Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.1-Sine/194-4.1.12

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**357**]. This is test number [194].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (357)	0.00 (0)
Mathematica	97.76 (349)	2.24 (8)
Fricas	85.43 (305)	14.57 (52)
Maxima	75.63 (270)	24.37 (87)
Maple	68.63 (245)	31.37 (112)
Giac	51.26 (183)	48.74 (174)
Reduce	38.66 (138)	61.34 (219)
Mupad	36.13 (129)	63.87 (228)
Sympy	32.21 (115)	67.79 (242)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

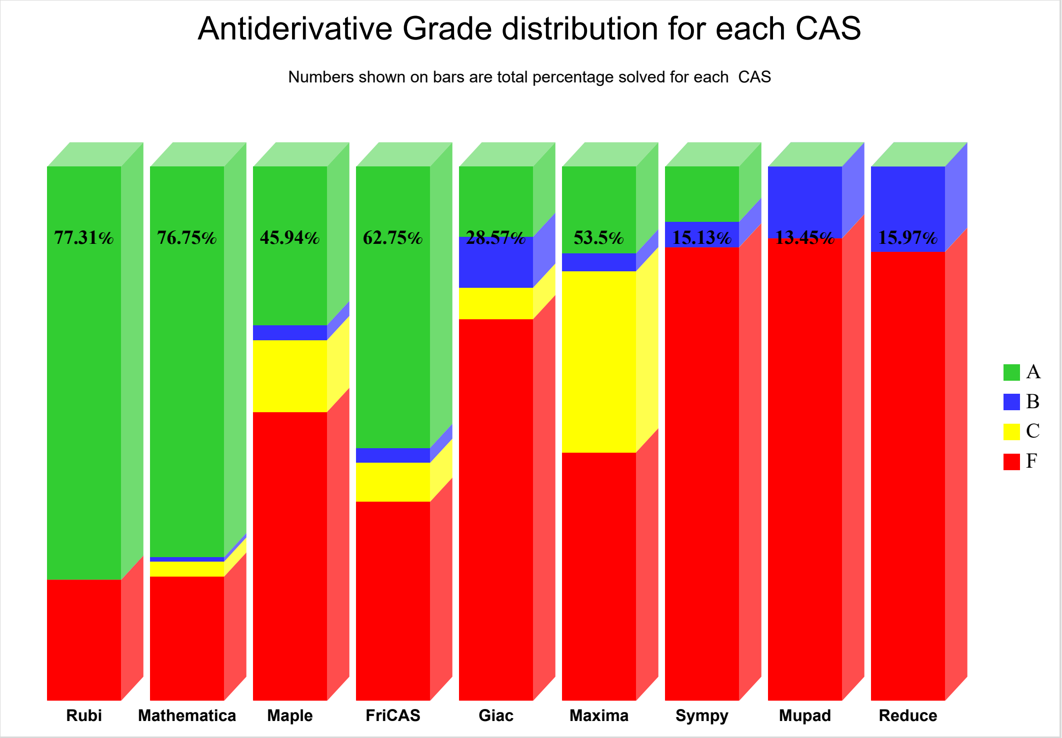
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

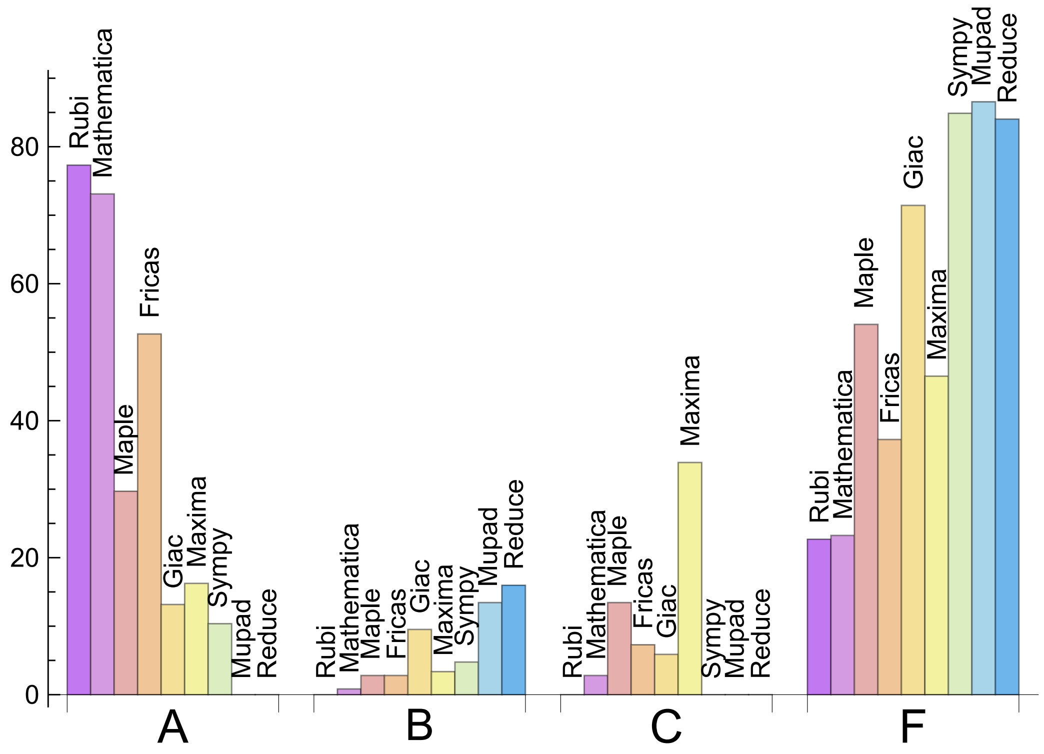
System	% A grade	% B grade	% C grade	% F grade
Rubi	77.311	0.000	0.000	22.689
Mathematica	73.109	0.840	2.801	23.249
Fricas	52.661	2.801	7.283	37.255
Maple	29.692	2.801	13.445	54.062
Maxima	16.246	3.361	33.894	46.499
Giac	13.165	9.524	5.882	71.429
Sympy	10.364	4.762	0.000	84.874
Mupad	0.000	13.445	0.000	86.555
Reduce	0.000	15.966	0.000	84.034

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	8	25.00	75.00	0.00
Fricas	52	100.00	0.00	0.00
Maxima	87	93.10	2.30	4.60
Maple	112	100.00	0.00	0.00
Giac	174	95.98	0.00	4.02
Reduce	219	100.00	0.00	0.00
Sympy	242	86.78	13.22	0.00
Mupad	228	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Rubi	0.41
Maxima	1.10
Maple	1.16
Giac	1.31
Reduce	1.65
Mathematica	3.10
Sympy	14.29
Mupad	36.19

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	38.55	1.08	22.00	1.10
Sympy	114.69	1.90	29.00	1.07
Mathematica	127.67	0.97	85.00	0.96
Rubi	128.32	0.95	91.00	1.00
Fricas	144.43	1.30	72.00	1.02
Maple	150.39	1.20	56.00	1.00
Giac	288.81	1.79	32.00	1.11
Maxima	410.60	12.66	90.50	1.10
Reduce	617.79	29.06	46.00	1.32

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

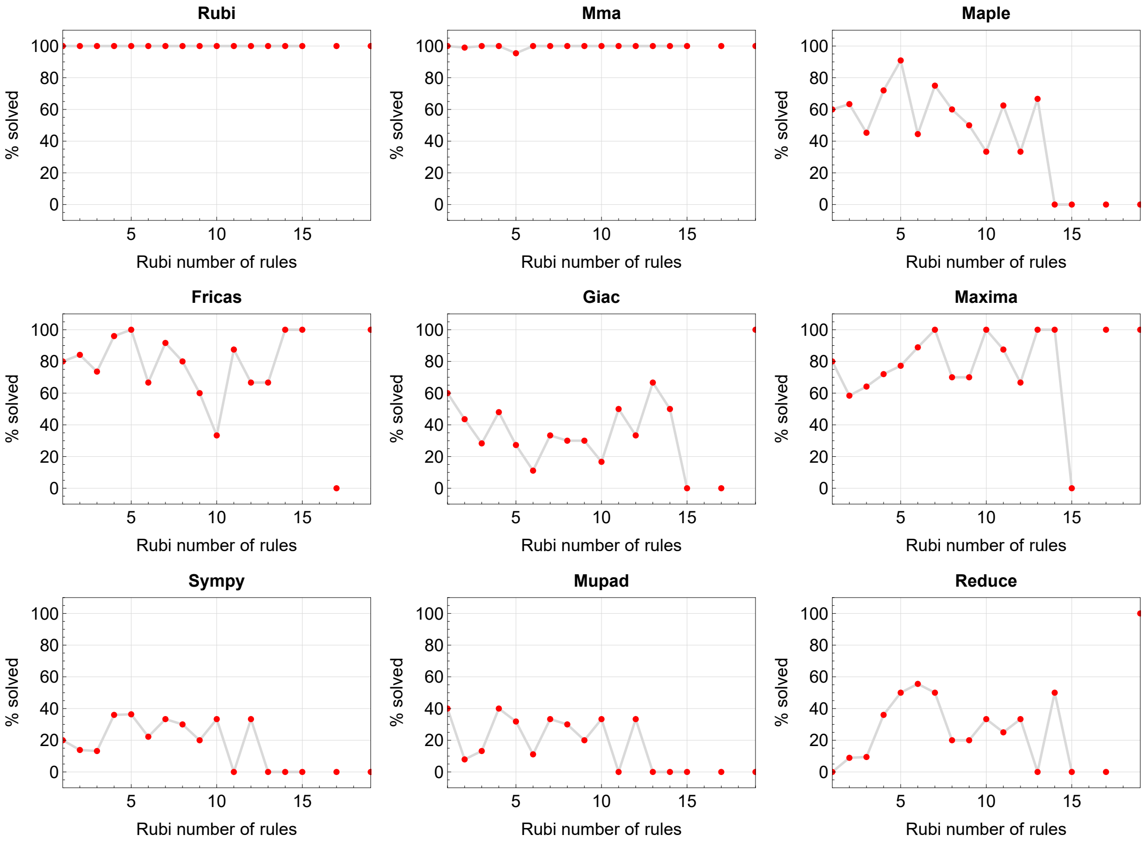


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

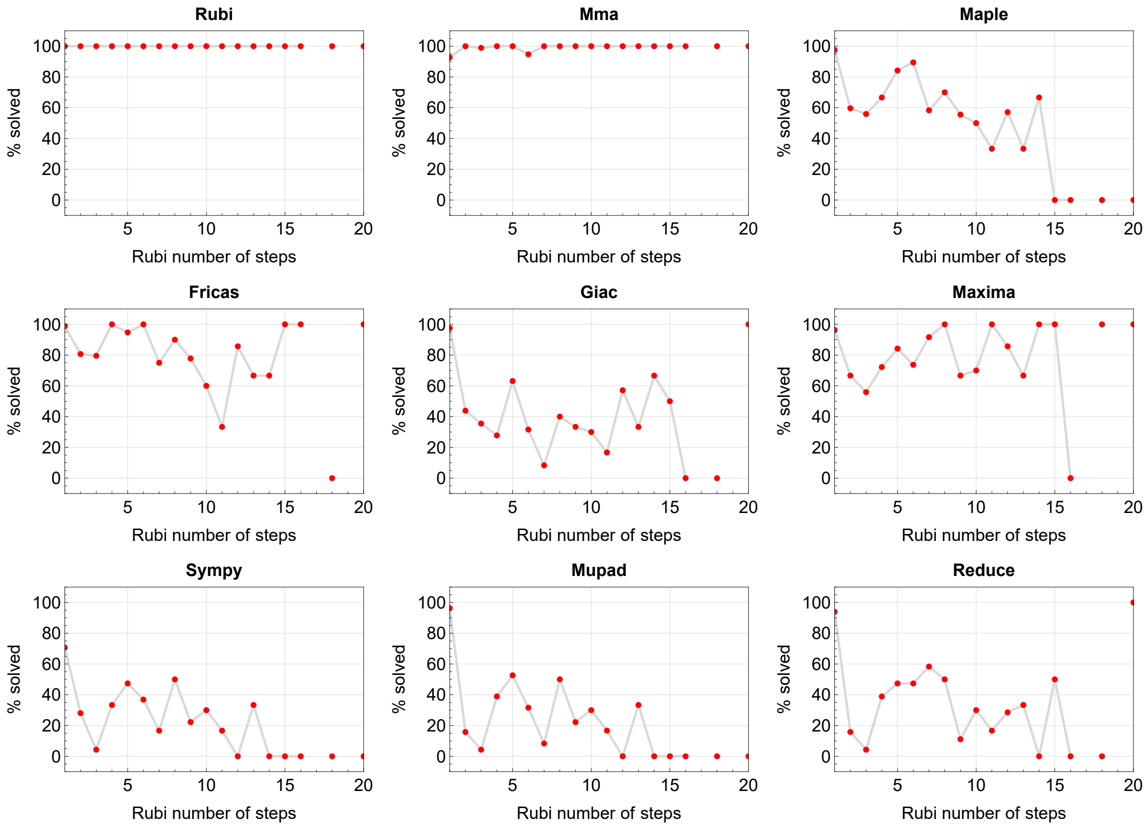


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

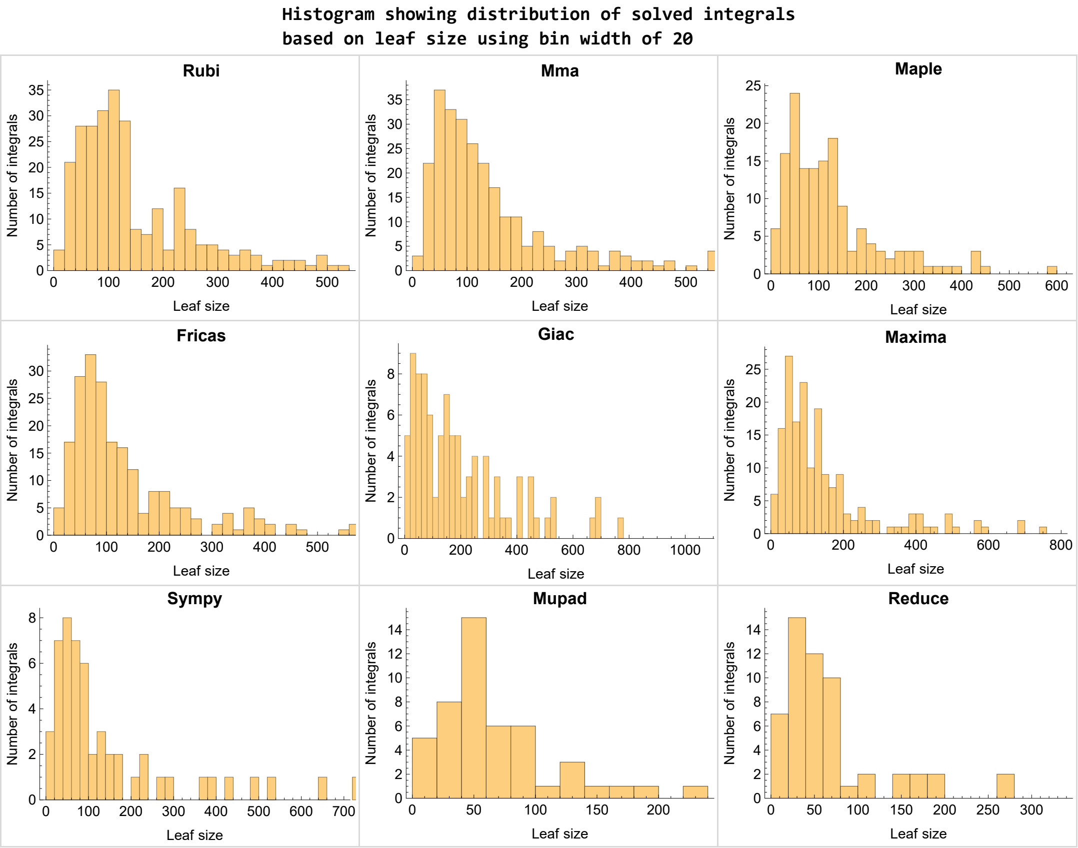


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

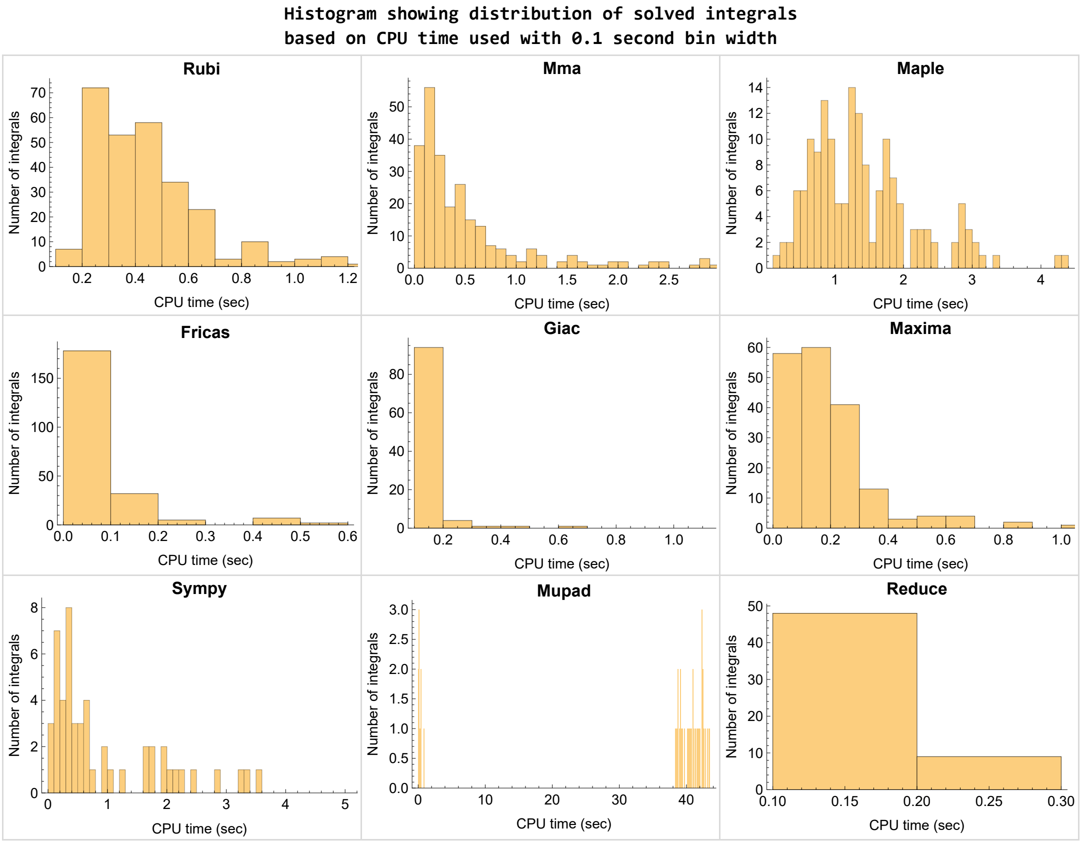


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

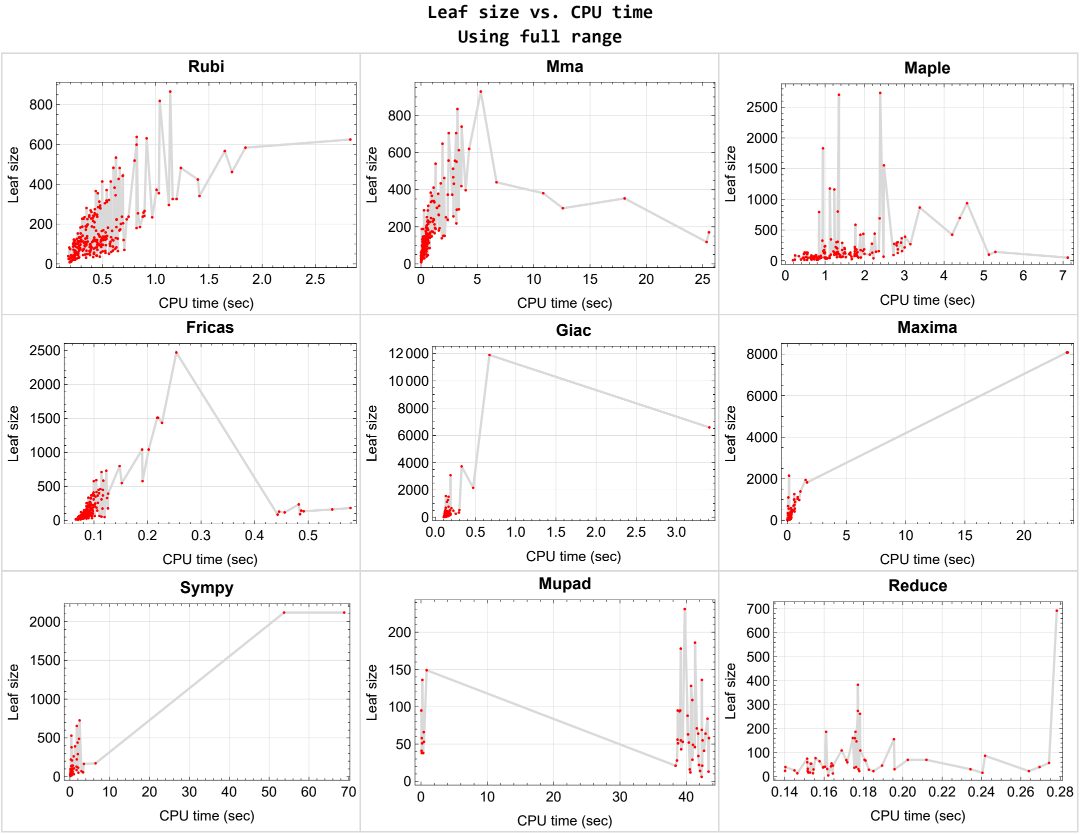


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 101, 102, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 195, 196, 205, 206, 215, 216, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{23, 25, 33, 125, 128, 224, 249, 250, 253, 257, 258\}$

Mathematica $\{133\}$

Maple $\{15, 16, 17, 26, 27, 221, 315, 323, 331, 339, 347, 355\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

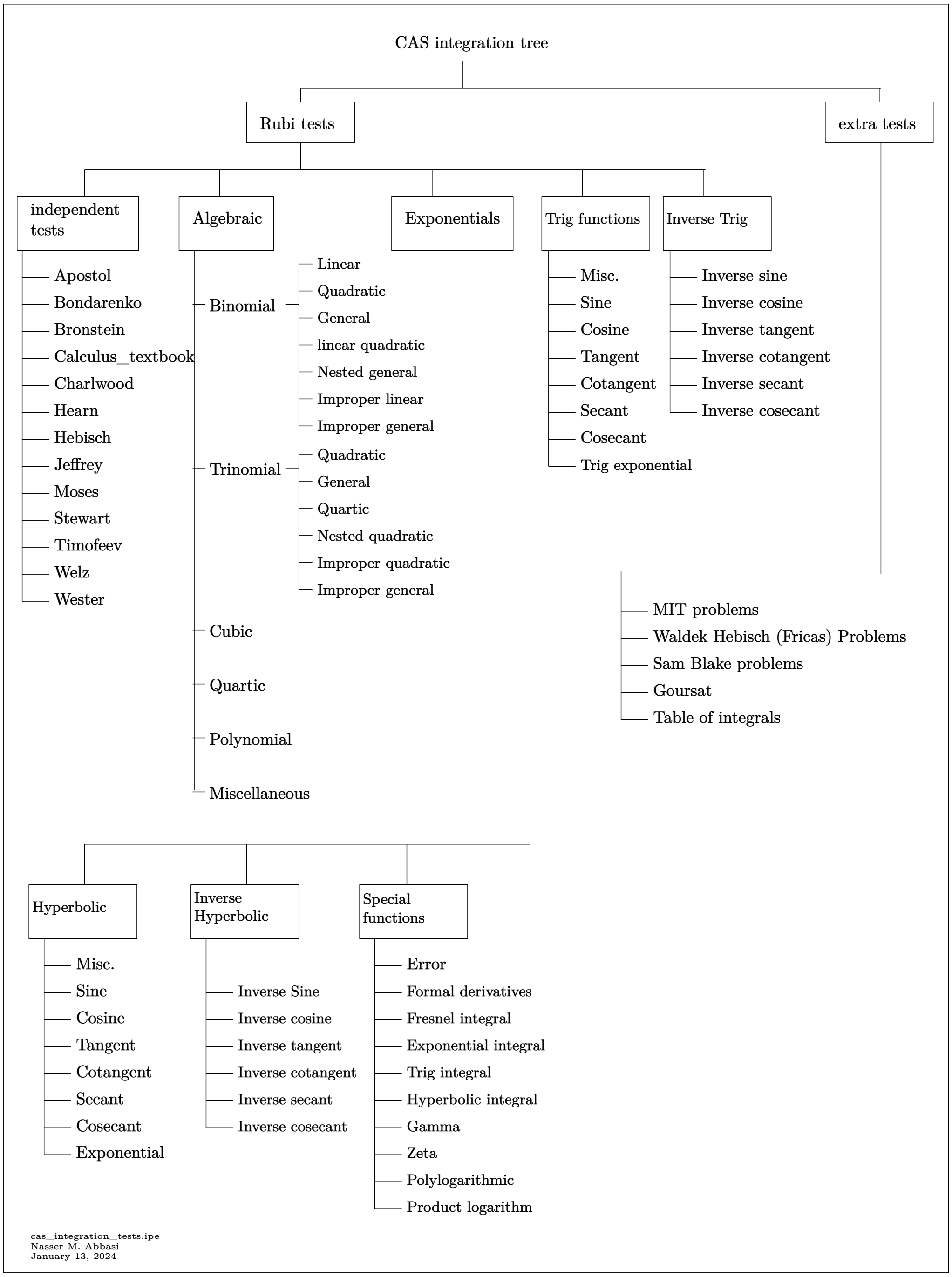
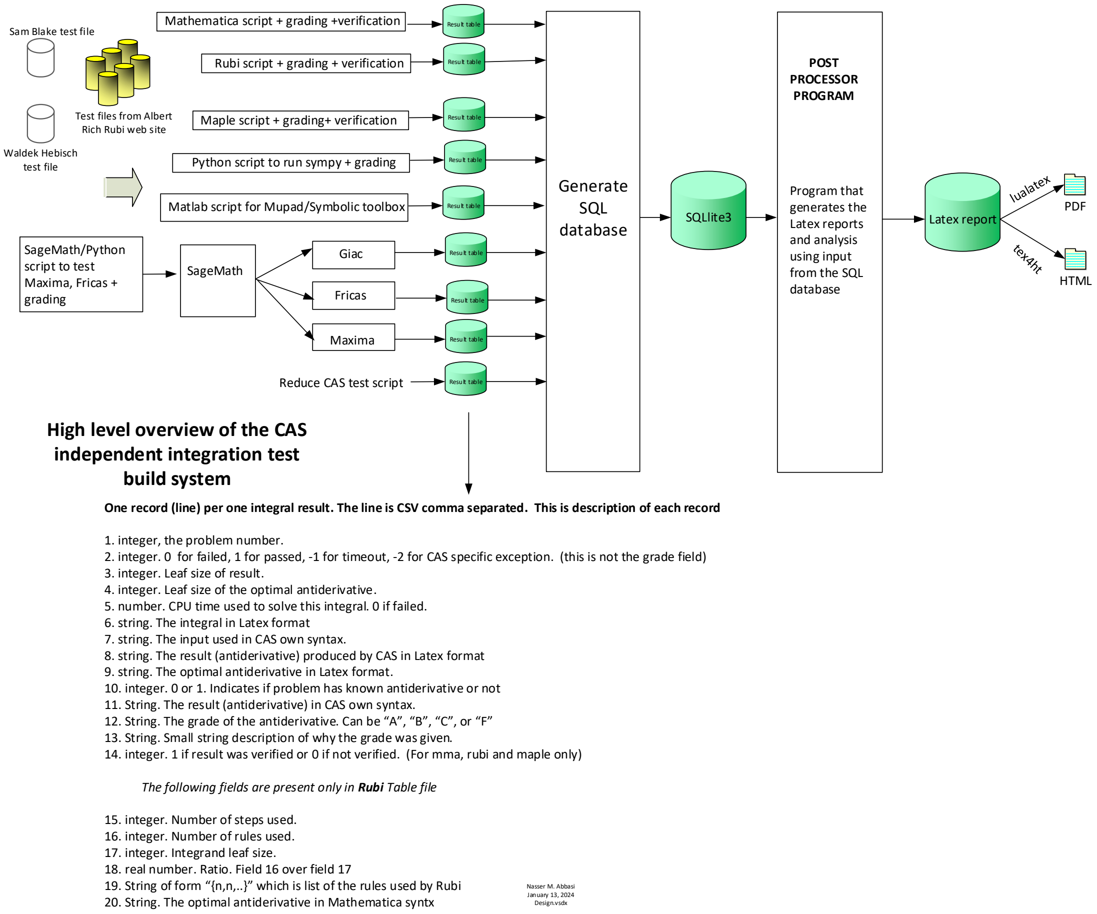


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 184, 187, 188, 189, 192, 194, 198, 199, 202, 204, 207, 208, 209, 213, 214, 218, 219, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { 183, 193, 203 }

C grade { 190, 191, 197, 210, 211, 212, 217, 220, 221, 222 }

F normal fail { 200, 201 }

F(-1) timedout fail { 282, 283, 285, 286, 304, 308 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 37, 45, 57, 58, 69, 70, 82, 90, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 156, 159, 160, 161, 162, 177, 178, 179, 189, 197, 198, 199, 200, 209, 212, 213, 214, 217, 218, 219, 222, 223, 224, 288, 289, 290, 291, 292, 294, 295, 296, 297 }

B grade { 153, 154, 155, 187, 188, 190, 191, 201, 207, 208 }

C grade { 15, 16, 17, 26, 27, 127, 139, 142, 165, 166, 167, 168, 210, 211, 220, 221, 293, 298, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 331, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 355 }

F normal fail { 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 131, 133, 140, 141, 143, 144, 171, 172, 173, 174, 182, 183, 184, 192, 193, 194, 202, 203, 204, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 45, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 90, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 187, 188, 189, 192, 193, 194, 197, 198, 199, 202, 204, 207, 208, 209, 212, 213, 214, 217, 218, 219, 222, 223, 224, 227, 228, 229, 230, 231, 237, 238, 245, 246, 247, 248, 255, 256, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 318, 319, 321, 334, 335, 336, 337, 338, 342, 343, 345 }

B grade { 35, 36, 43, 44, 81, 89, 182, 183, 184, 203 }

C grade { 190, 191, 200, 201, 210, 211, 220, 221, 315, 316, 317, 320, 322, 323, 324, 325, 331, 339, 340, 341, 344, 346, 347, 348, 349, 355 }

F normal fail { 131, 133, 139, 140, 141, 142, 143, 144, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 254, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 326, 327, 328, 329, 330, 332, 333, 350, 351, 352, 353, 354, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 12, 13, 14, 23, 24, 25, 33, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 107, 115, 122, 124, 125, 126, 127, 128, 145, 146, 189, 193, 194, 204, 209, 231, 238, 245, 255, 311, 312, 313, 314, 319, 321, 336, 343, 345 }

B grade { 37, 82, 187, 188, 192, 202, 203, 207, 208, 335, 337, 338 }

C grade { 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 59, 60, 71, 72, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 135, 136, 137, 138, 153, 154, 155, 156, 165, 166, 167, 168, 197, 198, 199, 212, 213, 214, 217, 218, 219, 222, 223, 224, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 288, 289, 290, 294, 295, 315, 316, 317, 320, 322, 323, 324, 325, 331, 339, 340, 341, 344, 346, 347, 348, 349, 355 }

F normal fail { 35, 36, 43, 52, 53, 54, 81, 98, 99, 100, 131, 133, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 200, 201, 210, 211, 220, 221, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 291, 292, 293, 296, 297, 298, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

F(-1) timedout fail { 45, 90 }

F(-2) exception fail { 44, 89, 93, 94 }

Giac

A grade { 2, 3, 4, 12, 13, 14, 15, 23, 24, 25, 26, 33, 34, 37, 45, 57, 58, 59, 69, 70, 71, 82, 90, 106, 107, 108, 115, 116, 117, 122, 124, 125, 126, 127, 128, 187, 188, 189, 208, 209, 227, 228, 229, 230, 231, 291, 296 }

B grade { 1, 5, 6, 16, 17, 27, 60, 72, 103, 104, 105, 109, 110, 111, 112, 113, 114, 118, 197, 198, 199, 207, 217, 218, 219, 288, 289, 290, 292, 293, 294, 295, 297, 298 }

C grade { 7, 8, 9, 18, 19, 20, 28, 29, 31, 32, 153, 154, 155, 156, 165, 166, 167, 168, 212, 213, 214 }

F normal fail { 10, 11, 21, 22, 30, 35, 36, 43, 44, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 119, 120, 121, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 200, 201, 202, 203, 204, 210, 211, 220, 221, 222, 223, 224, 232, 233, 234, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { 235, 236, 237, 238, 239, 240, 241 }

Mupad

A grade { }

B grade { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 33, 37, 45, 57, 58, 69, 70, 82, 90, 107, 108, 109, 110, 115, 116, 117, 118, 121, 122, 124, 125, 126, 127, 128, 153, 154, 155, 156, 162, 168, 189, 209, 311, 312, 313, 314, 319, 321 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 5, 6, 7, 8, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 106, 111, 112, 113, 114, 119, 120, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 165, 166, 167, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 187, 188, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 29, 31, 33, 34, 57, 58, 69, 70, 106, 107, 108, 109, 110, 114, 122, 124, 125, 126, 128, 187, 188, 189, 209, 311, 312, 313, 319 }

B grade { 7, 8, 28, 32, 37, 45, 82, 90, 115, 116, 117, 118, 127, 145, 146, 314, 321 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 30, 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 111, 112, 113, 119, 120, 121, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 164, 181, 186, 206, 227, 234, 235, 247, 248, 249, 256, 257, 258, 277, 281, 282, 283, 284, 285, 286, 287, 299, 303, 304, 305, 306, 307, 308, 309, 350, 351, 352 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 12, 14, 23, 24, 25, 33, 37, 45, 57, 58, 70, 82, 90, 107, 108, 110, 115, 122, 124, 125, 126, 127, 128, 145, 146, 187, 188, 189, 207, 208, 209, 227, 228, 229, 230, 231, 237, 238, 245, 246, 247, 255, 256, 311, 312, 313, 314, 319, 321, 335, 336, 337, 338, 345 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 106, 109, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	47	47	51	65	128	57	53
N.S.	1	1.00	0.89	0.82	0.82	0.89	1.14	2.25	1.00	0.93
time (sec)	N/A	0.239	0.156	0.835	0.040	0.086	0.393	0.115	0.274	0.447

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	37	40	49	75	40	38
N.S.	1	1.00	1.00	0.89	0.84	0.91	1.11	1.70	0.91	0.86
time (sec)	N/A	0.206	0.009	0.690	0.047	0.078	0.221	0.128	0.270	0.128

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	41	22	21	23	31	26	23	21
N.S.	1	1.00	1.64	0.88	0.84	0.92	1.24	1.04	0.92	0.84
time (sec)	N/A	0.183	0.054	0.727	0.035	0.078	0.085	0.113	0.264	42.401

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	29	50	27	0	32	21	0
N.S.	1	1.00	0.94	0.94	1.61	0.87	0.00	1.03	0.68	0.00
time (sec)	N/A	0.198	0.110	0.678	0.106	0.081	0.000	0.137	0.232	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	47	57	49	0	99	29	0
N.S.	1	1.00	0.91	0.89	1.08	0.92	0.00	1.87	0.55	0.00
time (sec)	N/A	0.254	0.147	0.727	0.108	0.078	0.000	0.112	0.197	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	86	65	58	63	0	204	58	0
N.S.	1	1.00	1.16	0.88	0.78	0.85	0.00	2.76	0.78	0.00
time (sec)	N/A	0.293	0.176	0.725	0.134	0.101	0.000	0.119	0.197	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	125	89	92	103	488	165	55	0
N.S.	1	1.00	1.03	0.74	0.76	0.85	4.03	1.36	0.45	0.00
time (sec)	N/A	0.302	0.316	0.774	0.059	0.096	2.287	0.117	0.215	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	68	75	86	223	145	23	0
N.S.	1	1.00	1.02	0.67	0.74	0.84	2.19	1.42	0.23	0.00
time (sec)	N/A	0.241	0.246	0.695	0.042	0.083	1.735	0.136	0.201	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	61	48	53	67	66	102	16	56
N.S.	1	1.00	0.82	0.65	0.72	0.91	0.89	1.38	0.22	0.76
time (sec)	N/A	0.198	0.187	0.564	0.042	0.081	0.182	0.132	0.211	38.673

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	66	81	78	0	0	50	0
N.S.	1	1.00	1.03	0.75	0.92	0.89	0.00	0.00	0.57	0.00
time (sec)	N/A	0.247	0.351	0.714	0.188	0.086	0.000	0.000	0.209	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	83	82	98	0	0	29	0
N.S.	1	1.00	1.04	0.73	0.72	0.86	0.00	0.00	0.25	0.00
time (sec)	N/A	0.274	0.288	0.757	0.197	0.084	0.000	0.000	0.228	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	167	122	111	106	121	209	284	156	149
N.S.	1	1.02	0.75	0.68	0.65	0.74	1.28	1.74	0.96	0.91
time (sec)	N/A	0.440	0.405	1.993	0.046	0.081	0.542	0.133	0.195	0.855

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	107	92	89	87	84	136	165	68	95
N.S.	1	1.05	0.90	0.87	0.85	0.82	1.33	1.62	0.67	0.93
time (sec)	N/A	0.325	0.239	1.800	0.059	0.082	0.321	0.114	0.178	38.701

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	62	52	52	52	53	95	57	60	51
N.S.	1	1.07	0.90	0.90	0.90	0.91	1.64	0.98	1.03	0.88
time (sec)	N/A	0.240	0.279	1.839	0.038	0.086	0.125	0.132	0.172	38.746

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	157	108	68	0	77	45	0
N.S.	1	1.00	0.96	2.12	1.46	0.92	0.00	1.04	0.61	0.00
time (sec)	N/A	0.292	0.187	2.940	0.153	0.075	0.000	0.146	0.160	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	116	217	124	112	0	226	883	0
N.S.	1	1.00	1.01	1.89	1.08	0.97	0.00	1.97	7.68	0.00
time (sec)	N/A	0.426	0.276	3.006	0.151	0.082	0.000	0.125	0.168	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	158	270	128	156	0	448	1358	0
N.S.	1	1.00	0.93	1.60	0.76	0.92	0.00	2.65	8.04	0.00
time (sec)	N/A	0.509	0.495	3.150	0.147	0.085	0.000	0.144	0.184	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	234	185	207	216	0	329	84	0
N.S.	1	1.00	0.95	0.75	0.84	0.87	0.00	1.33	0.34	0.00
time (sec)	N/A	0.450	0.603	1.985	0.148	0.089	0.000	0.129	0.177	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	191	138	171	176	0	283	47	0
N.S.	1	1.00	0.96	0.70	0.86	0.89	0.00	1.43	0.24	0.00
time (sec)	N/A	0.362	0.553	1.818	0.148	0.126	0.000	0.150	0.193	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	98	129	134	0	195	36	0
N.S.	1	1.00	0.96	0.64	0.84	0.88	0.00	1.27	0.24	0.00
time (sec)	N/A	0.296	0.355	1.632	0.192	0.081	0.000	0.142	0.167	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	184	134	170	159	0	0	50	0
N.S.	1	1.00	0.98	0.72	0.91	0.85	0.00	0.00	0.27	0.00
time (sec)	N/A	0.350	0.633	1.805	0.243	0.097	0.000	0.000	0.178	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	226	173	176	206	0	0	56	0
N.S.	1	1.00	0.95	0.72	0.74	0.86	0.00	0.00	0.23	0.00
time (sec)	N/A	0.419	0.732	1.895	0.243	0.101	0.000	0.000	0.187	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	130	75	85	79	79	143	138	109	94
N.S.	1	1.11	0.64	0.73	0.68	0.68	1.22	1.18	0.93	0.80
time (sec)	N/A	0.626	0.312	1.384	0.073	0.084	0.749	0.124	0.178	38.978

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	83	58	66	60	58	92	94	67	66
N.S.	1	1.05	0.73	0.84	0.76	0.73	1.16	1.19	0.85	0.84
time (sec)	N/A	0.399	0.180	1.665	0.039	0.080	0.385	0.116	0.181	0.416

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	23	33	26	27	26	46	26	37	28
N.S.	1	0.70	1.00	0.79	0.82	0.79	1.39	0.79	1.12	0.85
time (sec)	N/A	0.227	0.035	1.673	0.036	0.086	0.155	0.106	0.175	38.596

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	125	89	47	0	47	16	0
N.S.	1	1.00	0.93	2.27	1.62	0.85	0.00	0.85	0.29	0.00
time (sec)	N/A	0.267	0.088	2.832	0.153	0.120	0.000	0.131	0.160	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	185	97	90	0	186	717	0
N.S.	1	1.00	0.99	2.03	1.07	0.99	0.00	2.04	7.88	0.00
time (sec)	N/A	0.389	0.157	2.836	0.166	0.083	0.000	0.117	0.177	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	159	132	143	147	439	259	16	0
N.S.	1	1.00	0.85	0.70	0.76	0.78	2.34	1.38	0.09	0.00
time (sec)	N/A	0.401	0.488	1.309	0.127	0.082	1.930	0.129	0.172	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	117	99	112	120	129	185	12	0
N.S.	1	1.00	0.76	0.65	0.73	0.78	0.84	1.21	0.08	0.00
time (sec)	N/A	0.269	0.263	1.145	0.121	0.076	0.516	0.145	0.172	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	172	167	130	152	147	0	0	16	0
N.S.	1	1.02	0.99	0.77	0.90	0.88	0.00	0.00	0.10	0.00
time (sec)	N/A	0.345	0.488	1.316	0.326	0.088	0.000	0.000	0.177	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	58	97	51	116	97	12	0
N.S.	1	1.00	0.89	0.82	1.37	0.72	1.63	1.37	0.17	0.00
time (sec)	N/A	0.229	0.083	1.521	0.118	0.077	2.078	0.113	0.166	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	89	75	78	117	73	291	125	73	0
N.S.	1	1.06	0.89	0.93	1.39	0.87	3.46	1.49	0.87	0.00
time (sec)	N/A	0.281	0.176	1.742	0.126	0.078	1.990	0.135	0.182	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	35	67	50	55	52	95	52	77	55
N.S.	1	0.52	1.00	0.75	0.82	0.78	1.42	0.78	1.15	0.82
time (sec)	N/A	0.239	0.051	7.118	0.040	0.090	0.639	0.136	0.156	39.172

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	54	38	51	39	49	0
N.S.	1	1.00	0.93	0.89	1.23	0.86	1.16	0.89	1.11	0.00
time (sec)	N/A	0.272	0.138	2.220	0.073	0.077	2.128	0.124	0.155	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	341	289	0	0	1435	0	0	20	0
N.S.	1	0.94	0.80	0.00	0.00	3.96	0.00	0.00	0.06	0.00
time (sec)	N/A	1.411	0.270	0.000	0.000	0.227	0.000	0.000	0.165	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	237	188	0	0	1041	0	0	20	0
N.S.	1	0.97	0.77	0.00	0.00	4.25	0.00	0.00	0.08	0.00
time (sec)	N/A	0.879	0.077	0.000	0.000	0.190	0.000	0.000	0.162	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	54	48	48	8078	208	165	63	54	128
N.S.	1	1.12	1.00	1.00	168.29	4.33	3.44	1.31	1.12	2.67
time (sec)	N/A	0.273	0.134	0.440	23.684	0.089	3.507	0.134	0.153	40.715

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	19	15	20	38	20
N.S.	1	1.00	1.11	1.00	1.11	1.06	0.83	1.11	2.11	1.11
time (sec)	N/A	0.186	1.864	0.158	0.230	0.071	2.394	0.112	0.167	39.437

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	48	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	2.67	1.11
time (sec)	N/A	0.186	1.552	0.165	0.224	0.071	3.125	0.139	0.169	39.157

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11	1.11
time (sec)	N/A	0.189	0.916	0.155	0.331	0.073	1.840	0.126	0.175	38.596

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.14
time (sec)	N/A	0.167	0.037	0.152	0.212	0.071	0.801	0.146	0.160	38.596

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	45	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	2.50	1.11
time (sec)	N/A	0.184	0.817	0.163	0.233	0.070	2.990	0.137	0.164	38.842

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	663	625	513	0	0	2469	0	0	38	0
N.S.	1	0.94	0.77	0.00	0.00	3.72	0.00	0.00	0.06	0.00
time (sec)	N/A	2.828	2.845	0.000	0.000	0.254	0.000	0.000	0.162	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	326	302	0	0	1509	0	0	38	0
N.S.	1	1.01	0.93	0.00	0.00	4.66	0.00	0.00	0.12	0.00
time (sec)	N/A	1.197	1.428	0.000	0.000	0.218	0.000	0.000	0.164	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	99	91	131	0	366	2116	144	187	178
N.S.	1	1.09	1.00	1.44	0.00	4.02	23.25	1.58	2.05	1.96
time (sec)	N/A	0.385	0.238	0.524	0.000	0.117	53.750	0.114	0.176	39.163

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3466	45	17	20	112	20
N.S.	1	1.00	1.11	1.00	192.56	2.50	0.94	1.11	6.22	1.11
time (sec)	N/A	0.193	8.207	0.727	4.707	0.087	26.650	0.252	0.189	39.154

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3475	51	19	20	1331	20
N.S.	1	1.00	1.11	1.00	193.06	2.83	1.06	1.11	73.94	1.11
time (sec)	N/A	0.191	12.636	0.731	4.573	0.085	38.698	0.393	0.261	39.322

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	922	46	17	20	38	20
N.S.	1	1.00	1.11	1.00	51.22	2.56	0.94	1.11	2.11	1.11
time (sec)	N/A	0.188	5.064	0.743	0.492	0.082	39.002	0.190	0.162	38.565

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	3381	43	15	16	34	16
N.S.	1	1.00	1.14	1.00	241.50	3.07	1.07	1.14	2.43	1.14
time (sec)	N/A	0.167	5.290	0.813	4.067	0.079	17.666	0.164	0.153	38.746

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3486	51	19	20	128	20
N.S.	1	1.00	1.11	1.00	193.67	2.83	1.06	1.11	7.11	1.11
time (sec)	N/A	0.189	10.115	0.728	4.993	0.080	41.768	0.208	0.168	38.687

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20	1.10
time (sec)	N/A	0.189	1.386	0.159	0.570	0.089	10.736	0.396	0.172	38.692

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	373	0	0	323	0	0	178	0
N.S.	1	1.00	0.84	0.00	0.00	0.73	0.00	0.00	0.40	0.00
time (sec)	N/A	0.693	2.379	0.000	0.000	0.094	0.000	0.000	0.153	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	551	0	0	198	0	0	126	0
N.S.	1	1.00	1.97	0.00	0.00	0.71	0.00	0.00	0.45	0.00
time (sec)	N/A	0.481	3.152	0.000	0.000	0.090	0.000	0.000	0.158	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	98	0	0	49	0
N.S.	1	1.00	1.11	0.00	0.00	0.73	0.00	0.00	0.37	0.00
time (sec)	N/A	0.297	0.561	0.000	0.000	0.082	0.000	0.000	0.163	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	81	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	4.05	1.10
time (sec)	N/A	0.191	1.158	0.193	0.473	0.081	0.412	0.148	0.155	38.395

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	3886	48	19	22	226	22
N.S.	1	1.00	1.10	1.00	194.30	2.40	0.95	1.10	11.30	1.10
time (sec)	N/A	0.192	1.995	0.787	9.536	0.080	1.506	0.197	0.169	38.492

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	37	40	49	75	40	38
N.S.	1	1.00	1.00	0.89	0.84	0.91	1.11	1.70	0.91	0.86
time (sec)	N/A	0.231	0.016	0.828	0.039	0.083	0.393	0.107	0.160	0.341

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	41	22	21	23	31	26	23	21
N.S.	1	1.00	1.64	0.88	0.84	0.92	1.24	1.04	0.92	0.84
time (sec)	N/A	0.189	0.066	0.655	0.033	0.079	0.110	0.119	0.152	38.382

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	0	50	27	0	32	21	0
N.S.	1	1.00	0.94	0.00	1.61	0.87	0.00	1.03	0.68	0.00
time (sec)	N/A	0.205	0.128	0.000	0.112	0.076	0.000	0.113	0.166	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	0	57	49	0	99	29	0
N.S.	1	1.00	0.91	0.00	1.08	0.92	0.00	1.87	0.55	0.00
time (sec)	N/A	0.262	0.215	0.000	0.149	0.079	0.000	0.126	0.154	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	124	0	109	82	0	0	23	0
N.S.	1	1.00	1.11	0.00	0.97	0.73	0.00	0.00	0.21	0.00
time (sec)	N/A	0.268	0.293	0.000	0.185	0.084	0.000	0.000	0.150	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	108	0	93	63	0	0	21	0
N.S.	1	1.00	1.19	0.00	1.02	0.69	0.00	0.00	0.23	0.00
time (sec)	N/A	0.232	0.171	0.000	0.182	0.080	0.000	0.000	0.164	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	89	72	0	0	53	0
N.S.	1	1.00	1.19	0.00	0.88	0.71	0.00	0.00	0.52	0.00
time (sec)	N/A	0.252	0.302	0.000	0.188	0.084	0.000	0.000	0.157	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	143	0	91	101	0	0	29	0
N.S.	1	1.00	1.10	0.00	0.70	0.78	0.00	0.00	0.22	0.00
time (sec)	N/A	0.284	0.499	0.000	0.185	0.080	0.000	0.000	0.154	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	124	0	110	80	0	0	23	0
N.S.	1	1.00	1.17	0.00	1.04	0.75	0.00	0.00	0.22	0.00
time (sec)	N/A	0.235	0.243	0.000	0.179	0.082	0.000	0.000	0.171	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	138	0	85	61	0	0	16	0
N.S.	1	1.00	1.68	0.00	1.04	0.74	0.00	0.00	0.20	0.00
time (sec)	N/A	0.190	0.190	0.000	0.181	0.077	0.000	0.000	0.151	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	90	80	0	0	53	0
N.S.	1	1.00	1.19	0.00	0.89	0.79	0.00	0.00	0.52	0.00
time (sec)	N/A	0.225	0.258	0.000	0.189	0.084	0.000	0.000	0.152	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	146	0	91	101	0	0	29	0
N.S.	1	1.00	1.16	0.00	0.72	0.80	0.00	0.00	0.23	0.00
time (sec)	N/A	0.260	0.452	0.000	0.201	0.083	0.000	0.000	0.156	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	92	92	87	84	143	165	68	95
N.S.	1	1.00	0.86	0.86	0.81	0.79	1.34	1.54	0.64	0.89
time (sec)	N/A	0.353	0.296	2.725	0.045	0.086	0.530	0.162	0.152	39.069

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	62	52	52	52	53	99	57	60	51
N.S.	1	1.03	0.87	0.87	0.87	0.88	1.65	0.95	1.00	0.85
time (sec)	N/A	0.252	0.162	1.902	0.039	0.087	0.165	0.138	0.151	0.169

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	108	70	0	79	45	0
N.S.	1	1.00	0.89	0.00	1.35	0.88	0.00	0.99	0.56	0.00
time (sec)	N/A	0.291	0.216	0.000	0.163	0.095	0.000	0.129	0.168	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	116	0	124	112	0	226	883	0
N.S.	1	1.00	0.95	0.00	1.02	0.92	0.00	1.85	7.24	0.00
time (sec)	N/A	0.424	0.301	0.000	0.148	0.085	0.000	0.144	0.173	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	339	0	234	186	0	0	47	0
N.S.	1	1.00	1.36	0.00	0.94	0.75	0.00	0.00	0.19	0.00
time (sec)	N/A	0.433	0.987	0.000	0.222	0.105	0.000	0.000	0.161	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	251	0	199	141	0	0	43	0
N.S.	1	1.00	1.30	0.00	1.03	0.73	0.00	0.00	0.22	0.00
time (sec)	N/A	0.354	2.214	0.000	0.224	0.091	0.000	0.000	0.177	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	229	332	0	187	173	0	0	94	0
N.S.	1	0.99	1.44	0.00	0.81	0.75	0.00	0.00	0.41	0.00
time (sec)	N/A	0.408	1.193	0.000	0.224	0.116	0.000	0.000	0.162	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	283	292	0	194	232	0	0	56	0
N.S.	1	0.99	1.02	0.00	0.68	0.81	0.00	0.00	0.20	0.00
time (sec)	N/A	0.475	3.177	0.000	0.230	0.094	0.000	0.000	0.180	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	339	0	240	182	0	0	47	0
N.S.	1	1.00	1.43	0.00	1.01	0.77	0.00	0.00	0.20	0.00
time (sec)	N/A	0.376	1.141	0.000	0.237	0.097	0.000	0.000	0.173	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	228	0	192	139	0	0	36	0
N.S.	1	1.00	1.25	0.00	1.05	0.76	0.00	0.00	0.20	0.00
time (sec)	N/A	0.294	1.044	0.000	0.299	0.086	0.000	0.000	0.159	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	225	334	0	188	189	0	0	56	0
N.S.	1	0.99	1.47	0.00	0.83	0.83	0.00	0.00	0.25	0.00
time (sec)	N/A	0.359	0.884	0.000	0.229	0.091	0.000	0.000	0.174	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	275	294	0	193	233	0	0	56	0
N.S.	1	0.99	1.06	0.00	0.70	0.84	0.00	0.00	0.20	0.00
time (sec)	N/A	0.429	3.363	0.000	0.243	0.092	0.000	0.000	0.164	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	241	188	0	0	1041	0	0	20	0
N.S.	1	0.98	0.77	0.00	0.00	4.25	0.00	0.00	0.08	0.00
time (sec)	N/A	0.894	0.193	0.000	0.000	0.202	0.000	0.000	0.157	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	57	51	49	8078	208	172	64	55	136
N.S.	1	1.12	1.00	0.96	158.39	4.08	3.37	1.25	1.08	2.67
time (sec)	N/A	0.270	0.113	0.459	23.610	0.093	6.440	0.115	0.164	42.347

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	19	15	20	38	20
N.S.	1	1.00	1.11	1.00	1.11	1.06	0.83	1.11	2.11	1.11
time (sec)	N/A	0.181	1.845	0.171	0.315	0.075	2.581	0.117	0.158	40.482

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	48	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	2.67	1.11
time (sec)	N/A	0.185	1.563	0.160	0.316	0.076	4.470	0.135	0.158	40.858

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.177	0.718	0.165	0.297	0.068	2.038	0.141	0.160	40.541

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	45	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	2.50	1.11
time (sec)	N/A	0.188	0.736	0.152	0.320	0.072	2.669	0.123	0.157	39.835

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.14
time (sec)	N/A	0.163	0.023	0.148	0.295	0.077	0.690	0.115	0.157	39.936

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	23	17	20	48	20
N.S.	1	1.00	1.11	1.00	1.11	1.28	0.94	1.11	2.67	1.11
time (sec)	N/A	0.183	0.864	0.157	0.335	0.083	3.901	0.142	0.158	39.769

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	326	302	0	0	1509	0	0	38	0
N.S.	1	1.01	0.93	0.00	0.00	4.66	0.00	0.00	0.12	0.00
time (sec)	N/A	1.160	1.414	0.000	0.000	0.220	0.000	0.000	0.153	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	91	131	0	366	2116	146	187	186
N.S.	1	1.05	0.97	1.39	0.00	3.89	22.51	1.55	1.99	1.98
time (sec)	N/A	0.388	0.240	0.560	0.000	0.098	68.839	0.134	0.161	41.311

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	2696	45	17	20	112	20
N.S.	1	1.00	1.11	1.00	149.78	2.50	0.94	1.11	6.22	1.11
time (sec)	N/A	0.193	13.683	0.743	4.450	0.080	23.650	0.276	0.166	41.136

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	2705	51	19	20	1331	20
N.S.	1	1.00	1.11	1.00	150.28	2.83	1.06	1.11	73.94	1.11
time (sec)	N/A	0.191	20.426	0.719	4.436	0.091	46.662	0.434	0.239	41.323

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	0	44	15	18	36	18
N.S.	1	1.00	1.12	1.00	0.00	2.75	0.94	1.12	2.25	1.12
time (sec)	N/A	0.181	8.079	0.714	0.000	0.088	22.951	0.176	0.160	40.906

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	0	51	19	20	1445	20
N.S.	1	1.00	1.11	1.00	0.00	2.83	1.06	1.11	80.28	1.11
time (sec)	N/A	0.186	15.433	1.047	0.000	0.085	32.873	0.179	0.175	41.151

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	2171	43	15	16	34	16
N.S.	1	1.00	1.14	1.00	155.07	3.07	1.07	1.14	2.43	1.14
time (sec)	N/A	0.164	10.594	0.784	1.834	0.086	15.386	0.144	0.163	41.112

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	2171	51	19	20	133	20
N.S.	1	1.00	1.11	1.00	120.61	2.83	1.06	1.11	7.39	1.11
time (sec)	N/A	0.185	17.470	0.730	2.176	0.080	46.068	0.210	0.170	40.306

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20	1.10
time (sec)	N/A	0.186	1.260	0.164	0.528	0.089	17.128	0.433	0.160	39.954

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	373	0	0	347	0	0	178	0
N.S.	1	1.00	0.84	0.00	0.00	0.79	0.00	0.00	0.40	0.00
time (sec)	N/A	0.690	2.772	0.000	0.000	0.103	0.000	0.000	0.170	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	556	0	0	214	0	0	126	0
N.S.	1	1.00	1.95	0.00	0.00	0.75	0.00	0.00	0.44	0.00
time (sec)	N/A	0.460	3.013	0.000	0.000	0.089	0.000	0.000	0.156	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	106	0	0	49	0
N.S.	1	1.00	1.11	0.00	0.00	0.79	0.00	0.00	0.37	0.00
time (sec)	N/A	0.283	0.503	0.000	0.000	0.082	0.000	0.000	0.158	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	81	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	4.05	1.10
time (sec)	N/A	0.185	1.065	0.175	0.505	0.072	0.412	0.173	0.165	42.285

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2505	48	19	22	226	22
N.S.	1	1.00	1.10	1.00	125.25	2.40	0.95	1.10	11.30	1.10
time (sec)	N/A	0.183	2.055	0.816	3.957	0.085	1.427	0.184	0.171	40.562

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	70	73	86	66	0	400	16	0
N.S.	1	0.97	0.90	0.94	1.10	0.85	0.00	5.13	0.21	0.00
time (sec)	N/A	0.620	0.124	1.810	0.080	0.085	0.000	0.119	0.172	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	55	52	57	76	56	0	251	51	0
N.S.	1	0.92	0.87	0.95	1.27	0.93	0.00	4.18	0.85	0.00
time (sec)	N/A	0.529	0.074	1.402	0.078	0.078	0.000	0.117	0.158	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	32	38	58	34	0	132	12	0
N.S.	1	1.06	1.00	1.19	1.81	1.06	0.00	4.12	0.38	0.00
time (sec)	N/A	0.414	0.033	0.800	0.076	0.075	0.000	0.141	0.156	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	43	21	17	42	16	0
N.S.	1	1.00	1.00	1.05	2.05	1.00	0.81	2.00	0.76	0.00
time (sec)	N/A	0.243	0.060	0.829	0.075	0.072	0.400	0.116	0.168	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	14	14	14	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.17	1.17	1.17	1.17	1.00
time (sec)	N/A	0.208	0.022	0.480	0.031	0.066	0.264	0.111	0.154	40.616

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	50	33	29	48	33	29
N.S.	1	1.00	1.00	1.17	1.72	1.14	1.00	1.66	1.14	1.00
time (sec)	N/A	0.267	0.007	0.833	0.127	0.071	0.368	0.117	0.152	40.916

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	38	46	51	44	46	106	56	46
N.S.	1	1.09	0.84	1.02	1.13	0.98	1.02	2.36	1.24	1.02
time (sec)	N/A	0.356	0.065	0.858	0.079	0.070	0.489	0.138	0.162	41.288

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	71	61	52	50	52	61	191	75	64
N.S.	1	1.16	1.00	0.85	0.82	0.85	1.00	3.13	1.23	1.05
time (sec)	N/A	0.453	0.010	0.909	0.074	0.071	0.678	0.143	0.151	41.793

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	97	86	96	99	96	0	442	811	0
N.S.	1	0.97	0.86	0.96	0.99	0.96	0.00	4.42	8.11	0.00
time (sec)	N/A	0.384	0.206	1.232	0.083	0.078	0.000	0.120	0.159	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	67	65	76	87	77	0	283	763	0
N.S.	1	1.02	0.98	1.15	1.32	1.17	0.00	4.29	11.56	0.00
time (sec)	N/A	0.600	0.185	1.114	0.082	0.081	0.000	0.114	0.166	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	52	66	45	0	153	14	0
N.S.	1	1.00	1.00	1.27	1.61	1.10	0.00	3.73	0.34	0.00
time (sec)	N/A	0.448	0.098	1.020	0.076	0.084	0.000	0.118	0.153	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	36	51	31	31	65	18	0
N.S.	1	1.00	0.86	0.97	1.38	0.84	0.84	1.76	0.49	0.00
time (sec)	N/A	0.222	0.092	1.044	0.074	0.096	0.994	0.129	0.157	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	23	25	34	262	29	38	22
N.S.	1	1.00	1.03	0.74	0.81	1.10	8.45	0.94	1.23	0.71
time (sec)	N/A	0.228	0.079	0.821	0.033	0.073	0.953	0.111	0.159	41.907

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	42	68	60	391	77	18	41
N.S.	1	1.00	0.84	0.82	1.33	1.18	7.67	1.51	0.35	0.80
time (sec)	N/A	0.252	0.091	1.063	0.075	0.081	1.289	0.164	0.156	42.667

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	92	54	56	69	72	654	153	18	64
N.S.	1	1.06	0.62	0.64	0.79	0.83	7.52	1.76	0.21	0.74
time (sec)	N/A	0.341	0.146	1.299	0.078	0.075	1.732	0.135	0.159	42.892

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	65	67	68	90	726	255	18	84
N.S.	1	1.05	0.61	0.63	0.64	0.84	6.79	2.38	0.17	0.79
time (sec)	N/A	0.362	0.217	1.447	0.075	0.078	2.425	0.120	0.159	43.200

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	81	59	127	74	0	0	14	0
N.S.	1	1.10	1.01	0.74	1.59	0.92	0.00	0.00	0.18	0.00
time (sec)	N/A	0.337	0.153	0.635	0.079	0.082	0.000	0.000	0.150	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	43	21	0	0	18	0
N.S.	1	1.00	1.00	0.88	1.72	0.84	0.00	0.00	0.72	0.00
time (sec)	N/A	0.246	0.060	0.652	0.076	0.075	0.000	0.000	0.155	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	47	98	64	0	0	18	55
N.S.	1	1.00	0.81	0.63	1.31	0.85	0.00	0.00	0.24	0.73
time (sec)	N/A	0.278	0.120	0.629	0.146	0.110	0.000	0.000	0.161	42.448

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	20	17	17	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.33	1.13	1.13	0.87
time (sec)	N/A	0.203	0.020	0.546	0.026	0.067	0.468	0.104	0.152	43.345

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	89	65	74	85	0	0	18	0
N.S.	1	1.03	0.92	0.67	0.76	0.88	0.00	0.00	0.19	0.00
time (sec)	N/A	0.342	0.187	0.762	0.081	0.080	0.000	0.000	0.157	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	8	6	5	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	1.00	0.75	0.62	0.75
time (sec)	N/A	0.192	0.020	0.191	0.027	0.071	0.097	0.107	0.162	42.308

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	18	23	15	15	15	29	15	17	14
N.S.	1	0.86	1.10	0.71	0.71	0.71	1.38	0.71	0.81	0.67
time (sec)	N/A	0.216	0.034	0.792	0.049	0.076	0.159	0.125	0.153	42.072

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	13	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.59	0.73
time (sec)	N/A	0.232	0.030	0.237	0.029	0.075	0.088	0.131	0.164	40.526

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	76	41	19	30	37	379	30	43	34
N.S.	1	1.10	0.59	0.28	0.43	0.54	5.49	0.43	0.62	0.49
time (sec)	N/A	0.284	0.058	0.394	0.029	0.082	0.407	0.110	0.163	41.814

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	98	62	59	47	51	71	47	64	58
N.S.	1	1.13	0.71	0.68	0.54	0.59	0.82	0.54	0.74	0.67
time (sec)	N/A	0.490	0.066	0.829	0.030	0.080	2.893	0.110	0.158	43.414

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	23	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.28	1.11
time (sec)	N/A	0.186	1.846	0.227	1.559	0.092	10.721	0.857	0.168	42.126

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20	1.10
time (sec)	N/A	0.195	1.892	0.224	1.337	0.089	33.273	4.547	0.168	42.761

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	0	0	0	0	0	29	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.333	0.204	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	24	33	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	1.09	1.50	1.09
time (sec)	N/A	0.254	1.967	0.279	1.265	0.082	9.103	0.892	0.181	43.120

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	148	0	0	0	0	0	30	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.424	0.948	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	34	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.42	1.08
time (sec)	N/A	0.259	2.118	0.250	1.337	0.089	26.100	4.325	0.174	42.721

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	91	23	0	0	14	0
N.S.	1	1.00	0.92	0.96	3.64	0.92	0.00	0.00	0.56	0.00
time (sec)	N/A	0.255	0.079	0.676	0.166	0.078	0.000	0.000	0.158	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	100	36	0	0	16	0
N.S.	1	1.00	0.86	0.93	2.33	0.84	0.00	0.00	0.37	0.00
time (sec)	N/A	0.236	0.103	0.872	0.249	0.085	0.000	0.000	0.146	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	181	50	0	0	16	0
N.S.	1	1.00	0.81	0.78	2.70	0.75	0.00	0.00	0.24	0.00
time (sec)	N/A	0.283	0.129	0.983	0.267	0.077	0.000	0.000	0.156	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	189	62	0	0	16	0
N.S.	1	1.00	0.84	0.84	2.39	0.78	0.00	0.00	0.20	0.00
time (sec)	N/A	0.287	0.124	2.474	0.266	0.079	0.000	0.000	0.154	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	95	74	0	0	0	0	10	0
N.S.	1	1.00	1.09	0.85	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.225	0.105	0.340	0.000	0.000	0.000	0.000	0.156	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	0	0	0	0	0	12	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.271	0.312	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	177	0	0	0	0	0	12	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.312	0.419	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	118	110	0	0	0	0	14	0
N.S.	1	1.00	1.08	1.01	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.278	0.223	0.465	0.000	0.000	0.000	0.000	0.152	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	129	0	0	0	0	0	16	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.356	0.590	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	225	0	0	0	0	0	16	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.452	0.640	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	30	36	32	32	56	0	30	0
N.S.	1	0.94	0.86	1.03	0.91	0.91	1.60	0.00	0.86	0.00
time (sec)	N/A	0.280	0.079	0.799	0.038	0.090	3.288	0.000	0.155	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	32	29	35	29	29	56	0	29	0
N.S.	1	0.94	0.85	1.03	0.85	0.85	1.65	0.00	0.85	0.00
time (sec)	N/A	0.281	0.071	1.006	0.049	0.075	3.310	0.000	0.183	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	47	44	0	47	0	0	19	0
N.S.	1	0.91	1.02	0.96	0.00	1.02	0.00	0.00	0.41	0.00
time (sec)	N/A	0.444	0.086	0.867	0.000	0.080	0.000	0.000	0.177	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	67	58	66	0	53	0	0	54	0
N.S.	1	0.99	0.85	0.97	0.00	0.78	0.00	0.00	0.79	0.00
time (sec)	N/A	0.314	0.151	1.368	0.000	0.114	0.000	0.000	0.167	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	113	95	99	0	95	0	0	746	0
N.S.	1	0.99	0.83	0.87	0.00	0.83	0.00	0.00	6.54	0.00
time (sec)	N/A	0.411	0.214	5.130	0.000	0.095	0.000	0.000	0.182	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	65	68	65	0	68	0	0	62	0
N.S.	1	0.83	0.87	0.83	0.00	0.87	0.00	0.00	0.79	0.00
time (sec)	N/A	0.527	0.149	0.842	0.000	0.079	0.000	0.000	0.196	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	82	89	0	88	0	0	816	0
N.S.	1	0.98	0.85	0.92	0.00	0.91	0.00	0.00	8.41	0.00
time (sec)	N/A	0.350	0.198	1.376	0.000	0.092	0.000	0.000	0.181	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	165	141	144	0	143	0	0	803	0
N.S.	1	0.99	0.84	0.86	0.00	0.86	0.00	0.00	4.81	0.00
time (sec)	N/A	0.494	0.437	5.293	0.000	0.092	0.000	0.000	0.186	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	209	173	586	974	255	0	509	1101	231
N.S.	1	0.94	0.78	2.63	4.37	1.14	0.00	2.28	4.94	1.04
time (sec)	N/A	0.427	1.067	1.766	1.008	0.086	0.000	0.182	0.215	39.761

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	142	117	291	564	162	0	331	683	136
N.S.	1	0.95	0.78	1.94	3.76	1.08	0.00	2.21	4.55	0.91
time (sec)	N/A	0.315	0.653	1.431	0.633	0.087	0.000	0.169	0.206	0.213

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	66	120	271	80	0	233	53	58
N.S.	1	0.97	0.96	1.74	3.93	1.16	0.00	3.38	0.77	0.84
time (sec)	N/A	0.244	0.243	1.287	0.354	0.091	0.000	0.190	0.172	0.119

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	42	53	45	0	143	23	41
N.S.	1	1.00	1.00	1.08	1.36	1.15	0.00	3.67	0.59	1.05
time (sec)	N/A	0.179	0.019	0.464	0.043	0.077	0.000	0.127	0.165	0.087

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	31	29	20	31	20
N.S.	1	1.00	1.11	1.00	1.11	1.72	1.61	1.11	1.72	1.11
time (sec)	N/A	0.184	4.061	0.337	0.235	0.072	0.847	0.236	0.164	39.743

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	42	31	20	42	20
N.S.	1	1.00	1.11	1.00	1.11	2.33	1.72	1.11	2.33	1.11
time (sec)	N/A	0.180	7.375	0.329	0.246	0.080	2.333	2.437	0.191	40.077

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	311	440	277	0	398	0	0	0	0
N.S.	1	0.92	1.31	0.82	0.00	1.18	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	6.700	2.181	0.000	0.110	0.000	0.000	14.624	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	216	265	196	0	262	0	0	0	0
N.S.	1	0.93	1.14	0.84	0.00	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	1.566	1.756	0.000	0.093	0.000	0.000	0.676	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	112	95	101	0	130	0	0	53	0
N.S.	1	0.93	0.79	0.84	0.00	1.08	0.00	0.00	0.44	0.00
time (sec)	N/A	0.299	0.523	1.300	0.000	0.089	0.000	0.000	0.217	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	60	52	0	73	0	0	23	52
N.S.	1	0.98	1.00	0.87	0.00	1.22	0.00	0.00	0.38	0.87
time (sec)	N/A	0.245	0.041	0.668	0.000	0.081	0.000	0.000	0.187	40.376

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	31	26	20	31	20
N.S.	1	1.00	1.11	1.00	1.11	1.72	1.44	1.11	1.72	1.11
time (sec)	N/A	0.176	1.674	0.390	0.209	0.073	19.665	0.141	0.185	40.799

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	42	0	20	42	20
N.S.	1	1.00	1.11	1.00	1.11	2.33	0.00	1.11	2.33	1.11
time (sec)	N/A	0.176	25.849	0.407	0.218	0.084	0.000	0.280	0.200	46.432

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	321	218	690	1824	328	0	523	1146	0
N.S.	1	0.94	0.64	2.02	5.35	0.96	0.00	1.53	3.36	0.00
time (sec)	N/A	0.561	3.142	2.375	1.673	0.124	0.000	0.299	0.217	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	242	151	438	1038	208	0	345	713	0
N.S.	1	0.95	0.59	1.71	4.05	0.81	0.00	1.35	2.79	0.00
time (sec)	N/A	0.414	1.973	1.964	0.868	0.086	0.000	0.293	0.205	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	117	114	209	483	131	0	247	55	0
N.S.	1	0.96	0.93	1.71	3.96	1.07	0.00	2.02	0.45	0.00
time (sec)	N/A	0.301	0.705	1.289	0.441	0.084	0.000	0.254	0.181	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	87	69	89	0	151	24	95
N.S.	1	1.00	0.81	1.05	0.83	1.07	0.00	1.82	0.29	1.14
time (sec)	N/A	0.261	0.061	0.655	0.045	0.081	0.000	0.184	0.194	0.049

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	32	31	22	32	22
N.S.	1	1.00	1.10	1.00	1.10	1.60	1.55	1.10	1.60	1.10
time (sec)	N/A	0.176	9.115	0.396	0.266	0.079	0.844	0.376	0.179	39.889

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	43	32	22	43	22
N.S.	1	1.00	1.10	1.00	1.10	2.15	1.60	1.10	2.15	1.10
time (sec)	N/A	0.179	10.117	0.401	0.370	0.078	2.335	6.543	0.182	40.053

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	434	414	353	0	0	591	0	0	0	0
N.S.	1	0.95	0.81	0.00	0.00	1.36	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	18.073	0.000	0.000	0.104	0.000	0.000	0.257	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	269	300	0	0	409	0	0	127	0
N.S.	1	0.96	1.07	0.00	0.00	1.46	0.00	0.00	0.45	0.00
time (sec)	N/A	0.389	12.587	0.000	0.000	0.099	0.000	0.000	0.210	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	227	381	0	0	263	0	0	79	0
N.S.	1	0.97	1.62	0.00	0.00	1.12	0.00	0.00	0.34	0.00
time (sec)	N/A	0.323	10.849	0.000	0.000	0.093	0.000	0.000	0.179	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	0	0	113	0	0	36	0
N.S.	1	1.00	1.07	0.00	0.00	1.06	0.00	0.00	0.34	0.00
time (sec)	N/A	0.235	0.152	0.000	0.000	0.082	0.000	0.000	0.165	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	44	44	22	44	22
N.S.	1	1.00	1.10	1.00	1.10	2.20	2.20	1.10	2.20	1.10
time (sec)	N/A	0.184	24.597	0.450	0.329	0.075	0.988	2.689	0.206	39.389

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	55	46	22	55	22
N.S.	1	1.00	1.10	1.00	1.10	2.75	2.30	1.10	2.75	1.10
time (sec)	N/A	0.183	56.975	0.444	0.346	0.077	2.593	3.793	0.184	39.941

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	345	467	274	0	392	0	0	0	0
N.S.	1	0.93	1.26	0.74	0.00	1.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	2.864	2.743	0.000	0.126	0.000	0.000	0.976	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	184	242	150	0	233	0	0	93	0
N.S.	1	0.93	1.22	0.76	0.00	1.18	0.00	0.00	0.47	0.00
time (sec)	N/A	0.421	1.111	1.748	0.000	0.101	0.000	0.000	0.221	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	100	80	0	137	0	0	43	0
N.S.	1	1.05	0.95	0.76	0.00	1.30	0.00	0.00	0.41	0.00
time (sec)	N/A	0.343	0.189	0.892	0.000	0.088	0.000	0.000	0.179	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	51	27	22	51	22
N.S.	1	1.00	1.10	1.00	1.10	2.55	1.35	1.10	2.55	1.10
time (sec)	N/A	0.180	3.971	0.523	0.412	0.079	18.976	0.759	0.193	40.413

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	62	0	22	62	22
N.S.	1	1.00	1.10	1.00	1.10	3.10	0.00	1.10	3.10	1.10
time (sec)	N/A	0.180	32.520	0.521	0.441	0.083	0.000	1.080	0.201	42.982

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	313	620	0	0	576	0	0	0	0
N.S.	1	0.95	1.88	0.00	0.00	1.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	4.272	0.000	0.000	0.100	0.000	0.000	0.450	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	227	705	0	0	367	0	0	0	0
N.S.	1	0.97	3.00	0.00	0.00	1.56	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	2.462	0.000	0.000	0.095	0.000	0.000	1.453	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	203	0	0	185	0	0	66	0
N.S.	1	1.00	1.90	0.00	0.00	1.73	0.00	0.00	0.62	0.00
time (sec)	N/A	0.231	0.484	0.000	0.000	0.087	0.000	0.000	0.180	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	74	39	22	74	22
N.S.	1	1.00	1.10	1.00	1.10	3.70	1.95	1.10	3.70	1.10
time (sec)	N/A	0.178	5.370	0.530	0.555	0.070	70.163	4.906	0.221	41.342

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	85	0	22	85	22
N.S.	1	1.00	1.10	1.00	1.10	4.25	0.00	1.10	4.25	1.10
time (sec)	N/A	0.180	36.467	0.549	0.562	0.091	0.000	6.895	0.248	46.578

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	414	138	1161	1101	196	529	698	383	0
N.S.	1	1.01	0.34	2.83	2.69	0.48	1.29	1.70	0.93	0.00
time (sec)	N/A	0.578	1.846	1.234	0.091	0.088	0.371	0.129	0.177	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	189	85	347	348	86	221	218	147	0
N.S.	1	1.02	0.46	1.88	1.88	0.46	1.19	1.18	0.79	0.00
time (sec)	N/A	0.343	0.664	1.148	0.050	0.080	0.238	0.116	0.176	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	50	61	62	44	65	44	40	43
N.S.	1	0.96	0.93	1.13	1.15	0.81	1.20	0.81	0.74	0.80
time (sec)	N/A	0.272	0.066	0.520	0.039	0.075	0.195	0.134	0.176	39.230

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	258	238	793	0	250	0	0	21	0
N.S.	1	1.08	1.00	3.33	0.00	1.05	0.00	0.00	0.09	0.00
time (sec)	N/A	0.886	2.375	0.849	0.000	0.088	0.000	0.000	0.172	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	355	397	1831	0	416	0	0	32	0
N.S.	1	1.05	1.17	5.40	0.00	1.23	0.00	0.00	0.09	0.00
time (sec)	N/A	1.030	3.965	0.947	0.000	0.105	0.000	0.000	0.186	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	382	366	419	0	694	366	0	0	91	0
N.S.	1	0.96	1.10	0.00	1.82	0.96	0.00	0.00	0.24	0.00
time (sec)	N/A	0.438	3.584	0.000	0.525	0.100	0.000	0.000	0.203	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	281	705	0	375	223	0	0	55	0
N.S.	1	0.97	2.42	0.00	1.29	0.77	0.00	0.00	0.19	0.00
time (sec)	N/A	0.351	3.121	0.000	0.381	0.097	0.000	0.000	0.194	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	114	123	0	112	75	0	0	24	0
N.S.	1	0.99	1.07	0.00	0.97	0.65	0.00	0.00	0.21	0.00
time (sec)	N/A	0.273	0.175	0.000	0.217	0.084	0.000	0.000	0.172	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	29	34	22	32	22
N.S.	1	1.00	1.09	0.91	1.00	1.32	1.55	1.00	1.45	1.00
time (sec)	N/A	0.179	22.463	0.415	0.558	0.081	5.511	0.241	0.164	38.754

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	40	36	22	43	22
N.S.	1	1.00	1.09	0.91	1.00	1.82	1.64	1.00	1.95	1.00
time (sec)	N/A	0.180	30.838	0.415	0.729	0.081	36.385	0.286	0.197	39.637

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	631	557	696	877	393	0	6587	85	0
N.S.	1	1.03	0.91	1.14	1.44	0.64	0.00	10.78	0.14	0.00
time (sec)	N/A	0.915	2.445	4.397	0.663	0.098	0.000	3.409	0.177	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	318	367	295	407	203	0	2158	51	0
N.S.	1	1.06	1.22	0.98	1.35	0.67	0.00	7.17	0.17	0.00
time (sec)	N/A	0.538	0.967	2.915	0.279	0.100	0.000	0.470	0.179	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	84	124	104	0	413	22	0
N.S.	1	1.00	1.05	0.89	1.32	1.11	0.00	4.39	0.23	0.00
time (sec)	N/A	0.532	0.090	1.773	0.130	0.093	0.000	0.163	0.169	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	296	0	441	0	308	0	0	30	0
N.S.	1	1.07	0.00	1.60	0.00	1.12	0.00	0.00	0.11	0.00
time (sec)	N/A	1.125	0.000	2.250	0.000	0.114	0.000	0.000	0.189	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	372	0	2734	0	448	0	0	41	0
N.S.	1	1.06	0.00	7.81	0.00	1.28	0.00	0.00	0.12	0.00
time (sec)	N/A	1.009	0.000	2.389	0.000	0.117	0.000	0.000	0.191	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	371	463	0	993	584	0	0	154	0
N.S.	1	0.95	1.19	0.00	2.55	1.50	0.00	0.00	0.39	0.00
time (sec)	N/A	0.540	2.066	0.000	0.575	0.117	0.000	0.000	0.197	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	244	835	0	503	364	0	0	97	0
N.S.	1	0.97	3.33	0.00	2.00	1.45	0.00	0.00	0.39	0.00
time (sec)	N/A	0.377	3.235	0.000	0.373	0.107	0.000	0.000	0.179	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	114	166	0	151	152	0	0	45	0
N.S.	1	0.99	1.44	0.00	1.31	1.32	0.00	0.00	0.39	0.00
time (sec)	N/A	0.284	0.459	0.000	0.234	0.086	0.000	0.000	0.195	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	59	32	22	53	22
N.S.	1	1.00	1.09	0.91	1.00	2.68	1.45	1.00	2.41	1.00
time (sec)	N/A	0.180	17.661	0.644	0.802	0.097	47.294	0.383	0.174	40.327

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	70	0	22	64	22
N.S.	1	1.00	1.09	0.91	1.00	3.18	0.00	1.00	2.91	1.00
time (sec)	N/A	0.179	26.989	0.631	1.046	0.093	0.000	0.532	0.194	40.411

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	638	256	2704	2151	333	0	1555	692	0
N.S.	1	1.01	0.40	4.27	3.40	0.53	0.00	2.46	1.09	0.00
time (sec)	N/A	0.822	2.880	1.350	0.148	0.089	0.000	0.131	0.278	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	293	147	801	681	142	0	453	262	0
N.S.	1	1.02	0.51	2.78	2.36	0.49	0.00	1.57	0.91	0.00
time (sec)	N/A	0.479	0.849	1.319	0.065	0.086	0.000	0.146	0.178	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	134	120	58	94	82	69	69
N.S.	1	1.00	0.76	1.58	1.41	0.68	1.11	0.96	0.81	0.81
time (sec)	N/A	0.381	0.120	0.483	0.045	0.079	0.284	0.137	0.171	42.328

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	424	118	327	0	448	0	0	22	0
N.S.	1	1.07	0.30	0.83	0.00	1.13	0.00	0.00	0.06	0.00
time (sec)	N/A	1.396	25.345	0.932	0.000	0.109	0.000	0.000	0.205	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	567	180	1176	0	730	0	0	33	0
N.S.	1	1.02	0.32	2.12	0.00	1.32	0.00	0.00	0.06	0.00
time (sec)	N/A	1.649	1.585	1.120	0.000	0.123	0.000	0.000	0.447	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	482	432	366	561	308	0	770	61	0
N.S.	1	0.94	0.84	0.71	1.09	0.60	0.00	1.50	0.12	0.00
time (sec)	N/A	0.662	2.918	2.931	0.128	0.099	0.000	0.164	0.193	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	228	213	175	248	159	0	406	35	0
N.S.	1	0.94	0.88	0.72	1.02	0.65	0.00	1.67	0.14	0.00
time (sec)	N/A	0.395	1.197	0.976	0.063	0.094	0.000	0.144	0.176	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	129	114	86	92	98	0	170	14	0
N.S.	1	0.99	0.88	0.66	0.71	0.75	0.00	1.31	0.11	0.00
time (sec)	N/A	0.356	0.199	0.244	0.044	0.083	0.000	0.168	0.177	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	19	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.178	63.589	0.409	0.917	0.088	1.680	0.141	0.230	41.394

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	33	20	22	33	22
N.S.	1	1.00	1.09	0.91	1.00	1.50	0.91	1.00	1.50	1.00
time (sec)	N/A	0.175	58.753	0.419	1.607	0.085	10.206	0.171	0.460	41.539

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	866	929	936	1003	576	0	11903	85	0
N.S.	1	1.01	1.09	1.09	1.17	0.67	0.00	13.92	0.10	0.00
time (sec)	N/A	1.137	5.308	4.579	0.590	0.191	0.000	0.673	0.235	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	427	540	391	458	258	0	3727	51	0
N.S.	1	1.02	1.29	0.93	1.09	0.62	0.00	8.89	0.12	0.00
time (sec)	N/A	0.652	1.296	3.016	0.467	0.107	0.000	0.327	0.217	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	127	133	108	138	121	0	663	22	0
N.S.	1	0.93	0.98	0.79	1.01	0.89	0.00	4.88	0.16	0.00
time (sec)	N/A	0.644	0.122	1.745	0.138	0.085	0.000	0.199	0.194	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	462	170	156	0	548	0	0	30	0
N.S.	1	1.06	0.39	0.36	0.00	1.26	0.00	0.00	0.07	0.00
time (sec)	N/A	1.716	25.555	2.355	0.000	0.152	0.000	0.000	0.278	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	566	584	313	1554	0	798	0	0	41	0
N.S.	1	1.03	0.55	2.75	0.00	1.41	0.00	0.00	0.07	0.00
time (sec)	N/A	1.843	1.611	2.483	0.000	0.148	0.000	0.000	0.511	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	599	613	424	1260	461	0	0	85	0
N.S.	1	0.95	0.97	0.67	2.00	0.73	0.00	0.00	0.13	0.00
time (sec)	N/A	0.820	3.363	4.204	0.594	0.112	0.000	0.000	0.231	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	305	378	225	584	247	0	0	51	0
N.S.	1	0.96	1.19	0.71	1.84	0.78	0.00	0.00	0.16	0.00
time (sec)	N/A	0.524	1.525	1.749	0.314	0.102	0.000	0.000	0.204	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	146	105	219	143	0	0	22	0
N.S.	1	1.06	1.04	0.74	1.55	1.01	0.00	0.00	0.16	0.00
time (sec)	N/A	0.461	0.167	0.546	0.150	0.106	0.000	0.000	0.194	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	36	19	22	30	22
N.S.	1	1.00	1.09	0.91	1.00	1.64	0.86	1.00	1.36	1.00
time (sec)	N/A	0.183	63.705	0.652	0.701	0.082	3.116	0.198	0.291	42.143

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	47	20	22	41	22
N.S.	1	1.00	1.09	0.91	1.00	2.14	0.91	1.00	1.86	1.00
time (sec)	N/A	0.183	58.847	0.632	1.379	0.089	21.758	0.207	0.531	43.411

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	234	226	0	175	234	0	479	274	0
N.S.	1	0.81	0.78	0.00	0.61	0.81	0.00	1.66	0.95	0.00
time (sec)	N/A	0.968	0.801	0.000	0.250	0.482	0.000	0.224	0.177	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	168	111	0	193	143	0	286	160	0
N.S.	1	0.83	0.55	0.00	0.96	0.71	0.00	1.42	0.79	0.00
time (sec)	N/A	0.660	0.467	0.000	0.250	0.487	0.000	0.170	0.176	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	135	97	0	156	128	0	192	109	0
N.S.	1	0.84	0.61	0.00	0.98	0.80	0.00	1.20	0.68	0.00
time (sec)	N/A	0.534	0.492	0.000	0.235	0.446	0.000	0.190	0.169	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	72	70	0	129	84	0	90	46	0
N.S.	1	0.85	0.82	0.00	1.52	0.99	0.00	1.06	0.54	0.00
time (sec)	N/A	0.322	0.162	0.000	0.225	0.443	0.000	0.151	0.189	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	23	46	0	37	23	0
N.S.	1	1.00	1.00	0.00	0.55	1.10	0.00	0.88	0.55	0.00
time (sec)	N/A	0.239	0.188	0.000	0.059	0.076	0.000	0.121	0.178	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	88	85	0	126	0	0	0	41	0
N.S.	1	0.73	0.71	0.00	1.05	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.487	0.303	0.000	0.219	0.000	0.000	0.000	0.170	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	121	115	0	129	0	0	0	41	0
N.S.	1	0.69	0.66	0.00	0.74	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.594	0.353	0.000	0.345	0.000	0.000	0.000	0.176	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	185	184	0	129	0	0	0	59	0
N.S.	1	0.69	0.69	0.00	0.48	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.853	0.528	0.000	0.237	0.000	0.000	0.000	0.183	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	222	175	0	386	0	0	0	81	0
N.S.	1	0.83	0.66	0.00	1.45	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.633	0.953	0.000	0.351	0.000	0.000	0.000	0.198	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	182	160	0	424	0	0	0	26	0
N.S.	1	0.80	0.70	0.00	1.87	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.552	0.610	0.000	0.615	0.000	0.000	0.000	0.176	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	74	72	0	129	89	0	0	46	0
N.S.	1	0.83	0.81	0.00	1.45	1.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.393	0.083	0.000	0.227	0.485	0.000	0.000	0.164	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	23	46	0	0	23	0
N.S.	1	1.00	1.00	0.00	0.52	1.05	0.00	0.00	0.52	0.00
time (sec)	N/A	0.318	0.176	0.000	0.055	0.074	0.000	0.000	0.185	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	110	96	0	487	0	0	0	26	0
N.S.	1	0.83	0.72	0.00	3.66	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.403	0.240	0.000	0.349	0.000	0.000	0.000	0.185	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	143	133	0	380	0	0	0	41	0
N.S.	1	0.85	0.79	0.00	2.26	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.470	0.401	0.000	0.372	0.000	0.000	0.000	0.183	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	90	87	0	126	0	0	0	41	0
N.S.	1	0.71	0.69	0.00	1.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.574	0.359	0.000	0.235	0.000	0.000	0.000	0.170	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	180	208	0	129	0	0	0	34	0
N.S.	1	0.73	0.84	0.00	0.52	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.820	0.515	0.000	0.322	0.000	0.000	0.000	0.201	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	118	131	0	172	0	0	0	34	0
N.S.	1	0.70	0.78	0.00	1.02	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.603	0.322	0.000	0.236	0.000	0.000	0.000	0.189	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	85	88	0	155	0	0	0	34	0
N.S.	1	0.73	0.76	0.00	1.34	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.489	0.320	0.000	0.251	0.000	0.000	0.000	0.182	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	31	64	0	0	31	0
N.S.	1	1.00	0.93	0.00	0.69	1.42	0.00	0.00	0.69	0.00
time (sec)	N/A	0.248	0.175	0.000	0.058	0.072	0.000	0.000	0.196	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	75	72	0	171	116	0	0	70	0
N.S.	1	0.82	0.79	0.00	1.88	1.27	0.00	0.00	0.77	0.00
time (sec)	N/A	0.325	0.103	0.000	0.249	0.456	0.000	0.000	0.181	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	140	113	0	1389	160	0	0	161	0
N.S.	1	0.81	0.66	0.00	8.08	0.93	0.00	0.00	0.94	0.00
time (sec)	N/A	0.542	0.264	0.000	1.109	0.545	0.000	0.000	0.175	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	173	112	0	1943	181	0	0	67	0
N.S.	1	0.80	0.52	0.00	8.95	0.83	0.00	0.00	0.31	0.00
time (sec)	N/A	0.668	0.417	0.000	1.549	0.579	0.000	0.000	0.184	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	237	237	0	1120	0	0	0	70	0
N.S.	1	0.79	0.79	0.00	3.75	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.745	1.144	0.000	0.879	0.000	0.000	0.000	0.213	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	204	228	0	749	0	0	0	34	0
N.S.	1	0.78	0.87	0.00	2.86	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.695	0.568	0.000	0.605	0.000	0.000	0.000	0.200	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	120	113	0	129	0	0	0	34	0
N.S.	1	0.71	0.67	0.00	0.77	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.699	0.521	0.000	0.235	0.000	0.000	0.000	0.184	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	87	90	0	126	0	0	0	34	0
N.S.	1	0.71	0.74	0.00	1.03	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.599	0.369	0.000	0.228	0.000	0.000	0.000	0.186	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	140	136	0	383	0	0	0	34	0
N.S.	1	0.85	0.83	0.00	2.34	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.544	0.434	0.000	0.346	0.000	0.000	0.000	0.179	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	113	96	0	487	0	0	0	49	0
N.S.	1	0.80	0.68	0.00	3.45	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.445	0.291	0.000	0.470	0.000	0.000	0.000	0.190	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	31	64	0	0	31	0
N.S.	1	1.00	0.94	0.00	0.66	1.36	0.00	0.00	0.66	0.00
time (sec)	N/A	0.338	0.189	0.000	0.052	0.079	0.000	0.000	0.177	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	77	72	0	129	133	0	0	70	0
N.S.	1	0.81	0.76	0.00	1.36	1.40	0.00	0.00	0.74	0.00
time (sec)	N/A	0.421	0.114	0.000	0.247	0.492	0.000	0.000	0.203	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	185	165	0	408	0	0	0	67	0
N.S.	1	0.78	0.70	0.00	1.72	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.631	1.211	0.000	0.384	0.000	0.000	0.000	0.201	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	224	192	0	405	0	0	0	85	0
N.S.	1	0.81	0.69	0.00	1.46	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.728	1.652	0.000	0.373	0.000	0.000	0.000	0.188	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	22	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.22	1.11
time (sec)	N/A	0.175	11.577	0.157	0.293	0.075	7.752	0.147	0.180	40.280

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	503	483	384	0	0	0	0	0	18	0
N.S.	1	0.96	0.76	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.603	0.864	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	369	355	288	0	0	0	0	0	18	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.457	0.613	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	235	192	0	0	0	0	0	16	0
N.S.	1	0.97	0.79	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.332	0.577	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	0	0	0	0	0	14	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.242	0.130	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.169	1.714	0.155	0.352	0.081	1.789	0.157	0.185	40.093

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.169	1.434	0.138	0.381	0.076	5.269	0.139	0.171	39.815

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	411	0	0	0	0	0	27	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.802	1.119	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	313	0	0	0	0	0	27	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.614	0.645	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	215	0	0	0	0	0	25	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.427	0.635	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	20	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.232	0.316	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	25	22	17	22	25	22
N.S.	1	1.00	1.10	1.00	1.25	1.10	0.85	1.10	1.25	1.10
time (sec)	N/A	0.190	3.567	0.115	0.342	0.082	4.397	0.129	0.162	39.905

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	27	22	19	22	29	22
N.S.	1	1.00	1.10	1.00	1.35	1.10	0.95	1.10	1.45	1.10
time (sec)	N/A	0.186	3.041	0.109	0.348	0.087	19.737	0.143	0.169	39.474

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	856	819	648	0	0	0	0	0	55	0
N.S.	1	0.96	0.76	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.038	1.909	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	556	534	434	0	0	0	0	0	51	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.627	1.754	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	247	0	0	0	0	0	44	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.397	0.887	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	62	52	19	24	24	24
N.S.	1	1.00	1.09	1.00	2.82	2.36	0.86	1.09	1.09	1.09
time (sec)	N/A	0.185	5.290	0.932	1.017	0.086	18.744	0.225	200.015	39.799

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	67	52	20	24	58	24
N.S.	1	1.00	1.09	1.00	3.05	2.36	0.91	1.09	2.64	1.09
time (sec)	N/A	0.187	4.527	1.368	1.039	0.081	59.763	0.216	0.161	39.773

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	0	24	48	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	2.18	1.09
time (sec)	N/A	0.193	3.195	0.194	0.974	0.079	0.000	0.166	0.187	39.603

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	46	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	2.30	1.10
time (sec)	N/A	0.181	2.901	0.210	0.809	0.078	103.887	0.168	0.190	39.921

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	42	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	2.33	1.11
time (sec)	N/A	0.168	0.278	0.191	0.456	0.085	43.991	0.135	0.172	40.022

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	23	19	24	46	24
N.S.	1	1.00	1.09	1.00	1.09	1.05	0.86	1.09	2.09	1.09
time (sec)	N/A	0.189	2.562	0.214	0.440	0.080	104.346	0.151	0.172	40.323

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	27	0	24	53	24
N.S.	1	1.00	1.09	1.00	1.09	1.23	0.00	1.09	2.41	1.09
time (sec)	N/A	0.191	2.195	0.181	0.587	0.081	0.000	0.175	0.167	39.949

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	1509	54	0	24	20518	24
N.S.	1	1.00	0.00	1.00	68.59	2.45	0.00	1.09	932.64	1.09
time (sec)	N/A	0.189	0.000	0.908	4.993	0.090	0.000	0.230	0.820	39.931

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	0	20	1476	52	0	22	15251	22
N.S.	1	1.00	0.00	1.00	73.80	2.60	0.00	1.10	762.55	1.10
time (sec)	N/A	0.181	0.000	0.925	4.510	0.094	0.000	0.220	0.648	40.592

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1403	51	0	20	9964	20
N.S.	1	1.00	1.11	1.00	77.94	2.83	0.00	1.11	553.56	1.11
time (sec)	N/A	0.169	12.197	0.950	1.283	0.089	0.000	0.196	0.530	40.621

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	5041	53	0	24	136	24
N.S.	1	1.00	0.00	1.00	229.14	2.41	0.00	1.09	6.18	1.09
time (sec)	N/A	0.190	0.000	1.184	27.052	0.096	0.000	0.195	0.230	40.515

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	5369	59	0	24	152	24
N.S.	1	1.00	0.00	1.00	244.05	2.68	0.00	1.09	6.91	1.09
time (sec)	N/A	0.189	0.000	1.230	34.924	0.100	0.000	0.184	0.246	40.359

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	302	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	12.58	1.08
time (sec)	N/A	0.186	2.819	0.218	1.863	0.094	0.000	150.031	0.208	40.707

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	150	188	258	185	0	1263	687	0
N.S.	1	1.00	0.67	0.84	1.15	0.83	0.00	5.64	3.07	0.00
time (sec)	N/A	0.633	0.626	1.773	0.136	0.094	0.000	0.156	0.155	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	79	109	153	108	0	520	366	0
N.S.	1	1.00	0.67	0.92	1.30	0.92	0.00	4.41	3.10	0.00
time (sec)	N/A	0.416	0.289	1.352	0.108	0.081	0.000	0.124	0.150	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	50	43	65	40	0	137	18	0
N.S.	1	1.00	1.32	1.13	1.71	1.05	0.00	3.61	0.47	0.00
time (sec)	N/A	0.236	0.051	1.116	0.082	0.080	0.000	0.115	0.148	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	83	129	0	100	0	169	36	0
N.S.	1	1.03	0.79	1.23	0.00	0.95	0.00	1.61	0.34	0.00
time (sec)	N/A	0.450	0.223	1.634	0.000	0.084	0.000	0.160	0.150	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	85	131	0	129	0	339	87	0
N.S.	1	1.00	0.89	1.36	0.00	1.34	0.00	3.53	0.91	0.00
time (sec)	N/A	0.451	0.777	1.637	0.000	0.087	0.000	0.137	0.150	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	151	423	0	328	0	1501	177	0
N.S.	1	1.00	0.64	1.78	0.00	1.38	0.00	6.33	0.75	0.00
time (sec)	N/A	0.684	2.097	1.891	0.000	0.094	0.000	0.164	0.159	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	252	249	321	240	0	1125	1377	0
N.S.	1	1.00	0.99	0.98	1.26	0.94	0.00	4.43	5.42	0.00
time (sec)	N/A	0.834	0.525	2.803	0.158	0.094	0.000	0.144	0.155	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	105	100	137	103	0	305	40	0
N.S.	1	1.00	1.12	1.06	1.46	1.10	0.00	3.24	0.43	0.00
time (sec)	N/A	0.467	0.151	2.128	0.105	0.087	0.000	0.168	0.149	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	266	195	302	0	217	0	362	67	0
N.S.	1	1.03	0.75	1.17	0.00	0.84	0.00	1.40	0.26	0.00
time (sec)	N/A	0.895	0.457	2.832	0.000	0.094	0.000	0.177	0.152	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	201	263	292	0	271	0	686	0	0
N.S.	1	1.01	1.32	1.47	0.00	1.36	0.00	3.45	0.00	0.00
time (sec)	N/A	0.674	1.583	2.807	0.000	0.092	0.000	0.128	0.233	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	482	740	866	0	710	0	3078	0	0
N.S.	1	1.01	1.55	1.81	0.00	1.49	0.00	6.44	0.00	0.00
time (sec)	N/A	1.237	3.604	3.387	0.000	0.114	0.000	0.188	6.239	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	35	0	24	134	24
N.S.	1	1.00	1.09	1.00	1.09	1.59	0.00	1.09	6.09	1.09
time (sec)	N/A	0.196	1.282	0.618	2.974	0.081	0.000	0.596	0.152	39.689

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	24	15	22	82	22
N.S.	1	1.00	1.10	1.00	1.10	1.20	0.75	1.10	4.10	1.10
time (sec)	N/A	0.184	0.652	0.603	0.469	0.074	167.719	0.544	0.151	39.607

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	18	12	16	38	16
N.S.	1	1.00	1.14	1.00	1.14	1.29	0.86	1.14	2.71	1.14
time (sec)	N/A	0.168	0.042	0.338	0.198	0.076	27.141	0.483	0.146	40.035

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	24	15	22	82	22
N.S.	1	1.00	1.10	1.00	1.10	1.20	0.75	1.10	4.10	1.10
time (sec)	N/A	0.182	0.055	0.019	0.460	0.078	166.501	0.515	0.151	0.002

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	35	0	24	134	24
N.S.	1	1.00	1.09	1.00	1.09	1.59	0.00	1.09	6.09	1.09
time (sec)	N/A	0.194	0.061	0.017	2.766	0.109	0.000	0.562	0.151	0.002

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	1281	63	0	24	8294	24
N.S.	1	1.00	0.00	1.00	58.23	2.86	0.00	1.09	377.00	1.09
time (sec)	N/A	0.193	0.000	0.708	25.512	0.091	0.000	4.201	0.389	40.152

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1103	52	0	22	3980	22
N.S.	1	1.00	1.10	1.00	55.15	2.60	0.00	1.10	199.00	1.10
time (sec)	N/A	0.180	18.935	0.803	1.909	0.090	0.000	3.188	0.270	40.175

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	974	47	0	16	1167	16
N.S.	1	1.00	1.14	1.00	69.57	3.36	0.00	1.14	83.36	1.14
time (sec)	N/A	0.162	3.340	0.424	0.408	0.087	0.000	2.299	0.222	40.289

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1103	52	0	22	3980	22
N.S.	1	1.00	1.10	1.00	55.15	2.60	0.00	1.10	199.00	1.10
time (sec)	N/A	0.182	2.209	0.018	1.929	0.093	0.000	3.155	0.267	0.002

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	22	1281	63	0	24	8294	24
N.S.	1	1.00	0.00	1.00	58.23	2.86	0.00	1.09	377.00	1.09
time (sec)	N/A	0.195	0.000	0.019	24.272	0.094	0.000	4.273	0.364	0.002

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	26	0	24	836	24
N.S.	1	1.00	1.09	1.00	1.09	1.18	0.00	1.09	38.00	1.09
time (sec)	N/A	0.190	3.155	0.494	0.568	0.084	0.000	0.193	0.242	40.418

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	96	94	0	0	80	0	0	16	0
N.S.	1	0.83	0.82	0.00	0.00	0.70	0.00	0.00	0.14	0.00
time (sec)	N/A	0.504	0.220	0.000	0.000	0.087	0.000	0.000	0.158	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	83	47	151	146	74	129	0	54	109
N.S.	1	0.86	0.49	1.57	1.52	0.77	1.34	0.00	0.56	1.14
time (sec)	N/A	0.631	0.611	1.211	0.165	0.080	1.664	0.000	0.153	40.905

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	63	40	133	99	64	107	0	40	88
N.S.	1	0.85	0.54	1.80	1.34	0.86	1.45	0.00	0.54	1.19
time (sec)	N/A	0.503	0.502	1.227	0.192	0.080	1.018	0.000	0.140	40.194

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	30	117	60	55	70	0	24	63
N.S.	1	0.98	0.67	2.60	1.33	1.22	1.56	0.00	0.53	1.40
time (sec)	N/A	0.384	0.303	1.268	0.136	0.079	0.605	0.000	0.140	40.261

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	105	31	43	49	0	14	49
N.S.	1	1.00	1.00	4.20	1.24	1.72	1.96	0.00	0.56	1.96
time (sec)	N/A	0.244	0.113	1.786	0.115	0.078	0.394	0.000	0.146	41.002

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	36	36	106	42	57	0	0	16	0
N.S.	1	0.65	0.65	1.93	0.76	1.04	0.00	0.00	0.29	0.00
time (sec)	N/A	0.475	0.227	1.251	0.178	0.074	0.000	0.000	0.145	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	51	51	102	229	70	0	0	16	0
N.S.	1	0.66	0.66	1.32	2.97	0.91	0.00	0.00	0.21	0.00
time (sec)	N/A	0.562	0.487	1.289	0.166	0.092	0.000	0.000	0.146	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	70	69	123	256	90	0	0	43	0
N.S.	1	0.60	0.59	1.06	2.21	0.78	0.00	0.00	0.37	0.00
time (sec)	N/A	0.706	0.410	1.261	0.170	0.091	0.000	0.000	0.143	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	130	138	0	0	98	0	0	18	0
N.S.	1	0.85	0.90	0.00	0.00	0.64	0.00	0.00	0.12	0.00
time (sec)	N/A	0.511	0.639	0.000	0.000	0.079	0.000	0.000	0.157	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	38	135	32	67	85	0	31	71
N.S.	1	0.98	0.66	2.33	0.55	1.16	1.47	0.00	0.53	1.22
time (sec)	N/A	0.485	0.111	0.997	0.137	0.083	1.665	0.000	0.234	41.582

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	119	105	240	73	134	0	0	18	0
N.S.	1	0.77	0.68	1.55	0.47	0.86	0.00	0.00	0.12	0.00
time (sec)	N/A	0.506	0.567	1.002	0.147	0.098	0.000	0.000	0.236	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	119	16	51	63	0	16	53
N.S.	1	1.00	1.00	3.84	0.52	1.65	2.03	0.00	0.52	1.71
time (sec)	N/A	0.346	0.171	1.509	0.131	0.075	0.625	0.000	0.240	39.459

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	94	80	157	51	120	0	0	14	0
N.S.	1	0.80	0.68	1.34	0.44	1.03	0.00	0.00	0.12	0.00
time (sec)	N/A	0.340	0.173	0.953	0.138	0.082	0.000	0.000	0.149	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	50	47	128	47	67	0	0	18	0
N.S.	1	0.68	0.64	1.75	0.64	0.92	0.00	0.00	0.25	0.00
time (sec)	N/A	0.378	0.190	0.970	0.212	0.080	0.000	0.000	0.178	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	112	105	232	76	134	0	0	18	0
N.S.	1	0.83	0.78	1.72	0.56	0.99	0.00	0.00	0.13	0.00
time (sec)	N/A	0.484	0.477	0.990	0.218	0.087	0.000	0.000	0.166	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	64	67	119	52	86	0	0	18	0
N.S.	1	0.65	0.68	1.21	0.53	0.88	0.00	0.00	0.18	0.00
time (sec)	N/A	0.646	0.352	0.991	0.208	0.079	0.000	0.000	0.150	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	134	142	0	0	0	0	0	18	0
N.S.	1	0.85	0.90	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.581	0.796	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	120	129	0	0	0	0	0	18	0
N.S.	1	0.84	0.90	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.504	0.438	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	120	129	0	0	0	0	0	18	0
N.S.	1	0.84	0.90	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.487	0.425	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	120	129	0	0	0	0	0	16	0
N.S.	1	0.84	0.90	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.441	0.362	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	112	119	0	0	0	0	0	14	0
N.S.	1	0.83	0.88	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.321	0.176	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	50	47	131	144	70	0	0	18	0
N.S.	1	0.68	0.64	1.79	1.97	0.96	0.00	0.00	0.25	0.00
time (sec)	N/A	0.420	0.252	2.189	0.282	0.090	0.000	0.000	0.151	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	116	110	0	0	0	0	0	18	0
N.S.	1	0.83	0.79	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.438	0.343	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	120	114	0	0	0	0	0	18	0
N.S.	1	0.84	0.80	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.451	0.466	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	126	142	0	0	112	0	0	18	0
N.S.	1	0.75	0.84	0.00	0.00	0.66	0.00	0.00	0.11	0.00
time (sec)	N/A	0.539	0.787	0.000	0.000	0.093	0.000	0.000	0.256	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	121	79	208	286	111	0	0	87	0
N.S.	1	0.73	0.48	1.26	1.73	0.67	0.00	0.00	0.53	0.00
time (sec)	N/A	0.527	0.502	1.489	0.164	0.087	0.000	0.000	0.242	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	101	69	190	219	95	0	0	70	0
N.S.	1	0.73	0.50	1.37	1.58	0.68	0.00	0.00	0.50	0.00
time (sec)	N/A	0.464	0.441	1.497	0.153	0.082	0.000	0.000	0.212	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	66	55	174	162	82	0	0	41	0
N.S.	1	0.84	0.70	2.20	2.05	1.04	0.00	0.00	0.52	0.00
time (sec)	N/A	0.350	0.292	1.430	0.137	0.084	0.000	0.000	0.160	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	48	47	158	116	60	0	0	26	0
N.S.	1	0.87	0.85	2.87	2.11	1.09	0.00	0.00	0.47	0.00
time (sec)	N/A	0.249	0.144	1.951	0.123	0.082	0.000	0.000	0.145	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	99	56	50	119	52	63	0	0	18	0
N.S.	1	0.57	0.51	1.20	0.53	0.64	0.00	0.00	0.18	0.00
time (sec)	N/A	0.422	0.197	1.467	0.165	0.096	0.000	0.000	0.182	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	60	65	112	265	80	0	0	33	0
N.S.	1	0.70	0.76	1.30	3.08	0.93	0.00	0.00	0.38	0.00
time (sec)	N/A	0.584	0.252	1.608	0.193	0.086	0.000	0.000	0.150	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	101	85	137	296	104	0	0	527	0
N.S.	1	0.85	0.71	1.15	2.49	0.87	0.00	0.00	4.43	0.00
time (sec)	N/A	0.570	0.402	1.497	0.237	0.095	0.000	0.000	0.155	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	158	189	0	0	130	0	0	65	0
N.S.	1	0.76	0.90	0.00	0.00	0.62	0.00	0.00	0.31	0.00
time (sec)	N/A	0.530	1.298	0.000	0.000	0.088	0.000	0.000	0.160	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	81	67	200	47	96	0	0	20	0
N.S.	1	0.89	0.74	2.20	0.52	1.05	0.00	0.00	0.22	0.00
time (sec)	N/A	0.435	0.489	1.308	0.134	0.088	0.000	0.000	0.188	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	118	113	309	99	149	0	0	20	0
N.S.	1	0.61	0.58	1.58	0.51	0.76	0.00	0.00	0.10	0.00
time (sec)	N/A	0.423	0.441	1.319	0.137	0.094	0.000	0.000	0.155	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	61	55	182	28	72	0	0	32	0
N.S.	1	0.94	0.85	2.80	0.43	1.11	0.00	0.00	0.49	0.00
time (sec)	N/A	0.387	0.200	1.793	0.202	0.086	0.000	0.000	0.162	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	97	93	224	76	119	0	0	16	0
N.S.	1	0.66	0.63	1.51	0.51	0.80	0.00	0.00	0.11	0.00
time (sec)	N/A	0.295	0.123	1.268	0.140	0.091	0.000	0.000	0.152	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	115	64	60	145	55	73	0	0	20	0
N.S.	1	0.56	0.52	1.26	0.48	0.63	0.00	0.00	0.17	0.00
time (sec)	N/A	0.378	0.188	1.318	0.214	0.082	0.000	0.000	0.147	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	111	107	301	90	133	0	0	20	0
N.S.	1	0.84	0.81	2.28	0.68	1.01	0.00	0.00	0.15	0.00
time (sec)	N/A	0.576	0.316	1.286	0.209	0.092	0.000	0.000	0.158	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	84	79	134	64	100	0	0	39	0
N.S.	1	0.52	0.49	0.83	0.40	0.62	0.00	0.00	0.24	0.00
time (sec)	N/A	0.447	0.280	1.286	0.299	0.098	0.000	0.000	0.161	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	166	194	0	0	0	0	0	20	0
N.S.	1	0.76	0.89	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.591	1.245	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	137	161	0	0	0	0	0	20	0
N.S.	1	0.73	0.86	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.523	0.802	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	137	168	0	0	0	0	0	20	0
N.S.	1	0.73	0.89	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.505	0.780	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	137	160	0	0	0	0	0	18	0
N.S.	1	0.73	0.85	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.474	0.770	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	127	149	0	0	0	0	0	16	0
N.S.	1	0.71	0.84	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.340	0.288	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	121	70	63	147	153	78	0	0	20	0
N.S.	1	0.58	0.52	1.21	1.26	0.64	0.00	0.00	0.17	0.00
time (sec)	N/A	0.390	0.286	2.283	0.278	0.085	0.000	0.000	0.236	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	129	125	0	0	0	0	0	20	0
N.S.	1	0.72	0.69	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.494	0.634	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	133	129	0	0	0	0	0	20	0
N.S.	1	0.72	0.70	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.499	0.596	0.000	0.000	0.000	0.000	0.000	0.157	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [128] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	16	0.125
2	A	2	2	1.00	16	0.125
3	A	2	2	1.00	14	0.143
4	A	2	2	1.00	16	0.125
5	A	2	2	1.00	16	0.125
6	A	2	2	1.00	16	0.125
7	A	2	2	1.00	16	0.125
8	A	2	2	1.00	16	0.125
9	A	1	1	1.00	12	0.083
10	A	2	2	1.00	16	0.125
11	A	2	2	1.00	16	0.125
12	A	5	4	1.02	18	0.222
13	A	5	4	1.05	18	0.222
14	A	4	3	1.07	16	0.188
15	A	3	3	1.00	18	0.167
16	A	3	3	1.00	18	0.167
17	A	3	3	1.00	18	0.167
18	A	3	3	1.00	18	0.167
19	A	3	3	1.00	18	0.167
20	A	2	2	1.00	14	0.143
21	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	18	0.167
23	A	13	12	1.11	14	0.857
24	A	8	7	1.05	14	0.500
25	A	5	4	0.70	12	0.333
26	A	2	2	1.00	14	0.143
27	A	2	2	1.00	14	0.143
28	A	2	2	1.00	14	0.143
29	A	2	2	1.00	10	0.200
30	A	3	3	1.02	14	0.214
31	A	2	2	1.00	10	0.200
32	A	3	3	1.06	14	0.214
33	A	5	4	0.52	12	0.333
34	A	2	2	1.00	12	0.167
35	A	10	9	0.94	18	0.500
36	A	9	8	0.97	18	0.444
37	A	6	5	1.12	16	0.312
38	N/A	1	0	1.00	18	0.000
39	N/A	1	0	1.00	18	0.000
40	N/A	1	0	1.00	18	0.000
41	N/A	1	0	1.00	14	0.000
42	N/A	1	0	1.00	18	0.000
43	A	16	15	0.94	18	0.833
44	A	13	12	1.01	18	0.667
45	A	10	9	1.09	16	0.562
46	N/A	1	0	1.00	18	0.000
47	N/A	1	0	1.00	18	0.000
48	N/A	1	0	1.00	18	0.000
49	N/A	1	0	1.00	14	0.000
50	N/A	1	0	1.00	18	0.000
51	N/A	1	0	1.00	20	0.000
52	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	3	3	1.00	20	0.150
54	A	2	2	1.00	18	0.111
55	N/A	1	0	1.00	20	0.000
56	N/A	1	0	1.00	20	0.000
57	A	2	2	1.00	16	0.125
58	A	2	2	1.00	16	0.125
59	A	2	2	1.00	16	0.125
60	A	2	2	1.00	16	0.125
61	A	2	2	1.00	16	0.125
62	A	2	2	1.00	14	0.143
63	A	2	2	1.00	16	0.125
64	A	2	2	1.00	16	0.125
65	A	2	2	1.00	16	0.125
66	A	1	1	1.00	12	0.083
67	A	2	2	1.00	16	0.125
68	A	2	2	1.00	16	0.125
69	A	5	4	1.00	18	0.222
70	A	4	3	1.03	18	0.167
71	A	3	3	1.00	18	0.167
72	A	3	3	1.00	18	0.167
73	A	3	3	1.00	18	0.167
74	A	3	3	1.00	16	0.188
75	A	3	3	0.99	18	0.167
76	A	3	3	0.99	18	0.167
77	A	3	3	1.00	18	0.167
78	A	2	2	1.00	14	0.143
79	A	3	3	0.99	18	0.167
80	A	3	3	0.99	18	0.167
81	A	9	8	0.98	18	0.444
82	A	6	5	1.12	18	0.278
83	N/A	1	0	1.00	18	0.000
84	N/A	1	0	1.00	18	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	N/A	1	0	1.00	16	0.000
86	N/A	1	0	1.00	18	0.000
87	N/A	1	0	1.00	14	0.000
88	N/A	1	0	1.00	18	0.000
89	A	13	12	1.01	18	0.667
90	A	10	9	1.05	18	0.500
91	N/A	1	0	1.00	18	0.000
92	N/A	1	0	1.00	18	0.000
93	N/A	1	0	1.00	16	0.000
94	N/A	1	0	1.00	18	0.000
95	N/A	1	0	1.00	14	0.000
96	N/A	1	0	1.00	18	0.000
97	N/A	1	0	1.00	20	0.000
98	A	4	4	1.00	20	0.200
99	A	3	3	1.00	20	0.150
100	A	2	2	1.00	18	0.111
101	N/A	1	0	1.00	20	0.000
102	N/A	1	0	1.00	20	0.000
103	A	14	13	0.97	12	1.083
104	A	12	11	0.92	10	1.100
105	A	9	8	1.06	8	1.000
106	A	3	3	1.00	12	0.250
107	A	4	3	1.00	12	0.250
108	A	6	5	1.00	12	0.417
109	A	9	8	1.09	12	0.667
110	A	11	10	1.16	12	0.833
111	A	2	2	0.97	14	0.143
112	A	12	11	1.02	12	0.917
113	A	10	9	1.00	10	0.900
114	A	2	2	1.00	14	0.143
115	A	5	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	5	4	1.00	14	0.286
117	A	8	7	1.06	14	0.500
118	A	8	7	1.05	14	0.500
119	A	6	5	1.10	8	0.625
120	A	3	3	1.00	12	0.250
121	A	5	4	1.00	12	0.333
122	A	4	3	1.00	12	0.250
123	A	6	5	1.03	12	0.417
124	A	4	3	1.00	12	0.250
125	A	5	4	0.86	14	0.286
126	A	6	5	1.00	6	0.833
127	A	8	7	1.10	8	0.875
128	A	13	12	1.13	8	1.500
129	N/A	1	0	1.00	18	0.000
130	N/A	1	0	1.00	20	0.000
131	A	5	4	1.00	20	0.200
132	N/A	2	0	1.00	22	0.000
133	A	7	6	1.00	22	0.273
134	N/A	2	0	1.00	24	0.000
135	A	3	3	1.00	12	0.250
136	A	2	2	1.00	14	0.143
137	A	2	2	1.00	14	0.143
138	A	2	2	1.00	14	0.143
139	A	2	2	1.00	8	0.250
140	A	2	2	1.00	10	0.200
141	A	2	2	1.00	10	0.200
142	A	2	2	1.00	12	0.167
143	A	2	2	1.00	14	0.143
144	A	2	2	1.00	14	0.143
145	A	6	5	0.94	16	0.312
146	A	7	6	0.94	16	0.375
147	A	9	8	0.91	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	2	2	0.99	18	0.111
149	A	2	2	0.99	18	0.111
150	A	12	11	0.83	16	0.688
151	A	2	2	0.98	18	0.111
152	A	2	2	0.99	18	0.111
153	A	3	2	0.94	18	0.111
154	A	3	2	0.95	18	0.111
155	A	3	2	0.97	16	0.125
156	A	1	1	1.00	10	0.100
157	N/A	1	0	1.00	18	0.000
158	N/A	1	0	1.00	18	0.000
159	A	3	2	0.92	18	0.111
160	A	3	2	0.93	18	0.111
161	A	3	2	0.93	16	0.125
162	A	4	3	0.98	10	0.300
163	N/A	1	0	1.00	18	0.000
164	N/A	1	0	1.00	18	0.000
165	A	3	2	0.94	20	0.100
166	A	3	2	0.95	20	0.100
167	A	3	2	0.96	18	0.111
168	A	3	3	1.00	12	0.250
169	N/A	1	0	1.00	20	0.000
170	N/A	1	0	1.00	20	0.000
171	A	3	2	0.95	20	0.100
172	A	3	2	0.96	20	0.100
173	A	3	2	0.97	18	0.111
174	A	2	2	1.00	12	0.167
175	N/A	1	0	1.00	20	0.000
176	N/A	1	0	1.00	20	0.000
177	A	3	2	0.93	20	0.100
178	A	3	2	0.93	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	A	6	5	1.05	12	0.417
180	N/A	1	0	1.00	20	0.000
181	N/A	1	0	1.00	20	0.000
182	A	3	2	0.95	20	0.100
183	A	3	2	0.97	18	0.111
184	A	2	2	1.00	12	0.167
185	N/A	1	0	1.00	20	0.000
186	N/A	1	0	1.00	20	0.000
187	A	3	2	1.01	22	0.091
188	A	3	2	1.02	20	0.100
189	A	6	5	0.96	14	0.357
190	A	3	2	1.08	22	0.091
191	A	6	5	1.05	22	0.227
192	A	3	2	0.96	22	0.091
193	A	3	2	0.97	20	0.100
194	A	4	3	0.99	14	0.214
195	N/A	1	0	1.00	22	0.000
196	N/A	1	0	1.00	22	0.000
197	A	3	2	1.03	22	0.091
198	A	3	2	1.06	20	0.100
199	A	12	11	1.00	14	0.786
200	A	3	2	1.07	22	0.091
201	A	6	5	1.06	22	0.227
202	A	3	2	0.95	22	0.091
203	A	3	2	0.97	20	0.100
204	A	4	3	0.99	14	0.214
205	N/A	1	0	1.00	22	0.000
206	N/A	1	0	1.00	22	0.000
207	A	3	2	1.01	22	0.091
208	A	3	2	1.02	20	0.100
209	A	9	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
210	A	3	2	1.07	22	0.091
211	A	6	5	1.02	22	0.227
212	A	3	2	0.94	22	0.091
213	A	3	2	0.94	20	0.100
214	A	6	5	0.99	14	0.357
215	N/A	1	0	1.00	22	0.000
216	N/A	1	0	1.00	22	0.000
217	A	3	2	1.01	22	0.091
218	A	3	2	1.02	20	0.100
219	A	14	13	0.93	14	0.929
220	A	3	2	1.06	22	0.091
221	A	6	5	1.03	22	0.227
222	A	3	2	0.95	22	0.091
223	A	3	2	0.96	20	0.100
224	A	8	7	1.06	14	0.500
225	N/A	1	0	1.00	22	0.000
226	N/A	1	0	1.00	22	0.000
227	A	20	19	0.81	27	0.704
228	A	15	14	0.83	27	0.519
229	A	12	11	0.84	27	0.407
230	A	7	6	0.85	27	0.222
231	A	5	4	1.00	27	0.148
232	A	10	9	0.73	27	0.333
233	A	13	12	0.69	27	0.444
234	A	18	17	0.69	27	0.630
235	A	10	9	0.83	27	0.333
236	A	9	8	0.80	27	0.296
237	A	8	7	0.83	27	0.259
238	A	6	5	1.00	27	0.185
239	A	7	6	0.83	27	0.222
240	A	8	7	0.85	27	0.259
241	A	11	10	0.71	27	0.370

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
242	A	18	17	0.73	27	0.630
243	A	13	12	0.70	27	0.444
244	A	10	9	0.73	27	0.333
245	A	5	4	1.00	27	0.148
246	A	7	6	0.82	27	0.222
247	A	12	11	0.81	27	0.407
248	A	15	14	0.80	27	0.519
249	A	12	11	0.79	27	0.407
250	A	11	10	0.78	27	0.370
251	A	14	13	0.71	27	0.481
252	A	11	10	0.71	27	0.370
253	A	9	8	0.85	27	0.296
254	A	7	6	0.80	27	0.222
255	A	6	5	1.00	27	0.185
256	A	8	7	0.81	27	0.259
257	A	10	9	0.78	27	0.333
258	A	11	10	0.81	27	0.370
259	N/A	1	0	1.00	18	0.000
260	A	3	2	0.96	16	0.125
261	A	3	2	0.96	16	0.125
262	A	3	2	0.97	14	0.143
263	A	2	2	1.00	12	0.167
264	N/A	1	0	1.00	16	0.000
265	N/A	1	0	1.00	16	0.000
266	A	2	2	1.00	20	0.100
267	A	2	2	1.00	20	0.100
268	A	2	2	1.00	18	0.111
269	A	1	1	1.00	16	0.062
270	N/A	2	0	1.00	20	0.000
271	N/A	2	0	1.00	20	0.000
272	A	3	2	0.96	22	0.091
273	A	3	2	0.96	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
274	A	2	2	1.00	18	0.111
275	N/A	1	0	1.00	22	0.000
276	N/A	1	0	1.00	22	0.000
277	N/A	1	0	1.00	22	0.000
278	N/A	1	0	1.00	20	0.000
279	N/A	1	0	1.00	18	0.000
280	N/A	1	0	1.00	22	0.000
281	N/A	1	0	1.00	22	0.000
282	N/A	1	0	1.00	22	0.000
283	N/A	1	0	1.00	20	0.000
284	N/A	1	0	1.00	18	0.000
285	N/A	1	0	1.00	22	0.000
286	N/A	1	0	1.00	22	0.000
287	N/A	1	0	1.00	24	0.000
288	A	3	2	1.00	20	0.100
289	A	3	2	1.00	18	0.111
290	A	1	1	1.00	12	0.083
291	A	3	2	1.03	20	0.100
292	A	5	4	1.00	20	0.200
293	A	3	2	1.00	20	0.100
294	A	3	2	1.00	20	0.100
295	A	5	4	1.00	14	0.286
296	A	3	2	1.03	22	0.091
297	A	5	4	1.01	22	0.182
298	A	3	2	1.01	22	0.091
299	N/A	1	0	1.00	22	0.000
300	N/A	1	0	1.00	20	0.000
301	N/A	1	0	1.00	14	0.000
302	N/A	1	0	1.00	20	0.000
303	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	N/A	1	0	1.00	22	0.000
305	N/A	1	0	1.00	20	0.000
306	N/A	1	0	1.00	14	0.000
307	N/A	1	0	1.00	20	0.000
308	N/A	1	0	1.00	22	0.000
309	N/A	1	0	1.00	22	0.000
310	A	4	4	0.83	18	0.222
311	A	10	10	0.86	18	0.556
312	A	8	8	0.85	18	0.444
313	A	5	5	0.98	16	0.312
314	A	4	4	1.00	14	0.286
315	A	6	6	0.65	18	0.333
316	A	8	8	0.66	18	0.444
317	A	11	11	0.60	18	0.611
318	A	3	3	0.85	20	0.150
319	A	7	6	0.98	20	0.300
320	A	5	5	0.77	20	0.250
321	A	6	5	1.00	18	0.278
322	A	4	4	0.80	16	0.250
323	A	4	4	0.68	20	0.200
324	A	5	5	0.83	20	0.250
325	A	10	9	0.65	20	0.450
326	A	3	3	0.85	20	0.150
327	A	3	3	0.84	20	0.150
328	A	3	3	0.84	20	0.150
329	A	3	3	0.84	18	0.167
330	A	3	3	0.83	16	0.188
331	A	4	4	0.68	20	0.200
332	A	3	3	0.83	20	0.150
333	A	3	3	0.84	20	0.150
334	A	4	4	0.75	18	0.222
335	A	7	7	0.73	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	7	7	0.73	18	0.389
337	A	4	4	0.84	16	0.250
338	A	5	5	0.87	14	0.357
339	A	4	4	0.57	18	0.222
340	A	9	9	0.70	18	0.500
341	A	7	7	0.85	18	0.389
342	A	3	3	0.76	20	0.150
343	A	6	5	0.89	20	0.250
344	A	3	3	0.61	20	0.150
345	A	7	6	0.94	18	0.333
346	A	3	3	0.66	16	0.188
347	A	3	3	0.56	20	0.150
348	A	7	7	0.84	20	0.350
349	A	3	3	0.52	20	0.150
350	A	3	3	0.76	20	0.150
351	A	3	3	0.73	20	0.150
352	A	3	3	0.73	20	0.150
353	A	3	3	0.73	18	0.167
354	A	3	3	0.71	16	0.188
355	A	3	3	0.58	20	0.150
356	A	3	3	0.72	20	0.150
357	A	3	3	0.72	20	0.150

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}+\frac{b\cos\left(c+dx^2\right)}{d^3}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}$$

output

```
1/6*a*x^6+b*cos(d*x^2+c)/d^3-1/2*b*x^4*cos(d*x^2+c)/d+b*x^2*sin(d*x^2+c)/d
^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ad^3x^6-3b(-2+d^2x^4)\cos\left(c+dx^2\right)+6bdx^2\sin\left(c+dx^2\right)}{6d^3}$$

input

```
Integrate[x^5*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*d^3*x^6 - 3*b*(-2 + d^2*x^4)*Cos[c + d*x^2] + 6*b*d*x^2*Sin[c + d*x^2])
/(6*d^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \sin \left(c+d x^2\right)\right) \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(a x^5+b x^5 \sin \left(c+d x^2\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^6}{6}+\frac{b \cos \left(c+d x^2\right)}{d^3}+\frac{b x^2 \sin \left(c+d x^2\right)}{d^2}-\frac{b x^4 \cos \left(c+d x^2\right)}{2 d}$$

input

```
Int[x^5*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^6)/6 + (b*Cos[c + d*x^2])/d^3 - (b*x^4*Cos[c + d*x^2])/(2*d) + (b*x^2
*Sin[c + d*x^2])/d^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result
risch	$\frac{x^{6} a}{6}-\frac{b \left(x^{4} d^{2}-2\right) \cos \left(d\,x^{2}+c \right)}{2 d^{3}}+\frac{b\,x^{2} \sin \left(d\,x^{2}+c \right)}{d^{2}}$
parallelrisch	$\frac{\left(-3 x^{4} d^{2}+6\right) b \cos \left(d\,x^{2}+c \right)+x^{6} a\,d^{3}+6 b\,x^{2} \sin \left(d\,x^{2}+c \right) d -6 b}{6 d^{3}}$
default	$\frac{x^{6} a}{6}+b \left(-\frac{x^{4} \cos \left(d\,x^{2}+c \right)}{2 d}+\frac{\frac{x^{2} \sin \left(d\,x^{2}+c \right)}{d}+\frac{\cos \left(d\,x^{2}+c \right)}{d^{2}}}{d}\right)$
parts	$\frac{x^{6} a}{6}+b \left(-\frac{x^{4} \cos \left(d\,x^{2}+c \right)}{2 d}+\frac{\frac{x^{2} \sin \left(d\,x^{2}+c \right)}{d}+\frac{\cos \left(d\,x^{2}+c \right)}{d^{2}}}{d}\right)$
norman	$\frac{\frac{2 b}{d^{3}}+\frac{x^{6} a}{6}-\frac{b\,x^{4}}{2 d}+\frac{x^{6} a \tan \left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)^{2}}{6}+\frac{2 b\,x^{2} \tan \left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}{d^{2}}+\frac{b\,x^{4} \tan \left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)^{2}}{2 d}}{1+\tan \left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{\left(4 x^{8} d^{4}+65 x^{4} d^{2}-270\right) \left(a +b \sin \left(d\,x^{2}+c \right)\right)}{24 x^{2} d^{4}}-\frac{\left(17 x^{4} d^{2}-78\right) \left(5 x^{4} \left(a +b \sin \left(d\,x^{2}+c \right)\right)+2 x^{6} b d \cos \left(d\,x^{2}+c \right)\right)}{24 x^{6} d^{4}}+\frac{(x^{4} d^{2}-6) (20x}{}$

input
```
int(x^5*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a-1/2*b*(d^2*x^4-2)/d^3*cos(d*x^2+c)+b*x^2*sin(d*x^2+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ad^3x^6+6\,bdx^2\sin\left(dx^2+c\right)-3\left(bd^2x^4-2\,b\right)\cos\left(dx^2+c\right)}{6\,d^3}$$

input
```
integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/6*(a*d^3*x^6 + 6*b*d*x^2*sin(d*x^2 + c) - 3*(b*d^2*x^4 - 2*b)*cos(d*x^2
 + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int x^5 \left(a+b\sin\left(c+dx^2\right)\right)\,dx = \begin{cases} \frac{ax^6}{6} - \frac{bx^4\cos\left(c+dx^2\right)}{2d} + \frac{bx^2\sin\left(c+dx^2\right)}{d^2} + \frac{b\cos\left(c+dx^2\right)}{d^3} & \text{for } d \neq 0 \\ \frac{x^6(a+b\sin(c))}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(a+b*sin(d*x**2+c)),x)
```

output
```
Piecewise((a*x**6/6 - b*x**4*cos(c + d*x**2)/(2*d) + b*x**2*sin(c + d*x**2
)/d**2 + b*cos(c + d*x**2)/d**3, Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int x^5 \left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{1}{6}\,ax^6 + \frac{\left(2\,dx^2\sin\left(dx^2+c\right) - \left(d^2x^4-2\right)\cos\left(dx^2+c\right)\right)b}{2\,d^3}$$

input
```
integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/2*(2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*b/
d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.25

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=-\frac{\left((dx^2+c)^2b-2\,(dx^2+c)bc-2\,b\right)\cos\left(dx^2+c\right)}{2\,d^3}+\frac{((dx^2+c)b-bc)\sin\left(dx^2+c\right)}{d^3}+\frac{(dx^2+c)^3a-3\,(dx^2+c)^2ac}{6\,d^3}+\frac{(dx^2+c)ac^2-bc^2\cos\left(dx^2+c\right)}{2\,d^3}$$

input
```
integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
-1/2*((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c - 2*b)*cos(d*x^2 + c)/d^3 + ((d*
x^2 + c)*b - b*c)*sin(d*x^2 + c)/d^3 + 1/6*((d*x^2 + c)^3*a - 3*(d*x^2 + c
)^2*a*c)/d^3 + 1/2*((d*x^2 + c)*a*c^2 - b*c^2*cos(d*x^2 + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{a\,x^6}{6}+\frac{b\,\cos\left(d\,x^2+c\right)-\frac{b\,d^2\,x^4\,\cos\left(d\,x^2+c\right)}{2}+b\,d\,x^2\,\sin\left(d\,x^2+c\right)}{d^3}$$

input
```
int(x^5*(a + b*sin(c + d*x^2)),x)
```

output
```
(a*x^6)/6 + (b*cos(c + d*x^2) - (b*d^2*x^4*cos(c + d*x^2))/2 + b*d*x^2*sin
(c + d*x^2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{-3\cos\left(d\,x^2+c\right)b\,d^2x^4+6\cos\left(d\,x^2+c\right)b+6\sin\left(d\,x^2+c\right)bd\,x^2+a\,d^3x^6}{6d^3}$$

input

```
int(x^5*(a+b*sin(d*x^2+c)),x)
```

output

```
( - 3*cos(c + d*x**2)*b*d**2*x**4 + 6*cos(c + d*x**2)*b + 6*sin(c + d*x**2
)*b*d*x**2 + a*d**3*x**6)/(6*d**3)
```

3.2 $\int x^3(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2}$$

output
```
1/4*a*x^4-1/2*b*x^2*cos(d*x^2+c)/d+1/2*b*sin(d*x^2+c)/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2}$$

input
```
Integrate[x^3*(a + b*Sin[c + d*x^2]),x]
```

output
```
(a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx \arrow[d, "2010"] \\
\int \left(ax^3+bx^3\sin\left(c+dx^2\right)\right)dx \arrow[d, "2009"] \\
\frac{ax^4}{4}+\frac{b\sin\left(c+dx^2\right)}{2d^2}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}
\end{tikzcd}$$

input

```
Int[x^3*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result
risch	$\frac{a\,x^{4}}{4}-\frac{b\,x^{2}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{b\sin\left(d\,x^{2}+c\right)}{2d^{2}}$
default	$\frac{a\,x^{4}}{4}+b\left(-\frac{x^{2}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\sin\left(d\,x^{2}+c\right)}{2d^{2}}\right)$
parts	$\frac{a\,x^{4}}{4}+b\left(-\frac{x^{2}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\sin\left(d\,x^{2}+c\right)}{2d^{2}}\right)$
parallelrisch	$\frac{a\,x^{4}d^{2}-2x^{2}bd\cos\left(d\,x^{2}+c\right)+2b\sin\left(d\,x^{2}+c\right)}{4d^{2}}$
norman	$\frac{\frac{b\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}{d^{2}}+\frac{a\,x^{4}}{4}+\frac{a\,x^{4}\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)^{2}}{4}-\frac{b\,x^{2}}{2d}+\frac{b\,x^{2}\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)^{2}}{2d}}{1+\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{(4x^{4}d^{2}+35)(a+b\sin(d\,x^{2}+c))}{16d^{2}}-\frac{11(3x^{2}(a+b\sin(d\,x^{2}+c))+2x^{4}bd\cos(d\,x^{2}+c))}{16x^{2}d^{2}}+\frac{6x(a+b\sin(d\,x^{2}+c))+14x^{3}bd\cos(d}{16d^{2}x}$

input
```
int(x^3*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4-1/2*b*x^2*cos(d*x^2+c)/d+1/2*b*sin(d*x^2+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^{3}\left(a+b\sin\left(c+dx^{2}\right)\right)\,dx=\frac{ad^{2}x^{4}-2\,bdx^{2}\cos\left(dx^{2}+c\right)+2\,b\sin\left(dx^{2}+c\right)}{4\,d^{2}}$$

input
```
integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/4*(a*d^2*x^4 - 2*b*d*x^2*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2} & \text{for } d\neq 0\\ \frac{x^4(a+b\sin\left(c\right))}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(a+b*sin(d*x**2+c)),x)
```

output
```
Piecewise((a*x**4/4 - b*x**2*cos(c + d*x**2)/(2*d) + b*sin(c + d*x**2)/(2*
d**2), Ne(d, 0)), (x**4*(a + b*sin(c))/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{4}\,ax^4-\frac{(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right))b}{2\,d^2}$$

input
```
integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 - 1/2*(d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*b/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx$$
$$=\frac{\left(dx^2+c\right)^2a-2\,(dx^2+c)b\cos\left(dx^2+c\right)+2\,b\sin\left(dx^2+c\right)}{4\,d^2}-\frac{(dx^2+c)ac-bc\cos\left(dx^2+c\right)}{2\,d^2}$$

input
```
integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output

```
1/4*((d*x^2 + c)^2*a - 2*(d*x^2 + c)*b*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c)
)/d^2 - 1/2*((d*x^2 + c)*a*c - b*c*cos(d*x^2 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{a\,x^4}{4}+\frac{\frac{b\,\sin\left(d\,x^2+c\right)}{2}-\frac{b\,d\,x^2\,\cos\left(d\,x^2+c\right)}{2}}{d^2}$$

input

```
int(x^3*(a + b*sin(c + d*x^2)),x)
```

output

```
(a*x^4)/4 + ((b*sin(c + d*x^2))/2 - (b*d*x^2*cos(c + d*x^2))/2)/d^2
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{-2\cos\left(d\,x^2+c\right)bd\,x^2+2\sin\left(d\,x^2+c\right)b+a\,d^2x^4}{4d^2}$$

input

```
int(x^3*(a+b*sin(d*x^2+c)),x)
```

output

```
( - 2*cos(c + d*x**2)*b*d*x**2 + 2*sin(c + d*x**2)*b + a*d**2*x**4)/(4*d**
2)
```

3.3 $\int x(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}$$

output

```
1/2*a*x^2-1/2*b*cos(d*x^2+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\cos(c)\cos\left(dx^2\right)}{2d}+\frac{b\sin(c)\sin\left(dx^2\right)}{2d}$$

input

```
Integrate[x*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^2)/2 - (b*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sin[c]*Sin[d*x^2])/(2*d)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax+bx\sin\left(c+dx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}$$

input

```
Int[x*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^2)/2 - (b*Cos[c + d*x^2])/(2*d)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result
risch	$\frac{a\,x^2}{2}-\frac{b\cos(d\,x^2+c)}{2d}$
parts	$\frac{a\,x^2}{2}-\frac{b\cos(d\,x^2+c)}{2d}$
parallelrisch	$\frac{a\,x^2d-b\cos(d\,x^2+c)+b}{2d}$
derivativedivides	$\frac{(d\,x^2+c)a-b\cos(d\,x^2+c)}{2d}$
default	$\frac{(d\,x^2+c)a-b\cos(d\,x^2+c)}{2d}$
norman	$\frac{\frac{b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2}{d}+\frac{a\,x^2}{2}+\frac{a\,x^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2}{2}}{1+\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2}$
orering	$\frac{(4x^4d^2+5)(a+b\sin(d\,x^2+c))}{8x^2d^2}-\frac{5(a+b\sin(d\,x^2+c)+2x^2bd\cos(d\,x^2+c))}{8x^2d^2}+\frac{6bdx\cos(d\,x^2+c)-4x^3b\,d^2\sin(d\,x^2+c}{8d^2x}$

input
```
int(x*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2-1/2*b*cos(d*x^2+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{adx^2-b\cos\left(dx^2+c\right)}{2\,d}$$

input
```
integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x^2 - b*cos(d*x^2 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d} & \text{for } d\neq 0\\ \frac{x^2(a+b\sin\left(c\right))}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*sin(d*x**2+c)),x)
```

output
```
Piecewise((a*x**2/2 - b*cos(c + d*x**2)/(2*d), Ne(d, 0)), (x**2*(a + b*sin
(c))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{2}\,ax^2-\frac{b\cos\left(dx^2+c\right)}{2\,d}$$

input
```
integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 - 1/2*b*cos(d*x^2 + c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{(dx^2+c)a-b\cos\left(dx^2+c\right)}{2\,d}$$

input
```
integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
1/2*((d*x^2 + c)*a - b*cos(d*x^2 + c))/d
```

Mupad [B] (verification not implemented)

Time = 42.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\cos\left(d\,x^2+c\right)}{2\,d}$$

input

```
int(x*(a + b*sin(c + d*x^2)),x)
```

output

```
(a*x^2)/2 - (b*cos(c + d*x^2))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{-\cos\left(d\,x^2+c\right)b+ad\,x^2}{2d}$$

input

```
int(x*(a+b*sin(d*x^2+c)),x)
```

output

```
( - cos(c + d*x**2)*b + a*d*x**2)/(2*d)
```

3.4 $\int \frac{a+b\sin(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx = a\log(x)+\frac{1}{2}b\,\text{CosIntegral}\left(dx^2\right)\sin(c)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)$$

output
```
a*ln(x)+1/2*b*Ci(d*x^2)*sin(c)+1/2*b*cos(c)*Si(d*x^2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx = a\log(x)+\frac{1}{2}b\left(\text{CosIntegral}\left(dx^2\right)\sin(c)+\cos(c)\text{Si}\left(dx^2\right)\right)$$

input
```
Integrate[(a + b*Sin[c + d*x^2])/x,x]
```

output
```
a*Log[x] + (b*(CosIntegral[d*x^2]*Sin[c] + Cos[c]*SinIntegral[d*x^2]))/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x}+\frac{b\sin\left(c+dx^2\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+\frac{1}{2}b\sin(c)\,\text{CosIntegral}\left(dx^2\right)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)$$

input
```
Int[(a + b*Sin[c + d*x^2])/x,x]
```

output
```
a*Log[x] + (b*CosIntegral[d*x^2]*Sin[c])/2 + (b*Cos[c]*SinIntegral[d*x^2])
/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$a \ln \left(x \right)+b \left(\frac{\cos \left(c \right) \operatorname{Si}\left(d \,x^{2}\right)}{2}+\frac{\sin \left(c \right) \operatorname{Ci}\left(d \,x^{2}\right)}{2}\right)$	29
parts	$a \ln \left(x \right)+b \left(\frac{\cos \left(c \right) \operatorname{Si}\left(d \,x^{2}\right)}{2}+\frac{\sin \left(c \right) \operatorname{Ci}\left(d \,x^{2}\right)}{2}\right)$	29
risch	$a \ln \left(x \right)-\frac{\operatorname{csgn}\left(d \,x^{2}\right) {\mathrm e}^{-i c} \pi b}{4}+\frac{\operatorname{Si}\left(d \,x^{2}\right) {\mathrm e}^{-i c} b}{2}-\frac{i {\mathrm e}^{-i c} \operatorname{expIntegral}_{1}\left(-i d \,x^{2}\right) b}{4}+\frac{i b \,{\mathrm e}^{i c} \operatorname{expIntegral}_{1}\left(-i d \,x^{2}\right)}{4}$	71

input

```
int((a+b*sin(d*x^2+c))/x,x,method=_RETURNVERBOSE)
```

output

```
a*ln(x)+b*(1/2*cos(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx=\frac{1}{2}\,b\operatorname{Ci}\left(dx^2\right)\sin\left(c\right)+\frac{1}{2}\,b\cos\left(c\right)\operatorname{Si}\left(dx^2\right)+a\log\left(x\right)$$

input

```
integrate((a+b*sin(d*x^2+c))/x,x, algorithm="fricas")
```

output

```
1/2*b*cos_integral(d*x^2)*sin(c) + 1/2*b*cos(c)*sin_integral(d*x^2) + a*lo
g(x)
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx=\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx$$

input

```
integrate((a+b*sin(d*x**2+c))/x,x)
```

output
```
Integral((a + b*sin(c + d*x**2))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = -\frac{1}{4}\left(\left(i\operatorname{Ei}\left(i\,dx^2\right)-i\operatorname{Ei}\left(-i\,dx^2\right)\right)\cos(c)-\left(\operatorname{Ei}\left(i\,dx^2\right)+\operatorname{Ei}\left(-i\,dx^2\right)\right)\sin(c)\right)b + a\log(x)$$

input
```
integrate((a+b*sin(d*x^2+c))/x,x, algorithm="maxima")
```

output
```
-1/4*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2
))*sin(c))*b + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sin(c+dx^2)}{x}\,dx = \frac{1}{2}\,b\operatorname{Ci}\left(dx^2\right)\sin(c)+\frac{1}{2}\,b\cos(c)\operatorname{Si}\left(dx^2\right)+\frac{1}{2}\,a\log\left(dx^2\right)$$

input
```
integrate((a+b*sin(d*x^2+c))/x,x, algorithm="giac")
```

output
```
1/2*b*cos_integral(d*x^2)*sin(c) + 1/2*b*cos(c)*sin_integral(d*x^2) + 1/2*
a*log(d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx = a\,\ln\left(x\right) + \frac{b\,\sin\left(c\right)\,\mathrm{cosint}\left(d\,x^2\right)}{2} + \frac{b\,\cos\left(c\right)\,\mathrm{sinint}\left(d\,x^2\right)}{2}$$

input
```
int((a + b*sin(c + d*x^2))/x,x)
```

output
```
a*log(x) + (b*sin(c)*cosint(d*x^2))/2 + (b*cos(c)*sinint(d*x^2))/2
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx = \left(\int \frac{\sin\left(d\,x^2+c\right)}{x}dx\right)b + \log(x)\,a$$

input
```
int((a+b*sin(d*x^2+c))/x,x)
```

output
```
int(sin(c + d*x**2)/x,x)*b + log(x)*a
```

3.5 $\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = -\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\operatorname{CosIntegral}\left(dx^2\right) - \frac{b\sin(c+dx^2)}{2x^2}-\frac{1}{2}bd\sin(c)\operatorname{Si}\left(dx^2\right)$$

output
```
-1/2*a/x^2+1/2*b*d*cos(c)*Ci(d*x^2)-1/2*b*sin(d*x^2+c)/x^2-1/2*b*d*sin(c)*
Si(d*x^2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = -\frac{a-bdx^2\cos(c)\operatorname{CosIntegral}(dx^2)+b\sin(c+dx^2)+bdx^2\sin(c)\operatorname{Si}(dx^2)}{2x^2}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])/x^3,x]
```

output
```
-1/2*(a - b*d*x^2*Cos[c]*CosIntegral[d*x^2] + b*Sin[c + d*x^2] + b*d*x^2*S
in[c]*SinIntegral[d*x^2])/x^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^3}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^3}+\frac{b\sin\left(c+dx^2\right)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{2x^2}$$

input
```
Int[(a + b*Sin[c + d*x^2])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*d*Cos[c]*CosIntegral[d*x^2])/2 - (b*Sin[c + d*x^2])/(2*x^2
) - (b*d*Sin[c]*SinIntegral[d*x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a}{2x^2}+b\left(-\frac{\sin(d\,x^2+c)}{2x^2}+d\left(\frac{\cos(c)\,\mathrm{Ci}(d\,x^2)}{2}-\frac{\sin(c)\,\mathrm{Si}(d\,x^2)}{2}\right)\right)$
parts	$-\frac{a}{2x^2}+b\left(-\frac{\sin(d\,x^2+c)}{2x^2}+d\left(\frac{\cos(c)\,\mathrm{Ci}(d\,x^2)}{2}-\frac{\sin(c)\,\mathrm{Si}(d\,x^2)}{2}\right)\right)$
risch	$\frac{i\pi\ \mathrm{csgn}(d\,x^2)\mathrm{e}^{-ic}bd\,x^2-2i\,\mathrm{Si}(d\,x^2)\mathrm{e}^{-ic}bd\,x^2-bd\ \mathrm{expIntegral}_1(-id\,x^2)\mathrm{e}^{ic}x^2-\mathrm{e}^{-ic}\ \mathrm{expIntegral}_1(-id\,x^2)bd\,x^2-2b\sin(d\,x^2+c)-2}{4x^2}$

input
```
int((a+b*sin(d*x^2+c))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*(-1/2/x^2*sin(d*x^2+c)+d*(1/2*cos(c)*Ci(d*x^2)-1/2*sin(c)*Si(
d*x^2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = \frac{bdx^2\cos(c)\,\mathrm{Ci}(dx^2)-bdx^2\sin(c)\,\mathrm{Si}(dx^2)-b\sin(dx^2+c)-a}{2\,x^2}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*d*x^2*cos(c)*cos_integral(d*x^2) - b*d*x^2*sin(c)*sin_integral(d*x^
2) - b*sin(d*x^2 + c) - a)/x^2
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx=\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))/x**3,x)
```

output
```
Integral((a + b*sin(c + d*x**2))/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = \frac{1}{4}\left((\Gamma(-1,i\,dx^2)+\Gamma(-1,-i\,dx^2))\cos(c)-(i\,\Gamma(-1,i\,dx^2)-i\,\Gamma(-1,-i\,dx^2))\sin(c)\right)bd - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="maxima")
```

output
```
1/4*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*
x^2) - I*gamma(-1, -I*d*x^2))*sin(c))*b*d - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx$$
$$= \frac{(dx^2+c)bd^2\cos(c)\,\mathrm{Ci}(dx^2) - bcd^2\cos(c)\,\mathrm{Ci}(dx^2) - (dx^2+c)bd^2\sin(c)\,\mathrm{Si}(dx^2) + bcd^2\sin(c)\,\mathrm{Si}(dx^2) -}{2\,d^2x^2}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="giac")
```

output
```
1/2*((d*x^2 + c)*b*d^2*cos(c)*cos_integral(d*x^2) - b*c*d^2*cos(c)*cos_int
egral(d*x^2) - (d*x^2 + c)*b*d^2*sin(c)*sin_integral(d*x^2) + b*c*d^2*sin(
c)*sin_integral(d*x^2) - b*d^2*sin(d*x^2 + c) - a*d^2)/(d^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = \int \frac{a+b\,\sin(d\,x^2+c)}{x^3}\,dx$$

input
```
int((a + b*sin(c + d*x^2))/x^3,x)
```

output
```
int((a + b*sin(c + d*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx = \frac{2\left(\int \frac{\sin(d\,x^2+c)}{x^3}dx\right)b\,x^2 - a}{2x^2}$$

input
```
int((a+b*sin(d*x^2+c))/x^3,x)
```

output

```
(2*int(sin(c + d*x**2)/x**3,x)*b*x**2 - a)/(2*x**2)
```

3.6 $\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{bd\cos(c+dx^2)}{4x^2}-\frac{1}{4}bd^2\operatorname{CosIntegral}(dx^2)\sin(c)-\frac{b\sin(c+dx^2)}{4x^4}-\frac{1}{4}bd^2\cos(c)\operatorname{Si}(dx^2)$$

output
```
-1/4*a/x^4-1/4*b*d*cos(d*x^2+c)/x^2-1/4*b*d^2*Ci(d*x^2)*sin(c)-1/4*b*sin(d
*x^2+c)/x^4-1/4*b*d^2*cos(c)*Si(d*x^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{b\cos(dx^2)(dx^2\cos(c)+\sin(c))}{4x^4}+\frac{b(-\cos(c)+dx^2\sin(c))\sin(dx^2)}{4x^4}-\frac{1}{4}bd^2(\operatorname{CosIntegral}(dx^2)\sin(c)+\cos(c)\operatorname{Si}(dx^2))$$

input
```
Integrate[(a + b*Sin[c + d*x^2])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*Cos[d*x^2]*(d*x^2*Cos[c] + Sin[c]))/(4*x^4) + (b*(-Cos[c]
+ d*x^2*Sin[c])*Sin[d*x^2])/(4*x^4) - (b*d^2*(CosIntegral[d*x^2]*Sin[c] +
Cos[c]*SinIntegral[d*x^2]))/4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^5}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^2\right)}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{4x^4}-\frac{1}{4}bd^2\sin(c)\,\text{CosIntegral}\left(dx^2\right)-\frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{b\sin\left(c+dx^2\right)}{4x^4}$$

input
```
Int[(a + b*Sin[c + d*x^2])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*d*Cos[c + d*x^2])/(4*x^2) - (b*d^2*CosIntegral[d*x^2]*Sin[
c])/4 - (b*Sin[c + d*x^2])/(4*x^4) - (b*d^2*Cos[c]*SinIntegral[d*x^2])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a}{4x^{4}}+b\left(-\frac{\sin\left(d\,x^{2}+c\right)}{4x^{4}}+\frac{d\left(-\frac{\cos\left(d\,x^{2}+c\right)}{2x^{2}}-d\left(\frac{\cos\left(c\right)\,\mathrm{Si}\left(d\,x^{2}\right)}{2}+\frac{\sin\left(c\right)\,\mathrm{Ci}\left(d\,x^{2}\right)}{2}\right)\right)}{2}\right)$
parts	$-\frac{a}{4x^{4}}+b\left(-\frac{\sin\left(d\,x^{2}+c\right)}{4x^{4}}+\frac{d\left(-\frac{\cos\left(d\,x^{2}+c\right)}{2x^{2}}-d\left(\frac{\cos\left(c\right)\,\mathrm{Si}\left(d\,x^{2}\right)}{2}+\frac{\sin\left(c\right)\,\mathrm{Ci}\left(d\,x^{2}\right)}{2}\right)\right)}{2}\right)$
risch	$-\frac{-\pi\ \mathrm{csgn}\left(d\,x^{2}\right)\mathrm{e}^{-ic}b\,d^{2}x^{4}-i\mathrm{e}^{-ic}\ \mathrm{expIntegral}_{1}\left(-id\,x^{2}\right)b\,d^{2}x^{4}+ib\,d^{2}\ \mathrm{expIntegral}_{1}\left(-id\,x^{2}\right)\mathrm{e}^{ic}x^{4}+2\ \mathrm{Si}\left(d\,x^{2}\right)\mathrm{e}^{-ic}b\,d^{2}x^{4}+2x^{2}bd\,\mathrm{co}}{8x^{4}}$

input
```
int((a+b*sin(d*x^2+c))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4+b*(-1/4/x^4*sin(d*x^2+c)+1/2*d*(-1/2/x^2*cos(d*x^2+c)-d*(1/2*co
s(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = -\frac{bd^2x^4\operatorname{Ci}(dx^2)\sin(c)+bd^2x^4\cos(c)\operatorname{Si}(dx^2)+bdx^2\cos(dx^2+c)+b\sin(dx^2+c)+a}{4\,x^4}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="fricas")
```

output
```
-1/4*(b*d^2*x^4*cos_integral(d*x^2)*sin(c) + b*d^2*x^4*cos(c)*sin_integral
(d*x^2) + b*d*x^2*cos(d*x^2 + c) + b*sin(d*x^2 + c) + a)/x^4
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = \int \frac{a+b\sin(c+dx^2)}{x^5}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))/x**5,x)
```

output
```
Integral((a + b*sin(c + d*x**2))/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx = \frac{1}{4}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos(c)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin(c)\right)bd^2 - \frac{a}{4\,x^4}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="maxima")
```

output
```
1/4*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*
d*x^2) + gamma(-2, -I*d*x^2))*sin(c))*b*d^2 - 1/4*a/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(64) = 128$.

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.76

$$\int \frac{a + b\sin(c + dx^2)}{x^5}\, dx =$$
$$-\frac{(dx^2 + c)^2 bd^3 \operatorname{Ci}(dx^2)\sin(c) - 2(dx^2 + c)bcd^3 \operatorname{Ci}(dx^2)\sin(c) + bc^2d^3 \operatorname{Ci}(dx^2)\sin(c) + (dx^2 + c)^2 bd^3}{}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="giac")
```

output
```
-1/4*((d*x^2 + c)^2*b*d^3*cos_integral(d*x^2)*sin(c) - 2*(d*x^2 + c)*b*c*d
^3*cos_integral(d*x^2)*sin(c) + b*c^2*d^3*cos_integral(d*x^2)*sin(c) + (d*
x^2 + c)^2*b*d^3*cos(c)*sin_integral(d*x^2) - 2*(d*x^2 + c)*b*c*d^3*cos(c)
*sin_integral(d*x^2) + b*c^2*d^3*cos(c)*sin_integral(d*x^2) + (d*x^2 + c)*
b*d^3*cos(d*x^2 + c) - b*c*d^3*cos(d*x^2 + c) + b*d^3*sin(d*x^2 + c) + a*d
^3)/(((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sin(c + dx^2)}{x^5}\, dx = \int \frac{a + b\,\sin(d\,x^2 + c)}{x^5}\, dx$$

input
```
int((a + b*sin(c + d*x^2))/x^5,x)
```

output
```
int((a + b*sin(c + d*x^2))/x^5, x)
```

Reduce [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx$$
$$=\frac{-\cos\left(d\,x^2+c\right)bd\,x^2-2\left(\int \frac{\sin\left(d\,x^2+c\right)}{x}dx\right)b\,d^2x^4-\sin\left(d\,x^2+c\right)b-a}{4x^4}$$

input
```
int((a+b*sin(d*x^2+c))/x^5,x)
```

output
```
( - cos(c + d*x**2)*b*d*x**2 - 2*int(sin(c + d*x**2)/x,x)*b*d**2*x**4 - si
n(c + d*x**2)*b - a)/(4*x**4)
```

3.7 $\int x^4(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}-\frac{3b\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3b\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{4d^{5/2}}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}$$

output
```
1/5*a*x^5-1/2*b*x^3*cos(d*x^2+c)/d-3/8*b*2^(1/2)*Pi^(1/2)*cos(c)*FresnelS(
d^(1/2)*2^(1/2)/Pi^(1/2)*x)/d^(5/2)-3/8*b*2^(1/2)*Pi^(1/2)*FresnelC(d^(1/2
)*2^(1/2)/Pi^(1/2)*x)*sin(c)/d^(5/2)+3/4*b*x*sin(d*x^2+c)/d^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\begin{aligned}\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \frac{ax^5}{5}-\frac{bx\cos\left(dx^2\right)\left(2dx^2\cos(c)-3\sin(c)\right)}{4d^2}\\ &\quad-\frac{3b\sqrt{\frac{\pi}{2}}\left(\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{4d^{5/2}}\\ &\quad+\frac{bx\left(3\cos(c)+2dx^2\sin(c)\right)\sin\left(dx^2\right)}{4d^2}\end{aligned}$$

input

```
Integrate[x^4*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^5)/5 - (b*x*Cos[d*x^2]*(2*d*x^2*Cos[c] - 3*Sin[c]))/(4*d^2) - (3*b*Sq
rt[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt[2/
Pi]*x]*Sin[c]))/(4*d^(5/2)) + (b*x*(3*Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])
/(4*d^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(ax^4+bx^4\sin\left(c+dx^2\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^5}{5}-\frac{3\sqrt{\frac{\pi}{2}}b\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}b\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}$$

input

```
Int[x^4*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^5)/5 - (b*x^3*Cos[c + d*x^2])/(2*d) - (3*b*Sqrt[Pi/2]*Cos[c]*FresnelS
[Sqrt[d]*Sqrt[2/Pi]*x])/(4*d^(5/2)) - (3*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqr
t[2/Pi]*x]*Sin[c])/(4*d^(5/2)) + (3*b*x*Sin[c + d*x^2])/(4*d^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{a\,x^{5}}{5}+b\left(-\frac{x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\frac{3x\sin\left(d\,x^{2}+c\right)}{4d}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin\left(c\right)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}}{d}\right)$	89
parts	$\frac{a\,x^{5}}{5}+b\left(-\frac{x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\frac{3x\sin\left(d\,x^{2}+c\right)}{4d}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin\left(c\right)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}}{d}\right)$	89
risch	$\frac{a\,x^{5}}{5}-\frac{3ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right){\mathrm e}^{-ic}}{16d^{2}\sqrt{id}}+\frac{3ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right){\mathrm e}^{ic}}{16d^{2}\sqrt{-id}}-\frac{b\,x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{3bx\sin\left(d\,x^{2}+c\right)}{4d^{2}}$	100

input

```
int(x^4*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output

```
1/5*a*x^5+b*(-1/2/d*x^3*cos(d*x^2+c)+3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/
2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)+sin(c)*Fr
esnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{8\,ad^3x^5 - 20\,bd^2x^3\cos\left(dx^2+c\right) - 15\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right) - 15\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)}{40\,d^3}$$

input

```
integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output

```
1/40*(8*a*d^3*x^5 - 20*b*d^2*x^3*cos(d*x^2 + c) - 15*sqrt(2)*pi*b*sqrt(d/p
i)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) - 15*sqrt(2)*pi*b*sqrt(d/pi)*f
resnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 30*b*d*x*sin(d*x^2 + c))/d^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(126) = 252$.

