Sqrt[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]))/(2*b) + (d*((Cos[c +
(Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Co
s[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b)
 + (Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(2*b) - (Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*b)))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{e^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8b^2}-\frac{e^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8b^2}-\frac{e^{\frac{ibc+d\sqrt{a}}{b}}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^2*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^2*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I
*c))/b)*d-1/8/b^2*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b
*(I*d*x+I*c))/b)*d-1/8/a/b^2*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a
*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)+1/8/a/b^2*exp((I*b*c-d*(a*b)^(1/2)
)/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)-1/8/b^2*Ei(1,
-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*d-1/
8/b^2*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*b*c-d*(a*b)^(1/
2))/b)*d+1/8/a/b^2*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2
)*exp(-(I*b*c+d*(a*b)^(1/2))/b)-1/8/a/b^2*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*
d*x+I*c))/b)*(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)+I/d^3*(1/2/(-2*I*(I
*d*x+I*c)*b*c+(I*d*x+I*c)^2*b-d^2*a-b*c^2)/b/a*(-I*a*c*d^2-I*b*c^3-(I*d*x+
I*c)*a*d^2+b*(I*d*x+I*c)*c^2)*d^2+1/2*I*c^2*d^3*x/(-2*I*(I*d*x+I*c)*b*c+(I
*d*x+I*c)^2*b-d^2*a-b*c^2)/a-I*c*d^2*(I*(I*d*x+I*c)*b*c+d^2*a+b*c^2)/a/b/(
2*I*(I*d*x+I*c)*b*c-(I*d*x+I*c)^2*b+d^2*a+b*c^2))*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.80

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{4\,abdx\sin(dx+c) - \left(abd^2x^2 + a^2d^2 + (b^2x^2+ab)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} - \left(abd^2x^2 \cdots\right.}{\cdots}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sq
rt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*
x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e
^(I*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d
^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
+ a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-
I*c - sqrt(a*d^2/b)))/(a*b^3*d*x^2 + a^2*b^2*d)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \sin{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*sin(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x**2*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \sin{(dx+c)}}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 2*(cos(c)^2 + sin(c)^2)*x
*sin(d*x + c) + ((d*x^2*cos(c) - 2*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c
) - 2*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*s
in(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2
 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*
x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)
^2)*d^2)*sin(d*x + c)^2)*integrate(-(2*a*d*x*cos(d*x + c) - (3*b*x^2 - a)*
sin(d*x + c))/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*d^2*x^2 + a^3*d^2),
 x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*si
n(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^
2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x
^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(2*a*d*
x*cos(d*x + c) - (3*b*x^2 - a)*sin(d*x + c))/((b^3*d^2*x^6 + 3*a*b^2*d^2*x
^4 + 3*a^2*b*d^2*x^2 + a^3*d^2)*cos(d*x + c)^2 + (b^3*d^2*x^6 + 3*a*b^2*d^
2*x^4 + 3*a^2*b*d^2*x^2 + a^3*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) +
2*x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(c) + 2*x*cos(c))*sin(d*x + c)^2)*s
in(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 +
 a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2
 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2
)*d^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^2 \sin(dx+c)}{(bx^2+a)^2} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2\,\sin(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)\,x^2}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(x^2*sin(d*x+c)/(b*x^2+a)^2,x)
```

output
```
int((sin(c + d*x)*x**2)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.68 $\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$

output

```
1/4*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(1/2)/
b^(3/2)-1/4*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/(-a
)^(1/2)/b^(3/2)-1/2*sin(d*x+c)/b/(b*x^2+a)-1/4*d*sin(c+(-a)^(1/2)*d/b^(1/2
))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)+1/4*d*sin(c-(-a)^(1/2)
*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \frac{-\frac{ide^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{a}}+\frac{ide^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-\text{Exp}\right.}{\sqrt{a}}}{8b^{3/2}}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
(((-I)*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIn
tegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[
b] - I*d*x]))/Sqrt[a] + (I*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*
d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - ExpIntegralEi[
(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[a] - (4*Sqrt[b]*Cos[d*x]*Sin[c])/(a +
b*x^2) - (4*Sqrt[b]*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{3822}$$

$$\frac{d\int \frac{\cos(c+dx)}{bx^2+a}dx}{2b}-\frac{\sin(c+dx)}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{3815}$$

$$\frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{\sin(c+dx)}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{S}}{2\sqrt{-}}\right.}{2b}$$

$$\frac{\sin(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*Sin[c + d*x]/(b*(a + b*x^2)) + (d*((Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Cos
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sqr
t[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqr
t[b]) + (Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*Sqrt[-a]*Sqrt[b]) + (Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b])))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{id\sqrt{ab}\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8a\,b^2}+\frac{id\sqrt{ab}\,\mathrm{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}}{8a\,b^2}-$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*d*(a*b)^(1/2)/a/b^2*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b
)^(1/2)-b*(I*d*x+I*c))/b)+1/8*I*d*(a*b)^(1/2)/a/b^2*Ei(1,(I*b*c-d*(a*b)^(1
/2)-b*(I*d*x+I*c))/b)*exp((I*b*c-d*(a*b)^(1/2))/b)-1/8*I*d*(a*b)^(1/2)/a/b
^2*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))
/b)+1/8*I*d*(a*b)^(1/2)/a/b^2*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c-d
*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/d^2*(1/2/(2*I*(I*d*x+I*c)*b*c-(I*d*x+I*c)
^2*b+d^2*a+b*c^2)/b/a*(I*(I*d*x+I*c)*b*c+d^2*a+b*c^2)*d^2-1/2*c*d^3*x/(-2*
I*(I*d*x+I*c)*b*c+(I*d*x+I*c)^2*b-d^2*a-b*c^2)/a)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$$

$$= \frac{(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+(-$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt
(a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I
*c - sqrt(a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(-I*d*x - sqrt(a*d^
2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(-I*d*x +
 sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c))/(a*b^2*x^2 +
a^2*b)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\sin{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\sin{(dx+c)}}{(bx^2+a)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*
cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*
d*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)
^2)*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(
c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)
^2)*integrate(1/2*(3*b*x^2 - a)*cos(d*x + c)/(b^3*d*x^6 + 3*a*b^2*d*x^4 +
3*a^2*b*d*x^2 + a^3*d), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(
a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(
d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*
sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integra
te(1/2*(3*b*x^2 - a)*cos(d*x + c)/((b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*
x^2 + a^3*d)*cos(d*x + c)^2 + (b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*x^2 +
 a^3*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*
sin(c))*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos
(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c
)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^
2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\sin(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\,\sin(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x*sin(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin(dx+c)\,x}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(x*sin(d*x+c)/(b*x^2+a)^2,x)
```

output
```
int((sin(c + d*x)*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.69 $\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 476

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} + \frac{\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab}$$

output
```
-1/4*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/a/b-1/4*d*
cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/a/b+1/4*Ci((-a)^(
1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*Ci(
(-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(1/2)-1
/4*sin(d*x+c)/a/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)+1/4*sin(d*x+c)/a/b^(1/2)/((
-a)^(1/2)+b^(1/2)*x)-1/4*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1
/2)+d*x)/(-a)^(3/2)/b^(1/2)+1/4*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/
2)*d/b^(1/2)+d*x)/a/b+1/4*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1
/2)+d*x)/(-a)^(3/2)/b^(1/2)+1/4*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2
)*d/b^(1/2)+d*x)/a/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.59

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = \frac{\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\left(\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{b}+\frac{e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{Exp}\cdots}{\cdots}}{8a^{3/2}}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x^2)^2,x]
```

output
```
((E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)
/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - (Sqrt[b] + Sqrt[
a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/b + (E^(I*c - (Sqrt[a]*
d)/Sqrt[b])*((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi
[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - (Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sq
rt[a]*d)/Sqrt[b] + I*d*x]))/b + (4*Sqrt[a]*x*Cos[d*x]*Sin[c])/(a + b*x^2)
+ (4*Sqrt[a]*x*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*a^(3/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx$$

$$\Big\downarrow \ 3814$$

$$\int \left(-\frac{b\sin(c+dx)}{2a\left(-ab-b^2x^2\right)}-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sin\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab}-\frac{d\sin\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4ab}+\frac{d\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab}+\frac{\cos\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$

input
```
Int[Sin[c + d*x]/(a + b*x^2)^2,x]
```

output
```
-1/4*(d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x])/(a*b) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(4*a*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (S
qrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - (CosIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sin[c
 + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]/(4*a*Sqrt[b]*(
Sqrt[-a] + Sqrt[b]*x)) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Cos[c - (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt
[b]) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(4*a*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.03

method	result
derivativedivides	$d^3 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{2a\,d^2}-\frac{c}{2a\,d^2}\right)}{d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2} - \frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin \cdots}{4a\,d^2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)} \cdots \right.$
default	$d^3 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{2a\,d^2}-\frac{c}{2a\,d^2}\right)}{d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2} - \frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin \cdots}{4a\,d^2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)} \cdots \right.$
risch	$\frac{d\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8ab} + \frac{d\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8ab} - \frac{\sqrt{ab}\,\mathrm{e}^{\cdots}}{\cdots}$

input
```
int(sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*b*c*(d*x+c)
+b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(Si(d*x+c-(d*(-a*b)^
(1/2)+b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)
*sin((d*(-a*b)^(1/2)+b*c)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-b*c)/b+c)*(Si(d
*x+c+(d*(-a*b)^(1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)-Ci(d*x+c+(d*(-a*b
)^(1/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a
*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c
)/b)*cos((d*(-a*b)^(1/2)+b*c)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-b*
c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*cos((d*
(-a*b)^(1/2)-b*c)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.70

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{4\,abdx\sin(dx+c) - \left(abd^2x^2 + a^2d^2 - (b^2x^2+ab)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} - \left(abd^2x^2 + a\right.}{}$$

input
```
integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqr
t(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x
^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^
(I*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^
2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 +
 a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I
*c - sqrt(a*d^2/b)))/(a^2*b^2*d*x^2 + a^3*b*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x^2)^2,x)
```

output
```
int(sin(c + d*x)/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin\left(dx+c\right)}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(sin(d*x+c)/(b*x^2+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.70 $\int \frac{\sin(c+dx)}{x(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 435

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}$$

output

```
1/4*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(3/2)/
b^(1/2)-1/4*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/(-a
)^(3/2)/b^(1/2)+Ci(d*x)*sin(c)/a^2-1/2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-
(-a)^(1/2)*d/b^(1/2))/a^2-1/2*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2
)*d/b^(1/2))/a^2+1/2*sin(d*x+c)/a/(b*x^2+a)+cos(c)*Si(d*x)/a^2-1/2*cos(c+(
-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^2-1/4*d*sin(c+(-a)^(1
/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(1/2)-1/2*cos(c-
(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a^2+1/4*d*sin(c-(-a)^(1
/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \frac{\frac{i\sqrt{a}de^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{b}} - 2ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

output
```
((I*Sqrt[a]*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/
Sqrt[b] - I*d*x]))/Sqrt[b] - (2*I)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2
*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIn
tegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) - (I*Sqrt[a]*d*E^(I*c - (Sqrt[a]*d)
/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b])
+ I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[b] + (2*I)*E^
(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sq
rt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) +
 (4*a*Cos[d*x]*Sin[c])/(a + b*x^2) + (4*a*Cos[c]*Sin[d*x])/(a + b*x^2) + 8
*(CosIntegral[d*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/(8*a^2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^2}\,dx$$

$$\Big\downarrow \; 3826$$

$$\int \left(-\frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)}+\frac{\sin(c+dx)}{a^2x}-\frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}+\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin(c+dx)}{2a\left(a+bx^2\right)}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

output
```
(d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*(-a)^(3/2)*Sqrt[b]) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c]
)/a^2 - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt
[b]])/(2*a^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]
*d)/Sqrt[b]])/(2*a^2) + Sin[c + d*x]/(2*a*(a + b*x^2)) + (Cos[c]*SinIntegr
al[d*x])/a^2 + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqr
t[b] - d*x])/(2*a^2) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) + (d*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt
[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\sin(dx+c)d^2}{2a\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^2}-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{C}}{2a^2}$
default	$\frac{\sin(dx+c)d^2}{2a\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^2}-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\mathrm{C}}{2a^2}$
risch	$\frac{ie^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a^2}+\frac{ie^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8a\sqrt{ab}}-\frac{ie^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}}{}\right.}{8a\sqrt{ab}}$

input
```
int(sin(d*x+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(d*x+c)*d^2/a/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/a^2*(Si(d*x
)*cos(c)+Ci(d*x)*sin(c))-1/2/a^2*(Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*cos((d*
(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(1/2)+
b*c)/b))-1/2/a^2*(Si(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c
)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b))+1/4*d^2
/a/b/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d
*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*cos((d*(-a*b)^(1/2)
+b*c)/b))+1/4*d^2/a/b/((d*(-a*b)^(1/2)-b*c)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)
-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*cos(
(d*(-a*b)^(1/2)-b*c)/b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.74

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx =$$
$$-\frac{\left(-2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}-2i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}\right.}{}$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((-2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 2*I*a)*Ei(I*d*x - sqr
t(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (-2*I*b*x^2 - (I*b*x^2 + I*a)*sqrt(a
*d^2/b) - 2*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (2*I*
b*x^2 - (I*b*x^2 + I*a)*sqrt(a*d^2/b) + 2*I*a)*Ei(-I*d*x - sqrt(a*d^2/b))*
e^(-I*c + sqrt(a*d^2/b)) + (2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 2
*I*a)*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 8*(b*x^2 + a)*
cos_integral(d*x)*sin(c) - 8*(b*x^2 + a)*cos(c)*sin_integral(d*x) - 4*a*si
n(d*x + c))/(a^2*b*x^2 + a^3)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx = \int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x**2+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^2x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^2x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx = \int \frac{\sin(c+d\,x)}{x\,(b\,x^2+a)^2}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x^2)^2),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{b^2x^5+2ab\,x^3+a^2x}dx$$

input
```
int(sin(d*x+c)/x/(b*x^2+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)
```

3.71 $\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 501

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x^2 (a+bx^2)^2}\,dx = {} & \frac{d\cos(c)\operatorname{CosIntegral}(dx)}{a^2} \\
& + \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} \\
& + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2} \\
& + \frac{3\sqrt{b}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} \\
& - \frac{3\sqrt{b}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} \\
& - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} \\
& - \frac{d\sin(c)\operatorname{Si}(dx)}{a^2} + \frac{3\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} \\
& + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} \\
& + \frac{3\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}} \\
& - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}
\end{aligned}
$$

output
```
d*cos(c)*Ci(d*x)/a^2+1/4*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(
1/2)-d*x)/a^2+1/4*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*
x)/a^2+3/4*b^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2)
)/(-a)^(5/2)-3/4*b^(1/2)*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b
^(1/2))/(-a)^(5/2)-sin(d*x+c)/a^2/x+1/4*b^(1/2)*sin(d*x+c)/a^2/((-a)^(1/2)
-b^(1/2)*x)-1/4*b^(1/2)*sin(d*x+c)/a^2/((-a)^(1/2)+b^(1/2)*x)-d*sin(c)*Si(
d*x)/a^2-3/4*b^(1/2)*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+
d*x)/(-a)^(5/2)-1/4*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)
+d*x)/a^2+3/4*b^(1/2)*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+
d*x)/(-a)^(5/2)-1/4*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+
d*x)/a^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.66

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx = \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-\left(\left(3\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+\left(3\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

output
```
(E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((3*Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]
*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + (3*Sqrt[b] +
 Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + E^(I*c - (Sqrt[a
]*d)/Sqrt[b])*(-((3*Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInte
gralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (3*Sqrt[b] + Sqrt[a]*d)*ExpInteg
ralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - (4*Sqrt[a]*(2*a + 3*b*x^2)*Cos[d*x]*
Sin[c])/(x*(a + b*x^2)) - (4*Sqrt[a]*(2*a + 3*b*x^2)*Cos[c]*Sin[d*x])/(x*(
a + b*x^2)) + 8*Sqrt[a]*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinIntegral[d*
x]))/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(-\frac{b\sin(c+dx)}{a^2 (a+bx^2)}+\frac{\sin(c+dx)}{a^2x^2}-\frac{b\sin(c+dx)}{a (a+bx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}-\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a^2}-\frac{d\sin(c)\text{Si}(dx)}{a^2}-\frac{\sin(c+dx)}{a^2x}+\frac{3\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}}-\frac{3\sqrt{b}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

output
```
(d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b
]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*CosIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(5/2
)) - (3*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*
d)/Sqrt[b]])/(4*(-a)^(5/2)) - Sin[c + d*x]/(a^2*x) + (Sqrt[b]*Sin[c + d*x]
)/(4*a^2*(Sqrt[-a] - Sqrt[b]*x)) - (Sqrt[b]*Sin[c + d*x])/(4*a^2*(Sqrt[-a]
 + Sqrt[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^2 + (3*Sqrt[b]*Cos[c + (Sqr
t[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(5/2))
+ (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(4*a^2) + (3*Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]
*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2)) - (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{d\,\text{expIntegral}_1(-idx)\,e^{ic}}{2a^2}-\frac{d\,e^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8a^2}-\frac{d\,e^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}_1\left(-\frac{-ib}{}\right.}{8a^2}$
derivativedivides	$d\left(-\frac{b\left(-\frac{\text{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\text{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\text{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos\left(\right.}{}\right.}{a^2}\right.$
default	$d\left(-\frac{b\left(-\frac{\text{Si}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\cos\left(\frac{d\sqrt{-ab}+bc}{b}\right)+\text{Ci}\left(dx+c-\frac{d\sqrt{-ab}+bc}{b}\right)\sin\left(\frac{d\sqrt{-ab}+bc}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+bc}{b}+c\right)}-\frac{\text{Si}\left(dx+c+\frac{d\sqrt{-ab}-bc}{b}\right)\cos\left(\right.}{}\right.}{a^2}\right.$

input
```
int(sin(d*x+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a^2*Ei(1,-I*d*x)*exp(I*c)-1/8*d/a^2*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei
(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/8*d/a^2*exp((I*b*c-d*(a*b)^(1/
2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+3/8/a^2/(a*b)^(1/2)*e
xp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b-
3/8/a^2/(a*b)^(1/2)*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/
2)+b*(I*d*x+I*c))/b)*b-1/2*d/a^2*Ei(1,I*d*x)*exp(-I*c)-1/8*d/a^2*exp(-(I*b
*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/8*d/a^
2*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/
b)-3/8/a^2/(a*b)^(1/2)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^
(1/2)-b*(I*d*x+I*c))/b)*b+3/8/a^2/(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b
)*Ei(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)*b-1/2*(6*I*(I*d*x+I*c)*b*c-
3*(I*d*x+I*c)^2*b+2*d^2*a+3*b*c^2)/a^2/(2*I*(I*d*x+I*c)*b*c-(I*d*x+I*c)^2*
b+d^2*a+b*c^2)/x*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.80

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx$$

$$= \frac{8(abd^2x^3+a^2d^2x)\cos(c)\operatorname{Ci}(dx)+\left(abd^2x^3+a^2d^2x-3(b^2x^3+abx)\sqrt{\frac{ad^2}{b}}\right)\operatorname{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\vphantom{x}}\right.}}{}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(8*(a*b*d^2*x^3 + a^2*d^2*x)*cos(c)*cos_integral(d*x) + (a*b*d^2*x^3 +
 a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*
e^(I*c + sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)*s
qrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b*d^2
*x^3 + a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d
^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 +
a*b*x)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b))
- 8*(a*b*d^2*x^3 + a^2*d^2*x)*sin(c)*sin_integral(d*x) - 4*(3*a*b*d*x^2 +
2*a^2*d)*sin(d*x + c))/(a^3*b*d*x^3 + a^4*d*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx = \int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx$$

input
```
integrate(sin(d*x+c)/x**2/(b*x**2+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(x**2*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^2x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^2x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx = \int \frac{\sin(c+d\,x)}{x^2\,(b\,x^2+a)^2}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x^2)^2),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2 (a+bx^2)^2} \, dx = \int \frac{\sin(dx+c)}{b^2x^6+2ab\,x^4+a^2x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x^2+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x**2 + 2*a*b*x**4 + b**2*x**6),x)
```

3.72 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 476

$$\begin{aligned}\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3} \, dx = & -\frac{dx \cos(c+dx)}{8b^2 (a+bx^2)} + \frac{3d \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\ & - \frac{3d \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} \\ & - \frac{d^2 \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} \\ & - \frac{d^2 \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} - \frac{x^2 \sin(c+dx)}{4b (a+bx^2)^2} \\ & - \frac{\sin(c+dx)}{4b^2 (a+bx^2)} + \frac{d^2 \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} \\ & + \frac{3d \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\ & - \frac{d^2 \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3} \\ & + \frac{3d \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}\end{aligned}$$

output
```
-1/8*d*x*cos(d*x+c)/b^2/(b*x^2+a)+3/16*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-
a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(1/2)/b^(5/2)-3/16*d*cos(c-(-a)^(1/2)*d/b^(1/
2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2)-1/16*d^2*Ci((-a)^(1/2)
*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/b^3-1/16*d^2*Ci((-a)^(1/2)*d/b
^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/b^3-1/4*x^2*sin(d*x+c)/b/(b*x^2+a)
^2-1/4*sin(d*x+c)/b^2/(b*x^2+a)-1/16*d^2*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(
-a)^(1/2)*d/b^(1/2)+d*x)/b^3-3/16*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(
1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2)-1/16*d^2*cos(c-(-a)^(1/2)*d/b^(1/2)
)*Si((-a)^(1/2)*d/b^(1/2)+d*x)/b^3+3/16*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((
-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.69

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{-\frac{ide^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(-3\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{a}}+\frac{ide^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3\sqrt{b}+\sqrt{a}d\right)\right.}{}}{}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

output
```
(((-I)*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^((2*S
qrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (-3*Sqr
t[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[a] + (
I*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d
)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (-3*Sqrt[b] + S
qrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[a] - (4*b*Cos[
d*x]*(d*x*(a + b*x^2)*Cos[c] + 2*(a + 2*b*x^2)*Sin[c]))/(a + b*x^2)^2 + (4
*b*(-2*(a + 2*b*x^2)*Cos[c] + d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2
)^2)/(32*b^3)
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.50, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3824, 3822, 3815, 2009, 3825, 3815, 2009, 3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{\left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \text{3824}$$

$$\frac{\int \frac{x \sin(c+dx)}{\left(bx^2+a\right)^2}dx}{2b}+\frac{d\int \frac{x^2\cos(c+dx)}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{3822}$$

$$\frac{d\int \frac{x^2\cos(c+dx)}{\left(bx^2+a\right)^2}dx}{4b}+\frac{\frac{d\int \frac{\cos(c+dx)}{bx^2+a}dx}{2b}-\frac{\sin(c+dx)}{2b(a+bx^2)}}{2b}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{3815}$$

$$\frac{d\int \frac{x^2\cos(c+dx)}{\left(bx^2+a\right)^2}dx}{4b}+\frac{\frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{\sin(c+dx)}{2b(a+bx^2)}}{2b}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\int \frac{x^2\cos(c+dx)}{\left(bx^2+a\right)^2}dx}{4b}+\frac{\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}}{2b}}{2b}$$

$$\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{3825}$$

$$\frac{d\left(-\frac{d\int \frac{x\sin(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \frac{\cos(c+dx)}{bx^2+a}dx}{2b}-\frac{x\cos(c+dx)}{2b(a+bx^2)}\right)}{4b}+$$

$$\frac{\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right.}{2b}}{2b}$$

$$\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 3815$$

$$\frac{d\left(-\frac{d\int \frac{x\sin(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{x\cos(c+dx)}{2b(a+bx^2)}\right)}{4b}+$$

$$\frac{\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right.}{2b}}{2b}$$

$$\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\left(-\frac{d\int \frac{x\sin(c+dx)}{bx^2+a}dx}{2b}+\frac{\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\right.}{}}{2b}\right.}{4b}$$

$$\frac{\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right.}{2b}}{2b}$$

$$\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 3826$$

$$d\left(-\frac{d\int\left(\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{2b}+\frac{\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{}}{2b}\right.$$

$$\frac{}{4b}$$

$$\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2\sqrt{-a}\sqrt{b}}\right.}{2b}$$

$$\frac{}{2b}$$

$$\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$d\left(-\frac{d\left(\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\right.}{2b}\right.}{2b}\right.$$

$$\frac{d\left(\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2\sqrt{-a}\sqrt{b}}\right.}{2b}$$

$$\frac{}{2b}$$

$$\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

input

```
Int[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^2*Sin[c + d*x])/(b*(a + b*x^2)^2) + (d*(-1/2*(x*Cos[c + d*x])/(b*(
a + b*x^2)) - (d*((CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]])/(2*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (
Sqrt[-a]*d)/Sqrt[b]])/(2*b) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b)))/(2*b) + ((Cos[c + (Sqrt[-a]*d)/S
qrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (C
os[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*S
qrt[-a]*Sqrt[b]) + (Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) + (Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]))/(2*b)))/(4*b)
+ (-1/2*Sin[c + d*x]/(b*(a + b*x^2)) + (d*((Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (
Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*
Sqrt[b]) + (Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(2*Sqrt[-a]*Sqrt[b]) + (Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b])))/(2*b))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3825
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Cos[c + d*x], x], x] + Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{i d^{2} e^{\frac{i b c+d\sqrt{a b}}{b}} \operatorname{expIntegral}_{1}\left(\frac{i b c+d\sqrt{a b}-b(i d x+i c)}{b}\right)}{32 b^{3}}-\frac{i d^{2} e^{\frac{i b c-d\sqrt{a b}}{b}} \operatorname{expIntegral}_{1}\left(-\frac{-i b c+d\sqrt{a b}+b(i d x+i c)}{b}\right)}{32 b^{3}}-\frac{3i}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^3*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*I*d^2/b^3*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(
I*d*x+I*c))/b)-1/32*I*d^2/b^3*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*c+d
*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-3/32*I*d/a/b^3*(a*b)^(1/2)*exp((I*b*c+d*(a*
b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+3/32*I*d/a/b^3*(a
*b)^(1/2)*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/2)+b*(I*d*
x+I*c))/b)+1/32*I*d^2/b^3*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*
b)^(1/2)-b*(I*d*x+I*c))/b)+1/32*I*d^2/b^3*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei
(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-3/32*I*d/a/b^3*(a*b)^(1/2)*exp(
-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+3/3
2*I*d/a/b^3*(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(-I*b*c+d*(a*b)
^(1/2)+b*(I*d*x+I*c))/b)+1/8/a*(a*b*d^5*x^3+a^2*d^5*x)/b^2/(-b^2*d^4*x^4-2
*a*b*d^4*x^2-a^2*d^4)*cos(d*x+c)-1/8/d^2*(-4*a^2*b*d^6*x^2-2*a^3*d^6)/a^2/
b^2/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx =$$

$$-\frac{\left(-i\,ab^2d^2x^4 - 2i\,a^2bd^2x^2 - i\,a^3d^2 + 3\left(-i\,b^3x^4 - 2i\,ab^2x^2 - i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right.}}{}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*((-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 + 3*(-I*b^3*x^4 -
 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c
+ sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 + 3*(
I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/
b))*e^(I*c - sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 + I*a^3
*d^2 + 3*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x -
sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2
*x^2 + I*a^3*d^2 + 3*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))
*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*(a*b^2*d*x^3 + a^
2*b*d*x)*cos(d*x + c) + 8*(2*a*b^2*x^2 + a^2*b)*sin(d*x + c))/(a*b^5*x^4 +
 2*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*sin(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(3*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 3*d*
x^2*sin(c) - 12*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 3*d*x^2*sin(c
) - 12*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d
^2*x^3 - 12*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^3*cos(c)^2 + b^
3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)
*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)
^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2
 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x
*sin(d*x + c) + ((a*d^2 + 10*b)*x^2 - 2*a)*cos(d*x + c))/(b^4*d^3*x^8 + 4*
a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) - 2*(((
b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)
*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a
^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6
+ 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*
sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*int
egrate(3*(3*a*d*x*sin(d*x + c) + ((a*d^2 + 10*b)*x^2 - 2*a)*cos(d*x + c))/
((b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^
4*d^3)*cos(d*x + c)^2 + (b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4
 + 4*a^3*b*d^3*x^2 + a^4*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c) + ...
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3} dx = \int \frac{x^3 \sin(dx+c)}{(bx^2+a)^3} dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{x^3\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x^2)^3,x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{\sin\left(dx+c\right)x^3}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx$$

input
```
int(x^3*sin(d*x+c)/(b*x^2+a)^3,x)
```

output
```
int((sin(c + d*x)*x**3)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6)
,x)
```

3.73 $\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 746

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx = -\frac{d\cos(c+dx)}{8b^2(a+bx^2)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\sin(c+dx)}{4b(a+bx^2)^2} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$

output
```
-1/8*d*cos(d*x+c)/b^2/(b*x^2+a)-1/16*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)
^(1/2)*d/b^(1/2)-d*x)/a/b^2-1/16*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/
2)*d/b^(1/2)+d*x)/a/b^2+1/16*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)
*d/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d^2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c
-(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(5/2)-1/16*Ci((-a)^(1/2)*d/b^(1/2)-d*x
)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d^2*Ci((-a)^(1/2)*d/
b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(5/2)-1/16*sin(d*x+c
)/a/b^(3/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*sin(d*x+c)/a/b^(3/2)/((-a)^(1/2)+b
^(1/2)*x)-1/4*x*sin(d*x+c)/b/(b*x^2+a)^2-1/16*cos(c+(-a)^(1/2)*d/b^(1/2))*
Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)-1/16*d^2*cos(c+(-a)^(1/2)
*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2)+1/16*d*sin(c+
(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a/b^2+1/16*cos(c-(-a)^
(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)+1/16*d^2*
cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2
)+1/16*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.46 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.49

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(b-\sqrt{a}\sqrt{b}d-ad^2\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\left(b+\sqrt{a}\sqrt{b}d-ad^2\right)\text{ExpIntegralE}\right.}{}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

output
```
(E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((b - Sqrt[a]*Sqrt[b]*d - a*d^2)*E^((2*S
qrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - (b + Sq
rt[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + E^(
I*c - (Sqrt[a]*d)/Sqrt[b])*((b - Sqrt[a]*Sqrt[b]*d - a*d^2)*E^((2*Sqrt[a]*
d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - (b + Sqrt[a]*S
qrt[b]*d - a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - (4*Sqrt[a]
*Sqrt[b]*Cos[d*x]*(a*d*(a + b*x^2)*Cos[c] + b*x*(a - b*x^2)*Sin[c]))/(a +
b*x^2)^2 + (4*Sqrt[a]*Sqrt[b]*(b*x*(-a + b*x^2)*Cos[c] + a*d*(a + b*x^2)*S
in[c])*Sin[d*x])/(a + b*x^2)^2)/(32*a^(3/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3824, 3814, 2009, 3823, 3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{3824}$$

$$\frac{\int \frac{\sin(c+dx)}{(bx^2+a)^2}dx}{4b}+\frac{d\int \frac{x\cos(c+dx)}{(bx^2+a)^2}dx}{4b}-\frac{x\sin(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{3814}$$

$$\frac{\int \left(-\frac{b\sin(c+dx)}{2a(-b^2x^2-ab)}-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sin(c+dx)}{4a\left(bx+\sqrt{-a}\sqrt{b}\right)^2}\right)dx}{4b}+\frac{d\int \frac{x\cos(c+dx)}{(bx^2+a)^2}dx}{4b}-\frac{x\sin(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\int \frac{x\cos(c+dx)}{(bx^2+a)^2}dx}{4b}+\frac{-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos\left(\right.}{}}{}$$

$$\frac{x\sin(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{3823}$$

$$\frac{d\left(-\frac{d\int \frac{\sin(c+dx)}{bx^2+a}dx}{2b}-\frac{\cos(c+dx)}{2b(a+bx^2)}\right)}{4b}+\frac{-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos\left(\right.}{}}{}$$

$$\frac{x\sin(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{3814}$$

$$\frac{d\left(-\frac{d\int\left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{\cos(c+dx)}{2b(a+bx^2)}\right)}{4b}+$$

$$-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos(}{}$$

$$\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 2009

$$\frac{d\left(-\frac{d\left(-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd\right.}{2\sqrt{-a}\sqrt{b}}\right.}{2b}\right.}{4b}$$

$$-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos(}{}$$

$$\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

input

```
Int[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

output
```
-1/4*(x*Sin[c + d*x])/(b*(a + b*x^2)^2) + (-1/4*(d*Cos[c + (Sqrt[-a]*d)/Sq
rt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(a*b) - (d*Cos[c - (Sqrt[-
a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) + (CosInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3
/2)*Sqrt[b]) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*
d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sin[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a]
- Sqrt[b]*x)) + Sin[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Cos[c
 + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^
(3/2)*Sqrt[b]) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*a*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*Sin[c - (Sqrt[-a]*d)/
Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b))/(4*b) + (d*(-1/
2*Cos[c + d*x]/(b*(a + b*x^2)) - (d*(-1/2*(CosIntegral[(Sqrt[-a]*d)/Sqrt[b
] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(Sqrt[-a]*Sqrt[b]) + (CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqr
t[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b])))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3823
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
x] + Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.21

method	result
risch	$\frac{d^2 e^{\frac{ibc+d\sqrt{ab}}{b}} \sqrt{ab}\, \text{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a\,b^3} - \frac{d^2 \sqrt{ab}\, e^{\frac{ibc-d\sqrt{ab}}{b}}\, \text{expIntegral}_1\left(-\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)}{32a\,b^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^2*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*d^2/a/b^3*exp((I*b*c+d*(a*b)^(1/2))/b)*(a*b)^(1/2)*Ei(1,(I*b*c+d*(a*b
)^(1/2)-b*(I*d*x+I*c))/b)-1/32*d^2/a/b^3*(a*b)^(1/2)*exp((I*b*c-d*(a*b)^(1
/2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/32*d/a/b^2*exp((I*
b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32*d/a
/b^2*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c
))/b)-1/32/a^2/b^2*exp((I*b*c+d*(a*b)^(1/2))/b)*(a*b)^(1/2)*Ei(1,(I*b*c+d*
(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32/a^2/b^2*(a*b)^(1/2)*exp((I*b*c-d*(a*b)^
(1/2))/b)*Ei(1,-(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-1/32*d^2/a/b^3*(a*
b)^(1/2)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x
+I*c))/b)+1/32*d^2/a/b^3*(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(-
I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/32*d/a/b^2*exp(-(I*b*c+d*(a*b)^(1/
2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32*d/a/b^2*exp(-(I*b
*c-d*(a*b)^(1/2))/b)*Ei(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/32/a^2
/b^2*(a*b)^(1/2)*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-
b*(I*d*x+I*c))/b)-1/32/a^2/b^2*(a*b)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*E
i(1,(-I*b*c+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-1/8*d/a*(-a*b*d^4*x^2-a^2*d^4)
/b^2/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*cos(d*x+c)+1/8/d/b*(-a*b*d^5*x^3
+a^2*d^5*x)/a^2/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.81

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^3}\,dx =$$

$$-\frac{\left(ab^2d^2x^4+2\,a^2bd^2x^2+a^3d^2+(a^3d^2+(ab^2d^2-b^3)x^4-a^2b+2\,(a^2bd^2-ab^2)x^2)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\right.}\right.}{}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2
- b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x -
sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
+ a^3*d^2 - (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^
2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) +
 (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 - b^3)
*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(
a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^
3*d^2 - (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x
^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4
*(a^2*b*d^2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(a*b^2*d*x^3 - a^2*b*d*x)*sin(
d*x + c))/(a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**2*sin(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^2 \sin(dx+c)}{(bx^2+a)^3} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 4*(cos(c)^2 + sin(c)^2)*x
*sin(d*x + c) + ((d*x^2*cos(c) - 4*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c
) - 4*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*s
in(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*
cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*co
s(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2
+ a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 +
(a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos
(d*x + c) - 2*(5*b*x^2 - a)*sin(d*x + c))/(b^4*d^2*x^8 + 4*a*b^3*d^2*x^6 +
 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2), x) + 2*(((b^3*cos(c)^2 +
b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a
^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^
2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(
c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x
^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*
x*cos(d*x + c) - 2*(5*b*x^2 - a)*sin(d*x + c))/((b^4*d^2*x^8 + 4*a*b^3*d^2
*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2)*cos(d*x + c)^2 + (b^
4*d^2*x^8 + 4*a*b^3*d^2*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^
2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) + 4*x*cos(c))*cos(d*x + c)^2 + (d*
x^2*sin(c) + 4*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^...
```

Giac [F]

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{x^2\sin\left(dx+c\right)}{\left(bx^2+a\right)^3}\,dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^2\ \sin(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x^2)^3,x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)\,x^2}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx$$

input
```
int(x^2*sin(d*x+c)/(b*x^2+a)^3,x)
```

output
```
int((sin(c + d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6)
,x)
```

3.74 $\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 512

$$
\begin{aligned}
\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = & -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
& -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} \\
& +\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} \\
& +\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} \\
& +\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} \\
& -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} \\
& -\frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} \\
& +\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} \\
& -\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}
\end{aligned}
$$

output

```
-1/16*d*cos(d*x+c)/a/b^(3/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*d*cos(d*x+c)/a/b^
(3/2)/((-a)^(1/2)+b^(1/2)*x)-1/16*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1
/2)*d/b^(1/2)-d*x)/(-a)^(3/2)/b^(3/2)+1/16*d*cos(c-(-a)^(1/2)*d/b^(1/2))*C
i((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)+1/16*d^2*Ci((-a)^(1/2)*d/b^
(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/a/b^2+1/16*d^2*Ci((-a)^(1/2)*d/b^(1
/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/a/b^2-1/4*sin(d*x+c)/b/(b*x^2+a)^2+1/
16*d^2*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a/b^2+1/1
6*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b
^(3/2)+1/16*d^2*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a
/b^2-1/16*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^
(3/2)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.62

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{ide^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-\left(\left(\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+\left(\sqrt{b}+\sqrt{a}d\right)\operatorname{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-\right.\right.}{}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

output
```
(I*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[
a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + (Sqrt[b] +
 Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) - I*d*E^(I*c - (Sq
rt[a]*d)/Sqrt[b])*(-((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIn
tegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (Sqrt[b] + Sqrt[a]*d)*ExpInteg
ralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (4*Sqrt[a]*b*Cos[d*x]*(d*x*(a + b*x^
2)*Cos[c] - 2*a*Sin[c]))/(a + b*x^2)^2 - (4*Sqrt[a]*b*(2*a*Cos[c] + d*x*(a
 + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2)/(32*a^(3/2)*b^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{3822}$$

$$\frac{d\int \frac{\cos(c+dx)}{(bx^2+a)^2}dx}{4b} - \frac{\sin(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{3815}$$

$$\frac{d\int \left(-\frac{b\cos(c+dx)}{2a(-b^2x^2-ab)}-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\cos(c+dx)}{4a\left(bx+\sqrt{-a}\sqrt{b}\right)^2}\right)dx}{4b}-\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cos}{}\right.$$

$$\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

input

```
Int[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

output

```
-1/4*Sin[c + d*x]/(b*(a + b*x^2)^2) + (d*(-1/4*Cos[c + d*x]/(a*Sqrt[b]*(Sq
rt[-a] - Sqrt[b]*x)) + Cos[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) -
 (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*(-a)^(3/2)*Sqrt[b]) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CosIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) + (d*CosIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) - (d*Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b)
 - (Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(4*(-a)^(3/2)*Sqrt[b]) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) - (Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b])))/(4*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.24

method	result
risch	$\frac{id^2 e^{\frac{ibc+d\sqrt{ab}}{b}} \operatorname{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a\,b^2} + \frac{id^2 e^{\frac{ibc-d\sqrt{ab}}{b}} \operatorname{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a\,b^2} - \frac{id\, e^{\frac{ibc+}{}}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*I*d^2/a/b^2*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*
(I*d*x+I*c))/b)+1/32*I*d^2/a/b^2*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-
d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/32*I*d/a^2/b^2*exp((I*b*c+d*(a*b)^(1/2))
/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)+1/32*I*d/a^2/b
^2*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b
)*(a*b)^(1/2)-1/32*I*d^2/a/b^2*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b
)*exp(-(I*b*c-d*(a*b)^(1/2))/b)-1/32*I*d^2/a/b^2*Ei(1,-(I*b*c+d*(a*b)^(1/2
)-b*(I*d*x+I*c))/b)*exp(-(I*b*c+d*(a*b)^(1/2))/b)+1/32*I*d/a^2/b^2*Ei(1,-(
I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*(a*b)^
(1/2)-1/32*I*d/a^2/b^2*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(
I*b*c+d*(a*b)^(1/2))/b)*(a*b)^(1/2)-1/8*d^3/a*x*(b*d^2*x^2+a*d^2)/b/(-b^2*
d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*cos(d*x+c)+1/4*d^4/b/(-b^2*d^4*x^4-2*a*b*d^
4*x^2-a^2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.94

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{8\,a^2b\sin(dx+c) + \left(i\,ab^2d^2x^4 + 2i\,a^2bd^2x^2 + i\,a^3d^2 - (i\,b^3x^4 + 2i\,ab^2x^2 + i\,a^2b)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(8*a^2*b*sin(d*x + c) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 + I*a^3
*d^2 - (I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqr
t(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2
 + I*a^3*d^2 - (-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*
d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*I*a^2
*b*d^2*x^2 - I*a^3*d^2 - (-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2
/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x
^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 - (I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)
*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(a
*b^2*d*x^3 + a^2*b*d*x)*cos(d*x + c))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b
^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{x\sin(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*
cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*
d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^
2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((
b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin
(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(5*b*x^2 - a)*cos(d*x + c)/(b^4*d*
x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x) + 2*(((
b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin
(c)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2
*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*
x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(5*b*
x^2 - a)*cos(d*x + c)/((b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^
3*b*d*x^2 + a^4*d)*cos(d*x + c)^2 + (b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2
*d*x^4 + 4*a^3*b*d*x^2 + a^4*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*si
n(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^
2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^
2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*si...
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{x\sin(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{x\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x^2)^3,x)
```

output
```
int((x*sin(c + d*x))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx=\int \frac{\sin\left(dx+c\right)x}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx$$

input
```
int(x*sin(d*x+c)/(b*x^2+a)^3,x)
```

output
```
int((sin(c + d*x)*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)
```

3.75 $\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 856

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

output

```
1/16*d*cos(d*x+c)/(-a)^(3/2)/b/((-a)^(1/2)-b^(1/2)*x)+1/16*d*cos(d*x+c)/(-
a)^(3/2)/b/((-a)^(1/2)+b^(1/2)*x)-3/16*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-
a)^(1/2)*d/b^(1/2)-d*x)/a^2/b-3/16*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(
1/2)*d/b^(1/2)+d*x)/a^2/b-3/16*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/
2)*d/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*Ci((-a)^(1/2)*d/b^(1/2)+d*x)*sin
(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(3/2)+3/16*Ci((-a)^(1/2)*d/b^(1/2)-d
*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16*d^2*Ci((-a)^(1/2)*
d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*sin(d*x
+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)^2-3/16*sin(d*x+c)/a^2/b^(1/2
)/((-a)^(1/2)-b^(1/2)*x)+1/16*sin(d*x+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)+b^
(1/2)*x)^2+3/16*sin(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+b^(1/2)*x)+3/16*cos(c+(
-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)-1/16
*d^2*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/
b^(3/2)+3/16*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a
^2/b-3/16*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5
/2)/b^(1/2)+1/16*d^2*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d
*x)/(-a)^(3/2)/b^(3/2)+3/16*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/
b^(1/2)+d*x)/a^2/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.81 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.44

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3b-3\sqrt{a}\sqrt{b}d+ad^2\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\left(3b+3\sqrt{a}\sqrt{b}d+ad^2\right)\text{ExpInteg}\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x^2)^3,x]
```

output
```
(E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((3*b - 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^(
(2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - (3*
b + 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x
]) + E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((3*b - 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^
((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - (3
*b + 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*
x]) + (4*Sqrt[a]*Sqrt[b]*Cos[d*x]*(a*d*(a + b*x^2)*Cos[c] + b*x*(5*a + 3*b
*x^2)*Sin[c]))/(a + b*x^2)^2 - (4*Sqrt[a]*Sqrt[b]*(-(b*x*(5*a + 3*b*x^2)*C
os[c]) + a*d*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2)/(32*a^(5/2)*b^(3
/2))
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{3814}$$

$$\int \left(-\frac{3b\sin(c+dx)}{8a^2(-ab-b^2x^2)} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3} - \frac{}{8(-}\right.$$

$\downarrow$ 2009

$$\frac{\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}} + \frac{\cos(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cos(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{b}x+\sqrt{-a}\right)} - \frac{3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^2b} - \frac{3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b} - \frac{3\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^2b} + \frac{3\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b} - \frac{3\,\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{3\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)} - \frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)^2} - \frac{3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$

input
```
Int[Sin[c + d*x]/(a + b*x^2)^3,x]
```

output
```
(d*Cos[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x
])/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) - (3*d*Cos[c + (Sqrt[-a]*d)/Sq
rt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*d*Cos[c -
(Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b)
- (3*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
in[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) + (3*CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqr
t[b]) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/
Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(S
qrt[-a] - Sqrt[b]*x)^2) - (3*Sin[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqr
t[b]*x)) + Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) +
 (3*Sin[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (3*Cos[c + (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)
*Sqrt[b]) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*Cos[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt
[b]) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(16*(-a)^(3/2)*b^(3/2)) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*S...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.70

method	result
derivativedivides	$d^5\left(-\frac{\sin(dx+c)\left(5ac\,d^2-5a\,d^2(dx+c)+3b\,c^3-9b\,c^2(dx+c)+9bc(dx+c)^2-3b(dx+c)^3\right)}{8a^2d^4\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)^2}+\frac{\cos(dx+c}{8ab\,d^2\left(d^2a+b\,c^2-2bc(dx\right.}\right.$
default	$d^5\left(-\frac{\sin(dx+c)\left(5ac\,d^2-5a\,d^2(dx+c)+3b\,c^3-9b\,c^2(dx+c)+9bc(dx+c)^2-3b(dx+c)^3\right)}{8a^2d^4\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)^2}+\frac{\cos(dx+c}{8ab\,d^2\left(d^2a+b\,c^2-2bc(dx\right.}\right.$
risch	$-\frac{d^2\sqrt{ab}\,e^{\frac{ibc+d\sqrt{ab}}{b}}\operatorname{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2b^2}+\frac{d^2\sqrt{ab}\,e^{\frac{ibc-d\sqrt{ab}}{b}}\operatorname{expIntegral}_1\left(\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2b^2}+$

input
```
int(sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d^5*(-1/8*sin(d*x+c)*(5*a*c*d^2-5*a*d^2*(d*x+c)+3*b*c^3-9*b*c^2*(d*x+c)+9*
b*c*(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d^4/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^
2)^2+1/8*cos(d*x+c)/a/b/d^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/16*(
a*d^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/
2)+b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*si
n((d*(-a*b)^(1/2)+b*c)/b))-1/16*(a*d^2+3*b)/a^2/d^4/b^2/((d*(-a*b)^(1/2)-b
*c)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)-Ci(
d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b))-3/16/a^2/d^4/b*
(-Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d
*(-a*b)^(1/2)+b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b))-3/16/a^2/d^4/b*(Si(d*x+
c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b)+Ci(d*x+c+(d*(-a*b)^(
1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.71

$$\int\frac{\sin(c+dx)}{(a+bx^2)^3}\,dx=$$

$$-\frac{\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2-(a^3d^2+(ab^2d^2+3\,b^3)x^4+3\,a^2b+2\,(a^2bd^2+3\,ab^2)x^2)\sqrt{\frac{ad^2}{b}}\right)\operatorname{Ei}\left(\right.}{}$$

input
```
integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*((3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 + (a*b^2*
d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*E
i(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^
2*b*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2
*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*
c - sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3
*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sq
rt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (3*a*b^
2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x
^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqr
t(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(a^2*b*d^2*x^2 + a^3*d^2)*cos(d*x
 + c) - 4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*sin(d*x + c))/(a^3*b^3*d*x^4 + 2*a
^4*b^2*d*x^2 + a^5*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x^2)^3,x)
```

output
```
int(sin(c + d*x)/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx$$

input
```
int(sin(d*x+c)/(b*x^2+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)
```

3.76 $\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 730

$$\int \frac{\sin(c+dx)}{x(a+bx^2)^3}\,dx = \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{5d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{5d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} + \frac{\sin(c+dx)}{4a(a+bx^2)^2} + \frac{\sin(c+dx)}{2a^2(a+bx^2)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} - \frac{5d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} - \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} - \frac{5d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}$$

output
```
1/16*d*cos(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)-1/16*d*cos(d*x+c)/a^2
/b^(1/2)/((-a)^(1/2)+b^(1/2)*x)-5/16*d*cos(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)
^(1/2)*d/b^(1/2)-d*x)/(-a)^(5/2)/b^(1/2)+5/16*d*cos(c-(-a)^(1/2)*d/b^(1/2)
)*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)+Ci(d*x)*sin(c)/a^3-1/2*C
i((-a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/a^3-1/16*d^2*Ci((-
a)^(1/2)*d/b^(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/a^2/b-1/2*Ci((-a)^(1/2
)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/a^3-1/16*d^2*Ci((-a)^(1/2)*d/
b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/a^2/b+1/4*sin(d*x+c)/a/(b*x^2+a)^
2+1/2*sin(d*x+c)/a^2/(b*x^2+a)+cos(c)*Si(d*x)/a^3-1/2*cos(c+(-a)^(1/2)*d/b
^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^3-1/16*d^2*cos(c+(-a)^(1/2)*d/b^(1
/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^2/b+5/16*d*sin(c+(-a)^(1/2)*d/b^(1/2)
)*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)-1/2*cos(c-(-a)^(1/2)*d/
b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a^3-1/16*d^2*cos(c-(-a)^(1/2)*d/b^(1
/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a^2/b-5/16*d*sin(c-(-a)^(1/2)*d/b^(1/2))
*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 672, normalized size of antiderivative = 0.92

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^3}\,dx$$

$$= \frac{-\frac{4adx\cos(c+dx)}{a+bx^2} + \frac{4i\sqrt{a}de^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{b}} - 8ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{Exp}\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

output
```
((-4*a*d*x*Cos[c + d*x])/(a + b*x^2) + ((4*I)*Sqrt[a]*d*E^((-I)*c - (Sqrt[
a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt
[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[b] - (8*
I)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegral
Ei[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I
*d*x]) - (I*Sqrt[a]*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((Sqrt[b] - Sqrt[
a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d
*x]) + (Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/
b - ((4*I)*Sqrt[a]*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[
b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - ExpIntegralEi[(Sqrt[a]
*d)/Sqrt[b] + I*d*x]))/Sqrt[b] + (8*I)*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((
2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpI
ntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (I*Sqrt[a]*d*E^(I*c - (Sqrt[a]*d
)/Sqrt[b])*(-((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralE
i[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(
Sqrt[a]*d)/Sqrt[b] + I*d*x]))/b + 32*CosIntegral[d*x]*Sin[c] + (8*a*(3*a +
 2*b*x^2)*Sin[c + d*x])/(a + b*x^2)^2 + 32*Cos[c]*SinIntegral[d*x])/(32*a^
3)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(-\frac{bx\sin(c+dx)}{a^3\left(a+bx^2\right)}+\frac{\sin(c+dx)}{a^3x}-\frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)^2}-\frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}+\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}+\frac{\sin(c)\,\text{CosIntegral}(dx)}{a^3}+\frac{\cos(c)\text{Si}(dx)}{a^3}-\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}-\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}+\frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)}+\frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{5d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{5d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{5d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{5d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

output
```
(d*Cos[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Cos[c + d*x]
)/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (5*d*Cos[c + (Sqrt[-a]*d)/Sqrt
[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (5
*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(5/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^3) - (d^2*Cos
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^
2*b) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[
b]])/(2*a^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[
-a]*d)/Sqrt[b]])/(16*a^2*b) + Sin[c + d*x]/(4*a*(a + b*x^2)^2) + Sin[c + d
*x]/(2*a^2*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cos
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^
2*b) - (5*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (5*d*Sin[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)
*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 580, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\sin(dx+c)d^2\left(3d^2a+2b\,c^2-4bc(dx+c)+2b(dx+c)^2\right)}{4a^2\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)^2}-\frac{\cos(dx+c)d^3x}{8a^2\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(}{a^3}$
default	$\frac{\sin(dx+c)d^2\left(3d^2a+2b\,c^2-4bc(dx+c)+2b(dx+c)^2\right)}{4a^2\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)^2}-\frac{\cos(dx+c)d^3x}{8a^2\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(}{a^3}$
risch	$-\frac{i\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)d^2}{32b\,a^2}-\frac{i\mathrm{e}^{-ic}\,\mathrm{expIntegral}_1(-idx)}{2a^3}-\frac{i\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}_1\left(-\right.}{32b\,a^2}$

input
```
int(sin(d*x+c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(d*x+c)*d^2*(3*d^2*a+2*b*c^2-4*b*c*(d*x+c)+2*b*(d*x+c)^2)/a^2/(d^2*
a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2-1/8*cos(d*x+c)*d^3*x/a^2/(d^2*a+b*c^2
-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/16*(a*
d^2+8*b)/a^3/b*(Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/
b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(1/2)+b*c)/b))-1/16*(a*d
^2+8*b)/a^3/b*(Si(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b
)-Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b))+5/16*d^2/a
^2/b/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((d
*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*cos((d*(-a*b)^(1/2)
+b*c)/b))+5/16*d^2/a^2/b/((d*(-a*b)^(1/2)-b*c)/b+c)*(Si(d*x+c+(d*(-a*b)^(1
/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*c
os((d*(-a*b)^(1/2)-b*c)/b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 637, normalized size of antiderivative = 0.87

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{\left(-i\,a^3d^2-i\,(ab^2d^2+8\,b^3)x^4-8i\,a^2b-2i\,(a^2bd^2+8\,ab^2)x^2+5\,(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(\right.}{}$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*((-I*a^3*d^2 - I*(a*b^2*d^2 + 8*b^3)*x^4 - 8*I*a^2*b - 2*I*(a^2*b*d^
2 + 8*a*b^2)*x^2 + 5*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*
Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (-I*a^3*d^2 - I*(a*b^2
*d^2 + 8*b^3)*x^4 - 8*I*a^2*b - 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(-I*b^3*
x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^
(I*c - sqrt(a*d^2/b)) + (I*a^3*d^2 + I*(a*b^2*d^2 + 8*b^3)*x^4 + 8*I*a^2*b
 + 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b
)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*
a^3*d^2 + I*(a*b^2*d^2 + 8*b^3)*x^4 + 8*I*a^2*b + 2*I*(a^2*b*d^2 + 8*a*b^2
)*x^2 + 5*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x +
 sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 32*(b^3*x^4 + 2*a*b^2*x^2 + a^2
*b)*cos_integral(d*x)*sin(c) - 32*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cos(c)*s
in_integral(d*x) + 4*(a*b^2*d*x^3 + a^2*b*d*x)*cos(d*x + c) - 8*(2*a*b^2*x
^2 + 3*a^2*b)*sin(d*x + c))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^3}\,dx = \int \frac{\sin\left(c+dx\right)}{x\left(a+bx^2\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x**2+a)**3,x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x**2)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3 x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x\left(b\,x^2+a\right)^3}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x^2)^3),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}dx$$

input
```
int(sin(d*x+c)/x/(b*x^2+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)
```

3.77 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 875

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3} dx = \text{Too large to display}$$

output

```
1/16*d*cos(d*x+c)/(-a)^(5/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*d*cos(d*x+c)/(-a)
^(5/2)/((-a)^(1/2)+b^(1/2)*x)+d*cos(c)*Ci(d*x)/a^3+7/16*d*cos(c+(-a)^(1/2)
*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/a^3+7/16*d*cos(c-(-a)^(1/2)*d/b^(
1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/a^3-15/16*b^(1/2)*Ci((-a)^(1/2)*d/b^(1/
2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(7/2)+1/16*d^2*Ci((-a)^(1/2)*d/b^
(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(5/2)/b^(1/2)+15/16*b^(1/2)*Ci
((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(7/2)-1/16*d^2
*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(5/2)/b^(1/
2)-sin(d*x+c)/a^3/x-1/16*b^(1/2)*sin(d*x+c)/(-a)^(5/2)/((-a)^(1/2)-b^(1/2)
*x)^2+7/16*b^(1/2)*sin(d*x+c)/a^3/((-a)^(1/2)-b^(1/2)*x)+1/16*b^(1/2)*sin(
d*x+c)/(-a)^(5/2)/((-a)^(1/2)+b^(1/2)*x)^2-7/16*b^(1/2)*sin(d*x+c)/a^3/((-
a)^(1/2)+b^(1/2)*x)-d*sin(c)*Si(d*x)/a^3+15/16*b^(1/2)*cos(c+(-a)^(1/2)*d/
b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)-1/16*d^2*cos(c+(-a)^(1/2
)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)-7/16*d*sin(c
+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^3-15/16*b^(1/2)*cos
(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)+1/16*d^2*
cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2
)-7/16*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.76 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.68

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx$$
$$= \frac{8\sqrt{b}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-\left((7b-7\sqrt{\ }\right.\right.}{}}{}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

output
```
(8*Sqrt[b]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*E
xpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + ExpIntegralEi[(Sqrt[a]*d)/
Sqrt[b] - I*d*x]) + (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((7*b - 7*Sqrt[a]*
Sqrt[b]*d + a*d^2)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/S
qrt[b]) - I*d*x]) + (7*b + 7*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqr
t[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[b] + 8*Sqrt[b]*E^(I*c - (Sqrt[a]*d)/Sqrt[b
])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d
*x]) + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (E^(I*c - (Sqrt[a]*d)
/Sqrt[b])*(-((7*b - 7*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^((2*Sqrt[a]*d)/Sqrt[b])
*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (7*b + 7*Sqrt[a]*Sqrt[b]
*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[b] - (4*Sqrt
[a]*Cos[d*x]*(a*d*x*(a + b*x^2)*Cos[c] + (8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)
*Sin[c]))/(x*(a + b*x^2)^2) + (4*Sqrt[a]*(-((8*a^2 + 25*a*b*x^2 + 15*b^2*x
^4)*Cos[c]) + a*d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(x*(a + b*x^2)^2) + 32*S
qrt[a]*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinIntegral[d*x]))/(32*a^(7/2))
```

Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \text{ 3826}$$

$$\int \left(-\frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)}+\frac{\sin(c+dx)}{a^3x^2}-\frac{b\sin(c+dx)}{a^2\left(a+bx^2\right)^2}-\frac{b\sin(c+dx)}{a\left(a+bx^2\right)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{CosIntegral}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}+\frac{\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}+\frac{\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}+\frac{\cos(c+dx)d}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cos(c+dx)d}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)}+\frac{\cos(c)\,\text{CosIntegral}(dx)d}{a^3}+\frac{7\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)d}{16a^3}+\frac{7\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)d}{16a^3}-\frac{\sin(c)\text{Si}(dx)d}{a^3}+\frac{7\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)d}{16a^3}-\frac{7\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)d}{16a^3}-\frac{15\sqrt{b}\,\text{CosIntegral}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{7/2}}+\frac{15\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{7/2}}-\frac{\sin(c+dx)}{a^3x}+\frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2}+\frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)^2}-\frac{15\sqrt{b}\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}-\frac{15\sqrt{b}\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

output
```
(d*Cos[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])
/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3
+ (7*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(16*a^3) + (7*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*a^3) - (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) + (d^2*CosIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sq
rt[b]) + (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt
[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - Sin[c + d*
x]/(a^3*x) - (Sqrt[b]*Sin[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)^
2) + (7*Sqrt[b]*Sin[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*S
in[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*Sqrt[b]*Sin[c +
 d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^3 -
 (15*Sqrt[b]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x])/(16*(-a)^(7/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (7*d*Sin[c + (Sqrt
[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) - (15*S
qrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*(-a)^(7/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(S...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{d\,\mathrm{expIntegral}_1(-idx)\mathrm{e}^{ic}}{2a^3}+\frac{d^2\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\mathrm{expIntegral}_1\left(\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2\sqrt{ab}}-\frac{d^2\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\mathrm{expIntegral}_1\left(\frac{ibc-}{}\right.}{32a^2\sqrt{ab}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(sin(d*x+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a^3*Ei(1,-I*d*x)*exp(I*c)+1/32/a^2*d^2/(a*b)^(1/2)*exp((I*b*c+d*(a*
b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/32/a^2*d^2/(a*b
)^(1/2)*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*
c))/b)-7/32*d/a^3*exp((I*b*c+d*(a*b)^(1/2))/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b
*(I*d*x+I*c))/b)-7/32*d/a^3*exp((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*
b)^(1/2)-b*(I*d*x+I*c))/b)+15/32/a^3/(a*b)^(1/2)*exp((I*b*c+d*(a*b)^(1/2))
/b)*Ei(1,(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b-15/32/a^3/(a*b)^(1/2)*ex
p((I*b*c-d*(a*b)^(1/2))/b)*Ei(1,(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b-1
/2*d/a^3*Ei(1,I*d*x)*exp(-I*c)-1/32/a^2*d^2/(a*b)^(1/2)*exp(-(I*b*c+d*(a*b
)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32/a^2*d^2/(a*b
)^(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+
I*c))/b)-7/32*d/a^3*exp(-(I*b*c+d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/
2)-b*(I*d*x+I*c))/b)-7/32*d/a^3*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c
-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-15/32/a^3/(a*b)^(1/2)*exp(-(I*b*c+d*(a*b)
^(1/2))/b)*Ei(1,-(I*b*c+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b+15/32/a^3/(a*b)^
(1/2)*exp(-(I*b*c-d*(a*b)^(1/2))/b)*Ei(1,-(I*b*c-d*(a*b)^(1/2)-b*(I*d*x+I*
c))/b)*b+1/8/a^2*d^2*(b*d^3*x^3+a*d^3*x)/x/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2
*d^4)*cos(d*x+c)-1/8*(-15*b^2*d^4*x^4-25*a*b*d^4*x^2-8*a^2*d^4)/a^3/x/(-b^
2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 714, normalized size of antiderivative = 0.82

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/32*(32*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cos(c)*cos_integral
(d*x) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x - ((a*b^2*d^2 +
15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(
a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (7*a*b^2*d^2
*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2
*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(I*d*x +
 sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*
x^3 + 7*a^3*d^2*x - ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x
^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(
-I*c + sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x
+ ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15
*a^2*b)*x)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/
b)) - 32*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sin(c)*sin_integral
(d*x) - 4*(a^2*b*d^2*x^3 + a^3*d^2*x)*cos(d*x + c) - 4*(15*a*b^2*d*x^4 + 2
5*a^2*b*d*x^2 + 8*a^3*d)*sin(d*x + c))/(a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^
6*d*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/x**2/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\sin(c+d\,x)}{x^2\,(b\,x^2+a)^3}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x^2)^3),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^2)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{b^3x^8+3a\,b^2x^6+3a^2b\,x^4+a^3x^2}dx$$

input

```
int(sin(d*x+c)/x^2/(b*x^2+a)^3,x)
```

output

```
int(sin(c + d*x)/(a**3*x**2 + 3*a**2*b*x**4 + 3*a*b**2*x**6 + b**3*x**8),x
)
```

3.78 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 791

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/2*d*cos(d*x+c)/a^3/x-1/16*b^(1/2)*d*cos(d*x+c)/a^3/((-a)^(1/2)-b^(1/2)*
x)+1/16*b^(1/2)*d*cos(d*x+c)/a^3/((-a)^(1/2)+b^(1/2)*x)-9/16*b^(1/2)*d*cos
(c+(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(7/2)+9/16*b^(1
/2)*d*cos(c-(-a)^(1/2)*d/b^(1/2))*Ci((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)-
3*b*Ci(d*x)*sin(c)/a^4-1/2*d^2*Ci(d*x)*sin(c)/a^3+3/2*b*Ci((-a)^(1/2)*d/b^
(1/2)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/a^4+1/16*d^2*Ci((-a)^(1/2)*d/b^(1/2
)+d*x)*sin(c-(-a)^(1/2)*d/b^(1/2))/a^3+3/2*b*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*
sin(c+(-a)^(1/2)*d/b^(1/2))/a^4+1/16*d^2*Ci((-a)^(1/2)*d/b^(1/2)-d*x)*sin(
c+(-a)^(1/2)*d/b^(1/2))/a^3-1/2*sin(d*x+c)/a^3/x^2-1/4*b*sin(d*x+c)/a^2/(b
*x^2+a)^2-b*sin(d*x+c)/a^3/(b*x^2+a)-3*b*cos(c)*Si(d*x)/a^4-1/2*d^2*cos(c)
*Si(d*x)/a^3+3/2*b*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*
x)/a^4+1/16*d^2*cos(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*x)/
a^3+9/16*b^(1/2)*d*sin(c+(-a)^(1/2)*d/b^(1/2))*Si(-(-a)^(1/2)*d/b^(1/2)+d*
x)/(-a)^(7/2)+3/2*b*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*
x)/a^4+1/16*d^2*cos(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)/a
^3-9/16*b^(1/2)*d*sin(c-(-a)^(1/2)*d/b^(1/2))*Si((-a)^(1/2)*d/b^(1/2)+d*x)
/(-a)^(7/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.55

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx$$
$$= \frac{ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(24b-9\sqrt{a}\sqrt{b}d+ad^2\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(24b+9\sqrt{a}\sqrt{b}d+ad^2\right)\text{ExpIn}}{}$$

input
```
Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

output
```
(I*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((24*b - 9*Sqrt[a]*Sqrt[b]*d + a*d^2)*
E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] +
(24*b + 9*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I
*d*x]) - I*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((24*b - 9*Sqrt[a]*Sqrt[b]*d + a*
d^2)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*
x] + (24*b + 9*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b
] + I*d*x]) - (4*a*Cos[d*x]*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cos[c] +
2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Sin[c]))/(x^2*(a + b*x^2)^2) + (4*a*(-2*
(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Cos[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^
4)*Sin[c])*Sin[d*x])/(x^2*(a + b*x^2)^2) - 16*(6*b + a*d^2)*(CosIntegral[d
*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/(32*a^4)
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx$$
$$\Big\downarrow \; 3826$$

$$\int \left(\frac{3b^2x\sin(c+dx)}{a^4\left(a+bx^2\right)}-\frac{3b\sin(c+dx)}{a^4x}+\frac{2b^2x\sin(c+dx)}{a^3\left(a+bx^2\right)^2}+\frac{\sin(c+dx)}{a^3x^3}+\frac{b^2x\sin(c+dx)}{a^2\left(a+bx^2\right)^3}\right)dx$$

$\big\downarrow$ 2009

$$-\frac{3b\sin(c)\,\text{CosIntegral}(dx)}{a^4}+\frac{3b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}+\frac{3b\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}-\frac{3b\cos(c)\text{Si}(dx)}{a^4}-\frac{3b\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}+\frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}+\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}+\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}-\frac{d^2\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}+\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}-\frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)}-\frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a^3}-\frac{d^2\cos(c)\text{Si}(dx)}{2a^3}-\frac{\sin(c+dx)}{2a^3x^2}-\frac{d\cos(c+dx)}{2a^3x}-\frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2}-\frac{9\sqrt{b}d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}+\frac{9\sqrt{b}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}-\frac{9\sqrt{b}d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}-\frac{9\sqrt{b}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

input
```
Int[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

output
```
-1/2*(d*Cos[c + d*x])/(a^3*x) - (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a]
 - Sqrt[b]*x)) + (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x))
- (9*Sqrt[b]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(7/2)) + (9*Sqrt[b]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*C
osIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (3*b*CosIntegral
[d*x]*Sin[c])/a^4 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^3) + (3*b*CosIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^4) + (d
^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/
(16*a^3) + (3*b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*
d)/Sqrt[b]])/(2*a^4) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c
+ (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) - Sin[c + d*x]/(2*a^3*x^2) - (b*Sin[c +
d*x])/(4*a^2*(a + b*x^2)^2) - (b*Sin[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Co
s[c]*SinIntegral[d*x])/a^4 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) - (3*b*
Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*
a^4) - (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(16*a^3) - (9*Sqrt[b]*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) + (3*b*Cos[c - (Sqrt[-a]*d)/
Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) + (d^2*Cos[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) - (
9*Sqrt[b]*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 697, normalized size of antiderivative = 0.88

method	result
derivativedivides	$d^{2}\left(-\frac{\sin(dx+c)\left(2a^{2}d^{4}+9ab\,c^{2}d^{2}-18abc\,d^{2}(dx+c)+9ab\,d^{2}(dx+c)^{2}+6b^{2}c^{4}-24b^{2}c^{3}(dx+c)+36b^{2}c^{2}(dx+c)^{2}-24b^{2}c\right.}{4a^{3}d^{2}x^{2}\left(d^{2}a+b\,c^{2}-2bc(dx+c)+b(dx+c)^{2}\right)^{2}}\right.$
default	$d^{2}\left(-\frac{\sin(dx+c)\left(2a^{2}d^{4}+9ab\,c^{2}d^{2}-18abc\,d^{2}(dx+c)+9ab\,d^{2}(dx+c)^{2}+6b^{2}c^{4}-24b^{2}c^{3}(dx+c)+36b^{2}c^{2}(dx+c)^{2}-24b^{2}c\right.}{4a^{3}d^{2}x^{2}\left(d^{2}a+b\,c^{2}-2bc(dx+c)+b(dx+c)^{2}\right)^{2}}\right.$
risch	$-\frac{9 i d\, e^{-\frac{i b c+d\sqrt{a b}}{b}} \operatorname{expIntegral}_{1}\left(-\frac{i b c+d\sqrt{a b}-b(i d x+i c)}{b}\right) b}{32 a^{3}\sqrt{a b}}-\frac{9 i d\, e^{\frac{i b c+d\sqrt{a b}}{b}} \operatorname{expIntegral}_{1}\left(\frac{i b c+d\sqrt{a b}-b(i d x+i c)}{b}\right) b}{32 a^{3}\sqrt{a b}}-$

input
```
int(sin(d*x+c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(-1/4*sin(d*x+c)*(2*a^2*d^4+9*a*b*c^2*d^2-18*a*b*c*d^2*(d*x+c)+9*a*b*d
^2*(d*x+c)^2+6*b^2*c^4-24*b^2*c^3*(d*x+c)+36*b^2*c^2*(d*x+c)^2-24*b^2*c*(d
*x+c)^3+6*b^2*(d*x+c)^4)/a^3/d^2/x^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^
2)^2-1/8*cos(d*x+c)*(4*d^2*a+3*b*c^2-6*b*c*(d*x+c)+3*b*(d*x+c)^2)/a^3/d/x/
(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/2/a^4*(a*d^2+6*b)/d^2*(Si(d*x)*c
os(c)+Ci(d*x)*sin(c))+1/16*(a*d^2+24*b)/d^2/a^4*(Si(d*x+c-(d*(-a*b)^(1/2)+
b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+b*c)/b)*sin((
d*(-a*b)^(1/2)+b*c)/b))+1/16*(a*d^2+24*b)/d^2/a^4*(Si(d*x+c+(d*(-a*b)^(1/2
)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin
((d*(-a*b)^(1/2)-b*c)/b))-9/16/a^3/(-(d*(-a*b)^(1/2)+b*c)/b+c)*(-Si(d*x+c-
(d*(-a*b)^(1/2)+b*c)/b)*sin((d*(-a*b)^(1/2)+b*c)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+b*c)/b)*cos((d*(-a*b)^(1/2)+b*c)/b))-9/16/a^3/((d*(-a*b)^(1/2)-b*c)/b+c
)*(Si(d*x+c+(d*(-a*b)^(1/2)-b*c)/b)*sin((d*(-a*b)^(1/2)-b*c)/b)+Ci(d*x+c+(
d*(-a*b)^(1/2)-b*c)/b)*cos((d*(-a*b)^(1/2)-b*c)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 758, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{x^3 (a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*((I*(a*b^2*d^2 + 24*b^3)*x^6 + 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + I*(a
^3*d^2 + 24*a^2*b)*x^2 + 9*(-I*b^3*x^6 - 2*I*a*b^2*x^4 - I*a^2*b*x^2)*sqrt
(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (I*(a*b^2*d
^2 + 24*b^3)*x^6 + 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + I*(a^3*d^2 + 24*a^2*b)
*x^2 + 9*(I*b^3*x^6 + 2*I*a*b^2*x^4 + I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(I*d*x
 + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-I*(a*b^2*d^2 + 24*b^3)*x^6 -
 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 - I*(a^3*d^2 + 24*a^2*b)*x^2 + 9*(I*b^3*x^
6 + 2*I*a*b^2*x^4 + I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))
*e^(-I*c + sqrt(a*d^2/b)) + (-I*(a*b^2*d^2 + 24*b^3)*x^6 - 2*I*(a^2*b*d^2
+ 24*a*b^2)*x^4 - I*(a^3*d^2 + 24*a^2*b)*x^2 + 9*(-I*b^3*x^6 - 2*I*a*b^2*x
^4 - I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt
(a*d^2/b)) + 16*((a*b^2*d^2 + 6*b^3)*x^6 + 2*(a^2*b*d^2 + 6*a*b^2)*x^4 + (
a^3*d^2 + 6*a^2*b)*x^2)*cos_integral(d*x)*sin(c) + 16*((a*b^2*d^2 + 6*b^3)
*x^6 + 2*(a^2*b*d^2 + 6*a*b^2)*x^4 + (a^3*d^2 + 6*a^2*b)*x^2)*cos(c)*sin_i
ntegral(d*x) + 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)*cos(d*x + c)
+ 8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*sin(d*x + c))/(a^4*b^2*x^6 + 2*a^5
*b*x^4 + a^6*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3 (a+bx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/x**3/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^3\,{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int(sin(c + d*x)/(x^3*(a + b*x^2)^3),x)
```

output
```
int(sin(c + d*x)/(x^3*(a + b*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{b^3x^9+3a\,b^2x^7+3a^2b\,x^5+a^3x^3}dx$$

input
```
int(sin(d*x+c)/x^3/(b*x^2+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3*x**3 + 3*a**2*b*x**5 + 3*a*b**2*x**7 + b**3*x**9),x
)
```

3.79 $\int x^3(a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 156

$$\int x^3(a+bx^3)\sin(c+dx)\,dx=\frac{720b\cos(c+dx)}{d^7}+\frac{6ax\cos(c+dx)}{d^3}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+dx)}{d^3}-\frac{bx^6\cos(c+dx)}{d}-\frac{6a\sin(c+dx)}{d^4}+\frac{720bx\sin(c+dx)}{d^6}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{120bx^3\sin(c+dx)}{d^4}+\frac{6bx^5\sin(c+dx)}{d^2}$$

output
```
720*b*cos(d*x+c)/d^7+6*a*x*cos(d*x+c)/d^3-360*b*x^2*cos(d*x+c)/d^5-a*x^3*c
os(d*x+c)/d+30*b*x^4*cos(d*x+c)/d^3-b*x^6*cos(d*x+c)/d-6*a*sin(d*x+c)/d^4+
720*b*x*sin(d*x+c)/d^6+3*a*x^2*sin(d*x+c)/d^2-120*b*x^3*sin(d*x+c)/d^4+6*b
*x^5*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$\int x^3\left(a+bx^3\right)\sin(c+dx)\,dx$$
$$=\frac{-((ad^4x(-6+d^2x^2)+b(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+3d(ad^2(-2+d^2x^2)+2bx(1}{d^7}$$

input
```
Integrate[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(-((a*d^4*x*(-6 + d^2*x^2) + b*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6)
)*Cos[c + d*x]) + 3*d*(a*d^2*(-2 + d^2*x^2) + 2*b*x*(120 - 20*d^2*x^2 + d^
4*x^4))*Sin[c + d*x])/d^7
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^3\right)\sin(c+dx)\,dx$$
$$\Big\downarrow \ 3820$$
$$\int \left(ax^3\sin(c+dx)+bx^6\sin(c+dx)\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{ax^3\cos(c+dx)}{d}+\frac{720b\cos(c+dx)}{d^7}+\frac{720bx\sin(c+dx)}{d^6}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{120bx^3\sin(c+dx)}{d^4}+\frac{30bx^4\cos(c+dx)}{d^3}+\frac{6bx^5\sin(c+dx)}{d^2}-\frac{bx^6\cos(c+dx)}{d}$$

input
```
Int[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(720*b*Cos[c + d*x])/d^7 + (6*a*x*Cos[c + d*x])/d^3 - (360*b*x^2*Cos[c + d
*x])/d^5 - (a*x^3*Cos[c + d*x])/d + (30*b*x^4*Cos[c + d*x])/d^3 - (b*x^6*C
os[c + d*x])/d - (6*a*Sin[c + d*x])/d^4 + (720*b*x*Sin[c + d*x])/d^6 + (3*
a*x^2*Sin[c + d*x])/d^2 - (120*b*x^3*Sin[c + d*x])/d^4 + (6*b*x^5*Sin[c +
d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(b x^6 d^6+a d^6 x^3-30 b x^4 d^4-6 a d^4 x+360 x^2 d^2 b-720 b)\cos(dx+c)}{d^7}+\frac{3(2 b d^4 x^5+a d^4 x^2-40 b d^2 x^3-2 d^2 a+240 b x)\text{si}}{d^6}$
parallelrisch	$\frac{(x^3(b x^3+a) d^6-6 x(5 b x^3+a) d^4+360 x^2 d^2 b-1440 b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+6 d(x^2(2 b x^3+a) d^4+(-40 b x^3-2 a) d^2+240 b x)\tan}{d^7\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$\frac{6(2 b^2 d^6 x^9+3 a b d^6 x^6-50 b^2 d^4 x^7+a^2 d^6 x^3-42 a b d^4 x^4+480 b^2 d^2 x^5-4 a^2 d^4 x+300 a b d^2 x^2-720 x^3 b^2-360 a b)\sin(dx+c)}{d^8 x(b x^3+a)}$
norman	$\frac{\frac{1440 b}{d^7}+\frac{a x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{b x^6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{12 a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6 a x}{d^3}-\frac{a x^3}{d}-\frac{360 b x^2}{d^5}+\frac{30 b x^4}{d^3}-\frac{b x^6}{d}-\frac{6 a x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}+}{1+\tan}$
meijerg	$\frac{64 b\sin(c)\sqrt{\pi}\left(\frac{x(d^2)^{\frac{7}{2}}\left(\frac{21}{8}x^4 d^4-\frac{105}{2}x^2 d^2+315\right)\cos(dx)}{28\sqrt{\pi}\,d^6}-\frac{(d^2)^{\frac{7}{2}}\left(-\frac{7}{16}x^6 d^6+\frac{105}{8}x^4 d^4-\frac{315}{2}x^2 d^2+315\right)\sin(dx)}{28\sqrt{\pi}\,d^7}\right)}{d^6\sqrt{d^2}}+\frac{64 b\,c}{}$
parts	$-\frac{b x^6\cos(dx+c)}{d}-\frac{a x^3\cos(dx+c)}{d}+\frac{\frac{3 a c^2\sin(dx+c)}{d^2}-\frac{6 a c(\cos(dx+c)+(dx+c)\sin(dx+c))}{d^2}+\frac{3 a\left((dx+c)^2\sin(dx+c)-2\sin\right.}{d}}{}$
derivativedivides	$\frac{a c^3\cos(dx+c)+3 a c^2(\sin(dx+c)-\cos(dx+c)(dx+c))-3 a c\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{}$
default	$\frac{a c^3\cos(dx+c)+3 a c^2(\sin(dx+c)-\cos(dx+c)(dx+c))-3 a c\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{}$

input
```
int(x^3*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d^6*x^6+a*d^6*x^3-30*b*d^4*x^4-6*a*d^4*x+360*b*d^2*x^2-720*b)/d^7*cos(
d*x+c)+3/d^6*(2*b*d^4*x^5+a*d^4*x^2-40*b*d^2*x^3-2*a*d^2+240*b*x)*sin(d*x+
c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int x^3(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^6x^6+ad^6x^3-30\,bd^4x^4-6\,ad^4x+360\,bd^2x^2-720\,b)\cos(dx+c)-3\,(2\,bd^5x^5+ad^5x^2-40\,bd^3x^3}{d^7}$$

input
```
integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*
b)*cos(d*x + c) - 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 24
0*b*d*x)*sin(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$= \begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^6\cos(c+dx)}{d} + \frac{6bx^5\sin(c+dx)}{d^2} + \frac{30bx^4\cos(c+dx)}{d^3} - \\ \left(\frac{ax^4}{4} + \frac{bx^7}{7}\right)\sin(c) \end{cases}$$

input
```
integrate(x**3*(b*x**3+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos
(c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**6*cos(c + d*x)/d + 6*b*x**5*
sin(c + d*x)/d**2 + 30*b*x**4*cos(c + d*x)/d**3 - 120*b*x**3*sin(c + d*x)/
d**4 - 360*b*x**2*cos(c + d*x)/d**5 + 720*b*x*sin(c + d*x)/d**6 + 720*b*co
s(c + d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(156) = 312$.