Time = 2.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.03

$$\begin{aligned}\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx = {} & \frac{ax^5}{5} - \frac{5\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)} \\ & + \frac{\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} \\ & - \frac{21\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{32\Gamma\left(\frac{11}{4}\right)} \\ & + \frac{\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} \\ & - \frac{15\sqrt{2}\sqrt{\pi}b\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{128d^2\Gamma\left(\frac{9}{4}\right)} \\ & - \frac{63\sqrt{2}\sqrt{\pi}b\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{128d^2\Gamma\left(\frac{11}{4}\right)} \\ & + \frac{5bx^3\sqrt{\frac{1}{d}}\sin(c)\sin(dx^2)\Gamma\left(\frac{1}{4}\right)}{32\sqrt{d}\Gamma\left(\frac{9}{4}\right)} \\ & - \frac{21bx^3\sqrt{\frac{1}{d}}\cos(c)\cos(dx^2)\Gamma\left(\frac{3}{4}\right)}{32\sqrt{d}\Gamma\left(\frac{11}{4}\right)} \\ & + \frac{15bx\sqrt{\frac{1}{d}}\sin(c)\cos(dx^2)\Gamma\left(\frac{1}{4}\right)}{64d^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} \\ & + \frac{63bx\sqrt{\frac{1}{d}}\sin(dx^2)\cos(c)\Gamma\left(\frac{3}{4}\right)}{64d^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}\end{aligned}$$

input
```
integrate(x**4*(a+b*sin(d*x**2+c)),x)
```

output

```
a*x**5/5 - 5*sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)*sqr
t(d)*x/sqrt(pi))*gamma(1/4)/(32*gamma(9/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt
(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 - 21*sqrt(2)*sqrt(pi)*
b*x**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(3
2*gamma(11/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)
*sqrt(d)*x/sqrt(pi))/2 - 15*sqrt(2)*sqrt(pi)*b*sqrt(1/d)*sin(c)*fresnelc(s
qrt(2)*sqrt(d)*x/sqrt(pi))*gamma(1/4)/(128*d**2*gamma(9/4)) - 63*sqrt(2)*s
qrt(pi)*b*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)
/(128*d**2*gamma(11/4)) + 5*b*x**3*sqrt(1/d)*sin(c)*sin(d*x**2)*gamma(1/4)
/(32*sqrt(d)*gamma(9/4)) - 21*b*x**3*sqrt(1/d)*cos(c)*cos(d*x**2)*gamma(3/
4)/(32*sqrt(d)*gamma(11/4)) + 15*b*x*sqrt(1/d)*sin(c)*cos(d*x**2)*gamma(1/
4)/(64*d**(3/2)*gamma(9/4)) + 63*b*x*sqrt(1/d)*sin(d*x**2)*cos(c)*gamma(3/
4)/(64*d**(3/2)*gamma(11/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{1}{5}\,ax^5-\frac{\left(16\,d^3x^3\cos\left(dx^2+c\right)-24\,d^2x\sin\left(dx^2+c\right)+3\,\sqrt{2}\sqrt{\pi}\left(\left((i+1)\,\cos\left(c\right)-(i-1)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}x\right)\right.\right.}{32\,d^4}$$

input

```
integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output

```
1/5*a*x^5 - 1/32*(16*d^3*x^3*cos(d*x^2 + c) - 24*d^2*x*sin(d*x^2 + c) + 3*
sqrt(2)*sqrt(pi)*(((I + 1)*cos(c) - (I - 1)*sin(c))*erf(sqrt(I*d)*x) + (-(
I - 1)*cos(c) + (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*d^(3/2))*b/d^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.36

$$\begin{aligned}\int x^4(a+b\sin(c+dx^2))\,dx = \frac{1}{5}ax^5 &- \frac{3i\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{16\,d^2\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}} \\ &+ \frac{3i\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{16\,d^2\left(\frac{id}{|d|}+1\right)\sqrt{|d|}} \\ &+ \frac{i\,(2i\,bdx^3-3\,bx)e^{(i\,dx^2+i\,c)}}{8\,d^2} \\ &+ \frac{i\,(2i\,bdx^3+3\,bx)e^{(-i\,dx^2-i\,c)}}{8\,d^2}\end{aligned}$$

input

```
integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output

```
1/5*a*x^5 - 3/16*I*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)
*sqrt(abs(d)))*e^(I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d))) + 3/16*I*sqrt(
2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(
d^2*(I*d/abs(d) + 1)*sqrt(abs(d))) + 1/8*I*(2*I*b*d*x^3 - 3*b*x)*e^(I*d*x^
2 + I*c)/d^2 + 1/8*I*(2*I*b*d*x^3 + 3*b*x)*e^(-I*d*x^2 - I*c)/d^2
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\sin(c+dx^2))\,dx = \int x^4\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

input

```
int(x^4*(a + b*sin(c + d*x^2)),x)
```

output

```
int(x^4*(a + b*sin(c + d*x^2)), x)
```

Reduce [F]

$$\int x^4 \left(a + b \sin \left(c + dx^2\right)\right) \, dx$$
$$= \frac{-10 \cos \left(d\, x^2 + c\right) b d\, x^3 - 15 \left(\int \sin \left(d\, x^2 + c\right) dx\right) b + 15 \sin \left(d\, x^2 + c\right) b x + 4 a\, d^2 x^5}{20 d^2}$$

input
```
int(x^4*(a+b*sin(d*x^2+c)),x)
```

output
```
( - 10*cos(c + d*x**2)*b*d*x**3 - 15*int(sin(c + d*x**2),x)*b + 15*sin(c +
 d*x**2)*b*x + 4*a*d**2*x**5)/(20*d**2)
```

3.8 $\int x^2(a+b\sin(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{b\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{3/2}}$$

output

```
1/3*a*x^3-1/2*b*x*cos(d*x^2+c)/d+1/4*b*2^(1/2)*Pi^(1/2)*cos(c)*FresnelC(d^
(1/2)*2^(1/2)/Pi^(1/2)*x)/d^(3/2)-1/4*b*2^(1/2)*Pi^(1/2)*FresnelS(d^(1/2)*
2^(1/2)/Pi^(1/2)*x)*sin(c)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \frac{ax^3}{3}-\frac{bx\cos(c)\cos\left(dx^2\right)}{2d}\\ &\quad+\frac{b\sqrt{\frac{\pi}{2}}\left(\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{2d^{3/2}}\\ &\quad+\frac{bx\sin(c)\sin\left(dx^2\right)}{2d}\end{aligned}$$

input
```
Integrate[x^2*(a + b*Sin[c + d*x^2]),x]
```

output
```
(a*x^3)/3 - (b*x*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sqrt[Pi/2]*(Cos[c]*FresnelC
[Sqrt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]))/(2*d^(3/2
)) + (b*x*Sin[c]*Sin[d*x^2])/(2*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^2+bx^2\sin\left(c+dx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{\sqrt{\frac{\pi}{2}}b\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}b\sin(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{bx\cos\left(c+dx^2\right)}{2d}$$

input

```
Int[x^2*(a + b*Sin[c + d*x^2]),x]
```

output

```
(a*x^3)/3 - (b*x*Cos[c + d*x^2])/(2*d) + (b*Sqrt[Pi/2]*Cos[c]*FresnelC[Sqr
t[d]*Sqrt[2/Pi]*x])/(2*d^(3/2)) - (b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt[2/Pi
]*x]*Sin[c])/(2*d^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{a\,x^{3}}{3}+b\left(-\frac{x\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)-\sin\left(c\right)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{4d^{\frac{3}{2}}}\right)$	68
parts	$\frac{a\,x^{3}}{3}+b\left(-\frac{x\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)-\sin\left(c\right)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{4d^{\frac{3}{2}}}\right)$	68
risch	$\frac{b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{8d\sqrt{-id}}+\frac{b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{8d\sqrt{id}}+\frac{a\,x^{3}}{3}-\frac{bx\cos\left(d\,x^{2}+c\right)}{2d}$	81

input

```
int(x^2*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output

```
1/3*a*x^3+b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fr
esnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)-sin(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2
)*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{4\,ad^2x^3+3\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-3\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-6\,bdx\cos\left(dx^2+c\right)}{12\,d^2}$$

input
```
integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/12*(4*a*d^2*x^3 + 3*sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x
*sqrt(d/pi)) - 3*sqrt(2)*pi*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))
*sin(c) - 6*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(102) = 204.

Time = 1.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.19

$$\begin{aligned}\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx = {} & \frac{ax^3}{3}-\frac{bd^{\frac{3}{2}}x^5\sqrt{\frac{1}{d}}\cos\left(c\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} \\ & -\frac{b\sqrt{d}x^3\sqrt{\frac{1}{d}}\sin\left(c\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\right|-\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} \\ & +\frac{\sqrt{2}\sqrt{\pi}bx^2\sqrt{\frac{1}{d}}\sin\left(c\right)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} \\ & +\frac{\sqrt{2}\sqrt{\pi}bx^2\sqrt{\frac{1}{d}}\cos\left(c\right)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2}\end{aligned}$$

input
```
integrate(x**2*(a+b*sin(d*x**2+c)),x)
```

output
```
a*x**3/3 - b*d**(3/2)*x**5*sqrt(1/d)*cos(c)*gamma(3/4)*gamma(5/4)*hyper((3
/4, 5/4), (3/2, 7/4, 9/4), -d**2*x**4/4)/(8*gamma(7/4)*gamma(9/4)) - b*sqr
t(d)*x**3*sqrt(1/d)*sin(c)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5
/4, 7/4), -d**2*x**4/4)/(8*gamma(5/4)*gamma(7/4)) + sqrt(2)*sqrt(pi)*b*x**
2*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 + sqrt(2)*sqrt(p
i)*b*x**2*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^2 (a + b\sin(c + dx^2)) \, dx = \frac{1}{3} ax^3$$

$$- \frac{(8 d^2 x \cos(dx^2 + c) + \sqrt{2}\sqrt{\pi}(((i-1)\cos(c) + (i+1)\sin(c))\operatorname{erf}(\sqrt{i d} x) + (-(i+1)\cos(c) - (i -}{16 d^3}$$

input
```
integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 - 1/16*(8*d^2*x*cos(d*x^2 + c) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(
c) + (I + 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I + 1)*cos(c) - (I - 1)*sin(c))
*erf(sqrt(-I*d)*x))*d^(3/2))*b/d^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.42

$$\int x^2 (a + b\sin(c + dx^2)) \, dx = \frac{1}{3} ax^3 - \frac{bxe^{(i dx^2 + i c)}}{4 d} - \frac{bxe^{(-i dx^2 - i c)}}{4 d} - \frac{\sqrt{2}\sqrt{\pi} b \operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{i d}{|d|} + 1\right)\sqrt{|d|}\right)e^{(i c)}}{8 d\left(-\frac{i d}{|d|} + 1\right)\sqrt{|d|}} - \frac{\sqrt{2}\sqrt{\pi} b \operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{i d}{|d|} + 1\right)\sqrt{|d|}\right)e^{(-i c)}}{8 d\left(\frac{i d}{|d|} + 1\right)\sqrt{|d|}}$$

input
```
integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
1/3*a*x^3 - 1/4*b*x*e^(I*d*x^2 + I*c)/d - 1/4*b*x*e^(-I*d*x^2 - I*c)/d - 1
/8*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e
^(I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/8*sqrt(2)*sqrt(pi)*b*erf(-1/
2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d*(I*d/abs(d) + 1)*sq
rt(abs(d)))
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+dx^2))\,dx=\int x^2\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

input
```
int(x^2*(a + b*sin(c + d*x^2)),x)
```

output
```
int(x^2*(a + b*sin(c + d*x^2)), x)
```

Reduce [F]

$$\int x^2(a+b\sin(c+dx^2))\,dx=\left(\int \sin\left(d\,x^2+c\right)x^2dx\right)b+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a+b*sin(d*x^2+c)),x)
```

output
```
(3*int(sin(c + d*x**2)*x**2,x)*b + a*x**3)/3
```

3.9 $\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx=ax+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{b\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}$$

output
```
a*x+1/2*b*2^(1/2)*Pi^(1/2)*cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)/d^(
1/2)+1/2*b*2^(1/2)*Pi^(1/2)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)/d^
(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx=ax+\frac{b\sqrt{\frac{\pi}{2}}\left(\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{\sqrt{d}}$$

input
```
Integrate[a + b*Sin[c + d*x^2],x]
```

output
```
a*x + (b*Sqrt[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt
[d]*Sqrt[2/Pi]*x]*Sin[c]))/Sqrt[d]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{\sqrt{\frac{\pi}{2}}b\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}b\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}$$

input
```
Int[a + b*Sin[c + d*x^2],x]
```

output
```
a*x + (b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sqrt[d] + (b*Sq
rt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
default	$ax+\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos(c)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(c)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{2\sqrt{d}}$	48
parts	$ax+\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos(c)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(c)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{2\sqrt{d}}$	48
risch	$ax+\frac{ib\,\mathrm{e}^{-ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)}{4\sqrt{id}}-\frac{ib\,\mathrm{e}^{ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)}{4\sqrt{-id}}$	59

input
```
int(a+b*sin(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*2^(1/2)*Pi^(1/2)/d^(1/2)*(cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/
2)*x)+sin(c)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx$$
$$=\frac{\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)+2\,adx}{2\,d}$$

input
```
integrate(a+b*sin(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) + sq
rt(2)*pi*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 2*a*d*x)/
d
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \left(a+b\sin \left(c+dx^2\right)\right)\,dx=ax+\frac{\sqrt{2}\sqrt{\pi}b\left(\sin{(c)}C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)+\cos{(c)}S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{d}}}{2}$$

input
```
integrate(a+b*sin(d*x**2+c),x)
```

output
```
a*x + sqrt(2)*sqrt(pi)*b*(sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi)) + co
s(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi)))*sqrt(1/d)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \left(a+b\sin \left(c+dx^2\right)\right)\,dx=$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\,\cos{(c)}+(i-1)\,\sin{(c)}\right)\operatorname{erf}\left(\sqrt{i\,d}x\right)+\left((i-1)\,\cos{(c)}-(i+1)\,\sin{(c)}\right)\operatorname{erf}\bigl(\sqrt{-i\,d}}{8\sqrt{d}}$$

$$+ax$$

input
```
integrate(a+b*sin(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(c) + (I - 1)*sin(c))*erf(sqrt(I*d)*x)
 + ((I - 1)*cos(c) - (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*b/sqrt(d) + a*x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx = -\frac{1}{4}\left(-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}}+\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{\left(\frac{id}{|d|}+1\right)\sqrt{|d|}}\right)b + ax$$

input

```
integrate(a+b*sin(d*x^2+c),x, algorithm="giac")
```

output

```
-1/4*(-I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)
))*e^(I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))) + I*sqrt(2)*sqrt(pi)*erf(-1/2*
sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((I*d/abs(d) + 1)*sqrt(a
bs(d))))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 38.67 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx = a\,x+\frac{\sqrt{2}\,b\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi}}\right)\,\cos\left(c\right)}{2\,\sqrt{d}}+\frac{\sqrt{2}\,b\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi}}\right)\,\sin\left(c\right)}{2\,\sqrt{d}}$$

input

```
int(a + b*sin(c + d*x^2),x)
```

output

```
a*x + (2^(1/2)*b*pi^(1/2)*fresnels((2^(1/2)*d^(1/2)*x)/pi^(1/2))*cos(c))/(
2*d^(1/2)) + (2^(1/2)*b*pi^(1/2)*fresnelc((2^(1/2)*d^(1/2)*x)/pi^(1/2))*si
n(c))/(2*d^(1/2))
```

Reduce [F]

$$\int \left(a+b\sin \left(c+dx^2\right)\right) \, dx = \left(\int \sin \left(d\,x^2+c\right)dx\right)b+ax$$

input

```
int(a+b*sin(d*x^2+c),x)
```

output

```
int(sin(c + d*x**2),x)*b + a*x
```

3.10 $\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = -\frac{a}{x} + b\sqrt{d}\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - b\sqrt{d}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c) - \frac{b\sin(c+dx^2)}{x}$$

output

```
-a/x+b*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(c)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x
)-b*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)-b
*sin(d*x^2+c)/x
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = -\frac{a}{x} - \frac{b\cos(dx^2)\sin(c)}{x} + b\sqrt{d}\sqrt{2\pi}\left(\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) - \text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right) - \frac{b\cos(c)\sin(dx^2)}{x}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])/x^2,x]
```

output
```
-(a/x) - (b*Cos[d*x^2]*Sin[c])/x + b*Sqrt[d]*Sqrt[2*Pi]*(Cos[c]*FresnelC[S
qrt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]) - (b*Cos[c]*
Sin[d*x^2])/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^2\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{x}+\sqrt{2\pi}b\sqrt{d}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{2\pi}b\sqrt{d}\sin(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{b\sin\left(c+dx^2\right)}{x}$$

input
```
Int[(a + b*Sin[c + d*x^2])/x^2,x]
```

output
```
-(a/x) + b*Sqrt[d]*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] - b*Sq
rt[d]*Sqrt[2*Pi]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - (b*Sin[c + d*x^2]
)/x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result
default	$-\frac{a}{x}+b\left(-\frac{\sin\left(d\,x^{2}+c\right)}{x}+\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)-\sin\left(c\right)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)\right)$
parts	$-\frac{a}{x}+b\left(-\frac{\sin\left(d\,x^{2}+c\right)}{x}+\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\operatorname{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)-\sin\left(c\right)\operatorname{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)\right)$
risch	$\frac{b d\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-i d}\,x\right){\mathrm e}^{i c}}{2\sqrt{-i d}}+\frac{b d\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{i d}\,x\right){\mathrm e}^{-i c}}{2\sqrt{i d}}-\frac{a}{x}-\frac{b\sin\left(d\,x^{2}+c\right)}{x}$

input
```
int((a+b*sin(d*x^2+c))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*(-sin(d*x^2+c)/x+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(d^(1/2)*
2^(1/2)/Pi^(1/2)*x)-sin(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin\left(c+dx^{2}\right)}{x^{2}}\,dx = \frac{\sqrt{2}\pi bx\sqrt{\frac{d}{\pi}}\cos\left(c\right)\operatorname{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-\sqrt{2}\pi bx\sqrt{\frac{d}{\pi}}\operatorname{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-b\sin\left(dx^{2}+c\right)-a}{x}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="fricas")
```

output
```
(sqrt(2)*pi*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - sqrt
(2)*pi*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) - b*sin(d*x
^2 + c) - a)/x
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = \int \frac{a+b\sin(c+dx^2)}{x^2}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))/x**2,x)
```

output
```
Integral((a + b*sin(c + d*x**2))/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx = -\frac{\sqrt{dx^2}\big(\big((i-1)\sqrt{2}\Gamma(-\frac{1}{2}, i\,dx^2) - (i+1)\sqrt{2}\Gamma(-\frac{1}{2}, -i\,dx^2)\big)\cos(c) + \big((i+1)\sqrt{2}\Gamma(-\frac{1}{2}, i\,dx^2) - (i -}{8x} - \frac{a}{x}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="maxima")
```

output
```
-1/8*sqrt(d*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I + 1)*sqrt(2)*
gamma(-1/2, -I*d*x^2))*cos(c) + ((I + 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I
 - 1)*sqrt(2)*gamma(-1/2, -I*d*x^2))*sin(c))*b/x - a/x
```

Giac [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx=\int \frac{b\sin\left(dx^2+c\right)+a}{x^2}\,dx$$

input

```
integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="giac")
```

output

```
integrate((b*sin(d*x^2 + c) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\,\sin\left(d\,x^2+c\right)}{x^2}\,dx$$

input

```
int((a + b*sin(c + d*x^2))/x^2,x)
```

output

```
int((a + b*sin(c + d*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx=\frac{4\left(\int \frac{1}{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2+1}dx\right)bdx-\sin\left(d\,x^2+c\right)b-a-2bd\,x^2}{x}$$

input

```
int((a+b*sin(d*x^2+c))/x^2,x)
```

output

```
(4*int(1/(tan((c + d*x**2)/2)**2 + 1),x)*b*d*x - sin(c + d*x**2)*b - a - 2
*b*d*x**2)/x
```

3.11 $\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\begin{aligned}\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = {}& -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x} \\ & -\frac{2}{3}bd^{3/2}\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) \\ & -\frac{2}{3}bd^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{3x^3}\end{aligned}$$

output

```
-1/3*a/x^3-2/3*b*d*cos(d*x^2+c)/x-2/3*b*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(c)*Fr
esnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)-2/3*b*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC
(d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)-1/3*b*sin(d*x^2+c)/x^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = &-\frac{a}{3x^3}-\frac{b\cos\left(dx^2\right)\left(2dx^2\cos(c)+\sin(c)\right)}{3x^3}\\ &-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\left(\cos(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right.\\ &\left.+\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)\\ &+\frac{b(-\cos(c)+2dx^2\sin(c))\sin\left(dx^2\right)}{3x^3}\end{aligned}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*Cos[d*x^2]*(2*d*x^2*Cos[c] + Sin[c]))/(3*x^3) - (2*b*d^(3/
2)*Sqrt[2*Pi]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sq
rt[2/Pi]*x]*Sin[c]))/3 + (b*(-Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^4}+\frac{b\sin\left(c+dx^2\right)}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{b\sin\left(c+dx^2\right)}{3x^3}$$

input
```
Int[(a + b*Sin[c + d*x^2])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b*d*Cos[c + d*x^2])/(3*x) - (2*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]
*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/3 - (2*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt
[d]*Sqrt[2/Pi]*x]*Sin[c])/3 - (b*Sin[c + d*x^2])/(3*x^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

method	result
default	$-\frac{a}{3x^3}+b\left(-\frac{\sin\left(d\,x^2+c\right)}{3x^3}+\frac{2d\left(-\frac{\cos\left(d\,x^2+c\right)}{x}-\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\text{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(c)\,\text{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)\right)}{3}\right)$
parts	$-\frac{a}{3x^3}+b\left(-\frac{\sin\left(d\,x^2+c\right)}{3x^3}+\frac{2d\left(-\frac{\cos\left(d\,x^2+c\right)}{x}-\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\text{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(c)\,\text{FresnelC}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)\right)}{3}\right)$
risch	$-\frac{a}{3x^3}-\frac{ib\,d^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{3\sqrt{id}}+\frac{ib\,d^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{3\sqrt{-id}}-\frac{2bd\cos\left(d\,x^2+c\right)}{3x}-\frac{b\sin\left(d\,x^2+c\right)}{3x^3}$

input
```
int((a+b*sin(d*x^2+c))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*(-1/3*sin(d*x^2+c)/x^3+2/3*d*(-1/x*cos(d*x^2+c)-d^(1/2)*2^(1/
2)*Pi^(1/2)*(cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)+sin(c)*FresnelC(d
^(1/2)*2^(1/2)/Pi^(1/2)*x))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.86

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx =$$
$$-\frac{2\sqrt{2}\pi bdx^3\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+2\sqrt{2}\pi bdx^3\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)+2\,bdx^2\cos(dx^2+c)+b\,\mathrm{si}}{3\,x^3}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/
pi)) + 2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*s
in(c) + 2*b*d*x^2*cos(d*x^2 + c) + b*sin(d*x^2 + c) + a)/x^3
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx = \int \frac{a+b\sin(c+dx^2)}{x^4}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))/x**4,x)
```

output
```
Integral((a + b*sin(c + d*x**2))/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = -\frac{\sqrt{dx^2}\left(\left(-(i+1)\ \sqrt{2}\Gamma\left(-\frac{3}{2}, i\,dx^2\right) + (i-1)\ \sqrt{2}\Gamma\left(-\frac{3}{2}, -i\,dx^2\right)\right)\cos\left(c\right) + \left(\left((i-1)\ \sqrt{2}\Gamma\left(-\frac{3}{2}, i\,dx^2\right) - (i\right.\right.}{8\,x} - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="maxima")
```

output
```
-1/8*sqrt(d*x^2)*((-(I + 1)*sqrt(2)*gamma(-3/2, I*d*x^2) + (I - 1)*sqrt(2)
*gamma(-3/2, -I*d*x^2))*cos(c) + ((I - 1)*sqrt(2)*gamma(-3/2, I*d*x^2) - (
I + 1)*sqrt(2)*gamma(-3/2, -I*d*x^2))*sin(c))*b*d/x - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx = \int \frac{b\sin\left(dx^2+c\right)+a}{x^4}\,dx$$

input
```
integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^2 + c) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx=\int \frac{a+b\,\sin(d\,x^2+c)}{x^4}\,dx$$

input
```
int((a + b*sin(c + d*x^2))/x^4,x)
```

output
```
int((a + b*sin(c + d*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx=\frac{3\left(\int \frac{\sin(d\,x^2+c)}{x^4}dx\right)b\,x^3-a}{3x^3}$$

input
```
int((a+b*sin(d*x^2+c))/x^4,x)
```

output
```
(3*int(sin(c + d*x**2)/x**4,x)*b*x**3 - a)/(3*x**3)
```

3.12 $\int x^5(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 163

$$\begin{aligned}\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = {} & -\frac{b^2x^2}{8d^2}+\frac{a^2x^6}{6}+\frac{b^2x^6}{12}+\frac{2ab\cos\left(c+dx^2\right)}{d^3} \\ & -\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2} \\ & +\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{8d^3} \\ & -\frac{b^2x^4\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2}\end{aligned}$$

output

```
-1/8*b^2*x^2/d^2+1/6*a^2*x^6+1/12*b^2*x^6+2*a*b*cos(d*x^2+c)/d^3-a*b*x^4*c
os(d*x^2+c)/d+2*a*b*x^2*sin(d*x^2+c)/d^2+1/8*b^2*cos(d*x^2+c)*sin(d*x^2+c)
/d^3-1/4*b^2*x^4*cos(d*x^2+c)*sin(d*x^2+c)/d+1/4*b^2*x^2*sin(d*x^2+c)^2/d^
2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{8a^2d^3x^6+4b^2d^3x^6-48ab(-2+d^2x^4)\cos\left(c+dx^2\right)-6b^2dx^2\cos\left(2(c+dx^2)\right)+96abdx^2\sin\left(c+dx^2\right)+}{48d^3}$$

input
```
Integrate[x^5*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(8*a^2*d^3*x^6 + 4*b^2*d^3*x^6 - 48*a*b*(-2 + d^2*x^4)*Cos[c + d*x^2] - 6*
b^2*d*x^2*Cos[2*(c + d*x^2)] + 96*a*b*d*x^2*Sin[c + d*x^2] + 3*b^2*Sin[2*(
c + d*x^2)] - 6*b^2*d^2*x^4*Sin[2*(c + d*x^2)])/(48*d^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3860, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$\Big\downarrow \text{3860}$$
$$\frac{1}{2}\int x^4\left(a+b\sin\left(dx^2+c\right)\right)^2\,dx^2$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}\int x^4\left(a+b\sin\left(dx^2+c\right)\right)^2\,dx^2$$
$$\Big\downarrow \text{3798}$$
$$\frac{1}{2}\int\left(a^2x^4+b^2\sin^2\left(dx^2+c\right)x^4+2ab\sin\left(dx^2+c\right)x^4\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2x^6}{3}+\frac{4ab\cos\left(c+dx^2\right)}{d^3}+\frac{4abx^2\sin\left(c+dx^2\right)}{d^2}-\frac{2abx^4\cos\left(c+dx^2\right)}{d}+\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d^3}+\right.$$

input
```
Int[x^5*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(-1/4*(b^2*x^2)/d^2 + (a^2*x^6)/3 + (b^2*x^6)/6 + (4*a*b*Cos[c + d*x^2])/d
^3 - (2*a*b*x^4*Cos[c + d*x^2])/d + (4*a*b*x^2*Sin[c + d*x^2])/d^2 + (b^2*
Cos[c + d*x^2]*Sin[c + d*x^2])/(4*d^3) - (b^2*x^4*Cos[c + d*x^2]*Sin[c + d
*x^2])/(2*d) + (b^2*x^2*Sin[c + d*x^2]^2)/(2*d^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x^6a^2}{6}+\frac{b^2x^6}{12}-\frac{ab(x^4d^2-2)\cos(dx^2+c)}{d^3}+\frac{2ab\,x^2\sin(dx^2+c)}{d^2}-\frac{b^2x^2\cos(2dx^2+2c)}{8d^2}-\frac{b^2(2x^4d^2-1)\sin(2dx^2+2c)}{16d^3}$
parallelrisch	$\frac{(-6x^4d^2+3)b^2\sin(2dx^2+2c)-6b^2x^2\cos(2dx^2+2c)d-48ba(x^4d^2-2)\cos(dx^2+c)+8a^2d^3x^6+4b^2d^3x^6+96ab\,x^2\sin(dx^2+c}{48d^3}$
parts	$\frac{x^6a^2}{6}+b^2\left(\frac{x^6}{12}-\frac{x^4\sin(2dx^2+2c)}{8d}+\frac{-\frac{x^2\cos(2dx^2+2c)}{4d}+\frac{\sin(2dx^2+2c)}{8d^2}}{2d}\right)+2ab\left(-\frac{x^4\cos(dx^2+c)}{2d}+\right.$
default	$\frac{\left(a^2+\frac{b^2}{2}\right)x^6}{6}-\frac{b^2\left(\frac{x^4\sin(2dx^2+2c)}{4d}-\frac{-\frac{x^2\cos(2dx^2+2c)}{4d}+\frac{\sin(2dx^2+2c)}{8d^2}}{d}\right)}{2}+2ab\left(-\frac{x^4\cos(dx^2+c)}{2d}+\right.$
norman	$\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^6\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2+\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^4+\frac{ab\,x^4\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^4}{d}+\frac{b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{4d^3}-$
orering	$\frac{(128x^{12}d^6+2600x^8d^4-630x^4d^2-61425)(a+b\sin(dx^2+c))^2}{768x^6d^6}-\frac{5(136x^8d^4+258x^4d^2-5373)\left(5x^4(a+b\sin(dx^2+c))^2+4x\right.}{768x^{10}d^6}$

input
```
int(x^5*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a^2+1/12*b^2*x^6-a*b*(d^2*x^4-2)/d^3*cos(d*x^2+c)+2*a*b*x^2*sin(d*
x^2+c)/d^2-1/8*b^2*x^2/d^2*cos(2*d*x^2+2*c)-1/16*b^2*(2*d^2*x^4-1)/d^3*sin
(2*d*x^2+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{2\,(2\,a^2+b^2)d^3x^6-6\,b^2dx^2\cos\left(dx^2+c\right)^2+3\,b^2dx^2-24\,(abd^2x^4-2\,ab)\cos\left(dx^2+c\right)+3\,(16\,abdx^2-(2}{24\,d^3}$$

input
```
integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/24*(2*(2*a^2 + b^2)*d^3*x^6 - 6*b^2*d*x^2*cos(d*x^2 + c)^2 + 3*b^2*d*x^2
 - 24*(a*b*d^2*x^4 - 2*a*b)*cos(d*x^2 + c) + 3*(16*a*b*d*x^2 - (2*b^2*d^2*
x^4 - b^2)*cos(d*x^2 + c))*sin(d*x^2 + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.28

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^6}{6}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}+\frac{2ab\cos\left(c+dx^2\right)}{d^3}+\frac{b^2x^6\sin^2\left(c+dx^2\right)}{12}+\frac{b^2x^6\cos^2\left(c+dx^2\right)}{12}-\frac{b^2x^4\sin\left(c+dx^2\right)\,c}{4d}\\ \frac{x^6(a+b\sin(c))^2}{6}\end{cases}$$

input
```
integrate(x**5*(a+b*sin(d*x**2+c))**2,x)
```

output
```
Piecewise((a**2*x**6/6 - a*b*x**4*cos(c + d*x**2)/d + 2*a*b*x**2*sin(c + d
*x**2)/d**2 + 2*a*b*cos(c + d*x**2)/d**3 + b**2*x**6*sin(c + d*x**2)**2/12
 + b**2*x**6*cos(c + d*x**2)**2/12 - b**2*x**4*sin(c + d*x**2)*cos(c + d*x
**2)/(4*d) + b**2*x**2*sin(c + d*x**2)**2/(8*d**2) - b**2*x**2*cos(c + d*x
**2)**2/(8*d**2) + b**2*sin(c + d*x**2)*cos(c + d*x**2)/(8*d**3), Ne(d, 0)
), (x**6*(a + b*sin(c))**2/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{1}{6}\,a^2x^6+\frac{\left(2\,dx^2\sin\left(dx^2+c\right)-\left(d^2x^4-2\right)\cos\left(dx^2+c\right)\right)ab}{d^3}$$
$$+\frac{\left(4\,d^3x^6-6\,dx^2\cos\left(2\,dx^2+2\,c\right)-3\left(2\,d^2x^4-1\right)\sin\left(2\,dx^2+2\,c\right)\right)b^2}{48\,d^3}$$

input
```
integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 + (2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*a*b/
d^3 + 1/48*(4*d^3*x^6 - 6*d*x^2*cos(2*d*x^2 + 2*c) - 3*(2*d^2*x^4 - 1)*sin
(2*d*x^2 + 2*c))*b^2/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.74

$$
\begin{aligned}
\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx
&= -\frac{((dx^2+c)b^2-b^2c)\cos\left(2\,dx^2+2\,c\right)}{8\,d^3}\\
&\quad-\frac{\left((dx^2+c)^2ab-2\,(dx^2+c)abc-2\,ab\right)\cos\left(dx^2+c\right)}{d^3}\\
&\quad-\frac{\left(2\,(dx^2+c)^2b^2-4\,(dx^2+c)b^2c-b^2\right)\sin\left(2\,dx^2+2\,c\right)}{16\,d^3}\\
&\quad+\frac{2\,((dx^2+c)ab-abc)\sin\left(dx^2+c\right)}{d^3}\\
&\quad+\frac{2\,(dx^2+c)^3a^2+(dx^2+c)^3b^2-6\,(dx^2+c)^2a^2c-3\,(dx^2+c)^2b^2c}{12\,d^3}\\
&\quad+\frac{4\,(dx^2+c)a^2c^2+(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right))b^2c^2-8\,abc^2\cos\left(dx^2+c\right)}{8\,d^3}
\end{aligned}
$$

input
```
integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
-1/8*((d*x^2 + c)*b^2 - b^2*c)*cos(2*d*x^2 + 2*c)/d^3 - ((d*x^2 + c)^2*a*b
 - 2*(d*x^2 + c)*a*b*c - 2*a*b)*cos(d*x^2 + c)/d^3 - 1/16*(2*(d*x^2 + c)^2
*b^2 - 4*(d*x^2 + c)*b^2*c - b^2)*sin(2*d*x^2 + 2*c)/d^3 + 2*((d*x^2 + c)*
a*b - a*b*c)*sin(d*x^2 + c)/d^3 + 1/12*(2*(d*x^2 + c)^3*a^2 + (d*x^2 + c)^
3*b^2 - 6*(d*x^2 + c)^2*a^2*c - 3*(d*x^2 + c)^2*b^2*c)/d^3 + 1/8*(4*(d*x^2
 + c)*a^2*c^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2*c^2 - 8*a*b*c^2*c
os(d*x^2 + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

$$\int x^5 \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx$$
$$= \frac{\frac{3\,b^2\,\sin\left(2\,d\,x^2+2\,c\right)}{2}-96\,a\,b\,\sin\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2+4\,a^2\,d^3\,x^6+2\,b^2\,d^3\,x^6+3\,b^2\,d\,x^2\,\left(2\,\sin\left(d\,x^2+c\right)^2-1\right)-3\,b^2\,d}{24\,d^3}$$

input
```
int(x^5*(a + b*sin(c + d*x^2))^2,x)
```

output
```
((3*b^2*sin(2*c + 2*d*x^2))/2 - 96*a*b*sin(c/2 + (d*x^2)/2)^2 + 4*a^2*d^3*
x^6 + 2*b^2*d^3*x^6 + 3*b^2*d*x^2*(2*sin(c + d*x^2)^2 - 1) - 3*b^2*d^2*x^4
*sin(2*c + 2*d*x^2) + 24*a*b*d^2*x^4*(2*sin(c/2 + (d*x^2)/2)^2 - 1) + 48*a
*b*d*x^2*sin(c + d*x^2))/(24*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

$$\int x^5 \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx$$
$$= \frac{-6\cos\left(d\,x^2+c\right)\sin\left(d\,x^2+c\right)b^2d^2x^4+3\cos\left(d\,x^2+c\right)\sin\left(d\,x^2+c\right)b^2-24\cos\left(d\,x^2+c\right)ab\,d^2x^4+48\,c}{24}$$

input
```
int(x^5*(a+b*sin(d*x^2+c))^2,x)
```

output
```
( - 6*cos(c + d*x**2)*sin(c + d*x**2)*b**2*d**2*x**4 + 3*cos(c + d*x**2)*s
in(c + d*x**2)*b**2 - 24*cos(c + d*x**2)*a*b*d**2*x**4 + 48*cos(c + d*x**2
)*a*b + 6*sin(c + d*x**2)**2*b**2*d*x**2 + 48*sin(c + d*x**2)*a*b*d*x**2 +
 4*a**2*d**3*x**6 + 9*b**2*c + 2*b**2*d**3*x**6 - 3*b**2*d*x**2)/(24*d**3)
```

3.13 $\int x^3(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{ab\sin\left(c+dx^2\right)}{d^2}-\frac{b^2x^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2\sin^2\left(c+dx^2\right)}{8d^2}$$

output
```
1/4*a^2*x^4+1/8*b^2*x^4-a*b*x^2*cos(d*x^2+c)/d+a*b*sin(d*x^2+c)/d^2-1/4*b^
2*x^2*cos(d*x^2+c)*sin(d*x^2+c)/d+1/8*b^2*sin(d*x^2+c)^2/d^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{4a^2d^2x^4+2b^2d^2x^4-16abdx^2\cos\left(c+dx^2\right)-b^2\cos\left(2(c+dx^2)\right)+16ab\sin\left(c+dx^2\right)-2b^2dx^2\sin(2(c+}{16d^2}$$

input
```
Integrate[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

output

```
(4*a^2*d^2*x^4 + 2*b^2*d^2*x^4 - 16*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(
c + d*x^2)] + 16*a*b*Sin[c + d*x^2] - 2*b^2*d*x^2*Sin[2*(c + d*x^2)])/(16*
d^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3860, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2 dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{2}\int x^2 \left(a+b\sin\left(dx^2+c\right)\right)^2 dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int x^2 \left(a+b\sin\left(dx^2+c\right)\right)^2 dx^2$$

$$\Big\downarrow \text{ 3798}$$

$$\frac{1}{2}\int \left(a^2x^2+b^2\sin^2\left(dx^2+c\right)x^2+2ab\sin\left(dx^2+c\right)x^2\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^2x^4}{2}+\frac{2ab\sin\left(c+dx^2\right)}{d^2}-\frac{2abx^2\cos\left(c+dx^2\right)}{d}+\frac{b^2\sin^2\left(c+dx^2\right)}{4d^2}-\frac{b^2x^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{2d}+\frac{b^2}{4}\right.$$

input

```
Int[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

output

```
((a^2*x^4)/2 + (b^2*x^4)/4 - (2*a*b*x^2*Cos[c + d*x^2])/d + (2*a*b*Sin[c +
 d*x^2])/d^2 - (b^2*x^2*Cos[c + d*x^2]*Sin[c + d*x^2])/(2*d) + (b^2*Sin[c
+ d*x^2]^2)/(4*d^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

method	result
parts	$\frac{a^{2} x^{4}}{4}+b^{2} \left(\frac{x^{4}}{8}-\frac{x^{2} \sin \left(2 d \,x^{2}+2 c \right)}{8 d}-\frac{\cos \left(2 d \,x^{2}+2 c \right)}{16 d^{2}}\right)+2 a b \left(-\frac{x^{2} \cos \left(d \,x^{2}+c \right)}{2 d}+\frac{\sin \left(d \,x^{2}+c \right)}{2 d^{2}}\right)$
default	$\frac{\left(a^{2}+\frac{b^{2}}{2}\right) x^{4}}{4}-\frac{b^{2} \left(\frac{x^{2} \sin \left(2 d \,x^{2}+2 c \right)}{4 d}+\frac{\cos \left(2 d \,x^{2}+2 c \right)}{8 d^{2}}\right)}{2}+2 a b \left(-\frac{x^{2} \cos \left(d \,x^{2}+c \right)}{2 d}+\frac{\sin \left(d \,x^{2}+c \right)}{2 d^{2}}\right)$
risch	$\frac{a^{2} x^{4}}{4}+\frac{b^{2} x^{4}}{8}-\frac{a b \,x^{2} \cos \left(d \,x^{2}+c \right)}{d}+\frac{a b \sin \left(d \,x^{2}+c \right)}{d^{2}}-\frac{b^{2} \cos \left(2 d \,x^{2}+2 c \right)}{16 d^{2}}-\frac{b^{2} x^{2} \sin \left(2 d \,x^{2}+2 c \right)}{8 d}$
parallelrisch	$\frac{4 a^{2} d^{2} x^{4}+2 b^{2} d^{2} x^{4}-2 b^{2} x^{2} \sin \left(2 d \,x^{2}+2 c \right) d -16 a b \,x^{2} \cos \left(d \,x^{2}+c \right) d +16 \sin \left(d \,x^{2}+c \right) a b -b^{2} \cos \left(2 d \,x^{2}+2 c \right)+b^{2}}{16 d^{2}}$
norman	$\frac{\left(\frac{a^{2}}{4}+\frac{b^{2}}{8}\right) x^{4}+\left(\frac{a^{2}}{2}+\frac{b^{2}}{4}\right) x^{4} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{2}+\left(\frac{a^{2}}{4}+\frac{b^{2}}{8}\right) x^{4} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{4}+\frac{a b \,x^{2} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{4}}{d}+\frac{b^{2} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{2}}{2 d^{2}}-b^{2} x^{2} \mathrm{ta}}{\left(1+\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$
orering	$\frac{\left(64 x^{8} d^{4}+700 x^{4} d^{2}+1485\right) \left(a +b \sin \left(d \,x^{2}+c \right)\right)^{2}}{256 x^{4} d^{4}}-\frac{5 \left(44 x^{4} d^{2}+169\right) \left(3 x^{2} \left(a +b \sin \left(d \,x^{2}+c \right)\right)^{2}+4 x^{4} \left(a +b \sin \left(d \,x^{2}+c \right)\right) b d \cos (d}{256 d^{4} x^{6}}$

input
```
int(x^3*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+b^2*(1/8*x^4-1/8/d*x^2*sin(2*d*x^2+2*c)-1/16/d^2*cos(2*d*x^2+2
*c))+2*a*b*(-1/2/d*x^2*cos(d*x^2+c)+1/2/d^2*sin(d*x^2+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{(2\,a^2+b^2)d^2x^4-8\,abdx^2\cos\left(dx^2+c\right)-b^2\cos\left(dx^2+c\right)^2-2\,\left(b^2dx^2\cos\left(dx^2+c\right)-4\,ab\right)\sin\left(dx^2+c\right)}{8\,d^2}$$

input
```
integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/8*((2*a^2 + b^2)*d^2*x^4 - 8*a*b*d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 +
c)^2 - 2*(b^2*d*x^2*cos(d*x^2 + c) - 4*a*b)*sin(d*x^2 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^4}{4}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{ab\sin\left(c+dx^2\right)}{d^2}+\frac{b^2x^4\sin^2\left(c+dx^2\right)}{8}+\frac{b^2x^4\cos^2\left(c+dx^2\right)}{8}-\frac{b^2x^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}+\frac{b^2\sin^2(}{8d} \\ \frac{x^4(a+b\sin\left(c\right))^2}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*sin(d*x**2+c))**2,x)
```

output
```
Piecewise((a**2*x**4/4 - a*b*x**2*cos(c + d*x**2)/d + a*b*sin(c + d*x**2)/
d**2 + b**2*x**4*sin(c + d*x**2)**2/8 + b**2*x**4*cos(c + d*x**2)**2/8 - b
**2*x**2*sin(c + d*x**2)*cos(c + d*x**2)/(4*d) + b**2*sin(c + d*x**2)**2/(
8*d**2), Ne(d, 0)), (x**4*(a + b*sin(c))**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{4}\,a^2x^4-\frac{\left(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right)\right)ab}{d^2}+\frac{\left(2\,d^2x^4-2\,dx^2\sin\left(2\,dx^2+2\,c\right)-\cos\left(2\,dx^2+2\,c\right)\right)b^2}{16\,d^2}$$

input

```
integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
1/4*a^2*x^4 - (d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*a*b/d^2 + 1/16*(2*d^
2*x^4 - 2*d*x^2*sin(2*d*x^2 + 2*c) - cos(2*d*x^2 + 2*c))*b^2/d^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

$$=\frac{4\left(dx^2+c\right)^2a^2+2\left(dx^2+c\right)^2b^2-16\left(dx^2+c\right)ab\cos\left(dx^2+c\right)-2\left(dx^2+c\right)b^2\sin\left(2\,dx^2+2\,c\right)-b^2\cos(2}{16\,d^2}$$

$$-\frac{4\left(dx^2+c\right)a^2c+\left(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right)\right)b^2c-8\,abc\cos\left(dx^2+c\right)}{8\,d^2}$$

input

```
integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output

```
1/16*(4*(d*x^2 + c)^2*a^2 + 2*(d*x^2 + c)^2*b^2 - 16*(d*x^2 + c)*a*b*cos(d
*x^2 + c) - 2*(d*x^2 + c)*b^2*sin(2*d*x^2 + 2*c) - b^2*cos(2*d*x^2 + 2*c)
+ 16*a*b*sin(d*x^2 + c))/d^2 - 1/8*(4*(d*x^2 + c)*a^2*c + (2*d*x^2 + 2*c -
 sin(2*d*x^2 + 2*c))*b^2*c - 8*a*b*c*cos(d*x^2 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 38.70 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = -\frac{b^2\cos\left(d\,x^2+c\right)^2-2\,a^2\,d^2\,x^4-b^2\,d^2\,x^4-8\,a\,b\,\sin\left(d\,x^2+c\right)+8\,a\,b\,d\,x^2\,\cos\left(d\,x^2+c\right)+2\,b^2\,d\,x^2\,\cos(}{8\,d^2}$$

input
```
int(x^3*(a + b*sin(c + d*x^2))^2,x)
```

output
```
-(b^2*cos(c + d*x^2)^2 - 2*a^2*d^2*x^4 - b^2*d^2*x^4 - 8*a*b*sin(c + d*x^2
) + 8*a*b*d*x^2*cos(c + d*x^2) + 2*b^2*d*x^2*cos(c + d*x^2)*sin(c + d*x^2)
)/(8*d^2)
```

Reduce [F]

$$\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{-4\cos\left(d\,x^2+c\right)abd\,x^2+4\left(\int\sin\left(d\,x^2+c\right)^2x^3dx\right)b^2d^2+4\sin\left(d\,x^2+c\right)ab+a^2d^2x^4}{4d^2}$$

input
```
int(x^3*(a+b*sin(d*x^2+c))^2,x)
```

output
```
( - 4*cos(c + d*x**2)*a*b*d*x**2 + 4*int(sin(c + d*x**2)**2*x**3,x)*b**2*d
**2 + 4*sin(c + d*x**2)*a*b + a**2*d**2*x**4)/(4*d**2)
```

3.14 $\int x(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{4}\left(2a^2+b^2\right)x^2-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}$$

output
```
1/4*(2*a^2+b^2)*x^2-a*b*cos(d*x^2+c)/d-1/4*b^2*cos(d*x^2+c)*sin(d*x^2+c)/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=-\frac{-2(2a^2+b^2)\left(c+dx^2\right)+8ab\cos\left(c+dx^2\right)+b^2\sin\left(2(c+dx^2)\right)}{8d}$$

input
```
Integrate[x*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
-1/8*(-2*(2*a^2 + b^2)*(c + d*x^2) + 8*a*b*Cos[c + d*x^2] + b^2*Sin[2*(c +
 d*x^2)])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3860, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \left(a+b\sin\left(dx^2+c\right)\right)^2dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \left(a+b\sin\left(dx^2+c\right)\right)^2dx^2$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\left(2a^2+b^2\right)-\frac{2ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{2d}\right)$$

input
```
Int[x*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(((2*a^2 + b^2)*x^2)/2 - (2*a*b*Cos[c + d*x^2])/d - (b^2*Cos[c + d*x^2]*Si
n[c + d*x^2])/(2*d))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x^2a^2}{2}+\frac{x^2b^2}{4}-\frac{ab\cos(dx^2+c)}{d}-\frac{b^2\sin(2dx^2+2c)}{8d}$
parallelrisch	$\frac{4a^2dx^2+2x^2b^2d-8ab\cos(dx^2+c)-b^2\sin(2dx^2+2c)+8ab}{8d}$
parts	$\frac{x^2a^2}{2}+\frac{b^2\left(-\frac{\cos(dx^2+c)\sin(dx^2+c)}{2}+\frac{dx^2}{2}+\frac{c}{2}\right)}{2d}-\frac{ab\cos(dx^2+c)}{d}$
derivativedivides	$\frac{b^2\left(-\frac{\cos(dx^2+c)\sin(dx^2+c)}{2}+\frac{dx^2}{2}+\frac{c}{2}\right)-2ab\cos(dx^2+c)+a^2(dx^2+c)}{2d}$
default	$\frac{b^2\left(-\frac{\cos(dx^2+c)\sin(dx^2+c)}{2}+\frac{dx^2}{2}+\frac{c}{2}\right)-2ab\cos(dx^2+c)+a^2(dx^2+c)}{2d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^2+\left(a^2+\frac{b^2}{2}\right)x^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2+\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^4+\frac{2ab\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^4}{d}-\frac{b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{2d}+}{\left(1+\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2\right)^2}$
orering	$\frac{(64x^8d^4+100x^4d^2+135)(a+b\sin(dx^2+c))^2}{128x^6d^4}-\frac{5(20x^4d^2+27)\left((a+b\sin(dx^2+c))^2+4x^2(a+b\sin(dx^2+c))bd\cos(\right.}{128d^4x^6}$

input
```
int(x*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*a^2+1/4*x^2*b^2-a*b*cos(d*x^2+c)/d-1/8*b^2/d*sin(2*d*x^2+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{(2\,a^2+b^2)dx^2-b^2\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)-4\,ab\cos\left(dx^2+c\right)}{4\,d}$$

input
```
integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*d*x^2 - b^2*cos(d*x^2 + c)*sin(d*x^2 + c) - 4*a*b*cos(d
*x^2 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \begin{cases} \frac{a^2x^2}{2}-\frac{ab\cos\left(c+dx^2\right)}{d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4}+\frac{b^2x^2\cos^2\left(c+dx^2\right)}{4}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d} & \text{for } d\neq 0 \\ \frac{x^2(a+b\sin\left(c\right))^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*sin(d*x**2+c))**2,x)
```

output
```
Piecewise((a**2*x**2/2 - a*b*cos(c + d*x**2)/d + b**2*x**2*sin(c + d*x**2)
**2/4 + b**2*x**2*cos(c + d*x**2)**2/4 - b**2*sin(c + d*x**2)*cos(c + d*x*
*2)/(4*d), Ne(d, 0)), (x**2*(a + b*sin(c))**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{2}\,a^2x^2+\frac{(2\,dx^2-\sin\left(2\,dx^2+2\,c\right))b^2}{8\,d}-\frac{ab\cos\left(dx^2+c\right)}{d}$$

input
```
integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/8*(2*d*x^2 - sin(2*d*x^2 + 2*c))*b^2/d - a*b*cos(d*x^2 + c
)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{4\,(dx^2+c)a^2+(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right))b^2-8\,ab\cos\left(dx^2+c\right)}{8\,d}$$

input
```
integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
1/8*(4*(d*x^2 + c)*a^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2 - 8*a*b*
cos(d*x^2 + c))/d
```

Mupad [B] (verification not implemented)

Time = 38.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,x^2}{4}-\frac{b^2\,\sin\left(2\,d\,x^2+2\,c\right)}{8\,d}-\frac{a\,b\,\cos\left(d\,x^2+c\right)}{d}$$

input
```
int(x*(a + b*sin(c + d*x^2))^2,x)
```

output

```
(a^2*x^2)/2 + (b^2*x^2)/4 - (b^2*sin(2*c + 2*d*x^2))/(8*d) - (a*b*cos(c +
d*x^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{-\cos\left(d\,x^2+c\right)\sin\left(d\,x^2+c\right)b^2-4\cos\left(d\,x^2+c\right)ab+2a^2d\,x^2+4ab+b^2d\,x^2}{4d}$$

input

```
int(x*(a+b*sin(d*x^2+c))^2,x)
```

output

```
( - cos(c + d*x**2)*sin(c + d*x**2)*b**2 - 4*cos(c + d*x**2)*a*b + 2*a**2*
d*x**2 + 4*a*b + b**2*d*x**2)/(4*d)
```

3.15 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}\,dx = {} & -\frac{1}{4}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^2\right) \\ & +\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\,\text{CosIntegral}\left(dx^2\right)\sin(c) \\ & +ab\cos(c)\text{Si}\left(dx^2\right)+\frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)\end{aligned}$$

output

```
-1/4*b^2*cos(2*c)*Ci(2*d*x^2)+1/2*(2*a^2+b^2)*ln(x)+a*b*Ci(d*x^2)*sin(c)+a
*b*cos(c)*Si(d*x^2)+1/4*b^2*sin(2*c)*Si(2*d*x^2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}\,dx = {} & \frac{1}{2}\left(2a^2+b^2\right)\log(x)-\frac{1}{4}b\Big(b\cos(2c)\,\text{CosIntegral}\left(2dx^2\right) \\ & -4a\,\text{CosIntegral}\left(dx^2\right)\sin(c)-4a\cos(c)\text{Si}\left(dx^2\right) \\ & -b\sin(2c)\text{Si}\left(2dx^2\right)\Big)\end{aligned}$$

input

```
Integrate[(a + b*Sin[c + d*x^2])^2/x,x]
```

output

```
((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^2] - 4*a*CosIn
tegral[d*x^2]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^2] - b*Sin[2*c]*SinInteg
ral[2*d*x^2]))/4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(\frac{a^2}{x}+\frac{2ab\sin\left(c+dx^2\right)}{x}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+\frac{b^2}{2x}\right)dx$$

$$\Big\downarrow \text{6}$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x}+\frac{2ab\sin\left(c+dx^2\right)}{x}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\sin(c)\,\text{CosIntegral}\left(dx^2\right)+ab\cos(c)\text{Si}\left(dx^2\right)-\frac{1}{4}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^2\right)+\frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)$$

input

```
Int[(a + b*Sin[c + d*x^2])^2/x,x]
```

output

```
-1/4*(b^2*Cos[2*c]*CosIntegral[2*d*x^2]) + ((2*a^2 + b^2)*Log[x])/2 + a*b*
CosIntegral[d*x^2]*Sin[c] + a*b*Cos[c]*SinIntegral[d*x^2] + (b^2*Sin[2*c]*
SinIntegral[2*d*x^2])/4
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.94 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

method	result
risch	$-\frac{e^{-ic}\pi \operatorname{csgn}\left(d x^{2}\right) a b}{2}+e^{-ic} \operatorname{Si}\left(d x^{2}\right) a b-\frac{i e^{-ic} \operatorname{expIntegral}_{1}\left(-i d x^{2}\right) a b}{2}+\ln\left(x\right) a^{2}+\frac{\ln(x) b^{2}}{2}-\frac{i e^{-2ic}\pi \operatorname{csgn}(d x}{8}$

input
```
int((a+b*sin(d*x^2+c))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-I*c)*Pi*csgn(d*x^2)*a*b+exp(-I*c)*Si(d*x^2)*a*b-1/2*I*exp(-I*c)*
Ei(1,-I*d*x^2)*a*b+ln(x)*a^2+1/2*ln(x)*b^2-1/8*I*exp(-2*I*c)*Pi*csgn(d*x^2
)*b^2+1/4*I*exp(-2*I*c)*Si(2*d*x^2)*b^2+1/8*exp(-2*I*c)*Ei(1,-2*I*d*x^2)*b
^2+1/8*b^2*exp(2*I*c)*Ei(1,-2*I*d*x^2)+1/2*I*a*b*exp(I*c)*Ei(1,-I*d*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = {} & -\frac{1}{4}b^2\cos(2c)\operatorname{Ci}\left(2dx^2\right) \\ & + ab\operatorname{Ci}\left(dx^2\right)\sin(c) + \frac{1}{4}b^2\sin(2c)\operatorname{Si}\left(2dx^2\right) \\ & + ab\cos(c)\operatorname{Si}\left(dx^2\right) + \frac{1}{2}\left(2a^2+b^2\right)\log(x)\end{aligned}$$

input

```
integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="fricas")
```

output

```
-1/4*b^2*cos(2*c)*cos_integral(2*d*x^2) + a*b*cos_integral(d*x^2)*sin(c) +
 1/4*b^2*sin(2*c)*sin_integral(2*d*x^2) + a*b*cos(c)*sin_integral(d*x^2) +
 1/2*(2*a^2 + b^2)*log(x)
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx$$

input

```
integrate((a+b*sin(d*x**2+c))**2/x,x)
```

output

```
Integral((a + b*sin(c + d*x**2))**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx &= -\frac{1}{2}\left(\left(i\operatorname{Ei}(i\,dx^2)-i\operatorname{Ei}(-i\,dx^2)\right)\cos(c)-\left(\operatorname{Ei}(i\,dx^2)+\operatorname{Ei}(-i\,dx^2)\right)\sin(c)\right)ab\\ &\quad -\frac{1}{8}\left(\left(\operatorname{Ei}(2i\,dx^2)+\operatorname{Ei}(-2i\,dx^2)\right)\cos(2\,c)-\left(-i\operatorname{Ei}(2i\,dx^2)+i\operatorname{Ei}(-2i\,dx^2)\right)\sin(2\,c)-4\,\log(x)\right)b^2\\ &\quad +a^2\log(x)\end{aligned}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="maxima")
```

output
```
-1/2*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2
))*sin(c))*a*b - 1/8*((Ei(2*I*d*x^2) + Ei(-2*I*d*x^2))*cos(2*c) - (-I*Ei(2
*I*d*x^2) + I*Ei(-2*I*d*x^2))*sin(2*c) - 4*log(x))*b^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = &-\frac{1}{4}\,b^2\cos(2\,c)\operatorname{Ci}\left(2\,dx^2\right)+ab\operatorname{Ci}\left(dx^2\right)\sin(c)\\ &+ab\cos(c)\operatorname{Si}\left(dx^2\right)-\frac{1}{4}\,b^2\sin(2\,c)\operatorname{Si}\left(-2\,dx^2\right)\\ &+\frac{1}{2}\,a^2\log\left(dx^2\right)+\frac{1}{4}\,b^2\log\left(dx^2\right)\end{aligned}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="giac")
```

output
```
-1/4*b^2*cos(2*c)*cos_integral(2*d*x^2) + a*b*cos_integral(d*x^2)*sin(c) +
 a*b*cos(c)*sin_integral(d*x^2) - 1/4*b^2*sin(2*c)*sin_integral(-2*d*x^2)
+ 1/2*a^2*log(d*x^2) + 1/4*b^2*log(d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\,\sin(d\,x^2+c))^2}{x}\,dx$$

input
```
int((a + b*sin(c + d*x^2))^2/x,x)
```

output
```
int((a + b*sin(c + d*x^2))^2/x, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx=\left(\int \frac{\sin(d\,x^2+c)^2}{x}dx\right)b^2+2\left(\int \frac{\sin(d\,x^2+c)}{x}dx\right)ab+\log(x)\,a^2$$

input
```
int((a+b*sin(d*x^2+c))^2/x,x)
```

output
```
int(sin(c + d*x**2)**2/x,x)*b**2 + 2*int(sin(c + d*x**2)/x,x)*a*b + log(x)
*a**2
```

3.16 $\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx = & -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2(c+dx^2))}{4x^2} \\ & + abd\cos(c)\,\text{CosIntegral}\left(dx^2\right) \\ & + \frac{1}{2}b^2d\,\text{CosIntegral}\left(2dx^2\right)\sin(2c)-\frac{ab\sin(c+dx^2)}{x^2} \\ & - abd\sin(c)\text{Si}\left(dx^2\right)+\frac{1}{2}b^2d\cos(2c)\text{Si}\left(2dx^2\right)\end{aligned}$$

output

```
-1/4*(2*a^2+b^2)/x^2+1/4*b^2*cos(2*d*x^2+2*c)/x^2+a*b*d*cos(c)*Ci(d*x^2)+1
/2*b^2*d*Ci(2*d*x^2)*sin(2*c)-a*b*sin(d*x^2+c)/x^2-a*b*d*sin(c)*Si(d*x^2)+
1/2*b^2*d*cos(2*c)*Si(2*d*x^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$$
$$= \frac{-2a^2-b^2+b^2\cos(2(c+dx^2))+4abdx^2\cos(c)\,\mathrm{CosIntegral}(dx^2)+2b^2dx^2\,\mathrm{CosIntegral}(2dx^2)\sin(2c)-}{4x^2}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

output
```
(-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*d*x^2*Cos[c]*CosIntegral[d*
x^2] + 2*b^2*d*x^2*CosIntegral[2*d*x^2]*Sin[2*c] - 4*a*b*Sin[c + d*x^2] -
4*a*b*d*x^2*Sin[c]*SinIntegral[d*x^2] + 2*b^2*d*x^2*Cos[2*c]*SinIntegral[2
*d*x^2])/(4*x^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$$
$$\Big\downarrow \; 3884$$
$$\int \left(\frac{a^2}{x^3}+\frac{2ab\sin(c+dx^2)}{x^3}-\frac{b^2\cos(2c+2dx^2)}{2x^3}+\frac{b^2}{2x^3}\right)dx$$
$$\Big\downarrow \; 6$$
$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^3}+\frac{2ab\sin(c+dx^2)}{x^3}-\frac{b^2\cos(2c+2dx^2)}{2x^3}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{2a^2+b^2}{4x^2}+abd\cos(c)\,\mathrm{CosIntegral}\left(dx^2\right)-abd\sin(c)\mathrm{Si}\left(dx^2\right)-\frac{ab\sin\left(c+dx^2\right)}{x^2}+\frac{1}{2}b^2d\sin(2c)\,\mathrm{CosIntegral}\left(2dx^2\right)+\frac{1}{2}b^2d\cos(2c)\mathrm{Si}\left(2dx^2\right)+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}$$

input

```
Int[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

output

```
-1/4*(2*a^2 + b^2)/x^2 + (b^2*Cos[2*(c + d*x^2)])/(4*x^2) + a*b*d*Cos[c]*C
osIntegral[d*x^2] + (b^2*d*CosIntegral[2*d*x^2]*Sin[2*c])/2 - (a*b*Sin[c +
 d*x^2])/x^2 - a*b*d*Sin[c]*SinIntegral[d*x^2] + (b^2*d*Cos[2*c]*SinIntegr
al[2*d*x^2])/2
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.01 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{-2 i \mathrm{e}^{-i c} \pi \,\mathrm{csgn}\left(d\,x^{2}\right) a b d\,x^{2}+\mathrm{e}^{-2 i c} \pi \,\mathrm{csgn}\left(d\,x^{2}\right) b^{2} d\,x^{2}+4 i \mathrm{e}^{-i c}\,\mathrm{Si}\left(d\,x^{2}\right) a b d\,x^{2}+i\,\mathrm{expIntegral}_{1}\left(-2 i d\,x^{2}\right) \mathrm{e}^{-2 i c} b^{2} d\,x^{2}-i b^{2} d\,\mathrm{expIn}}{}$

input

```
int((a+b*sin(d*x^2+c))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*I*exp(-I*c)*Pi*csgn(d*x^2)*a*b*d*x^2+exp(-2*I*c)*Pi*csgn(d*x^2)*b
^2*d*x^2+4*I*exp(-I*c)*Si(d*x^2)*a*b*d*x^2+I*Ei(1,-2*I*d*x^2)*exp(-2*I*c)*
b^2*d*x^2-I*b^2*d*Ei(1,-2*I*d*x^2)*exp(2*I*c)*x^2-2*exp(-2*I*c)*Si(2*d*x^2
)*b^2*d*x^2+2*a*b*d*Ei(1,-I*d*x^2)*exp(I*c)*x^2+2*Ei(1,-I*d*x^2)*exp(-I*c)
*a*b*d*x^2+4*sin(d*x^2+c)*a*b-b^2*cos(2*d*x^2+2*c)+2*a^2+b^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$$
$$= \frac{2\,abdx^2\cos(c)\operatorname{Ci}(dx^2)+b^2dx^2\operatorname{Ci}(2\,dx^2)\sin(2\,c)+b^2dx^2\cos(2\,c)\operatorname{Si}(2\,dx^2)-2\,abdx^2\sin(c)\operatorname{Si}(dx^2)+}{2\,x^2}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*d*x^2*cos(c)*cos_integral(d*x^2) + b^2*d*x^2*cos_integral(2*d*x
^2)*sin(2*c) + b^2*d*x^2*cos(2*c)*sin_integral(2*d*x^2) - 2*a*b*d*x^2*sin(
c)*sin_integral(d*x^2) + b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2
 - b^2)/x^2
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx=\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))**2/x**3,x)
```

output
```
Integral((a + b*sin(c + d*x**2))**2/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3}\,dx$$

$$= \frac{1}{2}\left(\left(\Gamma\left(-1,i\,dx^2\right)+\Gamma\left(-1,-i\,dx^2\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1,i\,dx^2\right)-i\,\Gamma\left(-1,-i\,dx^2\right)\right)\sin\left(c\right)\right)abd$$

$$+ \frac{\left(\left(\left(i\,\Gamma(-1,2i\,dx^2)-i\,\Gamma(-1,-2i\,dx^2)\right)\cos\left(2\,c\right)+\left(\Gamma(-1,2i\,dx^2)+\Gamma(-1,-2i\,dx^2)\right)\sin\left(2\,c\right)\right)dx^2-1\right)b^2}{4\,x^2}$$

$$- \frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*
x^2) - I*gamma(-1, -I*d*x^2))*sin(c))*a*b*d + 1/4*(((I*gamma(-1, 2*I*d*x^2
) - I*gamma(-1, -2*I*d*x^2))*cos(2*c) + (gamma(-1, 2*I*d*x^2) + gamma(-1,
-2*I*d*x^2))*sin(2*c))*d*x^2 - 1)*b^2/x^2 - 1/2*a^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(108) = 216.

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.97

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3}\,dx$$

$$= \frac{4\left(dx^2+c\right)abd^2\cos\left(c\right)\mathrm{Ci}\left(dx^2\right)-4\,abcd^2\cos\left(c\right)\mathrm{Ci}\left(dx^2\right)+2\left(dx^2+c\right)b^2d^2\,\mathrm{Ci}\left(2\,dx^2\right)\sin\left(2\,c\right)-2\,b^2cd^2\,\mathrm{C}}{}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="giac")
```

output

```
1/4*(4*(d*x^2 + c)*a*b*d^2*cos(c)*cos_integral(d*x^2) - 4*a*b*c*d^2*cos(c)
*cos_integral(d*x^2) + 2*(d*x^2 + c)*b^2*d^2*cos_integral(2*d*x^2)*sin(2*c
) - 2*b^2*c*d^2*cos_integral(2*d*x^2)*sin(2*c) - 4*(d*x^2 + c)*a*b*d^2*sin
(c)*sin_integral(d*x^2) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^2) - 2*(d*x^
2 + c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^2) + 2*b^2*c*d^2*cos(2*c)*sin_
integral(-2*d*x^2) + b^2*d^2*cos(2*d*x^2 + 2*c) - 4*a*b*d^2*sin(d*x^2 + c)
 - 2*a^2*d^2 - b^2*d^2)/(d^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx=\int \frac{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}{x^3}\,dx$$

input

```
int((a + b*sin(c + d*x^2))^2/x^3,x)
```

output

```
int((a + b*sin(c + d*x^2))^2/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx=\text{Too large to display}$$

input

```
int((a+b*sin(d*x^2+c))^2/x^3,x)
```

output

```
( - 2*cos(c + d*x**2)*sin(c + d*x**2)*tan((c + d*x**2)/2)**4*a*b - 4*cos(c
 + d*x**2)*sin(c + d*x**2)*tan((c + d*x**2)/2)**2*a*b - 2*cos(c + d*x**2)*
sin(c + d*x**2)*a*b + 24*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**
4*x**3 + 2*tan((c + d*x**2)/2)**2*x**3 + x**3),x)*tan((c + d*x**2)/2)**4*b
**2*x**2 + 48*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*x**3 + 2*
tan((c + d*x**2)/2)**2*x**3 + x**3),x)*tan((c + d*x**2)/2)**2*b**2*x**2 +
24*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*x**3 + 2*tan((c + d*
x**2)/2)**2*x**3 + x**3),x)*b**2*x**2 + 24*int(tan((c + d*x**2)/2)**2/(tan
((c + d*x**2)/2)**4*x + 2*tan((c + d*x**2)/2)**2*x + x),x)*tan((c + d*x**2
)/2)**4*a*b*d*x**2 + 48*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4
*x + 2*tan((c + d*x**2)/2)**2*x + x),x)*tan((c + d*x**2)/2)**2*a*b*d*x**2
+ 24*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*x + 2*tan((c + d*x
**2)/2)**2*x + x),x)*a*b*d*x**2 + 24*int(1/(tan((c + d*x**2)/2)**4*x + 2*t
an((c + d*x**2)/2)**2*x + x),x)*tan((c + d*x**2)/2)**4*a*b*d*x**2 + 48*int
(1/(tan((c + d*x**2)/2)**4*x + 2*tan((c + d*x**2)/2)**2*x + x),x)*tan((c +
 d*x**2)/2)**2*a*b*d*x**2 + 24*int(1/(tan((c + d*x**2)/2)**4*x + 2*tan((c
+ d*x**2)/2)**2*x + x),x)*a*b*d*x**2 - 12*log(x)*tan((c + d*x**2)/2)**4*a*
b*d*x**2 - 24*log(x)*tan((c + d*x**2)/2)**2*a*b*d*x**2 - 12*log(x)*a*b*d*x
**2 - 8*sin(c + d*x**2)*tan((c + d*x**2)/2)**4*a*b - 16*sin(c + d*x**2)*ta
n((c + d*x**2)/2)**2*a*b - 8*sin(c + d*x**2)*a*b - 3*tan((c + d*x**2)/2...
```

3.17 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx &= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}\\ &\quad+\frac{1}{2}b^2d^2\cos(2c)\,\text{CosIntegral}\left(2dx^2\right)\\ &\quad-\frac{1}{2}abd^2\,\text{CosIntegral}\left(dx^2\right)\sin(c)\\ &\quad-\frac{ab\sin\left(c+dx^2\right)}{2x^4}-\frac{b^2d\sin\left(2\left(c+dx^2\right)\right)}{4x^2}\\ &\quad-\frac{1}{2}abd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{1}{2}b^2d^2\sin(2c)\text{Si}\left(2dx^2\right)\end{aligned}$$

output

```
-1/8*(2*a^2+b^2)/x^4-1/2*a*b*d*cos(d*x^2+c)/x^2+1/8*b^2*cos(2*d*x^2+2*c)/x
^4+1/2*b^2*d^2*cos(2*c)*Ci(2*d*x^2)-1/2*a*b*d^2*Ci(d*x^2)*sin(c)-1/2*a*b*s
in(d*x^2+c)/x^4-1/4*b^2*d*sin(2*d*x^2+2*c)/x^2-1/2*a*b*d^2*cos(c)*Si(d*x^2
)-1/2*b^2*d^2*sin(2*c)*Si(2*d*x^2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx =$$
$$-\frac{2a^2+b^2+4abdx^2\cos(c+dx^2)-b^2\cos(2(c+dx^2))-4b^2d^2x^4\cos(2c)\,\mathrm{CosIntegral}(2dx^2)+4abd^2x^4}{}$$

input

```
Integrate[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

output

```
-1/8*(2*a^2 + b^2 + 4*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] -
4*b^2*d^2*x^4*Cos[2*c]*CosIntegral[2*d*x^2] + 4*a*b*d^2*x^4*CosIntegral[d*
x^2]*Sin[c] + 4*a*b*Sin[c + d*x^2] + 2*b^2*d*x^2*Sin[2*(c + d*x^2)] + 4*a*
b*d^2*x^4*Cos[c]*SinIntegral[d*x^2] + 4*b^2*d^2*x^4*Sin[2*c]*SinIntegral[2
*d*x^2])/x^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$$
$$\Big\downarrow \text{3884}$$
$$\int \left(\frac{a^2}{x^5}+\frac{2ab\sin(c+dx^2)}{x^5}-\frac{b^2\cos(2c+2dx^2)}{2x^5}+\frac{b^2}{2x^5}\right)dx$$
$$\Big\downarrow \text{6}$$
$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^5}+\frac{2ab\sin(c+dx^2)}{x^5}-\frac{b^2\cos(2c+2dx^2)}{2x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2+b^2}{8x^4}-\frac{1}{2}abd^2\sin(c)\operatorname{CosIntegral}\left(dx^2\right)-\frac{1}{2}abd^2\cos(c)\operatorname{Si}\left(dx^2\right)-\frac{abd\cos\left(c+dx^2\right)}{2x^2}-\frac{ab\sin\left(c+dx^2\right)}{2x^4}+\frac{1}{2}b^2d^2\cos(2c)\operatorname{CosIntegral}\left(2dx^2\right)-\frac{1}{2}b^2d^2\sin(2c)\operatorname{Si}\left(2dx^2\right)-\frac{b^2d\sin\left(2\left(c+dx^2\right)\right)}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}$$

input

```
Int[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

output

```
-1/8*(2*a^2 + b^2)/x^4 - (a*b*d*Cos[c + d*x^2])/(2*x^2) + (b^2*Cos[2*(c +
d*x^2)])/(8*x^4) + (b^2*d^2*Cos[2*c]*CosIntegral[2*d*x^2])/2 - (a*b*d^2*Co
sIntegral[d*x^2]*Sin[c])/2 - (a*b*Sin[c + d*x^2])/(2*x^4) - (b^2*d*Sin[2*(
c + d*x^2)])/(4*x^2) - (a*b*d^2*Cos[c]*SinIntegral[d*x^2])/2 - (b^2*d^2*Si
n[2*c]*SinIntegral[2*d*x^2])/2
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.15 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{-2i\mathrm{e}^{-2ic}\pi\ \mathrm{csgn}\left(d\,x^{2}\right) b^{2} d^{2} x^{4}-2i\mathrm{e}^{-ic}\ \mathrm{expIntegral}_{1}\left(-id\,x^{2}\right) ab\,d^{2} x^{4}+4i\mathrm{e}^{-2ic}\ \mathrm{Si}\left(2d\,x^{2}\right) b^{2} d^{2} x^{4}+2iab\,d^{2}\ \mathrm{expIntegral}_{1}\left(-id\,x^{2}\right)\mathrm{e}^{ic}}{}$

input
```
int((a+b*sin(d*x^2+c))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*I*exp(-2*I*c)*Pi*csgn(d*x^2)*b^2*d^2*x^4-2*I*exp(-I*c)*Ei(1,-I*d*
x^2)*a*b*d^2*x^4+4*I*exp(-2*I*c)*Si(2*d*x^2)*b^2*d^2*x^4+2*I*a*b*d^2*Ei(1,
-I*d*x^2)*exp(I*c)*x^4-2*exp(-I*c)*Pi*csgn(d*x^2)*a*b*d^2*x^4+2*Ei(1,-2*I*
d*x^2)*exp(-2*I*c)*b^2*d^2*x^4+2*b^2*d^2*Ei(1,-2*I*d*x^2)*exp(2*I*c)*x^4+4
*exp(-I*c)*Si(d*x^2)*a*b*d^2*x^4+4*a*b*x^2*cos(d*x^2+c)*d+2*b^2*x^2*sin(2*
d*x^2+2*c)*d+4*sin(d*x^2+c)*a*b-b^2*cos(2*d*x^2+2*c)+2*a^2+b^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sin\left(c+dx^{2}\right)\right)^{2}}{x^{5}}\,dx = \frac{2\,b^{2}d^{2}x^{4}\cos\left(2\,c\right)\mathrm{Ci}\left(2\,dx^{2}\right)-2\,abd^{2}x^{4}\,\mathrm{Ci}\left(dx^{2}\right)\sin\left(c\right)-2\,b^{2}d^{2}x^{4}\sin\left(2\,c\right)\mathrm{Si}\left(2\,dx^{2}\right)-2\,abd^{2}x^{4}\cos\left(c\right)\mathrm{Si}\,(}{4\,x^{4}}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*d^2*x^4*cos(2*c)*cos_integral(2*d*x^2) - 2*a*b*d^2*x^4*cos_inte
gral(d*x^2)*sin(c) - 2*b^2*d^2*x^4*sin(2*c)*sin_integral(2*d*x^2) - 2*a*b*
d^2*x^4*cos(c)*sin_integral(d*x^2) - 2*a*b*d*x^2*cos(d*x^2 + c) + b^2*cos(
d*x^2 + c)^2 - a^2 - b^2 - 2*(b^2*d*x^2*cos(d*x^2 + c) + a*b)*sin(d*x^2 +
c))/x^4
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx=\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx$$

input

```
integrate((a+b*sin(d*x**2+c))**2/x**5,x)
```

output

```
Integral((a + b*sin(c + d*x**2))**2/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx\\&=\frac{1}{2}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos\left(c\right)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin\left(c\right)\right)abd^2\\&\quad-\frac{\left(4\left(\left(\Gamma(-2,2i\,dx^2)+\Gamma(-2,-2i\,dx^2)\right)\cos\left(2\,c\right)+\left(-i\,\Gamma(-2,2i\,dx^2)+i\,\Gamma(-2,-2i\,dx^2)\right)\sin\left(2\,c\right)\right)d^2x^4+\right.}{8\,x^4}\\&\quad-\frac{a^2}{4\,x^4}\end{aligned}$$

input

```
integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="maxima")
```

output

```
1/2*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*
d*x^2) + gamma(-2, -I*d*x^2))*sin(c))*a*b*d^2 - 1/8*(4*((gamma(-2, 2*I*d*x
^2) + gamma(-2, -2*I*d*x^2))*cos(2*c) + (-I*gamma(-2, 2*I*d*x^2) + I*gamma
(-2, -2*I*d*x^2))*sin(2*c))*d^2*x^4 + 1)*b^2/x^4 - 1/4*a^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(153) = 306$.

Time = 0.14 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.65

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$$
$$= \frac{4\,(dx^2+c)^2 b^2 d^3 \cos(2\,c)\,\mathrm{Ci}\,(2\,dx^2) - 8\,(dx^2+c) b^2 c d^3 \cos(2\,c)\,\mathrm{Ci}\,(2\,dx^2) + 4\,b^2 c^2 d^3 \cos(2\,c)\,\mathrm{Ci}\,(2\,dx^2) -}{}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="giac")
```

output
```
1/8*(4*(d*x^2 + c)^2*b^2*d^3*cos(2*c)*cos_integral(2*d*x^2) - 8*(d*x^2 + c
)*b^2*c*d^3*cos(2*c)*cos_integral(2*d*x^2) + 4*b^2*c^2*d^3*cos(2*c)*cos_in
tegral(2*d*x^2) - 4*(d*x^2 + c)^2*a*b*d^3*cos_integral(d*x^2)*sin(c) + 8*(
d*x^2 + c)*a*b*c*d^3*cos_integral(d*x^2)*sin(c) - 4*a*b*c^2*d^3*cos_integr
al(d*x^2)*sin(c) - 4*(d*x^2 + c)^2*a*b*d^3*cos(c)*sin_integral(d*x^2) + 8*
(d*x^2 + c)*a*b*c*d^3*cos(c)*sin_integral(d*x^2) - 4*a*b*c^2*d^3*cos(c)*si
n_integral(d*x^2) + 4*(d*x^2 + c)^2*b^2*d^3*sin(2*c)*sin_integral(-2*d*x^2
) - 8*(d*x^2 + c)*b^2*c*d^3*sin(2*c)*sin_integral(-2*d*x^2) + 4*b^2*c^2*d^
3*sin(2*c)*sin_integral(-2*d*x^2) - 4*(d*x^2 + c)*a*b*d^3*cos(d*x^2 + c) +
 4*a*b*c*d^3*cos(d*x^2 + c) - 2*(d*x^2 + c)*b^2*d^3*sin(2*d*x^2 + 2*c) + 2
*b^2*c*d^3*sin(2*d*x^2 + 2*c) + b^2*d^3*cos(2*d*x^2 + 2*c) - 4*a*b*d^3*sin
(d*x^2 + c) - 2*a^2*d^3 - b^2*d^3)/(((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2
)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = \int \frac{(a+b\,\sin(d\,x^2+c))^2}{x^5}\,dx$$

input
```
int((a + b*sin(c + d*x^2))^2/x^5,x)
```

output
```
int((a + b*sin(c + d*x^2))^2/x^5, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx = \text{Too large to display}$$

input

```
int((a+b*sin(d*x^2+c))^2/x^5,x)
```

output

```
( - 10*cos(c + d*x**2)*sin(c + d*x**2)*tan((c + d*x**2)/2)**4*a*b + 10*cos
(c + d*x**2)*sin(c + d*x**2)*tan((c + d*x**2)/2)**4*b**2*d*x**2 - 20*cos(c
 + d*x**2)*sin(c + d*x**2)*tan((c + d*x**2)/2)**2*a*b + 20*cos(c + d*x**2)
*sin(c + d*x**2)*tan((c + d*x**2)/2)**2*b**2*d*x**2 - 10*cos(c + d*x**2)*s
in(c + d*x**2)*a*b + 10*cos(c + d*x**2)*sin(c + d*x**2)*b**2*d*x**2 - 40*c
os(c + d*x**2)*tan((c + d*x**2)/2)**4*b**2 - 80*cos(c + d*x**2)*tan((c + d
*x**2)/2)**2*b**2 - 40*cos(c + d*x**2)*b**2 + 96*int(tan((c + d*x**2)/2)**
3/(tan((c + d*x**2)/2)**4*x**5 + 2*tan((c + d*x**2)/2)**2*x**5 + x**5),x)*
tan((c + d*x**2)/2)**4*a*b*x**4 + 192*int(tan((c + d*x**2)/2)**3/(tan((c +
 d*x**2)/2)**4*x**5 + 2*tan((c + d*x**2)/2)**2*x**5 + x**5),x)*tan((c + d*
x**2)/2)**2*a*b*x**4 + 96*int(tan((c + d*x**2)/2)**3/(tan((c + d*x**2)/2)*
*4*x**5 + 2*tan((c + d*x**2)/2)**2*x**5 + x**5),x)*a*b*x**4 - 480*int(tan(
(c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*x + 2*tan((c + d*x**2)/2)**2*x
+ x),x)*tan((c + d*x**2)/2)**4*b**2*d**2*x**4 - 960*int(tan((c + d*x**2)/2
)**2/(tan((c + d*x**2)/2)**4*x + 2*tan((c + d*x**2)/2)**2*x + x),x)*tan((c
 + d*x**2)/2)**2*b**2*d**2*x**4 - 480*int(tan((c + d*x**2)/2)**2/(tan((c +
 d*x**2)/2)**4*x + 2*tan((c + d*x**2)/2)**2*x + x),x)*b**2*d**2*x**4 + 96*
int(1/(tan((c + d*x**2)/2)**4*x**3 + 2*tan((c + d*x**2)/2)**2*x**3 + x**3)
,x)*tan((c + d*x**2)/2)**4*a*b*d*x**4 + 192*int(1/(tan((c + d*x**2)/2)**4*
x**3 + 2*tan((c + d*x**2)/2)**2*x**3 + x**3),x)*tan((c + d*x**2)/2)**2*...
```

3.18 $\int x^4(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 247

$$
\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = {} & \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d} \\
& -\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2} \\
& +\frac{3b^2\sqrt{\pi}\cos(2c)\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}} \\
& -\frac{3ab\sqrt{\frac{\pi}{2}}\cos(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}} \\
& -\frac{3ab\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{5/2}} \\
& -\frac{3b^2\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{64d^{5/2}} \\
& +\frac{3abx\sin\left(c+dx^2\right)}{2d^2}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d}
\end{aligned}
$$

output
```
1/10*(2*a^2+b^2)*x^5-a*b*x^3*cos(d*x^2+c)/d-3/32*b^2*x*cos(2*d*x^2+2*c)/d^
2+3/64*b^2*Pi^(1/2)*cos(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1/2))/d^(5/2)-3/4*a*
b*2^(1/2)*Pi^(1/2)*cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)/d^(5/2)-3/4
*a*b*2^(1/2)*Pi^(1/2)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)/d^(5/2)-
3/64*b^2*Pi^(1/2)*FresnelS(2*d^(1/2)*x/Pi^(1/2))*sin(2*c)/d^(5/2)+3/2*a*b*
x*sin(d*x^2+c)/d^2-1/8*b^2*x^3*sin(2*d*x^2+2*c)/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.95

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

$$=\frac{64a^2d^{5/2}x^5+32b^2d^{5/2}x^5-320abd^{3/2}x^3\cos\left(c+dx^2\right)-30b^2\sqrt{d}x\cos\left(2(c+dx^2)\right)+15b^2\sqrt{\pi}\cos(2c)\,\text{Fres}}{}$$

input
```
Integrate[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(64*a^2*d^(5/2)*x^5 + 32*b^2*d^(5/2)*x^5 - 320*a*b*d^(3/2)*x^3*Cos[c + d*x
^2] - 30*b^2*Sqrt[d]*x*Cos[2*(c + d*x^2)] + 15*b^2*Sqrt[Pi]*Cos[2*c]*Fresn
elC[(2*Sqrt[d]*x)/Sqrt[Pi]] - 240*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*S
qrt[2/Pi]*x] - 240*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] -
15*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 480*a*b*Sqrt[d
]*x*Sin[c + d*x^2] - 40*b^2*d^(3/2)*x^3*Sin[2*(c + d*x^2)])/(320*d^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

$$\Big\downarrow \; 3884$$

$$\int\left(a^2x^4+2abx^4\sin\left(c+dx^2\right)-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+\frac{b^2x^4}{2}\right)dx$$

$$\Big\downarrow \; 6$$

$$\int\left(x^4\left(a^2+\frac{b^2}{2}\right)+2abx^4\sin\left(c+dx^2\right)-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{3\sqrt{\frac{\pi}{2}}ab\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}ab\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2}-\frac{abx^3\cos\left(c+dx^2\right)}{d}+\frac{3\sqrt{\pi}b^2\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3\sqrt{\pi}b^2\sin(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d}$$

input

```
Int[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

output

```
((2*a^2 + b^2)*x^5)/10 - (a*b*x^3*Cos[c + d*x^2])/d - (3*b^2*x*Cos[2*c + 2
*d*x^2])/(32*d^2) + (3*b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[P
i]])/(64*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x
])/(2*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/
(2*d^(5/2)) - (3*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(
64*d^(5/2)) + (3*a*b*x*Sin[c + d*x^2])/(2*d^2) - (b^2*x^3*Sin[2*c + 2*d*x^
2])/(8*d)
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.75

method	result
parts	$\frac{x^{5}a^{2}}{5}+b^{2}\left(\frac{x^{5}}{10}-\frac{x^{3}\sin\left(2d\,x^{2}+2c\right)}{8d}+\frac{-\frac{3x\cos\left(2d\,x^{2}+2c\right)}{32d}+\frac{3\sqrt{\pi}\left(\cos\left(2c\right)\operatorname{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\sin\left(2c\right)\operatorname{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)}{64d^{\frac{3}{2}}}}{d}\right)+2a$
default	$\frac{\left(a^{2}+\frac{b^{2}}{2}\right)x^{5}}{5}-\frac{b^{2}\left(\frac{x^{3}\sin\left(2d\,x^{2}+2c\right)}{4d}-\frac{3\left(-\frac{x\cos\left(2d\,x^{2}+2c\right)}{4d}+\frac{\sqrt{\pi}\left(\cos\left(2c\right)\operatorname{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\sin\left(2c\right)\operatorname{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}\right)}{4d}\right)}{2}+2ab\Bigg($
risch	$\frac{x^{5}a^{2}}{5}-\frac{3iab\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{8d^{2}\sqrt{id}}+\frac{b^{2}x^{5}}{10}+\frac{3b^{2}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{256d^{2}\sqrt{id}}+\frac{3b^{2}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2id}\,x\right)\mathrm{e}^{2ic}}{128d^{2}\sqrt{-2id}}+\frac{3iab\sqrt{\pi}\,\operatorname{erf}(\sqrt{-}}{8d^{2}\sqrt{-id}}$

input
```
int(x^4*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*a^2+b^2*(1/10*x^5-1/8/d*x^3*sin(2*d*x^2+2*c)+3/8/d*(-1/4/d*x*cos(2
*d*x^2+2*c)+1/8/d^(3/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1/2))-
sin(2*c)*FresnelS(2*d^(1/2)*x/Pi^(1/2)))))+2*a*b*(-1/2/d*x^3*cos(d*x^2+c)+
3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(
d^(1/2)*2^(1/2)/Pi^(1/2)*x)+sin(c)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.87

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

$$=\frac{32\left(2a^2+b^2\right)d^3x^5-320abd^2x^3\cos\left(dx^2+c\right)-60b^2dx\cos\left(dx^2+c\right)^2-240\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\right.}{}$$

input

```
integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output

```
1/320*(32*(2*a^2 + b^2)*d^3*x^5 - 320*a*b*d^2*x^3*cos(d*x^2 + c) - 60*b^2*
d*x*cos(d*x^2 + c)^2 - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sq
rt(2)*x*sqrt(d/pi)) - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*
sqrt(d/pi))*sin(c) + 15*pi*b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/
pi)) - 15*pi*b^2*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 30*b^2*
d*x - 80*(b^2*d^2*x^3*cos(d*x^2 + c) - 6*a*b*d*x)*sin(d*x^2 + c))/d^3
```

Sympy [F]

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

input

```
integrate(x**4*(a+b*sin(d*x**2+c))**2,x)
```

output

```
Integral(x**4*(a + b*sin(c + d*x**2))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \frac{1}{5}\,a^2x^5$$
$$-\frac{\Big(16\,d^3x^3\cos\left(dx^2+c\right)-24\,d^2x\sin\left(dx^2+c\right)+3\,\sqrt{2}\sqrt{\pi}\Big(((i+1)\,\cos\left(c\right)-(i-1)\,\sin\left(c\right))\,\mathrm{erf}\left(\sqrt{i\,d}x\right)}{16\,d^4}$$
$$+\frac{\Big(256\,d^4x^5-320\,d^3x^3\sin\left(2\,dx^2+2\,c\right)-240\,d^2x\cos\left(2\,dx^2+2\,c\right)+15\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big((-(i-1)\,\cos\left(2\,c\right)}{2560\,d^4}$$

input
```
integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 - 1/16*(16*d^3*x^3*cos(d*x^2 + c) - 24*d^2*x*sin(d*x^2 + c) +
3*sqrt(2)*sqrt(pi)*(((I + 1)*cos(c) - (I - 1)*sin(c))*erf(sqrt(I*d)*x) + (
-(I - 1)*cos(c) + (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*d^(3/2))*a*b/d^4 + 1/
2560*(256*d^4*x^5 - 320*d^3*x^3*sin(2*d*x^2 + 2*c) - 240*d^2*x*cos(2*d*x^2
 + 2*c) + 15*4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(2*c) - (I + 1)*sin(2*
c))*erf(sqrt(2*I*d)*x) + ((I + 1)*cos(2*c) + (I - 1)*sin(2*c))*erf(sqrt(-2
*I*d)*x))*d^(3/2))*b^2/d^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.33

$$
\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \frac{1}{5}\,a^2x^5+\frac{1}{10}\,b^2x^5 \\
&\quad-\frac{3i\,\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{8\,d^2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} \\
&\quad+\frac{3i\,\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{8\,d^2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} \\
&\quad-\frac{3\,\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{(2i\,c)}}{128\,d^{\frac{5}{2}}\left(-\frac{i\,d}{|d|}+1\right)} \\
&\quad-\frac{3\,\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(\frac{i\,d}{|d|}+1\right)\right)e^{(-2i\,c)}}{128\,d^{\frac{5}{2}}\left(\frac{i\,d}{|d|}+1\right)} \\
&\quad-\frac{(-4i\,b^2dx^3+3\,b^2x)e^{(2i\,dx^2+2i\,c)}}{64\,d^2} \\
&\quad+\frac{i\,(2i\,abdx^3-3\,abx)e^{(i\,dx^2+i\,c)}}{4\,d^2} \\
&\quad+\frac{i\,(2i\,abdx^3+3\,abx)e^{(-i\,dx^2-i\,c)}}{4\,d^2} \\
&\quad-\frac{(4i\,b^2dx^3+3\,b^2x)e^{(-2i\,dx^2-2i\,c)}}{64\,d^2}
\end{aligned}
$$

input

```
integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output

```
1/5*a^2*x^5 + 1/10*b^2*x^5 - 3/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x
*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d
))) + 3/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(
abs(d)))*e^(-I*c)/(d^2*(I*d/abs(d) + 1)*sqrt(abs(d))) - 3/128*sqrt(pi)*b^2
*erf(-sqrt(d)*x*(-I*d/abs(d) + 1))*e^(2*I*c)/(d^(5/2)*(-I*d/abs(d) + 1)) -
 3/128*sqrt(pi)*b^2*erf(-sqrt(d)*x*(I*d/abs(d) + 1))*e^(-2*I*c)/(d^(5/2)*(
I*d/abs(d) + 1)) - 1/64*(-4*I*b^2*d*x^3 + 3*b^2*x)*e^(2*I*d*x^2 + 2*I*c)/d
^2 + 1/4*I*(2*I*a*b*d*x^3 - 3*a*b*x)*e^(I*d*x^2 + I*c)/d^2 + 1/4*I*(2*I*a*
b*d*x^3 + 3*a*b*x)*e^(-I*d*x^2 - I*c)/d^2 - 1/64*(4*I*b^2*d*x^3 + 3*b^2*x)
*e^(-2*I*d*x^2 - 2*I*c)/d^2
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a+b\sin(c+dx^2))^2\,dx = \int x^4\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2\,dx$$

input

```
int(x^4*(a + b*sin(c + d*x^2))^2,x)
```

output

```
int(x^4*(a + b*sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int x^4 (a+b\sin(c+dx^2))^2\,dx$$
$$= \frac{-10\cos\left(d\,x^2+c\right)abd\,x^3-15\left(\int\sin\left(d\,x^2+c\right)dx\right)ab+10\left(\int\sin\left(d\,x^2+c\right)^2x^4dx\right)b^2d^2+15\sin\left(d\,x^2+\right.}{10d^2}$$

input

```
int(x^4*(a+b*sin(d*x^2+c))^2,x)
```

output

```
( - 10*cos(c + d*x**2)*a*b*d*x**3 - 15*int(sin(c + d*x**2),x)*a*b + 10*int
(sin(c + d*x**2)**2*x**4,x)*b**2*d**2 + 15*sin(c + d*x**2)*a*b*x + 2*a**2*
d**2*x**5)/(10*d**2)
```

3.19 $\int x^2(a+b\sin(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 198

$$\begin{aligned}\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=&\ \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}\\&+\frac{ab\sqrt{\frac{\pi}{2}}\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}\\&+\frac{b^2\sqrt{\pi}\cos(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}\\&-\frac{ab\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{d^{3/2}}\\&+\frac{b^2\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{16d^{3/2}}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}\end{aligned}$$

output

```
1/6*(2*a^2+b^2)*x^3-a*b*x*cos(d*x^2+c)/d+1/2*a*b*2^(1/2)*Pi^(1/2)*cos(c)*F
resnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)/d^(3/2)+1/16*b^2*Pi^(1/2)*cos(2*c)*Fre
snelS(2*d^(1/2)*x/Pi^(1/2))/d^(3/2)-1/2*a*b*2^(1/2)*Pi^(1/2)*FresnelS(d^(1
/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)/d^(3/2)+1/16*b^2*Pi^(1/2)*FresnelC(2*d^(1/2
)*x/Pi^(1/2))*sin(2*c)/d^(3/2)-1/8*b^2*x*sin(2*d*x^2+2*c)/d
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{16a^2d^{3/2}x^3+8b^2d^{3/2}x^3-48ab\sqrt{d}x\cos\left(c+dx^2\right)+24ab\sqrt{2\pi}\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+3b^2\sqrt{\pi}\cos(2c}{}$$

input
```
Integrate[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(16*a^2*d^(3/2)*x^3 + 8*b^2*d^(3/2)*x^3 - 48*a*b*Sqrt[d]*x*Cos[c + d*x^2]
+ 24*a*b*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 3*b^2*Sqrt[Pi]
*Cos[2*c]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 24*a*b*Sqrt[2*Pi]*FresnelS[Sq
rt[d]*Sqrt[2/Pi]*x]*Sin[c] + 3*b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi
]]*Sin[2*c] - 6*b^2*Sqrt[d]*x*Sin[2*(c + d*x^2)])/(48*d^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(a^2x^2+2abx^2\sin\left(c+dx^2\right)-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+\frac{b^2x^2}{2}\right)dx$$

$$\Big\downarrow \text{6}$$

$$\int \left(x^2\left(a^2+\frac{b^2}{2}\right)+2abx^2\sin\left(c+dx^2\right)-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^3\left(2a^2+b^2\right)+\frac{\sqrt{\frac{\pi}{2}}ab\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}ab\sin(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{\sqrt{\pi}b^2\sin(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}+\frac{\sqrt{\pi}b^2\cos(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}$$

input

```
Int[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

output

```
((2*a^2 + b^2)*x^3)/6 - (a*b*x*Cos[c + d*x^2])/d + (a*b*Sqrt[Pi/2]*Cos[c]*
FresnelC[Sqrt[d]*Sqrt[2/Pi]*x])/d^(3/2) + (b^2*Sqrt[Pi]*Cos[2*c]*FresnelS[
(2*Sqrt[d]*x)/Sqrt[Pi]])/(16*d^(3/2)) - (a*b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*S
qrt[2/Pi]*x]*Sin[c])/d^(3/2) + (b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[P
i]]*Sin[2*c])/(16*d^(3/2)) - (b^2*x*Sin[2*c + 2*d*x^2])/(8*d)
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
 + (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

method	result
parts	$\frac{x^{3} a^{2}}{3}+b^{2} \left(\frac{x^{3}}{6}-\frac{x \sin \left(2 d \,x^{2}+2 c \right)}{8 d}+\frac{\sqrt{\pi}\, \left(\cos \left(2 c \right) \operatorname{FresnelS}\left(\frac{2 \sqrt{d}\, x}{\sqrt{\pi}}\right)+\sin \left(2 c \right) \operatorname{FresnelC}\left(\frac{2 \sqrt{d}\, x}{\sqrt{\pi}}\right)\right)}{16 d^{\frac{3}{2}}}\right)+2 a b \left(-\frac{x \cos \left(d \,x^{2}+\right.}{2 d}\right.$
default	$\frac{\left(a^{2}+\frac{b^{2}}{2}\right) x^{3}}{3}-\frac{b^{2} \left(\frac{x \sin \left(2 d \,x^{2}+2 c \right)}{4 d}-\frac{\sqrt{\pi}\, \left(\cos \left(2 c \right) \operatorname{FresnelS}\left(\frac{2 \sqrt{d}\, x}{\sqrt{\pi}}\right)+\sin \left(2 c \right) \operatorname{FresnelC}\left(\frac{2 \sqrt{d}\, x}{\sqrt{\pi}}\right)\right)}{8 d^{\frac{3}{2}}}\right)}{2}+2 a b \left(-\frac{x \cos \left(d \,x^{2}+c \right)}{2 d}+\right.$
risch	$\frac{i b^{2} \sqrt{\pi}\, \sqrt{2}\, \operatorname{erf}\left(\sqrt{2}\, \sqrt{i d}\, x \right) {\mathrm e}^{-2 i c}}{64 d \sqrt{i d}}-\frac{i b^{2} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-2 i d}\, x \right) {\mathrm e}^{2 i c}}{32 d \sqrt{-2 i d}}+\frac{a b \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-i d}\, x \right) {\mathrm e}^{i c}}{4 d \sqrt{-i d}}+\frac{a b \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{i d}\, x \right) {\mathrm e}^{-i c}}{4 d \sqrt{i d}}+\frac{x^{3} a^{2}}{3}+$

input
```
int(x^2*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a^2+b^2*(1/6*x^3-1/8/d*x*sin(2*d*x^2+2*c)+1/16/d^(3/2)*Pi^(1/2)*(c
os(2*c)*FresnelS(2*d^(1/2)*x/Pi^(1/2))+sin(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1
/2))))+2*a*b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*F
resnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)-sin(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/
2)*x)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.89

$$\int x^2 (a + b \sin(c + dx^2))^2 \, dx$$
$$= \frac{8\,(2\,a^2 + b^2)d^2x^3 + 24\,\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right) - 12\,b^2dx\cos(dx^2+c)\sin(dx^2+c) - 24\,\sqrt{2}\pi ab}{}$$

input
```
integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/48*(8*(2*a^2 + b^2)*d^2*x^3 + 24*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresne
l_cos(sqrt(2)*x*sqrt(d/pi)) - 12*b^2*d*x*cos(d*x^2 + c)*sin(d*x^2 + c) - 2
4*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + 3*p
i*b^2*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + 3*pi*b^2*sqrt(d/pi
)*fresnel_cos(2*x*sqrt(d/pi))*sin(2*c) - 48*a*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [F]

$$\int x^2 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int x^2 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral(x**2*(a + b*sin(c + d*x**2))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int x^2 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\frac{1}{3}\,a^2x^3$$
$$-\frac{\Big(8\,d^2x\cos\left(dx^2+c\right)+\sqrt{2}\sqrt{\pi}\Big(\left(\left(i-1\right)\,\cos\left(c\right)+\left(i+1\right)\,\sin\left(c\right)\right)\mathrm{erf}\left(\sqrt{i}\,dx\right)+\left(-\left(i+1\right)\,\cos\left(c\right)-(i-}{8\,d^3}$$
$$+\frac{\Big(64\,d^3x^3-48\,d^2x\sin\left(2\,dx^2+2\,c\right)+3\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left(\left(i+1\right)\,\cos\left(2\,c\right)-\left(i-1\right)\,\sin\left(2\,c\right)\right)\mathrm{erf}\left(\sqrt{2i}\,dx\right)+}{384\,d^3}$$

input
```
integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 - 1/8*(8*d^2*x*cos(d*x^2 + c) + sqrt(2)*sqrt(pi)*(((I - 1)*cos
(c) + (I + 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I + 1)*cos(c) - (I - 1)*sin(c)
)*erf(sqrt(-I*d)*x))*d^(3/2))*a*b/d^3 + 1/384*(64*d^3*x^3 - 48*d^2*x*sin(2
*d*x^2 + 2*c) + 3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*c) - (I - 1)*si
n(2*c))*erf(sqrt(2*I*d)*x) + (-(I - 1)*cos(2*c) + (I + 1)*sin(2*c))*erf(sq
rt(-2*I*d)*x))*d^(3/2))*b^2/d^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.43

$$\begin{aligned}\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = {} & \frac{1}{3}\,a^2x^3+\frac{1}{6}\,b^2x^3+\frac{i\,b^2xe^{\left(2i\,dx^2+2i\,c\right)}}{16\,d}-\frac{abxe^{\left(i\,dx^2+i\,c\right)}}{2\,d} \\ & -\frac{abxe^{\left(-i\,dx^2-i\,c\right)}}{2\,d}-\frac{i\,b^2xe^{\left(-2i\,dx^2-2i\,c\right)}}{16\,d} \\ & -\frac{\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{4\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} \\ & -\frac{\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{4\,d\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} \\ & +\frac{i\,\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{(2i\,c)}}{32\,d^{\frac{3}{2}}\left(-\frac{i\,d}{|d|}+1\right)} \\ & -\frac{i\,\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(\frac{i\,d}{|d|}+1\right)\right)e^{(-2i\,c)}}{32\,d^{\frac{3}{2}}\left(\frac{i\,d}{|d|}+1\right)}\end{aligned}$$

input
```
integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
1/3*a^2*x^3 + 1/6*b^2*x^3 + 1/16*I*b^2*x*e^(2*I*d*x^2 + 2*I*c)/d - 1/2*a*b
*x*e^(I*d*x^2 + I*c)/d - 1/2*a*b*x*e^(-I*d*x^2 - I*c)/d - 1/16*I*b^2*x*e^(
-2*I*d*x^2 - 2*I*c)/d - 1/4*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(-I*d/
abs(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/4
*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^
(-I*c)/(d*(I*d/abs(d) + 1)*sqrt(abs(d))) + 1/32*I*sqrt(pi)*b^2*erf(-sqrt(d
)*x*(-I*d/abs(d) + 1))*e^(2*I*c)/(d^(3/2)*(-I*d/abs(d) + 1)) - 1/32*I*sqrt
(pi)*b^2*erf(-sqrt(d)*x*(I*d/abs(d) + 1))*e^(-2*I*c)/(d^(3/2)*(I*d/abs(d)
+ 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+dx^2))^2\,dx=\int x^2\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2dx$$

input
```
int(x^2*(a + b*sin(c + d*x^2))^2,x)
```

output
```
int(x^2*(a + b*sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int x^2(a+b\sin(c+dx^2))^2\,dx=\left(\int\sin\left(d\,x^2+c\right)^2x^2dx\right)b^2+2\left(\int\sin\left(d\,x^2+c\right)x^2dx\right)ab+\frac{a^2x^3}{3}$$

input
```
int(x^2*(a+b*sin(d*x^2+c))^2,x)
```

output
```
(3*int(sin(c + d*x**2)**2*x**2,x)*b**2 + 6*int(sin(c + d*x**2)*x**2,x)*a*b
 + a**2*x**3)/3
```

3.20 $\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 153

$$\begin{aligned}\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = {} & \frac{1}{2}\left(2a^2+b^2\right)x-\frac{b^2\sqrt{\pi}\cos(2c)\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}} \\ & +\frac{ab\sqrt{2\pi}\cos(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}} \\ & +\frac{ab\sqrt{2\pi}\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}} \\ & +\frac{b^2\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)}{4\sqrt{d}}\end{aligned}$$

output

```
1/2*(2*a^2+b^2)*x-1/4*b^2*Pi^(1/2)*cos(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1/2))
/d^(1/2)+a*b*2^(1/2)*Pi^(1/2)*cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)/
d^(1/2)+a*b*2^(1/2)*Pi^(1/2)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)/d
^(1/2)+1/4*b^2*Pi^(1/2)*FresnelS(2*d^(1/2)*x/Pi^(1/2))*sin(2*c)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{4a^2\sqrt{d}x+2b^2\sqrt{d}x-b^2\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+4ab\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+4ab\sqrt{2\pi}\,\text{Fr}}{4\sqrt{d}}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(4*a^2*Sqrt[d]*x + 2*b^2*Sqrt[d]*x - b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqr
t[d]*x)/Sqrt[Pi]] + 4*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]
 + 4*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[Pi]*F
resnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx$$
$$\Big\downarrow \text{3838}$$
$$\int \left(a^2+2ab\sin \left(c+dx^2\right)-\frac{1}{2}b^2\cos \left(2c+2dx^2\right)+\frac{b^2}{2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{\sqrt{2\pi}ab\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{\sqrt{2\pi}ab\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}-\frac{\sqrt{\pi}b^2\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{\sqrt{\pi}b^2\sin(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}$$

input
```
Int[(a + b*Sin[c + d*x^2])^2,x]
```

output
```
((2*a^2 + b^2)*x)/2 - (b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[P
i]])/(4*Sqrt[d]) + (a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/
Sqrt[d] + (a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d] +
 (b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

method	result
parts	$a^2x+b^2\left(\frac{x}{2}-\frac{\sqrt{\pi}\left(\cos(2c)\,\mathrm{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)}{4\sqrt{d}}\right)+\frac{ab\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(c)\right.}{\sqrt{d}}$
default	$a^2x+\frac{b^2x}{2}-\frac{b^2\sqrt{\pi}\,\left(\cos(2c)\,\mathrm{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)}{4\sqrt{d}}+\frac{ab\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\,\mathrm{FresnelS}\left(\frac{\sqrt{d}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{Fre}\right.}{\sqrt{d}}$
risch	$a^2x+\frac{iab\,\mathrm{e}^{-ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)}{2\sqrt{id}}+\frac{b^2x}{2}-\frac{b^2\mathrm{e}^{-2ic}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)}{16\sqrt{id}}-\frac{b^2\mathrm{e}^{2ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2id}\,x\right)}{8\sqrt{-2id}}-\frac{iab\,\mathrm{e}^{ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-id}\right.}{2\sqrt{-id}}$

input
```
int((a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2*(1/2*x-1/4*Pi^(1/2)/d^(1/2)*(cos(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1
/2))-sin(2*c)*FresnelS(2*d^(1/2)*x/Pi^(1/2))))+a*b*2^(1/2)*Pi^(1/2)/d^(1/2
)*(cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)+sin(c)*FresnelC(d^(1/2)*2^(
1/2)/Pi^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{4\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-\pi b^2\sqrt{\frac{d}{\pi}}\cos\left(2\,c\right)\mathrm{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)+\pi}{4\,d}$$

input
```
integrate((a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))
+ 4*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) - p
i*b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) + pi*b^2*sqrt(d/pi)*
fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 2*(2*a^2 + b^2)*d*x)/d
```

Sympy [F]

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x**2))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\Big(\left(-(i+1)\,\cos(c)+(i-1)\,\sin(c)\right)\operatorname{erf}\left(\sqrt{i\,d}x\right)+\left((i-1)\,\cos(c)-(i+1)\,\sin(c)\right)\operatorname{erf}\left(\sqrt{-i\,d}\right.}{4\sqrt{d}}$$
$$+\,a^2x$$
$$+\frac{\Big(4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left((i-1)\,\cos(2\,c)+(i+1)\,\sin(2\,c)\right)\operatorname{erf}\left(\sqrt{2i\,d}x\right)+\left(-(i+1)\,\cos(2\,c)-(i-1)\,\sin(2\,c)\right)}{32\,d^2}$$

input
```
integrate((a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(c) + (I - 1)*sin(c))*erf(sqrt(I*d)*x)
 + ((I - 1)*cos(c) - (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*a*b/sqrt(d) + a^2*
x + 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*c) + (I + 1)*sin(2*c))*
erf(sqrt(2*I*d)*x) + (-(I + 1)*cos(2*c) - (I - 1)*sin(2*c))*erf(sqrt(-2*I*
d)*x))*d^(3/2) + 16*d^2*x)*b^2/d^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx =\; & \frac{i\sqrt{2}\sqrt{\pi}ab\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} \\ & -\frac{i\sqrt{2}\sqrt{\pi}ab\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} \\ & +\frac{\sqrt{\pi}b^2\operatorname{erf}\left(-\sqrt{d}x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{(2i\,c)}}{8\sqrt{d}\left(-\frac{i\,d}{|d|}+1\right)} \\ & +\frac{\sqrt{\pi}b^2\operatorname{erf}\left(-\sqrt{d}x\left(\frac{i\,d}{|d|}+1\right)\right)e^{(-2i\,c)}}{8\sqrt{d}\left(\frac{i\,d}{|d|}+1\right)}+\frac{1}{2}\left(2\,a^2+b^2\right)x\end{aligned}$$

input

```
integrate((a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output

```
1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d
)))*e^(I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/2*I*sqrt(2)*sqrt(pi)*a*b*
erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((I*d/abs(d) +
1)*sqrt(abs(d))) + 1/8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(-I*d/abs(d) + 1))*e^(2
*I*c)/(sqrt(d)*(-I*d/abs(d) + 1)) + 1/8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(I*d/a
bs(d) + 1))*e^(-2*I*c)/(sqrt(d)*(I*d/abs(d) + 1)) + 1/2*(2*a^2 + b^2)*x
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx=\int {\left(a+b\,\sin \left(d\,x^2+c\right)\right)}^2\,dx$$

input

```
int((a + b*sin(c + d*x^2))^2,x)
```

output

```
int((a + b*sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int \left(a+b\sin \left(c+dx^2\right)\right)^2\,dx=\left(\int \sin \left(d\,x^2+c\right)^2dx\right)b^2+2\left(\int \sin \left(d\,x^2+c\right)dx\right)ab+a^2x$$

input

```
int((a+b*sin(d*x^2+c))^2,x)
```

output

```
int(sin(c + d*x**2)**2,x)*b**2 + 2*int(sin(c + d*x**2),x)*a*b + a**2*x
```

3.21 $\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = {} & -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^2)}{2x} \\ & + 2ab\sqrt{d}\sqrt{2\pi}\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right) \\ & + b^2\sqrt{d}\sqrt{\pi}\cos(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right) \\ & - 2ab\sqrt{d}\sqrt{2\pi}\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c) \\ & + b^2\sqrt{d}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)-\frac{2ab\sin(c+dx^2)}{x}\end{aligned}$$

output

```
-1/2*(2*a^2+b^2)/x+1/2*b^2*cos(2*d*x^2+2*c)/x+2*a*b*d^(1/2)*2^(1/2)*Pi^(1/
2)*cos(c)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)+b^2*d^(1/2)*Pi^(1/2)*cos(2*
c)*FresnelS(2*d^(1/2)*x/Pi^(1/2))-2*a*b*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(
d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)+b^2*d^(1/2)*Pi^(1/2)*FresnelC(2*d^(1/2)
*x/Pi^(1/2))*sin(2*c)-2*a*b*sin(d*x^2+c)/x
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$$
$$= \frac{-2a^2-b^2+b^2\cos(2(c+dx^2))+4ab\sqrt{d}\sqrt{2\pi}x\cos(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+2b^2\sqrt{d}\sqrt{\pi}x\cos(2c)\,\text{Fresnel}}{}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

output
```
(-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*Cos[c]
*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*Cos[2*c]*Fresne
lS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*FresnelS[Sqrt[d]*S
qrt[2/Pi]*x]*Sin[c] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[d]*x)/Sqrt
[Pi]]*Sin[2*c] - 4*a*b*Sin[c + d*x^2])/(2*x)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$$
$$\Big\downarrow \text{3884}$$
$$\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx^2)}{x^2}-\frac{b^2\cos(2c+2dx^2)}{2x^2}+\frac{b^2}{2x^2}\right)dx$$
$$\Big\downarrow \text{6}$$
$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^2}+\frac{2ab\sin(c+dx^2)}{x^2}-\frac{b^2\cos(2c+2dx^2)}{2x^2}\right)dx$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{2a^2+b^2}{2x}+2\sqrt{2\pi}ab\sqrt{d}\cos(c)\operatorname{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-2\sqrt{2\pi}ab\sqrt{d}\sin(c)\operatorname{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2ab\sin\left(c+dx^2\right)}{x}+\sqrt{\pi}b^2\sqrt{d}\sin(2c)\operatorname{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+\sqrt{\pi}b^2\sqrt{d}\cos(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}$$

input
```
Int[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

output
```
-1/2*(2*a^2 + b^2)/x + (b^2*Cos[2*c + 2*d*x^2])/(2*x) + 2*a*b*Sqrt[d]*Sqrt
[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + b^2*Sqrt[d]*Sqrt[Pi]*Cos[2*
c]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 2*a*b*Sqrt[d]*Sqrt[2*Pi]*FresnelS[Sq
rt[d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[d]*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/S
qrt[Pi]]*Sin[2*c] - (2*a*b*Sin[c + d*x^2])/x
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
 + (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

method	result
parts	$-\frac{a^{2}}{x}+b^{2}\left(-\frac{1}{2x}+\frac{\cos\left(2d\,x^{2}+2c\right)}{2x}+\sqrt{d}\,\sqrt{\pi}\,\left(\cos\left(2c\right)\operatorname{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)+\sin\left(2c\right)\operatorname{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)\right.$
default	$-\frac{a^{2}+\frac{b^{2}}{2}}{x}-\frac{b^{2}\left(-\frac{\cos\left(2d\,x^{2}+2c\right)}{x}-2\sqrt{d}\,\sqrt{\pi}\,\left(\cos(2c)\operatorname{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)+\sin(2c)\operatorname{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)\right)}{2}+2ab\left(-\frac{\sin\left(d\,x^{2}+c\right)}{x}+\right.$
risch	$\frac{ib^{2}d\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{4\sqrt{id}}-\frac{ib^{2}d\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2id}\,x\right)\mathrm{e}^{2ic}}{2\sqrt{-2id}}+\frac{abd\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{\sqrt{-id}}+\frac{abd\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{\sqrt{id}}-\frac{a^{2}}{x}-$

input
```
int((a+b*sin(d*x^2+c))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*a^2+b^2*(-1/2/x+1/2/x*cos(2*d*x^2+2*c)+d^(1/2)*Pi^(1/2)*(cos(2*c)*Fre
snelS(2*d^(1/2)*x/Pi^(1/2))+sin(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1/2))))+2*a*
b*(-sin(d*x^2+c)/x+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(d^(1/2)*2^(1/
2)/Pi^(1/2)*x)-sin(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int\frac{\left(a+b\sin\left(c+dx^{2}\right)\right)^{2}}{x^{2}}\,dx$$
$$=\frac{2\sqrt{2}\pi abx\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-2\sqrt{2}\pi abx\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+\pi b^{2}x\sqrt{\frac{d}{\pi}}\cos\left(2\,c\right)\mathrm{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right)}{x}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="fricas")
```

output
```
(2*sqrt(2)*pi*a*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) -
2*sqrt(2)*pi*a*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + p
i*b^2*x*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + pi*b^2*x*sqrt(d/
pi)*fresnel_cos(2*x*sqrt(d/pi))*sin(2*c) + b^2*cos(d*x^2 + c)^2 - 2*a*b*si
n(d*x^2 + c) - a^2 - b^2)/x
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))**2/x**2,x)
```

output
```
Integral((a + b*sin(c + d*x**2))**2/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx =$$
$$-\frac{\sqrt{dx^2}\,(((i-1)\,\sqrt{2}\Gamma(-\frac{1}{2}, i\,dx^2) - (i+1)\,\sqrt{2}\Gamma(-\frac{1}{2}, -i\,dx^2))\cos(c) + ((i+1)\,\sqrt{2}\Gamma(-\frac{1}{2}, i\,dx^2) - (i -}{4\,x}$$
$$-\frac{(\sqrt{2}\sqrt{dx^2}\,((-(i+1)\,\sqrt{2}\Gamma(-\frac{1}{2}, 2i\,dx^2) + (i-1)\,\sqrt{2}\Gamma(-\frac{1}{2}, -2i\,dx^2))\cos(2\,c) + ((i-1)\,\sqrt{2}\Gamma(-\frac{1}{2}, 2i}{16\,x}$$
$$-\frac{a^2}{x}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

output
```
-1/4*sqrt(d*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I + 1)*sqrt(2)*
gamma(-1/2, -I*d*x^2))*cos(c) + ((I + 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I
 - 1)*sqrt(2)*gamma(-1/2, -I*d*x^2))*sin(c))*a*b/x - 1/16*(sqrt(2)*sqrt(d*
x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*d*x^2) + (I - 1)*sqrt(2)*gamma(-1/
2, -2*I*d*x^2))*cos(2*c) + ((I - 1)*sqrt(2)*gamma(-1/2, 2*I*d*x^2) - (I +
1)*sqrt(2)*gamma(-1/2, -2*I*d*x^2))*sin(2*c)) + 8)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\sin\left(dx^2+c\right)+a\right)^2}{x^2}\,dx$$

input

```
integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="giac")
```

output

```
integrate((b*sin(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}{x^2}\,dx$$

input

```
int((a + b*sin(c + d*x^2))^2/x^2,x)
```

output

```
int((a + b*sin(c + d*x^2))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx=\frac{\left(\int \frac{\sin\left(d\,x^2+c\right)^2}{x^2}dx\right)b^2x+2\left(\int \frac{\sin\left(d\,x^2+c\right)}{x^2}dx\right)abx-a^2}{x}$$

input

```
int((a+b*sin(d*x^2+c))^2/x^2,x)
```

output

```
(int(sin(c + d*x**2)**2/x**2,x)*b**2*x + 2*int(sin(c + d*x**2)/x**2,x)*a*b
*x - a**2)/x
```

3.22 $\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx = &-\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos(c+dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3}\\ &+\frac{4}{3}b^2d^{3/2}\sqrt{\pi}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)-\frac{4}{3}abd^{3/2}\sqrt{2\pi}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\\ &-\frac{4}{3}abd^{3/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{4}{3}b^2d^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)-\frac{2ab\sin(c+dx^2)}{3x^3}\end{aligned}$$

output
```
-1/6*(2*a^2+b^2)/x^3-4/3*a*b*d*cos(d*x^2+c)/x+1/6*b^2*cos(2*d*x^2+2*c)/x^3
+4/3*b^2*d^(3/2)*Pi^(1/2)*cos(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1/2))-4/3*a*b*
d^(3/2)*2^(1/2)*Pi^(1/2)*cos(c)*FresnelS(d^(1/2)*2^(1/2)/Pi^(1/2)*x)-4/3*a
*b*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(c)-4/
3*b^2*d^(3/2)*Pi^(1/2)*FresnelS(2*d^(1/2)*x/Pi^(1/2))*sin(2*c)-2/3*a*b*sin
(d*x^2+c)/x^3-2/3*b^2*d*sin(2*d*x^2+2*c)/x
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx =$$

$$-\frac{2a^2+b^2+8abdx^2\cos\left(c+dx^2\right)-b^2\cos\left(2(c+dx^2)\right)-8b^2d^{3/2}\sqrt{\pi}x^3\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+8abd}{}$$

input
```
Integrate[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

output
```
-1/6*(2*a^2 + b^2 + 8*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] -
8*b^2*d^(3/2)*Sqrt[Pi]*x^3*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]] + 8*a
*b*d^(3/2)*Sqrt[2*Pi]*x^3*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + 8*a*b*d^
(3/2)*Sqrt[2*Pi]*x^3*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + 8*b^2*d^(3/2)
*Sqrt[Pi]*x^3*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 4*a*b*Sin[c + d*
x^2] + 4*b^2*d*x^2*Sin[2*(c + d*x^2)])/x^3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ab\sin\left(c+dx^2\right)}{x^4}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^4}+\frac{b^2}{2x^4}\right)dx$$

$$\Big\downarrow \text{6}$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^4}+\frac{2ab\sin\left(c+dx^2\right)}{x^4}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2+b^2}{6x^3}-\frac{4}{3}\sqrt{2\pi}abd^{3/2}\sin(c)\,\text{FresnelC}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{4}{3}\sqrt{2\pi}abd^{3/2}\cos(c)\,\text{FresnelS}\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{4abd\cos\left(c+dx^2\right)}{3x}-\frac{2ab\sin\left(c+dx^2\right)}{3x^3}+\frac{4}{3}\sqrt{\pi}b^2d^{3/2}\cos(2c)\,\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)-\frac{4}{3}\sqrt{\pi}b^2d^{3/2}\sin(2c)\,\text{FresnelS}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)-\frac{2b^2d\sin\left(2c+2dx^2\right)}{3x}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}$$

input

```
Int[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

output

```
-1/6*(2*a^2 + b^2)/x^3 - (4*a*b*d*Cos[c + d*x^2])/(3*x) + (b^2*Cos[2*c + 2
*d*x^2])/(6*x^3) + (4*b^2*d^(3/2)*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)
/Sqrt[Pi]])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/P
i]*x]])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]
)/3 - (4*b^2*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/3
 - (2*a*b*Sin[c + d*x^2])/(3*x^3) - (2*b^2*d*Sin[2*c + 2*d*x^2])/(3*x)
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

method	result
parts	$-\frac{a^2}{3x^3}+b^2\left(-\frac{1}{6x^3}+\frac{\cos(2d\,x^2+2c)}{6x^3}+\frac{2d\,(-\frac{\sin(2d\,x^2+2c)}{x}+2\sqrt{d}\,\sqrt{\pi}\,(\cos(2c)\,\mathrm{FresnelC}(\frac{2\sqrt{d}\,x}{\sqrt{\pi}})-\sin(2c)\,\mathrm{FresnelS}(\frac{2\sqrt{d}\,x}{\sqrt{\pi}})}{3}\right.$
default	$-\frac{a^2+\frac{b^2}{2}}{3x^3}-\frac{b^2\left(-\frac{\cos(2d\,x^2+2c)}{3x^3}-\frac{4d\left(-\frac{\sin(2d\,x^2+2c)}{x}+2\sqrt{d}\,\sqrt{\pi}\left(\cos(2c)\,\mathrm{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{FresnelS}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)\right)}{3}\right)}{2}+2ab$
risch	$-\frac{2iab\,d^2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{3\sqrt{id}}-\frac{a^2}{3x^3}-\frac{b^2}{6x^3}+\frac{b^2d^2\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{3\sqrt{id}}+\frac{2b^2d^2\sqrt{\pi}\,\mathrm{erf}(\sqrt{-2id}\,x)\mathrm{e}^{2ic}}{3\sqrt{-2id}}+2iab\,d^2$

input
```
int((a+b*sin(d*x^2+c))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*a^2+b^2*(-1/6/x^3+1/6/x^3*cos(2*d*x^2+2*c)+2/3*d*(-1/x*sin(2*d*x^
2+2*c)+2*d^(1/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*d^(1/2)*x/Pi^(1/2))-sin(2*c
)*FresnelS(2*d^(1/2)*x/Pi^(1/2)))))+2*a*b*(-1/3*sin(d*x^2+c)/x^3+2/3*d*(-1
/x*cos(d*x^2+c)-d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(d^(1/2)*2^(1/2)/
Pi^(1/2)*x)+sin(c)*FresnelC(d^(1/2)*2^(1/2)/Pi^(1/2)*x))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx =$$

$$-\;4\sqrt{2}\pi abdx^3\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\pi abdx^3\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)-4\pi b^2dx^3\sqrt{\frac{d}{\pi}}\cos(2c)\,\mathrm{C}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(
d/pi)) + 4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi
))*sin(c) - 4*pi*b^2*d*x^3*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi))
 + 4*pi*b^2*d*x^3*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 4*a*b*
d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c)^2 + a^2 + b^2 + 2*(2*b^2*d*x^2*c
os(d*x^2 + c) + a*b)*sin(d*x^2 + c))/x^3
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx = \int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx$$

input
```
integrate((a+b*sin(d*x**2+c))**2/x**4,x)
```

output
```
Integral((a + b*sin(c + d*x**2))**2/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx =$$
$$-\frac{\sqrt{dx^2}\,\big(\big(-(i+1)\,\sqrt{2}\Gamma\big(-\tfrac{3}{2}, i\,dx^2\big)+(i-1)\,\sqrt{2}\Gamma\big(-\tfrac{3}{2}, -i\,dx^2\big)\big)\cos(c)+\big((i-1)\,\sqrt{2}\Gamma\big(-\tfrac{3}{2}, i\,dx^2\big)-(i}{4\,x}$$
$$-\frac{\Big(3\,\sqrt{2}\sqrt{dx^2}\,\big(\big(-(i-1)\,\sqrt{2}\Gamma\big(-\tfrac{3}{2}, 2i\,dx^2\big)+(i+1)\,\sqrt{2}\Gamma\big(-\tfrac{3}{2}, -2i\,dx^2\big)\big)\cos(2\,c)+\big(-(i+1)\,\sqrt{2}\Gamma\big(-}{24\,x^3}$$
$$-\frac{a^2}{3\,x^3}$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="maxima")
```

output
```
-1/4*sqrt(d*x^2)*((-(I + 1)*sqrt(2)*gamma(-3/2, I*d*x^2) + (I - 1)*sqrt(2)
*gamma(-3/2, -I*d*x^2))*cos(c) + ((I - 1)*sqrt(2)*gamma(-3/2, I*d*x^2) - (
I + 1)*sqrt(2)*gamma(-3/2, -I*d*x^2))*sin(c))*a*b*d/x - 1/24*(3*sqrt(2)*sq
rt(d*x^2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*d*x^2) + (I + 1)*sqrt(2)*gamm
a(-3/2, -2*I*d*x^2))*cos(2*c) + (-(I + 1)*sqrt(2)*gamma(-3/2, 2*I*d*x^2) +
 (I - 1)*sqrt(2)*gamma(-3/2, -2*I*d*x^2))*sin(2*c))*d*x^2 + 4)*b^2/x^3 - 1
/3*a^2/x^3
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx=\int \frac{(b\sin(dx^2+c)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^2 + c) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx=\int \frac{(a+b\,\sin(d\,x^2+c))^2}{x^4}\,dx$$

input
```
int((a + b*sin(c + d*x^2))^2/x^4,x)
```

output
```
int((a + b*sin(c + d*x^2))^2/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx = \frac{3\left(\int \frac{\sin(d\,x^2+c)^2}{x^4}dx\right)b^2x^3+6\left(\int \frac{\sin(d\,x^2+c)}{x^4}dx\right)ab\,x^3-a^2}{3x^3}$$

input

```
int((a+b*sin(d*x^2+c))^2/x^4,x)
```

output

```
(3*int(sin(c + d*x**2)**2/x**4,x)*b**2*x**3 + 6*int(sin(c + d*x**2)/x**4,x
)*a*b*x**3 - a**2)/(3*x**3)
```

3.23 $\int x^5 \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\begin{aligned}\int x^5 \sin^3(a+bx^2)\,dx = {} & \frac{7\cos(a+bx^2)}{9b^3} - \frac{x^4\cos(a+bx^2)}{3b} \\ & - \frac{\cos^3(a+bx^2)}{27b^3} + \frac{2x^2\sin(a+bx^2)}{3b^2} \\ & - \frac{x^4\cos(a+bx^2)\sin^2(a+bx^2)}{6b} + \frac{x^2\sin^3(a+bx^2)}{9b^2}\end{aligned}$$

output
```
7/9*cos(b*x^2+a)/b^3-1/3*x^4*cos(b*x^2+a)/b-1/27*cos(b*x^2+a)^3/b^3+2/3*x^
2*sin(b*x^2+a)/b^2-1/6*x^4*cos(b*x^2+a)*sin(b*x^2+a)^2/b+1/9*x^2*sin(b*x^2
+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int x^5 \sin^3(a+bx^2)\,dx$$
$$= \frac{-81(-2+b^2x^4)\cos(a+bx^2) + (-2+9b^2x^4)\cos(3(a+bx^2)) - 6bx^2(-27\sin(a+bx^2) + \sin(3(a+bx}{216b^3}$$

input
```
Integrate[x^5*Sin[a + b*x^2]^3,x]
```

output
```
(-81*(-2 + b^2*x^4)*Cos[a + b*x^2] + (-2 + 9*b^2*x^4)*Cos[3*(a + b*x^2)] -
 6*b*x^2*(-27*Sin[a + b*x^2] + Sin[3*(a + b*x^2)]))/(216*b^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3860, 3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sin^3\left(a+bx^2\right) dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int x^4 \sin^3\left(bx^2+a\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int x^4 \sin\left(bx^2+a\right)^3 dx^2$$

$$\Big\downarrow \text{3792}$$

$$\frac{1}{2}\left(-\frac{2\int \sin^3\left(bx^2+a\right) dx^2}{9b^2}+\frac{2}{3}\int x^4 \sin\left(bx^2+a\right) dx^2+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2\int \sin\left(bx^2+a\right)^3 dx^2}{9b^2}+\frac{2}{3}\int x^4 \sin\left(bx^2+a\right) dx^2+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{2}\left(\frac{2\int \left(1-x^4\right) d\cos\left(bx^2+a\right)}{9b^3}+\frac{2}{3}\int x^4 \sin\left(bx^2+a\right) dx^2+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+bx\right.}{3b}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}\int x^4\sin\left(bx^2+a\right)dx^2+\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{2\int x^2\cos\left(bx^2+a\right)dx^2}{b}-\frac{x^4\cos\left(a+bx^2\right)}{b}\right)+\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-x^4\sin^2(a$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{2\int x^2\sin\left(bx^2+a+\frac{\pi}{2}\right)dx^2}{b}-\frac{x^4\cos\left(a+bx^2\right)}{b}\right)+\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-x^4\sin$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{2\left(\frac{\int-\sin(bx^2+a)dx^2}{b}+\frac{x^2\sin(a+bx^2)}{b}\right)}{b}-\frac{x^4\cos\left(a+bx^2\right)}{b}\right)+\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{2\left(\frac{x^2\sin(a+bx^2)}{b}-\frac{\int\sin(bx^2+a)dx^2}{b}\right)}{b}-\frac{x^4\cos\left(a+bx^2\right)}{b}\right)+\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{2\left(\frac{x^2\sin(a+bx^2)}{b}-\frac{\int\sin(bx^2+a)dx^2}{b}\right)}{b}-\frac{x^4\cos\left(a+bx^2\right)}{b}\right)+\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}-$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{2}\left(\frac{2\left(\cos\left(a+bx^2\right)-\frac{x^6}{3}\right)}{9b^3}+\frac{2x^2\sin^3\left(a+bx^2\right)}{9b^2}+\frac{2}{3}\left(\frac{2\left(\frac{\cos(a+bx^2)}{b^2}+\frac{x^2\sin(a+bx^2)}{b}\right)}{b}-\frac{x^4\cos\left(a+bx^2\right)}{b}\right)-x^4\sin$$

input

```
Int[x^5*Sin[a + b*x^2]^3,x]
```

output

```
((2*(-1/3*x^6 + Cos[a + b*x^2]))/(9*b^3) - (x^4*Cos[a + b*x^2]*Sin[a + b*x
^2]^2)/(3*b) + (2*x^2*Sin[a + b*x^2]^3)/(9*b^2) + (2*(-((x^4*Cos[a + b*x^2
])/b) + (2*(Cos[a + b*x^2]/b^2 + (x^2*Sin[a + b*x^2])/b))/b))/3)/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{3\left(b^{2} x^{4}-2\right) \cos \left(b \,x^{2}+a \right)}{8 b^{3}}+\frac{3 x^{2} \sin \left(b \,x^{2}+a \right)}{4 b^{2}}+\frac{\left(9 b^{2} x^{4}-2\right) \cos \left(3 b \,x^{2}+3 a \right)}{216 b^{3}}-\frac{x^{2} \sin \left(3 b \,x^{2}+3 a \right)}{36 b^{2}}$
default	$-\frac{3 x^{4} \cos \left(b \,x^{2}+a \right)}{8 b}+\frac{\frac{3 x^{2} \sin \left(b \,x^{2}+a \right)}{4 b}+\frac{3 \cos \left(b \,x^{2}+a \right)}{4 b^{2}}}{b}+\frac{x^{4} \cos \left(3 b \,x^{2}+3 a \right)}{24 b}-\frac{\frac{x^{2} \sin \left(3 b \,x^{2}+3 a \right)}{6 b}+\frac{\cos \left(3 b \,x^{2}+3 a \right)}{18 b^{2}}}{6 b}$
orering	$\frac{5\left(648 b^{4} x^{8}+91 b^{2} x^{4}-1260\right) \sin \left(b \,x^{2}+a \right)^{3}}{1296 b^{6} x^{6}}-\frac{5\left(72 b^{4} x^{8}+155 b^{2} x^{4}-492\right)\left(5 x^{4} \sin \left(b \,x^{2}+a \right)^{3}+6 x^{6} \sin \left(b \,x^{2}+a \right)^{2} b \cos \left(b \,x^{2}+a \right)\right)}{1296 b^{6} x^{10}}+$

input
```
int(x^5*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*(b^2*x^4-2)/b^3*cos(b*x^2+a)+3/4*x^2*sin(b*x^2+a)/b^2+1/216*(9*b^2*x^
4-2)/b^3*cos(3*b*x^2+3*a)-1/36*x^2/b^2*sin(3*b*x^2+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int x^5 \sin^3\left(a+bx^2\right)\,dx$$

$$=\frac{\left(9\,b^2x^4-2\right)\cos\left(bx^2+a\right)^3-3\left(9\,b^2x^4-14\right)\cos\left(bx^2+a\right)-6\left(bx^2\cos\left(bx^2+a\right)^2-7\,bx^2\right)\sin\left(bx^2+a\right)}{54\,b^3}$$

input
```
integrate(x^5*sin(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/54*((9*b^2*x^4 - 2)*cos(b*x^2 + a)^3 - 3*(9*b^2*x^4 - 14)*cos(b*x^2 + a)
 - 6*(b*x^2*cos(b*x^2 + a)^2 - 7*b*x^2)*sin(b*x^2 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int x^5 \sin^3\left(a+bx^2\right)\,dx$$
$$=\begin{cases} -\frac{x^4\sin^2(a+bx^2)\cos(a+bx^2)}{2b}-\frac{x^4\cos^3(a+bx^2)}{3b}+\frac{7x^2\sin^3(a+bx^2)}{9b^2}+\frac{2x^2\sin(a+bx^2)\cos^2(a+bx^2)}{3b^2}+\frac{7\sin^2(a+bx^2)\cos(a+bx^2}{9b^3} \\ \frac{x^6\sin^3(a)}{6} \end{cases}$$

input
```
integrate(x**5*sin(b*x**2+a)**3,x)
```

output
```
Piecewise((-x**4*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**4*cos(a + b
*x**2)**3/(3*b) + 7*x**2*sin(a + b*x**2)**3/(9*b**2) + 2*x**2*sin(a + b*x*
*2)*cos(a + b*x**2)**2/(3*b**2) + 7*sin(a + b*x**2)**2*cos(a + b*x**2)/(9*
b**3) + 20*cos(a + b*x**2)**3/(27*b**3), Ne(b, 0)), (x**6*sin(a)**3/6, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int x^5 \sin^3\left(a+bx^2\right)\,dx =$$
$$-\frac{6\,bx^2\sin\left(3\,bx^2+3\,a\right)-162\,bx^2\sin\left(bx^2+a\right)-\left(9\,b^2x^4-2\right)\cos\left(3\,bx^2+3\,a\right)+81\left(b^2x^4-2\right)\cos\left(bx^2+\right.}{216\,b^3}$$

input
```
integrate(x^5*sin(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/216*(6*b*x^2*sin(3*b*x^2 + 3*a) - 162*b*x^2*sin(b*x^2 + a) - (9*b^2*x^4
 - 2)*cos(3*b*x^2 + 3*a) + 81*(b^2*x^4 - 2)*cos(b*x^2 + a))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\begin{aligned}\int x^5\sin^3\left(a+bx^2\right)\,dx=&-\frac{x^2\sin\left(3\,bx^2+3\,a\right)}{36\,b^2}+\frac{3\,x^2\sin\left(bx^2+a\right)}{4\,b^2}\\&+\frac{\left(\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)\right)a^2}{6\,b^3}\\&+\frac{\left(9\,(bx^2+a)^2-18\,(bx^2+a)a-2\right)\cos\left(3\,bx^2+3\,a\right)}{216\,b^3}\\&-\frac{3\,\left((bx^2+a)^2-2\,(bx^2+a)a-2\right)\cos\left(bx^2+a\right)}{8\,b^3}\end{aligned}$$

input
```
integrate(x^5*sin(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/36*x^2*sin(3*b*x^2 + 3*a)/b^2 + 3/4*x^2*sin(b*x^2 + a)/b^2 + 1/6*(cos(b
*x^2 + a)^3 - 3*cos(b*x^2 + a))*a^2/b^3 + 1/216*(9*(b*x^2 + a)^2 - 18*(b*x
^2 + a)*a - 2)*cos(3*b*x^2 + 3*a)/b^3 - 3/8*((b*x^2 + a)^2 - 2*(b*x^2 + a)
*a - 2)*cos(b*x^2 + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 38.98 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\begin{aligned}&\int x^5\sin^3\left(a+bx^2\right)\,dx\\&=\frac{\frac{3\,\cos\left(b\,x^2+a\right)}{4}-\frac{\cos\left(3\,b\,x^2+3\,a\right)}{108}+b\,\left(\frac{3\,x^2\,\sin\left(b\,x^2+a\right)}{4}-\frac{x^2\,\sin\left(3\,b\,x^2+3\,a\right)}{36}\right)+b^2\,\left(\frac{x^4\,\cos\left(3\,b\,x^2+3\,a\right)}{24}-\frac{3\,x^4\,\cos\left(b\,x^2+a\right)}{8}\right)}{b^3}\end{aligned}$$

input
```
int(x^5*sin(a + b*x^2)^3,x)
```

output
```
((3*cos(a + b*x^2))/4 - cos(3*a + 3*b*x^2)/108 + b*((3*x^2*sin(a + b*x^2))
/4 - (x^2*sin(3*a + 3*b*x^2))/36) + b^2*((x^4*cos(3*a + 3*b*x^2))/24 - (3*
x^4*cos(a + b*x^2))/8))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int x^5 \sin^3\left(a+bx^2\right) \, dx$$

$$= \frac{-9\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^2 b^2x^4+2\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^2-18\cos\left(b\,x^2+a\right)b^2x^4+40\cos\left(b\,x\right.}{54b^3}$$

input

```
int(x^5*sin(b*x^2+a)^3,x)
```

output

```
( - 9*cos(a + b*x**2)*sin(a + b*x**2)**2*b**2*x**4 + 2*cos(a + b*x**2)*sin
(a + b*x**2)**2 - 18*cos(a + b*x**2)*b**2*x**4 + 40*cos(a + b*x**2) + 6*si
n(a + b*x**2)**3*b*x**2 + 36*sin(a + b*x**2)*b*x**2 + 16)/(54*b**3)
```

3.24 $\int x^3 \sin^3\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = -\frac{x^2\cos\left(a+bx^2\right)}{3b}+\frac{\sin\left(a+bx^2\right)}{3b^2} -\frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b}+\frac{\sin^3\left(a+bx^2\right)}{18b^2}$$

output
```
-1/3*x^2*cos(b*x^2+a)/b+1/3*sin(b*x^2+a)/b^2-1/6*x^2*cos(b*x^2+a)*sin(b*x^
2+a)^2/b+1/18*sin(b*x^2+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = -\frac{27bx^2\cos\left(a+bx^2\right)-3bx^2\cos\left(3\left(a+bx^2\right)\right)-27\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)}{72b^2}$$

input
```
Integrate[x^3*Sin[a + b*x^2]^3,x]
```

output
```
-1/72*(27*b*x^2*Cos[a + b*x^2] - 3*b*x^2*Cos[3*(a + b*x^2)] - 27*Sin[a + b
*x^2] + Sin[3*(a + b*x^2)])/b^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3860, 3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin^3\left(a+bx^2\right)\, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int x^2 \sin^3\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int x^2 \sin\left(bx^2+a\right)^3 dx^2$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{2}\left(\frac{2}{3}\int x^2 \sin\left(bx^2+a\right)dx^2+\frac{\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{3}\int x^2 \sin\left(bx^2+a\right)dx^2+\frac{\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{\int \cos\left(bx^2+a\right)dx^2}{b}-\frac{x^2\cos\left(a+bx^2\right)}{b}\right)+\frac{\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{\int \sin\left(bx^2+a+\frac{\pi}{2}\right)dx^2}{b}-\frac{x^2\cos\left(a+bx^2\right)}{b}\right)+\frac{\sin^3\left(a+bx^2\right)}{9b^2}-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2}\left(\frac{\sin^3\left(a+bx^2\right)}{9b^2}+\frac{2}{3}\left(\frac{\sin\left(a+bx^2\right)}{b^2}-\frac{x^2\cos\left(a+bx^2\right)}{b}\right)-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b}\right)$$

input
```
Int[x^3*Sin[a + b*x^2]^3,x]
```

output
```
(-1/3*(x^2*Cos[a + b*x^2]*Sin[a + b*x^2]^2)/b + Sin[a + b*x^2]^3/(9*b^2) +
 (2*(-((x^2*Cos[a + b*x^2])/b) + Sin[a + b*x^2]/b^2))/3)/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result
default	$-\frac{3x^2\cos(bx^2+a)}{8b}+\frac{3\sin(bx^2+a)}{8b^2}+\frac{x^2\cos(3bx^2+3a)}{24b}-\frac{\sin(3bx^2+3a)}{72b^2}$
risch	$-\frac{3x^2\cos(bx^2+a)}{8b}+\frac{3\sin(bx^2+a)}{8b^2}+\frac{x^2\cos(3bx^2+3a)}{24b}-\frac{\sin(3bx^2+3a)}{72b^2}$
parallelrisch	$-\frac{-24i\left(-\ln\left(\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)-i\right)+\ln\left(\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)+i\right)\right)+24bx^2+27\cos(bx^2+a)bx^2-3bx^2\cos(3bx^2+3a)+\sin(3bx^2+3a)}{72b^2}$
norman	$\frac{\frac{x^2\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^4}{b}+\frac{2\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)}{3b^2}+\frac{16\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^3}{9b^2}+\frac{2\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^5}{3b^2}-\frac{x^2}{3b}-\frac{x^2\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^2}{b}+\frac{x^2\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^6}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^2\right)^3}$
orering	$\frac{5(40b^2x^4+27)\sin(bx^2+a)^3}{144x^4b^4}-\frac{(40b^2x^4+71)\left(3x^2\sin(bx^2+a)^3+6x^4\sin(bx^2+a)^2b\cos(bx^2+a)\right)}{144b^4x^6}+\frac{\frac{7x\sin(bx^2+a)^3}{12}+}{}$

input
```
int(x^3*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*x^2*cos(b*x^2+a)/b+3/8*sin(b*x^2+a)/b^2+1/24/b*x^2*cos(3*b*x^2+3*a)-1
/72/b^2*sin(3*b*x^2+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3\sin^3\left(a+bx^2\right)\,dx$$
$$=\frac{3\,bx^2\cos\left(bx^2+a\right)^3-9\,bx^2\cos\left(bx^2+a\right)-\left(\cos\left(bx^2+a\right)^2-7\right)\sin\left(bx^2+a\right)}{18\,b^2}$$

input
```
integrate(x^3*sin(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/18*(3*b*x^2*cos(b*x^2 + a)^3 - 9*b*x^2*cos(b*x^2 + a) - (cos(b*x^2 + a)^
2 - 7)*sin(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \begin{cases} -\frac{x^2\sin^2(a+bx^2)\cos(a+bx^2)}{2b} - \frac{x^2\cos^3(a+bx^2)}{3b} + \frac{7\sin^3(a+bx^2)}{18b^2} + \frac{\sin(a+bx^2)\cos^2(a+bx^2)}{3b^2} & \text{for } b \neq 0 \\ \frac{x^4\sin^3(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*sin(b*x**2+a)**3,x)
```

output
```
Piecewise((-x**2*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**2*cos(a + b
*x**2)**3/(3*b) + 7*sin(a + b*x**2)**3/(18*b**2) + sin(a + b*x**2)*cos(a +
 b*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sin(a)**3/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \frac{3\,bx^2\cos\left(3\,bx^2+3\,a\right) - 27\,bx^2\cos\left(bx^2+a\right) - \sin\left(3\,bx^2+3\,a\right) + 27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

input
```
integrate(x^3*sin(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/72*(3*b*x^2*cos(3*b*x^2 + 3*a) - 27*b*x^2*cos(b*x^2 + a) - sin(3*b*x^2 +
 3*a) + 27*sin(b*x^2 + a))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = -\frac{\left(\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)\right)a}{6\,b^2} + \frac{3\left(bx^2+a\right)\cos\left(3\,bx^2+3\,a\right)-27\left(bx^2+a\right)\cos\left(bx^2+a\right)-\sin\left(3\,bx^2+3\,a\right)+27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

input
```
integrate(x^3*sin(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))*a/b^2 + 1/72*(3*(b*x^2 + a)*cos
(3*b*x^2 + 3*a) - 27*(b*x^2 + a)*cos(b*x^2 + a) - sin(3*b*x^2 + 3*a) + 27*
sin(b*x^2 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \frac{\frac{7\,\sin\left(b\,x^2+a\right)}{18}-\frac{\cos\left(b\,x^2+a\right)^2\,\sin\left(b\,x^2+a\right)}{18}+b\left(\frac{x^2\,\cos\left(b\,x^2+a\right)^3}{6}-\frac{x^2\,\cos\left(b\,x^2+a\right)}{2}\right)}{b^2}$$

input
```
int(x^3*sin(a + b*x^2)^3,x)
```

output
```
((7*sin(a + b*x^2))/18 - (cos(a + b*x^2)^2*sin(a + b*x^2))/18 + b*((x^2*co
s(a + b*x^2)^3)/6 - (x^2*cos(a + b*x^2))/2))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx$$
$$=\frac{-3\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^2 b\,x^2-6\cos\left(b\,x^2+a\right) b\,x^2+\sin\left(b\,x^2+a\right)^3+6\sin\left(b\,x^2+a\right)-6a}{18b^2}$$

input
```
int(x^3*sin(b*x^2+a)^3,x)
```

output
```
( - 3*cos(a + b*x**2)*sin(a + b*x**2)**2*b*x**2 - 6*cos(a + b*x**2)*b*x**2
 + sin(a + b*x**2)**3 + 6*sin(a + b*x**2) - 6*a)/(18*b**2)
```

3.25 $\int x\sin^3\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x\sin^3\left(a+bx^2\right)\,dx=-\frac{\cos\left(a+bx^2\right)}{2b}+\frac{\cos^3\left(a+bx^2\right)}{6b}$$

output
```
-1/2*cos(b*x^2+a)/b+1/6*cos(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\sin^3\left(a+bx^2\right)\,dx=-\frac{3\cos\left(a+bx^2\right)}{8b}+\frac{\cos\left(3(a+bx^2)\right)}{24b}$$

input
```
Integrate[x*Sin[a + b*x^2]^3,x]
```

output
```
(-3*Cos[a + b*x^2])/(8*b) + Cos[3*(a + b*x^2)]/(24*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3860, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^3\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{2}\int \sin^3\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \sin\left(bx^2+a\right)^3dx^2$$

$$\Big\downarrow \; 3113$$

$$-\frac{\int \left(1-x^4\right)d\cos\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cos\left(a+bx^2\right)-\frac{x^6}{3}}{2b}$$

input

```
Int[x*Sin[a + b*x^2]^3,x]
```

output

```
-1/2*(-1/3*x^6 + Cos[a + b*x^2])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{\left(2+\sin\left(b\,x^{2}+a\right)^{2}\right)\cos\left(b\,x^{2}+a\right)}{6b}$
default	$-\frac{\left(2+\sin\left(b\,x^{2}+a\right)^{2}\right)\cos\left(b\,x^{2}+a\right)}{6b}$
parallelrisch	$\frac{-8-9\cos\left(b\,x^{2}+a\right)+\cos\left(3b\,x^{2}+3a\right)}{24b}$
risch	$-\frac{3\cos\left(b\,x^{2}+a\right)}{8b}+\frac{\cos\left(3b\,x^{2}+3a\right)}{24b}$
norman	$\frac{-\frac{2\tan\left(\frac{a}{2}+\frac{b\,x^{2}}{2}\right)^{2}}{b}-\frac{2}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{b\,x^{2}}{2}\right)^{2}\right)^{3}}$
orering	$\frac{5\left(8b^{2}x^{4}+3\right)\sin\left(b\,x^{2}+a\right)^{3}}{144b^{4}x^{6}}-\frac{5\left(8b^{2}x^{4}+3\right)\left(\sin\left(b\,x^{2}+a\right)^{3}+6x^{2}\sin\left(b\,x^{2}+a\right)^{2}b\cos\left(b\,x^{2}+a\right)\right)}{144b^{4}x^{6}}+18\sin\left(b\,x^{2}+a\right)^{2}bx\,\mathrm{co}$

input
```
int(x*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b*(2+sin(b*x^2+a)^2)*cos(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(a+bx^2\right)\,dx=\frac{\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)}{6\,b}$$

input
```
integrate(x*sin(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int x\sin^3\left(a+bx^2\right)\,dx=\begin{cases}-\frac{\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{\cos^3\left(a+bx^2\right)}{3b} & \text{for}\ b\neq 0\\ \frac{x^2\sin^3\left(a\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*sin(b*x**2+a)**3,x)
```

output
```
Piecewise((-sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - cos(a + b*x**2)**3/
(3*b), Ne(b, 0)), (x**2*sin(a)**3/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\sin^3\left(a+bx^2\right)\,dx=\frac{\cos\left(3\,bx^2+3\,a\right)-9\,\cos\left(bx^2+a\right)}{24\,b}$$

input

```
integrate(x*sin(b*x^2+a)^3,x, algorithm="maxima")
```

output

```
1/24*(cos(3*b*x^2 + 3*a) - 9*cos(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(a+bx^2\right)\,dx=\frac{\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)}{6\,b}$$

input

```
integrate(x*sin(b*x^2+a)^3,x, algorithm="giac")
```

output

```
1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 38.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x\sin^3\left(a+bx^2\right)\,dx=-\frac{3\,\cos\left(b\,x^2+a\right)-\cos\left(b\,x^2+a\right)^3}{6\,b}$$

input

```
int(x*sin(a + b*x^2)^3,x)
```

output

```
-(3*cos(a + b*x^2) - cos(a + b*x^2)^3)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int x\sin^3\left(a+bx^2\right)\,dx=\frac{-\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^2-2\cos\left(b\,x^2+a\right)+2}{6b}$$

input
```
int(x*sin(b*x^2+a)^3,x)
```

output
```
( - cos(a + b*x**2)*sin(a + b*x**2)**2 - 2*cos(a + b*x**2) + 2)/(6*b)
```

3.26 $\int \frac{\sin^3(a+bx^2)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \frac{3}{8}\,\text{CosIntegral}\left(bx^2\right)\sin(a) - \frac{1}{8}\,\text{CosIntegral}\left(3bx^2\right)\sin(3a) + \frac{3}{8}\cos(a)\text{Si}\left(bx^2\right) - \frac{1}{8}\cos(3a)\text{Si}\left(3bx^2\right)$$

output
```
3/8*Ci(b*x^2)*sin(a)-1/8*Ci(3*b*x^2)*sin(3*a)+3/8*cos(a)*Si(b*x^2)-1/8*cos
(3*a)*Si(3*b*x^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \frac{1}{8}\left(3\,\text{CosIntegral}\left(bx^2\right)\sin(a) - \text{CosIntegral}\left(3bx^2\right)\sin(3a) + 3\cos(a)\text{Si}\left(bx^2\right) - \cos(3a)\text{Si}\left(3bx^2\right)\right)$$

input
```
Integrate[Sin[a + b*x^2]^3/x,x]
```

output
```
(3*CosIntegral[b*x^2]*Sin[a] - CosIntegral[3*b*x^2]*Sin[3*a] + 3*Cos[a]*Si
nIntegral[b*x^2] - Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3\left(a+bx^2\right)}{x}dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(\frac{3\sin\left(a+bx^2\right)}{4x}-\frac{\sin\left(3a+3bx^2\right)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}\sin(a)\,\text{CosIntegral}\left(bx^2\right)-\frac{1}{8}\sin(3a)\,\text{CosIntegral}\left(3bx^2\right)+\frac{3}{8}\cos(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\cos(3a)\text{Si}\left(3bx^2\right)$$

input
```
Int[Sin[a + b*x^2]^3/x,x]
```

output
```
(3*CosIntegral[b*x^2]*Sin[a])/8 - (CosIntegral[3*b*x^2]*Sin[3*a])/8 + (3*C
os[a]*SinIntegral[b*x^2])/8 - (Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.83 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{i \mathrm{e}^{3 i a} \operatorname{expIntegral}_{1}\left(-3 i b \,x^{2}\right)}{16}+\frac{\mathrm{e}^{-3 i a} \operatorname{csgn}\left(b \,x^{2}\right) \pi}{16}-\frac{\mathrm{e}^{-3 i a} \operatorname{Si}\left(3 b \,x^{2}\right)}{8}+\frac{i \operatorname{expIntegral}_{1}\left(-3 i b \,x^{2}\right) \mathrm{e}^{-3 i a}}{16}-\frac{3 \,\mathrm{e}^{-i a} \operatorname{csgn}\left(b \,x^{2}\right) \pi}{16}$

input

```
int(sin(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

output

```
-1/16*I*exp(3*I*a)*Ei(1,-3*I*b*x^2)+1/16*exp(-3*I*a)*csgn(b*x^2)*Pi-1/8*ex
p(-3*I*a)*Si(3*b*x^2)+1/16*I*Ei(1,-3*I*b*x^2)*exp(-3*I*a)-3/16*exp(-I*a)*c
sgn(b*x^2)*Pi+3/8*exp(-I*a)*Si(b*x^2)-3/16*I*exp(-I*a)*Ei(1,-I*b*x^2)+3/16
*I*exp(I*a)*Ei(1,-I*b*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3\left(a+bx^2\right)}{x}\,dx = -\frac{1}{8}\,\operatorname{Ci}\left(3\,bx^2\right)\sin\left(3\,a\right)+\frac{3}{8}\,\operatorname{Ci}\left(bx^2\right)\sin\left(a\right) - \frac{1}{8}\,\cos\left(3\,a\right)\operatorname{Si}\left(3\,bx^2\right)+\frac{3}{8}\,\cos\left(a\right)\operatorname{Si}\left(bx^2\right)$$

input

```
integrate(sin(b*x^2+a)^3/x,x, algorithm="fricas")
```

output
```
-1/8*cos_integral(3*b*x^2)*sin(3*a) + 3/8*cos_integral(b*x^2)*sin(a) - 1/8
*cos(3*a)*sin_integral(3*b*x^2) + 3/8*cos(a)*sin_integral(b*x^2)
```

Sympy [F]

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \int \frac{\sin^3(a+bx^2)}{x}\,dx$$

input
```
integrate(sin(b*x**2+a)**3/x,x)
```

output
```
Integral(sin(a + b*x**2)**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{\sin^3(a+bx^2)}{x}\,dx = {} & \frac{1}{16}\left(i\,\mathrm{Ei}\left(3i\,bx^2\right) - i\,\mathrm{Ei}\left(-3i\,bx^2\right)\right)\cos(3\,a) \\ & - \frac{3}{16}\left(i\,\mathrm{Ei}\left(i\,bx^2\right) - i\,\mathrm{Ei}\left(-i\,bx^2\right)\right)\cos(a) \\ & - \frac{1}{16}\left(\mathrm{Ei}\left(3i\,bx^2\right) + \mathrm{Ei}\left(-3i\,bx^2\right)\right)\sin(3\,a) \\ & + \frac{3}{16}\left(\mathrm{Ei}\left(i\,bx^2\right) + \mathrm{Ei}\left(-i\,bx^2\right)\right)\sin(a)\end{aligned}$$

input
```
integrate(sin(b*x^2+a)^3/x,x, algorithm="maxima")
```

output
```
1/16*(I*Ei(3*I*b*x^2) - I*Ei(-3*I*b*x^2))*cos(3*a) - 3/16*(I*Ei(I*b*x^2) -
 I*Ei(-I*b*x^2))*cos(a) - 1/16*(Ei(3*I*b*x^2) + Ei(-3*I*b*x^2))*sin(3*a) +
 3/16*(Ei(I*b*x^2) + Ei(-I*b*x^2))*sin(a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = -\frac{1}{8}\operatorname{Ci}\left(3\,bx^2\right)\sin\left(3\,a\right)+\frac{3}{8}\operatorname{Ci}\left(bx^2\right)\sin\left(a\right) + \frac{3}{8}\cos\left(a\right)\operatorname{Si}\left(bx^2\right)+\frac{1}{8}\cos\left(3\,a\right)\operatorname{Si}\left(-3\,bx^2\right)$$

input
```
integrate(sin(b*x^2+a)^3/x,x, algorithm="giac")
```

output
```
-1/8*cos_integral(3*b*x^2)*sin(3*a) + 3/8*cos_integral(b*x^2)*sin(a) + 3/8
*cos(a)*sin_integral(b*x^2) + 1/8*cos(3*a)*sin_integral(-3*b*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \int \frac{\sin\left(b\,x^2+a\right)^3}{x}\,dx$$

input
```
int(sin(a + b*x^2)^3/x,x)
```

output
```
int(sin(a + b*x^2)^3/x, x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \int \frac{\sin\left(b\,x^2+a\right)^3}{x}dx$$

input
```
int(sin(b*x^2+a)^3/x,x)
```

output
```
int(sin(a + b*x**2)**3/x,x)
```

3.27 $\int \frac{\sin^3(a+bx^2)}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\begin{aligned}\int \frac{\sin^3(a+bx^2)}{x^3}\,dx &= \frac{3}{8}b\cos(a)\operatorname{CosIntegral}\left(bx^2\right) - \frac{3}{8}b\cos(3a)\operatorname{CosIntegral}\left(3bx^2\right)\\ &\quad - \frac{3\sin(a+bx^2)}{8x^2} + \frac{\sin(3(a+bx^2))}{8x^2}\\ &\quad - \frac{3}{8}b\sin(a)\operatorname{Si}\left(bx^2\right) + \frac{3}{8}b\sin(3a)\operatorname{Si}\left(3bx^2\right)\end{aligned}$$

output

```
3/8*b*cos(a)*Ci(b*x^2)-3/8*b*cos(3*a)*Ci(3*b*x^2)-3/8*sin(b*x^2+a)/x^2+1/8
*sin(3*b*x^2+3*a)/x^2-3/8*b*sin(a)*Si(b*x^2)+3/8*b*sin(3*a)*Si(3*b*x^2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \frac{3bx^2\cos(a)\operatorname{CosIntegral}(bx^2) - 3bx^2\cos(3a)\operatorname{CosIntegral}(3bx^2) - 3\sin(a+bx^2) + \sin(3(a+bx^2)) - 3b}{8x^2}$$

input

```
Integrate[Sin[a + b*x^2]^3/x^3,x]
```

output
```
(3*b*x^2*Cos[a]*CosIntegral[b*x^2] - 3*b*x^2*Cos[3*a]*CosIntegral[3*b*x^2]
 - 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)] - 3*b*x^2*Sin[a]*SinIntegral[b*x^
2] + 3*b*x^2*Sin[3*a]*SinIntegral[3*b*x^2])/(8*x^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^3}dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(\frac{3\sin\left(a+bx^2\right)}{4x^3}-\frac{\sin\left(3a+3bx^2\right)}{4x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}b\cos(a)\,\text{CosIntegral}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\,\text{CosIntegral}\left(3bx^2\right)-\frac{3}{8}b\sin(a)\text{Si}\left(bx^2\right)+\frac{3}{8}b\sin(3a)\text{Si}\left(3bx^2\right)-\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}$$

input
```
Int[Sin[a + b*x^2]^3/x^3,x]
```

output
```
(3*b*Cos[a]*CosIntegral[b*x^2])/8 - (3*b*Cos[3*a]*CosIntegral[3*b*x^2])/8
- (3*Sin[a + b*x^2])/(8*x^2) + Sin[3*(a + b*x^2)]/(8*x^2) - (3*b*Sin[a]*Si
nIntegral[b*x^2])/8 + (3*b*Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.84 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{3i\pi\,\mathrm{e}^{-3ia}\,\mathrm{csgn}\left(b\,x^{2}\right)b\,x^{2}-3i\pi\,\mathrm{e}^{-ia}\,\mathrm{csgn}\left(b\,x^{2}\right)b\,x^{2}-6i\mathrm{e}^{-3ia}\,\mathrm{Si}\left(3b\,x^{2}\right)b\,x^{2}+6i\mathrm{e}^{-ia}\,\mathrm{Si}\left(b\,x^{2}\right)b\,x^{2}-3\,\mathrm{expIntegral}_{1}\left(-3ib\,x^{2}\right)\mathrm{e}^{-3ia}b}{}$

input
```
int(sin(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(3*I*Pi*exp(-3*I*a)*csgn(b*x^2)*b*x^2-3*I*Pi*exp(-I*a)*csgn(b*x^2)*b
*x^2-6*I*exp(-3*I*a)*Si(3*b*x^2)*b*x^2+6*I*exp(-I*a)*Si(b*x^2)*b*x^2-3*Ei(
1,-3*I*b*x^2)*exp(-3*I*a)*b*x^2-3*exp(3*I*a)*b*Ei(1,-3*I*b*x^2)*x^2+3*exp(
I*a)*b*Ei(1,-I*b*x^2)*x^2+3*Ei(1,-I*b*x^2)*exp(-I*a)*b*x^2+6*sin(b*x^2+a)-
2*sin(3*b*x^2+3*a))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx =$$
$$-\frac{3\,bx^2\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^2\right)-3\,bx^2\cos\left(a\right)\mathrm{Ci}\left(bx^2\right)-3\,bx^2\sin\left(3\,a\right)\mathrm{Si}\left(3\,bx^2\right)+3\,bx^2\sin\left(a\right)\mathrm{Si}\left(bx^2\right)-4}{8\,x^2}$$

input
```
integrate(sin(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

output
```
-1/8*(3*b*x^2*cos(3*a)*cos_integral(3*b*x^2) - 3*b*x^2*cos(a)*cos_integral
(b*x^2) - 3*b*x^2*sin(3*a)*sin_integral(3*b*x^2) + 3*b*x^2*sin(a)*sin_inte
gral(b*x^2) - 4*(cos(b*x^2 + a)^2 - 1)*sin(b*x^2 + a))/x^2
```

Sympy [F]

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \int \frac{\sin^3(a+bx^2)}{x^3}\,dx$$

input
```
integrate(sin(b*x**2+a)**3/x**3,x)
```

output
```
Integral(sin(a + b*x**2)**3/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx =$$

$$-\frac{3}{16}\big(\big(\Gamma(-1,3i\,bx^2)+\Gamma(-1,-3i\,bx^2)\big)\cos(3\,a)-\big(\Gamma(-1,i\,bx^2)+\Gamma(-1,-i\,bx^2)\big)\cos(a)+\big(-i\,\Gamma(-1$$

input
```
integrate(sin(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

output
```
-3/16*((gamma(-1, 3*I*b*x^2) + gamma(-1, -3*I*b*x^2))*cos(3*a) - (gamma(-1
, I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) + (-I*gamma(-1, 3*I*b*x^2) + I*ga
mma(-1, -3*I*b*x^2))*sin(3*a) + (I*gamma(-1, I*b*x^2) - I*gamma(-1, -I*b*x
^2))*sin(a))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(80) = 160.

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.04

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx =$$

$$-\frac{3\,(bx^2+a)b^2\cos(3\,a)\,\mathrm{Ci}\,(3\,bx^2) - 3\,ab^2\cos(3\,a)\,\mathrm{Ci}\,(3\,bx^2) - 3\,(bx^2+a)b^2\cos(a)\,\mathrm{Ci}\,(bx^2) + 3\,ab^2\cos(}{}$$

input
```
integrate(sin(b*x^2+a)^3/x^3,x, algorithm="giac")
```

output
```
-1/8*(3*(b*x^2 + a)*b^2*cos(3*a)*cos_integral(3*b*x^2) - 3*a*b^2*cos(3*a)*
cos_integral(3*b*x^2) - 3*(b*x^2 + a)*b^2*cos(a)*cos_integral(b*x^2) + 3*a
*b^2*cos(a)*cos_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(a)*sin_integral(b*
x^2) - 3*a*b^2*sin(a)*sin_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(3*a)*sin
_integral(-3*b*x^2) - 3*a*b^2*sin(3*a)*sin_integral(-3*b*x^2) - b^2*sin(3*
b*x^2 + 3*a) + 3*b^2*sin(b*x^2 + a))/(b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \int \frac{\sin\left(b\,x^2+a\right)^3}{x^3}\,dx$$

input
```
int(sin(a + b*x^2)^3/x^3,x)
```

output
```
int(sin(a + b*x^2)^3/x^3, x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx = \text{Too large to display}$$

input

```
int(sin(b*x^2+a)^3/x^3,x)
```

output

```
( - 3*cos(a + b*x**2)*sin(a + b*x**2)*tan((a + b*x**2)/2)**6 - 9*cos(a + b
*x**2)*sin(a + b*x**2)*tan((a + b*x**2)/2)**4 - 9*cos(a + b*x**2)*sin(a +
b*x**2)*tan((a + b*x**2)/2)**2 - 3*cos(a + b*x**2)*sin(a + b*x**2) + 40*in
t(tan((a + b*x**2)/2)**3/(tan((a + b*x**2)/2)**6*x**3 + 3*tan((a + b*x**2)
/2)**4*x**3 + 3*tan((a + b*x**2)/2)**2*x**3 + x**3),x)*tan((a + b*x**2)/2)
**6*x**2 + 120*int(tan((a + b*x**2)/2)**3/(tan((a + b*x**2)/2)**6*x**3 + 3
*tan((a + b*x**2)/2)**4*x**3 + 3*tan((a + b*x**2)/2)**2*x**3 + x**3),x)*ta
n((a + b*x**2)/2)**4*x**2 + 120*int(tan((a + b*x**2)/2)**3/(tan((a + b*x**
2)/2)**6*x**3 + 3*tan((a + b*x**2)/2)**4*x**3 + 3*tan((a + b*x**2)/2)**2*x
**3 + x**3),x)*tan((a + b*x**2)/2)**2*x**2 + 40*int(tan((a + b*x**2)/2)**3
/(tan((a + b*x**2)/2)**6*x**3 + 3*tan((a + b*x**2)/2)**4*x**3 + 3*tan((a +
 b*x**2)/2)**2*x**3 + x**3),x)*x**2 - sin(a + b*x**2)**3*tan((a + b*x**2)/
2)**6 - 3*sin(a + b*x**2)**3*tan((a + b*x**2)/2)**4 - 3*sin(a + b*x**2)**3
*tan((a + b*x**2)/2)**2 - sin(a + b*x**2)**3 - 3*sin(a + b*x**2)*tan((a +
b*x**2)/2)**6 - 9*sin(a + b*x**2)*tan((a + b*x**2)/2)**4 - 9*sin(a + b*x**
2)*tan((a + b*x**2)/2)**2 - 3*sin(a + b*x**2) + 20*tan((a + b*x**2)/2)**3
+ 12*tan((a + b*x**2)/2))/(5*x**2*(tan((a + b*x**2)/2)**6 + 3*tan((a + b*x
**2)/2)**4 + 3*tan((a + b*x**2)/2)**2 + 1))
```

3.28 $\int x^2 \sin^3\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 188

$$
\begin{aligned}
\int x^2 \sin^3\left(a+bx^2\right)\,dx = &-\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b} \\
&+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}} \\
&-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}} \\
&-\frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{8b^{3/2}} \\
&+\frac{\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{24b^{3/2}}
\end{aligned}
$$

output

```
-3/8*x*cos(b*x^2+a)/b+1/24*x*cos(3*b*x^2+3*a)/b+3/16*2^(1/2)*Pi^(1/2)*cos(
a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x)/b^(3/2)-1/144*6^(1/2)*Pi^(1/2)*cos
(3*a)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*x)/b^(3/2)-3/16*2^(1/2)*Pi^(1/2)*F
resnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(a)/b^(3/2)+1/144*6^(1/2)*Pi^(1/2)*
FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*x)*sin(3*a)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx$$
$$= \frac{-54\sqrt{b}x\cos\left(a+bx^2\right)+6\sqrt{b}x\cos\left(3\left(a+bx^2\right)\right)+27\sqrt{2\pi}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\cos(3a)\operatorname{Fres}}{144b^{3/2}}$$

input
```
Integrate[x^2*Sin[a + b*x^2]^3,x]
```

output
```
(-54*Sqrt[b]*x*Cos[a + b*x^2] + 6*Sqrt[b]*x*Cos[3*(a + b*x^2)] + 27*Sqrt[2
*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]*FresnelC[
Sqrt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a
] + Sqrt[6*Pi]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(144*b^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx$$
$$\Big\downarrow \text{3884}$$
$$\int \left(\frac{3}{4}x^2\sin\left(a+bx^2\right)-\frac{1}{4}x^2\sin\left(3a+3bx^2\right)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}$$

input
```
Int[x^2*Sin[a + b*x^2]^3,x]
```

output
```
(-3*x*Cos[a + b*x^2])/(8*b) + (x*Cos[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi/2
]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[Pi/6]*Cos[3*a
]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*FresnelS[Sq
rt[b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) + (Sqrt[Pi/6]*FresnelS[Sqrt[b]*Sqr
t[6/Pi]*x]*Sin[3*a])/(24*b^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.70

method	result
default	$-\frac{3x\cos\left(b\,x^2+a\right)}{8b}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)-\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}}+\frac{x\cos\left(3b\,x^2+3a\right)}{24b}-\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\bigl(\cos($
risch	$-\frac{\mathrm{e}^{-3ia}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{288b\sqrt{ib}}-\frac{\mathrm{e}^{3ia}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-3ib}\,x\right)}{96b\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-ib}\,x\right)}{32b\sqrt{-ib}}+\frac{3\,\mathrm{e}^{-ia}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{ib}\,x\right)}{32b\sqrt{ib}}-\frac{3x\cos\left(b\,x^2+a\right)}{8b}$

input
```
int(x^2*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*x*cos(b*x^2+a)/b+3/16/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/
2)*2^(1/2)/Pi^(1/2)*x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x))+1/24*x
*cos(3*b*x^2+3*a)/b-1/144/b^(3/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)
*3^(1/2)*b^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx$$
$$= \frac{24\,bx\cos\left(bx^2+a\right)^3 - \sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right) + 27\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right) + \sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\bigg(}{144\,b^2}$$

input
```
integrate(x^2*sin(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/144*(24*b*x*cos(b*x^2 + a)^3 - sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_co
s(sqrt(6)*x*sqrt(b/pi)) + 27*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt
(2)*x*sqrt(b/pi)) + sqrt(6)*pi*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)
)*sin(3*a) - 27*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*si
n(a) - 72*b*x*cos(b*x^2 + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(194) = 388$.

Time = 1.93 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.34

$$
\begin{aligned}
\int x^2 \sin^3\left(a+bx^2\right)\,dx = &-\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} \\
&+\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(3a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} \\
&-\frac{3\sqrt{b}x^3\sqrt{\frac{1}{b}}\sin(a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} \\
&+\frac{\sqrt{b}x^3\sqrt{\frac{1}{b}}\sin(3a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} \\
&+\frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(a)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} \\
&-\frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(3a)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)}{24} \\
&+\frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(a)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} \\
&-\frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(3a)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)}{24}
\end{aligned}
$$

input

```
integrate(x**2*sin(b*x**2+a)**3,x)
```

output
```
-3*b**(3/2)*x**5*sqrt(1/b)*cos(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4),
(3/2, 7/4, 9/4), -b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) + 3*b**(3/2)*x**
5*sqrt(1/b)*cos(3*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/
4), -9*b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) - 3*sqrt(b)*x**3*sqrt(1/b)*
sin(a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4
/4)/(32*gamma(5/4)*gamma(7/4)) + sqrt(b)*x**3*sqrt(1/b)*sin(3*a)*gamma(1/4
)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*gamma(
5/4)*gamma(7/4)) + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fresnelc(sqrt(
2)*sqrt(b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*sin(3*a)*fresne
lc(sqrt(6)*sqrt(b)*x/sqrt(pi))/24 + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(
a)*fresnels(sqrt(2)*sqrt(b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b
)*cos(3*a)*fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi))/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int x^2\sin^3\left(a+bx^2\right)\,dx$$
$$=\frac{24\,b^2x\cos\left(3\,bx^2+3\,a\right)-216\,b^2x\cos\left(bx^2+a\right)+9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left((i-1)\,\cos\left(3\,a\right)+(i+1)\,\sin\left(3\,a\right)\right)\operatorname{erf}\Big(\sqrt{3}}{}$$

input
```
integrate(x^2*sin(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/576*(24*b^2*x*cos(3*b*x^2 + 3*a) - 216*b^2*x*cos(b*x^2 + a) + 9^(1/4)*sq
rt(2)*sqrt(pi)*(((I - 1)*cos(3*a) + (I + 1)*sin(3*a))*erf(sqrt(3*I*b)*x) +
 (-(I + 1)*cos(3*a) - (I - 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))*b^(3/2) - 27*
sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(I*b)*x) + (-(
I + 1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\begin{aligned}\int x^2\sin^3\left(a+bx^2\right)\,dx ={}& \frac{xe^{\left(3i\,bx^2+3i\,a\right)}}{48\,b} - \frac{3\,xe^{\left(i\,bx^2+i\,a\right)}}{16\,b} - \frac{3\,xe^{\left(-i\,bx^2-i\,a\right)}}{16\,b} \\ &+ \frac{xe^{\left(-3i\,bx^2-3i\,a\right)}}{48\,b} + \frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{6}\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(3i\,a)}}{288\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)} \\ &- \frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{32\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} \\ &- \frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{32\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} \\ &+ \frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{6}\sqrt{b}x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(-3i\,a)}}{288\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)}\end{aligned}$$

input

```
integrate(x^2*sin(b*x^2+a)^3,x, algorithm="giac")
```

output

```
1/48*x*e^(3*I*b*x^2 + 3*I*a)/b - 3/16*x*e^(I*b*x^2 + I*a)/b - 3/16*x*e^(-I
*b*x^2 - I*a)/b + 1/48*x*e^(-3*I*b*x^2 - 3*I*a)/b + 1/288*sqrt(6)*sqrt(pi)
*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3*I*a)/(b^(3/2)*(-I*b/ab
s(b) + 1)) - 3/32*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b) + 1)*sq
rt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) - 3/32*sqrt(2)*sqrt
(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/ab
s(b) + 1)*sqrt(abs(b))) + 1/288*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*
x*(I*b/abs(b) + 1))*e^(-3*I*a)/(b^(3/2)*(I*b/abs(b) + 1))
```

Mupad [**F(-1)**]

Timed out.

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx=\int x^2\sin\left(b\,x^2+a\right)^3\,dx$$

input
```
int(x^2*sin(a + b*x^2)^3,x)
```

output
```
int(x^2*sin(a + b*x^2)^3, x)
```

Reduce [**F**]

$$\int x^2 \sin^3\left(a+bx^2\right)\,dx=\int \sin\left(b\,x^2+a\right)^3 x^2dx$$

input
```
int(x^2*sin(b*x^2+a)^3,x)
```

output
```
int(sin(a + b*x**2)**3*x**2,x)
```

3.29 $\int \sin^3(a+bx^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 153

$$\begin{aligned}\int \sin^3\left(a+bx^2\right)\,dx = {} & \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} \\ & - \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}} \\ & + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}} \\ & - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{4\sqrt{b}}\end{aligned}$$

output

```
3/8*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x)/b^(1/2)-1
/24*6^(1/2)*Pi^(1/2)*cos(3*a)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*x)/b^(1/2)
+3/8*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(a)/b^(1/2)-
1/24*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*x)*sin(3*a)/b^(1/2
)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \sin^3\left(a+bx^2\right)\,dx = \frac{\sqrt{\frac{\pi}{6}}\left(3\sqrt{3}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)+3\sqrt{3}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right.}{4\sqrt{b}}$$

input
```
Integrate[Sin[a + b*x^2]^3,x]
```

output
```
(Sqrt[Pi/6]*(3*Sqrt[3]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Cos[3*a]*Fr
esnelS[Sqrt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[3]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Si
n[a] - FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a]))/(4*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(a+bx^2\right)dx$$

$$\Big\downarrow \text{ 3838}$$

$$\int \left(\frac{3}{4}\sin\left(a+bx^2\right)-\frac{1}{4}\sin\left(3a+3bx^2\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}$$

input
```
Int[Sin[a + b*x^2]^3,x]
```

output
```
(3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) - (Sqrt[P
i/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) + (3*Sqrt[Pi/2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelC[
Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

method	result
default	$\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{8\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(\cos(3a)\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\sin(3a)\,\mathrm{FresnelC}\right.}{24\sqrt{b}}$
risch	$\frac{i\mathrm{e}^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{16\sqrt{-3ib}}-\frac{i\mathrm{e}^{-3ia}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{48\sqrt{ib}}+\frac{3i\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{16\sqrt{ib}}-\frac{3i\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{16\sqrt{-ib}}$

input
```
int(sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x)+
sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)
/b^(1/2)*(cos(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int \sin^3\left(a+bx^2\right)\,dx =$$

$$-\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)-9\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)-}{24\,b}$$

input
```
integrate(sin(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/24*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) -
9*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)
*pi*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) - 9*sqrt(2)*pi*s
qrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \sin^3\left(a+bx^2\right)\,dx = \frac{3\sqrt{2}\sqrt{\pi}\left(\sin\left(a\right)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos\left(a\right)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8} - \frac{\sqrt{6}\sqrt{\pi}\left(\sin\left(3a\right)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos\left(3a\right)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{24}$$

input
```
integrate(sin(b*x**2+a)**3,x)
```

output
```
3*sqrt(2)*sqrt(pi)*(sin(a)*fresnelc(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*f
resnels(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/8 - sqrt(6)*sqrt(pi)*(sin(3
*a)*fresnelc(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnels(sqrt(6)*sqrt(
b)*x/sqrt(pi)))*sqrt(1/b)/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \sin^3\left(a+bx^2\right)\,dx = \frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left(-(i+1)\,\cos\left(3\,a\right)+(i-1)\,\sin\left(3\,a\right)\right)\operatorname{erf}\left(\sqrt{3i\,b}x\right)+\left((i-1)\,\cos\left(3\,a\right)-(i+1)\,\sin\left(3\,a\right)\right)\mathrm{er}}{}$$

input
```
integrate(sin(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(3*a) + (I - 1)*sin(3*a))*erf
(sqrt(3*I*b)*x) + ((I - 1)*cos(3*a) - (I + 1)*sin(3*a))*erf(sqrt(-3*I*b)*x
))*b^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sq
rt(I*b)*x) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))
/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \sin^3\left(a+bx^2\right)\,dx = &-\frac{i\,\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{6}\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(3i\,a)}}{48\,\sqrt{b}\left(-\frac{i\,b}{|b|}+1\right)}\\ &+\frac{3i\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{16\,\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}\\ &-\frac{3i\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{16\,\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}\\ &+\frac{i\,\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{6}\sqrt{b}x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(-3i\,a)}}{48\,\sqrt{b}\left(\frac{i\,b}{|b|}+1\right)}\end{aligned}$$

input
```
integrate(sin(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/48*I*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(
3*I*a)/(sqrt(b)*(-I*b/abs(b) + 1)) + 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt
(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((-I*b/abs(b) + 1)*sqrt(abs(
b))) - 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(ab
s(b)))*e^(-I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) + 1/48*I*sqrt(6)*sqrt(pi)*
erf(-1/2*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(I*b/abs(
b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3\left(a+bx^2\right)\,dx=\int \sin\left(b\,x^2+a\right)^3\,dx$$

input
```
int(sin(a + b*x^2)^3,x)
```

output
```
int(sin(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \sin^3\left(a+bx^2\right)\,dx=\int \sin\left(b\,x^2+a\right)^3\,dx$$

input
```
int(sin(b*x^2+a)^3,x)
```

output
```
int(sin(a + b*x**2)**3,x)
```

3.30 $\int \frac{\sin^3(a+bx^2)}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 168

$$\begin{aligned}\int \frac{\sin^3(a+bx^2)}{x^2}\,dx &= \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\\ &\quad-\frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\\ &\quad-\frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\\ &\quad+\frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)-\frac{\sin^3(a+bx^2)}{x}\end{aligned}$$

output

```
3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x)-1
/4*b^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*x)-
3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(a)+1
/4*b^(1/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*x)*sin(3*a)-
sin(b*x^2+a)^3/x
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx$$
$$= \frac{3\sqrt{b}\sqrt{2\pi}x\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{b}\sqrt{6\pi}x\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-3\sqrt{b}\sqrt{2\pi}x\,\text{FresnelS}\left(\right.}{4x}$$

input
```
Integrate[Sin[a + b*x^2]^3/x^2,x]
```

output
```
(3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[b]*Sq
rt[6*Pi]*x*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x] - 3*Sqrt[b]*Sqrt[2*Pi]*
x*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelS[Sq
rt[b]*Sqrt[6/Pi]*x]*Sin[3*a] - 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)])/(4*x
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3874, 5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx$$
$$\Big\downarrow \text{3874}$$
$$6b\int \cos(bx^2+a)\sin^2(bx^2+a)\,dx-\frac{\sin^3(a+bx^2)}{x}$$
$$\Big\downarrow \text{5085}$$
$$6b\int \left(\frac{1}{4}\cos(bx^2+a)-\frac{1}{4}\cos(3bx^2+3a)\right)dx-\frac{\sin^3(a+bx^2)}{x}$$
$$\Big\downarrow \text{2009}$$

$$6b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}+\frac{\sqrt{}}{}\right.$$

$$\frac{\sin^3\left(a+bx^2\right)}{x}$$

input
```
Int[Sin[a + b*x^2]^3/x^2,x]
```

output
```
6*b*((Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) - (Sqr
t[Pi/6]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) - (Sqrt[Pi/2]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) + (Sqrt[Pi/6]*FresnelS
[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])) - Sin[a + b*x^2]^3/x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3874
```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[Sin[a + b*x^
n]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[
m + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

method	result
default	$-\frac{3\sin(b\,x^2+a)}{4x}+\frac{3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)-\sin(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}\,x}{\sqrt{\pi}}\right)\right)}{4}+\frac{\sin(3b\,x^2+3a)}{4x}-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,(\mathrm{c}$
risch	$-\frac{\mathrm{e}^{-3ia}b\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{8\sqrt{ib}}-\frac{3\,\mathrm{e}^{3ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{8\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{8\sqrt{-ib}}+\frac{3\,\mathrm{e}^{-ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{8\sqrt{ib}}-\frac{3\sin(b\,x^2+}{4x}$

input
```
int(sin(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4/x*sin(b*x^2+a)+3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*
2^(1/2)/Pi^(1/2)*x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x))+1/4*sin(3
*b*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC(2^(1
/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)
*b^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx =$$

$$-\frac{\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\cos(3\,a)\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)-3\,\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin(3\,a}{4\,x}$$

input
```
integrate(sin(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

output
```
-1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_cos(sqrt(6)*x*sqrt(b/pi)) -
 3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt
(6)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)
*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a) - 4*(cos(b*x^2 +
 a)^2 - 1)*sin(b*x^2 + a))/x
```

Sympy [F]

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx = \int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx$$

input
```
integrate(sin(b*x**2+a)**3/x**2,x)
```

output
```
Integral(sin(a + b*x**2)**3/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx$$

$$= \frac{\sqrt{3}\sqrt{bx^2}\left(\left((i-1)\,\sqrt{2}\Gamma\left(-\tfrac{1}{2},3i\,bx^2\right)-(i+1)\,\sqrt{2}\Gamma\left(-\tfrac{1}{2},-3i\,bx^2\right)\right)\cos\left(3\,a\right)+\left((i+1)\,\sqrt{2}\Gamma\left(-\tfrac{1}{2},3i\,bx^2\right)\right.}{}$$

input
```
integrate(sin(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/32*(sqrt(3)*sqrt(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) - (I +
1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*cos(3*a) + ((I + 1)*sqrt(2)*gamma(-1/2
, 3*I*b*x^2) - (I - 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*sin(3*a)) - 3*sqrt
(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/
2, -I*b*x^2))*cos(a) + ((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I - 1)*sqr
t(2)*gamma(-1/2, -I*b*x^2))*sin(a)))/x
```

Giac [F]

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx = \int \frac{\sin(bx^2+a)^3}{x^2}\,dx$$

input
```
integrate(sin(b*x^2+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x^2 + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx = \int \frac{\sin(b\,x^2+a)^3}{x^2}\,dx$$

input
```
int(sin(a + b*x^2)^3/x^2,x)
```

output
```
int(sin(a + b*x^2)^3/x^2, x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx^2)}{x^2}\,dx = \int \frac{\sin(b\,x^2+a)^3}{x^2}dx$$

input
```
int(sin(b*x^2+a)^3/x^2,x)
```

output
```
int(sin(a + b*x**2)**3/x**2,x)
```

3.31 $\int x^2 \sin^3(x^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 71

$$\int x^2 \sin^3(x^2)\,dx = -\frac{1}{2}x\cos(x^2) + \frac{1}{6}x\cos^3(x^2) + \frac{3}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{24}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)$$

output
```
-1/2*x*cos(x^2)+1/6*x*cos(x^2)^3+3/16*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi
^(1/2)*x)-1/144*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x^2 \sin^3(x^2)\,dx = \frac{1}{144}\left(6x\left(-9\cos(x^2) + \cos(3x^2)\right) + 27\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) - \sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)\right)$$

input
```
Integrate[x^2*Sin[x^2]^3,x]
```

output
```
(6*x*(-9*Cos[x^2] + Cos[3*x^2]) + 27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] - S
qrt[6*Pi]*FresnelC[Sqrt[6/Pi]*x])/144
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^3\left(x^2\right) dx$$

$$\Big\downarrow \text{ 3884}$$

$$\int \left(\frac{3}{4} x^2 \sin\left(x^2\right) - \frac{1}{4} x^2 \sin\left(3 x^2\right)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{24}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{8} x \cos\left(x^2\right) + \frac{1}{24} x \cos\left(3x^2\right)$$

input
```
Int[x^2*Sin[x^2]^3,x]
```

output
```
(-3*x*Cos[x^2])/8 + (x*Cos[3*x^2])/24 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*
x])/8 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

method	result
default	$-\frac{3x\cos\left(x^{2}\right)}{8}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)}{16}+\frac{x\cos\left(3x^{2}\right)}{24}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{\sqrt{\pi}}\right)}{144}$
risch	$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-3i}\,x\right)}{96\sqrt{-3i}}+\frac{(-1)^{\frac{3}{4}}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(\sqrt{3}\,(-1)^{\frac{1}{4}}x\right)}{288}-\frac{3(-1)^{\frac{3}{4}}\sqrt{\pi}\,\operatorname{erf}\left((-1)^{\frac{1}{4}}x\right)}{32}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-i}\,x\right)}{32\sqrt{-i}}-\frac{3x\cos\left(x^{2}\right)}{8}+$

input
```
int(x^2*sin(x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*x*cos(x^2)+3/16*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*x)+1/24*x*
cos(3*x^2)-1/144*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2
)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int x^{2}\sin^{3}\left(x^{2}\right)\,dx=\frac{1}{6}\,x\cos\left(x^{2}\right)^{3}-\frac{1}{2}\,x\cos\left(x^{2}\right)-\frac{1}{144}\,\sqrt{6}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)+\frac{3}{16}\,\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)$$

input
```
integrate(x^2*sin(x^2)^3,x, algorithm="fricas")
```

output
```
1/6*x*cos(x^2)^3 - 1/2*x*cos(x^2) - 1/144*sqrt(6)*sqrt(pi)*fresnel_cos(sqr
t(6)*x/sqrt(pi)) + 3/16*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi))
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.63

$$\int x^2 \sin^3\left(x^2\right)\,dx = -\frac{15x\cos\left(x^2\right)\Gamma\left(\frac{5}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)} + \frac{5x\cos\left(3x^2\right)\Gamma\left(\frac{5}{4}\right)}{96\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)} - \frac{5\sqrt{6}\sqrt{\pi}C\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{576\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**2*sin(x**2)**3,x)
```

output
```
-15*x*cos(x**2)*gamma(5/4)/(32*gamma(9/4)) + 5*x*cos(3*x**2)*gamma(5/4)/(9
6*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*gamma(5/4
)/(64*gamma(9/4)) - 5*sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt(pi))*gamma(
5/4)/(576*gamma(9/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int x^2 \sin^3\left(x^2\right)\,dx = \frac{1}{24}\,x\cos\left(3\,x^2\right) - \frac{3}{8}\,x\cos\left(x^2\right) + \frac{1}{1152}\,\sqrt{\pi}\Bigg((2i-2)\,\sqrt{3}\sqrt{2}\,\mathrm{erf}\left(\sqrt{3i}x\right) - (2i+2)\,\sqrt{3}\sqrt{2}\,\mathrm{erf}\left(\sqrt{-3i}x\right) - (27i-27)\,\sqrt{2}\,\mathrm{erf}\Bigg(\bigg(\frac{1}{2}i + \frac{1}{2}$$

input
```
integrate(x^2*sin(x^2)^3,x, algorithm="maxima")
```

output
```
1/24*x*cos(3*x^2) - 3/8*x*cos(x^2) + 1/1152*sqrt(pi)*((2*I - 2)*sqrt(3)*sq
rt(2)*erf(sqrt(3*I)*x) - (2*I + 2)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27
*I - 27)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1
/2*I - 1/2)*sqrt(2)*x) + (27*I + 27)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)
*sqrt(2)*erf((-1)^(1/4)*x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\begin{aligned}\int x^2\sin^3\left(x^2\right)\,dx = {} & \left(\frac{1}{576}i+\frac{1}{576}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}x\right) \\ & -\left(\frac{1}{576}i-\frac{1}{576}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}x\right) \\ & -\left(\frac{3}{64}i+\frac{3}{64}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}x\right) \\ & +\left(\frac{3}{64}i-\frac{3}{64}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}x\right) \\ & +\frac{1}{48}xe^{\left(3i\,x^2\right)}-\frac{3}{16}xe^{\left(i\,x^2\right)}-\frac{3}{16}xe^{\left(-i\,x^2\right)}+\frac{1}{48}xe^{\left(-3i\,x^2\right)}\end{aligned}$$

input
```
integrate(x^2*sin(x^2)^3,x, algorithm="giac")
```

output
```
(1/576*I + 1/576)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) - (1/576*I
 - 1/576)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) - (3/64*I + 3/64)
*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) + (3/64*I - 3/64)*sqrt(2)*s
qrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) + 1/48*x*e^(3*I*x^2) - 3/16*x*e^(I*x
^2) - 3/16*x*e^(-I*x^2) + 1/48*x*e^(-3*I*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^3\left(x^2\right)\,dx=\int x^2\sin\left(x^2\right)^3\,dx$$

input
```
int(x^2*sin(x^2)^3,x)
```

output
```
int(x^2*sin(x^2)^3, x)
```

Reduce [F]

$$\int x^2 \sin^3\left(x^2\right)\,dx=\int \sin\left(x^2\right)^3x^2dx$$

input
```
int(x^2*sin(x^2)^3,x)
```

output
```
int(sin(x**2)**3*x**2,x)
```

3.32 $\int x^4 \cos\left(x^2\right) \sin^2\left(x^2\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int x^4 \cos\left(x^2\right) \sin^2\left(x^2\right)\,dx = \frac{1}{4}x\cos\left(x^2\right) - \frac{1}{12}x\cos^3\left(x^2\right) - \frac{3}{16}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) + \frac{1}{48}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right) + \frac{1}{6}x^3\sin^3\left(x^2\right)$$

output
```
1/4*x*cos(x^2)-1/12*x*cos(x^2)^3-3/32*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi
^(1/2)*x)+1/288*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*x)+1/6*x^3*sin(
x^2)^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int x^4 \cos\left(x^2\right) \sin^2\left(x^2\right)\,dx = \frac{1}{288}\left(-27\sqrt{2\pi}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right) + \sqrt{6\pi}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right) + 6x\left(9\cos\left(x^2\right) - \cos\left(3x^2\right) + 8x^2\sin^3\left(x^2\right)\right)\right)$$

input
```
Integrate[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

output
```
(-27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] + Sqrt[6*Pi]*FresnelC[Sqrt[6/Pi]*x]
 + 6*x*(9*Cos[x^2] - Cos[3*x^2] + 8*x^2*Sin[x^2]^3))/288
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3924, 3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sin^2\left(x^2\right) \cos\left(x^2\right) \, dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{1}{6} x^3 \sin^3\left(x^2\right) - \frac{1}{2} \int x^2 \sin^3\left(x^2\right) dx$$

$$\Big\downarrow \text{3884}$$

$$\frac{1}{6} x^3 \sin^3\left(x^2\right) - \frac{1}{2} \int \left(\frac{3}{4} x^2 \sin\left(x^2\right) - \frac{1}{4} x^2 \sin\left(3x^2\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{3}{8} \sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} x\right) + \frac{1}{24} \sqrt{\frac{\pi}{6}} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} x\right) + \frac{3}{8} x \cos\left(x^2\right) - \frac{1}{24} x \cos\left(3x^2\right)\right) + \frac{1}{6} x^3 \sin^3\left(x^2\right)$$

input
```
Int[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

output
```
((3*x*Cos[x^2])/8 - (x*Cos[3*x^2])/24 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*
x])/8 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/24)/2 + (x^3*Sin[x^2]^3)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

method	result
default	$\frac{x^{3} \sin\left(x^{2}\right)}{8}+\frac{3 x \cos\left(x^{2}\right)}{16}-\frac{3 \sqrt{2}\, \sqrt{\pi}\, \operatorname{FresnelC}\left(\frac{\sqrt{2}\, x}{\sqrt{\pi}}\right)}{32}-\frac{x^{3} \sin\left(3 x^{2}\right)}{24}-\frac{x \cos\left(3 x^{2}\right)}{48}+\frac{\sqrt{2}\, \sqrt{\pi}\, \sqrt{3}\, \operatorname{FresnelC}\left(\frac{\sqrt{2}\, \sqrt{3}\, x}{\sqrt{\pi}}\right)}{288}$
risch	$-\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{\pi}\, \sqrt{3}\, \operatorname{erf}\left(\sqrt{3}\, \left(-1\right)^{\frac{1}{4}} x\right)}{576}+\frac{3 \left(-1\right)^{\frac{3}{4}} \sqrt{\pi}\, \operatorname{erf}\left(\left(-1\right)^{\frac{1}{4}} x\right)}{64}-\frac{3 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-i}\, x\right)}{64 \sqrt{-i}}+\frac{\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-3 i}\, x\right)}{192 \sqrt{-3 i}}+\frac{3 x \cos\left(x^{2}\right)}{16}+ x^{3}$

input
```
int(x^4*cos(x^2)*sin(x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^3*sin(x^2)+3/16*x*cos(x^2)-3/32*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi
^(1/2)*x)-1/24*x^3*sin(3*x^2)-1/48*x*cos(3*x^2)+1/288*2^(1/2)*Pi^(1/2)*3^(
1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int x^4\cos\left(x^2\right)\sin^2\left(x^2\right)\,dx = -\frac{1}{12}\,x\cos\left(x^2\right)^3+\frac{1}{4}\,x\cos\left(x^2\right)+\frac{1}{288}\,\sqrt{6}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)-\frac{3}{32}\,\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)-\frac{1}{6}\left(x^3\cos\left(x^2\right)^2-x^3\right)\sin\left(x^2\right)$$

input
```
integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="fricas")
```

output
```
-1/12*x*cos(x^2)^3 + 1/4*x*cos(x^2) + 1/288*sqrt(6)*sqrt(pi)*fresnel_cos(s
qrt(6)*x/sqrt(pi)) - 3/32*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi))
 - 1/6*(x^3*cos(x^2)^2 - x^3)*sin(x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(82) = 164.

Time = 1.99 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.46

$$\int x^4\cos\left(x^2\right)\sin^2\left(x^2\right)\,dx = -\frac{9x^5\Gamma\left(-\frac{9}{4}\right)}{40\Gamma\left(-\frac{5}{4}\right)}+\frac{9x^3\sin\left(x^2\right)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)}-\frac{5x^3\sin\left(x^2\right)\Gamma\left(-\frac{5}{4}\right)}{16\Gamma\left(-\frac{1}{4}\right)}+\frac{3x^3\sin\left(3x^2\right)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)}+\frac{27x\cos\left(x^2\right)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)}-\frac{15x\cos\left(x^2\right)\Gamma\left(-\frac{5}{4}\right)}{32\Gamma\left(-\frac{1}{4}\right)}+\frac{3x\cos\left(3x^2\right)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)}+\frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{5}{4}\right)}{64\Gamma\left(-\frac{1}{4}\right)}-\frac{27\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{9}{4}\right)}{128\Gamma\left(-\frac{5}{4}\right)}-\frac{\sqrt{6}\sqrt{\pi}C\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{9}{4}\right)}{128\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate(x**4*cos(x**2)*sin(x**2)**2,x)
```

output
```
-9*x**5*gamma(-9/4)/(40*gamma(-5/4)) + 9*x**3*sin(x**2)*gamma(-9/4)/(32*ga
mma(-5/4)) - 5*x**3*sin(x**2)*gamma(-5/4)/(16*gamma(-1/4)) + 3*x**3*sin(3*
x**2)*gamma(-9/4)/(32*gamma(-5/4)) + 27*x*cos(x**2)*gamma(-9/4)/(64*gamma(
-5/4)) - 15*x*cos(x**2)*gamma(-5/4)/(32*gamma(-1/4)) + 3*x*cos(3*x**2)*gam
ma(-9/4)/(64*gamma(-5/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi
))*gamma(-5/4)/(64*gamma(-1/4)) - 27*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/s
qrt(pi))*gamma(-9/4)/(128*gamma(-5/4)) - sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)
*x/sqrt(pi))*gamma(-9/4)/(128*gamma(-5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int x^4\cos\left(x^2\right)\sin^2\left(x^2\right)\,dx$$
$$= -\frac{1}{24}x^3\sin\left(3x^2\right) + \frac{1}{8}x^3\sin\left(x^2\right) - \frac{1}{48}x\cos\left(3x^2\right) + \frac{3}{16}x\cos\left(x^2\right)$$
$$-\frac{1}{2304}\sqrt{\pi}\left((2i-2)\sqrt{3}\sqrt{2}\,\mathrm{erf}\left(\sqrt{3i}x\right) - (2i+2)\sqrt{3}\sqrt{2}\,\mathrm{erf}\left(\sqrt{-3i}x\right) - (27i-27)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right.\right.\right.$$

input
```
integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="maxima")
```

output
```
-1/24*x^3*sin(3*x^2) + 1/8*x^3*sin(x^2) - 1/48*x*cos(3*x^2) + 3/16*x*cos(x
^2) - 1/2304*sqrt(pi)*((2*I - 2)*sqrt(3)*sqrt(2)*erf(sqrt(3*I)*x) - (2*I +
 2)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I - 27)*sqrt(2)*erf((1/2*I + 1
/2)*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*x) + (27*I
+ 27)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqrt(2)*erf((-1)^(1/4)*x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.49

$$\begin{aligned}\int x^4 \cos\left(x^2\right)\sin^2\left(x^2\right)\,dx = & -\left(\frac{1}{1152}i + \frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{6}x\right)\\ & + \left(\frac{1}{1152}i - \frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{6}x\right)\\ & + \left(\frac{3}{128}i + \frac{3}{128}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}x\right)\\ & - \left(\frac{3}{128}i - \frac{3}{128}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}x\right)\\ & - \frac{1}{96}\left(-2i\,x^3 + x\right)e^{\left(3i\,x^2\right)} - \frac{1}{32}\left(2i\,x^3 - 3\,x\right)e^{\left(i\,x^2\right)}\\ & - \frac{1}{32}\left(-2i\,x^3 - 3\,x\right)e^{\left(-i\,x^2\right)} - \frac{1}{96}\left(2i\,x^3 + x\right)e^{\left(-3i\,x^2\right)}\end{aligned}$$

input
```
integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="giac")
```

output
```
-(1/1152*I + 1/1152)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) + (1/11
52*I - 1/1152)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) + (3/128*I +
 3/128)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) - (3/128*I - 3/128)*
sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) - 1/96*(-2*I*x^3 + x)*e^(3*
I*x^2) - 1/32*(2*I*x^3 - 3*x)*e^(I*x^2) - 1/32*(-2*I*x^3 - 3*x)*e^(-I*x^2)
 - 1/96*(2*I*x^3 + x)*e^(-3*I*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cos\left(x^2\right)\sin^2\left(x^2\right)\,dx = \int x^4\,\cos\left(x^2\right)\,{\sin\left(x^2\right)}^2\,dx$$

input
```
int(x^4*cos(x^2)*sin(x^2)^2,x)
```

output
```
int(x^4*cos(x^2)*sin(x^2)^2, x)
```

Reduce [F]

$$\int x^4 \cos \left(x^2\right) \sin^2 \left(x^2\right) \, dx$$
$$= \frac{\sqrt{2}\, \cos \left(x^2\right) \sin \left(x^2\right)^2 x + 2\sqrt{2}\, \cos \left(x^2\right) x - 2\sqrt{\pi}\, \mathit{fresnel\_c}\left(\frac{\sqrt{2}\, x}{\sqrt{\pi}}\right) - \sqrt{2}\left(\int \cos \left(x^2\right) \sin \left(x^2\right)^2 dx\right) + 2\sqrt{2}}{12\sqrt{2}}$$

input

```
int(x^4*cos(x^2)*sin(x^2)^2,x)
```

output

```
(sqrt(2)*cos(x**2)*sin(x**2)**2*x + 2*sqrt(2)*cos(x**2)*x - 2*sqrt(pi)*fre
snel_c((sqrt(2)*x)/sqrt(pi)) - sqrt(2)*int(cos(x**2)*sin(x**2)**2,x) + 2*s
qrt(2)*sin(x**2)**3*x**3)/(12*sqrt(2))
```

3.33 $\int x \sin^7(a+bx^2)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x \sin^7\left(a+bx^2\right)\, dx = -\frac{\cos\left(a+bx^2\right)}{2b} + \frac{\cos^3\left(a+bx^2\right)}{2b} - \frac{3\cos^5\left(a+bx^2\right)}{10b} + \frac{\cos^7\left(a+bx^2\right)}{14b}$$

output
```
-1/2*cos(b*x^2+a)/b+1/2*cos(b*x^2+a)^3/b-3/10*cos(b*x^2+a)^5/b+1/14*cos(b*
x^2+a)^7/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x \sin^7\left(a+bx^2\right)\, dx = -\frac{35\cos\left(a+bx^2\right)}{128b} + \frac{7\cos\left(3(a+bx^2)\right)}{128b} - \frac{7\cos\left(5(a+bx^2)\right)}{640b} + \frac{\cos\left(7(a+bx^2)\right)}{896b}$$

input
```
Integrate[x*Sin[a + b*x^2]^7,x]
```

output

```
(-35*Cos[a + b*x^2])/(128*b) + (7*Cos[3*(a + b*x^2)])/(128*b) - (7*Cos[5*(
a + b*x^2)])/(640*b) + Cos[7*(a + b*x^2)]/(896*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3860, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^7\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \sin^7\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(bx^2+a\right)^7dx^2$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(-x^{12}+3x^8-3x^4+1\right)d\cos\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos\left(a+bx^2\right)-\frac{x^{14}}{7}+\frac{3x^{10}}{5}-x^6}{2b}$$

input

```
Int[x*Sin[a + b*x^2]^7,x]
```

output

```
-1/2*(-x^6 + (3*x^10)/5 - x^14/7 + Cos[a + b*x^2])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 7.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\left(\frac{16}{5}+\sin\left(b\,x^{2}+a\right)^{6}+\frac{6\sin\left(b\,x^{2}+a\right)^{4}}{5}+\frac{8\sin\left(b\,x^{2}+a\right)^{2}}{5}\right)\cos\left(b\,x^{2}+a\right)}{14b}$	50
default	$-\frac{\left(\frac{16}{5}+\sin\left(b\,x^{2}+a\right)^{6}+\frac{6\sin\left(b\,x^{2}+a\right)^{4}}{5}+\frac{8\sin\left(b\,x^{2}+a\right)^{2}}{5}\right)\cos\left(b\,x^{2}+a\right)}{14b}$	50
parallelrisch	$\frac{-1024-1225\cos\left(b\,x^{2}+a\right)+245\cos\left(3b\,x^{2}+3a\right)-49\cos\left(5b\,x^{2}+5a\right)+5\cos\left(7b\,x^{2}+7a\right)}{4480b}$	57
risch	$-\frac{35\cos\left(b\,x^{2}+a\right)}{128b}+\frac{\cos\left(7b\,x^{2}+7a\right)}{896b}-\frac{7\cos\left(5b\,x^{2}+5a\right)}{640b}+\frac{7\cos\left(3b\,x^{2}+3a\right)}{128b}$	63
orering	Expression too large to display	1544

input
```
int(x*sin(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/14/b*(16/5+sin(b*x^2+a)^6+6/5*sin(b*x^2+a)^4+8/5*sin(b*x^2+a)^2)*cos(b*
x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{5\,\cos\left(bx^2+a\right)^7-21\,\cos\left(bx^2+a\right)^5+35\,\cos\left(bx^2+a\right)^3-35\,\cos\left(bx^2+a\right)}{70\,b}$$

input
```
integrate(x*sin(b*x^2+a)^7,x, algorithm="fricas")
```

output
```
1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*
cos(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int x\sin^7\left(a+bx^2\right)\,dx = \begin{cases} -\frac{\sin^6\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{\sin^4\left(a+bx^2\right)\cos^3\left(a+bx^2\right)}{b}-\frac{4\sin^2\left(a+bx^2\right)\cos^5\left(a+bx^2\right)}{5b}-\frac{8\cos^7\left(a+bx^2\right)}{35b} & \text{for}\ b\neq 0 \\ \frac{x^2\sin^7\left(a\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*sin(b*x**2+a)**7,x)
```

output
```
Piecewise((-sin(a + b*x**2)**6*cos(a + b*x**2)/(2*b) - sin(a + b*x**2)**4*
cos(a + b*x**2)**3/b - 4*sin(a + b*x**2)**2*cos(a + b*x**2)**5/(5*b) - 8*c
os(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sin(a)**7/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{5\cos\left(7\,bx^2+7\,a\right)-49\cos\left(5\,bx^2+5\,a\right)+245\cos\left(3\,bx^2+3\,a\right)-1225\cos\left(bx^2+a\right)}{4480\,b}$$

input
```
integrate(x*sin(b*x^2+a)^7,x, algorithm="maxima")
```

output
```
1/4480*(5*cos(7*b*x^2 + 7*a) - 49*cos(5*b*x^2 + 5*a) + 245*cos(3*b*x^2 + 3
*a) - 1225*cos(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sin^7\left(a+bx^2\right)\,dx = \frac{5\cos\left(bx^2+a\right)^7-21\cos\left(bx^2+a\right)^5+35\cos\left(bx^2+a\right)^3-35\cos\left(bx^2+a\right)}{70\,b}$$

input
```
integrate(x*sin(b*x^2+a)^7,x, algorithm="giac")
```

output
```
1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*
cos(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 39.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\sin^7\left(a+bx^2\right)\,dx$$
$$=\frac{245\,\cos\left(3\,b\,x^2+3\,a\right)-49\,\cos\left(5\,b\,x^2+5\,a\right)+5\,\cos\left(7\,b\,x^2+7\,a\right)-1225\,\cos\left(b\,x^2+a\right)}{4480\,b}$$

input
```
int(x*sin(a + b*x^2)^7,x)
```

output
```
(245*cos(3*a + 3*b*x^2) - 49*cos(5*a + 5*b*x^2) + 5*cos(7*a + 7*b*x^2) - 1
225*cos(a + b*x^2))/(4480*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int x\sin^7\left(a+bx^2\right)\,dx$$
$$=\frac{-5\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^6-6\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^4-8\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^2-16\cos}{70b}$$

input
```
int(x*sin(b*x^2+a)^7,x)
```

output
```
( - 5*cos(a + b*x**2)*sin(a + b*x**2)**6 - 6*cos(a + b*x**2)*sin(a + b*x**
2)**4 - 8*cos(a + b*x**2)*sin(a + b*x**2)**2 - 16*cos(a + b*x**2) + 16)/(7
0*b)
```

3.34 $\int \frac{(1+\sin(x^2))^2}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = -\frac{3}{4x^2}+\frac{\cos(2x^2)}{4x^2}+\text{CosIntegral}\left(x^2\right)-\frac{\sin(x^2)}{x^2}+\frac{\text{Si}(2x^2)}{2}$$

output
```
-3/4/x^2+1/4*cos(2*x^2)/x^2+Ci(x^2)-sin(x^2)/x^2+1/2*Si(2*x^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{-3+\cos(2x^2)+4x^2\,\text{CosIntegral}(x^2)-4\sin(x^2)+2x^2\text{Si}(2x^2)}{4x^2}$$

input
```
Integrate[(1 + Sin[x^2])^2/x^3,x]
```

output
```
(-3 + Cos[2*x^2] + 4*x^2*CosIntegral[x^2] - 4*Sin[x^2] + 2*x^2*SinIntegral
[2*x^2])/(4*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\sin(x^2)+1)^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 3884}$$

$$\int \left(\frac{3}{2x^3}+\frac{2\sin(x^2)}{x^3}-\frac{\cos(2x^2)}{2x^3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\text{CosIntegral}(x^2)+\frac{\text{Si}(2x^2)}{2}-\frac{3}{4x^2}-\frac{\sin(x^2)}{x^2}+\frac{\cos(2x^2)}{4x^2}$$

input

```
Int[(1 + Sin[x^2])^2/x^3,x]
```

output

```
-3/(4*x^2) + Cos[2*x^2]/(4*x^2) + CosIntegral[x^2] - Sin[x^2]/x^2 + SinInt
egral[2*x^2]/2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{3}{4x^2}+\frac{\cos(2x^2)}{4x^2}+\operatorname{Ci}\left(x^2\right)-\frac{\sin(x^2)}{x^2}+\frac{\operatorname{Si}(2x^2)}{2}$	39
parts	$-\frac{3}{4x^2}+\frac{\cos(2x^2)}{4x^2}+\operatorname{Ci}\left(x^2\right)-\frac{\sin(x^2)}{x^2}+\frac{\operatorname{Si}(2x^2)}{2}$	39
risch	$\operatorname{Ci}\left(x^2\right)-\frac{i\pi\ \operatorname{csgn}(ix^2)\operatorname{csgn}(x^2)}{2}+\frac{i\pi\ \operatorname{csgn}(ix^2)}{2}-\frac{3}{4x^2}-\frac{\pi\ \operatorname{csgn}(x^2)}{4}+\frac{\operatorname{Si}(2x^2)}{2}-\frac{\sin(x^2)}{x^2}+\frac{\cos(2x^2)}{4x^2}$	72

input
```
int((1+sin(x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/x^2+1/4*cos(2*x^2)/x^2+Ci(x^2)-sin(x^2)/x^2+1/2*Si(2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\left(1+\sin\left(x^2\right)\right)^2}{x^3}\,dx=\frac{2\,x^2\operatorname{Ci}\left(x^2\right)+x^2\operatorname{Si}\left(2\,x^2\right)+\cos\left(x^2\right)^2-2\,\sin\left(x^2\right)-2}{2\,x^2}$$

input
```
integrate((1+sin(x^2))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*x^2*cos_integral(x^2) + x^2*sin_integral(2*x^2) + cos(x^2)^2 - 2*si
n(x^2) - 2)/x^2
```

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{\left(1+\sin\left(x^2\right)\right)^2}{x^3}\,dx=-\log\left(x^2\right)+\frac{\log\left(x^4\right)}{2}+\operatorname{Ci}\left(x^2\right)+\frac{\operatorname{Si}\left(2x^2\right)}{2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{\cos\left(2x^2\right)}{4x^2}-\frac{3}{4x^2}$$

input

```
integrate((1+sin(x**2))**2/x**3,x)
```

output

```
-log(x**2) + log(x**4)/2 + Ci(x**2) + Si(2*x**2)/2 - sin(x**2)/x**2 + cos(
2*x**2)/(4*x**2) - 3/(4*x**2)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{x^2(i\,\Gamma(-1,2i\,x^2)-i\,\Gamma(-1,-2i\,x^2))-1}{4\,x^2} - \frac{1}{2\,x^2}+\frac{1}{2}\,\Gamma\left(-1,i\,x^2\right)+\frac{1}{2}\,\Gamma\left(-1,-i\,x^2\right)$$

input

```
integrate((1+sin(x^2))^2/x^3,x, algorithm="maxima")
```

output

```
1/4*(x^2*(I*gamma(-1, 2*I*x^2) - I*gamma(-1, -2*I*x^2)) - 1)/x^2 - 1/2/x^2
 + 1/2*gamma(-1, I*x^2) + 1/2*gamma(-1, -I*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(1+\sin(x^2))^2}{x^3}\,dx = \frac{4\,x^2\,\mathrm{Ci}\left(x^2\right)+2\,x^2\,\mathrm{Si}\left(2\,x^2\right)+\cos\left(2\,x^2\right)-4\,\sin\left(x^2\right)-3}{4\,x^2}$$

input

```
integrate((1+sin(x^2))^2/x^3,x, algorithm="giac")
```

output

```
1/4*(4*x^2*cos_integral(x^2) + 2*x^2*sin_integral(2*x^2) + cos(2*x^2) - 4*
sin(x^2) - 3)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sin\left(x^2\right)\right)^2}{x^3}\,dx = \operatorname{cosint}\left(x^2\right)+\frac{\operatorname{sinint}(2\,x^2)}{2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{\cos\left(x^2\right)^2}{2\,x^2}-\frac{1}{x^2}$$

input
```
int((sin(x^2) + 1)^2/x^3,x)
```

output
```
cosint(x^2) + sinint(2*x^2)/2 - sin(x^2)/x^2 + cos(x^2)^2/(2*x^2) - 1/x^2
```

Reduce [F]

$$\int \frac{\left(1+\sin\left(x^2\right)\right)^2}{x^3}\,dx = \frac{2\left(\int \frac{\sin\left(x^2\right)^2}{x^3}dx\right)x^2+4\left(\int \frac{\sin\left(x^2\right)}{x^3}dx\right)x^2-4\left(\int \frac{1}{x^3}dx\right)x^2-3}{2x^2}$$

input
```
int((1+sin(x^2))^2/x^3,x)
```

output
```
(2*int(sin(x**2)**2/x**3,x)*x**2 + 4*int(sin(x**2)/x**3,x)*x**2 - 4*int(1/
x**3,x)*x**2 - 3)/(2*x**2)
```

3.35 $\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 362

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx = -\frac{ix^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}+\frac{ix^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}$$
$$-\frac{x^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}+\frac{x^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}$$
$$-\frac{i\,\mathrm{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^3}+\frac{i\,\mathrm{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^3}$$

output

```
-1/2*I*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/
d+1/2*I*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/d-x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/d^2+x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/
2)/d^2-I*polylog(3,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/
2)/d^3+I*polylog(3,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/
2)/d^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx = \frac{-2dx^2\,\mathrm{PolyLog}\left(2,-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)-i\left(d^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)-d^2x^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)+2idx^2\,\mathrm{P}\right.}{2\sqrt{a^2-b^2}d^3}$$

input
```
Integrate[x^5/(a + b*Sin[c + d*x^2]),x]
```

output
```
(-2*d*x^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] -
I*(d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - d^2*x
^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*x^2*Po
lyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, (I*
b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])] - 2*PolyLog[3, (I*b*E^(I*(c +
d*x^2)))/(a + Sqrt[a^2 - b^2])]))/(2*Sqrt[a^2 - b^2]*d^3)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3860, 3042, 3804, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \frac{x^4}{a+b\sin(dx^2+c)}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{x^4}{a+b\sin(dx^2+c)}dx^2$$

$$\Big\downarrow \text{3804}$$

$$\int \frac{x^4 e^{i\left(c+dx^2\right)}}{2ae^{i\left(c+dx^2\right)}-ibe^{2i\left(c+dx^2\right)}+ib}dx^2$$

$$\Big\downarrow \text{2694}$$

$$\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^4}{2\left(a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^4}{2\left(a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^4}{a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^4}{a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\int x^2\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\int x^2\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{i\int\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^2}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{i\int\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)dx^2}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(3,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(3,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}$$

input

```
Int[x^5/(a + b*Sin[c + d*x^2]),x]
```

output

```
((-1/2*I)*b*((x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/
(b*d) - (2*((I*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2]
)])/d - PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])]/d^2))/(b
*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*((x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - (2*((I*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))
/(a + Sqrt[a^2 - b^2])])/d - PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
a^2 - b^2])]/d^2))/(b*d)))/Sqrt[a^2 - b^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^5}{a+b\sin\left(d\,x^2+c\right)}dx$$

input
```
int(x^5/(a+b*sin(d*x^2+c)),x)
```

output
```
int(x^5/(a+b*sin(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(300) = 600$.

Time = 0.23 (sec) , antiderivative size = 1435, normalized size of antiderivative = 3.96

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output

```
1/4*(2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(
d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) - 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 +
c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b + 1) - 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*
cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*
dilog((-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*si
n(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + b*c^2*sqrt(-(a^2 - b^2)
/b^2)*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) + b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) - 2*
I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - b*c^2*sqrt(-(a^
2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) + 2*I*a) - b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2
 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*d^
2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x
^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b) + (b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 +
c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2
 - b^2)/b^2) - b)/b) - (b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-...
```

Sympy [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx$$

input

```
integrate(x**5/(a+b*sin(d*x**2+c)),x)
```

output

```
Integral(x**5/(a + b*sin(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^5}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Giac [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^5}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^5}{a+b\,\sin\left(d\,x^2+c\right)}\,dx$$

input
```
int(x^5/(a + b*sin(c + d*x^2)),x)
```

output
```
int(x^5/(a + b*sin(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx = \int \frac{x^5}{\sin\left(d\,x^2+c\right)b+a}dx$$

input

```
int(x^5/(a+b*sin(d*x^2+c)),x)
```

output

```
int(x**5/(sin(c + d*x**2)*b + a),x)
```

3.36 $\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 245

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = -\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2} + \frac{\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2}$$

output

```
-1/2*I*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/
d+1/2*I*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/d-1/2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/d^2+1/2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/
2)/d^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = \frac{-idx^2\left(\log\left(1+\frac{ibe^{i\left(c+dx^2\right)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)\right)-\text{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^2\right)}}{-a+\sqrt{a^2-b^2}}\right)+\text{PolyLog}\left(2,\right.}{2\sqrt{a^2-b^2}d^2}$$

input
```
Integrate[x^3/(a + b*Sin[c + d*x^2]),x]
```

output
```
((-I)*d*x^2*(Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - Log
[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*
E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x
^2)))/(a + Sqrt[a^2 - b^2])])/(2*Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3860, 3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \frac{x^2}{a+b\sin(dx^2+c)}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{x^2}{a+b\sin(dx^2+c)}dx^2$$

$$\Big\downarrow \text{3804}$$

$$\int \frac{x^2 e^{i(c+dx^2)}}{2ae^{i(c+dx^2)}-ibe^{2i(c+dx^2)}+ib}dx^2$$

$$\Big\downarrow \; 2694$$

$$\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{2\left(a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{2\left(a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \; 2620$$

$$\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{\int \log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int \log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{ib\left(\frac{i\int \frac{\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{bd^2}+\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{i\int \frac{\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{bd^2}+\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}$$

input

```
Int[x^3/(a + b*Sin[c + d*x^2]),x]
```

output

```
((-1/2*I)*b*((x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/
(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(b*d
^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*((x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sq
rt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [F]

$$\int \frac{x^{3}}{a +b \sin \left(d \,x^{2}+c \right)}d x$$

input
```
int(x^3/(a+b*sin(d*x^2+c)),x)
```

output
```
int(x^3/(a+b*sin(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(199) = 398$.

Time = 0.19 (sec) , antiderivative size = 1041, normalized size of antiderivative = 4.25

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
-1/4*(b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + b*c*sqrt(-(a^2 - b^2)/b^2)*lo
g(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d
*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^
2)*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) - 2*I*a) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*s
in(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) -
a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^
2)/b^2) - b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c
) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2
 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-
(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/
b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b
*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*x^2 + b*c)*sqrt(-(a
^2 - b^2)/b^2)*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*...
```

Sympy [F]

$$\int \frac{x^3}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^3}{a+b\sin\left(c+dx^2\right)}\,dx$$

input
```
integrate(x**3/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral(x**3/(a + b*sin(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^3}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^3}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^3}{a+b\,\sin\left(d\,x^2+c\right)}\,dx$$

input
```
int(x^3/(a + b*sin(c + d*x^2)),x)
```

output
```
int(x^3/(a + b*sin(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx=\int \frac{x^3}{\sin\left(d\,x^2+c\right)b+a}dx$$

input
```
int(x^3/(a+b*sin(d*x^2+c)),x)
```

output
```
int(x**3/(sin(c + d*x**2)*b + a),x)
```

3.37 $\int \frac{x}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx=\frac{\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

output
```
arctan((b+a*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sin(c+dx^2)}\,dx=\frac{\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

input
```
Integrate[x/(a + b*Sin[c + d*x^2]),x]
```

output
```
ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3860, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sin\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{2}\int \frac{1}{a+b\sin\left(dx^2+c\right)}dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{1}{a+b\sin\left(dx^2+c\right)}dx^2$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{\int \frac{1}{ax^4+a+2b\tan\left(\frac{1}{2}\left(dx^2+c\right)\right)}d\tan\left(\frac{1}{2}\left(dx^2+c\right)\right)}{d}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{2\int \frac{1}{-x^4-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}\left(dx^2+c\right)\right)\right)}{d}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{2a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

input

```
Int[x/(a + b*Sin[c + d*x^2]),x]
```

output

```
ArcTan[(2*b + 2*a*Tan[(c + d*x^2)/2])/(2*Sqrt[a^2 - b^2])]/(Sqrt[a^2 - b^2
]*d)
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2a\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{d\sqrt{a^{2}-b^{2}}}$	48
default	$\frac{\arctan\left(\frac{2a\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{d\sqrt{a^{2}-b^{2}}}$	48
risch	$-\frac{\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}+\frac{i\sqrt{-a^{2}+b^{2}}\,a-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{2\sqrt{-a^{2}+b^{2}}\,d}+\frac{\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}+\frac{i\sqrt{-a^{2}+b^{2}}\,a+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{2\sqrt{-a^{2}+b^{2}}\,d}$	138

input

```
int(x/(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.33

$$\int \frac{x}{a+b\sin\left(c+dx^{2}\right)}\,dx = \left[-\frac{\sqrt{-a^{2}+b^{2}}\log\left(\frac{\left(2\,a^{2}-b^{2}\right)\cos\left(dx^{2}+c\right)^{2}-2\,ab\sin\left(dx^{2}+c\right)-a^{2}-b^{2}+2\,\left(a\cos\left(dx^{2}+c\right)\sin\left(dx^{2}+c\right)+b\cos\left(dx^{2}+c\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(dx^{2}+c\right)^{2}-2\,ab\sin\left(dx^{2}+c\right)-a^{2}-b^{2}}\right)}{4\,\left(a^{2}-b^{2}\right)d}, -\frac{\arctan\left(-\frac{a\sin\left(dx^{2}+c\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(dx^{2}+c\right)}\right)}{2\,\sqrt{a^{2}-b^{2}}d}\right]$$

input

```
integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x
^2 + c) - a^2 - b^2 + 2*(a*cos(d*x^2 + c)*sin(d*x^2 + c) + b*cos(d*x^2 + c
))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 -
b^2))/((a^2 - b^2)*d), -1/2*arctan(-(a*sin(d*x^2 + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x^2 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(37) = 74$.

Time = 3.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

$$\int \frac{x}{a + b\sin(c + dx^2)}\,dx = \begin{cases} \frac{\tilde{\infty}x^2}{\sin(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right)\right)}{2bd} & \text{for } a = 0 \\ \frac{x^2}{2(a+b\sin(c))} & \text{for } d = 0 \\ \frac{1}{bd\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) - bd} & \text{for } a = -b \\ -\frac{1}{bd\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) + bd} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^2}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*sin(d*x**2+c)),x)
```

output
```
Piecewise((zoo*x**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2
+ d*x**2/2))/(2*b*d), Eq(a, 0)), (x**2/(2*(a + b*sin(c))), Eq(d, 0)), (1/(
b*d*tan(c/2 + d*x**2/2) - b*d), Eq(a, -b)), (-1/(b*d*tan(c/2 + d*x**2/2) +
 b*d), Eq(a, b)), (log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)/(
2*d*sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**2/2) + b/a + sqrt(-a**2 + b**
2)/a)/(2*d*sqrt(-a**2 + b**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8078 vs. $2(43) = 86$.

Time = 23.68 (sec) , antiderivative size = 8078, normalized size of antiderivative = 168.29

$$\int \frac{x}{a + b\sin(c + dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/2*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^4*cos(c)*sin(c) - 4*(a^
2*b^4 - b^6)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c
)^3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 4*(3*(a^3*
b^3 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^
6)*cos(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)
^3 + 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2
*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^
4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 +
 3*((a^3*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*
x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5
 + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^
3 + a*b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a
*b^5)*cos(c)^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3
+ (2*a^5*b - 3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^
2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*
sin(c) - (a^3*b^3 - a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + ((4*a^4*b^2 - 5*
a^2*b^4 + b^6)*cos(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^
2 + 2*c))*sin(d*x^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^5*cos(c) - 4*a*b^4*cos(
d*x^2 + 2*c)^4*cos(c)*sin(c) + b^5*sin(d*x^2 + 2*c)^5*sin(c) + (b^5*cos(d*
x^2 + 2*c)*cos(c) + 4*a*b^4*cos(c)*sin(c))*sin(d*x^2 + 2*c)^4 + 2*((2*a...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{x}{a+b\sin\left(c+dx^2\right)}\,dx = \frac{\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

input

```
integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output

```
(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 40.71 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.67

$$\int \frac{x}{a+b\sin\left(c+dx^2\right)}\,dx = -\frac{\ln\left(-x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}-\frac{2\,x\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)-\ln\left(-x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}+\frac{2\,x\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{2\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

input

```
int(x/(a + b*sin(c + d*x^2)),x)
```

output

```
-(log(- x*exp(d*x^2*1i)*exp(c*1i)*2i - (2*x*(b*1i + a*exp(d*x^2*1i)*exp(c*
1i)))/((a + b)^(1/2)*(b - a)^(1/2))) - log((2*x*(b*1i + a*exp(d*x^2*1i)*ex
p(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2)) - x*exp(d*x^2*1i)*exp(c*1i)*2i))/(
2*d*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin\left(c+dx^2\right)}\,dx = \frac{\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)}$$

input

```
int(x/(a+b*sin(d*x^2+c)),x)
```

output

```
(sqrt(a**2 - b**2)*atan((tan((c + d*x**2)/2)*a + b)/sqrt(a**2 - b**2)))/(d
*(a**2 - b**2))
```

3.38 $\int \frac{1}{x(a+b\sin(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

output
```
Defer(Int)(1/x/(a+b*sin(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*Sin[c + d*x^2])),x]
```

output
```
Integrate[1/(x*(a + b*Sin[c + d*x^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}dx$$

$$\Big\downarrow \; 3908$$

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}dx$$

input
```
Int[1/(x*(a + b*Sin[c + d*x^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^{2}+c\right)\right)}dx$$

input
```
int(1/x/(a+b*sin(d*x^2+c)),x)
```

output
```
int(1/x/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^2+c\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*sin(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral(1/(x*(a + b*sin(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 39.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*sin(c + d*x^2))),x)
```

output
```
int(1/(x*(a + b*sin(c + d*x^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\frac{-\left(\int \frac{\sin\left(d\,x^2+c\right)}{\sin\left(d\,x^2+c\right)bx+ax}dx\right)b+\log(x)}{a}$$

input
```
int(1/x/(a+b*sin(d*x^2+c)),x)
```

output
```
( - int(sin(c + d*x**2)/(sin(c + d*x**2)*b*x + a*x),x)*b + log(x))/a
```

3.39 $\int \frac{1}{x^3(a+b\sin(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx = \text{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^3/(a+b*sin(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

input
```
Integrate[1/(x^3*(a + b*Sin[c + d*x^2])),x]
```

output
```
Integrate[1/(x^3*(a + b*Sin[c + d*x^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+b\sin(c+dx^2))}dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x^3 (a+b\sin(c+dx^2))}dx$$

input
```
Int[1/(x^3*(a + b*Sin[c + d*x^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+b\sin(d\,x^2+c))}dx$$

input
```
int(1/x^3/(a+b*sin(d*x^2+c)),x)
```

output
```
int(1/x^3/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*sin(d*x^2 + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

input
```
integrate(1/x**3/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral(1/(x**3*(a + b*sin(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{(b \sin (dx^2 + c) + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{(b \sin (dx^2 + c) + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 39.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx = \int \frac{1}{x^3 \, (a + b \, \sin (d \, x^2 + c))} \, dx$$

input
```
int(1/(x^3*(a + b*sin(c + d*x^2))),x)
```

output
```
int(1/(x^3*(a + b*sin(c + d*x^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \frac{1}{x^3 (a+b\sin(c+dx^2))}\,dx = \frac{-2\left(\int \frac{\sin(d\,x^2+c)}{\sin(d\,x^2+c)b\,x^3+a\,x^3}dx\right)b\,x^2-1}{2a\,x^2}$$

input
```
int(1/x^3/(a+b*sin(d*x^2+c)),x)
```

output
```
( - 2*int(sin(c + d*x**2)/(sin(c + d*x**2)*b*x**3 + a*x**3),x)*b*x**2 - 1)
/(2*a*x**2)
```

3.40 $\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx=\text{Int}\left(\frac{x^2}{a+b\sin\left(c+dx^2\right)},x\right)$$

output
```
Defer(Int)(x^2/(a+b*sin(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

input
```
Integrate[x^2/(a + b*Sin[c + d*x^2]),x]
```

output
```
Integrate[x^2/(a + b*Sin[c + d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$$

$$\Big\downarrow \text{ 3908}$$

$$\int \frac{x^2}{a+b\sin(c+dx^2)}dx$$

input
```
Int[x^2/(a + b*Sin[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sin(d\,x^2+c)}dx$$

input
```
int(x^2/(a+b*sin(d*x^2+c)),x)
```

output
```
int(x^2/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^2}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(x^2/(b*sin(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

input
```
integrate(x**2/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral(x**2/(a + b*sin(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx = \int \frac{x^2}{b\sin(dx^2+c)+a}\,dx$$

input
```
integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx = \int \frac{x^2}{b\sin(dx^2+c)+a}\,dx$$

input
```
integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 38.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx = \int \frac{x^2}{a+b\,\sin(d\,x^2+c)}\,dx$$

input
```
int(x^2/(a + b*sin(c + d*x^2)),x)
```

output
```
int(x^2/(a + b*sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b\sin(c + dx^2)}\,dx = \int \frac{x^2}{\sin(d\,x^2 + c)\,b + a}dx$$

input
```
int(x^2/(a+b*sin(d*x^2+c)),x)
```

output
```
int(x**2/(sin(c + d*x**2)*b + a),x)
```

3.41 $\int \frac{1}{a+b\sin(c+dx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \text{Int}\left(\frac{1}{a+b\sin(c+dx^2)},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx = \int \frac{1}{a+b\sin(c+dx^2)}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x^2])^(-1),x]
```

output
```
Integrate[(a + b*Sin[c + d*x^2])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3850}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx$$
$$\Big\downarrow \; 3850$$
$$\int \frac{1}{a+b\sin(c+dx^2)}dx$$

input
```
Int[(a + b*Sin[c + d*x^2])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(d\,x^2+c)}dx$$

input
```
int(1/(a+b*sin(d*x^2+c)),x)
```

output
```
int(1/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{1}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*sin(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

input
```
integrate(1/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral(1/(a + b*sin(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{b\sin(dx^2+c)+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{b\sin(dx^2+c)+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 38.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{a+b\,\sin(d\,x^2+c)}\,dx$$

input
```
int(1/(a + b*sin(c + d*x^2)),x)
```

output
```
int(1/(a + b*sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^2)}\,dx=\int \frac{1}{\sin\left(d\,x^2+c\right)b+a}dx$$

input
```
int(1/(a+b*sin(d*x^2+c)),x)
```

output
```
int(1/(sin(c + d*x**2)*b + a),x)
```

3.42 $\int \frac{1}{x^2(a+b\sin(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\mathrm{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^2])),x]
```

output
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^2\right)\right)} \, dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^2\right)\right)}dx$$

input

```
Int[1/(x^2*(a + b*Sin[c + d*x^2])),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b \sin \left(d \,x^2+c \right)\right)}dx$$

input

```
int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

output
```
int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^2+c\right)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*sin(d*x^2 + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral(1/(x**2*(a + b*sin(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^2))} \, dx = \int \frac{1}{(b\sin(dx^2 + c) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^2))} \, dx = \int \frac{1}{(b\sin(dx^2 + c) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 38.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\sin(c + dx^2))} \, dx = \int \frac{1}{x^2 \ (a + b\ \sin(d\,x^2 + c))} \, dx$$

input
```
int(1/(x^2*(a + b*sin(c + d*x^2))),x)
```

output
```
int(1/(x^2*(a + b*sin(c + d*x^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x^2 (a + b\sin (c + dx^2))}\, dx = \frac{-\left(\int \frac{\sin(d\,x^2+c)}{\sin(d\,x^2+c)b\,x^2+a\,x^2}dx\right) bx - 1}{ax}$$

input
```
int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

output
```
( - (int(sin(c + d*x**2)/(sin(c + d*x**2)*b*x**2 + a*x**2),x)*b*x + 1))/(a
*x)
```

3.43 $\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 663

$$
\begin{aligned}
\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = {} & \frac{ix^4}{2\left(a^2-b^2\right)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} \\
& - \frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} \\
& + \frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^3} \\
& - \frac{ax^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^2} \\
& + \frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^3} + \frac{ax^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^2} \\
& - \frac{ia\,\mathrm{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^3} + \frac{ia\,\mathrm{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^3} \\
& + \frac{bx^4\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}
\end{aligned}
$$

output
```
1/2*I*x^4/(a^2-b^2)/d-x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(
a^2-b^2)/d^2-1/2*I*a*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a
^2-b^2)^(3/2)/d-x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^
2)/d^2+1/2*I*a*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2
)^(3/2)/d+I*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/
d^3-a*x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3
/2)/d^2+I*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^
3+a*x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2
)/d^2-I*a*polylog(3,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3
/2)/d^3+I*a*polylog(3,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^
(3/2)/d^3+1/2*b*x^4*cos(d*x^2+c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx$$

$$= \frac{id^2x^4 - 2dx^2\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right) - \frac{iad^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2dx^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right) +}{}$$

input
```
Integrate[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(I*d^2*x^4 - 2*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2]
)] - (I*a*d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])
/Sqrt[a^2 - b^2] - 2*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])] + (I*a*d^2*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2
])])/Sqrt[a^2 - b^2] + (2*I - (2*a*d*x^2)/Sqrt[a^2 - b^2])*PolyLog[2, ((-I
)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + (2*I + (2*a*d*x^2)/Sqrt[a
^2 - b^2])*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] - ((2
*I)*a*PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2
- b^2] + ((2*I)*a*PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])
])/Sqrt[a^2 - b^2] + (b*d^2*x^4*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2
*(a^2 - b^2)*d^3)
```

Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3860, 3042, 3805, 3042, 3804, 2694, 27, 2620, 3011, 2720, 5030, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{2}\int \frac{x^4}{\left(a+b\sin\left(dx^2+c\right)\right)^2}dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{x^4}{\left(a+b\sin\left(dx^2+c\right)\right)^2}dx^2$$

$$\Big\downarrow \text{ 3805}$$

$$\frac{1}{2}\left(-\frac{2b\int \frac{x^2\cos\left(dx^2+c\right)}{a+b\sin\left(dx^2+c\right)}dx^2}{d\left(a^2-b^2\right)}+\frac{a\int \frac{x^4}{a+b\sin\left(dx^2+c\right)}dx^2}{a^2-b^2}+\frac{bx^4\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(-\frac{2b\int \frac{x^2\cos\left(dx^2+c\right)}{a+b\sin\left(dx^2+c\right)}dx^2}{d\left(a^2-b^2\right)}+\frac{a\int \frac{x^4}{a+b\sin\left(dx^2+c\right)}dx^2}{a^2-b^2}+\frac{bx^4\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{ 3804}$$

$$\frac{1}{2}\left(-\frac{2b\int \frac{x^2\cos\left(dx^2+c\right)}{a+b\sin\left(dx^2+c\right)}dx^2}{d\left(a^2-b^2\right)}+\frac{2a\int \frac{e^{i\left(dx^2+c\right)}x^4}{2e^{i\left(dx^2+c\right)}a-ibe^{2i\left(dx^2+c\right)}+ib}dx^2}{a^2-b^2}+\frac{bx^4\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{1}{2}\left(-\frac{2b\int\frac{x^2\cos(dx^2+c)}{a+b\sin(dx^2+c)}dx^2}{d\left(a^2-b^2\right)}+\frac{2a\left(\frac{ib\int\frac{e^{i\left(dx^2+c\right)}x^4}{2\left(a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i\left(dx^2+c\right)}x^4}{2\left(a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{bx^4\,\mathrm{c}}{d\left(a^2-b^2\right)(}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(-\frac{2b\int\frac{x^2\cos(dx^2+c)}{a+b\sin(dx^2+c)}dx^2}{d\left(a^2-b^2\right)}+\frac{2a\left(\frac{ib\int\frac{e^{i\left(dx^2+c\right)}x^4}{a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}-\frac{ib\int\frac{e^{i\left(dx^2+c\right)}x^4}{a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{bx^4\cos\left(c+\right.}{d\left(a^2-b^2\right)\left(a+b\,\mathrm{s}\right.}\right.$$

$$\Big\downarrow 2620$$

$$\frac{1}{2}\left(-\frac{2b\int\frac{x^2\cos(dx^2+c)}{a+b\sin(dx^2+c)}dx^2}{d\left(a^2-b^2\right)}+\frac{2a\left(\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\int x^2\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\int}{}\right.}{2\sqrt{a^2-}}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow 3011$$

$$\frac{1}{2}\Bigg(\frac{2a\Bigg(\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{i\int\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^2}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\Bigg(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-}{}}{a^2-b^2}$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}\left(\frac{2a\left(\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-$$

$$a^2-b^2$$

↓ 5030

$$\frac{1}{2}\Bigg(\frac{2a\Bigg(\frac{ib\Bigg(\frac{x^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\mathrm{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i(dx^2+c)}}{d^2}\right)}{bd}\Bigg)}{2\sqrt{a^2-b^2}}-\frac{ib\Bigg(\frac{x^4\log\Bigg(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-}}}{bd}}{}}{a^2-b^2}$$

$\downarrow$ 2620

$$\frac{1}{2}\Bigg(\frac{2a\Bigg(\frac{ib\Bigg(\frac{x^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i(dx^2+c)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i(dx^2+c)}}{d^2}\right)}{bd}\Bigg)}{2\sqrt{a^2-b^2}}-\frac{ib\Bigg(\frac{x^4\log\Big(1-\frac{ibe^{i(c+dx}}{a-\sqrt{a^2-}}}{bd}}{}}{a^2-b^2}$$

$\downarrow$ 2715

$$\frac{1}{2}\left(\frac{2a\left(\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx\right.}}{a-\sqrt{a^2-}}\right.}{bd}\right.}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 2838

$$\frac{1}{2}\left(\frac{2a\left(\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{2\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{d^2}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^4\log\left(1-\frac{ibe^{i\left(c+dx\right.}}{a-\sqrt{a^2-}}\right.}{bd}\right.}{}\right.}{a^2-b^2}\right.$$

↓ 7143

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{ix^4}{2b}\right)}{d\left(a^2-b^2\right)}\right.$$

input
```
Int[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
((-2*b*(((-1/2*I)*x^4)/b + (x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[
a^2 - b^2])])/(b*d) + (x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2
])])/(b*d^2) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])
])/(b*d^2)))/((a^2 - b^2)*d) + (2*a*(((-1/2*I)*b*((x^4*Log[1 - (I*b*E^(I*(
c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (2*((I*x^2*PolyLog[2, (I*b*E^
(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/d - PolyLog[3, (I*b*E^(I*(c + d*x
^2)))/(a - Sqrt[a^2 - b^2])]/d^2))/(b*d)))/Sqrt[a^2 - b^2] + ((I/2)*b*((x^
4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*((I*x
^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/d - PolyLog[
3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])]/d^2))/(b*d)))/Sqrt[a^2 -
 b^2]))/(a^2 - b^2) + (b*x^4*Cos[c + d*x^2])/((a^2 - b^2)*d*(a + b*Sin[c +
 d*x^2])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^{5}}{\left(a +b \sin \left(d \,x^{2}+c \right)\right)^{2}}d x$$

input
```
int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2469 vs. $2(565) = 1130$.

Time = 0.25 (sec) , antiderivative size = 2469, normalized size of antiderivative = 3.72

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/4*(2*(a^2*b - b^3)*d^2*x^4*cos(d*x^2 + c) + 2*(a*b^2*sin(d*x^2 + c) + a^
2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x^2 + c) + a*sin(d*x^2
+ c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b)
- 2*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a
*cos(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c
))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^
2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos
(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(a*b^2*si
n(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x^2 +
 c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^
2 - b^2)/b^2))/b) - 2*(-I*a^3 + I*a*b^2 + (-I*a^2*b + I*b^3)*sin(d*x^2 + c
) + (-I*a*b^2*d*x^2*sin(d*x^2 + c) - I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2)
)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*s
in(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*a^3 + I*a*b^2 +
(-I*a^2*b + I*b^3)*sin(d*x^2 + c) + (I*a*b^2*d*x^2*sin(d*x^2 + c) + I*a^2*
b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 +
 c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/
b + 1) - 2*(I*a^3 - I*a*b^2 + (I*a^2*b - I*b^3)*sin(d*x^2 + c) + (I*a*b^2*
d*x^2*sin(d*x^2 + c) + I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*
cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 +...
```

Sympy [F]

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(x**5/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral(x**5/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\sin(dx^2+c)+a)^2}\,dx$$

input

```
integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
(a*b*x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*x^4*cos(d*x^2 + c) + ((a^
2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2
 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)
*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b
- a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin
(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))*integrate(2*(2*a^2*d*
x^5*cos(d*x^2 + c)^2 + 2*a^2*d*x^5*sin(d*x^2 + c)^2 + a*b*d*x^5*sin(d*x^2
+ c) - 2*a*b*x^3*cos(d*x^2 + c) - (a*b*d*x^5*sin(d*x^2 + c) + 2*a*b*x^3*co
s(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a*b*d*x^5*cos(d*x^2 + c) - 2*a*b*x^3*s
in(d*x^2 + c) - 2*b^2*x^3)*sin(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*
x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*
cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2
 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 +
c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2
- b^4)*d)*cos(2*d*x^2 + 2*c)), x) + (a*b*x^4*sin(d*x^2 + c) + b^2*x^4)*sin
(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^
2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2
*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x
^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*
(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c...
```

Giac [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\sin(dx^2+c)+a)^2}\,dx$$

input

```
integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^5/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{x^5}{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

input
```
int(x^5/(a + b*sin(c + d*x^2))^2,x)
```

output
```
int(x^5/(a + b*sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{x^5}{\sin\left(d\,x^2+c\right)^2 b^2+2\sin\left(d\,x^2+c\right)ab+a^2}dx$$

input
```
int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(x**5/(sin(c + d*x**2)**2*b**2 + 2*sin(c + d*x**2)*a*b + a**2),x)
```

3.44 $\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 324

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = {} & -\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d}+\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d}\\ & -\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2\left(a^2-b^2\right)d^2}-\frac{a\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d^2}\\ & +\frac{a\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d^2}\\ & +\frac{bx^2\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}\end{aligned}$$

output

```
-1/2*I*a*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2
)/d+1/2*I*a*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(
3/2)/d-1/2*ln(a+b*sin(d*x^2+c))/(a^2-b^2)/d^2-1/2*a*polylog(2,I*b*exp(I*(d
*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/2*a*polylog(2,I*b*exp(
I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/2*b*x^2*cos(d*x^2+
c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \frac{-\frac{iadx^2\log\left(1+\frac{ibe^{i\left(c+dx^2\right)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{iadx^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{\log(a+b\sin(c+dx^2))}{a^2-b^2}-\frac{a\,\text{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^2\right)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\,\text{PolyLog}\left(2\right.}{\left(a^2-\right.}}{2d^2}$$

input
```
Integrate[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(((-I)*a*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a
^2 - b^2)^(3/2) + (I*a*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2
 - b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^2]]/(a^2 - b^2) - (a*
PolyLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)
^(3/2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (b*d*x^2*Cos[c + d*x^2])/((a^2 - b^2)*(a + b*Sin[c + d*x^
2])))/(2*d^2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3860, 3042, 3805, 3042, 3147, 16, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{2}\int \frac{x^2}{(a+b\sin(dx^2+c))^2}dx^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \frac{x^2}{\left(a+b\sin\left(dx^2+c\right)\right)^2}dx^2$$

$$\Big\downarrow \text{3805}$$

$$\frac{1}{2}\left(\frac{a\int \frac{x^2}{a+b\sin(dx^2+c)}dx^2}{a^2-b^2}-\frac{b\int \frac{\cos(dx^2+c)}{a+b\sin(dx^2+c)}dx^2}{d\left(a^2-b^2\right)}+\frac{bx^2\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{a\int \frac{x^2}{a+b\sin(dx^2+c)}dx^2}{a^2-b^2}-\frac{b\int \frac{\cos(dx^2+c)}{a+b\sin(dx^2+c)}dx^2}{d\left(a^2-b^2\right)}+\frac{bx^2\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{3147}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{a+b\sin(dx^2+c)}d\left(b\sin\left(dx^2+c\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{a\int \frac{x^2}{a+b\sin(dx^2+c)}dx^2}{a^2-b^2}+\frac{bx^2\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{a\int \frac{x^2}{a+b\sin(dx^2+c)}dx^2}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^2\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{3804}$$

$$\frac{1}{2}\left(\frac{2a\int \frac{e^{i\left(dx^2+c\right)}x^2}{2e^{i\left(dx^2+c\right)}a-ibe^{2i\left(dx^2+c\right)}+ib}dx^2}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^2\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2694}$$

$$\frac{1}{2}\left(\frac{2a\left(\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{2\left(a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{2\left(a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}\right)}dx^2}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^2}{d\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\Bigg(\frac{2a\left(\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{a-ibe^{i\left(dx^2+c\right)}+\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^2+c\right)}x^2}{a-ibe^{i\left(dx^2+c\right)}-\sqrt{a^2-b^2}}dx^2}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^2\cos(c}{d\left(a^2-b^2\right)(a+b}$$

$$\Big\downarrow 2620$$

$$\frac{1}{2}\Bigg(\frac{2a\left(\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{\int \log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int \log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)dx^2}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log}{}$$

$$\Big\downarrow 2715$$

$$\frac{1}{2}\left(\frac{2a\left(\frac{ib\left(\frac{i\int\frac{\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{bd^2}+\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{i\int\frac{\log\left(1-\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{x^2}de^{i\left(dx^2+c\right)}}{bd^2}+\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx\right.}}{a-\sqrt{a^2-}}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow 2838$$

$$\frac{1}{2}\left(\frac{2a\left(\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^2+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\log\right.$$

input
```
Int[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(-(Log[a + b*Sin[c + d*x^2]]/((a^2 - b^2)*d^2)) + (2*a*(((-1/2*I)*b*((x^2*
Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*PolyLog
[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b
^2] + ((I/2)*b*((x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])
])/(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(
b*d^2)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (b*x^2*Cos[c + d*x^2])/((a^2 - b^
2)*d*(a + b*Sin[c + d*x^2])))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\sin\left(d\,x^2+c\right)\right)^2}dx$$

input
```
int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(274) = 548$.

Time = 0.22 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.66

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output

```
1/4*(2*(a^2*b - b^3)*d*x^2*cos(d*x^2 + c) + (I*a*b^2*sin(d*x^2 + c) + I*a^
2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) +
 (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*
cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)
*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b
*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (I*a*b^2*sin(d*x^2 + c) + I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos
(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2
+ a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c)
 - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b) + (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(
d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2
+ c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b) - (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(
d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d
*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b...
```

Sympy [F]

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx$$

input

```
integrate(x**3/(a+b*sin(d*x**2+c))**2,x)
```

output

```
Integral(x**3/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{x^3}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^3/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{x^3}{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

input
```
int(x^3/(a + b*sin(c + d*x^2))^2,x)
```

output
```
int(x^3/(a + b*sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{x^3}{\sin\left(d\,x^2+c\right)^2 b^2+2\sin\left(d\,x^2+c\right) ab+a^2}dx$$

input
```
int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(x**3/(sin(c + d*x**2)**2*b**2 + 2*sin(c + d*x**2)*a*b + a**2),x)
```

3.45 $\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 91

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}$$

output

```
a*arctan((b+a*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+1/2
*b*cos(d*x^2+c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{\frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b\cos\left(c+dx^2\right)}{a+b\sin\left(c+dx^2\right)}}{2(a-b)(a+b)d}$$

input

```
Integrate[x/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
((2*a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
+ (b*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a - b)*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3860, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \frac{1}{\left(a+b\sin\left(dx^2+c\right)\right)^2}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\left(a+b\sin\left(dx^2+c\right)\right)^2}dx^2$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{2}\left(\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}-\frac{\int -\frac{a}{a+b\sin\left(dx^2+c\right)}dx^2}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{a}{a+b\sin\left(dx^2+c\right)}dx^2}{a^2-b^2}+\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{a+b\sin\left(dx^2+c\right)}dx^2}{a^2-b^2}+\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{a+b\sin\left(dx^2+c\right)}dx^2}{a^2-b^2}+\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{2}\left(\frac{2a\int \frac{1}{ax^4+a+2b\tan\left(\frac{1}{2}(dx^2+c)\right)}d\tan\left(\frac{1}{2}\left(dx^2+c\right)\right)}{d\left(a^2-b^2\right)}+\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}-\frac{4a\int \frac{1}{-x^4-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}\left(dx^2+c\right)\right)\right)}{d\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{2a\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx^2)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^2\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}\right)$$

input

```
Int[x/(a + b*Sin[c + d*x^2])^2,x]
```

output

```
((2*a*ArcTan[(2*b + 2*a*Tan[(c + d*x^2)/2])/(2*Sqrt[a^2 - b^2])])/((a^2 -
b^2)^(3/2)*d) + (b*Cos[c + d*x^2])/((a^2 - b^2)*d*(a + b*Sin[c + d*x^2])))
/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2 a+2b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{2d}$
default	$\frac{\frac{\frac{2b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2 a+2b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{2d}$
risch	$\frac{ib+a\,\mathrm{e}^{i\left(d\,x^2+c\right)}}{(a^2-b^2)d\left(b\,\mathrm{e}^{2i\left(d\,x^2+c\right)}-b+2ia\,\mathrm{e}^{i\left(d\,x^2+c\right)}\right)}-\frac{a\ln\left(\mathrm{e}^{i\left(d\,x^2+c\right)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{2\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{a\ln\left(\mathrm{e}^{i\left(d\,x^2+c\right)}+\frac{i\sqrt{-a^2}}{\sqrt{}}\right.}{2\sqrt{-a^2+b^2}\,(a+b}$

input
```
int(x/(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(2*(b^2/a/(a^2-b^2)*tan(1/2*d*x^2+1/2*c)+b/(a^2-b^2))/(tan(1/2*d*x^2
+1/2*c)^2*a+2*b*tan(1/2*d*x^2+1/2*c)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*
a*tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.02

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

$$=\left[\frac{\left(ab\sin\left(dx^2+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2-2\,\left(a\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)+b\cos\right.}{b^2\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2}\right.}{4\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx^2+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right.}\right.$$

$$\left.-\frac{\left(ab\sin\left(dx^2+c\right)+a^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx^2+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx^2+c\right)}\right)-\left(a^2b-b^3\right)\cos\left(dx^2+c\right)}{2\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx^2+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}\right]$$

input
```
integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output

```
[1/4*((a*b*sin(d*x^2 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2 - 2*(a*cos(d*x^2 + c)*sin(
d*x^2 + c) + b*cos(d*x^2 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2
*a*b*sin(d*x^2 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^2 + c))/((a^4*
b - 2*a^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/2
*((a*b*sin(d*x^2 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^2 + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x^2 + c))) - (a^2*b - b^3)*cos(d*x^2 + c))/((a^4*
b - 2*a^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(71) = 142$.

Time = 53.75 (sec) , antiderivative size = 2116, normalized size of antiderivative = 23.25

$$\int \frac{x}{\left(a + b\sin\left(c + dx^2\right)\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate(x/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Piecewise((zoo*x**2/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2
+ d*x**2/2)/(4*d) - 1/(4*d*tan(c/2 + d*x**2/2)))/b**2, Eq(a, 0)), (-3*tan(
c/2 + d*x**2/2)**2/(3*b**2*d*tan(c/2 + d*x**2/2)**3 - 9*b**2*d*tan(c/2 + d
*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) - 3*b**2*d) + 3*tan(c/2 + d*x**
2/2)/(3*b**2*d*tan(c/2 + d*x**2/2)**3 - 9*b**2*d*tan(c/2 + d*x**2/2)**2 +
9*b**2*d*tan(c/2 + d*x**2/2) - 3*b**2*d) - 2/(3*b**2*d*tan(c/2 + d*x**2/2)
**3 - 9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) - 3*b
**2*d), Eq(a, -b)), (-3*tan(c/2 + d*x**2/2)**2/(3*b**2*d*tan(c/2 + d*x**2/
2)**3 + 9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) + 3
*b**2*d) - 3*tan(c/2 + d*x**2/2)/(3*b**2*d*tan(c/2 + d*x**2/2)**3 + 9*b**2
*d*tan(c/2 + d*x**2/2)**2 + 9*b**2*d*tan(c/2 + d*x**2/2) + 3*b**2*d) - 2/(
3*b**2*d*tan(c/2 + d*x**2/2)**3 + 9*b**2*d*tan(c/2 + d*x**2/2)**2 + 9*b**2
*d*tan(c/2 + d*x**2/2) + 3*b**2*d), Eq(a, b)), (x**2/(2*(a + b*sin(c))**2)
, Eq(d, 0)), (a**3*log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)*t
an(c/2 + d*x**2/2)**2/(2*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2
+ 2*a**4*d*sqrt(-a**2 + b**2) + 4*a**3*b*d*sqrt(-a**2 + b**2)*tan(c/2 + d*
x**2/2) - 2*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 - 2*a**2
*b**2*d*sqrt(-a**2 + b**2) - 4*a*b**3*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**
2/2)) + a**3*log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)/(2*a**4
*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**2/2)**2 + 2*a**4*d*sqrt(-a**2 + b*...
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{\left(\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{(a^2d-b^2d)\sqrt{a^2-b^2}} + \frac{b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+ab}{(a^3d-ab^2d)\left(a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+a\right)}$$

input
```
integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2
*c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + (b^2*tan(
1/2*d*x^2 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^2 + 1/2*c)^2 +
 2*b*tan(1/2*d*x^2 + 1/2*c) + a))
```

Mupad [B] (verification not implemented)

Time = 39.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.96

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{\frac{b}{a^2-b^2}+\frac{b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{a\,\left(a^2-b^2\right)}}{d\,\left(a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2+2\,b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+a\right)} + \frac{a\,\mathrm{atan}\left(\frac{\left(a^2-b^2\right)\,\left(\frac{a^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{a\,\left(2\,a^2\,b-2\,b^3\right)}{2\,{\left(a+b\right)}^{3/2}\,\left(a^2-b^2\right)\,{\left(a-b\right)}^{3/2}}\right)}{a}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

input
```
int(x/(a + b*sin(c + d*x^2))^2,x)
```

output

```
(b/(a^2 - b^2) + (b^2*tan(c/2 + (d*x^2)/2))/(a*(a^2 - b^2)))/(d*(a + a*tan
(c/2 + (d*x^2)/2)^2 + 2*b*tan(c/2 + (d*x^2)/2))) + (a*atan(((a^2 - b^2)*((
a^2*tan(c/2 + (d*x^2)/2))/((a + b)^(3/2)*(a - b)^(3/2)) + (a*(2*a^2*b - 2*
b^3))/(2*(a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/a))/(d*(a + b)^(3/2)*(
a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin\left(d\,x^2+c\right)ab+2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2+\cos\left(d\,x^2+c\right)}{2d\left(\sin\left(d\,x^2+c\right)a^4b-2\sin\left(d\,x^2+c\right)a^2b^3+\sin\left(d\,x^2+c\right)b^5+a^5-2a^3b^2+a\,b^4\right)}$$

input

```
int(x/(a+b*sin(d*x^2+c))^2,x)
```

output

```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x**2)/2)*a + b)/sqrt(a**2 - b**2))*s
in(c + d*x**2)*a*b + 2*sqrt(a**2 - b**2)*atan((tan((c + d*x**2)/2)*a + b)/
sqrt(a**2 - b**2))*a**2 + cos(c + d*x**2)*a**2*b - cos(c + d*x**2)*b**3)/(
2*d*(sin(c + d*x**2)*a**4*b - 2*sin(c + d*x**2)*a**2*b**3 + sin(c + d*x**2
)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.46 $\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[1/(x*(a + b*Sin[c + d*x^2])^2),x]
```

output
```
Integrate[1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}dx$$

input
```
Int[1/(x*(a + b*Sin[c + d*x^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^2+c\right)\right)^2}dx$$

input
```
int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x*cos(d*x^2 + c)^2 - 2*a*b*x*sin(d*x^2 + c) - (a^2 + b^2)
*x), x)
```

Sympy [N/A]

Not integrable

Time = 26.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral(1/(x*(a + b*sin(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 3466, normalized size of antiderivative = 192.56

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x}\,dx$$

input

```
integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
(a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4
*cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b -
a*b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^
2)*sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4
 - a^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^2*cos(2*d*x^2
 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin
(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 +
(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b^6*cos(2*c)^2
 + b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*
x^2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 +
a*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2 - 2*(2*
((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*
x^2*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos
(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*
d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2*d*x^
2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3
*b^3)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2 + 2
*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)
*sin(c))*d*x^2*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3...
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 39.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

input
```
int(1/(x*(a + b*sin(c + d*x^2))^2),x)
```

output
```
int(1/(x*(a + b*sin(c + d*x^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 6.22

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$
$$=\frac{-\left(\int \frac{\sin(d\,x^2+c)^2}{\sin(d\,x^2+c)^2 b^2x+2\sin(d\,x^2+c)abx+a^2x}dx\right)b^2-2\left(\int \frac{\sin(d\,x^2+c)}{\sin(d\,x^2+c)^2 b^2x+2\sin(d\,x^2+c)abx+a^2x}dx\right)ab+\log(x)}{a^2}$$

input

```
int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

output

```
( - int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*b**2*x + 2*sin(c + d*x**2)*
a*b*x + a**2*x),x)*b**2 - 2*int(sin(c + d*x**2)/(sin(c + d*x**2)**2*b**2*x
 + 2*sin(c + d*x**2)*a*b*x + a**2*x),x)*a*b + log(x))/a**2
```

3.47 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
```

output
```
Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2}dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2}dx$$

input

```
Int[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \left(a+b\sin\left(d\,x^2+c\right)\right)^2}dx$$

input

```
int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x^3*cos(d*x^2 + c)^2 - 2*a*b*x^3*sin(d*x^2 + c) - (a^2 +
b^2)*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 38.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(1/x**3/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral(1/(x**3*(a + b*sin(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.57 (sec) , antiderivative size = 3475, normalized size of antiderivative = 193.06

$$\int \frac{1}{x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2 x^3}\,dx$$

input

```
integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
(a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4
*cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b -
a*b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^
2)*sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4
 - a^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^4*cos(2*d*x^2
 + 2*c)^2 + a^4*b^2*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin
(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 +
(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + (b^6*cos(2*c)^2
 + b^6*sin(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*
x^4*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 +
a*b^5)*d*x^4*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^4 - 2*(2*
((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*
x^4*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos
(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*
d*x^2) - 2*(a^2*b^4*d*x^4*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^4*sin(2*d*x^
2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3
*b^3)*d*x^4*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x^2 + 2
*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)
*sin(c))*d*x^4*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3...
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^2))^2} \, dx = \int \frac{1}{(b\sin(dx^2 + c) + a)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 39.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^2))^2} \, dx = \int \frac{1}{x^3 \left(a + b\,\sin(d\,x^2 + c)\right)^2} \, dx$$

input
```
int(1/(x^3*(a + b*sin(c + d*x^2))^2),x)
```

output
```
int(1/(x^3*(a + b*sin(c + d*x^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 1331, normalized size of antiderivative = 73.94

$$\int \frac{1}{x^3 (a + b\sin(c + dx^2))^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

output
```
(4*cos(c + d*x**2)*b**2 + 8*int(cos(c + d*x**2)/(sin(c + d*x**2)**2*b**2*x
**3 + 2*sin(c + d*x**2)*a*b*x**3 + a**2*x**3),x)*sin(c + d*x**2)*a*b**3*x*
*2 + 8*int(cos(c + d*x**2)/(sin(c + d*x**2)**2*b**2*x**3 + 2*sin(c + d*x**
2)*a*b*x**3 + a**2*x**3),x)*a**2*b**2*x**2 - 4*int(cos(c + d*x**2)/(sin(c
+ d*x**2)**2*b**2*x + 2*sin(c + d*x**2)*a*b*x + a**2*x),x)*sin(c + d*x**2)
*a**2*b**2*d*x**2 + 8*int(cos(c + d*x**2)/(sin(c + d*x**2)**2*b**2*x + 2*s
in(c + d*x**2)*a*b*x + a**2*x),x)*sin(c + d*x**2)*b**4*d*x**2 - 4*int(cos(
c + d*x**2)/(sin(c + d*x**2)**2*b**2*x + 2*sin(c + d*x**2)*a*b*x + a**2*x)
,x)*a**3*b*d*x**2 + 8*int(cos(c + d*x**2)/(sin(c + d*x**2)**2*b**2*x + 2*s
in(c + d*x**2)*a*b*x + a**2*x),x)*a*b**3*d*x**2 + 2*int(sin(c + d*x**2)**2
/(sin(c + d*x**2)**2*b**2*x**3 + 2*sin(c + d*x**2)*a*b*x**3 + a**2*x**3),x
)*sin(c + d*x**2)*a*b**3*x**2 + 2*int(sin(c + d*x**2)**2/(sin(c + d*x**2)*
*2*b**2*x**3 + 2*sin(c + d*x**2)*a*b*x**3 + a**2*x**3),x)*a**2*b**2*x**2 -
 4*int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*b**2*x + 2*sin(c + d*x**2)*a
*b*x + a**2*x),x)*sin(c + d*x**2)*b**4*d*x**2 - 4*int(sin(c + d*x**2)**2/(
sin(c + d*x**2)**2*b**2*x + 2*sin(c + d*x**2)*a*b*x + a**2*x),x)*a*b**3*d*
x**2 + 8*int(sin(c + d*x**2)/(sin(c + d*x**2)**2*b**2*x**3 + 2*sin(c + d*x
**2)*a*b*x**3 + a**2*x**3),x)*sin(c + d*x**2)*b**4*x**2 + 8*int(sin(c + d*
x**2)/(sin(c + d*x**2)**2*b**2*x**3 + 2*sin(c + d*x**2)*a*b*x**3 + a**2*x*
*3),x)*a*b**3*x**2 + 8*int((cos(c + d*x**2)*sin(c + d*x**2))/(sin(c + d...
```

3.48 $\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx=\text{Int}\left(\frac{x^2}{(a+b\sin(c+dx^2))^2},x\right)$$

output
```
Defer(Int)(x^2/(a+b*sin(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx$$

input
```
Integrate[x^2/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
Integrate[x^2/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

$$\Big\downarrow \ \text{3908}$$

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}dx$$

input

```
Int[x^2/(a + b*Sin[c + d*x^2])^2,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a+b\sin\left(d\,x^2+c\right)\right)^2}dx$$

input
```
int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(-x^2/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x
)
```

Sympy [N/A]

Not integrable

Time = 39.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(x**2/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral(x**2/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 922, normalized size of antiderivative = 51.22

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sin(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
(a*b*x*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*x*cos(d*x^2 + c) + ((a^2*b^
2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4
*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*s
in(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*
b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x
^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))*integrate((4*a^2*d*x^2*co
s(d*x^2 + c)^2 + 4*a^2*d*x^2*sin(d*x^2 + c)^2 + 2*a*b*d*x^2*sin(d*x^2 + c)
 - a*b*cos(d*x^2 + c) - (2*a*b*d*x^2*sin(d*x^2 + c) + a*b*cos(d*x^2 + c))*
cos(2*d*x^2 + 2*c) + (2*a*b*d*x^2*cos(d*x^2 + c) - a*b*sin(d*x^2 + c) - b^
2)*sin(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 -
a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x
^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*s
in(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d -
 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 +
2*c)), x) + (a*b*x*sin(d*x^2 + c) + b^2*x)*sin(2*d*x^2 + 2*c))/((a^2*b^2 -
 b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a
^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(
2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3
)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2
+ c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sin(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^2/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 38.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\,\sin(d\,x^2+c))^2}\,dx$$

input
```
int(x^2/(a + b*sin(c + d*x^2))^2,x)
```

output
```
int(x^2/(a + b*sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{x^2}{\sin(d\,x^2+c)^2\,b^2+2\sin(d\,x^2+c)\,ab+a^2}dx$$

input
```
int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

output

```
int(x**2/(sin(c + d*x**2)**2*b**2 + 2*sin(c + d*x**2)*a*b + a**2),x)
```

3.49 $\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x^2])^(-2),x]
```

output
```
Integrate[(a + b*Sin[c + d*x^2])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3850\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx^2))^2} \, dx$$

$$\Big\downarrow \text{3850}$$

$$\int \frac{1}{(a+b\sin(c+dx^2))^2} dx$$

input

```
Int[(a + b*Sin[c + d*x^2])^(-2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3850

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sin(d\,x^2+c))^2} dx$$

input

```
int(1/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(1/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 17.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x**2))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 3381, normalized size of antiderivative = 241.50

$$\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(b\sin(dx^2+c)+a)^2}\,dx$$

input

```
integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
(a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4
*cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b -
a*b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^
2)*sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4
 - a^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x*cos(2*d*x^2 +
 2*c)^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c
)^2)*d*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*s
in(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a
^2*b^4)*sin(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos
(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)
*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^
2*b^4 - b^6)*d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^
3 - a*b^5)*sin(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*
cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3
*b^3)*d*x*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*sin(c) +
(a^4*b^2 - a^2*b^4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(
2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2) + 2*((a^3*
b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*s...
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^2 + c) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 38.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

input
```
int(1/(a + b*sin(c + d*x^2))^2,x)
```

output
```
int(1/(a + b*sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\sin\left(d\,x^2+c\right)^2b^2+2\sin\left(d\,x^2+c\right)ab+a^2}dx$$

input
```
int(1/(a+b*sin(d*x^2+c))^2,x)
```

output

```
int(1/(sin(c + d*x**2)**2*b**2 + 2*sin(c + d*x**2)*a*b + a**2),x)
```

3.50 $\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
```

output
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2} dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2} dx$$

input

```
Int[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+b\sin(d\,x^2+c))^2} dx$$

input

```
int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x^2*cos(d*x^2 + c)^2 - 2*a*b*x^2*sin(d*x^2 + c) - (a^2 +
b^2)*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 41.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral(1/(x**2*(a + b*sin(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 3486, normalized size of antiderivative = 193.67

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x^2}\,dx$$

input

```
integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
(a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4
*cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b -
a*b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^
2)*sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4
 - a^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^3*cos(2*d*x^2
 + 2*c)^2 + a^4*b^2*d*x^3*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin
(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 +
(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2
 + b^6*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*
x^3*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 +
a*b^5)*d*x^3*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*
((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*
x^3*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^3*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos
(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*
d*x^2) - 2*(a^2*b^4*d*x^3*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^3*sin(2*d*x^
2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3
*b^3)*d*x^3*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^3)*cos(2*d*x^2 + 2
*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)
*sin(c))*d*x^3*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3...
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{(b\sin(dx^2+c)+a)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 38.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2}\,dx = \int \frac{1}{x^2 \left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b*sin(c + d*x^2))^2),x)
```

output
```
int(1/(x^2*(a + b*sin(c + d*x^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^2))^2}\,dx = \frac{-\left(\int \frac{\sin(d\,x^2+c)^2}{\sin(d\,x^2+c)^2 b^2x^2+2\sin(d\,x^2+c)ab\,x^2+a^2x^2}dx\right) b^2x - 2\left(\int \frac{\sin(d\,x^2+c)}{\sin(d\,x^2+c)^2 b^2x^2+2\sin(d\,x^2+c)ab\,x^2+a^2x^2}dx\right) abx - 1}{a^2x}$$

input

```
int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

output

```
( - int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*b**2*x**2 + 2*sin(c + d*x**
2)*a*b*x**2 + a**2*x**2),x)*b**2*x - 2*int(sin(c + d*x**2)/(sin(c + d*x**2
)**2*b**2*x**2 + 2*sin(c + d*x**2)*a*b*x**2 + a**2*x**2),x)*a*b*x - 1)/(a*
*2*x)
```

3.51 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx = \text{Int}\left((ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p, x\right)$$

output
```
Defer(Int)((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx = \int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]
```

output
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p \, dx$$

$$\Big\downarrow \; 3908$$

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p dx$$

input
```
Int[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m \left(a+b\sin \left(d\,x^2+c\right)\right)^p dx$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx=\int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 10.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**2+c))**p,x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**2))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx = \int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx = \int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 38.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx = \int (e\,x)^m \left(a+b\,\sin\left(d\,x^2+c\right)\right)^p \, dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^2))^p,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \, dx = e^m \left(\int x^m \left(\sin\left(d\,x^2+c\right)b+a\right)^p dx\right)$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

output
```
e**m*int(x**m*(sin(c + d*x**2)*b + a)**p,x)
```

3.52 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 444

$$
\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx
&= \frac{a(2a^2+3b^2)\,(ex)^{1+m}}{2e(1+m)} + \frac{3ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{16e} \\
&\quad - \frac{3ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{16e} \\
&\quad + \frac{3\;2^{-\frac{7}{2}-\frac{m}{2}}ab^2e^{2ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2idx^2\right)}{e} \\
&\quad + \frac{3\;2^{-\frac{7}{2}-\frac{m}{2}}ab^2e^{-2ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2idx^2\right)}{e} \\
&\quad - \frac{i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{3ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-3idx^2\right)}{16e} \\
&\quad + \frac{i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{-3ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},3idx^2\right)}{16e}
\end{aligned}
$$

output

```
1/2*a*(2*a^2+3*b^2)*(e*x)^(1+m)/e/(1+m)+3/16*I*b*(4*a^2+b^2)*exp(I*c)*(e*x
)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-I*d*x^2)/e-3/16*I*b*(4*a^
2+b^2)*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,I*d*x^2)/e/exp(I
*c)+3*2^(-7/2-1/2*m)*a*b^2*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*
GAMMA(1/2+1/2*m,-2*I*d*x^2)/e+3*2^(-7/2-1/2*m)*a*b^2*(e*x)^(1+m)*(I*d*x^2)
^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,2*I*d*x^2)/e/exp(2*I*c)-1/16*I*3^(-1/2-1/2*m
)*b^3*exp(3*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-3*I*
d*x^2)/e+1/16*I*3^(-1/2-1/2*m)*b^3*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMM
A(1/2+1/2*m,3*I*d*x^2)/e/exp(3*I*c)
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.84

$$
\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx = \frac{1}{16}ix(ex)^m \Bigg(&-\frac{8ia(2a^2+3b^2)}{1+m} \\
&+3b\left(4a^2+b^2\right)e^{ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-idx^2\right) \\
&-3b\left(4a^2+b^2\right)e^{-ic}\left(idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right) \\
&-3i2^{\frac{1}{2}-\frac{m}{2}}ab^2e^{2ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-2idx^2\right) \\
&-3i2^{\frac{1}{2}-\frac{m}{2}}ab^2e^{-2ic}\left(idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},2idx^2\right) \\
&-3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{3ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-3idx^2\right) \\
&+3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{-3ic}\left(idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},3idx^2\right)\Bigg)
\end{aligned}
$$

input

```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

output
```
(I/16)*x*(e*x)^m*(((-8*I)*a*(2*a^2 + 3*b^2))/(1 + m) + 3*b*(4*a^2 + b^2)*E
^(I*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-I)*d*x^2] - (3*b*(4*a^
2 + b^2)*(I*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, I*d*x^2])/E^(I*c) - (3*I)
*2^(1/2 - m/2)*a*b^2*E^((2*I)*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2
, (-2*I)*d*x^2] - ((3*I)*2^(1/2 - m/2)*a*b^2*(I*d*x^2)^(-1/2 - m/2)*Gamma[
(1 + m)/2, (2*I)*d*x^2])/E^((2*I)*c) - 3^(-1/2 - m/2)*b^3*E^((3*I)*c)*((-I
)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2] + (3^(-1/2 - m/2)*b^3
*(I*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (3*I)*d*x^2])/E^((3*I)*c))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3884, 6, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(a^3(ex)^m+3a^2b(ex)^m\sin\left(c+dx^2\right)-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)+\frac{3}{2}ab^2(ex)^m+\frac{3}{4}b^3(ex)^m\sin\left(c+dx^2\right)-$$

$$\Big\downarrow \text{6}$$

$$\int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m+3a^2b(ex)^m\sin\left(c+dx^2\right)-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)+\frac{3}{4}b^3(ex)^m\sin\left(c+dx^2\right)-\frac{1}{4}$$

$$\Big\downarrow \text{6}$$

$$\int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m+\left(3a^2b+\frac{3b^3}{4}\right)(ex)^m\sin\left(c+dx^2\right)-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)-\frac{1}{4}b^3(ex)^m\sin(3c$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ibe^{ic}\left(4a^2+b^2\right)\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-idx^2\right)}{16e}-\frac{3ibe^{-ic}\left(4a^2+b^2\right)\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},idx^2\right)}{16e}+\frac{a\left(2a^2+3b^2\right)(ex)^{m+1}}{2e(m+1)}+\frac{3ab^2e^{2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-2idx^2\right)}{e}+\frac{3ab^2e^{-2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},2idx^2\right)}{e}-\frac{ib^3e^{3ic}3^{-\frac{m}{2}-\frac{1}{2}}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-3idx^2\right)}{16e}+\frac{ib^3e^{-3ic}3^{-\frac{m}{2}-\frac{1}{2}}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},3idx^2\right)}{16e}$$

input

```
Int[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

output

```
(a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + (((3*I)/16)*b*(4*a^2 + b
^2)*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*
d*x^2])/e - (((3*I)/16)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/
2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c)) + (3*2^(-7/2 - m/2)*a*b^2*E^((2*
I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^
2])/e + (3*2^(-7/2 - m/2)*a*b^2*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma
[(1 + m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c)) - ((I/16)*3^(-1/2 - m/2)*b^3*E^(
(3*I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-3*I)*d
*x^2])/e + ((I/16)*3^(-1/2 - m/2)*b^3*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)
*Gamma[(1 + m)/2, (3*I)*d*x^2])/(e*E^((3*I)*c))
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int (ex)^m \left(a + b\sin\left(d\,x^2 + c\right)\right)^3 dx$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.73

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)^3\,dx = \frac{24\,(2\,a^3 + 3\,ab^2)(ex)^m\,dx + (b^3em + b^3e)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{3i\,d}{e^2}\right) - 3i\,c\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, 3i\,dx^2\right) - 9\,(i\,ab^2em + i\,ab^2e)}{}$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="fricas")
```

output
```
1/48*(24*(2*a^3 + 3*a*b^2)*(e*x)^m*d*x + (b^3*e*m + b^3*e)*e^(-1/2*(m - 1)
*log(3*I*d/e^2) - 3*I*c)*gamma(1/2*m + 1/2, 3*I*d*x^2) - 9*(I*a*b^2*e*m +
I*a*b^2*e)*e^(-1/2*(m - 1)*log(2*I*d/e^2) - 2*I*c)*gamma(1/2*m + 1/2, 2*I*
d*x^2) - 9*((4*a^2*b + b^3)*e*m + (4*a^2*b + b^3)*e)*e^(-1/2*(m - 1)*log(I
*d/e^2) - I*c)*gamma(1/2*m + 1/2, I*d*x^2) - 9*((4*a^2*b + b^3)*e*m + (4*a
^2*b + b^3)*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I*c)*gamma(1/2*m + 1/2, -I*
d*x^2) - 9*(-I*a*b^2*e*m - I*a*b^2*e)*e^(-1/2*(m - 1)*log(-2*I*d/e^2) + 2*
I*c)*gamma(1/2*m + 1/2, -2*I*d*x^2) + (b^3*e*m + b^3*e)*e^(-1/2*(m - 1)*lo
g(-3*I*d/e^2) + 3*I*c)*gamma(1/2*m + 1/2, -3*I*d*x^2))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**2+c))**3,x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**2))**3, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int \left(b\sin\left(dx^2+c\right)+a\right)^3 (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="maxima")
```

output
```
(e*x)^(m + 1)*a^3/(e*(m + 1)) + 1/8*(12*a*b^2*e^m*x*x^m - 12*(a*b^2*e^m*m
+ a*b^2*e^m)*integrate(x^m*cos(2*d*x^2 + 2*c), x) + 3*((4*a^2*b + b^3)*e^m
*m*sin(c) + (4*a^2*b + b^3)*e^m*sin(c))*integrate(x^m*cos(d*x^2), x) - 2*(
b^3*e^m*m + b^3*e^m)*integrate(x^m*sin(3*d*x^2 + 3*c), x) + 3*((4*a^2*b +
b^3)*e^m*m + (4*a^2*b + b^3)*e^m)*integrate(x^m*sin(d*x^2 + c), x) + 3*((4
*a^2*b + b^3)*e^m*m*cos(c) + (4*a^2*b + b^3)*e^m*cos(c))*integrate(x^m*sin
(d*x^2), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx = \int \left(b\sin\left(dx^2+c\right)+a\right)^3 (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^2 + c) + a)^3*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx = \int (e\,x)^m \left(a+b\,\sin\left(d\,x^2+c\right)\right)^3\,dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^2))^3,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^2))^3, x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx$$
$$= \frac{e^m\left(x^m a^3 x+6x^m a\,b^2 x-6\left(\int x^m dx\right) a\,b^2 m-6\left(\int x^m dx\right) a\,b^2+\left(\int x^m \sin\left(d\,x^2+c\right)^3 dx\right) b^3 m+\left(\int x^m \mathrm{s}\right.\right.}{}$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

output
```
(e**m*(x**m*a**3*x + 6*x**m*a*b**2*x - 6*int(x**m,x)*a*b**2*m - 6*int(x**m
,x)*a*b**2 + int(x**m*sin(c + d*x**2)**3,x)*b**3*m + int(x**m*sin(c + d*x*
*2)**3,x)*b**3 + 3*int(x**m*sin(c + d*x**2)**2,x)*a*b**2*m + 3*int(x**m*si
n(c + d*x**2)**2,x)*a*b**2 + 3*int(x**m*sin(c + d*x**2),x)*a**2*b*m + 3*in
t(x**m*sin(c + d*x**2),x)*a**2*b))/(m + 1)
```

3.53 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 279

$$
\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{2e} \\
&\quad - \frac{iabe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{2e} \\
&\quad + \frac{2^{-\frac{7}{2}-\frac{m}{2}}b^2e^{2ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2idx^2\right)}{e} \\
&\quad + \frac{2^{-\frac{7}{2}-\frac{m}{2}}b^2e^{-2ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2idx^2\right)}{e}
\end{aligned}
$$

output
```
1/2*(2*a^2+b^2)*(e*x)^(1+m)/e/(1+m)+1/2*I*a*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x
^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-I*d*x^2)/e-1/2*I*a*b*(e*x)^(1+m)*(I*d*x^
2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,I*d*x^2)/e/exp(I*c)+2^(-7/2-1/2*m)*b^2*exp
(2*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-2*I*d*x^2)/e+
2^(-7/2-1/2*m)*b^2*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,2*I*
d*x^2)/e/exp(2*I*c)
```

Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.97

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^2 \, dx$$
$$= \frac{2^{\frac{1}{2}(-7-m)} x(ex)^m \left(d^2x^4\right)^{\frac{1}{2}(-1-m)} \left(2^{\frac{7+m}{2}} a^2\left(d^2x^4\right)^{\frac{1+m}{2}} + 2^{\frac{5+m}{2}} b^2\left(d^2x^4\right)^{\frac{1+m}{2}} + b^2\left(idx^2\right)^{\frac{1+m}{2}} \cos(2c)\Gamma\left(\frac{1+m}{2}, -2i\right.\right.}{}$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
(2^((-7 - m)/2)*x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(2^((7 + m)/2)*a^2*(d^2*x
^4)^((1 + m)/2) + 2^((5 + m)/2)*b^2*(d^2*x^4)^((1 + m)/2) + b^2*(I*d*x^2)^
((1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*m*(I*d*x^2)^((1
+ m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*((-I)*d*x^2)^((1 + m
)/2)*Cos[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] + b^2*m*((-I)*d*x^2)^((1 + m)/
2)*Cos[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] - I*2^((5 + m)/2)*a*b*(1 + m)*((
-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*2
^((5 + m)/2)*a*b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2
]*(Cos[c] + I*Sin[c]) + I*b^2*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I
)*d*x^2]*Sin[2*c] + I*b^2*m*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*
d*x^2]*Sin[2*c] - I*b^2*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*
x^2]*Sin[2*c] - I*b^2*m*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*
x^2]*Sin[2*c]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^2 \, dx$$

$$\Big\downarrow \; 3884$$

$$\int \left(a^2(ex)^m+2ab(ex)^m\sin\left(c+dx^2\right)-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^2\right)+\frac{1}{2}b^2(ex)^m\right)dx$$

$$\Big\downarrow 6$$

$$\int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m+2ab(ex)^m\sin\left(c+dx^2\right)-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^2\right)\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{\left(2a^2+b^2\right)(ex)^{m+1}}{2e(m+1)}+\frac{iabe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-idx^2\right)}{2e}-$$
$$\frac{iabe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},idx^2\right)}{2e}+$$
$$\frac{b^2e^{2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-2idx^2\right)}{e}+$$
$$\frac{b^2e^{-2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},2idx^2\right)}{e}$$

input
```
Int[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

output
```
((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/2)*a*b*E^(I*c)*(e*x)^(1
+ m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/2)*a*
b*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*
c)) + (2^(-7/2 - m/2)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)
/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2])/e + (2^(-7/2 - m/2)*b^2*(e*x)^(1 + m)*
(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int (ex)^m \left(a+b\sin\left(d\,x^2+c\right)\right)^2 dx$$

input

```
int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

output

```
int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\, dx = \frac{8\,(2\,a^2+b^2)(ex)^m\, dx + (-i\,b^2em - i\,b^2e)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{2i\,d}{e^2}\right)-2i\,c\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2}, 2i\,dx^2\right) - 8\,(abem+abe)e^{(-}}{}$$

input

```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output

```
1/16*(8*(2*a^2 + b^2)*(e*x)^m*d*x + (-I*b^2*e*m - I*b^2*e)*e^(-1/2*(m - 1)
*log(2*I*d/e^2) - 2*I*c)*gamma(1/2*m + 1/2, 2*I*d*x^2) - 8*(a*b*e*m + a*b*
e)*e^(-1/2*(m - 1)*log(I*d/e^2) - I*c)*gamma(1/2*m + 1/2, I*d*x^2) - 8*(a*
b*e*m + a*b*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I*c)*gamma(1/2*m + 1/2, -I*
d*x^2) + (I*b^2*e*m + I*b^2*e)*e^(-1/2*(m - 1)*log(-2*I*d/e^2) + 2*I*c)*ga
mma(1/2*m + 1/2, -2*I*d*x^2))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx = \int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**2))**2, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx = \int \left(b\sin\left(dx^2+c\right)+a\right)^2 (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
(e*x)^(m + 1)*a^2/(e*(m + 1)) + 1/2*(b^2*e^m*x*x^m - (b^2*e^m*m + b^2*e^m)
*integrate(x^m*cos(2*d*x^2 + 2*c), x) + 4*(a*b*e^m*m + a*b*e^m)*integrate(
x^m*sin(d*x^2 + c), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx = \int \left(b\sin\left(dx^2+c\right)+a\right)^2 (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^2 + c) + a)^2*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^2 \, dx=\int {\left(e\,x\right)}^m\,{\left(a+b\,\sin \left(d\,x^2+c\right)\right)}^2 \,d x$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^2))^2,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^2 \, dx = \frac{e^m \bigg(x^m a^2 x+2x^m b^2 x-2\left(\int x^m d x\right) b^2 m-2\left(\int x^m d x\right) b^2+\bigg(\int x^m \sin \left(d\,x^2+c\right)^2 d x\bigg) b^2 m+\bigg(\int x^m \sin (d}{m+1}$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

output
```
(e**m*(x**m*a**2*x + 2*x**m*b**2*x - 2*int(x**m,x)*b**2*m - 2*int(x**m,x)*
b**2 + int(x**m*sin(c + d*x**2)**2,x)*b**2*m + int(x**m*sin(c + d*x**2)**2
,x)*b**2 + 2*int(x**m*sin(c + d*x**2),x)*a*b*m + 2*int(x**m*sin(c + d*x**2
),x)*a*b))/(m + 1)
```

3.54 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\begin{aligned}\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \frac{a(ex)^{1+m}}{e(1+m)}\\ &\quad+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{4e}\\ &\quad-\frac{ibe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{4e}\end{aligned}$$

output
```
a*(e*x)^(1+m)/e/(1+m)+1/4*I*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)
*GAMMA(1/2+1/2*m,-I*d*x^2)/e-1/4*I*b*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GA
MMA(1/2+1/2*m,I*d*x^2)/e/exp(I*c)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$= \frac{x(ex)^m\left(d^2x^4\right)^{\frac{1}{2}(-1-m)}\left(4a(d^2x^4)^{\frac{1+m}{2}}-ib(1+m)\left(-idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right)(\cos(c)-i\sin(c))+ib(1+}{4(1+m)}$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

output
```
(x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(4*a*(d^2*x^4)^((1 + m)/2) - I*b*(1 + m)
*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) +
I*b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c] + I
*Sin[c])))/(4*(1 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(a(ex)^m+b(ex)^m\sin\left(c+dx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(ex)^{m+1}}{e(m+1)}+\frac{ibe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-idx^2\right)}{4e}-\frac{ibe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},idx^2\right)}{4e}$$

input
```
Int[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

output
```
(a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/4)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2
)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/4)*b*(e*x)^(1 + m)*(I
*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int (ex)^m\left(a+b\sin\left(d\,x^2+c\right)\right)dx$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int (ex)^m\left(a+b\sin\left(c+dx^2\right)\right)\,dx = \frac{4\,(ex)^m\,adx-(bem+be)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{i\,d}{e^2}\right)-i\,c\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},i\,dx^2\right)-(bem+be)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-\frac{i\,d}{e^2}\right)+i\,c\right)}\Gamma\bigl(\frac{1}{2}}{4\,(dm+d)}$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/4*(4*(e*x)^m*a*d*x - (b*e*m + b*e)*e^(-1/2*(m - 1)*log(I*d/e^2) - I*c)*g
amma(1/2*m + 1/2, I*d*x^2) - (b*e*m + b*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) +
 I*c)*gamma(1/2*m + 1/2, -I*d*x^2))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**2+c)),x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**2)), x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx=\int \left(b\sin\left(dx^2+c\right)+a\right)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
b*e^m*integrate(x^m*sin(d*x^2 + c), x) + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx=\int \left(b\sin\left(dx^2+c\right)+a\right)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^2 + c) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)\,dx=\int {\left(e\,x\right)}^m\,\left(a+b\,\sin \left(d\,x^2+c\right)\right) \,dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^2)),x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^2)), x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)\,dx = \frac{e^m \left(x^m a x+\left(\int x^m \sin \left(d\,x^2+c\right)dx\right) b m+\left(\int x^m \sin \left(d\,x^2+c\right)dx\right) b\right)}{m+1}$$

input
```
int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

output
```
(e**m*(x**m*a*x + int(x**m*sin(c + d*x**2),x)*b*m + int(x**m*sin(c + d*x**
2),x)*b))/(m + 1)
```

3.55 $\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx=\operatorname{Int}\left(\frac{(ex)^m}{a+b\sin\left(c+dx^2\right)},x\right)$$

output
```
Defer(Int)((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

input
```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]),x]
```

output
```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \textcolor{blue}{3908}$$

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}dx$$

input

```
Int[(e*x)^m/(a + b*Sin[c + d*x^2]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{a+b\sin\left(d\,x^2+c\right)}dx$$

input

```
int((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

output
```
int((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx=\int \frac{(ex)^m}{b\sin(dx^2+c)+a}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx=\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$$

input
```
integrate((e*x)**m/(a+b*sin(d*x**2+c)),x)
```

output
```
Integral((e*x)**m/(a + b*sin(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{(ex)^m}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{(ex)^m}{b\sin\left(dx^2+c\right)+a}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 38.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{{\left(e\,x\right)}^m}{a+b\,\sin\left(d\,x^2+c\right)}\,dx$$

input
```
int((e*x)^m/(a + b*sin(c + d*x^2)),x)
```

output
```
int((e*x)^m/(a + b*sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.05

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx = \frac{e^m\left(x^m x-\left(\int \frac{x^m \sin\left(d\,x^2+c\right)}{\sin\left(d\,x^2+c\right)b+a}dx\right)bm-\left(\int \frac{x^m \sin\left(d\,x^2+c\right)}{\sin\left(d\,x^2+c\right)b+a}dx\right)b\right)}{a\,(m+1)}$$

input
```
int((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

output
```
(e**m*(x**m*x - int((x**m*sin(c + d*x**2))/(sin(c + d*x**2)*b + a),x)*b*m
- int((x**m*sin(c + d*x**2))/(sin(c + d*x**2)*b + a),x)*b))/(a*(m + 1))
```

3.56 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}dx \\ \Big\downarrow \textbf{3908} \\ \int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}dx \end{array}$$

input
```
Int[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{(a+b\sin(d\,x^2+c))^2}dx$$

input
```
int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

output
```
int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{(ex)^m}{(b\sin(dx^2+c)+a)^2}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(-(e*x)^m/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2
), x)
```

Sympy [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx$$

input
```
integrate((e*x)**m/(a+b*sin(d*x**2+c))**2,x)
```

output
```
Integral((e*x)**m/(a + b*sin(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 9.54 (sec) , antiderivative size = 3886, normalized size of antiderivative = 194.30

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx=\int \frac{(ex)^m}{(b\sin(dx^2+c)+a)^2}\,dx$$

input

```
integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
(a^3*b*e^m*x^m*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*e^m*x^m*cos(2*c)*si
n(2*d*x^2) - b^4*e^m*x^m*cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*e^m*x^m
*cos(d*x^2)*cos(c) - 2*(a^3*b - a*b^3)*e^m*x^m*sin(d*x^2)*sin(c) - (a*b^3*
e^m*x^m*cos(2*d*x^2)*cos(2*c) - a*b^3*e^m*x^m*sin(2*d*x^2)*sin(2*c) + 2*(a
^4 - a^2*b^2)*e^m*x^m*cos(c)*sin(d*x^2) + 2*(a^4 - a^2*b^2)*e^m*x^m*cos(d*
x^2)*sin(c) + (a^3*b - a*b^3)*e^m*x^m)*cos(d*x^2 + c) - (a^4*b^2*d*x*cos(2
*d*x^2 + 2*c)^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6
*sin(2*c)^2)*d*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2
+ (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2
 + b^6*sin(2*c)^2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*
cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4
*b^2 + a^2*b^4)*sin(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)
*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3
- a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^
2) - (a^2*b^4 - b^6)*d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*
b^4*d*x*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5
*b - a^3*b^3)*d*x*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*s
in(c) + (a^4*b^2 - a^2*b^4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2)...
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 38.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{{\left(e\,x\right)}^m}{{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^2}\,dx$$

input
```
int((e*x)^m/(a + b*sin(c + d*x^2))^2,x)
```

output
```
int((e*x)^m/(a + b*sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$
$$=\frac{e^m\left(x^m x-\left(\int \frac{x^m\sin\left(d\,x^2+c\right)^2}{\sin(d\,x^2+c)^2 b^2+2\sin(d\,x^2+c)ab+a^2}dx\right)b^2 m-\left(\int \frac{x^m\sin\left(d\,x^2+c\right)^2}{\sin(d\,x^2+c)^2 b^2+2\sin(d\,x^2+c)ab+a^2}dx\right)b^2-2\Bigg(\int \frac{}{\sin(d\,x^2}\right.}{a^2\left(m+1\right)}$$

input

```
int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

output

```
(e**m*(x**m*x - int((x**m*sin(c + d*x**2)**2)/(sin(c + d*x**2)**2*b**2 + 2
*sin(c + d*x**2)*a*b + a**2),x)*b**2*m - int((x**m*sin(c + d*x**2)**2)/(si
n(c + d*x**2)**2*b**2 + 2*sin(c + d*x**2)*a*b + a**2),x)*b**2 - 2*int((x**
m*sin(c + d*x**2))/(sin(c + d*x**2)**2*b**2 + 2*sin(c + d*x**2)*a*b + a**2
),x)*a*b*m - 2*int((x**m*sin(c + d*x**2))/(sin(c + d*x**2)**2*b**2 + 2*sin
(c + d*x**2)*a*b + a**2),x)*a*b))/(a**2*(m + 1))
```

3.57 $\int x^5(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2}$$

output
```
1/6*a*x^6-1/3*b*x^3*cos(d*x^3+c)/d+1/3*b*sin(d*x^3+c)/d^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2}$$

input
```
Integrate[x^5*(a + b*Sin[c + d*x^3]),x]
```

output
```
(a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2010, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$
$$\Big\downarrow \text{2010}$$
$$\int \left(ax^5+bx^5\sin\left(c+dx^3\right)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^6}{6}+\frac{b\sin\left(c+dx^3\right)}{3d^2}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}$$

input
```
Int[x^5*(a + b*Sin[c + d*x^3]),x]
```

output
```
(a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result
risch	$\frac{x^{6} a}{6}-\frac{b\,x^{3} \cos\left(d\,x^{3}+c\right)}{3 d}+\frac{b \sin\left(d\,x^{3}+c\right)}{3 d^{2}}$
parallelrisch	$\frac{x^{6} a\,d^{2}-2 x^{3} b d \cos\left(d\,x^{3}+c\right)+2 b \sin\left(d\,x^{3}+c\right)}{6 d^{2}}$
parts	$\frac{x^{6} a}{6}+\frac{\frac{2 b \tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}{3 d^{2}}-\frac{b\,x^{3}}{3 d}+\frac{b\,x^{3} \tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}{3 d}}{1+\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}$
norman	$\frac{\frac{x^{6} a}{6}+\frac{2 b \tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)}{3 d^{2}}-\frac{b\,x^{3}}{3 d}+\frac{x^{6} a \tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}{6}+\frac{b\,x^{3} \tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}{3 d}}{1+\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{\left(9 x^{6} d^{2}+88\right) \left(a+b \sin\left(d\,x^{3}+c\right)\right)}{54 d^{2}}-\frac{5 x^{4} \left(a+b \sin\left(d\,x^{3}+c\right)\right)+3 x^{7} b d \cos\left(d\,x^{3}+c\right)}{3 d^{2} x^{4}}+\frac{20 x^{3} \left(a+b \sin\left(d\,x^{3}+c\right)\right)+36 x^{6} b d \cos\left(d\,x^{3}\right.}{54 x^{3} d^{2}}$

input
```
int(x^5*(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a-1/3*b*x^3*cos(d*x^3+c)/d+1/3*b*sin(d*x^3+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^{5}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\frac{ad^{2}x^{6}-2\,bdx^{3}\cos\left(dx^{3}+c\right)+2\,b\sin\left(dx^{3}+c\right)}{6\,d^{2}}$$

input
```
integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/6*(a*d^2*x^6 - 2*b*d*x^3*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\begin{cases}\frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2} & \text{for } d\neq 0\\ \frac{x^6(a+b\sin(c))}{6} & \text{otherwise}\end{cases}$$

input

```
integrate(x**5*(a+b*sin(d*x**3+c)),x)
```

output

```
Piecewise((a*x**6/6 - b*x**3*cos(c + d*x**3)/(3*d) + b*sin(c + d*x**3)/(3*
d**2), Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{1}{6}\,ax^6-\frac{\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)b}{3\,d^2}$$

input

```
integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output

```
1/6*a*x^6 - 1/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*b/d^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

$$=\frac{\left(dx^3+c\right)^2a-2\left(dx^3+c\right)b\cos\left(dx^3+c\right)+2\,b\sin\left(dx^3+c\right)}{6\,d^2}-\frac{\left(dx^3+c\right)ac-bc\cos\left(dx^3+c\right)}{3\,d^2}$$

input

```
integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
1/6*((d*x^3 + c)^2*a - 2*(d*x^3 + c)*b*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c)
)/d^2 - 1/3*((d*x^3 + c)*a*c - b*c*cos(d*x^3 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{a\,x^6}{6}+\frac{\frac{b\,\sin\left(d\,x^3+c\right)}{3}-\frac{b\,d\,x^3\,\cos\left(d\,x^3+c\right)}{3}}{d^2}$$

input
```
int(x^5*(a + b*sin(c + d*x^3)),x)
```

output
```
(a*x^6)/6 + ((b*sin(c + d*x^3))/3 - (b*d*x^3*cos(c + d*x^3))/3)/d^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{-2\cos\left(d\,x^3+c\right)bd\,x^3+2\sin\left(d\,x^3+c\right)b+a\,d^2x^6}{6d^2}$$

input
```
int(x^5*(a+b*sin(d*x^3+c)),x)
```

output
```
( - 2*cos(c + d*x**3)*b*d*x**3 + 2*sin(c + d*x**3)*b + a*d**2*x**6)/(6*d**
2)
```

3.58 $\int x^2(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int x^2(a+b\sin(c+dx^3))\,dx=\frac{ax^3}{3}-\frac{b\cos(c+dx^3)}{3d}$$

output
```
1/3*a*x^3-1/3*b*cos(d*x^3+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int x^2(a+b\sin(c+dx^3))\,dx=\frac{ax^3}{3}-\frac{b\cos(c)\cos(dx^3)}{3d}+\frac{b\sin(c)\sin(dx^3)}{3d}$$

input
```
Integrate[x^2*(a + b*Sin[c + d*x^3]),x]
```

output
```
(a*x^3)/3 - (b*Cos[c]*Cos[d*x^3])/(3*d) + (b*Sin[c]*Sin[d*x^3])/(3*d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \sin \left(c+d x^3\right)\right) \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(a x^2+b x^2 \sin \left(c+d x^3\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^3}{3}-\frac{b \cos \left(c+d x^3\right)}{3 d}$$

input

```
Int[x^2*(a + b*Sin[c + d*x^3]),x]
```

output

```
(a*x^3)/3 - (b*Cos[c + d*x^3])/(3*d)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result
risch	$\frac{a\,x^{3}}{3}-\frac{b\cos\left(d\,x^{3}+c\right)}{3d}$
parts	$\frac{a\,x^{3}}{3}-\frac{b\cos\left(d\,x^{3}+c\right)}{3d}$
derivativedivides	$\frac{\left(d\,x^{3}+c\right)a-b\cos\left(d\,x^{3}+c\right)}{3d}$
default	$\frac{\left(d\,x^{3}+c\right)a-b\cos\left(d\,x^{3}+c\right)}{3d}$
parallelrisch	$\frac{a\,x^{3}d-b\cos\left(d\,x^{3}+c\right)-b}{3d}$
norman	$\frac{\frac{a\,x^{3}}{3}-\frac{2b}{3d}+\frac{a\,x^{3}\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}{3}}{1+\tan\left(\frac{d\,x^{3}}{2}+\frac{c}{2}\right)^{2}}$
orering	$\frac{\left(9x^{6}d^{2}+16\right)\left(a+b\sin\left(d\,x^{3}+c\right)\right)}{27x^{3}d^{2}}-\frac{2x\left(a+b\sin\left(d\,x^{3}+c\right)\right)+3x^{4}bd\cos\left(d\,x^{3}+c\right)}{3d^{2}x^{4}}+\frac{2a+2b\sin\left(d\,x^{3}+c\right)+18x^{3}bd\cos(d\,x}{27x^{3}d^{2}}$

input
```
int(x^2*(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3-1/3*b*cos(d*x^3+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{adx^3-b\cos\left(dx^3+c\right)}{3\,d}$$

input
```
integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/3*(a*d*x^3 - b*cos(d*x^3 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\begin{cases}\frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d} & \text{for } d\neq 0\\ \frac{x^3(a+b\sin(c))}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a+b*sin(d*x**3+c)),x)
```

output
```
Piecewise((a*x**3/3 - b*cos(c + d*x**3)/(3*d), Ne(d, 0)), (x**3*(a + b*sin
(c))/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{1}{3}\,ax^3-\frac{b\cos\left(dx^3+c\right)}{3\,d}$$

input
```
integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 - 1/3*b*cos(d*x^3 + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{(dx^3+c)a-b\cos\left(dx^3+c\right)}{3\,d}$$

input
```
integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
1/3*((d*x^3 + c)*a - b*cos(d*x^3 + c))/d
```

Mupad [B] (verification not implemented)

Time = 38.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,\cos\left(d\,x^3+c\right)}{3\,d}$$

input
```
int(x^2*(a + b*sin(c + d*x^3)),x)
```

output
```
(a*x^3)/3 - (b*cos(c + d*x^3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{-\cos\left(d\,x^3+c\right)b+ad\,x^3}{3d}$$

input
```
int(x^2*(a+b*sin(d*x^3+c)),x)
```

output
```
( - cos(c + d*x**3)*b + a*d*x**3)/(3*d)
```

3.59 $\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx = a\log(x)+\frac{1}{3}b\,\text{CosIntegral}\left(dx^3\right)\sin(c)+\frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

output

```
a*ln(x)+1/3*b*Ci(d*x^3)*sin(c)+1/3*b*cos(c)*Si(d*x^3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx = a\log(x)+\frac{1}{3}b\left(\text{CosIntegral}\left(dx^3\right)\sin(c)+\cos(c)\text{Si}\left(dx^3\right)\right)$$

input

```
Integrate[(a + b*Sin[c + d*x^3])/x,x]
```

output

```
a*Log[x] + (b*(CosIntegral[d*x^3]*Sin[c] + Cos[c]*SinIntegral[d*x^3]))/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x}+\frac{b\sin\left(c+dx^3\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+\frac{1}{3}b\sin(c)\,\text{CosIntegral}\left(dx^3\right)+\frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

input
```
Int[(a + b*Sin[c + d*x^3])/x,x]
```

output
```
a*Log[x] + (b*CosIntegral[d*x^3]*Sin[c])/3 + (b*Cos[c]*SinIntegral[d*x^3])
/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x}dx$$

input
```
int((a+b*sin(d*x^3+c))/x,x)
```

output
```
int((a+b*sin(d*x^3+c))/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx=\frac{1}{3}\,b\operatorname{Ci}\left(dx^3\right)\sin\left(c\right)+\frac{1}{3}\,b\cos\left(c\right)\operatorname{Si}\left(dx^3\right)+a\log\left(x\right)$$

input
```
integrate((a+b*sin(d*x^3+c))/x,x, algorithm="fricas")
```

output
```
1/3*b*cos_integral(d*x^3)*sin(c) + 1/3*b*cos(c)*sin_integral(d*x^3) + a*lo
g(x)
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx=\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))/x,x)
```

output
```
Integral((a + b*sin(c + d*x**3))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{a+b\sin(c+dx^3)}{x}\,dx$$
$$= -\frac{1}{6}\left(\left(i\,\mathrm{Ei}\left(i\,dx^3\right)-i\,\mathrm{Ei}\left(-i\,dx^3\right)\right)\cos(c)-\left(\mathrm{Ei}\left(i\,dx^3\right)+\mathrm{Ei}\left(-i\,dx^3\right)\right)\sin(c)\right)b$$
$$+ a\log(x)$$

input
```
integrate((a+b*sin(d*x^3+c))/x,x, algorithm="maxima")
```

output
```
-1/6*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3
))*sin(c))*b + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sin(c+dx^3)}{x}\,dx = \frac{1}{3}\,b\,\mathrm{Ci}\left(dx^3\right)\sin(c)+\frac{1}{3}\,b\cos(c)\,\mathrm{Si}\left(dx^3\right)+\frac{1}{3}\,a\log\left(dx^3\right)$$

input
```
integrate((a+b*sin(d*x^3+c))/x,x, algorithm="giac")
```

output
```
1/3*b*cos_integral(d*x^3)*sin(c) + 1/3*b*cos(c)*sin_integral(d*x^3) + 1/3*
a*log(d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx=a\,\ln\left(x\right)+\frac{b\,\sin\left(c\right)\,\mathrm{cosint}\left(d\,x^3\right)}{3}+\frac{b\,\cos\left(c\right)\,\mathrm{sinint}\left(d\,x^3\right)}{3}$$

input
```
int((a + b*sin(c + d*x^3))/x,x)
```

output
```
a*log(x) + (b*sin(c)*cosint(d*x^3))/3 + (b*cos(c)*sinint(d*x^3))/3
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx=\left(\int \frac{\sin\left(d\,x^3+c\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*sin(d*x^3+c))/x,x)
```

output
```
int(sin(c + d*x**3)/x,x)*b + log(x)*a
```

3.60 $\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = -\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\,\text{CosIntegral}\left(dx^3\right) - \frac{b\sin(c+dx^3)}{3x^3}-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)$$

output

```
-1/3*a/x^3+1/3*b*d*cos(c)*Ci(d*x^3)-1/3*b*sin(d*x^3+c)/x^3-1/3*b*d*sin(c)*
Si(d*x^3)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx = -\frac{a-bdx^3\cos(c)\,\text{CosIntegral}(dx^3)+b\sin(c+dx^3)+bdx^3\sin(c)\text{Si}(dx^3)}{3x^3}$$

input

```
Integrate[(a + b*Sin[c + d*x^3])/x^4,x]
```

output
```
-1/3*(a - b*d*x^3*Cos[c]*CosIntegral[d*x^3] + b*Sin[c + d*x^3] + b*d*x^3*S
in[c]*SinIntegral[d*x^3])/x^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(c+dx^3)}{x^4}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^4}+\frac{b\sin(c+dx^3)}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{CosIntegral}\left(dx^3\right)-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)-\frac{b\sin(c+dx^3)}{3x^3}$$

input
```
Int[(a + b*Sin[c + d*x^3])/x^4,x]
```

output
```
-1/3*a/x^3 + (b*d*Cos[c]*CosIntegral[d*x^3])/3 - (b*Sin[c + d*x^3])/(3*x^3
) - (b*d*Sin[c]*SinIntegral[d*x^3])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{4}}dx$$

input
```
int((a+b*sin(d*x^3+c))/x^4,x)
```

output
```
int((a+b*sin(d*x^3+c))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx = \frac{bdx^3\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-bdx^3\sin\left(c\right)\operatorname{Si}\left(dx^3\right)-b\sin\left(dx^3+c\right)-a}{3\,x^3}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="fricas")
```

output
```
1/3*(b*d*x^3*cos(c)*cos_integral(d*x^3) - b*d*x^3*sin(c)*sin_integral(d*x^
3) - b*sin(d*x^3 + c) - a)/x^3
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx = \int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx$$

input

```
integrate((a+b*sin(d*x**3+c))/x**4,x)
```

output

```
Integral((a + b*sin(c + d*x**3))/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx = \frac{1}{6}\left(\left(\Gamma\left(-1, i\,dx^3\right)+\Gamma\left(-1,-i\,dx^3\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1, i\,dx^3\right)-i\,\Gamma\left(-1,-i\,dx^3\right)\right)\sin\left(c\right)\right)bd - \frac{a}{3\,x^3}$$

input

```
integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="maxima")
```

output

```
1/6*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*
x^3) - I*gamma(-1, -I*d*x^3))*sin(c))*b*d - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(45) = 90.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx = \frac{(dx^3+c)bd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-bcd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-(dx^3+c)bd^2\sin\left(c\right)\operatorname{Si}\left(dx^3\right)+bcd^2\sin\left(c\right)\operatorname{Si}\left(dx^3\right) -}{3\,d^2x^3}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="giac")
```

output
```
1/3*((d*x^3 + c)*b*d^2*cos(c)*cos_integral(d*x^3) - b*c*d^2*cos(c)*cos_int
egral(d*x^3) - (d*x^3 + c)*b*d^2*sin(c)*sin_integral(d*x^3) + b*c*d^2*sin(
c)*sin_integral(d*x^3) - b*d^2*sin(d*x^3 + c) - a*d^2)/(d^2*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx=\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^4}\,dx$$

input
```
int((a + b*sin(c + d*x^3))/x^4,x)
```

output
```
int((a + b*sin(c + d*x^3))/x^4, x)
```

Reduce [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx=\frac{3\left(\int \frac{\sin\left(d\,x^3+c\right)}{x^4}dx\right)b\,x^3-a}{3x^3}$$

input
```
int((a+b*sin(d*x^3+c))/x^4,x)
```

output
```
(3*int(sin(c + d*x**3)/x**4,x)*b*x**3 - a)/(3*x**3)
```

3.61 $\int x^4(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}$$

output

```
1/5*a*x^5-1/3*b*x^2*cos(d*x^3+c)/d-1/9*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/
d/(-I*d*x^3)^(2/3)-1/9*b*x^2*GAMMA(2/3,I*d*x^3)/d/exp(I*c)/(I*d*x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

$$=\frac{dx^8\Big(3(d^2x^6)^{2/3}\left(3adx^3-5b\cos\left(c+dx^3\right)\right)-5b(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)-5b(idx^3)^{2/3}\Gamma\big(}{45\left(d^2x^6\right)^{5/3}}$$

input

```
Integrate[x^4*(a + b*Sin[c + d*x^3]),x]
```

output
```
(d*x^8*(3*(d^2*x^6)^(2/3)*(3*a*d*x^3 - 5*b*Cos[c + d*x^3]) - 5*b*((-I)*d*x
^3)^(2/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 5*b*(I*d*x^3)^(2/3)*Ga
mma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(45*(d^2*x^6)^(5/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\sin\left(c+dx^3\right)\right) dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^4+bx^4\sin\left(c+dx^3\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}$$

input
```
Int[x^4*(a + b*Sin[c + d*x^3]),x]
```

output
```
(a*x^5)/5 - (b*x^2*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x^2*Gamma[2/3, (-I)*
d*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (b*x^2*Gamma[2/3, I*d*x^3])/(9*d*E^(I*c
)*(I*d*x^3)^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^4\left(a+b\sin\left(d\,x^3+c\right)\right)dx$$

input
```
int(x^4*(a+b*sin(d*x^3+c)),x)
```

output
```
int(x^4*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx \\ = \frac{9\,ad^2x^5-15\,bdx^2\cos\left(dx^3+c\right)-5\left(-i\,b\cos\left(c\right)-b\sin\left(c\right)\right)\left(i\,d\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},i\,dx^3\right)-5\left(i\,b\cos\left(c\right)-b\sin\left(c\right)\right)\left(-\right.}{45\,d^2}$$

input
```
integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/45*(9*a*d^2*x^5 - 15*b*d*x^2*cos(d*x^3 + c) - 5*(-I*b*cos(c) - b*sin(c))
*(I*d)^(1/3)*gamma(2/3, I*d*x^3) - 5*(I*b*cos(c) - b*sin(c))*(-I*d)^(1/3)*
gamma(2/3, -I*d*x^3))/d^2
```

Sympy [F]

$$\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

input
```
integrate(x**4*(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(x**4*(a + b*sin(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{1}{5}\,ax^5$$
$$-\frac{\left(6\,dx^3\cos\left(dx^3+c\right)-\left(dx^3\right)^{\frac{1}{3}}\left(\left(\left(i\,\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i\,dx^3\right)+\left(-i\,\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(\sqrt{3}+\right.\right.\right.\right.}{18\,d^2x}$$

input
```
integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 - 1/18*(6*d*x^3*cos(d*x^3 + c) - (d*x^3)^(1/3)*(((I*sqrt(3) - 1)
*gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*cos(c) + ((s
qrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*sin(
c)))*b/(d^2*x)
```

Giac [F]

$$\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)x^4\,dx$$

input
```
integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^4\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

input

```
int(x^4*(a + b*sin(c + d*x^3)),x)
```

output

```
int(x^4*(a + b*sin(c + d*x^3)), x)
```

Reduce [F]

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\left(\int \sin\left(d\,x^3+c\right)x^4dx\right)b+\frac{a\,x^5}{5}$$

input

```
int(x^4*(a+b*sin(d*x^3+c)),x)
```

output

```
(5*int(sin(c + d*x**3)*x**4,x)*b + a*x**5)/5
```

3.62 $\int x(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^2}{2}+\frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}$$

output
```
1/2*a*x^2+1/6*I*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)-1/6*I*
b*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$
$$=\frac{x^2\left(3a(d^2x^6)^{2/3}+b(-idx^3)^{2/3}\,\Gamma\left(\frac{2}{3},idx^3\right)\left(-i\cos(c)-\sin(c)\right)+ib(idx^3)^{2/3}\,\Gamma\left(\frac{2}{3},-idx^3\right)\left(\cos(c)+i\sin(c\right.\right.}{6\left(d^2x^6\right)^{2/3}}$$

input
```
Integrate[x*(a + b*Sin[c + d*x^3]),x]
```

output

```
(x^2*(3*a*(d^2*x^6)^(2/3) + b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*((-I)
*Cos[c] - Sin[c]) + I*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I
*Sin[c])))/(6*(d^2*x^6)^(2/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax+bx\sin\left(c+dx^3\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}$$

input

```
Int[x*(a + b*Sin[c + d*x^3]),x]
```

output

```
(a*x^2)/2 + ((I/6)*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3
) - ((I/6)*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x\left(a+b\sin\left(d\,x^{3}+c\right)\right)dx$$

input
```
int(x*(a+b*sin(d*x^3+c)),x)
```

output
```
int(x*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int x\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$
$$=\frac{3\,adx^{2}-(b\cos(c)-i\,b\sin(c))(i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},i\,dx^{3}\right)-(b\cos(c)+i\,b\sin(c))(-i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)}{6\,d}$$

input
```
integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/6*(3*a*d*x^2 - (b*cos(c) - I*b*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) -
 (b*cos(c) + I*b*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3))/d
```

Sympy [F]

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

input
```
integrate(x*(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(x*(a + b*sin(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{1}{2}ax^2 - \frac{(dx^3)^{\frac{1}{3}}\,(((\sqrt{3}+i)\Gamma(\tfrac{2}{3},i\,dx^3)+(\sqrt{3}-i)\Gamma(\tfrac{2}{3},-i\,dx^3))\cos(c)-((i\sqrt{3}-1)\Gamma(\tfrac{2}{3},i\,dx^3)+(-i\sqrt{3}-}{12\,dx}$$

input
```
integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 - 1/12*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt
(3) - I)*gamma(2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(2/3, I*d*x^
3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*sin(c))*b/(d*x)
```