Time = 0.06 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.88

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$= \frac{ac^3\cos(dx+c) - \frac{bc^6\cos(dx+c)}{d^3} - 3((dx+c)\cos(dx+c) - \sin(dx+c))ac^2 + \frac{6((dx+c)\cos(dx+c)-\sin(dx+c))bc^5}{d^3}}{}$$

input
```
integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a*c^3*cos(d*x + c) - b*c^6*cos(d*x + c)/d^3 - 3*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^5/d^3
 + 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 15*
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^4/d^3 - ((
(d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c)
)*a + 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*s
in(d*x + c))*b*c^3/d^3 - 15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x +
 c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c^2/d^3 + 6*(((d*x + c
)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12
*(d*x + c)^2 + 24)*sin(d*x + c))*b*c/d^3 - (((d*x + c)^6 - 30*(d*x + c)^4
+ 360*(d*x + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 +
120*d*x + 120*c)*sin(d*x + c))*b/d^3)/d^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$= -\frac{(bd^6x^6+ad^6x^3-30\,bd^4x^4-6\,ad^4x+360\,bd^2x^2-720\,b)\cos(dx+c)}{d^7} + \frac{3\,(2\,bd^5x^5+ad^5x^2-40\,bd^3x^3-2\,ad^3+240\,bdx)\sin(dx+c)}{d^7}$$

input
```
integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b
)*cos(d*x + c)/d^7 + 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 +
 240*b*d*x)*sin(d*x + c)/d^7
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$= \frac{d^4\,(6\,a\,x\,\cos\left(c+d\,x\right)+30\,b\,x^4\,\cos\left(c+d\,x\right))+720\,b\,\cos\left(c+d\,x\right)-d^6\,(a\,x^3\,\cos\left(c+d\,x\right)+b\,x^6\,\cos\left(c+\right.}{}$$

input
```
int(x^3*sin(c + d*x)*(a + b*x^3),x)
```

output
```
(d^4*(6*a*x*cos(c + d*x) + 30*b*x^4*cos(c + d*x)) + 720*b*cos(c + d*x) - d
^6*(a*x^3*cos(c + d*x) + b*x^6*cos(c + d*x)) + d^5*(3*a*x^2*sin(c + d*x) +
 6*b*x^5*sin(c + d*x)) - d^3*(6*a*sin(c + d*x) + 120*b*x^3*sin(c + d*x)) +
 720*b*d*x*sin(c + d*x) - 360*b*d^2*x^2*cos(c + d*x))/d^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$= \frac{-\cos\left(dx+c\right)a\,d^6x^3+6\cos\left(dx+c\right)a\,d^4x-\cos\left(dx+c\right)b\,d^6x^6+30\cos\left(dx+c\right)b\,d^4x^4-360\cos\left(dx+\right.}{}$$

input
```
int(x^3*(b*x^3+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**6*x**3 + 6*cos(c + d*x)*a*d**4*x - cos(c + d*x)*b*d*
*6*x**6 + 30*cos(c + d*x)*b*d**4*x**4 - 360*cos(c + d*x)*b*d**2*x**2 + 720
*cos(c + d*x)*b + 3*sin(c + d*x)*a*d**5*x**2 - 6*sin(c + d*x)*a*d**3 + 6*s
in(c + d*x)*b*d**5*x**5 - 120*sin(c + d*x)*b*d**3*x**3 + 720*sin(c + d*x)*
b*d*x)/d**7
```

3.80 $\int x^2(a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int x^2(a+bx^3)\sin(c+dx)\,dx = \frac{2a\cos(c+dx)}{d^3}-\frac{120bx\cos(c+dx)}{d^5}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}+\frac{2ax\sin(c+dx)}{d^2}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{5bx^4\sin(c+dx)}{d^2}$$

output
```
2*a*cos(d*x+c)/d^3-120*b*x*cos(d*x+c)/d^5-a*x^2*cos(d*x+c)/d+20*b*x^3*cos(
d*x+c)/d^3-b*x^5*cos(d*x+c)/d+120*b*sin(d*x+c)/d^6+2*a*x*sin(d*x+c)/d^2-60
*b*x^2*sin(d*x+c)/d^4+5*b*x^4*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

$$\int x^2(a+bx^3)\sin(c+dx)\,dx = \frac{-d(ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\cos(c+dx)+(2ad^4x+5b(24-12d^2x^2+d^4x^4))\sin(c+}{d^6}$$

input
```
Integrate[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(-(d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x
]) + (2*a*d^4*x + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right) \sin(c+dx) \, dx$$

$$\Big\downarrow \text{ 3820}$$

$$\int \left(ax^2 \sin(c+dx)+bx^5 \sin(c+dx)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}-\frac{120bx\cos(c+dx)}{d^5}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{20bx^3\cos(c+dx)}{d^3}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{bx^5\cos(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(2*a*Cos[c + d*x])/d^3 - (120*b*x*Cos[c + d*x])/d^5 - (a*x^2*Cos[c + d*x])
/d + (20*b*x^3*Cos[c + d*x])/d^3 - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c +
 d*x])/d^6 + (2*a*x*Sin[c + d*x])/d^2 - (60*b*x^2*Sin[c + d*x])/d^4 + (5*b
*x^4*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\left(b\,d^4x^5+a\,d^4x^2-20b\,d^2x^3-2d^2a+120bx\right)\cos\left(dx+c\right)}{d^5}+\frac{\left(5b\,x^4d^4+2a\,d^4x-60x^2d^2b+120b\right)\sin\left(dx+c\right)}{d^6}$
parallelrisch	$\frac{d\left(x\left(b\,x^3+a\right)d^4-20x^2d^2b+120b\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\left(10b\,x^4+4ax\right)d^4-120x^2d^2b+240b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-d\left(x^2\left(b\,x^3+a\right)d^4\cdots\right.\right.}{d^6\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$\frac{2\left(5b^2d^4x^8+7ab\,d^4x^5-80b^2d^2x^6+2a^2d^4x^2-55ab\,d^2x^3+360b^2x^4-2a^2d^2+180abx\right)\sin\left(dx+c\right)}{d^6x\left(b\,x^3+a\right)}-\frac{\left(b\,d^4x^5+a\,d^4x^2-20\cdots\right.}{\cdots}$
norman	$\frac{\frac{4a}{d^3}+\frac{a\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{b\,x^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{a\,x^2}{d}+\frac{240b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^6}-\frac{120bx}{d^5}+\frac{20b\,x^3}{d^3}-\frac{b\,x^5}{d}+\frac{4ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}+\frac{120bx\tan\left(\cdots\right)}{d^5}\cdots}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
meijerg	$\frac{32b\sin\left(c\right)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}x^4d^4-\frac{45}{2}x^2d^2+45\right)\cos\left(dx\right)}{12\sqrt{\pi}}+\frac{xd\left(\frac{3}{8}x^4d^4-\frac{15}{2}x^2d^2+45\right)\sin\left(dx\right)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32b\cos\left(c\right)\sqrt{\pi}\left(-\frac{xd\left(\frac{7}{8}x^4\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{b\,x^5\cos\left(dx+c\right)}{d}-\frac{a\,x^2\cos\left(dx+c\right)}{d}+\frac{-\frac{2ac\sin\left(dx+c\right)}{d}+\frac{2a\left(\cos\left(dx+c\right)+\left(dx+c\right)\sin\left(dx+c\right)\right)}{d}+\frac{5b\,c^4\sin\left(dx+c\right)}{d^4}-\frac{20b\,c^3\left(\cos\left(\cdots\right)\right.}{\cdots}\cdots}{\cdots}$
derivativedivides	$\frac{-a\,c^2\cos\left(dx+c\right)-2ac\left(\sin\left(dx+c\right)-\cos\left(dx+c\right)\left(dx+c\right)\right)+a\left(-\left(dx+c\right)^2\cos\left(dx+c\right)+2\cos\left(dx+c\right)+2\left(dx+c\right)\sin\left(dx+c\right)\right)+\cdots}{\cdots}$
default	$\frac{-a\,c^2\cos\left(dx+c\right)-2ac\left(\sin\left(dx+c\right)-\cos\left(dx+c\right)\left(dx+c\right)\right)+a\left(-\left(dx+c\right)^2\cos\left(dx+c\right)+2\cos\left(dx+c\right)+2\left(dx+c\right)\sin\left(dx+c\right)\right)+\cdots}{\cdots}$

input
```
int(x^2*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^5*(b*d^4*x^5+a*d^4*x^2-20*b*d^2*x^3-2*a*d^2+120*b*x)*cos(d*x+c)+(5*b*
d^4*x^4+2*a*d^4*x-60*b*d^2*x^2+120*b)/d^6*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int x^2(a+bx^3)\sin(c+dx)\,dx =$$
$$-\frac{(bd^5x^5+ad^5x^2-20\,bd^3x^3-2\,ad^3+120\,bdx)\cos(dx+c)-(5\,bd^4x^4+2\,ad^4x-60\,bd^2x^2+120\,b)\sin}{d^6}$$

input
```
integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c
) - (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int x^2(a+bx^3)\sin(c+dx)\,dx$$
$$=\begin{cases} -\frac{ax^2\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+\frac{5bx^4\sin(c+dx)}{d^2}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{60bx^2\sin(c+dx)}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^6}{6}\right)\sin(c) \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c +
d*x)/d**3 - b*x**5*cos(c + d*x)/d + 5*b*x**4*sin(c + d*x)/d**2 + 20*b*x**3
*cos(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4 - 120*b*x*cos(c + d*x)/d*
*5 + 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*sin(c), Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(126) = 252.

Time = 0.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.59

$$\int x^2 (a + bx^3) \sin(c + dx) \, dx =$$

$$-\frac{ac^2 \cos(dx + c) - \frac{bc^5 \cos(dx+c)}{d^3} - 2((dx + c)\cos(dx + c) - \sin(dx + c))ac + \frac{5((dx+c)\cos(dx+c)-\sin(dx+c))b}{d^3}}{}$$

input
```
integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*c^2*cos(d*x + c) - b*c^5*cos(d*x + c)/d^3 - 2*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*a*c + 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^3
+ (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 10*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^3 + 10*(((d
*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*
b*c^2/d^3 - 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x
+ c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^3 + (((d*x + c)^5 - 20*(d*x + c)
^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)
*sin(d*x + c))*b/d^3)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int x^2 (a + bx^3) \sin(c + dx) \, dx = -\frac{(bd^5x^5 + ad^5x^2 - 20\, bd^3x^3 - 2\, ad^3 + 120\, bdx)\cos(dx + c)}{d^6} + \frac{(5\, bd^4x^4 + 2\, ad^4x - 60\, bd^2x^2 + 120\, b)\sin(dx + c)}{d^6}$$

input
```
integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)
/d^6 + (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c)/d^6
```

Mupad [B] (verification not implemented)

Time = 41.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^3)\sin(c+dx)\,dx$$
$$=\frac{120\,b\,\sin(c+d\,x)+d^4\,(5\,b\,x^4\,\sin(c+d\,x)+2\,a\,x\,\sin(c+d\,x))-d^5\,(a\,x^2\,\cos(c+d\,x)+b\,x^5\,\cos(c+d}{d^6}$$

input
```
int(x^2*sin(c + d*x)*(a + b*x^3),x)
```

output
```
(120*b*sin(c + d*x) + d^4*(5*b*x^4*sin(c + d*x) + 2*a*x*sin(c + d*x)) - d^
5*(a*x^2*cos(c + d*x) + b*x^5*cos(c + d*x)) + d^3*(2*a*cos(c + d*x) + 20*b
*x^3*cos(c + d*x)) - 60*b*d^2*x^2*sin(c + d*x) - 120*b*d*x*cos(c + d*x))/d
^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^3)\sin(c+dx)\,dx$$
$$=\frac{-\cos(dx+c)\,a\,d^5x^2+2\cos(dx+c)\,a\,d^3-\cos(dx+c)\,b\,d^5x^5+20\cos(dx+c)\,b\,d^3x^3-120\cos(dx+c}{d^6}$$

input
```
int(x^2*(b*x^3+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**5*x**2 + 2*cos(c + d*x)*a*d**3 - cos(c + d*x)*b*d**5
*x**5 + 20*cos(c + d*x)*b*d**3*x**3 - 120*cos(c + d*x)*b*d*x + 2*sin(c + d
*x)*a*d**4*x + 5*sin(c + d*x)*b*d**4*x**4 - 60*sin(c + d*x)*b*d**2*x**2 +
120*sin(c + d*x)*b)/d**6
```

3.81 $\int x(a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{24b\cos(c+dx)}{d^5}-\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{24bx\sin(c+dx)}{d^4}+\frac{4bx^3\sin(c+dx)}{d^2}$$

output
```
-24*b*cos(d*x+c)/d^5-a*x*cos(d*x+c)/d+12*b*x^2*cos(d*x+c)/d^3-b*x^4*cos(d*
x+c)/d+a*sin(d*x+c)/d^2-24*b*x*sin(d*x+c)/d^4+4*b*x^3*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int x(a+bx^3)\sin(c+dx)\,dx = \frac{-((ad^4x+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+d(ad^2+4bx(-6+d^2x^2))\sin(c+dx)}{d^5}$$

input
```
Integrate[x*(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(-((a*d^4*x + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + d*(a*d^2 + 4*
b*x*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)\sin(c+dx)\,dx$$
$$\Big\downarrow \text{3820}$$
$$\int (ax\sin(c+dx)+bx^4\sin(c+dx))\,dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

input
```
Int[x*(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(-24*b*Cos[c + d*x])/d^5 - (a*x*Cos[c + d*x])/d + (12*b*x^2*Cos[c + d*x])/
d^3 - (b*x^4*Cos[c + d*x])/d + (a*Sin[c + d*x])/d^2 - (24*b*x*Sin[c + d*x]
)/d^4 + (4*b*x^3*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(b\,x^4 d^4+a\,d^4 x-12x^2 d^2 b+24b\right)\cos(dx+c)}{d^5}+\frac{\left(4b\,d^2 x^3+d^2 a-24bx\right)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{\left(\left(b\,x^3+a\right)d^2-12bx\right)d^2 x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2d\left(\left(4b\,x^3+a\right)d^2-24bx\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-b\,x^4-ax\right)d^4+12x^2 d^2 b-48b}{d^5\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
norman	$\frac{\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{b\,x^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{48b}{d^5}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{ax}{d}+\frac{12b\,x^2}{d^3}-\frac{b\,x^4}{d}-\frac{48bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}-\frac{12b\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}+}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
orering	$\frac{2\left(4b^2 d^4 x^7+5ab\,d^4 x^4-36b^2 d^2 x^5+a^2 d^4 x-18ab\,d^2 x^2+48x^3 b^2+12ab\right)\sin(dx+c)}{d^6 x(b\,x^3+a)}-\frac{\left(b\,x^4 d^4+a\,d^4 x-12x^2 d^2 b+24b\right)\left(\left(b\right.\right.}{}$
parts	$-\frac{b\,x^4\cos(dx+c)}{d}-\frac{ax\cos(dx+c)}{d}+\frac{a\sin(dx+c)-\frac{4b\,c^3\sin(dx+c)}{d^3}+\frac{12b\,c^2(\cos(dx+c)+(dx+c)\sin(dx+c))}{d^3}-\frac{12bc\left((dx+c)\right.}{}}{}$
meijerg	$\frac{16b\sin(c)\sqrt{\pi}\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}x^4d^4-\frac{15}{2}x^2d^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\cos(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^4\right.}{}\right.}{}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{b\,c^4\cos(dx+c)}{d^3}-\frac{4b\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{6b\,c^2\left(-(dx+c)^2\cos\right.}{}}{}$
default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{b\,c^4\cos(dx+c)}{d^3}-\frac{4b\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{6b\,c^2\left(-(dx+c)^2\cos\right.}{}}{}$

input
```
int(x*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d^4*x^4+a*d^4*x-12*b*d^2*x^2+24*b)/d^5*cos(d*x+c)+1/d^4*(4*b*d^2*x^3+a
*d^2-24*b*x)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^4x^4+ad^4x-12\,bd^2x^2+24\,b)\cos(dx+c)-(4\,bd^3x^3+ad^3-24\,bdx)\sin(dx+c)}{d^5}$$

input
```
integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c) - (4*b*d^3*x^3
+ a*d^3 - 24*b*d*x)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int x(a+bx^3)\sin(c+dx)\,dx = \begin{cases} -\frac{ax\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}-\frac{24b\cos(c+dx)}{d^5} & \text{fo} \\ \left(\frac{ax^2}{2}+\frac{bx^5}{5}\right)\sin(c) & \text{ot} \end{cases}$$

input
```
integrate(x*(b*x**3+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**4*cos(c + d*x)
/d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin
(c + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5/5)
*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(95) = 190$.

Time = 0.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.36

$$\int x(a+bx^3)\sin(c+dx)\,dx$$
$$= \frac{ac\cos(dx+c) - \frac{bc^4\cos(dx+c)}{d^3} - ((dx+c)\cos(dx+c) - \sin(dx+c))a + \frac{4((dx+c)\cos(dx+c)-\sin(dx+c))bc^3}{d^3} - \frac{6}{}}{}$$

input
```
integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a*c*cos(d*x + c) - b*c^4*cos(d*x + c)/d^3 - ((d*x + c)*cos(d*x + c) - sin
(d*x + c))*a + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^3 - 6*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^3 + 4*(((
d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))
*b*c/d^3 - ((((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c
)^3 - 6*d*x - 6*c)*sin(d*x + c))*b/d^3)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x - 12\,bd^2x^2 + 24\,b)\cos(dx+c)}{d^5} + \frac{(4\,bd^3x^3 + ad^3 - 24\,bdx)\sin(dx+c)}{d^5}$$

input
```
integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c)/d^5 + (4*b*d^3*x
^3 + a*d^3 - 24*b*d*x)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 43.89 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{d^4\,\left(a\,x\,\cos\left(c+d\,x\right)+b\,x^4\,\cos\left(c+d\,x\right)\right)+24\,b\,\cos\left(c+d\,x\right)-d^3\,\left(a\,\sin\left(c+d\,x\right)+4\,b\,x^3\,\sin\left(c+d\,x\right)\right)}{d^5}$$

input
```
int(x*sin(c + d*x)*(a + b*x^3),x)
```

output
```
-(d^4*(a*x*cos(c + d*x) + b*x^4*cos(c + d*x)) + 24*b*cos(c + d*x) - d^3*(a
*sin(c + d*x) + 4*b*x^3*sin(c + d*x)) + 24*b*d*x*sin(c + d*x) - 12*b*d^2*x
^2*cos(c + d*x))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int x(a+bx^3)\sin(c+dx)\,dx = \frac{-\cos(dx+c)\,a\,d^4x-\cos(dx+c)\,b\,d^4x^4+12\cos(dx+c)\,b\,d^2x^2-24\cos(dx+c)\,b+\sin(dx+c)\,a\,d^3}{d^5}$$

input
```
int(x*(b*x^3+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**4*x - cos(c + d*x)*b*d**4*x**4 + 12*cos(c + d*x)*b*d
**2*x**2 - 24*cos(c + d*x)*b + sin(c + d*x)*a*d**3 + 4*sin(c + d*x)*b*d**3
*x**3 - 24*sin(c + d*x)*b*d*x)/d**5
```

3.82 $\int (a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4}+\frac{3bx^2\sin(c+dx)}{d^2}$$

output
```
-a*cos(d*x+c)/d+6*b*x*cos(d*x+c)/d^3-b*x^3*cos(d*x+c)/d-6*b*sin(d*x+c)/d^4
+3*b*x^2*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int (a+bx^3)\sin(c+dx)\,dx = \frac{-d(ad^2+bx(-6+d^2x^2))\cos(c+dx)+3b(-2+d^2x^2)\sin(c+dx)}{d^4}$$

input
```
Integrate[(a + b*x^3)*Sin[c + d*x],x]
```

output
```
(-(d*(a*d^2 + b*x*(-6 + d^2*x^2))*Cos[c + d*x]) + 3*b*(-2 + d^2*x^2)*Sin[c
 + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\sin(c+dx)\,dx$$

$$\Big\downarrow \text{3810}$$

$$\int \left(a\sin(c+dx)+bx^3\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

input
```
Int[(a + b*x^3)*Sin[c + d*x],x]
```

output
```
-((a*Cos[c + d*x])/d) + (6*b*x*Cos[c + d*x])/d^3 - (b*x^3*Cos[c + d*x])/d
- (6*b*Sin[c + d*x])/d^4 + (3*b*x^2*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3810
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(b\,d^2x^3+d^2a-6bx)\cos(dx+c)}{d^3}+\frac{3b(x^2d^2-2)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{dxb(x^2d^2-6)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+6b(x^2d^2-2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-b\,x^3-2a)d^3+6dxb}{d^4\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
parts	$-\frac{b\,x^3\cos(dx+c)}{d}-\frac{a\cos(dx+c)}{d}+\frac{3b\left(c^2\sin(dx+c)-2c(\cos(dx+c)+(dx+c)\sin(dx+c))+(dx+c)^2\sin(dx+c)-2\sin\right.}{d^4}$
orering	$\frac{6b(b\,d^2x^5+a\,d^2x^2-4b\,x^3-a)\sin(dx+c)}{d^4(b\,x^3+a)}-\frac{(b\,d^2x^3+d^2a-6bx)(3x^2b\sin(dx+c)+(b\,x^3+a)d\cos(dx+c))}{d^4(b\,x^3+a)}$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{b\,x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{12b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6bx}{d^3}-\frac{b\,x^3}{d}-\frac{6bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^3}+\frac{6b\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
meijerg	$\frac{8b\sin(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2d^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{xd\left(-\frac{x^2d^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\cos(c)\sqrt{\pi}\left(\frac{xd\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-1\right.}{}\right.}{d^4}$
derivativedivides	$\frac{-\cos(dx+c)a+\frac{b\,c^3\cos(dx+c)}{d^3}+\frac{3b\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}-\frac{3bc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c\right.}{d^3}}{d}$
default	$\frac{-\cos(dx+c)a+\frac{b\,c^3\cos(dx+c)}{d^3}+\frac{3b\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}-\frac{3bc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c\right.}{d^3}}{d}$

input
```
int((b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(b*d^2*x^3+a*d^2-6*b*x)*cos(d*x+c)+3/d^4*b*(d^2*x^2-2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^3x^3+ad^3-6\,bdx)\cos(dx+c)-3\,(bd^2x^2-2\,b)\sin(dx+c)}{d^4}$$

input
```
integrate((b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c) - 3*(b*d^2*x^2 - 2*b)*sin(d*x
 + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int (a+bx^3)\sin(c+dx)\,dx = \begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{3bx^2\sin(c+dx)}{d^2} + \frac{6bx\cos(c+dx)}{d^3} - \frac{6b\sin(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**3+a)*sin(d*x+c),x)
```

output
```
Piecewise((-a*cos(c + d*x)/d - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*
x)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*
x + b*x**4/4)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(68) = 136.

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.07

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{a\cos(dx+c) - \frac{bc^3\cos(dx+c)}{d^3} + \frac{3((dx+c)\cos(dx+c)-\sin(dx+c))bc^2}{d^3} - \frac{3\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc}{d^3} +}{d}$$

input
```
integrate((b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*cos(d*x + c) - b*c^3*cos(d*x + c)/d^3 + 3*((d*x + c)*cos(d*x + c) - si
n(d*x + c))*b*c^2/d^3 - 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*si
n(d*x + c))*b*c/d^3 + (((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x
+ c)^2 - 2)*sin(d*x + c))*b/d^3)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^3x^3+ad^3-6\,bdx)\cos(dx+c)}{d^4} + \frac{3\,(bd^2x^2-2\,b)\sin(dx+c)}{d^4}$$

input
```
integrate((b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c)/d^4 + 3*(b*d^2*x^2 - 2*b)*sin(
d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{6\,b\,\sin(c+d\,x)+d^3\,(a\,\cos(c+d\,x)+b\,x^3\,\cos(c+d\,x))-3\,b\,d^2\,x^2\,\sin(c+d\,x)-6\,b\,d\,x\,\cos(c+d\,x}{d^4}$$

input
```
int(sin(c + d*x)*(a + b*x^3),x)
```

output
```
-(6*b*sin(c + d*x) + d^3*(a*cos(c + d*x) + b*x^3*cos(c + d*x)) - 3*b*d^2*x
^2*sin(c + d*x) - 6*b*d*x*cos(c + d*x))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int (a + bx^3) \sin(c + dx)\, dx$$
$$= \frac{-\cos(dx + c)\, a\, d^3 - \cos(dx + c)\, b\, d^3 x^3 + 6 \cos(dx + c)\, bdx + 3 \sin(dx + c)\, b\, d^2 x^2 - 6 \sin(dx + c)\, b}{d^4}$$

input
```
int((b*x^3+a)*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a*d**3 - cos(c + d*x)*b*d**3*x**3 + 6*cos(c + d*x)*b*d*x
+ 3*sin(c + d*x)*b*d**2*x**2 - 6*sin(c + d*x)*b)/d**4
```

3.83 $\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = \frac{2b\cos(c+dx)}{d^3} - \frac{bx^2\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{2bx\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

output
```
2*b*cos(d*x+c)/d^3-b*x^2*cos(d*x+c)/d+a*Ci(d*x)*sin(c)+2*b*x*sin(d*x+c)/d^
2+a*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = a\,\text{CosIntegral}(dx)\sin(c) + \frac{b((2-d^2x^2)\cos(c+dx)+2dx\sin(c+dx))}{d^3} + a\cos(c)\text{Si}(dx)$$

input
```
Integrate[((a + b*x^3)*Sin[c + d*x])/x,x]
```

output
```
a*CosIntegral[d*x]*Sin[c] + (b*((2 - d^2*x^2)*Cos[c + d*x] + 2*d*x*Sin[c +
 d*x]))/d^3 + a*Cos[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a\sin(c+dx)}{x}+bx^2\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\sin(c)\,\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)*Sin[c + d*x])/x,x]
```

output
```
(2*b*Cos[c + d*x])/d^3 - (b*x^2*Cos[c + d*x])/d + a*CosIntegral[d*x]*Sin[c
] + (2*b*x*Sin[c + d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{e^{-ic}\pi\ \mathrm{csgn}(dx)a}{2}-\frac{ie^{-ic}\ \mathrm{expIntegral}_1(-idx)a}{2}+\frac{ia\,e^{ic}\ \mathrm{expIntegral}_1(-idx)}{2}-\frac{b\,x^2\cos(dx+c)}{d}+e^{-ic}\ \mathrm{Si}\,(d$
derivativedivides	$a(\mathrm{Si}\,(dx)\cos\,(c)+\mathrm{Ci}\,(dx)\sin\,(c))-\frac{3c^2b\cos(dx+c)}{d^3}-\frac{3bc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{(c}{}$
default	$a(\mathrm{Si}\,(dx)\cos\,(c)+\mathrm{Ci}\,(dx)\sin\,(c))-\frac{3c^2b\cos(dx+c)}{d^3}-\frac{3bc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{(c}{}$
meijerg	$\frac{4b\sin(c)\sqrt{\pi}\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3x^2d^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b\cos(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2d^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx}{2\sqrt{\pi}}\right.}{d^3}$

input
```
int((b*x^3+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-I*c)*Pi*csgn(d*x)*a-1/2*I*exp(-I*c)*Ei(1,-I*d*x)*a+1/2*I*a*exp(I
*c)*Ei(1,-I*d*x)-b*x^2*cos(d*x+c)/d+exp(-I*c)*Si(d*x)*a+2*b*x*sin(d*x+c)/d
^2+2*b*cos(d*x+c)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = \frac{ad^3\,\mathrm{Ci}\,(dx)\sin\,(c)+ad^3\cos\,(c)\,\mathrm{Si}\,(dx)+2\,bdx\sin\,(dx+c)-(bd^2x^2-2\,b)\cos\,(dx+c)}{d^3}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

output
```
(a*d^3*cos_integral(d*x)*sin(c) + a*d^3*cos(c)*sin_integral(d*x) + 2*b*d*x
*sin(d*x + c) - (b*d^2*x^2 - 2*b)*cos(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{(a + bx^3)\sin(c + dx)}{x}\,dx$$
$$= a\sin(c)\operatorname{Ci}(dx) + a\cos(c)\operatorname{Si}(dx) + bx^2\left(\begin{cases} x\sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right)$$
$$- 2b\left(\begin{cases} \frac{x^3\sin(c)}{3} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{x\sin(c+dx)}{d} + \frac{\cos(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2\cos(c)}{2} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**3+a)*sin(d*x+c)/x,x)
```

output
```
a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x**2*Piecewise((x*sin(c), Eq(d, 0)
), (-cos(c + d*x)/d, True)) - 2*b*Piecewise((x**3*sin(c)/3, Eq(d, 0)), (-P
iecewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*cos(c)/2,
 True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{(a + bx^3)\sin(c + dx)}{x}\,dx$$
$$= \frac{(a(-i\operatorname{Ei}(i\,dx) + i\operatorname{Ei}(-i\,dx))\cos(c) + a(\operatorname{Ei}(i\,dx) + \operatorname{Ei}(-i\,dx))\sin(c))d^3 + 4\,bdx\sin(dx + c) - 2\,(bd^2x^2}{2\,d^3}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

output
```
1/2*((a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(Ei(I*d*x) + Ei(-I*d*x))*
sin(c))*d^3 + 4*b*d*x*sin(d*x + c) - 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/d^3
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 510, normalized size of antiderivative = 8.95

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="giac")
```

output
```
-1/2*(2*b*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d^3*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*imag_part(cos_integral(-d*x))
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*sin_integral(d*x)*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*a*d^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c
) - 2*a*d^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*
d^2*x^2*tan(1/2*d*x)^2 - a*d^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2
 + a*d^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^3*sin_integr
al(d*x)*tan(1/2*d*x)^2 - 8*b*d^2*x^2*tan(1/2*d*x)*tan(1/2*c) - 2*b*d^2*x^2
*tan(1/2*c)^2 + a*d^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^3*im
ag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*sin_integral(d*x)*tan(1
/2*c)^2 - 2*a*d^3*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*real_p
art(cos_integral(-d*x))*tan(1/2*c) + 8*b*d*x*tan(1/2*d*x)^2*tan(1/2*c) + 8
*b*d*x*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b*d^2*x^2 - a*d^3*imag_part(cos_integ
ral(d*x)) + a*d^3*imag_part(cos_integral(-d*x)) - 2*a*d^3*sin_integral(d*x
) - 4*b*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*b*d*x*tan(1/2*d*x) - 8*b*d*x*tan(1
/2*c) + 4*b*tan(1/2*d*x)^2 + 16*b*tan(1/2*d*x)*tan(1/2*c) + 4*b*tan(1/2*c)
^2 - 4*b)/(d^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*tan(1/2*d*x)^2 + d^3*tan(
1/2*c)^2 + d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = a\,\mathrm{cosint}(d\,x)\,\sin(c) + a\,\mathrm{sinint}(d\,x)\,\cos(c) + \frac{b\,(2\,\cos(c+d\,x) - d^2\,x^2\,\cos(c+d\,x) + 2\,d\,x\,\sin(c+d\,x))}{d^3}$$

input
```
int((sin(c + d*x)*(a + b*x^3))/x,x)
```

output
```
a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) + (b*(2*cos(c + d*x) - d^2*x^2
*cos(c + d*x) + 2*d*x*sin(c + d*x)))/d^3
```

Reduce [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = \frac{-\cos(dx+c)\,b\,d^2x^2+2\cos(dx+c)\,b+\left(\int \frac{\sin(dx+c)}{x}dx\right)a\,d^3+2\sin(dx+c)\,bdx}{d^3}$$

input
```
int((b*x^3+a)*sin(d*x+c)/x,x)
```

output
```
( - cos(c + d*x)*b*d**2*x**2 + 2*cos(c + d*x)*b + int(sin(c + d*x)/x,x)*a*
d**3 + 2*sin(c + d*x)*b*d*x)/d**3
```

3.84 $\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = -\frac{bx\cos(c+dx)}{d}+ad\cos(c)\operatorname{CosIntegral}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}-ad\sin(c)\operatorname{Si}(dx)$$

output
```
-b*x*cos(d*x+c)/d+a*d*cos(c)*Ci(d*x)+b*sin(d*x+c)/d^2-a*sin(d*x+c)/x-a*d*s
in(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = -\frac{bx\cos(c+dx)}{d}+ad\cos(c)\operatorname{CosIntegral}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}-ad\sin(c)\operatorname{Si}(dx)$$

input
```
Integrate[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

output
```
-((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d
^2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a\sin(c+dx)}{x^2}+bx\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\cos(c)\,\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+\frac{b\sin(c+dx)}{d^2}-\frac{bx\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

output

```
-((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d
^2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)+\frac{3bc\cos(dx+c)}{d^3}+\frac{(2c+1)b(\sin(dx+c)-\cos(d}{d^3}\right.$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)+\frac{3bc\cos(dx+c)}{d^3}+\frac{(2c+1)b(\sin(dx+c)-\cos(d}{d^3}\right.$
risch	$-\frac{i\,\mathrm{expIntegral}_1(-idx)\sin(c)a\,d^3x-i\,\mathrm{expIntegral}_1(idx)\sin(c)a\,d^3x+\mathrm{expIntegral}_1(-idx)\cos(c)a\,d^3x+\mathrm{expIntegral}_1(i}{2d^2x}$
meijerg	$\frac{2b\sin(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\cos(c)\sqrt{\pi}\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sin(c)\sqrt{\pi}\,d^2\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right.}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}\right.}{4\sqrt{d^2}}$

input
```
int((b*x^3+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+3/d^3*b*c*cos(d*x+c)+
(2*c+1)/d^3*b*(sin(d*x+c)-cos(d*x+c)*(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int\frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx=\frac{ad^3x\cos\left(c\right)\mathrm{Ci}\left(dx\right)-ad^3x\sin\left(c\right)\mathrm{Si}\left(dx\right)-bdx^2\cos\left(dx+c\right)-\left(ad^2-bx\right)\sin\left(dx+c\right)}{d^2x}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
(a*d^3*x*cos(c)*cos_integral(d*x) - a*d^3*x*sin(c)*sin_integral(d*x) - b*d
*x^2*cos(d*x + c) - (a*d^2 - b*x)*sin(d*x + c))/(d^2*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)\sin{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a)*sin(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**3)*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \frac{(a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos{(c)}+a(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin{(c)})d^3-2\,bdx\cos{(dx+}}{2\,d^2}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
1/2*((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a*(-I*gamma(-1, I*
d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^3 - 2*b*d*x*cos(d*x + c) + 2*b*sin(d
*x + c))/d^2
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 489, normalized size of antiderivative = 8.73

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a*d^3*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a
*d^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3
*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x*sin_integr
al(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x*real_part(cos_integral(d*x))*t
an(1/2*d*x)^2 - a*d^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + a*d
^3*x*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x*real_part(cos_int
egral(-d*x))*tan(1/2*c)^2 + 2*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
3*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*x*imag_part(cos_inte
gral(-d*x))*tan(1/2*c) + 4*a*d^3*x*sin_integral(d*x)*tan(1/2*c) - a*d^3*x*
real_part(cos_integral(d*x)) - a*d^3*x*real_part(cos_integral(-d*x)) - 2*b
*d*x^2*tan(1/2*d*x)^2 - 8*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 4*a*d^2*tan(1/
2*d*x)^2*tan(1/2*c) - 2*b*d*x^2*tan(1/2*c)^2 - 4*a*d^2*tan(1/2*d*x)*tan(1/
2*c)^2 + 4*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*b*d*x^2 + 4*a*d^2*tan(1/2*d*x) + 4*a*d^2*tan(1/2*c) - 4*b*x*tan(1/2*d
*x) - 4*b*x*tan(1/2*c))/(d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*x*tan(1/2
*d*x)^2 + d^2*x*tan(1/2*c)^2 + d^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)}{x^2}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3))/x^2,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \frac{-\cos(dx+c)\,bdx + \left(\int \frac{\sin(dx+c)}{x^2}dx\right)a\,d^2 + \sin(dx+c)\,b}{d^2}$$

input
```
int((b*x^3+a)*sin(d*x+c)/x^2,x)
```

output
```
( - cos(c + d*x)*b*d*x + int(sin(c + d*x)/x**2,x)*a*d**2 + sin(c + d*x)*b)
/d**2
```

3.85 $\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = -\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)$$

output

```
-b*cos(d*x+c)/d-1/2*a*d*cos(d*x+c)/x-1/2*a*d^2*Ci(d*x)*sin(c)-1/2*a*sin(d*
x+c)/x^2-1/2*a*d^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \frac{1}{2}\left(-\frac{2b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{x}-ad^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{x^2}-ad^2\cos(c)\text{Si}(dx)\right)$$

input
```
Integrate[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

output
```
((-2*b*Cos[c + d*x])/d - (a*d*Cos[c + d*x])/x - a*d^2*CosIntegral[d*x]*Sin
[c] - (a*Sin[c + d*x])/x^2 - a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a\sin(c+dx)}{x^3}+b\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}ad^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

output
```
-((b*Cos[c + d*x])/d) - (a*d*Cos[c + d*x])/(2*x) - (a*d^2*CosIntegral[d*x]
*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\operatorname{Si}(dx)\cos(c)}{2}-\frac{\operatorname{Ci}(dx)\sin(c)}{2}\right)-\frac{b\cos(dx+c)}{d^3}\right)$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\operatorname{Si}(dx)\cos(c)}{2}-\frac{\operatorname{Ci}(dx)\sin(c)}{2}\right)-\frac{b\cos(dx+c)}{d^3}\right)$
risch	$\frac{id^2\cos(c)a\ \operatorname{expIntegral}_1(idx)}{4}-\frac{id^2\cos(c)a\ \operatorname{expIntegral}_1(-idx)}{4}+\frac{d^2\sin(c)a\ \operatorname{expIntegral}_1(idx)}{4}+\frac{d^2\sin(c)a\ \operatorname{exp}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d}+\frac{b\cos(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\sin(c)\sqrt{\pi}\,d^2\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6x^2d^2+4}{\sqrt{\pi}\,x^2d^2}+\right.}{8}$

input
```
int((b*x^3+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*
Ci(d*x)*sin(c))-b*cos(d*x+c)/d^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = -\frac{ad^3x^2\operatorname{Ci}(dx)\sin(c)+ad^3x^2\cos(c)\operatorname{Si}(dx)+ad\sin(dx+c)+(ad^2x+2\,bx^2)\cos(dx+c)}{2\,dx^2}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d^3*x^2*cos_integral(d*x)*sin(c) + a*d^3*x^2*cos(c)*sin_integral(d
*x) + a*d*sin(d*x + c) + (a*d^2*x + 2*b*x^2)*cos(d*x + c))/(d*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)\sin{(c+dx)}}{x^3}\,dx$$

input
```
integrate((b*x**3+a)*sin(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**3)*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1146, normalized size of antiderivative = 16.37

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*(((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)^3
+ (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)*sin(c)
^2 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c)^3 + (I*
exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) + ((exp_int
egral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_integral_e(3
, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c))*b*c^3/((d*x + c)^2*(cos(c)^2
 + sin(c)^2)*d^3 - 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^3 + (c^2*cos(c)
^2 + c^2*sin(c)^2)*d^3) - ((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(
3, -I*d*x))*cos(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -
I*d*x))*cos(c)*sin(c)^2 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I
*d*x))*sin(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x
))*cos(c) + ((exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)
^2 + exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c))*a/(c^2*
cos(c)^2 + c^2*sin(c)^2 + (d*x + c)^2*(cos(c)^2 + sin(c)^2) - 2*(c*cos(c)^
2 + c*sin(c)^2)*(d*x + c)) - (2*((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3
*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin
(c)^2)*(d*x + c))*cos(d*x + c)^3 - 3*(b*c^3*(exp_integral_e(4, I*d*x) + ex
p_integral_e(4, -I*d*x))*cos(c)^3 + b*c^3*(exp_integral_e(4, I*d*x) + exp_
integral_e(4, -I*d*x))*cos(c)*sin(c)^2 + b*c^3*(-I*exp_integral_e(4, I*d*x
) + I*exp_integral_e(4, -I*d*x))*sin(c)^3 + b*c^3*(exp_integral_e(4, I*...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 564, normalized size of antiderivative = 8.06