Giac [F]

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)x\,dx$$

input
```
integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

input
```
int(x*(a + b*sin(c + d*x^3)),x)
```

output
```
int(x*(a + b*sin(c + d*x^3)), x)
```

Reduce [F]

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\left(\int\sin\left(d\,x^3+c\right)x dx\right)b+\frac{a\,x^2}{2}$$

input
```
int(x*(a+b*sin(d*x^3+c)),x)
```

output
```
(2*int(sin(c + d*x**3)*x,x)*b + a*x**2)/2
```

3.63 $\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx = -\frac{a}{x} - \frac{bde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}} - \frac{bde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}} - \frac{b\sin(c+dx^3)}{x}$$

output
```
-a/x-1/2*b*d*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)-1/2*b*d*x^2
*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)-b*sin(d*x^3+c)/x
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx$$
$$= \frac{-ib(-idx^3)^{5/3}\,\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c)) + ib(idx^3)^{5/3}\,\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c)) - 2(d^2x^6)^{2/3}\,(a}{2x\left(d^2x^6\right)^{2/3}}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])/x^2,x]
```

output
```
((-I)*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(
I*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(2
/3)*(a + b*Sin[c + d*x^3]))/(2*x*(d^2*x^6)^(2/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^3\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}-\frac{be^{ic}dx^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}dx^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}$$

input
```
Int[(a + b*Sin[c + d*x^3])/x^2,x]
```

output
```
-(a/x) - (b*d*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(2/3)) -
 (b*d*x^2*Gamma[2/3, I*d*x^3])/(2*E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*
x^3])/x
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{2}}dx$$

input

```
int((a+b*sin(d*x^3+c))/x^2,x)
```

output

```
int((a+b*sin(d*x^3+c))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{2}}\,dx = \frac{(i\,bx\cos(c)+bx\sin(c))(i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},i\,dx^{3}\right)+(-i\,bx\cos(c)+bx\sin(c))(-i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)-2\,b\sin(dx^{3}}{2\,x}$$

input

```
integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="fricas")
```

output

```
1/2*((I*b*x*cos(c) + b*x*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) + (-I*b*x
*cos(c) + b*x*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3) - 2*b*sin(d*x^3 +
c) - 2*a)/x
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx = \int \frac{a+b\sin(c+dx^3)}{x^2}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))/x**2,x)
```

output
```
Integral((a + b*sin(c + d*x**3))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx =$$
$$-\frac{(dx^3)^{\frac{1}{3}}\,(((i\sqrt{3}-1)\Gamma(-\frac{1}{3}, i\,dx^3) + (-i\sqrt{3}-1)\Gamma(-\frac{1}{3}, -i\,dx^3))\cos(c) + ((\sqrt{3}+i)\Gamma(-\frac{1}{3}, i\,dx^3) + (}{12\,x}$$
$$-\frac{a}{x}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="maxima")
```

output
```
-1/12*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(-1/3, I*d*x^3) + (-I*sqrt(3) -
 1)*gamma(-1/3, -I*d*x^3))*cos(c) + ((sqrt(3) + I)*gamma(-1/3, I*d*x^3) +
(sqrt(3) - I)*gamma(-1/3, -I*d*x^3))*sin(c))*b/x - a/x
```

Giac [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx = \int \frac{b\sin(dx^3+c)+a}{x^2}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx=\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^2}\,dx$$

input
```
int((a + b*sin(c + d*x^3))/x^2,x)
```

output
```
int((a + b*sin(c + d*x^3))/x^2, x)
```

Reduce [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx=\frac{12\left(\int \frac{x}{\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2+1}dx\right)bdx-2\sin\left(d\,x^3+c\right)b-2a-3bd\,x^3}{2x}$$

input
```
int((a+b*sin(d*x^3+c))/x^2,x)
```

output
```
(12*int(x/(tan((c + d*x**3)/2)**2 + 1),x)*b*d*x - 2*sin(c + d*x**3)*b - 2*
a - 3*b*d*x**3)/(2*x)
```

3.64 $\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{3bd\cos(c+dx^3)}{4x}-\frac{3ibd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}} + \frac{3ibd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{b\sin(c+dx^3)}{4x^4}$$

output

```
-1/4*a/x^4-3/4*b*d*cos(d*x^3+c)/x-3/8*I*b*d^2*exp(I*c)*x^2*GAMMA(2/3,-I*d*
x^3)/(-I*d*x^3)^(2/3)+3/8*I*b*d^2*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3
)^(2/3)-1/4*b*sin(d*x^3+c)/x^4
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$$

$$= \frac{3bd^2x^6(idx^3)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)(-i\cos(c)+\sin(c))+3bd^2x^6(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)(i\cos(c)+\sin(c))-2(d}{8x^4\left(d^2x^6\right)^{2/3}}$$

input

```
Integrate[(a + b*Sin[c + d*x^3])/x^5,x]
```

output

```
(3*b*d^2*x^6*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
 + 3*b*d^2*x^6*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(I*Cos[c] + Sin[c])
- 2*(d^2*x^6)^(2/3)*(a + 3*b*d*x^3*Cos[c + d*x^3] + b*Sin[c + d*x^3]))/(8*
x^4*(d^2*x^6)^(2/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^5}dx$$

$$\Big\downarrow \textbf{2010}$$

$$\int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^3\right)}{x^5}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{a}{4x^4}-\frac{3ibe^{ic}d^2x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibe^{-ic}d^2x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}$$

input

```
Int[(a + b*Sin[c + d*x^3])/x^5,x]
```

output

```
-1/4*a/x^4 - (3*b*d*Cos[c + d*x^3])/(4*x) - (((3*I)/8)*b*d^2*E^(I*c)*x^2*G
amma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) + (((3*I)/8)*b*d^2*x^2*Gamma[2/3
, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^3])/(4*x^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^3+c\right)}{x^5}dx$$

input
```
int((a+b*sin(d*x^3+c))/x^5,x)
```

output
```
int((a+b*sin(d*x^3+c))/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^5}\,dx =$$
$$-\frac{6\,bdx^3\cos\left(dx^3+c\right)-3\left(bdx^4\cos\left(c\right)-i\,bdx^4\sin\left(c\right)\right)\left(i\,d\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},i\,dx^3\right)-3\left(bdx^4\cos\left(c\right)+i\,bdx^4\sin\left(c\right)}{8\,x^4}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="fricas")
```

output
```
-1/8*(6*b*d*x^3*cos(d*x^3 + c) - 3*(b*d*x^4*cos(c) - I*b*d*x^4*sin(c))*(I*
d)^(1/3)*gamma(2/3, I*d*x^3) - 3*(b*d*x^4*cos(c) + I*b*d*x^4*sin(c))*(-I*d
)^(1/3)*gamma(2/3, -I*d*x^3) + 2*b*sin(d*x^3 + c) + 2*a)/x^4
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = \int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))/x**5,x)
```

output
```
Integral((a + b*sin(c + d*x**3))/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.70

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$$
$$= \frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3}, i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3}, -i\,dx^3\right)\right)\cos(c)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3}, i\,dx^3\right)+\left(-i\sqrt{3}\right.\right.\right.}{12\,x}$$
$$-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="maxima")
```

output
```
1/12*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(-4/3, I*d*x^3) + (sqrt(3) - I)*ga
mma(-4/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(-4/3, I*d*x^3) + (-I*
sqrt(3) - 1)*gamma(-4/3, -I*d*x^3))*sin(c))*b*d/x - 1/4*a/x^4
```

Giac [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx = \int \frac{b\sin(dx^3+c)+a}{x^5}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx=\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^5}\,dx$$

input
```
int((a + b*sin(c + d*x^3))/x^5,x)
```

output
```
int((a + b*sin(c + d*x^3))/x^5, x)
```

Reduce [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx=\frac{4\left(\int \frac{\sin\left(d\,x^3+c\right)}{x^5}dx\right)b\,x^4-a}{4x^4}$$

input
```
int((a+b*sin(d*x^3+c))/x^5,x)
```

output
```
(4*int(sin(c + d*x**3)/x**5,x)*b*x**4 - a)/(4*x**4)
```

3.65 $\int x^3(a+b\sin(c+dx^3))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}$$

output

```
1/4*a*x^4-1/3*b*x*cos(d*x^3+c)/d-1/18*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/d/(
-I*d*x^3)^(1/3)-1/18*b*x*GAMMA(1/3,I*d*x^3)/d/exp(I*c)/(I*d*x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

$$=\frac{dx^7\Big(3\sqrt[3]{d^2x^6}(3adx^3-4b\cos\left(c+dx^3\right))-2b\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)-2b\sqrt[3]{idx^3}\Gamma\Big(\frac{1}{3},-idx^3}{36\left(d^2x^6\right)^{4/3}}$$

input

```
Integrate[x^3*(a + b*Sin[c + d*x^3]),x]
```

output

```
(d*x^7*(3*(d^2*x^6)^(1/3)*(3*a*d*x^3 - 4*b*Cos[c + d*x^3]) - 2*b*((-I)*d*x
^3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 2*b*(I*d*x^3)^(1/3)*Ga
mma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(36*(d^2*x^6)^(4/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \sin \left(c+d x^3\right)\right) \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(a x^3+b x^3 \sin \left(c+d x^3\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a x^4}{4}-\frac{b x \cos \left(c+d x^3\right)}{3 d}-\frac{b e^{i c} x \Gamma \left(\frac{1}{3},-i d x^3\right)}{18 d \sqrt[3]{-i d x^3}}-\frac{b e^{-i c} x \Gamma \left(\frac{1}{3},i d x^3\right)}{18 d \sqrt[3]{i d x^3}}$$

input

```
Int[x^3*(a + b*Sin[c + d*x^3]),x]
```

output

```
(a*x^4)/4 - (b*x*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^
3])/(18*d*((-I)*d*x^3)^(1/3)) - (b*x*Gamma[1/3, I*d*x^3])/(18*d*E^(I*c)*(I
*d*x^3)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^{3} \left(a +b \sin \left(d \,x^{3}+c \right)\right)d x$$

input
```
int(x^3*(a+b*sin(d*x^3+c)),x)
```

output
```
int(x^3*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int x^3 \left(a+b \sin \left(c+d x^3\right)\right) \, dx = \frac{9 \, a d^{2} x^{4} - 12 \, b d x \cos \left(d x^{3} + c\right) - 2 \, (-i \, b \cos \left(c\right) - b \sin \left(c\right)) (i \, d)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, i \, d x^{3}\right) - 2 \, (i \, b \cos \left(c\right) - b \sin \left(c\right)) (-}{36 \, d^{2}}$$

input
```
integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/36*(9*a*d^2*x^4 - 12*b*d*x*cos(d*x^3 + c) - 2*(-I*b*cos(c) - b*sin(c))*(
I*d)^(2/3)*gamma(1/3, I*d*x^3) - 2*(I*b*cos(c) - b*sin(c))*(-I*d)^(2/3)*ga
mma(1/3, -I*d*x^3))/d^2
```

Sympy [F]

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

input
```
integrate(x**3*(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(x**3*(a + b*sin(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{1}{4}ax^4$$
$$-\frac{(12\,(dx^3)^{\frac{1}{3}}\,x\cos\left(dx^3+c\right)+(((\sqrt{3}-i)\Gamma\left(\frac{1}{3},i\,dx^3\right)+(\sqrt{3}+i)\Gamma\left(\frac{1}{3},-i\,dx^3\right))\cos\left(c\right)+((-i\sqrt{3}-1)\Gamma}{36\,(dx^3)^{\frac{1}{3}}\,d}$$

input
```
integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 - 1/36*(12*(d*x^3)^(1/3)*x*cos(d*x^3 + c) + (((sqrt(3) - I)*gamm
a(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -I*d*x^3))*cos(c) + ((-I*sqrt(3
) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*d*x^3))*sin(c))
*x)*b/((d*x^3)^(1/3)*d)
```

Giac [F]

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int\left(b\sin\left(dx^3+c\right)+a\right)x^3\,dx$$

input
```
integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int x^3\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

input
```
int(x^3*(a + b*sin(c + d*x^3)),x)
```

output
```
int(x^3*(a + b*sin(c + d*x^3)), x)
```

Reduce [F]

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx=\left(\int\sin\left(d\,x^3+c\right)x^3dx\right)b+\frac{a\,x^4}{4}$$

input
```
int(x^3*(a+b*sin(d*x^3+c)),x)
```

output
```
(4*int(sin(c + d*x**3)*x**3,x)*b + a*x**4)/4
```

3.66 $\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx=ax+\frac{ibe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}$$

output

```
a*x+1/6*I*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/6*I*b*x*GAMM
A(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\begin{aligned}\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx&=ax-\frac{1}{2}ib\cos(c)\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}+\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\\&\quad+\frac{1}{2}b\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\sin(c)\end{aligned}$$

input

```
Integrate[a + b*Sin[c + d*x^3],x]
```

output
```
a*x - (I/2)*b*Cos[c]*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) +
 (x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3))) + (b*(-1/3*(x*Gamma[1/3, (-I
)*d*x^3])/((-I)*d*x^3)^(1/3) - (x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3))
)*Sin[c])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{ibe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}$$

input
```
Int[a + b*Sin[c + d*x^3],x]
```

output
```
a*x + ((I/6)*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - ((I/
6)*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sin\left(d\,x^{3}+c\right)\right)dx$$

input

```
int(a+b*sin(d*x^3+c),x)
```

output

```
int(a+b*sin(d*x^3+c),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int \left(a+b\sin\left(c+dx^{3}\right)\right)\,dx = \frac{6\,adx-\left(b\cos\left(c\right)-i\,b\sin\left(c\right)\right)\left(i\,d\right)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},i\,dx^{3}\right)-\left(b\cos\left(c\right)+i\,b\sin\left(c\right)\right)\left(-i\,d\right)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},-i\,dx^{3}\right)}{6\,d}$$

input

```
integrate(a+b*sin(d*x^3+c),x, algorithm="fricas")
```

output

```
1/6*(6*a*d*x - (b*cos(c) - I*b*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) - (
b*cos(c) + I*b*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3))/d
```

Sympy [F]

$$\int \left(a+b\sin\left(c+dx^{3}\right)\right)\,dx=\int \left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$

input

```
integrate(a+b*sin(d*x**3+c),x)
```

output

```
Integral(a + b*sin(c + d*x**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx$$
$$=\frac{\big(\big(\big(-i\sqrt{3}-1\big)\Gamma\big(\tfrac{1}{3},i\,dx^3\big)+\big(i\sqrt{3}-1\big)\Gamma\big(\tfrac{1}{3},-i\,dx^3\big)\big)\cos(c)-\big(\big(\sqrt{3}-i\big)\Gamma\big(\tfrac{1}{3},i\,dx^3\big)+\big(\sqrt{3}+i\big)\Gamma\big(\tfrac{1}{3},-i}{12\,\left(dx^3\right)^{\frac{1}{3}}}$$
$$+ax$$

input
```
integrate(a+b*sin(d*x^3+c),x, algorithm="maxima")
```

output
```
1/12*(((-I*sqrt(3) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -
I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(1/3, I*d*x^3) + (sqrt(3) + I)*gamm
a(1/3, -I*d*x^3))*sin(c))*b*x/(d*x^3)^(1/3) + a*x
```

Giac [F]

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int b\sin\left(dx^3+c\right)+a\,dx$$

input
```
integrate(a+b*sin(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(b*sin(d*x^3 + c) + a, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int a+b\,\sin\left(d\,x^3+c\right)\,dx$$

input
```
int(a + b*sin(c + d*x^3),x)
```

output
```
int(a + b*sin(c + d*x^3), x)
```

Reduce [F]

$$\int \left(a+b\sin \left(c+dx^3\right)\right)\, dx = \left(\int \sin \left(d\,x^3+c\right)dx\right) b+ax$$

input

```
int(a+b*sin(d*x^3+c),x)
```

output

```
int(sin(c + d*x**3),x)*b + a*x
```

3.67 $\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{bde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{2x^2}$$

output
```
-1/2*a/x^2-1/4*b*d*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/4*b*d
*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)-1/2*b*sin(d*x^3+c)/x^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx$$
$$=\frac{-ib\left(-idx^3\right)^{4/3}\Gamma\left(\frac{1}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)+ib\left(idx^3\right)^{4/3}\Gamma\left(\frac{1}{3},-idx^3\right)\left(\cos(c)+i\sin(c)\right)-2\sqrt[3]{d^2x^6}(a+}{4x^2\sqrt[3]{d^2x^6}}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])/x^3,x]
```

output

```
((-I)*b*((-I)*d*x^3)^(4/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(
I*d*x^3)^(4/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(1
/3)*(a + b*Sin[c + d*x^3]))/(4*x^2*(d^2*x^6)^(1/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^3}+\frac{b\sin\left(c+dx^3\right)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}-\frac{be^{ic}dx\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{be^{-ic}dx\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{2x^2}$$

input

```
Int[(a + b*Sin[c + d*x^3])/x^3,x]
```

output

```
-1/2*a/x^2 - (b*d*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/(4*((-I)*d*x^3)^(1/3))
 - (b*d*x*Gamma[1/3, I*d*x^3])/(4*E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*
x^3])/(2*x^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{3}}dx$$

input

```
int((a+b*sin(d*x^3+c))/x^3,x)
```

output

```
int((a+b*sin(d*x^3+c))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{3}}\,dx$$
$$= \frac{(i\,bx^{2}\cos\left(c\right)+bx^{2}\sin\left(c\right))(i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},i\,dx^{3}\right)+(-i\,bx^{2}\cos\left(c\right)+bx^{2}\sin\left(c\right))(-i\,d)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},-i\,dx^{3}\right)-2\,b\sin}{4\,x^{2}}$$

input

```
integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="fricas")
```

output

```
1/4*((I*b*x^2*cos(c) + b*x^2*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) + (-I
*b*x^2*cos(c) + b*x^2*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3) - 2*b*sin(
d*x^3 + c) - 2*a)/x^2
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx = \int \frac{a+b\sin(c+dx^3)}{x^3}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))/x**3,x)
```

output
```
Integral((a + b*sin(c + d*x**3))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx = \frac{(dx^3)^{\frac{2}{3}}\left(\left((\sqrt{3}-i)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+(\sqrt{3}+i)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos(c)-\left((i\sqrt{3}+1)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+(-i\sqrt{3}\right.\right.}{12\,x^2} - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="maxima")
```

output
```
1/12*(d*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-2/3, I*d*x^3) + (sqrt(3) + I)*ga
mma(-2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) + 1)*gamma(-2/3, I*d*x^3) + (-I*
sqrt(3) + 1)*gamma(-2/3, -I*d*x^3))*sin(c))*b/x^2 - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx = \int \frac{b\sin(dx^3+c)+a}{x^3}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx=\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^3}\,dx$$

input
```
int((a + b*sin(c + d*x^3))/x^3,x)
```

output
```
int((a + b*sin(c + d*x^3))/x^3, x)
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx=\frac{6\left(\int \frac{1}{\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2+1}dx\right)bd\,x^2-\sin\left(d\,x^3+c\right)b-a-3bd\,x^3}{2x^2}$$

input
```
int((a+b*sin(d*x^3+c))/x^3,x)
```

output
```
(6*int(1/(tan((c + d*x**3)/2)**2 + 1),x)*b*d*x**2 - sin(c + d*x**3)*b - a
- 3*b*d*x**3)/(2*x**2)
```

3.68 $\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{3bd\cos(c+dx^3)}{10x^2}-\frac{3ibd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}} + \frac{3ibd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin(c+dx^3)}{5x^5}$$

output

```
-1/5*a/x^5-3/10*b*d*cos(d*x^3+c)/x^2-3/20*I*b*d^2*exp(I*c)*x*GAMMA(1/3,-I*
d*x^3)/(-I*d*x^3)^(1/3)+3/20*I*b*d^2*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^
3)^(1/3)-1/5*b*sin(d*x^3+c)/x^5
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = \frac{3bd^2x^6\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3},-idx^3\right)(-i\cos(c)+\sin(c))+3bd^2x^6\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},idx^3\right)(i\cos(c)+\sin(c))-2\sqrt[3]{d^2x^6}(}{20x^5\sqrt[3]{d^2x^6}}$$

input

```
Integrate[(a + b*Sin[c + d*x^3])/x^6,x]
```

output
```
(3*b*d^2*x^6*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
 + 3*b*d^2*x^6*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(I*Cos[c] + Sin[c])
- 2*(d^2*x^6)^(1/3)*(2*a + 3*b*d*x^3*Cos[c + d*x^3] + 2*b*Sin[c + d*x^3]))
/(20*x^5*(d^2*x^6)^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^6}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^6}+\frac{b\sin\left(c+dx^3\right)}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{3ibe^{ic}d^2x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibe^{-ic}d^2x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}$$

input
```
Int[(a + b*Sin[c + d*x^3])/x^6,x]
```

output
```
-1/5*a/x^5 - (3*b*d*Cos[c + d*x^3])/(10*x^2) - (((3*I)/20)*b*d^2*E^(I*c)*x
*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (((3*I)/20)*b*d^2*x*Gamma[1/
3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^3])/(5*x^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{6}}dx$$

input
```
int((a+b*sin(d*x^3+c))/x^6,x)
```

output
```
int((a+b*sin(d*x^3+c))/x^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^6}\,dx = -\frac{6\,bdx^3\cos\left(dx^3+c\right) - 3\left(bdx^5\cos\left(c\right) - i\,bdx^5\sin\left(c\right)\right)\left(i\,d\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, i\,dx^3\right) - 3\left(bdx^5\cos\left(c\right) + i\,bdx^5\sin\left(c\right)}{20\,x^5}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(6*b*d*x^3*cos(d*x^3 + c) - 3*(b*d*x^5*cos(c) - I*b*d*x^5*sin(c))*(I
*d)^(2/3)*gamma(1/3, I*d*x^3) - 3*(b*d*x^5*cos(c) + I*b*d*x^5*sin(c))*(-I*
d)^(2/3)*gamma(1/3, -I*d*x^3) + 4*b*sin(d*x^3 + c) + 4*a)/x^5
```

Sympy [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx=\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))/x**6,x)
```

output
```
Integral((a + b*sin(c + d*x**3))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx = -\frac{(dx^3)^{\frac{2}{3}}\,(((-i\sqrt{3}-1)\Gamma(-\frac{5}{3},i\,dx^3)+(i\sqrt{3}-1)\Gamma(-\frac{5}{3},-i\,dx^3))\cos(c)-((\sqrt{3}-i)\Gamma(-\frac{5}{3},i\,dx^3)+(}{12\,x^2} - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="maxima")
```

output
```
-1/12*(d*x^3)^(2/3)*(((-I*sqrt(3) - 1)*gamma(-5/3, I*d*x^3) + (I*sqrt(3) -
 1)*gamma(-5/3, -I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(-5/3, I*d*x^3) +
(sqrt(3) + I)*gamma(-5/3, -I*d*x^3))*sin(c))*b*d/x^2 - 1/5*a/x^5
```

Giac [F]

$$\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx=\int \frac{b\sin(dx^3+c)+a}{x^6}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^6}\,dx=\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^6}\,dx$$

input
```
int((a + b*sin(c + d*x^3))/x^6,x)
```

output
```
int((a + b*sin(c + d*x^3))/x^6, x)
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^6}\,dx=\frac{5\left(\int \frac{\sin\left(d\,x^3+c\right)}{x^6}dx\right)b\,x^5-a}{5x^5}$$

input
```
int((a+b*sin(d*x^3+c))/x^6,x)
```

output
```
(5*int(sin(c + d*x**3)/x**6,x)*b*x**5 - a)/(5*x**5)
```

3.69 $\int x^5(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{b^2x^3\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}$$

output
```
1/6*a^2*x^6+1/12*b^2*x^6-2/3*a*b*x^3*cos(d*x^3+c)/d+2/3*a*b*sin(d*x^3+c)/d
^2-1/6*b^2*x^3*cos(d*x^3+c)*sin(d*x^3+c)/d+1/12*b^2*sin(d*x^3+c)^2/d^2
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$=\frac{4a^2d^2x^6+2b^2d^2x^6-16abdx^3\cos\left(c+dx^3\right)-b^2\cos\left(2(c+dx^3)\right)+16ab\sin\left(c+dx^3\right)-2b^2dx^3\sin\left(2(c+\right.}{24d^2}$$

input
```
Integrate[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

output
```
(4*a^2*d^2*x^6 + 2*b^2*d^2*x^6 - 16*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(
c + d*x^3)] + 16*a*b*Sin[c + d*x^3] - 2*b^2*d*x^3*Sin[2*(c + d*x^3)])/(24*
d^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3860, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \sin \left(c+d x^3\right)\right)^2 \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{3} \int x^3 \left(a+b \sin \left(d x^3+c\right)\right)^2 dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \int x^3 \left(a+b \sin \left(d x^3+c\right)\right)^2 dx^3$$

$$\Big\downarrow \text{3798}$$

$$\frac{1}{3} \int \left(a^2 x^3+b^2 \sin ^2\left(d x^3+c\right) x^3+2 a b \sin \left(d x^3+c\right) x^3\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^2 x^6}{2}+\frac{2 a b \sin \left(c+d x^3\right)}{d^2}-\frac{2 a b x^3 \cos \left(c+d x^3\right)}{d}+\frac{b^2 \sin ^2\left(c+d x^3\right)}{4 d^2}-\frac{b^2 x^3 \sin \left(c+d x^3\right) \cos \left(c+d x^3\right)}{2 d}+\frac{b^2 x}{4}\right.$$

input
```
Int[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

output
```
((a^2*x^6)/2 + (b^2*x^6)/4 - (2*a*b*x^3*Cos[c + d*x^3])/d + (2*a*b*Sin[c +
 d*x^3])/d^2 - (b^2*x^3*Cos[c + d*x^3]*Sin[c + d*x^3])/(2*d) + (b^2*Sin[c
+ d*x^3]^2)/(4*d^2))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x^6a^2}{6}+\frac{b^2x^6}{12}-\frac{2ab\,x^3\cos(d\,x^3+c)}{3d}+\frac{2ab\sin(d\,x^3+c)}{3d^2}-\frac{b^2\cos(2d\,x^3+2c)}{24d^2}-\frac{b^2x^3\sin(2d\,x^3+2c)}{12d}$
parallelrisch	$\frac{4a^2d^2x^6+2b^2d^2x^6-16ab\,x^3\cos(d\,x^3+c)d-2b^2x^3\sin(2d\,x^3+2c)d+16\sin(d\,x^3+c)ab-b^2\cos(2d\,x^3+2c)+b^2}{24d^2}$
parts	$\frac{x^6a^2}{6}+\frac{\frac{b^2x^6}{12}+\frac{x^6b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2}{6}+\frac{x^6b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^4}{12}+\frac{b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2}{3d^2}-\frac{b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+\frac{b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^3}{3d}}{\left(1+\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2\right)^2}+$
default	$\frac{b^2x^6}{6}+\frac{x^6a^2}{6}+\frac{-\frac{b^2x^6}{6}-\frac{x^6b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2}{3}-\frac{x^6b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^4}{6}+\frac{2b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2}{3d^2}-\frac{2b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+}{2\left(1+\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2\right)^2}$
norman	$\frac{\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^6\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2+\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^4+\frac{b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2}{3d^2}-\frac{b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+}{\left(1+\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2\right)^2}$
orering	$\frac{(81d^4x^{12}+990x^6d^2+2380)\left(a+b\sin(d\,x^3+c)\right)^2}{486x^6d^4}-\frac{(135x^6d^2+616)\left(5x^4\left(a+b\sin(d\,x^3+c)\right)^2+6x^7\left(a+b\sin(d\,x^3+c)\right)bd\cos\right.}{324d^4x^{10}}$

input
```
int(x^5*(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a^2+1/12*b^2*x^6-2/3*a*b*x^3*cos(d*x^3+c)/d+2/3*a*b*sin(d*x^3+c)/d
^2-1/24*b^2/d^2*cos(2*d*x^3+2*c)-1/12*b^2*x^3/d*sin(2*d*x^3+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{(2\,a^2+b^2)d^2x^6-8\,abdx^3\cos\left(dx^3+c\right)-b^2\cos\left(dx^3+c\right)^2-2\left(b^2dx^3\cos\left(dx^3+c\right)-4\,ab\right)\sin\left(dx^3+c\right)}{12\,d^2}$$

input
```
integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/12*((2*a^2 + b^2)*d^2*x^6 - 8*a*b*d*x^3*cos(d*x^3 + c) - b^2*cos(d*x^3 +
 c)^2 - 2*(b^2*d*x^3*cos(d*x^3 + c) - 4*a*b)*sin(d*x^3 + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.34

$$\int x^5 \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx$$
$$= \begin{cases} \frac{a^2x^6}{6} - \frac{2abx^3\cos\left(c+dx^3\right)}{3d} + \frac{2ab\sin\left(c+dx^3\right)}{3d^2} + \frac{b^2x^6\sin^2\left(c+dx^3\right)}{12} + \frac{b^2x^6\cos^2\left(c+dx^3\right)}{12} - \frac{b^2x^3\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d} - b^2\cos \\ \frac{x^6(a+b\sin\left(c\right))^2}{6} \end{cases}$$

input
```
integrate(x**5*(a+b*sin(d*x**3+c))**2,x)
```

output
```
Piecewise((a**2*x**6/6 - 2*a*b*x**3*cos(c + d*x**3)/(3*d) + 2*a*b*sin(c +
d*x**3)/(3*d**2) + b**2*x**6*sin(c + d*x**3)**2/12 + b**2*x**6*cos(c + d*x
**3)**2/12 - b**2*x**3*sin(c + d*x**3)*cos(c + d*x**3)/(6*d) - b**2*cos(c
+ d*x**3)**2/(12*d**2), Ne(d, 0)), (x**6*(a + b*sin(c))**2/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int x^5 \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx = \frac{1}{6}\,a^2x^6 - \frac{2\,\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)ab}{3\,d^2} + \frac{\left(2\,d^2x^6 - 2\,dx^3\sin\left(2\,dx^3+2\,c\right) - \cos\left(2\,dx^3+2\,c\right)\right)b^2}{24\,d^2}$$

input
```
integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 - 2/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*a*b/d^2 + 1/24*(
2*d^2*x^6 - 2*d*x^3*sin(2*d*x^3 + 2*c) - cos(2*d*x^3 + 2*c))*b^2/d^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.54

$$\int x^5 \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$= \frac{4\left(dx^3+c\right)^2a^2+2\left(dx^3+c\right)^2b^2-16\left(dx^3+c\right)ab\cos\left(dx^3+c\right)-2\left(dx^3+c\right)b^2\sin\left(2\,dx^3+2\,c\right)-b^2\cos\left(2\right.}{24\,d^2}$$
$$-\frac{4\left(dx^3+c\right)a^2c+\left(2\,dx^3+2\,c-\sin\left(2\,dx^3+2\,c\right)\right)b^2c-8\,abc\cos\left(dx^3+c\right)}{12\,d^2}$$

input
```
integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
1/24*(4*(d*x^3 + c)^2*a^2 + 2*(d*x^3 + c)^2*b^2 - 16*(d*x^3 + c)*a*b*cos(d
*x^3 + c) - 2*(d*x^3 + c)*b^2*sin(2*d*x^3 + 2*c) - b^2*cos(2*d*x^3 + 2*c)
+ 16*a*b*sin(d*x^3 + c))/d^2 - 1/12*(4*(d*x^3 + c)*a^2*c + (2*d*x^3 + 2*c
- sin(2*d*x^3 + 2*c))*b^2*c - 8*a*b*c*cos(d*x^3 + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 39.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int x^5 \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx =$$
$$-\frac{b^2\cos\left(d\,x^3+c\right)^2-2\,a^2\,d^2\,x^6-b^2\,d^2\,x^6-8\,a\,b\,\sin\left(d\,x^3+c\right)+8\,a\,b\,d\,x^3\,\cos\left(d\,x^3+c\right)+2\,b^2\,d\,x^3\,\cos\left(\right.}{12\,d^2}$$

input
```
int(x^5*(a + b*sin(c + d*x^3))^2,x)
```

output
```
-(b^2*cos(c + d*x^3)^2 - 2*a^2*d^2*x^6 - b^2*d^2*x^6 - 8*a*b*sin(c + d*x^3
) + 8*a*b*d*x^3*cos(c + d*x^3) + 2*b^2*d*x^3*cos(c + d*x^3)*sin(c + d*x^3)
)/(12*d^2)
```

Reduce [F]

$$\int x^5 \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{-4\cos\left(d\,x^3+c\right)abd\,x^3+6\left(\int\sin\left(d\,x^3+c\right)^2x^5dx\right)b^2d^2+4\sin\left(d\,x^3+c\right)ab+a^2d^2x^6}{6d^2}$$

input

```
int(x^5*(a+b*sin(d*x^3+c))^2,x)
```

output

```
( - 4*cos(c + d*x**3)*a*b*d*x**3 + 6*int(sin(c + d*x**3)**2*x**5,x)*b**2*d
**2 + 4*sin(c + d*x**3)*a*b + a**2*d**2*x**6)/(6*d**2)
```

3.70 $\int x^2(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{2ab\cos\left(c+dx^3\right)}{3d}-\frac{b^2\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}$$

output

```
1/6*(2*a^2+b^2)*x^3-2/3*a*b*cos(d*x^3+c)/d-1/6*b^2*cos(d*x^3+c)*sin(d*x^3+
c)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=-\frac{-2(2a^2+b^2)\left(c+dx^3\right)+8ab\cos\left(c+dx^3\right)+b^2\sin\left(2(c+dx^3)\right)}{12d}$$

input

```
Integrate[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
-1/12*(-2*(2*a^2 + b^2)*(c + d*x^3) + 8*a*b*Cos[c + d*x^3] + b^2*Sin[2*(c
+ d*x^3)])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3860, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \sin \left(c+d x^3\right)\right)^2 \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{3} \int \left(a+b \sin \left(d x^3+c\right)\right)^2 dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \int \left(a+b \sin \left(d x^3+c\right)\right)^2 dx^3$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{3} \left(\frac{1}{2} x^3 \left(2 a^2+b^2\right)-\frac{2 a b \cos \left(c+d x^3\right)}{d}-\frac{b^2 \sin \left(c+d x^3\right) \cos \left(c+d x^3\right)}{2 d}\right)$$

input

```
Int[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
(((2*a^2 + b^2)*x^3)/2 - (2*a*b*Cos[c + d*x^3])/d - (b^2*Cos[c + d*x^3]*Si
n[c + d*x^3])/(2*d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result
risch	$\frac{x^{3} a^{2}}{3}+\frac{x^{3} b^{2}}{6}-\frac{2 a b \cos \left(d \,x^{3}+c \right)}{3 d}-\frac{b^{2} \sin \left(2 d \,x^{3}+2 c \right)}{12 d}$
parallelrisch	$\frac{4 a^{2} d \,x^{3}+2 b^{2} d \,x^{3}-8 a b \cos \left(d \,x^{3}+c \right)-b^{2} \sin \left(2 d \,x^{3}+2 c \right)-8 a b}{12 d}$
parts	$\frac{x^{3} a^{2}}{3}+\frac{b^{2} \left(-\frac{\cos \left(d \,x^{3}+c \right) \sin \left(d \,x^{3}+c \right)}{2}+\frac{d \,x^{3}}{2}+\frac{c}{2}\right)}{3 d}-\frac{2 a b \cos \left(d \,x^{3}+c \right)}{3 d}$
derivativedivides	$\frac{b^{2} \left(-\frac{\cos \left(d \,x^{3}+c \right) \sin \left(d \,x^{3}+c \right)}{2}+\frac{d \,x^{3}}{2}+\frac{c}{2}\right)-2 a b \cos \left(d \,x^{3}+c \right)+a^{2} \left(d \,x^{3}+c \right)}{3 d}$
default	$\frac{b^{2} \left(-\frac{\cos \left(d \,x^{3}+c \right) \sin \left(d \,x^{3}+c \right)}{2}+\frac{d \,x^{3}}{2}+\frac{c}{2}\right)-2 a b \cos \left(d \,x^{3}+c \right)+a^{2} \left(d \,x^{3}+c \right)}{3 d}$
norman	$\frac{\left(\frac{a^{2}}{3}+\frac{b^{2}}{6}\right) x^{3}+\left(\frac{a^{2}}{3}+\frac{b^{2}}{6}\right) x^{3} \tan \left(\frac{d \,x^{3}}{2}+\frac{c}{2}\right)^{4}+\left(\frac{2 a^{2}}{3}+\frac{b^{2}}{3}\right) x^{3} \tan \left(\frac{d \,x^{3}}{2}+\frac{c}{2}\right)^{2}-\frac{4 a b}{3 d}-\frac{b^{2} \tan \left(\frac{d \,x^{3}}{2}+\frac{c}{2}\right)}{3 d}+\frac{b^{2} \tan \left(\frac{d \,x^{3}}{2}+\frac{c}{2}\right)^{3}}{3 d}}{\left(1+\tan \left(\frac{d \,x^{3}}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$
orering	$\frac{\left(81 d^{4} x^{12}+180 x^{6} d^{2}+280\right) \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{2}}{243 x^{9} d^{4}}-\frac{\left(15 x^{6} d^{2}+28\right) \left(2 x \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{2}+6 x^{4} \left(a +b \sin \left(d \,x^{3}+c \right)\right) b d \,co\right.}{36 d^{4} x^{10}}$

input
```
int(x^2*(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a^2+1/6*x^3*b^2-2/3*a*b*cos(d*x^3+c)/d-1/12*b^2/d*sin(2*d*x^3+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{(2\,a^2+b^2)dx^3-b^2\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-4\,ab\cos\left(dx^3+c\right)}{6\,d}$$

input
```
integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/6*((2*a^2 + b^2)*d*x^3 - b^2*cos(d*x^3 + c)*sin(d*x^3 + c) - 4*a*b*cos(d
*x^3 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \begin{cases} \frac{a^2x^3}{3}-\frac{2ab\cos\left(c+dx^3\right)}{3d}+\frac{b^2x^3\sin^2\left(c+dx^3\right)}{6}+\frac{b^2x^3\cos^2\left(c+dx^3\right)}{6}-\frac{b^2\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d} & \text{for}\ d\neq 0 \\ \frac{x^3(a+b\sin\left(c\right))^2}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a+b*sin(d*x**3+c))**2,x)
```

output
```
Piecewise((a**2*x**3/3 - 2*a*b*cos(c + d*x**3)/(3*d) + b**2*x**3*sin(c + d
*x**3)**2/6 + b**2*x**3*cos(c + d*x**3)**2/6 - b**2*sin(c + d*x**3)*cos(c
+ d*x**3)/(6*d), Ne(d, 0)), (x**3*(a + b*sin(c))**2/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{3}\,a^2x^3+\frac{(2\,dx^3-\sin\left(2\,dx^3+2\,c\right))b^2}{12\,d}-\frac{2\,ab\cos\left(dx^3+c\right)}{3\,d}$$

input
```
integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/12*(2*d*x^3 - sin(2*d*x^3 + 2*c))*b^2/d - 2/3*a*b*cos(d*x^
3 + c)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{4\,(dx^3+c)a^2+(2\,dx^3+2\,c-\sin\left(2\,dx^3+2\,c\right))b^2-8\,ab\cos\left(dx^3+c\right)}{12\,d}$$

input
```
integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
1/12*(4*(d*x^3 + c)*a^2 + (2*d*x^3 + 2*c - sin(2*d*x^3 + 2*c))*b^2 - 8*a*b
*cos(d*x^3 + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^3}{6}-\frac{b^2\,\sin\left(2\,d\,x^3+2\,c\right)}{12\,d}-\frac{2\,a\,b\,\cos\left(d\,x^3+c\right)}{3\,d}$$

input
```
int(x^2*(a + b*sin(c + d*x^3))^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^3)/6 - (b^2*sin(2*c + 2*d*x^3))/(12*d) - (2*a*b*cos(c
 + d*x^3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{-\cos\left(d\,x^3+c\right)\sin\left(d\,x^3+c\right)b^2-4\cos\left(d\,x^3+c\right)ab+2a^2d\,x^3+4ab+b^2d\,x^3}{6d}$$

input
```
int(x^2*(a+b*sin(d*x^3+c))^2,x)
```

output
```
( - cos(c + d*x**3)*sin(c + d*x**3)*b**2 - 4*cos(c + d*x**3)*a*b + 2*a**2*
d*x**3 + 4*a*b + b**2*d*x**3)/(6*d)
```

3.71 $\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = &-\frac{1}{6}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^3\right)\\&+\frac{1}{2}\left(2a^2+b^2\right)\log(x)+\frac{2}{3}ab\,\text{CosIntegral}\left(dx^3\right)\sin(c)\\&+\frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right)+\frac{1}{6}b^2\sin(2c)\text{Si}\left(2dx^3\right)\end{aligned}$$

output
```
-1/6*b^2*cos(2*c)*Ci(2*d*x^3)+1/2*(2*a^2+b^2)*ln(x)+2/3*a*b*Ci(d*x^3)*sin(
c)+2/3*a*b*cos(c)*Si(d*x^3)+1/6*b^2*sin(2*c)*Si(2*d*x^3)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = &\frac{1}{2}\left(2a^2+b^2\right)\log(x)-\frac{1}{6}b\Big(b\cos(2c)\,\text{CosIntegral}\left(2dx^3\right)\\&-4a\,\text{CosIntegral}\left(dx^3\right)\sin(c)-4a\cos(c)\text{Si}\left(dx^3\right)\\&-b\sin(2c)\text{Si}\left(2dx^3\right)\Big)\end{aligned}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^2/x,x]
```

output
```
((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^3] - 4*a*CosIn
tegral[d*x^3]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^3] - b*Sin[2*c]*SinInteg
ral[2*d*x^3]))/6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx$$

$$\Big\downarrow \; 3884$$

$$\int \left(\frac{a^2}{x}+\frac{2ab\sin\left(c+dx^3\right)}{x}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x}+\frac{b^2}{2x}\right)dx$$

$$\Big\downarrow \; 6$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x}+\frac{2ab\sin\left(c+dx^3\right)}{x}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+\frac{2}{3}ab\sin(c)\,\text{CosIntegral}\left(dx^3\right)+\frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right)-\frac{1}{6}b^2\cos(2c)\,\text{CosIntegral}\left(2dx^3\right)+\frac{1}{6}b^2\sin(2c)\text{Si}\left(2dx^3\right)$$

input
```
Int[(a + b*Sin[c + d*x^3])^2/x,x]
```

output
```
-1/6*(b^2*Cos[2*c]*CosIntegral[2*d*x^3]) + ((2*a^2 + b^2)*Log[x])/2 + (2*a
*b*CosIntegral[d*x^3]*Sin[c])/3 + (2*a*b*Cos[c]*SinIntegral[d*x^3])/3 + (b
^2*Sin[2*c]*SinIntegral[2*d*x^3])/6
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x}dx$$

input
```
int((a+b*sin(d*x^3+c))^2/x,x)
```

output
```
int((a+b*sin(d*x^3+c))^2/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = -\frac{1}{6}\,b^2\cos(2\,c)\,\mathrm{Ci}\left(2\,dx^3\right) + \frac{2}{3}\,ab\,\mathrm{Ci}\left(dx^3\right)\sin(c) + \frac{1}{6}\,b^2\sin(2\,c)\,\mathrm{Si}\left(2\,dx^3\right) + \frac{2}{3}\,ab\cos(c)\,\mathrm{Si}\left(dx^3\right) + \frac{1}{2}\left(2\,a^2+b^2\right)\log(x)$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="fricas")
```

output
```
-1/6*b^2*cos(2*c)*cos_integral(2*d*x^3) + 2/3*a*b*cos_integral(d*x^3)*sin(
c) + 1/6*b^2*sin(2*c)*sin_integral(2*d*x^3) + 2/3*a*b*cos(c)*sin_integral(
d*x^3) + 1/2*(2*a^2 + b^2)*log(x)
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2/x,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\begin{aligned}&\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx\\&= -\frac{1}{3}\left(\left(i\,\mathrm{Ei}\left(i\,dx^3\right)-i\,\mathrm{Ei}\left(-i\,dx^3\right)\right)\cos(c)-\left(\mathrm{Ei}\left(i\,dx^3\right)+\mathrm{Ei}\left(-i\,dx^3\right)\right)\sin(c)\right)ab\\&\quad-\frac{1}{12}\left(\left(\mathrm{Ei}\left(2i\,dx^3\right)+\mathrm{Ei}\left(-2i\,dx^3\right)\right)\cos(2c)-\left(-i\,\mathrm{Ei}\left(2i\,dx^3\right)+i\,\mathrm{Ei}\left(-2i\,dx^3\right)\right)\sin(2c)-6\log(x)\right)b^2\\&\quad+a^2\log(x)\end{aligned}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="maxima")
```

output
```
-1/3*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3
))*sin(c))*a*b - 1/12*((Ei(2*I*d*x^3) + Ei(-2*I*d*x^3))*cos(2*c) - (-I*Ei(
2*I*d*x^3) + I*Ei(-2*I*d*x^3))*sin(2*c) - 6*log(x))*b^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx = -\frac{1}{6}b^2\cos\left(2\,c\right)\operatorname{Ci}\left(2\,dx^3\right)+\frac{2}{3}ab\operatorname{Ci}\left(dx^3\right)\sin\left(c\right) + \frac{2}{3}ab\cos\left(c\right)\operatorname{Si}\left(dx^3\right)-\frac{1}{6}b^2\sin\left(2\,c\right)\operatorname{Si}\left(-2\,dx^3\right) + \frac{1}{3}a^2\log\left(dx^3\right)+\frac{1}{6}b^2\log\left(dx^3\right)$$

input

```
integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="giac")
```

output

```
-1/6*b^2*cos(2*c)*cos_integral(2*d*x^3) + 2/3*a*b*cos_integral(d*x^3)*sin(
c) + 2/3*a*b*cos(c)*sin_integral(d*x^3) - 1/6*b^2*sin(2*c)*sin_integral(-2
*d*x^3) + 1/3*a^2*log(d*x^3) + 1/6*b^2*log(d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx = \int \frac{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}{x}\,dx$$

input

```
int((a + b*sin(c + d*x^3))^2/x,x)
```

output

```
int((a + b*sin(c + d*x^3))^2/x, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx = \left(\int \frac{\sin(d\,x^3+c)^2}{x}dx\right)b^2 + 2\left(\int \frac{\sin(d\,x^3+c)}{x}dx\right)ab + \log(x)\,a^2$$

input

```
int((a+b*sin(d*x^3+c))^2/x,x)
```

output

```
int(sin(c + d*x**3)**2/x,x)*b**2 + 2*int(sin(c + d*x**3)/x,x)*a*b + log(x)
*a**2
```

3.72 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = {} & -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2(c+dx^3)\right)}{6x^3} \\ & +\frac{2}{3}abd\cos(c)\operatorname{CosIntegral}\left(dx^3\right) \\ & +\frac{1}{3}b^2d\operatorname{CosIntegral}\left(2dx^3\right)\sin(2c)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3} \\ & -\frac{2}{3}abd\sin(c)\operatorname{Si}\left(dx^3\right)+\frac{1}{3}b^2d\cos(2c)\operatorname{Si}\left(2dx^3\right)\end{aligned}$$

output

```
-1/6*(2*a^2+b^2)/x^3+1/6*b^2*cos(2*d*x^3+2*c)/x^3+2/3*a*b*d*cos(c)*Ci(d*x^
3)+1/3*b^2*d*Ci(2*d*x^3)*sin(2*c)-2/3*a*b*sin(d*x^3+c)/x^3-2/3*a*b*d*sin(c
)*Si(d*x^3)+1/3*b^2*d*cos(2*c)*Si(2*d*x^3)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = \frac{-2a^2-b^2+b^2\cos(2(c+dx^3))+4abdx^3\cos(c)\,\text{CosIntegral}(dx^3)+2b^2dx^3\,\text{CosIntegral}(2dx^3)\sin(2c)-}{6x^3}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

output
```
(-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^3)] + 4*a*b*d*x^3*Cos[c]*CosIntegral[d*
x^3] + 2*b^2*d*x^3*CosIntegral[2*d*x^3]*Sin[2*c] - 4*a*b*Sin[c + d*x^3] -
4*a*b*d*x^3*Sin[c]*SinIntegral[d*x^3] + 2*b^2*d*x^3*Cos[2*c]*SinIntegral[2
*d*x^3])/(6*x^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$

$$\Big\downarrow \text{ 3884}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ab\sin(c+dx^3)}{x^4}-\frac{b^2\cos(2c+2dx^3)}{2x^4}+\frac{b^2}{2x^4}\right)dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^4}+\frac{2ab\sin(c+dx^3)}{x^4}-\frac{b^2\cos(2c+2dx^3)}{2x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2+b^2}{6x^3}+\frac{2}{3}abd\cos(c)\,\text{CosIntegral}\left(dx^3\right)-\frac{2}{3}abd\sin(c)\text{Si}\left(dx^3\right)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}b^2d\sin(2c)\,\text{CosIntegral}\left(2dx^3\right)+\frac{1}{3}b^2d\cos(2c)\text{Si}\left(2dx^3\right)+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}$$

input
```
Int[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

output
```
-1/6*(2*a^2 + b^2)/x^3 + (b^2*Cos[2*(c + d*x^3)])/(6*x^3) + (2*a*b*d*Cos[c
]*CosIntegral[d*x^3])/3 + (b^2*d*CosIntegral[2*d*x^3]*Sin[2*c])/3 - (2*a*b
*Sin[c + d*x^3])/(3*x^3) - (2*a*b*d*Sin[c]*SinIntegral[d*x^3])/3 + (b^2*d*
Cos[2*c]*SinIntegral[2*d*x^3])/3
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a+b\sin\left(d\,x^3+c\right)\right)^2}{x^4}dx$$

input
```
int((a+b*sin(d*x^3+c))^2/x^4,x)
```

output
```
int((a+b*sin(d*x^3+c))^2/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx$$
$$=\frac{2\,abdx^3\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)+b^2dx^3\operatorname{Ci}\left(2\,dx^3\right)\sin\left(2\,c\right)+b^2dx^3\cos\left(2\,c\right)\operatorname{Si}\left(2\,dx^3\right)-2\,abdx^3\sin\left(c\right)\operatorname{Si}\left(dx^3\right)+}{3\,x^3}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="fricas")
```

output
```
1/3*(2*a*b*d*x^3*cos(c)*cos_integral(d*x^3) + b^2*d*x^3*cos_integral(2*d*x
^3)*sin(2*c) + b^2*d*x^3*cos(2*c)*sin_integral(2*d*x^3) - 2*a*b*d*x^3*sin(
c)*sin_integral(d*x^3) + b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2
 - b^2)/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2/x**4,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$

$$=\frac{1}{3}\left(\left(\Gamma(-1,i\,dx^3)+\Gamma(-1,-i\,dx^3)\right)\cos(c)-\left(i\,\Gamma(-1,i\,dx^3)-i\,\Gamma(-1,-i\,dx^3)\right)\sin(c)\right)abd$$

$$+\frac{(((i\,\Gamma(-1,2i\,dx^3)-i\,\Gamma(-1,-2i\,dx^3))\cos(2\,c)+(\Gamma(-1,2i\,dx^3)+\Gamma(-1,-2i\,dx^3))\sin(2\,c))dx^3-1)b^2}{6\,x^3}$$

$$-\frac{a^2}{3\,x^3}$$

input

```
integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="maxima")
```

output

```
1/3*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*
x^3) - I*gamma(-1, -I*d*x^3))*sin(c))*a*b*d + 1/6*(((I*gamma(-1, 2*I*d*x^3
) - I*gamma(-1, -2*I*d*x^3))*cos(2*c) + (gamma(-1, 2*I*d*x^3) + gamma(-1,
-2*I*d*x^3))*sin(2*c))*d*x^3 - 1)*b^2/x^3 - 1/3*a^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(109) = 218.

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.85

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$$

$$=\frac{4\,(dx^3+c)abd^2\cos(c)\operatorname{Ci}(dx^3)-4\,abcd^2\cos(c)\operatorname{Ci}(dx^3)+2\,(dx^3+c)b^2d^2\operatorname{Ci}(2\,dx^3)\sin(2\,c)-2\,b^2cd^2\,\mathrm{C}}{}$$

input

```
integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="giac")
```

output

```
1/6*(4*(d*x^3 + c)*a*b*d^2*cos(c)*cos_integral(d*x^3) - 4*a*b*c*d^2*cos(c)
*cos_integral(d*x^3) + 2*(d*x^3 + c)*b^2*d^2*cos_integral(2*d*x^3)*sin(2*c
) - 2*b^2*c*d^2*cos_integral(2*d*x^3)*sin(2*c) - 4*(d*x^3 + c)*a*b*d^2*sin
(c)*sin_integral(d*x^3) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^3) - 2*(d*x^
3 + c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^3) + 2*b^2*c*d^2*cos(2*c)*sin_
integral(-2*d*x^3) + b^2*d^2*cos(2*d*x^3 + 2*c) - 4*a*b*d^2*sin(d*x^3 + c)
 - 2*a^2*d^2 - b^2*d^2)/(d^2*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = \int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^4}\,dx$$

input
```
int((a + b*sin(c + d*x^3))^2/x^4,x)
```

output
```
int((a + b*sin(c + d*x^3))^2/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*sin(d*x^3+c))^2/x^4,x)
```

output

```
( - 2*cos(c + d*x**3)*sin(c + d*x**3)*tan((c + d*x**3)/2)**4*a*b - 4*cos(c
 + d*x**3)*sin(c + d*x**3)*tan((c + d*x**3)/2)**2*a*b - 2*cos(c + d*x**3)*
sin(c + d*x**3)*a*b + 36*int(tan((c + d*x**3)/2)**2/(tan((c + d*x**3)/2)**
4*x**4 + 2*tan((c + d*x**3)/2)**2*x**4 + x**4),x)*tan((c + d*x**3)/2)**4*b
**2*x**3 + 72*int(tan((c + d*x**3)/2)**2/(tan((c + d*x**3)/2)**4*x**4 + 2*
tan((c + d*x**3)/2)**2*x**4 + x**4),x)*tan((c + d*x**3)/2)**2*b**2*x**3 +
36*int(tan((c + d*x**3)/2)**2/(tan((c + d*x**3)/2)**4*x**4 + 2*tan((c + d*
x**3)/2)**2*x**4 + x**4),x)*b**2*x**3 + 36*int(tan((c + d*x**3)/2)**2/(tan
((c + d*x**3)/2)**4*x + 2*tan((c + d*x**3)/2)**2*x + x),x)*tan((c + d*x**3
)/2)**4*a*b*d*x**3 + 72*int(tan((c + d*x**3)/2)**2/(tan((c + d*x**3)/2)**4
*x + 2*tan((c + d*x**3)/2)**2*x + x),x)*tan((c + d*x**3)/2)**2*a*b*d*x**3
+ 36*int(tan((c + d*x**3)/2)**2/(tan((c + d*x**3)/2)**4*x + 2*tan((c + d*x
**3)/2)**2*x + x),x)*a*b*d*x**3 + 36*int(1/(tan((c + d*x**3)/2)**4*x + 2*t
an((c + d*x**3)/2)**2*x + x),x)*tan((c + d*x**3)/2)**4*a*b*d*x**3 + 72*int
(1/(tan((c + d*x**3)/2)**4*x + 2*tan((c + d*x**3)/2)**2*x + x),x)*tan((c +
 d*x**3)/2)**2*a*b*d*x**3 + 36*int(1/(tan((c + d*x**3)/2)**4*x + 2*tan((c
+ d*x**3)/2)**2*x + x),x)*a*b*d*x**3 - 18*log(x)*tan((c + d*x**3)/2)**4*a*
b*d*x**3 - 36*log(x)*tan((c + d*x**3)/2)**2*a*b*d*x**3 - 18*log(x)*a*b*d*x
**3 - 8*sin(c + d*x**3)*tan((c + d*x**3)/2)**4*a*b - 16*sin(c + d*x**3)*ta
n((c + d*x**3)/2)**2*a*b - 8*sin(c + d*x**3)*a*b - 3*tan((c + d*x**3)/2...
```

3.73 $\int x^4(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 249

$$\begin{aligned}\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=&\ \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}\\&-\frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}\\&+\frac{ib^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}\\&-\frac{ib^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{36\ 2^{2/3}d\left(idx^3\right)^{2/3}}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}\end{aligned}$$

output

```
1/10*(2*a^2+b^2)*x^5-2/3*a*b*x^2*cos(d*x^3+c)/d-2/9*a*b*exp(I*c)*x^2*GAMMA
(2/3,-I*d*x^3)/d/(-I*d*x^3)^(2/3)-2/9*a*b*x^2*GAMMA(2/3,I*d*x^3)/d/exp(I*c
)/(I*d*x^3)^(2/3)+1/72*I*b^2*exp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/
d/(-I*d*x^3)^(2/3)-1/72*I*b^2*x^2*GAMMA(2/3,2*I*d*x^3)*2^(1/3)/d/exp(2*I*c
)/(I*d*x^3)^(2/3)-1/12*b^2*x^2*sin(2*d*x^3+2*c)/d
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.36

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\frac{dx^8\left(72a^2dx^3(d^2x^6)^{2/3}+36b^2dx^3(d^2x^6)^{2/3}-240ab(d^2x^6)^{2/3}\cos\left(c+dx^3\right)+5i\sqrt[3]{2}b^2(idx^3)^{2/3}\cos(2c)\Gamma\left(\frac{2}{3},\right.\right.}{}$$

input
```
Integrate[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

output
```
(d*x^8*(72*a^2*d*x^3*(d^2*x^6)^(2/3) + 36*b^2*d*x^3*(d^2*x^6)^(2/3) - 240*
a*b*(d^2*x^6)^(2/3)*Cos[c + d*x^3] + (5*I)*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Cos
[2*c]*Gamma[2/3, (-2*I)*d*x^3] - (5*I)*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Cos[
2*c]*Gamma[2/3, (2*I)*d*x^3] - 80*a*b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^
3]*(Cos[c] - I*Sin[c]) - 80*a*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Co
s[c] + I*Sin[c]) - 5*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Gamma[2/3, (-2*I)*d*x^3]*
Sin[2*c] - 5*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*
c] - 30*b^2*(d^2*x^6)^(2/3)*Sin[2*(c + d*x^3)]))/(360*(d^2*x^6)^(5/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$\Big\downarrow \; 3884$$
$$\int \left(a^2x^4+2abx^4\sin\left(c+dx^3\right)-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+\frac{b^2x^4}{2}\right)dx$$
$$\Big\downarrow \; 6$$

$$\int \left(x^4\left(a^2+\frac{b^2}{2}\right)+2abx^4\sin\left(c+dx^3\right)-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}+\frac{ib^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{36\ 2^{2/3}d\left(idx^3\right)^{2/3}}$$

input

```
Int[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
((2*a^2 + b^2)*x^5)/10 - (2*a*b*x^2*Cos[c + d*x^3])/(3*d) - (2*a*b*E^(I*c)
*x^2*Gamma[2/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (2*a*b*x^2*Gamma[2
/3, I*d*x^3])/(9*d*E^(I*c)*(I*d*x^3)^(2/3)) + ((I/36)*b^2*E^((2*I)*c)*x^2*
Gamma[2/3, (-2*I)*d*x^3])/(2^(2/3)*d*((-I)*d*x^3)^(2/3)) - ((I/36)*b^2*x^2
*Gamma[2/3, (2*I)*d*x^3])/(2^(2/3)*d*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (b^2*x
^2*Sin[2*c + 2*d*x^3])/(12*d)
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
 + (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int x^{4} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{2}dx$$

input
```
int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.75

$$\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$= \frac{36\,(2\,a^2+b^2)d^2x^5-60\,b^2dx^2\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-240\,abdx^2\cos\left(dx^3+c\right)-5\left(b^2\cos\left(2\,c\right)-i\,b^2\right.}{}$$

input
```
integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/360*(36*(2*a^2 + b^2)*d^2*x^5 - 60*b^2*d*x^2*cos(d*x^3 + c)*sin(d*x^3 +
c) - 240*a*b*d*x^2*cos(d*x^3 + c) - 5*(b^2*cos(2*c) - I*b^2*sin(2*c))*(2*I
*d)^(1/3)*gamma(2/3, 2*I*d*x^3) - 80*(-I*a*b*cos(c) - a*b*sin(c))*(I*d)^(1
/3)*gamma(2/3, I*d*x^3) - 80*(I*a*b*cos(c) - a*b*sin(c))*(-I*d)^(1/3)*gamm
a(2/3, -I*d*x^3) - 5*(b^2*cos(2*c) + I*b^2*sin(2*c))*(-2*I*d)^(1/3)*gamma(
2/3, -2*I*d*x^3))/d^2
```

Sympy [F]

$$\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

input
```
integrate(x**4*(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(x**4*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

$$\int x^4 \big(a+b\sin\left(c+dx^3\right)\big)^2\,dx = \frac{1}{5}\,a^2x^5$$
$$-\frac{\Big(6\,dx^3\cos\left(dx^3+c\right)-\left(dx^3\right)^{\frac{1}{3}}\,\big(\big((i\sqrt{3}-1)\Gamma\big(\frac{2}{3},i\,dx^3\big)+\big(-i\sqrt{3}-1\big)\Gamma\big(\frac{2}{3},-i\,dx^3\big)\big)\cos\left(c\right)+\big((\sqrt{3}+}{9\,d^2x}$$
$$+\frac{\Big(72\,d^2x^6-60\,dx^3\sin\left(2\,dx^3+2\,c\right)-5\cdot 2^{\frac{1}{3}}\left(dx^3\right)^{\frac{1}{3}}\,\big(\big(\big(\sqrt{3}+i\big)\Gamma\big(\frac{2}{3},2i\,dx^3\big)+\big(\sqrt{3}-i\big)\Gamma\big(\frac{2}{3},-2i\,dx^3\big)\big)\,\mathrm{c}}{720\,d^2x}$$

input
```
integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 - 1/9*(6*d*x^3*cos(d*x^3 + c) - (d*x^3)^(1/3)*(((I*sqrt(3) - 1
)*gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*cos(c) + ((
sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*sin
(c))*a*b/(d^2*x) + 1/720*(72*d^2*x^6 - 60*d*x^3*sin(2*d*x^3 + 2*c) - 5*2^
(1/3)*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, 2*I*d*x^3) + (sqrt(3) - I)*
gamma(2/3, -2*I*d*x^3))*cos(2*c) + ((-I*sqrt(3) + 1)*gamma(2/3, 2*I*d*x^3)
 + (I*sqrt(3) + 1)*gamma(2/3, -2*I*d*x^3))*sin(2*c)))*b^2/(d^2*x)
```

Giac [F]

$$\int x^4 \big(a+b\sin\left(c+dx^3\right)\big)^2\,dx = \int \big(b\sin\left(dx^3+c\right)+a\big)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\sin(c+dx^3))^2\,dx=\int x^4\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2\,dx$$

input
```
int(x^4*(a + b*sin(c + d*x^3))^2,x)
```

output
```
int(x^4*(a + b*sin(c + d*x^3))^2, x)
```

Reduce [F]

$$\int x^4(a+b\sin(c+dx^3))^2\,dx=\left(\int \sin\left(d\,x^3+c\right)^2 x^4dx\right)b^2 \\ +2\left(\int \sin\left(d\,x^3+c\right)x^4dx\right)ab+\frac{a^2x^5}{5}$$

input
```
int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

output
```
(5*int(sin(c + d*x**3)**2*x**4,x)*b**2 + 10*int(sin(c + d*x**3)*x**4,x)*a*
b + a**2*x**5)/5
```

3.74 $\int x(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 193

$$
\begin{aligned}
\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx ={}& \frac{1}{4}\left(2a^2+b^2\right)x^2+\frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}} \\
&-\frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}} \\
&+\frac{b^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}
\end{aligned}
$$

output

```
1/4*(2*a^2+b^2)*x^2+1/3*I*a*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^
(2/3)-1/3*I*a*b*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)+1/24*b^2*e
xp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(2/3)+1/24*b^2*x^2*
GAMMA(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.30

$$\begin{aligned}
&\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx \\
&=\frac{1}{24}\left(6\left(2a^2+b^2\right)x^2-\frac{8ab\cos(c)\left(\sqrt[3]{-idx^3}\Gamma\left(\frac{2}{3},-idx^3\right)+\sqrt[3]{idx^3}\Gamma\left(\frac{2}{3},idx^3\right)\right)}{dx}\right. \\
&\quad-\frac{8abx^2\left(\left(idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)+\left(-idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)\right)\sin(c)}{\left(d^2x^6\right)^{2/3}} \\
&\quad+\frac{\sqrt[3]{2}b^2x^2\Gamma\left(\frac{2}{3},2idx^3\right)\left(\cos(2c)-i\sin(2c)\right)}{\left(idx^3\right)^{2/3}} \\
&\quad\left.+\frac{\sqrt[3]{2}b^2x^2\Gamma\left(\frac{2}{3},-2idx^3\right)\left(\cos(2c)+i\sin(2c)\right)}{\left(-idx^3\right)^{2/3}}\right)
\end{aligned}$$

input

```
Integrate[x*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
(6*(2*a^2 + b^2)*x^2 - (8*a*b*Cos[c]*(((-I)*d*x^3)^(1/3)*Gamma[2/3, (-I)*d
*x^3] + (I*d*x^3)^(1/3)*Gamma[2/3, I*d*x^3]))/(d*x) - (8*a*b*x^2*((I*d*x^3
)^(2/3)*Gamma[2/3, (-I)*d*x^3] + ((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3])*S
in[c])/(d^2*x^6)^(2/3) + (2^(1/3)*b^2*x^2*Gamma[2/3, (2*I)*d*x^3]*(Cos[2*c
] - I*Sin[2*c]))/(I*d*x^3)^(2/3) + (2^(1/3)*b^2*x^2*Gamma[2/3, (-2*I)*d*x^
3]*(Cos[2*c] + I*Sin[2*c]))/((-I)*d*x^3)^(2/3))/24
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$\Big\downarrow\ \text{3884}$$

$$\int\left(a^2x+2abx\sin\left(c+dx^3\right)-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+\frac{b^2x}{2}\right)dx$$

$$\Big\downarrow\ \text{6}$$

$$\int\left(x\left(a^2+\frac{b^2}{2}\right)+2abx\sin\left(c+dx^3\right)-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)\right)dx$$

$$\Big\downarrow\ \text{2009}$$

$$\frac{1}{4}x^2\left(2a^2+b^2\right)+\frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}}+\frac{b^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}$$

input

```
Int[x*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
((2*a^2 + b^2)*x^2)/4 + ((I/3)*a*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-
I)*d*x^3)^(2/3) - ((I/3)*a*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(
2/3)) + (b^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(12*2^(2/3)*((-I)*d
*x^3)^(2/3)) + (b^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(12*2^(2/3)*E^((2*I)*c)*(
I*d*x^3)^(2/3))
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int x\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}dx$$

input
```
int(x*(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(x*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int x\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}\,dx = \frac{6\,(2\,a^{2}+b^{2})dx^{2}+(-i\,b^{2}\cos\left(2\,c\right)-b^{2}\sin\left(2\,c\right))(2i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3},2i\,dx^{3}\right)-8\,(ab\cos\left(c\right)-i\,ab\sin\left(c\right))(i\,d)^{\frac{1}{3}}\,\Gamma(}{}$$

input
```
integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/24*(6*(2*a^2 + b^2)*d*x^2 + (-I*b^2*cos(2*c) - b^2*sin(2*c))*(2*I*d)^(1/
3)*gamma(2/3, 2*I*d*x^3) - 8*(a*b*cos(c) - I*a*b*sin(c))*(I*d)^(1/3)*gamma
(2/3, I*d*x^3) - 8*(a*b*cos(c) + I*a*b*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*
d*x^3) + (I*b^2*cos(2*c) - b^2*sin(2*c))*(-2*I*d)^(1/3)*gamma(2/3, -2*I*d*
x^3))/d
```

Sympy [F]

$$\int x\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}\,dx = \int x\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}\,dx$$

input
```
integrate(x*(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(x*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int x\bigl(a+b\sin\left(c+dx^3\right)\bigr)^2\,dx=\frac{1}{2}\,a^2x^2$$
$$-\frac{(dx^3)^{\frac{1}{3}}\,\Bigl(\Bigl(\bigl(\sqrt{3}+i\bigr)\Gamma\bigl(\tfrac{2}{3},i\,dx^3\bigr)+\bigl(\sqrt{3}-i\bigr)\Gamma\bigl(\tfrac{2}{3},-i\,dx^3\bigr)\Bigr)\cos\left(c\right)-\Bigl(\bigl(i\sqrt{3}-1\bigr)\Gamma\bigl(\tfrac{2}{3},i\,dx^3\bigr)+\bigl(-i\sqrt{3}-}{6\,dx}$$
$$+\frac{\Bigl(12\,dx^3-2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\,\Bigl(\Bigl(\bigl(i\sqrt{3}-1\bigr)\Gamma\bigl(\tfrac{2}{3},2i\,dx^3\bigr)+\bigl(-i\sqrt{3}-1\bigr)\Gamma\bigl(\tfrac{2}{3},-2i\,dx^3\bigr)\Bigr)\cos\left(2\,c\right)+\Bigl(\bigl(\sqrt{3}+i\bigr)\Gamma\bigl(\tfrac{2}{3}}{48\,dx}$$

input
```
integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 - 1/6*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqr
t(3) - I)*gamma(2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(2/3, I*d*x
^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*sin(c))*a*b/(d*x) + 1/48*(12*
d*x^3 - 2^(1/3)*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(2/3, 2*I*d*x^3) + (-
I*sqrt(3) - 1)*gamma(2/3, -2*I*d*x^3))*cos(2*c) + ((sqrt(3) + I)*gamma(2/3
, 2*I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -2*I*d*x^3))*sin(2*c)))*b^2/(d*x)
```

Giac [F]

$$\int x\bigl(a+b\sin\left(c+dx^3\right)\bigr)^2\,dx=\int\bigl(b\sin\left(dx^3+c\right)+a\bigr)^2x\,dx$$

input
```
integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2\,dx$$

input
```
int(x*(a + b*sin(c + d*x^3))^2,x)
```

output
```
int(x*(a + b*sin(c + d*x^3))^2, x)
```

Reduce [F]

$$\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\left(\int \sin\left(d\,x^3+c\right)^2 x dx\right)b^2+2\left(\int \sin\left(d\,x^3+c\right)x dx\right)ab+\frac{a^2x^2}{2}$$

input
```
int(x*(a+b*sin(d*x^3+c))^2,x)
```

output
```
(2*int(sin(c + d*x**3)**2*x,x)*b**2 + 4*int(sin(c + d*x**3)*x,x)*a*b + a**
2*x**2)/2
```

3.75 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 231

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^2}\,dx &= \frac{-2a^2-b^2}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{2x}-\frac{abde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{\left(-idx^3\right)^{2/3}}\\&\quad-\frac{abde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{\left(idx^3\right)^{2/3}}+\frac{ib^2de^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{2\ 2^{2/3}\left(-idx^3\right)^{2/3}}\\&\quad-\frac{ib^2de^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{2\ 2^{2/3}\left(idx^3\right)^{2/3}}-\frac{2ab\sin\left(c+dx^3\right)}{x}\end{aligned}$$

output

```
1/2*(-2*a^2-b^2)/x+1/2*b^2*cos(2*d*x^3+2*c)/x-a*b*d*exp(I*c)*x^2*GAMMA(2/3
,-I*d*x^3)/(-I*d*x^3)^(2/3)-a*b*d*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3
)^(2/3)+1/4*I*b^2*d*exp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3
)^(2/3)-1/4*I*b^2*d*x^2*GAMMA(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^
(2/3)-2*a*b*sin(d*x^3+c)/x
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$$
$$= \frac{-4a^2(d^2x^6)^{2/3} - 2b^2(d^2x^6)^{2/3} + 2b^2(d^2x^6)^{2/3}\cos(2(c+dx^3)) + \sqrt[3]{2}b^2(idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{2}{3},-2idx^3\right) + \sqrt[3]{}}{}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

output
```
(-4*a^2*(d^2*x^6)^(2/3) - 2*b^2*(d^2*x^6)^(2/3) + 2*b^2*(d^2*x^6)^(2/3)*Co
s[2*(c + d*x^3)] + 2^(1/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (-2*I)*
d*x^3] + 2^(1/3)*b^2*((-I)*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] -
 (4*I)*a*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (4
*I)*a*b*(I*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*2^(
1/3)*b^2*(I*d*x^3)^(5/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*c] - I*2^(1/3)*b^2
*((-I)*d*x^3)^(5/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 8*a*b*(d^2*x^6)^(2/
3)*Sin[c + d*x^3])/(4*x*(d^2*x^6)^(2/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$$
$$\Big\downarrow \text{3884}$$
$$\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx^3)}{x^2}-\frac{b^2\cos(2c+2dx^3)}{2x^2}+\frac{b^2}{2x^2}\right)dx$$
$$\Big\downarrow \text{6}$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^2}+\frac{2ab\sin\left(c+dx^3\right)}{x^2}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2+b^2}{2x}-\frac{2ab\sin\left(c+dx^3\right)}{x}-\frac{abe^{ic}dx^2\Gamma\left(\frac{2}{3},-idx^3\right)}{\left(-idx^3\right)^{2/3}}-\frac{abe^{-ic}dx^2\Gamma\left(\frac{2}{3},idx^3\right)}{\left(idx^3\right)^{2/3}}+\frac{b^2\cos\left(2c+2dx^3\right)}{2x}+\frac{ib^2e^{2ic}dx^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{2\ 2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}dx^2\Gamma\left(\frac{2}{3},2idx^3\right)}{2\ 2^{2/3}\left(idx^3\right)^{2/3}}$$

input
```
Int[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

output
```
-1/2*(2*a^2 + b^2)/x + (b^2*Cos[2*c + 2*d*x^3])/(2*x) - (a*b*d*E^(I*c)*x^2
*Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) - (a*b*d*x^2*Gamma[2/3, I*d*x^
3])/(E^(I*c)*(I*d*x^3)^(2/3)) + ((I/2)*b^2*d*E^((2*I)*c)*x^2*Gamma[2/3, (-
2*I)*d*x^3])/(2^(2/3)*((-I)*d*x^3)^(2/3)) - ((I/2)*b^2*d*x^2*Gamma[2/3, (2
*I)*d*x^3])/(2^(2/3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (2*a*b*Sin[c + d*x^3])
/x
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^2}\,dx$$

input
```
int((a+b*sin(d*x^3+c))^2/x^2,x)
```

output
```
int((a+b*sin(d*x^3+c))^2/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$$
$$= \frac{4\,b^2\cos(dx^3+c)^2 - 8\,ab\sin(dx^3+c) - (b^2x\cos(2\,c) - i\,b^2x\sin(2\,c))(2i\,d)^{\frac{1}{3}}\,\Gamma\left(\frac{2}{3}, 2i\,dx^3\right) - 4\,(-i\,abx\,\mathrm{co}}{}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*cos(d*x^3 + c)^2 - 8*a*b*sin(d*x^3 + c) - (b^2*x*cos(2*c) - I*b
^2*x*sin(2*c))*(2*I*d)^(1/3)*gamma(2/3, 2*I*d*x^3) - 4*(-I*a*b*x*cos(c) -
a*b*x*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) - 4*(I*a*b*x*cos(c) - a*b*x*
sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3) - (b^2*x*cos(2*c) + I*b^2*x*sin(
2*c))*(-2*I*d)^(1/3)*gamma(2/3, -2*I*d*x^3) - 4*a^2 - 4*b^2)/x
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2/x**2,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx =$$
$$-\frac{(dx^3)^{\frac{1}{3}}\left(\left((i\sqrt{3}-1)\Gamma\left(-\frac{1}{3}, i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3}, -i\,dx^3\right)\right)\cos(c)+\left((\sqrt{3}+i)\Gamma\left(-\frac{1}{3}, i\,dx^3\right)+(\right.\right.}{6\,x}$$
$$+\frac{\left(2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\left(\left((\sqrt{3}+i)\Gamma\left(-\frac{1}{3}, 2i\,dx^3\right)+(\sqrt{3}-i)\Gamma\left(-\frac{1}{3}, -2i\,dx^3\right)\right)\cos(2\,c)-\left((i\sqrt{3}-1)\Gamma\left(-\frac{1}{3}, 2i\,dx^3\right.\right.\right.\right.}{24\,x}$$
$$-\frac{a^2}{x}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="maxima")
```

output
```
-1/6*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(-1/3, I*d*x^3) + (-I*sqrt(3) -
1)*gamma(-1/3, -I*d*x^3))*cos(c) + ((sqrt(3) + I)*gamma(-1/3, I*d*x^3) + (
sqrt(3) - I)*gamma(-1/3, -I*d*x^3))*sin(c))*a*b/x + 1/24*(2^(1/3)*(d*x^3)^
(1/3)*(((sqrt(3) + I)*gamma(-1/3, 2*I*d*x^3) + (sqrt(3) - I)*gamma(-1/3, -
2*I*d*x^3))*cos(2*c) - ((I*sqrt(3) - 1)*gamma(-1/3, 2*I*d*x^3) + (-I*sqrt(
3) - 1)*gamma(-1/3, -2*I*d*x^3))*sin(2*c)) - 12)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \int \frac{(b\sin(dx^3+c)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \int \frac{(a+b\sin(d\,x^3+c))^2}{x^2}\,dx$$

input

```
int((a + b*sin(c + d*x^3))^2/x^2,x)
```

output

```
int((a + b*sin(c + d*x^3))^2/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx = \frac{\left(\int \frac{\sin(d\,x^3+c)^2}{x^2}dx\right) b^2x + 12\left(\int \frac{x}{\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2+1}dx\right) abdx - 2\left(\int \frac{1}{x^2}dx\right) b^2x - 2\sin(d\,x^3+c)\,ab - a^2 - 3abd}{x}$$

input

```
int((a+b*sin(d*x^3+c))^2/x^2,x)
```

output

```
(int(sin(c + d*x**3)**2/x**2,x)*b**2*x + 12*int(x/(tan((c + d*x**3)/2)**2
+ 1),x)*a*b*d*x - 2*int(1/x**2,x)*b**2*x - 2*sin(c + d*x**3)*a*b - a**2 -
3*a*b*d*x**3 - 2*b**2)/x
```

3.76 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 285

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx &= \frac{-2a^2-b^2}{8x^4}-\frac{3abd\cos(c+dx^3)}{2x}+\frac{b^2\cos(2c+2dx^3)}{8x^4}\\ &\quad-\frac{3iabd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iabd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}}\\ &\quad-\frac{3b^2d^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{4\ 2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{3b^2d^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{4\ 2^{2/3}\left(idx^3\right)^{2/3}}\\ &\quad-\frac{ab\sin(c+dx^3)}{2x^4}-\frac{3b^2d\sin(2c+2dx^3)}{4x}\end{aligned}$$

output

```
1/8*(-2*a^2-b^2)/x^4-3/2*a*b*d*cos(d*x^3+c)/x+1/8*b^2*cos(2*d*x^3+2*c)/x^4
-3/4*I*a*b*d^2*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)+3/4*I*a*b
*d^2*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)-3/8*b^2*d^2*exp(2*I*c
)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(2/3)-3/8*b^2*d^2*x^2*GAMMA
(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3)-1/2*a*b*sin(d*x^3+c)/x^
4-3/4*b^2*d*sin(2*d*x^3+2*c)/x
```

Mathematica [A] (verified)

Time = 3.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx =$$
$$-\frac{2a^2+b^2+12abdx^3\cos\left(c+dx^3\right)-b^2\cos\left(2(c+dx^3)\right)-3\sqrt[3]{2}b^2(idx^3)^{4/3}\cos(2c)\Gamma\left(\frac{2}{3},2idx^3\right)+6iab(id}{}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

output
```
-1/8*(2*a^2 + b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c + d*x^3)] -
 3*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] + (6*I)*a*
b*(I*d*x^3)^(4/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I*d
*x^3)^(2/3)*(d^2*x^6)^(1/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 3
*2^(1/3)*b^2*((-I)*d*x^3)^(4/3)*Gamma[2/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin
[2*c]) + (3*I)*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c
] + 4*a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/x^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}dx$$
$$\Big\downarrow \text{3884}$$
$$\int \left(\frac{a^2}{x^5}+\frac{2ab\sin\left(c+dx^3\right)}{x^5}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^5}+\frac{b^2}{2x^5}\right)dx$$
$$\Big\downarrow \text{6}$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^5}+\frac{2ab\sin\left(c+dx^3\right)}{x^5}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2+b^2}{8x^4}-\frac{3iabe^{ic}d^2x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iabe^{-ic}d^2x^2\Gamma\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}}-\frac{3abd\cos\left(c+dx^3\right)}{2x}-\frac{ab\sin\left(c+dx^3\right)}{2x^4}-\frac{3b^2e^{2ic}d^2x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{4\ 2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{3b^2e^{-2ic}d^2x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{4\ 2^{2/3}\left(idx^3\right)^{2/3}}-\frac{3b^2d\sin\left(2c+2dx^3\right)}{4x}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}$$

input
```
Int[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

output
```
-1/8*(2*a^2 + b^2)/x^4 - (3*a*b*d*Cos[c + d*x^3])/(2*x) + (b^2*Cos[2*c + 2
*d*x^3])/(8*x^4) - (((3*I)/4)*a*b*d^2*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/
((-I)*d*x^3)^(2/3) + (((3*I)/4)*a*b*d^2*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*
(I*d*x^3)^(2/3)) - (3*b^2*d^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(4
*2^(2/3)*((-I)*d*x^3)^(2/3)) - (3*b^2*d^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(4*
2^(2/3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (a*b*Sin[c + d*x^3])/(2*x^4) - (3*b
^2*d*Sin[2*c + 2*d*x^3])/(4*x)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a+b\sin\left(d\,x^3+c\right)\right)^2}{x^5}dx$$

input
```
int((a+b*sin(d*x^3+c))^2/x^5,x)
```

output
```
int((a+b*sin(d*x^3+c))^2/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx =$$
$$-\frac{12\,abdx^3\cos\left(dx^3+c\right)-2\,b^2\cos\left(dx^3+c\right)^2+3\left(-i\,b^2dx^4\cos\left(2\,c\right)-b^2dx^4\sin\left(2\,c\right)\right)\left(2i\,d\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},2i\,dx^3\right)}{}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="fricas")
```

output
```
-1/8*(12*a*b*d*x^3*cos(d*x^3 + c) - 2*b^2*cos(d*x^3 + c)^2 + 3*(-I*b^2*d*x
^4*cos(2*c) - b^2*d*x^4*sin(2*c))*(2*I*d)^(1/3)*gamma(2/3, 2*I*d*x^3) - 6*
(a*b*d*x^4*cos(c) - I*a*b*d*x^4*sin(c))*(I*d)^(1/3)*gamma(2/3, I*d*x^3) -
6*(a*b*d*x^4*cos(c) + I*a*b*d*x^4*sin(c))*(-I*d)^(1/3)*gamma(2/3, -I*d*x^3
) + 3*(I*b^2*d*x^4*cos(2*c) - b^2*d*x^4*sin(2*c))*(-2*I*d)^(1/3)*gamma(2/3
, -2*I*d*x^3) + 2*a^2 + 2*b^2 + 4*(3*b^2*d*x^3*cos(d*x^3 + c) + a*b)*sin(d
*x^3 + c))/x^4
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx=\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2/x**5,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.68

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx$$

$$=\frac{\left(dx^3\right)^{\frac{1}{3}} \left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3}, i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3}, -i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3}, i\,dx^3\right)+\left(-i\sqrt{3}\right.\right.\right.}{6\,x}$$

$$-\frac{\left(2\cdot 2^{\frac{1}{3}}\left(dx^3\right)^{\frac{1}{3}} \left(\left(\left(-i\sqrt{3}+1\right)\Gamma\left(-\frac{4}{3}, 2i\,dx^3\right)+\left(i\sqrt{3}+1\right)\Gamma\left(-\frac{4}{3}, -2i\,dx^3\right)\right)\cos\left(2\,c\right)-\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3},\right.\right.\right.\right.}{24\,x^4}$$

$$-\frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="maxima")
```

output
```
1/6*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(-4/3, I*d*x^3) + (sqrt(3) - I)*gam
ma(-4/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(-4/3, I*d*x^3) + (-I*s
qrt(3) - 1)*gamma(-4/3, -I*d*x^3))*sin(c))*a*b*d/x - 1/24*(2*2^(1/3)*(d*x^
3)^(1/3)*(((-I*sqrt(3) + 1)*gamma(-4/3, 2*I*d*x^3) + (I*sqrt(3) + 1)*gamma
(-4/3, -2*I*d*x^3))*cos(2*c) - ((sqrt(3) + I)*gamma(-4/3, 2*I*d*x^3) + (sq
rt(3) - I)*gamma(-4/3, -2*I*d*x^3))*sin(2*c))*d*x^3 + 3)*b^2/x^4 - 1/4*a^2
/x^4
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx=\int \frac{(b\sin(dx^3+c)+a)^2}{x^5}\,dx$$

input

```
integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="giac")
```

output

```
integrate((b*sin(d*x^3 + c) + a)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx=\int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^5}\,dx$$

input

```
int((a + b*sin(c + d*x^3))^2/x^5,x)
```

output

```
int((a + b*sin(c + d*x^3))^2/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx=\frac{4\left(\int \frac{\sin(d\,x^3+c)^2}{x^5}dx\right)b^2x^4+8\left(\int \frac{\sin(d\,x^3+c)}{x^5}dx\right)ab\,x^4-a^2}{4x^4}$$

input

```
int((a+b*sin(d*x^3+c))^2/x^5,x)
```

output

```
(4*int(sin(c + d*x**3)**2/x**5,x)*b**2*x**4 + 8*int(sin(c + d*x**3)/x**5,x
)*a*b*x**4 - a**2)/(4*x**4)
```

3.77 $\int x^3(a+b\sin(c+dx^3))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 237

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}$$

output

```
1/8*(2*a^2+b^2)*x^4-2/3*a*b*x*cos(d*x^3+c)/d-1/9*a*b*exp(I*c)*x*GAMMA(1/3,
-I*d*x^3)/d/(-I*d*x^3)^(1/3)-1/9*a*b*x*GAMMA(1/3,I*d*x^3)/d/exp(I*c)/(I*d*
x^3)^(1/3)+1/144*I*b^2*exp(2*I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/d/(-I*d*
x^3)^(1/3)-1/144*I*b^2*x*GAMMA(1/3,2*I*d*x^3)*2^(2/3)/d/exp(2*I*c)/(I*d*x^
3)^(1/3)-1/12*b^2*x*sin(2*d*x^3+2*c)/d
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.43

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$=\frac{dx^7\left(36a^2dx^3\sqrt[3]{d^2x^6}+18b^2dx^3\sqrt[3]{d^2x^6}-96ab\sqrt[3]{d^2x^6}\cos\left(c+dx^3\right)+i2^{2/3}b^2\sqrt[3]{idx^3}\cos(2c)\Gamma\left(\frac{1}{3},-2idx^3\right)-\right.}{}$$

input

```
Integrate[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
(d*x^7*(36*a^2*d*x^3*(d^2*x^6)^(1/3) + 18*b^2*d*x^3*(d^2*x^6)^(1/3) - 96*a
*b*(d^2*x^6)^(1/3)*Cos[c + d*x^3] + I*2^(2/3)*b^2*(I*d*x^3)^(1/3)*Cos[2*c]
*Gamma[1/3, (-2*I)*d*x^3] - I*2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Cos[2*c]*Gamm
a[1/3, (2*I)*d*x^3] - 16*a*b*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c
] - I*Sin[c]) - 16*a*b*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*
Sin[c]) - 2^(2/3)*b^2*(I*d*x^3)^(1/3)*Gamma[1/3, (-2*I)*d*x^3]*Sin[2*c] -
2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] - 12*b^2*(
d^2*x^6)^(1/3)*Sin[2*(c + d*x^3)]))/(144*(d^2*x^6)^(4/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(a^2x^3+2abx^3\sin\left(c+dx^3\right)-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+\frac{b^2x^3}{2}\right)dx$$

$$\Big\downarrow \text{6}$$

$$\int \left(x^3\left(a^2+\frac{b^2}{2}\right)+2abx^3\sin\left(c+dx^3\right)-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}x^4\left(2a^2+b^2\right)-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}+\frac{ib^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}$$

input
```
Int[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

output
```
((2*a^2 + b^2)*x^4)/8 - (2*a*b*x*Cos[c + d*x^3])/(3*d) - (a*b*E^(I*c)*x*Ga
mma[1/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(1/3)) - (a*b*x*Gamma[1/3, I*d*x^3
])/(9*d*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/72)*b^2*E^((2*I)*c)*x*Gamma[1/3, (-
2*I)*d*x^3])/(2^(1/3)*d*((-I)*d*x^3)^(1/3)) - ((I/72)*b^2*x*Gamma[1/3, (2*
I)*d*x^3])/(2^(1/3)*d*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (b^2*x*Sin[2*c + 2*d*
x^3])/(12*d)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int x^{3} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{2}d x$$

input
```
int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77

$$\int x^3(a+b\sin(c+dx^3))^2\,dx = \frac{18\,(2\,a^2+b^2)d^2x^4 - 24\,b^2dx\cos\left(dx^3+c\right)\sin\left(dx^3+c\right) - 96\,abdx\cos\left(dx^3+c\right) - (b^2\cos\left(2\,c\right) - i\,b^2\sin\left(2\right.}{}$$

input
```
integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/144*(18*(2*a^2 + b^2)*d^2*x^4 - 24*b^2*d*x*cos(d*x^3 + c)*sin(d*x^3 + c)
 - 96*a*b*d*x*cos(d*x^3 + c) - (b^2*cos(2*c) - I*b^2*sin(2*c))*(2*I*d)^(2/
3)*gamma(1/3, 2*I*d*x^3) - 16*(-I*a*b*cos(c) - a*b*sin(c))*(I*d)^(2/3)*gam
ma(1/3, I*d*x^3) - 16*(I*a*b*cos(c) - a*b*sin(c))*(-I*d)^(2/3)*gamma(1/3,
-I*d*x^3) - (b^2*cos(2*c) + I*b^2*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2*I
*d*x^3))/d^2
```

Sympy [F]

$$\int x^{3} \left(a + b \sin{\left(c + d x^{3} \right)}\right)^{2}\, dx = \int x^{3} \left(a + b \sin{\left(c + d x^{3} \right)}\right)^{2}\, dx$$

input
```
integrate(x**3*(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(x**3*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01

$$\int x^3 (a + b\sin(c + dx^3))^2 \, dx = \frac{1}{4} a^2 x^4$$

$$- \frac{2^{\frac{2}{3}} ((((i\sqrt{3} + 1)\Gamma(\frac{1}{3}, 2i\,dx^3) + (-i\sqrt{3} + 1)\Gamma(\frac{1}{3}, -2i\,dx^3)) \cos(2c) + ((\sqrt{3} - i)\Gamma(\frac{1}{3}, 2i\,dx^3) + (\sqrt{3}}{288\,(dx^3)^{\frac{1}{3}}\,d}$$

$$- \frac{(12\,(dx^3)^{\frac{1}{3}}\,x\cos(dx^3 + c) + (((\sqrt{3} - i)\Gamma(\frac{1}{3}, i\,dx^3) + (\sqrt{3} + i)\Gamma(\frac{1}{3}, -i\,dx^3)) \cos(c) + ((-i\sqrt{3} - 1)}{18\,(dx^3)^{\frac{1}{3}}\,d}$$

input
```
integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 - 1/288*2^(2/3)*((((I*sqrt(3) + 1)*gamma(1/3, 2*I*d*x^3) + (-I
*sqrt(3) + 1)*gamma(1/3, -2*I*d*x^3))*cos(2*c) + ((sqrt(3) - I)*gamma(1/3,
 2*I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -2*I*d*x^3))*sin(2*c))*x - 6*2^(1/3
)*(3*d*x^4 - 2*x*sin(2*d*x^3 + 2*c))*(d*x^3)^(1/3))*b^2/((d*x^3)^(1/3)*d)
- 1/18*(12*(d*x^3)^(1/3)*x*cos(d*x^3 + c) + (((sqrt(3) - I)*gamma(1/3, I*d
*x^3) + (sqrt(3) + I)*gamma(1/3, -I*d*x^3))*cos(c) + ((-I*sqrt(3) - 1)*gam
ma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*d*x^3))*sin(c))*x)*a*b/((
d*x^3)^(1/3)*d)
```

Giac [F]

$$\int x^3 (a + b\sin(c + dx^3))^2 \, dx = \int (b\sin(dx^3 + c) + a)^2 x^3 \, dx$$

input
```
integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + b\sin (c + dx^3))^2 \, dx = \int x^3 \, (a + b \, \sin (d \, x^3 + c))^2 \, dx$$

input
```
int(x^3*(a + b*sin(c + d*x^3))^2,x)
```

output
```
int(x^3*(a + b*sin(c + d*x^3))^2, x)
```

Reduce [F]

$$\int x^3 (a + b\sin (c + dx^3))^2 \, dx = \left(\int \sin (d \, x^3 + c)^2 \, x^3 dx\right) b^2 + 2\left(\int \sin (d \, x^3 + c) \, x^3 dx\right) ab + \frac{a^2 x^4}{4}$$

input
```
int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

output
```
(4*int(sin(c + d*x**3)**2*x**3,x)*b**2 + 8*int(sin(c + d*x**3)*x**3,x)*a*b
 + a**2*x**4)/4
```

3.78 $\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx = \frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}$$

output

```
1/2*(2*a^2+b^2)*x+1/3*I*a*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3
)-1/3*I*a*b*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)+1/24*b^2*exp(2*I
*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*x^3)^(1/3)+1/24*b^2*x*GAMMA(1/3,
2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\frac{1}{24}x\Bigg(&12\left(2a^2+b^2\right)\\&-8iab\cos(c)\left(-\frac{\Gamma\left(\frac{1}{3},-idx^3\right)}{\sqrt[3]{-idx^3}}+\frac{\Gamma\left(\frac{1}{3},idx^3\right)}{\sqrt[3]{idx^3}}\right)\\&+8ab\left(-\frac{\Gamma\left(\frac{1}{3},-idx^3\right)}{\sqrt[3]{-idx^3}}-\frac{\Gamma\left(\frac{1}{3},idx^3\right)}{\sqrt[3]{idx^3}}\right)\sin(c)\\&+\frac{2^{2/3}b^2\Gamma\left(\frac{1}{3},2idx^3\right)\left(\cos(2c)-i\sin(2c)\right)}{\sqrt[3]{idx^3}}\\&+\frac{2^{2/3}b^2\Gamma\left(\frac{1}{3},-2idx^3\right)\left(\cos(2c)+i\sin(2c)\right)}{\sqrt[3]{-idx^3}}\Bigg)\end{aligned}$$

input

```
Integrate[(a + b*Sin[c + d*x^3])^2,x]
```

output

```
(x*(12*(2*a^2 + b^2) - (8*I)*a*b*Cos[c]*(-(Gamma[1/3, (-I)*d*x^3]/((-I)*d*
x^3)^(1/3)) + Gamma[1/3, I*d*x^3]/(I*d*x^3)^(1/3)) + 8*a*b*(-(Gamma[1/3, (
-I)*d*x^3]/((-I)*d*x^3)^(1/3)) - Gamma[1/3, I*d*x^3]/(I*d*x^3)^(1/3))*Sin[
c] + (2^(2/3)*b^2*Gamma[1/3, (2*I)*d*x^3]*(Cos[2*c] - I*Sin[2*c]))/(I*d*x^
3)^(1/3) + (2^(2/3)*b^2*Gamma[1/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]))/
((-I)*d*x^3)^(1/3)))/24
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$\Big\downarrow 3838$$

$$\int \left(a^2+2ab\sin\left(c+dx^3\right)-\frac{1}{2}b^2\cos\left(2c+2dx^3\right)+\frac{b^2}{2}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}$$

input
```
Int[(a + b*Sin[c + d*x^3])^2,x]
```

output
```
((2*a^2 + b^2)*x)/2 + ((I/3)*a*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d
*x^3)^(1/3) - ((I/3)*a*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3))
+ (b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(12*2^(1/3)*((-I)*d*x^3)^(1
/3)) + (b^2*x*Gamma[1/3, (2*I)*d*x^3])/(12*2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(
1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Maple [F]

$$\int \left(a+b\sin\left(d\,x^3+c\right)\right)^2dx$$

input
```
int((a+b*sin(d*x^3+c))^2,x)
```

output
```
int((a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$
$$=\underline{12\,(2\,a^2+b^2)dx+(-i\,b^2\cos\left(2\,c\right)-b^2\sin\left(2\,c\right))(2i\,d)^{\frac{2}{3}}\,\Gamma\left(\tfrac{1}{3},2i\,dx^3\right)-8\,(ab\cos\left(c\right)-i\,ab\sin\left(c\right))(i\,d)^{\frac{2}{3}}\,\Gamma\big(}$$

input
```
integrate((a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/24*(12*(2*a^2 + b^2)*d*x + (-I*b^2*cos(2*c) - b^2*sin(2*c))*(2*I*d)^(2/3
)*gamma(1/3, 2*I*d*x^3) - 8*(a*b*cos(c) - I*a*b*sin(c))*(I*d)^(2/3)*gamma(
1/3, I*d*x^3) - 8*(a*b*cos(c) + I*a*b*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d
*x^3) + (I*b^2*cos(2*c) - b^2*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2*I*d*x
^3))/d
```

Sympy [F]

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$= \frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos(c)-\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\right.\right.\right.}{6\,(dx^3)^{\frac{1}{3}}}$$

$$+\frac{2^{\frac{2}{3}}\left(\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\cos(2\,c)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(i\sqrt{3}\right.\right.\right.\right.}{48\,(dx^3)^{\frac{1}{3}}}$$

$$+a^2x$$

input
```
integrate((a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
1/6*(((-I*sqrt(3) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I
*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(1/3, I*d*x^3) + (sqrt(3) + I)*gamma
(1/3, -I*d*x^3))*sin(c))*a*b*x/(d*x^3)^(1/3) + 1/48*2^(2/3)*((((sqrt(3) -
I)*gamma(1/3, 2*I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -2*I*d*x^3))*cos(2*c)
+ ((-I*sqrt(3) - 1)*gamma(1/3, 2*I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -2*
I*d*x^3))*sin(2*c))*x + 12*2^(1/3)*(d*x^3)^(1/3)*x)*b^2/(d*x^3)^(1/3) + a^
2*x
```

Giac [F]

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin \left(c+dx^3\right)\right)^2\,dx=\int \left(a+b\,\sin \left(d\,x^3+c\right)\right)^2\,dx$$

input
```
int((a + b*sin(c + d*x^3))^2,x)
```

output
```
int((a + b*sin(c + d*x^3))^2, x)
```

Reduce [F]

$$\int \left(a+b\sin \left(c+dx^3\right)\right)^2\,dx=\left(\int \sin \left(d\,x^3+c\right)^2dx\right)b^2+2\left(\int \sin \left(d\,x^3+c\right)dx\right)ab+a^2x$$

input
```
int((a+b*sin(d*x^3+c))^2,x)
```

output
```
int(sin(c + d*x**3)**2,x)*b**2 + 2*int(sin(c + d*x**3),x)*a*b + a**2*x
```

3.79 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 227

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx = \frac{-2a^2-b^2}{4x^2}+\frac{b^2\cos(2c+2dx^3)}{4x^2}-\frac{abde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}} - \frac{abde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}} + \frac{ib^2de^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}} - \frac{ib^2de^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{idx^3}} - \frac{ab\sin(c+dx^3)}{x^2}$$

output
```
1/4*(-2*a^2-b^2)/x^2+1/4*b^2*cos(2*d*x^3+2*c)/x^2-1/2*a*b*d*exp(I*c)*x*GAM
MA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/2*a*b*d*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/
(I*d*x^3)^(1/3)+1/8*I*b^2*d*exp(2*I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I
*d*x^3)^(1/3)-1/8*I*b^2*d*x*GAMMA(1/3,2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x
^3)^(1/3)-a*b*sin(d*x^3+c)/x^2
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.47

$$
\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx &= \frac{-2a^2-b^2}{4x^2}+\frac{b^2\cos(2c)\cos\left(2dx^3\right)}{4x^2} \\
&\quad +\frac{3}{2}abd\cos(c)\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right) \\
&\quad -\frac{ab\cos\left(dx^3\right)\sin(c)}{x^2} \\
&\quad +\frac{3}{2}iabd\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}+\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\sin(c) \\
&\quad -\frac{b^2\left(idx^3\right)^{2/3}\Gamma\left(\frac{1}{3},2idx^3\right)\left(\cos(2c)-i\sin(2c)\right)}{4\sqrt[3]{2}x^2} \\
&\quad +\frac{ib^2dx\Gamma\left(\frac{1}{3},-2idx^3\right)\left(\cos(2c)+i\sin(2c)\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}} \\
&\quad -\frac{ab\cos(c)\sin\left(dx^3\right)}{x^2}-\frac{b^2\sin(2c)\sin\left(2dx^3\right)}{4x^2}
\end{aligned}
$$

input

```
Integrate[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

output

```
(-2*a^2 - b^2)/(4*x^2) + (b^2*Cos[2*c]*Cos[2*d*x^3])/(4*x^2) + (3*a*b*d*Co
s[c]*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - (x*Gamma[1/3, I
*d*x^3])/(3*(I*d*x^3)^(1/3))))/2 - (a*b*Cos[d*x^3]*Sin[c])/x^2 + ((3*I)/2)
*a*b*d*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (x*Gamma[1/3,
 I*d*x^3])/(3*(I*d*x^3)^(1/3)))*Sin[c] - (b^2*(I*d*x^3)^(2/3)*Gamma[1/3, (
2*I)*d*x^3]*(Cos[2*c] - I*Sin[2*c]))/(4*2^(1/3)*x^2) + ((I/4)*b^2*d*x*Gamm
a[1/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]))/(2^(1/3)*((-I)*d*x^3)^(1/3))
 - (a*b*Cos[c]*Sin[d*x^3])/x^2 - (b^2*Sin[2*c]*Sin[2*d*x^3])/(4*x^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{3884}$$

$$\int \left(\frac{a^2}{x^3}+\frac{2ab\sin\left(c+dx^3\right)}{x^3}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^3}+\frac{b^2}{2x^3}\right)dx$$

$$\Big\downarrow \text{6}$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^3}+\frac{2ab\sin\left(c+dx^3\right)}{x^3}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2+b^2}{4x^2}-\frac{abe^{ic}dx\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abe^{-ic}dx\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}-\frac{ab\sin\left(c+dx^3\right)}{x^2}+\frac{ib^2e^{2ic}dx\Gamma\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}dx\Gamma\left(\frac{1}{3},2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{idx^3}}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}$$

input

```
Int[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

output

```
-1/4*(2*a^2 + b^2)/x^2 + (b^2*Cos[2*c + 2*d*x^3])/(4*x^2) - (a*b*d*E^(I*c)
*x*Gamma[1/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(1/3)) - (a*b*d*x*Gamma[1/3, I*
d*x^3])/(2*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/4)*b^2*d*E^((2*I)*c)*x*Gamma[1/3
, (-2*I)*d*x^3])/(2^(1/3)*((-I)*d*x^3)^(1/3)) - ((I/4)*b^2*d*x*Gamma[1/3,
(2*I)*d*x^3])/(2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (a*b*Sin[c + d*x^3])
/x^2
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*sin(d*x^3+c))^2/x^3,x)
```

output
```
int((a+b*sin(d*x^3+c))^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}{x^{3}}\,dx$$
$$=\frac{4\,b^{2}\cos\left(dx^{3}+c\right)^{2}-8\,ab\sin\left(dx^{3}+c\right)-\left(b^{2}x^{2}\cos\left(2\,c\right)-i\,b^{2}x^{2}\sin\left(2\,c\right)\right)\left(2i\,d\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},2i\,dx^{3}\right)-4\left(-i\,abx^{2}\right.}{}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(4*b^2*cos(d*x^3 + c)^2 - 8*a*b*sin(d*x^3 + c) - (b^2*x^2*cos(2*c) - I
*b^2*x^2*sin(2*c))*(2*I*d)^(2/3)*gamma(1/3, 2*I*d*x^3) - 4*(-I*a*b*x^2*cos
(c) - a*b*x^2*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) - 4*(I*a*b*x^2*cos(c
) - a*b*x^2*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^3) - (b^2*x^2*cos(2*c)
+ I*b^2*x^2*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/3, -2*I*d*x^3) - 4*a^2 - 4*b^
2)/x^2
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2/x**3,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx &= \frac{(dx^3)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3}, i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3}, -i\,dx^3\right)\right)\cos(c)-\left(\left(i\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3}, i\,dx^3\right)+\left(-i\sqrt{3}\right.\right.\right.}{6\,x^2} \\ &\quad - \frac{\left(2^{\frac{2}{3}}(dx^3)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3}, 2i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3}, -2i\,dx^3\right)\right)\cos(2\,c)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3}, 2i\right.\right.\right.\right.}{24\,x^2} \\ &\quad - \frac{a^2}{2\,x^2}\end{aligned}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="maxima")
```

output
```
1/6*(d*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-2/3, I*d*x^3) + (sqrt(3) + I)*gam
ma(-2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) + 1)*gamma(-2/3, I*d*x^3) + (-I*s
qrt(3) + 1)*gamma(-2/3, -I*d*x^3))*sin(c))*a*b/x^2 - 1/24*(2^(2/3)*(d*x^3)
^(2/3)*(((-I*sqrt(3) - 1)*gamma(-2/3, 2*I*d*x^3) + (I*sqrt(3) - 1)*gamma(-
2/3, -2*I*d*x^3))*cos(2*c) - ((sqrt(3) - I)*gamma(-2/3, 2*I*d*x^3) + (sqrt
(3) + I)*gamma(-2/3, -2*I*d*x^3))*sin(2*c)) + 6)*b^2/x^2 - 1/2*a^2/x^2
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx=\int \frac{(b\sin(dx^3+c)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx=\int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^3}\,dx$$

input
```
int((a + b*sin(c + d*x^3))^2/x^3,x)
```

output
```
int((a + b*sin(c + d*x^3))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx = \frac{2\left(\int \frac{\sin\left(d\,x^3+c\right)^2}{x^3}dx\right) b^2x^2 + 4\left(\int \frac{\sin\left(d\,x^3+c\right)}{x^3}dx\right) ab\,x^2 - a^2}{2x^2}$$

input

```
int((a+b*sin(d*x^3+c))^2/x^3,x)
```

output

```
(2*int(sin(c + d*x**3)**2/x**3,x)*b**2*x**2 + 4*int(sin(c + d*x**3)/x**3,x
)*a*b*x**2 - a**2)/(2*x**2)
```

3.80 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 277

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx &= \frac{-2a^2-b^2}{10x^5}-\frac{3abd\cos(c+dx^3)}{5x^2}+\frac{b^2\cos(2c+2dx^3)}{10x^5}\\ &\quad-\frac{3iabd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}}\\ &\quad-\frac{3b^2d^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{3b^2d^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{idx^3}}\\ &\quad-\frac{2ab\sin(c+dx^3)}{5x^5}-\frac{3b^2d\sin(2c+2dx^3)}{10x^2}\end{aligned}$$

output

```
1/10*(-2*a^2-b^2)/x^5-3/5*a*b*d*cos(d*x^3+c)/x^2+1/10*b^2*cos(2*d*x^3+2*c)
/x^5-3/10*I*a*b*d^2*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)+3/10*I
*a*b*d^2*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)-3/20*b^2*d^2*exp(2*
I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*x^3)^(1/3)-3/20*b^2*d^2*x*GAMMA
(1/3,2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^(1/3)-2/5*a*b*sin(d*x^3+c)/x^
5-3/10*b^2*d*sin(2*d*x^3+2*c)/x^2
```

Mathematica [A] (verified)

Time = 3.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx =$$

$$-\frac{4a^2+2b^2+12abdx^3\cos(c+dx^3)-2b^2\cos(2(c+dx^3))-3\,2^{2/3}b^2(idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{1}{3},2idx^3\right)+6ia}{}$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

output
```
-1/20*(4*a^2 + 2*b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - 2*b^2*Cos[2*(c + d*x^
3)] - 3*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[1/3, (2*I)*d*x^3] + (6*
I)*a*b*(I*d*x^3)^(5/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b
*(I*d*x^3)^(1/3)*(d^2*x^6)^(2/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]
) - 3*2^(2/3)*b^2*((-I)*d*x^3)^(5/3)*Gamma[1/3, (-2*I)*d*x^3]*(Cos[2*c] +
I*Sin[2*c]) + (3*I)*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Gamma[1/3, (2*I)*d*x^3]*Si
n[2*c] + 8*a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/x^5
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx$$

$$\Big\downarrow \; 3884$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ab\sin(c+dx^3)}{x^6}-\frac{b^2\cos(2c+2dx^3)}{2x^6}+\frac{b^2}{2x^6}\right)dx$$

$$\Big\downarrow \; 6$$

$$\int \left(\frac{a^2+\frac{b^2}{2}}{x^6}+\frac{2ab\sin\left(c+dx^3\right)}{x^6}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2+b^2}{10x^5}-\frac{3iabe^{ic}d^2x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabe^{-ic}d^2x\Gamma\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}-\frac{3b^2e^{2ic}d^2x\Gamma\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{3b^2e^{-2ic}d^2x\Gamma\left(\frac{1}{3},2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{idx^3}}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{3b^2d\sin\left(2c+2dx^3\right)}{10x^2}$$

input

```
Int[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

output

```
-1/10*(2*a^2 + b^2)/x^5 - (3*a*b*d*Cos[c + d*x^3])/(5*x^2) + (b^2*Cos[2*c
+ 2*d*x^3])/(10*x^5) - (((3*I)/10)*a*b*d^2*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3
])/((-I)*d*x^3)^(1/3) + (((3*I)/10)*a*b*d^2*x*Gamma[1/3, I*d*x^3])/(E^(I*c
)*(I*d*x^3)^(1/3)) - (3*b^2*d^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(1
0*2^(1/3)*((-I)*d*x^3)^(1/3)) - (3*b^2*d^2*x*Gamma[1/3, (2*I)*d*x^3])/(10*
2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (2*a*b*Sin[c + d*x^3])/(5*x^5) - (3
*b^2*d*Sin[2*c + 2*d*x^3])/(10*x^2)
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a + b\sin\left(d\,x^3 + c\right)\right)^2}{x^6}dx$$

input
```
int((a+b*sin(d*x^3+c))^2/x^6,x)
```

output
```
int((a+b*sin(d*x^3+c))^2/x^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int \frac{\left(a + b\sin\left(c + dx^3\right)\right)^2}{x^6}\,dx =$$

$$-\frac{12\,abdx^3\cos\left(dx^3+c\right) - 4\,b^2\cos\left(dx^3+c\right)^2 + 3\left(-i\,b^2dx^5\cos\left(2\,c\right) - b^2dx^5\sin\left(2\,c\right)\right)\left(2i\,d\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, 2i\,dx^3\right)}{}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="fricas")
```

output
```
-1/20*(12*a*b*d*x^3*cos(d*x^3 + c) - 4*b^2*cos(d*x^3 + c)^2 + 3*(-I*b^2*d*
x^5*cos(2*c) - b^2*d*x^5*sin(2*c))*(2*I*d)^(2/3)*gamma(1/3, 2*I*d*x^3) - 6
*(a*b*d*x^5*cos(c) - I*a*b*d*x^5*sin(c))*(I*d)^(2/3)*gamma(1/3, I*d*x^3) -
 6*(a*b*d*x^5*cos(c) + I*a*b*d*x^5*sin(c))*(-I*d)^(2/3)*gamma(1/3, -I*d*x^
3) + 3*(I*b^2*d*x^5*cos(2*c) - b^2*d*x^5*sin(2*c))*(-2*I*d)^(2/3)*gamma(1/
3, -2*I*d*x^3) + 4*a^2 + 4*b^2 + 4*(3*b^2*d*x^3*cos(d*x^3 + c) + 2*a*b)*si
n(d*x^3 + c))/x^5
```

Sympy [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = \int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx$$

input
```
integrate((a+b*sin(d*x**3+c))**2/x**6,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**2/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx = -\frac{(dx^3)^{\frac{2}{3}}\,(((-i\sqrt{3}-1)\Gamma(-\frac{5}{3}, i\,dx^3) + (i\sqrt{3}-1)\Gamma(-\frac{5}{3}, -i\,dx^3))\cos(c) - ((\sqrt{3}-i)\Gamma(-\frac{5}{3}, i\,dx^3) + (}{6\,x^2} - \frac{\Big(5\cdot 2^{\frac{2}{3}}(dx^3)^{\frac{2}{3}}\,(((\sqrt{3}-i)\Gamma(-\frac{5}{3}, 2i\,dx^3) + (\sqrt{3}+i)\Gamma(-\frac{5}{3}, -2i\,dx^3))\cos(2\,c) + ((-i\sqrt{3}-1)\Gamma(-\frac{5}{3}, 2}{60\,x^5} - \frac{a^2}{5\,x^5}$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="maxima")
```

output
```
-1/6*(d*x^3)^(2/3)*(((-I*sqrt(3) - 1)*gamma(-5/3, I*d*x^3) + (I*sqrt(3) -
1)*gamma(-5/3, -I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(-5/3, I*d*x^3) + (
sqrt(3) + I)*gamma(-5/3, -I*d*x^3))*sin(c))*a*b*d/x^2 - 1/60*(5*2^(2/3)*(d
*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-5/3, 2*I*d*x^3) + (sqrt(3) + I)*gamma(-
5/3, -2*I*d*x^3))*cos(2*c) + ((-I*sqrt(3) - 1)*gamma(-5/3, 2*I*d*x^3) + (I
*sqrt(3) - 1)*gamma(-5/3, -2*I*d*x^3))*sin(2*c))*d*x^3 + 6)*b^2/x^5 - 1/5*
a^2/x^5
```

Giac [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx=\int \frac{(b\sin(dx^3+c)+a)^2}{x^6}\,dx$$

input
```
integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx=\int \frac{(a+b\,\sin(d\,x^3+c))^2}{x^6}\,dx$$

input
```
int((a + b*sin(c + d*x^3))^2/x^6,x)
```

output
```
int((a + b*sin(c + d*x^3))^2/x^6, x)
```

Reduce [F]

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx=\frac{5\left(\int \frac{\sin(d\,x^3+c)^2}{x^6}dx\right)b^2x^5+10\left(\int \frac{\sin(d\,x^3+c)}{x^6}dx\right)ab\,x^5-a^2}{5x^5}$$

input
```
int((a+b*sin(d*x^3+c))^2/x^6,x)
```

output
```
(5*int(sin(c + d*x**3)**2/x**6,x)*b**2*x**5 + 10*int(sin(c + d*x**3)/x**6,
x)*a*b*x**5 - a**2)/(5*x**5)
```

3.81 $\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 245

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = -\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}+\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} - \frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}+\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}$$

output

```
-1/3*I*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/
d+1/3*I*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/d-1/3*polylog(2,I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
/d^2+1/3*polylog(2,I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/
2)/d^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx$$
$$=\frac{-idx^3\left(\log\left(1+\frac{ibe^{i\left(c+dx^3\right)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)\right)-\text{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^3\right)}}{-a+\sqrt{a^2-b^2}}\right)+\text{PolyLog}\left(2,\frac{ibe^{i(}}{a+\sqrt{}}\right.}{3\sqrt{a^2-b^2}d^2}$$

input
```
Integrate[x^5/(a + b*Sin[c + d*x^3]),x]
```

output
```
((-I)*d*x^3*(Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] - Log
[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*
E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x
^3)))/(a + Sqrt[a^2 - b^2])])/(3*Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3860, 3042, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx$$
$$\Big\downarrow \text{3860}$$
$$\frac{1}{3}\int \frac{x^3}{a+b\sin\left(dx^3+c\right)}dx^3$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{3}\int \frac{x^3}{a+b\sin\left(dx^3+c\right)}dx^3$$
$$\Big\downarrow \text{3804}$$

$$\frac{2}{3}\int \frac{e^{i\left(dx^3+c\right)}x^3}{2e^{i\left(dx^3+c\right)}a-ibe^{2i\left(dx^3+c\right)}+ib}dx^3$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2}{3}\left(\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{2\left(a-ibe^{i\left(dx^3+c\right)}+\sqrt{a^2-b^2}\right)}dx^3}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{2\left(a-ibe^{i\left(dx^3+c\right)}-\sqrt{a^2-b^2}\right)}dx^3}{\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\left(\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{a-ibe^{i\left(dx^3+c\right)}+\sqrt{a^2-b^2}}dx^3}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{a-ibe^{i\left(dx^3+c\right)}-\sqrt{a^2-b^2}}dx^3}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2}{3}\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{\int \log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^3}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int \log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a-\sqrt{a^2-b^2}}\right)dx^3}{bd}\right)}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{2}{3}\left(\frac{ib\left(\frac{i\int \frac{\log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^3}de^{i\left(dx^3+c\right)}}{bd^2}+\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{i\int \frac{\log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{x^3}de^{i\left(dx^3+c\right)}}{bd^2}+\frac{x^3\log\left(1-\frac{ibe^{i}}{a-}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{2}{3}\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^3+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{i\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(dx^3+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)$$

input
```
Int[x^5/(a + b*Sin[c + d*x^3]),x]
```

output
```
(2*(((-1/2*I)*b*((x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2]
)])/(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2]])/
(b*d^2)))/Sqrt[a^2 - b^2] + ((I/2)*b*((x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))
/(a + Sqrt[a^2 - b^2])])/(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a
+ Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b^2]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [F]

$$\int \frac{x^5}{a + b\sin\left(d\,x^3 + c\right)}dx$$

input
```
int(x^5/(a+b*sin(d*x^3+c)),x)
```

output
```
int(x^5/(a+b*sin(d*x^3+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(199) = 398$.

Time = 0.20 (sec) , antiderivative size = 1041, normalized size of antiderivative = 4.25

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input

```
integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output

```
-1/6*(b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + b*c*sqrt(-(a^2 - b^2)/b^2)*lo
g(2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^3 + c) + 2*I*b*sin(d
*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^
2)*log(-2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) - 2*I*a) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*s
in(d*x^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) -
a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^
2)/b^2) - b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c
) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3
 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(
a^2 - b^2)/b^2) - b)/b + 1) + (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-
(I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3
 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/
b^2)*log(-(I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b
*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*x^3 + b*c)*sqrt(-(a
^2 - b^2)/b^2)*log(-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*...
```

Sympy [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx$$

input
```
integrate(x**5/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(x**5/(a + b*sin(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x^5}{b\sin\left(dx^3+c\right)+a}\,dx$$

input
```
integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

Giac [F]

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x^5}{b\sin\left(dx^3+c\right)+a}\,dx$$

input
```
integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx=\int \frac{x^5}{a+b\,\sin\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^5/(a + b*sin(c + d*x^3)),x)
```

output
```
int(x^5/(a + b*sin(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx=\int \frac{x^5}{\sin\left(d\,x^3+c\right)b+a}dx$$

input
```
int(x^5/(a+b*sin(d*x^3+c)),x)
```

output
```
int(x**5/(sin(c + d*x**3)*b + a),x)
```

3.82 $\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx = \frac{2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}$$

output
```
2/3*arctan((b+a*tan(1/2*d*x^3+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx = \frac{2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}$$

input
```
Integrate[x^2/(a + b*Sin[c + d*x^3]),x]
```

output
```
(2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*
d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3860, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sin\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{3}\int \frac{1}{a+b\sin\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{1}{a+b\sin\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{2\int \frac{1}{ax^6+a+2b\tan\left(\frac{1}{2}\left(dx^3+c\right)\right)}d\tan\left(\frac{1}{2}\left(dx^3+c\right)\right)}{3d}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{4\int \frac{1}{-x^6-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}\left(dx^3+c\right)\right)\right)}{3d}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$$

input

```
Int[x^2/(a + b*Sin[c + d*x^3]),x]
```

output

```
(2*ArcTan[(2*b + 2*a*Tan[(c + d*x^3)/2])/(2*Sqrt[a^2 - b^2])])/(3*Sqrt[a^2
 - b^2]*d)
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$	49
default	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$	49
risch	$-\frac{\ln\left(\mathrm{e}^{i\left(d\,x^3+c\right)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{3\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{i\left(d\,x^3+c\right)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{3\sqrt{-a^2+b^2}\,d}$	138

input
```
int(x^2/(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.08

$$\int \frac{x^2}{a+b\sin\left(c+dx^3\right)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2+2\,\left(a\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)+b\cos\left(dx^3+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2}\right)}{6\,(a^2-b^2)d},\right.$$
$$\left.-\frac{\arctan\left(-\frac{a\sin\left(dx^3+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx^3+c\right)}\right)}{3\,\sqrt{a^2-b^2}d}\right]$$

input
```
integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output

```
[-1/6*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x
^3 + c) - a^2 - b^2 + 2*(a*cos(d*x^3 + c)*sin(d*x^3 + c) + b*cos(d*x^3 + c
))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 -
b^2))/((a^2 - b^2)*d), -1/3*arctan(-(a*sin(d*x^3 + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x^3 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(41) = 82$.

Time = 6.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.37

$$\int \frac{x^2}{a + b\sin(c + dx^3)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x^3}{\sin(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right)\right)}{3bd} & \text{for } a = 0 \\ \frac{x^3}{3(a + b\sin(c))} & \text{for } d = 0 \\ \frac{2}{3bd\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) - 3bd} & \text{for } a = -b \\ -\frac{2}{3bd\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) + 3bd} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2 + b^2}}{a}\right)}{3d\sqrt{-a^2 + b^2}} - \frac{\log\left(\tan\left(\frac{c}{2} + \frac{dx^3}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2 + b^2}}{a}\right)}{3d\sqrt{-a^2 + b^2}} & \text{otherwise} \end{cases}$$

input

```
integrate(x**2/(a+b*sin(d*x**3+c)),x)
```

output

```
Piecewise((zoo*x**3/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2
+ d*x**3/2))/(3*b*d), Eq(a, 0)), (x**3/(3*(a + b*sin(c))), Eq(d, 0)), (2/(
3*b*d*tan(c/2 + d*x**3/2) - 3*b*d), Eq(a, -b)), (-2/(3*b*d*tan(c/2 + d*x**
3/2) + 3*b*d), Eq(a, b)), (log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b*
*2)/a)/(3*d*sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**3/2) + b/a + sqrt(-a*
*2 + b**2)/a)/(3*d*sqrt(-a**2 + b**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8078 vs. $2(44) = 88$.

Time = 23.61 (sec) , antiderivative size = 8078, normalized size of antiderivative = 158.39

$$\int \frac{x^2}{a + b\sin\left(c + dx^3\right)}\,dx = \text{Too large to display}$$

input

```
integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output

```
1/3*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^4*cos(c)*sin(c) - 4*(a^
2*b^4 - b^6)*cos(c)*sin(d*x^3 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c
)^3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 4*(3*(a^3*
b^3 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^
6)*cos(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)
^3 + 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2
*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^
4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 +
 3*((a^3*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*
x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5
 + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^
3 + a*b^5)*cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a
*b^5)*cos(c)^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3
+ (2*a^5*b - 3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^
2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*
sin(c) - (a^3*b^3 - a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + ((4*a^4*b^2 - 5*
a^2*b^4 + b^6)*cos(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^
3 + 2*c))*sin(d*x^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^5*cos(c) - 4*a*b^4*cos(
d*x^3 + 2*c)^4*cos(c)*sin(c) + b^5*sin(d*x^3 + 2*c)^5*sin(c) + (b^5*cos(d*
x^3 + 2*c)*cos(c) + 4*a*b^4*cos(c)*sin(c))*sin(d*x^3 + 2*c)^4 + 2*((2*a...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx=\frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{3\,\sqrt{a^2-b^2}d}$$

input
```
integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 42.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx=$$
$$-\frac{\ln\left(-x^2\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}-\frac{2\,x^2\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)-\ln\left(-x^2\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}+\frac{2\,x^2\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{3\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

input
```
int(x^2/(a + b*sin(c + d*x^3)),x)
```

output
```
-(log(- x^2*exp(d*x^3*1i)*exp(c*1i)*2i - (2*x^2*(b*1i + a*exp(d*x^3*1i)*ex
p(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2))) - log((2*x^2*(b*1i + a*exp(d*x^3*
1i)*exp(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2)) - x^2*exp(d*x^3*1i)*exp(c*1i
)*2i))/(3*d*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{a+b\sin\left(c+dx^3\right)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)}$$

input

```
int(x^2/(a+b*sin(d*x^3+c)),x)
```

output

```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x**3)/2)*a + b)/sqrt(a**2 - b**2)))/
(3*d*(a**2 - b**2))
```

3.83 $\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\mathrm{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

output
```
Defer(Int)(1/x/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*Sin[c + d*x^3])),x]
```

output
```
Integrate[1/(x*(a + b*Sin[c + d*x^3])), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b \sin (c+d x^3))} \, dx$$

$$\Big\downarrow \; 3908$$

$$\int \frac{1}{x (a+b \sin (c+d x^3))}dx$$

input
```
Int[1/(x*(a + b*Sin[c + d*x^3])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x (a+b \sin (d x^3+c))}dx$$

input
```
int(1/x/(a+b*sin(d*x^3+c)),x)
```

output
```
int(1/x/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*sin(d*x^3 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(1/(x*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 40.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*sin(c + d*x^3))),x)
```

output
```
int(1/(x*(a + b*sin(c + d*x^3))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\frac{-\left(\int \frac{\sin\left(d\,x^3+c\right)}{\sin\left(d\,x^3+c\right)bx+ax}dx\right)b+\log(x)}{a}$$

input
```
int(1/x/(a+b*sin(d*x^3+c)),x)
```

output
```
( - int(sin(c + d*x**3)/(sin(c + d*x**3)*b*x + a*x),x)*b + log(x))/a
```

3.84 $\int \frac{1}{x^4(a+b\sin(c+dx^3))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

output

```
Defer(Int)(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

input

```
Integrate[1/(x^4*(a + b*Sin[c + d*x^3])),x]
```

output

```
Integrate[1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b \sin \left(c+d x^3\right)\right)} \, dx$$

$$\Big\downarrow \; 3908$$

$$\int \frac{1}{x^4 \left(a+b \sin \left(c+d x^3\right)\right)}dx$$

input
```
Int[1/(x^4*(a + b*Sin[c + d*x^3])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a+b \sin \left(d \,x^3+c \right)\right)}dx$$

input
```
int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

output
```
int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^4*sin(d*x^3 + c) + a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 4.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

input
```
integrate(1/x**4/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(1/(x**4*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 40.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)}\,dx$$

input
```
int(1/(x^4*(a + b*sin(c + d*x^3))),x)
```

output
```
int(1/(x^4*(a + b*sin(c + d*x^3))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \frac{1}{x^4 \left(a+b\sin \left(c+dx^3\right)\right)}\, dx = \frac{-3\left(\int \frac{\sin \left(d\,x^3+c\right)}{\sin \left(d\,x^3+c\right) b\,x^4+a\,x^4}dx\right) b\,x^3-1}{3a\,x^3}$$

input
```
int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

output
```
( - 3*int(sin(c + d*x**3)/(sin(c + d*x**3)*b*x**4 + a*x**4),x)*b*x**3 - 1)
/(3*a*x**3)
```

3.85 $\int \frac{x}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx = \text{Int}\left(\frac{x}{a+b\sin(c+dx^3)},x\right)$$

output
```
Defer(Int)(x/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx = \int \frac{x}{a+b\sin(c+dx^3)}\,dx$$

input
```
Integrate[x/(a + b*Sin[c + d*x^3]),x]
```

output
```
Integrate[x/(a + b*Sin[c + d*x^3]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$
$$\Big\downarrow \; 3908$$
$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}dx$$

input
```
Int[x/(a + b*Sin[c + d*x^3]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sin\left(d\,x^{3}+c\right)}dx$$

input
```
int(x/(a+b*sin(d*x^3+c)),x)
```

output
```
int(x/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral(x/(b*sin(d*x^3 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{a+b\sin(c+dx^3)}\,dx$$

input
```
integrate(x/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(x/(a + b*sin(c + d*x**3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 40.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx=\int \frac{x}{a+b\,\sin(d\,x^3+c)}\,dx$$

input
```
int(x/(a + b*sin(c + d*x^3)),x)
```

output
```
int(x/(a + b*sin(c + d*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{a+b\sin(c+dx^3)}\,dx = \int \frac{x}{\sin(d\,x^3+c)\,b+a}dx$$

input
```
int(x/(a+b*sin(d*x^3+c)),x)
```

output
```
int(x/(sin(c + d*x**3)*b + a),x)
```

3.86 $\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^3])),x]
```

output
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b\sin(c+dx^3))}dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x^2 (a+b\sin(c+dx^3))}dx$$

input
```
Int[1/(x^2*(a + b*Sin[c + d*x^3])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+b\sin(d\,x^3+c))}dx$$

input
```
int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

output
```
int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2 (a + b\sin(c + dx^3))}\,dx = \int \frac{1}{(b\sin(dx^3 + c) + a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*sin(d*x^3 + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a + b\sin(c + dx^3))}\,dx = \int \frac{1}{x^2 (a + b\sin(c + dx^3))}\,dx$$

input
```
integrate(1/x**2/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(1/(x**2*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^3\right)\right)}\, dx=\int \frac{1}{(b \sin \left(d x^3+c\right)+a) x^2}\, dx$$

input

```
integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output

```
integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^3\right)\right)}\, dx=\int \frac{1}{(b \sin \left(d x^3+c\right)+a) x^2}\, dx$$

input

```
integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output

```
integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 39.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^3\right)\right)}\, dx=\int \frac{1}{x^2\, \left(a+b\, \sin \left(d\, x^3+c\right)\right)}\, dx$$

input

```
int(1/(x^2*(a + b*sin(c + d*x^3))),x)
```

output
```
int(1/(x^2*(a + b*sin(c + d*x^3))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\frac{-\left(\int \frac{\sin\left(d\,x^3+c\right)}{\sin\left(d\,x^3+c\right)b\,x^2+a\,x^2}dx\right)bx-1}{ax}$$

input
```
int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

output
```
( - (int(sin(c + d*x**3)/(sin(c + d*x**3)*b*x**2 + a*x**2),x)*b*x + 1))/(a
*x)
```

3.87 $\int \frac{1}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \text{Int}\left(\frac{1}{a+b\sin(c+dx^3)},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{a+b\sin(c+dx^3)}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^(-1),x]
```

output
```
Integrate[(a + b*Sin[c + d*x^3])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3850\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

$$\Big\downarrow \; 3850$$

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}dx$$

input
```
Int[(a + b*Sin[c + d*x^3])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin\left(d\,x^3+c\right)}dx$$

input
```
int(1/(a+b*sin(d*x^3+c)),x)
```

output
```
int(1/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx = \int \frac{1}{b\sin\left(dx^3+c\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*sin(d*x^3 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx = \int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

input
```
integrate(1/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(1/(a + b*sin(c + d*x**3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin(c+dx^3)}\,dx = \int \frac{1}{a+b\,\sin\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(a + b*sin(c + d*x^3)),x)
```

output
```
int(1/(a + b*sin(c + d*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{1}{\sin\left(d\,x^3+c\right)b+a}dx$$

input
```
int(1/(a+b*sin(d*x^3+c)),x)
```

output
```
int(1/(sin(c + d*x**3)*b + a),x)
```

3.88 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)} \, dx=\text{Int}\left(\frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)} \, dx=\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)} \, dx$$

input
```
Integrate[1/(x^3*(a + b*Sin[c + d*x^3])),x]
```

output
```
Integrate[1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))} \, dx$$
$$\Big\downarrow \; 3908$$
$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))} dx$$

input
```
Int[1/(x^3*(a + b*Sin[c + d*x^3])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b\sin(d\,x^3 + c))} dx$$

input
```
int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

output
```
int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{(b\sin\left(dx^3+c\right)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*sin(d*x^3 + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

input
```
integrate(1/x**3/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral(1/(x**3*(a + b*sin(c + d*x**3))), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))} \, dx = \int \frac{1}{(b\sin(dx^3 + c) + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))} \, dx = \int \frac{1}{(b\sin(dx^3 + c) + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 39.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\sin(c + dx^3))} \, dx = \int \frac{1}{x^3 \, (a + b \, \sin(d\,x^3 + c))} \, dx$$

input
```
int(1/(x^3*(a + b*sin(c + d*x^3))),x)
```

output
```
int(1/(x^3*(a + b*sin(c + d*x^3))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\frac{-2\left(\int \frac{\sin\left(d\,x^3+c\right)}{\sin\left(d\,x^3+c\right)b\,x^3+a\,x^3}dx\right)b\,x^2-1}{2a\,x^2}$$

input
```
int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

output
```
( - 2*int(sin(c + d*x**3)/(sin(c + d*x**3)*b*x**3 + a*x**3),x)*b*x**2 - 1)
/(2*a*x**2)
```

3.89 $\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 324

$$\begin{aligned}\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx &= -\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}+\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}\\ &\quad-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3\left(a^2-b^2\right)d^2}-\frac{a\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d^2}\\ &\quad+\frac{a\,\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d^2}\\ &\quad+\frac{bx^3\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}\end{aligned}$$

output

```
-1/3*I*a*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2
)/d+1/3*I*a*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(
3/2)/d-1/3*ln(a+b*sin(d*x^3+c))/(a^2-b^2)/d^2-1/3*a*polylog(2,I*b*exp(I*(d
*x^3+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/3*a*polylog(2,I*b*exp(
I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/3*b*x^3*cos(d*x^3+
c)/(a^2-b^2)/d/(a+b*sin(d*x^3+c))
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{-\frac{iadx^3\log\left(1+\frac{ibe^{i\left(c+dx^3\right)}}{-a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{iadx^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{a^2-b^2}-\frac{a\operatorname{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^3\right)}}{-a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\operatorname{PolyLog}(2}{(a^2-}}{3d^2}$$

input
```
Integrate[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

output
```
(((-I)*a*d*x^3*Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a
^2 - b^2)^(3/2) + (I*a*d*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2
 - b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^3]]/(a^2 - b^2) - (a*
PolyLog[2, ((-I)*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)
^(3/2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (b*d*x^3*Cos[c + d*x^3])/((a^2 - b^2)*(a + b*Sin[c + d*x^
3])))/(3*d^2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3860, 3042, 3805, 3042, 3147, 16, 3804, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{3}\int \frac{x^3}{\left(a+b\sin\left(dx^3+c\right)\right)^2}dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{x^3}{\left(a+b\sin\left(dx^3+c\right)\right)^2}dx^3$$

$$\Big\downarrow \ 3805$$

$$\frac{1}{3}\left(\frac{a\int \frac{x^3}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}-\frac{b\int \frac{\cos(dx^3+c)}{a+b\sin(dx^3+c)}dx^3}{d\left(a^2-b^2\right)}+\frac{bx^3\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(\frac{a\int \frac{x^3}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}-\frac{b\int \frac{\cos(dx^3+c)}{a+b\sin(dx^3+c)}dx^3}{d\left(a^2-b^2\right)}+\frac{bx^3\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

$$\Big\downarrow \ 3147$$

$$\frac{1}{3}\left(-\frac{\int \frac{1}{a+b\sin(dx^3+c)}d\Big(b\sin\left(dx^3+c\right)\Big)}{d^2\left(a^2-b^2\right)}+\frac{a\int \frac{x^3}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}+\frac{bx^3\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{a\int \frac{x^3}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^3\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

$$\Big\downarrow \ 3804$$

$$\frac{1}{3}\left(\frac{2a\int \frac{e^{i\left(dx^3+c\right)}x^3}{2e^{i\left(dx^3+c\right)}a-ibe^{2i\left(dx^3+c\right)}+ib}dx^3}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^3\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

$$\Big\downarrow \ 2694$$

$$\frac{1}{3}\left(\frac{2a\left(\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{2\left(a-ibe^{i\left(dx^3+c\right)}+\sqrt{a^2-b^2}\right)}dx^3}{\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{2\left(a-ibe^{i\left(dx^3+c\right)}-\sqrt{a^2-b^2}\right)}dx^3}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^3}{d\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{2a\left(\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{a-ibe^{i\left(dx^3+c\right)}+\sqrt{a^2-b^2}}dx^3}{2\sqrt{a^2-b^2}}-\frac{ib\int \frac{e^{i\left(dx^3+c\right)}x^3}{a-ibe^{i\left(dx^3+c\right)}-\sqrt{a^2-b^2}}dx^3}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{d^2\left(a^2-b^2\right)}+\frac{bx^3\cos\left(c\right.}{d\left(a^2-b^2\right)\left(a+b\right.}\right.$$

$$\Big\downarrow 2620$$

$$\frac{1}{3}\left(\frac{2a\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{\int\log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a+\sqrt{a^2-b^2}}\right)dx^3}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int\log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a-\sqrt{a^2-b^2}}\right)dx^3}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log}{}\right.$$

$$\Big\downarrow 2715$$

$$\frac{1}{3}\left(\frac{2a\left(\frac{ib\left(\frac{i\int \frac{\log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{x^3}de^{i\left(dx^3+c\right)}}{bd^2}+\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{i\int \frac{\log\left(1-\frac{ibe^{i\left(dx^3+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{x^3}de^{i\left(dx^3+c\right)}}{bd^2}+\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx}{a-\sqrt{a^2-}}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \color{blue}{2838}$$

$$\frac{1}{3}\left(\frac{2a\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^3+c\right)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd}-\frac{i\,\text{PolyLog}\left(2,\frac{ibe^{i\left(dx^3+c\right)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{\log}{}\right.$$

input
```
Int[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

output
```
(-(Log[a + b*Sin[c + d*x^3]]/((a^2 - b^2)*d^2)) + (2*a*(((-1/2*I)*b*((x^3*
Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (I*PolyLog
[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(b*d^2)))/Sqrt[a^2 - b
^2] + ((I/2)*b*((x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])
])/(b*d) - (I*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])])/(
b*d^2)))/Sqrt[a^2 - b^2]))/(a^2 - b^2) + (b*x^3*Cos[c + d*x^3])/((a^2 - b^
2)*d*(a + b*Sin[c + d*x^3])))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [F]

$$\int \frac{x^5}{\left(a+b\sin\left(d\,x^3+c\right)\right)^2}dx$$

input
```
int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(274) = 548$.

Time = 0.22 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.66

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output

```
1/6*(2*(a^2*b - b^3)*d*x^3*cos(d*x^3 + c) + (I*a*b^2*sin(d*x^3 + c) + I*a^
2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) +
 (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1
) + (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*
cos(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)
*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b
*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (I*a*b^2*sin(d*x^3 + c) + I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos
(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b + 1) - (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3
+ a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3 + c)
 - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 -
 b^2)/b^2) - b)/b) + (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(
d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3 + c) - a*sin(d*x^3
+ c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)
/b) - (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqr
t(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(
d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d
*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b...
```

Sympy [F]

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input

```
integrate(x**5/(a+b*sin(d*x**3+c))**2,x)
```

output

```
Integral(x**5/(a + b*sin(c + d*x**3))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x^5}{(b\sin(dx^3+c)+a)^2}\,dx$$

input
```
integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate(x^5/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x^5}{(a+b\,\sin(d\,x^3+c))^2}\,dx$$

input
```
int(x^5/(a + b*sin(c + d*x^3))^2,x)
```

output
```
int(x^5/(a + b*sin(c + d*x^3))^2, x)
```

Reduce [F]

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x^5}{\sin(d\,x^3+c)^2\,b^2+2\sin(d\,x^3+c)\,ab+a^2}dx$$

input

```
int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

output

```
int(x**5/(sin(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*b + a**2),x)
```

3.90 $\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} + \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}$$

output

```
2/3*a*arctan((b+a*tan(1/2*d*x^3+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d
+1/3*b*cos(d*x^3+c)/(a^2-b^2)/d/(a+b*sin(d*x^3+c))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{\frac{2a\arctan\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b\cos\left(c+dx^3\right)}{a+b\sin\left(c+dx^3\right)}}{3(a-b)(a+b)d}$$

input

```
Integrate[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

output

```
((2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
+ (b*Cos[c + d*x^3])/(a + b*Sin[c + d*x^3]))/(3*(a - b)*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3860, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{3}\int \frac{1}{(a+b\sin(dx^3+c))^2}dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{(a+b\sin(dx^3+c))^2}dx^3$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{3}\left(\frac{b\cos(c+dx^3)}{d(a^2-b^2)(a+b\sin(c+dx^3))}-\frac{\int -\frac{a}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{a}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}+\frac{b\cos(c+dx^3)}{d(a^2-b^2)(a+b\sin(c+dx^3))}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{a\int \frac{1}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}+\frac{b\cos(c+dx^3)}{d(a^2-b^2)(a+b\sin(c+dx^3))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{a\int \frac{1}{a+b\sin(dx^3+c)}dx^3}{a^2-b^2}+\frac{b\cos(c+dx^3)}{d(a^2-b^2)(a+b\sin(c+dx^3))}\right)$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{3}\left(\frac{2a\int \frac{1}{ax^6+a+2b\tan\left(\frac{1}{2}(dx^3+c)\right)}d\tan\left(\frac{1}{2}\left(dx^3+c\right)\right)}{d\left(a^2-b^2\right)}+\frac{b\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{b\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}-\frac{4a\int \frac{1}{-x^6-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}\left(dx^3+c\right)\right)\right)}{d\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{2a\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx^3)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^3\right)}{d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}\right)$$

input
```
Int[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

output
```
((2*a*ArcTan[(2*b + 2*a*Tan[(c + d*x^3)/2])/(2*Sqrt[a^2 - b^2])])/((a^2 -
b^2)^(3/2)*d) + (b*Cos[c + d*x^3])/((a^2 - b^2)*d*(a + b*Sin[c + d*x^3])))
/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)^2 a+2b\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{3d}$
default	$\frac{\frac{\frac{2b^2\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)^2 a+2b\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{d x^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{3d}$
risch	$\frac{\frac{2ib}{3}+\frac{2a\,\mathrm{e}^{i\left(d x^3+c\right)}}{3}}{(a^2-b^2)d\left(b\,\mathrm{e}^{2i\left(d x^3+c\right)}-b+2ia\,\mathrm{e}^{i\left(d x^3+c\right)}\right)}-\frac{a\ln\left(\mathrm{e}^{i\left(d x^3+c\right)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{3\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{a\ln\left(\mathrm{e}^{i\left(d x^3+c\right)}+\frac{i\sqrt{-a^2}}{\sqrt{}}\right.}{3\sqrt{-a^2+b^2}\,(a+b}$

input
```
int(x^2/(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*(b^2/a/(a^2-b^2)*tan(1/2*d*x^3+1/2*c)+b/(a^2-b^2))/(tan(1/2*d*x^3
+1/2*c)^2*a+2*b*tan(1/2*d*x^3+1/2*c)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*
a*tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.89

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

$$=\left[\frac{\left(ab\sin\left(dx^3+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2-2\,\left(a\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)+b\cos\right.}{b^2\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2}\right.}{6\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx^3+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right.}\right.$$

$$\left.-\frac{\left(ab\sin\left(dx^3+c\right)+a^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx^3+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx^3+c\right)}\right)-\left(a^2b-b^3\right)\cos\left(dx^3+c\right)}{3\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx^3+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}\right]$$

input
```
integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
[1/6*((a*b*sin(d*x^3 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2 - 2*(a*cos(d*x^3 + c)*sin(
d*x^3 + c) + b*cos(d*x^3 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2
*a*b*sin(d*x^3 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^3 + c))/((a^4*
b - 2*a^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/3
*((a*b*sin(d*x^3 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^3 + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x^3 + c))) - (a^2*b - b^3)*cos(d*x^3 + c))/((a^4*
b - 2*a^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(75) = 150$.

Time = 68.84 (sec) , antiderivative size = 2116, normalized size of antiderivative = 22.51

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2/(a+b*sin(d*x**3+c))**2,x)
```

output

```
Piecewise((zoo*x**3/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2
+ d*x**3/2)/(6*d) - 1/(6*d*tan(c/2 + d*x**3/2)))/b**2, Eq(a, 0)), (-6*tan(
c/2 + d*x**3/2)**2/(9*b**2*d*tan(c/2 + d*x**3/2)**3 - 27*b**2*d*tan(c/2 +
d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2) - 9*b**2*d) + 6*tan(c/2 + d*x
**3/2)/(9*b**2*d*tan(c/2 + d*x**3/2)**3 - 27*b**2*d*tan(c/2 + d*x**3/2)**2
 + 27*b**2*d*tan(c/2 + d*x**3/2) - 9*b**2*d) - 4/(9*b**2*d*tan(c/2 + d*x**
3/2)**3 - 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2)
 - 9*b**2*d), Eq(a, -b)), (-6*tan(c/2 + d*x**3/2)**2/(9*b**2*d*tan(c/2 + d
*x**3/2)**3 + 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**
3/2) + 9*b**2*d) - 6*tan(c/2 + d*x**3/2)/(9*b**2*d*tan(c/2 + d*x**3/2)**3
+ 27*b**2*d*tan(c/2 + d*x**3/2)**2 + 27*b**2*d*tan(c/2 + d*x**3/2) + 9*b**
2*d) - 4/(9*b**2*d*tan(c/2 + d*x**3/2)**3 + 27*b**2*d*tan(c/2 + d*x**3/2)*
*2 + 27*b**2*d*tan(c/2 + d*x**3/2) + 9*b**2*d), Eq(a, b)), (x**3/(3*(a + b
*sin(c))**2), Eq(d, 0)), (a**3*log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2
+ b**2)/a)*tan(c/2 + d*x**3/2)**2/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d
*x**3/2)**2 + 3*a**4*d*sqrt(-a**2 + b**2) + 6*a**3*b*d*sqrt(-a**2 + b**2)*
tan(c/2 + d*x**3/2) - 3*a**2*b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)
**2 - 3*a**2*b**2*d*sqrt(-a**2 + b**2) - 6*a*b**3*d*sqrt(-a**2 + b**2)*tan
(c/2 + d*x**3/2)) + a**3*log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b**2
)/a)/(3*a**4*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x**3/2)**2 + 3*a**4*d*sqr...
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Timed out}$$

input

```
integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx = \frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{3\,(a^2d-b^2d)\sqrt{a^2-b^2}} + \frac{2\left(b^2\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+ab\right)}{3\,(a^3d-ab^2d)\left(a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+a\right)}$$

input
```
integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
 1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + 2/3*(
b^2*tan(1/2*d*x^3 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^3 + 1/
2*c)^2 + 2*b*tan(1/2*d*x^3 + 1/2*c) + a))
```

Mupad [B] (verification not implemented)

Time = 41.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.98

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx = \frac{\frac{2\,b}{a^2-b^2}+\frac{2\,b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{a\,(a^2-b^2)}}{d\,\left(3\,a\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)^2+6\,b\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+3\,a\right)} + \frac{2\,a\,\mathrm{atan}\left(\frac{3\,(a^2-b^2)\,\left(\frac{2\,a^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3\,(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,\left(3\,a^2\,b-3\,b^3\right)}{9\,(a+b)^{3/2}\,\left(a^2-b^2\right)\,(a-b)^{3/2}}\right)}{2\,a}\right)}{3\,d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(x^2/(a + b*sin(c + d*x^3))^2,x)
```

output
```
((2*b)/(a^2 - b^2) + (2*b^2*tan(c/2 + (d*x^3)/2))/(a*(a^2 - b^2)))/(d*(3*a
 + 3*a*tan(c/2 + (d*x^3)/2)^2 + 6*b*tan(c/2 + (d*x^3)/2))) + (2*a*atan((3*
(a^2 - b^2)*((2*a^2*tan(c/2 + (d*x^3)/2))/(3*(a + b)^(3/2)*(a - b)^(3/2))
+ (2*a*(3*a^2*b - 3*b^3))/(9*(a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/(2
*a)))/(3*d*(a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx$$
$$=\frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)\sin(d\,x^3+c)\,ab+2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2+\cos(d\,x^3+c)}{3d\left(\sin(d\,x^3+c)\,a^4b-2\sin(d\,x^3+c)\,a^2b^3+\sin(d\,x^3+c)\,b^5+a^5-2a^3b^2+a\,b^4\right)}$$

input
```
int(x^2/(a+b*sin(d*x^3+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x**3)/2)*a + b)/sqrt(a**2 - b**2))*s
in(c + d*x**3)*a*b + 2*sqrt(a**2 - b**2)*atan((tan((c + d*x**3)/2)*a + b)/
sqrt(a**2 - b**2))*a**2 + cos(c + d*x**3)*a**2*b - cos(c + d*x**3)*b**3)/(
3*d*(sin(c + d*x**3)*a**4*b - 2*sin(c + d*x**3)*a**2*b**3 + sin(c + d*x**3
)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.91 $\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

output

```
Defer(Int)(1/x/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input

```
Integrate[1/(x*(a + b*Sin[c + d*x^3])^2),x]
```

output

```
Integrate[1/(x*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx$$

$$\Big\downarrow \text{3908}$$

$$\int \frac{1}{x \left(a+b \sin \left(c+d x^3\right)\right)^2}dx$$

input

```
Int[1/(x*(a + b*Sin[c + d*x^3])^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a+b \sin \left(d \,x^{3}+c \right)\right)^{2}}d x$$

input

```
int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x*cos(d*x^3 + c)^2 - 2*a*b*x*sin(d*x^3 + c) - (a^2 + b^2)
*x), x)
```

Sympy [N/A]

Not integrable

Time = 23.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(1/(x*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 2696, normalized size of antiderivative = 149.78

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x}\,dx$$

input

```
integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d
*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) -
2*a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin
(c) - a*b)*cos(d*x^3 + c) + 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^
4)*sin(2*c)^2)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 -
 a^2*b^2)*sin(c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^
2*b^2 - b^4)*sin(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*co
s(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*
d*x^3*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2
- b^4)*d*x^3 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
(2*c)*sin(c))*d*x^3*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*
b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*
x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^
3)*sin(2*c)*sin(c))*d*x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
- (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^
3*sin(2*c))*sin(2*d*x^3))*integrate(-2*(b^4*cos(2*c)*sin(2*d*x^3) + b^4*co
s(2*d*x^3)*sin(2*c) - 2*(a^3*b - a*b^3)*cos(d*x^3)*cos(c) + 2*(a^3*b - a*b
^3)*sin(d*x^3)*sin(c) + (a^3*b*d*x^3*sin(d*x^3 + c) - a^3*b*cos(d*x^3 + c)
)*cos(2*d*x^3 + 2*c) + (a^3*b - a*b^3 + (a*b^3*d*x^3*sin(2*c) + a*b^3*cos(
2*c))*cos(2*d*x^3) - 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - (a^4 - a^2*b^2)*...
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2 x}\,dx$$

input

```
integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output

```
integrate(1/((b*sin(d*x^3 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 41.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

input

```
int(1/(x*(a + b*sin(c + d*x^3))^2),x)
```

output

```
int(1/(x*(a + b*sin(c + d*x^3))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 6.22

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$
$$=\frac{-\left(\int \frac{\sin\left(d\,x^3+c\right)^2}{\sin\left(d\,x^3+c\right)^2 b^2 x+2\sin\left(d\,x^3+c\right) a b x+a^2 x}dx\right) b^2-2\left(\int \frac{\sin\left(d\,x^3+c\right)}{\sin\left(d\,x^3+c\right)^2 b^2 x+2\sin\left(d\,x^3+c\right) a b x+a^2 x}dx\right) a b+\log(x)}{a^2}$$

input
```
int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

output
```
( - int(sin(c + d*x**3)**2/(sin(c + d*x**3)**2*b**2*x + 2*sin(c + d*x**3)*
a*b*x + a**2*x),x)*b**2 - 2*int(sin(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x
 + 2*sin(c + d*x**3)*a*b*x + a**2*x),x)*a*b + log(x))/a**2
```

3.92 $\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
```

output
```
Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx$$
$$\Big\downarrow \; 3908$$
$$\int \frac{1}{x^4 (a + b\sin(c + dx^3))^2} dx$$

input
```
Int[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a + b\sin(d\,x^3 + c))^2} dx$$

input
```
int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^4 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx=\int \frac{1}{\left(b \sin \left(d x^3+c\right)+a\right)^2 x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x^4*cos(d*x^3 + c)^2 - 2*a*b*x^4*sin(d*x^3 + c) - (a^2 +
b^2)*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 46.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx=\int \frac{1}{x^4 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx$$

input
```
integrate(1/x**4/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(1/(x**4*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.44 (sec) , antiderivative size = 2705, normalized size of antiderivative = 150.28

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^4}\,dx$$

input

```
integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d
*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) -
2*a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin
(c) - a*b)*cos(d*x^3 + c) + 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^
4)*sin(2*c)^2)*d*x^6*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 -
 a^2*b^2)*sin(c)^2)*d*x^6*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^
2*b^2 - b^4)*sin(2*c)^2)*d*x^6*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*co
s(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*
d*x^6*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2
- b^4)*d*x^6 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
(2*c)*sin(c))*d*x^6*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*
b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*sin(d*
x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^
3)*sin(2*c)*sin(c))*d*x^6*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
- (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^6*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^
6*sin(2*c))*sin(2*d*x^3))*integrate(-2*(2*b^4*cos(2*c)*sin(2*d*x^3) + 2*b^
4*cos(2*d*x^3)*sin(2*c) - 4*(a^3*b - a*b^3)*cos(d*x^3)*cos(c) + 4*(a^3*b -
 a*b^3)*sin(d*x^3)*sin(c) + (a^3*b*d*x^3*sin(d*x^3 + c) - 2*a^3*b*cos(d*x^
3 + c))*cos(2*d*x^3 + 2*c) + (2*a^3*b - 2*a*b^3 + (a*b^3*d*x^3*sin(2*c) +
2*a*b^3*cos(2*c))*cos(2*d*x^3) - 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - 2*(a...
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 (a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(b\sin(dx^3+c)+a)^2 x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 41.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 (a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{x^4 \left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

input
```
int(1/(x^4*(a + b*sin(c + d*x^3))^2),x)
```

output
```
int(1/(x^4*(a + b*sin(c + d*x^3))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 1331, normalized size of antiderivative = 73.94

$$\int \frac{1}{x^4 (a+b\sin(c+dx^3))^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

output

```
(4*cos(c + d*x**3)*b**2 + 12*int(cos(c + d*x**3)/(sin(c + d*x**3)**2*b**2*
x**4 + 2*sin(c + d*x**3)*a*b*x**4 + a**2*x**4),x)*sin(c + d*x**3)*a*b**3*x
**3 + 12*int(cos(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x**4 + 2*sin(c + d*x
**3)*a*b*x**4 + a**2*x**4),x)*a**2*b**2*x**3 - 6*int(cos(c + d*x**3)/(sin(
c + d*x**3)**2*b**2*x + 2*sin(c + d*x**3)*a*b*x + a**2*x),x)*sin(c + d*x**
3)*a**2*b**2*d*x**3 + 12*int(cos(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x +
2*sin(c + d*x**3)*a*b*x + a**2*x),x)*sin(c + d*x**3)*b**4*d*x**3 - 6*int(c
os(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x + 2*sin(c + d*x**3)*a*b*x + a**2
*x),x)*a**3*b*d*x**3 + 12*int(cos(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x +
 2*sin(c + d*x**3)*a*b*x + a**2*x),x)*a*b**3*d*x**3 + 3*int(sin(c + d*x**3
)**2/(sin(c + d*x**3)**2*b**2*x**4 + 2*sin(c + d*x**3)*a*b*x**4 + a**2*x**
4),x)*sin(c + d*x**3)*a*b**3*x**3 + 3*int(sin(c + d*x**3)**2/(sin(c + d*x*
*3)**2*b**2*x**4 + 2*sin(c + d*x**3)*a*b*x**4 + a**2*x**4),x)*a**2*b**2*x*
*3 - 6*int(sin(c + d*x**3)**2/(sin(c + d*x**3)**2*b**2*x + 2*sin(c + d*x**
3)*a*b*x + a**2*x),x)*sin(c + d*x**3)*b**4*d*x**3 - 6*int(sin(c + d*x**3)*
*2/(sin(c + d*x**3)**2*b**2*x + 2*sin(c + d*x**3)*a*b*x + a**2*x),x)*a*b**
3*d*x**3 + 12*int(sin(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x**4 + 2*sin(c
+ d*x**3)*a*b*x**4 + a**2*x**4),x)*sin(c + d*x**3)*b**4*x**3 + 12*int(sin(
c + d*x**3)/(sin(c + d*x**3)**2*b**2*x**4 + 2*sin(c + d*x**3)*a*b*x**4 + a
**2*x**4),x)*a*b**3*x**3 + 12*int((cos(c + d*x**3)*sin(c + d*x**3))/(si...
```

3.93 $\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \text{Int}\left(\frac{x}{(a+b\sin(c+dx^3))^2},x\right)$$

output

```
Defer(Int)(x/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx$$

input

```
Integrate[x/(a + b*Sin[c + d*x^3])^2,x]
```

output

```
Integrate[x/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\sin(c+dx^3))^2} \, dx$$

$$\Big\downarrow \ 3908$$

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}dx$$

input
```
Int[x/(a + b*Sin[c + d*x^3])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sin(d\,x^3+c))^2}dx$$

input
```
int(x/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(x/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \frac{x}{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}\,dx=\int \frac{x}{\left(b\sin\left(dx^{3}+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-x/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}\,dx=\int \frac{x}{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}\,dx$$

input
```
integrate(x/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(x/(a + b*sin(c + d*x**3))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{x}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{x}{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

input
```
int(x/(a + b*sin(c + d*x^3))^2,x)
```

output
```
int(x/(a + b*sin(c + d*x^3))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{x}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{x}{\sin(d\,x^3+c)^2\,b^2+2\sin(d\,x^3+c)\,ab+a^2}dx$$

input
```
int(x/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(x/(sin(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*b + a**2),x)
```

3.94 $\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 15.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
```

output
```
Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx$$
$$\Big\downarrow \ 3908$$
$$\int \frac{1}{x^2 \left(a+b \sin \left(c+d x^3\right)\right)^2}dx$$

input

```
Int[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{2}}d x$$

input

```
int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^2 \left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x^2*cos(d*x^3 + c)^2 - 2*a*b*x^2*sin(d*x^3 + c) - (a^2 +
b^2)*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 32.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(1/(x**2*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+b\sin(c+dx^3))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(b\sin(dx^3+c)+a)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin(d*x^3 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 41.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{x^2\,(a+b\,\sin(d\,x^3+c))^2}\,dx$$

input
```
int(1/(x^2*(a + b*sin(c + d*x^3))^2),x)
```

output

```
int(1/(x^2*(a + b*sin(c + d*x^3))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1445, normalized size of antiderivative = 80.28

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\text{Too large to display}$$

input

```
int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

output

```
( - 2*cos(c + d*x**3)*a**2*b**2 - 2*int(cos(c + d*x**3)/(sin(c + d*x**3)**
2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*sin(c + d*x**3)*a
**3*b**3*x - 2*int(cos(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x**2 + 2*sin(c
 + d*x**3)*a*b*x**2 + a**2*x**2),x)*a**4*b**2*x - int(sin(c + d*x**3)**2/(
sin(c + d*x**3)**2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*
sin(c + d*x**3)*a**3*b**3*x - int(sin(c + d*x**3)**2/(sin(c + d*x**3)**2*b
**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*a**4*b**2*x - 2*int(
sin(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2
 + a**2*x**2),x)*sin(c + d*x**3)*a**4*b**2*x + 2*int(sin(c + d*x**3)/(sin(
c + d*x**3)**2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*sin(
c + d*x**3)*a**2*b**4*x - 2*int(sin(c + d*x**3)/(sin(c + d*x**3)**2*b**2*x
**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*a**5*b*x + 2*int(sin(c +
d*x**3)/(sin(c + d*x**3)**2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*
x**2),x)*a**3*b**3*x - 2*int((cos(c + d*x**3)*sin(c + d*x**3))/(sin(c + d*
x**3)**2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*sin(c + d*
x**3)*a**2*b**4*x - 2*int((cos(c + d*x**3)*sin(c + d*x**3))/(sin(c + d*x**
3)**2*b**2*x**2 + 2*sin(c + d*x**3)*a*b*x**2 + a**2*x**2),x)*a**3*b**3*x +
 6*int((cos(c + d*x**3)*x)/(sin(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*
b + a**2),x)*sin(c + d*x**3)*a**2*b**4*d*x + 6*int((cos(c + d*x**3)*x)/(si
n(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*b + a**2),x)*a**3*b**3*d*x ...
```

3.95 $\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx = \operatorname{Int}\left(\frac{1}{(a+b\sin(c+dx^3))^2},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x^3])^(-2),x]
```

output
```
Integrate[(a + b*Sin[c + d*x^3])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3850}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{1}{(a+b\sin(c+dx^3))^2}dx \\ \Big\downarrow \text{3850} \\ \int \frac{1}{(a+b\sin(c+dx^3))^2}dx \end{array}$$

input
```
Int[(a + b*Sin[c + d*x^3])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sin(d\,x^3+c))^2}dx$$

input
```
int(1/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(1/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \frac{1}{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}\,dx=\int \frac{1}{\left(b\sin\left(dx^{3}+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [N/A]

Not integrable

Time = 15.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}\,dx=\int \frac{1}{\left(a+b\sin\left(c+dx^{3}\right)\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x**3))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 2171, normalized size of antiderivative = 155.07

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{1}{(b\sin(dx^3+c)+a)^2}\,dx$$

input

```
integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d
*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) -
2*a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin
(c) - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^
4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 -
 a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^
2*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*co
s(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*
d*x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2
- b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
(2*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*
b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*
x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^
3)*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
- (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^
2*sin(2*c))*sin(2*d*x^3))*integrate(-2/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*
cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin
(c) - (2*a*b - (3*a*b*d*x^3*sin(2*c) + 2*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3
*a^2*d*x^3*cos(c) - 2*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 2*a
*b*sin(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*sin(d...
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{1}{(b\sin(dx^3+c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 41.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{1}{(a+b\,\sin(d\,x^3+c))^2}\,dx$$

input
```
int(1/(a + b*sin(c + d*x^3))^2,x)
```

output
```
int(1/(a + b*sin(c + d*x^3))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{1}{\sin(d\,x^3+c)^2\,b^2+2\sin(d\,x^3+c)\,ab+a^2}dx$$

input
```
int(1/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(1/(sin(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*b + a**2),x)
```

3.96 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \mathrm{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

output

```
Defer(Int)(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

input

```
Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
```

output

```
Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx$$
$$\Big\downarrow \text{3908}$$
$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)^2}dx$$

input
```
Int[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 {\left(a +b \sin \left(d \,x^{3}+c \right)\right)}^{2}}d x$$

input
```
int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^3 (a + b\sin (c + dx^3))^2} \, dx = \int \frac{1}{(b\sin (dx^3 + c) + a)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x^3*cos(d*x^3 + c)^2 - 2*a*b*x^3*sin(d*x^3 + c) - (a^2 +
b^2)*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 46.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a + b\sin (c + dx^3))^2} \, dx = \int \frac{1}{x^3 (a + b\sin (c + dx^3))^2} \, dx$$

input
```
integrate(1/x**3/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral(1/(x**3*(a + b*sin(c + d*x**3))**2), x)
```

Maxima [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 2171, normalized size of antiderivative = 120.61

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx=\int \frac{1}{{\left(b \sin \left(d x^{3} + c\right) + a\right)}^{2} x^{3}} \, dx$$

input

```
integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d
*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) -
2*a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin
(c) - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^
4)*sin(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 -
 a^2*b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^
2*b^2 - b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*co
s(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*
d*x^5*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^3)*sin(c) + (a^2*b^2
- b^4)*d*x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
(2*c)*sin(c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^5*cos(2*c) - 2*((a^3*
b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*sin(d*
x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^
3)*sin(2*c)*sin(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
- (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^
5*sin(2*c))*sin(2*d*x^3))*integrate(-2/3*(10*a*b*cos(d*x^3)*cos(c) + 5*b^2
*cos(2*c)*sin(2*d*x^3) + 5*b^2*cos(2*d*x^3)*sin(2*c) - 10*a*b*sin(d*x^3)*s
in(c) - (5*a*b - (3*a*b*d*x^3*sin(2*c) + 5*a*b*cos(2*c))*cos(2*d*x^3) - 2*
(3*a^2*d*x^3*cos(c) - 5*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 5
*a*b*sin(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*sin...
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx=\int \frac{1}{\left(b \sin \left(d x^3+c\right)+a\right)^2 x^3} \, dx$$

input

```
integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output

```
integrate(1/((b*sin(d*x^3 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 40.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx=\int \frac{1}{x^3 \left(a+b\,\sin \left(d\,x^3+c\right)\right)^2} \, dx$$

input

```
int(1/(x^3*(a + b*sin(c + d*x^3))^2),x)
```

output

```
int(1/(x^3*(a + b*sin(c + d*x^3))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 7.39

$$\int \frac{1}{x^3 \left(a+b \sin \left(c+d x^3\right)\right)^2} \, dx = \frac{-2\left(\int \frac{\sin\left(d\,x^3+c\right)^2}{\sin\left(d\,x^3+c\right)^2 b^2 x^3+2\sin\left(d\,x^3+c\right) a b\,x^3+a^2 x^3}d x\right) b^2 x^2-4\left(\int \frac{\sin\left(d\,x^3+c\right)}{\sin\left(d\,x^3+c\right)^2 b^2 x^3+2\sin\left(d\,x^3+c\right) a b\,x^3+a^2 x^3}d x\right) a b\,x^2-1}{2a^2 x^2}$$

input
```
int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

output
```
( - 2*int(sin(c + d*x**3)**2/(sin(c + d*x**3)**2*b**2*x**3 + 2*sin(c + d*x
**3)*a*b*x**3 + a**2*x**3),x)*b**2*x**2 - 4*int(sin(c + d*x**3)/(sin(c + d
*x**3)**2*b**2*x**3 + 2*sin(c + d*x**3)*a*b*x**3 + a**2*x**3),x)*a*b*x**2
- 1)/(2*a**2*x**2)
```

3.97 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx=\text{Int}\left((ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p,x\right)$$

output

```
Defer(Int)((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

input

```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

output

```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p\, dx \\ \Big\downarrow \ 3908 \\ \int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p dx \end{array}$$

input
```
Int[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(e x \right)^{m} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{p}d x$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^p \, dx = \int (ex)^m \left(b\sin\left(dx^3 + c\right) + a\right)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 17.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^p \, dx = \int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^p \, dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**3+c))**p,x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**3))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx=\int (ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p\,dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx=\int (ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p\,dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 39.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx=\int (e\,x)^m \left(a+b\,\sin\left(d\,x^3+c\right)\right)^p\,dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^3))^p,x)
```

output

```
int((e*x)^m*(a + b*sin(c + d*x^3))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^p \, dx = e^m \left(\int x^m \left(\sin \left(d\,x^3+c\right) b+a\right)^p dx\right)$$

input

```
int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

output

```
e**m*int(x**m*(sin(c + d*x**3)*b + a)**p,x)
```

3.98 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 442

$$
\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx
&= \frac{a(2a^2+3b^2)\,(ex)^{1+m}}{2e(1+m)} + \frac{ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{8e} \\
&\quad - \frac{ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{8e} \\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}ab^2e^{2ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-2idx^3\right)}{e} \\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}ab^2e^{-2ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},2idx^3\right)}{e} \\
&\quad - \frac{i3^{-\frac{4}{3}-\frac{m}{3}}b^3e^{3ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-3idx^3\right)}{8e} \\
&\quad + \frac{i3^{-\frac{4}{3}-\frac{m}{3}}b^3e^{-3ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},3idx^3\right)}{8e}
\end{aligned}
$$

output
```
1/2*a*(2*a^2+3*b^2)*(e*x)^(1+m)/e/(1+m)+1/8*I*b*(4*a^2+b^2)*exp(I*c)*(e*x)
^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/8*I*b*(4*a^2+
b^2)*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c
)+2^(-7/3-1/3*m)*a*b^2*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMM
A(1/3+1/3*m,-2*I*d*x^3)/e+2^(-7/3-1/3*m)*a*b^2*(e*x)^(1+m)*(I*d*x^3)^(-1/3
-1/3*m)*GAMMA(1/3+1/3*m,2*I*d*x^3)/e/exp(2*I*c)-1/8*I*3^(-4/3-1/3*m)*b^3*e
xp(3*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-3*I*d*x^3)/
e+1/8*I*3^(-4/3-1/3*m)*b^3*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/
3*m,3*I*d*x^3)/e/exp(3*I*c)
```

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.84

$$\begin{aligned}\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx = \frac{1}{24}ix(ex)^m\Bigg(&-\frac{12ia(2a^2+3b^2)}{1+m}\\&+3b\left(4a^2+b^2\right)e^{ic}\left(-idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-idx^3\right)\\&-3b\left(4a^2+b^2\right)e^{-ic}\left(idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)\\&-3i2^{\frac{2}{3}-\frac{m}{3}}ab^2e^{2ic}\left(-idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-2idx^3\right)\\&-3i2^{\frac{2}{3}-\frac{m}{3}}ab^2e^{-2ic}\left(idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},2idx^3\right)\\&-3^{-\frac{1}{3}-\frac{m}{3}}b^3e^{3ic}\left(-idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-3idx^3\right)\\&+3^{-\frac{1}{3}-\frac{m}{3}}b^3e^{-3ic}\left(idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},3idx^3\right)\Bigg)\end{aligned}$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

output
```
(I/24)*x*(e*x)^m*(((-12*I)*a*(2*a^2 + 3*b^2))/(1 + m) + 3*b*(4*a^2 + b^2)*
E^(I*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (-I)*d*x^3] - (3*b*(4*a
^2 + b^2)*(I*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, I*d*x^3])/E^(I*c) - (3*I
)*2^(2/3 - m/3)*a*b^2*E^((2*I)*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/
3, (-2*I)*d*x^3] - ((3*I)*2^(2/3 - m/3)*a*b^2*(I*d*x^3)^(-1/3 - m/3)*Gamma
[(1 + m)/3, (2*I)*d*x^3])/E^((2*I)*c) - 3^(-1/3 - m/3)*b^3*E^((3*I)*c)*((-
I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3] + (3^(-1/3 - m/3)*b^
3*(I*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (3*I)*d*x^3])/E^((3*I)*c))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3884, 6, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3 \, dx$$

$$\Big\downarrow \text{3884}$$

$$\int \Big(a^3(ex)^m+3a^2b(ex)^m\sin\left(c+dx^3\right)-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right)+\frac{3}{2}ab^2(ex)^m+\frac{3}{4}b^3(ex)^m\sin\left(c+dx^3\right)-$$

$$\Big\downarrow \text{6}$$

$$\int \Big(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m+3a^2b(ex)^m\sin\left(c+dx^3\right)-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right)+\frac{3}{4}b^3(ex)^m\sin\left(c+dx^3\right)-\frac{1}{4}$$

$$\Big\downarrow \text{6}$$

$$\int \Big(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m+\left(3a^2b+\frac{3b^3}{4}\right)(ex)^m\sin\left(c+dx^3\right)-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right)-\frac{1}{4}b^3(ex)^m\sin(3c$$

$$\Big\downarrow \text{2009}$$

$$\frac{ibe^{ic}\left(4a^2+b^2\right)\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-idx^3\right)}{8e}-\frac{ibe^{-ic}\left(4a^2+b^2\right)\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},idx^3\right)}{8e}+\frac{a\left(2a^2+3b^2\right)(ex)^{m+1}}{2e(m+1)}+\frac{ab^2e^{2ic}2^{-\frac{m}{3}-\frac{7}{3}}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-2idx^3\right)}{e}+\frac{ab^2e^{-2ic}2^{-\frac{m}{3}-\frac{7}{3}}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},2idx^3\right)}{e}-\frac{ib^3e^{3ic}3^{-\frac{m}{3}-\frac{4}{3}}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-3idx^3\right)}{8e}+\frac{ib^3e^{-3ic}3^{-\frac{m}{3}-\frac{4}{3}}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},3idx^3\right)}{8e}$$

input

```
Int[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

output

```
(a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/8)*b*(4*a^2 + b^2)*E
^(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3
])/e - ((I/8)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(
1 + m)/3, I*d*x^3])/(e*E^(I*c)) + (2^(-7/3 - m/3)*a*b^2*E^((2*I)*c)*(e*x)^
(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3])/e + (2^(
-7/3 - m/3)*a*b^2*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2
*I)*d*x^3])/(e*E^((2*I)*c)) - ((I/8)*3^(-4/3 - m/3)*b^3*E^((3*I)*c)*(e*x)^
(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3])/e + ((I/
8)*3^(-4/3 - m/3)*b^3*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3
, (3*I)*d*x^3])/(e*E^((3*I)*c))
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int \left(e x \right)^{m} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{3}d x$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.79

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^3 \, dx$$

$$= \frac{36\,(2\,a^{3}+3\,ab^{2})(ex)^{m}\,dx+(b^{3}e^{2}m+b^{3}e^{2})e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{3i\,d}{e^{3}}\right)-3i\,c\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},3i\,dx^{3}\right)-9\,(i\,ab^{2}e^{2}m+i\,ab}{}$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="fricas")
```

output
```
1/72*(36*(2*a^3 + 3*a*b^2)*(e*x)^m*d*x + (b^3*e^2*m + b^3*e^2)*e^(-1/3*(m
- 2)*log(3*I*d/e^3) - 3*I*c)*gamma(1/3*m + 1/3, 3*I*d*x^3) - 9*(I*a*b^2*e^
2*m + I*a*b^2*e^2)*e^(-1/3*(m - 2)*log(2*I*d/e^3) - 2*I*c)*gamma(1/3*m + 1
/3, 2*I*d*x^3) - 9*((4*a^2*b + b^3)*e^2*m + (4*a^2*b + b^3)*e^2)*e^(-1/3*(
m - 2)*log(I*d/e^3) - I*c)*gamma(1/3*m + 1/3, I*d*x^3) - 9*((4*a^2*b + b^3
)*e^2*m + (4*a^2*b + b^3)*e^2)*e^(-1/3*(m - 2)*log(-I*d/e^3) + I*c)*gamma(
1/3*m + 1/3, -I*d*x^3) - 9*(-I*a*b^2*e^2*m - I*a*b^2*e^2)*e^(-1/3*(m - 2)*
log(-2*I*d/e^3) + 2*I*c)*gamma(1/3*m + 1/3, -2*I*d*x^3) + (b^3*e^2*m + b^3
*e^2)*e^(-1/3*(m - 2)*log(-3*I*d/e^3) + 3*I*c)*gamma(1/3*m + 1/3, -3*I*d*x
^3))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^3 \, dx=\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^3 \, dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**3+c))**3,x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**3))**3, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^3 \, dx=\int \left(b\sin \left(dx^3+c\right)+a\right)^3(ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="maxima")
```

output
```
(e*x)^(m + 1)*a^3/(e*(m + 1)) + 1/8*(12*a*b^2*e^m*x*x^m - 12*(a*b^2*e^m*m
+ a*b^2*e^m)*integrate(x^m*cos(2*d*x^3 + 2*c), x) + 3*((4*a^2*b + b^3)*e^m
*m*sin(c) + (4*a^2*b + b^3)*e^m*sin(c))*integrate(x^m*cos(d*x^3), x) - 2*(
b^3*e^m*m + b^3*e^m)*integrate(x^m*sin(3*d*x^3 + 3*c), x) + 3*((4*a^2*b +
b^3)*e^m*m + (4*a^2*b + b^3)*e^m)*integrate(x^m*sin(d*x^3 + c), x) + 3*((4
*a^2*b + b^3)*e^m*m*cos(c) + (4*a^2*b + b^3)*e^m*cos(c))*integrate(x^m*sin
(d*x^3), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^3 \, dx=\int \left(b\sin \left(dx^3+c\right)+a\right)^3(ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)^3*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx=\int (e\,x)^m\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^3\,dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^3))^3,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^3))^3, x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx$$
$$=\frac{e^m\left(x^m a^3 x+6x^m a\,b^2 x-6\left(\int x^m dx\right)a\,b^2 m-6\left(\int x^m dx\right)a\,b^2+\left(\int x^m\sin\left(d\,x^3+c\right)^3dx\right)b^3 m+\left(\int x^m s\right.}{}$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

output
```
(e**m*(x**m*a**3*x + 6*x**m*a*b**2*x - 6*int(x**m,x)*a*b**2*m - 6*int(x**m
,x)*a*b**2 + int(x**m*sin(c + d*x**3)**3,x)*b**3*m + int(x**m*sin(c + d*x*
*3)**3,x)*b**3 + 3*int(x**m*sin(c + d*x**3)**2,x)*a*b**2*m + 3*int(x**m*si
n(c + d*x**3)**2,x)*a*b**2 + 3*int(x**m*sin(c + d*x**3),x)*a**2*b*m + 3*in
t(x**m*sin(c + d*x**3),x)*a**2*b))/(m + 1)
```

3.99 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 285

$$
\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{3e} \\
&\quad - \frac{iabe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{3e} \\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}b^2e^{2ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-2idx^3\right)}{3e} \\
&\quad + \frac{2^{-\frac{7}{3}-\frac{m}{3}}b^2e^{-2ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},2idx^3\right)}{3e}
\end{aligned}
$$

output

```
1/2*(2*a^2+b^2)*(e*x)^(1+m)/e/(1+m)+1/3*I*a*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x
^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/3*I*a*b*(e*x)^(1+m)*(I*d*x^
3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c)+1/3*2^(-7/3-1/3*m)*b^2
*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-2*I*d*x^3
)/e+1/3*2^(-7/3-1/3*m)*b^2*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/
3*m,2*I*d*x^3)/e/exp(2*I*c)
```

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.95

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$= \frac{2^{\frac{1}{3}(-7-m)}x(ex)^m\left(d^2x^6\right)^{\frac{1}{3}(-1-m)}\Big(3\,2^{\frac{7+m}{3}}a^2\left(d^2x^6\right)^{\frac{1+m}{3}}+3\,2^{\frac{4+m}{3}}b^2\left(d^2x^6\right)^{\frac{1+m}{3}}+b^2\left(idx^3\right)^{\frac{1+m}{3}}\cos(2c)\Gamma\big(\frac{1+m}{3},}{}$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```

output
```
(2^((-7 - m)/3)*x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(3*2^((7 + m)/3)*a^2*(d^2
*x^6)^((1 + m)/3) + 3*2^((4 + m)/3)*b^2*(d^2*x^6)^((1 + m)/3) + b^2*(I*d*x
^3)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*m*(I*d*x^3)^
((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*((-I)*d*x^3)^((1
 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] + b^2*m*((-I)*d*x^3)^((1 +
 m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] - I*2^((7 + m)/3)*a*b*(1 + m
)*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) +
 I*2^((7 + m)/3)*a*b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d
*x^3]*(Cos[c] + I*Sin[c]) + I*b^2*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (
-2*I)*d*x^3]*Sin[2*c] + I*b^2*m*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2
*I)*d*x^3]*Sin[2*c] - I*b^2*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I
)*d*x^3]*Sin[2*c] - I*b^2*m*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I
)*d*x^3]*Sin[2*c]))/(3*(1 + m))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3884, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

$$\Big\downarrow \text{ 3884}$$

$$\int \left(a^2(ex)^m+2ab(ex)^m\sin\left(c+dx^3\right)-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right)+\frac{1}{2}b^2(ex)^m\right)dx$$

$$\Big\downarrow 6$$

$$\int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m+2ab(ex)^m\sin\left(c+dx^3\right)-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right)\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{\left(2a^2+b^2\right)(ex)^{m+1}}{2e(m+1)}+\frac{iabe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-idx^3\right)}{3e}-\frac{iabe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},idx^3\right)}{3e}+\frac{b^2e^{2ic}2^{-\frac{m}{3}-\frac{7}{3}}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-2idx^3\right)}{3e}+\frac{b^2e^{-2ic}2^{-\frac{m}{3}-\frac{7}{3}}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},2idx^3\right)}{3e}$$

input

```
Int[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```

output

```
((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/3)*a*b*E^(I*c)*(e*x)^(1
+ m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/3)*a*
b*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*
c)) + (2^(-7/3 - m/3)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)
/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3])/(3*e) + (2^(-7/3 - m/3)*b^2*(e*x)^(1 +
 m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3])/(3*e*E^((2*I)*c)
)
```

Defintions of rubi rules used

rule 6

```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int (ex)^{m} \left(a+b\sin \left(d\,x^{3}+c\right)\right)^{2}dx$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.75

$$\int (ex)^m \left(a+b\sin \left(c+dx^3\right)\right)^2 \, dx$$

$$= \frac{12\,(2\,a^2+b^2)(ex)^m\,dx+(-i\,b^2e^2m-i\,b^2e^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{2i\,d}{e^3}\right)-2i\,c\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},2i\,dx^3\right)-8\,(abe^2m+abe}{}$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
1/24*(12*(2*a^2 + b^2)*(e*x)^m*d*x + (-I*b^2*e^2*m - I*b^2*e^2)*e^(-1/3*(m
 - 2)*log(2*I*d/e^3) - 2*I*c)*gamma(1/3*m + 1/3, 2*I*d*x^3) - 8*(a*b*e^2*m
 + a*b*e^2)*e^(-1/3*(m - 2)*log(I*d/e^3) - I*c)*gamma(1/3*m + 1/3, I*d*x^3
) - 8*(a*b*e^2*m + a*b*e^2)*e^(-1/3*(m - 2)*log(-I*d/e^3) + I*c)*gamma(1/3
*m + 1/3, -I*d*x^3) + (I*b^2*e^2*m + I*b^2*e^2)*e^(-1/3*(m - 2)*log(-2*I*d
/e^3) + 2*I*c)*gamma(1/3*m + 1/3, -2*I*d*x^3))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \,dx = \int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \,dx$$

input

```
integrate((e*x)**m*(a+b*sin(d*x**3+c))**2,x)
```

output

```
Integral((e*x)**m*(a + b*sin(c + d*x**3))**2, x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \,dx = \int \left(b\sin\left(dx^3+c\right)+a\right)^2 (ex)^m \,dx$$

input

```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
(e*x)^(m + 1)*a^2/(e*(m + 1)) + 1/2*(b^2*e^m*x*x^m - (b^2*e^m*m + b^2*e^m)
*integrate(x^m*cos(2*d*x^3 + 2*c), x) + 4*(a*b*e^m*m + a*b*e^m)*integrate(
x^m*sin(d*x^3 + c), x))/(m + 1)
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \,dx = \int \left(b\sin\left(dx^3+c\right)+a\right)^2 (ex)^m \,dx$$

input

```
integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output

```
integrate((b*sin(d*x^3 + c) + a)^2*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx = \int (e\,x)^m \left(a+b\,\sin\left(d\,x^3+c\right)\right)^2 dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^3))^2,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^3))^2, x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx$$
$$= \frac{e^m \left(x^m a^2 x + 2x^m b^2 x - 2\left(\int x^m dx\right) b^2 m - 2\left(\int x^m dx\right) b^2 + \left(\int x^m \sin\left(d\,x^3+c\right)^2 dx\right) b^2 m + \left(\int x^m \sin (d\right.\right.}{m+1}$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

output
```
(e**m*(x**m*a**2*x + 2*x**m*b**2*x - 2*int(x**m,x)*b**2*m - 2*int(x**m,x)*
b**2 + int(x**m*sin(c + d*x**3)**2,x)*b**2*m + int(x**m*sin(c + d*x**3)**2
,x)*b**2 + 2*int(x**m*sin(c + d*x**3),x)*a*b*m + 2*int(x**m*sin(c + d*x**3
),x)*a*b))/(m + 1)
```

3.100 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\frac{a(ex)^{1+m}}{e(1+m)}+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{6e}$$

output
```
a*(e*x)^(1+m)/e/(1+m)+1/6*I*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)
*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/6*I*b*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GA
MMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

$$=\frac{x(ex)^m\left(d^2x^6\right)^{\frac{1}{3}(-1-m)}\left(6a\left(d^2x^6\right)^{\frac{1+m}{3}}-ib(1+m)\left(-idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)(\cos(c)-i\sin(c))+ib(1+\right.}{6(1+m)}$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

output
```
(x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(6*a*(d^2*x^6)^((1 + m)/3) - I*b*(1 + m)
*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) +
I*b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(Cos[c] + I
*Sin[c])))/(6*(1 + m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(a(ex)^m+b(ex)^m\sin\left(c+dx^3\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(ex)^{m+1}}{e(m+1)}+\frac{ibe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{3},idx^3\right)}{6e}$$

input
```
Int[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

output
```
(a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/6)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^3
)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/6)*b*(e*x)^(1 + m)*(I
*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int (ex)^m \left(a+b\sin\left(d\,x^3+c\right)\right) dx$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

output
```
int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\, dx$$
$$= \frac{6\,(ex)^m\,adx - (be^2m+be^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{i\,d}{e^3}\right)-i\,c\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},i\,dx^3\right) - (be^2m+be^2)e^{\left(-\frac{1}{3}\,(m-2)\log\left(-\frac{i\,d}{e^3}\right)+i\,c\right)}}{6\,(dm+d)}$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/6*(6*(e*x)^m*a*d*x - (b*e^2*m + b*e^2)*e^(-1/3*(m - 2)*log(I*d/e^3) - I*
c)*gamma(1/3*m + 1/3, I*d*x^3) - (b*e^2*m + b*e^2)*e^(-1/3*(m - 2)*log(-I*
d/e^3) + I*c)*gamma(1/3*m + 1/3, -I*d*x^3))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

input
```
integrate((e*x)**m*(a+b*sin(d*x**3+c)),x)
```

output
```
Integral((e*x)**m*(a + b*sin(c + d*x**3)), x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
b*e^m*integrate(x^m*sin(d*x^3 + c), x) + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx=\int \left(b\sin\left(dx^3+c\right)+a\right)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x^3 + c) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) \, dx = \int {\left(e\,x\right)}^m\,\left(a+b\,\sin\left(d\,x^3+c\right)\right) \,d x$$

input
```
int((e*x)^m*(a + b*sin(c + d*x^3)),x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^3)), x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) \, dx$$
$$= \frac{e^m \left(x^m a x+\left(\int x^m \sin\left(d\,x^3+c\right)dx\right) b m+\left(\int x^m \sin\left(d\,x^3+c\right)dx\right) b\right)}{m+1}$$

input
```
int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

output
```
(e**m*(x**m*a*x + int(x**m*sin(c + d*x**3),x)*b*m + int(x**m*sin(c + d*x**
3),x)*b))/(m + 1)
```

3.101 $\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \mathrm{Int}\left(\frac{(ex)^m}{a+b\sin(c+dx^3)},x\right)$$

output
```
Defer(Int)((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

input
```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
```

output
```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3908\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 3908}$$

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}dx$$

input

```
Int[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{a+b\sin(d\,x^3+c)}dx$$

input

```
int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

output
```
int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$$

input
```
integrate((e*x)**m/(a+b*sin(d*x**3+c)),x)
```

output
```
Integral((e*x)**m/(a + b*sin(c + d*x**3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(ex)^m}{b\sin(dx^3+c)+a}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

output
```
integrate((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 42.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx=\int \frac{(e\,x)^m}{a+b\,\sin(d\,x^3+c)}\,dx$$

input
```
int((e*x)^m/(a + b*sin(c + d*x^3)),x)
```

output
```
int((e*x)^m/(a + b*sin(c + d*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.05

$$\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx = \frac{e^m\left(x^m x-\left(\int \frac{x^m \sin(d\,x^3+c)}{\sin(d\,x^3+c)b+a}dx\right)bm-\left(\int \frac{x^m \sin(d\,x^3+c)}{\sin(d\,x^3+c)b+a}dx\right)b\right)}{a\,(m+1)}$$

input
```
int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

output
```
(e**m*(x**m*x - int((x**m*sin(c + d*x**3))/(sin(c + d*x**3)*b + a),x)*b*m
- int((x**m*sin(c + d*x**3))/(sin(c + d*x**3)*b + a),x)*b))/(a*(m + 1))
```

3.102 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \text{Int}\left(\frac{(ex)^m}{(a+b\sin(c+dx^3))^2},x\right)$$

output

```
Defer(Int)((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

input

```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
```

output

```
Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}dx \\ \Big\downarrow \text{3908} \\ \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}dx \end{array}$$

input
```
Int[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{(a+b\sin(d\,x^3+c))^2}dx$$

input
```
int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

output
```
int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{(ex)^m}{(b\sin(dx^3+c)+a)^2}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

output
```
integral(-(e*x)^m/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2
), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx=\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

input
```
integrate((e*x)**m/(a+b*sin(d*x**3+c))**2,x)
```

output
```
Integral((e*x)**m/(a + b*sin(c + d*x**3))**2, x)
```

Maxima [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 2505, normalized size of antiderivative = 125.25

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(b\sin(dx^3+c)+a)^2}\,dx$$

input

```
integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

output

```
1/3*(4*a*b*e^m*x^m*cos(d*x^3)*cos(c) + 2*b^2*e^m*x^m*cos(2*c)*sin(2*d*x^3)
 + 2*b^2*e^m*x^m*cos(2*d*x^3)*sin(2*c) - 4*a*b*e^m*x^m*sin(d*x^3)*sin(c) +
 2*(a*b*e^m*x^m*cos(2*d*x^3)*cos(2*c) - 2*a^2*e^m*x^m*cos(c)*sin(d*x^3) -
a*b*e^m*x^m*sin(2*d*x^3)*sin(2*c) - 2*a^2*e^m*x^m*cos(d*x^3)*sin(c) - a*b*
e^m*x^m)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)
*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a
^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*
b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(
c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*
x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 -
b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2
*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b
- a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^
3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)
*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) -
(a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*
sin(2*c))*sin(2*d*x^3))*integrate(2/3*((b^2*e^m*m*sin(2*c) - 2*b^2*e^m*sin
(2*c))*x^m*cos(2*d*x^3) + 2*(a*b*e^m*m*cos(c) - 2*a*b*e^m*cos(c))*x^m*cos(
d*x^3) + (b^2*e^m*m*cos(2*c) - 2*b^2*e^m*cos(2*c))*x^m*sin(2*d*x^3) - 2*(a
*b*e^m*m*sin(c) - 2*a*b*e^m*sin(c))*x^m*sin(d*x^3) - ((3*a*b*d*e^m*x^3*...
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{(ex)^m}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

input
```
integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{(e\,x)^m}{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

input
```
int((e*x)^m/(a + b*sin(c + d*x^3))^2,x)
```

output
```
int((e*x)^m/(a + b*sin(c + d*x^3))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{e^m\left(x^m x-\left(\int \frac{x^m\sin\left(d x^3+c\right)^2}{\sin(d x^3+c)^2 b^2+2\sin(d x^3+c)ab+a^2}dx\right)b^2 m-\left(\int \frac{x^m\sin\left(d x^3+c\right)^2}{\sin(d x^3+c)^2 b^2+2\sin(d x^3+c)ab+a^2}dx\right)b^2-2\left(\int \frac{}{\sin(d x^3}\right.\right.}{a^2\left(m+1\right)}$$

input

```
int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

output

```
(e**m*(x**m*x - int((x**m*sin(c + d*x**3)**2)/(sin(c + d*x**3)**2*b**2 + 2
*sin(c + d*x**3)*a*b + a**2),x)*b**2*m - int((x**m*sin(c + d*x**3)**2)/(si
n(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*b + a**2),x)*b**2 - 2*int((x**
m*sin(c + d*x**3))/(sin(c + d*x**3)**2*b**2 + 2*sin(c + d*x**3)*a*b + a**2
),x)*a*b*m - 2*int((x**m*sin(c + d*x**3))/(sin(c + d*x**3)**2*b**2 + 2*sin
(c + d*x**3)*a*b + a**2),x)*a*b))/(a**2*(m + 1))
```

3.103 $\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int x^2\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)+\frac{1}{6}b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

output
```
1/6*b*x^2*cos(a+b/x)+1/6*b^3*cos(a)*Ci(b/x)-1/6*b^2*x*sin(a+b/x)+1/3*x^3*s
in(a+b/x)-1/6*b^3*sin(a)*Si(b/x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int x^2\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{6}\left(b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)+x\left(bx\cos\left(a+\frac{b}{x}\right)-b^2\sin\left(a+\frac{b}{x}\right)+2x^2\sin\left(a+\frac{b}{x}\right)\right)-b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

input
```
Integrate[x^2*Sin[a + b/x],x]
```

output

```
(b^3*Cos[a]*CosIntegral[b/x] + x*(b*x*Cos[a + b/x] - b^2*Sin[a + b/x] + 2*
x^2*Sin[a + b/x]) - b^3*Sin[a]*SinIntegral[b/x])/6
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin\left(a+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{3860}$$

$$-\int x^4 \sin\left(a+\frac{b}{x}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^4 \sin\left(a+\frac{b}{x}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) - \frac{1}{3}b\int x^3 \cos\left(a+\frac{b}{x}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) - \frac{1}{3}b\int x^3 \sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) - \frac{1}{3}b\left(\frac{1}{2}b\int -x^2 \sin\left(a+\frac{b}{x}\right) d\frac{1}{x} - \frac{1}{2}x^2 \cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) - \frac{1}{3}b\left(-\frac{1}{2}b\int x^2 \sin\left(a+\frac{b}{x}\right) d\frac{1}{x} - \frac{1}{2}x^2 \cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\int x^2\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\left(b\int x\cos\left(a+\frac{b}{x}\right)d\frac{1}{x}-x\sin\left(a+\frac{b}{x}\right)\right)-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\left(b\int x\sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-x\sin\left(a+\frac{b}{x}\right)\right)-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\int x\cos\left(\frac{b}{x}\right)d\frac{1}{x}-\sin(a)\int x\sin\left(\frac{b}{x}\right)d\frac{1}{x}\right)-x\sin\left(a+\frac{b}{x}\right)\right)-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\int x\sin\left(\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-\sin(a)\int x\sin\left(\frac{b}{x}\right)d\frac{1}{x}\right)-x\sin\left(a+\frac{b}{x}\right)\right)-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\int x\sin\left(\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)-x\sin\left(a+\frac{b}{x}\right)\right)-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)-\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)-x\sin\left(a+\frac{b}{x}\right)\right)-\frac{1}{2}x^2\cos\left(a+\frac{b}{x}\right)\right)$$

input
```
Int[x^2*Sin[a + b/x],x]
```

output
```
(x^3*Sin[a + b/x])/3 - (b*(-1/2*(x^2*Cos[a + b/x]) - (b*(-(x*Sin[a + b/x])
 + b*(Cos[a]*CosIntegral[b/x] - Sin[a]*SinIntegral[b/x])))/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-b^{3}\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^{3}}{3b^{3}}-\frac{\cos\left(a+\frac{b}{x}\right)x^{2}}{6b^{2}}+\frac{\sin\left(a+\frac{b}{x}\right)x}{6b}+\frac{\operatorname{Si}\left(\frac{b}{x}\right)\sin(a)}{6}-\frac{\operatorname{Ci}\left(\frac{b}{x}\right)\cos(a)}{6}\right)$
default	$-b^{3}\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^{3}}{3b^{3}}-\frac{\cos\left(a+\frac{b}{x}\right)x^{2}}{6b^{2}}+\frac{\sin\left(a+\frac{b}{x}\right)x}{6b}+\frac{\operatorname{Si}\left(\frac{b}{x}\right)\sin(a)}{6}-\frac{\operatorname{Ci}\left(\frac{b}{x}\right)\cos(a)}{6}\right)$
risch	$\frac{i\pi\ \operatorname{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b^{3}}{12}-\frac{i\operatorname{Si}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b^{3}}{6}-\frac{\operatorname{expIntegral}_{1}\left(-\frac{ib}{x}\right)\mathrm{e}^{-ia}b^{3}}{12}-\frac{\mathrm{e}^{ia}\operatorname{expIntegral}_{1}\left(-\frac{ib}{x}\right)b^{3}}{12}+\frac{x^{2}b\cos\left(\frac{ax+}{x}\right.}{6}$
parts	$b\,x^{2}\operatorname{Si}\left(\frac{b}{x}\right)\sin(a)-b\,x^{2}\operatorname{Ci}\left(\frac{b}{x}\right)\cos(a)+x^{3}\sin\left(a+\frac{b}{x}\right)+2b\left(-\cos(a)\,b^{2}\left(-\frac{x^{2}\operatorname{Ci}\left(\frac{b}{x}\right)}{2b^{2}}-\right.\right.$
meijerg	$-\frac{b^{3}\sqrt{\pi}\cos(a)\left(-\frac{8x^{2}}{\sqrt{\pi}\,b^{2}}-\frac{4\left(2\gamma-\frac{11}{3}-2\ln(x)+2\ln(b)\right)}{3\sqrt{\pi}}+\frac{8x^{2}\left(-\frac{55b^{2}}{2x^{2}}+45\right)}{45\sqrt{\pi}\,b^{2}}+\frac{8\gamma}{3\sqrt{\pi}}+\frac{8\ln(2)}{3\sqrt{\pi}}+\frac{8\ln\left(\frac{b}{2x}\right)}{3\sqrt{\pi}}-\frac{8x^{2}\cos\left(\frac{b}{x}\right)}{3\sqrt{\pi}\,b^{2}}-\frac{16x^{3}(}{}\right.}{16}$

input
```
int(x^2*sin(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
-b^3*(-1/3*sin(a+b/x)/b^3*x^3-1/6*cos(a+b/x)/b^2*x^2+1/6*sin(a+b/x)/b*x+1/
6*Si(b/x)*sin(a)-1/6*Ci(b/x)*cos(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int x^{2}\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{6}\,b^{3}\cos(a)\operatorname{Ci}\left(\frac{b}{x}\right)-\frac{1}{6}\,b^{3}\sin(a)\operatorname{Si}\left(\frac{b}{x}\right)+\frac{1}{6}\,bx^{2}\cos\left(\frac{ax+b}{x}\right)-\frac{1}{6}\left(b^{2}x-2\,x^{3}\right)\sin\left(\frac{ax+b}{x}\right)$$

input
```
integrate(x^2*sin(a+b/x),x, algorithm="fricas")
```

output
```
1/6*b^3*cos(a)*cos_integral(b/x) - 1/6*b^3*sin(a)*sin_integral(b/x) + 1/6*
b*x^2*cos((a*x + b)/x) - 1/6*(b^2*x - 2*x^3)*sin((a*x + b)/x)
```

Sympy [F]

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx = \int x^2 \sin\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(x**2*sin(a+b/x),x)
```

output
```
Integral(x**2*sin(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\begin{aligned}\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx &= \frac{1}{12}\left(\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin(a)\right)b^3 \\ &\quad+\frac{1}{6}\,bx^2\cos\left(\frac{ax+b}{x}\right)-\frac{1}{6}\,(b^2x-2\,x^3)\sin\left(\frac{ax+b}{x}\right)\end{aligned}$$

input
```
integrate(x^2*sin(a+b/x),x, algorithm="maxima")
```

output
```
1/12*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) + (I*Ei(I*b/x) - I*Ei(-I*b/x))*sin(a
))*b^3 + 1/6*b*x^2*cos((a*x + b)/x) - 1/6*(b^2*x - 2*x^3)*sin((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 400, normalized size of antiderivative = 5.13

$$\int x^2 \sin\left(a + \frac{b}{x}\right) \, dx$$
$$= \frac{a^3b^4 \cos(a)\,\mathrm{Ci}\left(-a + \frac{ax+b}{x}\right) + a^3b^4 \sin(a)\,\mathrm{Si}\left(a - \frac{ax+b}{x}\right) - \frac{3\,(ax+b)a^2b^4 \cos(a)\,\mathrm{Ci}\left(-a+\frac{ax+b}{x}\right)}{x} - \frac{3\,(ax+b)a^2b^4 \sin(a)\,\mathrm{Si}(}{x}}{}$$

input
```
integrate(x^2*sin(a+b/x),x, algorithm="giac")
```

output
```
1/6*(a^3*b^4*cos(a)*cos_integral(-a + (a*x + b)/x) + a^3*b^4*sin(a)*sin_in
tegral(a - (a*x + b)/x) - 3*(a*x + b)*a^2*b^4*cos(a)*cos_integral(-a + (a*
x + b)/x)/x - 3*(a*x + b)*a^2*b^4*sin(a)*sin_integral(a - (a*x + b)/x)/x +
 3*(a*x + b)^2*a*b^4*cos(a)*cos_integral(-a + (a*x + b)/x)/x^2 + a^2*b^4*s
in((a*x + b)/x) + 3*(a*x + b)^2*a*b^4*sin(a)*sin_integral(a - (a*x + b)/x)
/x^2 + a*b^4*cos((a*x + b)/x) - (a*x + b)^3*b^4*cos(a)*cos_integral(-a + (
a*x + b)/x)/x^3 - 2*(a*x + b)*a*b^4*sin((a*x + b)/x)/x - (a*x + b)^3*b^4*s
in(a)*sin_integral(a - (a*x + b)/x)/x^3 - (a*x + b)*b^4*cos((a*x + b)/x)/x
 - 2*b^4*sin((a*x + b)/x) + (a*x + b)^2*b^4*sin((a*x + b)/x)/x^2)/((a^3 -
3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin\left(a + \frac{b}{x}\right) \, dx = \int x^2 \sin\left(a + \frac{b}{x}\right) \, dx$$

input
```
int(x^2*sin(a + b/x),x)
```

output
```
int(x^2*sin(a + b/x), x)
```

Reduce [F]

$$\int x^2 \sin\left(a+\frac{b}{x}\right)\,dx = \int \sin\left(\frac{ax+b}{x}\right)x^2dx$$

input
```
int(x^2*sin(a+b/x),x)
```

output
```
int(sin((a*x + b)/x)*x**2,x)
```

3.104 $\int x \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int x \sin\left(a+\frac{b}{x}\right)\,dx = \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) + \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

output
```
1/2*b*x*cos(a+b/x)+1/2*b^2*Ci(b/x)*sin(a)+1/2*x^2*sin(a+b/x)+1/2*b^2*cos(a
)*Si(b/x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x \sin\left(a+\frac{b}{x}\right)\,dx = \frac{1}{2}\left(b^2\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) + x\left(b\cos\left(a+\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)\right)+b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

input
```
Integrate[x*Sin[a + b/x],x]
```

output

```
(b^2*CosIntegral[b/x]*Sin[a] + x*(b*Cos[a + b/x] + x*Sin[a + b/x]) + b^2*C
os[a]*SinIntegral[b/x])/2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3860, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin\left(a+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{3860}$$

$$-\int x^3\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\int x^2\cos\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\int x^2\sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(b\int -x\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}-x\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(-b\int x\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}-x\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(-b\int x\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}-x\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(-b\left(\sin(a)\int x\cos\left(\frac{b}{x}\right)d\frac{1}{x}+\cos(a)\int x\sin\left(\frac{b}{x}\right)d\frac{1}{x}\right)-x\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(-b\left(\sin(a)\int x\sin\left(\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+\cos(a)\int x\sin\left(\frac{b}{x}\right)d\frac{1}{x}\right)-x\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(-b\left(\sin(a)\int x\sin\left(\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+\cos(a)\text{Si}\left(\frac{b}{x}\right)\right)-x\cos\left(a+\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(-b\left(\sin(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)+\cos(a)\text{Si}\left(\frac{b}{x}\right)\right)-x\cos\left(a+\frac{b}{x}\right)\right)$$

input
```
Int[x*Sin[a + b/x],x]
```

output
```
(x^2*Sin[a + b/x])/2 - (b*(-(x*Cos[a + b/x]) - b*(CosIntegral[b/x]*Sin[a]
+ Cos[a]*SinIntegral[b/x])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-b^2\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^2}{2b^2}-\frac{\cos\left(a+\frac{b}{x}\right)x}{2b}-\frac{\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)}{2}-\frac{\operatorname{Ci}\left(\frac{b}{x}\right)\sin(a)}{2}\right)$
default	$-b^2\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^2}{2b^2}-\frac{\cos\left(a+\frac{b}{x}\right)x}{2b}-\frac{\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)}{2}-\frac{\operatorname{Ci}\left(\frac{b}{x}\right)\sin(a)}{2}\right)$
risch	$-\frac{\mathrm{e}^{-ia}\pi\ \mathrm{csgn}\left(\frac{b}{x}\right)b^2}{4}+\frac{\mathrm{e}^{-ia}\operatorname{Si}\left(\frac{b}{x}\right)b^2}{2}-\frac{i\,\mathrm{expIntegral}_1\left(-\frac{ib}{x}\right)\mathrm{e}^{-ia}b^2}{4}+\frac{ib^2\,\mathrm{expIntegral}_1\left(-\frac{ib}{x}\right)\mathrm{e}^{ia}}{4}+\frac{bx\cos\left(\frac{ax}{\right.}}{2}$
parts	$bx\ \operatorname{Si}\left(\frac{b}{x}\right)\sin(a)-bx\ \operatorname{Ci}\left(\frac{b}{x}\right)\cos(a)+x^2\sin\left(a+\frac{b}{x}\right)+b\left(-\cos(a)\,b\left(-\frac{x\ \operatorname{Ci}\left(\frac{b}{x}\right)}{b}-\frac{\cos\left(\right.}{}\right.\right.$
meijerg	$-\frac{b^2\sqrt{\pi}\ \cos(a)\left(-\frac{4x\cos\left(\frac{b}{x}\right)}{b\sqrt{\pi}}-\frac{4x^2\sin\left(\frac{b}{x}\right)}{b^2\sqrt{\pi}}-\frac{4\ \operatorname{Si}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{8}-\frac{b^2\sqrt{\pi}\ \sin(a)\left(-\frac{4x^2}{\sqrt{\pi}\,b^2}-\frac{2\left(2\gamma-3-2\ln(x)+\ln\left(b^2\right)\right)}{\sqrt{\pi}}+\frac{4x^2\left(-\frac{9b}{2a}\right.}{3\sqrt{\pi}}\right.}{}$

input
```
int(x*sin(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
-b^2*(-1/2*sin(a+b/x)/b^2*x^2-1/2*cos(a+b/x)/b*x-1/2*cos(a)*Si(b/x)-1/2*Ci
(b/x)*sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\frac{1}{2}\,b^2\operatorname{Ci}\left(\frac{b}{x}\right)\sin(a)+\frac{1}{2}\,b^2\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)$$

input
```
integrate(x*sin(a+b/x),x, algorithm="fricas")
```

output
```
1/2*b^2*cos_integral(b/x)*sin(a) + 1/2*b^2*cos(a)*sin_integral(b/x) + 1/2*
b*x*cos((a*x + b)/x) + 1/2*x^2*sin((a*x + b)/x)
```

Sympy [F]

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\int x\sin\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(x*sin(a+b/x),x)
```

output
```
Integral(x*sin(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

$$\begin{aligned}\int x\sin\left(a+\frac{b}{x}\right)\,dx
&=\frac{1}{4}\left(\left(-i\operatorname{Ei}\left(\frac{ib}{x}\right)+i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a)+\left(\operatorname{Ei}\left(\frac{ib}{x}\right)+\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)\right)b^2\\
&\quad+\frac{1}{2}bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}x^2\sin\left(\frac{ax+b}{x}\right)\end{aligned}$$

input
```
integrate(x*sin(a+b/x),x, algorithm="maxima")
```

output
```
1/4*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a
))*b^2 + 1/2*b*x*cos((a*x + b)/x) + 1/2*x^2*sin((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.18

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx$$
$$= \frac{a^2b^3\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a) - a^2b^3\cos(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right) - \frac{2(ax+b)ab^3\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)}{x} + \frac{2(ax+b)ab^3\cos(a)\operatorname{Si}\left(a\right.}{x}}{2\left(a^2-\frac{2(a}{}\right.}$$

input
```
integrate(x*sin(a+b/x),x, algorithm="giac")
```

output
```
1/2*(a^2*b^3*cos_integral(-a + (a*x + b)/x)*sin(a) - a^2*b^3*cos(a)*sin_in
tegral(a - (a*x + b)/x) - 2*(a*x + b)*a*b^3*cos_integral(-a + (a*x + b)/x)
*sin(a)/x + 2*(a*x + b)*a*b^3*cos(a)*sin_integral(a - (a*x + b)/x)/x - a*b
^3*cos((a*x + b)/x) + (a*x + b)^2*b^3*cos_integral(-a + (a*x + b)/x)*sin(a
)/x^2 - (a*x + b)^2*b^3*cos(a)*sin_integral(a - (a*x + b)/x)/x^2 + (a*x +
b)*b^3*cos((a*x + b)/x)/x + b^3*sin((a*x + b)/x))/((a^2 - 2*(a*x + b)*a/x
+ (a*x + b)^2/x^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx = \int x\,\sin\left(a+\frac{b}{x}\right)\,dx$$

input
```
int(x*sin(a + b/x),x)
```

output
```
int(x*sin(a + b/x), x)
```

Reduce [F]

$$\int x\sin\left(a+\frac{b}{x}\right)\,dx=\frac{\cos\left(\frac{ax+b}{x}\right)bx}{2}-\frac{\left(\int\frac{\sin\left(\frac{ax+b}{x}\right)}{x}dx\right)b^2}{2}+\frac{\sin\left(\frac{ax+b}{x}\right)x^2}{2}$$

input
```
int(x*sin(a+b/x),x)
```

output
```
(cos((a*x + b)/x)*b*x - int(sin((a*x + b)/x)/x,x)*b**2 + sin((a*x + b)/x)*
x**2)/2
```

3.105 $\int \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \sin\left(a+\frac{b}{x}\right)\,dx=-b\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)+b\sin(a)\mathrm{Si}\left(\frac{b}{x}\right)$$

output
```
-b*cos(a)*Ci(b/x)+x*sin(a+b/x)+b*sin(a)*Si(b/x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sin\left(a+\frac{b}{x}\right)\,dx=-b\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)+b\sin(a)\mathrm{Si}\left(\frac{b}{x}\right)$$

input
```
Integrate[Sin[a + b/x],x]
```

output
```
-(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3842, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{3842}$$

$$-\int x^2\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\sin\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$x\sin\left(a+\frac{b}{x}\right)-b\int x\cos\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$x\sin\left(a+\frac{b}{x}\right)-b\int x\sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3784}$$

$$x\sin\left(a+\frac{b}{x}\right)-b\left(\cos(a)\int x\cos\left(\frac{b}{x}\right)d\frac{1}{x}-\sin(a)\int x\sin\left(\frac{b}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$x\sin\left(a+\frac{b}{x}\right)-b\left(\cos(a)\int x\sin\left(\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-\sin(a)\int x\sin\left(\frac{b}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3780}$$

$$x\sin\left(a+\frac{b}{x}\right)-b\left(\cos(a)\int x\sin\left(\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3783}$$

$$x\sin\left(a+\frac{b}{x}\right)-b\left(\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)-\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

input

```
Int[Sin[a + b/x],x]
```

output

```
x*Sin[a + b/x] - b*(Cos[a]*CosIntegral[b/x] - Sin[a]*SinIntegral[b/x])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-b\left(-\frac{\sin\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)+\operatorname{Ci}\left(\frac{b}{x}\right)\cos\left(a\right)\right)$
default	$-b\left(-\frac{\sin\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)+\operatorname{Ci}\left(\frac{b}{x}\right)\cos\left(a\right)\right)$
risch	$\frac{\mathrm{e}^{ia}\,\mathrm{expIntegral}_1\left(-\frac{ib}{x}\right)b}{2}-\frac{i\pi\;\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b}{2}+i\,\operatorname{Si}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b+\frac{\mathrm{expIntegral}_1\left(-\frac{ib}{x}\right)\mathrm{e}^{-ia}b}{2}+x\sin\left(\frac{ax+b}{x}\right.$
meijerg	$-\frac{\cos(a)\sqrt{\pi}\,b\left(\frac{4\gamma-4-4\ln(x)+4\ln(b)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}-\frac{4x\sin\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b}+\frac{4\;\operatorname{Ci}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{4}-\frac{\sin(a)\sqrt{\pi}\sqrt{b^2}\left(-\frac{4x\,b^2}{(}\right.}{}$

input
```
int(sin(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
-b*(-sin(a+b/x)/b*x-Si(b/x)*sin(a)+Ci(b/x)*cos(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \sin\left(a+\frac{b}{x}\right)\,dx=-b\cos\left(a\right)\operatorname{Ci}\left(\frac{b}{x}\right)+b\sin\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)+x\sin\left(\frac{ax+b}{x}\right)$$

input
```
integrate(sin(a+b/x),x, algorithm="fricas")
```

output
```
-b*cos(a)*cos_integral(b/x) + b*sin(a)*sin_integral(b/x) + x*sin((a*x + b)
/x)
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{x}\right)\,dx=\int \sin\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(sin(a+b/x),x)
```

output
```
Integral(sin(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.81

$$\begin{aligned}&\int \sin\left(a+\frac{b}{x}\right)\,dx\\&=-\frac{1}{2}\left(\left(\operatorname{Ei}\left(\frac{i\,b}{x}\right)+\operatorname{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)-\left(-i\operatorname{Ei}\left(\frac{i\,b}{x}\right)+i\operatorname{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b\\&\quad+x\sin\left(\frac{ax+b}{x}\right)\end{aligned}$$

input
```
integrate(sin(a+b/x),x, algorithm="maxima")
```

output
```
-1/2*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) - (-I*Ei(I*b/x) + I*Ei(-I*b/x))*sin(
a))*b + x*sin((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(32) = 64$.

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \sin\left(a+\frac{b}{x}\right)\,dx =$$

$$-\frac{ab^2\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)+ab^2\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)-\frac{(ax+b)b^2\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)}{x}-\frac{(ax+b)b^2\sin(a)\operatorname{Si}\left(a-\frac{ax}{x}\right.}{x}}{\left(a-\frac{ax+b}{x}\right)b}$$

input
```
integrate(sin(a+b/x),x, algorithm="giac")
```

output
```
-(a*b^2*cos(a)*cos_integral(-a + (a*x + b)/x) + a*b^2*sin(a)*sin_integral(
a - (a*x + b)/x) - (a*x + b)*b^2*cos(a)*cos_integral(-a + (a*x + b)/x)/x -
 (a*x + b)*b^2*sin(a)*sin_integral(a - (a*x + b)/x)/x + b^2*sin((a*x + b)/
x))/((a - (a*x + b)/x)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{x}\right)\,dx = \int \sin\left(a+\frac{b}{x}\right)\,dx$$

input
```
int(sin(a + b/x),x)
```

output
```
int(sin(a + b/x), x)
```

Reduce [F]

$$\int \sin \left(a+\frac{b}{x}\right) \, dx=\int \sin \left(\frac{ax+b}{x}\right)dx$$

input

```
int(sin(a+b/x),x)
```

output

```
int(sin((a*x + b)/x),x)
```

3.106 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\operatorname{CosIntegral}\left(\frac{b}{x}\right)\sin(a)-\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

output

```
-Ci(b/x)*sin(a)-cos(a)*Si(b/x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\operatorname{CosIntegral}\left(\frac{b}{x}\right)\sin(a)-\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

input

```
Integrate[Sin[a + b/x]/x,x]
```

output

```
-(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3858, 3856, 3857}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{x}\right)}{x} \, dx$$

$$\Big\downarrow \text{3858}$$

$$\sin(a) \int \frac{\cos \left(\frac{b}{x}\right)}{x}dx+\cos(a) \int \frac{\sin \left(\frac{b}{x}\right)}{x}dx$$

$$\Big\downarrow \text{3856}$$

$$\sin(a) \int \frac{\cos \left(\frac{b}{x}\right)}{x}dx-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

$$\Big\downarrow \text{3857}$$

$$\sin(a) \left(-\text{CosIntegral}\left(\frac{b}{x}\right)\right)-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

input
```
Int[Sin[a + b/x]/x,x]
```

output
```
-(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3857
```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3858
```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Sin[c]   Int[Cos[d*
x^n]/x, x], x] + Simp[Cos[c]   Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n},
 x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)-\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)$	22
default	$-\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)-\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)$	22
risch	$-\frac{i\mathrm{e}^{ia}\operatorname{expIntegral}_{1}\left(-\frac{ib}{x}\right)}{2}+\frac{\mathrm{e}^{-ia}\pi\ \operatorname{csgn}\left(\frac{b}{x}\right)}{2}-\mathrm{e}^{-ia}\operatorname{Si}\left(\frac{b}{x}\right)+\frac{i\operatorname{expIntegral}_{1}\left(-\frac{ib}{x}\right)\mathrm{e}^{-ia}}{2}$	63
meijerg	$-\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)-\frac{\sqrt{\pi}\,\sin(a)\left(\frac{2\gamma-2\ln(x)+\ln\left(b^{2}\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}+\frac{2\ \operatorname{Ci}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{2}$	72

input
```
int(sin(a+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-Ci(b/x)*sin(a)-cos(a)*Si(b/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int\frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx=-\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)-\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)$$

input
```
integrate(sin(a+b/x)/x,x, algorithm="fricas")
```

output
```
-cos_integral(b/x)*sin(a) - cos(a)*sin_integral(b/x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\sin(a)\operatorname{Ci}\left(\frac{b}{x}\right) - \cos(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

input
```
integrate(sin(a+b/x)/x,x)
```

output
```
-sin(a)*Ci(b/x) - cos(a)*Si(b/x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{1}{2}\left(i\operatorname{Ei}\left(\frac{ib}{x}\right) - i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a) - \frac{1}{2}\left(\operatorname{Ei}\left(\frac{ib}{x}\right) + \operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)$$

input
```
integrate(sin(a+b/x)/x,x, algorithm="maxima")
```

output
```
1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*cos(a) - 1/2*(Ei(I*b/x) + Ei(-I*b/x))*sin
(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{b\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a) - b\cos(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{b}$$

input
```
integrate(sin(a+b/x)/x,x, algorithm="giac")
```

output
```
-(b*cos_integral(-a + (a*x + b)/x)*sin(a) - b*cos(a)*sin_integral(a - (a*x
 + b)/x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx=-\sin\left(a\right)\,\mathrm{cosint}\left(\frac{b}{x}\right)-\cos\left(a\right)\,\mathrm{sinint}\left(\frac{b}{x}\right)$$

input
```
int(sin(a + b/x)/x,x)
```

output
```
- sin(a)*cosint(b/x) - cos(a)*sinint(b/x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx=\int \frac{\sin\left(\frac{ax+b}{x}\right)}{x}dx$$

input
```
int(sin(a+b/x)/x,x)
```

output
```
int(sin((a*x + b)/x)/x,x)
```

3.107 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

output
```
cos(a+b/x)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

input
```
Integrate[Sin[a + b/x]/x^2,x]
```

output
```
Cos[a + b/x]/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \sin\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \sin\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

input

```
Int[Sin[a + b/x]/x^2,x]
```

output

```
Cos[a + b/x]/b
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$	13
default	$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$	13
risch	$\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$	15
parallelrisch	$\frac{-1+\cos\left(\frac{ax+b}{x}\right)}{b}$	17
norman	$\frac{2}{b\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{2}\right)}$	23
meijerg	$-\frac{\sqrt{\pi}\,\cos\left(a\right)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}-\frac{\sin\left(a\right)\sin\left(\frac{b}{x}\right)}{b}$	40
orering	$-\frac{2x\sin\left(a+\frac{b}{x}\right)}{b^{2}}-\frac{x^{4}\left(-\frac{b\cos\left(a+\frac{b}{x}\right)}{x^{4}}-\frac{2\sin\left(a+\frac{b}{x}\right)}{x^{3}}\right)}{b^{2}}$	52

input

```
int(sin(a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output

```
cos(a+b/x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

input
```
integrate(sin(a+b/x)/x^2,x, algorithm="fricas")
```

output
```
cos((a*x + b)/x)/b
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\begin{cases}\frac{\cos\left(a+\frac{b}{x}\right)}{b} & \text{for } b\neq 0\\ -\frac{\sin\left(a\right)}{x} & \text{otherwise}\end{cases}$$

input
```
integrate(sin(a+b/x)/x**2,x)
```

output
```
Piecewise((cos(a + b/x)/b, Ne(b, 0)), (-sin(a)/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

input
```
integrate(sin(a+b/x)/x^2,x, algorithm="maxima")
```

output
```
cos(a + b/x)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

input
```
integrate(sin(a+b/x)/x^2,x, algorithm="giac")
```

output
```
cos((a*x + b)/x)/b
```

Mupad [B] (verification not implemented)

Time = 40.62 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

input
```
int(sin(a + b/x)/x^2,x)
```

output
```
cos(a + b/x)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

input
```
int(sin(a+b/x)/x^2,x)
```

output
```
cos((a*x + b)/x)/b
```

3.108 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

output
```
cos(a+b/x)/b/x-sin(a+b/x)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

input
```
Integrate[Sin[a + b/x]/x^3,x]
```

output
```
Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\int \cos\left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\int \sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

input
```
Int[Sin[a + b/x]/x^3,x]
```

output
```
Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

method	result	size
risch	$\frac{\cos\left(\frac{ax+b}{x}\right)}{bx}-\frac{\sin\left(\frac{ax+b}{x}\right)}{b^{2}}$	34
parallelrisch	$\frac{b\cos\left(\frac{ax+b}{x}\right)-x\sin\left(\frac{ax+b}{x}\right)}{b^{2}x}$	34
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)}{b^{2}}$	42
default	$-\frac{\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)}{b^{2}}$	42
orering	$-\frac{4\sin\left(a+\frac{b}{x}\right)}{b^{2}}-\frac{x^{4}\left(-\frac{b\cos\left(a+\frac{b}{x}\right)}{x^{5}}-\frac{3\sin\left(a+\frac{b}{x}\right)}{x^{4}}\right)}{b^{2}}$	51
norman	$\frac{\frac{x}{b}-\frac{2x^{2}\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^{2}}-\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{2}}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{2}\right)x^{2}}$	66
meijerg	$-\frac{2\sqrt{\pi}\,\cos\left(a\right)\left(-\frac{b\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}+\frac{\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}}\right)}{b^{2}}-\frac{2\sqrt{\pi}\,\sin\left(a\right)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^{2}}$	81

input
```
int(sin(a+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/b/x*cos((a*x+b)/x)-1/b^2*sin((a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^{3}}\,dx=\frac{b\cos\left(\frac{ax+b}{x}\right)-x\sin\left(\frac{ax+b}{x}\right)}{b^{2}x}$$

input
```
integrate(sin(a+b/x)/x^3,x, algorithm="fricas")
```

output
```
(b*cos((a*x + b)/x) - x*sin((a*x + b)/x))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = \begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b/x)/x**3,x)
```

output
```
Piecewise((cos(a + b/x)/(b*x) - sin(a + b/x)/b**2, Ne(b, 0)), (-sin(a)/(2*
x**2), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{\left(i\,\Gamma\left(2,\frac{i\,b}{x}\right) - i\,\Gamma\left(2,-\frac{i\,b}{x}\right)\right)\cos(a) + \left(\Gamma\left(2,\frac{i\,b}{x}\right) + \Gamma\left(2,-\frac{i\,b}{x}\right)\right)\sin(a)}{2\,b^2}$$

input
```
integrate(sin(a+b/x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*((I*gamma(2, I*b/x) - I*gamma(2, -I*b/x))*cos(a) + (gamma(2, I*b/x) +
 gamma(2, -I*b/x))*sin(a))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{a\cos\left(\frac{ax+b}{x}\right) - \frac{(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x} + \sin\left(\frac{ax+b}{x}\right)}{b^2}$$

input
```
integrate(sin(a+b/x)/x^3,x, algorithm="giac")
```

output
```
-(a*cos((a*x + b)/x) - (a*x + b)*cos((a*x + b)/x)/x + sin((a*x + b)/x))/b^
2
```

Mupad [B] (verification not implemented)

Time = 40.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\cos\left(a+\frac{b}{x}\right)}{b\,x} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

input
```
int(sin(a + b/x)/x^3,x)
```

output
```
cos(a + b/x)/(b*x) - sin(a + b/x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\cos\left(\frac{ax+b}{x}\right)b - \sin\left(\frac{ax+b}{x}\right)x}{b^2x}$$

input
```
int(sin(a+b/x)/x^3,x)
```

output
```
(cos((a*x + b)/x)*b - sin((a*x + b)/x)*x)/(b**2*x)
```

3.109 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}$$

output
```
-2*cos(a+b/x)/b^3+cos(a+b/x)/b/x^2-2*sin(a+b/x)/b^2/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{(b^2-2x^2)\cos\left(a+\frac{b}{x}\right)-2bx\sin\left(a+\frac{b}{x}\right)}{b^3x^2}$$

input
```
Integrate[Sin[a + b/x]/x^4,x]
```

output
```
((b^2 - 2*x^2)*Cos[a + b/x] - 2*b*x*Sin[a + b/x])/(b^3*x^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3860, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{x}\right)}{x^4} \, dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \frac{\sin \left(a+\frac{b}{x}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin \left(a+\frac{b}{x}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\cos \left(a+\frac{b}{x}\right)}{b x^2}-\frac{2 \int \frac{\cos \left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos \left(a+\frac{b}{x}\right)}{b x^2}-\frac{2 \int \frac{\sin \left(a+\frac{b}{x}+\frac{\pi }{2}\right)}{x}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\cos \left(a+\frac{b}{x}\right)}{b x^2}-\frac{2 \left(\frac{\int -\sin \left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}+\frac{\sin \left(a+\frac{b}{x}\right)}{b x}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cos \left(a+\frac{b}{x}\right)}{b x^2}-\frac{2 \left(\frac{\sin \left(a+\frac{b}{x}\right)}{b x}-\frac{\int \sin \left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\left(\frac{\sin\left(a+\frac{b}{x}\right)}{bx}-\frac{\int \sin\left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\left(\frac{\cos\left(a+\frac{b}{x}\right)}{b^2}+\frac{\sin\left(a+\frac{b}{x}\right)}{bx}\right)}{b}$$

input

```
Int[Sin[a + b/x]/x^4,x]
```

output

```
Cos[a + b/x]/(b*x^2) - (2*(Cos[a + b/x]/b^2 + Sin[a + b/x]/(b*x)))/b
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(b^2-2x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2}-\frac{2\sin\left(\frac{ax+b}{x}\right)}{b^2x}$
parallelrisch	$\frac{(b^2-2x^2)\cos\left(\frac{ax+b}{x}\right)-2\sin\left(\frac{ax+b}{x}\right)bx+2x^2}{b^3x^2}$
orering	$-\frac{2(3b^2-4x^2)\sin\left(a+\frac{b}{x}\right)}{x\,b^4}-\frac{(b^2-2x^2)x^4\left(-\frac{b\cos\left(a+\frac{b}{x}\right)}{x^6}-\frac{4\sin\left(a+\frac{b}{x}\right)}{x^5}\right)}{b^4}$
norman	$\frac{\frac{x}{b}+\frac{4x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{b^3}-\frac{4x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2\right)x^3}$
derivativedivides	$-\frac{-a^2\cos\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{b^3}$
default	$-\frac{-a^2\cos\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{b^3}$
meijerg	$-\frac{4\sqrt{\pi}\cos(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{b^2}{2x^2}+1\right)\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^3}-\frac{4\sqrt{\pi}\sin(a)\sqrt{b^2}\Bigg(\frac{(b^2)^{\frac{3}{2}}\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x\,b^2}-\frac{(b^2)^{\frac{3}{2}}\left(-\frac{3b^2}{2x^2}+3\right)\sin(}{6\sqrt{\pi}\,b^3}}{b^4}$

input
```
int(sin(a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(b^2-2*x^2)/b^3/x^2*cos((a*x+b)/x)-2/b^2/x*sin((a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int\frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=-\frac{2\,bx\sin\left(\frac{ax+b}{x}\right)-(b^2-2\,x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

input
```
integrate(sin(a+b/x)/x^4,x, algorithm="fricas")
```

output
```
-(2*b*x*sin((a*x + b)/x) - (b^2 - 2*x^2)*cos((a*x + b)/x))/(b^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = \begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x} - \frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b/x)/x**4,x)
```

output
```
Piecewise((cos(a + b/x)/(b*x**2) - 2*sin(a + b/x)/(b**2*x) - 2*cos(a + b/x
)/b**3, Ne(b, 0)), (-sin(a)/(3*x**3), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{\left(\Gamma\left(3,\frac{ib}{x}\right)+\Gamma\left(3,-\frac{ib}{x}\right)\right)\cos(a) - \left(i\,\Gamma\left(3,\frac{ib}{x}\right) - i\,\Gamma\left(3,-\frac{ib}{x}\right)\right)\sin(a)}{2\,b^3}$$

input
```
integrate(sin(a+b/x)/x^4,x, algorithm="maxima")
```

output
```
-1/2*((gamma(3, I*b/x) + gamma(3, -I*b/x))*cos(a) - (I*gamma(3, I*b/x) - I
*gamma(3, -I*b/x))*sin(a))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(45) = 90.

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx$$

$$= \frac{a^2\cos\left(\frac{ax+b}{x}\right) - \frac{2\,(ax+b)a\cos\left(\frac{ax+b}{x}\right)}{x} + 2\,a\sin\left(\frac{ax+b}{x}\right) + \frac{(ax+b)^2\cos\left(\frac{ax+b}{x}\right)}{x^2} - \frac{2\,(ax+b)\sin\left(\frac{ax+b}{x}\right)}{x} - 2\,\cos\left(\frac{ax+b}{x}\right)}{b^3}$$

input
```
integrate(sin(a+b/x)/x^4,x, algorithm="giac")
```

output
```
(a^2*cos((a*x + b)/x) - 2*(a*x + b)*a*cos((a*x + b)/x)/x + 2*a*sin((a*x +
b)/x) + (a*x + b)^2*cos((a*x + b)/x)/x^2 - 2*(a*x + b)*sin((a*x + b)/x)/x
- 2*cos((a*x + b)/x))/b^3
```

Mupad [B] (verification not implemented)

Time = 41.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{b^2\,\cos\left(a+\frac{b}{x}\right)-2\,b\,x\,\sin\left(a+\frac{b}{x}\right)}{b^3\,x^2}-\frac{2\,\cos\left(a+\frac{b}{x}\right)}{b^3}$$

input
```
int(sin(a + b/x)/x^4,x)
```

output
```
(b^2*cos(a + b/x) - 2*b*x*sin(a + b/x))/(b^3*x^2) - (2*cos(a + b/x))/b^3
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{-4\cos\left(\frac{ax+b}{x}\right)+\left(\int \frac{\sin\left(\frac{ax+b}{x}\right)}{x^4}dx\right)b^3+4\left(\int \frac{\sin\left(\frac{ax+b}{x}\right)}{x^2}dx\right)b}{b^3}$$

input
```
int(sin(a+b/x)/x^4,x)
```

output
```
( - 4*cos((a*x + b)/x) + int(sin((a*x + b)/x)/x**4,x)*b**3 + 4*int(sin((a*
x + b)/x)/x**2,x)*b)/b**3
```

3.110 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 61

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}$$

output
```
cos(a+b/x)/b/x^3-6*cos(a+b/x)/b^3/x+6*sin(a+b/x)/b^4-3*sin(a+b/x)/b^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx=\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}$$

input
```
Integrate[Sin[a + b/x]/x^5,x]
```

output
```
Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (
3*Sin[a + b/x])/(b^2*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3860, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\int \frac{\sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right)}{x^2}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\left(\frac{2\int -\frac{\sin\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{b}+\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\left(\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\left(\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\int\frac{\sin\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{b}\right)}{b}$$

$$\Big\downarrow 3777$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\left(\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\left(\frac{\int\cos\left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}-\frac{\cos\left(a+\frac{b}{x}\right)}{bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow 3042$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\left(\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\left(\frac{\int\sin\left(a+\frac{b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}}{b}-\frac{\cos\left(a+\frac{b}{x}\right)}{bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow 3117$$

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\left(\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\left(\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cos\left(a+\frac{b}{x}\right)}{bx}\right)}{b}\right)}{b}$$

input

```
Int[Sin[a + b/x]/x^5,x]
```

output

```
Cos[a + b/x]/(b*x^3) - (3*(Sin[a + b/x]/(b*x^2) - (2*(-(Cos[a + b/x]/(b*x)
) + Sin[a + b/x]/b^2))/b))/b
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{(b^3-6b\,x^2)\cos\left(\frac{ax+b}{x}\right)-3x\sin\left(\frac{ax+b}{x}\right)(b^2-2x^2)}{b^4x^3}$
risch	$\frac{(b^2-6x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^3}-\frac{3(b^2-2x^2)\sin\left(\frac{ax+b}{x}\right)}{x^2b^4}$
orering	$-\frac{4(2b^2-9x^2)\sin\left(a+\frac{b}{x}\right)}{x^2b^4}-\frac{(b^2-6x^2)x^4\left(-\frac{b\cos\left(a+\frac{b}{x}\right)}{x^7}-\frac{5\sin\left(a+\frac{b}{x}\right)}{x^6}\right)}{b^4}$
norman	$\frac{\frac{x}{b}-\frac{6x^3}{b^3}+\frac{12x^4\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^4}+\frac{6x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{b^3}-\frac{6x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2\right)x^4}$
meijerg	$-\frac{8\sqrt{\pi}\,\cos(a)\left(\frac{b\left(-\frac{5b^2}{2x^2}+15\right)\cos\left(\frac{b}{x}\right)}{20\sqrt{\pi}\,x}-\frac{\left(-\frac{15b^2}{2x^2}+15\right)\sin\left(\frac{b}{x}\right)}{20\sqrt{\pi}}\right)}{b^4}-\frac{8\sqrt{\pi}\,\sin(a)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3b^2}{2x^2}+3\right)\cos\left(\frac{b}{x}\right)}{4\sqrt{\pi}}-\frac{b\left(-\frac{b^2}{2x^2}+3\right.}{4\sqrt{\pi}}\right.}{b^4}$
derivativedivides	$-\frac{a^3\cos\left(a+\frac{b}{x}\right)+3a^2\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-3a\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\right.\right.}{b^4}$
default	$-\frac{a^3\cos\left(a+\frac{b}{x}\right)+3a^2\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-3a\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\right.\right.}{b^4}$

input
```
int(sin(a+b/x)/x^5,x,method=_RETURNVERBOSE)
```

output

```
((b^3-6*b*x^2)*cos((a*x+b)/x)-3*x*sin((a*x+b)/x)*(b^2-2*x^2))/b^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{(b^3-6\,bx^2)\cos\left(\frac{ax+b}{x}\right)-3\,(b^2x-2\,x^3)\sin\left(\frac{ax+b}{x}\right)}{b^4x^3}$$

input

```
integrate(sin(a+b/x)/x^5,x, algorithm="fricas")
```

output

```
((b^3 - 6*b*x^2)*cos((a*x + b)/x) - 3*(b^2*x - 2*x^3)*sin((a*x + b)/x))/(b
^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = \begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} & \text{for } b\neq 0 \\ -\frac{\sin(a)}{4x^4} & \text{otherwise} \end{cases}$$

input

```
integrate(sin(a+b/x)/x**5,x)
```

output

```
Piecewise((cos(a + b/x)/(b*x**3) - 3*sin(a + b/x)/(b**2*x**2) - 6*cos(a +
b/x)/(b**3*x) + 6*sin(a + b/x)/b**4, Ne(b, 0)), (-sin(a)/(4*x**4), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{\left(i\,\Gamma\left(4,\frac{i\,b}{x}\right) - i\,\Gamma\left(4,-\frac{i\,b}{x}\right)\right)\cos\left(a\right) + \left(\Gamma\left(4,\frac{i\,b}{x}\right) + \Gamma\left(4,-\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^4}$$

input
```
integrate(sin(a+b/x)/x^5,x, algorithm="maxima")
```

output
```
1/2*((I*gamma(4, I*b/x) - I*gamma(4, -I*b/x))*cos(a) + (gamma(4, I*b/x) +
gamma(4, -I*b/x))*sin(a))/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(61) = 122.

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.13

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx =$$

$$-\frac{a^3\cos\left(\frac{ax+b}{x}\right) - \frac{3\,(ax+b)a^2\cos\left(\frac{ax+b}{x}\right)}{x} + 3\,a^2\sin\left(\frac{ax+b}{x}\right) - 6\,a\cos\left(\frac{ax+b}{x}\right) + \frac{3\,(ax+b)^2a\cos\left(\frac{ax+b}{x}\right)}{x^2} - \frac{6\,(ax+b)a\sin\bigl(}{x}}{b^4}$$

input
```
integrate(sin(a+b/x)/x^5,x, algorithm="giac")
```

output
```
-(a^3*cos((a*x + b)/x) - 3*(a*x + b)*a^2*cos((a*x + b)/x)/x + 3*a^2*sin((a
*x + b)/x) - 6*a*cos((a*x + b)/x) + 3*(a*x + b)^2*a*cos((a*x + b)/x)/x^2 -
 6*(a*x + b)*a*sin((a*x + b)/x)/x - (a*x + b)^3*cos((a*x + b)/x)/x^3 + 6*(
a*x + b)*cos((a*x + b)/x)/x + 3*(a*x + b)^2*sin((a*x + b)/x)/x^2 - 6*sin((
a*x + b)/x))/b^4
```

Mupad [B] (verification not implemented)

Time = 41.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\sin \left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{6\,\sin \left(a+\frac{b}{x}\right)}{b^4}-\frac{6\,b\,x^2\,\cos \left(a+\frac{b}{x}\right)-b^3\,\cos \left(a+\frac{b}{x}\right)+3\,b^2\,x\,\sin \left(a+\frac{b}{x}\right)}{b^4\,x^3}$$

input

```
int(sin(a + b/x)/x^5,x)
```

output

```
(6*sin(a + b/x))/b^4 - (6*b*x^2*cos(a + b/x) - b^3*cos(a + b/x) + 3*b^2*x*
sin(a + b/x))/(b^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{\sin \left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{\cos \left(\frac{ax+b}{x}\right) b^3-6\cos \left(\frac{ax+b}{x}\right) b\,x^2-3\sin \left(\frac{ax+b}{x}\right) b^2x+6\sin \left(\frac{ax+b}{x}\right) x^3-6a\,x^3}{b^4x^3}$$

input

```
int(sin(a+b/x)/x^5,x)
```

output

```
(cos((a*x + b)/x)*b**3 - 6*cos((a*x + b)/x)*b*x**2 - 3*sin((a*x + b)/x)*b*
*2*x + 6*sin((a*x + b)/x)*x**3 - 6*a*x**3)/(b**4*x**3)
```

3.111 $\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 100

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx=\frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2a+\frac{2b}{x}\right)-\frac{1}{6}x^3\cos\left(2a+\frac{2b}{x}\right)+\frac{2}{3}b^3\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a)+\frac{1}{6}bx^2\sin\left(2a+\frac{2b}{x}\right)+\frac{2}{3}b^3\cos(2a)\,\text{Si}\left(\frac{2b}{x}\right)$$

output

```
1/6*x^3+1/3*b^2*x*cos(2*a+2*b/x)-1/6*x^3*cos(2*a+2*b/x)+2/3*b^3*Ci(2*b/x)*
sin(2*a)+1/6*b*x^2*sin(2*a+2*b/x)+2/3*b^3*cos(2*a)*Si(2*b/x)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx=\frac{1}{6}\left(4b^3\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a)+x\left(x^2+2b^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-x^2\cos\left(2\left(a+\frac{b}{x}\right)\right)+bx\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)+4b^3\cos(2a)\,\text{Si}\left(\frac{2b}{x}\right)\right)$$

input

```
Integrate[x^2*Sin[a + b/x]^2,x]
```

output

```
(4*b^3*CosIntegral[(2*b)/x]*Sin[2*a] + x*(x^2 + 2*b^2*Cos[2*(a + b/x)] - x
^2*Cos[2*(a + b/x)] + b*x*Sin[2*(a + b/x)]) + 4*b^3*Cos[2*a]*SinIntegral[(
2*b)/x])/6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3906, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{x^2}{2}-\frac{1}{2}x^2 \cos\left(2a+\frac{2b}{x}\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}b^3 \sin(2a) \text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{2}{3}b^3 \cos(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{3}b^2 x \cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3 \cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2 \sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{x^3}{6}$$

input

```
Int[x^2*Sin[a + b/x]^2,x]
```

output

```
x^3/6 + (b^2*x*Cos[2*(a + b/x)])/3 - (x^3*Cos[2*(a + b/x)])/6 + (2*b^3*Cos
Integral[(2*b)/x]*Sin[2*a])/3 + (b*x^2*Sin[2*(a + b/x)])/6 + (2*b^3*Cos[2*
a]*SinIntegral[(2*b)/x])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-b^{3}\left(-\frac{x^{3}}{6b^{3}}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^{3}}{6b^{3}}-\frac{\sin\left(2a+\frac{2b}{x}\right)x^{2}}{6b^{2}}-\frac{\cos\left(2a+\frac{2b}{x}\right)x}{3b}-\frac{2\,\mathrm{Si}\left(\frac{2b}{x}\right)\cos\left(2a\right)}{3}-\frac{2\,\mathrm{Ci}\left(\frac{2b}{x}\right)\sin\left(2a\right)}{3}\right.$
default	$-b^{3}\left(-\frac{x^{3}}{6b^{3}}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^{3}}{6b^{3}}-\frac{\sin\left(2a+\frac{2b}{x}\right)x^{2}}{6b^{2}}-\frac{\cos\left(2a+\frac{2b}{x}\right)x}{3b}-\frac{2\,\mathrm{Si}\left(\frac{2b}{x}\right)\cos\left(2a\right)}{3}-\frac{2\,\mathrm{Ci}\left(\frac{2b}{x}\right)\sin\left(2a\right)}{3}\right.$
risch	$-\frac{\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}b^{3}}{3}+\frac{2\,\mathrm{Si}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}b^{3}}{3}-\frac{i\,\mathrm{expIntegral}_{1}\left(-\frac{2ib}{x}\right)\mathrm{e}^{-2ia}b^{3}}{3}+\frac{ib^{3}\,\mathrm{expIntegral}_{1}\left(-\frac{2ib}{x}\right)\mathrm{e}^{2ia}}{3}+$

input
```
int(x^2*sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^3*(-1/6/b^3*x^3+1/6*cos(2*a+2*b/x)/b^3*x^3-1/6*sin(2*a+2*b/x)/b^2*x^2-1
/3*cos(2*a+2*b/x)/b*x-2/3*Si(2*b/x)*cos(2*a)-2/3*Ci(2*b/x)*sin(2*a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x^{2}\sin^{2}\left(a+\frac{b}{x}\right)\,dx=&\ \frac{2}{3}\,b^{3}\,\mathrm{Ci}\left(\frac{2\,b}{x}\right)\sin\left(2\,a\right)+\frac{1}{3}\,bx^{2}\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)\\&+\frac{2}{3}\,b^{3}\cos\left(2\,a\right)\mathrm{Si}\left(\frac{2\,b}{x}\right)-\frac{1}{3}\,b^{2}x\\&+\frac{1}{3}\,x^{3}+\frac{1}{3}\,\left(2\,b^{2}x-x^{3}\right)\cos\left(\frac{ax+b}{x}\right)^{2}\end{aligned}$$

input
```
integrate(x^2*sin(a+b/x)^2,x, algorithm="fricas")
```

output
```
2/3*b^3*cos_integral(2*b/x)*sin(2*a) + 1/3*b*x^2*cos((a*x + b)/x)*sin((a*x
 + b)/x) + 2/3*b^3*cos(2*a)*sin_integral(2*b/x) - 1/3*b^2*x + 1/3*x^3 + 1/
3*(2*b^2*x - x^3)*cos((a*x + b)/x)^2
```

Sympy [F]

$$\int x^2\sin^2\left(a+\frac{b}{x}\right)\,dx=\int x^2\sin^2\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(x**2*sin(a+b/x)**2,x)
```

output
```
Integral(x**2*sin(a + b/x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int x^2\sin^2\left(a+\frac{b}{x}\right)\,dx= -\frac{1}{3}\left(\left(i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b^3 +\frac{1}{6}\,bx^2\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{6}\,x^3+\frac{1}{6}\,(2\,b^2x-x^3)\cos\left(\frac{2\,(ax+b)}{x}\right)$$

input
```
integrate(x^2*sin(a+b/x)^2,x, algorithm="maxima")
```

output
```
-1/3*((I*Ei(2*I*b/x) - I*Ei(-2*I*b/x))*cos(2*a) - (Ei(2*I*b/x) + Ei(-2*I*b
/x))*sin(2*a))*b^3 + 1/6*b*x^2*sin(2*(a*x + b)/x) + 1/6*x^3 + 1/6*(2*b^2*x
 - x^3)*cos(2*(a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(88) = 176$.

Time = 0.12 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.42

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx$$
$$= \frac{4\,a^3b^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin\left(2\,a\right)-4\,a^3b^4\cos\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2b^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2}{x}}{}$$

input
```
integrate(x^2*sin(a+b/x)^2,x, algorithm="giac")
```

output
```
1/6*(4*a^3*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a) - 4*a^3*b^4*cos
(2*a)*sin_integral(2*a - 2*(a*x + b)/x) - 12*(a*x + b)*a^2*b^4*cos_integra
l(-2*a + 2*(a*x + b)/x)*sin(2*a)/x + 12*(a*x + b)*a^2*b^4*cos(2*a)*sin_int
egral(2*a - 2*(a*x + b)/x)/x - 2*a^2*b^4*cos(2*(a*x + b)/x) + 12*(a*x + b)
^2*a*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a)/x^2 - 12*(a*x + b)^2*
a*b^4*cos(2*a)*sin_integral(2*a - 2*(a*x + b)/x)/x^2 + 4*(a*x + b)*a*b^4*c
os(2*(a*x + b)/x)/x - 4*(a*x + b)^3*b^4*cos_integral(-2*a + 2*(a*x + b)/x)
*sin(2*a)/x^3 + a*b^4*sin(2*(a*x + b)/x) + 4*(a*x + b)^3*b^4*cos(2*a)*sin_
integral(2*a - 2*(a*x + b)/x)/x^3 + b^4*cos(2*(a*x + b)/x) - 2*(a*x + b)^2
*b^4*cos(2*(a*x + b)/x)/x^2 - (a*x + b)*b^4*sin(2*(a*x + b)/x)/x - b^4)/((
a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx = \int x^2 \sin\left(a+\frac{b}{x}\right)^2\,dx$$

input
```
int(x^2*sin(a + b/x)^2,x)
```

output
```
int(x^2*sin(a + b/x)^2, x)
```

Reduce [F]

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right) dx = \text{Too large to display}$$

input

```
int(x^2*sin(a+b/x)^2,x)
```

output

```
( - cos((a*x + b)/x)*sin((a*x + b)/x)*tan((a*x + b)/(2*x))**4*b*x**2 - 2*c
os((a*x + b)/x)*sin((a*x + b)/x)*tan((a*x + b)/(2*x))**2*b*x**2 - cos((a*x
 + b)/x)*sin((a*x + b)/x)*b*x**2 + 8*cos((a*x + b)/x)*tan((a*x + b)/(2*x))
**4*x**3 + 16*cos((a*x + b)/x)*tan((a*x + b)/(2*x))**2*x**3 + 8*cos((a*x +
 b)/x)*x**3 + 16*int(tan((a*x + b)/(2*x))/(tan((a*x + b)/(2*x))**4*x + 2*t
an((a*x + b)/(2*x))**2*x + x),x)*tan((a*x + b)/(2*x))**4*b**3 + 32*int(tan
((a*x + b)/(2*x))/(tan((a*x + b)/(2*x))**4*x + 2*tan((a*x + b)/(2*x))**2*x
 + x),x)*tan((a*x + b)/(2*x))**2*b**3 + 16*int(tan((a*x + b)/(2*x))/(tan((
a*x + b)/(2*x))**4*x + 2*tan((a*x + b)/(2*x))**2*x + x),x)*b**3 + 32*int((
tan((a*x + b)/(2*x))*x)/(tan((a*x + b)/(2*x))**4 + 2*tan((a*x + b)/(2*x))*
*2 + 1),x)*tan((a*x + b)/(2*x))**4*b + 64*int((tan((a*x + b)/(2*x))*x)/(ta
n((a*x + b)/(2*x))**4 + 2*tan((a*x + b)/(2*x))**2 + 1),x)*tan((a*x + b)/(2
*x))**2*b + 32*int((tan((a*x + b)/(2*x))*x)/(tan((a*x + b)/(2*x))**4 + 2*t
an((a*x + b)/(2*x))**2 + 1),x)*b - sin((a*x + b)/x)**2*tan((a*x + b)/(2*x)
)**4*x**3 - 2*sin((a*x + b)/x)**2*tan((a*x + b)/(2*x))**2*x**3 - sin((a*x
+ b)/x)**2*x**3 - 4*sin((a*x + b)/x)*tan((a*x + b)/(2*x))**4*b*x**2 - 8*si
n((a*x + b)/x)*tan((a*x + b)/(2*x))**2*b*x**2 - 4*sin((a*x + b)/x)*b*x**2
+ 3*tan((a*x + b)/(2*x))**4*b**2*x + 8*tan((a*x + b)/(2*x))**4*x**3 + 6*ta
n((a*x + b)/(2*x))**2*b**2*x + 16*tan((a*x + b)/(2*x))**2*x**3 - 5*b**2*x
- 8*x**3)/(9*(tan((a*x + b)/(2*x))**4 + 2*tan((a*x + b)/(2*x))**2 + 1))
```

3.112 $\int x \sin^2\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=-b^2\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2a+\frac{2b}{x}\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)$$

output

```
-b^2*cos(2*a)*Ci(2*b/x)+1/2*x^2*sin(a+b/x)^2+1/2*b*x*sin(2*a+2*b/x)+b^2*si
n(2*a)*Si(2*b/x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=-b^2\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{1}{4}x\left(x-x\cos\left(2\left(a+\frac{b}{x}\right)\right)+2b\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)$$

input

```
Integrate[x*Sin[a + b/x]^2,x]
```

output
```
-(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x*(x - x*Cos[2*(a + b/x)] + 2*b*Si
n[2*(a + b/x)]))/4 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3874, 5084, 3854, 3842, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^2\left(a+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{3874}$$

$$b\int \cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)dx+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)$$

$$\Big\downarrow \text{5084}$$

$$\frac{1}{2}b\int \sin\left(2\left(a+\frac{b}{x}\right)\right)dx+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)$$

$$\Big\downarrow \text{3854}$$

$$\frac{1}{2}b\int \sin\left(2a+\frac{2b}{x}\right)dx+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)$$

$$\Big\downarrow \text{3842}$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\int x^2\sin\left(2a+\frac{2b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\int x^2\sin\left(2a+\frac{2b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(2b\int x\cos\left(2a+\frac{2b}{x}\right)d\frac{1}{x}-x\sin\left(2a+\frac{2b}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(2b\int x\sin\left(2a+\frac{2b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-x\sin\left(2a+\frac{2b}{x}\right)\right)$$

$$\Big\downarrow \ 3784$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(2b\left(\cos(2a)\int x\cos\left(\frac{2b}{x}\right)d\frac{1}{x}-\sin(2a)\int x\sin\left(\frac{2b}{x}\right)d\frac{1}{x}\right)-x\sin\left(2a+\frac{2b}{x}\right)\right)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(2b\left(\cos(2a)\int x\sin\left(\frac{2b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-\sin(2a)\int x\sin\left(\frac{2b}{x}\right)d\frac{1}{x}\right)-x\sin\left(2a+\frac{2b}{x}\right)\right)$$

$$\Big\downarrow \ 3780$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(2b\left(\cos(2a)\int x\sin\left(\frac{2b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}-\sin(2a)\text{Si}\left(\frac{2b}{x}\right)\right)-x\sin\left(2a+\frac{2b}{x}\right)\right)$$

$$\Big\downarrow \ 3783$$

$$\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\left(2b\left(\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)-\sin(2a)\text{Si}\left(\frac{2b}{x}\right)\right)-x\sin\left(2a+\frac{2b}{x}\right)\right)$$

input

```
Int[x*Sin[a + b/x]^2,x]
```

output

```
(x^2*Sin[a + b/x]^2)/2 - (b*(-(x*Sin[2*a + (2*b)/x]) + 2*b*(Cos[2*a]*CosIn
tegral[(2*b)/x] - Sin[2*a]*SinIntegral[(2*b)/x])))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

rule 3854
```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

rule 3874
```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[Sin[a + b*x^
n]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[
m + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

rule 5084
```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Sin[
2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-b^{2} \left(-\frac{x^{2}}{4 b^{2}}+\frac{\cos \left(2 a +\frac{2 b}{x}\right) x^{2}}{4 b^{2}}-\frac{\sin \left(2 a +\frac{2 b}{x}\right) x}{2 b}-\sin \left(2 a \right) \operatorname{Si}\left(\frac{2 b}{x}\right)+\cos \left(2 a \right) \operatorname{Ci}\left(\frac{2 b}{x}\right)\right)$
default	$-b^{2} \left(-\frac{x^{2}}{4 b^{2}}+\frac{\cos \left(2 a +\frac{2 b}{x}\right) x^{2}}{4 b^{2}}-\frac{\sin \left(2 a +\frac{2 b}{x}\right) x}{2 b}-\sin \left(2 a \right) \operatorname{Si}\left(\frac{2 b}{x}\right)+\cos \left(2 a \right) \operatorname{Ci}\left(\frac{2 b}{x}\right)\right)$
risch	$-\frac{i \pi \,\operatorname{csgn}\left(\frac{b}{x}\right) {\mathrm e}^{-2 i a} b^{2}}{2}+i \operatorname{Si}\left(\frac{2 b}{x}\right) {\mathrm e}^{-2 i a} b^{2}+\frac{\operatorname{expIntegral}_{1}\left(-\frac{2 i b}{x}\right) {\mathrm e}^{-2 i a} b^{2}}{2}+\frac{{\mathrm e}^{2 i a} \operatorname{expIntegral}_{1}\left(-\frac{2 i b}{x}\right) b^{2}}{2}+$

input
```
int(x*sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2*(-1/4/b^2*x^2+1/4*cos(2*a+2*b/x)/b^2*x^2-1/2*sin(2*a+2*b/x)/b*x-sin(2
*a)*Si(2*b/x)+cos(2*a)*Ci(2*b/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int x \sin^{2}\left(a+\frac{b}{x}\right)\,dx=-\frac{1}{2}\,x^{2}\cos\left(\frac{ax+b}{x}\right)^{2}-b^{2}\cos\left(2\,a\right)\operatorname{Ci}\left(\frac{2\,b}{x}\right)$$
$$+bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+b^{2}\sin\left(2\,a\right)\operatorname{Si}\left(\frac{2\,b}{x}\right)+\frac{1}{2}\,x^{2}$$

input
```
integrate(x*sin(a+b/x)^2,x, algorithm="fricas")
```

output
```
-1/2*x^2*cos((a*x + b)/x)^2 - b^2*cos(2*a)*cos_integral(2*b/x) + b*x*cos((
a*x + b)/x)*sin((a*x + b)/x) + b^2*sin(2*a)*sin_integral(2*b/x) + 1/2*x^2
```

Sympy [F]

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=\int x\sin^2\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(x*sin(a+b/x)**2,x)
```

output
```
Integral(x*sin(a + b/x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=-\frac{1}{2}\left(\left(\operatorname{Ei}\left(\frac{2i\,b}{x}\right)+\operatorname{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)+\left(i\operatorname{Ei}\left(\frac{2i\,b}{x}\right)-i\operatorname{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b^2-\frac{1}{4}\,x^2\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{2}\,bx\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{4}\,x^2$$

input
```
integrate(x*sin(a+b/x)^2,x, algorithm="maxima")
```

output
```
-1/2*((Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + (I*Ei(2*I*b/x) - I*Ei(-2*I*b
/x))*sin(2*a))*b^2 - 1/4*x^2*cos(2*(a*x + b)/x) + 1/2*b*x*sin(2*(a*x + b)/
x) + 1/4*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.29

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx =$$

$$-\frac{4\,a^2b^3\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)+4\,a^2b^3\sin\left(2\,a\right)\operatorname{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{8\,(ax+b)ab^3\cos(2\,a)\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b}{x}\right.}{x}}{}$$

input
```
integrate(x*sin(a+b/x)^2,x, algorithm="giac")
```

output
```
-1/4*(4*a^2*b^3*cos(2*a)*cos_integral(-2*a + 2*(a*x + b)/x) + 4*a^2*b^3*si
n(2*a)*sin_integral(2*a - 2*(a*x + b)/x) - 8*(a*x + b)*a*b^3*cos(2*a)*cos_
integral(-2*a + 2*(a*x + b)/x)/x - 8*(a*x + b)*a*b^3*sin(2*a)*sin_integral
(2*a - 2*(a*x + b)/x)/x + 4*(a*x + b)^2*b^3*cos(2*a)*cos_integral(-2*a + 2
*(a*x + b)/x)/x^2 + 2*a*b^3*sin(2*(a*x + b)/x) + 4*(a*x + b)^2*b^3*sin(2*a
)*sin_integral(2*a - 2*(a*x + b)/x)/x^2 + b^3*cos(2*(a*x + b)/x) - 2*(a*x
+ b)*b^3*sin(2*(a*x + b)/x)/x - b^3)/((a^2 - 2*(a*x + b)*a/x + (a*x + b)^2
/x^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx = \int x\sin\left(a+\frac{b}{x}\right)^2\,dx$$

input
```
int(x*sin(a + b/x)^2,x)
```

output
```
int(x*sin(a + b/x)^2, x)
```

Reduce [F]

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx=\text{Too large to display}$$

input
```
int(x*sin(a+b/x)^2,x)
```

output
```
( - 2*cos((a*x + b)/x)*sin((a*x + b)/x)*tan((a*x + b)/(2*x))**4*b*x - 4*co
s((a*x + b)/x)*sin((a*x + b)/x)*tan((a*x + b)/(2*x))**2*b*x - 2*cos((a*x +
 b)/x)*sin((a*x + b)/x)*b*x + 8*cos((a*x + b)/x)*tan((a*x + b)/(2*x))**4*x
**2 + 16*cos((a*x + b)/x)*tan((a*x + b)/(2*x))**2*x**2 + 8*cos((a*x + b)/x
)*x**2 + 32*int(tan((a*x + b)/(2*x))/(tan((a*x + b)/(2*x))**4 + 2*tan((a*x
 + b)/(2*x))**2 + 1),x)*tan((a*x + b)/(2*x))**4*b + 64*int(tan((a*x + b)/(
2*x))/(tan((a*x + b)/(2*x))**4 + 2*tan((a*x + b)/(2*x))**2 + 1),x)*tan((a*
x + b)/(2*x))**2*b + 32*int(tan((a*x + b)/(2*x))/(tan((a*x + b)/(2*x))**4
+ 2*tan((a*x + b)/(2*x))**2 + 1),x)*b - 16*int(1/(tan((a*x + b)/(2*x))**4*
x + 2*tan((a*x + b)/(2*x))**2*x + x),x)*tan((a*x + b)/(2*x))**4*b**2 - 32*
int(1/(tan((a*x + b)/(2*x))**4*x + 2*tan((a*x + b)/(2*x))**2*x + x),x)*tan
((a*x + b)/(2*x))**2*b**2 - 16*int(1/(tan((a*x + b)/(2*x))**4*x + 2*tan((a
*x + b)/(2*x))**2*x + x),x)*b**2 + 6*log(x)*tan((a*x + b)/(2*x))**4*b**2 +
 12*log(x)*tan((a*x + b)/(2*x))**2*b**2 + 6*log(x)*b**2 - sin((a*x + b)/x)
**2*tan((a*x + b)/(2*x))**4*x**2 - 2*sin((a*x + b)/x)**2*tan((a*x + b)/(2*
x))**2*x**2 - sin((a*x + b)/x)**2*x**2 - 8*sin((a*x + b)/x)*tan((a*x + b)/
(2*x))**4*b*x - 16*sin((a*x + b)/x)*tan((a*x + b)/(2*x))**2*b*x - 8*sin((a
*x + b)/x)*b*x + 8*tan((a*x + b)/(2*x))**4*x**2 + 16*tan((a*x + b)/(2*x))*
*2*x**2 - 8*x**2)/(6*(tan((a*x + b)/(2*x))**4 + 2*tan((a*x + b)/(2*x))**2
+ 1))
```

3.113 $\int \sin^2\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx=-b\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a)+x\sin^2\left(a+\frac{b}{x}\right)-b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)$$

output
```
-b*Ci(2*b/x)*sin(2*a)+x*sin(a+b/x)^2-b*cos(2*a)*Si(2*b/x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx=-b\,\text{CosIntegral}\left(\frac{2b}{x}\right)\sin(2a)+x\sin^2\left(a+\frac{b}{x}\right)-b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)$$

input
```
Integrate[Sin[a + b/x]^2,x]
```

output
```
-(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInte
gral[(2*b)/x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3842, 3042, 3794, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(a+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{3842}$$

$$-\int x^2\sin^2\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\sin\left(a+\frac{b}{x}\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{3794}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-2b\int \frac{1}{2}x\sin\left(2a+\frac{2b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-b\int x\sin\left(2a+\frac{2b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-b\int x\sin\left(2a+\frac{2b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3784}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-b\left(\sin(2a)\int x\cos\left(\frac{2b}{x}\right)d\frac{1}{x}+\cos(2a)\int x\sin\left(\frac{2b}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-b\left(\sin(2a)\int x\sin\left(\frac{2b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+\cos(2a)\int x\sin\left(\frac{2b}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3780}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-b\left(\sin(2a)\int x\sin\left(\frac{2b}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+\cos(2a)\text{Si}\left(\frac{2b}{x}\right)\right)$$

$$\Big\downarrow \text{3783}$$

$$x\sin^2\left(a+\frac{b}{x}\right)-b\left(\sin(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\cos(2a)\text{Si}\left(\frac{2b}{x}\right)\right)$$

input

```
Int[Sin[a + b/x]^2,x]
```

output

```
x*Sin[a + b/x]^2 - b*(CosIntegral[(2*b)/x]*Sin[2*a] + Cos[2*a]*SinIntegral
[(2*b)/x])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

method	result
derivativedivides	$-b\left(-\frac{x}{2b}+\frac{\cos\left(2a+\frac{2b}{x}\right)x}{2b}+\operatorname{Si}\left(\frac{2b}{x}\right)\cos\left(2a\right)+\operatorname{Ci}\left(\frac{2b}{x}\right)\sin\left(2a\right)\right)$
default	$-b\left(-\frac{x}{2b}+\frac{\cos\left(2a+\frac{2b}{x}\right)x}{2b}+\operatorname{Si}\left(\frac{2b}{x}\right)\cos\left(2a\right)+\operatorname{Ci}\left(\frac{2b}{x}\right)\sin\left(2a\right)\right)$
risch	$\frac{\pi\ \operatorname{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}b}{2}-\operatorname{Si}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}b+\frac{i\mathrm{e}^{-2ia}\ \mathrm{expIntegral}_1\left(-\frac{2ib}{x}\right)b}{2}-\frac{ib\ \mathrm{expIntegral}_1\left(-\frac{2ib}{x}\right)\mathrm{e}^{2ia}}{2}+\frac{x}{2}-$

input
```
int(sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-b*(-1/2*x/b+1/2*cos(2*a+2*b/x)/b*x+Si(2*b/x)*cos(2*a)+Ci(2*b/x)*sin(2*a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx = -x\cos\left(\frac{ax+b}{x}\right)^2 - b\,\mathrm{Ci}\left(\frac{2\,b}{x}\right)\sin\left(2\,a\right) - b\cos\left(2\,a\right)\mathrm{Si}\left(\frac{2\,b}{x}\right) + x$$

input
```
integrate(sin(a+b/x)^2,x, algorithm="fricas")
```

output
```
-x*cos((a*x + b)/x)^2 - b*cos_integral(2*b/x)*sin(2*a) - b*cos(2*a)*sin_in
tegral(2*b/x) + x
```

Sympy [F]

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx = \int \sin^2\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(sin(a+b/x)**2,x)
```

output
```
Integral(sin(a + b/x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\begin{aligned}\int \sin^2\left(a+\frac{b}{x}\right)\,dx =\\ &-\frac{1}{2}\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)+\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b\\ &-\frac{1}{2}\,x\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{2}\,x\end{aligned}$$

input
```
integrate(sin(a+b/x)^2,x, algorithm="maxima")
```

output
```
-1/2*((-I*Ei(2*I*b/x) + I*Ei(-2*I*b/x))*cos(2*a) + (Ei(2*I*b/x) + Ei(-2*I*
b/x))*sin(2*a))*b - 1/2*x*cos(2*(a*x + b)/x) + 1/2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.73

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx =$$

$$-\frac{2\,ab^2\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin\left(2\,a\right)-2\,ab^2\cos\left(2\,a\right)\operatorname{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{2\,(ax+b)b^2\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x}}{2\left(a-\frac{ax+b}{x}\right)b}$$

input
```
integrate(sin(a+b/x)^2,x, algorithm="giac")
```

output
```
-1/2*(2*a*b^2*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a) - 2*a*b^2*cos(2*
a)*sin_integral(2*a - 2*(a*x + b)/x) - 2*(a*x + b)*b^2*cos_integral(-2*a +
 2*(a*x + b)/x)*sin(2*a)/x + 2*(a*x + b)*b^2*cos(2*a)*sin_integral(2*a - 2
*(a*x + b)/x)/x - b^2*cos(2*(a*x + b)/x) + b^2)/((a - (a*x + b)/x)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx = \int \sin\left(a+\frac{b}{x}\right)^2\,dx$$

input
```
int(sin(a + b/x)^2,x)
```

output
```
int(sin(a + b/x)^2, x)
```

Reduce [F]

$$\int \sin^2\left(a+\frac{b}{x}\right)\,dx=\int \sin\left(\frac{ax+b}{x}\right)^2\,dx$$

input
```
int(sin(a+b/x)^2,x)
```

output
```
int(sin((a*x + b)/x)**2,x)
```

3.114 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\frac{1}{2}\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)$$

output

```
1/2*cos(2*a)*Ci(2*b/x)+1/2*ln(x)-1/2*sin(2*a)*Si(2*b/x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\frac{1}{2}\left(\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)+\log(x)-\sin(2a)\text{Si}\left(\frac{2b}{x}\right)\right)$$

input

```
Integrate[Sin[a + b/x]^2/x,x]
```

output

```
(Cos[2*a]*CosIntegral[(2*b)/x] + Log[x] - Sin[2*a]*SinIntegral[(2*b)/x])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{1}{2x}-\frac{\cos\left(2a+\frac{2b}{x}\right)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\cos(2a)\,\text{CosIntegral}\left(\frac{2b}{x}\right)-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}$$

input
```
Int[Sin[a + b/x]^2/x,x]
```

output
```
(Cos[2*a]*CosIntegral[(2*b)/x])/2 + Log[x]/2 - (Sin[2*a]*SinIntegral[(2*b)
/x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{b}{x}\right)}{2}-\frac{\sin(2a)\operatorname{Si}\left(\frac{2b}{x}\right)}{2}+\frac{\cos(2a)\operatorname{Ci}\left(\frac{2b}{x}\right)}{2}$	36
default	$-\frac{\ln\left(\frac{b}{x}\right)}{2}-\frac{\sin(2a)\operatorname{Si}\left(\frac{2b}{x}\right)}{2}+\frac{\cos(2a)\operatorname{Ci}\left(\frac{2b}{x}\right)}{2}$	36
risch	$\frac{i\mathrm{e}^{-2ia}\pi\operatorname{csgn}\left(\frac{b}{x}\right)}{4}-\frac{i\mathrm{e}^{-2ia}\operatorname{Si}\left(\frac{2b}{x}\right)}{2}-\frac{\mathrm{e}^{-2ia}\operatorname{expIntegral}_1\left(-\frac{2ib}{x}\right)}{4}-\frac{\mathrm{e}^{2ia}\operatorname{expIntegral}_1\left(-\frac{2ib}{x}\right)}{4}+\frac{\ln(x)}{2}$	68

input

```
int(sin(a+b/x)^2/x,x,method=_RETURNVERBOSE)
```

output

```
-1/2*ln(b/x)-1/2*sin(2*a)*Si(2*b/x)+1/2*cos(2*a)*Ci(2*b/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\frac{1}{2}\cos\left(2\,a\right)\operatorname{Ci}\left(\frac{2\,b}{x}\right)-\frac{1}{2}\sin\left(2\,a\right)\operatorname{Si}\left(\frac{2\,b}{x}\right)+\frac{1}{2}\log\left(x\right)$$

input

```
integrate(sin(a+b/x)^2/x,x, algorithm="fricas")
```

output

```
1/2*cos(2*a)*cos_integral(2*b/x) - 1/2*sin(2*a)*sin_integral(2*b/x) + 1/2*
log(x)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\frac{\log\left(x\right)}{2}-\frac{\sin\left(2a\right)\operatorname{Si}\left(\frac{2b}{x}\right)}{2}+\frac{\cos\left(2a\right)\operatorname{Ci}\left(\frac{2b}{x}\right)}{2}$$

input

```
integrate(sin(a+b/x)**2/x,x)
```

output

```
log(x)/2 - sin(2*a)*Si(2*b/x)/2 + cos(2*a)*Ci(2*b/x)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx&=\frac{1}{4}\left(\operatorname{Ei}\left(\frac{2i\,b}{x}\right)+\operatorname{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)\\&\quad+\frac{1}{4}\left(i\operatorname{Ei}\left(\frac{2i\,b}{x}\right)-i\operatorname{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)+\frac{1}{2}\log\left(x\right)\end{aligned}$$

input

```
integrate(sin(a+b/x)^2/x,x, algorithm="maxima")
```

output

```
1/4*(Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + 1/4*(I*Ei(2*I*b/x) - I*Ei(-2*I
*b/x))*sin(2*a) + 1/2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\begin{aligned}&\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx\\&=\frac{b\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)+b\sin\left(2\,a\right)\operatorname{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-b\log\left(-a+\frac{ax+b}{x}\right)}{2\,b}\end{aligned}$$

input
```
integrate(sin(a+b/x)^2/x,x, algorithm="giac")
```

output
```
1/2*(b*cos(2*a)*cos_integral(-2*a + 2*(a*x + b)/x) + b*sin(2*a)*sin_integr
al(2*a - 2*(a*x + b)/x) - b*log(-a + (a*x + b)/x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x}\,dx$$

input
```
int(sin(a + b/x)^2/x,x)
```

output
```
int(sin(a + b/x)^2/x, x)
```

Reduce [F]

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx=\int \frac{\sin\left(\frac{ax+b}{x}\right)^2}{x}dx$$

input
```
int(sin(a+b/x)^2/x,x)
```

output
```
int(sin((a*x + b)/x)**2/x,x)
```

3.115 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} \, dx = -\frac{1}{2x}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}$$

output
```
-1/2/x+1/2*cos(a+b/x)*sin(a+b/x)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} \, dx = -\frac{a+\frac{b}{x}}{2b}+\frac{\sin\left(2\left(a+\frac{b}{x}\right)\right)}{4b}$$

input
```
Integrate[Sin[a + b/x]^2/x^2,x]
```

output
```
-1/2*(a + b/x)/b + Sin[2*(a + b/x)]/(4*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3860, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}dx$$

$$\Big\downarrow 3860$$

$$-\int \sin^2\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow 3042$$

$$-\int \sin\left(a+\frac{b}{x}\right)^2d\frac{1}{x}$$

$$\Big\downarrow 3115$$

$$\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}-\frac{\int 1d\frac{1}{x}}{2}$$

$$\Big\downarrow 24$$

$$\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}-\frac{1}{2x}$$

input
```
Int[Sin[a + b/x]^2/x^2,x]
```

output
```
-1/2*1/x + (Cos[a + b/x]*Sin[a + b/x])/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{1}{2x}+\frac{\sin\left(\frac{2ax+2b}{x}\right)}{4b}$
parallelrisch	$\frac{-2b+x\sin\left(\frac{2ax+2b}{x}\right)}{4bx}$
derivativedivides	$-\frac{-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}}{b}$
default	$-\frac{-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}}{b}$
norman	$\frac{-\frac{1}{2}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{2}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{4}}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{3}}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^{2}\right)^{2}x}$
orering	$-\frac{\left(b^{2}+2x^{2}\right)\sin\left(a+\frac{b}{x}\right)^{2}}{x\,b^{2}}-\frac{7x^{4}\left(-\frac{2\sin\left(a+\frac{b}{x}\right)b\cos\left(a+\frac{b}{x}\right)}{x^{4}}-\frac{2\sin\left(a+\frac{b}{x}\right)^{2}}{x^{3}}\right)}{4b^{2}}-\frac{x^{5}\left(\frac{2b^{2}\cos\left(a+\frac{b}{x}\right)^{2}}{x^{6}}+\frac{12\sin\left(a+\frac{b}{x}\right)b\cos}{x^{5}}\right.}{4}$

input
```
int(sin(a+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x+1/4/b*sin(2*(a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{x\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-b}{2\,bx}$$

input
```
integrate(sin(a+b/x)^2/x^2,x, algorithm="fricas")
```

output
```
1/2*(x*cos((a*x + b)/x)*sin((a*x + b)/x) - b)/(b*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(20) = 40$.

Time = 0.95 (sec) , antiderivative size = 262, normalized size of antiderivative = 8.45

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx$$
$$=\begin{cases} -\frac{b\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx}-\frac{2b\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx}-\frac{b}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2} \\ -\frac{\sin^2(a)}{x} \end{cases}$$

input
```
integrate(sin(a+b/x)**2/x**2,x)
```

output

```
Piecewise((-b*tan(a/2 + b/(2*x))**4/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*t
an(a/2 + b/(2*x))**2 + 2*b*x) - 2*b*tan(a/2 + b/(2*x))**2/(2*b*x*tan(a/2 +
 b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - b/(2*b*x*tan(a/2 + b
/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - 2*x*tan(a/2 + b/(2*x))
**3/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) +
2*x*tan(a/2 + b/(2*x))/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2
*x))**2 + 2*b*x), Ne(b, 0)), (-sin(a)**2/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = \frac{x\sin\left(\frac{2\,(ax+b)}{x}\right)-2\,b}{4\,bx}$$

input

```
integrate(sin(a+b/x)^2/x^2,x, algorithm="maxima")
```

output

```
1/4*(x*sin(2*(a*x + b)/x) - 2*b)/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\frac{2\,(ax+b)}{x}-\sin\left(\frac{2\,(ax+b)}{x}\right)}{4\,b}$$

input

```
integrate(sin(a+b/x)^2/x^2,x, algorithm="giac")
```

output

```
-1/4*(2*(a*x + b)/x - sin(2*(a*x + b)/x))/b
```

Mupad [B] (verification not implemented)

Time = 41.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b}-\frac{1}{2\,x}$$

input
```
int(sin(a + b/x)^2/x^2,x)
```

output
```
sin(2*a + (2*b)/x)/(4*b) - 1/(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx=\frac{\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)x-ax-b}{2bx}$$

input
```
int(sin(a+b/x)^2/x^2,x)
```

output
```
(cos((a*x + b)/x)*sin((a*x + b)/x)*x - a*x - b)/(2*b*x)
```

3.116 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx=-\frac{1}{4x^2}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}$$

output
```
-1/4/x^2+1/2*cos(a+b/x)*sin(a+b/x)/b/x-1/4*sin(a+b/x)^2/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{x^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-2b\left(b-x\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{8b^2x^2}$$

input
```
Integrate[Sin[a + b/x]^2/x^3,x]
```

output
```
(x^2*Cos[2*(a + b/x)] - 2*b*(b - x*Sin[2*(a + b/x)]))/(8*b^2*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3860, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{1}{2}\int \frac{1}{x}d\frac{1}{x}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}-\frac{1}{4x^2}$$

input

```
Int[Sin[a + b/x]^2/x^3,x]
```

output

```
-1/4*1/x^2 + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x) - Sin[a + b/x]^2/(4*b^2)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{1}{4x^2}+\frac{\cos\left(\frac{2ax+2b}{x}\right)}{8b^2}+\frac{\sin\left(\frac{2ax+2b}{x}\right)}{4bx}$
parallelrisch	$\frac{2bx\sin\left(\frac{2ax+2b}{x}\right)+x^2\cos\left(\frac{2ax+2b}{x}\right)-2b^2-x^2}{8x^2b^2}$
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\sin\left(a+\frac{b}{x}\right)^2}{4}-a\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)}{b^2}$
default	$-\frac{\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\sin\left(a+\frac{b}{x}\right)^2}{4}-a\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)}{b^2}$
norman	$\frac{-\frac{1}{4}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{b^2}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{2}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^4}{4}-\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^3}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2\right)^2x^2}$
orering	$-\frac{\left(b^2+5x^2\right)\sin\left(a+\frac{b}{x}\right)^2}{2x^2b^2}-\frac{5x^4\left(-\frac{2\sin\left(a+\frac{b}{x}\right)b\cos\left(a+\frac{b}{x}\right)}{x^5}-\frac{3\sin\left(a+\frac{b}{x}\right)^2}{x^4}\right)}{4b^2}-x^5\Big(\frac{2b^2\cos\left(a+\frac{b}{x}\right)^2}{x^7}+\frac{16\sin\left(a+\frac{b}{x}\right)b\cos}{x^6}$

input
```
int(sin(a+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2+1/8/b^2*cos(2*(a*x+b)/x)+1/4/b/x*sin(2*(a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int\frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{2\,x^2\cos\left(\frac{ax+b}{x}\right)^2+4\,bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-2\,b^2-x^2}{8\,b^2x^2}$$

input
```
integrate(sin(a+b/x)^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*x^2*cos((a*x + b)/x)^2 + 4*b*x*cos((a*x + b)/x)*sin((a*x + b)/x) -
2*b^2 - x^2)/(b^2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(37) = 74$.

Time = 1.29 (sec) , antiderivative size = 391, normalized size of antiderivative = 7.67

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx$$
$$= \begin{cases} -\frac{b^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2} - \frac{2b^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2} - \frac{b^2}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x} \\ -\frac{\sin^2(a)}{2x^2} \end{cases}$$

input
```
integrate(sin(a+b/x)**2/x**3,x)
```

output
```
Piecewise((-b**2*tan(a/2 + b/(2*x))**4/(4*b**2*x**2*tan(a/2 + b/(2*x))**4
+ 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 2*b**2*tan(a/2 + b/(2
*x))**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x)
)**2 + 4*b**2*x**2) - b**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**
2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*b*x*tan(a/2 + b/(2*x))**3/(4*b*
*2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2
*x**2) + 4*b*x*tan(a/2 + b/(2*x))/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b
**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*x**2*tan(a/2 + b/(2*x))*
*2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2
+ 4*b**2*x**2), Ne(b, 0)), (-sin(a)**2/(2*x**2), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx$$
$$= \frac{\left(\left(\Gamma\left(2, \frac{2i\,b}{x}\right) + \Gamma\left(2, -\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right) - \left(i\,\Gamma\left(2, \frac{2i\,b}{x}\right) - i\,\Gamma\left(2, -\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^2 - 4\,b^2}{16\,b^2x^2}$$

input
```
integrate(sin(a+b/x)^2/x^3,x, algorithm="maxima")
```

output

```
1/16*(((gamma(2, 2*I*b/x) + gamma(2, -2*I*b/x))*cos(2*a) - (I*gamma(2, 2*I
*b/x) - I*gamma(2, -2*I*b/x))*sin(2*a))*x^2 - 4*b^2)/(b^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx$$
$$= -\frac{2\,a\sin\left(\frac{2\,(ax+b)}{x}\right) - \frac{4\,(ax+b)a}{x} - \frac{2\,(ax+b)\sin\left(\frac{2\,(ax+b)}{x}\right)}{x} + \frac{2\,(ax+b)^2}{x^2} - \cos\left(\frac{2\,(ax+b)}{x}\right)}{8\,b^2}$$

input

```
integrate(sin(a+b/x)^2/x^3,x, algorithm="giac")
```

output

```
-1/8*(2*a*sin(2*(a*x + b)/x) - 4*(a*x + b)*a/x - 2*(a*x + b)*sin(2*(a*x +
b)/x)/x + 2*(a*x + b)^2/x^2 - cos(2*(a*x + b)/x))/b^2
```

Mupad [B] (verification not implemented)

Time = 42.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\cos\left(2\,a+\frac{2\,b}{x}\right)}{8\,b^2} - \frac{1}{4\,x^2} + \frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b\,x}$$

input

```
int(sin(a + b/x)^2/x^3,x)
```

output

```
cos(2*a + (2*b)/x)/(8*b^2) - 1/(4*x^2) + sin(2*a + (2*b)/x)/(4*b*x)
```

Reduce [F]

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx = \int \frac{\sin\left(\frac{ax+b}{x}\right)^2}{x^3}dx$$

input

```
int(sin(a+b/x)^2/x^3,x)
```

output

```
int(sin((a*x + b)/x)**2/x**3,x)
```

3.117 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{1}{6x^3}+\frac{1}{4b^2x}-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}$$

output

```
-1/6/x^3+1/4/b^2/x-1/4*cos(a+b/x)*sin(a+b/x)/b^3+1/2*cos(a+b/x)*sin(a+b/x)
/b/x^2-1/2*sin(a+b/x)^2/b^2/x
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{-4b^3+6bx^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-3(-2b^2x+x^3)\sin\left(2\left(a+\frac{b}{x}\right)\right)}{24b^3x^3}$$

input

```
Integrate[Sin[a + b/x]^2/x^4,x]
```

output
```
(-4*b^3 + 6*b*x^2*Cos[2*(a + b/x)] - 3*(-2*b^2*x + x^3)*Sin[2*(a + b/x)])/
(24*b^3*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3860, 3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\int \sin^2\left(a+\frac{b}{x}\right)d\frac{1}{x}}{2b^2}-\frac{1}{2}\int \frac{1}{x^2}d\frac{1}{x}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int \sin^2\left(a+\frac{b}{x}\right)d\frac{1}{x}}{2b^2}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{1}{6x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(a+\frac{b}{x}\right)^2d\frac{1}{x}}{2b^2}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{1}{6x^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1d\frac{1}{x}}{2}-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}}{2b^2}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{1}{6x^3}$$

$$\Big\downarrow 24$$

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}+\frac{\frac{1}{2x}-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}}{2b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{1}{6x^3}$$

input
```
Int[Sin[a + b/x]^2/x^4,x]
```

output
```
-1/6*1/x^3 + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x^2) - Sin[a + b/x]^2/(2*b^2
*x) + (1/(2*x) - (Cos[a + b/x]*Sin[a + b/x])/(2*b))/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{1}{6x^3}+\frac{\cos\left(\frac{2ax+2b}{x}\right)}{4b^2x}+\frac{(2b^2-x^2)\sin\left(\frac{2ax+2b}{x}\right)}{8b^3x^2}$
parallelrisch	$\frac{6x^2b\cos\left(\frac{2ax+2b}{x}\right)+6b^2x\sin\left(\frac{2ax+2b}{x}\right)-3x^3\sin\left(\frac{2ax+2b}{x}\right)-4b^3}{24x^3b^3}$
norman	$\frac{-\frac{1}{6}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{3}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^4}{6}+\frac{x^2}{4b^2}-\frac{x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2b^3}+\frac{x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^3}{2b^3}-\frac{3x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{2b^2}+\frac{x^2\tan(}{4}}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2\right)^2x^3}$
orering	$-\frac{(b^4+8x^2b^2-9x^4)\sin\left(a+\frac{b}{x}\right)^2}{3x^3b^4}-\frac{x^4(26b^2-33x^2)\left(-\frac{2\sin\left(a+\frac{b}{x}\right)b\cos\left(a+\frac{b}{x}\right)}{x^6}-\frac{4\sin\left(a+\frac{b}{x}\right)^2}{x^5}\right)}{24b^4}-\frac{(2b^2-3x^2)x^5\left(\frac{2b}{}\right.}{}$
derivativedivides	$-\frac{a^2\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-2a\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\sin\left(a+\frac{b}{x}\right)^2}{4}\right)+\left(a-\right.}{b^3}$
default	$-\frac{a^2\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-2a\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\sin\left(a+\frac{b}{x}\right)^2}{4}\right)+\left(a-\right.}{b^3}$

input
```
int(sin(a+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^3+1/4/b^2/x*cos(2*(a*x+b)/x)+1/8*(2*b^2-x^2)/b^3/x^2*sin(2*(a*x+b)/
x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{6\,bx^2\cos\left(\frac{ax+b}{x}\right)^2 - 2\,b^3 - 3\,bx^2 + 3\,(2\,b^2x - x^3)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{12\,b^3x^3}$$

input
```
integrate(sin(a+b/x)^2/x^4,x, algorithm="fricas")
```

output
```
1/12*(6*b*x^2*cos((a*x + b)/x)^2 - 2*b^3 - 3*b*x^2 + 3*(2*b^2*x - x^3)*cos
((a*x + b)/x)*sin((a*x + b)/x))/(b^3*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(68) = 136.

Time = 1.73 (sec) , antiderivative size = 654, normalized size of antiderivative = 7.52

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b/x)**2/x**4,x)
```

output

```
Piecewise((-2*b**3*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))*
*4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 4*b**3*tan(a/2 +
 b/(2*x))**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 +
b/(2*x))**2 + 12*b**3*x**3) - 2*b**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 +
 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 12*b**2*x*tan(a/2 +
b/(2*x))**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b
/(2*x))**2 + 12*b**3*x**3) + 12*b**2*x*tan(a/2 + b/(2*x))/(12*b**3*x**3*ta
n(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) +
 3*b*x**2*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b
**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 18*b*x**2*tan(a/2 + b/(2*
x))**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x
))**2 + 12*b**3*x**3) + 3*b*x**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 6*x**3*tan(a/2 + b/(2*x)
)**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))
**2 + 12*b**3*x**3) - 6*x**3*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 + b/
(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3), Ne(b, 0)),
 (-sin(a)**2/(3*x**3), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$$
$$= \frac{3\left(\left(-i\,\Gamma\left(3,\frac{2i\,b}{x}\right)+i\,\Gamma\left(3,-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(\Gamma\left(3,\frac{2i\,b}{x}\right)+\Gamma\left(3,-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^3-16\,b^3}{96\,b^3x^3}$$

input

```
integrate(sin(a+b/x)^2/x^4,x, algorithm="maxima")
```

output

```
1/96*(3*((-I*gamma(3, 2*I*b/x) + I*gamma(3, -2*I*b/x))*cos(2*a) - (gamma(3
, 2*I*b/x) + gamma(3, -2*I*b/x))*sin(2*a))*x^3 - 16*b^3)/(b^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$$

$$= \frac{6\,a^2\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2}{x}-6\,a\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{12\,(ax+b)^2a}{x^2}+\frac{6\,(ax+b)\cos\left(\frac{2\,(ax+b)}{x}\right)}{x}}{24\,b^3}$$

input

```
integrate(sin(a+b/x)^2/x^4,x, algorithm="giac")
```

output

```
1/24*(6*a^2*sin(2*(a*x + b)/x) - 12*(a*x + b)*a^2/x - 6*a*cos(2*(a*x + b)/
x) - 12*(a*x + b)*a*sin(2*(a*x + b)/x)/x + 12*(a*x + b)^2*a/x^2 + 6*(a*x +
 b)*cos(2*(a*x + b)/x)/x + 6*(a*x + b)^2*sin(2*(a*x + b)/x)/x^2 - 4*(a*x +
 b)^3/x^3 - 3*sin(2*(a*x + b)/x))/b^3
```

Mupad [B] (verification not implemented)

Time = 42.89 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{\frac{b\,x^2\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{4}-\frac{b^3}{6}+\frac{b^2\,x\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{4}}{b^3\,x^3}-\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{8\,b^3}$$

input

```
int(sin(a + b/x)^2/x^4,x)
```

output

```
((b*x^2*cos(2*a + (2*b)/x))/4 - b^3/6 + (b^2*x*sin(2*a + (2*b)/x))/4)/(b^3
*x^3) - sin(2*a + (2*b)/x)/(8*b^3)
```

Reduce [F]

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx=\int \frac{\sin\left(\frac{ax+b}{x}\right)^2}{x^4}dx$$

input
```
int(sin(a+b/x)^2/x^4,x)
```

output
```
int(sin((a*x + b)/x)**2/x**4,x)
```

3.118 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\begin{aligned}\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = &-\frac{1}{8x^4}+\frac{3}{8b^2x^2}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3}\\ &-\frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x}+\frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}\end{aligned}$$

output

```
-1/8/x^4+3/8/b^2/x^2+1/2*cos(a+b/x)*sin(a+b/x)/b/x^3-3/4*cos(a+b/x)*sin(a+
b/x)/b^3/x+3/8*sin(a+b/x)^2/b^4-3/4*sin(a+b/x)^2/b^2/x^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\begin{aligned}&\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx\\ &= -\frac{3(-2b^2x^2+x^4)\cos\left(2\left(a+\frac{b}{x}\right)\right)+2b\left(b^3+(-2b^2x+3x^3)\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{16b^4x^4}\end{aligned}$$

input

```
Integrate[Sin[a + b/x]^2/x^5,x]
```

output
```
-1/16*(3*(-2*b^2*x^2 + x^4)*Cos[2*(a + b/x)] + 2*b*(b^3 + (-2*b^2*x + 3*x^
3)*Sin[2*(a + b/x)]))/(b^4*x^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3860, 3042, 3792, 15, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}dx$$

$$\Big\downarrow \text{3860}$$

$$-\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{3792}$$

$$\frac{3\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{2b^2}-\frac{1}{2}\int \frac{1}{x^3}d\frac{1}{x}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{2b^2}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}-\frac{1}{8x^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x}d\frac{1}{x}}{2b^2}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}-\frac{1}{8x^4}$$

$$\Big\downarrow \text{3791}$$

$$\frac{3\left(\frac{1}{2}\int \frac{1}{x}d\frac{1}{x}+\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}\right)}{2b^2}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}-\frac{1}{8x^4}$$

$$\Big\downarrow 15$$

$$-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{3\left(\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}+\frac{1}{4x^2}\right)}{2b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}-\frac{1}{8x^4}$$

input
```
Int[Sin[a + b/x]^2/x^5,x]
```

output
```
-1/8*1/x^4 + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x^3) - (3*Sin[a + b/x]^2)/(4
*b^2*x^2) + (3*(1/(4*x^2) - (Cos[a + b/x]*Sin[a + b/x])/(2*b*x) + Sin[a +
b/x]^2/(4*b^2)))/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{1}{8x^4}+\frac{3(2b^2-x^2)\cos\left(\frac{2ax+2b}{x}\right)}{16x^2b^4}+\frac{(2b^2-3x^2)\sin\left(\frac{2ax+2b}{x}\right)}{8b^3x^3}$
parallelrisch	$\frac{(6x^2b^2-3x^4)\cos\left(\frac{2ax+2b}{x}\right)+(4b^3x-6b\,x^3)\sin\left(\frac{2ax+2b}{x}\right)-2b^4+3x^4}{16x^4b^4}$
orering	$-\frac{(2b^4+25x^2b^2-63x^4)\sin\left(a+\frac{b}{x}\right)^2}{8x^4b^4}-\frac{x^4(8b^2-21x^2)\left(-\frac{2\sin\left(a+\frac{b}{x}\right)b\cos\left(a+\frac{b}{x}\right)}{x^7}-\frac{5\sin\left(a+\frac{b}{x}\right)^2}{x^6}\right)}{8b^4}-(b^2-3x^2)x^5\Big($
norman	$\frac{-\frac{1}{8}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{4}-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^4}{8}+\frac{3x^2}{8b^2}-\frac{3x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2b^3}+\frac{3x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^3}{2b^3}-\frac{9x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2}{4b^2}+3x^2}{\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x}\right)^2\right)^2x^4}$
derivativedivides	$-\frac{-a^3\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)+3a^2\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\sin\left(a+\frac{b}{x}\right)^2}{4}\right)-3}{}$
default	$-\frac{-a^3\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)+3a^2\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\sin\left(a+\frac{b}{x}\right)^2}{4}\right)-3}{}$

input
```
int(sin(a+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8/x^4+3/16/x^2*(2*b^2-x^2)/b^4*cos(2*(a*x+b)/x)+1/8/b^3*(2*b^2-3*x^2)/x
^3*sin(2*(a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{2\,b^4+6\,b^2x^2-3\,x^4-6\,(2\,b^2x^2-x^4)\cos\left(\frac{ax+b}{x}\right)^2-4\,(2\,b^3x-3\,bx^3)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{16\,b^4x^4}$$

input
```
integrate(sin(a+b/x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/16*(2*b^4 + 6*b^2*x^2 - 3*x^4 - 6*(2*b^2*x^2 - x^4)*cos((a*x + b)/x)^2
- 4*(2*b^3*x - 3*b*x^3)*cos((a*x + b)/x)*sin((a*x + b)/x))/(b^4*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(92) = 184.

Time = 2.42 (sec) , antiderivative size = 726, normalized size of antiderivative = 6.79

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b/x)**2/x**5,x)
```

output
```
Piecewise((-b**4*tan(a/2 + b/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4
+ 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 2*b**4*tan(a/2 + b/(
2*x))**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*
x))**2 + 8*b**4*x**4) - b**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*
x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 8*b**3*x*tan(a/2 + b/(2*x))**3
/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 +
 8*b**4*x**4) + 8*b**3*x*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x)
)**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3*b**2*x**2*tan
(a/2 + b/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a
/2 + b/(2*x))**2 + 8*b**4*x**4) - 18*b**2*x**2*tan(a/2 + b/(2*x))**2/(8*b*
*4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**
4*x**4) + 3*b**2*x**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*ta
n(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 12*b*x**3*tan(a/2 + b/(2*x))**3/(8*b*
*4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**
4*x**4) - 12*b*x**3*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4
+ 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 12*x**4*tan(a/2 + b/
(2*x))**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4), Ne(b, 0)), (-sin(a)**2/(4*x**4), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{\left(\left(\Gamma\left(4,\frac{2ib}{x}\right)+\Gamma\left(4,-\frac{2ib}{x}\right)\right)\cos\left(2\,a\right)-\left(i\,\Gamma\left(4,\frac{2ib}{x}\right)-i\,\Gamma\left(4,-\frac{2ib}{x}\right)\right)\sin\left(2\,a\right)\right)x^4+8\,b^4}{64\,b^4x^4}$$

input
```
integrate(sin(a+b/x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/64*(((gamma(4, 2*I*b/x) + gamma(4, -2*I*b/x))*cos(2*a) - (I*gamma(4, 2*
I*b/x) - I*gamma(4, -2*I*b/x))*sin(2*a))*x^4 + 8*b^4)/(b^4*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(95) = 190$.

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.38

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx =$$

$$-\frac{4\,a^3\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{8\,(ax+b)a^3}{x}-6\,a^2\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{12\,(ax+b)^2a^2}{x^2}+\frac{12\,(ax+b)a\cos\Big(}{x}}{}$$

input
```
integrate(sin(a+b/x)^2/x^5,x, algorithm="giac")
```

output
```
-1/16*(4*a^3*sin(2*(a*x + b)/x) - 8*(a*x + b)*a^3/x - 6*a^2*cos(2*(a*x + b
)/x) - 12*(a*x + b)*a^2*sin(2*(a*x + b)/x)/x + 12*(a*x + b)^2*a^2/x^2 + 12
*(a*x + b)*a*cos(2*(a*x + b)/x)/x - 6*a*sin(2*(a*x + b)/x) + 12*(a*x + b)^
2*a*sin(2*(a*x + b)/x)/x^2 - 8*(a*x + b)^3*a/x^3 - 6*(a*x + b)^2*cos(2*(a*
x + b)/x)/x^2 - 4*(a*x + b)^3*sin(2*(a*x + b)/x)/x^3 + 6*(a*x + b)*sin(2*(
a*x + b)/x)/x + 2*(a*x + b)^4/x^4 + 3*cos(2*(a*x + b)/x))/b^4
```

Mupad [B] (verification not implemented)

Time = 43.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{3\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{16\,b^4} - \frac{\frac{b^4}{8}-\frac{3\,b^2\,x^2\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{8}+\frac{3\,b\,x^3\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{8}-\frac{b^3\,x\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{4}}{b^4\,x^4}$$

input
```
int(sin(a + b/x)^2/x^5,x)
```

output
```
- (3*cos(2*a + (2*b)/x))/(16*b^4) - (b^4/8 - (3*b^2*x^2*cos(2*a + (2*b)/x)
)/8 + (3*b*x^3*sin(2*a + (2*b)/x))/8 - (b^3*x*sin(2*a + (2*b)/x))/4)/(b^4*
x^4)
```

Reduce [F]

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx=\int \frac{\sin\left(\frac{ax+b}{x}\right)^2}{x^5}dx$$

input

```
int(sin(a+b/x)^2/x^5,x)
```

output

```
int(sin((a*x + b)/x)**2/x**5,x)
```

3.119 $\int \sin\left(a+\frac{b}{x^2}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int \sin\left(a+\frac{b}{x^2}\right) dx = -\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a) + x\sin\left(a+\frac{b}{x^2}\right)$$

output

```
-b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)+b^(1
/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)+x*sin(a+b
/x^2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sin\left(a+\frac{b}{x^2}\right) dx = x\cos\left(\frac{b}{x^2}\right)\sin(a) - \sqrt{b}\sqrt{2\pi}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) - \text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right) + x\cos(a)\sin\left(\frac{b}{x^2}\right)$$

input
```
Integrate[Sin[a + b/x^2],x]
```

output
```
x*Cos[b/x^2]*Sin[a] - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi])/x] - FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) + x*Cos[a]*Sin[b/x^2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3840, 3868, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{b}{x^2}\right)dx$$

$$\Big\downarrow \text{3840}$$

$$-\int x^2\sin\left(a+\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3868}$$

$$x\sin\left(a+\frac{b}{x^2}\right)-2b\int \cos\left(a+\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3835}$$

$$x\sin\left(a+\frac{b}{x^2}\right)-2b\left(\cos(a)\int \cos\left(\frac{b}{x^2}\right)d\frac{1}{x}-\sin(a)\int \sin\left(\frac{b}{x^2}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3832}$$

$$x\sin\left(a+\frac{b}{x^2}\right)-2b\left(\cos(a)\int \cos\left(\frac{b}{x^2}\right)d\frac{1}{x}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}\right)$$

$$\Big\downarrow \text{3833}$$

$$x\sin\left(a+\frac{b}{x^2}\right)-2b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}\right)$$

input

```
Int[Sin[a + b/x^2],x]
```

output

```
-2*b*((Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b] - (Sqrt
[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]) + x*Sin[a + b/x^2
]
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3840

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[-f^(-1)   Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(
e + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &
& EqQ[n, -2]
```

rule 3868

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result
derivativedivides	$x \sin\left(a+\frac{b}{x^2}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$
default	$x \sin\left(a+\frac{b}{x^2}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$
risch	$-\frac{e^{ia} b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{2\sqrt{-ib}}-\frac{e^{-ia} b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{x}\right)}{2\sqrt{ib}}+x\sin\left(\frac{a\,x^2+b}{x^2}\right)$
meijerg	$-\frac{\cos(a)\sqrt{\pi}\,\sqrt{2}\,\sqrt{b}\left(-\frac{4\sqrt{2}\,x\sin\left(\frac{b}{x^2}\right)}{\sqrt{b}\,\sqrt{\pi}}+8\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{8}-\frac{\sin(a)\sqrt{\pi}\,\sqrt{2}\,(b^2)^{\frac{1}{4}}\left(-\frac{4x\sqrt{2}\,\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}\,(b^2)^{\frac{1}{4}}}-\frac{8\sqrt{b}\,\mathrm{Fresne}}{(b^2}\right.}{8}$

input
```
int(sin(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
x*sin(a+b/x^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/P
i^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx=-\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)+x\sin\left(\frac{ax^2+b}{x^2}\right)$$

input
```
integrate(sin(a+b/x^2),x, algorithm="fricas")
```

output
```
-sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) + sqrt(2)*
pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) + x*sin((a*x^2 + b)
/x^2)
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{x^2}\right)dx=\int \sin\left(a+\frac{b}{x^2}\right)dx$$

input
```
integrate(sin(a+b/x**2),x)
```

output
```
Integral(sin(a + b/x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.59

$$\int \sin\left(a+\frac{b}{x^2}\right)dx$$
$$= \frac{\sqrt{2}\Bigl(2\sqrt{2}bx^2\sqrt{\frac{1}{x^4}}\sin\left(\frac{ax^2+b}{x^2}\right)+\Bigl(\Bigl((i-1)\sqrt{\pi}\Bigl(\operatorname{erf}\Bigl(\sqrt{\frac{ib}{x^2}}\Bigr)-1\Bigr)-(i+1)\sqrt{\pi}\Bigl(\operatorname{erf}\Bigl(\sqrt{-\frac{ib}{x^2}}\Bigr)-1\Bigr)\Bigr)\cos}{4bx}$$

input
```
integrate(sin(a+b/x^2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(2*sqrt(2)*b*x^2*sqrt(x^(-4))*sin((a*x^2 + b)/x^2) + (((I - 1)
*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2))
 - 1))*cos(a) + ((I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1)*sqrt(
pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*b*(b^2/x^4)^(1/4))*sqrt(x^4)/(b*x)
```

Giac [F]

$$\int \sin \left(a+\frac{b}{x^2}\right)\,dx=\int \sin \left(a+\frac{b}{x^2}\right)\,dx$$

input
```
integrate(sin(a+b/x^2),x, algorithm="giac")
```

output
```
integrate(sin(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+\frac{b}{x^2}\right)\,dx=\int \sin \left(a+\frac{b}{x^2}\right)\,dx$$

input
```
int(sin(a + b/x^2),x)
```

output
```
int(sin(a + b/x^2), x)
```

Reduce [F]

$$\int \sin \left(a+\frac{b}{x^2}\right)\,dx=\int \sin \left(\frac{a\,x^2+b}{x^2}\right)dx$$

input
```
int(sin(a+b/x^2),x)
```

output
```
int(sin((a*x**2 + b)/x**2),x)
```

3.120 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx=-\frac{1}{2}\,\text{CosIntegral}\left(\frac{b}{x^2}\right)\sin(a)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

output
```
-1/2*Ci(b/x^2)*sin(a)-1/2*cos(a)*Si(b/x^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx=\frac{1}{2}\left(-\,\text{CosIntegral}\left(\frac{b}{x^2}\right)\sin(a)-\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\right)$$

input
```
Integrate[Sin[a + b/x^2]/x,x]
```

output
```
(-(CosIntegral[b/x^2]*Sin[a]) - Cos[a]*SinIntegral[b/x^2])/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3858, 3856, 3857}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}dx$$

$$\Big\downarrow \text{ 3858}$$

$$\sin(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}dx+\cos(a)\int \frac{\sin\left(\frac{b}{x^2}\right)}{x}dx$$

$$\Big\downarrow \text{ 3856}$$

$$\sin(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}dx-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

$$\Big\downarrow \text{ 3857}$$

$$-\frac{1}{2}\sin(a)\,\text{CosIntegral}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

input
```
Int[Sin[a + b/x^2]/x,x]
```

output
```
-1/2*(CosIntegral[b/x^2]*Sin[a]) - (Cos[a]*SinIntegral[b/x^2])/2
```

Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3857
```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3858
```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Sin[c]   Int[Cos[d*
x^n]/x, x], x] + Simp[Cos[c]   Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n},
 x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{Ci}\left(\frac{b}{x^{2}}\right) \sin \left(a \right)}{2}-\frac{\cos \left(a \right) \operatorname{Si}\left(\frac{b}{x^{2}}\right)}{2}$	22
default	$-\frac{\operatorname{Ci}\left(\frac{b}{x^{2}}\right) \sin \left(a \right)}{2}-\frac{\cos \left(a \right) \operatorname{Si}\left(\frac{b}{x^{2}}\right)}{2}$	22
risch	$-\frac{i {\mathrm e}^{i a} \operatorname{expIntegral}_{1}\left(-\frac{i b}{x^{2}}\right)}{4}+\frac{\pi \,\operatorname{csgn}\left(\frac{b}{x^{2}}\right) {\mathrm e}^{-i a}}{4}-\frac{\operatorname{Si}\left(\frac{b}{x^{2}}\right) {\mathrm e}^{-i a}}{2}+\frac{i \operatorname{expIntegral}_{1}\left(-\frac{i b}{x^{2}}\right) {\mathrm e}^{-i a}}{4}$	63
meijerg	$-\frac{\cos \left(a \right) \operatorname{Si}\left(\frac{b}{x^{2}}\right)}{2}-\frac{\sqrt{\pi}\, \sin \left(a \right) \left(\frac{2 \gamma -4 \ln \left(x \right)+\ln \left(b^{2}\right)}{\sqrt{\pi}}-\frac{2 \gamma}{\sqrt{\pi}}-\frac{2 \ln \left(2\right)}{\sqrt{\pi}}-\frac{2 \ln \left(\frac{b}{2 x^{2}}\right)}{\sqrt{\pi}}+\frac{2 \,\operatorname{Ci}\left(\frac{b}{x^{2}}\right)}{\sqrt{\pi}}\right)}{4}$	72

input
```
int(sin(a+b/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Ci(b/x^2)*sin(a)-1/2*cos(a)*Si(b/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sin \left(a+\frac{b}{x^2}\right)}{x} \, dx=-\frac{1}{2} \, \operatorname{Ci}\left(\frac{b}{x^{2}}\right) \sin \left(a\right) - \frac{1}{2} \, \cos \left(a\right) \operatorname{Si}\left(\frac{b}{x^{2}}\right)$$

input
```
integrate(sin(a+b/x^2)/x,x, algorithm="fricas")
```

output
```
-1/2*cos_integral(b/x^2)*sin(a) - 1/2*cos(a)*sin_integral(b/x^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

input
```
integrate(sin(a+b/x**2)/x,x)
```

output
```
Integral(sin(a + b/x**2)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = \frac{1}{4}\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x^2}\right) - i\,\mathrm{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\cos(a) - \frac{1}{4}\left(\mathrm{Ei}\left(\frac{i\,b}{x^2}\right) + \mathrm{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\sin(a)$$

input
```
integrate(sin(a+b/x^2)/x,x, algorithm="maxima")
```

output
```
1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x^2))*cos(a) - 1/4*(Ei(I*b/x^2) + Ei(-I*b/x
^2))*sin(a)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

input
```
integrate(sin(a+b/x^2)/x,x, algorithm="giac")
```

output
```
integrate(sin(a + b/x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{\sin (a)\ \mathrm{cosint}\left(\frac{b}{x^2}\right)}{2} - \frac{\cos (a)\ \mathrm{sinint}\left(\frac{b}{x^2}\right)}{2}$$

input
```
int(sin(a + b/x^2)/x,x)
```

output
```
- (sin(a)*cosint(b/x^2))/2 - (cos(a)*sinint(b/x^2))/2
```

Reduce [F]

$$\int \frac{\sin \left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sin \left(\frac{a\,x^2+b}{x^2}\right)}{x}dx$$

input
```
int(sin(a+b/x^2)/x,x)
```

output
```
int(sin((a*x**2 + b)/x**2)/x,x)
```

3.121 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}$$

output

```
-1/2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)/b^(1/2)-
1/2*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right)}{\sqrt{b}}$$

input

```
Integrate[Sin[a + b/x^2]/x^2,x]
```

output

```
-((Sqrt[Pi/2]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + FresnelC[(Sqrt[b]
*Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3864, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{x^2}\right)}{x^2} \, dx$$

$$\Big\downarrow \; 3864$$

$$-\int \sin \left(a+\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \; 3834$$

$$-\sin(a)\int \cos\left(\frac{b}{x^2}\right)d\frac{1}{x}-\cos(a)\int \sin\left(\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \; 3832$$

$$-\sin(a)\int \cos\left(\frac{b}{x^2}\right)d\frac{1}{x}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

$$\Big\downarrow \; 3833$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

input

```
Int[Sin[a + b/x^2]/x^2,x]
```

output

```
-((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) - (Sqrt[Pi
/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3864

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Simp[2/n   Subst[
Int[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
 n/2 - 1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right) \operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin\left(a\right) \operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	47
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right) \operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin\left(a\right) \operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	47
meijerg	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\cos\left(a\right) \operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)}{2\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sin\left(a\right)}{2\sqrt{b}}$	56
risch	$\frac{i\,\mathrm{e}^{i a}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-i b}}{x}\right)}{4\sqrt{-i b}}-\frac{i\,\mathrm{e}^{-i a}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{i b}}{x}\right)}{4\sqrt{i b}}$	58

input

```
int(sin(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)
+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)}{2\,b}$$

input
```
integrate(sin(a+b/x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqr
t(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a))/b
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b/x**2)/x**2,x)
```

output
```
Integral(sin(a + b/x**2)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{x^4}\left(\left((i+1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right)-(i-1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos(a)+\left(-(i-1)\sqrt{\pi}\left(\mathrm{e}\right.\right.\right.}{8\,bx}$$

input
```
integrate(sin(a+b/x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(((I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1)*sqrt(pi
)*(erf(sqrt(-I*b/x^2)) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/x^2
)) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*sqrt(x^4)*(b
^2/x^4)^(1/4)/(b*x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b/x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/x^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 42.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\cos\left(a\right)}{2\,\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\sin\left(a\right)}{2\,\sqrt{b}}$$

input
```
int(sin(a + b/x^2)/x^2,x)
```

output
```
- (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*cos(a))/(2*b^
(1/2)) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*sin(a)
)/(2*b^(1/2))
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=\int \frac{\sin\left(\frac{a\,x^2+b}{x^2}\right)}{x^2}dx$$

input
```
int(sin(a+b/x^2)/x^2,x)
```

output
```
int(sin((a*x**2 + b)/x**2)/x**2,x)
```

3.122 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} \, dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

output
```
1/2*cos(a+b/x^2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} \, dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

input
```
Integrate[Sin[a + b/x^2]/x^3,x]
```

output
```
Cos[a + b/x^2]/(2*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3860}$$

$$-\frac{1}{2}\int \sin\left(a+\frac{b}{x^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sin\left(a+\frac{b}{x^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

input
```
Int[Sin[a + b/x^2]/x^3,x]
```

output
```
Cos[a + b/x^2]/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$	14
risch	$\frac{\cos\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$	18
parallelrisch	$\frac{-1+\cos\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$	20
norman	$\frac{1}{b\left(1+\tan\left(\frac{a}{2}+\frac{b}{2x^2}\right)^2\right)}$	22
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}-\frac{\sin(a)\sin\left(\frac{b}{x^2}\right)}{2b}$	40
orering	$-\frac{3x^2\sin\left(a+\frac{b}{x^2}\right)}{4b^2}-\frac{x^6\left(-\frac{2b\cos\left(a+\frac{b}{x^2}\right)}{x^6}-\frac{3\sin\left(a+\frac{b}{x^2}\right)}{x^4}\right)}{4b^2}$	54

input
```
int(sin(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(a+b/x^2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx=\frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

input
```
integrate(sin(a+b/x^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*cos((a*x^2 + b)/x^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx=\begin{cases}\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b\neq 0\\ -\frac{\sin\left(a\right)}{2x^2} & \text{otherwise}\end{cases}$$

input
```
integrate(sin(a+b/x**2)/x**3,x)
```

output
```
Piecewise((cos(a + b/x**2)/(2*b), Ne(b, 0)), (-sin(a)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx=\frac{\cos\left(a+\frac{b}{x^2}\right)}{2\,b}$$

input
```
integrate(sin(a+b/x^2)/x^3,x, algorithm="maxima")
```

output
```
1/2*cos(a + b/x^2)/b
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

input
```
integrate(sin(a+b/x^2)/x^3,x, algorithm="giac")
```

output
```
1/2*cos((a*x^2 + b)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 43.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \frac{\cos\left(a+\frac{b}{x^2}\right)}{2\,b}$$

input
```
int(sin(a + b/x^2)/x^3,x)
```

output
```
cos(a + b/x^2)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \frac{\cos\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$$

input
```
int(sin(a+b/x^2)/x^3,x)
```

output
```
cos((a*x**2 + b)/x**2)/(2*b)
```

3.123 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx=\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}$$

output

```
1/2*cos(a+b/x^2)/b/x-1/4*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/
Pi^(1/2)/x)/b^(3/2)+1/4*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
/x)*sin(a)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx=\frac{2\sqrt{b}\cos\left(a+\frac{b}{x^2}\right)-\sqrt{2\pi}x\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{2\pi}x\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{4b^{3/2}x}$$

input
```
Integrate[Sin[a + b/x^2]/x^4,x]
```

output
```
(2*Sqrt[b]*Cos[a + b/x^2] - Sqrt[2*Pi]*x*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/x] + Sqrt[2*Pi]*x*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(4*b^(3/2)*
x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3890, 3866, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}dx$$

$$\Big\downarrow \text{3890}$$

$$-\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3866}$$

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\int \cos\left(a+\frac{b}{x^2}\right)d\frac{1}{x}}{2b}$$

$$\Big\downarrow \text{3835}$$

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\cos(a)\int \cos\left(\frac{b}{x^2}\right)d\frac{1}{x}-\sin(a)\int \sin\left(\frac{b}{x^2}\right)d\frac{1}{x}}{2b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\cos(a)\int \cos\left(\frac{b}{x^2}\right)d\frac{1}{x}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}}{2b}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}}{2b}$$

input

```
Int[Sin[a + b/x^2]/x^4,x]
```

output

```
Cos[a + b/x^2]/(2*b*x) - ((Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])
/x])/Sqrt[b] - (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b
])/(2*b)
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3866

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3890

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\cos\left(a+\frac{b}{x^{2}}\right)}{2 b x}-\frac{\sqrt{2}\, \sqrt{\pi}\, \left(\cos\left(a\right) \mathrm{FresnelC}\left(\frac{\sqrt{b}\, \sqrt{2}}{\sqrt{\pi}\, x}\right)-\sin\left(a\right) \mathrm{FresnelS}\left(\frac{\sqrt{b}\, \sqrt{2}}{\sqrt{\pi}\, x}\right)\right)}{4 b^{\frac{3}{2}}}$
default	$\frac{\cos\left(a+\frac{b}{x^{2}}\right)}{2 b x}-\frac{\sqrt{2}\, \sqrt{\pi}\, \left(\cos\left(a\right) \mathrm{FresnelC}\left(\frac{\sqrt{b}\, \sqrt{2}}{\sqrt{\pi}\, x}\right)-\sin\left(a\right) \mathrm{FresnelS}\left(\frac{\sqrt{b}\, \sqrt{2}}{\sqrt{\pi}\, x}\right)\right)}{4 b^{\frac{3}{2}}}$
risch	$-\frac{{\mathrm e}^{i a} \sqrt{\pi}\, \mathrm{erf}\left(\frac{\sqrt{-i b}}{x}\right)}{8 b \sqrt{-i b}}-\frac{{\mathrm e}^{-i a} \sqrt{\pi}\, \mathrm{erf}\left(\frac{\sqrt{i b}}{x}\right)}{8 b \sqrt{i b}}+\frac{\cos\left(\frac{a \,x^{2}+b}{x^{2}}\right)}{2 b x}$
meijerg	$-\frac{\sqrt{\pi}\, \cos\left(a\right) \sqrt{2}\, \left(-\frac{\sqrt{2}\, \sqrt{b}\, \cos\left(\frac{b}{x^{2}}\right)}{2 \sqrt{\pi}\, x}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{b}\, \sqrt{2}}{\sqrt{\pi}\, x}\right)}{2}\right)}{2 b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\, \sin\left(a\right) \sqrt{2}\, \left(b^{2}\right)^{\frac{1}{4}} \left(\frac{\sqrt{2}\, \left(b^{2}\right)^{\frac{3}{4}} \sin\left(\frac{b}{x^{2}}\right)}{2 \sqrt{\pi}\, x b}-\frac{\left(b^{2}\right)^{\frac{3}{4}} \mathrm{Fresne}}{2 b}\right.}{2 b^{2}}$

input
```
int(sin(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(a+b/x^2)/b/x-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)
*2^(1/2)/Pi^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$
$$= -\frac{\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)-\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)-2\,b\cos\left(\frac{ax^2+b}{x^2}\right)}{4\,b^2x}$$

input
```
integrate(sin(a+b/x^2)/x^4,x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) - s
qrt(2)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) - 2*b*cos(
(a*x^2 + b)/x^2))/(b^2*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx=\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

input
```
integrate(sin(a+b/x**2)/x**4,x)
```

output
```
Integral(sin(a + b/x**2)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx =$$

$$-\frac{\sqrt{2}(x^4)^{\frac{3}{2}}\big(\big((i-1)\,\Gamma\big(\frac{3}{2},\frac{i\,b}{x^2}\big)-(i+1)\,\Gamma\big(\frac{3}{2},-\frac{i\,b}{x^2}\big)\big)\cos{(a)}+\big((i+1)\,\Gamma\big(\frac{3}{2},\frac{i\,b}{x^2}\big)-(i-1)\,\Gamma\big(\frac{3}{2},-\frac{i\,b}{x^2}\big)\big)\sin}{8\,b^3x^3}$$

input
```
integrate(sin(a+b/x^2)/x^4,x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(((I - 1)*gamma(3/2, I*b/x^2) - (I + 1)*gamma(3/2, -I*b/x^2))
*cos(a) + ((I + 1)*gamma(3/2, I*b/x^2) - (I - 1)*gamma(3/2, -I*b/x^2))*sin
(a))*(x^4)^(3/2)*(b^2/x^4)^(3/4)/(b^3*x^3)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx=\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

input
```
integrate(sin(a+b/x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(sin(a + b/x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx=\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

input
```
int(sin(a + b/x^2)/x^4,x)
```

output
```
int(sin(a + b/x^2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx=\int \frac{\sin\left(\frac{a\,x^2+b}{x^2}\right)}{x^4}dx$$

input
```
int(sin(a+b/x^2)/x^4,x)
```

output
```
int(sin((a*x**2 + b)/x**2)/x**4,x)
```

3.124 $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

output

```
-2*cos(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

input

```
Integrate[Sin[Sqrt[x]]/Sqrt[x],x]
```

output

```
-2*Cos[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3860, 3042, 3118\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{3860}$$

$$2\int \sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3118}$$

$$-2\cos\left(\sqrt{x}\right)$$

input
```
Int[Sin[Sqrt[x]]/Sqrt[x],x]
```

output
```
-2*Cos[Sqrt[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\cos\left(\sqrt{x}\right)$	7
default	$-2\cos\left(\sqrt{x}\right)$	7
meijerg	$2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\sqrt{x}\right)}{\sqrt{\pi}}\right)$	19

input

```
int(sin(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2*cos(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=-2\,\cos\left(\sqrt{x}\right)$$

input

```
integrate(sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output

```
-2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

input
```
integrate(sin(x**(1/2))/x**(1/2),x)
```

output
```
-2*cos(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
-2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
-2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 42.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\cos\left(\sqrt{x}\right)$$

input

```
int(sin(x^(1/2))/x^(1/2),x)
```

output

```
-2*cos(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\sqrt{x}\right)$$

input

```
int(sin(x^(1/2))/x^(1/2),x)
```

output

```
 - 2*cos(sqrt(x))
```

3.125 $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=-2\cos\left(\sqrt{x}\right)+\frac{2}{3}\cos^3\left(\sqrt{x}\right)$$

output
```
-2*cos(x^(1/2))+2/3*cos(x^(1/2))^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=-\frac{3}{2}\cos\left(\sqrt{x}\right)+\frac{1}{6}\cos\left(3\sqrt{x}\right)$$

input
```
Integrate[Sin[Sqrt[x]]^3/Sqrt[x],x]
```

output
```
(-3*Cos[Sqrt[x]])/2 + Cos[3*Sqrt[x]]/6
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3860, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 3860}$$

$$2\int \sin^3\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \sin\left(\sqrt{x}\right)^3d\sqrt{x}$$

$$\Big\downarrow \text{ 3113}$$

$$-2\int (1-x)d\cos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\cos\left(\sqrt{x}\right)-\frac{x^{3/2}}{3}\right)$$

input
```
Int[Sin[Sqrt[x]]^3/Sqrt[x],x]
```

output
```
-2*(-1/3*x^(3/2) + Cos[Sqrt[x]])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{2\left(2+\sin\left(\sqrt{x}\right)^{2}\right)\cos\left(\sqrt{x}\right)}{3}$	15
default	$-\frac{2\left(2+\sin\left(\sqrt{x}\right)^{2}\right)\cos\left(\sqrt{x}\right)}{3}$	15

input

```
int(sin(x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*(2+sin(x^(1/2))^2)*cos(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{3}\cos\left(\sqrt{x}\right)^3 - 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sin^2\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right) - \frac{4\cos^3\left(\sqrt{x}\right)}{3}$$

input
```
integrate(sin(x**(1/2))**3/x**(1/2),x)
```

output
```
-2*sin(sqrt(x))**2*cos(sqrt(x)) - 4*cos(sqrt(x))**3/3
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{3}\cos\left(\sqrt{x}\right)^3 - 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{3}\cos\left(\sqrt{x}\right)^3 - 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="giac")
```

output
```
2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 42.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\cos\left(\sqrt{x}\right)\left(\cos\left(\sqrt{x}\right)^2 - 3\right)}{3}$$

input
```
int(sin(x^(1/2))^3/x^(1/2),x)
```

output
```
(2*cos(x^(1/2))*(cos(x^(1/2))^2 - 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)^2}{3} - \frac{4\cos\left(\sqrt{x}\right)}{3} + \frac{4}{3}$$

input
```
int(sin(x^(1/2))^3/x^(1/2),x)
```

output
```
(2*( - cos(sqrt(x))*sin(sqrt(x))**2 - 2*cos(sqrt(x)) + 2))/3
```

3.126 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

output
```
-2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

input
```
Integrate[Sin[Sqrt[x]],x]
```

output
```
-2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3842, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{ 3842}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3777}$$

$$2\left(\int \cos\left(\sqrt{x}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$2\left(\int \sin\left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 3117}$$

$$2\left(\sin\left(\sqrt{x}\right)-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

input