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a*d^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
a*d^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^3*x^2*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^2*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^2*imag_part(cos_int
egral(d*x))*tan(1/2*d*x)^2 + a*d^3*x^2*imag_part(cos_integral(-d*x))*tan(1
/2*d*x)^2 - 2*a*d^3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^3*x^2*imag_
part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^3*x^2*imag_part(cos_integral(-d
*x))*tan(1/2*c)^2 + 2*a*d^3*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*a*d^3*x
^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*x^2*real_part(cos_int
egral(-d*x))*tan(1/2*c) - 2*a*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*x^
2*imag_part(cos_integral(d*x)) + a*d^3*x^2*imag_part(cos_integral(-d*x)) -
 2*a*d^3*x^2*sin_integral(d*x) - 4*b*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a
*d^2*x*tan(1/2*d*x)^2 + 8*a*d^2*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d^2*x*tan(
1/2*c)^2 + 4*b*x^2*tan(1/2*d*x)^2 + 16*b*x^2*tan(1/2*d*x)*tan(1/2*c) + 4*a
*d*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x^2*tan(1/2*c)^2 + 4*a*d*tan(1/2*d*x)*t
an(1/2*c)^2 - 2*a*d^2*x - 4*b*x^2 - 4*a*d*tan(1/2*d*x) - 4*a*d*tan(1/2*c))
/(d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*x^2*tan(1/2*d*x)^2 + d*x^2*tan(1/2
*c)^2 + d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx=\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)}{x^3}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3))/x^3,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$$
$$= \frac{-\cos(dx+c)\,a\,d^2x - 2\cos(dx+c)\,b\,x^2 - \left(\int \frac{\sin(dx+c)}{x}dx\right)a\,d^3x^2 - \sin(dx+c)\,ad}{2d\,x^2}$$

input
```
int((b*x^3+a)*sin(d*x+c)/x^3,x)
```

output
```
( - cos(c + d*x)*a*d**2*x - 2*cos(c + d*x)*b*x**2 - int(sin(c + d*x)/x,x)*
a*d**3*x**2 - sin(c + d*x)*a*d)/(2*d*x**2)
```

3.86 $\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{ad\cos(c+dx)}{6x^2} - \frac{1}{6}ad^3\cos(c)\,\mathrm{CosIntegral}(dx) + b\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} + \frac{ad^2\sin(c+dx)}{6x} + b\cos(c)\mathrm{Si}(dx) + \frac{1}{6}ad^3\sin(c)\mathrm{Si}(dx)$$

output
```
-1/6*a*d*cos(d*x+c)/x^2-1/6*a*d^3*cos(c)*Ci(d*x)+b*Ci(d*x)*sin(c)-1/3*a*si
n(d*x+c)/x^3+1/6*a*d^2*sin(d*x+c)/x+b*cos(c)*Si(d*x)+1/6*a*d^3*sin(c)*Si(d
*x)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = b\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{a\cos(dx)\left(-dx\cos(c)-2\sin(c)+d^2x^2\sin(c)\right)}{6x^3} + \frac{a(-2\cos(c)+d^2x^2\cos(c)+dx\sin(c))\sin(dx)}{6x^3} + b\cos(c)\mathrm{Si}(dx) - \frac{1}{6}ad^3(\cos(c)\,\mathrm{CosIntegral}(dx)-\sin(c)\mathrm{Si}(dx))$$

input
```
Integrate[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

output
```
b*CosIntegral[d*x]*Sin[c] + (a*Cos[d*x]*(-(d*x*Cos[c]) - 2*Sin[c] + d^2*x^
2*Sin[c]))/(6*x^3) + (a*(-2*Cos[c] + d^2*x^2*Cos[c] + d*x*Sin[c])*Sin[d*x]
)/(6*x^3) + b*Cos[c]*SinIntegral[d*x] - (a*d^3*(Cos[c]*CosIntegral[d*x] -
Sin[c]*SinIntegral[d*x]))/6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \; 3820$$

$$\int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+b\sin(c)\,\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

input
```
Int[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

output
```
-1/6*(a*d*Cos[c + d*x])/x^2 - (a*d^3*Cos[c]*CosIntegral[d*x])/6 + b*CosInt
egral[d*x]*Sin[c] - (a*Sin[c + d*x])/(3*x^3) + (a*d^2*Sin[c + d*x])/(6*x)
+ b*Cos[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{b(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\,\text{s}}{d^3}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{b(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\,\text{s}}{d^3}\right.$
risch	$\frac{\cos(c)a\ \text{expIntegral}_1(-idx)d^3}{12}-\frac{i\cos(c)\,\text{expIntegral}_1(idx)b}{2}+\frac{\cos(c)a\ \text{expIntegral}_1(idx)d^3}{12}+\frac{i\cos(c)\,\text{expIntegra}}{2}$
meijerg	$\frac{b\sin(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\ \text{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos\left(c\right)\text{Si}\left(dx\right)+\frac{a\sin(c)\sqrt{\pi}\,d^4\left(-\frac{8(}{}\right.}{}$

input
```
int((b*x^3+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+
1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+b/d^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c
)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{adx\cos(dx+c)+(ad^3x^3\operatorname{Ci}(dx)-6\,bx^3\operatorname{Si}(dx))\cos(c)-(ad^2x^2-2\,a)\sin(dx+c)-(ad^3x^3\operatorname{Si}(dx)}{6\,x^3}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a*d*x*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) - 6*b*x^3*sin_inte
gral(d*x))*cos(c) - (a*d^2*x^2 - 2*a)*sin(d*x + c) - (a*d^3*x^3*sin_integr
al(d*x) + 6*b*x^3*cos_integral(d*x))*sin(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x**3+a)*sin(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**3)*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^6-6\,(b(i\,\Gamma(-3,}{}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3,
I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^6 - 6*(b*(I*gamma(-3, I*d*x) - I*g
amma(-3, -I*d*x))*cos(c) + b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c)
)*d^3)*x^3 + 2*b*d*x*sin(d*x + c) + 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/(d^3
*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 796, normalized size of antiderivative = 8.75

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

output
```
1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a
*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*
x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*
sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/
2*d*x)^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3
*real_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_in
tegral(d*x))*tan(1/2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/
2*c) + 4*a*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 6*b*x^3*imag_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*x^3*imag_part(cos_integral(
-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*rea
l_part(cos_integral(-d*x)) - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*
x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*
d*x)*tan(1/2*c)^2 + 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 -
6*b*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 12*b*x^3*sin_integr
al(d*x)*tan(1/2*d*x)^2 - 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2
 + 6*b*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 12*b*x^3*sin_in...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)}{x^4}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3))/x^4,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$$

$$= \frac{-6\cos(dx+c)\,b\,d^2x^2+12\cos(dx+c)\,b-72\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)b\,x^3+4\left(\int \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^3+x^3}dx\right)a}{6d^3x^3}$$

input
```
int((b*x^3+a)*sin(d*x+c)/x^4,x)
```

output
```
( - 6*cos(c + d*x)*b*d**2*x**2 + 12*cos(c + d*x)*b - 72*int(tan((c + d*x)/
2)**2/(tan((c + d*x)/2)**2*x**4 + x**4),x)*b*x**3 + 4*int(1/(tan((c + d*x)
/2)**2*x**3 + x**3),x)*a*d**4*x**3 - 2*sin(c + d*x)*a*d**3 - 6*sin(c + d*x
)*b*d*x + a*d**4*x - 12*b)/(6*d**3*x**3)
```

3.87 $\int x(a+bx^3)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\begin{aligned}\int x(a+bx^3)^2\sin(c+dx)\,dx = &-\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}\\&-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}\\&-\frac{840b^2x^3\cos(c+dx)}{d^5}\\&-\frac{2abx^4\cos(c+dx)}{d}+\frac{42b^2x^5\cos(c+dx)}{d^3}\\&-\frac{b^2x^7\cos(c+dx)}{d}-\frac{5040b^2\sin(c+dx)}{d^8}\\&+\frac{a^2\sin(c+dx)}{d^2}-\frac{48abx\sin(c+dx)}{d^4}\\&+\frac{2520b^2x^2\sin(c+dx)}{d^6}+\frac{8abx^3\sin(c+dx)}{d^2}\\&-\frac{210b^2x^4\sin(c+dx)}{d^4}+\frac{7b^2x^6\sin(c+dx)}{d^2}\end{aligned}$$

output

```
-48*a*b*cos(d*x+c)/d^5+5040*b^2*x*cos(d*x+c)/d^7-a^2*x*cos(d*x+c)/d+24*a*b
*x^2*cos(d*x+c)/d^3-840*b^2*x^3*cos(d*x+c)/d^5-2*a*b*x^4*cos(d*x+c)/d+42*b
^2*x^5*cos(d*x+c)/d^3-b^2*x^7*cos(d*x+c)/d-5040*b^2*sin(d*x+c)/d^8+a^2*sin
(d*x+c)/d^2-48*a*b*x*sin(d*x+c)/d^4+2520*b^2*x^2*sin(d*x+c)/d^6+8*a*b*x^3*
sin(d*x+c)/d^2-210*b^2*x^4*sin(d*x+c)/d^4+7*b^2*x^6*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx$$
$$=\frac{-d(a^2d^6x+2abd^2(24-12d^2x^2+d^4x^4)+b^2x(-5040+840d^2x^2-42d^4x^4+d^6x^6))\cos(c+dx)+(a^2d^6}{d^8}$$

input
```
Integrate[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

output
```
(-(d*(a^2*d^6*x + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*x*(-5040 + 8
40*d^2*x^2 - 42*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + (a^2*d^6 + 8*a*b*d^4*x
*(-6 + d^2*x^2) + 7*b^2*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Sin[c
 + d*x])/d^8
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx$$
$$\Big\downarrow \text{3820}$$
$$\int \left(a^2x\sin(c+dx)+2abx^4\sin(c+dx)+b^2x^7\sin(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}-\frac{48ab\cos(c+dx)}{d^5}-\frac{48abx\sin(c+dx)}{d^4}+\frac{24abx^2\cos(c+dx)}{d^3}+\frac{8abx^3\sin(c+dx)}{d^2}-\frac{2abx^4\cos(c+dx)}{d}-\frac{5040b^2\sin(c+dx)}{d^8}+\frac{5040b^2x\cos(c+dx)}{d^7}+\frac{2520b^2x^2\sin(c+dx)}{d^6}-\frac{840b^2x^3\cos(c+dx)}{d^5}-\frac{210b^2x^4\sin(c+dx)}{d^4}+\frac{42b^2x^5\cos(c+dx)}{d^3}+\frac{7b^2x^6\sin(c+dx)}{d^2}-\frac{b^2x^7\cos(c+dx)}{d}$$

input
```
Int[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

output
```
(-48*a*b*Cos[c + d*x])/d^5 + (5040*b^2*x*Cos[c + d*x])/d^7 - (a^2*x*Cos[c
+ d*x])/d + (24*a*b*x^2*Cos[c + d*x])/d^3 - (840*b^2*x^3*Cos[c + d*x])/d^5
 - (2*a*b*x^4*Cos[c + d*x])/d + (42*b^2*x^5*Cos[c + d*x])/d^3 - (b^2*x^7*C
os[c + d*x])/d - (5040*b^2*Sin[c + d*x])/d^8 + (a^2*Sin[c + d*x])/d^2 - (4
8*a*b*x*Sin[c + d*x])/d^4 + (2520*b^2*x^2*Sin[c + d*x])/d^6 + (8*a*b*x^3*S
in[c + d*x])/d^2 - (210*b^2*x^4*Sin[c + d*x])/d^4 + (7*b^2*x^6*Sin[c + d*x
])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{(b^2d^6x^7+2ab\,d^6x^4-42b^2d^4x^5+a^2d^6x-24ab\,d^4x^2+840b^2d^2x^3+48b\,d^2a-5040b^2x)\cos(dx+c)}{d^7}+\frac{(7b^2x^6d^6+8ab\,d^6}{}$
parallelrisch	$\frac{d\left(x(b\,x^3+a)^2d^6+(-42b^2x^5-24ab\,x^2)d^4+(840x^3b^2+96ab)d^2-5040b^2x\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+((14b^2x^6+16ab\,x^3+2a^2)d^6}{d^8\left(1+\right.}$
orering	$\frac{2(7b^3d^6x^{10}+15a\,b^2d^6x^7-252b^3d^4x^8+9a^2b\,d^6x^4-234a\,b^2d^4x^5+4200b^3d^2x^6+a^3d^6x-36a^2b\,d^4x^2+1848a\,b^2d^2x^3-201}{d^8x(b\,x^3+a)}$
norman	$\frac{\frac{96ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^5}+\frac{b^2x^7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{(a^2d^6-5040b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d^7}-\frac{840b^2x^3}{d^5}+\frac{42b^2x^5}{d^3}-\frac{b^2x^7}{d}-\frac{(a^2d^6-5040b^2)x}{d^7}+\frac{2(}{}}{}$
meijerg	$\frac{128b^2\sin(c)\sqrt{\pi}\left(\frac{315}{8\sqrt{\pi}}-\frac{\left(-\frac{7}{16}x^6d^6+\frac{105}{8}x^4d^4-\frac{315}{2}x^2d^2+315\right)\cos(dx)}{8\sqrt{\pi}}-\frac{xd\left(-\frac{1}{16}x^6d^6+\frac{21}{8}x^4d^4-\frac{105}{2}x^2d^2+315\right)\sin(dx)}{8\sqrt{\pi}}\right)}{d^8}+$
parts	$-\frac{b^2x^7\cos(dx+c)}{d}-\frac{2ab\,x^4\cos(dx+c)}{d}-\frac{a^2x\cos(dx+c)}{d}+\frac{a^2\sin(dx+c)-\frac{8ab\,c^3\sin(dx+c)}{d^3}+\frac{24ab\,c^2(\cos(dx+c)+(dx}{d^3}}{}$
derivativedivides	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^4\cos(dx+c)}{d^3}-\frac{8ab\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{12ab\,c^2\left(-(dx\right.}{}}{}$
default	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^4\cos(dx+c)}{d^3}-\frac{8ab\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{12ab\,c^2\left(-(dx\right.}{}}{}$

input
```
int(x*(b*x^3+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^7*(b^2*d^6*x^7+2*a*b*d^6*x^4-42*b^2*d^4*x^5+a^2*d^6*x-24*a*b*d^4*x^2+
840*b^2*d^2*x^3+48*a*b*d^2-5040*b^2*x)*cos(d*x+c)+(7*b^2*d^6*x^6+8*a*b*d^6
*x^3-210*b^2*d^4*x^4+a^2*d^6-48*a*b*d^4*x+2520*b^2*d^2*x^2-5040*b^2)/d^8*s
in(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx = -\frac{(b^2d^7x^7+2\,abd^7x^4-42\,b^2d^5x^5-24\,abd^5x^2+840\,b^2d^3x^3+48\,abd^3+(a^2d^7-5040\,b^2d)x)\cos{(dx+c)}}{d^8}$$

input
```
integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 - 24*a*b*d^5*x^2 + 840*b^2
*d^3*x^3 + 48*a*b*d^3 + (a^2*d^7 - 5040*b^2*d)*x)*cos(d*x + c) - (7*b^2*d^
6*x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^
2*d^2*x^2 - 5040*b^2)*sin(d*x + c))/d^8
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.21

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx = \begin{cases} -\frac{a^2x\cos{(c+dx)}}{d}+\frac{a^2\sin{(c+dx)}}{d^2}-\frac{2abx^4\cos{(c+dx)}}{d}+\frac{8abx^3\sin{(c+dx)}}{d^2}+\frac{24abx^2\cos{(c+dx)}}{d^3}-\frac{48abx\sin{(c+dx)}}{d^4}-\frac{48ab\cos{(c+}}{d^5} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^5}{5}+\frac{b^2x^8}{8}\right)\sin{(c)} \end{cases}$$

input
```
integrate(x*(b*x**3+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**4*co
s(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**2 + 24*a*b*x**2*cos(c + d*x)/d**
3 - 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos(c + d*x)/d**5 - b**2*x**7*cos(
c + d*x)/d + 7*b**2*x**6*sin(c + d*x)/d**2 + 42*b**2*x**5*cos(c + d*x)/d**
3 - 210*b**2*x**4*sin(c + d*x)/d**4 - 840*b**2*x**3*cos(c + d*x)/d**5 + 25
20*b**2*x**2*sin(c + d*x)/d**6 + 5040*b**2*x*cos(c + d*x)/d**7 - 5040*b**2
*sin(c + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8)
*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(235) = 470$.

Time = 0.09 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.82

$$\int x(a+bx^3)^2 \sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
(a^2*c*cos(d*x + c) + b^2*c^7*cos(d*x + c)/d^6 - 2*a*b*c^4*cos(d*x + c)/d^
3 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 7*((d*x + c)*cos(d*x + c
) - sin(d*x + c))*b^2*c^6/d^6 + 8*((d*x + c)*cos(d*x + c) - sin(d*x + c))*
a*b*c^3/d^3 + 21*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c
))*b^2*c^5/d^6 - 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x
+ c))*a*b*c^2/d^3 - 35*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x
 + c)^2 - 2)*sin(d*x + c))*b^2*c^4/d^6 + 8*(((d*x + c)^3 - 6*d*x - 6*c)*co
s(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^3 + 35*(((d*x + c)^
4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(
d*x + c))*b^2*c^3/d^6 - 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c
) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^3 - 21*(((d*x + c)^5
 - 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d
*x + c)^2 + 24)*sin(d*x + c))*b^2*c^2/d^6 + 7*(((d*x + c)^6 - 30*(d*x + c)
^4 + 360*(d*x + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3
 + 120*d*x + 120*c)*sin(d*x + c))*b^2*c/d^6 - (((d*x + c)^7 - 42*(d*x + c)
^5 + 840*(d*x + c)^3 - 5040*d*x - 5040*c)*cos(d*x + c) - 7*((d*x + c)^6 -
30*(d*x + c)^4 + 360*(d*x + c)^2 - 720)*sin(d*x + c))*b^2/d^6)/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int x(a+bx^3)^2 \sin(c+dx)\,dx = -\frac{(b^2d^7x^7 + 2\,abd^7x^4 - 42\,b^2d^5x^5 + a^2d^7x - 24\,abd^5x^2 + 840\,b^2d^3x^3 + 48\,abd^3 - 5040\,b^2dx)\cos(dx+c)}{d^8} + \frac{(7\,b^2d^6x^6 + 8\,abd^6x^3 - 210\,b^2d^4x^4 + a^2d^6 - 48\,abd^4x + 2520\,b^2d^2x^2 - 5040\,b^2)\sin(dx+c)}{d^8}$$

input
```
integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-(b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 + a^2*d^7*x - 24*a*b*d^5*x^
2 + 840*b^2*d^3*x^3 + 48*a*b*d^3 - 5040*b^2*d*x)*cos(d*x + c)/d^8 + (7*b^2
*d^6*x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520
*b^2*d^2*x^2 - 5040*b^2)*sin(d*x + c)/d^8
```

Mupad [B] (verification not implemented)

Time = 42.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x(a+bx^3)^2\sin(c+dx)\,dx &= \frac{42\,b^2\,x^5\,\cos(c+d\,x)+24\,a\,b\,x^2\,\cos(c+d\,x)}{d^3} \\ &\quad -\frac{b^2\,x^7\,\cos(c+d\,x)+a^2\,x\,\cos(c+d\,x)+2\,a\,b\,x^4\,\cos(c+d\,x)}{d} \\ &\quad -\frac{840\,b^2\,x^3\,\cos(c+d\,x)+48\,a\,b\,\cos(c+d\,x)}{d^5} \\ &\quad +\frac{a^2\,\sin(c+d\,x)+7\,b^2\,x^6\,\sin(c+d\,x)+8\,a\,b\,x^3\,\sin(c+d\,x)}{d^2} \\ &\quad -\frac{210\,b^2\,x^4\,\sin(c+d\,x)+48\,a\,b\,x\,\sin(c+d\,x)}{d^4}-\frac{5040\,b^2\,\sin(c+d\,x)}{d^8} \\ &\quad +\frac{2520\,b^2\,x^2\,\sin(c+d\,x)}{d^6}+\frac{5040\,b^2\,x\,\cos(c+d\,x)}{d^7}\end{aligned}$$

input
```
int(x*sin(c + d*x)*(a + b*x^3)^2,x)
```

output
```
(42*b^2*x^5*cos(c + d*x) + 24*a*b*x^2*cos(c + d*x))/d^3 - (b^2*x^7*cos(c +
 d*x) + a^2*x*cos(c + d*x) + 2*a*b*x^4*cos(c + d*x))/d - (840*b^2*x^3*cos(
c + d*x) + 48*a*b*cos(c + d*x))/d^5 + (a^2*sin(c + d*x) + 7*b^2*x^6*sin(c
+ d*x) + 8*a*b*x^3*sin(c + d*x))/d^2 - (210*b^2*x^4*sin(c + d*x) + 48*a*b*
x*sin(c + d*x))/d^4 - (5040*b^2*sin(c + d*x))/d^8 + (2520*b^2*x^2*sin(c +
d*x))/d^6 + (5040*b^2*x*cos(c + d*x))/d^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx$$
$$= \frac{-\cos(dx+c)\,a^2d^7x - 2\cos(dx+c)\,ab\,d^7x^4 + 24\cos(dx+c)\,ab\,d^5x^2 - 48\cos(dx+c)\,ab\,d^3 - \cos(dx+}{}$$

input
```
int(x*(b*x^3+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**7*x - 2*cos(c + d*x)*a*b*d**7*x**4 + 24*cos(c + d
*x)*a*b*d**5*x**2 - 48*cos(c + d*x)*a*b*d**3 - cos(c + d*x)*b**2*d**7*x**7
 + 42*cos(c + d*x)*b**2*d**5*x**5 - 840*cos(c + d*x)*b**2*d**3*x**3 + 5040
*cos(c + d*x)*b**2*d*x + sin(c + d*x)*a**2*d**6 + 8*sin(c + d*x)*a*b*d**6*
x**3 - 48*sin(c + d*x)*a*b*d**4*x + 7*sin(c + d*x)*b**2*d**6*x**6 - 210*si
n(c + d*x)*b**2*d**4*x**4 + 2520*sin(c + d*x)*b**2*d**2*x**2 - 5040*sin(c
+ d*x)*b**2)/d**8
```

3.88 $\int (a+bx^3)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 188

$$\int (a+bx^3)^2 \sin(c+dx)\,dx = \frac{720b^2\cos(c+dx)}{d^7} - \frac{a^2\cos(c+dx)}{d} + \frac{12abx\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} - \frac{2abx^3\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{b^2x^6\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} + \frac{720b^2x\sin(c+dx)}{d^6} + \frac{6abx^2\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{6b^2x^5\sin(c+dx)}{d^2}$$

output
```
720*b^2*cos(d*x+c)/d^7-a^2*cos(d*x+c)/d+12*a*b*x*cos(d*x+c)/d^3-360*b^2*x^
2*cos(d*x+c)/d^5-2*a*b*x^3*cos(d*x+c)/d+30*b^2*x^4*cos(d*x+c)/d^3-b^2*x^6*
cos(d*x+c)/d-12*a*b*sin(d*x+c)/d^4+720*b^2*x*sin(d*x+c)/d^6+6*a*b*x^2*sin(
d*x+c)/d^2-120*b^2*x^3*sin(d*x+c)/d^4+6*b^2*x^5*sin(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int (a+bx^3)^2 \sin(c+dx)\,dx = \frac{-((a^2d^6+2abd^4x(-6+d^2x^2)+b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+6bd(ad^2(-2+d^2}{d^7}$$

input
```
Integrate[(a + b*x^3)^2*Sin[c + d*x],x]
```

output
```
(-((a^2*d^6 + 2*a*b*d^4*x*(-6 + d^2*x^2) + b^2*(-720 + 360*d^2*x^2 - 30*d^
4*x^4 + d^6*x^6))*Cos[c + d*x]) + 6*b*d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 -
 20*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^7
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^2 \sin(c+dx)\,dx$$

$$\Big\downarrow \text{ 3810}$$

$$\int \left(a^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^6\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}+\frac{720b^2\cos(c+dx)}{d^7}+\frac{720b^2x\sin(c+dx)}{d^6}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{30b^2x^4\cos(c+dx)}{d^3}+\frac{6b^2x^5\sin(c+dx)}{d^2}-\frac{b^2x^6\cos(c+dx)}{d}$$

input
```
Int[(a + b*x^3)^2*Sin[c + d*x],x]
```

output
```
(720*b^2*Cos[c + d*x])/d^7 - (a^2*Cos[c + d*x])/d + (12*a*b*x*Cos[c + d*x]
)/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 - (2*a*b*x^3*Cos[c + d*x])/d + (30*
b^2*x^4*Cos[c + d*x])/d^3 - (b^2*x^6*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x
])/d^4 + (720*b^2*x*Sin[c + d*x])/d^6 + (6*a*b*x^2*Sin[c + d*x])/d^2 - (12
0*b^2*x^3*Sin[c + d*x])/d^4 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3810
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(b^2x^6d^6+2ab\,d^6x^3-30b^2x^4d^4+a^2d^6-12ab\,d^4x+360x^2d^2b^2-720b^2)\cos(dx+c)}{d^7}+\frac{6b(b\,d^4x^5+a\,d^4x^2-20b\,d^2x^3-2}{d^6}$
parallelrisch	$\frac{2d^2\left(x^2\left(\frac{b\,x^3}{2}+a\right)d^4+(-15b\,x^3-6a)\,d^2+180bx\right)bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+12db(x^2(b\,x^3+a)\,d^4+(-20b\,x^3-2a)\,d^2+120bx)\tan}{d^7\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$\frac{12b(b^2d^6x^8+2ab\,d^6x^5-25b^2d^4x^6+a^2d^6x^2-17ab\,d^4x^3+240b^2x^4d^2-a^2d^4+60ab\,d^2x-360x^2b^2)\sin(dx+c)}{d^8(b\,x^3+a)}-\frac{(b^2x^6d}{}$
norman	$\frac{\frac{b^2x^6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{2a^2d^6-1440b^2}{d^7}-\frac{360b^2x^2}{d^5}+\frac{30b^2x^4}{d^3}-\frac{b^2x^6}{d}-\frac{24ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{1440b^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^6}+\frac{360b^2x^2\tan\left(\frac{dx}{2}+\right.}{d^5}}{}$
meijerg	$\frac{64b^2\sin(c)\sqrt{\pi}\left(\frac{x\left(d^2\right)^{\frac{7}{2}}\left(\frac{21}{8}x^4d^4-\frac{105}{2}x^2d^2+315\right)\cos(dx)}{28\sqrt{\pi}\,d^6}-\frac{\left(d^2\right)^{\frac{7}{2}}\left(-\frac{7}{16}x^6d^6+\frac{105}{8}x^4d^4-\frac{315}{2}x^2d^2+315\right)\sin(dx)}{28\sqrt{\pi}\,d^7}\right)}{d^6\sqrt{d^2}}+\frac{64b}{}$
parts	$-\frac{b^2x^6\cos(dx+c)}{d}-\frac{2ab\,x^3\cos(dx+c)}{d}-\frac{a^2\cos(dx+c)}{d}+\frac{6b\left(a\,c^2\sin(dx+c)-2ac(\cos(dx+c)+(dx+c)\sin(dx+c)\right.}{}$
derivativedivides	$\frac{-\cos(dx+c)a^2+\frac{2ab\,c^3\cos(dx+c)}{d^3}+\frac{6ab\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}-\frac{6abc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin\right.}{d^3}}{}$
default	$\frac{-\cos(dx+c)a^2+\frac{2ab\,c^3\cos(dx+c)}{d^3}+\frac{6ab\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}-\frac{6abc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin\right.}{d^3}}{}$

input
```
int((b*x^3+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^6*x^6+2*a*b*d^6*x^3-30*b^2*d^4*x^4+a^2*d^6-12*a*b*d^4*x+360*b^2*d^
2*x^2-720*b^2)/d^7*cos(d*x+c)+6*b/d^6*(b*d^4*x^5+a*d^4*x^2-20*b*d^2*x^3-2*
a*d^2+120*b*x)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

$$\int \left(a+bx^3\right)^2\sin(c+dx)\,dx = -\frac{(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2)\cos{(dx+c)}-6\,(b^2d^5x^5+a}{d^7}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-((b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x +
 360*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c) - 6*(b^2*d^5*x^5 + a*b*d^5*x^2 -
20*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.20

$$\int (a+bx^3)^2 \sin(c+dx)\,dx$$
$$= \begin{cases} -\frac{a^2\cos(c+dx)}{d} - \frac{2abx^3\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{12abx\cos(c+dx)}{d^3} - \frac{12ab\sin(c+dx)}{d^4} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{6b^2x^5\sin(c+}{d^2} \\ \left(a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7}\right)\sin(c) \end{cases}$$

input
```
integrate((b*x**3+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*s
in(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 -
 b**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 + 30*b**2*x**4*c
os(c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**2*x**2*cos(c +
 d*x)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c + d*x)/d**7, Ne
(d, 0)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(188) = 376.

Time = 0.07 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.60

$$\int (a+bx^3)^2 \sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output

```
-(a^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^6 - 2*a*b*c^3*cos(d*x + c)/d^3
 - 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^5/d^6 + 6*((d*x + c)*co
s(d*x + c) - sin(d*x + c))*a*b*c^2/d^3 + 15*(((d*x + c)^2 - 2)*cos(d*x + c
) - 2*(d*x + c)*sin(d*x + c))*b^2*c^4/d^6 - 6*(((d*x + c)^2 - 2)*cos(d*x +
 c) - 2*(d*x + c)*sin(d*x + c))*a*b*c/d^3 - 20*(((d*x + c)^3 - 6*d*x - 6*c
)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c^3/d^6 + 2*(((d*x
+ c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b
/d^3 + 15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)
^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2*c^2/d^6 - 6*(((d*x + c)^5 - 20*(d*x +
c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 2
4)*sin(d*x + c))*b^2*c/d^6 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c
)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*
c)*sin(d*x + c))*b^2/d^6)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int (a+bx^3)^2 \sin(c+dx)\,dx = -\frac{(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2)\cos{(dx+c)}}{d^7} + \frac{6\,(b^2d^5x^5+abd^5x^2-20\,b^2d^3x^3-2\,abd^3+120\,b^2dx)\sin{(dx+c)}}{d^7}$$

input

```
integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

output

```
-(b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x +
360*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c)/d^7 + 6*(b^2*d^5*x^5 + a*b*d^5*x^2
 - 20*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c)/d^7
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^2 \sin(c+dx)\, dx = \frac{\cos(c+d\,x)\,(720\,b^2-a^2\,d^6)}{d^7} - \frac{b^2\,x^6\,\cos(c+d\,x)}{d} + \frac{30\,b^2\,x^4\,\cos(c+d\,x)}{d^3} - \frac{360\,b^2\,x^2\,\cos(c+d\,x)}{d^5} + \frac{6\,b^2\,x^5\,\sin(c+d\,x)}{d^2} - \frac{120\,b^2\,x^3\,\sin(c+d\,x)}{d^4} - \frac{12\,a\,b\,\sin(c+d\,x)}{d^4} + \frac{720\,b^2\,x\,\sin(c+d\,x)}{d^6} - \frac{2\,a\,b\,x^3\,\cos(c+d\,x)}{d} + \frac{6\,a\,b\,x^2\,\sin(c+d\,x)}{d^2} + \frac{12\,a\,b\,x\,\cos(c+d\,x)}{d^3}$$

input
```
int(sin(c + d*x)*(a + b*x^3)^2,x)
```

output
```
(cos(c + d*x)*(720*b^2 - a^2*d^6))/d^7 - (b^2*x^6*cos(c + d*x))/d + (30*b^
2*x^4*cos(c + d*x))/d^3 - (360*b^2*x^2*cos(c + d*x))/d^5 + (6*b^2*x^5*sin(
c + d*x))/d^2 - (120*b^2*x^3*sin(c + d*x))/d^4 - (12*a*b*sin(c + d*x))/d^4
 + (720*b^2*x*sin(c + d*x))/d^6 - (2*a*b*x^3*cos(c + d*x))/d + (6*a*b*x^2*
sin(c + d*x))/d^2 + (12*a*b*x*cos(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99

$$\int (a+bx^3)^2 \sin(c+dx)\, dx = \frac{-\cos(dx+c)\,a^2d^6 - 2\cos(dx+c)\,ab\,d^6x^3 + 12\cos(dx+c)\,ab\,d^4x - \cos(dx+c)\,b^2d^6x^6 + 30\cos(dx+}{}$$

input
```
int((b*x^3+a)^2*sin(d*x+c),x)
```

output
```
( - cos(c + d*x)*a**2*d**6 - 2*cos(c + d*x)*a*b*d**6*x**3 + 12*cos(c + d*x
)*a*b*d**4*x - cos(c + d*x)*b**2*d**6*x**6 + 30*cos(c + d*x)*b**2*d**4*x**
4 - 360*cos(c + d*x)*b**2*d**2*x**2 + 720*cos(c + d*x)*b**2 + 6*sin(c + d*
x)*a*b*d**5*x**2 - 12*sin(c + d*x)*a*b*d**3 + 6*sin(c + d*x)*b**2*d**5*x**
5 - 120*sin(c + d*x)*b**2*d**3*x**3 + 720*sin(c + d*x)*b**2*d*x)/d**7
```

3.89 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\begin{aligned}\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx = {} & \frac{4ab\cos(c+dx)}{d^3} - \frac{120b^2x\cos(c+dx)}{d^5} - \frac{2abx^2\cos(c+dx)}{d} \\ & + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5\cos(c+dx)}{d} \\ & + a^2\,\text{CosIntegral}(dx)\sin(c) + \frac{120b^2\sin(c+dx)}{d^6} \\ & + \frac{4abx\sin(c+dx)}{d^2} - \frac{60b^2x^2\sin(c+dx)}{d^4} \\ & + \frac{5b^2x^4\sin(c+dx)}{d^2} + a^2\cos(c)\text{Si}(dx)\end{aligned}$$

output
```
4*a*b*cos(d*x+c)/d^3-120*b^2*x*cos(d*x+c)/d^5-2*a*b*x^2*cos(d*x+c)/d+20*b^
2*x^3*cos(d*x+c)/d^3-b^2*x^5*cos(d*x+c)/d+a^2*Ci(d*x)*sin(c)+120*b^2*sin(d
*x+c)/d^6+4*a*b*x*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/d^4+5*b^2*x^4*sin(d
*x+c)/d^2+a^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx = -\frac{b(2ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\cos(c+dx)}{d^5} + a^2\,\text{CosIntegral}(dx)\sin(c) + \frac{b(4ad^4x+5b(24-12d^2x^2+d^4x^4))\sin(c+dx)}{d^6} + a^2\cos(c)\text{Si}(dx)$$

input
```
Integrate[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

output
```
-((b*(2*a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d
*x])/d^5) + a^2*CosIntegral[d*x]*Sin[c] + (b*(4*a*d^4*x + 5*b*(24 - 12*d^2
*x^2 + d^4*x^4))*Sin[c + d*x])/d^6 + a^2*Cos[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a^2\sin(c+dx)}{x}+2abx^2\sin(c+dx)+b^2x^5\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2\sin(c)\,\text{CosIntegral}(dx)+a^2\cos(c)\text{Si}(dx)+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}+\frac{120b^2\sin(c+dx)}{d^6}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{60b^2x^2\sin(c+dx)}{d^4}+\frac{20b^2x^3\cos(c+dx)}{d^3}+\frac{5b^2x^4\sin(c+dx)}{d^2}-\frac{b^2x^5\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

output
```
(4*a*b*Cos[c + d*x])/d^3 - (120*b^2*x*Cos[c + d*x])/d^5 - (2*a*b*x^2*Cos[c
 + d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (b^2*x^5*Cos[c + d*x])/d + a^
2*CosIntegral[d*x]*Sin[c] + (120*b^2*Sin[c + d*x])/d^6 + (4*a*b*x*Sin[c +
d*x])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (5*b^2*x^4*Sin[c + d*x])/d^2 +
 a^2*Cos[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{b^2x^5\cos(dx+c)}{d}+\frac{5b^2x^4\sin(dx+c)}{d^2}+\frac{ia^2\mathrm{e}^{ic}\,\text{expIntegral}_1(-idx)}{2}-\frac{ia^2\mathrm{e}^{-ic}\,\text{expIntegral}_1(idx)}{2}-\frac{2ab\,x^2\cos(}{d}$
meijerg	$\frac{32b^2\sin(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}x^4d^4-\frac{45}{2}x^2d^2+45\right)\cos(dx)}{12\sqrt{\pi}}+\frac{xd\left(\frac{3}{8}x^4d^4-\frac{15}{2}x^2d^2+45\right)\sin(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32b^2\cos(c)\sqrt{\pi}\left(-\frac{xd\left(\frac{7}{8}\right.}{}\right.}{}$
derivativedivides	$a^2(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))-\frac{6ab\,c^2\cos(dx+c)}{d^3}-\frac{6abc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+$
default	$a^2(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))-\frac{6ab\,c^2\cos(dx+c)}{d^3}-\frac{6abc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-b^2*x^5*cos(d*x+c)/d+5*b^2*x^4*sin(d*x+c)/d^2+1/2*I*a^2*exp(I*c)*Ei(1,-I*
d*x)-1/2*I*a^2*exp(-I*c)*Ei(1,I*d*x)-2*a*b*x^2*cos(d*x+c)/d+20*b^2*x^3*cos
(d*x+c)/d^3+4*a*b*x*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/d^4+4*a*b*cos(d*x
+c)/d^3-120*b^2*x*cos(d*x+c)/d^5+120*b^2*sin(d*x+c)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx$$
$$=\frac{a^2d^6\,\mathrm{Ci}\,(dx)\sin(c)+a^2d^6\cos(c)\,\mathrm{Si}\,(dx)-(b^2d^5x^5+2\,abd^5x^2-20\,b^2d^3x^3-4\,abd^3+120\,b^2dx)\cos(dx+}{d^6}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

output
```
(a^2*d^6*cos_integral(d*x)*sin(c) + a^2*d^6*cos(c)*sin_integral(d*x) - (b^
2*d^5*x^5 + 2*a*b*d^5*x^2 - 20*b^2*d^3*x^3 - 4*a*b*d^3 + 120*b^2*d*x)*cos(
d*x + c) + (5*b^2*d^4*x^4 + 4*a*b*d^4*x - 60*b^2*d^2*x^2 + 120*b^2)*sin(d*
x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx$$

$$= a^2 \sin(c)\operatorname{Ci}(dx) + a^2\cos(c)\operatorname{Si}(dx) + 2abx^2\left(\begin{cases} x\sin(c) & \text{for}\ d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right) - 4ab\left(\begin{cases} \frac{x^3\sin(c)}{3} & \text{for}\ d=0 \\ -\frac{\begin{cases} \frac{x\sin(c+dx)}{d} + \frac{\cos(c+dx)}{d^2} & \text{for}\ d\neq 0 \\ \frac{x^2\cos(c)}{2} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right) + b^2x^5\left(\begin{cases} x\sin(c) & \text{for}\ d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right) - 5b^2\left(\begin{cases} \frac{x^6\sin(c)}{6} & \text{for}\ d=0 \\ -\frac{\begin{cases} \frac{x^4\sin(c+dx)}{d} + \frac{4x^3\cos(c+dx)}{d^2} - \frac{12x^2\sin(c+dx)}{d^3} - \frac{24x\cos(c+dx)}{d^4} + \frac{24\sin(c+dx)}{d^5} & \text{for}\ d\neq 0 \\ \frac{x^5\cos(c)}{5} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**3+a)**2*sin(d*x+c)/x,x)
```

output
```
a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x**2*Piecewise((x*sin(c)
, Eq(d, 0)), (-cos(c + d*x)/d, True)) - 4*a*b*Piecewise((x**3*sin(c)/3, Eq
(d, 0)), (-Piecewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x*
*2*cos(c)/2, True))/d, True)) + b**2*x**5*Piecewise((x*sin(c), Eq(d, 0)),
(-cos(c + d*x)/d, True)) - 5*b**2*Piecewise((x**6*sin(c)/6, Eq(d, 0)), (-P
iecewise((x**4*sin(c + d*x)/d + 4*x**3*cos(c + d*x)/d**2 - 12*x**2*sin(c +
 d*x)/d**3 - 24*x*cos(c + d*x)/d**4 + 24*sin(c + d*x)/d**5, Ne(d, 0)), (x*
*5*cos(c)/5, True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx = \frac{(a^2(-i\,\mathrm{Ei}(i\,dx)+i\,\mathrm{Ei}(-i\,dx))\cos{(c)}+a^2(\mathrm{Ei}(i\,dx)+\mathrm{Ei}(-i\,dx))\sin{(c)})d^6-2\,(b^2d^5x^5+2\,abd^5x^2-20\,b}{2\,d^6}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

output
```
1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*
x))*sin(c))*d^6 - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 - 20*b^2*d^3*x^3 - 4*a*b*
d^3 + 120*b^2*d*x)*cos(d*x + c) + 2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x - 60*b^2*
d^2*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 921, normalized size of antiderivative = 5.72

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(2*b^2*d^5*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*b^2*d^5*x^5*tan
(1/2*d*x + 1/2*c)^2 - 2*b^2*d^5*x^5*tan(1/2*c)^2 + 20*b^2*d^4*x^4*tan(1/2*
d*x + 1/2*c)*tan(1/2*c)^2 + 4*a*b*d^5*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 - a^2*d^6*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)^2 + a^2*d^6*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2 - 2*a^2*d^6*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 -
 2*b^2*d^5*x^5 + 2*a^2*d^6*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c) + 2*a^2*d^6*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/
2*c)^2*tan(1/2*c) - 40*b^2*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2
0*b^2*d^4*x^4*tan(1/2*d*x + 1/2*c) + 4*a*b*d^5*x^2*tan(1/2*d*x + 1/2*c)^2
+ a^2*d^6*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2 - a^2*d^6*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^6*sin_integra
l(d*x)*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*d^5*x^2*tan(1/2*c)^2 - a^2*d^6*imag_
part(cos_integral(d*x))*tan(1/2*c)^2 + a^2*d^6*imag_part(cos_integral(-d*x
))*tan(1/2*c)^2 - 2*a^2*d^6*sin_integral(d*x)*tan(1/2*c)^2 - 40*b^2*d^3*x^
3*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^6*real_part(cos_integral(d*x))*tan(1/2*
c) + 2*a^2*d^6*real_part(cos_integral(-d*x))*tan(1/2*c) + 40*b^2*d^3*x^3*t
an(1/2*c)^2 + 16*a*b*d^4*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a*b*d^5*x
^2 + a^2*d^6*imag_part(cos_integral(d*x)) - a^2*d^6*imag_part(cos_integral
(-d*x)) + 2*a^2*d^6*sin_integral(d*x) - 240*b^2*d^2*x^2*tan(1/2*d*x + 1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx=\int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)^2}{x}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3)^2)/x,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3)^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx$$

$$= \frac{-2\cos(dx+c)\,ab\,d^5x^2 + 4\cos(dx+c)\,ab\,d^3 - \cos(dx+c)\,b^2d^5x^5 + 20\cos(dx+c)\,b^2d^3x^3 - 120\cos(dx+c)\,(dx}{}$$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x,x)
```

output
```
( - 2*cos(c + d*x)*a*b*d**5*x**2 + 4*cos(c + d*x)*a*b*d**3 - cos(c + d*x)*
b**2*d**5*x**5 + 20*cos(c + d*x)*b**2*d**3*x**3 - 120*cos(c + d*x)*b**2*d*
x + int(sin(c + d*x)/x,x)*a**2*d**6 + 4*sin(c + d*x)*a*b*d**4*x + 5*sin(c
+ d*x)*b**2*d**4*x**4 - 60*sin(c + d*x)*b**2*d**2*x**2 + 120*sin(c + d*x)*
b**2)/d**6
```

3.90 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} \, dx = -\frac{24b^2\cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)$$

output
```
-24*b^2*cos(d*x+c)/d^5-2*a*b*x*cos(d*x+c)/d+12*b^2*x^2*cos(d*x+c)/d^3-b^2*
x^4*cos(d*x+c)/d+a^2*d*cos(c)*Ci(d*x)+2*a*b*sin(d*x+c)/d^2-a^2*sin(d*x+c)/
x-24*b^2*x*sin(d*x+c)/d^4+4*b^2*x^3*sin(d*x+c)/d^2-a^2*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} \, dx = -\frac{24b^2\cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)$$

input
```
Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

output
```
(-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c
+ d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (
2*a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d
^4 + (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} \, dx$$

$$\Big\downarrow \; 3820$$

$$\int \left(\frac{a^2\sin(c+dx)}{x^2} + 2abx\sin(c+dx) + b^2x^4\sin(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2 d\cos(c)\,\text{CosIntegral}(dx) - a^2 d\sin(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{x} + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} - \frac{24b^2\cos(c+dx)}{d^5} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{12b^2x^2\cos(c+dx)}{d^3} + \frac{4b^2x^3\sin(c+dx)}{d^2} - \frac{b^2x^4\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

output
```
(-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c
 + d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (
2*a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d
^4 + (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.41 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{-\sin(c)\pi \,\mathrm{csgn}(dx)a^2d^6x+2\sin(c)\,\mathrm{Si}(dx)a^2d^6x-i\cos(c)\pi \,\mathrm{csgn}(dx)a^2d^6x+2\cos(dx+c)b^2d^4x^5+2i\cos(c)\,\mathrm{Si}(dx)a^2d}{}$
meijerg	$\frac{16b^2\sin(c)\sqrt{\pi}\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}x^4d^4-\frac{15}{2}x^2d^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b^2\cos(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}\right.}{}\right.}{}$
derivativedivides	$d\left(-\frac{15b^2c^4\cos(dx+c)}{d^6}+\frac{(5c^4+4c^3+3c^2+2c+1)b^2\left(-(dx+c)^4\cos(dx+c)+4(dx+c)^3\sin(dx+c)+12(dx+c)^2\cos(d\right.}{d^6}\right.$
default	$d\left(-\frac{15b^2c^4\cos(dx+c)}{d^6}+\frac{(5c^4+4c^3+3c^2+2c+1)b^2\left(-(dx+c)^4\cos(dx+c)+4(dx+c)^3\sin(dx+c)+12(dx+c)^2\cos(d\right.}{d^6}\right.$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x/d^5*(-sin(c)*Pi*csgn(d*x)*a^2*d^6*x+2*sin(c)*Si(d*x)*a^2*d^6*x-I*co
s(c)*Pi*csgn(d*x)*a^2*d^6*x+2*cos(d*x+c)*b^2*d^4*x^5+2*I*cos(c)*Si(d*x)*a^
2*d^6*x+2*cos(c)*Ei(1,-I*d*x)*a^2*d^6*x-8*sin(d*x+c)*b^2*d^3*x^4+4*cos(d*x
+c)*a*b*d^4*x^2+2*sin(d*x+c)*a^2*d^5-24*cos(d*x+c)*b^2*d^2*x^3-4*sin(d*x+c
)*a*b*d^3*x+48*sin(d*x+c)*b^2*d*x^2+48*cos(d*x+c)*b^2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+bx^3\right)^2\sin(c+dx)}{x^2}\,dx$$
$$=\frac{a^2d^6x\cos\left(c\right)\mathrm{Ci}\left(dx\right)-a^2d^6x\sin\left(c\right)\mathrm{Si}\left(dx\right)-\left(b^2d^4x^5+2\,abd^4x^2-12\,b^2d^2x^3+24\,b^2x\right)\cos\left(dx+c\right)+(4}{d^5x}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
(a^2*d^6*x*cos(c)*cos_integral(d*x) - a^2*d^6*x*sin(c)*sin_integral(d*x) -
 (b^2*d^4*x^5 + 2*a*b*d^4*x^2 - 12*b^2*d^2*x^3 + 24*b^2*x)*cos(d*x + c) +
(4*b^2*d^3*x^4 - a^2*d^5 + 2*a*b*d^3*x - 24*b^2*d*x^2)*sin(d*x + c))/(d^5*
x)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx$$

input
```
integrate((b*x**3+a)**2*sin(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**3)**2*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \frac{(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c))d^6-2\,(b^2d^4x^4+2}{2\,d^5}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1
, I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^4*x^4 + 2*a*b*d^4*x
 - 12*b^2*d^2*x^2 + 24*b^2)*cos(d*x + c) + 4*(2*b^2*d^3*x^3 + a*b*d^3 - 12
*b^2*d*x)*sin(d*x + c))/d^5
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 2038, normalized size of antiderivative = 14.06

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

output
```
1/2*(2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^
2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x)^2 - 2*a^2*d^6*x*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_part(cos_integral(-d*x))
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_integr
al(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*d^4*x^5*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2*d^4*x^5*tan(1/2*d*x)^2*tan(1/2*
c)^2 + a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x)^2 + a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x)^2 - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^2*d^4*x^5*tan(1/2*d*x)^2 - 2*a
^2*d^6*x*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) +
2*a^2*d^6*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx=\int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^2}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^2,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx$$
$$=\frac{-2\cos(dx+c)\,ab\,d^4x-\cos(dx+c)\,b^2d^4x^4+12\cos(dx+c)\,b^2d^2x^2-24\cos(dx+c)\,b^2+\left(\int \frac{\sin(dx+c)}{x^2}d\right.}{d^5}$$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^2,x)
```

output
```
( - 2*cos(c + d*x)*a*b*d**4*x - cos(c + d*x)*b**2*d**4*x**4 + 12*cos(c + d
*x)*b**2*d**2*x**2 - 24*cos(c + d*x)*b**2 + int(sin(c + d*x)/x**2,x)*a**2*
d**5 + 2*sin(c + d*x)*a*b*d**3 + 4*sin(c + d*x)*b**2*d**3*x**3 - 24*sin(c
+ d*x)*b**2*d*x)/d**5
```

3.91 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} \, dx = -\frac{2ab\cos(c+dx)}{d} - \frac{a^2 d\cos(c+dx)}{2x} + \frac{6b^2 x\cos(c+dx)}{d^3} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{1}{2}a^2d^2\,\text{CosIntegral}(dx)\sin(c) - \frac{6b^2\sin(c+dx)}{d^4} - \frac{a^2\sin(c+dx)}{2x^2} + \frac{3b^2x^2\sin(c+dx)}{d^2} - \frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)$$

output
```
-2*a*b*cos(d*x+c)/d-1/2*a^2*d*cos(d*x+c)/x+6*b^2*x*cos(d*x+c)/d^3-b^2*x^3*
cos(d*x+c)/d-1/2*a^2*d^2*Ci(d*x)*sin(c)-6*b^2*sin(d*x+c)/d^4-1/2*a^2*sin(d
*x+c)/x^2+3*b^2*x^2*sin(d*x+c)/d^2-1/2*a^2*d^2*cos(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} dx = \frac{1}{2}\left(-\frac{4ab\cos(c+dx)}{d} - \frac{a^2 d\cos(c+dx)}{x} + \frac{12b^2 x\cos(c+dx)}{d^3} - \frac{2b^2x^3\cos(c+dx)}{d} - a^2d^2 \operatorname{CosIntegral}(dx)\sin(c) - \frac{12b^2\sin(c+dx)}{d^4} - \frac{a^2\sin(c+dx)}{x^2} + \frac{6b^2x^2\sin(c+dx)}{d^2} - a^2d^2\cos(c)\operatorname{Si}(dx)\right)$$

input
```
Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

output
```
((-4*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x + (12*b^2*x*Cos[c + d*x]
)/d^3 - (2*b^2*x^3*Cos[c + d*x])/d - a^2*d^2*CosIntegral[d*x]*Sin[c] - (12
*b^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/x^2 + (6*b^2*x^2*Sin[c + d*x])
/d^2 - a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} dx$$

$$\Big\downarrow \text{3820}$$

$$\int \left(\frac{a^2\sin(c+dx)}{x^3} + 2ab\sin(c+dx) + b^2x^3\sin(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a^2d^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}-\frac{2ab\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

output
```
(-2*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/(2*x) + (6*b^2*x*Cos[c + d*
x])/d^3 - (b^2*x^3*Cos[c + d*x])/d - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 -
 (6*b^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/(2*x^2) + (3*b^2*x^2*Sin[c
+ d*x])/d^2 - (a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.53 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{-\cos(c)\pi\ \mathrm{csgn}(dx)a^2d^6x^2+2\cos(c)\,\mathrm{Si}(dx)a^2d^6x^2+i\sin(c)\pi\ \mathrm{csgn}(dx)a^2d^6x^2-2i\sin(c)\,\mathrm{Si}(dx)a^2d^6x^2-2\sin(c)\,\mathrm{expI}}{}$
derivativedivides	$d^2\left(\frac{20b^2c^3\cos(dx+c)}{d^6}-\frac{6c\,b^2(6c^2+3c+1)\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^6}-\frac{2ab\cos(d}{d^3}\right.$
default	$d^2\left(\frac{20b^2c^3\cos(dx+c)}{d^6}-\frac{6c\,b^2(6c^2+3c+1)\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^6}-\frac{2ab\cos(d}{d^3}\right.$
meijerg	$\frac{8b^2\sin(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2d^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{xd\left(-\frac{x^2d^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\cos(c)\sqrt{\pi}\left(\frac{xd\left(-\frac{5x^2d^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-\right.}{}\right.}{d^4}$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2/d^4*(-cos(c)*Pi*csgn(d*x)*a^2*d^6*x^2+2*cos(c)*Si(d*x)*a^2*d^6*x^
2+I*sin(c)*Pi*csgn(d*x)*a^2*d^6*x^2-2*I*sin(c)*Si(d*x)*a^2*d^6*x^2-2*sin(c
)*Ei(1,-I*d*x)*a^2*d^6*x^2+4*cos(d*x+c)*b^2*d^3*x^5-12*sin(d*x+c)*b^2*d^2*
x^4+2*cos(d*x+c)*a^2*d^5*x+8*cos(d*x+c)*a*b*d^3*x^2+2*sin(d*x+c)*a^2*d^4-2
4*cos(d*x+c)*b^2*d*x^3+24*sin(d*x+c)*b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx = -\frac{a^2d^6x^2\,\mathrm{Ci}\,(dx)\sin(c)+a^2d^6x^2\cos(c)\,\mathrm{Si}\,(dx)+(2\,b^2d^3x^5+a^2d^5x+4\,abd^3x^2-12\,b^2dx^3)\cos(dx+c)}{2\,d^4x^2}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*d^6*x^2*cos_integral(d*x)*sin(c) + a^2*d^6*x^2*cos(c)*sin_integr
al(d*x) + (2*b^2*d^3*x^5 + a^2*d^5*x + 4*a*b*d^3*x^2 - 12*b^2*d*x^3)*cos(d
*x + c) - (6*b^2*d^2*x^4 - a^2*d^4 - 12*b^2*x^2)*sin(d*x + c))/(d^4*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx=\int \frac{(a+bx^3)^2\sin{(c+dx)}}{x^3}\,dx$$

input
```
integrate((b*x**3+a)**2*sin(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**3)**2*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx$$
$$=\frac{(a^2(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx))\cos{(c)}+a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin{(c)})d^6-2\,(b^2d^3x^3+2\,ab}{2\,d^4}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/2*((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-
2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^3*x^3 + 2*a*b*d^3 -
6*b^2*d*x)*cos(d*x + c) + 6*(b^2*d^2*x^2 - 2*b^2)*sin(d*x + c))/d^4
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 2171, normalized size of antiderivative = 15.29

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d
*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^6*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)
 - 2*a^2*d^6*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*d*x)^2*tan(1/2*c) + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/
2*c)^2*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x
))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^6*x^2*imag_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c)^2 + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 -
2*a^2*d^6*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c) - 2*a^2*d^6*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c) - 2*a^2*d^6*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx=\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)^2}{x^3}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^3,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx$$
$$= \frac{-\cos(dx+c)\,a^2d^5x - 4\cos(dx+c)\,ab\,d^3x^2 - 2\cos(dx+c)\,b^2d^3x^5 + 12\cos(dx+c)\,b^2d\,x^3 - \left(\int \frac{\sin(dx+}{x}\right.}{2d^4x^2}$$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^3,x)
```

output
```
( - cos(c + d*x)*a**2*d**5*x - 4*cos(c + d*x)*a*b*d**3*x**2 - 2*cos(c + d*
x)*b**2*d**3*x**5 + 12*cos(c + d*x)*b**2*d*x**3 - int(sin(c + d*x)/x,x)*a*
*2*d**6*x**2 - sin(c + d*x)*a**2*d**4 + 6*sin(c + d*x)*b**2*d**2*x**4 - 12
*sin(c + d*x)*b**2*x**2)/(2*d**4*x**2)
```

3.92 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx = \frac{2b^2\cos(c+dx)}{d^3} - \frac{a^2d\cos(c+dx)}{6x^2} - \frac{b^2x^2\cos(c+dx)}{d} - \frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx) + 2ab\,\text{CosIntegral}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{3x^3} + \frac{a^2d^2\sin(c+dx)}{6x} + \frac{2b^2x\sin(c+dx)}{d^2} + 2ab\cos(c)\text{Si}(dx) + \frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)$$

output
```
2*b^2*cos(d*x+c)/d^3-1/6*a^2*d*cos(d*x+c)/x^2-b^2*x^2*cos(d*x+c)/d-1/6*a^2
*d^3*cos(c)*Ci(d*x)+2*a*b*Ci(d*x)*sin(c)-1/3*a^2*sin(d*x+c)/x^3+1/6*a^2*d^
2*sin(d*x+c)/x+2*b^2*x*sin(d*x+c)/d^2+2*a*b*cos(c)*Si(d*x)+1/6*a^2*d^3*sin
(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} \, dx = \frac{1}{6} \left(\frac{12b^2 \cos(c+dx)}{d^3} - \frac{a^2 d \cos(c+dx)}{x^2} - \frac{6b^2 x^2 \cos(c+dx)}{d} - a \operatorname{CosIntegral}(dx) \left(ad^3 \cos(c) - 12b \sin(c)\right) - \frac{2a^2 \sin(c+dx)}{x^3} + \frac{a^2 d^2 \sin(c+dx)}{x} + \frac{12b^2 x \sin(c+dx)}{d^2} + a\left(12b \cos(c) + ad^3 \sin(c)\right) \operatorname{Si}(dx) \right)$$

input
```
Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

output
```
((12*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/x^2 - (6*b^2*x^2*Cos[c +
 d*x])/d - a*CosIntegral[d*x]*(a*d^3*Cos[c] - 12*b*Sin[c]) - (2*a^2*Sin[c
+ d*x])/x^3 + (a^2*d^2*Sin[c + d*x])/x + (12*b^2*x*Sin[c + d*x])/d^2 + a*(
12*b*Cos[c] + a*d^3*Sin[c])*SinIntegral[d*x])/6
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} \, dx$$

$$\Big\downarrow \text{ 3820}$$

$$\int \left(\frac{a^2 \sin(c+dx)}{x^4} + \frac{2ab \sin(c+dx)}{x} + b^2 x^2 \sin(c+dx) \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2ab\sin(c)\,\text{CosIntegral}(dx)+2ab\cos(c)\text{Si}(dx)+\frac{2b^2\cos(c+dx)}{d^3}+\frac{2b^2x\sin(c+dx)}{d^2}-\frac{b^2x^2\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

output
```
(2*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/(6*x^2) - (b^2*x^2*Cos[c +
 d*x])/d - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 + 2*a*b*CosIntegral[d*x]*Si
n[c] - (a^2*Sin[c + d*x])/(3*x^3) + (a^2*d^2*Sin[c + d*x])/(6*x) + (2*b^2*
x*Sin[c + d*x])/d^2 + 2*a*b*Cos[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinI
ntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.30

method	result
derivativedivides	$d^3\left(\frac{2ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^3}-\frac{15b^2c^2\cos(dx+c)}{d^6}-\frac{6c\,b^2(4c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^6}+\frac{(10c\right.$
default	$d^3\left(\frac{2ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^3}-\frac{15b^2c^2\cos(dx+c)}{d^6}-\frac{6c\,b^2(4c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^6}+\frac{(10c\right.$
risch	$-\frac{\pi\ \text{csgn}(dx)\sin(c)a^2d^6x^3-2\ \text{Si}(dx)\sin(c)a^2d^6x^3-12i\pi\ \text{csgn}(dx)\sin(c)ab\,d^3x^3+i\pi\ \text{csgn}(dx)\cos(c)a^2d^6x^3+12\pi\ \text{csg}}{}$
meijerg	$\frac{4b^2\sin(c)\sqrt{\pi}\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3x^2d^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b^2\cos(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2d^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin}{2\sqrt{}}\right.}{d^3}$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*(2/d^3*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-15/d^6*b^2*c^2*cos(d*x+c)-6
*c*b^2*(4*c+1)/d^6*(sin(d*x+c)-cos(d*x+c)*(d*x+c))+(10*c^2+4*c+1)/d^6*b^2*
(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a^2*(-1/3*sin(d*
x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(d*x)*sin(c)-
1/6*Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^4}\,dx =$$
$$-\frac{(6\,b^2d^2x^5+a^2d^4x-12\,b^2x^3)\cos(dx+c)+(a^2d^6x^3\,\text{Ci}(dx)-12\,abd^3x^3\,\text{Si}(dx))\cos(c)-(a^2d^5x^2+12}{6\,d^3x^3}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/6*((6*b^2*d^2*x^5 + a^2*d^4*x - 12*b^2*x^3)*cos(d*x + c) + (a^2*d^6*x^3
*cos_integral(d*x) - 12*a*b*d^3*x^3*sin_integral(d*x))*cos(c) - (a^2*d^5*x
^2 + 12*b^2*d*x^4 - 2*a^2*d^3)*sin(d*x + c) - (a^2*d^6*x^3*sin_integral(d*
x) + 12*a*b*d^3*x^3*cos_integral(d*x))*sin(c))/(d^3*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x**3+a)**2*sin(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**3)**2*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4}\,dx = -\frac{((a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)-a^2(i\,\Gamma(-3,i\,dx)-i\,\Gamma(-3,-i\,dx))\sin(c))d^6+12\,(ab(-i\,\Gamma(\cdots}{\cdots}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) - a^2*(I*gamma(-
3, I*d*x) - I*gamma(-3, -I*d*x))*sin(c))*d^6 + 12*(a*b*(-I*gamma(-3, I*d*x
) + I*gamma(-3, -I*d*x))*cos(c) - a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x
))*sin(c))*d^3)*x^3 + 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 2*b^2*x^3 - 4*a*b)*
cos(d*x + c) - 4*(b^2*d*x^4 - a*b*d*x)*sin(d*x + c))/(d^3*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 1181, normalized size of antiderivative = 7.82

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

output
```
1/12*(a^2*d^6*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + a^2*d^6*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*a^2*d^6*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*
a^2*d^6*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^
2*d^6*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^6*x^3*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^6*x^3*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2 + a^2*d^6*x^3*real_part(cos_integral(d*x))*tan(1/2*
c)^2 + a^2*d^6*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^6*
x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^6*x^3*imag_part(cos_
integral(-d*x))*tan(1/2*c) + 4*a^2*d^6*x^3*sin_integral(d*x)*tan(1/2*c) -
12*b^2*d^2*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b*d^3*x^3*imag_part(cos_
integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^3*x^3*imag_part(cos_
integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a*b*d^3*x^3*sin_integral(
d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^3*real_part(cos_integral(d*x)
) - a^2*d^6*x^3*real_part(cos_integral(-d*x)) - 4*a^2*d^5*x^2*tan(1/2*d*x)
^2*tan(1/2*c) + 24*a*b*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2
*tan(1/2*c) + 24*a*b*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*
tan(1/2*c) - 4*a^2*d^5*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 12*b^2*d^2*x^5*tan(
1/2*d*x)^2 + 12*a*b*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 -
12*a*b*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*a*b*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^4}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^4}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^4,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^4}\,dx$$
$$= \frac{-12\cos(dx+c)\,ab\,d^2x^2+24\cos(dx+c)\,ab-6\cos(dx+c)\,b^2d^2x^5+12\cos(dx+c)\,b^2x^3-144\left(\int \frac{\text{t}}{\text{tan}(}\right.}{}$$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^4,x)
```

output
```
( - 12*cos(c + d*x)*a*b*d**2*x**2 + 24*cos(c + d*x)*a*b - 6*cos(c + d*x)*b
**2*d**2*x**5 + 12*cos(c + d*x)*b**2*x**3 - 144*int(tan((c + d*x)/2)**2/(t
an((c + d*x)/2)**2*x**4 + x**4),x)*a*b*x**3 + 4*int(1/(tan((c + d*x)/2)**2
*x**3 + x**3),x)*a**2*d**4*x**3 - 2*sin(c + d*x)*a**2*d**3 - 12*sin(c + d*
x)*a*b*d*x + 12*sin(c + d*x)*b**2*d*x**4 + a**2*d**4*x - 24*a*b - 12*b**2*
x**3)/(6*d**3*x**3)
```

3.93 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} dx =& -\frac{a^2 d \cos(c+dx)}{12x^3} + \frac{a^2 d^3 \cos(c+dx)}{24x} \\ & - \frac{b^2 x \cos(c+dx)}{d} + 2abd \cos(c) \operatorname{CosIntegral}(dx) \\ & + \frac{1}{24} a^2 d^4 \operatorname{CosIntegral}(dx) \sin(c) + \frac{b^2 \sin(c+dx)}{d^2} \\ & - \frac{a^2 \sin(c+dx)}{4x^4} + \frac{a^2 d^2 \sin(c+dx)}{24x^2} - \frac{2ab \sin(c+dx)}{x} \\ & + \frac{1}{24} a^2 d^4 \cos(c) \operatorname{Si}(dx) - 2abd \sin(c) \operatorname{Si}(dx)\end{aligned}$$

output
```
-1/12*a^2*d*cos(d*x+c)/x^3+1/24*a^2*d^3*cos(d*x+c)/x-b^2*x*cos(d*x+c)/d+2*
a*b*d*cos(c)*Ci(d*x)+1/24*a^2*d^4*Ci(d*x)*sin(c)+b^2*sin(d*x+c)/d^2-1/4*a^
2*sin(d*x+c)/x^4+1/24*a^2*d^2*sin(d*x+c)/x^2-2*a*b*sin(d*x+c)/x+1/24*a^2*d
^4*cos(c)*Si(d*x)-2*a*b*d*sin(c)*Si(d*x)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} dx = \frac{1}{24}\left(-\frac{2a^2d\cos(c+dx)}{x^3} + \frac{a^2d^3\cos(c+dx)}{x} - \frac{24b^2x\cos(c+dx)}{d} + ad\,\text{CosIntegral}(dx)\left(48b\cos(c)+ad^3\sin(c)\right) + \frac{24b^2\sin(c+dx)}{d^2} - \frac{6a^2\sin(c+dx)}{x^4} + \frac{a^2d^2\sin(c+dx)}{x^2} - \frac{48ab\sin(c+dx)}{x} + ad\left(ad^3\cos(c)-48b\sin(c)\right)\text{Si}(dx)\right)$$

input
```
Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

output
```
((-2*a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/x - (24*b^2*x*Cos[c
+ d*x])/d + a*d*CosIntegral[d*x]*(48*b*Cos[c] + a*d^3*Sin[c]) + (24*b^2*Si
n[c + d*x])/d^2 - (6*a^2*Sin[c + d*x])/x^4 + (a^2*d^2*Sin[c + d*x])/x^2 -
(48*a*b*Sin[c + d*x])/x + a*d*(a*d^3*Cos[c] - 48*b*Sin[c])*SinIntegral[d*x
])/24
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3820, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} dx$$

$$\Big\downarrow \; 3820$$

$$\int \left(\frac{a^2\sin(c+dx)}{x^5} + \frac{2ab\sin(c+dx)}{x^2} + b^2x\sin(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}a^2d^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^3\cos(c+dx)}{24x}+\frac{a^2d^2\sin(c+dx)}{24x^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos(c+dx)}{12x^3}+2abd\cos(c)\,\text{CosIntegral}(dx)-2abd\sin(c)\text{Si}(dx)-\frac{2ab\sin(c+dx)}{x}+\frac{b^2\sin(c+dx)}{d^2}-\frac{b^2x\cos(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

output
```
-1/12*(a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/(24*x) - (b^2*x*Co
s[c + d*x])/d + 2*a*b*d*Cos[c]*CosIntegral[d*x] + (a^2*d^4*CosIntegral[d*x
]*Sin[c])/24 + (b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/(4*x^4) + (a^2*
d^2*Sin[c + d*x])/(24*x^2) - (2*a*b*Sin[c + d*x])/x + (a^2*d^4*Cos[c]*SinI
ntegral[d*x])/24 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3820
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

method	result
derivativedivides	$d^4\left(\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)}{d^3}+\frac{6b^2c\cos(dx+c)}{d^6}+\frac{(5c+1)b^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^6}\right.$
default	$d^4\left(\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)}{d^3}+\frac{6b^2c\cos(dx+c)}{d^6}+\frac{(5c+1)b^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^6}\right.$
risch	$-\cos(c)\pi\ \mathrm{csgn}(dx)a^2d^6x^4-96i\cos(c)\,\mathrm{Si}(dx)ab\,d^3x^4+2\cos(c)\,\mathrm{Si}(dx)a^2d^6x^4+i\sin(c)\pi\ \mathrm{csgn}(dx)a^2d^6x^4+48\sin(c)\pi\ \mathrm{cs}$
meijerg	$\frac{2b^2\sin(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{xd\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\cos(c)\sqrt{\pi}\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{d^2ab\sin(c)\sqrt{\pi}\left(-\frac{4d^2\cos(x}{x(d^2)^{\frac{3}{2}}}\right.}{2\sqrt{d^2}}$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*(2/d^3*a*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+6/d^6*b^2*c
*cos(d*x+c)+(5*c+1)/d^6*b^2*(sin(d*x+c)-cos(d*x+c)*(d*x+c))+a^2*(-1/4*sin(
d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^2/x^2+1/24*cos(d*
x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx$$
$$=\frac{(a^2d^5x^3-24\,b^2dx^5-2\,a^2d^3x)\cos(dx+c)+(a^2d^6x^4\,\mathrm{Si}(dx)+48\,abd^3x^4\,\mathrm{Ci}(dx))\cos(c)+(a^2d^4x^2-48}{24\,d^2x^4}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
1/24*((a^2*d^5*x^3 - 24*b^2*d*x^5 - 2*a^2*d^3*x)*cos(d*x + c) + (a^2*d^6*x
^4*sin_integral(d*x) + 48*a*b*d^3*x^4*cos_integral(d*x))*cos(c) + (a^2*d^4
*x^2 - 48*a*b*d^2*x^3 + 24*b^2*x^4 - 6*a^2*d^2)*sin(d*x + c) + (a^2*d^6*x^
4*cos_integral(d*x) - 48*a*b*d^3*x^4*sin_integral(d*x))*sin(c))/(d^2*x^4)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx=\int \frac{(a+bx^3)^2\sin{(c+dx)}}{x^5}\,dx$$

input
```
integrate((b*x**3+a)**2*sin(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x**3)**2*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx = \frac{((a^2(-i\,\Gamma(-4,i\,dx)+i\,\Gamma(-4,-i\,dx))\cos{(c)}-a^2(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin{(c)})d^7-48\,(ab(\Gamma(-4,}{}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
1/2*(((a^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c) - a^2*(gamma
(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^7 - 48*(a*b*(gamma(-4, I*d*x) +
 gamma(-4, -I*d*x))*cos(c) + a*b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x
))*sin(c))*d^4)*x^4 - 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 12*a*b)*cos(d*x + c
) + 2*(b^2*d*x^4 - 4*a*b*d*x)*sin(d*x + c))/(d^3*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1255, normalized size of antiderivative = 7.51

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

output
```
-1/48*(a^2*d^6*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2 - a^2*d^6*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^6*
x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^6*x^4
*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^6*x^4*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^6*x^4*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*d*x)^2
 + a^2*d^6*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x^4*ima
g_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^6*x^4*sin_integral(d*x)*
tan(1/2*c)^2 - 2*a^2*d^6*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a
^2*d^6*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) - 2*a^2*d^5*x^3*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^4*imag_part(cos_integral(d*x)) + a^2*d^
6*x^4*imag_part(cos_integral(-d*x)) - 2*a^2*d^6*x^4*sin_integral(d*x) + 96
*a*b*d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 96*a
*b*d^3*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 192*a
*b*d^3*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^5*x^3*tan
(1/2*d*x)^2 - 48*a*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 -
 48*a*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 8*a^2*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx=\int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^5}\,dx$$

input
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^5,x)
```

output
```
int((sin(c + d*x)*(a + b*x^3)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx$$
$$=\frac{-24\cos(dx+c)\,ab\,d^2x^2+144\cos(dx+c)\,ab-12\cos(dx+c)\,b^2d^2x^5-6\left(\int \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^4+x^4}dx\right)a^2d^4x}{}$$

input
```
int((b*x^3+a)^2*sin(d*x+c)/x^5,x)
```

output
```
( - 24*cos(c + d*x)*a*b*d**2*x**2 + 144*cos(c + d*x)*a*b - 12*cos(c + d*x)
*b**2*d**2*x**5 - 6*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**2*x**4 + x*
*4),x)*a**2*d**4*x**4 + 1152*int(1/(tan((c + d*x)/2)**2*x**5 + x**5),x)*a*
b*x**4 - 3*sin(c + d*x)*a**2*d**3 - 48*sin(c + d*x)*a*b*d*x + 12*sin(c + d
*x)*b**2*d*x**4 - a**2*d**4*x + 144*a*b - 12*b**2*c*d*x**4)/(12*d**3*x**4)
```

3.94 $\int \frac{x^4 \sin(c+dx)}{a+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}
&\int \frac{x^4 \sin(c+dx)}{a+bx^3} \, dx \\
&= -\frac{x\cos(c+dx)}{bd} + \frac{a^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\
&+ \frac{(-1)^{2/3}a^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\
&- \frac{\sqrt[3]{-1}a^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\
&+ \frac{\sin(c+dx)}{bd^2} - \frac{(-1)^{2/3}a^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\
&+ \frac{a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}} \\
&- \frac{\sqrt[3]{-1}a^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}
\end{aligned}$$

output
```
-x*cos(d*x+c)/b/d+1/3*a^(2/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^
(1/3))/b^(5/3)+1/3*(-1)^(2/3)*a^(2/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)
*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Ci((-1
)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)
+sin(d*x+c)/b/d^2+1/3*(-1)^(2/3)*a^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)+1/3*a^(2/3)*cos(c-a^(1/3)
*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*cos(c
-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.62

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3} \, dx = \frac{-iad^2 \text{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-}{\#1}\right.}{}$$

input
```
Integrate[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
((-I)*a*d^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)]
- I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*
(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] + I*a*d^2*RootSu
m[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*
(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c
+ d*#1]*SinIntegral[d*(x - #1)])/#1 & ] + 6*b*(-(d*x*Cos[c + d*x]) + Sin[c
 + d*x]))/(6*b^2*d^2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3} \, dx$$

$$\Big\downarrow \ 3826$$

$$\int \left(\frac{x \sin(c+dx)}{b} - \frac{ax \sin(c+dx)}{b\,(a+bx^3)}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^{2/3} \sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{(-1)^{2/3} a^{2/3} \sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{(-1)^{2/3} a^{2/3} \cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right) \text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} + \frac{a^{2/3} \cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{\sin(c+dx)}{bd^2} - \frac{x\cos(c+dx)}{bd}$$

input
```
Int[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
-((x*Cos[c + d*x])/(b*d)) + (a^(2/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x
]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + ((-1)^(2/3)*a^(2/3)*CosInteg
ral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) +
Sin[c + d*x]/(b*d^2) - ((-1)^(2/3)*a^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)) +
(a^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{d^3 c^4 \left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)} \frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^3 c^4 \left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)} \frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	Expression too large to display

input
```
int(x^4*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^5*(1/3*d^3*c^4/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+C
i(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
))-4/3*d^3*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(
d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
+2*d^3*c^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*
x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+4
*d^3*c/b*cos(d*x+c)+4/3/b^2*d^3*c*sum((-3*_R1^2*b*c+3*_R1*b*c^2+a*d^3-b*c^
3)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+(-3*cos(d*x+c)*d^3*c+
d^3*(sin(d*x+c)-cos(d*x+c)*(d*x+c)))/b-1/3/b^2*d^3*sum((-6*_R1^2*b*c^2+_R1
*a*d^3+8*_R1*b*c^3+3*a*c*d^3-3*b*c^4)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)
*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+
a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.07

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx$$

$$= \frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)}-\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(\right.}{}$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-
I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^
3/b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1
))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(s
qrt(3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a
*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*E
i(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3
)*(-I*sqrt(3) + 1) + I*c) - 12*d*x*cos(d*x + c) + 2*I*(-I*a*d^3/b)^(2/3)*E
i(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/
b)^(2/3)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 12*
sin(d*x + c))/(b*d^2)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \sin{(c+dx)}}{a+bx^3}\,dx$$

input
```
integrate(x**4*sin(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x**4*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \sin{(dx+c)}}{bx^3+a}\,dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^4*cos(d*x + c) - (cos(c)^2 + sin(c)^2)*x^3
*sin(d*x + c) + ((d*x^4*cos(c) + x^3*sin(c))*cos(d*x + c)^2 + (d*x^4*cos(c
) + x^3*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c
)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^
2 + b*sin(c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*i
ntegrate(-3/2*(a*d*x^3*cos(d*x + c) - a*x^2*sin(d*x + c))/(b^2*d^2*x^6 + 2
*a*b*d^2*x^3 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*c
os(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2
*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-3/2*(a*d*
x^3*cos(d*x + c) - a*x^2*sin(d*x + c))/((b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2
*d^2)*cos(d*x + c)^2 + (b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*sin(d*x + c
)^2), x) + ((d*x^4*sin(c) - x^3*cos(c))*cos(d*x + c)^2 + (d*x^4*sin(c) - x
^3*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2
*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*si
n(c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^4\sin(c+dx)}{a+bx^3}\,dx=\int \frac{x^4\sin{(dx+c)}}{bx^3+a}\,dx$$

input
```
integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^4*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sin(c+dx)}{a+bx^3}\,dx=\int \frac{x^4\,\sin{(c+d\,x)}}{b\,x^3+a}\,dx$$

input
```
int((x^4*sin(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^4*sin(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \frac{-\cos(dx+c)\,dx - \left(\int \frac{\sin(dx+c)x}{b\,x^3+a}dx\right) a\,d^2 + \sin(dx+c)}{b\,d^2}$$

input
```
int(x^4*sin(d*x+c)/(b*x^3+a),x)
```

output
```
( - cos(c + d*x)*d*x - int((sin(c + d*x)*x)/(a + b*x**3),x)*a*d**2 + sin(c
 + d*x))/(b*d**2)
```

3.95 $\int \frac{x^3\sin(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 357

$$\begin{aligned}
&\int \frac{x^3\sin(c+dx)}{a+bx^3}\,dx \\
&= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt[3]{a}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} \\
&\quad + \frac{\sqrt[3]{-1}\sqrt[3]{a}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} \\
&\quad - \frac{(-1)^{2/3}\sqrt[3]{a}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} \\
&\quad - \frac{\sqrt[3]{-1}\sqrt[3]{a}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} \\
&\quad - \frac{\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} \\
&\quad - \frac{(-1)^{2/3}\sqrt[3]{a}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}
\end{aligned}$$

output
```
-cos(d*x+c)/b/d-1/3*a^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1
/3))/b^(4/3)+1/3*(-1)^(1/3)*a^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*s
in(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Ci((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)+1
/3*(-1)^(1/3)*a^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a
^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*a^(1/3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1
/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)
*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.61

$$\int \frac{x^3\sin(c+dx)}{a+bx^3}\,dx = -\frac{6b\cos(c+dx)+iad\,\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-}{\#1^2}\right.}{}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
-1/6*(6*b*Cos[c + d*x] + I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIn
tegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d
*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2
 & ] - I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)]
 + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d
*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3} \, dx$$

$$\Big\downarrow \ 3826$$

$$\int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\,(a+bx^3)}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt[3]{a}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\sqrt[3]{a}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{a}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\cos(c+dx)}{bd}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
-(Cos[c + d*x]/(b*d)) - (a^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Si
n[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*CosIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) - ((-1
)^(1/3)*a^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^
(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cos[c - (a^(1/3)*d
)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^(2/
3)*a^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{d^3 c^3 \Big(\sum_{\_R1=\operatorname{RootOf}(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3)} \frac{-\operatorname{Si}(-dx+\_R1-c)\cos(\_R1)+\operatorname{Ci}(dx-\_R1+c)\sin(\ldots}{\_R1^2-2\_R1c+c^2}}{3b}$
default	$-\frac{d^3 c^3 \Big(\sum_{\_R1=\operatorname{RootOf}(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3)} \frac{-\operatorname{Si}(-dx+\_R1-c)\cos(\_R1)+\operatorname{Ci}(dx-\_R1+c)\sin(\ldots}{\_R1^2-2\_R1c+c^2}}{3b}$
risch	$-\frac{i\Big(\sum_{\_R1=\operatorname{RootOf}(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3\_Zbc^2)} \frac{e^{-\_R1}\operatorname{expIntegral}_1(idx+ic-\_R1)}{-2ic\_R1+\_R1^2-c^2}\Big)c^3}{6db}+\frac{i\Big(\sum_{\_R1=\operatorname{Root}\ldots}}{}$

input
```
int(x^3*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-1/3*d^3*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+
Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))+d^3*c^2/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x
-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^
3*c/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^3/b*co
s(d*x+c)-1/3/b^2*d^3*sum((-3*_R1^2*b*c+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_
R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.10