```
Int[Sin[Sqrt[x]],x]
```

output

```
2*(-(Sqrt[x]*Cos[Sqrt[x]]) + Sin[Sqrt[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
default	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos(\sqrt{x})}{2\sqrt{\pi}}+\frac{\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	28

input
```
int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \, \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \, \sin \left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="fricas")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x} \cos \left(\sqrt{x}\right) + 2 \sin \left(\sqrt{x}\right)$$

input
```
integrate(sin(x**(1/2)),x)
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \, \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \, \sin \left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="maxima")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 40.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = 2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*( - sqrt(x)*cos(sqrt(x)) + sin(sqrt(x)))
```

3.127 $\int \sin^2\left(\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3\sqrt[3]{x}}{4}+\frac{x}{2}+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) -\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)$$

output

```
-3/4*x^(1/3)+1/2*x+3/4*cos(x^(1/3))*sin(x^(1/3))-3/2*x^(2/3)*cos(x^(1/3))*
sin(x^(1/3))+3/2*x^(1/3)*sin(x^(1/3))^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = \frac{1}{8}\left(4x-6\sqrt[3]{x}\cos\left(2\sqrt[3]{x}\right)+\left(3-6x^{2/3}\right)\sin\left(2\sqrt[3]{x}\right)\right)$$

input

```
Integrate[Sin[x^(1/3)]^2,x]
```

output

```
(4*x - 6*x^(1/3)*Cos[2*x^(1/3)] + (3 - 6*x^(2/3))*Sin[2*x^(1/3)])/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3842, 3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(\sqrt[3]{x}\right)dx$$

$$\Big\downarrow \text{3842}$$

$$3\int x^{2/3}\sin^2\left(\sqrt[3]{x}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{3042}$$

$$3\int x^{2/3}\sin\left(\sqrt[3]{x}\right)^2d\sqrt[3]{x}$$

$$\Big\downarrow \text{3792}$$

$$3\left(\frac{1}{2}\int x^{2/3}d\sqrt[3]{x}-\frac{1}{2}\int \sin^2\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-\frac{1}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{1}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{15}$$

$$3\left(-\frac{1}{2}\int \sin^2\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-\frac{1}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{6}+\frac{1}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$3\left(-\frac{1}{2}\int \sin\left(\sqrt[3]{x}\right)^2d\sqrt[3]{x}-\frac{1}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{6}+\frac{1}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$3\left(\frac{1}{2}\left(\frac{1}{2}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)-\frac{\int 1d\sqrt[3]{x}}{2}\right)-\frac{1}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{6}+\frac{1}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{24}$$

$$3\left(-\frac{1}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{6}+\frac{1}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)+\frac{1}{2}\left(\frac{1}{2}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)-\frac{\sqrt[3]{x}}{2}\right)\right)$$

input
```
Int[Sin[x^(1/3)]^2,x]
```

output
```
3*(x/6 - (x^(2/3)*Cos[x^(1/3)]*Sin[x^(1/3)])/2 + (x^(1/3)*Sin[x^(1/3)]^2)/
2 + (-1/2*x^(1/3) + (Cos[x^(1/3)]*Sin[x^(1/3)])/2)/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

method	result	size
meijerg	$\frac{3x^{\frac{5}{3}}\operatorname{hypergeom}\left(\left[1,\frac{5}{2}\right],\left[\frac{3}{2},2,\frac{7}{2}\right],-x^{\frac{2}{3}}\right)}{5}$	19
derivativedivides	$3x^{\frac{2}{3}}\left(-\frac{\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{x^{\frac{1}{3}}}{2}\right)-\frac{3x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}}{4}-x$	52
default	$3x^{\frac{2}{3}}\left(-\frac{\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{x^{\frac{1}{3}}}{2}\right)-\frac{3x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}}{4}-x$	52

input

```
int(sin(x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output

```
3/5*x^(5/3)*hypergeom([1,5/2],[3/2,2,7/2],-x^(2/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \sin^{2}\left(\sqrt[3]{x}\right)\,dx=-\frac{3}{4}\left(2\,x^{\frac{2}{3}}-1\right)\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)-\frac{3}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}+\frac{1}{2}\,x+\frac{3}{4}\,x^{\frac{1}{3}}$$

input

```
integrate(sin(x^(1/3))^2,x, algorithm="fricas")
```

output

```
-3/4*(2*x^(2/3) - 1)*cos(x^(1/3))*sin(x^(1/3)) - 3/2*x^(1/3)*cos(x^(1/3))^
2 + 1/2*x + 3/4*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(66) = 132$.

Time = 0.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 5.49

$$\begin{aligned}\int \sin^2\left(\sqrt[3]{x}\right)\,dx &= \frac{12x^{\frac{2}{3}}\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad- \frac{12x^{\frac{2}{3}}\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad- \frac{3\sqrt[3]{x}\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad+ \frac{18\sqrt[3]{x}\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad- \frac{3\sqrt[3]{x}}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad+ \frac{2x\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad+ \frac{4x\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad+ \frac{2x}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad- \frac{6\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4} \\ &\quad+ \frac{6\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+4}\end{aligned}$$

input

```
integrate(sin(x**(1/3))**2,x)
```

output

```
12*x**(2/3)*tan(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**
2 + 4) - 12*x**(2/3)*tan(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3
)/2)**2 + 4) - 3*x**(1/3)*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan
(x**(1/3)/2)**2 + 4) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**
4 + 8*tan(x**(1/3)/2)**2 + 4) - 3*x**(1/3)/(4*tan(x**(1/3)/2)**4 + 8*tan(x
**(1/3)/2)**2 + 4) + 2*x*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(
x**(1/3)/2)**2 + 4) + 4*x*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 + 8*tan
(x**(1/3)/2)**2 + 4) + 2*x/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 +
4) - 6*tan(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4
) + 6*tan(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

input

```
integrate(sin(x^(1/3))^2,x, algorithm="maxima")
```

output

```
-3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = -\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

input

```
integrate(sin(x^(1/3))^2,x, algorithm="giac")
```

output

```
-3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Mupad [B] (verification not implemented)

Time = 41.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx=\frac{x}{2}+\frac{3\,\sin\left(2\,x^{1/3}\right)}{8}-\frac{3\,x^{1/3}\,\cos\left(2\,x^{1/3}\right)}{4}-\frac{3\,x^{2/3}\,\sin\left(2\,x^{1/3}\right)}{4}$$

input
```
int(sin(x^(1/3))^2,x)
```

output
```
x/2 + (3*sin(2*x^(1/3)))/8 - (3*x^(1/3)*cos(2*x^(1/3)))/4 - (3*x^(2/3)*sin
(2*x^(1/3)))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx=-\frac{3x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)^2}{2}-\frac{3x^{\frac{1}{3}}}{4}+\frac{x}{2}$$

input
```
int(sin(x^(1/3))^2,x)
```

output
```
( - 6*x**(2/3)*cos(x**(1/3))*sin(x**(1/3)) + 3*cos(x**(1/3))*sin(x**(1/3))
 + 6*x**(1/3)*sin(x**(1/3))**2 - 3*x**(1/3) + 2*x)/4
```

3.128 $\int \sin^3\left(\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 87

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{14}{3}\cos\left(\sqrt[3]{x}\right) - 2x^{2/3}\cos\left(\sqrt[3]{x}\right) - \frac{2}{9}\cos^3\left(\sqrt[3]{x}\right) + 4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)$$

output
```
14/3*cos(x^(1/3))-2*x^(2/3)*cos(x^(1/3))-2/9*cos(x^(1/3))^3+4*x^(1/3)*sin(
x^(1/3))-x^(2/3)*cos(x^(1/3))*sin(x^(1/3))^2+2/3*x^(1/3)*sin(x^(1/3))^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(-81\left(-2+x^{2/3}\right)\cos\left(\sqrt[3]{x}\right) + \left(-2+9x^{2/3}\right)\cos\left(3\sqrt[3]{x}\right) - 6\sqrt[3]{x}\left(-27\sin\left(\sqrt[3]{x}\right)+\sin\left(3\sqrt[3]{x}\right)\right)\right)$$

input
```
Integrate[Sin[x^(1/3)]^3,x]
```

output
```
(-81*(-2 + x^(2/3))*Cos[x^(1/3)] + (-2 + 9*x^(2/3))*Cos[3*x^(1/3)] - 6*x^(
1/3)*(-27*Sin[x^(1/3)] + Sin[3*x^(1/3)]))/36
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3842, 3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(\sqrt[3]{x}\right)dx$$

$$\Big\downarrow \text{3842}$$

$$3\int x^{2/3}\sin^3\left(\sqrt[3]{x}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{3042}$$

$$3\int x^{2/3}\sin\left(\sqrt[3]{x}\right)^3d\sqrt[3]{x}$$

$$\Big\downarrow \text{3792}$$

$$3\left(\frac{2}{3}\int x^{2/3}\sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-\frac{2}{9}\int \sin^3\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$3\left(\frac{2}{3}\int x^{2/3}\sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-\frac{2}{9}\int \sin\left(\sqrt[3]{x}\right)^3d\sqrt[3]{x}-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3113}$$

$$3\left(\frac{2}{3}\int x^{2/3}\sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}+\frac{2}{9}\int \left(1-x^{2/3}\right)d\cos\left(\sqrt[3]{x}\right)-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2}{3}\int x^{2/3}\sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+\frac{2}{9}\left(\cos\left(\sqrt[3]{x}\right)-\frac{x}{3}\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$3\Bigg(\frac{2}{3}\Bigg(2\int \sqrt[3]{x}\cos\left(\sqrt[3]{x}\right)d\sqrt[3]{x}-x^{2/3}\cos\left(\sqrt[3]{x}\right)\Bigg)-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+\frac{2}{9}\Bigg(\cos\left(\sqrt[3]{x}\right)-$$

$$\Big\downarrow \textbf{3042}$$

$$3\Bigg(\frac{2}{3}\Bigg(2\int \sqrt[3]{x}\sin\left(\sqrt[3]{x}+\frac{\pi}{2}\right)d\sqrt[3]{x}-x^{2/3}\cos\left(\sqrt[3]{x}\right)\Bigg)-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+\frac{2}{9}\Bigg(\cos($$

$$\Big\downarrow \textbf{3777}$$

$$3\Bigg(\frac{2}{3}\Bigg(2\Bigg(\int -\sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}+\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)\Bigg)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\Bigg)-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)$$

$$\Big\downarrow \textbf{25}$$

$$3\Bigg(\frac{2}{3}\Bigg(2\Bigg(\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-\int \sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}\Bigg)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\Bigg)-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+$$

$$\Big\downarrow \textbf{3042}$$

$$3\Bigg(\frac{2}{3}\Bigg(2\Bigg(\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-\int \sin\left(\sqrt[3]{x}\right)d\sqrt[3]{x}\Bigg)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\Bigg)-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+$$

$$\Big\downarrow \textbf{3118}$$

$$3\Bigg(-\frac{1}{3}x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{3}\Big(2\left(\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)+\cos\left(\sqrt[3]{x}\right)\right)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\Big)+\frac{2}{9}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+\frac{2}{9}\Big(\cos($$

input

```
Int[Sin[x^(1/3)]^3,x]
```

output

```
3*((2*(-1/3*x + Cos[x^(1/3)]))/9 - (x^(2/3)*Cos[x^(1/3)]*Sin[x^(1/3)]^2)/3
 + (2*x^(1/3)*Sin[x^(1/3)]^3)/9 + (2*(-(x^(2/3)*Cos[x^(1/3)]) + 2*(Cos[x^(
1/3)] + x^(1/3)*Sin[x^(1/3)])))/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-x^{\frac{2}{3}}\left(2+\sin\left(x^{\frac{1}{3}}\right)^{2}\right)\cos\left(x^{\frac{1}{3}}\right)+4\cos\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)^{3}}{3}+$
default	$-x^{\frac{2}{3}}\left(2+\sin\left(x^{\frac{1}{3}}\right)^{2}\right)\cos\left(x^{\frac{1}{3}}\right)+4\cos\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)^{3}}{3}+$

input
```
int(sin(x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
-x^(2/3)*(2+sin(x^(1/3))^2)*cos(x^(1/3))+4*cos(x^(1/3))+4*x^(1/3)*sin(x^(1
/3))+2/3*x^(1/3)*sin(x^(1/3))^3+2/9*(2+sin(x^(1/3))^2)*cos(x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \sin^{3}\left(\sqrt[3]{x}\right)\,dx=\frac{1}{9}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)^{3}-\frac{1}{3}\left(9\,x^{\frac{2}{3}}-14\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{2}{3}\left(x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}-7\,x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(sin(x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/9*(9*x^(2/3) - 2)*cos(x^(1/3))^3 - 1/3*(9*x^(2/3) - 14)*cos(x^(1/3)) - 2
/3*(x^(1/3)*cos(x^(1/3))^2 - 7*x^(1/3))*sin(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = 3x^{\frac{2}{3}}\left(\frac{\cos^3\left(\sqrt[3]{x}\right)}{3} - \cos\left(\sqrt[3]{x}\right)\right) - 2\sqrt[3]{x}\left(-\frac{\sin^3\left(\sqrt[3]{x}\right)}{3} - 2\sin\left(\sqrt[3]{x}\right)\right) - \frac{2\cos^3\left(\sqrt[3]{x}\right)}{9} + \frac{14\cos\left(\sqrt[3]{x}\right)}{3}$$

input
```
integrate(sin(x**(1/3))**3,x)
```

output
```
3*x**(2/3)*(cos(x**(1/3))**3/3 - cos(x**(1/3))) - 2*x**(1/3)*(-sin(x**(1/3
))**3/3 - 2*sin(x**(1/3))) - 2*cos(x**(1/3))**3/9 + 14*cos(x**(1/3))/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(9\,x^{\frac{2}{3}} - 2\right)\cos\left(3\,x^{\frac{1}{3}}\right) - \frac{9}{4}\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) - \frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right) + \frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(sin(x^(1/3))^3,x, algorithm="maxima")
```

output
```
1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6
*x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(9\,x^{\frac{2}{3}} - 2\right)\cos\left(3\,x^{\frac{1}{3}}\right) - \frac{9}{4}\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) - \frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right) + \frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

input

```
integrate(sin(x^(1/3))^3,x, algorithm="giac")
```

output

```
1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6
*x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 43.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = \frac{14\cos\left(x^{1/3}\right)}{3} - 3\,x^{2/3}\cos\left(x^{1/3}\right) + \frac{14\,x^{1/3}\sin\left(x^{1/3}\right)}{3} - \frac{2\cos\left(x^{1/3}\right)^3}{9} + x^{2/3}\cos\left(x^{1/3}\right)^3 - \frac{2\,x^{1/3}\cos\left(x^{1/3}\right)^2\sin\left(x^{1/3}\right)}{3}$$

input

```
int(sin(x^(1/3))^3,x)
```

output

```
(14*cos(x^(1/3)))/3 - 3*x^(2/3)*cos(x^(1/3)) + (14*x^(1/3)*sin(x^(1/3)))/3
 - (2*cos(x^(1/3))^3)/9 + x^(2/3)*cos(x^(1/3))^3 - (2*x^(1/3)*cos(x^(1/3))
^2*sin(x^(1/3)))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \sin^3\left(\sqrt[3]{x}\right)\,dx = -x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)^2 - 2x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right) + \frac{2\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)^2}{9} + \frac{40\cos\left(x^{\frac{1}{3}}\right)}{9} + \frac{2x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)^3}{3} + 4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right) + \frac{16}{9}$$

input
```
int(sin(x^(1/3))^3,x)
```

output
```
( - 9*x**(2/3)*cos(x**(1/3))*sin(x**(1/3))**2 - 18*x**(2/3)*cos(x**(1/3))
+ 2*cos(x**(1/3))*sin(x**(1/3))**2 + 40*cos(x**(1/3)) + 6*x**(1/3)*sin(x**
(1/3))**3 + 36*x**(1/3)*sin(x**(1/3)) + 16)/9
```

3.129 $\int (ex)^m (b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \text{Int}((ex)^m (b\sin(c+dx^n))^p, x)$$

output

```
Defer(Int)((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(c+dx^n))^p \, dx$$

input

```
Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

output

```
Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (b\sin(c+dx^n))^p\,dx$$

$$\Big\downarrow \text{3908}$$

$$\int (ex)^m (b\sin(c+dx^n))^p\,dx$$

input
```
Int[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m (b\sin(c+d\,x^n))^p dx$$

input
```
int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

output
```
int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m\,(b\sin{(c+dx^n)})^p\,dx=\int (ex)^m\,(b\sin{(dx^n+c)})^p\,dx$$

input
```
integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 10.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m\,(b\sin{(c+dx^n)})^p\,dx=\int (b\sin{(c+dx^n)})^p\,(ex)^m\,dx$$

input
```
integrate((e*x)**m*(b*sin(c+d*x**n))**p,x)
```

output
```
Integral((b*sin(c + d*x**n))**p*(e*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c))^p \, dx$$

input

```
integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

output

```
integrate((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c))^p \, dx$$

input

```
integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="giac")
```

output

```
integrate((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 42.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (b\,\sin(c+d\,x^n))^p\,(e\,x)^m \, dx$$

input

```
int((b*sin(c + d*x^n))^p*(e*x)^m,x)
```

output
```
int((b*sin(c + d*x^n))^p*(e*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int (ex)^m (b\sin (c+dx^n))^p \, dx = e^m b^p \left(\int x^m \sin (x^n d+c)^p \, dx\right)$$

input
```
int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

output
```
e**m*b**p*int(x**m*sin(x**n*d + c)**p,x)
```

3.130 $\int (ex)^m (a+b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \text{Int}\left((ex)^m (a+b\sin(c+dx^n))^p, x\right)$$

output
```
Defer(Int)((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (ex)^m (a+b\sin(c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+b\sin(c+dx^n))^p\,dx$$

$$\Big\downarrow \text{3908}$$

$$\int (ex)^m (a+b\sin(c+dx^n))^p dx$$

input

```
Int[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3908

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\sin(c+d\,x^n))^p dx$$

input

```
int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

output

```
int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c)+a)^p \, dx$$

input

```
integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

output

```
integral((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 33.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (ex)^m (a+b\sin(c+dx^n))^p \, dx$$

input

```
integrate((e*x)**m*(a+b*sin(c+d*x**n))**p,x)
```

output

```
Integral((e*x)**m*(a + b*sin(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c)+a)^p \, dx$$

input

```
integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

output

```
integrate((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(dx^n+c)+a)^p \, dx$$

input

```
integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

output

```
integrate((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 42.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sin(c+dx^n))^p \, dx = \int (e\,x)^m \left(a+b\,\sin\left(c+d\,x^n\right)\right)^p dx$$

input

```
int((e*x)^m*(a + b*sin(c + d*x^n))^p,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*x^n))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=e^m\left(\int x^m(\sin\left(x^nd+c\right)b+a)^p dx\right)$$

input
```
int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

output
```
e**m*int(x**m*(sin(x**n*d + c)*b + a)**p,x)
```

3.131 $\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx = \frac{x^{-n}(ex)^n \cos(c+dx^n) \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cos^2(c+dx^n)}}$$

output
```
(e*x)^n*cos(c+d*x^n)*hypergeom([1/2, 1/2*p+1/2],[3/2+1/2*p],sin(c+d*x^n)^2
)*(b*sin(c+d*x^n))^(p+1)/b/d/e/n/(p+1)/(x^n)/(cos(c+d*x^n)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx = \frac{x^{1-n}(ex)^{-1+n}\sqrt{\cos^2(c+dx^n)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^p \tan(c+}{dn(1+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*Sqrt[Cos[c + d*x^n]^2]*Hypergeometric2F1[1/2, (1
 + p)/2, (3 + p)/2, Sin[c + d*x^n]^2]*(b*Sin[c + d*x^n])^p*Tan[c + d*x^n])
/(d*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3862, 3860, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (b\sin (c+dx^n))^p \, dx$$

$$\Big\downarrow \text{3862}$$

$$\frac{x^{-n}(ex)^n \int x^{n-1}(b\sin (dx^n+c))^p dx}{e}$$

$$\Big\downarrow \text{3860}$$

$$\frac{x^{-n}(ex)^n \int (b\sin (dx^n+c))^p dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n \int (b\sin (dx^n+c))^p dx^n}{en}$$

$$\Big\downarrow \text{3122}$$

$$\frac{x^{-n}(ex)^n \cos (c+dx^n) (b\sin (c+dx^n))^{p+1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2 (dx^n+c)\right)}{bden(p+1)\sqrt{\cos^2 (c+dx^n)}}$$

input
```
Int[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

output
```
((e*x)^n*Cos[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[c
 + d*x^n]^2]*(b*Sin[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cos[c +
 d*x^n]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int (ex)^{-1+n} \left(b\sin\left(c+d\,x^{n}\right)\right)^{p} dx$$

input
```
int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} (b\sin (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\sin (dx^n+c))^p \, dx$$

input

```
integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

output

```
integral((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n} (b\sin (c+dx^n))^p \, dx = \int (b\sin (c+dx^n))^p (ex)^{n-1} \, dx$$

input

```
integrate((e*x)**(-1+n)*(b*sin(c+d*x**n))**p,x)
```

output

```
Integral((b*sin(c + d*x**n))**p*(e*x)**(n - 1), x)
```

Maxima [F]

$$\int (ex)^{-1+n} (b\sin (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\sin (dx^n+c))^p \, dx$$

input

```
integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

output

```
integrate((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\sin(dx^n+c))^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx=\int \left(b\,\sin\left(c+d\,x^n\right)\right)^p\,{\left(e\,x\right)}^{n-1}\,dx$$

input
```
int((b*sin(c + d*x^n))^p*(e*x)^(n - 1),x)
```

output
```
int((b*sin(c + d*x^n))^p*(e*x)^(n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx=\frac{e^n b^p\left(\int \frac{x^n\sin(x^n d+c)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

output
```
(e**n*b**p*int((x**n*sin(x**n*d + c)**p)/x,x))/e
```

3.132 $\int (ex)^{-1+2n} (b\sin (c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (ex)^{-1+2n} (b\sin (c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\mathrm{Int}(x^{-1+2n}(b\sin (c+dx^n))^p, x)}{e}$$

output
```
(e*x)^(2*n)*Defer(Int)(x^(-1+2*n)*(b*sin(c+d*x^n))^p,x)/e/(x^(2*n))
```

Mathematica [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b\sin (c+dx^n))^p \, dx = \int (ex)^{-1+2n} (b\sin (c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3862, 3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} \left(b \sin \left(c+dx^n\right)\right)^p \, dx$$

$$\Big\downarrow \text{ 3862}$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(b \sin (dx^n+c))^p dx}{e}$$

$$\Big\downarrow \text{ 3908}$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(b \sin (dx^n+c))^p dx}{e}$$

input
```
Int[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n} \left(b \sin \left(c+d\, x^{n}\right)\right)^{p}dx$$

input
```
int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx = \int (ex)^{2\,n-1}(b\sin(dx^n+c))^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 9.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx = \int (b\sin(c+dx^n))^p\,(ex)^{2n-1}\,dx$$

input
```
integrate((e*x)**(-1+2*n)*(b*sin(c+d*x**n))**p,x)
```

output
```
Integral((b*sin(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

Maxima [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{2n-1}(b\sin(dx^n+c))^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (ex)^{2n-1}(b\sin(dx^n+c))^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 43.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sin(c+dx^n))^p\,dx=\int (b\,\sin(c+d\,x^n))^p\,(e\,x)^{2\,n-1}\,dx$$

input
```
int((b*sin(c + d*x^n))^p*(e*x)^(2*n - 1),x)
```

output
```
int((b*sin(c + d*x^n))^p*(e*x)^(2*n - 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int (ex)^{-1+2n} \left(b \sin \left(c+dx^n\right)\right)^p \, dx = \frac{e^{2n} b^p \left(\int \frac{x^{2n} \sin (x^n d+c)^p}{x} dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

output
```
(e**(2*n)*b**p*int((x**(2*n)*sin(x**n*d + c)**p)/x,x))/e
```

3.133 $\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx = -\frac{\sqrt{2}x^{-n}(ex)^n \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx^n)),\frac{b(1-\sin(c+dx^n))}{a+b}\right)\cos(c+dx^n)\,(a+b\sin(c+d}{den\sqrt{1+\sin(c+dx^n)}}$$

output
```
-2^(1/2)*(e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-sin(c+d*x^n))/(a+b),1/2-1/2*
sin(c+d*x^n))*cos(c+d*x^n)*(a+b*sin(c+d*x^n))^p/d/e/n/(x^n)/(1+sin(c+d*x^n
))^(1/2)/(((a+b*sin(c+d*x^n))/(a+b))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx = \frac{x^{-n}(ex)^n \operatorname{AppellF1}\left(1+p,\frac{1}{2},\frac{1}{2},2+p,\frac{a+b\sin(c+dx^n)}{a-b},\frac{a+b\sin(c+dx^n)}{a+b}\right)\sec(c+dx^n)\sqrt{-\frac{b(-1+\sin(c+dx^n))}{a+b}}\sqrt{\frac{b(1+}{}}}{bden(1+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

output

```
((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sin[c + d*x^n])/(a - b),
(a + b*Sin[c + d*x^n])/(a + b)]*Sec[c + d*x^n]*Sqrt[-((b*(-1 + Sin[c + d*x
^n]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x^n]))/(-a + b)]*(a + b*Sin[c + d*x
^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3862, 3860, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} \left(a+b\sin\left(c+dx^n\right)\right)^p\, dx$$

$$\Big\downarrow \text{3862}$$

$$\frac{x^{-n}(ex)^n\int x^{n-1}(a+b\sin(dx^n+c))^p dx}{e}$$

$$\Big\downarrow \text{3860}$$

$$\frac{x^{-n}(ex)^n\int (a+b\sin(dx^n+c))^p dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n\int (a+b\sin(dx^n+c))^p dx^n}{en}$$

$$\Big\downarrow \text{3144}$$

$$\frac{x^{-n}(ex)^n\cos(c+dx^n)\int \frac{(a+b\sin(dx^n+c))^p}{\sqrt{1-\sin(dx^n+c)}\sqrt{\sin(dx^n+c)+1}}d\sin(dx^n+c)}{den\sqrt{1-\sin(c+dx^n)}\sqrt{\sin(c+dx^n)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{x^{-n}(ex)^n\cos(c+dx^n)\left(a+b\sin(c+dx^n)\right)^p\left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p}\int \frac{\left(\frac{a}{a+b}+\frac{b\sin(dx^n+c)}{a+b}\right)^p}{\sqrt{1-\sin(dx^n+c)}\sqrt{\sin(dx^n+c)+1}}d\sin(dx^n+c)}{den\sqrt{1-\sin(c+dx^n)}\sqrt{\sin(c+dx^n)+1}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}x^{-n}(ex)^n\cos\left(c+dx^n\right)\left(a+b\sin\left(c+dx^n\right)\right)^p\left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\sin\left(dx^n+c\right))\right.}{den\sqrt{\sin\left(c+dx^n\right)+1}}$$

input

```
Int[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

output

```
-((Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Sin[c + d*x^n])/2, (b*
(1 - Sin[c + d*x^n]))/(a + b)]*Cos[c + d*x^n]*(a + b*Sin[c + d*x^n])^p)/(d
*e*n*x^n*Sqrt[1 + Sin[c + d*x^n]]*((a + b*Sin[c + d*x^n])/(a + b))^p))
```

Defintions of rubi rules used

rule 155

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int (ex)^{-1+n} \left(a+b\sin\left(c+d\,x^{n}\right)\right)^{p}dx$$

input
```
int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{n-1}\left(b\sin\left(dx^{n}+c\right)+a\right)^{p}dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{n-1}(a+b\sin(c+dx^n))^p\,dx$$

input
```
integrate((e*x)**(-1+n)*(a+b*sin(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(n - 1)*(a + b*sin(c + d*x**n))**p, x)
```

Maxima [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\sin(dx^n+c)+a)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}(a+b\sin(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\sin(dx^n+c)+a)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a+b\sin (c+dx^n))^p \, dx = \int (e\,x)^{n-1}\,(a+b\,\sin (c+d\,x^n))^p\,dx$$

input
```
int((e*x)^(n - 1)*(a + b*sin(c + d*x^n))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b*sin(c + d*x^n))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n} (a+b\sin (c+dx^n))^p \, dx = \frac{e^n \left(\int \frac{x^n (\sin(x^n d+c)b+a)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

output
```
(e**n*int((x**n*(sin(x**n*d + c)*b + a)**p)/x,x))/e
```

3.134 $\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\text{Int}\left(x^{-1+2n}(a+b\sin(c+dx^n))^p, x\right)}{e}$$

output
```
(e*x)^(2*n)*Defer(Int)(x^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)/e/(x^(2*n))
```

Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx = \int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3862, 3908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} \left(a+b \sin \left(c+dx^n\right)\right)^p \, dx$$

$$\Big\downarrow \text{ 3862}$$

$$\frac{x^{-2n} (ex)^{2n} \int x^{2n-1} (a+b \sin (dx^n+c))^p dx}{e}$$

$$\Big\downarrow \text{ 3908}$$

$$\frac{x^{-2n} (ex)^{2n} \int x^{2n-1} (a+b \sin (dx^n+c))^p dx}{e}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3908
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n} \left(a+b\sin \left(c+d\,x^{n}\right)\right)^{p}dx$$

input
```
int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx = \int (ex)^{2\,n-1}(b\sin(dx^n+c)+a)^p \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 26.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx = \int (ex)^{2n-1}(a+b\sin(c+dx^n))^p \, dx$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*sin(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(2*n - 1)*(a + b*sin(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{2n-1}\left(b\sin\left(dx^n+c\right)+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{2n-1}\left(b\sin\left(dx^n+c\right)+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 42.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\int (e\,x)^{2\,n-1}\,\left(a+b\,\sin\left(c+d\,x^n\right)\right)^p\,dx$$

input
```
int((e*x)^(2*n - 1)*(a + b*sin(c + d*x^n))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b*sin(c + d*x^n))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx = \frac{e^{2n}\left(\int \frac{x^{2n}(\sin(x^n d+c)b+a)^p}{x} dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

output
```
(e**(2*n)*int((x**(2*n)*(sin(x**n*d + c)*b + a)**p)/x,x))/e
```

3.135 $\int \frac{\sin(a+bx^n)}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sin(a+bx^n)}{x} \, dx = \frac{\text{CosIntegral}(bx^n)\sin(a)}{n} + \frac{\cos(a)\text{Si}(bx^n)}{n}$$

output
```
Ci(b*x^n)*sin(a)/n+cos(a)*Si(b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sin(a+bx^n)}{x} \, dx = \frac{\text{CosIntegral}(bx^n)\sin(a) + \cos(a)\text{Si}(bx^n)}{n}$$

input
```
Integrate[Sin[a + b*x^n]/x,x]
```

output
```
(CosIntegral[b*x^n]*Sin[a] + Cos[a]*SinIntegral[b*x^n])/n
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3858, 3856, 3857}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx^n)}{x}dx$$

$$\Big\downarrow \text{ 3858}$$

$$\sin(a)\int \frac{\cos(bx^n)}{x}dx+\cos(a)\int \frac{\sin(bx^n)}{x}dx$$

$$\Big\downarrow \text{ 3856}$$

$$\sin(a)\int \frac{\cos(bx^n)}{x}dx+\frac{\cos(a)\text{Si}(bx^n)}{n}$$

$$\Big\downarrow \text{ 3857}$$

$$\frac{\sin(a)\,\text{CosIntegral}(bx^n)}{n}+\frac{\cos(a)\text{Si}(bx^n)}{n}$$

input
```
Int[Sin[a + b*x^n]/x,x]
```

output
```
(CosIntegral[b*x^n]*Sin[a])/n + (Cos[a]*SinIntegral[b*x^n])/n
```

Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3857
```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3858
```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Sin[c]   Int[Cos[d*
x^n]/x, x], x] + Simp[Cos[c]   Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n},
 x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\operatorname{Si}(b\,x^{n})\cos(a)+\operatorname{Ci}(b\,x^{n})\sin(a)}{n}$	24
default	$\frac{\operatorname{Si}(b\,x^{n})\cos(a)+\operatorname{Ci}(b\,x^{n})\sin(a)}{n}$	24
risch	$\frac{i\mathrm{e}^{ia}\operatorname{expIntegral}_{1}(-ib\,x^{n})}{2n}-\frac{\mathrm{e}^{-ia}\pi\ \operatorname{csgn}(b\,x^{n})}{2n}+\frac{\mathrm{e}^{-ia}\operatorname{Si}(b\,x^{n})}{n}-\frac{i\mathrm{e}^{-ia}\operatorname{expIntegral}_{1}(-ib\,x^{n})}{2n}$	74
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma+2n\ln(x)+\ln\left(b^{2}\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b\,x^{n}}{2}\right)}{\sqrt{\pi}}+\frac{2\ \operatorname{Ci}(b\,x^{n})}{\sqrt{\pi}}\right)\sin(a)}{2n}+\frac{\cos(a)\operatorname{Si}(b\,x^{n})}{n}$	78

input
```
int(sin(a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(Si(b*x^n)*cos(a)+Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int\frac{\sin\left(a+bx^{n}\right)}{x}\,dx=\frac{\operatorname{Ci}\left(bx^{n}\right)\sin\left(a\right)+\cos\left(a\right)\operatorname{Si}\left(bx^{n}\right)}{n}$$

input
```
integrate(sin(a+b*x^n)/x,x, algorithm="fricas")
```

output
```
(cos_integral(b*x^n)*sin(a) + cos(a)*sin_integral(b*x^n))/n
```

Sympy [F]

$$\int \frac{\sin(a+bx^n)}{x}\,dx=\int \frac{\sin(a+bx^n)}{x}\,dx$$

input
```
integrate(sin(a+b*x**n)/x,x)
```

output
```
Integral(sin(a + b*x**n)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.64

$$\int \frac{\sin(a+bx^n)}{x}\,dx =$$

$$-\frac{\left(i\operatorname{Ei}(i\,bx^n)-i\operatorname{Ei}(-i\,bx^n)+i\operatorname{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\operatorname{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a)-\left(\operatorname{Ei}(i\,bx^n)+\operatorname{Ei}(-i\right.}{4\,n}$$

input
```
integrate(sin(a+b*x^n)/x,x, algorithm="maxima")
```

output
```
-1/4*((I*Ei(I*b*x^n) - I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x))))
- I*Ei(-I*b*e^(n*conjugate(log(x)))))*cos(a) - (Ei(I*b*x^n) + Ei(-I*b*x^n)
 + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjugate(log(x)))))*sin
(a))/n
```

Giac [F]

$$\int \frac{\sin(a+bx^n)}{x}\,dx = \int \frac{\sin(bx^n+a)}{x}\,dx$$

input
```
integrate(sin(a+b*x^n)/x,x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx^n)}{x}\,dx = \int \frac{\sin(a+b\,x^n)}{x}\,dx$$

input
```
int(sin(a + b*x^n)/x,x)
```

output
```
int(sin(a + b*x^n)/x, x)
```

Reduce [F]

$$\int \frac{\sin(a+bx^n)}{x}\,dx = \int \frac{\sin(x^nb+a)}{x}dx$$

input
```
int(sin(a+b*x^n)/x,x)
```

output
```
int(sin(x**n*b + a)/x,x)
```

3.136 $\int \frac{\sin^2(a+bx^n)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx=-\frac{\cos(2a)\,\text{CosIntegral}\,(2bx^n)}{2n}+\frac{\log(x)}{2}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}$$

output
```
-1/2*cos(2*a)*Ci(2*b*x^n)/n+1/2*ln(x)+1/2*sin(2*a)*Si(2*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx=\frac{-\cos(2a)\,\text{CosIntegral}\,(2bx^n)+n\log(x)+\sin(2a)\text{Si}(2bx^n)}{2n}$$

input
```
Integrate[Sin[a + b*x^n]^2/x,x]
```

output
```
(-(Cos[2*a]*CosIntegral[2*b*x^n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^
n])/(2*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+bx^n\right)}{x}dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{1}{2x}-\frac{\cos\left(2a+2bx^n\right)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)}{2n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}+\frac{\log(x)}{2}$$

input
```
Int[Sin[a + b*x^n]^2/x,x]
```

output
```
-1/2*(Cos[2*a]*CosIntegral[2*b*x^n])/n + Log[x]/2 + (Sin[2*a]*SinIntegral[
2*b*x^n])/(2*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{\ln(b\,x^n)}{2}+\frac{\operatorname{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\operatorname{Ci}(2b\,x^n)\cos(2a)}{2}}{n}$
default	$\frac{\frac{\ln(b\,x^n)}{2}+\frac{\operatorname{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\operatorname{Ci}(2b\,x^n)\cos(2a)}{2}}{n}$
risch	$-\frac{i\mathrm{e}^{-2ia}\pi\ \operatorname{csgn}(b\,x^n)}{4n}+\frac{i\mathrm{e}^{-2ia}\operatorname{Si}(2b\,x^n)}{2n}+\frac{\mathrm{e}^{-2ia}\operatorname{expIntegral}_1(-2ib\,x^n)}{4n}+\frac{\mathrm{e}^{2ia}\operatorname{expIntegral}_1(-2ib\,x^n)}{4n}+\frac{\ln(x)}{2}$

input
```
int(sin(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*ln(b*x^n)+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx=-\frac{\cos(2\,a)\operatorname{Ci}(2\,bx^n)-n\log(x)-\sin(2\,a)\operatorname{Si}(2\,bx^n)}{2\,n}$$

input
```
integrate(sin(a+b*x^n)^2/x,x, algorithm="fricas")
```

output
```
-1/2*(cos(2*a)*cos_integral(2*b*x^n) - n*log(x) - sin(2*a)*sin_integral(2*
b*x^n))/n
```

Sympy [F]

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx=\int \frac{\sin^2(a+bx^n)}{x}\,dx$$

input
```
integrate(sin(a+b*x**n)**2/x,x)
```

output
```
Integral(sin(a + b*x**n)**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx =$$

$$-\frac{\left(\mathrm{Ei}(2i\,bx^n)+\mathrm{Ei}(-2i\,bx^n)+\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(2\,a)-4\,n\log(x)-\Big(-i}{8\,n}$$

input
```
integrate(sin(a+b*x^n)^2/x,x, algorithm="maxima")
```

output
```
-1/8*((Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))
+ Ei(-2*I*b*e^(n*conjugate(log(x)))))*cos(2*a) - 4*n*log(x) - (-I*Ei(2*I*b
*x^n) + I*Ei(-2*I*b*x^n) - I*Ei(2*I*b*e^(n*conjugate(log(x))) + I*Ei(-2*I
*b*e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Giac [F]

$$\int \frac{\sin^2(a+bx^n)}{x}\,dx = \int \frac{\sin(bx^n+a)^2}{x}\,dx$$

input
```
integrate(sin(a+b*x^n)^2/x,x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2\left(a+bx^n\right)}{x}\,dx=\int \frac{\sin\left(a+b\,x^n\right)^2}{x}\,dx$$

input

```
int(sin(a + b*x^n)^2/x,x)
```

output

```
int(sin(a + b*x^n)^2/x, x)
```

Reduce [F]

$$\int \frac{\sin^2\left(a+bx^n\right)}{x}\,dx=\int \frac{\sin\left(x^nb+a\right)^2}{x}dx$$

input

```
int(sin(a+b*x^n)^2/x,x)
```

output

```
int(sin(x**n*b + a)**2/x,x)
```

3.137 $\int \frac{\sin^3(a+bx^n)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \frac{3\,\mathrm{CosIntegral}(bx^n)\sin(a)}{4n} - \frac{\mathrm{CosIntegral}(3bx^n)\sin(3a)}{4n} + \frac{3\cos(a)\mathrm{Si}(bx^n)}{4n} - \frac{\cos(3a)\mathrm{Si}(3bx^n)}{4n}$$

output

```
3/4*Ci(b*x^n)*sin(a)/n-1/4*Ci(3*b*x^n)*sin(3*a)/n+3/4*cos(a)*Si(b*x^n)/n-1
/4*cos(3*a)*Si(3*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = \frac{3\,\mathrm{CosIntegral}(bx^n)\sin(a) - \mathrm{CosIntegral}(3bx^n)\sin(3a) + 3\cos(a)\mathrm{Si}(bx^n) - \cos(3a)\mathrm{Si}(3bx^n)}{4n}$$

input

```
Integrate[Sin[a + b*x^n]^3/x,x]
```

output
```
(3*CosIntegral[b*x^n]*Sin[a] - CosIntegral[3*b*x^n]*Sin[3*a] + 3*Cos[a]*Si
nIntegral[b*x^n] - Cos[3*a]*SinIntegral[3*b*x^n])/(4*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3906, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx^n)}{x}dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{3\sin(a+bx^n)}{4x}-\frac{\sin(3a+3bx^n)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sin(a)\operatorname{CosIntegral}(bx^n)}{4n}-\frac{\sin(3a)\operatorname{CosIntegral}(3bx^n)}{4n}+\frac{3\cos(a)\operatorname{Si}(bx^n)}{4n}-\frac{\cos(3a)\operatorname{Si}(3bx^n)}{4n}$$

input
```
Int[Sin[a + b*x^n]^3/x,x]
```

output
```
(3*CosIntegral[b*x^n]*Sin[a])/(4*n) - (CosIntegral[3*b*x^n]*Sin[3*a])/(4*n
) + (3*Cos[a]*SinIntegral[b*x^n])/(4*n) - (Cos[3*a]*SinIntegral[3*b*x^n])/
(4*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{\mathrm{Si}(3b\,x^{n})\cos(3a)}{4}-\frac{\mathrm{Ci}(3b\,x^{n})\sin(3a)}{4}+\frac{3\ \mathrm{Si}(b\,x^{n})\cos(a)}{4}+\frac{3\ \mathrm{Ci}(b\,x^{n})\sin(a)}{4}}{n}$
default	$\frac{-\frac{\mathrm{Si}(3b\,x^{n})\cos(3a)}{4}-\frac{\mathrm{Ci}(3b\,x^{n})\sin(3a)}{4}+\frac{3\ \mathrm{Si}(b\,x^{n})\cos(a)}{4}+\frac{3\ \mathrm{Ci}(b\,x^{n})\sin(a)}{4}}{n}$
risch	$-\frac{i\mathrm{e}^{3ia}\,\mathrm{expIntegral}_1(-3ib\,x^{n})}{8n}+\frac{\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(b\,x^{n})}{8n}-\frac{\mathrm{e}^{-3ia}\,\mathrm{Si}(3b\,x^{n})}{4n}+\frac{i\mathrm{e}^{-3ia}\,\mathrm{expIntegral}_1(-3ib\,x^{n})}{8n}-$

input
```
int(sin(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/4*Si(3*b*x^n)*cos(3*a)-1/4*Ci(3*b*x^n)*sin(3*a)+3/4*Si(b*x^n)*cos(
a)+3/4*Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx = -\frac{\mathrm{Ci}\left(3\,bx^{n}\right)\sin\left(3\,a\right)-3\ \mathrm{Ci}\left(bx^{n}\right)\sin\left(a\right)+\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx^{n}\right)-3\ \cos\left(a\right)\mathrm{Si}\left(bx^{n}\right)}{4\,n}$$

input
```
integrate(sin(a+b*x^n)^3/x,x, algorithm="fricas")
```

output
```
-1/4*(cos_integral(3*b*x^n)*sin(3*a) - 3*cos_integral(b*x^n)*sin(a) + cos(
3*a)*sin_integral(3*b*x^n) - 3*cos(a)*sin_integral(b*x^n))/n
```

Sympy [F]

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx=\int \frac{\sin^3(a+bx^n)}{x}\,dx$$

input
```
integrate(sin(a+b*x**n)**3/x,x)
```

output
```
Integral(sin(a + b*x**n)**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.70

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx$$

$$= \frac{\left(i\,\mathrm{Ei}(3i\,bx^n) - i\,\mathrm{Ei}(-3i\,bx^n) + i\,\mathrm{Ei}\left(3i\,be^{\left(n\overline{\log(x)}\right)}\right) - i\,\mathrm{Ei}\left(-3i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(3\,a) - 3\left(i\,\mathrm{Ei}(i\,bx^n) -\right.}{}$$

input
```
integrate(sin(a+b*x^n)^3/x,x, algorithm="maxima")
```

output
```
1/16*((I*Ei(3*I*b*x^n) - I*Ei(-3*I*b*x^n) + I*Ei(3*I*b*e^(n*conjugate(log(
x)))) - I*Ei(-3*I*b*e^(n*conjugate(log(x)))))*cos(3*a) - 3*(I*Ei(I*b*x^n)
- I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x)))) - I*Ei(-I*b*e^(n*conj
ugate(log(x)))))*cos(a) - (Ei(3*I*b*x^n) + Ei(-3*I*b*x^n) + Ei(3*I*b*e^(n*
conjugate(log(x)))) + Ei(-3*I*b*e^(n*conjugate(log(x)))))*sin(3*a) + 3*(Ei
(I*b*x^n) + Ei(-I*b*x^n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*
conjugate(log(x)))))*sin(a))/n
```

Giac [F]

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx=\int \frac{\sin(bx^n+a)^3}{x}\,dx$$

input
```
integrate(sin(a+b*x^n)^3/x,x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx=\int \frac{\sin(a+b\,x^n)^3}{x}\,dx$$

input
```
int(sin(a + b*x^n)^3/x,x)
```

output
```
int(sin(a + b*x^n)^3/x, x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx=\int \frac{\sin(x^n b+a)^3}{x}dx$$

input
```
int(sin(a+b*x^n)^3/x,x)
```

output
```
int(sin(x**n*b + a)**3/x,x)
```

3.138 $\int \frac{\sin^4(a+bx^n)}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{\sin^4(a+bx^n)}{x} \, dx = -\frac{\cos(2a)\operatorname{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\operatorname{CosIntegral}(4bx^n)}{8n} + \frac{3\log(x)}{8} + \frac{\sin(2a)\operatorname{Si}(2bx^n)}{2n} - \frac{\sin(4a)\operatorname{Si}(4bx^n)}{8n}$$

output

```
-1/2*cos(2*a)*Ci(2*b*x^n)/n+1/8*cos(4*a)*Ci(4*b*x^n)/n+3/8*ln(x)+1/2*sin(2
*a)*Si(2*b*x^n)/n-1/8*sin(4*a)*Si(4*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{\sin^4(a+bx^n)}{x} \, dx = \frac{3\log(x)}{8} + \frac{-4\cos(2a)\operatorname{CosIntegral}(2bx^n) + \cos(4a)\operatorname{CosIntegral}(4bx^n) + 4\sin(2a)\operatorname{Si}(2bx^n) - \sin(4a)\operatorname{Si}(4bx^n)}{8n}$$

input

```
Integrate[Sin[a + b*x^n]^4/x,x]
```

output
```
(3*Log[x])/8 + (-4*Cos[2*a]*CosIntegral[2*b*x^n] + Cos[4*a]*CosIntegral[4*
b*x^n] + 4*Sin[2*a]*SinIntegral[2*b*x^n] - Sin[4*a]*SinIntegral[4*b*x^n])/
(8*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4\left(a+bx^n\right)}{x}dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(-\frac{\cos\left(2a+2bx^n\right)}{2x}+\frac{\cos\left(4a+4bx^n\right)}{8x}+\frac{3}{8x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)}{2n}+\frac{\cos(4a)\,\text{CosIntegral}\left(4bx^n\right)}{8n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}-\frac{\sin(4a)\text{Si}(4bx^n)}{8n}+\frac{3\log(x)}{8}$$

input
```
Int[Sin[a + b*x^n]^4/x,x]
```

output
```
-1/2*(Cos[2*a]*CosIntegral[2*b*x^n])/n + (Cos[4*a]*CosIntegral[4*b*x^n])/(
8*n) + (3*Log[x])/8 + (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*Si
nIntegral[4*b*x^n])/(8*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{3\ln\left(b\,x^{n}\right)}{8}-\frac{\operatorname{Si}\left(4b\,x^{n}\right)\sin\left(4a\right)}{8}+\frac{\operatorname{Ci}\left(4b\,x^{n}\right)\cos\left(4a\right)}{8}+\frac{\operatorname{Si}\left(2b\,x^{n}\right)\sin\left(2a\right)}{2}-\frac{\operatorname{Ci}\left(2b\,x^{n}\right)\cos\left(2a\right)}{2}}{n}$
default	$\frac{\frac{3\ln\left(b\,x^{n}\right)}{8}-\frac{\operatorname{Si}\left(4b\,x^{n}\right)\sin\left(4a\right)}{8}+\frac{\operatorname{Ci}\left(4b\,x^{n}\right)\cos\left(4a\right)}{8}+\frac{\operatorname{Si}\left(2b\,x^{n}\right)\sin\left(2a\right)}{2}-\frac{\operatorname{Ci}\left(2b\,x^{n}\right)\cos\left(2a\right)}{2}}{n}$
risch	$\frac{i\mathrm{e}^{-4ia}\pi\ \operatorname{csgn}\left(b\,x^{n}\right)}{16n}-\frac{i\mathrm{e}^{-4ia}\operatorname{Si}\left(4b\,x^{n}\right)}{8n}-\frac{\mathrm{e}^{-4ia}\operatorname{expIntegral}_{1}\left(-4ib\,x^{n}\right)}{16n}-\frac{\mathrm{e}^{4ia}\operatorname{expIntegral}_{1}\left(-4ib\,x^{n}\right)}{16n}+\frac{3\ln\left(x\right)}{8}$

input
```
int(sin(a+b*x^n)^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(3/8*ln(b*x^n)-1/8*Si(4*b*x^n)*sin(4*a)+1/8*Ci(4*b*x^n)*cos(4*a)+1/2*S
i(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int\frac{\sin^{4}\left(a+bx^{n}\right)}{x}\,dx$$
$$=\frac{\cos\left(4\,a\right)\operatorname{Ci}\left(4\,bx^{n}\right)-4\,\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^{n}\right)+3\,n\log\left(x\right)-\sin\left(4\,a\right)\operatorname{Si}\left(4\,bx^{n}\right)+4\,\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^{n}\right)}{8\,n}$$

input
```
integrate(sin(a+b*x^n)^4/x,x, algorithm="fricas")
```

output
```
1/8*(cos(4*a)*cos_integral(4*b*x^n) - 4*cos(2*a)*cos_integral(2*b*x^n) + 3
*n*log(x) - sin(4*a)*sin_integral(4*b*x^n) + 4*sin(2*a)*sin_integral(2*b*x
^n))/n
```

Sympy [F]

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \int \frac{\sin^4(a+bx^n)}{x}\,dx$$

input
```
integrate(sin(a+b*x**n)**4/x,x)
```

output
```
Integral(sin(a + b*x**n)**4/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.39

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \frac{\left(\mathrm{Ei}(4i\,bx^n)+\mathrm{Ei}(-4i\,bx^n)+\mathrm{Ei}\left(4i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-4i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(4\,a)-4\left(\mathrm{Ei}(2i\,bx^n)+\mathrm{Ei}(\right.}{}$$

input
```
integrate(sin(a+b*x^n)^4/x,x, algorithm="maxima")
```

output
```
1/32*((Ei(4*I*b*x^n) + Ei(-4*I*b*x^n) + Ei(4*I*b*e^(n*conjugate(log(x)))
+ Ei(-4*I*b*e^(n*conjugate(log(x)))))*cos(4*a) - 4*(Ei(2*I*b*x^n) + Ei(-2*
I*b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x))) + Ei(-2*I*b*e^(n*conjugate(lo
g(x)))))*cos(2*a) + 12*n*log(x) + (I*Ei(4*I*b*x^n) - I*Ei(-4*I*b*x^n) + I*
Ei(4*I*b*e^(n*conjugate(log(x))) - I*Ei(-4*I*b*e^(n*conjugate(log(x)))))*
sin(4*a) - 4*(I*Ei(2*I*b*x^n) - I*Ei(-2*I*b*x^n) + I*Ei(2*I*b*e^(n*conjuga
te(log(x))) - I*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Giac [F]

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx=\int \frac{\sin(bx^n+a)^4}{x}\,dx$$

input
```
integrate(sin(a+b*x^n)^4/x,x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx=\int \frac{\sin(a+b\,x^n)^4}{x}\,dx$$

input
```
int(sin(a + b*x^n)^4/x,x)
```

output
```
int(sin(a + b*x^n)^4/x, x)
```

Reduce [F]

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx=\int \frac{\sin(x^nb+a)^4}{x}dx$$

input
```
int(sin(a+b*x^n)^4/x,x)
```

output
```
int(sin(x**n*b + a)**4/x,x)
```

3.139 $\int \sin\left(a+bx^n\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 87

$$\int \sin\left(a+bx^n\right)\,dx = \frac{ie^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

output

```
1/2*I*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-1/2*I*x*GAMMA(1/
n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \sin\left(a+bx^n\right)\,dx = \frac{ix\left(b^2x^{2n}\right)^{-1/n}\left(-\left(-ibx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)\left(\cos(a)-i\sin(a)\right)+\left(ibx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)\left(\cos(a)+i\sin(a)\right)\right)}{2n}$$

input

```
Integrate[Sin[a + b*x^n],x]
```

output
```
((I/2)*x*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[a] - I*Sin[a])
) + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Sin[a])))/(n*(b
^2*x^(2*n))^n^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3846, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{3846}$$

$$\frac{1}{2}i\int e^{-ibx^n-ia}dx-\frac{1}{2}i\int e^{ibx^n+ia}dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{ie^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

input
```
Int[Sin[a + b*x^n],x]
```

output
```
((I/2)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - ((I/
2)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

rule 3846
```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Simp[I/2   I
nt[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Simp[I/2   Int[E^(c*I + d*I*(e +
f*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{1}{2n}\right],-\frac{x^{2n}b^{2}}{4}\right)\sin\left(a\right)+\frac{b\,x^{1+n}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{2n}\right],-\frac{x^{2n}b^{2}}{4}\right)\cos\left(a\right)}{1+n}$

input
```
int(sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2/n],[1/2,1+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)+b/(1+n)*x^(1+n)
*hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Fricas [F]

$$\int \sin\left(a+bx^{n}\right)\,dx=\int \sin\left(bx^{n}+a\right)\,dx$$

input
```
integrate(sin(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(sin(b*x^n + a), x)
```

Sympy [F]

$$\int \sin\left(a+bx^n\right)\,dx=\int \sin\left(a+bx^n\right)dx$$

input
```
integrate(sin(a+b*x**n),x)
```

output
```
Integral(sin(a + b*x**n), x)
```

Maxima [F]

$$\int \sin\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)\,dx$$

input
```
integrate(sin(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(sin(b*x^n + a), x)
```

Giac [F]

$$\int \sin\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)\,dx$$

input
```
integrate(sin(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx^n)\,dx=\int \sin\left(a+b\,x^n\right)\,dx$$

input

```
int(sin(a + b*x^n),x)
```

output

```
int(sin(a + b*x^n), x)
```

Reduce [F]

$$\int \sin(a+bx^n)\,dx=\int \sin(x^n b+a)\,dx$$

input

```
int(sin(a+b*x^n),x)
```

output

```
int(sin(x**n*b + a),x)
```

3.140 $\int \sin^2(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 100

$$\int \sin^2(a+bx^n)\,dx = \frac{x}{2} + \frac{2^{-2-\frac{1}{n}} e^{2ia} x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n} + \frac{2^{-2-\frac{1}{n}} e^{-2ia} x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}$$

output

```
1/2*x+2^(-2-1/n)*exp(2*I*a)*x*GAMMA(1/n,-2*I*b*x^n)/n/((-I*b*x^n)^(1/n))+2
^(-2-1/n)*x*GAMMA(1/n,2*I*b*x^n)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \sin^2(a+bx^n)\,dx = \frac{x\left(2n+2^{-1/n}e^{2ia}(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)+2^{-1/n}e^{-2ia}(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)\right)}{4n}$$

input

```
Integrate[Sin[a + b*x^n]^2,x]
```

output
```
(x*(2*n + (E^((2*I)*a)*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b*x^n)
^n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^n^(-
1))))/(4*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(a+bx^n\right)dx$$

$$\Big\downarrow \text{3848}$$

$$\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(2a+2bx^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2ia}2^{-\frac{1}{n}-2}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

input
```
Int[Sin[a + b*x^n]^2,x]
```

output
```
x/2 + (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)
*b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)
*a)*n*(I*b*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3848
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Maple [F]

$$\int \sin \left(a+b\,x^{n}\right)^{2}d x$$

input
```
int(sin(a+b*x^n)^2,x)
```

output
```
int(sin(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \sin ^2\left(a+b x^n\right) \, dx=\int \sin \left(b x^{n} + a\right)^{2} \,d x$$

input
```
integrate(sin(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(-cos(b*x^n + a)^2 + 1, x)
```

Sympy [F]

$$\int \sin^2(a+bx^n)\,dx=\int \sin^2(a+bx^n)\,dx$$

input
```
integrate(sin(a+b*x**n)**2,x)
```

output
```
Integral(sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \sin^2(a+bx^n)\,dx=\int \sin{(bx^n+a)}^2\,dx$$

input
```
integrate(sin(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/2*integrate(cos(2*b*x^n + 2*a), x)
```

Giac [F]

$$\int \sin^2(a+bx^n)\,dx=\int \sin{(bx^n+a)}^2\,dx$$

input
```
integrate(sin(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2\left(a+bx^n\right)\,dx=\int \sin\left(a+b\,x^n\right)^2\,dx$$

input
```
int(sin(a + b*x^n)^2,x)
```

output
```
int(sin(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \sin^2\left(a+bx^n\right)\,dx=\int \sin\left(x^n b+a\right)^2 dx$$

input
```
int(sin(a+b*x^n)^2,x)
```

output
```
int(sin(x**n*b + a)**2,x)
```

3.141 $\int \sin^3\left(a+bx^n\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 187

$$\begin{aligned}\int \sin^3\left(a+bx^n\right)\,dx &= \frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}\\ &\quad-\frac{i3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n}\\ &\quad+\frac{i3^{-1/n}e^{-3ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}\end{aligned}$$

output

```
3/8*I*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-3/8*I*x*GAMMA(1/
n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))-1/8*I*exp(3*I*a)*x*GAMMA(1/n,-3*I*
b*x^n)/(3^(1/n))/n/((-I*b*x^n)^(1/n))+1/8*I*x*GAMMA(1/n,3*I*b*x^n)/(3^(1/n
))/exp(3*I*a)/n/((I*b*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \sin^3(a+bx^n)\,dx$$
$$= \frac{i3^{-1/n}e^{-3ia}x(b^2x^{2n})^{-1/n}\Big(3^{1+\frac{1}{n}}e^{4ia}(ibx^n)^{\frac{1}{n}}\Gamma\big(\frac{1}{n},-ibx^n\big)-3^{1+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\Gamma\big(\frac{1}{n},ibx^n\big)-e^{6ia}(ibx^n)^{\frac{1}{n}}\Gamma(}{8n}$$

input
```
Integrate[Sin[a + b*x^n]^3,x]
```

output
```
((I/8)*x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b
*x^n] - 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x
^n] - E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x
^n)^n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*
n))^n^(-1))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx^n)\,dx$$
$$\Big\downarrow \text{3848}$$
$$\int \left(\frac{3}{4}\sin(a+bx^n)-\frac{1}{4}\sin(3a+3bx^n)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3ie^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}+\frac{ie^{-3ia}3^{-1/n}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}$$

input

```
Int[Sin[a + b*x^n]^3,x]
```

output

```
(((3*I)/8)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) -
(((3*I)/8)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1)) - ((I/8)
*E^((3*I)*a)*x*Gamma[n^(-1), (-3*I)*b*x^n])/(3^n^(-1)*n*((-I)*b*x^n)^n^(-1
)) + ((I/8)*x*Gamma[n^(-1), (3*I)*b*x^n])/(3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n
)^n^(-1))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3848

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Maple [F]

$$\int \sin \left(a+b\,x^{n}\right)^{3}d x$$

input

```
int(sin(a+b*x^n)^3,x)
```

output

```
int(sin(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \sin^{3}\left(a+bx^{n}\right)\,dx=\int \sin\left(bx^{n}+a\right)^{3}\,dx$$

input

```
integrate(sin(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a), x)
```

Sympy [F]

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin^3\left(a+bx^n\right)dx$$

input
```
integrate(sin(a+b*x**n)**3,x)
```

output
```
Integral(sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)^3\,dx$$

input
```
integrate(sin(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate(sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin\left(bx^n+a\right)^3\,dx$$

input
```
integrate(sin(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin\left(a+b\,x^n\right)^3\,dx$$

input
```
int(sin(a + b*x^n)^3,x)
```

output
```
int(sin(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \sin^3\left(a+bx^n\right)\,dx=\int \sin\left(x^nb+a\right)^3dx$$

input
```
int(sin(a+b*x^n)^3,x)
```

output
```
int(sin(x**n*b + a)**3,x)
```

3.142 $\int x^m \sin(a+bx^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int x^m \sin(a+bx^n)\,dx = \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n} - \frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}$$

output
```
1/2*I*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-1/
2*I*x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08

$$\int x^m \sin(a+bx^n)\,dx$$

$$= \frac{ix^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\,\Big(-(-ibx^n)^{\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)(\cos(a)+}{2n}$$

input
```
Integrate[x^m*Sin[a + b*x^n],x]
```

output
```
((I/2)*x^(1 + m)*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n]*(Co
s[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n]*(Co
s[a] + I*Sin[a])))/(n*(b^2*x^(2*n))^((1 + m)/n))
```

Rubi [A] (verified)

Time $= 0.28$ (sec) , antiderivative size $= 109$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used $= \{3904, 2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin \left(a+b x^n\right) \, dx$$

$$\Big\downarrow \text{3904}$$

$$\frac{1}{2} i \int e^{-i b x^n-i a} x^m dx-\frac{1}{2} i \int e^{i b x^n+i a} x^m dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{i e^{i a} x^{m+1} \left(-i b x^n\right)^{-\frac{m+1}{n}} \Gamma \left(\frac{m+1}{n},-i b x^n\right)}{2 n}-\frac{i e^{-i a} x^{m+1} \left(i b x^n\right)^{-\frac{m+1}{n}} \Gamma \left(\frac{m+1}{n},i b x^n\right)}{2 n}$$

input
```
Int[x^m*Sin[a + b*x^n],x]
```

output
```
((I/2)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^((1
 + m)/n)) - ((I/2)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^
n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 2648

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],-\frac{x^{2n} b^{2}}{4}\right) \sin(a)}{1+m}+\frac{b\,x^{1+m+n} \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n}\right],-\frac{x^{2n} b^{2}}{4}\right) \mathrm{c}}{1+m+n}$

input

```
int(x^m*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

output

```
1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],-1/4*x^(2*
n)*b^2)*sin(a)+b/(1+m+n)*x^(1+m+n)*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+
1/2/n*m+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Fricas [F]

$$\int x^{m} \sin\left(a+bx^{n}\right)\,dx=\int x^{m} \sin\left(bx^{n}+a\right)\,dx$$

input

```
integrate(x^m*sin(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(x^m*sin(b*x^n + a), x)
```

Sympy [F]

$$\int x^m \sin(a + bx^n)\,dx = \int x^m \sin(a + bx^n)\,dx$$

input
```
integrate(x**m*sin(a+b*x**n),x)
```

output
```
Integral(x**m*sin(a + b*x**n), x)
```

Maxima [F]

$$\int x^m \sin(a + bx^n)\,dx = \int x^m \sin(bx^n + a)\,dx$$

input
```
integrate(x^m*sin(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x^m*sin(b*x^n + a), x)
```

Giac [F]

$$\int x^m \sin(a + bx^n)\,dx = \int x^m \sin(bx^n + a)\,dx$$

input
```
integrate(x^m*sin(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^m*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin(a+bx^n)\,dx = \int x^m\,\sin\left(a+b\,x^n\right)\,dx$$

input
```
int(x^m*sin(a + b*x^n),x)
```

output
```
int(x^m*sin(a + b*x^n), x)
```

Reduce [F]

$$\int x^m \sin(a+bx^n)\,dx = \int x^m \sin(x^n b+a)\,dx$$

input
```
int(x^m*sin(a+b*x^n),x)
```

output
```
int(x**m*sin(x**n*b + a),x)
```

3.143 $\int x^m \sin^2(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 139

$$\int x^m \sin^2(a+bx^n)\,dx = \frac{x^{1+m}}{2(1+m)} + \frac{2^{-\frac{1+m+2n}{n}} e^{2ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right)}{n} + \frac{2^{-\frac{1+m+2n}{n}} e^{-2ia} x^{1+m} (ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2ibx^n\right)}{n}$$

output
```
x^(1+m)/(2+2*m)+exp(2*I*a)*x^(1+m)*GAMMA((1+m)/n,-2*I*b*x^n)/(2^((1+m+2*n)
/n))/n/((-I*b*x^n)^((1+m)/n))+x^(1+m)*GAMMA((1+m)/n,2*I*b*x^n)/(2^((1+m+2*
n)/n))/exp(2*I*a)/n/((I*b*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int x^m \sin^2(a+bx^n)\,dx$$
$$= \frac{x^{1+m}\Big(2n + 2^{-\frac{1+m}{n}} e^{2ia} (1+m) (-ibx^n)^{-\frac{1+m}{n}} \Gamma\big(\frac{1+m}{n}, -2ibx^n\big) + 2^{-\frac{1+m}{n}} e^{-2ia} (1+m) (ibx^n)^{-\frac{1+m}{n}} \Gamma\big(\frac{1+m}{n}, 2i}{4(1+m)n}$$

input
```
Integrate[x^m*Sin[a + b*x^n]^2,x]
```

output

```
(x^(1 + m)*(2*n + (E^((2*I)*a)*(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^n])/(2^
((1 + m)/n)*((-I)*b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, (2*I)*b*
x^n])/(2^((1 + m)/n)*E^((2*I)*a)*(I*b*x^n)^((1 + m)/n))))/(4*(1 + m)*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^2\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{ 3906}$$

$$\int \left(\frac{x^m}{2}-\frac{1}{2}x^m \cos\left(2a+2bx^n\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(-ibx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(ibx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2ibx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

input

```
Int[x^m*Sin[a + b*x^n]^2,x]
```

output

```
x^(1 + m)/(2*(1 + m)) + (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x
^n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1 + m)*Gamma[(
1 + m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 +
 m)/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [F]

$$\int x^m \sin\left(a+b\,x^n\right)^2 dx$$

input
```
int(x^m*sin(a+b*x^n)^2,x)
```

output
```
int(x^m*sin(a+b*x^n)^2,x)
```

Fricas [F]

$$\int x^m \sin^2\left(a+bx^n\right)\,dx = \int x^m \sin\left(bx^n+a\right)^2\,dx$$

input
```
integrate(x^m*sin(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(-x^m*cos(b*x^n + a)^2 + x^m, x)
```

Sympy [F]

$$\int x^m \sin^2(a+bx^n)\,dx = \int x^m \sin^2(a+bx^n)\,dx$$

input
```
integrate(x**m*sin(a+b*x**n)**2,x)
```

output
```
Integral(x**m*sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^m \sin^2(a+bx^n)\,dx = \int x^m \sin(bx^n+a)^2\,dx$$

input
```
integrate(x^m*sin(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/2*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))/(m + 1)
```

Giac [F]

$$\int x^m \sin^2(a+bx^n)\,dx = \int x^m \sin(bx^n+a)^2\,dx$$

input
```
integrate(x^m*sin(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin^2\left(a+bx^n\right)\,dx=\int x^m\,\sin\left(a+b\,x^n\right)^2\,dx$$

input
```
int(x^m*sin(a + b*x^n)^2,x)
```

output
```
int(x^m*sin(a + b*x^n)^2, x)
```

Reduce [F]

$$\int x^m \sin^2\left(a+bx^n\right)\,dx=\int x^m\sin\left(x^n b+a\right)^2dx$$

input
```
int(x^m*sin(a+b*x^n)^2,x)
```

output
```
int(x**m*sin(x**n*b + a)**2,x)
```

3.144 $\int x^m \sin^3\left(a+bx^n\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 237

$$\int x^m \sin^3\left(a+bx^n\right)\,dx = \frac{3ie^{ia}x^{1+m}\left(-ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{1+m}\left(ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n} - \frac{i3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}\left(-ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3ibx^n\right)}{8n} + \frac{i3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}\left(ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3ibx^n\right)}{8n}$$

output

```
3/8*I*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-3/
8*I*x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))-1/8*I*
exp(3*I*a)*x^(1+m)*GAMMA((1+m)/n,-3*I*b*x^n)/(3^((1+m)/n))/n/((-I*b*x^n)^(
(1+m)/n))+1/8*I*x^(1+m)*GAMMA((1+m)/n,3*I*b*x^n)/(3^((1+m)/n))/exp(3*I*a)/
n/((I*b*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95

$$\int x^m \sin^3(a+bx^n)\,dx = \frac{i3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\,\Bigl(3^{\frac{1+m+n}{n}}e^{4ia}(ibx^n)^{\frac{1+m}{n}}\,\Gamma(\frac{1+m}{n},-ibx^n)-3^{\frac{1+m+n}{n}}e^{2ia}(-ibx^n)^{\frac{1+m}{n}}\,\Gamma(\frac{1+m}{n},ibx}{8n}$$

input
```
Integrate[x^m*Sin[a + b*x^n]^3,x]
```

output
```
((I/8)*x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamm
a[(1 + m)/n, (-I)*b*x^n] - 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1
 + m)/n)*Gamma[(1 + m)/n, I*b*x^n] - E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamm
a[(1 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*
I)*b*x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^3(a+bx^n)\,dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{3}{4}x^m\sin(a+bx^n)-\frac{1}{4}x^m\sin(3a+3bx^n)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-3ibx^n\right)}{8n}+\frac{ie^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},3ibx^n\right)}{8n}$$

input
```
Int[x^m*Sin[a + b*x^n]^3,x]
```

output
```
(((3*I)/8)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)
^((1 + m)/n)) - (((3*I)/8)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n
*(I*b*x^n)^((1 + m)/n)) - ((I/8)*E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-
3*I)*b*x^n])/(3^((1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + ((I/8)*x^(1 + m)
*Gamma[(1 + m)/n, (3*I)*b*x^n])/(3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1
 + m)/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [F]

$$\int x^m \sin\left(a + b\,x^n\right)^3 dx$$

input
```
int(x^m*sin(a+b*x^n)^3,x)
```

output
```
int(x^m*sin(a+b*x^n)^3,x)
```

Fricas [F]

$$\int x^m \sin^3(a+bx^n)\ dx = \int x^m \sin(bx^n+a)^3\ dx$$

input
```
integrate(x^m*sin(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(-(x^m*cos(b*x^n + a)^2 - x^m)*sin(b*x^n + a), x)
```

Sympy [F]

$$\int x^m \sin^3(a+bx^n)\ dx = \int x^m \sin^3(a+bx^n)\, dx$$

input
```
integrate(x**m*sin(a+b*x**n)**3,x)
```

output
```
Integral(x**m*sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^m \sin^3(a+bx^n)\ dx = \int x^m \sin(bx^n+a)^3\ dx$$

input
```
integrate(x^m*sin(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^m \sin^3(a+bx^n)\,dx = \int x^m \sin(bx^n+a)^3\,dx$$

input
```
integrate(x^m*sin(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^m*sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sin^3(a+bx^n)\,dx = \int x^m \sin\left(a+b\,x^n\right)^3\,dx$$

input
```
int(x^m*sin(a + b*x^n)^3,x)
```

output
```
int(x^m*sin(a + b*x^n)^3, x)
```

Reduce [F]

$$\int x^m \sin^3(a+bx^n)\,dx = \int x^m \sin(x^n b+a)^3\,dx$$

input
```
int(x^m*sin(a+b*x^n)^3,x)
```

output
```
int(x**m*sin(x**n*b + a)**3,x)
```

3.145 $\int x^{-1+2n}\sin(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int x^{-1+2n}\sin(a+bx^n)\,dx=-\frac{x^n\cos(a+bx^n)}{bn}+\frac{\sin(a+bx^n)}{b^2n}$$

output
```
-x^n*cos(a+b*x^n)/b/n+sin(a+b*x^n)/b^2/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int x^{-1+2n}\sin(a+bx^n)\,dx=\frac{-bx^n\cos(a+bx^n)+\sin(a+bx^n)}{b^2n}$$

input
```
Integrate[x^(-1 + 2*n)*Sin[a + b*x^n],x]
```

output
```
(-(b*x^n*Cos[a + b*x^n]) + Sin[a + b*x^n])/(b^2*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\sin\left(a+bx^n\right)dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{\int x^n\sin\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int x^n\sin\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{\int\cos(bx^n+a)dx^n}{b}-\frac{x^n\cos(a+bx^n)}{b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int\sin\left(bx^n+a+\frac{\pi}{2}\right)dx^n}{b}-\frac{x^n\cos(a+bx^n)}{b}}{n}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{\sin(a+bx^n)}{b^2}-\frac{x^n\cos(a+bx^n)}{b}}{n}$$

input
```
Int[x^(-1 + 2*n)*Sin[a + b*x^n],x]
```

output
```
(-((x^n*Cos[a + b*x^n])/b) + Sin[a + b*x^n]/b^2)/n
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^{n} \cos\left(a+b\,x^{n}\right)}{b n}+\frac{\sin\left(a+b\,x^{n}\right)}{b^{2} n}$	36
default	$\frac{\sin\left(a+b\,x^{n}\right)-\left(a+b\,x^{n}\right) \cos\left(a+b\,x^{n}\right)+a \cos\left(a+b\,x^{n}\right)}{n\,b^{2}}$	44
meijerg	$\frac{2 \sqrt{\pi}\ \operatorname{MeijerG}\left(\left[\left[1\right],\left[\right]\right],\left[\left[1\right],\left[\frac{3}{2},0\right]\right],\frac{x^{2 n} b^{2}}{4}\right) \sin\left(a\right)}{b^{2} n}+\frac{2 \sqrt{\pi}\,\left(-\frac{x^{n} b \cos\left(b\,x^{n}\right)}{2 \sqrt{\pi}}+\frac{\sin\left(b\,x^{n}\right)}{2 \sqrt{\pi}}\right) \cos\left(a\right)}{b^{2} n}$	76

input
```
int(x^(-1+2*n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-x^n*cos(a+b*x^n)/b/n+sin(a+b*x^n)/b^2/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=-\frac{bx^n\cos\left(bx^n+a\right)-\sin\left(bx^n+a\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

output
```
-(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 3.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=\begin{cases}\log\left(x\right)\sin\left(a\right) & \text{for } b=0\wedge n=0\\ \frac{xx^{2n-1}\sin\left(a\right)}{2n} & \text{for } b=0\\ \log\left(x\right)\sin\left(a+b\right) & \text{for } n=0\\ -\frac{x^n\cos\left(a+bx^n\right)}{bn}+\frac{\sin\left(a+bx^n\right)}{b^2n} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+2*n)*sin(a+b*x**n),x)
```

output
```
Piecewise((log(x)*sin(a), Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)*sin(a)/(2*
n), Eq(b, 0)), (log(x)*sin(a + b), Eq(n, 0)), (-x**n*cos(a + b*x**n)/(b*n)
 + sin(a + b*x**n)/(b**2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=-\frac{bx^n\cos\left(bx^n+a\right)-\sin\left(bx^n+a\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="maxima")
```

output
```
-(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Giac [F]

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=\int x^{2\,n-1}\sin\left(bx^n+a\right)\,dx$$

input
```
integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=\int x^{2\,n-1}\,\sin\left(a+b\,x^n\right)\,dx$$

input
```
int(x^(2*n - 1)*sin(a + b*x^n),x)
```

output
```
int(x^(2*n - 1)*sin(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=\frac{-x^n\cos\left(x^nb+a\right)b+\sin\left(x^nb+a\right)}{b^2n}$$

input
```
int(x^(-1+2*n)*sin(a+b*x^n),x)
```

output
```
( - x**n*cos(x**n*b + a)*b + sin(x**n*b + a))/(b**2*n)
```

3.146 $\int x^{-1+2n} \cos \left(a+b x^n\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int x^{-1+2n} \cos \left(a+b x^n\right) \, dx=\frac{\cos \left(a+b x^n\right)}{b^2 n}+\frac{x^n \sin \left(a+b x^n\right)}{b n}$$

output

```
cos(a+b*x^n)/b^2/n+x^n*sin(a+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n} \cos \left(a+b x^n\right) \, dx=\frac{\cos \left(a+b x^n\right)+b x^n \sin \left(a+b x^n\right)}{b^2 n}$$

input

```
Integrate[x^(-1 + 2*n)*Cos[a + b*x^n],x]
```

output

```
(Cos[a + b*x^n] + b*x^n*Sin[a + b*x^n])/(b^2*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3861, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1} \cos \left(a+b x^n\right) \, dx$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{\int x^n \cos \left(b x^n+a\right) dx^n}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int x^n \sin \left(b x^n+a+\frac{\pi}{2}\right) dx^n}{n}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{\int -\sin \left(b x^n+a\right) dx^n}{b}+\frac{x^n \sin \left(a+b x^n\right)}{b}}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{x^n \sin \left(a+b x^n\right)}{b}-\frac{\int \sin \left(b x^n+a\right) dx^n}{b}}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{x^n \sin \left(a+b x^n\right)}{b}-\frac{\int \sin \left(b x^n+a\right) dx^n}{b}}{n}$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\frac{\cos \left(a+b x^n\right)}{b^2}+\frac{x^n \sin \left(a+b x^n\right)}{b}}{n}$$

input

```
Int[x^(-1 + 2*n)*Cos[a + b*x^n],x]
```

output

```
(Cos[a + b*x^n]/b^2 + (x^n*Sin[a + b*x^n])/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{\cos(a+b\,x^{n})}{b^{2}n}+\frac{x^{n}\sin(a+b\,x^{n})}{bn}$	35
default	$\frac{\cos(a+b\,x^{n})+(a+b\,x^{n})\sin(a+b\,x^{n})-a\sin(a+b\,x^{n})}{n\,b^{2}}$	44
meijerg	$\frac{2\sqrt{\pi}\ \mathrm{MeijerG}\left([[1],[]],[[1],[\frac{3}{2},0]],\frac{x^{2n}b^{2}}{4}\right)\cos(a)}{b^{2}n}-\frac{2\sqrt{\pi}\left(-\frac{x^{n}b\cos(b\,x^{n})}{2\sqrt{\pi}}+\frac{\sin(b\,x^{n})}{2\sqrt{\pi}}\right)\sin(a)}{b^{2}n}$	76

input
```
int(x^(-1+2*n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
cos(a+b*x^n)/b^2/n+x^n*sin(a+b*x^n)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\frac{bx^n\sin\left(bx^n+a\right)+\cos\left(bx^n+a\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="fricas")
```

output
```
(b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 3.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx=\begin{cases}\log\left(x\right)\cos\left(a\right) & \text{for } b=0\wedge n=0\\ \frac{xx^{2n-1}\cos\left(a\right)}{2n} & \text{for } b=0\\ \log\left(x\right)\cos\left(a+b\right) & \text{for } n=0\\ \frac{x^n\sin\left(a+bx^n\right)}{bn}+\frac{\cos\left(a+bx^n\right)}{b^2n} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+2*n)*cos(a+b*x**n),x)
```

output
```
Piecewise((log(x)*cos(a), Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)*cos(a)/(2*
n), Eq(b, 0)), (log(x)*cos(a + b), Eq(n, 0)), (x**n*sin(a + b*x**n)/(b*n)
+ cos(a + b*x**n)/(b**2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}\cos\left(a+bx^{n}\right)\,dx=\frac{bx^{n}\sin\left(bx^{n}+a\right)+\cos\left(bx^{n}+a\right)}{b^{2}n}$$

input
```
integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="maxima")
```

output
```
(b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Giac [F]

$$\int x^{-1+2n}\cos\left(a+bx^{n}\right)\,dx=\int x^{2\,n-1}\cos\left(bx^{n}+a\right)\,dx$$

input
```
integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)*cos(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\cos\left(a+bx^{n}\right)\,dx=\int x^{2\,n-1}\,\cos\left(a+b\,x^{n}\right)\,dx$$

input
```
int(x^(2*n - 1)*cos(a + b*x^n),x)
```

output
```
int(x^(2*n - 1)*cos(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}\cos\left(a+bx^{n}\right)\,dx=\frac{\cos\left(x^{n}b+a\right)+x^{n}\sin\left(x^{n}b+a\right)b}{b^{2}n}$$

input

```
int(x^(-1+2*n)*cos(a+b*x^n),x)
```

output

```
(cos(x**n*b + a) + x**n*sin(x**n*b + a)*b)/(b**2*n)
```

3.147 $\int x^{-1-n} \sin \left(a+b x^n\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int x^{-1-n} \sin \left(a+b x^n\right) \, dx=\frac{b \cos (a) \operatorname{CosIntegral}\left(b x^n\right)}{n}-\frac{x^{-n} \sin \left(a+b x^n\right)}{n}-\frac{b \sin (a) \operatorname{Si}\left(b x^n\right)}{n}$$

output
```
b*cos(a)*Ci(b*x^n)/n-sin(a+b*x^n)/n/(x^n)-b*sin(a)*Si(b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int x^{-1-n} \sin \left(a+b x^n\right) \, dx =\frac{x^{-n} \left(b x^n \cos (a) \operatorname{CosIntegral}\left(b x^n\right)-\sin \left(a+b x^n\right)-b x^n \sin (a) \operatorname{Si}\left(b x^n\right)\right)}{n}$$

input
```
Integrate[x^(-1 - n)*Sin[a + b*x^n],x]
```

output
```
(b*x^n*Cos[a]*CosIntegral[b*x^n] - Sin[a + b*x^n] - b*x^n*Sin[a]*SinIntegr
al[b*x^n])/(n*x^n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3860, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\sin\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{\int x^{-2n}\sin\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int x^{-2n}\sin\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{3778}$$

$$\frac{b\int x^{-n}\cos\left(bx^n+a\right)dx^n-x^{-n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int x^{-n}\sin\left(bx^n+a+\frac{\pi}{2}\right)dx^n-x^{-n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3784}$$

$$\frac{b\left(\cos(a)\int x^{-n}\cos\left(bx^n\right)dx^n-\sin(a)\int x^{-n}\sin\left(bx^n\right)dx^n\right)-x^{-n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\left(\cos(a)\int x^{-n}\sin\left(bx^n+\frac{\pi}{2}\right)dx^n-\sin(a)\int x^{-n}\sin\left(bx^n\right)dx^n\right)-x^{-n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3780}$$

$$\frac{b\left(\cos(a)\int x^{-n}\sin\left(bx^n+\frac{\pi}{2}\right)dx^n-\sin(a)\text{Si}(bx^n)\right)-x^{-n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3783}$$

$$\frac{b(\cos(a)\,\text{CosIntegral}\left(bx^n\right)-\sin(a)\text{Si}(bx^n))-x^{-n}\sin\left(a+bx^n\right)}{n}$$

input
```
Int[x^(-1 - n)*Sin[a + b*x^n],x]
```

output
```
(-(Sin[a + b*x^n]/x^n) + b*(Cos[a]*CosIntegral[b*x^n] - Sin[a]*SinIntegral
[b*x^n]))/n
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result
default	$\frac{b\left(-\frac{\sin\left(a+b\,x^{n}\right) x^{-n}}{b}-\operatorname{Si}\left(b\,x^{n}\right) \sin\left(a\right)+\operatorname{Ci}\left(b\,x^{n}\right) \cos\left(a\right)\right)}{n}$
risch	$-\frac{b\,{\mathrm e}^{i a} \operatorname{expIntegral}_{1}\left(-i b\,x^{n}\right)}{2 n}+\frac{i b\,{\mathrm e}^{-i a} \pi \,\operatorname{csgn}\left(b\,x^{n}\right)}{2 n}-\frac{i b\,{\mathrm e}^{-i a} \operatorname{Si}\left(b\,x^{n}\right)}{n}-\frac{b\,{\mathrm e}^{-i a} \operatorname{expIntegral}_{1}\left(-i b\,x^{n}\right)}{2 n}-\frac{\sin\left(a+b\,x^{n}\right) x^{-n}}{n}$

input

```
int(x^(-1-n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

output

```
1/n*b*(-sin(a+b*x^n)/b/(x^n)-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\frac{bx^{n}\cos\left(a\right)\operatorname{Ci}\left(bx^{n}\right)-bx^{n}\sin\left(a\right)\operatorname{Si}\left(bx^{n}\right)-\sin\left(bx^{n}+a\right)}{nx^{n}}$$

input

```
integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="fricas")
```

output

```
(b*x^n*cos(a)*cos_integral(b*x^n) - b*x^n*sin(a)*sin_integral(b*x^n) - sin
(b*x^n + a))/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\int x^{-n-1}\sin\left(a+bx^{n}\right)dx$$

input

```
integrate(x**(-1-n)*sin(a+b*x**n),x)
```

output

```
Integral(x**(-n - 1)*sin(a + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-n}\sin\left(a+bx^n\right)\,dx=\int x^{-n-1}\sin\left(bx^n+a\right)\,dx$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Giac [F]

$$\int x^{-1-n}\sin\left(a+bx^n\right)\,dx=\int x^{-n-1}\sin\left(bx^n+a\right)\,dx$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sin\left(a+bx^n\right)\,dx=\int \frac{\sin\left(a+b\,x^n\right)}{x^{n+1}}\,dx$$

input
```
int(sin(a + b*x^n)/x^(n + 1),x)
```

output
```
int(sin(a + b*x^n)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}\sin\left(a+bx^{n}\right)\,dx=\int \frac{\sin\left(x^{n}b+a\right)}{x^{n}x}dx$$

input

```
int(x^(-1-n)*sin(a+b*x^n),x)
```

output

```
int(sin(x**n*b + a)/(x**n*x),x)
```

3.148 $\int x^{-1-n}\sin^2(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx=-\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2a+2bx^n)}{2n}+\frac{b\,\text{CosIntegral}(2bx^n)\sin(2a)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}$$

output
```
-1/2/n/(x^n)+1/2*cos(2*a+2*b*x^n)/n/(x^n)+b*Ci(2*b*x^n)*sin(2*a)/n+b*cos(2
*a)*Si(2*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx=\frac{x^{-n}(-1+\cos(2(a+bx^n))+2bx^n\,\text{CosIntegral}(2bx^n)\sin(2a)+2bx^n\cos(2a)\text{Si}(2bx^n))}{2n}$$

input
```
Integrate[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

output
```
(-1 + Cos[2*(a + b*x^n)] + 2*b*x^n*CosIntegral[2*b*x^n]*Sin[2*a] + 2*b*x^n
*Cos[2*a]*SinIntegral[2*b*x^n])/(2*n*x^n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{3906, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\sin^2\left(a+bx^n\right)\,dx$$
$$\Big\downarrow \text{3906}$$
$$\int \left(\frac{x^{-n-1}}{2}-\frac{1}{2}x^{-n-1}\cos\left(2a+2bx^n\right)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{b\sin(2a)\,\text{CosIntegral}\left(2bx^n\right)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}+\frac{x^{-n}\cos\left(2(a+bx^n)\right)}{2n}-\frac{x^{-n}}{2n}$$

input
```
Int[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

output
```
-1/2*1/(n*x^n) + Cos[2*(a + b*x^n)]/(2*n*x^n) + (b*CosIntegral[2*b*x^n]*Si
n[2*a])/n + (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

method	result
default	$-\frac{x^{-n}}{2n}-\frac{b\left(-\frac{\cos\left(2a+2b\,x^{n}\right)x^{-n}}{2b}-\mathrm{Si}(2b\,x^{n})\cos(2a)-\mathrm{Ci}(2b\,x^{n})\sin(2a)\right)}{n}$
risch	$-\frac{(b\,\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(b\,x^{n})x^{n}+ib\,\mathrm{e}^{-2ia}\ \mathrm{expIntegral}_{1}(-2ib\,x^{n})x^{n}-ib\,\mathrm{e}^{2ia}\ \mathrm{expIntegral}_{1}(-2ib\,x^{n})x^{n}-2b\,\mathrm{e}^{-2ia}\ \mathrm{Si}(2b\,x^{n})x^{n}-\cos(2a+2b}{2n}$

input
```
int(x^(-1-n)*sin(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/n/(x^n)-1/n*b*(-1/2*cos(2*a+2*b*x^n)/b/(x^n)-Si(2*b*x^n)*cos(2*a)-Ci(
2*b*x^n)*sin(2*a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int x^{-1-n}\sin^{2}\left(a+bx^{n}\right)\,dx$$
$$=\frac{bx^{n}\,\mathrm{Ci}\left(2\,bx^{n}\right)\sin\left(2\,a\right)+bx^{n}\cos\left(2\,a\right)\mathrm{Si}\left(2\,bx^{n}\right)+\cos\left(bx^{n}+a\right)^{2}-1}{nx^{n}}$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
(b*x^n*cos_integral(2*b*x^n)*sin(2*a) + b*x^n*cos(2*a)*sin_integral(2*b*x^
n) + cos(b*x^n + a)^2 - 1)/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n} \sin^2\left(a+bx^n\right)\,dx = \int x^{-n-1} \sin^2\left(a+bx^n\right)dx$$

input
```
integrate(x**(-1-n)*sin(a+b*x**n)**2,x)
```

output
```
Integral(x**(-n - 1)*sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^{-1-n} \sin^2\left(a+bx^n\right)\,dx = \int x^{-n-1} \sin\left(bx^n+a\right)^2\,dx$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/2*(n*x^n*integrate(cos(2*b*x^n + 2*a)/(x*x^n), x) + 1)/(n*x^n)
```

Giac [F]

$$\int x^{-1-n} \sin^2\left(a+bx^n\right)\,dx = \int x^{-n-1} \sin\left(bx^n+a\right)^2\,dx$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sin^2\left(a+bx^n\right)\,dx=\int \frac{\sin\left(a+b\,x^n\right)^2}{x^{n+1}}\,dx$$

input
```
int(sin(a + b*x^n)^2/x^(n + 1),x)
```

output
```
int(sin(a + b*x^n)^2/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}\sin^2\left(a+bx^n\right)\,dx=\frac{x^n\left(\int\frac{\sin(x^nb+a)^2}{x^nx}dx\right)n-2x^n\left(\int\frac{1}{x^nx}dx\right)n-2}{x^nn}$$

input
```
int(x^(-1-n)*sin(a+b*x^n)^2,x)
```

output
```
(x**n*int(sin(x**n*b + a)**2/(x**n*x),x)*n - 2*x**n*int(1/(x**n*x),x)*n -
2)/(x**n*n)
```

3.149 $\int x^{-1-n} \sin^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 114

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\frac{3b\cos(a)\operatorname{CosIntegral}\left(bx^n\right)}{4n}-\frac{3b\cos(3a)\operatorname{CosIntegral}\left(3bx^n\right)}{4n}-\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3a+3bx^n\right)}{4n}-\frac{3b\sin(a)\operatorname{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\operatorname{Si}(3bx^n)}{4n}$$

output
```
3/4*b*cos(a)*Ci(b*x^n)/n-3/4*b*cos(3*a)*Ci(3*b*x^n)/n-3/4*sin(a+b*x^n)/n/(
x^n)+1/4*sin(3*a+3*b*x^n)/n/(x^n)-3/4*b*sin(a)*Si(b*x^n)/n+3/4*b*sin(3*a)*
Si(3*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{x^{-n}(3bx^n\cos(a)\operatorname{CosIntegral}\left(bx^n\right)-3bx^n\cos(3a)\operatorname{CosIntegral}\left(3bx^n\right)-3\sin\left(a+bx^n\right)+\sin\left(3(a+bx^n\right)}{4n}$$

input
```
Integrate[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

output
```
(3*b*x^n*Cos[a]*CosIntegral[b*x^n] - 3*b*x^n*Cos[3*a]*CosIntegral[3*b*x^n]
 - 3*Sin[a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b*x^n*Sin[a]*SinIntegral[b*x^
n] + 3*b*x^n*Sin[3*a]*SinIntegral[3*b*x^n])/(4*n*x^n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\sin^3\left(a+bx^n\right)dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{3}{4}x^{-n-1}\sin\left(a+bx^n\right)-\frac{1}{4}x^{-n-1}\sin\left(3a+3bx^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\cos(a)\operatorname{CosIntegral}\left(bx^n\right)}{4n}-\frac{3b\cos(3a)\operatorname{CosIntegral}\left(3bx^n\right)}{4n}-\frac{3b\sin(a)\operatorname{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\operatorname{Si}(3bx^n)}{4n}-\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3(a+bx^n)\right)}{4n}$$

input
```
Int[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

output
```
(3*b*Cos[a]*CosIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*CosIntegral[3*b*x^n]
)/(4*n) - (3*Sin[a + b*x^n])/(4*n*x^n) + Sin[3*(a + b*x^n)]/(4*n*x^n) - (3
*b*Sin[a]*SinIntegral[b*x^n])/(4*n) + (3*b*Sin[3*a]*SinIntegral[3*b*x^n])/
(4*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

method	result
default	$\frac{3b\left(-\frac{\sin(a+b\,x^{n})x^{-n}}{b}-\mathrm{Si}(b\,x^{n})\sin(a)+\mathrm{Ci}(b\,x^{n})\cos(a)\right)}{4n}-\frac{3b\left(-\frac{\sin(3a+3b\,x^{n})x^{-n}}{3b}-\mathrm{Si}(3b\,x^{n})\sin(3a)+\mathrm{Ci}(3b\,x^{n})\cos(3a)\right)}{4n}$
risch	$-\frac{(3ib\,\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(b\,x^{n})x^{n}-3ib\,\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(b\,x^{n})x^{n}-6ib\,\mathrm{e}^{-3ia}\ \mathrm{Si}(3b\,x^{n})x^{n}+6ib\,\mathrm{e}^{-ia}\ \mathrm{Si}(b\,x^{n})x^{n}-3b\,\mathrm{e}^{3ia}\ \mathrm{expIntegral}_{1}(-3ib\,x^{n})x}{}$

input
```
int(x^(-1-n)*sin(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
3/4/n*b*(-sin(a+b*x^n)/b/(x^n)-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))-3/4/n*b*
(-1/3*sin(3*a+3*b*x^n)/b/(x^n)-Si(3*b*x^n)*sin(3*a)+Ci(3*b*x^n)*cos(3*a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int x^{-1-n}\sin^{3}\left(a+bx^{n}\right)\,dx =$$
$$-\frac{3\,bx^{n}\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^{n}\right)-3\,bx^{n}\cos\left(a\right)\mathrm{Ci}\left(bx^{n}\right)-3\,bx^{n}\sin\left(3\,a\right)\mathrm{Si}\left(3\,bx^{n}\right)+3\,bx^{n}\sin\left(a\right)\mathrm{Si}\left(bx^{n}\right)-4}{4\,nx^{n}}$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/4*(3*b*x^n*cos(3*a)*cos_integral(3*b*x^n) - 3*b*x^n*cos(a)*cos_integral
(b*x^n) - 3*b*x^n*sin(3*a)*sin_integral(3*b*x^n) + 3*b*x^n*sin(a)*sin_inte
gral(b*x^n) - 4*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-n-1}\sin^3\left(a+bx^n\right)dx$$

input
```
integrate(x**(-1-n)*sin(a+b*x**n)**3,x)
```

output
```
Integral(x**(-n - 1)*sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-n-1}\sin\left(bx^n+a\right)^3\,dx$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-n-1}\sin\left(bx^n+a\right)^3\,dx$$

input
```
integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\int\frac{\sin\left(a+b\,x^n\right)^3}{x^{n+1}}\,dx$$

input
```
int(sin(a + b*x^n)^3/x^(n + 1),x)
```

output
```
int(sin(a + b*x^n)^3/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx=\text{Too large to display}$$

input
```
int(x^(-1-n)*sin(a+b*x^n)^3,x)
```

output
```
(2*( - 3*cos(x**n*b + a)*sin(x**n*b + a)*tan((x**n*b + a)/2)**6 - 9*cos(x*
*n*b + a)*sin(x**n*b + a)*tan((x**n*b + a)/2)**4 - 9*cos(x**n*b + a)*sin(x
**n*b + a)*tan((x**n*b + a)/2)**2 - 3*cos(x**n*b + a)*sin(x**n*b + a) + 20
*x**n*int(tan((x**n*b + a)/2)**3/(x**n*tan((x**n*b + a)/2)**6*x + 3*x**n*t
an((x**n*b + a)/2)**4*x + 3*x**n*tan((x**n*b + a)/2)**2*x + x**n*x),x)*tan
((x**n*b + a)/2)**6*n + 60*x**n*int(tan((x**n*b + a)/2)**3/(x**n*tan((x**n
*b + a)/2)**6*x + 3*x**n*tan((x**n*b + a)/2)**4*x + 3*x**n*tan((x**n*b + a
)/2)**2*x + x**n*x),x)*tan((x**n*b + a)/2)**4*n + 60*x**n*int(tan((x**n*b
+ a)/2)**3/(x**n*tan((x**n*b + a)/2)**6*x + 3*x**n*tan((x**n*b + a)/2)**4*
x + 3*x**n*tan((x**n*b + a)/2)**2*x + x**n*x),x)*tan((x**n*b + a)/2)**2*n
+ 20*x**n*int(tan((x**n*b + a)/2)**3/(x**n*tan((x**n*b + a)/2)**6*x + 3*x*
*n*tan((x**n*b + a)/2)**4*x + 3*x**n*tan((x**n*b + a)/2)**2*x + x**n*x),x)
*n - sin(x**n*b + a)**3*tan((x**n*b + a)/2)**6 - 3*sin(x**n*b + a)**3*tan(
(x**n*b + a)/2)**4 - 3*sin(x**n*b + a)**3*tan((x**n*b + a)/2)**2 - sin(x**
n*b + a)**3 - 3*sin(x**n*b + a)*tan((x**n*b + a)/2)**6 - 9*sin(x**n*b + a)
*tan((x**n*b + a)/2)**4 - 9*sin(x**n*b + a)*tan((x**n*b + a)/2)**2 - 3*sin
(x**n*b + a) + 20*tan((x**n*b + a)/2)**3 + 12*tan((x**n*b + a)/2)))/(5*x**
n*n*(tan((x**n*b + a)/2)**6 + 3*tan((x**n*b + a)/2)**4 + 3*tan((x**n*b + a
)/2)**2 + 1))
```

3.150 $\int x^{-1-2n} \sin\left(a+bx^n\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\begin{aligned}\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx=&-\frac{bx^{-n}\cos\left(a+bx^n\right)}{2n}-\frac{b^2\,\mathrm{CosIntegral}\left(bx^n\right)\sin(a)}{2n}\\&-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}-\frac{b^2\cos(a)\mathrm{Si}(bx^n)}{2n}\end{aligned}$$

output
```
-1/2*b*cos(a+b*x^n)/n/(x^n)-1/2*b^2*Ci(b*x^n)*sin(a)/n-1/2*sin(a+b*x^n)/n/
(x^(2*n))-1/2*b^2*cos(a)*Si(b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx=\\&-\frac{x^{-2n}\left(bx^n\cos\left(a+bx^n\right)+b^2x^{2n}\,\mathrm{CosIntegral}\left(bx^n\right)\sin(a)+\sin\left(a+bx^n\right)+b^2x^{2n}\cos(a)\mathrm{Si}(bx^n)\right)}{2n}\end{aligned}$$

input
```
Integrate[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

output

```
-1/2*(b*x^n*Cos[a + b*x^n] + b^2*x^(2*n)*CosIntegral[b*x^n]*Sin[a] + Sin[a
 + b*x^n] + b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n])/(n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {3860, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1} \sin\left(a+bx^n\right) dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{\int x^{-3n} \sin\left(bx^n+a\right) dx^n}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int x^{-3n} \sin\left(bx^n+a\right) dx^n}{n}$$

$$\Big\downarrow \text{3778}$$

$$\frac{\frac{1}{2}b\int x^{-2n} \cos\left(bx^n+a\right) dx^n - \frac{1}{2}x^{-2n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}b\int x^{-2n} \sin\left(bx^n+a+\frac{\pi}{2}\right) dx^n - \frac{1}{2}x^{-2n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3778}$$

$$\frac{\frac{1}{2}b\left(b\int -x^{-n} \sin\left(bx^n+a\right) dx^n - x^{-n}\cos\left(a+bx^n\right)\right) - \frac{1}{2}x^{-2n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}b\left(x^{-n}\left(-\cos\left(a+bx^n\right)\right) - b\int x^{-n} \sin\left(bx^n+a\right) dx^n\right) - \frac{1}{2}x^{-2n}\sin\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}b\left(x^{-n}(-\cos (a+bx^n))-b\int x^{-n}\sin (bx^n+a)\,dx^n\right)-\frac{1}{2}x^{-2n}\sin (a+bx^n)}{n}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\frac{1}{2}b\left(x^{-n}(-\cos (a+bx^n))-b\left(\sin(a)\int x^{-n}\cos (bx^n)\,dx^n+\cos(a)\int x^{-n}\sin (bx^n)\,dx^n\right)\right)-\frac{1}{2}x^{-2n}\sin (a+bx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}b\left(x^{-n}(-\cos (a+bx^n))-b\left(\sin(a)\int x^{-n}\sin \left(bx^n+\frac{\pi}{2}\right)dx^n+\cos(a)\int x^{-n}\sin (bx^n)\,dx^n\right)\right)-\frac{1}{2}x^{-2n}\sin (a+bx}{n}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\frac{1}{2}b\left(x^{-n}(-\cos (a+bx^n))-b\left(\sin(a)\int x^{-n}\sin \left(bx^n+\frac{\pi}{2}\right)dx^n+\cos(a)\text{Si}(bx^n)\right)\right)-\frac{1}{2}x^{-2n}\sin (a+bx^n)}{n}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\frac{1}{2}b(x^{-n}(-\cos (a+bx^n))-b(\sin(a)\,\text{CosIntegral}\,(bx^n)+\cos(a)\text{Si}(bx^n)))-\frac{1}{2}x^{-2n}\sin (a+bx^n)}{n}$$

input

```
Int[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

output

```
(-1/2*Sin[a + b*x^n]/x^(2*n) + (b*(-(Cos[a + b*x^n]/x^n) - b*(CosIntegral[
b*x^n]*Sin[a] + Cos[a]*SinIntegral[b*x^n])))/2)/n
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
default	$\frac{b^{2} \left(-\frac{\sin \left(a +b \,x^{n}\right) x^{-2 n}}{2 b^{2}}-\frac{\cos \left(a +b \,x^{n}\right) x^{-n}}{2 b}-\frac{\operatorname{Si}\left(b \,x^{n}\right) \cos \left(a \right)}{2}-\frac{\operatorname{Ci}\left(b \,x^{n}\right) \sin \left(a \right)}{2}\right)}{n}$
risch	$-\frac{(-b^{2} {\mathrm e}^{-i a} \pi \,\operatorname{csgn}(b \,x^{n}) x^{2 n}-i b^{2} {\mathrm e}^{-i a} \operatorname{expIntegral}_{1}(-i b \,x^{n}) x^{2 n}+i b^{2} {\mathrm e}^{i a} \operatorname{expIntegral}_{1}(-i b \,x^{n}) x^{2 n}+2 b^{2} {\mathrm e}^{-i a} \operatorname{Si}(b \,x^{n}) x^{2 n}+2 x^{n} \cos(}{4 n}$
meijerg	$\frac{b^{2} \sqrt{\pi}\, \left(-\frac{x^{2 \left(\frac{-1-2 n}{2 n}+\frac{1}{2 n}\right) n} 2^{-\frac{-1-2 n}{n}-\frac{1}{n}}}{\sqrt{\pi}\, b^{2}}+\frac{\left(-1\right)^{-\frac{-1-2 n}{2 n}-\frac{1}{2 n}} \left(-\Psi \left(1-\frac{-1-2 n}{2 n}-\frac{1}{2 n}\right)-\Psi \left(\frac{1}{2}-\frac{-1-2 n}{2 n}-\frac{1}{2 n}\right)+2 n \ln \left(x \right)-2 \ln \left(2\right)+\ln \left(b^{2}\right)\right) \sqrt{}}{2 \sqrt{\pi}\, \Gamma \left(-\frac{-1-2 n}{n}-\frac{1}{n}\right)}\right.}{}$

input
```
int(x^(-1-2*n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*b^2*(-1/2*sin(a+b*x^n)/b^2/(x^n)^2-1/2*cos(a+b*x^n)/b/(x^n)-1/2*Si(b*x
^n)*cos(a)-1/2*Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx = -\frac{b^2x^{2\,n}\operatorname{Ci}\left(bx^n\right)\sin\left(a\right)+b^2x^{2\,n}\cos\left(a\right)\operatorname{Si}\left(bx^n\right)+bx^n\cos\left(bx^n+a\right)+\sin\left(bx^n+a\right)}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^(2*n)*cos_integral(b*x^n)*sin(a) + b^2*x^(2*n)*cos(a)*sin_inte
gral(b*x^n) + b*x^n*cos(b*x^n + a) + sin(b*x^n + a))/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx = \int x^{-2n-1}\sin\left(a+bx^n\right)dx$$

input
```
integrate(x**(-1-2*n)*sin(a+b*x**n),x)
```

output
```
Integral(x**(-2*n - 1)*sin(a + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)\,dx$$

input
```
integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Giac [F]

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)\,dx$$

input
```
integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx=\int\frac{\sin\left(a+b\,x^n\right)}{x^{2\,n+1}}\,dx$$

input
```
int(sin(a + b*x^n)/x^(2*n + 1),x)
```

output
```
int(sin(a + b*x^n)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}\sin\left(a+bx^{n}\right)\,dx = \frac{-x^{n}\cos\left(x^{n}b+a\right)b-x^{2n}\left(\int\frac{\sin(x^{n}b+a)}{x}dx\right)b^{2}n-\sin\left(x^{n}b+a\right)}{2x^{2n}n}$$

input

```
int(x^(-1-2*n)*sin(a+b*x^n),x)
```

output

```
( - (x**n*cos(x**n*b + a)*b + x**(2*n)*int(sin(x**n*b + a)/x,x)*b**2*n + s
in(x**n*b + a)))/(2*x**(2*n)*n)
```

3.151 $\int x^{-1-2n} \sin^2\left(a+bx^n\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 97

$$\begin{aligned}\int x^{-1-2n} \sin^2\left(a+bx^n\right) \, dx = & -\frac{x^{-2n}}{4n}+\frac{x^{-2n} \cos\left(2a+2bx^n\right)}{4n} \\ & +\frac{b^2 \cos(2a) \operatorname{CosIntegral}\left(2bx^n\right)}{n} \\ & -\frac{bx^{-n} \sin\left(2a+2bx^n\right)}{2n}-\frac{b^2 \sin(2a) \operatorname{Si}(2bx^n)}{n}\end{aligned}$$

output
```
-1/4/n/(x^(2*n))+1/4*cos(2*a+2*b*x^n)/n/(x^(2*n))+b^2*cos(2*a)*Ci(2*b*x^n)
/n-1/2*b*sin(2*a+2*b*x^n)/n/(x^n)-b^2*sin(2*a)*Si(2*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\begin{aligned}&\int x^{-1-2n} \sin^2\left(a+bx^n\right) \, dx \\ &= \frac{x^{-2n}\left(-1+\cos\left(2(a+bx^n)\right)+4b^2x^{2n}\cos(2a)\operatorname{CosIntegral}\left(2bx^n\right)-2bx^n\sin\left(2(a+bx^n)\right)-4b^2x^{2n}\sin(2a\right.}{4n}\end{aligned}$$

input
```
Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

output
```
(-1 + Cos[2*(a + b*x^n)] + 4*b^2*x^(2*n)*Cos[2*a]*CosIntegral[2*b*x^n] - 2
*b*x^n*Sin[2*(a + b*x^n)] - 4*b^2*x^(2*n)*Sin[2*a]*SinIntegral[2*b*x^n])/(
4*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}\sin^2\left(a+bx^n\right)dx$$

$$\Big\downarrow \text{3906}$$

$$\int \left(\frac{1}{2}x^{-2n-1}-\frac{1}{2}x^{-2n-1}\cos\left(2a+2bx^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)}{n}-\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}-\frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n}+\frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n}-\frac{x^{-2n}}{4n}$$

input
```
Int[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

output
```
-1/4*1/(n*x^(2*n)) + Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) + (b^2*Cos[2*a]*CosI
ntegral[2*b*x^n])/n - (b*Sin[2*(a + b*x^n)])/(2*n*x^n) - (b^2*Sin[2*a]*Sin
Integral[2*b*x^n])/n
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

method	result
default	$-\frac{x^{-2n}}{4n}-\frac{2b^{2}\left(-\frac{x^{-2n}\cos\left(2a+2b\,x^{n}\right)}{8b^{2}}+\frac{\sin\left(2a+2b\,x^{n}\right)x^{-n}}{4b}+\frac{\operatorname{Si}\left(2b\,x^{n}\right)\sin\left(2a\right)}{2}-\frac{\operatorname{Ci}\left(2b\,x^{n}\right)\cos\left(2a\right)}{2}\right)}{n}$
risch	$\frac{(2ib^{2}\mathrm{e}^{-2ia}\pi\ \operatorname{csgn}(b\,x^{n})x^{2n}-4ib^{2}\mathrm{e}^{-2ia}\ \operatorname{Si}(2b\,x^{n})x^{2n}-2b^{2}\mathrm{e}^{2ia}\ \operatorname{expIntegral}_{1}(-2ib\,x^{n})x^{2n}-2b^{2}\mathrm{e}^{-2ia}\ \operatorname{expIntegral}_{1}(-2ib\,x^{n})x^{2n}-2\,\mathrm{s}}{4n}$

input

```
int(x^(-1-2*n)*sin(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/4/n/(x^n)^2-2/n*b^2*(-1/8/b^2/(x^n)^2*cos(2*a+2*b*x^n)+1/4*sin(2*a+2*b*
x^n)/b/(x^n)+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int x^{-1-2n}\sin^{2}\left(a+bx^{n}\right)\,dx$$
$$=\frac{2\,b^{2}x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^{n}\right)-2\,b^{2}x^{2\,n}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^{n}\right)-2\,bx^{n}\cos\left(bx^{n}+a\right)\sin\left(bx^{n}+a\right)+\cos\left(bx^{n}+a\right.}{2\,nx^{2\,n}}$$

input

```
integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^(2*n)*cos(2*a)*cos_integral(2*b*x^n) - 2*b^2*x^(2*n)*sin(2*a)
*sin_integral(2*b*x^n) - 2*b*x^n*cos(b*x^n + a)*sin(b*x^n + a) + cos(b*x^n
 + a)^2 - 1)/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\int x^{-2n-1}\sin^2\left(a+bx^n\right)dx$$

input
```
integrate(x**(-1-2*n)*sin(a+b*x**n)**2,x)
```

output
```
Integral(x**(-2*n - 1)*sin(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)^2\,dx$$

input
```
integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*n*x^(2*n)*integrate(cos(2*b*x^n + 2*a)/(x*x^(2*n)), x) + 1)/(n*x^(
2*n))
```

Giac [F]

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)^2\,dx$$

input
```
integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)*sin(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\int \frac{\sin\left(a+b\,x^n\right)^2}{x^{2\,n+1}}\,dx$$

input
```
int(sin(a + b*x^n)^2/x^(2*n + 1),x)
```

output
```
int(sin(a + b*x^n)^2/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx=\text{Too large to display}$$

input
```
int(x^(-1-2*n)*sin(a+b*x^n)^2,x)
```

output

```
(2*x**n*cos(x**n*b + a)*sin(x**n*b + a)*tan((x**n*b + a)/2)**4*b + 4*x**n*
cos(x**n*b + a)*sin(x**n*b + a)*tan((x**n*b + a)/2)**2*b + 2*x**n*cos(x**n
*b + a)*sin(x**n*b + a)*b - 8*cos(x**n*b + a)*tan((x**n*b + a)/2)**4 - 16*
cos(x**n*b + a)*tan((x**n*b + a)/2)**2 - 8*cos(x**n*b + a) + 32*x**(2*n)*i
nt(tan((x**n*b + a)/2)/(x**n*tan((x**n*b + a)/2)**4*x + 2*x**n*tan((x**n*b
 + a)/2)**2*x + x**n*x),x)*tan((x**n*b + a)/2)**4*b*n + 64*x**(2*n)*int(ta
n((x**n*b + a)/2)/(x**n*tan((x**n*b + a)/2)**4*x + 2*x**n*tan((x**n*b + a)
/2)**2*x + x**n*x),x)*tan((x**n*b + a)/2)**2*b*n + 32*x**(2*n)*int(tan((x*
*n*b + a)/2)/(x**n*tan((x**n*b + a)/2)**4*x + 2*x**n*tan((x**n*b + a)/2)**
2*x + x**n*x),x)*b*n - 16*x**(2*n)*int(1/(tan((x**n*b + a)/2)**4*x + 2*tan
((x**n*b + a)/2)**2*x + x),x)*tan((x**n*b + a)/2)**4*b**2*n - 32*x**(2*n)*
int(1/(tan((x**n*b + a)/2)**4*x + 2*tan((x**n*b + a)/2)**2*x + x),x)*tan((
x**n*b + a)/2)**2*b**2*n - 16*x**(2*n)*int(1/(tan((x**n*b + a)/2)**4*x + 2
*tan((x**n*b + a)/2)**2*x + x),x)*b**2*n + 6*x**(2*n)*log(x)*tan((x**n*b +
 a)/2)**4*b**2*n + 12*x**(2*n)*log(x)*tan((x**n*b + a)/2)**2*b**2*n + 6*x*
*(2*n)*log(x)*b**2*n + 8*x**n*sin(x**n*b + a)*tan((x**n*b + a)/2)**4*b + 1
6*x**n*sin(x**n*b + a)*tan((x**n*b + a)/2)**2*b + 8*x**n*sin(x**n*b + a)*b
 + sin(x**n*b + a)**2*tan((x**n*b + a)/2)**4 + 2*sin(x**n*b + a)**2*tan((x
**n*b + a)/2)**2 + sin(x**n*b + a)**2 - 8*tan((x**n*b + a)/2)**4 - 16*tan(
(x**n*b + a)/2)**2 + 8)/(6*x**(2*n)*n*(tan((x**n*b + a)/2)**4 + 2*tan((...
```

3.152 $\int x^{-1-2n} \sin^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 167

$$
\begin{aligned}
\int x^{-1-2n} \sin^3(a+bx^n)\,dx = {} & -\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3a+3bx^n)}{8n} \\
& -\frac{3b^2\operatorname{CosIntegral}(bx^n)\sin(a)}{8n} \\
& +\frac{9b^2\operatorname{CosIntegral}(3bx^n)\sin(3a)}{8n} \\
& -\frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3a+3bx^n)}{8n} \\
& -\frac{3b^2\cos(a)\operatorname{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\operatorname{Si}(3bx^n)}{8n}
\end{aligned}
$$

output

```
-3/8*b*cos(a+b*x^n)/n/(x^n)+3/8*b*cos(3*a+3*b*x^n)/n/(x^n)-3/8*b^2*Ci(b*x^
n)*sin(a)/n+9/8*b^2*Ci(3*b*x^n)*sin(3*a)/n-3/8*sin(a+b*x^n)/n/(x^(2*n))+1/
8*sin(3*a+3*b*x^n)/n/(x^(2*n))-3/8*b^2*cos(a)*Si(b*x^n)/n+9/8*b^2*cos(3*a)
*Si(3*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{x^{-2n}(-3bx^n\cos\left(a+bx^n\right)+3bx^n\cos\left(3(a+bx^n)\right)-3b^2x^{2n}\,\text{CosIntegral}\left(bx^n\right)\sin(a)+9b^2x^{2n}\,\text{CosIntegra}}{8n}$$

input
```
Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

output
```
(-3*b*x^n*Cos[a + b*x^n] + 3*b*x^n*Cos[3*(a + b*x^n)] - 3*b^2*x^(2*n)*CosI
ntegral[b*x^n]*Sin[a] + 9*b^2*x^(2*n)*CosIntegral[3*b*x^n]*Sin[3*a] - 3*Si
n[a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n
] + 9*b^2*x^(2*n)*Cos[3*a]*SinIntegral[3*b*x^n])/(8*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}\sin^3\left(a+bx^n\right)\,dx$$
$$\Big\downarrow \text{3906}$$
$$\int \left(\frac{3}{4}x^{-2n-1}\sin\left(a+bx^n\right)-\frac{1}{4}x^{-2n-1}\sin\left(3a+3bx^n\right)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3b^2\sin(a)\,\text{CosIntegral}\left(bx^n\right)}{8n}+\frac{9b^2\sin(3a)\,\text{CosIntegral}\left(3bx^n\right)}{8n}-\frac{3b^2\cos(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n}-\frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n}+\frac{x^{-2n}\sin\left(3(a+bx^n)\right)}{8n}-\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\cos\left(3(a+bx^n)\right)}{8n}$$

input
```
Int[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

output
```
(-3*b*Cos[a + b*x^n])/(8*n*x^n) + (3*b*Cos[3*(a + b*x^n)])/(8*n*x^n) - (3*
b^2*CosIntegral[b*x^n]*Sin[a])/(8*n) + (9*b^2*CosIntegral[3*b*x^n]*Sin[3*a
])/(8*n) - (3*Sin[a + b*x^n])/(8*n*x^(2*n)) + Sin[3*(a + b*x^n)]/(8*n*x^(2
*n)) - (3*b^2*Cos[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Cos[3*a]*SinIntegr
al[3*b*x^n])/(8*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

method	result
default	$\frac{3b^2\left(-\frac{\sin(a+b\,x^n)x^{-2n}}{2b^2}-\frac{\cos(a+b\,x^n)x^{-n}}{2b}-\frac{\mathrm{Si}(b\,x^n)\cos(a)}{2}-\frac{\mathrm{Ci}(b\,x^n)\sin(a)}{2}\right)}{4n}-\frac{9b^2\left(-\frac{\sin(3a+3b\,x^n)x^{-2n}}{18b^2}-\frac{\cos(3a+3b\,x^n)x^{-n}}{6b}-\mathrm{Si}(3\right.}{4n}$
risch	$-\frac{(-3ib^2\mathrm{e}^{-ia}\,\mathrm{expIntegral}_1(-ib\,x^n)x^{2n}-3b^2\mathrm{e}^{-ia}\pi\,\mathrm{csgn}(b\,x^n)x^{2n}-9ib^2\mathrm{e}^{3ia}\,\mathrm{expIntegral}_1(-3ib\,x^n)x^{2n}+9ib^2\mathrm{e}^{-3ia}\,\mathrm{expIntegral}_1(-}{}$

input
```
int(x^(-1-2*n)*sin(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
3/4/n*b^2*(-1/2*sin(a+b*x^n)/b^2/(x^n)^2-1/2*cos(a+b*x^n)/b/(x^n)-1/2*Si(b
*x^n)*cos(a)-1/2*Ci(b*x^n)*sin(a))-9/4/n*b^2*(-1/18*sin(3*a+3*b*x^n)/b^2/(
x^n)^2-1/6*cos(3*a+3*b*x^n)/b/(x^n)-1/2*Si(3*b*x^n)*cos(3*a)-1/2*Ci(3*b*x^
n)*sin(3*a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx$$
$$=\frac{12\,bx^n\cos\left(bx^n+a\right)^3+9\,b^2x^{2\,n}\operatorname{Ci}\left(3\,bx^n\right)\sin\left(3\,a\right)-3\,b^2x^{2\,n}\operatorname{Ci}\left(bx^n\right)\sin\left(a\right)+9\,b^2x^{2\,n}\cos\left(3\,a\right)\operatorname{Si}\left(3\,bx^n\right)}{8\,nx^{2\,n}}$$

input

```
integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

output

```
1/8*(12*b*x^n*cos(b*x^n + a)^3 + 9*b^2*x^(2*n)*cos_integral(3*b*x^n)*sin(3
*a) - 3*b^2*x^(2*n)*cos_integral(b*x^n)*sin(a) + 9*b^2*x^(2*n)*cos(3*a)*si
n_integral(3*b*x^n) - 3*b^2*x^(2*n)*cos(a)*sin_integral(b*x^n) - 12*b*x^n*
cos(b*x^n + a) + 4*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-2n-1}\sin^3\left(a+bx^n\right)dx$$

input

```
integrate(x**(-1-2*n)*sin(a+b*x**n)**3,x)
```

output

```
Integral(x**(-2*n - 1)*sin(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)^3\,dx$$

input

```
integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="maxima")
```

output

```
integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int x^{-2\,n-1}\sin\left(bx^n+a\right)^3\,dx$$

input
```
integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\int\frac{\sin\left(a+b\,x^n\right)^3}{x^{2\,n+1}}\,dx$$

input
```
int(sin(a + b*x^n)^3/x^(2*n + 1),x)
```

output
```
int(sin(a + b*x^n)^3/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx=\text{Too large to display}$$

input
```
int(x^(-1-2*n)*sin(a+b*x^n)^3,x)
```

output

```
( - 9*cos(x**n*b + a)*sin(x**n*b + a)*tan((x**n*b + a)/2)**6 - 27*cos(x**n
*b + a)*sin(x**n*b + a)*tan((x**n*b + a)/2)**4 - 27*cos(x**n*b + a)*sin(x*
*n*b + a)*tan((x**n*b + a)/2)**2 - 9*cos(x**n*b + a)*sin(x**n*b + a) + 160
*x**(2*n)*int(tan((x**n*b + a)/2)**3/(x**(2*n)*tan((x**n*b + a)/2)**6*x +
3*x**(2*n)*tan((x**n*b + a)/2)**4*x + 3*x**(2*n)*tan((x**n*b + a)/2)**2*x
+ x**(2*n)*x),x)*tan((x**n*b + a)/2)**6*n + 480*x**(2*n)*int(tan((x**n*b +
 a)/2)**3/(x**(2*n)*tan((x**n*b + a)/2)**6*x + 3*x**(2*n)*tan((x**n*b + a)
/2)**4*x + 3*x**(2*n)*tan((x**n*b + a)/2)**2*x + x**(2*n)*x),x)*tan((x**n*
b + a)/2)**4*n + 480*x**(2*n)*int(tan((x**n*b + a)/2)**3/(x**(2*n)*tan((x*
*n*b + a)/2)**6*x + 3*x**(2*n)*tan((x**n*b + a)/2)**4*x + 3*x**(2*n)*tan((
x**n*b + a)/2)**2*x + x**(2*n)*x),x)*tan((x**n*b + a)/2)**2*n + 160*x**(2*
n)*int(tan((x**n*b + a)/2)**3/(x**(2*n)*tan((x**n*b + a)/2)**6*x + 3*x**(2
*n)*tan((x**n*b + a)/2)**4*x + 3*x**(2*n)*tan((x**n*b + a)/2)**2*x + x**(2
*n)*x),x)*n + 2*sin(x**n*b + a)**3*tan((x**n*b + a)/2)**6 + 6*sin(x**n*b +
 a)**3*tan((x**n*b + a)/2)**4 + 6*sin(x**n*b + a)**3*tan((x**n*b + a)/2)**
2 + 2*sin(x**n*b + a)**3 - 24*sin(x**n*b + a)*tan((x**n*b + a)/2)**6 - 72*
sin(x**n*b + a)*tan((x**n*b + a)/2)**4 - 72*sin(x**n*b + a)*tan((x**n*b +
a)/2)**2 - 24*sin(x**n*b + a) + 30*tan((x**n*b + a)/2)**5 + 80*tan((x**n*b
 + a)/2)**3 + 66*tan((x**n*b + a)/2))/(20*x**(2*n)*n*(tan((x**n*b + a)/2)*
*6 + 3*tan((x**n*b + a)/2)**4 + 3*tan((x**n*b + a)/2)**2 + 1))
```

3.153 $\int (e+fx)^3 \sin \left(b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 223

$$\begin{aligned}\int (e+fx)^3 \sin \left(b(c+dx)^2\right) \, dx = & -\frac{3f(de-cf)^2 \cos \left(b(c+dx)^2\right)}{2bd^4} \\ & -\frac{3f^2(de-cf)(c+dx) \cos \left(b(c+dx)^2\right)}{2bd^4} \\ & -\frac{f^3(c+dx)^2 \cos \left(b(c+dx)^2\right)}{2bd^4} \\ & +\frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} \\ & +\frac{(de-cf)^3\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} \\ & +\frac{f^3 \sin \left(b(c+dx)^2\right)}{2b^2d^4}\end{aligned}$$

output

```
-3/2*f*(-c*f+d*e)^2*cos(b*(d*x+c)^2)/b/d^4-3/2*f^2*(-c*f+d*e)*(d*x+c)*cos(
b*(d*x+c)^2)/b/d^4-1/2*f^3*(d*x+c)^2*cos(b*(d*x+c)^2)/b/d^4+3/4*f^2*(-c*f+
d*e)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(3/2)/d
^4+1/2*(-c*f+d*e)^3*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c))/b^(1/2)/d^4+1/2*f^3*sin(b*(d*x+c)^2)/b^2/d^4
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int (e+fx)^3\sin \left(b(c+dx)^2\right)\,dx$$
$$=\frac{-4bf(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2))\cos \left(b(c+dx)^2\right)-6\sqrt{b}f^2(-de+cf)\sqrt{2\pi}\,\mathrm{FresnelC}}{8b^2d^4}$$

input
```
Integrate[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

output
```
(-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Co
s[b*(c + d*x)^2] - 6*Sqrt[b]*f^2*(-(d*e) + c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)] + 4*b^(3/2)*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelS[Sqrt
[b]*Sqrt[2/Pi]*(c + d*x)] + 4*f^3*Sin[b*(c + d*x)^2])/(8*b^2*d^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3\sin \left(b(c+dx)^2\right)\,dx$$

$$\Big\downarrow \; 3914$$

$$\frac{\int \left(\sin \left(b(c+dx)^2\right)(de-cf)^3+3f(c+dx)\sin \left(b(c+dx)^2\right)(de-cf)^2+3f^2(c+dx)^2\sin \left(b(c+dx)^2\right)(de-cf)}{d^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\sqrt{\frac{\pi}{2}}f^2(de-cf)\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}}+\frac{f^3\sin\left(b(c+dx)^2\right)}{2b^2}-\frac{3f^2(c+dx)(de-cf)\cos\left(b(c+dx)^2\right)}{2b}+\frac{\sqrt{\frac{\pi}{2}}(de-cf)^3\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+d}{\sqrt{b}}}{d^4}$$

input

```
Int[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

output

```
((-3*f*(d*e - c*f)^2*Cos[b*(c + d*x)^2])/(2*b) - (3*f^2*(d*e - c*f)*(c + d
*x)*Cos[b*(c + d*x)^2])/(2*b) - (f^3*(c + d*x)^2*Cos[b*(c + d*x)^2])/(2*b)
 + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(
2*b^(3/2)) + ((d*e - c*f)^3*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*
x)])/Sqrt[b] + (f^3*Sin[b*(c + d*x)^2])/(2*b^2))/d^4
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(194) = 388$.

Time = 1.77 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.63

method	result
default	$-\frac{f^3x^2\cos\left(x^2d^2b+2cdxb+b\,c^2\right)}{2d^2b}-\frac{f^3c\left(-\frac{x\cos\left(x^2d^2b+2cdxb+b\,c^2\right)}{2d^2b}-\frac{c\left(-\frac{\cos\left(x^2d^2b+2cdxb+b\,c^2\right)}{2d^2b}-\frac{c\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\left(b\,d^2x+cdb\right.}{\sqrt{\pi}\,\sqrt{d^2b}}\right.}{2d\sqrt{d^2b}}\right.}{d}\right.}{d}$
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^3}{4\sqrt{-ib}\,d}-\frac{if^3c^3\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^4\sqrt{-ib}}+\frac{3f^3c\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8d^4b\sqrt{-ib}}-\frac{3ie^2fc\sqrt{\pi}\ \mathrm{erf}\left(-d\surd\right.}{4d^2\surd-}$
parts	Expression too large to display

input
```
int((f*x+e)^3*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^3/d^2/b*x^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^3*c/d*(-1/2/d^2/b*x*co
s(b*d^2*x^2+2*b*c*d*x+b*c^2)-c/d*(-1/2/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2
)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)
^(1/2)*(b*d^2*x+b*c*d)))+1/4/d^2/b*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelC
(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))+f^3/d^2/b*(1/2/d^2/b*sin
(b*d^2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))-3/2*e*f^2/d^2/b*x*cos(b
*d^2*x^2+2*b*c*d*x+b*c^2)-3*e*f^2*c/d*(-1/2/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+
b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d
^2*b)^(1/2)*(b*d^2*x+b*c*d)))+3/4*e*f^2/d^2/b*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/
2)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-3/2*e^2*f/d^2/
b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-3/2*e^2*f*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1
/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*P
i^(1/2)/(d^2*b)^(1/2)*e^3*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x
+b*c*d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.14

$$\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx$$

$$=\frac{2\,df^3\sin\left(bd^2x^2+2\,bcdx+bc^2\right)+3\,\sqrt{2}\pi(def^2-cf^3)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+2\,\sqrt{2}\pi(bd^3e^3-3\,bcd^2e^2}{}$$

input
```
integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2) + 3*sqrt(2)*pi*(d*e*f^2 -
c*f^3)*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) + 2*
sqrt(2)*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqr
t(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3
*x^2 + 3*b*d^3*e^2*f - 3*b*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*
c*d^2*f^3)*x)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^5)
```

Sympy [F]

$$\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx=\int (e+fx)^3\sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

input
```
integrate((f*x+e)**3*sin(b*(d*x+c)**2),x)
```

output
```
Integral((e + f*x)**3*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 974, normalized size of antiderivative = 4.37

$$\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*sqrt(pi)*e^3*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I -
1)*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d) - 3/8*(2*d*x*(e^(I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
- sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt
(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(er
f(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*e^
2*f/(b*d^3*x + b*c*d^2) + 3/8*(4*b*c*d*x*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 4*b*c^2*(e^(I*b*d^2*
x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
 sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt
(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(er
f(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2 - (I - 1)*sqrt(2
)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(
3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*e*f^2/(b^2*d^4*x + b^2*c*d^3)
 - 1/8*(6*b*c^3*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2
 - 2*I*b*c*d*x - I*b*c^2)) + 2*(3*b*c^2*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - I*gamma(2, I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) + I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c
^2))*d*x + 2*c*(-I*gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*ga...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.28

$$\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx =$$

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3-3i\,def^2+3i\,cf^3\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-3\,bd^2e}{}}{8\,d^3}$$

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3+3i\,def^2-3i\,cf^3\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-3\,b}{}}{8\,d^3}$$

input
```
integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(I*sqrt(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2
- 2*b*c^3*f^3 - 3*I*d*e*f^2 + 3*I*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b
*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1
)*b) + 2*I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c/d) - 3*b*c
*d*f^3*(-I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I*b*c^2*f^3
- f^3)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)/(b^2*d))/d^3 - 1/8*(-I*sqr
t(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2 - 2*b*c^3*f
^3 + 3*I*d*e*f^2 - 3*I*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(
b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*
I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c/d) - 3*b*c*d*f^3*(-
I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I*b*c^2*f^3 + f^3)*e^
(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b^2*d))/d^3
```

Mupad [B] (verification not implemented)

Time = 39.76 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int (e+fx)^3\sin\left(b(c+dx)^2\right)\,dx\\
&=\frac{f^3\sin\left(b\,(c+d\,x)^2\right)}{2\,b^2\,d^4}-\frac{\cos\left(b\,(c+d\,x)^2\right)\,\left(c^2\,f^3-3\,c\,d\,e\,f^2+3\,d^2\,e^2\,f\right)}{2\,b\,d^4}\\
&\quad-\frac{f^3\,x^2\,\cos\left(b\,(c+d\,x)^2\right)}{2\,b\,d^2}+\frac{x\,\cos\left(b\,(c+d\,x)^2\right)\,\left(c\,f^3-3\,d\,e\,f^2\right)}{2\,b\,d^3}\\
&\quad-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\left(c^3\,f^3-3\,c^2\,d\,e\,f^2+3\,c\,d^2\,e^2\,f-d^3\,e^3\right)}{2\,\sqrt{b}\,d^4}\\
&\quad-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\left(3\,c\,f^3-3\,d\,e\,f^2\right)}{4\,b^{3/2}\,d^4}
\end{aligned}
$$

input
```
int(sin(b*(c + d*x)^2)*(e + f*x)^3,x)
```

output
```
(f^3*sin(b*(c + d*x)^2))/(2*b^2*d^4) - (cos(b*(c + d*x)^2)*(c^2*f^3 + 3*d^
2*e^2*f - 3*c*d*e*f^2))/(2*b*d^4) - (f^3*x^2*cos(b*(c + d*x)^2))/(2*b*d^2)
 + (x*cos(b*(c + d*x)^2)*(c*f^3 - 3*d*e*f^2))/(2*b*d^3) - (2^(1/2)*pi^(1/2
)*fresnels((2^(1/2)*b^(1/2)*(c + d*x))/pi^(1/2))*(c^3*f^3 - d^3*e^3 + 3*c*
d^2*e^2*f - 3*c^2*d*e*f^2))/(2*b^(1/2)*d^4) - (2^(1/2)*pi^(1/2)*fresnelc((
2^(1/2)*b^(1/2)*(c + d*x))/pi^(1/2))*(3*c*f^3 - 3*d*e*f^2))/(4*b^(3/2)*d^4
)
```

Reduce [F]

$$\int (e+fx)^3 \sin \left(b(c+dx)^2\right) \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^3*sin(b*(d*x+c)^2),x)
```

output
```
( - 3*cos(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c**2*f**3 + 9*cos(b*c**2 + 2
*b*c*d*x + b*d**2*x**2)*b*c*d*e*f**2 + 3*cos(b*c**2 + 2*b*c*d*x + b*d**2*x
**2)*b*c*d*f**3*x - 9*cos(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*d**2*e**2*f
- 9*cos(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*d**2*e*f**2*x - 3*cos(b*c**2 +
 2*b*c*d*x + b*d**2*x**2)*b*d**2*f**3*x**2 + 24*int(x**2/(tan((b*c**2 + 2*
b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c**3*d**3*f**3 - 72*int(x**2/(ta
n((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c**2*d**4*e*f**2 +
 72*int(x**2/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c*
d**5*e**2*f - 24*int(x**2/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 +
1),x)*b**3*d**6*e**3 - 24*int(1/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)
**2 + 1),x)*b**3*c**5*d*f**3 + 72*int(1/(tan((b*c**2 + 2*b*c*d*x + b*d**2*
x**2)/2)**2 + 1),x)*b**3*c**4*d**2*e*f**2 - 72*int(1/(tan((b*c**2 + 2*b*c*
d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c**3*d**3*e**2*f + 24*int(1/(tan((b*
c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c**2*d**4*e**3 - 18*int
(1/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b*c*d*f**3 + 18*i
nt(1/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b*d**2*e*f**2 +
 6*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b**2*c**4*f**3 - 18*sin(b*c**2 +
2*b*c*d*x + b*d**2*x**2)*b**2*c**3*d*e*f**2 - 6*sin(b*c**2 + 2*b*c*d*x + b
*d**2*x**2)*b**2*c**3*d*f**3*x + 18*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*
b**2*c**2*d**2*e**2*f + 18*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b**2*c...
```

3.154 $\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\begin{aligned}\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx = & -\frac{f(de-cf)\cos \left(b(c+dx)^2\right)}{bd^3} \\ & -\frac{f^2(c+dx)\cos \left(b(c+dx)^2\right)}{2bd^3} \\ & +\frac{f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} \\ & +\frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}\end{aligned}$$

output

```
-f*(-c*f+d*e)*cos(b*(d*x+c)^2)/b/d^3-1/2*f^2*(d*x+c)*cos(b*(d*x+c)^2)/b/d^
3+1/4*f^2*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(3
/2)/d^3+1/2*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c))/b^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.78

$$\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx$$
$$= \frac{-2\sqrt{b}f(2de-cf+dfx)\cos \left(b(c+dx)^2\right)+f^2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+2b(de-cf)^2\sqrt{2\pi}\,\text{Fres}}{4b^{3/2}d^3}$$

input
```
Integrate[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

output
```
(-2*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[b*(c + d*x)^2] + f^2*Sqrt[2*Pi]*Fr
esnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + 2*b*(d*e - c*f)^2*Sqrt[2*Pi]*Fresne
lS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(4*b^(3/2)*d^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx$$
$$\Big\downarrow \text{3914}$$
$$\frac{\int \left(\sin \left(b(c+dx)^2\right)(de-cf)^2+2f(c+dx)\sin \left(b(c+dx)^2\right)(de-cf)+f^2(c+dx)^2\sin \left(b(c+dx)^2\right)\right)d(c+dx)}{d^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{\sqrt{\frac{\pi}{2}}f^2\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}(de-cf)^2\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}}-\frac{f(de-cf)\cos \left(b(c+dx)^2\right)}{b}-\frac{f^2(c+dx)\cos \left(b(c+dx)^2\right)}{2b}}{d^3}$$

input
```
Int[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

output
```
(-((f*(d*e - c*f)*Cos[b*(c + d*x)^2])/b) - (f^2*(c + d*x)*Cos[b*(c + d*x)^
2])/(2*b) + (f^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(2*b^(
3/2)) + ((d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/
Sqrt[b])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(127) = 254$.

Time = 1.43 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.94

method	result
default	$-\frac{f^2 x \cos\left(x^2 d^2 b+2 c d x b+b\,c^2\right)}{2 d^2 b}-\frac{f^2 c\left(-\frac{\cos\left(x^2 d^2 b+2 c d x b+b\,c^2\right)}{2 d^2 b}-\frac{c\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^2 x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^2 b}}\right)}{2 d\sqrt{d^2 b}}\right)}{d}+\frac{f^2\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\right.}{4 d^2 b\sqrt{}}$
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)\sqrt{\pi}\,e^2}{4\sqrt{-i b}\,d}+\frac{i f^2 c^2\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)}{4 d^3\sqrt{-i b}}-\frac{f^2\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)}{8 b\,d^3\sqrt{-i b}}-\frac{i e f c\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-i b}\right.}{2 d^2\sqrt{-i b}}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^2 x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^2 b}}\right)x^2 f^2}{2\sqrt{d^2 b}}+\frac{\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^2 x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^2 b}}\right)e f x}{\sqrt{d^2 b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e^2\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^2 x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^2 b}}\right)}{2\sqrt{d^2 b}}-$

input
```
int((f*x+e)^2*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^2/d^2/b*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^2*c/d*(-1/2/d^2/b*cos(b*
d^2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2
^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))+1/4*f^2/d^2/b*2^(1/2)*Pi^(
1/2)/(d^2*b)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)
)-e*f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-e*f*c/d*2^(1/2)*Pi^(1/2)/(d^2*b
)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/
2)*Pi^(1/2)/(d^2*b)^(1/2)*e^2*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d
^2*x+b*c*d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx$$
$$=\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}f^2\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+2\sqrt{2}\pi(bd^2e^2-2\,bcdef+bc^2f^2)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-2\,(bd^2f^2x+}{4\,b^2d^4}$$

input
```
integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*pi*sqrt(b*d^2/pi)*f^2*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x
 + c)/d) + 2*sqrt(2)*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(b*d^2/p
i)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*f^2*x + 2*b*
d^2*e*f - b*c*d*f^2)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^4)
```

Sympy [F]

$$\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx = \int (e+fx)^2 \sin \left(bc^2+2bcdx+bd^2x^2\right) dx$$

input
```
integrate((f*x+e)**2*sin(b*(d*x+c)**2),x)
```

output
```
Integral((e + f*x)**2*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.76

$$\int (e+fx)^2 \sin \left(b(c+dx)^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

output

```
1/8*sqrt(2)*sqrt(pi)*e^2*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I -
1)*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d) - 1/4*(2*d*x*(e^(I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
- sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt
(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(er
f(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*e*
f/(b*d^3*x + b*c*d^2) + 1/8*(4*b*c*d*x*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b
*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 4*b*c^2*(e^(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - s
qrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I
*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(
sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2 - (I - 1)*sqrt(2)*
gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/
2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*f^2/(b^2*d^4*x + b^2*c*d^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.21

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx$$
$$=\frac{\frac{\sqrt{2}\sqrt{\pi}\left(-2i\,bd^2e^2+4i\,bcdef-2i\,bc^2f^2-f^2\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}}{bd}}{8\,d^2}$$
$$+\frac{\frac{\sqrt{2}\sqrt{\pi}\left(2i\,bd^2e^2-4i\,bcdef+2i\,bc^2f^2-f^2\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}}{bd}}{8\,d^2}$$

input

```
integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*(-2*I*b*d^2*e^2 + 4*I*b*c*d*e*f - 2*I*b*c^2*f^2 - f^
2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sq
rt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(d*f^2*(x + c/d) + 2*d*e*f -
2*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)/(b*d))/d^2 + 1/8*(sqrt(2
)*sqrt(pi)*(2*I*b*d^2*e^2 - 4*I*b*c*d*e*f + 2*I*b*c^2*f^2 - f^2)*erf(-1/2*
sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(
-I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(d*f^2*(x + c/d) + 2*d*e*f - 2*c*f^2)*e
^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx = \frac{\cos\left(b\,(c+d\,x)^2\right)\,\left(c\,f^2-2\,d\,e\,f\right)}{2\,b\,d^3} - \frac{f^2\,x\,\cos\left(b\,(c+d\,x)^2\right)}{2\,b\,d^2} + \frac{\sqrt{2}\,f^2\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)}{4\,b^{3/2}\,d^3} + \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2\right)}{2\,\sqrt{b}\,d^3}$$

input
```
int(sin(b*(c + d*x)^2)*(e + f*x)^2,x)
```

output
```
(cos(b*(c + d*x)^2)*(c*f^2 - 2*d*e*f))/(2*b*d^3) - (f^2*x*cos(b*(c + d*x)^
2))/(2*b*d^2) + (2^(1/2)*f^2*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2)*(c + d*x))
/pi^(1/2)))/(4*b^(3/2)*d^3) + (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2)*
(c + d*x))/pi^(1/2))*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f))/(2*b^(1/2)*d^3)
```

Reduce [F]

$$\int (e+fx)^2\sin\left(b(c+dx)^2\right)\,dx$$

$$= \frac{12\sin\left(b\,d^2x^2+2bcdx+b\,c^2\right)b\,c^2def+6\sin\left(b\,d^2x^2+2bcdx+b\,c^2\right)b\,c^2d\,f^2x+24b^2c^3d^2efx-8b^2c\,d^4ef}{}$$

input

```
int((f*x+e)^2*sin(b*(d*x+c)^2),x)
```

output

```
(3*cos(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*c*f**2 - 6*cos(b*c**2 + 2*b*c*d*x
 + b*d**2*x**2)*d*e*f - 3*cos(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*d*f**2*x -
 24*int(x**2/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**2*c*
*2*d**3*f**2 + 48*int(x**2/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 +
 1),x)*b**2*c*d**4*e*f - 24*int(x**2/(tan((b*c**2 + 2*b*c*d*x + b*d**2*x**
2)/2)**2 + 1),x)*b**2*d**5*e**2 + 24*int(1/(tan((b*c**2 + 2*b*c*d*x + b*d*
*2*x**2)/2)**2 + 1),x)*b**2*c**4*d*f**2 - 48*int(1/(tan((b*c**2 + 2*b*c*d*
x + b*d**2*x**2)/2)**2 + 1),x)*b**2*c**3*d**2*e*f + 24*int(1/(tan((b*c**2
+ 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**2*c**2*d**3*e**2 + 6*int(1/(ta
n((b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*d*f**2 - 6*sin(b*c**2 +
 2*b*c*d*x + b*d**2*x**2)*b*c**3*f**2 + 12*sin(b*c**2 + 2*b*c*d*x + b*d**2
*x**2)*b*c**2*d*e*f + 6*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c**2*d*f**
2*x - 6*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c*d**2*e**2 - 12*sin(b*c**
2 + 2*b*c*d*x + b*d**2*x**2)*b*c*d**2*e*f*x + 6*sin(b*c**2 + 2*b*c*d*x + b
*d**2*x**2)*b*d**3*e**2*x - 12*b**2*c**4*d*f**2*x + 24*b**2*c**3*d**2*e*f*
x - 12*b**2*c**2*d**3*e**2*x + 4*b**2*c**2*d**3*f**2*x**3 - 8*b**2*c*d**4*
e*f*x**3 + 4*b**2*d**5*e**2*x**3 - 3*c*f**2 + 6*d*e*f - 3*d*f**2*x)/(6*b*d
**3)
```

3.155 $\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx = -\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}$$

output

```
-1/2*f*cos(b*(d*x+c)^2)/b/d^2+1/2*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx = \frac{-f\cos\left(b(c+dx)^2\right)+\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2bd^2}$$

input

```
Integrate[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

output
```
(-(f*Cos[b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]
*Sqrt[2/Pi]*(c + d*x)])/(2*b*d^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx$$

$$\Big\downarrow \; 3914$$

$$\frac{\int \left((de-cf)\sin\left(b(c+dx)^2\right)+f(c+dx)\sin\left(b(c+dx)^2\right)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{\frac{\pi}{2}}(de-cf)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}}-\frac{f\cos\left(b(c+dx)^2\right)}{2b}}{d^2}$$

input
```
Int[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

output
```
(-1/2*(f*Cos[b*(c + d*x)^2])/b + ((d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)])/Sqrt[b])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 1.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

method	result
default	$-\frac{f\cos\left(x^{2} d^{2} b+2 c d x b+b\,c^{2}\right)}{2 d^{2} b}-\frac{f c\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right)}{2 d\sqrt{d^{2} b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right)}{2\sqrt{d^{2} b}}$
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)\sqrt{\pi}\,e}{4\sqrt{-i b}\,d}-\frac{i f c\sqrt{\pi}\ \mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)}{4 d^{2}\sqrt{-i b}}+\frac{i e\sqrt{\pi}\ \mathrm{erf}\left(d\sqrt{i b}\,x+\frac{i b c}{\sqrt{i b}}\right)}{4 d\sqrt{i b}}-\frac{i f c\sqrt{\pi}\ \mathrm{erf}\left(d\sqrt{i b}\,x+\frac{i b c}{\sqrt{i b}}\right)}{4 d^{2}\sqrt{i b}}-$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right) f x}{2\sqrt{d^{2} b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right)}{2\sqrt{d^{2} b}}-$

input
```
int((f*x+e)*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1/2*f*c/d*2^(1/2)*Pi^(1/2)/(d^
2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^
(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*e*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*
d^2*x+b*c*d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}(de-cf)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right) - df\cos\left(bd^2x^2+2\,bcdx+bc^2\right)}{2\,bd^3}$$

input
```
integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*fresnel_sin(sqrt(2)*sqrt(b*d^2/
pi)*(d*x + c)/d) - d*f*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b*d^3)
```

Sympy [F]

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\,dx = \int (e+fx)\sin \left(bc^2+2bcdx+bd^2x^2\right)dx$$

input
```
integrate((f*x+e)*sin(b*(d*x+c)**2),x)
```

output
```
Integral((e + f*x)*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.93

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}e\left((i+1)\,\mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+(i-1)\,\mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\sqrt{b}d}$$

$$- \frac{\left(2\,dx\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right) - \sqrt{bd^2x^2+2\,bcdx+bc^2}(-(i+1)\,\sqrt{2}\sqrt{\pi}(\mathrm{erf}\,(\sqrt{i}}{}$$

input
```
integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*sqrt(pi)*e*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)
*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d) - 1/8*(2*d*x*(e^(I*b*d^2*x
^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I
*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(
sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*f/(b
*d^3*x + b*c*d^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.38

$$
\begin{aligned}
\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx
&= -\frac{\frac{\sqrt{2}\sqrt{\pi}(i\,de-i\,cf)\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}}{bd}}{4\,d} \\
&\quad - \frac{\frac{\sqrt{2}\sqrt{\pi}(-i\,de+i\,cf)\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}}{bd}}{4\,d}
\end{aligned}
$$

input
```
integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/4*(sqrt(2)*sqrt(pi)*(I*d*e - I*c*f)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d
^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1))
 + f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)/(b*d))/d - 1/4*(sqrt(2)*sqrt
(pi)*(-I*d*e + I*c*f)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\,dx=-\frac{f\,\cos \left(b\,{\left(c+d\,x\right)}^2\right)}{2\,b\,d^2}-\frac{\sqrt{2}\,\sqrt{\pi }\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi }}\right)\,\left(c\,f-d\,e\right)}{2\,\sqrt{b}\,d^2}$$

input
```
int(sin(b*(c + d*x)^2)*(e + f*x),x)
```

output
```
- (f*cos(b*(c + d*x)^2))/(2*b*d^2) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b
^(1/2)*(c + d*x))/pi^(1/2))*(c*f - d*e))/(2*b^(1/2)*d^2)
```

Reduce [F]

$$\int (e+fx)\sin \left(b(c+dx)^2\right)\,dx=\left(\int \sin \left(b\,d^{2}x^{2}+2bcdx+b\,c^{2}\right)dx\right)e+\left(\int \sin \left(b\,d^{2}x^{2}+2bcdx+b\,c^{2}\right)xdx\right)f$$

input
```
int((f*x+e)*sin(b*(d*x+c)^2),x)
```

output
```
int(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2),x)*e + int(sin(b*c**2 + 2*b*c*d*
x + b*d**2*x**2)*x,x)*f
```

3.156 $\int \sin \left(b(c+dx)^2\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{\frac{\pi}{2}}\, \text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

output
```
1/2*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{\frac{\pi}{2}}\, \text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

input
```
Integrate[Sin[b*(c + d*x)^2],x]
```

output
```
(Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(b(c+dx)^2\right) \, dx$$

$$\Big\downarrow \ 3832$$

$$\frac{\sqrt{\frac{\pi}{2}} \text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

input
```
Int[Sin[b*(c + d*x)^2],x]
```

output
```
(Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right)}{2 \sqrt{d^{2} b}}$	42
risch	$\frac{i \sqrt{\pi}\,\text{erf}\left(d \sqrt{i b}\, x+\frac{i b c}{\sqrt{i b}}\right)}{4 d \sqrt{i b}}+\frac{i \sqrt{\pi}\,\text{erf}\left(-d \sqrt{-i b}\, x+\frac{i b c}{\sqrt{-i b}}\right)}{4 d \sqrt{-i b}}$	77

input
```
int(sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)
*(b*d^2*x+b*c*d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \sin\left(b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)}{2\,bd^2}$$

input
```
integrate(sin(b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)
/d)/(b*d^2)
```

Sympy [F]

$$\int \sin\left(b(c+dx)^2\right)\,dx = \int \sin\left(b(c+dx)^2\right)dx$$

input
```
integrate(sin(b*(d*x+c)**2),x)
```

output
```
Integral(sin(b*(c + d*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \sin \left(b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\left((i+1)\,\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+(i-1)\,\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\,\sqrt{b}d}$$

input
```
integrate(sin(b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*sqrt(pi)*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)*e
rf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.67

$$\int \sin \left(b(c+dx)^2\right)\,dx = -\frac{i\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\,\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)} + \frac{i\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\,\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}$$

input
```
integrate(sin(b*(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4
) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt(2
)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x +
c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \sin \left(b(c+dx)^2\right) \, dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,b\,d\,\sqrt{\frac{1}{b\,d^2}}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}$$

input
```
int(sin(b*(c + d*x)^2),x)
```

output
```
(2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b*d*(1/(b*d^2))^(1/2)*(c + d*x))/pi^(1
/2))*(1/(b*d^2))^(1/2))/2
```

Reduce [F]

$$\int \sin \left(b(c+dx)^2\right) \, dx = \int \sin \left(b\,d^2x^2+2bcdx+b\,c^2\right) dx$$

input
```
int(sin(b*(d*x+c)^2),x)
```

output
```
int(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2),x)
```

3.157 $\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(b(c+dx)^2\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(b*(d*x+c)^2)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[b*(c + d*x)^2]/(e + f*x),x]
```

output
```
Integrate[Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx} \, dx$$

$$\Big\downarrow \ 3918$$

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}dx$$

input
```
Int[Sin[b*(c + d*x)^2]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(b(dx+c)^2\right)}{fx+e}dx$$

input
```
int(sin(b*(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(b*(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^2b\right)}{fx+e}\,dx$$

input
```
integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left(bc^2+2bcdx+bd^2x^2\right)}{e+fx}\,dx$$

input
```
integrate(sin(b*(d*x+c)**2)/(f*x+e),x)
```

output
```
Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^2 b\right)}{fx+e}\,dx$$

input
```
integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^2 b\right)}{fx+e}\,dx$$

input
```
integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 39.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left(b\,{\left(c+d\,x\right)}^2\right)}{e+f\,x}\,dx$$

input
```
int(sin(b*(c + d*x)^2)/(e + f*x),x)
```

output
```
int(sin(b*(c + d*x)^2)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin \left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left(b\,d^2x^2+2bcdx+b\,c^2\right)}{fx+e}dx$$

input
```
int(sin(b*(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x),x)
```

3.158 $\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx=\operatorname{Int}\left(\frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} \, dx$$
$$\Big\downarrow \ 3918$$
$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} dx$$

input
```
Int[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(b \left(d x +c \right)^{2}\right)}{\left(f x +e \right)^{2}}d x$$

input
```
int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sin (b(c+dx)^2)}{(e+fx)^2}\,dx=\int \frac{\sin \left((dx+c)^2 b\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin (b(c+dx)^2)}{(e+fx)^2}\,dx=\int \frac{\sin (bc^2+2bcdx+bd^2x^2)}{(e+fx)^2}\,dx$$

input
```
integrate(sin(b*(d*x+c)**2)/(f*x+e)**2,x)
```

output
```
Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^2b\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^2b\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(b\,{\left(c+d\,x\right)}^2\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input
```
int(sin(b*(c + d*x)^2)/(e + f*x)^2,x)
```

output
```
int(sin(b*(c + d*x)^2)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(b\,d^2x^2+2bcdx+b\,c^2\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.159 $\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 337

$$\begin{aligned}\int (e+fx)^3\sin\left(\frac{b}{(c+dx)^2}\right)\,dx &= \frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4}\\ &+\frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4}\\ &-\frac{3bf(de-cf)^2\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4}\\ &-\frac{\sqrt{b}(de-cf)^3\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4}\\ &+\frac{2b^{3/2}f^2(de-cf)\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4}\\ &+\frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4}\\ &+\frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4}\\ &+\frac{f^2(de-cf)(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4}\\ &+\frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4}+\frac{b^2f^3\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}\end{aligned}$$

output
```
2*b*f^2*(-c*f+d*e)*(d*x+c)*cos(b/(d*x+c)^2)/d^4+1/4*b*f^3*(d*x+c)^2*cos(b/
(d*x+c)^2)/d^4-3/2*b*f*(-c*f+d*e)^2*Ci(b/(d*x+c)^2)/d^4-b^(1/2)*(-c*f+d*e)
^3*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))/d^4+2*b^(3/
2)*f^2*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c))/d^4+(-c*f+d*e)^3*(d*x+c)*sin(b/(d*x+c)^2)/d^4+3/2*f*(-c*f+d*e)^2*(d*x+
c)^2*sin(b/(d*x+c)^2)/d^4+f^2*(-c*f+d*e)*(d*x+c)^3*sin(b/(d*x+c)^2)/d^4+1/
4*f^3*(d*x+c)^4*sin(b/(d*x+c)^2)/d^4+1/4*b^2*f^3*Si(b/(d*x+c)^2)/d^4
```

Mathematica [A] (verified)

Time = 6.70 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.31

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$

$$= \frac{8bcdef^2\cos\left(\frac{b}{(c+dx)^2}\right) - 7bc^2f^3\cos\left(\frac{b}{(c+dx)^2}\right) + 8bd^2ef^2x\cos\left(\frac{b}{(c+dx)^2}\right) - 6bcdf^3x\cos\left(\frac{b}{(c+dx)^2}\right) + bd^2f^3x}{}$$

input
```
Integrate[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

output
```
(8*b*c*d*e*f^2*Cos[b/(c + d*x)^2] - 7*b*c^2*f^3*Cos[b/(c + d*x)^2] + 8*b*d
^2*e*f^2*x*Cos[b/(c + d*x)^2] - 6*b*c*d*f^3*x*Cos[b/(c + d*x)^2] + b*d^2*f
^3*x^2*Cos[b/(c + d*x)^2] - 6*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)^2]
 - 4*Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)] + 8*b^(3/2)*d*e*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x
)] - 8*b^(3/2)*c*f^3*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] +
 4*c*d^3*e^3*Sin[b/(c + d*x)^2] - 6*c^2*d^2*e^2*f*Sin[b/(c + d*x)^2] + 4*c
^3*d*e*f^2*Sin[b/(c + d*x)^2] - c^4*f^3*Sin[b/(c + d*x)^2] + 4*d^4*e^3*x*S
in[b/(c + d*x)^2] + 6*d^4*e^2*f*x^2*Sin[b/(c + d*x)^2] + 4*d^4*e*f^2*x^3*S
in[b/(c + d*x)^2] + d^4*f^3*x^4*Sin[b/(c + d*x)^2] + b^2*f^3*SinIntegral[b
/(c + d*x)^2])/(4*d^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin\left(\frac{b}{(c+dx)^2}\right)(de-cf)^3+3f(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)(de-cf)^2+3f^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)(de-cf)+f^3(c}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{2\pi}b^{3/2}f^2(de-cf)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+\frac{1}{4}b^2f^3\operatorname{Si}\left(\frac{b}{(c+dx)^2}\right)-\frac{3}{2}bf(de-cf)^2\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)+f^2(c+d}{}$$

input

```
Int[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

output

```
(2*b*f^2*(d*e - c*f)*(c + d*x)*Cos[b/(c + d*x)^2] + (b*f^3*(c + d*x)^2*Cos
[b/(c + d*x)^2])/4 - (3*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)^2])/2 -
Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]
+ 2*b^(3/2)*f^2*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c +
d*x)] + (d*e - c*f)^3*(c + d*x)*Sin[b/(c + d*x)^2] + (3*f*(d*e - c*f)^2*(c
 + d*x)^2*Sin[b/(c + d*x)^2])/2 + f^2*(d*e - c*f)*(c + d*x)^3*Sin[b/(c + d
*x)^2] + (f^3*(c + d*x)^4*Sin[b/(c + d*x)^2])/4 + (b^2*f^3*SinIntegral[b/(
c + d*x)^2])/4)/d^4
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-(cf-de)^{3}(dx+c)\sin\left(\frac{b}{(dx+c)^{2}}\right)+(cf-de)^{3}\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\ \operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+\frac{3f(cf-de)^{2}(dx+c)^{2}\sin\left(\frac{b}{(dx+c)^{2}}\right)}{2}-\frac{3f(}{}}{}$
default	$\frac{-(cf-de)^{3}(dx+c)\sin\left(\frac{b}{(dx+c)^{2}}\right)+(cf-de)^{3}\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\ \operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+\frac{3f(cf-de)^{2}(dx+c)^{2}\sin\left(\frac{b}{(dx+c)^{2}}\right)}{2}-\frac{3f(}{}}{}$
risch	$\frac{b\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^{3}f^{3}}{2d^{4}\sqrt{-ib}}-\frac{3b\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^{2}e\,f^{2}}{2d^{3}\sqrt{-ib}}+\frac{3b\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c\,e^{2}f}{2d^{2}\sqrt{-ib}}-\frac{b\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e^{3}}{2d\sqrt{-ib}}+\frac{3b\ \operatorname{expInteg}}{}$
parts	Expression too large to display

input
```
int((f*x+e)^3*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-(c*f-d*e)^3*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)^3*b^(1/2)*2^(1/2)*P
i^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+3/2*f*(c*f-d*e)^2*(d*x+
c)^2*sin(b/(d*x+c)^2)-3/2*f*(c*f-d*e)^2*b*Ci(b/(d*x+c)^2)-f^2*(c*f-d*e)*(d
*x+c)^3*sin(b/(d*x+c)^2)+2*f^2*(c*f-d*e)*b*(-(d*x+c)*cos(b/(d*x+c)^2)-b^(1
/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/4*f^3*(
d*x+c)^4*sin(b/(d*x+c)^2)-1/2*f^3*b*(-1/2*(d*x+c)^2*cos(b/(d*x+c)^2)-1/2*b
*Si(b/(d*x+c)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.18

$$\int (e+fx)^{3}\sin\left(\frac{b}{(c+dx)^{2}}\right)\,dx$$

$$=\frac{b^{2}f^{3}\operatorname{Si}\left(\frac{b}{d^{2}x^{2}+2\,cdx+c^{2}}\right)-4\,\sqrt{2}\pi(d^{4}e^{3}-3\,cd^{3}e^{2}f+3\,c^{2}d^{2}ef^{2}-c^{3}df^{3})\sqrt{\frac{b}{\pi d^{2}}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)+8\,\sqrt{2}\pi(bd^{2}ef}{}$$

input
```
integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

output

```
1/4*(b^2*f^3*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - 4*sqrt(2)*pi*(d^4
*e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2 - c^3*d*f^3)*sqrt(b/(pi*d^2))*fresn
el_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 8*sqrt(2)*pi*(b*d^2*e*f^2 -
 b*c*d*f^3)*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x +
 c)) + (b*d^2*f^3*x^2 + 8*b*c*d*e*f^2 - 7*b*c^2*f^3 + 2*(4*b*d^2*e*f^2 - 3
*b*c*d*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 6*(b*d^2*e^2*f - 2*b*c*d
*e*f^2 + b*c^2*f^3)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + (d^4*f^3*x
^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 4*c*d^3*e^3 - 6*c^2
*d^2*e^2*f + 4*c^3*d*e*f^2 - c^4*f^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^
4
```

Sympy [F]

$$\int (e+fx)^3\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (e+fx)^3\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

input

```
integrate((f*x+e)**3*sin(b/(d*x+c)**2),x)
```

output

```
Integral((e + f*x)**3*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)^3\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)^3\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input

```
integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*(4*d^3*integrate(1/4*((4*b*c^3*d*e*f^2 - 3*b*c^4*f^3 - 6*(b*d^4*e^2*f
 - 2*b*c*d^3*e*f^2 + b*c^2*d^2*f^3)*x^2 - 4*(b*d^4*e^3 - 3*b*c^2*d^2*e*f^2
 + 2*b*c^3*d*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b^2*d^2*f^3*x^2 +
 2*(4*b^2*d^2*e*f^2 - 3*b^2*c*d*f^3)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/
(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3), x) + 4*d^3*integrate(1/4*
((4*b*c^3*d*e*f^2 - 3*b*c^4*f^3 - 6*(b*d^4*e^2*f - 2*b*c*d^3*e*f^2 + b*c^2
*d^2*f^3)*x^2 - 4*(b*d^4*e^3 - 3*b*c^2*d^2*e*f^2 + 2*b*c^3*d*f^3)*x)*cos(b
/(d^2*x^2 + 2*c*d*x + c^2)) + (b^2*d^2*f^3*x^2 + 2*(4*b^2*d^2*e*f^2 - 3*b^
2*c*d*f^3)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/((d^6*x^3 + 3*c*d^5*x^2 +
3*c^2*d^4*x + c^3*d^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^6*x^3 + 3*c
*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
- (b*d*f^3*x^2 + 2*(4*b*d*e*f^2 - 3*b*c*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x +
 c^2)) - (d^3*f^3*x^4 + 4*d^3*e*f^2*x^3 + 6*d^3*e^2*f*x^2 + 4*d^3*e^3*x)*s
in(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^3
```

Giac [F]

$$\int (e+fx)^3\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)^3\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin(b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^3\,dx$$

input
```
int(sin(b/(c + d*x)^2)*(e + f*x)^3,x)
```

output
```
int(sin(b/(c + d*x)^2)*(e + f*x)^3, x)
```

Reduce [F]

$$\int (e+fx)^3 \sin \left(\frac{b}{(c+dx)^2}\right) \, dx = \text{too large to display}$$

input

```
int((f*x+e)^3*sin(b/(d*x+c)^2),x)
```

output

```
( - 120*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**6*f**3 - 120*cos(b/(c*
*2 + 2*c*d*x + d**2*x**2))*b**2*c**5*d*e*f**2 - 456*cos(b/(c**2 + 2*c*d*x
+ d**2*x**2))*b**2*c**5*d*f**3*x - 456*cos(b/(c**2 + 2*c*d*x + d**2*x**2))
*b**2*c**4*d**2*e*f**2*x - 624*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*
*4*d**2*f**3*x**2 - 624*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**3*d**3
*e*f**2*x**2 - 336*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**3*d**3*f**3
*x**3 - 336*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**2*d**4*e*f**2*x**3
 - 24*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**2*d**4*f**3*x**4 - 24*co
s(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*d**5*e*f**2*x**4 + 24*cos(b/(c**2
 + 2*c*d*x + d**2*x**2))*b**2*c*d**5*f**3*x**5 + 24*cos(b/(c**2 + 2*c*d*x
+ d**2*x**2))*b**2*d**6*e*f**2*x**5 - 2250*cos(b/(c**2 + 2*c*d*x + d**2*x*
*2))*c**10*f**3 - 1980*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**9*d*e*f**2 -
 9000*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**9*d*f**3*x + 630*cos(b/(c**2
+ 2*c*d*x + d**2*x**2))*c**8*d**2*e**2*f - 7920*cos(b/(c**2 + 2*c*d*x + d*
*2*x**2))*c**8*d**2*e*f**2*x - 13500*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c
**8*d**2*f**3*x**2 + 240*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**3*e**
3 + 2520*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**3*e**2*f*x - 11880*co
s(b/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**3*e*f**2*x**2 - 9000*cos(b/(c**2
 + 2*c*d*x + d**2*x**2))*c**7*d**3*f**3*x**3 + 960*cos(b/(c**2 + 2*c*d*x +
 d**2*x**2))*c**6*d**4*e**3*x + 3780*cos(b/(c**2 + 2*c*d*x + d**2*x**2)...
```

3.160 $\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 233

$$
\begin{aligned}
\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx &= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} \\
&\quad - \frac{bf(de-cf)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3} \\
&\quad - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} \\
&\quad + \frac{2b^{3/2}f^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3} \\
&\quad + \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} \\
&\quad + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} \\
&\quad + \frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3}
\end{aligned}
$$

output
```
2/3*b*f^2*(d*x+c)*cos(b/(d*x+c)^2)/d^3-b*f*(-c*f+d*e)*Ci(b/(d*x+c)^2)/d^3-
b^(1/2)*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d
*x+c))/d^3+2/3*b^(3/2)*f^2*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)/(d*x+c))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(b/(d*x+c)^2)/d^3+f*(-c*f+d*e)*(d
*x+c)^2*sin(b/(d*x+c)^2)/d^3+1/3*f^2*(d*x+c)^3*sin(b/(d*x+c)^2)/d^3
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.14

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$

$$= \frac{2bcf^2\cos\left(\frac{b}{(c+dx)^2}\right)+2bdf^2x\cos\left(\frac{b}{(c+dx)^2}\right)+3bf(-de+cf)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)-3\sqrt{b}(de-cf)^2\sqrt{2\pi}}{}$$

input
```
Integrate[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

output
```
(2*b*c*f^2*Cos[b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[b/(c + d*x)^2] + 3*b*f*(-(
d*e) + c*f)*CosIntegral[b/(c + d*x)^2] - 3*Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi
]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*b^(3/2)*f^2*Sqrt[2*Pi]*Fres
nelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*c*d^2*e^2*Sin[b/(c + d*x)^2] - 3*
c^2*d*e*f*Sin[b/(c + d*x)^2] + c^3*f^2*Sin[b/(c + d*x)^2] + 3*d^3*e^2*x*Si
n[b/(c + d*x)^2] + 3*d^3*e*f*x^2*Sin[b/(c + d*x)^2] + d^3*f^2*x^3*Sin[b/(c
 + d*x)^2])/(3*d^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin\left(\frac{b}{(c+dx)^2}\right)(de-cf)^2 + 2f(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)(de-cf) + f^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\right) d(c+dx)}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2}{3}\sqrt{2\pi}b^{3/2}f^2 \operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) - bf(de-cf)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right) - \sqrt{2\pi}\sqrt{b}(de-cf)^2 \operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) +}{}$$

input
```
Int[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

output
```
((2*b*f^2*(c + d*x)*Cos[b/(c + d*x)^2])/3 - b*f*(d*e - c*f)*CosIntegral[b/
(c + d*x)^2] - Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)] + (2*b^(3/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(
c + d*x)])/3 + (d*e - c*f)^2*(c + d*x)*Sin[b/(c + d*x)^2] + f*(d*e - c*f)*
(c + d*x)^2*Sin[b/(c + d*x)^2] + (f^2*(c + d*x)^3*Sin[b/(c + d*x)^2])/3)/d
^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{-(cf-de)^{2}(dx+c)\sin\left(\frac{b}{(dx+c)^{2}}\right)+(cf-de)^{2}\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+f(cf-de)(dx+c)^{2}\sin\left(\frac{b}{(dx+c)^{2}}\right)-f}{d^{3}}$
default	$-\frac{-(cf-de)^{2}(dx+c)\sin\left(\frac{b}{(dx+c)^{2}}\right)+(cf-de)^{2}\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+f(cf-de)(dx+c)^{2}\sin\left(\frac{b}{(dx+c)^{2}}\right)-f}{d^{3}}$
risch	$-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)c^{2}f^{2}}{2d^{3}\sqrt{-ib}}+\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)cef}{d^{2}\sqrt{-ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e^{2}}{2d\sqrt{-ib}}-\frac{b\,\mathrm{expIntegral}_{1}\left(-\frac{ib}{(dx+c)^{2}}\right)c\,f^{2}}{2d^{3}}+b\ \mathrm{exp}$
parts	$-\frac{\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)f^{2}x^{2}}{d}+\sin\left(\frac{b}{(dx+c)^{2}}\right)f^{2}x^{3}-\frac{2\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)efx}{d}+$

input
```
int((f*x+e)^2*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(-(c*f-d*e)^2*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)^2*b^(1/2)*2^(1/2)*
Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+f*(c*f-d*e)*(d*x+c)^2*
sin(b/(d*x+c)^2)-f*(c*f-d*e)*b*Ci(b/(d*x+c)^2)-1/3*f^2*(d*x+c)^3*sin(b/(d*
x+c)^2)+2/3*f^2*b*(-(d*x+c)*cos(b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12

$$\int (e+fx)^{2}\sin\left(\frac{b}{(c+dx)^{2}}\right)\,dx$$
$$=\frac{2\sqrt{2}\pi bdf^{2}\sqrt{\frac{b}{\pi d^{2}}}\,\mathrm{S}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)-3\sqrt{2}\pi(d^{3}e^{2}-2\,cd^{2}ef+c^{2}df^{2})\sqrt{\frac{b}{\pi d^{2}}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)+2\,(bdf^{2}x+bcf^{2})}{}$$

input
```
integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(2)*pi*b*d*f^2*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(p
i*d^2))/(d*x + c)) - 3*sqrt(2)*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sqrt
(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 2*(b*d*f^
2*x + b*c*f^2)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*d*e*f - b*c*f^2)*co
s_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + (d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3
*d^3*e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin(b/(d^2*x^2 + 2*c*d*x
 + c^2)))/d^3
```

Sympy [F]

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (e+fx)^2\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

input
```
integrate((f*x+e)**2*sin(b/(d*x+c)**2),x)
```

output
```
Integral((e + f*x)**2*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)^2\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

output

```
1/3*(2*b*f^2*x*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*d^2*integrate(1/3*(2*b
^2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f -
b*c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos(b/(d^2*x^2 + 2*c*d*x
 + c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2), x) - 3*d^2*inte
grate(1/3*(2*b^2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3
*(b*d^3*e*f - b*c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos(b/(d^2
*x^2 + 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*c
os(b/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x +
 c^3*d^2)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d^2*e
*f*x^2 + 3*d^2*e^2*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Giac [F]

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)^2\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input

```
integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="giac")
```

output

```
integrate((f*x + e)^2*sin(b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^2\,dx$$

input

```
int(sin(b/(c + d*x)^2)*(e + f*x)^2,x)
```

output

```
int(sin(b/(c + d*x)^2)*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2 \sin \left(\frac{b}{(c+dx)^2}\right) \, dx = \text{too large to display}$$

input

```
int((f*x+e)^2*sin(b/(d*x+c)^2),x)
```

output

```
( - 60*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**5*f**2 - 228*cos(b/(c**
2 + 2*c*d*x + d**2*x**2))*b**2*c**4*d*f**2*x - 312*cos(b/(c**2 + 2*c*d*x +
 d**2*x**2))*b**2*c**3*d**2*f**2*x**2 - 168*cos(b/(c**2 + 2*c*d*x + d**2*x
**2))*b**2*c**2*d**3*f**2*x**3 - 12*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b*
*2*c*d**4*f**2*x**4 + 12*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d**5*f**
2*x**5 - 990*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**9*f**2 + 630*cos(b/(c*
*2 + 2*c*d*x + d**2*x**2))*c**8*d*e*f - 3960*cos(b/(c**2 + 2*c*d*x + d**2*
x**2))*c**8*d*f**2*x + 360*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**2*e
**2 + 2520*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**2*e*f*x - 5940*cos(
b/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**2*f**2*x**2 + 1440*cos(b/(c**2 + 2
*c*d*x + d**2*x**2))*c**6*d**3*e**2*x + 3780*cos(b/(c**2 + 2*c*d*x + d**2*
x**2))*c**6*d**3*e*f*x**2 - 3960*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**6*
d**3*f**2*x**3 + 2160*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**5*d**4*e**2*x
**2 + 2520*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c**5*d**4*e*f*x**3 - 990*co
s(b/(c**2 + 2*c*d*x + d**2*x**2))*c**5*d**4*f**2*x**4 + 1440*cos(b/(c**2 +
 2*c*d*x + d**2*x**2))*c**4*d**5*e**2*x**3 + 630*cos(b/(c**2 + 2*c*d*x + d
**2*x**2))*c**4*d**5*e*f*x**4 + 360*cos(b/(c**2 + 2*c*d*x + d**2*x**2))*c*
*3*d**6*e**2*x**4 - 72*int(x**4/(tan(b/(2*c**2 + 4*c*d*x + 2*d**2*x**2))**
2*c**5 + 5*tan(b/(2*c**2 + 4*c*d*x + 2*d**2*x**2))**2*c**4*d*x + 10*tan(b/
(2*c**2 + 4*c*d*x + 2*d**2*x**2))**2*c**3*d**2*x**2 + 10*tan(b/(2*c**2 ...
```

3.161 $\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = -\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

output
```
-1/2*b*f*Ci(b/(d*x+c)^2)/d^2-b^(1/2)*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*FresnelC(
b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))/d^2+(-c*f+d*e)*(d*x+c)*sin(b/(d*x+c)^2)/
d^2+1/2*f*(d*x+c)^2*sin(b/(d*x+c)^2)/d^2
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx =$$

$$-\frac{bf\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)+2\sqrt{b}(de-cf)\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+(c+dx)(-2de+cf-dfx)\sin\Big(}{2d^2}$$

input
```
Integrate[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

output
```
-1/2*(b*f*CosIntegral[b/(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + (c + d*x)*(-2*d*e + c*f - d*f*x)*
Sin[b/(c + d*x)^2])/d^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$$

$$\Big\downarrow \; 3914$$

$$\frac{\int \left((de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)+f(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{2}bf\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)-\sqrt{2\pi}\sqrt{b}(de-cf)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+(c+dx)(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)+\frac{1}{2}f(c+}{d^2}$$

input
```
Int[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

output
```
(-1/2*(b*f*CosIntegral[b/(c + d*x)^2]) - Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + (d*e - c*f)*(c + d*x)*Sin[b/(c +
d*x)^2] + (f*(c + d*x)^2*Sin[b/(c + d*x)^2])/2)/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+\frac{f(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{fb\,\mathrm{Ci}\left(\frac{b}{(dx+c)^2}\right.}{2}}{d^2}$
default	$\frac{-(cf-de)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)+\frac{f(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{fb\,\mathrm{Ci}\left(\frac{b}{(dx+c)^2}\right.}{2}}{d^2}$
risch	$\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)cf}{2d^2\sqrt{-ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)e}{2d\sqrt{-ib}}+\frac{b\,\mathrm{expIntegral}_1\left(-\frac{ib}{(dx+c)^2}\right)f}{4d^2}+\frac{b\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\sqrt{\pi}\,cf}{2d^2\sqrt{ib}}-\frac{b\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)}{2d\sqrt{ib}}$
parts	$-\frac{\sqrt{b}\,\sqrt{2}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\sqrt{\pi}\,fx}{d}+\sin\left(\frac{b}{(dx+c)^2}\right)f\,x^2-\frac{\sqrt{b}\,\sqrt{2}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\sqrt{\pi}\,e}{d}+\sin\left(\right.$

input
```
int((f*x+e)*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-(c*f-d*e)*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(1
/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+1/2*f*(d*x+c)^2*sin(b/(d*x+
c)^2)-1/2*f*b*Ci(b/(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx =$$

$$-\frac{2\sqrt{2}\pi(d^2e-cdf)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+bf\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)-(d^2fx^2+2\,d^2ex+2\,cde-c^2f)\sin\left(\frac{}{d^2}\right.}{2\,d^2}$$

input
```
integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*
sqrt(b/(pi*d^2))/(d*x + c)) + b*f*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)
) - (d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*sin(b/(d^2*x^2 + 2*c*d*x + c
^2)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (e+fx)\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)\,dx$$

input
```
integrate((f*x+e)*sin(b/(d*x+c)**2),x)
```

output
```
Integral((e + f*x)*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (fx+e)\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(f*x^2 + 2*e*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(1/2*(b*d*
f*x^2 + 2*b*d*e*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos(b/(d^2
*x^2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d
^2*x^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin
(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Giac [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int (fx+e)\sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)\,dx$$

input
```
int(sin(b/(c + d*x)^2)*(e + f*x),x)
```

output
```
int(sin(b/(c + d*x)^2)*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\left(\int \sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)dx\right)e+\left(\int \sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)xdx\right)f$$

input
```
int((f*x+e)*sin(b/(d*x+c)^2),x)
```

output
```
int(sin(b/(c**2 + 2*c*d*x + d**2*x**2)),x)*e + int(sin(b/(c**2 + 2*c*d*x +
 d**2*x**2))*x,x)*f
```

3.162 $\int \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \sin\left(\frac{b}{(c+dx)^2}\right) dx = -\frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

output
```
-b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))/d+(d*
x+c)*sin(b/(d*x+c)^2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \sin\left(\frac{b}{(c+dx)^2}\right) dx = -\frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

input
```
Integrate[Sin[b/(c + d*x)^2],x]
```

output
```
-((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
 d*x)*Sin[b/(c + d*x)^2])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3840, 3868, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(\frac{b}{(c+dx)^2}\right) dx$$

$$\Big\downarrow \text{3840}$$

$$-\frac{\int (c+dx)^2 \sin \left(\frac{b}{(c+dx)^2}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{2b \int \cos \left(\frac{b}{(c+dx)^2}\right) d\frac{1}{c+dx} - (c+dx) \sin \left(\frac{b}{(c+dx)^2}\right)}{d}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{\sqrt{2\pi}\sqrt{b}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) - (c+dx) \sin \left(\frac{b}{(c+dx)^2}\right)}{d}$$

input
```
Int[Sin[b/(c + d*x)^2],x]
```

output
```
-((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] - (c + d*x)
*Sin[b/(c + d*x)^2])/d)
```

Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3840
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[-f^(-1)   Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(
e + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &
& EqQ[n, -2]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)}{d}$	52
default	$-\frac{-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)}{d}$	52
risch	$-\frac{b\sqrt{\pi}\ \mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)}{2d\sqrt{ib}}-\frac{b\sqrt{\pi}\ \mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}-\frac{(-dx-c)\sin\left(\frac{b}{(dx+c)^2}\right)}{d}$	85

input
```
int(sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-(d*x+c)*sin(b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*
2^(1/2)/Pi^(1/2)/(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \sin \left(\frac{b}{(c+dx)^2}\right) dx = -\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\, \mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right) - (dx+c)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{d}$$

input
```
integrate(sin(b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*
x + c)) - (d*x + c)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int \sin \left(\frac{b}{(c+dx)^2}\right) dx = \int \sin \left(\frac{b}{(c+dx)^2}\right) dx$$

input
```
integrate(sin(b/(d*x+c)**2),x)
```

output
```
Integral(sin(b/(c + d*x)**2), x)
```

Maxima [F]

$$\int \sin \left(\frac{b}{(c+dx)^2}\right) dx = \int \sin \left(\frac{b}{(dx+c)^2}\right) dx$$

input
```
integrate(sin(b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3), x) + b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/((
d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^
2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d^2*x^2 + 2*c*d*x + c
^2))^2), x) + x*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
```

Giac [F]

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate(sin(b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(sin(b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 40.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx=\frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)\,(c+d\,x)}{d}-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{\pi}\,(c+d\,x)}\right)}{d}$$

input
```
int(sin(b/(c + d*x)^2),x)
```

output
```
(sin(b/(c + d*x)^2)*(c + d*x))/d - (2^(1/2)*b^(1/2)*pi^(1/2)*fresnelc((2^(
1/2)*b^(1/2))/(pi^(1/2)*(c + d*x))))/d
```

Reduce [F]

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)dx$$

input
```
int(sin(b/(d*x+c)^2),x)
```

output
```
int(sin(b/(c**2 + 2*c*d*x + d**2*x**2)),x)
```

3.163 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \text{Int}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(b/(d*x+c)^2)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[b/(c + d*x)^2]/(e + f*x),x]
```

output
```
Integrate[Sin[b/(c + d*x)^2]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx$$

$$\Big\downarrow \ 3918$$

$$\int \frac{\sin \left(\frac{b}{(c+dx)^2}\right)}{e+fx}dx$$

input
```
Int[Sin[b/(c + d*x)^2]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}dx$$

input
```
int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

input
```
integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 19.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)}{e+fx}\,dx$$

input
```
integrate(sin(b/(d*x+c)**2)/(f*x+e),x)
```

output
```
Integral(sin(b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

input
```
integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

input
```
integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 40.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(\frac{b}{{\left(c+d\,x\right)}^2}\right)}{e+f\,x}\,dx$$

input
```
int(sin(b/(c + d*x)^2)/(e + f*x),x)
```

output
```
int(sin(b/(c + d*x)^2)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)}{fx+e}dx$$

input
```
int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x),x)
```

3.164 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\text{Int}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{\sin \left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}dx$$

input

```
int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

output

```
int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

input

```
integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

output

```
integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \text{Timed out}$$

input

```
integrate(sin(b/(d*x+c)**2)/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 46.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)}{\left(e+f\,x\right)^2}\,dx$$

input
```
int(sin(b/(c + d*x)^2)/(e + f*x)^2,x)
```

output
```
int(sin(b/(c + d*x)^2)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin(b/(c**2 + 2*c*d*x + d**2*x**2))/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.165 $\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 341

$$\begin{aligned}
&\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx \\
&= -\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4} \\
&\quad-\frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4} \\
&\quad+\frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4} \\
&\quad+\frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4} \\
&\quad+\frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^4} \\
&\quad-\frac{3f^2(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{2b^{3/2}d^4}+\frac{f^3\sin\left(a+b(c+dx)^2\right)}{2b^2d^4}
\end{aligned}$$

output
```
-3/2*f*(-c*f+d*e)^2*cos(a+b*(d*x+c)^2)/b/d^4-3/2*f^2*(-c*f+d*e)*(d*x+c)*co
s(a+b*(d*x+c)^2)/b/d^4-1/2*f^3*(d*x+c)^2*cos(a+b*(d*x+c)^2)/b/d^4+3/4*f^2*
(-c*f+d*e)*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c))/b^(3/2)/d^4+1/2*(-c*f+d*e)^3*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c))/b^(1/2)/d^4+1/2*(-c*f+d*e)^3*2^(1/2)*Pi^(1/2)*Fr
esnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))*sin(a)/b^(1/2)/d^4-3/4*f^2*(-c*f+
d*e)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))*sin(a)/b^
(3/2)/d^4+1/2*f^3*sin(a+b*(d*x+c)^2)/b^2/d^4
```

Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.64

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx$$

$$= \frac{-4bf\left(c^2f^2-cdf(3e+fx)+d^2\left(3e^2+3efx+f^2x^2\right)\right)\cos\left(a+b(c+dx)^2\right)+2\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{FresnelS}}{}$$

input
```
Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

output
```
(-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Co
s[a + b*(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*S
qrt[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - 3*f^2*Sin[a]) + 2*Sqrt[b]
*(d*e - c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(3*f^2*Cos[
a] + 2*b*(d*e - c*f)^2*Sin[a]) + 4*f^3*Sin[a + b*(c + d*x)^2])/(8*b^2*d^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^2\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin \left(b(c+dx)^2+a\right)(de-cf)^3+3f(c+dx)\sin \left(b(c+dx)^2+a\right)(de-cf)^2+3f^2(c+dx)^2\sin \left(b(c+dx)^2+\right.\right.}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\sqrt{\frac{\pi}{2}}f^2\cos(a)(de-cf)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}f^2\sin(a)(de-cf)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}}+\frac{f^3\sin\left(a+b(c+dx)^2\right)}{2b^2}-\frac{3f^2(c+dx)(}{}}{}$$

input
```
Int[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

output
```
((-3*f*(d*e - c*f)^2*Cos[a + b*(c + d*x)^2])/(2*b) - (3*f^2*(d*e - c*f)*(c
 + d*x)*Cos[a + b*(c + d*x)^2])/(2*b) - (f^3*(c + d*x)^2*Cos[a + b*(c + d*
x)^2])/(2*b) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[
2/Pi]*(c + d*x)])/(2*b^(3/2)) + ((d*e - c*f)^3*Sqrt[Pi/2]*Cos[a]*FresnelS[
Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/Sqrt[b] + ((d*e - c*f)^3*Sqrt[Pi/2]*Fresnel
C[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/Sqrt[b] - (3*f^2*(d*e - c*f)*Sqrt[
Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b^(3/2)) + (f^3*Si
n[a + b*(c + d*x)^2])/(2*b^2))/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.02

method	result
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^{3}\mathrm{e}^{ia}}{4\sqrt{-ib}\,d}-\frac{if^{3}\mathrm{e}^{ia}c^{3}\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^{4}\sqrt{-ib}}+\frac{3f^{3}\mathrm{e}^{ia}c\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8d^{4}b\sqrt{-ib}}-\frac{3if\,e^{2}\mathrm{e}^{ia}c}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int((f*x+e)^3*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))/(-I*b)^(1/2)/d*Pi^(1/2)*e^
3*exp(I*a)-1/4*I*f^3*exp(I*a)*c^3/d^4*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^
(1/2)*x+I*b*c/(-I*b)^(1/2))+3/8*f^3*exp(I*a)*c/d^4/b*Pi^(1/2)/(-I*b)^(1/2)
*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))-3/4*I*f*e^2*exp(I*a)*c/d^2*Pi^(
1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))+3/4*I*f^2*e*ex
p(I*a)*c^2/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1
/2))-3/8*f^2*e*exp(I*a)/b/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+
I*b*c/(-I*b)^(1/2))+1/4*I*exp(-I*a)*e^3*Pi^(1/2)/d/(I*b)^(1/2)*erf(d*(I*b)
^(1/2)*x+I*b*c/(I*b)^(1/2))-1/4*I*f^3*exp(-I*a)*c^3/d^4*Pi^(1/2)/(I*b)^(1/
2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))-3/8*f^3*exp(-I*a)*c/d^4/b*Pi^(1/
2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))-3/4*I*f*e^2*exp(-I*a
)*c/d^2*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))+3/4*I*
f^2*e*exp(-I*a)*c^2/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*
b)^(1/2))+3/8*f^2*e*exp(-I*a)/b/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)
*x+I*b*c/(I*b)^(1/2))-1/2*f/b*(d^2*f^2*x^2-c*d*f^2*x+3*d^2*e*f*x+c^2*f^2-3
*c*d*e*f+3*d^2*e^2)/d^4*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2*f^3/b^2/d^4*s
in(b*d^2*x^2+2*b*c*d*x+b*c^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx$$

$$=\frac{2\,df^3\sin\left(bd^2x^2+2\,bcdx+bc^2+a\right)+\sqrt{2}(3\,\pi(def^2-cf^3)\cos\left(a\right)+2\,\pi(bd^3e^3-3\,bcd^2e^2f+3\,bc^2def^2-}{}$$

input
```
integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a) + sqrt(2)*(3*pi*(d*e*f
^2 - c*f^3)*cos(a) + 2*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 -
 b*c^3*f^3)*sin(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x
 + c)/d) + sqrt(2)*(2*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 -
b*c^3*f^3)*cos(a) - 3*pi*(d*e*f^2 - c*f^3)*sin(a))*sqrt(b*d^2/pi)*fresnel_
sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3*x^2 + 3*b*d^3*e^2*f
 - 3*b*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2*f^3)*x)*cos(b*
d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^2*d^5)
```

Sympy [F]

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx = \int (e+fx)^3 \sin\left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

input
```
integrate((f*x+e)**3*sin(a+b*(d*x+c)**2),x)
```

output
```
Integral((e + f*x)**3*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 1824, normalized size of antiderivative = 5.35

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output

```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I
*b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c
)/sqrt(-I*b)))*e^3/(sqrt(b)*d) - 3/8*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^
2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
))*sin(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sq
rt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt
(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a)
 + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2
)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x -
I*b*c^2)) - 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) +
e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*
I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))
*c)*e^2*f/(b*d^3*x + b*c*d^2) + 3/8*(4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)
)*sin(a))*b*c*d*x + 4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 -
 sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.53

$$\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx =$$

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3+3i\,def^2-3i\,cf^3\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)\right.}{}}{8\,d^3}$$

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}\left(2\,bd^3e^3-6\,bcd^2e^2f+6\,bc^2def^2-2\,bc^3f^3-3i\,def^2+3i\,cf^3\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(-i\,bd^2f^3\left(x+\frac{c}{d}\right)^2-\right.}{}}{8\,d^3}$$

input
```
integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(-I*sqrt(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*f^2
 - 2*b*c^3*f^3 + 3*I*d*e*f^2 - 3*I*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I
*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b
^2*d^4) + 1)*b) + 2*I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I*c
/d) - 3*b*c*d*f^3*(-I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3*I
*b*c^2*f^3 + f^3)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b^2*d))/d
^3 - 1/8*(I*sqrt(2)*sqrt(pi)*(2*b*d^3*e^3 - 6*b*c*d^2*e^2*f + 6*b*c^2*d*e*
f^2 - 2*b*c^3*f^3 - 3*I*d*e*f^2 + 3*I*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*
(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt
(b^2*d^4) + 1)*b) + 2*I*(-I*b*d^2*f^3*(x + c/d)^2 - 3*b*d^2*e*f^2*(I*x + I
*c/d) - 3*b*c*d*f^3*(-I*x - I*c/d) - 3*I*b*d^2*e^2*f + 6*I*b*c*d*e*f^2 - 3
*I*b*c^2*f^3 - f^3)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b^2*d)
)/d^3
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^2\right)\,(e+f\,x)^3\,dx$$

input
```
int(sin(a + b*(c + d*x)^2)*(e + f*x)^3,x)
```

output
```
int(sin(a + b*(c + d*x)^2)*(e + f*x)^3, x)
```

Reduce [F]

$$\int (e+fx)^3\sin\left(a+b(c+dx)^2\right)\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^3*sin(a+b*(d*x+c)^2),x)
```

output

```
( - 3*cos(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c**2*f**3 + 9*cos(a + b*
c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c*d*e*f**2 + 3*cos(a + b*c**2 + 2*b*c*d*
x + b*d**2*x**2)*b*c*d*f**3*x - 9*cos(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2
)*b*d**2*e**2*f - 9*cos(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*d**2*e*f**
2*x - 3*cos(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*d**2*f**3*x**2 + 24*in
t(x**2/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c**3
*d**3*f**3 - 72*int(x**2/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2
 + 1),x)*b**3*c**2*d**4*e*f**2 + 72*int(x**2/(tan((a + b*c**2 + 2*b*c*d*x
+ b*d**2*x**2)/2)**2 + 1),x)*b**3*c*d**5*e**2*f - 24*int(x**2/(tan((a + b*
c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*d**6*e**3 - 24*int(1/(t
an((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c**5*d*f**3 +
 72*int(1/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**3*c
**4*d**2*e*f**2 - 72*int(1/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)*
*2 + 1),x)*b**3*c**3*d**3*e**2*f + 24*int(1/(tan((a + b*c**2 + 2*b*c*d*x +
 b*d**2*x**2)/2)**2 + 1),x)*b**3*c**2*d**4*e**3 - 18*int(1/(tan((a + b*c**
2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b*c*d*f**3 + 18*int(1/(tan((a +
 b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b*d**2*e*f**2 + 6*sin(a +
 b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b**2*c**4*f**3 - 18*sin(a + b*c**2 + 2*
b*c*d*x + b*d**2*x**2)*b**2*c**3*d*e*f**2 - 6*sin(a + b*c**2 + 2*b*c*d*x +
 b*d**2*x**2)*b**2*c**3*d*f**3*x + 18*sin(a + b*c**2 + 2*b*c*d*x + b*d*...
```

3.166 $\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 256

$$\begin{aligned}
&\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right)\,dx \\
&= -\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} \\
&\quad + \frac{f^2\sqrt{\frac{\pi}{2}}\cos(a)\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3} \\
&\quad + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} \\
&\quad + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^3} \\
&\quad - \frac{f^2\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{2b^{3/2}d^3}
\end{aligned}$$

output
```
-f*(-c*f+d*e)*cos(a+b*(d*x+c)^2)/b/d^3-1/2*f^2*(d*x+c)*cos(a+b*(d*x+c)^2)/
b/d^3+1/4*f^2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c))/b^(3/2)/d^3+1/2*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/
2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(1/2)/d^3+1/2*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))*sin(a)/b^(1/2)/d^3-1/4*f^2*2^(
1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))*sin(a)/b^(3/2)/d^
3
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.59

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$
$$=\frac{-4\sqrt{b}f(2de-cf+dfx)\cos\left(a+b(c+dx)^2\right)+2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\left(2b(de-cf)^2\cos(a)-\right.}{8b^{3/2}d^3}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

output
```
(-4*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[a + b*(c + d*x)^2] + 2*Sqrt[2*Pi]*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - f^2*Sin
[a]) + 2*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(f^2*Cos[a] + 2
*b*(d*e - c*f)^2*Sin[a]))/(8*b^(3/2)*d^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin \left(b(c+dx)^2+a\right) (de-cf)^2+2f(c+dx) \sin \left(b(c+dx)^2+a\right) (de-cf)+f^2(c+dx)^2 \sin \left(b(c+dx)^2+a\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{\frac{\pi}{2}} f^2 \cos(a) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}} f^2 \sin(a) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}} \sin(a)(de-cf)^2 \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}}+\sqrt{\frac{\pi}{2}}}{d^3}$$

input

```
Int[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

output

```
(-((f*(d*e - c*f)*Cos[a + b*(c + d*x)^2])/b) - (f^2*(c + d*x)*Cos[a + b*(c
 + d*x)^2])/(2*b) + (f^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
+ d*x)])/(2*b^(3/2)) + ((d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*S
qrt[2/Pi]*(c + d*x)])/Sqrt[b] + ((d*e - c*f)^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]
*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/Sqrt[b] - (f^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b^(3/2)))/d^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.71

method	result
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e^{2}\mathrm{e}^{ia}}{4\sqrt{-ib}\,d}+\frac{if^{2}\mathrm{e}^{ia}c^{2}\sqrt{\pi}\;\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^{3}\sqrt{-ib}}-\frac{f^{2}\mathrm{e}^{ia}\sqrt{\pi}\;\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{8b\,d^{3}\sqrt{-ib}}-\frac{ife\,\mathrm{e}^{ia}c\sqrt{\pi}\;\mathrm{e}}{}$
default	$-\frac{f^{2}x\cos\left(x^{2}d^{2}b+2cdxb+b\,c^{2}+a\right)}{2d^{2}b}-\frac{f^{2}c\left(-\frac{\cos\left(x^{2}d^{2}b+2cdxb+b\,c^{2}+a\right)}{2d^{2}b}-\frac{c\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{b^{2}c^{2}d^{2}-d^{2}b\left(b\,c^{2}+a\right)}{d^{2}b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\left(b\,d^{2}x+c\right.}{\sqrt{\pi}\,\sqrt{d^{2}b}}\right.\right.}{2}\right.}{d}$
parts	Expression too large to display

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))/(-I*b)^(1/2)/d*Pi^(1/2)*e^
2*exp(I*a)+1/4*I*f^2*exp(I*a)*c^2/d^3*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^
(1/2)*x+I*b*c/(-I*b)^(1/2))-1/8*f^2*exp(I*a)/b/d^3*Pi^(1/2)/(-I*b)^(1/2)*e
rf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))-1/2*I*f*e*exp(I*a)*c/d^2*Pi^(1/2)
/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))+1/4*I*exp(-I*a)*e^
2*Pi^(1/2)/d/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))+1/4*I*f^2*
exp(-I*a)*c^2/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/
2))+1/8*f^2*exp(-I*a)/b/d^3*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c
/(I*b)^(1/2))-1/2*I*f*e*exp(-I*a)*c/d^2*Pi^(1/2)/(I*b)^(1/2)*erf(d*(I*b)^(
1/2)*x+I*b*c/(I*b)^(1/2))+2*(1/2*I*f^2*(1/2*I*x/d^2/b-1/2*I/b/d^3*c)-1/2*e
*f/d^2/b)*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.81

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$

$$=\frac{\sqrt{2}(\pi f^2\cos(a)+2\pi(bd^2e^2-2bcdef+bc^2f^2)\sin(a))\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-\sqrt{2}(\pi f^2\sin(a)-2\pi(b}{}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(pi*f^2*cos(a) + 2*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*s
in(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - sq
rt(2)*(pi*f^2*sin(a) - 2*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a))*
sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*
f^2*x + 2*b*d^2*e*f - b*c*d*f^2)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(
b^2*d^4)
```

Sympy [F]

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx=\int (e+fx)^2\sin\left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

input
```
integrate((f*x+e)**2*sin(a+b*(d*x+c)**2),x)
```

output
```
Integral((e + f*x)**2*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 1038, normalized size of antiderivative = 4.05

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I
*b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c
)/sqrt(-I*b)))*e^2/(sqrt(b)*d) - 1/4*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^
2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
))*sin(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sq
rt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt
(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a)
 + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2
)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x -
I*b*c^2)) - 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) +
e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*
I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))
*c)*e*f/(b*d^3*x + b*c*d^2) + 1/8*(4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*
c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x
^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*
sin(a))*b*c*d*x + 4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 - s
qrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sq...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.35

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$

$$= \frac{\frac{\sqrt{2}\sqrt{\pi}\left(2i\,bd^2e^2-4i\,bcdef+2i\,bc^2f^2-f^2\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right.}}{bd}}{8\,d^2}$$

$$+\frac{\frac{\sqrt{2}\sqrt{\pi}\left(-2i\,bd^2e^2+4i\,bcdef-2i\,bc^2f^2-f^2\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(-i\,bd^2x^2-2i\,bcdx\right.}}{bd}}{8\,d^2}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*(2*I*b*d^2*e^2 - 4*I*b*c*d*e*f + 2*I*b*c^2*f^2 - f^2
)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(
I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(d*f^2*(x + c/d) + 2
*d*e*f - 2*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b*d))/d^2
 + 1/8*(sqrt(2)*sqrt(pi)*(-2*I*b*d^2*e^2 + 4*I*b*c*d*e*f - 2*I*b*c^2*f^2 -
 f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*
e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(d*f^2*(x + c/d)
+ 2*d*e*f - 2*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))
/d^2
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^2\right)\,(e+f\,x)^2\,dx$$

input
```
int(sin(a + b*(c + d*x)^2)*(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^2)*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2\sin\left(a+b(c+dx)^2\right)\,dx$$

$$= \frac{24b^2c^3d^2efx-8b^2cd^4efx^3-12\sin\left(bd^2x^2+2bcdx+bc^2+a\right)bcd^2efx-3df^2x+6def+6\Bigg(\int \frac{}{\tan(\frac{1}{2}bd}}{}$$

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^2),x)
```

output

```
(3*cos(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*c*f**2 - 6*cos(a + b*c**2 + 2
*b*c*d*x + b*d**2*x**2)*d*e*f - 3*cos(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2
)*d*f**2*x - 24*int(x**2/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2
 + 1),x)*b**2*c**2*d**3*f**2 + 48*int(x**2/(tan((a + b*c**2 + 2*b*c*d*x +
b*d**2*x**2)/2)**2 + 1),x)*b**2*c*d**4*e*f - 24*int(x**2/(tan((a + b*c**2
+ 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**2*d**5*e**2 + 24*int(1/(tan((a
 + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**2*c**4*d*f**2 - 48*i
nt(1/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),x)*b**2*c**3*d
**2*e*f + 24*int(1/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/2)**2 + 1),
x)*b**2*c**2*d**3*e**2 + 6*int(1/(tan((a + b*c**2 + 2*b*c*d*x + b*d**2*x**
2)/2)**2 + 1),x)*d*f**2 - 6*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c*
*3*f**2 + 12*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c**2*d*e*f + 6*si
n(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*b*c**2*d*f**2*x - 6*sin(a + b*c**2
 + 2*b*c*d*x + b*d**2*x**2)*b*c*d**2*e**2 - 12*sin(a + b*c**2 + 2*b*c*d*x
+ b*d**2*x**2)*b*c*d**2*e*f*x + 6*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2
)*b*d**3*e**2*x - 12*b**2*c**4*d*f**2*x + 24*b**2*c**3*d**2*e*f*x - 12*b**
2*c**2*d**3*e**2*x + 4*b**2*c**2*d**3*f**2*x**3 - 8*b**2*c*d**4*e*f*x**3 +
 4*b**2*d**5*e**2*x**3 - 3*c*f**2 + 6*d*e*f - 3*d*f**2*x)/(6*b*d**3)
```

3.167 $\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx = -\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d^2}$$

output
```
-1/2*f*cos(a+b*(d*x+c)^2)/b/d^2+1/2*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*cos(a)*Fre
snelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(1/2)/d^2+1/2*(-c*f+d*e)*2^(1/2)
*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))*sin(a)/b^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$$
$$=\frac{-f\cos\left(a+b(c+dx)^2\right)+\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+\sqrt{b}(de-cf)\sqrt{2\pi}\,\text{Fresn}}{2bd^2}$$

input
```
Integrate[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

output
```
(-(f*Cos[a + b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fresn
elS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Fresnel
C[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b*d^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$$
$$\Big\downarrow \text{3914}$$
$$\frac{\int \left((de-cf)\sin\left(b(c+dx)^2+a\right)+f(c+dx)\sin\left(b(c+dx)^2+a\right)\right)d(c+dx)}{d^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}}-\frac{f\cos\left(a+b(c+dx)^2\right)}{2b}}{d^2}$$

input
```
Int[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

output
```
(-1/2*(f*Cos[a + b*(c + d*x)^2])/b + ((d*e - c*f)*Sqrt[Pi/2]*Cos[a]*Fresne
lS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/Sqrt[b] + ((d*e - c*f)*Sqrt[Pi/2]*Fresne
lC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/Sqrt[b])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.71

method	result
risch	$\frac{i\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)\sqrt{\pi}\,e\,\mathrm{e}^{ia}}{4\sqrt{-ib}\,d}-\frac{if\,\mathrm{e}^{ia}c\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^{2}\sqrt{-ib}}+\frac{i\mathrm{e}^{-ia}e\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}-\frac{if\,\mathrm{e}^{-ia}c\sqrt{\pi}\,\mathrm{erf}\left(d\sqrt{\right.}}{4d^{2}\sqrt{ib}}$
default	$-\frac{f\cos\left(x^{2}d^{2}b+2cdxb+b\,c^{2}+a\right)}{2d^{2}b}-\frac{fc\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{b^{2}c^{2}d^{2}-d^{2}b\left(b\,c^{2}+a\right)}{d^{2}b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2}x+cdb\right)}{\sqrt{\pi}\,\sqrt{d^{2}b}}\right)-\sin\left(\frac{b^{2}c^{2}d^{2}-d^{2}b\left(b\,c^{2}+a\right)}{d^{2}b}\right.\right.}{2d\sqrt{d^{2}b}}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\cos\left(\frac{b^{2}c^{2}d^{2}-d^{2}b\left(b\,c^{2}+a\right)}{d^{2}b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2}x+cdb\right)}{\sqrt{\pi}\,\sqrt{d^{2}b}}\right)fx}{2\sqrt{d^{2}b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sin\left(\frac{b^{2}c^{2}d^{2}-d^{2}b\left(b\,c^{2}+a\right)}{d^{2}b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\left(b\,d^{2}x+cdb\right)}{\sqrt{\pi}\,\sqrt{d^{2}b}}\right.}{2\sqrt{d^{2}b}}$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I*b)^(1/2))/(-I*b)^(1/2)/d*Pi^(1/2)*e*
exp(I*a)-1/4*I*f*exp(I*a)*c/d^2*Pi^(1/2)/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*
x+I*b*c/(-I*b)^(1/2))+1/4*I*exp(-I*a)*e*Pi^(1/2)/d/(I*b)^(1/2)*erf(d*(I*b)
^(1/2)*x+I*b*c/(I*b)^(1/2))-1/4*I*f*exp(-I*a)*c/d^2*Pi^(1/2)/(I*b)^(1/2)*e
rf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/2))-1/2*f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+
b*c^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int (e+fx)\sin \left(a+b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}(de-cf)\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}(de-cf)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)\sin(a)-df\cos(b}{2\,bd^3}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*cos(a)*fresnel_sin(sqrt(2)*sqrt
(b*d^2/pi)*(d*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*fresnel_co
s(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d)*sin(a) - d*f*cos(b*d^2*x^2 + 2*b*c*d
*x + b*c^2 + a))/(b*d^3)
```

Sympy [F]

$$\int (e+fx)\sin \left(a+b(c+dx)^2\right)\,dx = \int (e+fx)\sin \left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)**2),x)
```

output
```
Integral((e + f*x)*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.96

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\,\cos\left(a\right)+(i-1)\,\sin\left(a\right)\right)\mathrm{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+\left(-(i-1)\,\cos\left(a\right)+(i+1)\,\sin\left(a\right)\right)\mathrm{erf}\left(\frac{i\right.}{}\right.}{8\,\sqrt{b}d}$$

$$-\frac{\left(2\left(\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)\cos\left(a\right)-\left(-i\,e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+i\,e^{\left(-i\,bd^2x^2-2i\,bcdx\right.}\right.\right.\right.}{}$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output

```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I
*b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c
)/sqrt(-I*b)))*e/(sqrt(b)*d) - 1/8*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b
*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*
x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
*sin(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt
(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2
)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) +
 ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
 - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2)) - 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^
(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*
b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c
)*f/(b*d^3*x + b*c*d^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.02

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$$
$$= -\frac{\frac{\sqrt{2}\sqrt{\pi}(-i\,de+i\,cf)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2+i\,a\right)}}{bd}}{4\,d}$$
$$-\frac{\frac{\sqrt{2}\sqrt{\pi}(i\,de-i\,cf)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2-i\,a\right)}}{bd}}{4\,d}$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

output

```
-1/4*(sqrt(2)*sqrt(pi)*(-I*d*e + I*c*f)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b
*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2
*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b*d))/d - 1
/4*(sqrt(2)*sqrt(pi)*(I*d*e - I*c*f)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2
/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4
) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^2\right)\,\left(e+f\,x\right)\,dx$$

input

```
int(sin(a + b*(c + d*x)^2)*(e + f*x),x)
```

output

```
int(sin(a + b*(c + d*x)^2)*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx = \left(\int \sin\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)dx\right)e + \left(\int \sin\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)xdx\right)f$$

input

```
int((f*x+e)*sin(a+b*(d*x+c)^2),x)
```

output

```
int(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2),x)*e + int(sin(a + b*c**2 +
2*b*c*d*x + b*d**2*x**2)*x,x)*f
```

3.168 $\int \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \sin\left(a+b(c+dx)^2\right)\,dx=\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d}$$

output

```
1/2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))/b^(
1/2)/d+1/2*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c))*sin
(a)/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \sin\left(a+b(c+dx)^2\right)\,dx=\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)\right)}{\sqrt{b}d}$$

input
```
Integrate[Sin[a + b*(c + d*x)^2],x]
```

output
```
(Sqrt[Pi/2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + FresnelC[Sqrt
[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a]))/(Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+b(c+dx)^2\right) dx$$

$$\Big\downarrow \text{3834}$$

$$\sin(a)\int \cos \left(b(c+dx)^2\right) dx+\cos(a)\int \sin \left(b(c+dx)^2\right) dx$$

$$\Big\downarrow \text{3832}$$

$$\sin(a)\int \cos \left(b(c+dx)^2\right) dx+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

input
```
Int[Sin[a + b*(c + d*x)^2],x]
```

output
```
(Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d) + (
Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d)
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

method	result	siz
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-i a}\,\mathrm{erf}\left(d\sqrt{i b}\,x+\frac{i b c}{\sqrt{i b}}\right)}{4 d\sqrt{i b}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{i a}\,\mathrm{erf}\left(-d\sqrt{-i b}\,x+\frac{i b c}{\sqrt{-i b}}\right)}{4 d\sqrt{-i b}}$	87
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{b^{2} c^{2} d^{2}-d^{2} b\left(b\,c^{2}+a\right)}{d^{2} b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right)-\sin\left(\frac{b^{2} c^{2} d^{2}-d^{2} b\left(b\,c^{2}+a\right)}{d^{2} b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\left(b\,d^{2} x+c d b\right)}{\sqrt{\pi}\,\sqrt{d^{2} b}}\right)\right)}{2\sqrt{d^{2} b}}$	13

input
```
int(sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*exp(-I*a)/d/(I*b)^(1/2)*erf(d*(I*b)^(1/2)*x+I*b*c/(I*b)^(1/
2))+1/4*I*Pi^(1/2)*exp(I*a)/d/(-I*b)^(1/2)*erf(-d*(-I*b)^(1/2)*x+I*b*c/(-I
*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \sin \left(a+b(c+dx)^2\right) \, dx = \frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\cos (a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)\sin (a)}{2\,bd^2}$$

input
```
integrate(sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(
d*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)
*(d*x + c)/d)*sin(a))/(b*d^2)
```

Sympy [F]

$$\int \sin \left(a+b(c+dx)^2\right) \, dx = \int \sin \left(a+b(c+dx)^2\right) dx$$

input
```
integrate(sin(a+b*(d*x+c)**2),x)
```

output
```
Integral(sin(a + b*(c + d*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \sin \left(a+b(c+dx)^2\right) \, dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos (a)+(i-1)\sin (a)\right)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+\left(-(i-1)\cos (a)+(i+1)\sin (a)\right)\operatorname{erf}\left(\right.\right.}{8\sqrt{b}d}$$

input

```
integrate(sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output

```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I
*b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c
)/sqrt(-I*b)))/(sqrt(b)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

$$\int \sin\left(a+b(c+dx)^2\right)\,dx = \frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia)}}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)} - \frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia)}}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}$$

input

```
integrate(sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

output

```
1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4
) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4
*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
1)*(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \sin\left(a+b(c+dx)^2\right)\,dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{b\,d^2}}\,\left(b\,x\,d^2+b\,c\,d\right)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2} + \frac{\sqrt{2}\,\sqrt{\pi}\,\sin(a)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{b\,d^2}}\,\left(b\,x\,d^2+b\,c\,d\right)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}$$

input
```
int(sin(a + b*(c + d*x)^2),x)
```

output
```
(2^(1/2)*pi^(1/2)*cos(a)*fresnels((2^(1/2)*(1/(b*d^2))^(1/2)*(b*c*d + b*d^
2*x))/pi^(1/2))*(1/(b*d^2))^(1/2))/2 + (2^(1/2)*pi^(1/2)*sin(a)*fresnelc((
2^(1/2)*(1/(b*d^2))^(1/2)*(b*c*d + b*d^2*x))/pi^(1/2))*(1/(b*d^2))^(1/2))/
2
```

Reduce [F]

$$\int \sin \left(a+b(c+dx)^2\right)\,dx=\int \sin \left(b\,d^2x^2+2bcdx+b\,c^2+a\right)dx$$

input
```
int(sin(a+b*(d*x+c)^2),x)
```

output
```
int(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2),x)
```

3.169 $\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(a+b(c+dx)^2\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 9.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx} \, dx$$

$$\Big\downarrow \text{ 3918}$$

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}dx$$

input
```
Int[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(dx+c)^2\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sin \left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^2b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\sin \left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left(a+bc^2+2bcdx+bd^2x^2\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**2)/(f*x+e),x)
```

output
```
Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^2b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^2b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 39.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^2\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b*(c + d*x)^2)/(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^2)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sin \left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin \left(b\,d^2x^2+2bcdx+b\,c^2+a\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x),x)
```

3.170 $\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \ 3918$$

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b\left(dx+c\right)^{2}\right)}{\left(fx+e\right)^{2}}dx$$

input
```
int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^2b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^2*x^2 + 2*e*f*x + e^2),
 x)
```

Sympy [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2}\,dx = \int \frac{\sin(a+bc^2+2bcdx+bd^2x^2)}{(e+fx)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**2)/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^2b+a\right)}{(fx+e)^2}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

output

```
integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 6.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^2b+a\right)}{(fx+e)^2}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

output

```
integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^2\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input

```
int(sin(a + b*(c + d*x)^2)/(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^2)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\sin \left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(b\,d^2x^2+2bcdx+b\,c^2+a\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e**2 + 2*e*f*x + f**2*x**2)
,x)
```

3.171 $\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 434

$$
\begin{aligned}
\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right) \, dx = & -\frac{f^2(de-cf) \cos \left(a+b(c+dx)^3\right)}{bd^4} \\
& -\frac{f^3(c+dx) \cos \left(a+b(c+dx)^3\right)}{3bd^4} \\
& -\frac{e^{ia} f^3(c+dx) \Gamma \left(\frac{1}{3},-ib(c+dx)^3\right)}{18bd^4 \sqrt[3]{-ib(c+dx)^3}} \\
& +\frac{ie^{ia}(de-cf)^3(c+dx) \Gamma \left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^4 \sqrt[3]{-ib(c+dx)^3}} \\
& -\frac{e^{-ia} f^3(c+dx) \Gamma \left(\frac{1}{3},ib(c+dx)^3\right)}{18bd^4 \sqrt[3]{ib(c+dx)^3}} \\
& -\frac{ie^{-ia}(de-cf)^3(c+dx) \Gamma \left(\frac{1}{3},ib(c+dx)^3\right)}{6d^4 \sqrt[3]{ib(c+dx)^3}} \\
& +\frac{ie^{ia} f(de-cf)^2(c+dx)^2 \Gamma \left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4 \left(-ib(c+dx)^3\right)^{2/3}} \\
& -\frac{ie^{-ia} f(de-cf)^2(c+dx)^2 \Gamma \left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4 \left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}
$$

output
```
-f^2*(-c*f+d*e)*cos(a+b*(d*x+c)^3)/b/d^4-1/3*f^3*(d*x+c)*cos(a+b*(d*x+c)^3
)/b/d^4-1/18*exp(I*a)*f^3*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/b/d^4/(-I*b*(d
*x+c)^3)^(1/3)+1/6*I*exp(I*a)*(-c*f+d*e)^3*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^
3)/d^4/(-I*b*(d*x+c)^3)^(1/3)-1/18*f^3*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/b/
d^4/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)-1/6*I*(-c*f+d*e)^3*(d*x+c)*GAMMA(1/3,I*
b*(d*x+c)^3)/d^4/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)+1/2*I*exp(I*a)*f*(-c*f+d*e
)^2*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/d^4/(-I*b*(d*x+c)^3)^(2/3)-1/2*I*f
*(-c*f+d*e)^2*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^4/exp(I*a)/(I*b*(d*x+c)
^3)^(2/3)
```

Mathematica [A] (verified)

Time = 18.07 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.81

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right)\,dx$$

$$= \frac{-6f^2(3de-2cf+dfx)\cos\left(a+bc^3\right)\cos\left(bdx\left(3c^2+3cdx+d^2x^2\right)\right)+\frac{(c+dx)\left(-\left(\left(f^3+3ib(de-cf)^3\right)\sqrt[3]{ib(c+dx}\right.\right.}{}}{}$$

input
```
Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```

output
```
(-6*f^2*(3*d*e - 2*c*f + d*f*x)*Cos[a + b*c^3]*Cos[b*d*x*(3*c^2 + 3*c*d*x
+ d^2*x^2)] + ((c + d*x)*(-((f^3 + (3*I)*b*(d*e - c*f)^3)*(I*b*(c + d*x)^3
)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3]) - (9*I)*b*f*(d*e - c*f)^2*(c + d*x)*G
amma[2/3, I*b*(c + d*x)^3])*(Cos[a] - I*Sin[a]))/(I*b*(c + d*x)^3)^(2/3) +
 ((c + d*x)*(-((f^3 - (3*I)*b*(d*e - c*f)^3)*((-I)*b*(c + d*x)^3)^(1/3)*Ga
mma[1/3, (-I)*b*(c + d*x)^3]) + (9*I)*b*f*(d*e - c*f)^2*(c + d*x)*Gamma[2/
3, (-I)*b*(c + d*x)^3])*(Cos[a] + I*Sin[a]))/((-I)*b*(c + d*x)^3)^(2/3) +
6*f^2*(3*d*e - 2*c*f + d*f*x)*Sin[a + b*c^3]*Sin[b*d*x*(3*c^2 + 3*c*d*x +
d^2*x^2)])/(18*b*d^4)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right) \, dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin\left(b(c+dx)^3+a\right)(de-cf)^3+3f(c+dx)\sin\left(b(c+dx)^3+a\right)(de-cf)^2+3f^2(c+dx)^2\sin\left(b(c+dx)^3+\right.\right.}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{b}+\frac{ie^{ia}f(c+dx)^2(de-cf)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{2\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{2\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de-c}{6\sqrt[3]{-ib}}$$

input

```
Int[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```

output

```
(-((f^2*(d*e - c*f)*Cos[a + b*(c + d*x)^3])/b) - (f^3*(c + d*x)*Cos[a + b*
(c + d*x)^3])/(3*b) - (E^(I*a)*f^3*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3
])/(18*b*((-I)*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*(d*e - c*f)^3*(c + d
*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/((-I)*b*(c + d*x)^3)^(1/3) - (f^3*(c +
 d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(18*b*E^(I*a)*(I*b*(c + d*x)^3)^(1/3))
- ((I/6)*(d*e - c*f)^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(E^(I*a)*(I*
b*(c + d*x)^3)^(1/3)) + ((I/2)*E^(I*a)*f*(d*e - c*f)^2*(c + d*x)^2*Gamma[2
/3, (-I)*b*(c + d*x)^3])/((-I)*b*(c + d*x)^3)^(2/3) - ((I/2)*f*(d*e - c*f)
^2*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(E^(I*a)*(I*b*(c + d*x)^3)^(2/
3)))/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)^3 \sin\left(a+b(dx+c)^3\right) dx$$

input
```
int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
```

output
```
int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.36

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right)\, dx =$$

$$-\frac{(i\,bd^3)^{\frac{2}{3}}\,((3\,bd^3e^3-9\,bcd^2e^2f+9\,bc^2def^2-3\,bc^3f^3-i\,f^3)\cos(a)-(3i\,bd^3e^3-9i\,bcd^2e^2f+9i\,bc^2de}{}$$

input
```
integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output

```
-1/18*((I*b*d^3)^(2/3)*((3*b*d^3*e^3 - 9*b*c*d^2*e^2*f + 9*b*c^2*d*e*f^2 -
 3*b*c^3*f^3 - I*f^3)*cos(a) - (3*I*b*d^3*e^3 - 9*I*b*c*d^2*e^2*f + 9*I*b*
c^2*d*e*f^2 - 3*I*b*c^3*f^3 + f^3)*sin(a))*gamma(1/3, I*b*d^3*x^3 + 3*I*b*
c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*((3*b*d^3*e^3 - 9*
b*c*d^2*e^2*f + 9*b*c^2*d*e*f^2 - 3*b*c^3*f^3 + I*f^3)*cos(a) - (-3*I*b*d^
3*e^3 + 9*I*b*c*d^2*e^2*f - 9*I*b*c^2*d*e*f^2 + 3*I*b*c^3*f^3 + f^3)*sin(a
))*gamma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) +
9*(I*b*d^3)^(1/3)*((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*cos(a) +
(-I*b*d^3*e^2*f + 2*I*b*c*d^2*e*f^2 - I*b*c^2*d*f^3)*sin(a))*gamma(2/3, I*
b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + 9*(-I*b*d^3)^(1/3
)*((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*cos(a) + (I*b*d^3*e^2*f -
 2*I*b*c*d^2*e*f^2 + I*b*c^2*d*f^3)*sin(a))*gamma(2/3, -I*b*d^3*x^3 - 3*I*
b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + 6*(b*d^3*f^3*x + 3*b*d^3*e*f^2 -
2*b*c*d^2*f^3)*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(
b^2*d^6)
```

Sympy [F]

$$\int (e+fx)^3\sin\left(a+b(c+dx)^3\right)\,dx$$
$$=\int (e+fx)^3\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)dx$$

input

```
integrate((f*x+e)**3*sin(a+b*(d*x+c)**3),x)
```

output

```
Integral((e + f*x)**3*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*
d**3*x**3), x)
```

Maxima [F]

$$\int (e+fx)^3\sin\left(a+b(c+dx)^3\right)\,dx=\int (fx+e)^3\sin\left((dx+c)^3b+a\right)\,dx$$

input
```
integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int (e+fx)^3\sin\left(a+b(c+dx)^3\right)\,dx=\int (fx+e)^3\sin\left((dx+c)^3b+a\right)\,dx$$

input
```
integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3\sin\left(a+b(c+dx)^3\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)^3\,dx$$

input
```
int(sin(a + b*(c + d*x)^3)*(e + f*x)^3,x)
```

output
```
int(sin(a + b*(c + d*x)^3)*(e + f*x)^3, x)
```

Reduce [F]

$$\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right) \, dx = \text{too large to display}$$

input
```
int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
```

output
```
(16*cos(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*tan((a
+ b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/2)**2*c**2*f**3 -
 24*cos(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*tan((a
+ b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/2)**2*c*d*e*f**2
- 8*cos(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*tan((a
+ b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/2)**2*c*d*f**3*x
+ 16*cos(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*c**2*f
**3 - 24*cos(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*c*
d*e*f**2 - 8*cos(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3
)*c*d*f**3*x - 72*int(x**3/(tan((a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x*
*2 + b*d**3*x**3)/2)**2 + 1),x)*tan((a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**
2*x**2 + b*d**3*x**3)/2)**2*b**2*c**4*d**4*f**3 + 216*int(x**3/(tan((a + b
*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/2)**2 + 1),x)*tan((a
 + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/2)**2*b**2*c**2*
d**6*e**2*f - 144*int(x**3/(tan((a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x*
*2 + b*d**3*x**3)/2)**2 + 1),x)*tan((a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**
2*x**2 + b*d**3*x**3)/2)**2*b**2*c*d**7*e**3 - 72*int(x**3/(tan((a + b*c**
3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/2)**2 + 1),x)*b**2*c**4*
d**4*f**3 + 216*int(x**3/(tan((a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2
 + b*d**3*x**3)/2)**2 + 1),x)*b**2*c**2*d**6*e**2*f - 144*int(x**3/(tan...
```

3.172 $\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 280

$$\begin{aligned}\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = & -\frac{f^2 \cos \left(a+b(c+dx)^3\right)}{3bd^3} \\ & + \frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3 \sqrt[3]{-ib(c+dx)^3}} \\ & - \frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^3 \sqrt[3]{ib(c+dx)^3}} \\ & + \frac{ie^{ia}f(de-cf)(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{3d^3 \left(-ib(c+dx)^3\right)^{2/3}} \\ & - \frac{ie^{-ia}f(de-cf)(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{3d^3 \left(ib(c+dx)^3\right)^{2/3}}\end{aligned}$$

output

```
-1/3*f^2*cos(a+b*(d*x+c)^3)/b/d^3+1/6*I*exp(I*a)*(-c*f+d*e)^2*(d*x+c)*GAMM
A(1/3,-I*b*(d*x+c)^3)/d^3/(-I*b*(d*x+c)^3)^(1/3)-1/6*I*(-c*f+d*e)^2*(d*x+c
)*GAMMA(1/3,I*b*(d*x+c)^3)/d^3/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)+1/3*I*exp(I*
a)*f*(-c*f+d*e)*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/d^3/(-I*b*(d*x+c)^3)^(
2/3)-1/3*I*f*(-c*f+d*e)*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^3/exp(I*a)/(I
*b*(d*x+c)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 12.59 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07

$$\int (e+fx)^2\sin \left(a+b(c+dx)^3\right)\,dx$$
$$=\frac{-\frac{2f^2\cos(a+bc^3)\cos\left(bdx\left(3c^2+3cdx+d^2x^2\right)\right)}{b}+\frac{(de-cf)(c+dx)\left((de-cf)\sqrt[3]{ib(c+dx)^3}\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)+2f(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)\right)}{(ib(c+dx)^3)^{2/3}}}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

output
```
((-2*f^2*Cos[a + b*c^3]*Cos[b*d*x*(3*c^2 + 3*c*d*x + d^2*x^2)])/b + ((d*e
- c*f)*(c + d*x)*((d*e - c*f)*(I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c +
d*x)^3] + 2*f*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^3])*((-I)*Cos[a] - Sin[a]
))/(I*b*(c + d*x)^3)^(2/3) + ((d*e - c*f)*(c + d*x)*((d*e - c*f)*((-I)*b*(
c + d*x)^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x)^3] + 2*f*(c + d*x)*Gamma[2/3
, (-I)*b*(c + d*x)^3])*(I*Cos[a] - Sin[a]))/((-I)*b*(c + d*x)^3)^(2/3) + (
2*f^2*Sin[a + b*c^3]*Sin[b*d*x*(3*c^2 + 3*c*d*x + d^2*x^2)])/b)/(6*d^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\sin \left(a+b(c+dx)^3\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin \left(b(c+dx)^3+a\right)(de-cf)^2+2f(c+dx)\sin \left(b(c+dx)^3+a\right)(de-cf)+f^2(c+dx)^2\sin \left(b(c+dx)^3+a\right.\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{ie^{ia}f(c+dx)^2(de-cf)\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{3(-ib(c+dx)^3)^{2/3}} - \frac{ie^{-ia}f(c+dx)^2(de-cf)\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{3(ib(c+dx)^3)^{2/3}} + \frac{ie^{ia}(c+dx)(de-cf)^2\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6\sqrt[3]{-ib(c+dx)^3}} - \frac{ie^{-ia}(c+dx)}{6}}{d^3}$$

input
```
Int[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

output
```
(-1/3*(f^2*Cos[a + b*(c + d*x)^3])/b + ((I/6)*E^(I*a)*(d*e - c*f)^2*(c + d
*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/((-I)*b*(c + d*x)^3)^(1/3) - ((I/6)*(d
*e - c*f)^2*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(E^(I*a)*(I*b*(c + d*x)
^3)^(1/3)) + ((I/3)*E^(I*a)*f*(d*e - c*f)*(c + d*x)^2*Gamma[2/3, (-I)*b*(c
 + d*x)^3])/((-I)*b*(c + d*x)^3)^(2/3) - ((I/3)*f*(d*e - c*f)*(c + d*x)^2*
Gamma[2/3, I*b*(c + d*x)^3])/(E^(I*a)*(I*b*(c + d*x)^3)^(2/3)))/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+b(dx+c)^3\right) dx$$

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)
```

output
```
int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.46

$$\int (e+fx)^2\sin\left(a+b(c+dx)^3\right)\,dx =$$

$$-\frac{2\,d^2f^2\cos\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)+\left(i\,bd^3\right)^{\frac{2}{3}}\left(\left(d^2e^2-2\,cdef+c^2f^2\right)\cos\left(a\right)-\left(i\,d^2e^2-\right.\right.}{}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*(2*d^2*f^2*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) +
 (I*b*d^3)^(2/3)*((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*cos(a) - (I*d^2*e^2 - 2*
I*c*d*e*f + I*c^2*f^2)*sin(a))*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 +
3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*((d^2*e^2 - 2*c*d*e*f + c^2*f^
2)*cos(a) - (-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2)*sin(a))*gamma(1/3, -I*b
*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + 2*(I*b*d^3)^(1/3)*
((d^2*e*f - c*d*f^2)*cos(a) + (-I*d^2*e*f + I*c*d*f^2)*sin(a))*gamma(2/3,
I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + 2*(-I*b*d^3)^(1
/3)*((d^2*e*f - c*d*f^2)*cos(a) + (I*d^2*e*f - I*c*d*f^2)*sin(a))*gamma(2/
3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^5)
```

Sympy [F]

$$\int (e+fx)^2\sin\left(a+b(c+dx)^3\right)\,dx$$
$$=\int\left(e+fx\right)^2\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)dx$$

input
```
integrate((f*x+e)**2*sin(a+b*(d*x+c)**3),x)
```

output
```
Integral((e + f*x)**2*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*
d**3*x**3), x)
```

Maxima [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \int (fx+e)^2 \sin \left((dx+c)^3 b+a\right) \, dx$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \int (fx+e)^2 \sin \left((dx+c)^3 b+a\right) \, dx$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^3\right) \, dx = \int \sin \left(a+b\,(c+d\,x)^3\right) \, (e+f\,x)^2 \, dx$$

input
```
int(sin(a + b*(c + d*x)^3)*(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^3)*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2\sin\left(a+b(c+dx)^3\right)\,dx = \left(\int \sin\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)dx\right)e^2 + \left(\int \sin\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)x^2dx\right)f^2 + 2\left(\int \sin\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)xdx\right)ef$$

input

```
int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)
```

output

```
int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3),x)*e**2
 + int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*x**2
,x)*f**2 + 2*int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*
x**3)*x,x)*e*f
```

3.173 $\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 235

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx &= \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}} \\ &\quad - \frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}} \\ &\quad + \frac{ie^{ia}f(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{6d^2\left(-ib(c+dx)^3\right)^{2/3}} \\ &\quad - \frac{ie^{-ia}f(c+dx)^2\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)}{6d^2\left(ib(c+dx)^3\right)^{2/3}}\end{aligned}$$

output

```
1/6*I*exp(I*a)*(-c*f+d*e)*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/d^2/(-I*b*(d*x
+c)^3)^(1/3)-1/6*I*(-c*f+d*e)*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/d^2/exp(I*a
)/(I*b*(d*x+c)^3)^(1/3)+1/6*I*exp(I*a)*f*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^
3)/d^2/(-I*b*(d*x+c)^3)^(2/3)-1/6*I*f*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d
^2/exp(I*a)/(I*b*(d*x+c)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 10.85 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.62

$$
\begin{aligned}
&\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx\\
&=-\frac{ie\cos(a)\left(-\frac{(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{3\sqrt[3]{-ib(c+dx)^3}}+\frac{(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{3\sqrt[3]{ib(c+dx)^3}}\right)}{2d}\\
&\quad+\frac{e\left(-\frac{(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{3\sqrt[3]{-ib(c+dx)^3}}-\frac{(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{3\sqrt[3]{ib(c+dx)^3}}\right)\sin(a)}{2d}\\
&\quad+\frac{f(c+dx)\left(c\sqrt[3]{-ib(c+dx)^3}\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)-(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)\right)(-i\cos(a)+\sin(a))}{6d^2\left(-ib(c+dx)^3\right)^{2/3}}\\
&\quad+\frac{f(c+dx)\left(c\sqrt[3]{ib(c+dx)^3}\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)-(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^3\right)\right)(i\cos(a)+\sin(a))}{6d^2\left(ib(c+dx)^3\right)^{2/3}}
\end{aligned}
$$

input

```
Integrate[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
```

output

```
((-1/2*I)*e*Cos[a]*(-1/3*((c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/((-I)*
b*(c + d*x)^3)^(1/3) + ((c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(3*(I*b*(c
+ d*x)^3)^(1/3))))/d + (e*(-1/3*((c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])
/((-I)*b*(c + d*x)^3)^(1/3) - ((c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(3*(
I*b*(c + d*x)^3)^(1/3)))*Sin[a])/(2*d) + (f*(c + d*x)*(c*((-I)*b*(c + d*x)
^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x)^3] - (c + d*x)*Gamma[2/3, (-I)*b*(c
+ d*x)^3])*((-I)*Cos[a] + Sin[a]))/(6*d^2*((-I)*b*(c + d*x)^3)^(2/3)) + (f
*(c + d*x)*(c*(I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3] - (c + d
*x)*Gamma[2/3, I*b*(c + d*x)^3])*(I*Cos[a] + Sin[a]))/(6*d^2*(I*b*(c + d*x
)^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left((de-cf)\sin\left(b(c+dx)^3+a\right)+f(c+dx)\sin\left(b(c+dx)^3+a\right)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{ie^{ia}(c+dx)(de-cf)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)(de-cf)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6\sqrt[3]{ib(c+dx)^3}}+\frac{ie^{ia}f(c+dx)^2\Gamma\left(\frac{2}{3},-ib(c+dx)^3\right)}{6\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2\Gamma\left(\frac{2}{3},ib(c\right.}{6\left(ib(c+dx)^3\right)^{2/3}}}{d^2}$$

input

```
Int[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
```

output

```
(((I/6)*E^(I*a)*(d*e - c*f)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/((-I
)*b*(c + d*x)^3)^(1/3) - ((I/6)*(d*e - c*f)*(c + d*x)*Gamma[1/3, I*b*(c +
d*x)^3])/(E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*f*(c + d*x)^2*
Gamma[2/3, (-I)*b*(c + d*x)^3])/((-I)*b*(c + d*x)^3)^(2/3) - ((I/6)*f*(c +
 d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(E^(I*a)*(I*b*(c + d*x)^3)^(2/3)))/d^
2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)\sin\left(a+b(dx+c)^3\right)dx$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

output
```
int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.12

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx =$$

$$-\frac{(i\,bd^3)^{\frac{2}{3}}\left((de-cf)\cos(a)-(i\,de-i\,cf)\sin(a)\right)\Gamma\left(\frac{1}{3}, i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+(-i\,bd^3}{}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*((I*b*d^3)^(2/3)*((d*e - c*f)*cos(a) - (I*d*e - I*c*f)*sin(a))*gamma(
1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)
^(2/3)*((d*e - c*f)*cos(a) - (-I*d*e + I*c*f)*sin(a))*gamma(1/3, -I*b*d^3*
x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + (I*b*d^3)^(1/3)*(d*f*co
s(a) - I*d*f*sin(a))*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*
d*x + I*b*c^3) + (-I*b*d^3)^(1/3)*(d*f*cos(a) + I*d*f*sin(a))*gamma(2/3, -
I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^4)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int (e+fx)\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)**3),x)
```

output
```
Integral((e + f*x)*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**
3*x**3), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int (fx+e)\sin\left((dx+c)^3b+a\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int (fx+e)\sin\left((dx+c)^3b+a\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)\,dx$$

input
```
int(sin(a + b*(c + d*x)^3)*(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^3)*(e + f*x), x)
```

Reduce [F]

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx&=\left(\int \sin\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)dx\right)e\\&\quad+\left(\int \sin\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3\right.\right.\\&\qquad\left.\left.+a\right)xdx\right)f\end{aligned}$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

output
```
int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3),x)*e +
int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*x,x)*f
```

3.174 $\int \sin \left (a+b(c+d x)^3\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \sin \left (a+b(c+d x)^3\right) \, dx=\frac {i e^{i a} (c+d x) \Gamma \left (\frac {1}{3},-i b (c+d x)^3\right)}{6 d \sqrt [3]{-i b (c+d x)^3}}-\frac {i e^{-i a} (c+d x) \Gamma \left (\frac {1}{3},i b (c+d x)^3\right)}{6 d \sqrt [3]{i b (c+d x)^3}}$$

output
```
1/6*I*exp(I*a)*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/d/(-I*b*(d*x+c)^3)^(1/3)-
1/6*I*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/d/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \sin \left (a+b(c+d x)^3\right) \, dx =\frac {i (c+d x) \left (-\sqrt [3]{-i b (c+d x)^3} \Gamma \left (\frac {1}{3},i b (c+d x)^3\right) (\cos (a)-i \sin (a))+\sqrt [3]{i b (c+d x)^3} \Gamma \left (\frac {1}{3},-i b (c+d x)^3\right) (\mathrm{cos} \right.}{6 d \sqrt [3]{b^2 (c+d x)^6}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^3],x]
```

output
```
((I/6)*(c + d*x)*(-(((-I)*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3]
*(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x
)^3]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^6)^(1/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3836, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+b(c+dx)^3\right)\,dx$$

$$\Big\downarrow \text{3836}$$

$$\frac{1}{2}i\int e^{-ib(c+dx)^3-ia}dx-\frac{1}{2}i\int e^{ib(c+dx)^3+ia}dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

input
```
Int[Sin[a + b*(c + d*x)^3],x]
```

output
```
((I/6)*E^(I*a)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d*((-I)*b*(c + d
*x)^3)^(1/3)) - ((I/6)*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d*E^(I*a)*(
I*b*(c + d*x)^3)^(1/3))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

rule 3836
```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Simp[I/2   I
nt[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Simp[I/2   Int[E^(c*I + d*I*(e +
f*x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Maple [F]

$$\int \sin\left(a+b(dx+c)^3\right)dx$$

input
```
int(sin(a+b*(d*x+c)^3),x)
```

output
```
int(sin(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \sin\left(a+b(c+dx)^3\right)\,dx =$$

$$-\frac{(i\,bd^3)^{\frac{2}{3}}(\cos(a)-i\,\sin(a))\Gamma\left(\frac{1}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+(-i\,bd^3)^{\frac{2}{3}}(\cos(a)+i\,\sin(a)}{6\,bd^3}$$

input
```
integrate(sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*((I*b*d^3)^(2/3)*(cos(a) - I*sin(a))*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c
*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*(cos(a) + I*sin(a))
*gamma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*
d^3)
```

Sympy [F]

$$\int \sin \left(a+b(c+dx)^3\right) \, dx=\int \sin \left(a+b(c+dx)^3\right) dx$$

input
```
integrate(sin(a+b*(d*x+c)**3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**3), x)
```

Maxima [F]

$$\int \sin \left(a+b(c+dx)^3\right) \, dx=\int \sin \left((dx+c)^3 b+a\right) \, dx$$

input
```
integrate(sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int \sin \left(a+b(c+dx)^3\right) \, dx=\int \sin \left((dx+c)^3 b+a\right) \, dx$$

input
```
integrate(sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+b(c+dx)^3\right) \, dx=\int \sin \left(a+b\,(c+d\,x)^3\right) \,dx$$

input
```
int(sin(a + b*(c + d*x)^3),x)
```

output
```
int(sin(a + b*(c + d*x)^3), x)
```

Reduce [F]

$$\int \sin \left(a+b(c+dx)^3\right) \, dx=\int \sin \left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)dx$$

input
```
int(sin(a+b*(d*x+c)^3),x)
```

output
```
int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3),x)
```

3.175 $\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 24.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx} \, dx$$

$$\Big\downarrow \; 3918$$

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx}dx$$

input
```
Int[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(dx+c)^3\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

output
```
int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin \left(\left(dx+c\right)^3b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f*x + e
), x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin \left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**3)/(f*x+e),x)
```

output
```
Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
 + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx = \int \frac{\sin\left((dx+c)^3b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx = \int \frac{\sin\left((dx+c)^3b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 39.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+b(c+dx)^3)}{e+fx}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^3\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b*(c + d*x)^3)/(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^3)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

output
```
int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e + f*
x),x)
```

3.176 $\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx=\text{Int}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 56.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{(e+fx)^2} \, dx$$
$$\Big\downarrow \text{3918}$$
$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b \left(d x+c\right)^{3}\right)}{\left(f x+e\right)^{2}}d x$$

input
```
int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

output
```
int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left({\left(dx+c\right)}^{3}b+a\right)}{{\left(fx+e\right)}^{2}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f^2*x^2
 + 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+bc^{3}+3bc^{2}dx+3bcd^{2}x^{2}+bd^{3}x^{3}\right)}{{\left(e+fx\right)}^{2}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**3)/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
 + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^3b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^3b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 39.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^3\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input
```
int(sin(a + b*(c + d*x)^3)/(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^3)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{\sin \left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

output
```
int(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e**2 +
 2*e*f*x + f**2*x**2),x)
```

3.177 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 371

$$\begin{aligned}\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx &= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}\\
&- \frac{bf(de-cf)\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}\\
&- \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}\\
&+ \frac{2b^{3/2}f^2\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}\\
&+ \frac{2b^{3/2}f^2\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{3d^3}\\
&+ \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^3}\\
&+ \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}\\
&+ \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}\\
&+ \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}\\
&+ \frac{bf(de-cf)\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{d^3}\end{aligned}$$

output
```
2/3*b*f^2*(d*x+c)*cos(a+b/(d*x+c)^2)/d^3-b*f*(-c*f+d*e)*cos(a)*Ci(b/(d*x+c
)^2)/d^3-b^(1/2)*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(
1/2)/Pi^(1/2)/(d*x+c))/d^3+2/3*b^(3/2)*f^2*2^(1/2)*Pi^(1/2)*cos(a)*Fresnel
S(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))/d^3+2/3*b^(3/2)*f^2*2^(1/2)*Pi^(1/2)*F
resnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)/d^3+b^(1/2)*(-c*f+d*e)^2*
2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)/d^3+(-c
*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^2)/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+b/(d
*x+c)^2)/d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^2)/d^3+b*f*(-c*f+d*e)*sin(a
)*Si(b/(d*x+c)^2)/d^3
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.26

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \frac{2bcf^2\cos\left(a+\frac{b}{(c+dx)^2}\right)+2bdf^2x\cos\left(a+\frac{b}{(c+dx)^2}\right)+3bf(-de+cf)\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)+2b^{3/}}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]
```

output
```
(2*b*c*f^2*Cos[a + b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[a + b/(c + d*x)^2] + 3
*b*f*(-(d*e) + c*f)*Cos[a]*CosIntegral[b/(c + d*x)^2] + 2*b^(3/2)*f^2*Sqrt
[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*Sqrt[b]*d^2*e^2
*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 6*Sqrt[b]*c*
d*e*f*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 3*Sqrt[
b]*c^2*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + Sq
rt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*(-3*(d*e - c*f)^
2*Cos[a] + 2*b*f^2*Sin[a]) + 3*c*d^2*e^2*Sin[a + b/(c + d*x)^2] - 3*c^2*d*
e*f*Sin[a + b/(c + d*x)^2] + c^3*f^2*Sin[a + b/(c + d*x)^2] + 3*d^3*e^2*x*
Sin[a + b/(c + d*x)^2] + 3*d^3*e*f*x^2*Sin[a + b/(c + d*x)^2] + d^3*f^2*x^
3*Sin[a + b/(c + d*x)^2] + 3*b*d*e*f*Sin[a]*SinIntegral[b/(c + d*x)^2] - 3
*b*c*f^2*Sin[a]*SinIntegral[b/(c + d*x)^2])/(3*d^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(\sin\left(a+\frac{b}{(c+dx)^2}\right)(de-cf)^2+2f(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)(de-cf)+f^2(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\right)d(c+}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2}{3}\sqrt{2\pi}b^{3/2}f^2\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+\frac{2}{3}\sqrt{2\pi}b^{3/2}f^2\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)-bf\cos(a)(de-cf)\,\text{CosIntegra}}{}$$

input
```
Int[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]
```

output
```
((2*b*f^2*(c + d*x)*Cos[a + b/(c + d*x)^2])/3 - b*f*(d*e - c*f)*Cos[a]*Cos
Integral[b/(c + d*x)^2] - Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*Cos[a]*FresnelC
[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + (2*b^(3/2)*f^2*Sqrt[2*Pi]*Cos[a]*Fresne
lS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]*FresnelC
[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/3 + Sqrt[b]*(d*e - c*f)^2*Sqrt[2*
Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + (d*e - c*f)^2*(c + d
*x)*Sin[a + b/(c + d*x)^2] + f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)
^2] + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^2])/3 + b*f*(d*e - c*f)*Sin[a]*
SinIntegral[b/(c + d*x)^2])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{-(cf-de)^{2}(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2}}\right)+(cf-de)^{2}\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+}\right.\right.}{}$
default	$-\frac{-(cf-de)^{2}(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2}}\right)+(cf-de)^{2}\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+}\right.\right.}{}$
risch	$-\frac{e^{ia}b\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\sqrt{\pi}\,c^{2}f^{2}}{2d^{3}\sqrt{-ib}}+\frac{e^{ia}b\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\sqrt{\pi}\,cef}{d^{2}\sqrt{-ib}}-\frac{e^{ia}b\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\sqrt{\pi}\,e^{2}}{2d\sqrt{-ib}}-\frac{e^{ia}b\operatorname{expIntegral}_{1}\left(-\frac{ib}{(dx+c)^{2}}\right)}{2d^{3}}$
parts	Expression too large to display

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(-(c*f-d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^2)+(c*f-d*e)^2*b^(1/2)*2^(1/2
)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*Fresn
elS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+f*(c*f-d*e)*(d*x+c)^2*sin(a+b/(d*x+
c)^2)-2*f*(c*f-d*e)*b*(1/2*cos(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^
2))-1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^2)+2/3*f^2*b*(-(d*x+c)*cos(a+b/(d*x+
c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(
d*x+c))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.06

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx =$$
$$-\frac{3\,(bdef-bcf^2)\cos(a)\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)-\sqrt{2}(2\,\pi bdf^2\sin(a)-3\,\pi(d^3e^2-2\,cd^2ef+c^2df^2)\cos(a))\sqrt{}}{}$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*(3*(b*d*e*f - b*c*f^2)*cos(a)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2
)) - sqrt(2)*(2*pi*b*d*f^2*sin(a) - 3*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^
2)*cos(a))*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x +
c)) - sqrt(2)*(2*pi*b*d*f^2*cos(a) + 3*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f
^2)*sin(a))*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x +
 c)) - 3*(b*d*e*f - b*c*f^2)*sin(a)*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^
2)) - 2*(b*d*f^2*x + b*c*f^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2
*x^2 + 2*c*d*x + c^2)) - (d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*c*
d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(
d^2*x^2 + 2*c*d*x + c^2)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \int (e+fx)^2 \sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)\,dx$$

input
```
integrate((f*x+e)**2*sin(a+b/(d*x+c)**2),x)
```

output
```
Integral((e + f*x)**2*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(2*b*f^2*x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
+ c^2)) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*c*d^
2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x
 + c^3*d^2), x) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f
- b*c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*
c^2*d^3*x + c^3*d^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*
c*d*x + c^2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*sin((a*d
^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f
^2*x^3 + 3*d^2*e*f*x^2 + 3*d^2*e^2*x)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Giac [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(a + b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^2\,dx$$

input

```
int(sin(a + b/(c + d*x)^2)*(e + f*x)^2,x)
```

output

```
int(sin(a + b/(c + d*x)^2)*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\text{too large to display}$$

input

```
int((f*x+e)^2*sin(a+b/(d*x+c)^2),x)
```

output

```
( - 60*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x
**2))*b**2*c**5*f**2 - 228*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**
2 + 2*c*d*x + d**2*x**2))*b**2*c**4*d*f**2*x - 312*cos((a*c**2 + 2*a*c*d*x
 + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c**3*d**2*f**2*x**2
 - 168*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x
**2))*b**2*c**2*d**3*f**2*x**3 - 12*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*d**4*f**2*x**4 + 12*cos((a*c**2
+ 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d**5*f**
2*x**5 - 990*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
d**2*x**2))*c**9*f**2 + 630*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*c**8*d*e*f - 3960*cos((a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**8*d*f**2*x + 360*cos((a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**2*e
**2 + 2520*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))*c**7*d**2*e*f*x - 5940*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*c**7*d**2*f**2*x**2 + 1440*cos((a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**6*d**3*e**2*x
 + 3780*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*
x**2))*c**6*d**3*e*f*x**2 - 3960*cos((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*c**6*d**3*f**2*x**3 + 2160*cos((a*c**2 ...
```

3.178 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 198

$$\begin{aligned}\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = &-\frac{bf\cos(a)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}\\ &-\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}\\ &+\frac{\sqrt{b}(de-cf)\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d^2}\\ &+\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}\\ &+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}\\ &+\frac{bf\sin(a)\operatorname{Si}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*b*f*cos(a)*Ci(b/(d*x+c)^2)/d^2-b^(1/2)*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*co
s(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))/d^2+b^(1/2)*(-c*f+d*e)*2^(
1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)/d^2+(-c*f+
d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)/d^2
+1/2*b*f*sin(a)*Si(b/(d*x+c)^2)/d^2
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.22

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$$

$$=\frac{-bf\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)-2\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+2\sqrt{b}de\sqrt{2\pi}\,\text{FresnelS}}{}$$

input
```
Integrate[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

output
```
(-(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2]) - 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*P
i]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*Sqrt[b]*d*e*Sqrt[2*
Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 2*Sqrt[b]*c*f*Sqrt[2
*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 2*c*d*e*Sin[a + b/(
c + d*x)^2] - c^2*f*Sin[a + b/(c + d*x)^2] + 2*d^2*e*x*Sin[a + b/(c + d*x)
^2] + d^2*f*x^2*Sin[a + b/(c + d*x)^2] + b*f*Sin[a]*SinIntegral[b/(c + d*x
)^2])/(2*d^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left((de-cf)\sin\left(a+\frac{b}{(c+dx)^2}\right)+f(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}bf\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)-\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)\,\text{Fres}}{d^2}$$

input

```
Int[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

output

```
(-1/2*(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2]) - Sqrt[b]*(d*e - c*f)*Sqrt[2
*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + Sqrt[b]*(d*e - c*f)
*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + (d*e - c*f)*
(c + d*x)*Sin[a + b/(c + d*x)^2] + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^2])/
2 + (b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/2)/d^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d^2}$
default	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d^2}$
risch	$\frac{\mathrm{e}^{ia}b\ \mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\sqrt{\pi}\,cf}{2d^2\sqrt{-ib}}-\frac{\mathrm{e}^{ia}b\ \mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\sqrt{\pi}\,e}{2d\sqrt{-ib}}+\frac{\mathrm{e}^{ia}b\ \mathrm{expIntegral}_1\left(-\frac{ib}{(dx+c)^2}\right)f}{4d^2}+\frac{\mathrm{e}^{-ia}b\ \mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\sqrt{\pi}\,cf}{2d^2\sqrt{ib}}-$
parts	$-\frac{\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)fx}{d}+\frac{\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\sin(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)fx}{d}-\frac{\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\cos(a)\,\mathrm{Fre}}{d}$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^
(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)-f*b*(
1/2*cos(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx =$$

$$-\frac{2\sqrt{2}\pi(d^2e-cdf)\sqrt{\frac{b}{\pi d^2}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-2\sqrt{2}\pi(d^2e-cdf)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)\sin(a)+bf\cos}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqr
t(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) - 2*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(
pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))*sin(a) + b*f*co
s(a)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - b*f*sin(a)*sin_integral(b
/(d^2*x^2 + 2*c*d*x + c^2)) - (d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*si
n((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)**2),x)
```

output
```
Integral((e + f*x)*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(f*x^2 + 2*e*x)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c
^2*d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^2*x^2 +
 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(a + b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)\,dx$$

input
```
int(sin(a + b/(c + d*x)^2)*(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^2)*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\left(\int \sin\left(\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}\right)dx\right)e+\left(\int \sin\left(\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}\right)xdx\right)f$$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^2),x)
```

output
```
int(sin((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
)),x)*e + int(sin((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))*x,x)*f
```

3.179 $\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = -\frac{\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{\sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

output

```
-b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)
)/d+b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*si
n(a)/d+(d*x+c)*sin(a+b/(d*x+c)^2)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)dx$$
$$=\frac{-\sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+\sqrt{b}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

input
```
Integrate[Sin[a + b/(c + d*x)^2],x]
```

output
```
(-(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]) + S
qrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + (c + d
*x)*Sin[a + b/(c + d*x)^2])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3840, 3868, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{3840}$$
$$-\frac{\int (c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)d\frac{1}{c+dx}}{d}$$
$$\Big\downarrow \text{3868}$$
$$-\frac{2b\int \cos\left(a+\frac{b}{(c+dx)^2}\right)d\frac{1}{c+dx}-(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$
$$\Big\downarrow \text{3835}$$

$$-\frac{2b\left(\cos(a)\int\cos\left(\frac{b}{(c+dx)^2}\right)d\frac{1}{c+dx}-\sin(a)\int\sin\left(\frac{b}{(c+dx)^2}\right)d\frac{1}{c+dx}\right)-(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

$$\Big\downarrow \textbf{3832}$$

$$-\frac{2b\left(\cos(a)\int\cos\left(\frac{b}{(c+dx)^2}\right)d\frac{1}{c+dx}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{\sqrt{b}}\right)-(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

$$\Big\downarrow \textbf{3833}$$

$$-\frac{2b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{\sqrt{b}}\right)-(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

input

```
Int[Sin[a + b/(c + d*x)^2],x]
```

output

```
-((2*b*((Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/Sqrt[
b] - (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/Sqrt[b])
 - (c + d*x)*Sin[a + b/(c + d*x)^2])/d)
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3840

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[-f^(-1)   Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(
e + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &
& EqQ[n, -2]
```

rule 3868

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{-(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2}}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d}$	80
default	$-\frac{-(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2}}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d}$	80
risch	$-\frac{b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\mathrm{e}^{-ia}}{2d\sqrt{ib}}-\frac{b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\mathrm{e}^{ia}}{2d\sqrt{-ib}}-\frac{(-dx-c)\sin\left(\frac{a\,d^{2}x^{2}+2acdx+a\,c^{2}+b}{(dx+c)^{2}}\right)}{d}$	115

input

```
int(sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
-1/d*(-(d*x+c)*sin(a+b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*Fresnel
C(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/
2)/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = -\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\cos(a)\,C\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\,S\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)\sin(a)-(dx+c)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right.}{d}$$

input
```
integrate(sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^
2))/(d*x + c)) - sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(
b/(pi*d^2))/(d*x + c))*sin(a) - (d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c
^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**2),x)
```

output
```
Integral(sin(a + b/(c + d*x)**2), x)
```

Maxima [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \int \sin\left(a+\frac{b}{(dx+c)^2}\right)\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*d*integrate(x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
 + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x) + b*d*integrate(x*c
os((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)
/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*
sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
+ x*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
```

Giac [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int \sin \left(a+\frac{b}{\left(dx+c\right)^2}\right)\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int \sin \left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)\,dx$$

input
```
int(sin(a + b/(c + d*x)^2),x)
```

output
```
int(sin(a + b/(c + d*x)^2), x)
```

Reduce [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx=\int \sin\left(\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}\right)dx$$

input

```
int(sin(a+b/(d*x+c)^2),x)
```

output

```
int(sin((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
)),x)
```

3.180 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}dx$$

input
```
Int[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}dx$$

input
```
int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)
)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 18.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**2)/(f*x+e),x)
```

output
```
Integral(sin(a + b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 40.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)}{e+f\,x} \,d x$$

input
```
int(sin(a + b/(c + d*x)^2)/(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^2)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin \left(\frac{a \,d^{2} x^{2}+2 a c d x +a \,c^{2}+b}{d^{2} x^{2}+2 c d x +c^{2}}\right)}{f x +e}d x$$

input
```
int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

output
```
int(sin((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))/(e + f*x),x)
```

3.181 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 32.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2} \, dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}dx$$

input
```
int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)
)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b/(d*x+c)**2)/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 42.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input
```
int(sin(a + b/(c + d*x)^2)/(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^2)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(sin((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.182 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 330

$$
\begin{aligned}
\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx
&= -\frac{bf^2\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}\\
&\quad-\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}\\
&\quad+\frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3}\\
&\quad-\frac{ie^{ia}(de-cf)^2\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3}\\
&\quad+\frac{ie^{-ia}(de-cf)^2\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3}\\
&\quad+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3}+\frac{bf^2\sin(a)\mathrm{Si}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}
\end{aligned}
$$

output
```
-1/3*b*f^2*cos(a)*Ci(b/(d*x+c)^3)/d^3-1/3*I*exp(I*a)*f*(-c*f+d*e)*(-I*b/(d
*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-I*b/(d*x+c)^3)/d^3+1/3*I*f*(-c*f+d*e)
*(I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,I*b/(d*x+c)^3)/d^3/exp(I*a)-1/
6*I*exp(I*a)*(-c*f+d*e)^2*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-I*b/(
d*x+c)^3)/d^3+1/6*I*(-c*f+d*e)^2*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,
I*b/(d*x+c)^3)/d^3/exp(I*a)+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^3)/d^3+1/3*b
*f^2*sin(a)*Si(b/(d*x+c)^3)/d^3
```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.88

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$$

$$=\frac{\frac{3bdef\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{-\frac{ib}{(c+dx)^3}}}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{\frac{ib}{(c+dx)^3}}}\right)}{c+dx}-\frac{3bcf^2\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{-\frac{ib}{(c+dx)^3}}}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{\sqrt[3]{\frac{ib}{(c+dx)^3}}}\right)}{c+dx}-3i(de-cf)^2\sqrt[3]{}}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

output
```
((3*b*d*e*f*Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3
)^(1/3) + Gamma[1/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(1/3)))/(c + d
*x) - (3*b*c*f^2*Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d
*x)^3)^(1/3) + Gamma[1/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(1/3)))/(
c + d*x) - (3*I)*(d*e - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[2
/3, (I*b)/(c + d*x)^3]*(Cos[a] - I*Sin[a]) + (3*I)*(d*e - c*f)^2*(((-I)*b)
/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[2/3, ((-I)*b)/(c + d*x)^3]*(Cos[a] + I
*Sin[a]) + 2*(c + d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x
 + f^2*x^2))*Cos[b/(c + d*x)^3]*Sin[a] + ((3*I)*b*d*e*f*(Gamma[1/3, ((-I)*
b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(1/3) - Gamma[1/3, (I*b)/(c + d*x)^
3]/((I*b)/(c + d*x)^3)^(1/3))*Sin[a])/(c + d*x) - ((3*I)*b*c*f^2*(Gamma[1/
3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(1/3) - Gamma[1/3, (I*b)/(
c + d*x)^3]/((I*b)/(c + d*x)^3)^(1/3))*Sin[a])/(c + d*x) + 2*(c + d*x)*(c^
2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos[a]*Sin[b/
(c + d*x)^3] - 2*b*f^2*(Cos[a]*CosIntegral[b/(c + d*x)^3] - Sin[a]*SinInte
gral[b/(c + d*x)^3]))/(6*d^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right) dx$$

$$\Big\downarrow \text{ 3914}$$

$$\frac{\int \left(\sin\left(a+\frac{b}{(c+dx)^3}\right)(de-cf)^2+2f(c+dx)\sin\left(a+\frac{b}{(c+dx)^3}\right)(de-cf)+f^2(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\right)d(c+}{d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{3}bf^2\cos(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^3}\right)-\frac{1}{3}ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)+\frac{1}{3}ie^{-ia}f(c+$$

input
```
Int[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

output
```
(-1/3*(b*f^2*Cos[a]*CosIntegral[b/(c + d*x)^3]) - (I/3)*E^(I*a)*f*(d*e - c
*f)*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I)*b)/(c + d*x
)^3] + ((I/3)*f*(d*e - c*f)*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2
/3, (I*b)/(c + d*x)^3])/E^(I*a) - (I/6)*E^(I*a)*(d*e - c*f)^2*(((-I)*b)/(c
 + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3] + ((I/6)*(d*e
 - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (I*b)/(c + d*x)^
3])/E^(I*a) + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^3])/3 + (b*f^2*Sin[a]*S
inIntegral[b/(c + d*x)^3])/3)/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^3}\right) dx$$

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

output
```
int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(264) = 528.

Time = 0.10 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.75

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx =$$

$$-\frac{2\,bf^2\cos(a)\,\mathrm{Ci}\left(\frac{b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-2\,bf^2\sin(a)\,\mathrm{Si}\left(\frac{b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+3\left(\left(i\,d^3ef-i\,cd^2f^2\right)\mathrm{co}\right.}{}$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*(2*b*f^2*cos(a)*cos_integral(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c
^3)) - 2*b*f^2*sin(a)*sin_integral(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)) + 3*((I*d^3*e*f - I*c*d^2*f^2)*cos(a) + (d^3*e*f - c*d^2*f^2)*sin(a)
)*(I*b/d^3)^(2/3)*gamma(1/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
) + 3*((-I*d^3*e*f + I*c*d^2*f^2)*cos(a) + (d^3*e*f - c*d^2*f^2)*sin(a))*(
-I*b/d^3)^(2/3)*gamma(1/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
 + 3*((I*d^3*e^2 - 2*I*c*d^2*e*f + I*c^2*d*f^2)*cos(a) + (d^3*e^2 - 2*c*d^
2*e*f + c^2*d*f^2)*sin(a))*(I*b/d^3)^(1/3)*gamma(2/3, I*b/(d^3*x^3 + 3*c*d
^2*x^2 + 3*c^2*d*x + c^3)) + 3*((-I*d^3*e^2 + 2*I*c*d^2*e*f - I*c^2*d*f^2)
*cos(a) + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sin(a))*(-I*b/d^3)^(1/3)*gam
ma(2/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*(d^3*f^2*x^3 +
 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin((a
*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = \int (e+fx)^2 \sin\left(a+\frac{b}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)\,dx$$

input
```
integrate((f*x+e)**2*sin(a+b/(d*x+c)**3),x)
```

output
```
Integral((e + f*x)**2*sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*
x**3)), x)
```

Maxima [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c
^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate
(1/2*(b*d*f^2*x^3 + 3*b*d*e*f*x^2 + 3*b*d*e^2*x)*cos((a*d^3*x^3 + 3*a*c*d^
2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + integrat
e(1/2*(b*d*f^2*x^3 + 3*b*d*e*f*x^2 + 3*b*d*e^2*x)*cos((a*d^3*x^3 + 3*a*c*d
^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3
))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a*d^3*x
^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c
^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^
4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*
c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x)
```

Giac [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin(a + b/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)\,{\left(e+f\,x\right)}^2\,dx$$

input
```
int(sin(a + b/(c + d*x)^3)*(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^3)*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

output

```
(12*int(x**3/(tan((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 +
 b)/(2*c**3 + 6*c**2*d*x + 6*c*d**2*x**2 + 2*d**3*x**3))**2*c**4 + 4*tan((
a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(2*c**3 + 6*c**
2*d*x + 6*c*d**2*x**2 + 2*d**3*x**3))**2*c**3*d*x + 6*tan((a*c**3 + 3*a*c*
*2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(2*c**3 + 6*c**2*d*x + 6*c*d**
2*x**2 + 2*d**3*x**3))**2*c**2*d**2*x**2 + 4*tan((a*c**3 + 3*a*c**2*d*x +
3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(2*c**3 + 6*c**2*d*x + 6*c*d**2*x**2 +
2*d**3*x**3))**2*c*d**3*x**3 + tan((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**
2 + a*d**3*x**3 + b)/(2*c**3 + 6*c**2*d*x + 6*c*d**2*x**2 + 2*d**3*x**3))*
*2*d**4*x**4 + c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4),x)*b*c**4*d**4*f**2 + 36*int(x**3/(tan((a*c**3 + 3*a*c**2*d*x + 3*a
*c*d**2*x**2 + a*d**3*x**3 + b)/(2*c**3 + 6*c**2*d*x + 6*c*d**2*x**2 + 2*d
**3*x**3))**2*c**4 + 4*tan((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d*
*3*x**3 + b)/(2*c**3 + 6*c**2*d*x + 6*c*d**2*x**2 + 2*d**3*x**3))**2*c**3*
d*x + 6*tan((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(2
*c**3 + 6*c**2*d*x + 6*c*d**2*x**2 + 2*d**3*x**3))**2*c**2*d**2*x**2 + 4*t
an((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(2*c**3 + 6
*c**2*d*x + 6*c*d**2*x**2 + 2*d**3*x**3))**2*c*d**3*x**3 + tan((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(2*c**3 + 6*c**2*d*x + 6*
c*d**2*x**2 + 2*d**3*x**3))**2*d**4*x**4 + c**4 + 4*c**3*d*x + 6*c**2*d...
```

3.183 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 235

$$\begin{aligned}\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx &= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2} \\ &\quad+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2} \\ &\quad-\frac{ie^{ia}(de-cf)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2} \\ &\quad+\frac{ie^{-ia}(de-cf)\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}\end{aligned}$$

output

```
-1/6*I*exp(I*a)*f*(-I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-I*b/(d*x+c)
^3)/d^2+1/6*I*f*(I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,I*b/(d*x+c)^3)/
d^2/exp(I*a)-1/6*I*exp(I*a)*(-c*f+d*e)*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMM
A(-1/3,-I*b/(d*x+c)^3)/d^2+1/6*I*(-c*f+d*e)*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*
GAMMA(-1/3,I*b/(d*x+c)^3)/d^2/exp(I*a)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 705 vs. 2(235) = 470.

Time = 2.46 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.00

$$
\begin{aligned}
&\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx \\
&= \frac{3bf\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)}\right)}{4d^2} \\
&\quad+\frac{3be\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)}{2d} \\
&\quad-\frac{3bcf\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)}{2d^2} \\
&\quad+\frac{e(c+dx)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)}{d}+\frac{f(-c+dx)(c+dx)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)}{2d^2} \\
&\quad+\frac{3ibf\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)}-\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)}\right)\sin(a)}{4d^2} \\
&\quad+\frac{3ibe\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)\sin(a)}{2d} \\
&\quad-\frac{3ibcf\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)\sin(a)}{2d^2} \\
&\quad+\frac{e(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{d}+\frac{f(-c+dx)(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{2d^2}
\end{aligned}
$$

input
```
Integrate[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

output
```
(3*b*f*Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^
(1/3)*(c + d*x)) + Gamma[1/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(1
/3)*(c + d*x))))/(4*d^2) + (3*b*e*Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]
/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) + Gamma[2/3, (I*b)/(c + d*x)
^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2)))/(2*d) - (3*b*c*f*Cos[a]*(G
amma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^
2) + Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^
2)))/(2*d^2) + (e*(c + d*x)*Cos[b/(c + d*x)^3]*Sin[a])/d + (f*(-c + d*x)*(
c + d*x)*Cos[b/(c + d*x)^3]*Sin[a])/(2*d^2) + (((3*I)/4)*b*f*(Gamma[1/3, (
(-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)) - Gamma[1/
3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)))*Sin[a])/d^2
 + (((3*I)/2)*b*e*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)
^3)^(2/3)*(c + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)
^3)^(2/3)*(c + d*x)^2))*Sin[a])/d - (((3*I)/2)*b*c*f*(Gamma[2/3, ((-I)*b)/
(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) - Gamma[2/3, (I*
b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2))*Sin[a])/d^2 + (
e*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^3])/d + (f*(-c + d*x)*(c + d*x)*Cos[a]*
Sin[b/(c + d*x)^3])/(2*d^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left((de-cf)\sin\left(a+\frac{b}{(c+dx)^3}\right)+f(c+dx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{6}ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)+\frac{1}{6}ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{d^2}$$

input
```
Int[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

output
```
((-1/6*I)*E^(I*a)*f*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, (
(-I)*b)/(c + d*x)^3] + ((I/6)*f*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamm
a[-2/3, (I*b)/(c + d*x)^3])/E^(I*a) - (I/6)*E^(I*a)*(d*e - c*f)*(((-I)*b)/
(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3] + ((I/6)*(d
*e - c*f)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (I*b)/(c + d*x)^
3])/E^(I*a))/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)dx$$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

output
```
int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(176) = 352$.

Time = 0.09 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.56

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$$
$$= \frac{(-i\,d^2f\cos{(a)}-d^2f\sin{(a)})\left(\frac{ib}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+(i\,d^2f\cos{(a)}-d^2f\sin{(a)})\left(-\frac{ib}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\right.}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/4*((-I*d^2*f*cos(a) - d^2*f*sin(a))*(I*b/d^3)^(2/3)*gamma(1/3, I*b/(d^3*
x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + (I*d^2*f*cos(a) - d^2*f*sin(a))*(-
I*b/d^3)^(2/3)*gamma(1/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
- 2*((I*d^2*e - I*c*d*f)*cos(a) + (d^2*e - c*d*f)*sin(a))*(I*b/d^3)^(1/3)*
gamma(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*((-I*d^2*e +
 I*c*d*f)*cos(a) + (d^2*e - c*d*f)*sin(a))*(-I*b/d^3)^(1/3)*gamma(2/3, -I*
b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d^2*f*x^2 + 2*d^2*e*x +
2*c*d*e - c^2*f)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)
/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = \int (e+fx)\sin\left(a+\frac{b}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)**3),x)
```

output
```
Integral((e + f*x)*sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3)), x)
```

Maxima [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

input

```
integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

output

```
1/2*(f*x^2 + 2*e*x)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 +
 b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(3/4*(b*d*f*x^2
+ 2*b*d*e*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^
3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2
*x^2 + 4*c^3*d*x + c^4), x) + integrate(3/4*(b*d*f*x^2 + 2*b*d*e*x)*cos((a
*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x +
 c^4)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^
2 + 4*c^3*d*x + c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3
+ b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x)
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

input

```
integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

output

```
integrate((f*x + e)*sin(a + b/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{{(c+d\,x)}^3}\right)\,(e+f\,x)\,dx$$

input
```
int(sin(a + b/(c + d*x)^3)*(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^3)*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\text{too large to display}$$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

output

```
(294*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**11*f + 210*cos((a*c**3 + 3*
a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d
**2*x**2 + d**3*x**3))*c**10*d*e + 1764*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c
*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3))*c**10*d*f*x + 1260*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d
**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**9*d**2*e
*x + 4410*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/
(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**9*d**2*f*x**2 + 3150*c
os((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c
**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**8*d**3*e*x**2 + 5880*cos((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*c**8*d**3*f*x**3 + 4200*cos((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*c**7*d**4*e*x**3 + 4410*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c
*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3))*c**7*d**4*f*x**4 + 3150*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2
+ a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**6*d
**5*e*x**4 + 1764*cos((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x*
*3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**6*d**5*f*x*...
```

3.184 $\int \sin \left(a+\frac{b}{(c+dx)^3}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \sin \left(a+\frac{b}{(c+dx)^3}\right) \, dx = -\frac{ie^{ia}\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d} + \frac{ie^{-ia}\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}$$

output

```
-1/6*I*exp(I*a)*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-I*b/(d*x+c)^3)/
d+1/6*I*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,I*b/(d*x+c)^3)/d/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.90

$$\int \sin \left(a+\frac{b}{(c+dx)^3}\right) \, dx = \frac{b\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)+2(c+dx)^3\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)+ib\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/}}\right.}{2d(c+dx)^2}$$

input
```
Integrate[Sin[a + b/(c + d*x)^3],x]
```

output
```
(b*Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(2/3) +
 Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)) + 2*(c + d*x)^3*
Cos[b/(c + d*x)^3]*Sin[a] + I*b*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b
)/(c + d*x)^3)^(2/3) - Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(
2/3))*Sin[a] + 2*(c + d*x)^3*Cos[a]*Sin[b/(c + d*x)^3])/(2*d*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3846, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{3846}$$

$$\frac{1}{2}i\int e^{-ia-\frac{ib}{(c+dx)^3}}dx-\frac{1}{2}i\int e^{ia+\frac{ib}{(c+dx)^3}}dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}$$

input
```
Int[Sin[a + b/(c + d*x)^3],x]
```

output
```
((-1/6*I)*E^(I*a)*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)
*b)/(c + d*x)^3])/d + ((I/6)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/
3, (I*b)/(c + d*x)^3])/(d*E^(I*a))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

rule 3846
```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Simp[I/2   I
nt[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Simp[I/2   Int[E^(c*I + d*I*(e +
f*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Maple [F]

$$\int \sin\left(a + \frac{b}{\left(dx+c\right)^{3}}\right) dx$$

input
```
int(sin(a+b/(d*x+c)^3),x)
```

output
```
int(sin(a+b/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(77) = 154.

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int \sin\left(a + \frac{b}{(c+dx)^3}\right)\,dx$$
$$= \frac{\left(-i\,d\cos\left(a\right) - d\sin\left(a\right)\right)\left(\frac{i\,b}{d^{3}}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, \frac{i\,b}{d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}}\right) + \left(i\,d\cos\left(a\right) - d\sin\left(a\right)\right)\left(-\frac{i\,b}{d^{3}}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, - \quad d^{3}x^{3}+3\,cd\right.}{2\,d}$$

input
```
integrate(sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/2*((-I*d*cos(a) - d*sin(a))*(I*b/d^3)^(1/3)*gamma(2/3, I*b/(d^3*x^3 + 3*
c*d^2*x^2 + 3*c^2*d*x + c^3)) + (I*d*cos(a) - d*sin(a))*(-I*b/d^3)^(1/3)*g
amma(2/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d*x + c)*si
n((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2
*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^3}\right) dx=\int \sin \left(a+\frac{b}{\left(c+dx\right)^3}\right) dx$$

input
```
integrate(sin(a+b/(d*x+c)**3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**3), x)
```

Maxima [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^3}\right) dx=\int \sin \left(a+\frac{b}{\left(dx+c\right)^3}\right) dx$$

input
```
integrate(sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

output
```
3*b*d*integrate(1/2*x*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3
 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 +
6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + 3*b*d*integrate(1/2*x*cos((a*d^3*x^
3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^
2*d*x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*c
os((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^
2*x^2 + 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c
^3*d*x + c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x) + x*sin((a*d^3*x^3 + 3*a*c
*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c
^3))
```

Giac [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^3}\right)\,dx$$

input
```
int(sin(a + b/(c + d*x)^3),x)
```

output
```
int(sin(a + b/(c + d*x)^3), x)
```

Reduce [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx=\int \sin\left(\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right)dx$$

input
```
int(sin(a+b/(d*x+c)^3),x)
```

output
```
int(sin((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)),x)
```

3.185 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\operatorname{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx},x\right)$$

output

```
Defer(Int)(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 5.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

input

```
Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
```

output

```
Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$
$$\Big\downarrow 3918$$
$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}dx$$

input
```
Int[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}dx$$

input

```
int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

output

```
int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
```

output

```
integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 70.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**3)/(f*x+e),x)
```

output
```
Integral(sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(e + f
*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 41.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b/(c + d*x)^3)/(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^3)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin \left(\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right)}{fx+e}dx$$

input
```
int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

output
```
int(sin((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(e + f*x),x)
```

3.186 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 36.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2} \, dx$$

$$\Big\downarrow \; 3918$$

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}dx$$

input

```
int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

output

```
int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.25

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
```

output

```
integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx = \text{Timed out}$$

input

```
integrate(sin(a+b/(d*x+c)**3)/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 6.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 46.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)}{{\left(e+f\,x\right)}^2} \,d x$$

input

```
int(sin(a + b/(c + d*x)^3)/(e + f*x)^2,x)
```

output

```
int(sin(a + b/(c + d*x)^3)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.25

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right)}{f^2x^2+2efx+e^2}dx$$

input

```
int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

output

```
int(sin((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.187 $\int (e+fx)^2 \sin \left(a+b\sqrt{c+dx}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 410

$$
\begin{aligned}
\int (e+fx)^2 \sin \left(a+b\sqrt{c+dx}\right) \, dx = & -\frac{240f^2\sqrt{c+dx}\cos \left(a+b\sqrt{c+dx}\right)}{b^5d^3} \\
& +\frac{24f(de-cf)\sqrt{c+dx}\cos \left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\
& -\frac{2(de-cf)^2\sqrt{c+dx}\cos \left(a+b\sqrt{c+dx}\right)}{bd^3} \\
& +\frac{40f^2(c+dx)^{3/2}\cos \left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\
& -\frac{4f(de-cf)(c+dx)^{3/2}\cos \left(a+b\sqrt{c+dx}\right)}{bd^3} \\
& -\frac{2f^2(c+dx)^{5/2}\cos \left(a+b\sqrt{c+dx}\right)}{bd^3} \\
& +\frac{240f^2\sin \left(a+b\sqrt{c+dx}\right)}{b^6d^3} \\
& -\frac{24f(de-cf)\sin \left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\
& +\frac{2(de-cf)^2\sin \left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\
& -\frac{120f^2(c+dx)\sin \left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\
& +\frac{12f(de-cf)(c+dx)\sin \left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\
& +\frac{10f^2(c+dx)^2\sin \left(a+b\sqrt{c+dx}\right)}{b^2d^3}
\end{aligned}
$$

output

```
-240*f^2*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(1/2))/b^5/d^3+24*f*(-c*f+d*e)*(d*x
+c)^(1/2)*cos(a+b*(d*x+c)^(1/2))/b^3/d^3-2*(-c*f+d*e)^2*(d*x+c)^(1/2)*cos(
a+b*(d*x+c)^(1/2))/b/d^3+40*f^2*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b^3/d
^3-4*f*(-c*f+d*e)*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b/d^3-2*f^2*(d*x+c)
^(5/2)*cos(a+b*(d*x+c)^(1/2))/b/d^3+240*f^2*sin(a+b*(d*x+c)^(1/2))/b^6/d^3
-24*f*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/2))/b^4/d^3+2*(-c*f+d*e)^2*sin(a+b*(d*
x+c)^(1/2))/b^2/d^3-120*f^2*(d*x+c)*sin(a+b*(d*x+c)^(1/2))/b^4/d^3+12*f*(-
c*f+d*e)*(d*x+c)*sin(a+b*(d*x+c)^(1/2))/b^2/d^3+10*f^2*(d*x+c)^2*sin(a+b*(
d*x+c)^(1/2))/b^2/d^3
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.34

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{-2b\sqrt{c+dx}(120f^2+b^4d^2(e+fx)^2-4b^2f(3de+2cf+5dfx))\cos\left(a+b\sqrt{c+dx}\right)+2(120f^2-12b^2f}{b^6d^3}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*b*Sqrt[c + d*x]*(120*f^2 + b^4*d^2*(e + f*x)^2 - 4*b^2*f*(3*d*e + 2*c*
f + 5*d*f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(120*f^2 - 12*b^2*f*(4*c*f + d*
(e + 5*f*x)) + b^4*d*(e + f*x)*(4*c*f + d*(e + 5*f*x)))*Sin[a + b*Sqrt[c +
 d*x]])/(b^6*d^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$\Big\downarrow \; 3912$$
$$\frac{2\int \left(\frac{f^2\sin\left(a+b\sqrt{c+dx}\right)(c+dx)^{5/2}}{d^2}+\frac{2f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)(c+dx)^{3/2}}{d^2}+\frac{(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)\sqrt{c+dx}}{d^2}\right)d\sqrt{c+dx}}{d}$$
$$\Big\downarrow \; 2009$$
$$2\left(\frac{120f^2\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^2}-\frac{120f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^2}-\frac{12f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{60f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+12f\sqrt{c+dx}(\right.$$

input
```
Int[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-120*f^2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^5*d^2) + (12*f*(d
*e - c*f)*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^2) - ((d*e - c*f)
^2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^2) + (20*f^2*(c + d*x)^(3/
2)*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*f*(d*e - c*f)*(c + d*x)^(3/2)*
Cos[a + b*Sqrt[c + d*x]])/(b*d^2) - (f^2*(c + d*x)^(5/2)*Cos[a + b*Sqrt[c
+ d*x]])/(b*d^2) + (120*f^2*Sin[a + b*Sqrt[c + d*x]])/(b^6*d^2) - (12*f*(d
*e - c*f)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2) + ((d*e - c*f)^2*Sin[a + b*S
qrt[c + d*x]])/(b^2*d^2) - (60*f^2*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^
4*d^2) + (6*f*(d*e - c*f)*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^2) +
(5*f^2*(c + d*x)^2*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^2)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. 2(374) = 748.

Time = 1.23 (sec) , antiderivative size = 1161, normalized size of antiderivative = 2.83

method	result	size
parts	Expression too large to display	1161
derivativedivides	Expression too large to display	1246
default	Expression too large to display	1246

input
```
int((f*x+e)^2*sin(a+(d*x+c)^(1/2)*b),x,method=_RETURNVERBOSE)
```

output
```
-2/d/b*(d*x+c)^(1/2)*cos(a+(d*x+c)^(1/2)*b)*f^2*x^2-4/d/b*(d*x+c)^(1/2)*co
s(a+(d*x+c)^(1/2)*b)*e*f*x-2/d/b*(d*x+c)^(1/2)*cos(a+(d*x+c)^(1/2)*b)*e^2+
2/d/b^2*sin(a+(d*x+c)^(1/2)*b)*f^2*x^2+4/d/b^2*sin(a+(d*x+c)^(1/2)*b)*e*f*
x+2/d/b^2*sin(a+(d*x+c)^(1/2)*b)*e^2-8/d^3/b^4*f*(3/b^2*a^2*f*(sin(a+(d*x+
c)^(1/2)*b)-(a+(d*x+c)^(1/2)*b)*cos(a+(d*x+c)^(1/2)*b))-3/b^2*a*f*(-(a+(d*
x+c)^(1/2)*b)^2*cos(a+(d*x+c)^(1/2)*b)+2*cos(a+(d*x+c)^(1/2)*b)+2*(a+(d*x+
c)^(1/2)*b)*sin(a+(d*x+c)^(1/2)*b))-6/b^2*a^2*f*((a+(d*x+c)^(1/2)*b)^2*sin
(a+(d*x+c)^(1/2)*b)-2*sin(a+(d*x+c)^(1/2)*b)+2*(a+(d*x+c)^(1/2)*b)*cos(a+(
d*x+c)^(1/2)*b))+4/b^2*a^3*f*(cos(a+(d*x+c)^(1/2)*b)+(a+(d*x+c)^(1/2)*b)*s
in(a+(d*x+c)^(1/2)*b))+4/b^2*a*f*((a+(d*x+c)^(1/2)*b)^3*sin(a+(d*x+c)^(1/2
)*b)+3*(a+(d*x+c)^(1/2)*b)^2*cos(a+(d*x+c)^(1/2)*b)-6*cos(a+(d*x+c)^(1/2)*
b)-6*(a+(d*x+c)^(1/2)*b)*sin(a+(d*x+c)^(1/2)*b))-c*f*(sin(a+(d*x+c)^(1/2)*
b)-(a+(d*x+c)^(1/2)*b)*cos(a+(d*x+c)^(1/2)*b))-c*f*a*cos(a+(d*x+c)^(1/2)*b
)+d*e*(sin(a+(d*x+c)^(1/2)*b)-(a+(d*x+c)^(1/2)*b)*cos(a+(d*x+c)^(1/2)*b))+
d*e*a*cos(a+(d*x+c)^(1/2)*b)+c*f*((a+(d*x+c)^(1/2)*b)^2*sin(a+(d*x+c)^(1/2
)*b)-2*sin(a+(d*x+c)^(1/2)*b)+2*(a+(d*x+c)^(1/2)*b)*cos(a+(d*x+c)^(1/2)*b)
)-d*e*((a+(d*x+c)^(1/2)*b)^2*sin(a+(d*x+c)^(1/2)*b)-2*sin(a+(d*x+c)^(1/2)*
b)+2*(a+(d*x+c)^(1/2)*b)*cos(a+(d*x+c)^(1/2)*b))+a^2*c*f*sin(a+(d*x+c)^(1/
2)*b)-a^2*d*e*sin(a+(d*x+c)^(1/2)*b)-2*c*f*a*(cos(a+(d*x+c)^(1/2)*b)+(a+(d
*x+c)^(1/2)*b)*sin(a+(d*x+c)^(1/2)*b))+2*d*e*a*(cos(a+(d*x+c)^(1/2)*b)+...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.48

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left(\left(b^5d^2f^2x^2+b^5d^2e^2-12\,b^3def-8\left(b^3c-15\,b\right)f^2+2\left(b^5d^2ef-10\,b^3df^2\right)x\right)\sqrt{dx+c}\cos\left(\sqrt{dx+c}b\right.}{}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-2*((b^5*d^2*f^2*x^2 + b^5*d^2*e^2 - 12*b^3*d*e*f - 8*(b^3*c - 15*b)*f^2 +
 2*(b^5*d^2*e*f - 10*b^3*d*f^2)*x)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a)
- (5*b^4*d^2*f^2*x^2 + b^4*d^2*e^2 + 4*(b^4*c - 3*b^2)*d*e*f - 24*(2*b^2*c
 - 5)*f^2 + 2*(3*b^4*d^2*e*f + 2*(b^4*c - 15*b^2)*d*f^2)*x)*sin(sqrt(d*x +
 c)*b + a))/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.29

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$= \begin{cases} \left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin(a) \\ \left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin\left(a+b\sqrt{c}\right) \\ -\frac{2e^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{4efx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2f^2x^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{8cef\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+ \end{cases}$$

input
```
integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise(((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a), Eq(b, 0) & (Eq(b, 0)
| Eq(d, 0))), ((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a + b*sqrt(c)), Eq(d,
 0)), (-2*e**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 4*e*f*x*sqrt
(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 2*f**2*x**2*sqrt(c + d*x)*cos(a
 + b*sqrt(c + d*x))/(b*d) + 8*c*e*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) +
 8*c*f**2*x*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e**2*sin(a + b*sqrt(c
 + d*x))/(b**2*d) + 12*e*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d) + 10*f**2*x
**2*sin(a + b*sqrt(c + d*x))/(b**2*d) + 16*c*f**2*sqrt(c + d*x)*cos(a + b*
sqrt(c + d*x))/(b**3*d**3) + 24*e*f*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))
/(b**3*d**2) + 40*f**2*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d**2
) - 96*c*f**2*sin(a + b*sqrt(c + d*x))/(b**4*d**3) - 24*e*f*sin(a + b*sqrt
(c + d*x))/(b**4*d**2) - 120*f**2*x*sin(a + b*sqrt(c + d*x))/(b**4*d**2) -
 240*f**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**5*d**3) + 240*f**2*si
n(a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. 2(374) = 748.

Time = 0.09 (sec) , antiderivative size = 1101, normalized size of antiderivative = 2.69

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output

```
2*(a*e^2*cos(sqrt(d*x + c)*b + a) - 2*a*c*e*f*cos(sqrt(d*x + c)*b + a)/d +
 a*c^2*f^2*cos(sqrt(d*x + c)*b + a)/d^2 - ((sqrt(d*x + c)*b + a)*cos(sqrt(
d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e^2 + 2*((sqrt(d*x + c)*b + a)
*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c*e*f/d - ((sqrt(d*x
 + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c^2*f^2/
d^2 + 2*a^3*e*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) - 2*a^3*c*f^2*cos(sqrt(d*
x + c)*b + a)/(b^2*d^2) - 6*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a
) - sin(sqrt(d*x + c)*b + a))*a^2*e*f/(b^2*d) + 6*((sqrt(d*x + c)*b + a)*c
os(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*c*f^2/(b^2*d^2) +
a^5*f^2*cos(sqrt(d*x + c)*b + a)/(b^4*d^2) + 6*(((sqrt(d*x + c)*b + a)^2 -
 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b
 + a))*a*e*f/(b^2*d) - 5*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) -
 sin(sqrt(d*x + c)*b + a))*a^4*f^2/(b^4*d^2) - 6*(((sqrt(d*x + c)*b + a)^2
 - 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)
*b + a))*a*c*f^2/(b^2*d^2) - 2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)
*b - 6*a)*cos(sqrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(s
qrt(d*x + c)*b + a))*e*f/(b^2*d) + 10*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(s
qrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a^
3*f^2/(b^4*d^2) + 2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*c
os(sqrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x ...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.70

$$\int (e+fx)^2\sin\left(a+b\sqrt{c+dx}\right)\,dx=\text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*((sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*
e^2/b - 2*e*f*(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b +
 a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 +
6*sqrt(d*x + c)*b)*cos(sqrt(d*x + c)*b + a)/b^2 - (b^2*c - 3*(sqrt(d*x + c
)*b + a)^2 + 6*(sqrt(d*x + c)*b + a)*a - 3*a^2 + 6)*sin(sqrt(d*x + c)*b +
a)/b^2)/(b*d) + f^2*(((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(
d*x + c)*b + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x
+ c)*b + a)*a^2*b^2*c + 2*a^3*b^2*c + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*
x + c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b +
 a)^2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 + 12*(sqrt(d*x + c)*b + a)*b
^2*c - 12*a*b^2*c - 20*(sqrt(d*x + c)*b + a)^3 + 60*(sqrt(d*x + c)*b + a)^
2*a - 60*(sqrt(d*x + c)*b + a)*a^2 + 20*a^3 + 120*sqrt(d*x + c)*b)*cos(sqr
t(d*x + c)*b + a)/b^4 - (b^4*c^2 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c + 12*(s
qrt(d*x + c)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)^4 - 20
*(sqrt(d*x + c)*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sqrt(d*x
 + c)*b + a)*a^3 + 5*a^4 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sq
rt(d*x + c)*b + a)*a - 60*a^2 + 120)*sin(sqrt(d*x + c)*b + a)/b^4)/(b*d^2)
)/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+b\sqrt{c+dx}\right) \, dx = \int \sin \left(a+b\,\sqrt{c+d\,x}\right)\,{\left(e+f\,x\right)}^2 \,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/2))*(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(1/2))*(e + f*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.93

$$\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$= \frac{-2\sqrt{dx+c}\,\cos\left(\sqrt{dx+c}\,b+a\right)b^5d^2e^2-4\sqrt{dx+c}\,\cos\left(\sqrt{dx+c}\,b+a\right)b^5d^2efx-2\sqrt{dx+c}\,\cos\left(\sqrt{dx}\right.}{}$$

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x)
```

output
```
(2*( - sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a)*b**5*d**2*e**2 - 2*sqrt(c +
d*x)*cos(sqrt(c + d*x)*b + a)*b**5*d**2*e*f*x - sqrt(c + d*x)*cos(sqrt(c +
 d*x)*b + a)*b**5*d**2*f**2*x**2 + 8*sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a
)*b**3*c*f**2 + 12*sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a)*b**3*d*e*f + 20*
sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a)*b**3*d*f**2*x - 120*sqrt(c + d*x)*c
os(sqrt(c + d*x)*b + a)*b*f**2 + 4*sin(sqrt(c + d*x)*b + a)*b**4*c*d*e*f +
 4*sin(sqrt(c + d*x)*b + a)*b**4*c*d*f**2*x + sin(sqrt(c + d*x)*b + a)*b**
4*d**2*e**2 + 6*sin(sqrt(c + d*x)*b + a)*b**4*d**2*e*f*x + 5*sin(sqrt(c +
d*x)*b + a)*b**4*d**2*f**2*x**2 - 48*sin(sqrt(c + d*x)*b + a)*b**2*c*f**2
- 12*sin(sqrt(c + d*x)*b + a)*b**2*d*e*f - 60*sin(sqrt(c + d*x)*b + a)*b**
2*d*f**2*x + 120*sin(sqrt(c + d*x)*b + a)*f**2))/(b**6*d**3)
```

3.188 $\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\begin{aligned}\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} \\ &- \frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} \\ &- \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} \\ &- \frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} \\ &+ \frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} \\ &+ \frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}\end{aligned}$$

output

```
12*f*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(1/2))/b^3/d^2-2*(-c*f+d*e)*(d*x+c)^(1/
2)*cos(a+b*(d*x+c)^(1/2))/b/d^2-2*f*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b
/d^2-12*f*sin(a+b*(d*x+c)^(1/2))/b^4/d^2+2*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/2
))/b^2/d^2+6*f*(d*x+c)*sin(a+b*(d*x+c)^(1/2))/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$$

$$= \frac{-2b\sqrt{c+dx}(-6f+b^2d(e+fx))\cos\left(a+b\sqrt{c+dx}\right)+2(-6f+b^2(2cf+d(e+3fx)))\sin\left(a+b\sqrt{c+\right.}{b^4d^2}$$

input
```
Integrate[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*b*Sqrt[c + d*x]*(-6*f + b^2*d*(e + f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*
(-6*f + b^2*(2*c*f + d*(e + 3*f*x)))*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{2\int \left(\frac{f\sin\left(a+b\sqrt{c+dx}\right)(c+dx)^{3/2}}{d}+\frac{(de-cf)\sin\left(a+b\sqrt{c+dx}\right)\sqrt{c+dx}}{d}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{6f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d}+\frac{6f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d}+\frac{(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{3f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}-\frac{\sqrt{c+dx}(de-cf)\cos(a-}{bd}\right.}{d}$$

input
```
Int[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((6*f*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d) - ((d*e - c*f)*Sq
rt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d) - (f*(c + d*x)^(3/2)*Cos[a + b
*Sqrt[c + d*x]])/(b*d) - (6*f*Sin[a + b*Sqrt[c + d*x]])/(b^4*d) + ((d*e -
c*f)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d) + (3*f*(c + d*x)*Sin[a + b*Sqrt[c +
 d*x]])/(b^2*d)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(167) = 334$.

Time = 1.15 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{2\sqrt{dx+c}\,\cos(a+\sqrt{dx+c}\,b) f x}{db}-\frac{2\sqrt{dx+c}\,\cos(a+\sqrt{dx+c}\,b) e}{db}+\frac{2\sin(a+\sqrt{dx+c}\,b) f x}{d\,b^{2}}+\frac{2\sin(a+\sqrt{dx+c}\,b) e}{d\,b^{2}}-$
derivativedivides	$-2cfa\cos(a+\sqrt{dx+c}\,b)+2dea\cos(a+\sqrt{dx+c}\,b)-2cf(\sin(a+\sqrt{dx+c}\,b)-(a+\sqrt{dx+c}\,b)\cos(a+\sqrt{dx+c}\,b))+2de(\sin($
default	$-2cfa\cos(a+\sqrt{dx+c}\,b)+2dea\cos(a+\sqrt{dx+c}\,b)-2cf(\sin(a+\sqrt{dx+c}\,b)-(a+\sqrt{dx+c}\,b)\cos(a+\sqrt{dx+c}\,b))+2de(\sin($

input
```
int((f*x+e)*sin(a+(d*x+c)^(1/2)*b),x,method=_RETURNVERBOSE)
```

output
```
-2/d/b*(d*x+c)^(1/2)*cos(a+(d*x+c)^(1/2)*b)*f*x-2/d/b*(d*x+c)^(1/2)*cos(a+
(d*x+c)^(1/2)*b)*e+2/d/b^2*sin(a+(d*x+c)^(1/2)*b)*f*x+2/d/b^2*sin(a+(d*x+c
)^(1/2)*b)*e-2/d/b^2*f*(2*a/d/b^2*(cos(a+(d*x+c)^(1/2)*b)+(a+(d*x+c)^(1/2)
*b)*sin(a+(d*x+c)^(1/2)*b)-a*sin(a+(d*x+c)^(1/2)*b))-2/d/b^2*((a+(d*x+c)^(
1/2)*b)^2*sin(a+(d*x+c)^(1/2)*b)-2*sin(a+(d*x+c)^(1/2)*b)+2*(a+(d*x+c)^(1/
2)*b)*cos(a+(d*x+c)^(1/2)*b)-a*(cos(a+(d*x+c)^(1/2)*b)+(a+(d*x+c)^(1/2)*b)
*sin(a+(d*x+c)^(1/2)*b)))+2/d/b^2*(sin(a+(d*x+c)^(1/2)*b)-(a+(d*x+c)^(1/2)
*b)*cos(a+(d*x+c)^(1/2)*b)+a*cos(a+(d*x+c)^(1/2)*b)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int(e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\,\Big(\left(b^3dfx+b^3de-6\,bf\right)\sqrt{dx+c}\cos\left(\sqrt{dx+c}b+a\right)-\left(3\,b^2dfx+b^2de+2\,(b^2c-3)f\right)\sin\Big(\sqrt{dx+c}b}{b^4d^2}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-2*((b^3*d*f*x + b^3*d*e - 6*b*f)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) -
 (3*b^2*d*f*x + b^2*d*e + 2*(b^2*c - 3)*f)*sin(sqrt(d*x + c)*b + a))/(b^4*
d^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.19

$$\int(e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases}\left(ex+\frac{fx^2}{2}\right)\sin\left(a\right) \\ \left(ex+\frac{fx^2}{2}\right)\sin\left(a+b\sqrt{c}\right) \\ -\frac{2e\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2fx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{4cf\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2e\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{6fx\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}\end{cases}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise(((e*x + f*x**2/2)*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), ((e
*x + f*x**2/2)*sin(a + b*sqrt(c)), Eq(d, 0)), (-2*e*sqrt(c + d*x)*cos(a +
b*sqrt(c + d*x))/(b*d) - 2*f*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d
) + 4*c*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e*sin(a + b*sqrt(c + d*
x))/(b**2*d) + 6*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d) + 12*f*sqrt(c + d*x
)*cos(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*f*sin(a + b*sqrt(c + d*x))/(b*
*4*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(167) = 334$.

Time = 0.05 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.88

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$$
$$= \frac{2\left(ae\cos\left(\sqrt{dx+c}b+a\right)-\frac{acf\cos\left(\sqrt{dx+c}b+a\right)}{d}-\left(\left(\sqrt{dx+c}b+a\right)\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+\right.\right.\right.}{}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
2*(a*e*cos(sqrt(d*x + c)*b + a) - a*c*f*cos(sqrt(d*x + c)*b + a)/d - ((sqr
t(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e +
 ((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a
))*c*f/d + a^3*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) - 3*((sqrt(d*x + c)*b +
a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*f/(b^2*d) + 3*
(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)
*b + a)*sin(sqrt(d*x + c)*b + a))*a*f/(b^2*d) - (((sqrt(d*x + c)*b + a)^3
- 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b
+ a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*f/(b^2*d))/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx =$$

$$-\frac{2\left(\frac{\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)e}{b}-\frac{f\left(\frac{\left(\left(\sqrt{dx+c}b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}b+a\right)^3+3\left(\sqrt{dx+c}b+a\right)^2a-3\left(\sqrt{dx+c}b+a\right)a^2+a^3+6\sqrt{}}{b^2}\right.}{}\right.}{bd}$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output

```
-2*((sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*
e/b - f*(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3
+ 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 + 6*sqrt
(d*x + c)*b)*cos(sqrt(d*x + c)*b + a)/b^2 - (b^2*c - 3*(sqrt(d*x + c)*b +
a)^2 + 6*(sqrt(d*x + c)*b + a)*a - 3*a^2 + 6)*sin(sqrt(d*x + c)*b + a)/b^2
)/(b*d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = \int \sin\left(a+b\sqrt{c+d\,x}\right)\,\left(e+f\,x\right)\,dx$$

input

```
int(sin(a + b*(c + d*x)^(1/2))*(e + f*x),x)
```

output

```
int(sin(a + b*(c + d*x)^(1/2))*(e + f*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.79

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$$

$$= \frac{-2\sqrt{dx+c}\,\cos\big(\sqrt{dx+c}\,b+a\big)\,b^3de-2\sqrt{dx+c}\,\cos\big(\sqrt{dx+c}\,b+a\big)\,b^3dfx+12\sqrt{dx+c}\,\cos\big(\sqrt{dx+}{}$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x)
```

output
```
(2*( - sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a)*b**3*d*e - sqrt(c + d*x)*cos
(sqrt(c + d*x)*b + a)*b**3*d*f*x + 6*sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a
)*b*f + 2*sin(sqrt(c + d*x)*b + a)*b**2*c*f + sin(sqrt(c + d*x)*b + a)*b**
2*d*e + 3*sin(sqrt(c + d*x)*b + a)*b**2*d*f*x - 6*sin(sqrt(c + d*x)*b + a)
*f))/(b**4*d**2)
```

3.189 $\int \sin \left(a+b\sqrt{c+dx}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sin \left(a+b\sqrt{c+dx}\right) \, dx = -\frac{2\sqrt{c+dx}\cos \left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\sin \left(a+b\sqrt{c+dx}\right)}{b^2d}$$

output
```
-2*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(1/2))/b/d+2*sin(a+b*(d*x+c)^(1/2))/b^2/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \sin \left(a+b\sqrt{c+dx}\right) \, dx = \frac{-2b\sqrt{c+dx}\cos \left(a+b\sqrt{c+dx}\right)+2\sin \left(a+b\sqrt{c+dx}\right)}{b^2d}$$

input
```
Integrate[Sin[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*b*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]] + 2*Sin[a + b*Sqrt[c + d*x]])
/(b^2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3842, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+b\sqrt{c+dx}\right) \, dx$$

$$\Big\downarrow \, 3842$$

$$\frac{2\int \sqrt{c+dx}\sin \left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \, 3042$$

$$\frac{2\int \sqrt{c+dx}\sin \left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \, 3777$$

$$\frac{2\left(\frac{\int \cos\left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{b}-\frac{\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b}\right)}{d}$$

$$\Big\downarrow \, 3042$$

$$\frac{2\left(\frac{\int \sin\left(a+b\sqrt{c+dx}+\frac{\pi}{2}\right)d\sqrt{c+dx}}{b}-\frac{\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b}\right)}{d}$$

$$\Big\downarrow \, 3117$$

$$\frac{2\left(\frac{\sin\left(a+b\sqrt{c+dx}\right)}{b^2}-\frac{\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b}\right)}{d}$$

input

```
Int[Sin[a + b*Sqrt[c + d*x]],x]
```

output

```
(2*(-((Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/b) + Sin[a + b*Sqrt[c + d*x
]]/b^2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{2\sin\left(a+\sqrt{dx+c}\,b\right)-2\left(a+\sqrt{dx+c}\,b\right)\cos\left(a+\sqrt{dx+c}\,b\right)+2a\cos\left(a+\sqrt{dx+c}\,b\right)}{b^{2}d}$	61
default	$\frac{2\sin\left(a+\sqrt{dx+c}\,b\right)-2\left(a+\sqrt{dx+c}\,b\right)\cos\left(a+\sqrt{dx+c}\,b\right)+2a\cos\left(a+\sqrt{dx+c}\,b\right)}{b^{2}d}$	61

input
```
int(sin(a+(d*x+c)^(1/2)*b),x,method=_RETURNVERBOSE)
```

output
```
2/d/b^2*(sin(a+(d*x+c)^(1/2)*b)-(a+(d*x+c)^(1/2)*b)*cos(a+(d*x+c)^(1/2)*b)
+a*cos(a+(d*x+c)^(1/2)*b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx=-\frac{2\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(
b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\begin{cases} x\sin\left(a\right) & \text{for } b=0\wedge\left(b=0\vee d=0\right) \\ x\sin\left(a+b\sqrt{c}\right) & \text{for } d=0 \\ -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sin(a + b*sqrt(
c)), Eq(d, 0)), (-2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) + 2*sin(a
 + b*sqrt(c + d*x))/(b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left(\left(\sqrt{dx+c}b+a\right)\cos\left(\sqrt{dx+c}b+a\right)-a\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
-2*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - a*cos(sqrt(d*x + c)*b
 + a) - sin(sqrt(d*x + c)*b + a))/(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(
b^2*d)
```

Mupad [B] (verification not implemented)

Time = 39.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sin \left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\left(\sin\left(a+b\,\sqrt{c+d\,x}\right)-b\,\cos\left(a+b\,\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

input
```
int(sin(a + b*(c + d*x)^(1/2)),x)
```

output
```
(2*(sin(a + b*(c + d*x)^(1/2)) - b*cos(a + b*(c + d*x)^(1/2))*(c + d*x)^(1
/2)))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \sin \left(a+b\sqrt{c+dx}\right) \, dx = \frac{-2\sqrt{dx+c}\,\cos \left(\sqrt{dx+c}\,b+a\right) b+2\sin \left(\sqrt{dx+c}\,b+a\right)}{b^2 d}$$

input
```
int(sin(a+b*(d*x+c)^(1/2)),x)
```

output
```
(2*( - sqrt(c + d*x)*cos(sqrt(c + d*x)*b + a)*b + sin(sqrt(c + d*x)*b + a)
))/(b**2*d)
```

3.190 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$
\begin{aligned}
\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = {} & \frac{\operatorname{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f} \\
& + \frac{\operatorname{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f} \\
& - \frac{\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\operatorname{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f} \\
& + \frac{\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\operatorname{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{f}
\end{aligned}
$$

output

```
Ci(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))*sin(a-b*(c*f-d*e)^(1/2)/f^(1
/2))/f+Ci(b*(c*f-d*e)^(1/2)/f^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*(c*f-d*e)^(1/
2)/f^(1/2))/f+cos(a+b*(c*f-d*e)^(1/2)/f^(1/2))*Si(-b*(c*f-d*e)^(1/2)/f^(1/
2)+b*(d*x+c)^(1/2))/f+cos(a-b*(c*f-d*e)^(1/2)/f^(1/2))*Si(b*(c*f-d*e)^(1/2
)/f^(1/2)+b*(d*x+c)^(1/2))/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$
$$= \frac{ie^{-i\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}\left(\text{ExpIntegralEi}\left(-ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)-e^{2i\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}\text{ExpIntegralEi}\left(ib\left(-\right.\right.\right.}{}$$

input
```
Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

output
```
((I/2)*(ExpIntegralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x
])] - E^((2*I)*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*ExpIntegralEi[I*b*(-(
Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])] + E^(((2*I)*b*Sqrt[-(d*e) +
c*f])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
 d*x])] - E^((2*I)*a)*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt
[c + d*x])]))/(E^(I*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*f)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{2\int \left(\frac{d\sin\left(a+b\sqrt{c+dx}\right)}{2\sqrt{f}\left(\sqrt{cf-de}+\sqrt{f}\sqrt{c+dx}\right)}-\frac{d\sin\left(a+b\sqrt{c+dx}\right)}{2\sqrt{f}\left(\sqrt{cf-de}-\sqrt{f}\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\Bigg(\frac{d\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{\sqrt{cf-de}b}{\sqrt{f}}+\sqrt{c+dx}b\right)}{2f}+\frac{d\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f}-\frac{d\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}(}{2f}}{d}$$

input
```
Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

output
```
(2*((d*CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]]*Sin[a
 - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]])/(2*f) + (d*CosIntegral[(b*Sqrt[-(d*e)
+ c*f])/Sqrt[f] - b*Sqrt[c + d*x]]*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]
)/(2*f) - (d*Cos[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-
(d*e) + c*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f) + (d*Cos[a - (b*Sqrt[-(d*e
) + c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c +
 d*x]])/(2*f)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(197) = 394$.

Time = 0.85 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.33

method	result
derivativedivides	$-\frac{b^{2}\left(af+\sqrt{b^{2}c\,f^{2}-b^{2}def}\right)\left(-\operatorname{Si}\left(-\sqrt{dx+c}\,b-a+\frac{af+\sqrt{b^{2}c\,f^{2}-b^{2}def}}{f}\right)\cos\left(\frac{af+\sqrt{b^{2}c\,f^{2}-b^{2}def}}{f}\right)+\operatorname{Ci}\left(\sqrt{dx+c}\,b+a-\frac{af+\sqrt{b^2}}{}\right.\right.}{f^{2}\left(-\frac{af+\sqrt{b^{2}c\,f^{2}-b^{2}def}}{f}+a\right)}$
default	$-\frac{b^{2}\left(af+\sqrt{b^{2}c\,f^{2}-b^{2}def}\right)\left(-\operatorname{Si}\left(-\sqrt{dx+c}\,b-a+\frac{af+\sqrt{b^{2}c\,f^{2}-b^{2}def}}{f}\right)\cos\left(\frac{af+\sqrt{b^{2}c\,f^{2}-b^{2}def}}{f}\right)+\operatorname{Ci}\left(\sqrt{dx+c}\,b+a-\frac{af+\sqrt{b^2}}{}\right.\right.}{f^{2}\left(-\frac{af+\sqrt{b^{2}c\,f^{2}-b^{2}def}}{f}+a\right)}$

input
```
int(sin(a+(d*x+c)^(1/2)*b)/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(-1/2*b^2*(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/(-(a*f+(b^2*c*f^2-b^
2*d*e*f)^(1/2))/f+a)*(-Si(-(d*x+c)^(1/2)*b-a+(a*f+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci((d*x+c)^(1/2)*b+a-(a*f
+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))
+1/2*b^2*(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/((-a*f+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f+a)*(-Si(-(d*x+c)^(1/2)*b-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
f)*cos((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci((d*x+c)^(1/2)*b+a+(-a*f+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-b
^2*a*(-1/2/f/(-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-(d*x+c)^(1/2)*
b-a+(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f)+Ci((d*x+c)^(1/2)*b+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f
+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-1/2/f/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2)
)/f+a)*(-Si(-(d*x+c)^(1/2)*b-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((
-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci((d*x+c)^(1/2)*b+a+(-a*f+(b^2*c*f^2
-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$
$$=\frac{-i\,\operatorname{Ei}\left(i\,\sqrt{dx+c}b-\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(i\,a+\sqrt{\frac{b^2de-b^2cf}{f}}\right)}-i\,\operatorname{Ei}\left(i\,\sqrt{dx+c}b+\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(i\,a-\sqrt{\frac{b^2de-b^2cf}{f}}\right)}+i}{2}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

output
```
1/2*(-I*Ei(I*sqrt(d*x + c)*b - sqrt((b^2*d*e - b^2*c*f)/f))*e^(I*a + sqrt(
(b^2*d*e - b^2*c*f)/f)) - I*Ei(I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f
)/f))*e^(I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + I*Ei(-I*sqrt(d*x + c)*b - sq
rt((b^2*d*e - b^2*c*f)/f))*e^(-I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + I*Ei(-
I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/f))*e^(-I*a - sqrt((b^2*d*e -
 b^2*c*f)/f)))/f
```

Sympy [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e),x)
```

output
```
Integral(sin(a + b*sqrt(c + d*x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin \left(\sqrt{dx+c}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\, dx = \int \frac{\sin \left(\sqrt{dx+c}b+a\right)}{fx+e}\, dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\, dx = \int \frac{\sin \left(a+b\,\sqrt{c+d\,x}\right)}{e+f\,x}\, dx$$

input
```
int(sin(a + b*(c + d*x)^(1/2))/(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/2))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{e+fx}\, dx = \int \frac{\sin \left(\sqrt{dx+c}\,b+a\right)}{fx+e} dx$$

input
```
int(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x)
```

output
```
int(sin(sqrt(c + d*x)*b + a)/(e + f*x),x)
```

3.191 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 339

$$\begin{aligned}\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx &= \frac{bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} \\ &\quad -\frac{bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} \\ &\quad -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)}+\frac{bd\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} \\ &\quad +\frac{bd\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}\end{aligned}$$

output

```
1/2*b*d*cos(a+b*(c*f-d*e)^(1/2)/f^(1/2))*Ci(b*(c*f-d*e)^(1/2)/f^(1/2)-b*(d
*x+c)^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)-1/2*b*d*cos(a-b*(c*f-d*e)^(1/2)/f^(1/
2))*Ci(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)-
sin(a+b*(d*x+c)^(1/2))/f/(f*x+e)-1/2*b*d*sin(a+b*(c*f-d*e)^(1/2)/f^(1/2))*
Si(-b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)+1/2
*b*d*sin(a-b*(c*f-d*e)^(1/2)/f^(1/2))*Si(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+
c)^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.17

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \frac{ide^{-ia}\left(-\frac{2e^{-ib\sqrt{c+dx}}\sqrt{f}}{de+dfx}-\frac{ibe^{-\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{ExpIntegralEi}\left(-ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{-de+cf}}+\frac{ibe^{\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{ExpIntegralEi}\left(-ib\left(\frac{\sqrt{-de+}}{\sqrt{f}}\right.\right.}{\sqrt{-de+cf}}\right.}{}$$

input
```
Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

output
```
((I/4)*d*((-2*Sqrt[f])/(E^(I*b*Sqrt[c + d*x])*(d*e + d*f*x)) - (I*b*ExpInt
egralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/(E^((I*b*
Sqrt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]) + (I*b*E^((I*b*Sqrt[-(d*e
) + c*f])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt
[c + d*x])])/Sqrt[-(d*e) + c*f] + E^((2*I)*a)*((2*E^(I*b*Sqrt[c + d*x])*Sq
rt[f])/(d*e + d*f*x) - (I*b*E^((I*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*ExpIntegr
alEi[I*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/Sqrt[-(d*e) + c
*f] + (I*b*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x])]
)/(E^((I*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]))))/(E^(I*a)*f^
(3/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3912, 27, 3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{2\int \frac{d^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{27}$$

$$2d\int \frac{\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{(de-cf+f(c+dx))^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{3822}$$

$$2d\left(\frac{b\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{de-cf+f(c+dx)}d\sqrt{c+dx}}{2f}-\frac{\sin\left(a+b\sqrt{c+dx}\right)}{2f(f(c+dx)-cf+de)}\right)$$

$$\Big\downarrow \text{3815}$$

$$2d\left(\frac{b\int \left(\frac{\sqrt{cf-de}\cos\left(a+b\sqrt{c+dx}\right)}{2(de-cf)\left(\sqrt{cf-de}-\sqrt{f}\sqrt{c+dx}\right)}+\frac{\sqrt{cf-de}\cos\left(a+b\sqrt{c+dx}\right)}{2(de-cf)\left(\sqrt{cf-de}+\sqrt{f}\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{2f}-\frac{\sin\left(a+b\sqrt{c+dx}\right)}{2f(f(c+dx)-cf+de)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2d\left(\frac{b\left(\frac{\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2\sqrt{f}\sqrt{cf-de}}-\frac{\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{\sqrt{cf-de}b}{\sqrt{f}}+\sqrt{c+dx}b\right)}{2\sqrt{f}\sqrt{cf-de}}+\frac{\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}(}{2\sqrt{f}\sqrt{}}\right.}{2f}\right.$$

input
```
Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

output
```
2*d*(-1/2*Sin[a + b*Sqrt[c + d*x]]/(f*(d*e - c*f + f*(c + d*x))) + (b*((Co
s[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c*f])/S
qrt[f] - b*Sqrt[c + d*x]])/(2*Sqrt[f]*Sqrt[-(d*e) + c*f]) - (Cos[a - (b*Sq
rt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*
Sqrt[c + d*x]])/(2*Sqrt[f]*Sqrt[-(d*e) + c*f]) + (Sin[a + (b*Sqrt[-(d*e) +
 c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] - b*Sqrt[c + d*
x]])/(2*Sqrt[f]*Sqrt[-(d*e) + c*f]) + (Sin[a - (b*Sqrt[-(d*e) + c*f])/Sqrt
[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/(2*Sqr
t[f]*Sqrt[-(d*e) + c*f])))/(2*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(273) = 546$.

Time = 0.95 (sec) , antiderivative size = 1831, normalized size of antiderivative = 5.40

method	result	size
derivativedivides	Expression too large to display	1831
default	Expression too large to display	1831

input
```
int(sin(a+(d*x+c)^(1/2)*b)/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d/b^2*(sin(a+(d*x+c)^(1/2)*b)*(-1/2*a*b^2/(c*f-d*e)*(a+(d*x+c)^(1/2)*b)+
1/2*b^2*(-b^2*c*f+b^2*d*e+a^2*f)/(c*f-d*e)/f)/(-c*f*b^2+d*e*b^2+a^2*f-2*a*
f*(a+(d*x+c)^(1/2)*b)+f*(a+(d*x+c)^(1/2)*b)^2)+1/4*a*b^2/(c*f-d*e)/f/(-(a*
f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-(d*x+c)^(1/2)*b-a+(a*f+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci((d*x+c)
^(1/2)*b+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*
e*f)^(1/2))/f))+1/4*a*b^2/(c*f-d*e)/f/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
f+a)*(-Si(-(d*x+c)^(1/2)*b-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a
*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci((d*x+c)^(1/2)*b+a+(-a*f+(b^2*c*f^2-b
^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4*b^2*(-c
*f*b^2+d*e*b^2+a^2*f-a*(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2)))/(c*f-d*e)/f^2/(-
(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(Si(-(d*x+c)^(1/2)*b-a+(a*f+(b^2*c*
f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci((d*x+
c)^(1/2)*b+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/f))+1/4*b^2*(-c*f*b^2+d*e*b^2+a^2*f+a*(-a*f+(b^2*c*f^2-b^2*d
*e*f)^(1/2)))/(c*f-d*e)/f^2/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(
-(d*x+c)^(1/2)*b-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*
f^2-b^2*d*e*f)^(1/2))/f)+Ci((d*x+c)^(1/2)*b+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^
(1/2))/f)*cos((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-a*b^4*(sin(a+(d*x+c)^
(1/2)*b)*(-1/2/b^2/(c*f-d*e)*(a+(d*x+c)^(1/2)*b)+1/2*a/b^2/(c*f-d*e))/(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.23

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{(-i\,dfx-i\,de)\sqrt{\frac{b^2de-b^2cf}{f}}\operatorname{Ei}\left(i\sqrt{dx+c}b-\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia+\sqrt{\frac{b^2de-b^2cf}{f}}\right)}+(i\,dfx+i\,de)\sqrt{\frac{b^2de-b^2cf}{f}}\operatorname{Ei}}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

output

```
-1/4*((-I*d*f*x - I*d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(I*sqrt(d*x + c)*b
- sqrt((b^2*d*e - b^2*c*f)/f))*e^(I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + (I*
d*f*x + I*d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(I*sqrt(d*x + c)*b + sqrt((b^
2*d*e - b^2*c*f)/f))*e^(I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + (I*d*f*x + I*
d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(-I*sqrt(d*x + c)*b - sqrt((b^2*d*e - b
^2*c*f)/f))*e^(-I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + (-I*d*f*x - I*d*e)*sq
rt((b^2*d*e - b^2*c*f)/f)*Ei(-I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)
/f))*e^(-I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + 4*(d*e - c*f)*sin(sqrt(d*x +
 c)*b + a))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{\left(e+fx\right)^{2}}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e)**2,x)
```

output

```
Integral(sin(a + b*sqrt(c + d*x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(\sqrt{dx+c}b+a\right)}{\left(fx+e\right)^{2}}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
```

output

```
integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(\sqrt{dx+c}b+a\right)}{\left(fx+e\right)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+b\,\sqrt{c+d\,x}\right)}{\left(e+f\,x\right)^2}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/2))/(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(1/2))/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin \left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(\sqrt{dx+c}\,b+a\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x)
```

output
```
int(sin(sqrt(c + d*x)*b + a)/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.192 $\int (e+fx)^2 \sin \left(a+b(c+dx)^{3/2}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 382

$$\begin{aligned}\int (e+fx)^2 \sin \left(a+b(c+dx)^{3/2}\right) \, dx = &-\frac{4f(de-cf)\sqrt{c+dx}\cos \left(a+b(c+dx)^{3/2}\right)}{3bd^3} \\ &-\frac{2f^2(c+dx)^{3/2}\cos \left(a+b(c+dx)^{3/2}\right)}{3bd^3} \\ &-\frac{2e^{ia}f(de-cf)\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}} \\ &-\frac{2e^{-ia}f(de-cf)\sqrt{c+dx}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}} \\ &+\frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^3\left(-ib(c+dx)^{3/2}\right)^{2/3}} \\ &-\frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^3\left(ib(c+dx)^{3/2}\right)^{2/3}}+\frac{2f^2\sin \left(a+b(c+dx)^{3/2}\right)}{3b^2d^3}\end{aligned}$$

output
```
-4/3*f*(-c*f+d*e)*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(3/2))/b/d^3-2/3*f^2*(d*x+
c)^(3/2)*cos(a+b*(d*x+c)^(3/2))/b/d^3-2/9*exp(I*a)*f*(-c*f+d*e)*(d*x+c)^(1
/2)*GAMMA(1/3,-I*b*(d*x+c)^(3/2))/b/d^3/(-I*b*(d*x+c)^(3/2))^(1/3)-2/9*f*(
-c*f+d*e)*(d*x+c)^(1/2)*GAMMA(1/3,I*b*(d*x+c)^(3/2))/b/d^3/exp(I*a)/(I*b*(
d*x+c)^(3/2))^(1/3)+1/3*I*exp(I*a)*(-c*f+d*e)^2*(d*x+c)*GAMMA(2/3,-I*b*(d*
x+c)^(3/2))/d^3/(-I*b*(d*x+c)^(3/2))^(2/3)-1/3*I*(-c*f+d*e)^2*(d*x+c)*GAMM
A(2/3,I*b*(d*x+c)^(3/2))/d^3/exp(I*a)/(I*b*(d*x+c)^(3/2))^(2/3)+2/3*f^2*si
n(a+b*(d*x+c)^(3/2))/b^2/d^3
```

Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.10

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{3/2}\right)\,dx =$$

$$-\frac{i\bigg((\cos(a)+i\sin(a))\bigg(\frac{f^2\cos(b(c+dx)^{3/2})}{b^2}-\frac{(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{2f(de-cf)(c+dx)^2\Gamma\left(\frac{4}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{4/3}}+}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

output
```
((-1/3*I)*((Cos[a] + I*Sin[a])*((f^2*Cos[b*(c + d*x)^(3/2)])/b^2 - ((d*e -
 c*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3
/2))^(2/3) - (2*f*(d*e - c*f)*(c + d*x)^2*Gamma[4/3, (-I)*b*(c + d*x)^(3/2
)])/((-I)*b*(c + d*x)^(3/2))^(4/3) + (I*f^2*Sin[b*(c + d*x)^(3/2)])/b^2 +
(f^2*(c + d*x)^(3/2)*((-I)*Cos[b*(c + d*x)^(3/2)] + Sin[b*(c + d*x)^(3/2)]
))/b) - (Cos[a] - I*Sin[a])*((f^2*Cos[b*(c + d*x)^(3/2)])/b^2 - ((d*e - c*
f)^2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(I*b*(c + d*x)^(3/2))^(2/3
) - (2*f*(d*e - c*f)*(c + d*x)^2*Gamma[4/3, I*b*(c + d*x)^(3/2)])/(I*b*(c
+ d*x)^(3/2))^(4/3) - (I*f^2*Sin[b*(c + d*x)^(3/2)])/b^2 + (f^2*(c + d*x)^
(3/2)*(I*Cos[b*(c + d*x)^(3/2)] + Sin[b*(c + d*x)^(3/2)]))/b)))/d^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{3/2}\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{2\int\left(f^2\sin\left(b(c+dx)^{3/2}+a\right)(c+dx)^{5/2}+2f(de-cf)\sin\left(b(c+dx)^{3/2}+a\right)(c+dx)^{3/2}+(de-cf)^2\sin\left(b(c+\right.\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{f^2\sin\left(a+b(c+dx)^{3/2}\right)}{3b^2}-\frac{2f\sqrt{c+dx}(de-cf)\cos\left(a+b(c+dx)^{3/2}\right)}{3b}-\frac{e^{ia}f\sqrt{c+dx}(de-cf)\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9b\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{e^{-ia}f\sqrt{c+dx}(de-cf)\Gamma\left(\frac{1}{\right.}{9b\sqrt[3]{ib(c+dx}}\right.}{d}$$

input
```
Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

output
```
(2*((-2*f*(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b) - (f
^2*(c + d*x)^(3/2)*Cos[a + b*(c + d*x)^(3/2)])/(3*b) - (E^(I*a)*f*(d*e - c
*f)*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*((-I)*b*(c + d*
x)^(3/2))^(1/3)) - (f*(d*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(
3/2)])/(9*b*E^(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/6)*E^(I*a)*(d*e - c
*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3/2
))^(2/3) - ((I/6)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])
/(E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3)) + (f^2*Sin[a + b*(c + d*x)^(3/2)])/
(3*b^2)))/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+b(dx+c)^{\frac{3}{2}}\right) dx$$

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

output
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\,dx = \frac{6\,f^2\sin\left((bdx+bc)\sqrt{dx+c}+a\right) - 2\left(\left(-i\,def+i\,cf^2\right)\cos(a) - \left(def-cf^2\right)\sin(a)\right)(i\,b)^{\frac{2}{3}}\,\Gamma}{}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

output
```
1/9*(6*f^2*sin((b*d*x + b*c)*sqrt(d*x + c) + a) - 2*((-I*d*e*f + I*c*f^2)*
cos(a) - (d*e*f - c*f^2)*sin(a))*(I*b)^(2/3)*gamma(1/3, (I*b*d*x + I*b*c)*
sqrt(d*x + c)) - 2*((I*d*e*f - I*c*f^2)*cos(a) - (d*e*f - c*f^2)*sin(a))*(
-I*b)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)) - 6*(b*d*f^2*x +
2*b*d*e*f - b*c*f^2)*sqrt(d*x + c)*cos((b*d*x + b*c)*sqrt(d*x + c) + a) -
3*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a) + (-I*b*d^2*e^2 + 2*I*b*c*
d*e*f - I*b*c^2*f^2)*sin(a))*(I*b)^(1/3)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt
(d*x + c)) - 3*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a) + (I*b*d^2*e^
2 - 2*I*b*c*d*e*f + I*b*c^2*f^2)*sin(a))*(-I*b)^(1/3)*gamma(2/3, (-I*b*d*x
 - I*b*c)*sqrt(d*x + c)))/(b^2*d^3)
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\,dx = \int (e+fx)^2 \sin\left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)dx$$

input
```
integrate((f*x+e)**2*sin(a+b*(d*x+c)**(3/2)),x)
```

output
```
Integral((e + f*x)**2*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(290) = 580.