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\right.}{}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*((I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a
*d^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(
3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1
/3)*(I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(
1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(I*sqrt
(3) - 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d
^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*
a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + 2*(I*a*d^3/b)^(1/3)*Ei(-I*d
*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*cos(d*x + c))/(b
*d)
```

Sympy [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3} dx = \int \frac{x^3 \sin(c+dx)}{a+bx^3} dx$$

input
```
integrate(x**3*sin(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x**3*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3} dx = \int \frac{x^3 \sin(dx+c)}{bx^3+a} dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x^3*cos(d*x + c) + (x^3*cos(d*x + c)^2*cos(c)
+ x^3*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 6*(((a*b*cos(c)^2 + a*b*sin(
c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b*cos(
c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)
^2)*integrate(1/2*x^2*cos(d*x + c)/(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d), x) -
 6*(((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d
)*cos(d*x + c)^2 + ((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 +
a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x^2*cos(d*x + c)/((b^2*d*x^
6 + 2*a*b*d*x^3 + a^2*d)*cos(d*x + c)^2 + (b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d
)*sin(d*x + c)^2), x) + (x^3*cos(d*x + c)^2*sin(c) + x^3*sin(d*x + c)^2*si
n(c))*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*
sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)
^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \sin(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^3*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \sin(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \frac{-\cos(dx+c) - \left(\int \frac{\sin(dx+c)}{b\,x^3+a}dx\right) ad}{bd}$$

input
```
int(x^3*sin(d*x+c)/(b*x^3+a),x)
```

output
```
( - (cos(c + d*x) + int(sin(c + d*x)/(a + b*x**3),x)*a*d))/(b*d)
```

3.96 $\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 281

$$
\begin{aligned}
\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx &= \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} \\
&+ \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} \\
&+ \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} \\
&- \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} \\
&+ \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b} \\
&+ \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}
\end{aligned}
$$

output
```
1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b+1/3*Ci((-1)^(1/3)
*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Ci((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b+1/3*cos
(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b+1
/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b+1/3*cos(c-(-1)^(2/
3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.66

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx = \frac{i(\text{RootSum}[a+b\#1^3\&, \cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1)) - i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1}{}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
((I/6)*(RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
#1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] - RootSum[a + b*#1^3 & ,
Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c +
d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegra
l[d*(x - #1)] & ]))/b
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx$$

$\Big\downarrow$ 3826

$$\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}-\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b
) + (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)])/(3*b) + (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b) - (Cos[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/
(3*b) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(3*b) + (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{d^3c^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)}\frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^3c^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)}\frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$\frac{ic^2\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3\_Zbc^2\right)}\frac{e^{-\_R1}\text{expIntegral}_1\left(idx+ic-\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}-\frac{ic^2\left(\sum_{\_R1=\text{Root}}\right.}{}$

input
```
int(x^2*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(1/3*d^3*c^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+C
i(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
))-2/3*d^3*c/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*
x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1
/3*d^3/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R
1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx$$
$$= \frac{i \operatorname{Ei}\left(-i\,dx + \frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)} - i \operatorname{Ei}\left(i\,dx + \frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)}{}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I
*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + I*Ei(-I*
d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))
*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - I*Ei(I*d*x + (-I*a*d^
3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + I*Ei(-I*d*x + (I*a*d^3/b)^(1/3)
)*e^(-I*c - (I*a*d^3/b)^(1/3)))/b
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx$$

input
```
integrate(x**2*sin(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x**2*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \sin(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + (cos(c)^2 + sin(c)^2)*x*s
in(d*x + c) + ((d*x^2*cos(c) - x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c) -
x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d
^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*
sin(c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integra
te(-1/2*(3*a*d*x*cos(d*x + c) - (2*b*x^3 - a)*sin(d*x + c))/(b^2*d^2*x^6 +
 2*a*b*d^2*x^3 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a
*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d
^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(3*
a*d*x*cos(d*x + c) - (2*b*x^3 - a)*sin(d*x + c))/((b^2*d^2*x^6 + 2*a*b*d^2
*x^3 + a^2*d^2)*cos(d*x + c)^2 + (b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*s
in(d*x + c)^2), x) + ((d*x^2*sin(c) + x*cos(c))*cos(d*x + c)^2 + (d*x^2*si
n(c) + x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^
2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2
+ b*sin(c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \sin(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^2*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^2\,\sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin\left(dx+c\right)x^2}{b\,x^3+a}dx$$

input
```
int(x^2*sin(d*x+c)/(b*x^3+a),x)
```

output
```
int((sin(c + d*x)*x**2)/(a + b*x**3),x)
```

3.97 $\int \frac{x\sin(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 343

$$\begin{aligned}\int \frac{x\sin(c+dx)}{a+bx^3}\,dx =& -\frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}\\
&-\frac{(-1)^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}\\
&+\frac{\sqrt[3]{-1}\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}\\
&+\frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&+\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\end{aligned}$$

output
```
-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/
3*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)
*d/b^(1/3))/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)
+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*c
os(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a
^(1/3)/b^(2/3)-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(1
/3)/b^(2/3)+1/3*(-1)^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/
3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - RootSum[a + b*#1^3
 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Si
n[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinI
ntegral[d*(x - #1)])/#1 & ]))/b
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{(-1)^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x^3),x]
```

output
```
-1/3*(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])
/(a^(1/3)*b^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((
-1)^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Cos[c + ((-1)
^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*
x])/(3*a^(1/3)*b^(2/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/
3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cos[c - ((-1)^(2/3
)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(
3*a^(1/3)*b^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{d^3\left(\sum_{\_\_R1=\operatorname{RootOf}\left(b\_\_Z^3-3bc\_\_Z^2+3\_\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{\_\_R1\left(-\operatorname{Si}\left(-dx+\_\_R1-c\right)\cos\left(\_\_R1\right)+\operatorname{Ci}\left(dx-\_\_R1+c\right)\sin\ldots}{\_\_R1^2-2\_\_R1c+c^2}\right.}{3b}$
default	$\frac{d^3\left(\sum_{\_\_R1=\operatorname{RootOf}\left(b\_\_Z^3-3bc\_\_Z^2+3\_\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{\_\_R1\left(-\operatorname{Si}\left(-dx+\_\_R1-c\right)\cos\left(\_\_R1\right)+\operatorname{Ci}\left(dx-\_\_R1+c\right)\sin\ldots}{\_\_R1^2-2\_\_R1c+c^2}\right.}{3b}$
risch	$\frac{d\left(\sum_{\_\_R1=\operatorname{RootOf}\left(-3i\_\_Z^2bc-id^3a+ib\,c^3+b\_\_Z^3-3\_\_Zb\,c^2\right)}\frac{\_\_R1\,e^{-\_\_R1}\operatorname{expIntegral}_1\left(idx+ic-\_\_R1\right)}{-2ic\_\_R1+\_\_R1^2-c^2}\right)}{6b}-\frac{idc\left(\sum_{\_\_R1=\ldots}\right.}{\ldots}$

input
```
int(x*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/3*d^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(
d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
-1/3*d^3*c/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R
1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.10

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx =$$

$$-\frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}(\sqrt{3}+i)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)}-\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}(\sqrt{3}+i)\mathrm{I}}{}$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(
-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d
^3/b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(
sqrt(3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*
a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*
Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/
3)*(-I*sqrt(3) + 1) + I*c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x + (-I*a*d^3/b
)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*Ei(-I*d*x +
(I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F]

$$\int \frac{x \sin(c+dx)}{a+bx^3} \, dx = \int \frac{x \sin{(c+dx)}}{a+bx^3} \, dx$$

input
```
integrate(x*sin(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x \sin(c+dx)}{a+bx^3} \, dx = \int \frac{x \sin{(dx+c)}}{bx^3+a} \, dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*
cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d*x^
3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^
2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(2*b
*x^3 - a)*cos(d*x + c)/(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d), x) + 2*(((b*cos(
c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + (
(b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c
)^2)*integrate(1/2*(2*b*x^3 - a)*cos(d*x + c)/((b^2*d*x^6 + 2*a*b*d*x^3 +
a^2*d)*cos(d*x + c)^2 + (b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sin(d*x + c)^2),
 x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/
(((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x +
 c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*si
n(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{x\sin(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{x\,\sin(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x^3),x)
```

output
```
int((x*sin(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(dx+c)\,x}{b\,x^3+a}dx$$

input
```
int(x*sin(d*x+c)/(b*x^3+a),x)
```

output
```
int((sin(c + d*x)*x)/(a + b*x**3),x)
```

3.98 $\int \frac{\sin(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 343

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+bx^3}\,dx &= \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} \\ &\quad - \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} \\ &\quad + \frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} \\ &\quad + \frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} \\ &\quad + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}} \\ &\quad + \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\end{aligned}$$

output
```
1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/3
*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+
d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*co
s(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^
(2/3)/b^(1/3)+1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(2/
3)/b^(1/3)+1/3*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3
)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#}{\#1^2}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x^3),x]
```

output
```
((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - RootSum[a + b*#1
^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*
Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*Si
nIntegral[d*(x - #1)])/#1^2 & ]))/b
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{3814}$$

$$\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\sqrt[3]{-1}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[Sin[c + d*x]/(a + b*x^3),x]
```

output
```
(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a
^(2/3)*b^(1/3)) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)
^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])
/(3*a^(2/3)*b^(1/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*
d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a
^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.25

method	result
derivativedivides	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$-\frac{id^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3\_Zbc^2\right)}\frac{e^{\_R1}\mathrm{expIntegral}_1\left(-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}+\frac{id^2\left(\sum_{\_R1=\mathrm{R}}\right.}{}$

input
```
int(sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\sqrt{3}+1)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(-i\sqrt{3}-1)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\sqrt{3}+1)-i\,c\right)}+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\sqrt{3}+1)}{}$$

input
```
integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*((I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*
(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*
d^3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3)
 - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3
)*(-I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d
^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*
a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(I*a*d^3/b)^(1/3)*Ei(-I*d
*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(c+dx)}{a+bx^3}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin{(c+d\,x)}}{b\,x^3+a}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x^3),x)
```

output
```
int(sin(c + d*x)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin{(dx+c)}}{b\,x^3+a}dx$$

input
```
int(sin(d*x+c)/(b*x^3+a),x)
```

output
```
int(sin(c + d*x)/(a + b*x**3),x)
```

3.99 $\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 301

$$\begin{aligned}\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx &= \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} \\ &\quad - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} \\ &\quad - \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} \\ &\quad + \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\ &\quad - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} \\ &\quad - \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}\end{aligned}$$

output
```
Ci(d*x)*sin(c)/a-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a-
1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))/a-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))/a+cos(c)*Si(d*x)/a-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1
)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d
/b^(1/3)+d*x)/a-1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1
/3)*d/b^(1/3)+d*x)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \frac{-i\mathrm{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\,\mathrm{CosIntegral}(d(x-\#1))-i\,\mathrm{CosIntegral}(d(x-\#1))\sin(c+d\#1\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

output
```
((-I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*Cos
Integral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1
)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + I*RootSum[a + b*#1^3 & ,
Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c +
d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegra
l[d*(x - #1)] & ] + 6*CosIntegral[d*x]*Sin[c] + 6*Cos[c]*SinIntegral[d*x])
/(6*a)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)}\,dx$$

$\Big\downarrow$ 3826

$$\int \left(\frac{\sin(c+dx)}{ax}-\frac{bx^2\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$\Big\downarrow$ 2009

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}+\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}+\frac{\sin(c)\operatorname{CosIntegral}(dx)}{a}+\frac{\cos(c)\operatorname{Si}(dx)}{a}$$

input

```
Int[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

output

```
(CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[
c - (a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[(
(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)])/(3*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cos[c
 - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (C
os[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x])/(3*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.29

method	result
derivativedivides	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\left(-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right.\right.}{3a}$
default	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\left(-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right.\right.}{3a}$
risch	$\frac{ie^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a}-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3\_Zb\,c^2\right)}\mathrm{e}^{\_R1}\,\mathrm{expIntegral}_1\left(-idx-ic+\_\right.\right.}{6a}$

input
```
int(sin(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/3/a*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(
d*x-_R1+c)*sin(_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.02

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)}\,dx$$
$$= \frac{-i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right.}{}$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(-I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^
3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-
I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - I*Ei(-I
*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I
*sqrt(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1)
)*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + I*Ei(I*d*x + (-I*a*d
^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - I*Ei(-I*d*x + (I*a*d^3/b)^(1/3
))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 6*cos_integral(d*x)*sin(c) + 6*cos(c)*si
n_integral(d*x))/a
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\sin\left(c+dx\right)}{x\,(a+bx^3)}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x**3+a),x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\sin(c+d\,x)}{x\,(b\,x^3+a)}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x^3)),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\sin\left(dx+c\right)}{b\,x^4+ax}dx$$

input
```
int(sin(d*x+c)/x/(b*x^3+a),x)
```

output
```
int(sin(c + d*x)/(a*x + b*x**4),x)
```

3.100 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 380

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx &= \frac{d\cos(c)\operatorname{CosIntegral}(dx)}{a} \\
&+ \frac{\sqrt[3]{b}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
&+ \frac{(-1)^{2/3}\sqrt[3]{b}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
&- \frac{\sqrt[3]{-1}\sqrt[3]{b}\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
&- \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\operatorname{Si}(dx)}{a} \\
&- \frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\
&+ \frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} \\
&- \frac{\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}\end{aligned}$$

output
```
d*cos(c)*Ci(d*x)/a+1/3*b^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b
^(1/3))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x
)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*Ci((-
1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3
)-sin(d*x+c)/a/x-d*sin(c)*Si(d*x)/a+1/3*(-1)^(2/3)*b^(1/3)*cos(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)+1/3*b^(
1/3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)-1/3*(-1)^(
1/3)*b^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b
^(1/3)+d*x)/a^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.61

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$$
$$= \frac{6dx\cos(c)\,\text{CosIntegral}(dx) - ix\text{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1)) - i\,\text{CosIntegral}(d(x-\#1))\,\text{si}}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

output
```
(6*d*x*Cos[c]*CosIntegral[d*x] - I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]
*CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos
[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)]
)/#1 & ] + I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1
)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral
[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - 6*Sin[c + d
*x] - 6*d*x*Sin[c]*SinIntegral[d*x])/(6*a*x)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(\frac{\sin(c+dx)}{ax^2}-\frac{bx\sin(c+dx)}{a(a+bx^3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a}-\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sin(c+dx)}{ax}$$

input
```
Int[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

output
```
(d*Cos[c]*CosIntegral[d*x])/a + (b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CosI
ntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*
d)/b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(
1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)
) - Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - ((-1)^(2/3)*b^(1/
3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*Si
nIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Co
s[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b
^(1/3) + d*x])/(3*a^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.31

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)}{adx}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_\right.}{-\_R1+c}}{3a}\right.$
default	$d\left(-\frac{\sin(dx+c)}{adx}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zbc^2+ad^3-bc^3\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_\right.}{-\_R1+c}}{3a}\right.$
risch	$-\frac{d\,\mathrm{expIntegral}_1(-idx)\mathrm{e}^{ic}}{2a}+\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3\_Zbc^2\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{expIntegral}_1\left(-idx-ic+\_\right.}{-ic+\_R1}\right.}{6a}$

input
```
int(sin(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d*(-sin(d*x+c)/a/d/x+1/3/a*sum(1/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x
-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/
a*(-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.18

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*(12*a*d^3*x*cos(c)*cos_integral(d*x) - 12*a*d^3*x*sin(c)*sin_integral
(d*x) + 2*I*(-I*a*d^3/b)^(2/3)*b*x*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c -
 (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*b*x*Ei(-I*d*x + (I*a*d^3/b)^(
1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*a*d^2*sin(d*x + c) + (I*a*d^3/b)^(
2/3)*(sqrt(3)*b*x + I*b*x)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/b)^(2/3)*
(sqrt(3)*b*x + I*b*x)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt
(3)*b*x - I*b*x)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2
*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3)*b
*x - I*b*x)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*
a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c))/(a^2*d^2*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$$

input
```
integrate(sin(d*x+c)/x**2/(b*x**3+a),x)
```

output
```
Integral(sin(c + d*x)/(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\sin(c+d\,x)}{x^2\,(b\,x^3+a)}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x^3)),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{b\,x^5+a\,x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x^3+a),x)
```

output
```
int(sin(c + d*x)/(a*x**2 + b*x**5),x)
```

3.101 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 408

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx &= -\frac{d\cos(c+dx)}{2ax}-\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a}\\
&\quad-\frac{b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\
&\quad+\frac{\sqrt[3]{-1}b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\
&\quad-\frac{(-1)^{2/3}b^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\
&\quad-\frac{\sin(c+dx)}{2ax^2}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}\\
&\quad-\frac{\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}\\
&\quad-\frac{b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}\\
&\quad-\frac{(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}\end{aligned}$$

output
```
-1/2*d*cos(d*x+c)/a/x-1/2*d^2*Ci(d*x)*sin(c)/a-1/3*b^(2/3)*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*Ci((-1)
^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-
1/3*(-1)^(2/3)*b^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/
3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/2*sin(d*x+c)/a/x^2-1/2*d^2*cos(c)*Si(d*x)/
a+1/3*(-1)^(1/3)*b^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3
)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/3*b^(2/3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a
^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*cos(c-(-1)^(2/3)*a^(1
/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.62

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \frac{-ix^2\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-}{\#1^2}\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

output
```
((-I)*x^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + I*x^2*RootSum[
a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x
 - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c +
d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - 3*(d*x*Cos[c + d*x] + d^2*x^2*Co
sIntegral[d*x]*Sin[c] + Sin[c + d*x] + d^2*x^2*Cos[c]*SinIntegral[d*x]))/(
6*a*x^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3826, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}dx$$

$$\Big\downarrow \text{3826}$$

$$\int \left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a(a+bx^3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{\sqrt[3]{-1}b^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{d^2\sin(c)\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

input
```
Int[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*(d*Cos[c + d*x])/(a*x) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^(2/
3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3
*a^(5/3)) + ((-1)^(1/3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)
*b^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sin[c + d*x]/(2*a*x^2) - (d^2*Cos[
c]*SinIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/
3)) - (b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3
) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.33

method	result
derivativedivides	$d^2\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_\right.}{\_R1^2-2\_R1c+c^2}}{3a}\right.$
default	$d^2\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_\right.}{\_R1^2-2\_R1c+c^2}}{3a}\right.$
risch	$-\frac{id^2\,\mathrm{expIntegral}_1(-idx)\mathrm{e}^{ic}}{4a}+\frac{id^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3\_Zbc^2\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{expIntegral}_1\left(-idx-\right.}{-2ic\_R1+\_R1}\right.}{6a}$

input
```
int(sin(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d^2*(-1/3/a*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1
+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(-
1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*s
in(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.20

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(6*a*d^3*x^2*cos_integral(d*x)*sin(c) + 6*a*d^3*x^2*cos(c)*sin_integ
ral(d*x) - 2*(-I*a*d^3/b)^(1/3)*b*x^2*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*
c - (-I*a*d^3/b)^(1/3)) - 2*(I*a*d^3/b)^(1/3)*b*x^2*Ei(-I*d*x + (I*a*d^3/b
)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 6*a*d^2*x*cos(d*x + c) - (-I*sqrt(
3)*b*x^2 - b*x^2)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*
sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*sqrt(3
)*b*x^2 - b*x^2)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*
sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*sqrt(3
)*b*x^2 - b*x^2)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sq
rt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) - (I*sqrt(3)*
b*x^2 - b*x^2)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqr
t(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + 6*a*d*sin(d
*x + c))/(a^2*d*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\sin{(c+dx)}}{x^3(a+bx^3)}\,dx$$

input
```
integrate(sin(d*x+c)/x**3/(b*x**3+a),x)
```

output
```
Integral(sin(c + d*x)/(x**3*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\sin{(dx+c)}}{(bx^3+a)x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx^3+a)x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(b\,x^3+a\right)}\,dx$$

input
```
int(sin(c + d*x)/(x^3*(a + b*x^3)),x)
```

output
```
int(sin(c + d*x)/(x^3*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \frac{\sin\left(dx+c\right)}{b\,x^6+a\,x^3}dx$$

input
```
int(sin(d*x+c)/x^3/(b*x^3+a),x)
```

output
```
int(sin(c + d*x)/(a*x**3 + b*x**6),x)
```

3.102 $\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 714

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \text{Too large to display}$$

output

```
-1/9*(-1)^(2/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)/a^(1/3)/b^(5/3)-1/9*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)
*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*(-1)^(1/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*Ci(a^(
1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3
)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c
-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/3*x*sin(d*x+c)/b/(b*x^3+a
)-1/9*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3
)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*d*sin(c+(-1)^(1/3)*a^(1/3)
*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*cos(
c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*d*sin(c
-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*(-1)^(2/
3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x
)/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si(
(-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.54

$$\int \frac{x^3\sin(c+dx)}{(a+bx^3)^2}\,dx$$
$$= \frac{\text{RootSum}\Big[a+b\#1^3\&,\ \frac{i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))+\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)+\cos(c+d\#1)\text{Si}(d(x-\#1))-}{}}{}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

output
```
(RootSum[a + b*#1^3 & , (I*Cos[c + d*#1]*CosIntegral[d*(x - #1)] + CosInte
gral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I
*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x
- #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#
1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1
)/#1^2 & ] + RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d*(x -
 #1)] + CosIntegral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[
d*(x - #1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*Co
sIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 +
I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral
[d*(x - #1)]*#1)/#1^2 & ] - (6*b*x*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3814, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sin(c+dx)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 3824}$$

$$\frac{\int \frac{\sin(c+dx)}{bx^3+a}dx}{3b}+\frac{d\int \frac{x\cos(c+dx)}{bx^3+a}dx}{3b}-\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 3814}$$

$$\frac{\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right)dx}{3b}+\frac{d\int \frac{x\cos(c+dx)}{bx^3+a}dx}{3b}-\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\int \frac{x\cos(c+dx)}{bx^3+a}dx}{3b}+\frac{\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{3a}}{} \frac{x\sin(c+dx)}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 3827}$$

$$\frac{d\int \left(-\frac{\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}-\frac{(-1)^{2/3}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}\right)}\right)dx}{3b}+\frac{\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{3a}}{} \frac{x\sin(c+dx)}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{3a}}{d\left(-\frac{(-1)^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/}}{}\right.}{}\right.}$$

$$\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

output
```
-1/3*(x*Sin[c + d*x])/(b*(a + b*x^3)) + ((CosIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) - ((-1)^(1/3)*Cos
Integral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3)*a^
(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3
)*b^(1/3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegr
al[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cos[c - (
a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(
1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((
-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)))/(3*b) + (d*(-1/3
*((-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(1/3)*b^(2/3)) - (Cos[c - (a^(1/3)*d)/b^(
1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^
(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) - ((-1)^(2/3)*Sin[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/
(3*a^(1/3)*b^(2/3)) + (Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) - ((-1)^(1/3)*Sin[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^
(1/3)*b^(2/3))))/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.06 (sec) , antiderivative size = 1184, normalized size of antiderivative = 1.66

method	result	size
derivativedivides	Expression too large to display	1184
default	Expression too large to display	1184
risch	Expression too large to display	1379

input
```
int(x^3*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-d^6*c^3*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3
*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1
*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x
+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c
+3*_Z*b*c^2+a*d^3-b*c^3)))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-c^3*d^3/a*(d*x+
c))/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/3*c^2*d^3/
a/b*sum((c+_R1)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c^2*
d^3/a/b*sum(_RR1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_
RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(-
2*c^2*d^3/a*(d*x+c)^2+3*c^3*d^3/a*(d*x+c)+c*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3-
b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-2/3*c^2*d^3/a/b*sum(_R1
/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/3*c*d^3/a/b^2*sum((-2
*_RR1^2*b*c+3*_RR1*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-
c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*
b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-1/3*d^3*(a*d^3+5*b*c^3
)/a/b*(d*x+c)-2/3*c*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*
b*c*(d*x+c)^2+b*(d*x+c)^3)+1/9*d^3/a/b^2*sum((3*_R1*b*c^2+a*d^3-b*c^3)/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.94

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a
*d^3/b)^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3))*E
i(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(I*sqrt(3) + 1) - I*c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^
3/b)^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(
I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)
*(I*sqrt(3) + 1) + I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3
/b)^(2/3) - (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(-
I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-
I*sqrt(3) + 1) - I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/
b)^(2/3) - (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I
*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(
-I*sqrt(3) + 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) - (b*x^3 + a)*(
-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1
/3)) - 2*((b*x^3 + a)*(I*a*d^3/b)^(2/3) - (b*x^3 + a)*(I*a*d^3/b)^(1/3))*E
i(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*b^2*d*x^3 +
 a^2*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate(x**3*sin(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^2} \, dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 3*d*
x^2*sin(c) - 12*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 3*d*x^2*sin(c
) - 12*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d
^2*x^3 - 12*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^2*cos(c)^2 + b^
2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c
)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d
^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin
(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x*sin(d*x + c) + (a*d^2*x^2
 + 10*b*x^3 - 2*a)*cos(d*x + c))/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*
d^3*x^3 + a^3*d^3), x) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*
b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*co
s(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 +
a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)
*integrate(3*(3*a*d*x*sin(d*x + c) + (a*d^2*x^2 + 10*b*x^3 - 2*a)*cos(d*x
+ c))/((b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)*cos(d*x
 + c)^2 + (b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)*sin(
d*x + c)^2), x) + ((d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*cos(d*x
 + c)^2 + (d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*sin(d*x + c)^2)*
sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2
+ a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + ...
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^2} \, dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^3 \sin(c+d\,x)}{(b\,x^3+a)^2} \, dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x^3)^2,x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{\sin(dx+c)\,x^3}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x^3*sin(d*x+c)/(b*x^3+a)^2,x)
```

output
```
int((sin(c + d*x)*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.103 $\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}
\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = &-\frac{\sqrt[3]{-1} d \cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\
&+\frac{d \cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\
&+\frac{(-1)^{2/3} d \cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\
&-\frac{\sin(c+dx)}{3b(a+bx^3)} \\
&-\frac{\sqrt[3]{-1} d \sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\
&-\frac{d \sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\
&-\frac{(-1)^{2/3} d \sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}
\end{aligned}$$

output
```
-1/9*(-1)^(1/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)/a^(2/3)/b^(4/3)+1/9*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)
*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)-1/3*sin(d*
x+c)/b/(b*x^3+a)+1/9*(-1)^(1/3)*d*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-
(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)-1/9*d*sin(c-a^(1/3)*d/b^
(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)-1/9*(-1)^(2/3)*d*sin(c-(-
1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b
^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.58

$$\int \frac{x^2\sin(c+dx)}{(a+bx^3)^2}\,dx = \frac{d\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1)}{\#1^2}\right.}{}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

output
```
(d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosIn
tegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
 - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + d*RootSum[a + b*#1^3
& , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin
[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIn
tegral[d*(x - #1)])/#1^2 & ] - (6*b*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} dx$$

$$\Big\downarrow \text{3822}$$

$$\frac{d\int \frac{\cos(c+dx)}{bx^3+a}dx}{3b} - \frac{\sin(c+dx)}{3b(a+bx^3)}$$

$$\Big\downarrow \text{3815}$$

$$\frac{d\int \left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right)dx}{3b} - \frac{\sin(c+dx)}{3b(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{\sqrt[3]{-1}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/}}{}\right.}{}\right.}{} \ \frac{\sin(c+dx)}{3b(a+bx^3)}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

output
```
-1/3*Sin[c + d*x]/(b*(a + b*x^3)) + (d*(-1/3*((-1)^(1/3)*Cos[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/
(a^(2/3)*b^(1/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2
/3)*b^(1/3)) - ((-1)^(1/3)*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) - (Sin[c -
 (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b
^(1/3)) - ((-1)^(2/3)*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3))))/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.75 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.22

method	result	size
derivativedivides	Expression too large to display	823
default	Expression too large to display	823
risch	Expression too large to display	926

input
```
int(x^2*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(d^6*c^2*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*
c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+
_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+
3*_Z*b*c^2+a*d^3-b*c^3)))+sin(d*x+c)*(-2/3*c*d^3/a*(d*x+c)^2+2/3*c^2*d^3/a
*(d*x+c))/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-2/9*c*
d^3/a/b*sum((c+_R1)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_
R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-2/9*
c*d^3/a/b*sum(_RR1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos
(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*
(2/3*c*d^3/a*(d*x+c)^2-c^2*d^3/a*(d*x+c)-1/3*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3
-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9*c*d^3/a/b*sum(_R1/
(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=
RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b^2*sum((-2*_R
R1^2*b*c+3*_RR1*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*
sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c
^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*((-I*b*x^3 + sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x +
 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(
3) + 1) - I*c) + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(-I*a*d^3/b)^(1/3)*
Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1
/3)*(I*sqrt(3) + 1) + I*c) + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d
^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a
*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I
*a)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*
e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(I*b*x^3 + I*a)*(-I*
a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3))
 - 2*(-I*b*x^3 - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(
-I*c - (I*a*d^3/b)^(1/3)) - 12*a*sin(d*x + c))/(a*b^2*x^3 + a^2*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*sin(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^2 \sin(dx+c)}{(bx^3+a)^2} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 4*(cos(c)^2 + sin(c)^2)*x
*sin(d*x + c) + ((d*x^2*cos(c) - 4*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c
) - 4*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*s
in(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2
 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*
x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)
^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c) - 2*(5*b*x^3 - a
)*sin(d*x + c))/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^2*b*d^2*x^3 + a^3*d^2
), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*
sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((
b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2
*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*
d*x*cos(d*x + c) - 2*(5*b*x^3 - a)*sin(d*x + c))/((b^3*d^2*x^9 + 3*a*b^2*d
^2*x^6 + 3*a^2*b*d^2*x^3 + a^3*d^2)*cos(d*x + c)^2 + (b^3*d^2*x^9 + 3*a*b^
2*d^2*x^6 + 3*a^2*b*d^2*x^3 + a^3*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c
) + 4*x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(c) + 4*x*cos(c))*sin(d*x + c)^
2)*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)
^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x +
c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(
c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^2 \sin(dx+c)}{(bx^3+a)^2} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^2 \sin(c+d\,x)}{(b\,x^3+a)^2} \, dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x^3)^2,x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{\sin(dx+c)\,x^2}{b^2x^6+2ab\,x^3+a^2} dx$$

input
```
int(x^2*sin(d*x+c)/(b*x^3+a)^2,x)
```

output
```
int((sin(c + d*x)*x**2)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.104 $\int \frac{x \sin(c+dx)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 691

$$\int \frac{x \sin(c+dx)}{(a+bx^3)^2} dx = \text{Too large to display}$$

output

```
-1/9*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)
-d*x)/a/b-1/9*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9
*d*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x
)/a/b-1/9*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/
3)-1/9*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^
(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^
(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/3*sin(d*x
+c)/a/b/x-1/3*sin(d*x+c)/b/x/(b*x^3+a)-1/9*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(
1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+1/9*
d*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x
)/a/b-1/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/
3)+1/9*d*sin(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a/b+1/9*(-1)^(
1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d
*x)/a^(4/3)/b^(2/3)+1/9*d*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3
)*a^(1/3)*d/b^(1/3)+d*x)/a/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.59

$$\int \frac{x \sin(c+dx)}{(a+bx^3)^2} \, dx =$$
$$-\frac{(a+bx^3)\,\mathrm{RootSum}\Big[a+b\#1^3\&, \frac{-i\cos(c+d\#1)\,\mathrm{CosIntegral}(d(x-\#1))-\mathrm{CosIntegral}(d(x-\#1))\sin(c+d\#1)-\cos(c+d\#1}{}}{}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

output
```
-1/18*((a + b*x^3)*RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[
d*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c + d*#1]*SinInt
egral[d*(x - #1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*
#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]
*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIn
tegral[d*(x - #1)]*#1)/#1 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , (I*Cos[
c + d*#1]*CosIntegral[d*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1]
- Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*Sin[c + d*#1]*SinIntegral[d*(x
 - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(
x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 -
 d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] - 6*b*x^2*Sin[c + d*x]
)/(a*b*(a + b*x^3))
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3824, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(c+dx)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \ 3824$$

$$\frac{d\int \frac{\cos(c+dx)}{x(bx^3+a)}dx}{3b}-\frac{\int \frac{\sin(c+dx)}{x^2(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx\,(a+bx^3)}$$

$$\Big\downarrow \ 3826$$

$$\frac{d\int \frac{\cos(c+dx)}{x(bx^3+a)}dx}{3b}-\frac{\int \left(\frac{\sin(c+dx)}{ax^2}-\frac{bx\sin(c+dx)}{a(bx^3+a)}\right)dx}{3b}-\frac{\sin(c+dx)}{3bx\,(a+bx^3)}$$

$$\Big\downarrow \ 2009$$

$$\frac{d\int \frac{\cos(c+dx)}{x(bx^3+a)}dx}{3b}-\frac{\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\right.}{}}{}-\frac{\sin(c+dx)}{3bx\,(a+bx^3)}$$

$$\Big\downarrow \ 3827$$

$$\frac{d\int \left(\frac{\cos(c+dx)}{ax}-\frac{bx^2\cos(c+dx)}{a(bx^3+a)}\right)dx}{3b}-\frac{\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\right.}{}}{}-\frac{\sin(c+dx)}{3bx\,(a+bx^3)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c\right.}{}}{}$$

$$\frac{d\left(-\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIn}}{3a}\right.}{}$$

$$\frac{\sin(c+dx)}{3bx\,(a+bx^3)}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

output
```
-1/3*Sin[c + d*x]/(b*x*(a + b*x^3)) - ((d*Cos[c]*CosIntegral[d*x])/a + (b^
(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])
/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - ((-1)^(1
/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - Sin[c + d*x]/(a*x) - (d*Sin[c]*
SinIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (
b^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x
])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)))/(3*b) +
 (d*((Cos[c]*CosIntegral[d*x])/a - (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cos[c - (a^(
1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cos[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
 + d*x])/(3*a) - (Sin[c]*SinIntegral[d*x])/a - (Sin[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) + (
Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a)
 + (Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x])/(3*a)))/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.52 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{\sin(dx+c)\left(\frac{d^3(dx+c)^2}{3a}-\frac{c\,d^3(dx+c)}{3a}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{\left(c+\_R1\right)\left(-\mathrm{Si}(\right.}{}\right.}{9ab}}{}$
default	$\frac{\frac{\sin(dx+c)\left(\frac{d^3(dx+c)^2}{3a}-\frac{c\,d^3(dx+c)}{3a}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{\left(c+\_R1\right)\left(-\mathrm{Si}(\right.}{}\right.}{9ab}}{}$
risch	$\frac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3\_Zb\,c^2\right)}\frac{\left(i\_R1+c-2i\right)\mathrm{e}^{-\_R1}\,\mathrm{expIntegral}_1\left(-idx-ic+\_R1\right)}{2ic\_R1-\_R1^2+c^2}\right)}{18ab}+$

input
```
int(x*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(sin(d*x+c)*(1/3*d^3/a*(d*x+c)^2-1/3*c*d^3/a*(d*x+c))/(a*d^3-b*c^3+3
*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/9*d^3/a/b*sum((c+_R1)/(_R1^2
-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b*sum(_RR1/(-_RR1+c)
*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^6*c*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/
3*c/a/d^3)/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a
/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*s
in(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b
*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 655, normalized size of antiderivative = 0.95

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*a*b*d^2*x^2*sin(d*x + c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (-I*b^2*x
^3 - I*a*b - sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(
I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1
) - I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b + sqrt(3)*(b^2*
x^3 + a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt
(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (2*a*b*d^3*x^
3 + 2*a^2*d^3 - (-I*b^2*x^3 - I*a*b + sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)
^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^
3 + I*a*b - sqrt(3)*(b^2*x^3 + a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-
I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
1) + I*c) - 2*(a*b*d^3*x^3 + a^2*d^3 + (I*b^2*x^3 + I*a*b)*(-I*a*d^3/b)^(2
/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(a*b*
d^3*x^3 + a^2*d^3 + (-I*b^2*x^3 - I*a*b)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + (I
*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a^2*b^2*d^2*x^3 + a^3*b*d^
2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx=\text{Timed out}$$

input
```
integrate(x*sin(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx=\int \frac{x\sin{(dx+c)}}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*
cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*
d*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)
^2)*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(
c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)
^2)*integrate(1/2*(5*b*x^3 - a)*cos(d*x + c)/(b^3*d*x^9 + 3*a*b^2*d*x^6 +
3*a^2*b*d*x^3 + a^3*d), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(
a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(
d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*
sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integra
te(1/2*(5*b*x^3 - a)*cos(d*x + c)/((b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*
x^3 + a^3*d)*cos(d*x + c)^2 + (b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^3 +
 a^3*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*
sin(c))*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos
(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c
)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^
2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\sin(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\,\sin(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x^3)^2,x)
```

output
```
int((x*sin(c + d*x))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx=\int \frac{\sin(dx+c)\,x}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x*sin(d*x+c)/(b*x^3+a)^2,x)
```

output
```
int((sin(c + d*x)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.105 $\int \frac{\sin(c+dx)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 735

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2} dx = \text{Too large to display}$$

output
```
1/9*(-1)^(2/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)
*d/b^(1/3)-d*x)/a^(4/3)/b^(2/3)+1/9*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*
d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*(-1)^(1/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*d
/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+2/9*Ci(a^(1
/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-2/9*(-1)^(1/3)
*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/
a^(5/3)/b^(1/3)+2/9*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-
(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/3*sin(d*x+c)/a/b/x^2-1/3*s
in(d*x+c)/b/x^2/(b*x^3+a)-2/9*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*
sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^(4/3)/b^(2/3)+2/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(
5/3)/b^(1/3)-1/9*d*sin(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(4
/3)/b^(2/3)+2/9*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/
3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+1/9*(-1)^(1/3)*d*sin(c-(-1)^(2/3
)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.55

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{-2i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-2\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-2\cos(c+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x^3)^2,x]
```

output
```
-1/18*((a + b*x^3)*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*
SinIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*
Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin
[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d
*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (a + b*x^3)*RootSum[a + b*#1^3
 & , ((2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #
1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c
+ d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*
#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinI
ntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2
& ] - 6*b*x*Sin[c + d*x])/(a*b*(a + b*x^3))
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 830, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3812, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx$$

$\Big\downarrow$ 3812

$$-\frac{2\int \frac{\sin(c+dx)}{x^3(bx^3+a)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^2(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^2(a+bx^3)}$$

$\Big\downarrow$ 3826

$$-\frac{2\int \left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a(bx^3+a)}\right)dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^2(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^2(a+bx^3)}$$

$\Big\downarrow$ 2009

$$\frac{d\int \frac{\cos(c+dx)}{x^2(bx^3+a)}dx}{3b}-$$

$$2\left(-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\text{si}\ldots}{\ldots}\right.$$

$$\frac{\sin(c+dx)}{3bx^2(a+bx^3)}$$

$\Big\downarrow$ 3827

$$\frac{d\int \left(\frac{\cos(c+dx)}{ax^2}-\frac{bx\cos(c+dx)}{a(bx^3+a)}\right)dx}{3b}-$$

$$2\left(-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\text{si}\ldots}{\ldots}\right.$$

$$\frac{\sin(c+dx)}{3bx^2(a+bx^3)}$$

$\Big\downarrow$ 2009

$$-\frac{\sin(c+dx)}{3bx^2(bx^3+a)}-$$

$$2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\text{CosIntegr}\ldots}{\ldots}\right.$$

$$d\left(-\frac{\cos(c+dx)}{ax}+\frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\ldots\right.$$

input
```
Int[Sin[c + d*x]/(a + b*x^3)^2,x]
```

output
```
-1/3*Sin[c + d*x]/(b*x^2*(a + b*x^3)) - (2*(-1/2*(d*Cos[c + d*x])/(a*x) -
(d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^(2/3)*CosIntegral[(a^(1/3)*d)/b^(
1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) + ((-1)^(1/3)*b^(2/3
)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*CosIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a
^(5/3)) - Sin[c + d*x]/(2*a*x^2) - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - (
(-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cos[c - (a^(1/3
)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^
(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))))/(3*b) + (d*(-(Cos[c + d*x]/(a
*x)) + ((-1)^(2/3)*b^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c -
 (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))
- ((-1)^(1/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - (d*CosIntegral[d*x]*S
in[c])/a - (d*Cos[c]*SinIntegral[d*x])/a + ((-1)^(2/3)*b^(1/3)*Sin[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(3*a^(4/3)) - (b^(1/3)*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3812
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Simp[
(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
 - Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x],
x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.34

method	result
derivativedivides	$d^5\left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{-\mathrm{Si}(}{}\right.}{9\ldots}\right.$
default	$d^5\left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\frac{-\mathrm{Si}(}{}\right.}{9\ldots}\right.$
risch	$-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3\_Zb\,c^2\right)}\frac{\left(i\_R1+c-2i\right)e^{\_R1}\,\mathrm{expIntegral}_1\left(-idx-ic+\_R1\right)}{2ic\_R1-\_R1^2+c^2}\right)}{18ba}-$

input
```
int(sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^5*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3*b*c^2*(d*x+
c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si
(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c
+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin
(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+
a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.91

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a
*d^3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3)
)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(
1/3)*(I*sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*
a*d^3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/
3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b
)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I
*a*d^3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3
))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*
a*d^3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3
))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^
(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) + 2*(b*x
^3 + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*
d^3/b)^(1/3)) - 2*((b*x^3 + a)*(I*a*d^3/b)^(2/3) + 2*(b*x^3 + a)*(I*a*d^3/
b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a^
2*b*d*x^3 + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2} dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2} dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2} dx$$

input
```
integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2} dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2} dx$$

input
```
integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx=\int \frac{\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x^3)^2,x)
```

output
```
int(sin(c + d*x)/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx=\int \frac{\sin\left(dx+c\right)}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(sin(d*x+c)/(b*x^3+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.106 $\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 693

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

output
```
1/9*(-1)^(1/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)
*d/b^(1/3)-d*x)/a^(5/3)/b^(1/3)-1/9*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*
d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*d
/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+Ci(d*x)*sin
(c)/a^2-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^2-1/3*Ci(
(-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-
1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3
))/a^2+1/3*sin(d*x+c)/a/b/x^3-1/3*sin(d*x+c)/b/x^3/(b*x^3+a)+cos(c)*Si(d*x
)/a^2-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(
1/3)+d*x)/a^2-1/9*(-1)^(1/3)*d*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1
)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/3*cos(c-a^(1/3)*d/b^(1/3)
)*Si(a^(1/3)*d/b^(1/3)+d*x)/a^2+1/9*d*sin(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*
d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si(
(-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2+1/9*(-1)^(2/3)*d*sin(c-(-1)^(2/3)*a^
(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.64

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{1}{2}i\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

output
```
((-1/2*I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x
- #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + (I/2)*RootSum[a + b*#
1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*
Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*Si
nIntegral[d*(x - #1)] & ] - (a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*Cos
Integral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1
^2 & ])/(6*b) - (a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(
x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIn
tegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(6*b
) + (a*Cos[d*x]*Sin[c])/(a + b*x^3) + 3*CosIntegral[d*x]*Sin[c] + (a*Cos[c
]*Sin[d*x])/(a + b*x^3) + 3*Cos[c]*SinIntegral[d*x])/(3*a^2)
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \ 3824$$

$$\frac{d\int \frac{\cos(c+dx)}{x^3(bx^3+a)}dx}{3b}-\frac{\int \frac{\sin(c+dx)}{x^4(bx^3+a)}dx}{b}-\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 3826$$

$$-\frac{\int \left(\frac{b^2\sin(c+dx)x^2}{a^2(bx^3+a)}-\frac{b\sin(c+dx)}{a^2x}+\frac{\sin(c+dx)}{ax^4}\right)dx}{b}+\frac{d\int \frac{\cos(c+dx)}{x^3(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{d\int \frac{\cos(c+dx)}{x^3(bx^3+a)}dx}{3b}-\frac{-\frac{b\sin(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}+\frac{b\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}+\frac{b\sin\ldots}{\ldots}}{\ldots}$$

$$\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 3827$$

$$\frac{d\int \left(\frac{\cos(c+dx)}{ax^3}-\frac{b\cos(c+dx)}{a(bx^3+a)}\right)dx}{3b}-\frac{-\frac{b\sin(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}+\frac{b\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}+\frac{b\sin\ldots}{\ldots}}{\ldots}$$

$$\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sin(c+dx)}{3bx^3\left(bx^3+a\right)}-$$

$$\frac{-\frac{\cos(c)\,\text{CosIntegral}(dx)d^3}{6a}+\frac{\sin(c)\text{Si}(dx)d^3}{6a}+\frac{\sin(c+dx)d^2}{6ax}-\frac{\cos(c+dx)d}{6ax^2}-\frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2}+\frac{b\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\ldots}{3a^2}\ldots}{\ldots}$$

$$\frac{d\left(-\frac{\cos(c)\,\text{CosIntegral}(dx)d^2}{2a}+\frac{\sin(c)\text{Si}(dx)d^2}{2a}+\frac{\sin(c+dx)d}{2ax}-\frac{\cos(c+dx)}{2ax^2}+\frac{\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{\ldots}}{\sqrt[3]{b}}\ldots\right)}{3a^{5/3}}\ldots\right.}{\ldots}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

output
```
-1/3*Sin[c + d*x]/(b*x^3*(a + b*x^3)) - (-1/6*(d*Cos[c + d*x])/(a*x^2) - (
d^3*Cos[c]*CosIntegral[d*x])/(6*a) - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^
2) + (b*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) + (b*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b
^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - Sin[c + d
*x]/(3*a*x^3) + (d^2*Sin[c + d*x])/(6*a*x) - (b*Cos[c]*SinIntegral[d*x])/a
^2 + (d^3*Sin[c]*SinIntegral[d*x])/(6*a) - (b*Cos[c + ((-1)^(1/3)*a^(1/3)*
d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + (
b*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*
a^2) + (b*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2))/b + (d*(-1/2*Cos[c + d*x]/(a*x^2) - (d
^2*Cos[c]*CosIntegral[d*x])/(2*a) + ((-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3
)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
3*a^(5/3)) - (b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(
5/3)) + (d*Sin[c + d*x])/(2*a*x) + (d^2*Sin[c]*SinIntegral[d*x])/(2*a) + (
(-1)^(1/3)*b^(2/3)*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) + (b^(2/3)*Sin[c - (a^(...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.34

method	result
derivativedivides	$\frac{\sin(dx+c)d^3}{3a\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\left(-\mathrm{Si}\left(-d\right.\right.}{3}$
default	$\frac{\sin(dx+c)d^3}{3a\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)}\left(-\mathrm{Si}\left(-d\right.\right.}{3}$
risch	$\frac{ie^{ic}\,\mathrm{expIntegral}_1(-idx)}{2a^2} - \frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3\_Zb\,c^2\right)}\frac{\left(-id^3a-6i\_R1bc+3\_R1^2b-3\right.}{-2ic\_}\right.}{18a^2b}$

input
```
int(sin(d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(d*x+c)*d^3/a/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c
)^3)-1/3/a^2*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(Si(d*x)*cos(c)+Ci(d*x)
*sin(c))-1/9*d^3/a/b*sum(1/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1
)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.83

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*((-6*I*b*x^3 + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3
) - 6*I*a)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a
*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 + sqrt(3)*(b
*x^3 + a) - I*a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) +
(-6*I*b*x^3 + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3) - 6*
I*a)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)
^(1/3)*(-I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 - sqrt(3)*(b*x^3 +
 a) - I*a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(
I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-3*
I*b*x^3 + (-I*b*x^3 - I*a)*(-I*a*d^3/b)^(1/3) - 3*I*a)*Ei(I*d*x + (-I*a*d^
3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(3*I*b*x^3 + (I*b*x^3 + I*a)*
(I*a*d^3/b)^(1/3) + 3*I*a)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d
^3/b)^(1/3)) + 36*(b*x^3 + a)*cos_integral(d*x)*sin(c) + 36*(b*x^3 + a)*co
s(c)*sin_integral(d*x) + 12*a*sin(d*x + c))/(a^2*b*x^3 + a^3)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx = \int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x**3+a)**2,x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^2x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^2x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x\,\left(b\,x^3+a\right)^2}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x^3)^2),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{b^2x^7+2ab\,x^4+a^2x}dx$$

input
```
int(sin(d*x+c)/x/(b*x^3+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)
```

3.107 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 712

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \text{Too large to display}$$

output
```
d*cos(c)*Ci(d*x)/a^2+1/9*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/
3)*a^(1/3)*d/b^(1/3)-d*x)/a^2+1/9*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*d/
b^(1/3)+d*x)/a^2+1/9*d*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(2/3)*a
^(1/3)*d/b^(1/3)+d*x)/a^2+4/9*b^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1
/3)*d/b^(1/3))/a^(7/3)+4/9*(-1)^(2/3)*b^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1
/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-4/9*(-1)^(1/3)*b^(1/3
)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
/a^(7/3)+1/3*sin(d*x+c)/a/b/x^4-4/3*sin(d*x+c)/a^2/x-1/3*sin(d*x+c)/b/x^4/
(b*x^3+a)-d*sin(c)*Si(d*x)/a^2+4/9*(-1)^(2/3)*b^(1/3)*cos(c+(-1)^(1/3)*a^(
1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)-1/9*d*sin(c+
(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2+4/
9*b^(1/3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)-1/9*d
*sin(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^2-4/9*(-1)^(1/3)*b^(
1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d
*x)/a^(7/3)-1/9*d*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3
)*d/b^(1/3)+d*x)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.85 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.62

$$\int \frac{\sin(c+dx)}{x^2 (a+bx^3)^2} \, dx =$$
$$-\frac{(3a+4bx^3)\cos(dx)\sin(c)+(3a+4bx^3)\cos(c)\sin(dx)-\frac{1}{6}x(a+bx^3)\left(18d\cos(c)\,\mathrm{CosIntegral}(dx)+\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

output
```
-1/3*((3*a + 4*b*x^3)*Cos[d*x]*Sin[c] + (3*a + 4*b*x^3)*Cos[c]*Sin[d*x] -
(x*(a + b*x^3)*(18*d*Cos[c]*CosIntegral[d*x] + RootSum[a + b*#1^3 & , ((-4
*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*(x - #1)]*Sin[
c + d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + (4*I)*Sin[c + d*#1]*
SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d
*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[
d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] + Root
Sum[a + b*#1^3 & , ((4*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosInt
egral[d*(x - #1)]*Sin[c + d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
- (4*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[
c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x -
#1)]*#1)/#1 & ] - 18*d*Sin[c]*SinIntegral[d*x]))/6)/(a^2*x*(a + b*x^3))
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx$$

$\downarrow$ 3824

$$-\frac{4\int \frac{\sin(c+dx)}{x^5(bx^3+a)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^4(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}$$

$\downarrow$ 3826

$$-\frac{4\int \left(\frac{x\sin(c+dx)b^2}{a^2(bx^3+a)}-\frac{\sin(c+dx)b}{a^2x^2}+\frac{\sin(c+dx)}{ax^5}\right)dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^4(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}$$

$\downarrow$ 2009

$$\frac{d\int \frac{\cos(c+dx)}{x^4(bx^3+a)}dx}{3b}-\frac{4\left(-\frac{b^{4/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{7/3}}-\frac{(-1)^{2/3}b^{4/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{7/3}}+\frac{\sqrt[3]{-1}b^{4/3}\text{si}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}$$

$\downarrow$ 3827

$$\frac{d\int \left(\frac{b^2\cos(c+dx)x^2}{a^2(bx^3+a)}-\frac{b\cos(c+dx)}{a^2x}+\frac{\cos(c+dx)}{ax^4}\right)dx}{3b}-\frac{4\left(-\frac{b^{4/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{7/3}}-\frac{(-1)^{2/3}b^{4/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{7/3}}+\frac{\sqrt[3]{-1}b^{4/3}\text{si}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}$$

$\downarrow$ 2009

$$-\frac{\sin(c+dx)}{3bx^4\,(bx^3+a)}-$$

$$\frac{4\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\,\text{CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(d}{a^2}\right.}{}$$

$$\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)d^3}{6a}+\frac{\cos(c)\text{Si}(dx)d^3}{6a}+\frac{\cos(c+dx)d^2}{6ax}+\frac{\sin(c+dx)d}{6ax^2}-\frac{\cos(c+dx)}{3ax^3}-\frac{b\cos(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{b\cos\left(c+\frac{\sqrt[3]{-}}{}\right.}{}\right.}{}$$

input

```
Int[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

output

```
-1/3*Sin[c + d*x]/(b*x^4*(a + b*x^3)) - (4*(-1/12*(d*Cos[c + d*x])/(a*x^3)
 + (d^3*Cos[c + d*x])/(24*a*x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (d^4*
CosIntegral[d*x]*Sin[c])/(24*a) - (b^(4/3)*CosIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(7/3)) - ((-1)^(2/3)*b^(4/3)*Co
sIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)])/(3*a^(7/3)) + ((-1)^(1/3)*b^(4/3)*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(7/
3)) - Sin[c + d*x]/(4*a*x^4) + (d^2*Sin[c + d*x])/(24*a*x^2) + (b*Sin[c +
d*x])/(a^2*x) + (d^4*Cos[c]*SinIntegral[d*x])/(24*a) + (b*d*Sin[c]*SinInte
gral[d*x])/a^2 + ((-1)^(2/3)*b^(4/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(7/3)) - (b^(4/
3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3
*a^(7/3)) + ((-1)^(1/3)*b^(4/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Si
nIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(7/3))))/(3*b) + (d*
(-1/3*Cos[c + d*x]/(a*x^3) + (d^2*Cos[c + d*x])/(6*a*x) - (b*Cos[c]*CosInt
egral[d*x])/a^2 + (b*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + (b*Cos[c - (a^(1/3)*d)/b^(
1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + (b*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(3*a^2) + (d^3*CosIntegral[d*x]*Sin[c])/(6*a) + (d*Sin[c + d*x])/(6*a...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.67 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.40

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)\left(\frac{4b(dx+c)^3}{3a^2}-\frac{4cb(dx+c)^2}{a^2}+\frac{4c^2b(dx+c)}{a^2}+\frac{3a\,d^3-4b\,c^3}{3a^2}\right)}{dx\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)}+\frac{4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+\right.}\right.}{}\right.$
default	$d\left(-\frac{\sin(dx+c)\left(\frac{4b(dx+c)^3}{3a^2}-\frac{4cb(dx+c)^2}{a^2}+\frac{4c^2b(dx+c)}{a^2}+\frac{3a\,d^3-4b\,c^3}{3a^2}\right)}{dx\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)}+\frac{4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+\right.}\right.}{}\right.$
risch	$-\frac{d\,\mathrm{expIntegral}_1(-idx)\mathrm{e}^{ic}}{2a^2}-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3\_Zb\,c^2\right)}\frac{\left(-ic+\_R1-4\right)\mathrm{e}^{\_R1}\,\mathrm{expInt}}{-ic+\_}\right.}{18a^2}$

input
```
int(sin(d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*(-sin(d*x+c)*(4/3*b/a^2*(d*x+c)^3-4*c*b/a^2*(d*x+c)^2+4*c^2*b/a^2*(d*x+c
)+1/3*(3*a*d^3-4*b*c^3)/a^2)/d/x/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c
)^2+b*(d*x+c)^3)+4/9/a^2*sum(1/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_
R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/
a^2*sum(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1),_RR1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(-Si(d*x)*sin(c)+Ci(d*x)*cos
(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.01

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/18*(18*(a*b*d^3*x^4 + a^2*d^3*x)*cos(c)*cos_integral(d*x) + (a*b*d^3*x^4
 + a^2*d^3*x - 2*(-I*b^2*x^4 - I*a*b*x - sqrt(3)*(b^2*x^4 + a*b*x))*(I*a*d
^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I
*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 2*(I*b
^2*x^4 + I*a*b*x + sqrt(3)*(b^2*x^4 + a*b*x))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x
 + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*s
qrt(3) + 1) + I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 2*(-I*b^2*x^4 - I*a*b*x +
sqrt(3)*(b^2*x^4 + a*b*x))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^
(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) +
(a*b*d^3*x^4 + a^2*d^3*x - 2*(I*b^2*x^4 + I*a*b*x - sqrt(3)*(b^2*x^4 + a*b
*x))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1)
)*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (a*b*d^3*x^4 + a^2*d
^3*x - 4*(-I*b^2*x^4 - I*a*b*x)*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + (-I*a*d^3/b
)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (a*b*d^3*x^4 + a^2*d^3*x - 4*(I*b^
2*x^4 + I*a*b*x)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c
 - (I*a*d^3/b)^(1/3)) - 18*(a*b*d^3*x^4 + a^2*d^3*x)*sin(c)*sin_integral(d
*x) - 6*(4*a*b*d^2*x^3 + 3*a^2*d^2)*sin(d*x + c))/(a^3*b*d^2*x^4 + a^4*d^2
*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2 (a+bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/x**2/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \int \frac{\sin(c+d\,x)}{x^2\,(b\,x^3+a)^2}\,dx$$

input
```
int(sin(c + d*x)/(x^2*(a + b*x^3)^2),x)
```

output
```
int(sin(c + d*x)/(x^2*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx = \int \frac{\sin\left(dx+c\right)}{b^2x^8+2ab\,x^5+a^2x^2}dx$$

input
```
int(sin(d*x+c)/x^2/(b*x^3+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x**2 + 2*a*b*x**5 + b**2*x**8),x)
```

3.108 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 800

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx = \text{Too large to display}$$

output
```
-1/2*d*cos(d*x+c)/a^2/x-1/9*(-1)^(2/3)*b^(1/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(7/3)-1/9*b^(1/3)*d*cos(
c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)+1/9*(-1)^(1/3)*b^(1
/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+
d*x)/a^(7/3)-1/2*d^2*Ci(d*x)*sin(c)/a^2-5/9*b^(2/3)*Ci(a^(1/3)*d/b^(1/3)+d
*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(8/3)+5/9*(-1)^(1/3)*b^(2/3)*Ci((-1)^(1/3)*
a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)-5/9*(-1
)^(2/3)*b^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1
/3)*d/b^(1/3))/a^(8/3)+1/3*sin(d*x+c)/a/b/x^5-5/6*sin(d*x+c)/a^2/x^2-1/3*s
in(d*x+c)/b/x^5/(b*x^3+a)-1/2*d^2*cos(c)*Si(d*x)/a^2+5/9*(-1)^(1/3)*b^(2/3
)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x
)/a^(8/3)+1/9*(-1)^(2/3)*b^(1/3)*d*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(
-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)-5/9*b^(2/3)*cos(c-a^(1/3)*d/b^(
1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)+1/9*b^(1/3)*d*sin(c-a^(1/3)*d/b^(1
/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)-5/9*(-1)^(2/3)*b^(2/3)*cos(c-(-1)^(
2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)-1/9*(
-1)^(1/3)*b^(1/3)*d*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1
/3)*d/b^(1/3)+d*x)/a^(7/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 1.00 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.59

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx$$
$$= \frac{\text{RootSum}\left[a+b\#1^3\&,\frac{-5i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-5\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-5\cos(c+d\#1)\text{Si}(d(x-\#\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

output
```
(RootSum[a + b*#1^3 & , ((-5*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 5*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] + (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosI
ntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*
d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d
*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3 & , ((5*I)*Cos[c + d*#1]*CosI
ntegral[d*(x - #1)] - 5*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c +
d*#1]*SinIntegral[d*(x - #1)] - (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1
)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin
[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - (3*(3*a*d*x*Cos[c + d*x]
 + 3*b*d*x^4*Cos[c + d*x] + 3*d^2*x^2*(a + b*x^3)*CosIntegral[d*x]*Sin[c]
+ 3*a*Sin[c + d*x] + 5*b*x^3*Sin[c + d*x] + 3*d^2*x^2*(a + b*x^3)*Cos[c]*S
inIntegral[d*x]))/(x^2*(a + b*x^3)))/(18*a^2)
```

Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 1059, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3824, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx$$

$\Big\downarrow$ 3824

$$-\frac{5\int \frac{\sin(c+dx)}{x^6(bx^3+a)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^5(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}$$

$\Big\downarrow$ 3826

$$-\frac{5\int \left(\frac{\sin(c+dx)b^2}{a^2(bx^3+a)}-\frac{\sin(c+dx)b}{a^2x^3}+\frac{\sin(c+dx)}{ax^6}\right)dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^5(bx^3+a)}dx}{3b}-\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}$$

$\Big\downarrow$ 2009

$$\frac{d\int \frac{\cos(c+dx)}{x^5(bx^3+a)}dx}{3b}-$$

$$5\left(\frac{b^{5/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{8/3}}-\frac{\sqrt[3]{-1}b^{5/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{8/3}}+\frac{(-1)^{2/3}b^{5/3}\sin\left(\right.}{}\right.$$

$$\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}$$

$\Big\downarrow$ 3827

$$\frac{d\int \left(\frac{x\cos(c+dx)b^2}{a^2(bx^3+a)}-\frac{\cos(c+dx)b}{a^2x^2}+\frac{\cos(c+dx)}{ax^5}\right)dx}{3b}-$$

$$5\left(\frac{b^{5/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{8/3}}-\frac{\sqrt[3]{-1}b^{5/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{8/3}}+\frac{(-1)^{2/3}b^{5/3}\sin\left(\right.}{}\right.$$

$$\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}$$

$\Big\downarrow$ 2009

$$-\frac{\sin(c+dx)}{3bx^5\,(bx^3+a)}-$$

$$\frac{5\left(\frac{\cos(c)\,\mathrm{CosIntegral}(dx)d^5}{120a}-\frac{\sin(c)\mathrm{Si}(dx)d^5}{120a}-\frac{\sin(c+dx)d^4}{120ax}+\frac{\cos(c+dx)d^3}{120ax^2}+\frac{b\,\mathrm{CosIntegral}(dx)\sin(c)d^2}{2a^2}+\frac{\sin(c+dx)d^2}{60ax^3}+\frac{b\cos(c)\mathrm{Si}}{2a^2}\right.}{}$$

$$d\left(\frac{\cos(c)\,\mathrm{CosIntegral}(dx)d^4}{24a}-\frac{\sin(c)\mathrm{Si}(dx)d^4}{24a}-\frac{\sin(c+dx)d^3}{24ax}+\frac{\cos(c+dx)d^2}{24ax^2}+\frac{b\,\mathrm{CosIntegral}(dx)\sin(c)d}{a^2}+\frac{\sin(c+dx)d}{12ax^3}+\frac{b\cos(c)\mathrm{Si}(d}{a^2}\right.$$

input
```
Int[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/3*Sin[c + d*x]/(b*x^5*(a + b*x^3)) - (5*(-1/20*(d*Cos[c + d*x])/(a*x^4)
 + (d^3*Cos[c + d*x])/(120*a*x^2) + (b*d*Cos[c + d*x])/(2*a^2*x) + (d^5*Co
s[c]*CosIntegral[d*x])/(120*a) + (b*d^2*CosIntegral[d*x]*Sin[c])/(2*a^2) +
 (b^(5/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(3*a^(8/3)) - ((-1)^(1/3)*b^(5/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/
b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(8/3)) + ((-1
)^(2/3)*b^(5/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(8/3)) - Sin[c + d*x]/(5*a*x^5) + (d
^2*Sin[c + d*x])/(60*a*x^3) + (b*Sin[c + d*x])/(2*a^2*x^2) - (d^4*Sin[c +
d*x])/(120*a*x) + (b*d^2*Cos[c]*SinIntegral[d*x])/(2*a^2) - (d^5*Sin[c]*Si
nIntegral[d*x])/(120*a) + ((-1)^(1/3)*b^(5/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*
d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(8/3))
 + (b^(5/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x])/(3*a^(8/3)) + ((-1)^(2/3)*b^(5/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(8/3))))/(3
*b) + (d*(-1/4*Cos[c + d*x]/(a*x^4) + (d^2*Cos[c + d*x])/(24*a*x^2) + (b*C
os[c + d*x])/(a^2*x) + (d^4*Cos[c]*CosIntegral[d*x])/(24*a) - ((-1)^(2/3)*
b^(4/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^
(1/3)*d)/b^(1/3) - d*x])/(3*a^(7/3)) - (b^(4/3)*Cos[c - (a^(1/3)*d)/b^(1/3
)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(7/3)) + ((-1)^(1/3)*b^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{i d^2 \operatorname{expIntegral}_1(-idx)\mathrm{e}^{ic}}{4a^2} - \frac{i d^2 \left(\sum_{\_R1=\operatorname{RootOf}\left(-3i\_Z^2 bc - id^3 a + ib\,c^3 + b\_Z^3 - 3\_Zb\,c^2\right)} \frac{\left(-ic+\_R1-5\right)\mathrm{e}^{\_R1} \exp}{-2ic\_R1}\right.}{18a^2}$
derivativedivides	$d^2 \left(-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)} \frac{-\operatorname{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\operatorname{Ci}\left(dx-\_R1+c\right)\sin\left(\_\right.}{\_R1^2-2\_R1c+c^2}}{3a^2}\right.$
default	$d^2 \left(-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3bc\_Z^2+3\_Zb\,c^2+a\,d^3-b\,c^3\right)} \frac{-\operatorname{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\operatorname{Ci}\left(dx-\_R1+c\right)\sin\left(\_\right.}{\_R1^2-2\_R1c+c^2}}{3a^2}\right.$

input

```
int(sin(d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/4*I*d^2/a^2*Ei(1,-I*d*x)*exp(I*c)-1/18*I*d^2/a^2*sum((-I*c+_R1-5)/(-2*I
*c*_R1+_R1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*
d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/4*I*d^2/a^2*Ei(1,I*d*x)*exp(-I*c)-1/18
*I*d^2/a^2*sum((-I*c+_R1+5)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*
c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/2*(-
b*d^4*x^4-a*d^4*x)/a^2/x^2/(b*d^3*x^3+a*d^3)*cos(d*x+c)-1/6*(5*b*d^3*x^3+3
*a*d^3)/a^2/x^2/(b*d^3*x^3+a*d^3)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{x^3 (a+bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b)^
(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b
)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d
^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*
x^5 + I*a*b*x^2))*(-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(I*b
^2*x^5 + I*a*b*x^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((b^
2*x^5 + a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(2/3) + 5*
(b^2*x^5 + a*b*x^2 + sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(1/3))*
Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(-I*sqrt(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*
a*b*x^2))*(-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(-I*b^2*x^5
- I*a*b*x^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqr
t(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((b^2*x^5
 + a*b*x^2)*(-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2)*(-I*a*d^3/b)^(1/3))
*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((b^2*x^5
 + a*b*x^2)*(I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2)*(I*a*d^3/b)^(1/3))*E
i(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 18*(a*b*d^3*x
^5 + a^2*d^3*x^2)*cos_integral(d*x)*sin(c) + 18*(a*b*d^3*x^5 + a^2*d^3*x^2
)*cos(c)*sin_integral(d*x) + 18*(a*b*d^2*x^4 + a^2*d^2*x)*cos(d*x + c) ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/x**3/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x^3}\,dx$$

input
```
integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx = \int \frac{\sin(c+d\,x)}{x^3\,(b\,x^3+a)^2}\,dx$$

input
```
int(sin(c + d*x)/(x^3*(a + b*x^3)^2),x)
```

output
```
int(sin(c + d*x)/(x^3*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{b^2x^9+2ab\,x^6+a^2x^3}dx$$

input
```
int(sin(d*x+c)/x^3/(b*x^3+a)^2,x)
```

output
```
int(sin(c + d*x)/(a**2*x**3 + 2*a*b*x**6 + b**2*x**9),x)
```

3.109 $\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 772

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

output
```
1/18*d*cos(d*x+c)/a/b^2/x-1/18*d*cos(d*x+c)/b^2/x/(b*x^3+a)+1/27*Ci(a^(1/3
)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Ci(a^(1
/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(1/3)*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b
^(4/3)+1/54*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/a/b^2+1/27*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Ci((-1)^(2/3)
*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/18*sin
(d*x+c)/a/b^2/x^2-1/6*x*sin(d*x+c)/b/(b*x^3+a)^2-1/18*sin(d*x+c)/b^2/x^2/(
b*x^3+a)-1/27*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3
)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*cos(c+(-1)^(1/3)*a^(1/3)
*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/27*cos(c-a^(1/3)
*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*cos(c-a^(1/
3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/27*(-1)^(2/3)*cos(c-(-1)^(
2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/
3)+1/54*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^
(1/3)+d*x)/a/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.72 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.59

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx$$
$$= \frac{i\text{RootSum}\left[a+b\#1^3\&, \frac{2\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-2i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-2i\cos(c+d\#1)\text{Si}(d(x-\#\ldots}{}\right.}{}$$

input
```
Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

output
```
(I*RootSum[a + b*#1^3 & , (2*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - (2*I)
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (2*I)*Cos[c + d*#1]*SinIntegral[d
*(x - #1)] - 2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d^2*Cos[c + d*#1]*C
osIntegral[d*(x - #1)]*#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*
#1^2 - I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1
]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - I*RootSum[a + b*#1^3 & , (2*Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + (2*I)*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] + (2*I)*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - 2*Sin[c + d*#1]*Sin
Integral[d*(x - #1)] + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + I*
d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 + I*d^2*Cos[c + d*#1]*SinIn
tegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/
#1^2 & ] + (6*b*x*(d*x*(a + b*x^3)*Cos[c + d*x] + (-2*a + b*x^3)*Sin[c + d
*x]))/(a + b*x^3)^2)/(108*a*b^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1592 vs. 2(772) = 1544.

Time = 3.69 (sec) , antiderivative size = 1592, normalized size of antiderivative = 2.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3824, 3812, 3825, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx$$

$\Big\downarrow$ 3824

$$\frac{\int \frac{\sin(c+dx)}{(bx^3+a)^2}dx}{6b}+\frac{d\int \frac{x\cos(c+dx)}{(bx^3+a)^2}dx}{6b}-\frac{x\sin(c+dx)}{6b\,(a+bx^3)^2}$$

$\Big\downarrow$ 3812

$$\frac{d\int \frac{x\cos(c+dx)}{(bx^3+a)^2}dx}{6b}+\frac{-\frac{2\int \frac{\sin(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^2(a+bx^3)}}{6b}-\frac{x\sin(c+dx)}{6b\,(a+bx^3)^2}$$

$\Big\downarrow$ 3825

$$\frac{-\frac{2\int \frac{\sin(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^2(a+bx^3)}}{6b}+\frac{d\left(-\frac{d\int \frac{\sin(c+dx)}{x\left(bx^3+a\right)}dx}{3b}-\frac{\int \frac{\cos(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cos(c+dx)}{3bx(a+bx^3)}\right)}{6b}-\frac{x\sin(c+dx)}{6b\,(a+bx^3)^2}$$

$\Big\downarrow$ 3826

$$\frac{-\frac{2\int \left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^2(a+bx^3)}}{6b}+\frac{d\left(-\frac{d\int \left(\frac{\sin(c+dx)}{ax}-\frac{bx^2\sin(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{\int \frac{\cos(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cos(c+dx)}{3bx(a+bx^3)}\right)}{6b}-\frac{x\sin(c+dx)}{6b\,(a+bx^3)^2}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{x\sin(c+dx)}{6b\left(bx^3+a\right)^2}+\frac{d\left(-\frac{\cos(c+dx)}{3bx(bx^3+a)}-\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)}{a}-\frac{\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\cdots\right.}{\cdots}\right.}{\cdots}}{-\frac{\sin(c+dx)}{3bx^2(bx^3+a)}-\frac{2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{CosIntegr}\cdots}{\cdots}\right.}{\cdots}}$$

$\downarrow$ 3827

$$\frac{-\frac{x\sin(c+dx)}{6b\left(bx^3+a\right)^2}+\frac{d\left(-\frac{\cos(c+dx)}{3bx(bx^3+a)}-\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)}{a}-\frac{\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\cdots\right.}{\cdots}\right.}{\cdots}}{-\frac{\sin(c+dx)}{3bx^2(bx^3+a)}-\frac{2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{CosIntegr}\cdots}{\cdots}\right.}{\cdots}}$$

$\downarrow$ 2009

$$-\frac{x\sin(c+dx)}{6b\,(bx^3+a)^2}+$$

$$d\left(-\frac{\cos(c+dx)}{3bx(bx^3+a)}-\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)}{a}-\frac{\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\cdots\right.}{\cdots}\right.$$

$$-\frac{\sin(c+dx)}{3bx^2(bx^3+a)}-\frac{2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{CosIntegr}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Int[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*(x*Sin[c + d*x])/(b*(a + b*x^3)^2) + (d*(-1/3*Cos[c + d*x]/(b*x*(a +
b*x^3)) - (d*((CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a^(1/3)*d)/b^(1/
3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1)^(1/3)*a
^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) -
(CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)])/(3*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/
(3*a) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(3*a) - (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a)))/(3*b) - (-(Cos[c + d*x]/(a*x)) + ((-
1)^(2/3)*b^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)
^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*
d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1
/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - (d*CosIntegral[d*x]*Sin[c])/a -
 (d*Cos[c]*SinIntegral[d*x])/a + ((-1)^(2/3)*b^(1/3)*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a
^(4/3)) - (b^(1/3)*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^
(1/3) + d*x])/(3*a^(4/3)) + ((-1)^(1/3)*b^(1/3)*Sin[c - ((-1)^(2/3)*a^(1/3
)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3812
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Simp[
(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
 - Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x],
x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3825
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Cos[c + d*x], x], x] + Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.96 (sec) , antiderivative size = 1337, normalized size of antiderivative = 1.73

method	result	size
risch	Expression too large to display	1337
derivativedivides	Expression too large to display	2035
default	Expression too large to display	2035

input
```
int(x^3*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*I/d/a^2/b*c^3*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+
_R1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I
*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/36*I/d/a^2/b^2*c^2*sum((-2*I*b*_R1*c^2+4*I*_R
1^2*b+2*I*b*c^2+_R1^2*b*c+a*d^3-b*c^3-4*I*_R1*b+2*_R1*b*c+6*b*c)/(2*I*c*_R
1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a
+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/36*I/d/a^2/b^2*c*sum((I*_R1*a*d^3+2*I*_R1*b
*c^3-8*I*_R1^2*b*c-2*I*a*d^3+2*I*b*c^3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b
*c-10*_R1*b*c^2-2*b*c^2)/(2*I*c*_R1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c
),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))-1/108*I/d/a
^2/b^2*sum((I*_R1*a*c*d^3+2*I*_R1*b*c^4-12*I*_R1^2*b*c^2-6*I*a*c*d^3+6*I*b
*c^4+_R1^2*a*d^3-_R1^2*b*c^3-a*c^2*d^3+c^5*b+12*I*b*_R1*c^2-18*_R1*b*c^3-2
*a*d^3+2*b*c^3)/(2*I*c*_R1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=Roo
tOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))-1/108*I/d/a^2/b*c^3*
sum((-2*I*c*_R1-6*I*c+_R1^2-c^2+6*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)
*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*
b*c^2))-1/36*I/d/a^2/b^2*c^2*sum((-2*I*b*_R1*c^2-4*I*_R1^2*b-2*I*b*c^2+_R1
^2*b*c+a*d^3-b*c^3-4*I*_R1*b-2*_R1*b*c+6*b*c)/(2*I*c*_R1-_R1^2+c^2)*exp(-_
R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*
_Z*b*c^2))-1/36*I/d/a^2/b^2*c*sum((I*_R1*a*d^3+2*I*_R1*b*c^3+8*I*_R1^2*b*c
+2*I*a*d^3-2*I*b*c^3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c+10*_R1*b*c^2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.15

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*((I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b
^2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/b
)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d
^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3
 - I*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a
*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a
*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b + sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*E
i(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)
*(-I*sqrt(3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d
^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3
- I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(
3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3
*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*(
-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1
/3)) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 2*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*
c - (I*a*d^3/b)^(1/3)) + 6*(a*b^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) +
6*(a*b^2*d*x^4 - 2*a^2*b*d*x)*sin(d*x + c))/(a^2*b^4*d*x^6 + 2*a^3*b^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*sin(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(6*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 6*d*
x^2*sin(c) - 42*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 6*d*x^2*sin(c
) - 42*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d
^2*x^3 - 42*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^3*cos(c)^2 + b^
3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)
*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)
^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3
 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3/2*(18*a*
d*x*sin(d*x + c) + (3*a*d^2*x^2 + 112*b*x^3 - 14*a)*cos(d*x + c))/(b^4*d^3
*x^12 + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3),
x) - 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2
*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2
)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^
2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x +
 c)^2)*integrate(3/2*(18*a*d*x*sin(d*x + c) + (3*a*d^2*x^2 + 112*b*x^3 - 1
4*a)*cos(d*x + c))/((b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 +
4*a^3*b*d^3*x^3 + a^4*d^3)*cos(d*x + c)^2 + (b^4*d^3*x^12 + 4*a*b^3*d^3*x^
9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3)*sin(d*x + c)^2), x) ...
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^3} \, dx$$

input
```
integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3\,\sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x^3*sin(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x^3*sin(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)\,x^3}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx$$

input
```
int(x^3*sin(d*x+c)/(b*x^3+a)^3,x)
```

output
```
int((sin(c + d*x)*x**3)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9)
,x)
```

3.110 $\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 777

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

output

```
1/18*d*cos(d*x+c)/a/b^2/x^2-1/18*d*cos(d*x+c)/b^2/x^2/(b*x^3+a)-1/27*(-1)^
(1/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3
)-d*x)/a^(5/3)/b^(4/3)+1/27*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*d/b^(1/3
)+d*x)/a^(5/3)/b^(4/3)+1/27*(-1)^(2/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3
))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-1/54*d^2*Ci(a^(1/3
)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(2/3)*
d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))/a^(4/3)/b^(5/3)+1/54*(-1)^(1/3)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x
)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/6*sin(d*x+c)/b/(b*
x^3+a)^2-1/54*(-1)^(2/3)*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^
(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)+1/27*(-1)^(1/3)*d*sin(c+(-1)^
(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(
4/3)-1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b
^(5/3)-1/27*d*sin(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b
^(4/3)+1/54*(-1)^(1/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/
3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)-1/27*(-1)^(2/3)*d*sin(c-(-1)^(2/
3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx$$
$$= \frac{id\text{RootSum}\left[a+b\#1^3\&, \frac{-2i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-2\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-2\cos(c+d\#1)\text{Si}(d(x-\right.}{}$$

input
```
Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

output
```
(I*d*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)]
- 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*
(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*
CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1
- I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegr
al[d*(x - #1)]*#1)/#1^2 & ] - I*d*RootSum[a + b*#1^3 & , ((2*I)*Cos[c + d*
#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*
Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIntegral[d*
(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d
*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1
 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (6*b*Cos[d*x]*(d
*x*(a + b*x^3)*Cos[c] - 3*a*Sin[c]))/(a + b*x^3)^2 - (6*b*(3*a*Cos[c] + d*
x*(a + b*x^3)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a*b^2)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3822, 3813, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sin(c+dx)}{\left(a+bx^3\right)^3}\,dx$$

$\Big\downarrow$ 3822

$$\frac{d\int \frac{\cos(c+dx)}{\left(bx^3+a\right)^2}dx}{6b}-\frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 3813

$$\frac{d\left(-\frac{2\int \frac{\cos(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{d\int \frac{\sin(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cos(c+dx)}{3bx^2\left(a+bx^3\right)}\right)}{6b}-\frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 3826

$$\frac{d\left(-\frac{2\int \frac{\cos(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{d\int \left(\frac{\sin(c+dx)}{ax^2}-\frac{bx\sin(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{\cos(c+dx)}{3bx^2\left(a+bx^3\right)}\right)}{6b}-\frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2009

$$d\left(-\frac{2\int \frac{\cos(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{d\left(\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\right.}{}\right.$$

$$\frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 3827

$$d\left(-\frac{2\int\left(\frac{\cos(c+dx)}{ax^3}-\frac{b\cos(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{d\left(\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-\cdots}}{\cdots}\right.}{3a^{4/3}}\cdots}{\cdots}\right.\cdots$$

$$\frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

$$d\left(-\frac{\cos(c+dx)}{3bx^2\left(bx^3+a\right)}-\frac{d\left(\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a}+\frac{\sqrt[3]{b}\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(\cdots\right.}{3a^{4/3}}\cdots}{\cdots}\right.\cdots$$

$$\frac{\sin(c+dx)}{6b\left(bx^3+a\right)^2}$$

input
```
Int[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*Sin[c + d*x]/(b*(a + b*x^3)^2) + (d*(-1/3*Cos[c + d*x]/(b*x^2*(a + b*
x^3)) - (d*((d*Cos[c]*CosIntegral[d*x])/a + (b^(1/3)*CosIntegral[(a^(1/3)*
d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*
b^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CosIntegral[((-
1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
)/(3*a^(4/3)) - Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - ((-1)
^(2/3)*b^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*d)
/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3
)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))))/(3*b) - (2*(-1/2*Cos[c + d*x]/(a*
x^2) - (d^2*Cos[c]*CosIntegral[d*x])/(2*a) + ((-1)^(1/3)*b^(2/3)*Cos[c + (
(-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(3*a^(5/3)) - (b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x
])/(3*a^(5/3)) + (d*Sin[c + d*x])/(2*a*x) + (d^2*Sin[c]*SinIntegral[d*x])/
(2*a) + ((-1)^(1/3)*b^(2/3)*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) + (b^(2/3)*Sin...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3813
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))), x] + (-Simp[
(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Cos[c + d*x])/x^n, x], x]
 + Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x],
x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.17 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.18

method	result	size
risch	Expression too large to display	918
derivativedivides	Expression too large to display	1396
default	Expression too large to display	1396

input
```
int(x^2*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/108*I/a^2/b*c^2*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_
R1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*
b*c^3+b*_Z^3-3*_Z*b*c^2))-1/54*I/a^2/b^2*c*sum((-2*I*b*_R1*c^2+4*I*_R1^2*b
+2*I*b*c^2+_R1^2*b*c+a*d^3-b*c^3-4*I*_R1*b+2*_R1*b*c+6*b*c)/(2*I*c*_R1-_R1
^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*
c^3+b*_Z^3-3*_Z*b*c^2))-1/108*I/a^2/b^2*sum((I*_R1*a*d^3+2*I*_R1*b*c^3-8*I
*_R1^2*b*c-2*I*a*d^3+2*I*b*c^3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c-10*_R
1*b*c^2-2*b*c^2)/(2*I*c*_R1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=Ro
otOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/108*I/a^2/b*c^2*s
um((-2*I*c*_R1-6*I*c+_R1^2-c^2+6*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*
Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b
*c^2))+1/54*I/a^2/b^2*c*sum((-2*I*b*_R1*c^2-4*I*_R1^2*b-2*I*b*c^2+_R1^2*b*
c+a*d^3-b*c^3-4*I*_R1*b-2*_R1*b*c+6*b*c)/(2*I*c*_R1-_R1^2+c^2)*exp(-_R1)*E
i(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*
c^2))+1/108*I/a^2/b^2*sum((I*_R1*a*d^3+2*I*_R1*b*c^3+8*I*_R1^2*b*c+2*I*a*d
^3-2*I*b*c^3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c+10*_R1*b*c^2-2*b*c^2)/(
2*I*c*_R1-_R1^2+c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*
c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/18*(a*b*d^7*x^4+a^2*d^7*x)/a^2/b/(
b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*cos(d*x+c)-1/6*d^6/b/(b^2*d^6*x^6+2*a*b
*d^6*x^3+a^2*d^6)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.20

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/216*(((-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
 a^2))*(I*a*d^3/b)^(2/3) - 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^
(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) +
((I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(
-I*a*d^3/b)^(2/3) - 2*(-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6
 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((-I
*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a
*d^3/b)^(2/3) - 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*
a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sq
rt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + ((I*b^2*x^6
 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)
^(2/3) - 2*(-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^
3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3)
 - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((I*b^2*x^6 +
 2*I*a*b*x^3 + I*a^2)*(-I*a*d^3/b)^(2/3) + 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*
a^2)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3
/b)^(1/3)) - 2*((-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(I*a*d^3/b)^(2/3) + 2*(
-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\, dx = \text{Timed out}$$

input
```
integrate(x**2*sin(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2 \sin(c + dx)}{(a + bx^3)^3} \, dx = \int \frac{x^2 \sin(dx + c)}{(bx^3 + a)^3} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 7*(cos(c)^2 + sin(c)^2)*x
*sin(d*x + c) + ((d*x^2*cos(c) - 7*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c
) - 7*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*s
in(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*
cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*co
s(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2
 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 +
 (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(9*a*d*x
*cos(d*x + c) - 7*(8*b*x^3 - a)*sin(d*x + c))/(b^4*d^2*x^12 + 4*a*b^3*d^2*
x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 2*(((b^3*cos(c)
^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 +
 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^
2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2
*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*
d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/
2*(9*a*d*x*cos(d*x + c) - 7*(8*b*x^3 - a)*sin(d*x + c))/((b^4*d^2*x^12 + 4
*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2)*cos(d*x +
c)^2 + (b^4*d^2*x^12 + 4*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x
^3 + a^4*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) + 7*x*cos(c))*cos(d*x +
 c)^2 + (d*x^2*sin(c) + 7*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((...
```

Giac [F]

$$\int \frac{x^2 \sin(c + dx)}{(a + bx^3)^3} \, dx = \int \frac{x^2 \sin(dx + c)}{(bx^3 + a)^3} \, dx$$

input
```
integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^2 \sin(c+d\,x)}{(b\,x^3+a)^3} \, dx$$

input
```
int((x^2*sin(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x^2*sin(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{\sin(dx+c)\,x^2}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3} dx$$

input
```
int(x^2*sin(d*x+c)/(b*x^3+a)^3,x)
```

output
```
int((sin(c + d*x)*x**2)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9)
,x)
```

3.111 $\int \frac{x \sin(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 1141

$$\int \frac{x \sin(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

output
```
1/18*d*cos(d*x+c)/a/b^2/x^3-1/18*d*cos(d*x+c)/b^2/x^3/(b*x^3+a)-2/27*d*cos
(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^2/
b-2/27*d*cos(c-a^(1/3)*d/b^(1/3))*Ci(a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*c
os(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^
2/b-2/27*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3
)+1/54*d^2*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4
/3)-2/27*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*
a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*(-1)^(1/3)*d^2*Ci((-1)^(1/3)*a^(1/
3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27
*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))/a^(7/3)/b^(2/3)+1/54*(-1)^(2/3)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(
1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/18*sin(d*x
+c)/a/b^2/x^4+2/9*sin(d*x+c)/a^2/b/x-1/6*sin(d*x+c)/b/x/(b*x^3+a)^2+1/18*s
in(d*x+c)/b^2/x^4/(b*x^3+a)-2/27*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(
1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-1/54*(-1)^(1/3
)*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)
+d*x)/a^(5/3)/b^(4/3)+2/27*d*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^
(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3
)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1
/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+2/27*d*sin(c-a^(1/3)*d/b^(1/3))*Si(a...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.65 (sec) , antiderivative size = 698, normalized size of antiderivative = 0.61

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/108*(RootSum[a + b*#1^3 & , ((-I)*a*d^2*Cos[c + d*#1]*CosIntegral[d*(x
- #1)] - a*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - a*d^2*Cos[c + d*#1]
*SinIntegral[d*(x - #1)] + I*a*d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] -
 (4*I)*b*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 4*b*CosIntegral[d*(x -
 #1)]*Sin[c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (4
*I)*b*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4*b*d*Cos[c + d*#1]*CosIn
tegral[d*(x - #1)]*#1^2 - (4*I)*b*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*
#1^2 - (4*I)*b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - 4*b*d*Sin[c
+ d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (I
*a*d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - a*d^2*CosIntegral[d*(x - #1
)]*Sin[c + d*#1] - a*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*a*d^2*S
in[c + d*#1]*SinIntegral[d*(x - #1)] + (4*I)*b*Cos[c + d*#1]*CosIntegral[d
*(x - #1)]*#1 - 4*b*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - 4*b*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)]*#1 - (4*I)*b*Sin[c + d*#1]*SinIntegral[d*(x
 - #1)]*#1 + 4*b*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + (4*I)*b*d*
CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 + (4*I)*b*d*Cos[c + d*#1]*SinIn
tegral[d*(x - #1)]*#1^2 - 4*b*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2
)/#1^2 & ] - (6*b*Cos[d*x]*(a*d*(a + b*x^3)*Cos[c] + b*x^2*(7*a + 4*b*x^3)
*Sin[c]))/(a + b*x^3)^2 - (6*b*(b*x^2*(7*a + 4*b*x^3)*Cos[c] - a*d*(a + b*
x^3)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(a^2*b^2)
```

Rubi [A] (verified)

Time = 3.97 (sec) , antiderivative size = 1802, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3824, 3824, 3825, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{3824}$$

$$\frac{d\int \frac{\cos(c+dx)}{x(bx^3+a)^2}dx}{6b}-\frac{\int \frac{\sin(c+dx)}{x^2(bx^3+a)^2}dx}{6b}-\frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{3824}$$

$$\frac{d\int \frac{\cos(c+dx)}{x(bx^3+a)^2}dx}{6b}-\frac{-\frac{4\int \frac{\sin(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^4\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^4(a+bx^3)}}{6b}-\frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 3825$$

$$\frac{d\left(-\frac{d\int \frac{\sin(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{\int \frac{\cos(c+dx)}{x^4\left(bx^3+a\right)}dx}{b}-\frac{\cos(c+dx)}{3bx^3(a+bx^3)}\right)}{6b}-\frac{-\frac{4\int \frac{\sin(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^4\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^4(a+bx^3)}}{6b}-\frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 3826$$

$$-\frac{-\frac{4\int \left(\frac{x\sin(c+dx)b^2}{a^2\left(bx^3+a\right)}-\frac{\sin(c+dx)b}{a^2x^2}+\frac{\sin(c+dx)}{ax^5}\right)dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^4\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^4(a+bx^3)}}{6b}+\frac{d\left(-\frac{d\int \left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{\int \frac{\cos(c+dx)}{x^4\left(bx^3+a\right)}dx}{b}-\frac{\cos(c+dx)}{3bx^3(a+bx^3)}\right)}{6b}-\frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sin(c+dx)}{6bx\left(bx^3+a\right)^2}+$$

$$d\left(-\frac{\cos(c+dx)}{3bx^3(bx^3+a)}-\frac{d\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\text{CosI}\ldots}{\ldots}\right.}{\ldots}\right.$$

$$-\frac{\sin(c+dx)}{3bx^4(bx^3+a)}-\frac{4\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\text{CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(dx)d}{a^2}-\ldots\right.}{\ldots}$$

$$\Big\downarrow \; 3827$$

$$-\frac{\sin(c+dx)}{6bx\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\cos(c+dx)}{3bx^3(bx^3+a)}-\frac{d\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{CosI}}{}\right.}{}\right.}{-\frac{\sin(c+dx)}{3bx^4(bx^3+a)}-\frac{4\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\,\text{CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(dx)d}{a^2}-\right.}{}}$$

$\downarrow$ 2009

$$-\frac{\sin(c+dx)}{6bx\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\cos(c+dx)}{3bx^3(bx^3+a)}-\frac{d\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a}-\frac{\cos(c)\text{Si}(dx)d^2}{2a}-\frac{\cos(c+dx)d}{2ax}-\frac{b^{2/3}\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{CosI}}{}\right.}{}\right.}{-\frac{\sin(c+dx)}{3bx^4(bx^3+a)}-\frac{4\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\,\text{CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(dx)d}{a^2}-\right.}{}}$$

input
```
Int[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*Sin[c + d*x]/(b*x*(a + b*x^3)^2) + (d*(-1/3*Cos[c + d*x]/(b*x^3*(a +
b*x^3)) - (d*(-1/2*(d*Cos[c + d*x])/(a*x) - (d^2*CosIntegral[d*x]*Sin[c])/
(2*a) - (b^(2/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d
)/b^(1/3)])/(3*a^(5/3)) + ((-1)^(1/3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3))
 - ((-1)^(2/3)*b^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*S
in[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sin[c + d*x]/(2*a*x^
2) - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cos[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(3*a^(5/3)) - (b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(
1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(3*a^(5/3))))/(3*b) - (-1/3*Cos[c + d*x]/(a*x^3) + (d^2*Cos[c + d*x])/(6*
a*x) - (b*Cos[c]*CosIntegral[d*x])/a^2 + (b*Cos[c + ((-1)^(1/3)*a^(1/3)*d)
/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + (b*
Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^
2) + (b*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^
(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + (d^3*CosIntegral[d*x]*Sin[c])/(6*a) + (
d*Sin[c + d*x])/(6*a*x^2) + (d^3*Cos[c]*SinIntegral[d*x])/(6*a) + (b*Sin[c
]*SinIntegral[d*x])/a^2 + (b*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Si...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3825
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Cos[c + d*x], x], x] + Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.55 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.53

method	result
risch	$\frac{idc\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^{2}bc-id^{3}a+ib\,c^{3}+b\_Z^{3}-3\_Zb\,c^{2}\right)}\frac{\left(-2ic\_R1+\_R1^{2}-c^{2}+6ic-6\_R1+10\right)e^{\_R1}\,\mathrm{expIntegra}}{-2ic\_R1+\_R1^{2}-c^{2}}\right.}{108a^{2}b}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*I*d/a^2/b*c*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_R
1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b
*c^3+b*_Z^3-3*_Z*b*c^2))+1/108*I*d/a^2/b^2*sum((-2*I*b*_R1*c^2+4*I*_R1^2*b
+2*I*b*c^2+_R1^2*b*c+a*d^3-b*c^3-4*I*_R1*b+2*_R1*b*c+6*b*c)/(2*I*c*_R1-_R1
^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*
c^3+b*_Z^3-3*_Z*b*c^2))-1/108*I*d/a^2/b*c*sum((-2*I*c*_R1-6*I*c+_R1^2-c^2+
6*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(
-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))-1/108*I*d/a^2/b^2*sum((-
2*I*b*_R1*c^2-4*I*_R1^2*b-2*I*b*c^2+_R1^2*b*c+a*d^3-b*c^3-4*I*_R1*b-2*_R1*
b*c+6*b*c)/(2*I*c*_R1-_R1^2+c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(
-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/18*d^4/a*(b*d^3*x^3+a*
d^3)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*cos(d*x+c)-1/18*d*(-4*b*d^5*x^5
-7*a*d^5*x^2)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1319, normalized size of antiderivative = 1.16

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/216*((8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(I*b^3*x^6 + 2
*I*a*b^2*x^3 + I*a^2*b + sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(I*a*d^3
/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(I*a*b^2*
d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1
/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3)
 + 1) - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(-I*b^3
*x^6 - 2*I*a*b^2*x^3 - I*a^2*b - sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*
(-I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*
(I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(-I*a*d^3/b)^(1/3))*Ei(
I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)
*(I*sqrt(3) + 1) + I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3
+ 4*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b - sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*(I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 +
 sqrt(3)*(-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(I*a*d^3/b)^(
1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b
)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*
a^3*d^3 + 4*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b + sqrt(3)*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b))*(-I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +
 a^3*d^3 + sqrt(3)*(-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(-I
*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\sin(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*
cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*
d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^
2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((
b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin
(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(8*b*x^3 - a)*cos(d*x + c)/(b^4*d*
x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x) + 2*((
(b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*
d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*si
n(c)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^
2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d
*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(8*b
*x^3 - a)*cos(d*x + c)/((b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*
a^3*b*d*x^3 + a^4*d)*cos(d*x + c)^2 + (b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*
b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2
*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*s
in(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(
c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x +
c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2...
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\sin(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\,\sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x*sin(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x*sin(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)\,x}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx$$

input
```
int(x*sin(d*x+c)/(b*x^3+a)^3,x)
```

output
```
int((sin(c + d*x)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)
```

3.112 $\int \frac{\sin(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 1161

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

output
```
5/27*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)+5/
27*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)-1/9*
sin(d*x+c)/a/b^2/x^5+5/18*sin(d*x+c)/a^2/b/x^2-1/6*sin(d*x+c)/b/x^2/(b*x^3
+a)^2+1/9*sin(d*x+c)/b^2/x^5/(b*x^3+a)-1/9*(-1)^(2/3)*d*sin(c+(-1)^(1/3)*a
^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/
9*(-1)^(1/3)*d*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d
/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-1/9*(-1)^(1/3)*d*cos(c-(-1)^(2/3)*a^(1/3)*d/
b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/9*(-1)^(2/
3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d
*x)/a^(7/3)/b^(2/3)-5/27*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si
(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)-1/54*d^2*cos(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b+5/27*(-
1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))/a^(8/3)/b^(1/3)-1/54*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(
c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b-5/27*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/54*
d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))/a^2/b+5/27*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)
*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)-1/54*d^2*cos(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-1/9*d*sin(c-a^(1/...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.58

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[c + d*x]/(a + b*x^3)^3,x]
```

output
```
(((-I)*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
 (10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + (10*I)*Cos[c + d*#1]*SinIn
tegral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (6*I)*d*Co
s[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x - #1)]*Sin[c
 + d*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (6*I)*d*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosIntegral[d*(x -
 #1)]*#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 - I*d^2*Cos[
c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(
x - #1)]*#1^2)/#1^2 & ])/b + (I*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*
CosIntegral[d*(x - #1)] - (10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (
10*I)*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral
[d*(x - #1)] + (6*I)*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x
- #1)]*#1 - (6*I)*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c +
 d*#1]*CosIntegral[d*(x - #1)]*#1^2 + I*d^2*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1]*#1^2 + I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[
c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ])/b - (6*x*Cos[d*x]*(d*x*(
a + b*x^3)*Cos[c] - (8*a + 5*b*x^3)*Sin[c]))/(a + b*x^3)^2 + (6*x*((8*a +
5*b*x^3)*Cos[c] + d*x*(a + b*x^3)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a^
2)
```

Rubi [A] (verified)

Time = 4.19 (sec) , antiderivative size = 2020, normalized size of antiderivative = 1.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3812, 3824, 3825, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \text{3812}$$

$$-\frac{\int \frac{\sin(c+dx)}{x^3(bx^3+a)^2}dx}{3b}+\frac{d\int \frac{\cos(c+dx)}{x^2(bx^3+a)^2}dx}{6b}-\frac{\sin(c+dx)}{6bx^2(a+bx^3)^2}$$

$$\Big\downarrow \text{3824}$$

$$\frac{d\int \frac{\cos(c+dx)}{x^2(bx^3+a)^2}dx}{6b} - \frac{-\frac{5\int \frac{\sin(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b} - \frac{\sin(c+dx)}{3bx^5(a+bx^3)}}{3b} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}$$

$\big\downarrow$ 3825

$$-\frac{-\frac{5\int \frac{\sin(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b} - \frac{\sin(c+dx)}{3bx^5(a+bx^3)}}{3b} + \frac{d\left(-\frac{4\int \frac{\cos(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b} - \frac{d\int \frac{\sin(c+dx)}{x^4\left(bx^3+a\right)}dx}{3b} - \frac{\cos(c+dx)}{3bx^4(a+bx^3)}\right)}{6b} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}$$

$\big\downarrow$ 3826

$$-\frac{-\frac{5\int \left(\frac{\sin(c+dx)b^2}{a^2\left(bx^3+a\right)} - \frac{\sin(c+dx)b}{a^2x^3} + \frac{\sin(c+dx)}{ax^6}\right)dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b} - \frac{\sin(c+dx)}{3bx^5(a+bx^3)}}{3b} + \frac{d\left(-\frac{d\int \left(\frac{b^2\sin(c+dx)x^2}{a^2\left(bx^3+a\right)} - \frac{b\sin(c+dx)}{a^2x} + \frac{\sin(c+dx)}{ax^4}\right)dx}{3b} - \frac{4\int \frac{\cos(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b} - \frac{\cos(c+dx)}{3bx^4(a+bx^3)}\right)}{6b} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}$$

$\big\downarrow$ 2009

$$-\frac{\sin(c+dx)}{6bx^2\left(bx^3+a\right)^2} + \frac{d\left(-\frac{\cos(c+dx)}{3bx^4(bx^3+a)} - \frac{d\left(-\frac{\cos(c)\operatorname{CosIntegral}(dx)d^3}{6a} + \frac{\sin(c)\operatorname{Si}(dx)d^3}{6a} + \frac{\sin(c+dx)d^2}{6ax} - \frac{\cos(c+dx)d}{6ax^2} - \frac{b\operatorname{CosIntegral}(dx)\sin(c)}{a^2} + \frac{b\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\ldots}{3a^2}\ldots\right.}{\ldots}\right.}{\ldots}$$

$$-\frac{\sin(c+dx)}{3bx^5(bx^3+a)} - \frac{5\left(\frac{\cos(c)\operatorname{CosIntegral}(dx)d^5}{120a} - \frac{\sin(c)\operatorname{Si}(dx)d^5}{120a} - \frac{\sin(c+dx)d^4}{120ax} + \frac{\cos(c+dx)d^3}{120ax^2} + \frac{b\operatorname{CosIntegral}(dx)\sin(c)d^2}{2a^2} + \frac{\sin(c+dx)d^2}{60ax^3} + \frac{b\cos(c)\operatorname{Si}(dx)d\ldots}{2a^2}\ldots\right.}{\ldots}$$

$\big\downarrow$ 3827

$$-\frac{\sin(c+dx)}{6bx^2\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\cos(c+dx)}{3bx^4(bx^3+a)}-\frac{d\left(-\frac{\cos(c)\,\text{CosIntegral}(dx)d^3}{6a}+\frac{\sin(c)\text{Si}(dx)d^3}{6a}+\frac{\sin(c+dx)d^2}{6ax}-\frac{\cos(c+dx)d}{6ax^2}-\frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2}+\frac{b\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\right.}{}\right)}{}$$

$$-\frac{\sin(c+dx)}{3bx^5(bx^3+a)}-\frac{5\left(\frac{\cos(c)\,\text{CosIntegral}(dx)d^5}{120a}-\frac{\sin(c)\text{Si}(dx)d^5}{120a}-\frac{\sin(c+dx)d^4}{120ax}+\frac{\cos(c+dx)d^3}{120ax^2}+\frac{b\,\text{CosIntegral}(dx)\sin(c)d^2}{2a^2}+\frac{\sin(c+dx)d^2}{60ax^3}+\frac{b\cos(c)\text{Si}(dx)d}{2a^2}\right.}{}$$

$\downarrow$ 2009

$$-\frac{\sin(c+dx)}{6bx^2\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\cos(c+dx)}{3bx^4(bx^3+a)}-\frac{d\left(-\frac{\cos(c)\,\text{CosIntegral}(dx)d^3}{6a}+\frac{\sin(c)\text{Si}(dx)d^3}{6a}+\frac{\sin(c+dx)d^2}{6ax}-\frac{\cos(c+dx)d}{6ax^2}-\frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2}+\frac{b\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\right.}{}\right)}{}$$

$$-\frac{\sin(c+dx)}{3bx^5(bx^3+a)}-\frac{5\left(\frac{\cos(c)\,\text{CosIntegral}(dx)d^5}{120a}-\frac{\sin(c)\text{Si}(dx)d^5}{120a}-\frac{\sin(c+dx)d^4}{120ax}+\frac{\cos(c+dx)d^3}{120ax^2}+\frac{b\,\text{CosIntegral}(dx)\sin(c)d^2}{2a^2}+\frac{\sin(c+dx)d^2}{60ax^3}+\frac{b\cos(c)\text{Si}(dx)d}{2a^2}\right.}{}$$

input
```
Int[Sin[c + d*x]/(a + b*x^3)^3,x]
```

output
```
-1/6*Sin[c + d*x]/(b*x^2*(a + b*x^3)^2) + (d*(-1/3*Cos[c + d*x]/(b*x^4*(a
+ b*x^3)) - (d*(-1/6*(d*Cos[c + d*x])/(a*x^2) - (d^3*Cos[c]*CosIntegral[d*
x])/(6*a) - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*CosIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^2) + (b*CosIntegral[((-1)
^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/
(3*a^2) + (b*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - Sin[c + d*x]/(3*a*x^3) + (d^2*Sin[
c + d*x])/(6*a*x) - (b*Cos[c]*SinIntegral[d*x])/a^2 + (d^3*Sin[c]*SinInteg
ral[d*x])/(6*a) - (b*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + (b*Cos[c - (a^(1/3)*d)/b^(
1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + (b*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(3*a^2)))/(3*b) - (4*(-1/4*Cos[c + d*x]/(a*x^4) + (d^2*Cos[c + d*x])/(24
*a*x^2) + (b*Cos[c + d*x])/(a^2*x) + (d^4*Cos[c]*CosIntegral[d*x])/(24*a)
- ((-1)^(2/3)*b^(4/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(7/3)) - (b^(4/3)*Cos[c - (a^(
1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(7/3)) + ((-
1)^(1/3)*b^(4/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(7/3)) + (b*d*CosIntegral[d*x]*Sin[
c])/a^2 + (d*Sin[c + d*x])/(12*a*x^3) - (d^3*Sin[c + d*x])/(24*a*x) + (...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3812
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Simp[
(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
 - Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x],
x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3825

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Cos[c + d*x], x], x] + Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.98 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.29

method	result
risch	$-\frac{id^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3\_Zbc^2\right)}\frac{\left(-2ic\_R1+\_R1^2-c^2+6ic-6\_R1+10\right)e^{-\_R1}\,\mathrm{expInte}}{-2ic\_R1+\_R1^2-c^2}\right.}{108a^2b}$
derivativedivides	$d^8\left(-\frac{\sin(dx+c)\left(8acd^3-8ad^3(dx+c)-5bc^4+20bc^3(dx+c)-30bc^2(dx+c)^2+20bc(dx+c)^3-5b(dx+c)^4\right)}{18a^2d^6\left(ad^3-bc^3+3bc^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{}{18a^2a}\right.$
default	$d^8\left(-\frac{\sin(dx+c)\left(8acd^3-8ad^3(dx+c)-5bc^4+20bc^3(dx+c)-30bc^2(dx+c)^2+20bc(dx+c)^3-5b(dx+c)^4\right)}{18a^2d^6\left(ad^3-bc^3+3bc^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{}{18a^2a}\right.$

input
```
int(sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/108*I*d^2/a^2/b*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_
R1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*
b*c^3+b*_Z^3-3*_Z*b*c^2))+1/108*I*d^2/a^2/b*sum((-2*I*c*_R1-6*I*c+_R1^2-c^
2+6*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootO
f(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*_Z*b*c^2))+1/18*d^2*(-b*d^5*x^5-a
*d^5*x^2)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*cos(d*x+c)+1/18*d^2*(5*b
*d^4*x^4+8*a*d^4*x)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1223, normalized size of antiderivative = 1.05

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*((-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + 3*(b^3*x^6 + 2*
a*b^2*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^
3/b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*
a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (I*a*
b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a
^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)^(2/3) +
 5*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 -
I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3
) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3*x
^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sq
rt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(2/3) + 5*(b^3*x
^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*
(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*a*b^2*d^3*x^6 + 2*I*a^2*
b*d^3*x^3 + I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3
*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(-I*a*d^3/b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2
*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)
^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^
3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx=\int \frac{\sin(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx=\int \frac{\sin(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx=\int \frac{\sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int(sin(c + d*x)/(a + b*x^3)^3,x)
```

output
```
int(sin(c + d*x)/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx$$

input
```
int(sin(d*x+c)/(b*x^3+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)
```

3.113 $\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1163

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

output
```
-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+
d*x)/a^3-1/6*sin(d*x+c)/a/b^2/x^6+1/3*sin(d*x+c)/a^2/b/x^3-1/6*sin(d*x+c)/
b/x^3/(b*x^3+a)^2+1/6*sin(d*x+c)/b^2/x^6/(b*x^3+a)-4/27*(-1)^(1/3)*d*sin(c
+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/
3)/b^(1/3)+1/54*(-1)^(2/3)*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1
)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+4/27*(-1)^(2/3)*d*sin(c-(-1
)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^
(1/3)-1/54*(-1)^(1/3)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^
(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/54*(-1)^(2/3)*d^2*Ci((-1)^(1/3)
*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3
)-1/54*(-1)^(1/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^
(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-4/27*(-1)^(2/3)*d*cos(c-(-1)^(2/3)*a^
(1/3)*d/b^(1/3))*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)+4/27
*(-1)^(1/3)*d*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Ci((-1)^(1/3)*a^(1/3)*d/
b^(1/3)-d*x)/a^(8/3)/b^(1/3)-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(
c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^3-1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3)
)*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^3-1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(
1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^3-1/3*Ci(a^(1/3)*d/b^(1/3)
+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^3-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)
*d/b^(1/3)+d*x)/a^3+4/27*d*sin(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 2109, normalized size of antiderivative = 1.81

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

output
```
(-6*a^2*b*d*x*Cos[c + d*x] - 6*a*b^2*d*x^4*Cos[c + d*x] - (18*I)*b*(a + b*
x^3)^2*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*Co
sIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + (18*I)*b*(a + b*x^3)^2*R
ootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegra
l[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Si
n[c + d*#1]*SinIntegral[d*(x - #1)] & ] - 6*a^3*d*RootSum[a + b*#1^3 & , (
Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c +
d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegra
l[d*(x - #1)])/#1^2 & ] - 12*a^2*b*d*x^3*RootSum[a + b*#1^3 & , (Cos[c + d
*#1]*CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I
*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x -
#1)])/#1^2 & ] - 6*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIn
tegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d
*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2
 & ] - 6*a^3*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1
)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral
[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - 12*a^2*b*
d*x^3*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*Co
sIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x ...
```

Rubi [A] (verified)

Time = 4.98 (sec) , antiderivative size = 2229, normalized size of antiderivative = 1.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3824, 3824, 3825, 3826, 2009, 3827, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{ 3824}$$

$$\frac{d\int \frac{\cos(c+dx)}{x^3(bx^3+a)^2}dx}{6b}-\frac{\int \frac{\sin(c+dx)}{x^4(bx^3+a)^2}dx}{2b}-\frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 3824}$$

$$\frac{d\int \frac{\cos(c+dx)}{x^3(bx^3+a)^2}dx}{6b}-\frac{-\frac{2\int \frac{\sin(c+dx)}{x^7\left(bx^3+a\right)}dx}{b}+\frac{d\int \frac{\cos(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^6(a+bx^3)}}{2b}-\frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{3825}$$

$$-\frac{-\frac{2\int \frac{\sin(c+dx)}{x^7\left(bx^3+a\right)}dx}{b}+\frac{d\int \frac{\cos(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^6(a+bx^3)}}{2b}+\frac{d\left(-\frac{5\int \frac{\cos(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b}-\frac{d\int \frac{\sin(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b}-\frac{\cos(c+dx)}{3bx^5(a+bx^3)}\right)}{6b}-\frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{3826}$$

$$\frac{d\left(-\frac{d\int \left(\frac{x\sin(c+dx)b^2}{a^2\left(bx^3+a\right)}-\frac{\sin(c+dx)b}{a^2x^2}+\frac{\sin(c+dx)}{ax^5}\right)dx}{3b}-\frac{5\int \frac{\cos(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b}-\frac{\cos(c+dx)}{3bx^5(a+bx^3)}\right)}{6b}-\frac{-\frac{2\int \left(-\frac{x^2\sin(c+dx)b^3}{a^3\left(bx^3+a\right)}+\frac{\sin(c+dx)b^2}{a^3x}-\frac{\sin(c+dx)b}{a^2x^4}+\frac{\sin(c+dx)}{ax^7}\right)dx}{b}+\frac{d\int \frac{\cos(c+dx)}{x^6\left(bx^3+a\right)}dx}{3b}-\frac{\sin(c+dx)}{3bx^6(a+bx^3)}}{2b}-\frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(c+dx)}{6bx^3\left(bx^3+a\right)^2}+$$

$$d\left(-\frac{\cos(c+dx)}{3bx^5(bx^3+a)}-\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\text{CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(dx)}{a^2}\right.}{}\right.$$

$$-\frac{\sin(c+dx)}{3bx^6(bx^3+a)}-\frac{2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^6}{720a}-\frac{\cos(c)\text{Si}(dx)d^6}{720a}-\frac{\cos(c+dx)d^5}{720ax}-\frac{\sin(c+dx)d^4}{720ax^2}+\frac{\cos(c+dx)d^3}{360ax^3}+\frac{b\cos(c)\text{CosIntegral}(dx)d^3}{6a^2}-\frac{b\sin(c)\text{Si}(dx)}{6a^2}\right.}{}$$

$$\Big\downarrow \text{3827}$$

$$-\frac{\sin(c+dx)}{6bx^3\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\cos(c+dx)}{3bx^5(bx^3+a)}-\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\text{ CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(dx\ldots}{a^2}\right.}{}\right.}{}$$

$$-\frac{\sin(c+dx)}{3bx^6(bx^3+a)}-\frac{2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^6}{720a}-\frac{\cos(c)\text{Si}(dx)d^6}{720a}-\frac{\cos(c+dx)d^5}{720ax}-\frac{\sin(c+dx)d^4}{720ax^2}+\frac{\cos(c+dx)d^3}{360ax^3}+\frac{b\cos(c)\text{ CosIntegral}(dx)d^3}{6a^2}-\frac{b\sin(c)\text{Si}(dx\ldots}{6a^2}\right.}{}$$

$\downarrow$ 2009

$$-\frac{\sin(c+dx)}{6bx^3\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\cos(c+dx)}{3bx^5(bx^3+a)}-\frac{d\left(\frac{\text{CosIntegral}(dx)\sin(c)d^4}{24a}+\frac{\cos(c)\text{Si}(dx)d^4}{24a}+\frac{\cos(c+dx)d^3}{24ax}+\frac{\sin(c+dx)d^2}{24ax^2}-\frac{\cos(c+dx)d}{12ax^3}-\frac{b\cos(c)\text{ CosIntegral}(dx)d}{a^2}+\frac{b\sin(c)\text{Si}(dx\ldots}{a^2}\right.}{}\right.}{}$$

$$-\frac{\sin(c+dx)}{3bx^6(bx^3+a)}-\frac{2\left(-\frac{\text{CosIntegral}(dx)\sin(c)d^6}{720a}-\frac{\cos(c)\text{Si}(dx)d^6}{720a}-\frac{\cos(c+dx)d^5}{720ax}-\frac{\sin(c+dx)d^4}{720ax^2}+\frac{\cos(c+dx)d^3}{360ax^3}+\frac{b\cos(c)\text{ CosIntegral}(dx)d^3}{6a^2}-\frac{b\sin(c)\text{Si}(dx\ldots}{6a^2}\right.}{}$$

input
```
Int[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

output
```
-1/6*Sin[c + d*x]/(b*x^3*(a + b*x^3)^2) + (d*(-1/3*Cos[c + d*x]/(b*x^5*(a
+ b*x^3)) - (d*(-1/12*(d*Cos[c + d*x])/(a*x^3) + (d^3*Cos[c + d*x])/(24*a*
x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (d^4*CosIntegral[d*x]*Sin[c])/(24
*a) - (b^(4/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/
b^(1/3)])/(3*a^(7/3)) - ((-1)^(2/3)*b^(4/3)*CosIntegral[((-1)^(1/3)*a^(1/3
)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(7/3)) +
 ((-1)^(1/3)*b^(4/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(7/3)) - Sin[c + d*x]/(4*a*x^4)
 + (d^2*Sin[c + d*x])/(24*a*x^2) + (b*Sin[c + d*x])/(a^2*x) + (d^4*Cos[c]*
SinIntegral[d*x])/(24*a) + (b*d*Sin[c]*SinIntegral[d*x])/a^2 + ((-1)^(2/3)
*b^(4/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a
^(1/3)*d)/b^(1/3) - d*x])/(3*a^(7/3)) - (b^(4/3)*Cos[c - (a^(1/3)*d)/b^(1/
3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(7/3)) + ((-1)^(1/3)*b^(4
/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x])/(3*a^(7/3))))/(3*b) - (5*(-1/5*Cos[c + d*x]/(a*x^5) +
 (d^2*Cos[c + d*x])/(60*a*x^3) + (b*Cos[c + d*x])/(2*a^2*x^2) - (d^4*Cos[c
 + d*x])/(120*a*x) + (b*d^2*Cos[c]*CosIntegral[d*x])/(2*a^2) - ((-1)^(1/3)
*b^(5/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a
^(1/3)*d)/b^(1/3) - d*x])/(3*a^(8/3)) + (b^(5/3)*Cos[c - (a^(1/3)*d)/b^(1/
3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(8/3)) + ((-1)^(2/3)*b...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3824
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Sin[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3825
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p + 1)
*Cos[c + d*x], x], x] + Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*x^n
)^(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1]
 && IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

rule 3826
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

rule 3827
```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.14 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.31

method	result
derivativedivides	$\frac{\sin(dx+c)d^3\left(3a\,d^3-2b\,c^3+6b\,c^2(dx+c)-6bc(dx+c)^2+2b(dx+c)^3\right)}{6a^2\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{\cos(dx+c)d^4x}{18a^2\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b\right.}$
default	$\frac{\sin(dx+c)d^3\left(3a\,d^3-2b\,c^3+6b\,c^2(dx+c)-6bc(dx+c)^2+2b(dx+c)^3\right)}{6a^2\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{\cos(dx+c)d^4x}{18a^2\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b\right.}$
risch	$\frac{ie^{ic}\,\text{expIntegral}_1(-idx)}{2a^3}-\frac{i\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3\_Zb\,c^2\right)}\frac{\left(i\_R1a\,d^3+ac\,d^3-8id^3a-36i\_\right.}{}\right.}{108a^3b}$

input
```
int(sin(d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(d*x+c)*d^3*(3*a*d^3-2*b*c^3+6*b*c^2*(d*x+c)-6*b*c*(d*x+c)^2+2*b*(d
*x+c)^3)/a^2/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1
/18*cos(d*x+c)*d^4*x/a^2/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d
*x+c)^3)+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/54/a^3/b*sum((a*d^3+18*_R
1*b-18*b*c)/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-4/27*d^3/a^2/b*sum(1/(_
RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_
RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1113, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/216*((-36*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3
 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) - 8*(-I*
b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*
d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(
I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 3
6*I*a^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3
 + a^2))*(-I*a*d^3/b)^(2/3) - 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)
*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3
/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*
c) + (-36*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3 +
 I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) - 8*(-I*b^
2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^
3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a
*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36*
I*a^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
 a^2))*(-I*a*d^3/b)^(2/3) - 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b
)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c)
 - 2*(-18*I*b^2*x^6 - 36*I*a*b*x^3 - 18*I*a^2 + (-I*b^2*x^6 - 2*I*a*b*x^3
- I*a^2)*(-I*a*d^3/b)^(2/3) + 8*(-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(-I*...
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx=\int \frac{\sin\left(c+dx\right)}{x\left(a+bx^3\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x**3+a)**3,x)
```

output
```
Integral(sin(c + d*x)/(x*(a + b*x**3)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^3x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^3x}\,dx$$

input
```
integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x\,\left(b\,x^3+a\right)^3}\,dx$$

input
```
int(sin(c + d*x)/(x*(a + b*x^3)^3),x)
```

output
```
int(sin(c + d*x)/(x*(a + b*x^3)^3), x)
```

Reduce [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{b^3x^{10}+3a\,b^2x^7+3a^2b\,x^4+a^3x}dx$$

input
```
int(sin(d*x+c)/x/(b*x^3+a)^3,x)
```

output
```
int(sin(c + d*x)/(a**3*x + 3*a**2*b*x**4 + 3*a*b**2*x**7 + b**3*x**10),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file