Time = 0.52 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.82

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

output
```
-1/18*(3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^
(3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sq
rt(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -
I*(d*x + c)^(3/2)*b))*sin(a))*e^2/(sqrt(d*x + c)*b) - 6*((d*x + c)^(3/2)*b
)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*ga
mma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*
x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a
))*c*e*f/(sqrt(d*x + c)*b*d) + 3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)
*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(
3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*s
qrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))*c^2*f^2/(sqrt(d*x +
c)*b*d^2) + 2*(12*((d*x + c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3
/2)*b + a) + sqrt(d*x + c)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)^(3/2)*b)
 + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(a) + ((-I*sqrt(3) -
 1)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x +
 c)^(3/2)*b))*sin(a)))*e*f/(((d*x + c)^(3/2)*b)^(1/3)*b*d) - 2*(12*((d*x +
 c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) + sqrt(d*x + c
)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) + I)*gamma(1/
3, -I*(d*x + c)^(3/2)*b))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*(d*x + c
)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(a)))...
```

Giac [F]

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int (fx+e)^2\sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin \left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin \left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)\,{\left(e+f\,x\right)}^2 \,d x$$

input
```
int(sin(a + b*(c + d*x)^(3/2))*(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(3/2))*(e + f*x)^2, x)
```

Reduce [F]

$$\begin{aligned}&\int (e+fx)^2\sin \left(a\right.\\&\left.+b(c+dx)^{3/2}\right)\,dx=\left(\int \sin \left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)dx\right)e^2\\&+\left(\int \sin \left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)x^2dx\right)f^2\\&+2\left(\int \sin \left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)xdx\right)ef\end{aligned}$$

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

output
```
int(sin(sqrt(c + d*x)*b*c + sqrt(c + d*x)*b*d*x + a),x)*e**2 + int(sin(sqr
t(c + d*x)*b*c + sqrt(c + d*x)*b*d*x + a)*x**2,x)*f**2 + 2*int(sin(sqrt(c
+ d*x)*b*c + sqrt(c + d*x)*b*d*x + a)*x,x)*e*f
```

3.193 $\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 291

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx &= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}\\ &\quad-\frac{e^{ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{e^{-ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{ib(c+dx)^{3/2}}}\\ &\quad+\frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}\\ &\quad-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}\end{aligned}$$

output

```
-2/3*f*(d*x+c)^(1/2)*cos(a+b*(d*x+c)^(3/2))/b/d^2-1/9*exp(I*a)*f*(d*x+c)^(
1/2)*GAMMA(1/3,-I*b*(d*x+c)^(3/2))/b/d^2/(-I*b*(d*x+c)^(3/2))^(1/3)-1/9*f*
(d*x+c)^(1/2)*GAMMA(1/3,I*b*(d*x+c)^(3/2))/b/d^2/exp(I*a)/(I*b*(d*x+c)^(3/
2))^(1/3)+1/3*I*exp(I*a)*(-c*f+d*e)*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/
d^2/(-I*b*(d*x+c)^(3/2))^(2/3)-1/3*I*(-c*f+d*e)*(d*x+c)*GAMMA(2/3,I*b*(d*x
+c)^(3/2))/d^2/exp(I*a)/(I*b*(d*x+c)^(3/2))^(2/3)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 705 vs. $2(291) = 582$.

Time = 3.12 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.42

$$
\begin{aligned}
\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx &= -\frac{2f\sqrt{c+dx}\cos(a)\cos\left(b(c+dx)^{3/2}\right)}{3bd^2} \\
&+ \frac{f\cos(a)\left(-\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)}{6bd^2} \\
&- \frac{ie\cos(a)\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}+\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{2d} \\
&+ \frac{icf\cos(a)\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}+\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{2d^2} \\
&+ \frac{if\left(-\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}}+\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)\sin(a)}{6bd^2} \\
&+ \frac{e\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)\sin(a)}{2d} \\
&- \frac{cf\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)\sin(a)}{2d^2} \\
&+ \frac{2f\sqrt{c+dx}\sin(a)\sin\left(b(c+dx)^{3/2}\right)}{3bd^2}
\end{aligned}
$$

input

```
Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

output

```
(-2*f*Sqrt[c + d*x]*Cos[a]*Cos[b*(c + d*x)^(3/2)])/(3*b*d^2) + (f*Cos[a]*(
(-2*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)
^(3/2))^(1/3)) - (2*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b
*(c + d*x)^(3/2))^(1/3))))/(6*b*d^2) - ((I/2)*e*Cos[a]*((-2*(c + d*x)*Gamm
a[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c
 + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3))))
/d + ((I/2)*c*f*Cos[a]*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/
(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)
^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d^2 + ((I/6)*f*((-2*Sqrt[c + d*
x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(1/3))
+ (2*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2
))^(1/3)))*Sin[a])/(b*d^2) + (e*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)
^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*(c + d*x)*Gamma[2/3, I*b*
(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))*Sin[a])/(2*d) - (c*f*((
-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2
))^(2/3)) - (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x
)^(3/2))^(2/3)))*Sin[a])/(2*d^2) + (2*f*Sqrt[c + d*x]*Sin[a]*Sin[b*(c + d*
x)^(3/2)])/(3*b*d^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{2\int \left(f\sin\left(b(c+dx)^{3/2}+a\right)(c+dx)^{3/2}+(de-cf)\sin\left(b(c+dx)^{3/2}+a\right)\sqrt{c+dx}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{ie^{ia}(c+dx)(de-cf)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{6\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)(de-cf)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{6\left(ib(c+dx)^{3/2}\right)^{2/3}}-\frac{f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3b}-\frac{e^{ia}f\sqrt{c+dx}\Gamma\left(\frac{1}{3},-\right.}{18b\sqrt[3]{-ib(c-}}\right.}{d^2}$$

input
```
Int[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

output
```
(2*(-1/3*(f*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/b - (E^(I*a)*f*Sqrt[
c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(18*b*((-I)*b*(c + d*x)^(3/2)
)^(1/3)) - (f*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(18*b*E^(I*a)
*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/6)*E^(I*a)*(d*e - c*f)*(c + d*x)*Gamma
[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3/2))^(2/3) - ((I/6)*(d*
e - c*f)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(E^(I*a)*(I*b*(c + d*x
)^(3/2))^(2/3))))/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)dx$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

output
```
int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.77

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx =$$

$$-\frac{6\sqrt{dx+c}bf\cos\left((bdx+bc)\sqrt{dx+c}+a\right)-(if\cos(a)+f\sin(a))(ib)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},(ibdx+ibc)\sqrt{dx+c}\right)-(}{}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

output
```
-1/9*(6*sqrt(d*x + c)*b*f*cos((b*d*x + b*c)*sqrt(d*x + c) + a) - (I*f*cos(
a) + f*sin(a))*(I*b)^(2/3)*gamma(1/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) - (
-I*f*cos(a) + f*sin(a))*(-I*b)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)*sqrt(d*
x + c)) + 3*((b*d*e - b*c*f)*cos(a) + (-I*b*d*e + I*b*c*f)*sin(a))*(I*b)^(
1/3)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) + 3*((b*d*e - b*c*f)*cos(
a) + (I*b*d*e - I*b*c*f)*sin(a))*(-I*b)^(1/3)*gamma(2/3, (-I*b*d*x - I*b*c
)*sqrt(d*x + c)))/(b^2*d^2)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx = \int (e+fx)\sin\left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)**(3/2)),x)
```

output
```
Integral((e + f*x)*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.29

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

output

```
-1/18*(3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^
(3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sq
rt(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -
I*(d*x + c)^(3/2)*b))*sin(a))*e/(sqrt(d*x + c)*b) - 3*((d*x + c)^(3/2)*b)^
(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamm
a(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x
+ c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))
*c*f/(sqrt(d*x + c)*b*d) + (12*((d*x + c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos
((d*x + c)^(3/2)*b + a) + sqrt(d*x + c)*(((sqrt(3) - I)*gamma(1/3, I*(d*x
+ c)^(3/2)*b) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(a) + (
(-I*sqrt(3) - 1)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1
/3, -I*(d*x + c)^(3/2)*b))*sin(a)))*f/(((d*x + c)^(3/2)*b)^(1/3)*b*d))/d
```

Giac [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx = \int (fx+e)\sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

output

```
integrate((f*x + e)*sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)\,\left(e+f\,x\right)\,dx$$

input
```
int(sin(a + b*(c + d*x)^(3/2))*(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(3/2))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx=\left(\int \sin\left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)dx\right)e+\left(\int \sin\left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)xdx\right)f$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

output
```
int(sin(sqrt(c + d*x)*b*c + sqrt(c + d*x)*b*d*x + a),x)*e + int(sin(sqrt(c
 + d*x)*b*c + sqrt(c + d*x)*b*d*x + a)*x,x)*f
```

3.194 $\int \sin \left(a+b(c+dx)^{3/2}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sin \left(a+b(c+dx)^{3/2}\right) \, dx = \frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

output
```
1/3*I*exp(I*a)*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d/(-I*b*(d*x+c)^(3/2)
)^(2/3)-1/3*I*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d/exp(I*a)/(I*b*(d*x+c)
^(3/2))^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \sin \left(a+b(c+dx)^{3/2}\right) \, dx = \frac{i(c+dx)\left(-\left(-ib(c+dx)^{3/2}\right)^{2/3}\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)(\cos(a)-i\sin(a))+\left(ib(c+dx)^{3/2}\right)^{2/3}}{3d\left(b^2(c+dx)^3\right)^{2/3}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(3/2)],x]
```

output

```
((I/3)*(c + d*x)*(-(((-I)*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, I*b*(c + d*x
)^(3/2)]*(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, (-I
)*b*(c + d*x)^(3/2)]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3844, 3870, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+b(c+dx)^{3/2}\right)dx$$

$$\Big\downarrow \text{3844}$$

$$\frac{2\int \sqrt{c+dx}\sin\left(b(c+dx)^{3/2}+a\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{3870}$$

$$\frac{2\left(\frac{1}{2}i\int e^{-ib(c+dx)^{3/2}-ia}\sqrt{c+dx}d\sqrt{c+dx}-\frac{1}{2}i\int e^{ib(c+dx)^{3/2}+ia}\sqrt{c+dx}d\sqrt{c+dx}\right)}{d}$$

$$\Big\downarrow \text{2648}$$

$$\frac{2\left(\frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{6\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{6\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{d}$$

input

```
Int[Sin[a + b*(c + d*x)^(3/2)],x]
```

output

```
(2*(((I/6)*E^(I*a)*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(
c + d*x)^(3/2))^(2/3) - ((I/6)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/
(E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3))))/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3844
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Simp[k/f   Subst[Int[x^(k - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

rule 3870
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Maple [F]

$$\int \sin\left(a+b(dx+c)^{\frac{3}{2}}\right)dx$$

input
```
int(sin(a+b*(d*x+c)^(3/2)),x)
```

output
```
int(sin(a+b*(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int \sin\left(a+b(c+dx)^{3/2}\right)dx =$$

$$-\frac{(i b)^{\frac{1}{3}}(\cos(a)-i\sin(a))\Gamma\left(\frac{2}{3},(i bdx+i bc)\sqrt{dx+c}\right)+(-i b)^{\frac{1}{3}}(\cos(a)+i\sin(a))\Gamma\left(\frac{2}{3},(-i bdx-i bc)\sqrt{}\right.}{3bd}$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

output
```
-1/3*((I*b)^(1/3)*(cos(a) - I*sin(a))*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*
x + c)) + (-I*b)^(1/3)*(cos(a) + I*sin(a))*gamma(2/3, (-I*b*d*x - I*b*c)*s
qrt(d*x + c)))/(b*d)
```

Sympy [F]

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx = \int \sin\left(a+b(c+dx)^{\frac{3}{2}}\right)dx$$

input
```
integrate(sin(a+b*(d*x+c)**(3/2)),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx =$$

$$-\frac{\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i\,(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i\,(dx+c)^{\frac{3}{2}}b\right)\right)\cos\left(a\right)-\left(\left(i\sqrt{3}-1\right)\Gamma\right.\right.}{6\sqrt{dx+c}bd}$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

output
```
-1/6*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2
)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3
) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d
*x + c)^(3/2)*b))*sin(a))/(sqrt(d*x + c)*b*d)
```

Giac [F]

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{3/2}\right)\,dx$$

input
```
int(sin(a + b*(c + d*x)^(3/2)),x)
```

output
```
int(sin(a + b*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx=\int \sin\left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)dx$$

input
```
int(sin(a+b*(d*x+c)^(3/2)),x)
```

output
```
int(sin(sqrt(c + d*x)*b*c + sqrt(c + d*x)*b*d*x + a),x)
```

3.195 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\operatorname{Int}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 22.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}dx$$

input
```
Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

output
```
int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 5.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e),x)
```

output
```
Integral(sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 38.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(3/2))/(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(3/2))/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin\left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

output
```
int(sin(sqrt(c + d*x)*b*c + sqrt(c + d*x)*b*d*x + a)/(e + f*x),x)
```

3.196 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\text{Int}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 30.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)}{\left(fx+e\right)^{2}}dx$$

input
```
int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

output
```
int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f^2*x^2 + 2*e*f*x + e^2), x
)
```

Sympy [N/A]

Not integrable

Time = 36.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin \left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)}{(e+fx)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x))/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left((dx+c)^{\frac{3}{2}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 39.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+b\,(c+d\,x)^{3/2}\right)}{(e+f\,x)^2}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(3/2))/(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(3/2))/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(\sqrt{dx+c}\,bc+\sqrt{dx+c}\,bdx+a\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

output
```
int(sin(sqrt(c + d*x)*b*c + sqrt(c + d*x)*b*d*x + a)/(e**2 + 2*e*f*x + f**
2*x**2),x)
```

3.197 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 611

$$
\begin{aligned}
\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx = {} & \frac{b^5f^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} \\
& -\frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} \\
& +\frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} \\
& -\frac{b^3f^2(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3} \\
& +\frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} \\
& +\frac{bf^2(c+dx)^{5/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3} \\
& +\frac{b^6f^2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{360d^3} \\
& -\frac{b^4f(de-cf)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{6d^3} \\
& +\frac{b^2(de-cf)^2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^3} \\
& +\frac{b^4f^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} \\
& -\frac{b^2f(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} \\
& +\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} \\
& -\frac{b^2f^2(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3} \\
& +\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} \\
& +\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} \\
& +\frac{b^6f^2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3} \\
& -\frac{b^4f(de-cf)\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3} \\
& b^2(de-cf)^2\cos(a)\mathrm{Si}\Big(\frac{b}{\quad}
\end{aligned}
$$

output

```
1/360*b^5*f^2*(d*x+c)^(1/2)*cos(a+b/(d*x+c)^(1/2))/d^3-1/6*b^3*f*(-c*f+d*e
)*(d*x+c)^(1/2)*cos(a+b/(d*x+c)^(1/2))/d^3+b*(-c*f+d*e)^2*(d*x+c)^(1/2)*co
s(a+b/(d*x+c)^(1/2))/d^3-1/180*b^3*f^2*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2)
)/d^3+1/3*b*f*(-c*f+d*e)*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^3+1/15*b*f
^2*(d*x+c)^(5/2)*cos(a+b/(d*x+c)^(1/2))/d^3+1/360*b^6*f^2*Ci(b/(d*x+c)^(1/
2))*sin(a)/d^3-1/6*b^4*f*(-c*f+d*e)*Ci(b/(d*x+c)^(1/2))*sin(a)/d^3+b^2*(-c
*f+d*e)^2*Ci(b/(d*x+c)^(1/2))*sin(a)/d^3+1/360*b^4*f^2*(d*x+c)*sin(a+b/(d*
x+c)^(1/2))/d^3-1/6*b^2*f*(-c*f+d*e)*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^3+(-
c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^3-1/60*b^2*f^2*(d*x+c)^2*sin(a
+b/(d*x+c)^(1/2))/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^3+1/
3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(1/2))/d^3+1/360*b^6*f^2*cos(a)*Si(b/(d*x+
c)^(1/2))/d^3-1/6*b^4*f*(-c*f+d*e)*cos(a)*Si(b/(d*x+c)^(1/2))/d^3+b^2*(-c*
f+d*e)^2*cos(a)*Si(b/(d*x+c)^(1/2))/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 557, normalized size of antiderivative = 0.91

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$

$$= \frac{ie^{-ia}\Bigg(e^{-\frac{ib}{\sqrt{c+dx}}}\sqrt{c+dx}(-ib^5f^2+b^4f^2\sqrt{c+dx}+2ib^3f(30de-29cf+dfx)-6b^2f\sqrt{c+dx}(10de-9cf}{}$$

input

```
Integrate[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```

output
```
((I/720)*((Sqrt[c + d*x]*((-I)*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] + (2*I)*b^3
*f*(30*d*e - 29*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f
*x) + 120*Sqrt[c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*
x + f^2*x^2)) - (24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2
+ 5*e*f*x + f^2*x^2))))/E^((I*b)/Sqrt[c + d*x]) - E^(I*(2*a + b/Sqrt[c + d
*x]))*Sqrt[c + d*x]*(I*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] - (2*I)*b^3*f*(30*d
*e - 29*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x) + 12
0*Sqrt[c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*
x^2)) + (24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*e*f*
x + f^2*x^2))) + b^2*(360*d^2*e^2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*
c + 360*c^2)*f^2)*ExpIntegralEi[((-I)*b)/Sqrt[c + d*x]] - b^2*E^((2*I)*a)*
(360*d^2*e^2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^2)*f^2)*Exp
IntegralEi[(I*b)/Sqrt[c + d*x]]))/(d^3*E^(I*a))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{2\int \left(\frac{f^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)(c+dx)^{7/2}}{d^2}+\frac{2f(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)(c+dx)^{5/2}}{d^2}+\frac{(de-cf)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)(c+dx)^{3/2}}{d^2}\right)d\frac{1}{\sqrt{c+dx}}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{b^6f^2\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{720d^2}-\frac{b^6f^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{720d^2}-\frac{b^5f^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{720d^2}+\frac{b^4f\sin(a)(de-cf)\,\text{CosIntegral}\Big(\frac{b}{\sqrt{c+}}}{12d^2}$$

input

```
Int[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```

output

```
(-2*(-1/720*(b^5*f^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b^3*f*
(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(12*d^2) - (b*(d*e - c
*f)^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(2*d^2) + (b^3*f^2*(c + d*x)
^(3/2)*Cos[a + b/Sqrt[c + d*x]])/(360*d^2) - (b*f*(d*e - c*f)*(c + d*x)^(3
/2)*Cos[a + b/Sqrt[c + d*x]])/(6*d^2) - (b*f^2*(c + d*x)^(5/2)*Cos[a + b/S
qrt[c + d*x]])/(30*d^2) - (b^6*f^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(7
20*d^2) + (b^4*f*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(12*d^2)
 - (b^2*(d*e - c*f)^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(2*d^2) - (b^4*
f^2*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(720*d^2) + (b^2*f*(d*e - c*f)*(c
+ d*x)*Sin[a + b/Sqrt[c + d*x]])/(12*d^2) - ((d*e - c*f)^2*(c + d*x)*Sin[a
 + b/Sqrt[c + d*x]])/(2*d^2) + (b^2*f^2*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x
]])/(120*d^2) - (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(2*d^
2) - (f^2*(c + d*x)^3*Sin[a + b/Sqrt[c + d*x]])/(6*d^2) - (b^6*f^2*Cos[a]*
SinIntegral[b/Sqrt[c + d*x]])/(720*d^2) + (b^4*f*(d*e - c*f)*Cos[a]*SinInt
egral[b/Sqrt[c + d*x]])/(12*d^2) - (b^2*(d*e - c*f)^2*Cos[a]*SinIntegral[b
/Sqrt[c + d*x]])/(2*d^2)))/d
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{2b^{2}\left(-2b^{2}c\,f^{2}\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)^{2}}{4b^{4}}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)^{\frac{3}{2}}}{12b^{3}}+\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{24b^{2}}+\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+}}{24b}\right.\right.}{}$
default	$-\frac{2b^{2}\left(-2b^{2}c\,f^{2}\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)^{2}}{4b^{4}}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)^{\frac{3}{2}}}{12b^{3}}+\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{24b^{2}}+\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+}}{24b}\right.\right.}{}$
parts	Expression too large to display

input

```
int((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output

```
-2/d^3*b^2*(-2*b^2*c*f^2*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*c
os(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d
*x+c)+1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))
*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(a))-2*c*d*e*f*(-1/2*sin(a+b/(d*x+c)^(
1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x
+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*sin(a))+2*b^2*d*e*f*(-1/4*sin(a+
b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*cos(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/
2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)+1/24*cos(a+b/(d*x+c)^(1/2))/b*(
d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(
a))+c^2*f^2*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(
1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2
))*sin(a))+d^2*e^2*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d
*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+
c)^(1/2))*sin(a))+b^4*f^2*(-1/6*sin(a+b/(d*x+c)^(1/2))/b^6*(d*x+c)^3-1/30*
cos(a+b/(d*x+c)^(1/2))/b^5*(d*x+c)^(5/2)+1/120*sin(a+b/(d*x+c)^(1/2))/b^4*
(d*x+c)^2+1/360*cos(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)-1/720*sin(a+b/(d*
x+c)^(1/2))/b^2*(d*x+c)-1/720*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/720
*Si(b/(d*x+c)^(1/2))*cos(a)-1/720*Ci(b/(d*x+c)^(1/2))*sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.64

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$$
$$= \frac{(360\, b^2 d^2 e^2 - 60\, (b^4 + 12\, b^2 c) d e f + (b^6 + 60\, b^4 c + 360\, b^2 c^2) f^2) \operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right) \sin(a) + (360\, b^2 d^2 e^2 - 60\, (b^4}{}$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
1/360*((360*b^2*d^2*e^2 - 60*(b^4 + 12*b^2*c)*d*e*f + (b^6 + 60*b^4*c + 36
0*b^2*c^2)*f^2)*cos_integral(b/sqrt(d*x + c))*sin(a) + (360*b^2*d^2*e^2 -
60*(b^4 + 12*b^2*c)*d*e*f + (b^6 + 60*b^4*c + 360*b^2*c^2)*f^2)*cos(a)*sin
_integral(b/sqrt(d*x + c)) + (24*b*d^2*f^2*x^2 + 360*b*d^2*e^2 - 60*(b^3 +
 10*b*c)*d*e*f + (b^5 + 58*b^3*c + 264*b*c^2)*f^2 + 2*(60*b*d^2*e*f - (b^3
 + 36*b*c)*d*f^2)*x)*sqrt(d*x + c)*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*
x + c)) + (120*d^3*f^2*x^3 + 360*c*d^2*e^2 - 60*(b^2*c + 6*c^2)*d*e*f + (b
^4*c + 54*b^2*c^2 + 120*c^3)*f^2 - 6*(b^2*d^2*f^2 - 60*d^3*e*f)*x^2 - (60*
b^2*d^2*e*f - 360*d^3*e^2 - (b^4 + 48*b^2*c)*d*f^2)*x)*sin((a*d*x + a*c +
sqrt(d*x + c)*b)/(d*x + c)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = \int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$$

input
```
integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/2)),x)
```

output
```
Integral((e + f*x)**2*sin(a + b/sqrt(c + d*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.44

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

output

```
1/720*(360*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a)
+ (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*
x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(
d*x + c)*a + b)/sqrt(d*x + c)))*e^2 - 720*(((-I*Ei(I*b/sqrt(d*x + c)) + I*
Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x
 + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x
 + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*e*f/d + 3
60*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*
b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b
*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)
*a + b)/sqrt(d*x + c)))*c^2*f^2/d^2 + 60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei
(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x +
 c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(
d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*sin((sq
rt(d*x + c)*a + b)/sqrt(d*x + c)))*e*f/d - 60*(((I*Ei(I*b/sqrt(d*x + c)) -
 I*Ei(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(
d*x + c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((
sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*si
n((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*f^2/d^2 + (((-I*Ei(I*b/sqrt(d*x
+ c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6587 vs. $2(537) = 1074$.

Time = 3.41 (sec) , antiderivative size = 6587, normalized size of antiderivative = 10.78

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

output

```
1/360*(360*(a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
*sin(a) - a^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x +
 c)) - 2*(sqrt(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a +
b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos
(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) +
(sqrt(d*x + c)*a + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d
*x + c))*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integra
l(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x
 + c)*a + b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)
*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(
d*x + c)))*e^2/((a^2 - 2*(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x
 + c)*a + b)^2/(d*x + c))*b) - 60*(a^4*b^5*cos_integral(-a + (sqrt(d*x + c
)*a + b)/sqrt(d*x + c))*sin(a) - a^4*b^5*cos(a)*sin_integral(a - (sqrt(d*x
 + c)*a + b)/sqrt(d*x + c)) - 4*(sqrt(d*x + c)*a + b)*a^3*b^5*cos_integral
(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(sqrt(
d*x + c)*a + b)*a^3*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt
(d*x + c))/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*cos_integral(
-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 12*a^4*b^3*c*
cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 6*(sqrt(d*
x + c)*a + b)^2*a^2*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/s...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,{\left(e+f\,x\right)}^2\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/2))*(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(1/2))*(e + f*x)^2, x)
```

Reduce [F]

$$\begin{aligned}\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=&\left(\int \sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)dx\right)e^2\\&+\left(\int \sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)x^2dx\right)f^2\\&+2\left(\int \sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)xdx\right)ef\end{aligned}$$

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x)
```

output
```
int(sin((sqrt(c + d*x)*a + b)/sqrt(c + d*x)),x)*e**2 + int(sin((sqrt(c + d
*x)*a + b)/sqrt(c + d*x))*x**2,x)*f**2 + 2*int(sin((sqrt(c + d*x)*a + b)/s
qrt(c + d*x))*x,x)*e*f
```

3.198 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$
\begin{aligned}
\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = & -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} \\
& +\frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} \\
& +\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2} \\
& -\frac{b^4f\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{12d^2} \\
& +\frac{b^2(de-cf)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d^2} \\
& -\frac{b^2f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} \\
& +\frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} \\
& +\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2} \\
& -\frac{b^4f\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2} \\
& +\frac{b^2(de-cf)\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}
\end{aligned}
$$

output

```
-1/12*b^3*f*(d*x+c)^(1/2)*cos(a+b/(d*x+c)^(1/2))/d^2+b*(-c*f+d*e)*(d*x+c)^
(1/2)*cos(a+b/(d*x+c)^(1/2))/d^2+1/6*b*f*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/
2))/d^2-1/12*b^4*f*Ci(b/(d*x+c)^(1/2))*sin(a)/d^2+b^2*(-c*f+d*e)*Ci(b/(d*x
+c)^(1/2))*sin(a)/d^2-1/12*b^2*f*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^2+(-c*f+
d*e)*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^(1
/2))/d^2-1/12*b^4*f*cos(a)*Si(b/(d*x+c)^(1/2))/d^2+b^2*(-c*f+d*e)*cos(a)*S
i(b/(d*x+c)^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.22

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \frac{e\sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)\left(b\cos(a)+\sqrt{c+dx}\sin(a)\right)}{d}$$

$$+\frac{f\sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)\left(-b^3\cos(a)-12bc\cos(a)+2b(c+dx)\cos(a)-b^2\sqrt{c+dx}\sin(a)-12c\sqrt{c+d}\right.}{12d^2}$$

$$+\frac{e\sqrt{c+dx}\left(\sqrt{c+dx}\cos(a)-b\sin(a)\right)\sin\left(\frac{b}{\sqrt{c+dx}}\right)}{d}$$

$$+\frac{f\sqrt{c+dx}\left(-b^2\sqrt{c+dx}\cos(a)-12c\sqrt{c+dx}\cos(a)+6(c+dx)^{3/2}\cos(a)+b^3\sin(a)+12bc\sin(a)-\right.}{12d^2}$$

$$+\frac{b^2e\left(\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$-\frac{b^2(b^2+12c)\,f\left(\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{12d^2}$$

input
```
Integrate[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

output
```
(e*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(b*Cos[a] + Sqrt[c + d*x]*Sin[a]))/d
 + (f*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(-(b^3*Cos[a]) - 12*b*c*Cos[a] +
2*b*(c + d*x)*Cos[a] - b^2*Sqrt[c + d*x]*Sin[a] - 12*c*Sqrt[c + d*x]*Sin[a
] + 6*(c + d*x)^(3/2)*Sin[a]))/(12*d^2) + (e*Sqrt[c + d*x]*(Sqrt[c + d*x]*
Cos[a] - b*Sin[a])*Sin[b/Sqrt[c + d*x]])/d + (f*Sqrt[c + d*x]*(-(b^2*Sqrt[
c + d*x]*Cos[a]) - 12*c*Sqrt[c + d*x]*Cos[a] + 6*(c + d*x)^(3/2)*Cos[a] +
b^3*Sin[a] + 12*b*c*Sin[a] - 2*b*(c + d*x)*Sin[a])*Sin[b/Sqrt[c + d*x]])/(
12*d^2) + (b^2*e*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] + Cos[a]*SinIntegral
[b/Sqrt[c + d*x]]))/d - (b^2*(b^2 + 12*c)*f*(CosIntegral[b/Sqrt[c + d*x]]*
Sin[a] + Cos[a]*SinIntegral[b/Sqrt[c + d*x]]))/(12*d^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{2\int \left(\frac{f\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)(c+dx)^{5/2}}{d}+\frac{(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)(c+dx)^{3/2}}{d}\right)d\frac{1}{\sqrt{c+dx}}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{b^4f\sin(a)\operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{24d}+\frac{b^4f\cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{24d}+\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{24d}-\frac{b^2\sin(a)(de-cf)\operatorname{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{2d}-\right.$$

input
```
Int[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

output
```
(-2*((b^3*f*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(24*d) - (b*(d*e - c*f
)*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(2*d) - (b*f*(c + d*x)^(3/2)*Cos
[a + b/Sqrt[c + d*x]])/(12*d) + (b^4*f*CosIntegral[b/Sqrt[c + d*x]]*Sin[a]
)/(24*d) - (b^2*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(2*d) + (
b^2*f*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(24*d) - ((d*e - c*f)*(c + d*x)*
Sin[a + b/Sqrt[c + d*x]])/(2*d) - (f*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])
/(4*d) + (b^4*f*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(24*d) - (b^2*(d*e -
c*f)*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(2*d)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2b^2\left(-cf\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)+de\left(-\frac{\sin\left(\right.}{}\right.\right.}{}$
default	$-\frac{2b^2\left(-cf\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)+de\left(-\frac{\sin\left(\right.}{}\right.\right.}{}$
parts	$\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)x^2f+\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)xe+\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)cfx}{d}+\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)ce}{d}+\frac{b\cos\left(a\right.}{}$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/d^2*b^2*(-c*f*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x
+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)
^(1/2))*sin(a))+d*e*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(
d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x
+c)^(1/2))*sin(a))+f*b^2*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*c
os(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d
*x+c)+1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))
*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.67

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \frac{(12\,b^2de-(b^4+12\,b^2c)f)\,\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)+(12\,b^2de-(b^4+12\,b^2c)f)\cos(a)\,\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)+(2\,bdfx+}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
1/12*((12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos_integral(b/sqrt(d*x + c))*sin(
a) + (12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos(a)*sin_integral(b/sqrt(d*x + c)
) + (2*b*d*f*x + 12*b*d*e - (b^3 + 10*b*c)*f)*sqrt(d*x + c)*cos((a*d*x + a
*c + sqrt(d*x + c)*b)/(d*x + c)) + (6*d^2*f*x^2 + 12*c*d*e - (b^2*c + 6*c^
2)*f - (b^2*d*f - 12*d^2*e)*x)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x +
c)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)**(1/2)),x)
```

output
```
Integral((e + f*x)*sin(a + b/sqrt(c + d*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.35

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$= \frac{12\bigg(\bigg(\bigg(-i\,\text{Ei}\Big(\frac{ib}{\sqrt{dx+c}}\Big)+i\,\text{Ei}\Big(-\frac{ib}{\sqrt{dx+c}}\Big)\bigg)\cos(a)+\bigg(\text{Ei}\Big(\frac{ib}{\sqrt{dx+c}}\Big)+\text{Ei}\Big(-\frac{ib}{\sqrt{dx+c}}\Big)\bigg)\sin(a)\bigg)b^2+2\sqrt{dx+c}\,b}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/24*(12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) +
(Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x
+ c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*
x + c)*a + b)/sqrt(d*x + c)))*e - 12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I
*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)
))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)
) + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*f/d + (((I*Ei(
I*b/sqrt(d*x + c)) - I*Ei(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x +
 c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x
 + c)^(3/2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2
 - 6*(d*x + c)^2)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f/d)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2158 vs. $2(263) = 526$.

Time = 0.47 (sec) , antiderivative size = 2158, normalized size of antiderivative = 7.17

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

output

```
1/12*(12*(a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*s
in(a) - a^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c
)) - 2*(sqrt(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)
/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a
)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (s
qrt(d*x + c)*a + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x
 + c))*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(
a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*
x + c)))*e/((a^2 - 2*(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c
)*a + b)^2/(d*x + c))*b) - (a^4*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b
)/sqrt(d*x + c))*sin(a) - a^4*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)) - 4*(sqrt(d*x + c)*a + b)*a^3*b^5*cos_integral(-a + (
sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(sqrt(d*x + c
)*a + b)*a^3*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x +
c))/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*cos_integral(-a + (s
qrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 12*a^4*b^3*c*cos_int
egral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 6*(sqrt(d*x + c)*
a + b)^2*a^2*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,(e+f\,x)\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/2))*(e + f*x),x)
```

output

```
int(sin(a + b/(c + d*x)^(1/2))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx = \left(\int \sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)dx\right)e + \left(\int \sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)xdx\right)f$$

input

```
int((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x)
```

output

```
int(sin((sqrt(c + d*x)*a + b)/sqrt(c + d*x)),x)*e + int(sin((sqrt(c + d*x)
*a + b)/sqrt(c + d*x))*x,x)*f
```

3.199 $\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 94

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx=\frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}$$

output
```
b*(d*x+c)^(1/2)*cos(a+b/(d*x+c)^(1/2))/d+b^2*Ci(b/(d*x+c)^(1/2))*sin(a)/d+
(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d+b^2*cos(a)*Si(b/(d*x+c)^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$
$$=\frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)+b^2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+c\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)+dx\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)+}{d}$$

input
```
Integrate[Sin[a + b/Sqrt[c + d*x]],x]
```

output
```
(b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]] + b^2*CosIntegral[b/Sqrt[c + d*x
]]*Sin[a] + c*Sin[a + b/Sqrt[c + d*x]] + d*x*Sin[a + b/Sqrt[c + d*x]] + b^
2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3842, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{3842}$$

$$-\frac{2\int (c+dx)^{3/2}\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int (c+dx)^{3/2}\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\left(\frac{1}{2}b\int (c+dx)\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}-\frac{1}{2}(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{1}{2}b\int (c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt{c+dx}}-\frac{1}{2}(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\left(\frac{1}{2}b\left(b\int -\sqrt{c+dx}\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}-\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)-\frac{1}{2}(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(\frac{1}{2}b\left(-b\int \sqrt{c+dx}\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}-\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)-\frac{1}{2}(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow 3042$$

$$-\frac{2\left(\frac{1}{2}b\left(-b\int \sqrt{c+dx}\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}-\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)-\frac{1}{2}(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow 3784$$

$$-\frac{2\left(\frac{1}{2}b\left(-b\left(\sin(a)\int \sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}+\cos(a)\int \sqrt{c+dx}\sin\left(\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}\right)-\sqrt{c+dx}\cos\left(a+\right.\right.\right.}{d}$$

$$\Big\downarrow 3042$$

$$-\frac{2\left(\frac{1}{2}b\left(-b\left(\sin(a)\int \sqrt{c+dx}\sin\left(\frac{b}{\sqrt{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt{c+dx}}+\cos(a)\int \sqrt{c+dx}\sin\left(\frac{b}{\sqrt{c+dx}}\right)d\frac{1}{\sqrt{c+dx}}\right)-\sqrt{c+dx}\cos\right.\right.}{d}$$

$$\Big\downarrow 3780$$

$$-\frac{2\left(\frac{1}{2}b\left(-b\left(\sin(a)\int \sqrt{c+dx}\sin\left(\frac{b}{\sqrt{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt{c+dx}}+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)-\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)-\frac{1}{2}(c\right.}{d}$$

$$\Big\downarrow 3783$$

$$-\frac{2\left(\frac{1}{2}b\left(-b\left(\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)-\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)\right)-\frac{1}{2}(c+dx)\sin\left(a+\right.\right.}{d}$$

input
```
Int[Sin[a + b/Sqrt[c + d*x]],x]
```

output
```
(-2*(-1/2*((c + d*x)*Sin[a + b/Sqrt[c + d*x]]) + (b*(-(Sqrt[c + d*x]*Cos[a
 + b/Sqrt[c + d*x]]) - b*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] + Cos[a]*Sin
Integral[b/Sqrt[c + d*x]])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2b^2\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)}{d}$	84
default	$-\frac{2b^2\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{2}\right)}{d}$	84

input
```
int(sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*b^2*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2
))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*
sin(a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$=\frac{b^2\,\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)+b^2\cos(a)\,\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)+\sqrt{dx+c}b\cos\left(\frac{adx+ac+\sqrt{dx+c}b}{dx+c}\right)+(dx+c)\sin\left(\frac{adx+ac+\sqrt{dx}}{dx+c}\right.}{d}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
(b^2*cos_integral(b/sqrt(d*x + c))*sin(a) + b^2*cos(a)*sin_integral(b/sqrt
(d*x + c)) + sqrt(d*x + c)*b*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)
) + (d*x + c)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/2)),x)
```

output
```
Integral(sin(a + b/sqrt(c + d*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.32

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$$
$$=\frac{\Bigl(\Bigl(-i\,\mathrm{Ei}\Bigl(\frac{i\,b}{\sqrt{dx+c}}\Bigr)+i\,\mathrm{Ei}\Bigl(-\frac{i\,b}{\sqrt{dx+c}}\Bigr)\Bigr)\cos(a)+\Bigl(\mathrm{Ei}\Bigl(\frac{i\,b}{\sqrt{dx+c}}\Bigr)+\mathrm{Ei}\Bigl(-\frac{i\,b}{\sqrt{dx+c}}\Bigr)\Bigr)\sin(a)\Bigr)b^2+2\sqrt{dx+c}b\cos}{2\,d}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I
*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*
b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c
)*a + b)/sqrt(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(84) = 168$.

Time = 0.16 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.39

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \frac{a^2b^3\,\mathrm{Ci}\left(-a+\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right)\sin\left(a\right)-a^2b^3\cos\left(a\right)\mathrm{Si}\left(a-\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right)-\frac{2\left(\sqrt{dx+c}a+b\right)ab^3\,\mathrm{Ci}\left(-a+\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right)\sin(a)}{\sqrt{dx+c}}+}{}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
(a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - a
^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*(s
qrt(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x
 + c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a)*sin_int
egral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (sqrt(d*x +
 c)*a + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*si
n(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(a - (sqrt
(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x + c)*a + b
)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))/
((a^2 - 2*(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c)*a + b)^2/
(d*x + c))*b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = \int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/2)),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx=\int \sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)dx$$

input

```
int(sin(a+b/(d*x+c)^(1/2)),x)
```

output

```
int(sin((sqrt(c + d*x)*a + b)/sqrt(c + d*x)),x)
```

3.200 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 276

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx ={}& -\frac{2\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} \\
&+\frac{\mathrm{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f} \\
&+\frac{\mathrm{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f} \\
&-\frac{2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f} \\
&-\frac{\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\mathrm{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{f} \\
&+\frac{\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\mathrm{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{f}
\end{aligned}
$$

output
```
-2*Ci(b/(d*x+c)^(1/2))*sin(a)/f+Ci(b*f^(1/2)/(c*f-d*e)^(1/2)+b/(d*x+c)^(1/
2))*sin(a-b*f^(1/2)/(c*f-d*e)^(1/2))/f+Ci(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x
+c)^(1/2))*sin(a+b*f^(1/2)/(c*f-d*e)^(1/2))/f-2*cos(a)*Si(b/(d*x+c)^(1/2))
/f-cos(a+b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)
^(1/2))/f+cos(a-b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)+b/
(d*x+c)^(1/2))/f
```

Mathematica [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x), x]
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

$$\Big\downarrow \; 3912$$

$$-\frac{2\int \left(\frac{d\sqrt{c+dx}\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{d(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{f\sqrt{c+dx}\left(f+\frac{de-cf}{c+dx}\right)}\right)d\frac{1}{\sqrt{c+dx}}}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(-\frac{d\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2f}-\frac{d\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2f}+\frac{d\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+}}\right.}{f}\right.}{d}$$

input
```
Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

output
```
(-2*((d*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/f - (d*CosIntegral[(b*Sqrt[f]
)/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]]*Sin[a - (b*Sqrt[f])/Sqrt[-(d*e) +
c*f]])/(2*f) - (d*CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c +
d*x]]*Sin[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/(2*f) + (d*Cos[a]*SinIntegr
al[b/Sqrt[c + d*x]])/f + (d*Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinInt
egral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c + d*x]])/(2*f) - (d*Cos[a
- (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*
f] + b/Sqrt[c + d*x]])/(2*f)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.60

method	result
derivativedivides	$-2b^2\Bigg(-\frac{-\operatorname{Si}\left(-\frac{b}{\sqrt{dx+c}}-a+\frac{cfa-dea+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)\cos\left(\frac{cfa-dea+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)+\operatorname{Ci}\Big(\frac{b}{\sqrt{dx+c}}+a-\frac{cfa-dea}{}}{2f\,b^2}$
default	$-2b^2\Bigg(-\frac{-\operatorname{Si}\left(-\frac{b}{\sqrt{dx+c}}-a+\frac{cfa-dea+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)\cos\left(\frac{cfa-dea+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)+\operatorname{Ci}\Big(\frac{b}{\sqrt{dx+c}}+a-\frac{cfa-dea}{}}{2f\,b^2}$

input
```
int(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-2*b^2*(-1/2/f/b^2*(-Si(-b/(d*x+c)^(1/2)-a+(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))*cos((c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d
*e))+Ci(b/(d*x+c)^(1/2)+a-(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d
*e))*sin((c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-1/2/f/b^2*(
Si(b/(d*x+c)^(1/2)+a+(-c*f*a+d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))
*cos((-c*f*a+d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1
/2)+a+(-c*f*a+d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-c*f*a+d*
e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/f/b^2*(Ci(b/(d*x+c)^(1/2))*
sin(a)+Si(b/(d*x+c)^(1/2))*cos(a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.12

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

$$= \frac{-i\,\mathrm{Ei}\left(-\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c)-i\sqrt{dx+c}b}{dx+c}\right)e^{\left(ia+\sqrt{\frac{b^2f}{de-cf}}\right)}-i\,\mathrm{Ei}\left(\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c)+i\sqrt{dx+c}b}{dx+c}\right)e^{\left(ia-\sqrt{\frac{b^2f}{de-cf}}\right)}+i\,\mathrm{Ei}\Bigg(-}{}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

output
```
1/2*(-I*Ei(-(sqrt(b^2*f/(d*e - c*f))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x +
 c))*e^(I*a + sqrt(b^2*f/(d*e - c*f))) - I*Ei((sqrt(b^2*f/(d*e - c*f))*(d*
x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(b^2*f/(d*e - c*f))) +
 I*Ei(-(sqrt(b^2*f/(d*e - c*f))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*
e^(-I*a + sqrt(b^2*f/(d*e - c*f))) + I*Ei((sqrt(b^2*f/(d*e - c*f))*(d*x +
c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(b^2*f/(d*e - c*f))) - 4*
cos_integral(b/sqrt(d*x + c))*sin(a) - 4*cos(a)*sin_integral(b/sqrt(d*x +
c)))/f
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e),x)
```

output
```
Integral(sin(a + b/sqrt(c + d*x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+\frac{b}{\sqrt{dx+c}}\right)}{fx+e}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
```

output

```
integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+\frac{b}{\sqrt{c+d\,x}}\right)}{e+f\,x}\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/2))/(e + f*x),x)
```

output

```
int(sin(a + b/(c + d*x)^(1/2))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin \left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{fx+e}dx$$

input

```
int(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x)
```

output

```
int(sin((sqrt(c + d*x)*a + b)/sqrt(c + d*x))/(e + f*x),x)
```

3.201 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 350

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx =& -\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}}\\
&+\frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}}\\
&+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)}\\
&-\frac{bd\sin\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}}\\
&-\frac{bd\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\operatorname{Si}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}}\end{aligned}$$

output
```
-1/2*b*d*cos(a+b*f^(1/2)/(c*f-d*e)^(1/2))*Ci(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(
d*x+c)^(1/2))/f^(1/2)/(c*f-d*e)^(3/2)+1/2*b*d*cos(a-b*f^(1/2)/(c*f-d*e)^(1
/2))*Ci(b*f^(1/2)/(c*f-d*e)^(1/2)+b/(d*x+c)^(1/2))/f^(1/2)/(c*f-d*e)^(3/2)
+(d*x+c)*sin(a+b/(d*x+c)^(1/2))/(-c*f+d*e)/(f*x+e)-1/2*b*d*sin(a+b*f^(1/2)
/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))/f^(1/2)/(c
*f-d*e)^(3/2)-1/2*b*d*sin(a-b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d
*e)^(1/2)+b/(d*x+c)^(1/2))/f^(1/2)/(c*f-d*e)^(3/2)
```

Mathematica [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2, x]
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3912, 27, 3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$$

$$\Big\downarrow \; 3912$$

$$-\frac{2\int \frac{d^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{\sqrt{c+dx}\left(f+\frac{d\left(e-\frac{cf}{d}\right)}{c+dx}\right)^2}d\frac{1}{\sqrt{c+dx}}}{d}$$

$$\Big\downarrow \; 27$$

$$-2d\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{\sqrt{c+dx}\left(f+\frac{de-cf}{c+dx}\right)^2}d\frac{1}{\sqrt{c+dx}}$$

$$\Big\downarrow \; 3822$$

$$-2d\left(\frac{b\int \frac{\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{f+\frac{de-cf}{c+dx}}d\frac{1}{\sqrt{c+dx}}}{2(de-cf)}-\frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2(de-cf)\left(\frac{de-cf}{c+dx}+f\right)}\right)$$

$$\Big\downarrow \; 3815$$

$$-2d\left(\frac{b\int \left(\frac{\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\left(\sqrt{f}-\frac{\sqrt{cf-de}}{\sqrt{c+dx}}\right)}+\frac{\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\left(\sqrt{f}+\frac{\sqrt{cf-de}}{\sqrt{c+dx}}\right)}\right)d\frac{1}{\sqrt{c+dx}}}{2(de-cf)}-\frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2(de-cf)\left(\frac{de-cf}{c+dx}+f\right)}\right)$$

$$\Big\downarrow \; 2009$$

$$-2d\left(\frac{b\left(-\frac{\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\sqrt{cf-de}}+\frac{\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\sqrt{cf-de}}-\frac{\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\Big(}{2\sqrt{f}\sqrt{cf-}}\right.}{2(de-cf)}\right.$$

input
```
Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

output
```
-2*d*(-1/2*Sin[a + b/Sqrt[c + d*x]]/((d*e - c*f)*(f + (d*e - c*f)/(c + d*x
))) + (b*(-1/2*(Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqr
t[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c + d*x]])/(Sqrt[f]*Sqrt[-(d*e) + c*f])
+ (Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[-(
d*e) + c*f] + b/Sqrt[c + d*x]])/(2*Sqrt[f]*Sqrt[-(d*e) + c*f]) - (Sin[a +
(b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f]
 - b/Sqrt[c + d*x]])/(2*Sqrt[f]*Sqrt[-(d*e) + c*f]) - (Sin[a - (b*Sqrt[f])
/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c
 + d*x]])/(2*Sqrt[f]*Sqrt[-(d*e) + c*f])))/(2*(d*e - c*f)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2733 vs. $2(284) = 568$.

Time = 2.39 (sec) , antiderivative size = 2734, normalized size of antiderivative = 7.81

method	result	size
derivativedivides	Expression too large to display	2734
default	Expression too large to display	2734

input
```
int(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*d*b^2*(sin(a+b/(d*x+c)^(1/2))*(-1/2*a/f/b^2*(a+b/(d*x+c)^(1/2))+1/2*(a^
2*c*f-a^2*d*e-b^2*f)/f/b^2/(c*f-d*e))/(a^2*c*f-a^2*d*e-2*a*c*f*(a+b/(d*x+c
)^(1/2))+2*a*d*e*(a+b/(d*x+c)^(1/2))+c*f*(a+b/(d*x+c)^(1/2))^2-d*e*(a+b/(d
*x+c)^(1/2))^2-b^2*f)+1/4*a/f/b^2/(c*f*a-d*e*a-c*(c*f*a-d*e*a+(b^2*c*f^2-b
^2*d*e*f)^(1/2))/(c*f-d*e)*f+d*e*(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))
/(c*f-d*e))*(-Si(-b/(d*x+c)^(1/2)-a+(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))*cos((c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci
(b/(d*x+c)^(1/2)+a-(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*si
n((c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4*a/f/b^2/(c*f*a
-d*e*a+c*(-c*f*a+d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*f-d*e*(-c*f*
a+d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/2)+a+(-c*
f*a+d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-c*f*a+d*e*a+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-c*f*a+d*e*a+(b^2*
c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-c*f*a+d*e*a+(b^2*c*f^2-b^2*d*e*f)
^(1/2))/(c*f-d*e)))+1/4*(a^2*c*f-a^2*d*e-a*c*f*(c*f*a-d*e*a+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/(c*f-d*e)+a*d*e*(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(
c*f-d*e)-b^2*f)/(c*f-d*e)/b^2/f/(c*f*a-d*e*a-c*(c*f*a-d*e*a+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/(c*f-d*e)*f+d*e*(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(
c*f-d*e))*(Si(-b/(d*x+c)^(1/2)-a+(c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))
/(c*f-d*e))*sin((c*f*a-d*e*a+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.28

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{(i\,dfx+i\,de)\sqrt{\frac{b^2f}{de-cf}}\mathrm{Ei}\left(-\frac{\sqrt{\frac{b^2f}{de-cf}}(dx+c)-i\sqrt{dx+c}b}{dx+c}\right)e^{\left(i\,a+\sqrt{\frac{b^2f}{de-cf}}\right)}+(-i\,dfx-i\,de)\sqrt{\frac{b^2f}{de-cf}}\mathrm{Ei}\left(\frac{\sqrt{\frac{b^2f}{de-cf}}(d\right.}{}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/4*((I*d*f*x + I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(-(sqrt(b^2*f/(d*e - c*f
))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a + sqrt(b^2*f/(d*e - c*
f))) + (-I*d*f*x - I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei((sqrt(b^2*f/(d*e - c*
f))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(b^2*f/(d*e - c
*f))) + (-I*d*f*x - I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(-(sqrt(b^2*f/(d*e -
c*f))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a + sqrt(b^2*f/(d*e
- c*f))) + (I*d*f*x + I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei((sqrt(b^2*f/(d*e -
 c*f))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(b^2*f/(d*e
 - c*f))) - 4*(d*f*x + c*f)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c))
)/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{\left(e+fx\right)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b/sqrt(c + d*x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{\left(fx+e\right)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)}{(e+f\,x)^2}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/2))/(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(1/2))/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x)
```

output
```
int(sin((sqrt(c + d*x)*a + b)/sqrt(c + d*x))/(e**2 + 2*e*f*x + f**2*x**2),
x)
```

3.202 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 390

$$
\begin{aligned}
\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx &= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} \\
&\quad - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\
&\quad + \frac{2ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\
&\quad - \frac{ie^{ia}(de-cf)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\
&\quad + \frac{ie^{-ia}(de-cf)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} \\
&\quad + \frac{b^2f^2\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{3/2}}\right)\sin(a)}{3d^3} \\
&\quad + \frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{b^2f^2\cos(a)\mathrm{Si}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}
\end{aligned}
$$

output
```
1/3*b*f^2*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(3/2))/d^3-2/3*I*exp(I*a)*f*(-c*f+
d*e)*(-I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,-I*b/(d*x+c)^(3/2))/d
^3+2/3*I*f*(-c*f+d*e)*(I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,I*b/(
d*x+c)^(3/2))/d^3/exp(I*a)-1/3*I*exp(I*a)*(-c*f+d*e)^2*(-I*b/(d*x+c)^(3/2)
)^(2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^(3/2))/d^3+1/3*I*(-c*f+d*e)^2*(I*b
/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/2))/d^3/exp(I*a)+1
/3*b^2*f^2*Ci(b/(d*x+c)^(3/2))*sin(a)/d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c
)^(3/2))/d^3+1/3*b^2*f^2*cos(a)*Si(b/(d*x+c)^(3/2))/d^3
```

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.19

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \frac{i\left((\cos(a)-i\sin(a))\left(b^2f^2\,\text{ExpIntegralEi}\left(-\frac{ib}{(c+dx)^{3/2}}\right)+4f(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}\right.\right.}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

output
```
((I/6)*((Cos[a] - I*Sin[a])*(b^2*f^2*ExpIntegralEi[((-I)*b)/(c + d*x)^(3/2
)] + 4*f*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
 (I*b)/(c + d*x)^(3/2)] + 2*(d*e - c*f)^2*((I*b)/(c + d*x)^(3/2))^(2/3)*(c
 + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)] - I*b*f^2*(c + d*x)^(3/2)*(Cos[
b/(c + d*x)^(3/2)] - I*Sin[b/(c + d*x)^(3/2)]) + f^2*(c + d*x)^3*(Cos[b/(c
 + d*x)^(3/2)] - I*Sin[b/(c + d*x)^(3/2)])) - (Cos[a] + I*Sin[a])*(b^2*f^2
*ExpIntegralEi[(I*b)/(c + d*x)^(3/2)] + 4*f*(d*e - c*f)*(((-I)*b)/(c + d*x
)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, ((-I)*b)/(c + d*x)^(3/2)] + 2*(d*e
- c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*b)/(
c + d*x)^(3/2)] + I*b*f^2*(c + d*x)^(3/2)*(Cos[b/(c + d*x)^(3/2)] + I*Sin[
b/(c + d*x)^(3/2)]) + f^2*(c + d*x)^3*(Cos[b/(c + d*x)^(3/2)] + I*Sin[b/(c
 + d*x)^(3/2)]))))/d^3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{2\int \Big(f^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)(c+dx)^{5/2}+2f(de-cf)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)(c+dx)^{3/2}+(de-cf)^2\sin\Big(a+\frac{b}{(c+dx}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$2\Big(\frac{1}{6}b^2f^2\sin(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{3/2}}\right)+\frac{1}{6}b^2f^2\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{3/2}}\right)-\frac{1}{3}ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf$$

input
```
Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

output
```
(2*((b*f^2*(c + d*x)^(3/2)*Cos[a + b/(c + d*x)^(3/2)])/6 - (I/3)*E^(I*a)*f
*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, ((-I
)*b)/(c + d*x)^(3/2)] + ((I/3)*f*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(4/3)
*(c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/E^(I*a) - (I/6)*E^(I*a)*(
d*e - c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*
b)/(c + d*x)^(3/2)] + ((I/6)*(d*e - c*f)^2*((I*b)/(c + d*x)^(3/2))^(2/3)*(
c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/E^(I*a) + (b^2*f^2*CosIntegra
l[b/(c + d*x)^(3/2)]*Sin[a])/6 + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^(3/2
)])/6 + (b^2*f^2*Cos[a]*SinIntegral[b/(c + d*x)^(3/2)])/6))/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right) dx$$

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
```

output
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.50

$$\int (e+fx)^2 \sin\left(a + \frac{b}{(c+dx)^{3/2}}\right) dx = 2\,b^2 f^2 \operatorname{Ci}\left(\frac{\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)\sin(a) + 2\,b^2 f^2 \cos(a) \operatorname{Si}\left(\frac{\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right) - 3\,((i\,d^2e^2 - 2i\,cde$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

output

```
1/6*(2*b^2*f^2*cos_integral(sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))*sin
(a) + 2*b^2*f^2*cos(a)*sin_integral(sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c
^2)) - 3*((I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*cos(a) + (d^2*e^2 - 2*c*d*
e*f + c^2*f^2)*sin(a))*(I*b)^(2/3)*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2 +
 2*c*d*x + c^2)) - 3*((-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2)*cos(a) + (d^2
*e^2 - 2*c*d*e*f + c^2*f^2)*sin(a))*(-I*b)^(2/3)*gamma(1/3, -I*sqrt(d*x +
c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(b*d*f^2*x + 9*b*d*e*f - 8*b*c*f^2)*sq
rt(d*x + c)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2
 + 2*c*d*x + c^2)) - 9*((b*d*e*f - b*c*f^2)*cos(a) + (-I*b*d*e*f + I*b*c*f
^2)*sin(a))*(I*b)^(1/3)*gamma(2/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x +
c^2)) - 9*((b*d*e*f - b*c*f^2)*cos(a) + (I*b*d*e*f - I*b*c*f^2)*sin(a))*(-
I*b)^(1/3)*gamma(2/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d
^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3
*f^2)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c
*d*x + c^2)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (e+fx)^2 \sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)dx$$

input

```
integrate((f*x+e)**2*sin(a+b/(d*x+c)**(3/2)),x)
```

output

```
Integral((e + f*x)**2*sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(296) = 592$.

Time = 0.58 (sec) , antiderivative size = 993, normalized size of antiderivative = 2.55

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

output

```
1/12*(3*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*
a + b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2))
+ (sqrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) -
1)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x
+ c)^(3/2)))*sin(a))*b)*e^2/(sqrt(d*x + c)*(b/(d*x + c)^(3/2))^(1/3)) - 6*
(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(
d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(
3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma
(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/
2)))*sin(a))*b)*c*e*f/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3)) + 3*(4*(
d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x
+ c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) +
 I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3
, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2)))
*sin(a))*b)*c^2*f^2/(sqrt(d*x + c)*d^2*(b/(d*x + c)^(3/2))^(1/3)) + 2*(2*(
d*x + c)^3*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 2*(d*x + c)^(3/2
)*b*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + ((-I*Ei(I*b/(d*x + c)^(
3/2)) + I*Ei(-I*b/(d*x + c)^(3/2)))*cos(a) + (Ei(I*b/(d*x + c)^(3/2)) + Ei
(-I*b/(d*x + c)^(3/2)))*sin(a))*b^2)*f^2/d^2 + 3*(4*(d*x + c)^3*(b/(d*x +
c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x ...
```

Giac [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)\,dx$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

output

```
integrate((f*x + e)^2*sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{3/2}}\right)\,{\left(e+f\,x\right)}^2\,dx$$

input
```
int(sin(a + b/(c + d*x)^(3/2))*(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(3/2))*(e + f*x)^2, x)
```

Reduce [F]

$$\begin{aligned}&\int (e+fx)^2\sin\bigg(a\\&+\frac{b}{(c+dx)^{3/2}}\bigg)\,dx=\left(\int \sin\left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)dx\right)e^2\\&+\left(\int \sin\left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)x^2dx\right)f^2\\&+2\left(\int \sin\left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)xdx\right)ef\end{aligned}$$

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
```

output
```
int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x + b)/(sqrt(c + d*x)*c + s
qrt(c + d*x)*d*x)),x)*e**2 + int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*
d*x + b)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x))*x**2,x)*f**2 + 2*int(sin((
sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x + b)/(sqrt(c + d*x)*c + sqrt(c + d
*x)*d*x))*x,x)*e*f
```

3.203 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 251

$$
\begin{aligned}
\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx = \\
&-\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} \\
&+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} \\
&-\frac{ie^{ia}(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} \\
&+\frac{ie^{-ia}(de-cf)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}
\end{aligned}
$$

output

```
-1/3*I*exp(I*a)*f*(-I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,-I*b/(d*
x+c)^(3/2))/d^2+1/3*I*f*(I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,I*b
/(d*x+c)^(3/2))/d^2/exp(I*a)-1/3*I*exp(I*a)*(-c*f+d*e)*(-I*b/(d*x+c)^(3/2)
)^(2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^(3/2))/d^2+1/3*I*(-c*f+d*e)*(I*b/(
d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/2))/d^2/exp(I*a)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 835 vs. $2(251) = 502$.

Time = 3.24 (sec) , antiderivative size = 835, normalized size of antiderivative = 3.33

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \text{Too large to display}$$

input

```
Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

output

```
(3*b*e*Cos[a]*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c +
d*x)^(3/2))^(1/3)*Sqrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(
3*((I*b)/(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x]))))/(4*d) - (3*b*c*f*Cos[a]*(
(2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/
3)*Sqrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d
*x)^(3/2))^(1/3)*Sqrt[c + d*x]))))/(4*d^2) + (((9*I)/8)*b^2*f*Cos[a]*((2*Ga
mma[2/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c
 + d*x)) - (2*Gamma[2/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2)
)^(2/3)*(c + d*x))))/d^2 + (e*(c + d*x)*Cos[b/(c + d*x)^(3/2)]*Sin[a])/d +
 (((3*I)/4)*b*e*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c
+ d*x)^(3/2))^(1/3)*Sqrt[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])
/(3*((I*b)/(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d - (((3*I)/4)*b
*c*f*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2
))^(1/3)*Sqrt[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/
(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d^2 - (9*b^2*f*((2*Gamma[2/
3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x
)) + (2*Gamma[2/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3
)*(c + d*x)))*Sin[a])/(8*d^2) + (f*Sqrt[c + d*x]*Cos[b/(c + d*x)^(3/2)]*(3
*b*Cos[a] - 2*c*Sqrt[c + d*x]*Sin[a] + (c + d*x)^(3/2)*Sin[a]))/(2*d^2) +
(e*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^(3/2)])/d + (f*Sqrt[c + d*x]*(-2*c*...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx$$

$$\Big\downarrow \textcolor{blue}{3914}$$

$$\frac{2\int \left(f\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)(c+dx)^{3/2}+(de-cf)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\sqrt{c+dx}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$2\Bigg(-\frac{1}{6}ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)+\frac{1}{6}ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\Gamma\Big(-\frac{2}{3}$$

input
```
Int[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

output
```
(2*((-1/6*I)*E^(I*a)*f*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[
-4/3, ((-I)*b)/(c + d*x)^(3/2)] + ((I/6)*f*((I*b)/(c + d*x)^(3/2))^(4/3)*(
c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/E^(I*a) - (I/6)*E^(I*a)*(d*
e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*b)/(
c + d*x)^(3/2)] + ((I/6)*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*
x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/E^(I*a)))/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)dx$$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

output
```
int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(176) = 352.

Time = 0.11 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.45

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{6\sqrt{dx+c}bf\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)-2\left(\left(i\,de-i\,cf\right)\cos\left(a\right)+\left(de-cf\right)\sin\left(a\right)}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

output

```
1/4*(6*sqrt(d*x + c)*b*f*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c
)*b)/(d^2*x^2 + 2*c*d*x + c^2)) - 2*((I*d*e - I*c*f)*cos(a) + (d*e - c*f)*
sin(a))*(I*b)^(2/3)*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)
) - 2*((-I*d*e + I*c*f)*cos(a) + (d*e - c*f)*sin(a))*(-I*b)^(2/3)*gamma(1/
3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*f*cos(a) - I*b*f*s
in(a))*(I*b)^(1/3)*gamma(2/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))
 - 3*(b*f*cos(a) + I*b*f*sin(a))*(-I*b)^(1/3)*gamma(2/3, -I*sqrt(d*x + c)*
b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)
*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x
+ c^2)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)dx$$

input

```
integrate((f*x+e)*sin(a+b/(d*x+c)**(3/2)),x)
```

output

```
Integral((e + f*x)*sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(176) = 352$.

Time = 0.37 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.00

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\text{Too large to display}$$

input

```
integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

output
```
1/8*(2*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a
 + b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) +
 (sqrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1
)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x +
 c)^(3/2)))*sin(a))*b)*e/(sqrt(d*x + c)*(b/(d*x + c)^(3/2))^(1/3)) - 2*(4*
(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x
 + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3)
+ I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/
3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2))
)*sin(a))*b)*c*f/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3)) + (4*(d*x + c
)^3*(b/(d*x + c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2))
 + 12*(d*x + c)^(3/2)*b*(b/(d*x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a +
 b)/(d*x + c)^(3/2)) - 3*(((sqrt(3) + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) +
 (sqrt(3) - I)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1
)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x +
 c)^(3/2)))*sin(a))*b^2)*f/((d*x + c)*d*(b/(d*x + c)^(3/2))^(2/3)))/d
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)\,(e+f\,x)\,dx$$

input
```
int(sin(a + b/(c + d*x)^(3/2))*(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^(3/2))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx=\left(\int \sin\left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)dx\right)e+\left(\int \sin\left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)xdx\right)f$$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

output
```
int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x + b)/(sqrt(c + d*x)*c + s
qrt(c + d*x)*d*x)),x)*e + int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x
 + b)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x))*x,x)*f
```

3.204 $\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) \, dx=-\frac{ie^{ia}\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}+\frac{ie^{-ia}\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

output
```
-1/3*I*exp(I*a)*(-I*b/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)
^(3/2))/d+1/3*I*(I*b/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(
3/2))/d/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \frac{b\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)(\cos(a)-i\sin(a))+b\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{2d\sqrt[3]{\frac{b^2}{(c+dx)^3}}\sqrt{c+d}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(3/2)],x]
```

output
```
(b*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, (I*b)/(c + d*x)^(3/2)]*(Cos
[a] - I*Sin[a]) + b*((I*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, ((-I)*b)/(c +
 d*x)^(3/2)]*(Cos[a] + I*Sin[a]) + 2*(b^2/(c + d*x)^3)^(1/3)*(c + d*x)^(3/
2)*Sin[a + b/(c + d*x)^(3/2)])/(2*d*(b^2/(c + d*x)^3)^(1/3)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3844, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx$$

$$\Big\downarrow \text{3844}$$

$$\frac{2\int \sqrt{c+dx}\sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{3904}$$

$$\frac{2\left(\frac{1}{2}i\int e^{-ia-\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}d\sqrt{c+dx}-\frac{1}{2}i\int e^{ia+\frac{ib}{(c+dx)^{3/2}}}\sqrt{c+dx}d\sqrt{c+dx}\right)}{d}$$

$$\Big\downarrow \text{2648}$$

$$\frac{2\left(\frac{1}{6}ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)-\frac{1}{6}ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)\right)}{d}$$

input

```
Int[Sin[a + b/(c + d*x)^(3/2)],x]
```

output

```
(2*((-1/6*I)*E^(I*a)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3
, ((-I)*b)/(c + d*x)^(3/2)] + ((I/6)*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*
x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/E^(I*a)))/d
```

Defintions of rubi rules used

rule 2648

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3844

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Simp[k/f   Subst[Int[x^(k - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

rule 3904

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Maple [F]

$$\int \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)dx$$

input
```
int(sin(a+b/(d*x+c)^(3/2)),x)
```

output
```
int(sin(a+b/(d*x+c)^(3/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{(i\,b)^{\frac{2}{3}}\,(-i\,\cos(a)-\sin(a))\Gamma\left(\frac{1}{3},\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)+(-i\,b)^{\frac{2}{3}}\,(i\,\cos(a)-\sin(a))\Gamma\Big(\frac{1}{3},-}{2\,d}$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

output
```
1/2*((I*b)^(2/3)*(-I*cos(a) - sin(a))*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^
2 + 2*c*d*x + c^2)) + (-I*b)^(2/3)*(I*cos(a) - sin(a))*gamma(1/3, -I*sqrt(
d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d*x + c)*sin((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \int \sin\left(a+\frac{b}{(c+dx)^{\frac{3}{2}}}\right)dx$$

input
```
integrate(sin(a+b/(d*x+c)**(3/2)),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31

$$\int \sin\left(a + \frac{b}{(c+dx)^{3/2}}\right) dx = \frac{4\,(dx+c)^{\frac{3}{2}}\left(\frac{b}{(dx+c)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{3}{2}}a+b}{(dx+c)^{\frac{3}{2}}}\right) + \Bigg(\Bigg((\sqrt{3}-i)\Gamma\left(\frac{1}{3}, \frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right) + (\sqrt{3}+i)\Gamma\Bigg(}{4\sqrt{dx}}$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

output
```
1/4*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a +
b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (s
qrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*g
amma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)
^(3/2)))*sin(a))*b)/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3))
```

Giac [F]

$$\int \sin\left(a + \frac{b}{(c+dx)^{3/2}}\right) dx = \int \sin\left(a + \frac{b}{(dx+c)^{\frac{3}{2}}}\right) dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx=\int \sin \left(a+\frac{b}{{\left(c+d\,x\right)}^{3/2}}\right) \,d x$$

input
```
int(sin(a + b/(c + d*x)^(3/2)),x)
```

output
```
int(sin(a + b/(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx=\int \sin \left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right) dx$$

input
```
int(sin(a+b/(d*x+c)^(3/2)),x)
```

output
```
int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x + b)/(sqrt(c + d*x)*c + s
qrt(c + d*x)*d*x)),x)
```

3.205 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 17.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx} \, dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}dx$$

input
```
Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}dx$$

input
```
int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

output
```
int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 +
2*c*d*x + c^2))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 47.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e),x)
```

output
```
Integral(sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x)))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 40.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+\frac{b}{{\left(c+d\,x\right)}^{3/2}}\right)}{e+f\,x} \,d x$$

input
```
int(sin(a + b/(c + d*x)^(3/2))/(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^(3/2))/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin \left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)}{fx+e}dx$$

input
```
int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

output
```
int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x + b)/(sqrt(c + d*x)*c + s
qrt(c + d*x)*d*x))/(e + f*x),x)
```

3.206 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\operatorname{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 26.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}dx$$
$$\Big\downarrow \text{3918}$$
$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^{2}}dx$$

input
```
int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

output
```
int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.18

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^{2}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 +
2*c*d*x + c^2))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{3/2}}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(3/2))/(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(3/2))/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.91

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(\frac{\sqrt{dx+c}\,ac+\sqrt{dx+c}\,adx+b}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

output
```
int(sin((sqrt(c + d*x)*a*c + sqrt(c + d*x)*a*d*x + b)/(sqrt(c + d*x)*c + s
qrt(c + d*x)*d*x))/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.207 $\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 633

$$
\begin{aligned}
\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = &-\frac{120960f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} \\
&+\frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&-\frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&+\frac{60480f^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} \\
&-\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+\frac{120f(de-cf)(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&-\frac{5040f^2(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&-\frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+\frac{168f^2(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&-\frac{3f^2(c+dx)^{8/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+\frac{720f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&-\frac{120960f^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} \\
&+\frac{6(de-cf)^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&-\frac{360f(de-cf)(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+\frac{20160f^2(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&+\frac{30f(de-cf)(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&-\frac{1008f^2(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}
\end{aligned}
$$

$$
24f^2(c+dx)^{7/3}\sin\left(a+b\sqrt[3]{c+dx}\right)
$$

output
```
-120960*f^2*cos(a+b*(d*x+c)^(1/3))/b^9/d^3+6*(-c*f+d*e)^2*cos(a+b*(d*x+c)^
(1/3))/b^3/d^3-720*f*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d
^3+60480*f^2*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^7/d^3-3*(-c*f+d*e)^2*(
d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d^3+120*f*(-c*f+d*e)*(d*x+c)*cos(a+b
*(d*x+c)^(1/3))/b^3/d^3-5040*f^2*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(1/3))/b^5/
d^3-6*f*(-c*f+d*e)*(d*x+c)^(5/3)*cos(a+b*(d*x+c)^(1/3))/b/d^3+168*f^2*(d*x
+c)^2*cos(a+b*(d*x+c)^(1/3))/b^3/d^3-3*f^2*(d*x+c)^(8/3)*cos(a+b*(d*x+c)^(
1/3))/b/d^3+720*f*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*f^2*(d*
x+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^8/d^3+6*(-c*f+d*e)^2*(d*x+c)^(1/3)*sin
(a+b*(d*x+c)^(1/3))/b^2/d^3-360*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b*(d*x+c)
^(1/3))/b^4/d^3+20160*f^2*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^6/d^3+30*f*(-c*
f+d*e)*(d*x+c)^(4/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^3-1008*f^2*(d*x+c)^(5/3)
*sin(a+b*(d*x+c)^(1/3))/b^4/d^3+24*f^2*(d*x+c)^(7/3)*sin(a+b*(d*x+c)^(1/3)
)/b^2/d^3
```

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.40

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{-3\Big(40320f^2-20160b^2f^2(c+dx)^{2/3}+b^8d^2(c+dx)^{2/3}(e+fx)^2+240b^4f\sqrt[3]{c+dx}(6cf+d(e+7fx))-}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-3*(40320*f^2 - 20160*b^2*f^2*(c + d*x)^(2/3) + b^8*d^2*(c + d*x)^(2/3)*(
e + f*x)^2 + 240*b^4*f*(c + d*x)^(1/3)*(6*c*f + d*(e + 7*f*x)) - 2*b^6*(9*
c^2*f^2 + 18*c*d*f*(e + 2*f*x) + d^2*(e^2 + 20*e*f*x + 28*f^2*x^2)))*Cos[a
 + b*(c + d*x)^(1/3)] + 6*b*(-20160*f^2*(c + d*x)^(1/3) - 12*b^4*f*(c + d*
x)^(2/3)*(5*d*e + 9*c*f + 14*d*f*x) + b^6*d*(c + d*x)^(1/3)*(e + f*x)*(3*c
*f + d*(e + 4*f*x)) + 120*b^2*f*(27*c*f + d*(e + 28*f*x)))*Sin[a + b*(c +
d*x)^(1/3)])/(b^9*d^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right) dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3\int \left(\frac{f^2 \sin\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{8/3}}{d^2}+\frac{2f(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{5/3}}{d^2}+\frac{(de-cf)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{2/3}}{d^2}\right)d\sqrt[3]{}}{d}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{40320f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^2}-\frac{40320f^2\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^2}+\frac{20160f^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^2}+\frac{240f(de-c}{}\right.$$

input
```
Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

output

```
(3*((-40320*f^2*Cos[a + b*(c + d*x)^(1/3)])/(b^9*d^2) + (2*(d*e - c*f)^2*C
os[a + b*(c + d*x)^(1/3)])/(b^3*d^2) - (240*f*(d*e - c*f)*(c + d*x)^(1/3)*
Cos[a + b*(c + d*x)^(1/3)])/(b^5*d^2) + (20160*f^2*(c + d*x)^(2/3)*Cos[a +
 b*(c + d*x)^(1/3)])/(b^7*d^2) - ((d*e - c*f)^2*(c + d*x)^(2/3)*Cos[a + b*
(c + d*x)^(1/3)])/(b*d^2) + (40*f*(d*e - c*f)*(c + d*x)*Cos[a + b*(c + d*x
)^(1/3)])/(b^3*d^2) - (1680*f^2*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)]
)/(b^5*d^2) - (2*f*(d*e - c*f)*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])
/(b*d^2) + (56*f^2*(c + d*x)^2*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^2) - (f^
2*(c + d*x)^(8/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^2) + (240*f*(d*e - c*f)
*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2) - (40320*f^2*(c + d*x)^(1/3)*Sin[a
+ b*(c + d*x)^(1/3)])/(b^8*d^2) + (2*(d*e - c*f)^2*(c + d*x)^(1/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^2*d^2) - (120*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a
+ b*(c + d*x)^(1/3)])/(b^4*d^2) + (6720*f^2*(c + d*x)*Sin[a + b*(c + d*x)^
(1/3)])/(b^6*d^2) + (10*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^
(1/3)])/(b^2*d^2) - (336*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(
b^4*d^2) + (8*f^2*(c + d*x)^(7/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^2)))/
d
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2703 vs. $2(573) = 1146$.

Time = 1.35 (sec) , antiderivative size = 2704, normalized size of antiderivative = 4.27

method	result	size
derivativedivides	Expression too large to display	2704
default	Expression too large to display	2704
parts	Expression too large to display	3867

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d^3/b^3*(-2*a*c^2*f^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+
b*(d*x+c)^(1/3)))-1/b^6*a^8*f^2*cos(a+b*(d*x+c)^(1/3))+70/b^6*a^4*f^2*(-(a
+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(a+b
*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a+b
*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-2/b^3*c*f^2
*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*si
n(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a
+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*
(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*f^2*(-(a+b*(d*x+c)^
(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1
/3))*sin(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*f^2*(-(a+b*(d*x+c)^(1/3))^3*cos(a+
b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*sin(a+b*
(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2*a*d^2*e^2*(
sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-8/b^6*a
^7*f^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))
-a^2*d^2*e^2*cos(a+b*(d*x+c)^(1/3))-a^2*c^2*f^2*cos(a+b*(d*x+c)^(1/3))+28/
b^6*a^2*f^2*(-(a+b*(d*x+c)^(1/3))^6*cos(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^
(1/3))^5*sin(a+b*(d*x+c)^(1/3))+30*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(
1/3))-120*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))-360*(a+b*(d*x+c)^(1
/3))^2*cos(a+b*(d*x+c)^(1/3))+720*cos(a+b*(d*x+c)^(1/3))+720*(a+b*(d*x+...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.53

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3\left(\left(56\,b^6d^2f^2x^2+2\,b^6d^2e^2+36\,b^6cdef+18\,(b^6c^2-2240)f^2+8\,(5\,b^6d^2ef+9\,b^6cdf^2)x-(b^8d^2f^2x^2+\right.\right.}{}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
3*((56*b^6*d^2*f^2*x^2 + 2*b^6*d^2*e^2 + 36*b^6*c*d*e*f + 18*(b^6*c^2 - 22
40)*f^2 + 8*(5*b^6*d^2*e*f + 9*b^6*c*d*f^2)*x - (b^8*d^2*f^2*x^2 + 2*b^8*d
^2*e*f*x + b^8*d^2*e^2 - 20160*b^2*f^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*f^2
*x + b^4*d*e*f + 6*b^4*c*f^2)*(d*x + c)^(1/3))*cos((d*x + c)^(1/3)*b + a)
+ 2*(3360*b^3*d*f^2*x + 120*b^3*d*e*f + 3240*b^3*c*f^2 - 12*(14*b^5*d*f^2*
x + 5*b^5*d*e*f + 9*b^5*c*f^2)*(d*x + c)^(2/3) + (4*b^7*d^2*f^2*x^2 + b^7*
d^2*e^2 + 3*b^7*c*d*e*f - 20160*b*f^2 + (5*b^7*d^2*e*f + 3*b^7*c*d*f^2)*x)
*(d*x + c)^(1/3))*sin((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [F]

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

input
```
integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral((e + f*x)**2*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2151 vs. $2(573) = 1146$.

Time = 0.15 (sec) , antiderivative size = 2151, normalized size of antiderivative = 3.40

$$\int (e+fx)^2 \sin \left(a+b\sqrt[3]{c+dx}\right) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-3*(a^2*e^2*cos((d*x + c)^(1/3)*b + a) - 2*a^2*c*e*f*cos((d*x + c)^(1/3)*b
 + a)/d + a^2*c^2*f^2*cos((d*x + c)^(1/3)*b + a)/d^2 - 2*(((d*x + c)^(1/3)
*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a*e^2 + 4
*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)
*b + a))*a*c*e*f/d - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a)
 - sin((d*x + c)^(1/3)*b + a))*a*c^2*f^2/d^2 - 2*a^5*e*f*cos((d*x + c)^(1/
3)*b + a)/(b^3*d) + 2*a^5*c*f^2*cos((d*x + c)^(1/3)*b + a)/(b^3*d^2) + (((
(d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1
/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*e^2 + 10*(((d*x + c)^(1/3)*b + a)*c
os((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*e*f/(b^3*d) -
2*((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x +
c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c*e*f/d - 10*(((d*x + c)^(1/3)
*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*c*f^2
/(b^3*d^2) + ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) -
 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c^2*f^2/d^2 + a^8*f
^2*cos((d*x + c)^(1/3)*b + a)/(b^6*d^2) - 20*((((d*x + c)^(1/3)*b + a)^2 -
 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(
1/3)*b + a))*a^3*e*f/(b^3*d) - 8*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1
/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^7*f^2/(b^6*d^2) + 20*((((d*x +
c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. $2(573) = 1146$.

Time $= 0.13$ (sec) , antiderivative size $= 1555$, normalized size of antiderivative $= 2.46$

$$\int (e+fx)^2 \sin \left(a+b\sqrt[3]{c+dx}\right)\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
3*(e^2*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)
*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b +
 a)/b^2) + 2*e*f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b
+ a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*
b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)
^2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/
3)*b + a)^3 - 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*
a^2 - 20*a^3 - 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b + a)/b^5 - (2*
((d*x + c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 +
20*((d*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d
*x + c)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*
x + c)^(1/3)*b + a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/b^5)/d -
f^2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*
c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3
)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c
)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*
b^3*c + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d
*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x +
 c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^
(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a^8 - 2*b^6*c^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(e+f\,x)^2\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))*(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))*(e + f*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.09

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x)
```

output

```
(3*( - (c + d*x)**(2/3)*cos((c + d*x)**(1/3)*b + a)*b**8*d**2*e**2 - 2*(c
+ d*x)**(2/3)*cos((c + d*x)**(1/3)*b + a)*b**8*d**2*e*f*x - (c + d*x)**(2/
3)*cos((c + d*x)**(1/3)*b + a)*b**8*d**2*f**2*x**2 + 20160*(c + d*x)**(2/3
)*cos((c + d*x)**(1/3)*b + a)*b**2*f**2 - 1440*(c + d*x)**(1/3)*cos((c + d
*x)**(1/3)*b + a)*b**4*c*f**2 - 240*(c + d*x)**(1/3)*cos((c + d*x)**(1/3)*
b + a)*b**4*d*e*f - 1680*(c + d*x)**(1/3)*cos((c + d*x)**(1/3)*b + a)*b**4
*d*f**2*x + 18*cos((c + d*x)**(1/3)*b + a)*b**6*c**2*f**2 + 36*cos((c + d*
x)**(1/3)*b + a)*b**6*c*d*e*f + 72*cos((c + d*x)**(1/3)*b + a)*b**6*c*d*f*
*2*x + 2*cos((c + d*x)**(1/3)*b + a)*b**6*d**2*e**2 + 40*cos((c + d*x)**(1
/3)*b + a)*b**6*d**2*e*f*x + 56*cos((c + d*x)**(1/3)*b + a)*b**6*d**2*f**2
*x**2 - 40320*cos((c + d*x)**(1/3)*b + a)*f**2 - 216*(c + d*x)**(2/3)*sin(
(c + d*x)**(1/3)*b + a)*b**5*c*f**2 - 120*(c + d*x)**(2/3)*sin((c + d*x)**
(1/3)*b + a)*b**5*d*e*f - 336*(c + d*x)**(2/3)*sin((c + d*x)**(1/3)*b + a)
*b**5*d*f**2*x + 6*(c + d*x)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b**7*c*d*e
*f + 6*(c + d*x)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b**7*c*d*f**2*x + 2*(c
 + d*x)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b**7*d**2*e**2 + 10*(c + d*x)**
(1/3)*sin((c + d*x)**(1/3)*b + a)*b**7*d**2*e*f*x + 8*(c + d*x)**(1/3)*sin
((c + d*x)**(1/3)*b + a)*b**7*d**2*f**2*x**2 - 40320*(c + d*x)**(1/3)*sin(
(c + d*x)**(1/3)*b + a)*b*f**2 + 6480*sin((c + d*x)**(1/3)*b + a)*b**3*c*f
**2 + 240*sin((c + d*x)**(1/3)*b + a)*b**3*d*e*f + 6720*sin((c + d*x)**...
```

3.208 $\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 288

$$\begin{aligned}\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = {} & \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ & - \frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} \\ & - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ & + \frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ & - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ & + \frac{360f\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} \\ & + \frac{6(de-cf)\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\ & - \frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\ & + \frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}\end{aligned}$$

output
```
6*(-c*f+d*e)*cos(a+b*(d*x+c)^(1/3))/b^3/d^2-360*f*(d*x+c)^(1/3)*cos(a+b*(d
*x+c)^(1/3))/b^5/d^2-3*(-c*f+d*e)*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d
^2+60*f*(d*x+c)*cos(a+b*(d*x+c)^(1/3))/b^3/d^2-3*f*(d*x+c)^(5/3)*cos(a+b*(
d*x+c)^(1/3))/b/d^2+360*f*sin(a+b*(d*x+c)^(1/3))/b^6/d^2+6*(-c*f+d*e)*(d*x
+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^2-180*f*(d*x+c)^(2/3)*sin(a+b*(d*x+
c)^(1/3))/b^4/d^2+15*f*(d*x+c)^(4/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.51

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{-3b\Big(120f\sqrt[3]{c+dx}+b^4d(c+dx)^{2/3}(e+fx)-2b^2(9cf+d(e+10fx))\Big)\cos\left(a+b\sqrt[3]{c+dx}\right)+3\Big(2b^4d}{b^6d^2}$$

input
```
Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-3*b*(120*f*(c + d*x)^(1/3) + b^4*d*(c + d*x)^(2/3)*(e + f*x) - 2*b^2*(9*
c*f + d*(e + 10*f*x)))*Cos[a + b*(c + d*x)^(1/3)] + 3*(2*b^4*d*e*(c + d*x)
^(1/3) + f*(120 - 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*
x)))*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$\Big\downarrow \text{ 3912}$$

$$\frac{3\int \left(\frac{f\sin\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{5/3}}{d}+\frac{(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{2/3}}{d}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$3\Bigg(\frac{120f\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d}-\frac{120f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d}-\frac{60f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d}+\frac{2(de-cf)\cos\Big(a+b\sqrt[3]{c-}}{b^3d}$$

input

```
Int[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output

```
(3*((2*(d*e - c*f)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (120*f*(c + d*x)^
(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d) - ((d*e - c*f)*(c + d*x)^(2/3)*C
os[a + b*(c + d*x)^(1/3)])/(b*d) + (20*f*(c + d*x)*Cos[a + b*(c + d*x)^(1/
3)])/(b^3*d) - (f*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) + (120
*f*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d) + (2*(d*e - c*f)*(c + d*x)^(1/3)*Si
n[a + b*(c + d*x)^(1/3)])/(b^2*d) - (60*f*(c + d*x)^(2/3)*Sin[a + b*(c + d
*x)^(1/3)])/(b^4*d) + (5*f*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^
2*d)))/d
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(258) = 516$.

Time = 1.32 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.78

method	result	size
derivativedivides	Expression too large to display	801
default	Expression too large to display	801
parts	Expression too large to display	1288

input

```
int((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output

```
3/d^2/b^3*(a^2*c*f*cos(a+b*(d*x+c)^(1/3))-a^2*d*e*cos(a+b*(d*x+c)^(1/3))+2
*a*c*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))
-2*a*d*e*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)
))-c*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1
/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d*e*(-(a+b*(d*x+c)^(1/3
))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))
*sin(a+b*(d*x+c)^(1/3)))+1/b^3*a^5*f*cos(a+b*(d*x+c)^(1/3))+5/b^3*a^4*f*(s
in(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-10/b^3*a
^3*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3
))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*f*(-(a+b*(d*x+
c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)
^(1/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/
3)))-5/b^3*a*f*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+
c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c
)^(1/3))-24*cos(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^
(1/3)))+1/b^3*f*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x
+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+
c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x
+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.49

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3\left(\left(20\,b^3dfx+2\,b^3de+18\,b^3cf-120\,(dx+c)^{\frac{1}{3}}bf-(b^5dfx+b^5de)(dx+c)^{\frac{2}{3}}\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\right.}{b^6d^2}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
3*((20*b^3*d*f*x + 2*b^3*d*e + 18*b^3*c*f - 120*(d*x + c)^(1/3)*b*f - (b^5
*d*f*x + b^5*d*e)*(d*x + c)^(2/3))*cos((d*x + c)^(1/3)*b + a) - (60*(d*x +
 c)^(2/3)*b^2*f - (5*b^4*d*f*x + 2*b^4*d*e + 3*b^4*c*f)*(d*x + c)^(1/3) -
120*f)*sin((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral((e + f*x)*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(258) = 516.

Time = 0.06 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.36

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-3*(a^2*e*cos((d*x + c)^(1/3)*b + a) - a^2*c*f*cos((d*x + c)^(1/3)*b + a)/
d - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^
(1/3)*b + a))*a*e + 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a)
- sin((d*x + c)^(1/3)*b + a))*a*c*f/d - a^5*f*cos((d*x + c)^(1/3)*b + a)/(
b^3*d) + ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*(
(d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*e + 5*(((d*x + c)^(1/3)
*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*f/(b^
3*d) - ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d
*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c*f/d - 10*((((d*x + c)^(
1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*
sin((d*x + c)^(1/3)*b + a))*a^3*f/(b^3*d) + 10*((((d*x + c)^(1/3)*b + a)^3
 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(
1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*f/(b^3*d) - 5*((((d*x +
 c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3
)*b + a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((
d*x + c)^(1/3)*b + a))*a*f/(b^3*d) + ((((d*x + c)^(1/3)*b + a)^5 - 20*((d*
x + c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)
*b + a) - 5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24
)*sin((d*x + c)^(1/3)*b + a))*f/(b^3*d))/(b^3*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.57

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$= \frac{3\left(e\left(\frac{2\,(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b} - \frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^{2}-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^{2}-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^{2}}\right) + \frac{f\left(\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^{2}b^{3}\right.}{}\right.}{}\right.}{}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
3*(e*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*b
 + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a
)/b^2) + f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a
*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)
^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3
 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b +
 a)^3 - 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 -
20*a^3 - 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b + a)/b^5 - (2*((d*x
+ c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d
*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c
)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x + c)
^(1/3)*b + a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/b^5)/d)/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(e+f\,x)\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))*(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))*(e + f*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.91

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$=\frac{-3(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^5de-3(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^5dfx-360(dx+c)^{\frac{1}{3}}\cos\Big(\Big(}{}$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x)
```

output
```
(3*( - (c + d*x)**(2/3)*cos((c + d*x)**(1/3)*b + a)*b**5*d*e - (c + d*x)**
(2/3)*cos((c + d*x)**(1/3)*b + a)*b**5*d*f*x - 120*(c + d*x)**(1/3)*cos((c
 + d*x)**(1/3)*b + a)*b*f + 18*cos((c + d*x)**(1/3)*b + a)*b**3*c*f + 2*co
s((c + d*x)**(1/3)*b + a)*b**3*d*e + 20*cos((c + d*x)**(1/3)*b + a)*b**3*d
*f*x - 60*(c + d*x)**(2/3)*sin((c + d*x)**(1/3)*b + a)*b**2*f + 3*(c + d*x
)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b**4*c*f + 2*(c + d*x)**(1/3)*sin((c
+ d*x)**(1/3)*b + a)*b**4*d*e + 5*(c + d*x)**(1/3)*sin((c + d*x)**(1/3)*b
+ a)*b**4*d*f*x + 120*sin((c + d*x)**(1/3)*b + a)*f))/(b**6*d**2)
```

3.209 $\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

output
```
6*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d+
6*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{\left(6-3b^2(c+dx)^{2/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+6b\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
((6 - 3*b^2*(c + d*x)^(2/3))*Cos[a + b*(c + d*x)^(1/3)] + 6*b*(c + d*x)^(1
/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3842, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+b \sqrt[3]{c+d x}\right) \, dx$$

$$\Big\downarrow \text{3842}$$

$$\frac{3 \int (c+d x)^{2/3} \sin \left(a+b \sqrt[3]{c+d x}\right)d\sqrt[3]{c+d x}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3 \int (c+d x)^{2/3} \sin \left(a+b \sqrt[3]{c+d x}\right)d\sqrt[3]{c+d x}}{d}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3 \left(\frac{2 \int \sqrt[3]{c+d x} \cos \left(a+b \sqrt[3]{c+d x}\right)d\sqrt[3]{c+d x}}{b}-\frac{(c+d x)^{2/3} \cos \left(a+b \sqrt[3]{c+d x}\right)}{b}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3 \left(\frac{2 \int \sqrt[3]{c+d x} \sin \left(a+b \sqrt[3]{c+d x}+\frac{\pi }{2}\right)d\sqrt[3]{c+d x}}{b}-\frac{(c+d x)^{2/3} \cos \left(a+b \sqrt[3]{c+d x}\right)}{b}\right)}{d}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\left(\frac{2\left(\frac{\int -\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}+\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d}$$

$$\Big\downarrow 25$$

$$\frac{3\left(\frac{2\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\int \sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(\frac{2\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\int \sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d}$$

$$\Big\downarrow 3118$$

$$\frac{3\left(\frac{2\left(\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2}+\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d}$$

input

```
Int[Sin[a + b*(c + d*x)^(1/3)],x]
```

output

```
(3*(-(((c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/b) + (2*(Cos[a + b*(c +
 d*x)^(1/3)]/b^2 + ((c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/b))/b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{-3a^{2}\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-\left(a+b(dx+c)^{\frac{1}{3}}\right)\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3\left(a+b(dx+c)^{\frac{1}{3}}\right)^{2}\cos(a+b}{d\,b^{3}}$
default	$\frac{-3a^{2}\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-\left(a+b(dx+c)^{\frac{1}{3}}\right)\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3\left(a+b(dx+c)^{\frac{1}{3}}\right)^{2}\cos(a+b}{d\,b^{3}}$

input
```
int(sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d/b^3*(-a^2*cos(a+b*(d*x+c)^(1/3))-2*a*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x
+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(
1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
3*(2*(d*x + c)^(1/3)*b*sin((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2 -
 2)*cos((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \begin{cases} x\sin(a) & \text{for } b = 0 \wedge (b = 0 \vee a \\ x\sin\left(a+b\sqrt[3]{c}\right) & \text{for } d = 0 \\ -\frac{3(c+dx)^{\frac{2}{3}}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3)),x)
```

output
```
Piecewise((x*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sin(a + b*c**(1
/3)), Eq(d, 0)), (-3*(c + d*x)**(2/3)*cos(a + b*(c + d*x)**(1/3))/(b*d) +
6*(c + d*x)**(1/3)*sin(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cos(a + b*(c +
 d*x)**(1/3))/(b**3*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(a^2\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)a+}{b^3d}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-3*(a^2*cos((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x +
 c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a + (((d*x + c)^(1/3)*b + a
)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x +
 c)^(1/3)*b + a))/(b^3*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$=\frac{3\left(\frac{2\,(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}-\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)}{bd}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
3*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - ((((d*x + c)^(1/3)*b +
a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b
^2)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 42.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3\left(2\cos\left(a+b\,(c+d\,x)^{1/3}\right)+2\,b\,\sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+d\,x)^{1/3}-b^2\,\cos\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+\right.}{b^3\,d}$$

input
```
int(sin(a + b*(c + d*x)^(1/3)),x)
```

output
```
(3*(2*cos(a + b*(c + d*x)^(1/3)) + 2*b*sin(a + b*(c + d*x)^(1/3))*(c + d*x
)^(1/3) - b^2*cos(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3)))/(b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{-3(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}\,b+a\right)b^2+6\cos\left((dx+c)^{\frac{1}{3}}\,b+a\right)+6(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}\,b+a\right)b}{b^3d}$$

input
```
int(sin(a+b*(d*x+c)^(1/3)),x)
```

output
```
(3*( - (c + d*x)**(2/3)*cos((c + d*x)**(1/3)*b + a)*b**2 + 2*cos((c + d*x)
**(1/3)*b + a) + 2*(c + d*x)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b))/(b**3*
d)
```

3.210 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 396

$$
\begin{aligned}
&\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx \\
&= \frac{\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f} \\
&\quad+\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f} \\
&\quad+\frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f} \\
&\quad-\frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f} \\
&\quad+\frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f} \\
&\quad+\frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}
\end{aligned}
$$

output

```
Ci(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*sin(a-b*(-c*f+d*e)^(1/3)/f^
(1/3))/f+Ci((-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)-b*(d*x+c)^(1/3))*sin(a+(
-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f+Ci((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f
^(1/3)+b*(d*x+c)^(1/3))*sin(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f+cos
(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(-(-1)^(1/3)*b*(-c*f+d*e)^(1/3
)/f^(1/3)+b*(d*x+c)^(1/3))/f+cos(a-b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(b*(-c*f+
d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+cos(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/
f^(1/3))*Si((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 25.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.30

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$
$$= \frac{i\left(\text{RootSum}\left[de-cf+f\#1^3\&,e^{-ia-ib\#1}\,\text{ExpIntegralEi}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]-\text{RootSum}\left[de-c\right.\right.}{2f}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

output
```
((I/2)*(RootSum[d*e - c*f + f*#1^3 & , E^((-I)*a - I*b*#1)*ExpIntegralEi[(
-I)*b*((c + d*x)^(1/3) - #1)] & ] - RootSum[d*e - c*f + f*#1^3 & , E^(I*a
+ I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #1)] & ]))/f
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$
$$\Big\downarrow \text{3912}$$
$$\frac{3\int\left(-\frac{d\sin\left(a+b\sqrt[3]{c+dx}\right)}{3f^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{de-cf}-\sqrt[3]{f}\sqrt[3]{c+dx}\right)}+\frac{d\sin\left(a+b\sqrt[3]{c+dx}\right)}{3f^{2/3}\left(\sqrt[3]{de-cf}+\sqrt[3]{f}\sqrt[3]{c+dx}\right)}+\frac{d\sin\left(a+b\sqrt[3]{c+dx}\right)}{3f^{2/3}\left((-1)^{2/3}\sqrt[3]{de-cf}+\sqrt[3]{f}\sqrt[3]{\right.}}\right.}{d}$$
$$\Big\downarrow \text{2009}$$

$$3\left(\frac{d\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{de-cf}b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}b\right)}{3f}+\frac{d\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-}}{\sqrt[3]{f}}\right.}{3f}\right.$$

input
```
Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

output
```
(3*((d*CosIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]*Sin[
a - (b*(d*e - c*f)^(1/3))/f^(1/3)])/(3*f) + (d*CosIntegral[((-1)^(1/3)*b*(
d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)]*Sin[a + ((-1)^(1/3)*b*(d*e
- c*f)^(1/3))/f^(1/3)])/(3*f) + (d*CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(
1/3))/f^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3)
)/f^(1/3)])/(3*f) - (d*Cos[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*S
inIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/
(3*f) + (d*Cos[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[(b*(d*e - c*
f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f) + (d*Cos[a - ((-1)^(2/3)*b*(
d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^
(1/3) + b*(c + d*x)^(1/3)])/(3*f)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{b^3 a^2 \left(\sum_{\_R1=\mathrm{RootOf}\left(-b^3cf+b^3de+f\_Z^3-3af\_Z^2+3a^2f\_Z-a^3f\right)} \frac{-\operatorname{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\cos\left(\_R1\right)+\operatorname{Ci}\left(b(dx+c\right.}{\_R1^2-2\_R1a+a^2}\right.}{f}$
default	$\frac{b^3 a^2 \left(\sum_{\_R1=\mathrm{RootOf}\left(-b^3cf+b^3de+f\_Z^3-3af\_Z^2+3a^2f\_Z-a^3f\right)} \frac{-\operatorname{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\cos\left(\_R1\right)+\operatorname{Ci}\left(b(dx+c\right.}{\_R1^2-2\_R1a+a^2}\right.}{f}$

input

```
int(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x,method=_RETURNVERBOSE)
```

output

```
3/b^3*(1/3*b^3*a^2/f*sum(1/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a
)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e
+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))-2/3*b^3*a/f*sum(_R1/(_R1^2-2*_R1*a+a
^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R
1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+1/3*b
^3/f*sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+C
i(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z
^2*a*f+3*_Z*a^2*f-a^3*f)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.13

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$

$$= \frac{i\operatorname{Ei}\left(-i\left(dx+c\right)^{\frac{1}{3}}b+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{ib^3de-ib^3cf}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{ib^3de-ib^3cf}{f}\right)^{\frac{1}{3}}-ia\right)}+i\operatorname{Ei}\left(-i\left(dx+c\right)^{\frac{1}{3}}b\right.}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

output
```
1/2*(I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*
c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) -
I*a) + I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3
*c*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3)
 - I*a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*d*e + I*
b^3*c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/
3) + I*a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*d*e +
I*b^3*c*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^
(1/3) + I*a) - I*Ei(I*(d*x + c)^(1/3)*b + ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/
3))*e^(I*a - ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)) + I*Ei(-I*(d*x + c)^(1/3)
*b + ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3))*e^(-I*a - ((I*b^3*d*e - I*b^3*c*f)
/f)^(1/3)))/f
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))/(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}\,b+a\right)}{fx+e}\,dx$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x)
```

output
```
int(sin((c + d*x)**(1/3)*b + a)/(e + f*x),x)
```

3.211 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 555

$$\begin{aligned}
&\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx \\
&= -\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} \\
&\quad+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} \\
&\quad+\frac{(-1)^{2/3}bd\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} \\
&\quad-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} \\
&\quad-\frac{\sqrt[3]{-1}bd\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} \\
&\quad-\frac{bd\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} \\
&\quad-\frac{(-1)^{2/3}bd\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}
\end{aligned}$$

output
```
-1/3*(-1)^(1/3)*b*d*cos(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))*Ci((-1)^(
1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)-b*(d*x+c)^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)+
1/3*b*d*cos(a-b*(-c*f+d*e)^(1/3)/f^(1/3))*Ci(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*
(d*x+c)^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)+1/3*(-1)^(2/3)*b*d*cos(a-(-1)^(2/3
)*b*(-c*f+d*e)^(1/3)/f^(1/3))*Ci((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(
d*x+c)^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-sin(a+b*(d*x+c)^(1/3))/f/(f*x+e)+1/
3*(-1)^(1/3)*b*d*sin(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(-(-1)^(1/
3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/
3*b*d*sin(a-b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d
*x+c)^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*(-1)^(2/3)*b*d*sin(a-(-1)^(2/3)*
b*(-c*f+d*e)^(1/3)/f^(1/3))*Si((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*
x+c)^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 1.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.32

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$$

$$= \frac{\frac{3ie^{-i\left(a+b\sqrt[3]{c+dx}\right)}\left(-1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)f}{e+fx} + bd\mathrm{RootSum}\left[de-cf+f\#1^3\&, \frac{e^{-ia-ib\#1}\,\mathrm{ExpIntegralEi}\left(-ib\left(\sqrt[3]{c+dx}-\right.\right.}{\#1^2}\right.}{6f^2}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

output
```
(((3*I)*(-1 + E^((2*I)*(a + b*(c + d*x)^(1/3))))*f)/(E^(I*(a + b*(c + d*x)
^(1/3)))*(e + f*x)) + b*d*RootSum[d*e - c*f + f*#1^3 & , (E^((-I)*a - I*b*
#1)*ExpIntegralEi[(-I)*b*((c + d*x)^(1/3) - #1)])/#1^2 & ] + b*d*RootSum[d
*e - c*f + f*#1^3 & , (E^(I*a + I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3)
 - #1)])/#1^2 & ])/(6*f^2)
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3912, 27, 3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3912}$$

$$\frac{3\int \frac{d^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$3d\int \frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{(de-cf+f(c+dx))^2}d\sqrt[3]{c+dx}$$

$$\Big\downarrow \text{ 3822}$$

$$3d\left(\frac{b\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{de-cf+f(c+dx)}d\sqrt[3]{c+dx}}{3f}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{3f(f(c+dx)-cf+de)}\right)$$

$$\Big\downarrow \text{ 3815}$$

$$3d\Bigg(\frac{b\int \Bigg(-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(de-cf)^{2/3}\left(-\sqrt[3]{de-cf}-\sqrt[3]{f}\sqrt[3]{c+dx}\right)}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(de-cf)^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{f}\sqrt[3]{c+dx}-\sqrt[3]{de-cf}\right)}-\frac{}{3(de-cf)^{2/3}\Big(-}}{3f}$$

$$\Big\downarrow \text{ 2009}$$

$$3d\Bigg(\frac{b\Bigg(-\frac{\sqrt[3]{-1}\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3\sqrt[3]{f}(de-cf)^{2/3}}+\frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegra}}{3\sqrt[3]{f}(de}}{}$$

input
```
Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

output
```
3*d*(-1/3*Sin[a + b*(c + d*x)^(1/3)]/(f*(d*e - c*f + f*(c + d*x))) + (b*(-
1/3*((-1)^(1/3)*Cos[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*CosInteg
ral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(f^(1/3
)*(d*e - c*f)^(2/3)) + (Cos[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*CosIntegral
[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(1/3)*(d*e - c*f
)^(2/3)) + ((-1)^(2/3)*Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*C
osIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/
(3*f^(1/3)*(d*e - c*f)^(2/3)) - ((-1)^(1/3)*Sin[a + ((-1)^(1/3)*b*(d*e - c
*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) -
 b*(c + d*x)^(1/3)])/(3*f^(1/3)*(d*e - c*f)^(2/3)) - (Sin[a - (b*(d*e - c*
f)^(1/3))/f^(1/3)]*SinIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)
^(1/3)])/(3*f^(1/3)*(d*e - c*f)^(2/3)) - ((-1)^(2/3)*Sin[a - ((-1)^(2/3)*b
*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/
f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(1/3)*(d*e - c*f)^(2/3))))/(3*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 1176, normalized size of antiderivative = 2.12

method	result	size
derivativedivides	Expression too large to display	1176
default	Expression too large to display	1176

input
```
int(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
3*d/b^3*(b^6*a^2*(sin(a+b*(d*x+c)^(1/3))*(1/3/b^3/(c*f-d*e)*(a+b*(d*x+c)^(
1/3))-1/3*a/b^3/(c*f-d*e))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x+c)^(1/
3))+3*a*f*(a+b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)-2/9/b^3/f*sum(1/(
c*f-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d
*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f
+3*_Z*a^2*f-a^3*f))-1/9/b^3/f*sum(1/(-_RR1+a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/
3)+_RR1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^
3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)))+sin(a+b*(d*x+c)^(1/3))
*(-2/3*a*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))^2+2/3*a^2*b^3/(c*f-d*e)*(a+b*(d
*x+c)^(1/3)))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x+c)^(1/3))+3*a*f*(a+
b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)+2/9*a*b^3/f*sum((_R1+a)/(c*f-d
*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)
^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z
*a^2*f-a^3*f))+2/9*a*b^3/f*sum(_RR1/(-_RR1+a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/
3)+_RR1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^
3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+sin(a+b*(d*x+c)^(1/3))*
(2/3*a*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))^2-a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^
(1/3))+1/3*b^3*(b^3*c*f-b^3*d*e+a^3*f)/f/(c*f-d*e))/(b^3*c*f-b^3*d*e+a^3*f
-3*a^2*f*(a+b*(d*x+c)^(1/3))+3*a*f*(a+b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1
/3))^3)-2/9*a*b^3/f*sum(_R1/(c*f-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.32

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/12*((I*d*f*x + I*d*e - sqrt(3)*(d*f*x + d*e))*((I*b^3*d*e - I*b^3*c*f)/
f)^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*d*e - I*b^
3*c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3)
- I*a) + (I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*((I*b^3*d*e - I*b^3*c*f
)/f)^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*d*e - I*b
^3*c*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3
) - I*a) + (-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e))*((-I*b^3*d*e + I*b^3
*c*f)/f)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*d*e
+ I*b^3*c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)
^(1/3) + I*a) + (-I*d*f*x - I*d*e - sqrt(3)*(d*f*x + d*e))*((-I*b^3*d*e +
I*b^3*c*f)/f)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*
d*e + I*b^3*c*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*
f)/f)^(1/3) + I*a) - 2*(-I*d*f*x - I*d*e)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/
3)*Ei(I*(d*x + c)^(1/3)*b + ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3))*e^(I*a - (
(-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)) - 2*(I*d*f*x + I*d*e)*((I*b^3*d*e - I*b
^3*c*f)/f)^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + ((I*b^3*d*e - I*b^3*c*f)/f)^(1/
3))*e^(-I*a - ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3)) + 12*(d*e - c*f)*sin((d*x
 + c)^(1/3)*b + a))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F]

$$\int \frac{\sin \left(a + b\sqrt[3]{c + dx}\right)}{(e + fx)^2}\, dx = \int \frac{\sin \left(a + b\sqrt[3]{c + dx}\right)}{\left(e + fx\right)^{2}}\, dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(e+f\,x)^2}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))/(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^{\frac{1}{3}}\,b+a\right)}{f^2x^2+2efx+e^2}dx$$

input

```
int(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x)
```

output

```
int(sin((c + d*x)**(1/3)*b + a)/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.212 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 513

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3}$$
$$-\frac{3(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$
$$+\frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3}$$
$$-\frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3}$$
$$-\frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$
$$+\frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}$$
$$+\frac{315f^2\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3}$$
$$+\frac{315f^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{16b^{9/2}d^3}$$
$$-\frac{3(de-cf)^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^3}$$
$$-\frac{315f^2\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3}$$
$$+\frac{6f(de-cf)(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3}$$
$$+\frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3}$$

output
```
6*f*(-c*f+d*e)*cos(a+b*(d*x+c)^(2/3))/b^3/d^3-3/2*(-c*f+d*e)^2*(d*x+c)^(1/
3)*cos(a+b*(d*x+c)^(2/3))/b/d^3+105/8*f^2*(d*x+c)*cos(a+b*(d*x+c)^(2/3))/b
^3/d^3-3*f*(-c*f+d*e)*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))/b/d^3-3/2*f^2*(
d*x+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))/b/d^3+3/4*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2
)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))/b^(3/2)/d^3+315/
32*f^2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/3))/b^(9/2)/d^3+315/32*f^2*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/3))*sin(a)/b^(9/2)/d^3-3/4*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*
FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)/b^(3/2)/d^3-315/16
*f^2*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^4/d^3+6*f*(-c*f+d*e)*(d*x+c)^(
2/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d^3+21/4*f^2*(d*x+c)^(5/3)*sin(a+b*(d*x+c)
^(2/3))/b^2/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.84

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{2/3}\right)\,dx =$$

$$-\frac{3i\Big((\cos(a)+i\sin(a))\Big((1+i)\left(-105if^2+8b^3(de-cf)^2\right)\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{(1+i)\sqrt{b}\sqrt[3]{c+dx}}{\sqrt{2}}\right)+2\sqrt{b}\Big(-105f^2\sqrt[3]{c}}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(((-3*I)/64)*((Cos[a] + I*Sin[a])*((1 + I)*((-105*I)*f^2 + 8*b^3*(d*e - c*
f)^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]] + 2*Sqrt[
b]*(-105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 +
 4*b^2*f*(c + d*x)^(2/3)*(8*d*e - c*f + 7*d*f*x) + (2*I)*b*f*(16*d*e + 19*
c*f + 35*d*f*x))*(Cos[b*(c + d*x)^(2/3)] + I*Sin[b*(c + d*x)^(2/3)])) + (2
*Sqrt[b]*(105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x
)^2 + 4*b^2*f*(c + d*x)^(2/3)*(-8*d*e + c*f - 7*d*f*x) + (2*I)*b*f*(16*d*e
 + 19*c*f + 35*d*f*x)) + (1 + I)*((105*I)*f^2 + 8*b^3*(d*e - c*f)^2)*Sqrt[
Pi/2]*Erf[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]]*(Cos[b*(c + d*x)^(2/3
)] + I*Sin[b*(c + d*x)^(2/3)]))*(Cos[a + b*(c + d*x)^(2/3)] - I*Sin[a + b*
(c + d*x)^(2/3)])))/(b^(9/2)*d^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right) \, dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{3\int \left(f^2 \sin \left(a+b(c+dx)^{2/3}\right)(c+dx)^{8/3}+2f(de-cf)\sin \left(a+b(c+dx)^{2/3}\right)(c+dx)^{5/3}+(de-cf)^2 \sin (a+}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$3\Bigg(\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^2\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}}+\frac{105\sqrt{\frac{\pi}{2}}f^2\sin(a)\operatorname{FresnelC}(}{16b^{9/2}}$$

input

```
Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

output

```
(3*((2*f*(d*e - c*f)*Cos[a + b*(c + d*x)^(2/3)])/b^3 - ((d*e - c*f)^2*(c +
 d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b) + (35*f^2*(c + d*x)*Cos[a +
b*(c + d*x)^(2/3)])/(8*b^3) - (f*(d*e - c*f)*(c + d*x)^(4/3)*Cos[a + b*(c
+ d*x)^(2/3)])/b - (f^2*(c + d*x)^(7/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b)
+ ((d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(
1/3)])/(2*b^(3/2)) + (105*f^2*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi
]*(c + d*x)^(1/3)])/(16*b^(9/2)) + (105*f^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sq
rt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(16*b^(9/2)) - ((d*e - c*f)^2*Sqrt[Pi/2]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)) - (105*f
^2*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)])/(16*b^4) + (2*f*(d*e - c*f)
*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)])/b^2 + (7*f^2*(c + d*x)^(5/3)*
Sin[a + b*(c + d*x)^(2/3)])/(4*b^2)))/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{3f^{2}(dx+c)^{\frac{7}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{21f^{2}\left(\frac{(dx+c)^{\frac{5}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}-\frac{5\left(-\frac{(dx+c)\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3(dx+c)^{\frac{1}{3}}\sin\left(a+b\right.}{4b}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{3f^{2}(dx+c)^{\frac{7}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{21f^{2}\left(\frac{(dx+c)^{\frac{5}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}-\frac{5\left(-\frac{(dx+c)\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3(dx+c)^{\frac{1}{3}}\sin\left(a+b\right.}{4b}\right.}{}\right.}{}}{}$
parts	Expression too large to display

input
```
int((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d^3*(-1/2*f^2/b*(d*x+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))+7/2*f^2/b*(1/2/b*(d
*x+c)^(5/3)*sin(a+b*(d*x+c)^(2/3))-5/2/b*(-1/2/b*(d*x+c)*cos(a+b*(d*x+c)^(
2/3))+3/2/b*(1/2/b*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(2/3))-1/4/b^(3/2)*2^(1/2
)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))+sin(a)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))))))+(c*f-d*e)*f/b*(d*x+c
)^(4/3)*cos(a+b*(d*x+c)^(2/3))-4*(c*f-d*e)*f/b*(1/2/b*(d*x+c)^(2/3)*sin(a+
b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))-1/2*(c*f-d*e)^2/b*(d*x+c)
^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4*(c*f-d*e)^2/b^(3/2)*2^(1/2)*Pi^(1/2)*(co
s(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))-sin(a)*FresnelS(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.60

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\left(\sqrt{2}(105\,\pi f^2\sin(a)+8\,\pi(b^3d^2e^2-2\,b^3cdef+b^3c^2f^2)\cos(a))\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)+\right.}{}$$

input
```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
3/32*(sqrt(2)*(105*pi*f^2*sin(a) + 8*pi*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3
*c^2*f^2)*cos(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi
)) + sqrt(2)*(105*pi*f^2*cos(a) - 8*pi*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*
c^2*f^2)*sin(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi)
) + 4*(35*b^2*d*f^2*x + 16*b^2*d*e*f + 19*b^2*c*f^2 - 4*(b^4*d^2*f^2*x^2 +
 2*b^4*d^2*e*f*x + b^4*d^2*e^2)*(d*x + c)^(1/3))*cos((d*x + c)^(2/3)*b + a
) - 2*(105*(d*x + c)^(1/3)*b*f^2 - 4*(7*b^3*d*f^2*x + 8*b^3*d*e*f - b^3*c*
f^2)*(d*x + c)^(2/3))*sin((d*x + c)^(2/3)*b + a))/(b^5*d^3)
```

Sympy [F]

$$\int (e+fx)^2\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\int (e+fx)^2\sin \left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

input

```
integrate((f*x+e)**2*sin(a+b*(d*x+c)**(2/3)),x)
```

output

```
Integral((e + f*x)**2*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.09

$$\int (e+fx)^2\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

output

```
-3/128*(8*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x +
c)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/
3)*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a)
)*e^2/b^3 - 16*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d
*x + c)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c
)^(1/3)*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b
 + a))*c*e*f/(b^3*d) + 8*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(
a))*erf((d*x + c)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*er
f((d*x + c)^(1/3)*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x +
c)^(2/3)*b + a))*c^2*f^2/(b^3*d^2) - 128*(2*(d*x + c)^(2/3)*b*sin((d*x + c
)^(2/3)*b + a) - ((d*x + c)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*e*f
/(b^3*d) + 128*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c
)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*c*f^2/(b^3*d^2) - (105*sqrt(2
)*sqrt(pi)*(((I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(I*b
)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*b
^(3/2) - 16*(4*(d*x + c)^(7/3)*b^5 - 35*(d*x + c)*b^3)*cos((d*x + c)^(2/3)
*b + a) + 56*(4*(d*x + c)^(5/3)*b^4 - 15*(d*x + c)^(1/3)*b^2)*sin((d*x + c
)^(2/3)*b + a))*f^2/(b^6*d^2))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.50

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
-3/64*(8*e^2*(sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(
b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)
*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*
e^(-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x +
 c)^(2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)
 - 16*(sqrt(2)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) +
1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt
(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(
-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(
d*x + c)^(1/3)*b^2*c - 2*(d*x + c)^(2/3)*b - 2*I)*e^(I*(d*x + c)^(2/3)*b +
 I*a)/b^3 + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c + 2*(d*x
+ c)^(2/3)*b - 2*I)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b^3)*e*f/d + f^2*(I*sqr
t(2)*sqrt(pi)*(-8*I*b^3*c^2 - 105)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/
abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^4*(-I*b/abs(b) + 1)*sqrt(abs(b))) + I
*sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 + 105)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I
*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^4*(I*b/abs(b) + 1)*sqrt(abs(b)))
- 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^(4/3)*b^3*c + 8*I*(d*x + c
)^(1/3)*b^3*c^2 - 28*(d*x + c)^(5/3)*b^2 + 32*(d*x + c)^(2/3)*b^2*c + 70*(
-I*d*x - I*c)*b + 32*I*b*c + 105*(d*x + c)^(1/3))*e^(I*(d*x + c)^(2/3)*b +
 I*a)/b^4 - 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^(4/3)*b^3*c +...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(e+f\,x\right)}^2\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))*(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \left(\int \sin \left((dx+c)^{\frac{2}{3}} b+a\right) dx\right) e^2 + \left(\int \sin \left((dx+c)^{\frac{2}{3}} b+a\right) x^2 dx\right) f^2 + 2\left(\int \sin \left((dx+c)^{\frac{2}{3}} b+a\right) x dx\right) ef$$

input

```
int((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x)
```

output

```
int(sin((c + d*x)**(2/3)*b + a),x)*e**2 + int(sin((c + d*x)**(2/3)*b + a)*
x**2,x)*f**2 + 2*int(sin((c + d*x)**(2/3)*b + a)*x,x)*e*f
```

3.213 $\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 243

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx &= \frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2}\\
&-\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}\\
&-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}\\
&+\frac{3(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}\\
&-\frac{3(de-cf)\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d^2}\\
&+\frac{3f(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^2}\end{aligned}$$

output
```
3*f*cos(a+b*(d*x+c)^(2/3))/b^3/d^2-3/2*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b*(d
*x+c)^(2/3))/b/d^2-3/2*f*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))/b/d^2+3/4*(-
c*f+d*e)*2^(1/2)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)
^(1/3))/b^(3/2)/d^2-3/4*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/
2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)/b^(3/2)/d^2+3*f*(d*x+c)^(2/3)*sin(a+b*(d
*x+c)^(2/3))/b^2/d^2
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.88

$$\int (e+fx)\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\frac{3\Big(4f\cos\left(a+b(c+dx)^{2/3}\right)-2b^2de\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)-2b^2dfx\sqrt[3]{c+dx}\cos\left(a\right.}{}$$

input

```
Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output

```
(3*(4*f*Cos[a + b*(c + d*x)^(2/3)] - 2*b^2*d*e*(c + d*x)^(1/3)*Cos[a + b*(
c + d*x)^(2/3)] - 2*b^2*d*f*x*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] +
 b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*
x)^(1/3)] - b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
+ d*x)^(1/3)]*Sin[a] + 4*b*f*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)]))/
(4*b^3*d^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{3\int \left(f\sin\left(a+b(c+dx)^{2/3}\right)(c+dx)^{5/3}+(de-cf)\sin\left(a+b(c+dx)^{2/3}\right)(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}}+\frac{f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3}+ f(c+\right.}{d^2}$$

input

```
Int[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output

```
(3*((f*Cos[a + b*(c + d*x)^(2/3)])/b^3 - ((d*e - c*f)*(c + d*x)^(1/3)*Cos[
a + b*(c + d*x)^(2/3)])/(2*b) - (f*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(2/
3)])/(2*b) + ((d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)])/(2*b^(3/2)) - ((d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqr
t[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)) + (f*(c + d*x)^(2/3)*Sin[a +
b*(c + d*x)^(2/3)])/b^2))/d^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{3f(dx+c)^{\frac{4}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{6f\left(\frac{(dx+c)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b^2}\right)}{b}+\frac{3(cf-de)(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c\right.}{2b}}{d^2}$
default	$\frac{-\frac{3f(dx+c)^{\frac{4}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{6f\left(\frac{(dx+c)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b^2}\right)}{b}+\frac{3(cf-de)(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c\right.}{2b}}{d^2}$
parts	$-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)fx}{2db}-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)e}{2db}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}}{\sqrt{\pi}}\right.}{4d\,b^{\frac{3}{2}}}$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d^2*(-1/2*f/b*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))+2*f/b*(1/2/b*(d*x+c)^
(2/3)*sin(a+b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))+1/2*(c*f-d*e)
/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))-1/4*(c*f-d*e)/b^(3/2)*2^(1/2)*Pi^(
1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))-sin(a)*Fresn
elS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.65

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\left(\sqrt{2}\pi(bde-bcf)\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\pi(bde-bcf)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)\right.\right.}{}$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
3/4*(sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x
 + c)^(1/3)*sqrt(b/pi)) - sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*fresnel_si
n(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi))*sin(a) + 4*(d*x + c)^(2/3)*b*f*sin((
d*x + c)^(2/3)*b + a) - 2*((b^2*d*f*x + b^2*d*e)*(d*x + c)^(1/3) - 2*f)*co
s((d*x + c)^(2/3)*b + a))/(b^3*d^2)
```

Sympy [F]

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \int (e+fx)\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

input
```
integrate((f*x+e)*sin(a+b*(d*x+c)**(2/3)),x)
```

output
```
Integral((e + f*x)*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\left(\frac{\left(\sqrt{2}\sqrt{\pi}\left(((i-1)\cos(a)+(i+1)\sin(a))\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{ib}\right)+(-(i+1)\cos(a)-(i-1)\sin(a))\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}+8(dx+c)^{\frac{1}{3}}b^{2}\cos\left((dx\right.\right.}{b^{3}}\right.}{}$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

output

```
-3/16*((sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^
(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*
sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*e
/b^3 - (sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^
(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*
sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*c
*f/(b^3*d) - 8*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c
)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*f/(b^3*d))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.67

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Too large to display}$$

input

```
integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

output

```
-3/8*(e*(sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) +
1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt
(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I
*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(
2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b) - (s
qrt(2)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(
abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*c*e
rf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b
*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)
^(1/3)*b^2*c - 2*(d*x + c)^(2/3)*b - 2*I)*e^(I*(d*x + c)^(2/3)*b + I*a)/b^
3 + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c + 2*(d*x + c)^(2/
3)*b - 2*I)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b^3)*f/d)/d
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,\left(e+f\,x\right)\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))*(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\left(\int \sin \left((dx+c)^{\frac{2}{3}}b+a\right)dx\right)e+\left(\int \sin \left((dx+c)^{\frac{2}{3}}b+a\right)xdx\right)f$$

input
```
int((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a),x)*e + int(sin((c + d*x)**(2/3)*b + a)*x,x
)*f
```

3.214 $\int \sin \left(a+b(c+dx)^{2/3}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 130

$$\int \sin \left(a+b(c+dx)^{2/3}\right) \, dx=-\frac{3\sqrt[3]{c+dx}\cos \left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{2b^{3/2}d}$$

output

```
-3/2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+3/4*2^(1/2)*Pi^(1/2)*cos(a)*
FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))/b^(3/2)/d-3/4*2^(1/2)*Pi^
(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)/b^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \sin \left(a+b(c+dx)^{2/3}\right) \, dx = -\frac{3\left(2\sqrt{b}\sqrt[3]{c+dx}\cos \left(a+b(c+dx)^{2/3}\right)-\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\right.\right.}{4b^{3/2}d}$$

input

```
Integrate[Sin[a + b*(c + d*x)^(2/3)],x]
```

output

```
(-3*(2*Sqrt[b]*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] - Sqrt[2*Pi]*Cos
[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)] + Sqrt[2*Pi]*FresnelS[Sqr
t[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(4*b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3844, 3866, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+b(c+dx)^{2/3}\right)dx$$

$$\Big\downarrow \text{3844}$$

$$\frac{3\int (c+dx)^{2/3}\sin \left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{3866}$$

$$\frac{3\left(\frac{\int \cos \left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}-\frac{\sqrt[3]{c+dx}\cos \left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d}$$

$$\Big\downarrow \text{3835}$$

$$\frac{3\left(\frac{\cos (a)\int \cos \left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\sin (a)\int \sin \left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}-\frac{\sqrt[3]{c+dx}\cos \left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d}$$

$$\Big\downarrow \text{3832}$$

$$\frac{3\left(\frac{\cos (a)\int \cos \left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\frac{\sqrt{\frac{\pi}{2}}\sin (a)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}}{2b}-\frac{\sqrt[3]{c+dx}\cos \left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d}$$

$$\Big\downarrow \text{3833}$$

$$\frac{3\left(\frac{\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}}{2b}-\frac{\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d}$$

input
```
Int[Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(3*(-1/2*((c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/b + ((Sqrt[Pi/2]*Cos
[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/Sqrt[b] - (Sqrt[Pi/2]*Fr
esnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/Sqrt[b])/(2*b)))/d
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835
```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3844
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Simp[k/f   Subst[Int[x^(k - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}}{\sqrt{\pi}}\right)-\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}}{d}$	86
default	$\frac{-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}}{\sqrt{\pi}}\right)-\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}}{d}$	86

input
```
int(sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d*(-1/2/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4/b^(3/2)*2^(1/2)*Pi^(1
/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))-sin(a)*Fresne
lS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\left(\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin(a)-2\,(dx+\right.}{4\,b^{2}d}$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
3/4*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt
(b/pi)) - sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b
/pi))*sin(a) - 2*(d*x + c)^(1/3)*b*cos((d*x + c)^(2/3)*b + a))/(b^2*d)
```

Sympy [F]

$$\int \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \int \sin \left(a+b(c+dx)^{\frac{2}{3}}\right) dx$$

input
```
integrate(sin(a+b*(d*x+c)**(2/3)),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int \sin \left(a+b(c+dx)^{2/3}\right) \, dx =$$

$$-\frac{3 \left(\sqrt{2}\sqrt{\pi}\left(\left((i-1)\cos(a)+(i+1)\sin(a)\right)\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{i\,b}\right)+\left(-(i+1)\cos(a)-(i-1)\sin(a)\right)\mathrm{er}\right.\right.}{16\,b^3 d}$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
-3/16*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(
1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*s
qrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))/(b
^3*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$\int \sin \left(a+b(c+dx)^{2/3}\right)\,dx = -\frac{3\left(\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{2\,(dx+c)^{\frac{1}{3}}e^{\left(i\,(dx+c)^{\frac{2}{3}}b+i\,a\right)}}{b}\right.}{8\,d}$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
-3/8*(sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*
sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi
)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)
/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3
)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+b(c+dx)^{2/3}\right)\,dx = \int \sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right) \,d x$$

input
```
int(sin(a + b*(c + d*x)^(2/3)),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3)), x)
```

Reduce [F]

$$\int \sin \left(a+b(c+dx)^{2/3}\right) \, dx = \int \sin \left((dx+c)^{\frac{2}{3}} b+a\right) dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a),x)
```

3.215 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 63.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx} \, dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}dx$$

input
```
Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin \left(a+b(dx+c)^{\frac{2}{3}}\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

output
```
int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 41.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))/(e + f*x),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin \left(\left(dx+c\right)^{\frac{2}{3}} b+a\right)}{fx+e}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a)/(e + f*x),x)
```

3.216 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 58.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(fx+e)^2}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

output
```
int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin((d*x + c)^(2/3)*b + a)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e+fx)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 41.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))/(e + f*x)^2,x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a)/(e**2 + 2*e*f*x + f**2*x**2),x)
```

3.217 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 855

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \text{Too large to display}$$

output
```
1/120*b^5*f*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/d^3-1/120960*b
^7*f^2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^3+1/2*b*(-c*f+d*e)^2*(d*x+c)
^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^3-1/60*b^3*f*(-c*f+d*e)*(d*x+c)*cos(a+b/(d
*x+c)^(1/3))/d^3+1/20160*b^5*f^2*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(1/3))/d^3+
1/5*b*f*(-c*f+d*e)*(d*x+c)^(5/3)*cos(a+b/(d*x+c)^(1/3))/d^3-1/1008*b^3*f^2
*(d*x+c)^2*cos(a+b/(d*x+c)^(1/3))/d^3+1/24*b*f^2*(d*x+c)^(8/3)*cos(a+b/(d*
x+c)^(1/3))/d^3-1/120960*b^9*f^2*cos(a)*Ci(b/(d*x+c)^(1/3))/d^3+1/2*b^3*(-
c*f+d*e)^2*cos(a)*Ci(b/(d*x+c)^(1/3))/d^3+1/120*b^6*f*(-c*f+d*e)*Ci(b/(d*x
+c)^(1/3))*sin(a)/d^3+1/120960*b^8*f^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3)
)/d^3-1/2*b^2*(-c*f+d*e)^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d^3+1/120*
b^4*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(1/3))/d^3-1/60480*b^6*f^2*
(d*x+c)*sin(a+b/(d*x+c)^(1/3))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^(1
/3))/d^3-1/20*b^2*f*(-c*f+d*e)*(d*x+c)^(4/3)*sin(a+b/(d*x+c)^(1/3))/d^3+1/
5040*b^4*f^2*(d*x+c)^(5/3)*sin(a+b/(d*x+c)^(1/3))/d^3+f*(-c*f+d*e)*(d*x+c)
^2*sin(a+b/(d*x+c)^(1/3))/d^3-1/168*b^2*f^2*(d*x+c)^(7/3)*sin(a+b/(d*x+c)^
(1/3))/d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(1/3))/d^3+1/120*b^6*f*(-c*f+
d*e)*cos(a)*Si(b/(d*x+c)^(1/3))/d^3+1/120960*b^9*f^2*sin(a)*Si(b/(d*x+c)^(
1/3))/d^3-1/2*b^3*(-c*f+d*e)^2*sin(a)*Si(b/(d*x+c)^(1/3))/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.31 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.09

$$\int (e+fx)^2 \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right) \, dx = \text{Too large to display}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

output
```
((-1/241920*I)*((Cos[a] + I*Sin[a])*((60480*I)*b^3*d^2*e^2*ExpIntegralEi[(
I*b)/(c + d*x)^(1/3)] + 1008*b^6*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)
] - (120960*I)*b^3*c*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - I*b^9*f^
2*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - 1008*b^6*c*f^2*ExpIntegralEi[(I*b
)/(c + d*x)^(1/3)] + (60480*I)*b^3*c^2*f^2*ExpIntegralEi[(I*b)/(c + d*x)^(
1/3)] + (c + d*x)^(1/3)*(b^8*f^2 - I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(
c + d*x)^(2/3) + (24*I)*b^3*f*(c + d*x)^(2/3)*(-84*d*e + 79*c*f - 5*d*f*x)
 + (6*I)*b^5*f*(168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*
d*e - 41*c*f + d*f*x) + 40320*(c + d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x)
 + d^2*(3*e^2 + 3*e*f*x + f^2*x^2)) + (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f
^2 - 2*c*d*f*(48*e + 7*f*x) + d^2*(60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b
^2*(383*c^2*f^2 - 2*c*d*f*(399*e + 16*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f
^2*x^2)))*(Cos[b/(c + d*x)^(1/3)] + I*Sin[b/(c + d*x)^(1/3)])) - ((c + d*x
)^(1/3)*(b^8*f^2 + I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*x)^(2/3) -
 (6*I)*b^5*f*(168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*
e - 41*c*f + d*f*x) + (24*I)*b^3*f*(c + d*x)^(2/3)*(84*d*e - 79*c*f + 5*d*
f*x) + 40320*(c + d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3
*e*f*x + f^2*x^2)) - (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d*f*(48*
e + 7*f*x) + d^2*(60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2*f^2 -
 2*c*d*f*(399*e + 16*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2))) + I*...
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \left(\frac{f^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)(c+dx)^{10/3}}{d^2}+\frac{2f(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)(c+dx)^{7/3}}{d^2}+\frac{(de-cf)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)(c+dx)^{4/3}}{d^2}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(\frac{f^2\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)b^9}{362880d^2}-\frac{f^2\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)b^9}{362880d^2}-\frac{f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)b^8}{362880d^2}+\frac{f^2(c+dx)^{2/3}\cos\left(\right.}{362}\right.}{}$$

input
```
Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

output
```
(-3*(-1/360*(b^5*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])
/d^2 + (b^7*f^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(362880*d^2) -
 (b*(d*e - c*f)^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(6*d^2) + (b
^3*f*(d*e - c*f)*(c + d*x)*Cos[a + b/(c + d*x)^(1/3)])/(180*d^2) - (b^5*f^
2*(c + d*x)^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(60480*d^2) - (b*f*(d*e - c*
f)*(c + d*x)^(5/3)*Cos[a + b/(c + d*x)^(1/3)])/(15*d^2) + (b^3*f^2*(c + d*
x)^2*Cos[a + b/(c + d*x)^(1/3)])/(3024*d^2) - (b*f^2*(c + d*x)^(8/3)*Cos[a
 + b/(c + d*x)^(1/3)])/(72*d^2) + (b^9*f^2*Cos[a]*CosIntegral[b/(c + d*x)^
(1/3)])/(362880*d^2) - (b^3*(d*e - c*f)^2*Cos[a]*CosIntegral[b/(c + d*x)^(
1/3)])/(6*d^2) - (b^6*f*(d*e - c*f)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])
/(360*d^2) - (b^8*f^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(362880*
d^2) + (b^2*(d*e - c*f)^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(6*d
^2) - (b^4*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(360*
d^2) + (b^6*f^2*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(181440*d^2) - ((d*e
 - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(3*d^2) + (b^2*f*(d*e - c*
f)*(c + d*x)^(4/3)*Sin[a + b/(c + d*x)^(1/3)])/(60*d^2) - (b^4*f^2*(c + d*
x)^(5/3)*Sin[a + b/(c + d*x)^(1/3)])/(15120*d^2) - (f*(d*e - c*f)*(c + d*x
)^2*Sin[a + b/(c + d*x)^(1/3)])/(3*d^2) + (b^2*f^2*(c + d*x)^(7/3)*Sin[a +
 b/(c + d*x)^(1/3)])/(504*d^2) - (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^(1/3
)])/(9*d^2) - (b^6*f*(d*e - c*f)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	Expression too large to display	936
default	Expression too large to display	936
parts	Expression too large to display	2775

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3/d^3*b^3*(2*b^3*d*e*f*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/30*co
s(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/3))/b^4*(d
*x+c)^(4/3)+1/360*cos(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+b/(d*x+c)
^(1/3))/b^2*(d*x+c)^(2/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/7
20*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))-2*b^3*c*f^
2*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/30*cos(a+b/(d*x+c)^(1/3))/b
^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/3))/b^4*(d*x+c)^(4/3)+1/360*cos(
a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2
/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/720*Si(b/(d*x+c)^(1/3))
*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))-2*c*d*e*f*(-1/3*sin(a+b/(d*x+c)^
(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+
b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(
d*x+c)^(1/3))*cos(a))+d^2*e^2*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6
*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*
x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+
c^2*f^2*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3)
)/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d
*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+b^6*f^2*(-1/9*sin(a+b/
(d*x+c)^(1/3))/b^9*(d*x+c)^3-1/72*cos(a+b/(d*x+c)^(1/3))/b^8*(d*x+c)^(8/3)
+1/504*sin(a+b/(d*x+c)^(1/3))/b^7*(d*x+c)^(7/3)+1/3024*cos(a+b/(d*x+c)^...
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.67

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx =$$

$$-\frac{\left(120\, b^3d^2f^2x^2 + 2016\, b^3cdef - 1896\, b^3c^2f^2 + 48\,(42\, b^3d^2ef - 37\, b^3cdf^2)x - (5040\, bd^2f^2x^2 + 60480\, b\right.}{}$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
-1/120960*((120*b^3*d^2*f^2*x^2 + 2016*b^3*c*d*e*f - 1896*b^3*c^2*f^2 + 48
*(42*b^3*d^2*e*f - 37*b^3*c*d*f^2)*x - (5040*b*d^2*f^2*x^2 + 60480*b*d^2*e
^2 - 96768*b*c*d*e*f - (b^7 - 41328*b*c^2)*f^2 + 2016*(12*b*d^2*e*f - 7*b*
c*d*f^2)*x)*(d*x + c)^(2/3) - 6*(b^5*d*f^2*x + 168*b^5*d*e*f - 167*b^5*c*f
^2)*(d*x + c)^(1/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - ((
60480*b^3*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*cos(a)
 + 1008*(b^6*d*e*f - b^6*c*f^2)*sin(a))*cos_integral(b/(d*x + c)^(1/3)) -
(40320*d^3*f^2*x^3 + 120960*d^3*e*f*x^2 + 120960*c*d^2*e^2 - 120960*c^2*d*
e*f - 2*(b^6*c - 20160*c^3)*f^2 - 2*(b^6*d*f^2 - 60480*d^3*e^2)*x + 24*(b^
4*d*f^2*x + 42*b^4*d*e*f - 41*b^4*c*f^2)*(d*x + c)^(2/3) - (720*b^2*d^2*f^
2*x^2 + 60480*b^2*d^2*e^2 - 114912*b^2*c*d*e*f - (b^8 - 55152*b^2*c^2)*f^2
 + 288*(21*b^2*d^2*e*f - 16*b^2*c*d*f^2)*x)*(d*x + c)^(1/3))*sin((a*d*x +
a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - (1008*(b^6*d*e*f - b^6*c*f^2)*cos(a)
 - (60480*b^3*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*si
n(a))*sin_integral(b/(d*x + c)^(1/3)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = \int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx$$

input
```
integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/3)),x)
```

output
```
Integral((e + f*x)**2*sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.17

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

output

```
1/241920*(60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos
(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3
 + 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((
d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3)))*e^2 - 120960*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*
cos(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*
b^3 + 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2
*((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3)))*c*e*f/d + 60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1
/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*si
n(a))*b^3 + 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
)) - 2*((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*
x + c)^(1/3)))*c^2*f^2/d^2 + 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*
b/(d*x + c)^(1/3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^
(1/3)))*sin(a))*b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x +
 c)^(5/3)*b)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(
2/3)*b^4 - 6*(d*x + c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3)))*e*f/d - 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(
-I*b/(d*x + c)^(1/3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x +
c)^(1/3)))*sin(a))*b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11903 vs. 2(739) = 1478.

Time = 0.67 (sec) , antiderivative size = 11903, normalized size of antiderivative = 13.92

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

output

```
1/120960*(60480*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)
/(d*x + c)^(1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(
-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x +
c)^(1/3)*a + b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3)
 + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^
4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x +
 c) - ((d*x + c)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)
/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin
(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*
x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d
*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x +
c)^(1/3)*a + b)/(d*x + c)^(1/3)))*e^2/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^
2/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x
+ c)^(1/3)*a + b)^3/(d*x + c))*b) + 1008*(a^6*b^7*cos_integral(-a + ((d...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)\,{\left(e+f\,x\right)}^2\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/3))*(e + f*x)^2,x)
```

output

```
int(sin(a + b/(c + d*x)^(1/3))*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right) \, dx = \left(\int \sin \left(\frac{(dx+c)^{\frac{1}{3}} a+b}{(dx+c)^{\frac{1}{3}}}\right) dx\right) e^2 + \left(\int \sin \left(\frac{(dx+c)^{\frac{1}{3}} a+b}{(dx+c)^{\frac{1}{3}}}\right) x^2 dx\right) f^2 + 2\left(\int \sin \left(\frac{(dx+c)^{\frac{1}{3}} a+b}{(dx+c)^{\frac{1}{3}}}\right) x dx\right) e f$$

input

```
int((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x)
```

output

```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3)),x)*e**2 + int(sin(((c +
 d*x)**(1/3)*a + b)/(c + d*x)**(1/3))*x**2,x)*f**2 + 2*int(sin(((c + d*x)*
*(1/3)*a + b)/(c + d*x)**(1/3))*x,x)*e*f
```

3.218 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 419

$$
\begin{aligned}
\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = {} & \frac{b^5 f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} \\
& + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
& - \frac{b^3 f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} \\
& + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} \\
& + \frac{b^3(de-cf)\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
& + \frac{b^6 f\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{240d^2} \\
& - \frac{b^2(de-cf)\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
& + \frac{b^4 f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} \\
& + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} \\
& - \frac{b^2 f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2} \\
& + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} \\
& + \frac{b^6 f\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} \\
& - \frac{b^3(de-cf)\sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}
\end{aligned}
$$

output
```
1/240*b^5*f*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/d^2+1/2*b*(-c*f+d*e)*(d*x
+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^2-1/120*b^3*f*(d*x+c)*cos(a+b/(d*x+c)^(
1/3))/d^2+1/10*b*f*(d*x+c)^(5/3)*cos(a+b/(d*x+c)^(1/3))/d^2+1/2*b^3*(-c*f+
d*e)*cos(a)*Ci(b/(d*x+c)^(1/3))/d^2+1/240*b^6*f*Ci(b/(d*x+c)^(1/3))*sin(a)
/d^2-1/2*b^2*(-c*f+d*e)*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d^2+1/240*b^4
*f*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(1/3))/d^2+(-c*f+d*e)*(d*x+c)*sin(a+b/(d*
x+c)^(1/3))/d^2-1/40*b^2*f*(d*x+c)^(4/3)*sin(a+b/(d*x+c)^(1/3))/d^2+1/2*f*
(d*x+c)^2*sin(a+b/(d*x+c)^(1/3))/d^2+1/240*b^6*f*cos(a)*Si(b/(d*x+c)^(1/3)
)/d^2-1/2*b^3*(-c*f+d*e)*sin(a)*Si(b/(d*x+c)^(1/3))/d^2
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.29

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

$$= \frac{e\sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)\left(b\sqrt[3]{c+dx}\cos(a)-b^2\sin(a)+2(c+dx)^{2/3}\sin(a)\right)}{2d}$$

$$+\frac{f\sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)\left(b^5\cos(a)-120bc\sqrt[3]{c+dx}\cos(a)-2b^3(c+dx)^{2/3}\cos(a)+24b(c+dx)^{4/3}\right.}{}$$

$$+\frac{e\sqrt[3]{c+dx}\left(-b^2\cos(a)+2(c+dx)^{2/3}\cos(a)-b\sqrt[3]{c+dx}\sin(a)\right)\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

$$+\frac{f\sqrt[3]{c+dx}\left(120b^2c\cos(a)+b^4\sqrt[3]{c+dx}\cos(a)-240c(c+dx)^{2/3}\cos(a)-6b^2(c+dx)\cos(a)+120(c+\right.}{}$$

$$+\frac{b^3e\left(\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)-\sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{2d}$$

$$+\frac{b^3f\left(-120c\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)+b^3\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+\right.\right.}{240d^2}$$

input
```
Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

output
```
(e*(c + d*x)^(1/3)*Cos[b/(c + d*x)^(1/3)]*(b*(c + d*x)^(1/3)*Cos[a] - b^2*
Sin[a] + 2*(c + d*x)^(2/3)*Sin[a]))/(2*d) + (f*(c + d*x)^(1/3)*Cos[b/(c +
d*x)^(1/3)]*(b^5*Cos[a] - 120*b*c*(c + d*x)^(1/3)*Cos[a] - 2*b^3*(c + d*x)
^(2/3)*Cos[a] + 24*b*(c + d*x)^(4/3)*Cos[a] + 120*b^2*c*Sin[a] + b^4*(c +
d*x)^(1/3)*Sin[a] - 240*c*(c + d*x)^(2/3)*Sin[a] - 6*b^2*(c + d*x)*Sin[a]
+ 120*(c + d*x)^(5/3)*Sin[a]))/(240*d^2) + (e*(c + d*x)^(1/3)*(-(b^2*Cos[a
]) + 2*(c + d*x)^(2/3)*Cos[a] - b*(c + d*x)^(1/3)*Sin[a])*Sin[b/(c + d*x)^
(1/3)])/(2*d) + (f*(c + d*x)^(1/3)*(120*b^2*c*Cos[a] + b^4*(c + d*x)^(1/3)
*Cos[a] - 240*c*(c + d*x)^(2/3)*Cos[a] - 6*b^2*(c + d*x)*Cos[a] + 120*(c +
 d*x)^(5/3)*Cos[a] - b^5*Sin[a] + 120*b*c*(c + d*x)^(1/3)*Sin[a] + 2*b^3*(
c + d*x)^(2/3)*Sin[a] - 24*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)
])/(240*d^2) + (b^3*e*(Cos[a]*CosIntegral[b/(c + d*x)^(1/3)] - Sin[a]*SinI
ntegral[b/(c + d*x)^(1/3)]))/(2*d) + (b^3*f*(-120*c*Cos[a]*CosIntegral[b/(
c + d*x)^(1/3)] + b^3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] + b^3*Cos[a]*S
inIntegral[b/(c + d*x)^(1/3)] + 120*c*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)
]))/(240*d^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \left(\frac{f\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)(c+dx)^{7/3}}{d}+\frac{(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)(c+dx)^{4/3}}{d}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(-\frac{b^6 f\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{720d}-\frac{b^6 f\cos(a)\,\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{720d}-\frac{b^5 f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{720d}-\frac{b^4 f(c+dx)^{2/3}\sin\left(\right.}{720}\right.}{}$$

input

```
Int[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

output

```
(-3*(-1/720*(b^5*f*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/d - (b*(d*e
 - c*f)*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(6*d) + (b^3*f*(c + d*
x)*Cos[a + b/(c + d*x)^(1/3)])/(360*d) - (b*f*(c + d*x)^(5/3)*Cos[a + b/(c
 + d*x)^(1/3)])/(30*d) - (b^3*(d*e - c*f)*Cos[a]*CosIntegral[b/(c + d*x)^(
1/3)])/(6*d) - (b^6*f*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(720*d) + (b^
2*(d*e - c*f)*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(6*d) - (b^4*f*(
c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(720*d) - ((d*e - c*f)*(c + d*x
)*Sin[a + b/(c + d*x)^(1/3)])/(3*d) + (b^2*f*(c + d*x)^(4/3)*Sin[a + b/(c
+ d*x)^(1/3)])/(120*d) - (f*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])/(6*d)
- (b^6*f*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(720*d) + (b^3*(d*e - c*f)
*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(6*d)))/d
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{3b^3\left(-cf\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a}{6}\right.\right.}{}$
default	$-\frac{3b^3\left(-cf\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a}{6}\right.\right.}{}$
parts	Expression too large to display

input
```
int((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3/d^2*b^3*(-c*f*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x
+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/
6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+d*e*(-1/3*sin
(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/
3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(
a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+f*b^3*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*
(d*x+c)^2-1/30*cos(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x
+c)^(1/3))/b^4*(d*x+c)^(4/3)+1/360*cos(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/72
0*sin(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*
(d*x+c)^(1/3)-1/720*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*s
in(a)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.62

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$
$$=\frac{\left((dx+c)^{\frac{1}{3}}b^5f-2\,b^3dfx-2\,b^3cf+24\,(bdfx+5\,bde-4\,bcf)(dx+c)^{\frac{2}{3}}\right)\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)+(b^6f\,\mathrm{s}}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
1/240*(((d*x + c)^(1/3)*b^5*f - 2*b^3*d*f*x - 2*b^3*c*f + 24*(b*d*f*x + 5*
b*d*e - 4*b*c*f)*(d*x + c)^(2/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d
*x + c)) + (b^6*f*sin(a) + 120*(b^3*d*e - b^3*c*f)*cos(a))*cos_integral(b/
(d*x + c)^(1/3)) + ((d*x + c)^(2/3)*b^4*f + 120*d^2*f*x^2 + 240*d^2*e*x +
240*c*d*e - 120*c^2*f - 6*(b^2*d*f*x + 20*b^2*d*e - 19*b^2*c*f)*(d*x + c)^
(1/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) + (b^6*f*cos(a) -
120*(b^3*d*e - b^3*c*f)*sin(a))*sin_integral(b/(d*x + c)^(1/3)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)**(1/3)),x)
```

output
```
Integral((e + f*x)*sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.09

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
1/480*(120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) +
 (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*
(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x +
 c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))
*e - 120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (
I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d
*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c
)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*c
*f/d + (((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*cos(a)
+ (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^6 + 2*((d
*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*cos(((d*x + c)
^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*x + c)^(4/3
)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*f/d
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3727 vs. 2(357) = 714.

Time = 0.33 (sec) , antiderivative size = 3727, normalized size of antiderivative = 8.89

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

output

```
1/240*(120*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a +
((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(1
/3)*a + b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_i
ntegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*
((d*x + c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos
(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) -
 ((d*x + c)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a
+ b)/(d*x + c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3)) - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)
/(d*x + c)^(1/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*
x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c
)^(1/3)*a + b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x +
c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3)))*e/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x
+ c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1
/3)*a + b)^3/(d*x + c))*b) + (a^6*b^7*cos_integral(-a + ((d*x + c)^(1/3...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)\,\left(e+f\,x\right)\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/3))*(e + f*x),x)
```

output

```
int(sin(a + b/(c + d*x)^(1/3))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\left(\int \sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)dx\right)e+\left(\int \sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)xdx\right)f$$

input

```
int((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x)
```

output

```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3)),x)*e + int(sin(((c + d*
x)**(1/3)*a + b)/(c + d*x)**(1/3))*x,x)*f
```

3.219 $\int \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 136

$$\begin{aligned}\int \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right) \, dx = {} & \frac{b(c+dx)^{2/3} \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} \\ & + \frac{b^3 \cos(a) \operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} \\ & - \frac{b^2 \sqrt[3]{c+dx} \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} \\ & + \frac{(c+dx) \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} - \frac{b^3 \sin(a) \operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}\end{aligned}$$

output

```
1/2*b*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d+1/2*b^3*cos(a)*Ci(b/(d*x+c)^(
1/3))/d-1/2*b^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d+(d*x+c)*sin(a+b/(d*
x+c)^(1/3))/d-1/2*b^3*sin(a)*Si(b/(d*x+c)^(1/3))/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$
$$=\frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+2c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+2dx\,\text{s}}{2d}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(1/3)],x]
```

output
```
(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^3*Cos[a]*CosIntegral[b/(
c + d*x)^(1/3)] + 2*c*Sin[a + b/(c + d*x)^(1/3)] + 2*d*x*Sin[a + b/(c + d*
x)^(1/3)] - b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] - b^3*Sin[a]*Si
nIntegral[b/(c + d*x)^(1/3)])/(2*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3842, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$
$$\Big\downarrow \text{3842}$$
$$-\frac{3\int (c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{3\int (c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\left(\frac{1}{3}b\int (c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{3}(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{1}{3}b\int (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{3}(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\Bigg(\frac{1}{3}b\Bigg(\frac{1}{2}b\int -(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)-\frac{1}{3}(c+dx)\sin}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\Bigg(\frac{1}{3}b\Bigg(-\frac{1}{2}b\int (c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)-\frac{1}{3}(c+dx)\sin}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\Bigg(\frac{1}{3}b\Bigg(-\frac{1}{2}b\int (c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)-\frac{1}{3}(c+dx)\sin}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\Bigg(\frac{1}{3}b\Bigg(-\frac{1}{2}b\Bigg(b\int \sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-\frac{1}{2}(c+dx)^{2/3}\cos}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\Bigg(\frac{1}{3}b\Bigg(-\frac{1}{2}b\Bigg(b\int \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-\frac{1}{2}(c+dx)^{2/3}}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{3\Bigg(\frac{1}{3}b\Bigg(-\frac{1}{2}b\Bigg(b\Bigg(\cos(a)\int \sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int \sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}\Bigg)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+d}\right.\right.\right.\right.\right.}{d}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{3\left(\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\sqrt[3]{c+dx}\sin\left(a+\right.\right.\right.\right.}{d}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{3\left(\frac{1}{3}b\left(-\frac{1}{2}b\left(b\left(\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)-\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\frac{1}{2}\right.\right.}{d}$$

input

```
Int[Sin[a + b/(c + d*x)^(1/3)],x]
```

output

```
(-3*(-1/3*((c + d*x)*Sin[a + b/(c + d*x)^(1/3)]) + (b*(-1/2*((c + d*x)^(2/
3)*Cos[a + b/(c + d*x)^(1/3)]) - (b*(-((c + d*x)^(1/3)*Sin[a + b/(c + d*x)
^(1/3)]) + b*(Cos[a]*CosIntegral[b/(c + d*x)^(1/3)] - Sin[a]*SinIntegral[b
/(c + d*x)^(1/3)])))/2))/3))/d
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{3b^3\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{6}-\mathrm{Ci}$
default	$-\frac{3b^3\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{6}-\mathrm{Ci}$

input
```
int(sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3/d*b^3*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3
))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(
d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \frac{b^3\cos(a)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) - b^3\sin(a)\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) + (dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right) - \Big((dx+c)^{\frac{1}{3}}b^2 - 2}{2d}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
1/2*(b^3*cos(a)*cos_integral(b/(d*x + c)^(1/3)) - b^3*sin(a)*sin_integral(
b/(d*x + c)^(1/3)) + (d*x + c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)*
b)/(d*x + c)) - ((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin((a*d*x + a*c + (d*
x + c)^(2/3)*b)/(d*x + c)))/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3)),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \frac{\left(\left(\mathrm{Ei}\left(\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos\left(a\right)+\left(i\,\mathrm{Ei}\left(\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin\left(a\right)\right)b^{3}+2\,(dx+}{4\,d}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
1/4*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*Ei(
I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*x +
c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/
3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(114) = 228.

Time = 0.20 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.88

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

output

```
1/2*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a + ((d*x +
 c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(1/3)*a +
 b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_integral
(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*((d*x +
 c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos(a)*cos
_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x
+ c)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c)^(1/3)
*a + b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x + c)^(1/3
)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1/3)*a +
 b)/(d*x + c)^(1/3)))/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x + c)^(1/3
) + 3*((d*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b
)^3/(d*x + c))*b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/3)),x)
```

output

```
int(sin(a + b/(c + d*x)^(1/3)), x)
```

Reduce [F]

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx=\int \sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3)),x)
```

output
```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3)),x)
```

3.220 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 434

$$
\begin{aligned}
&\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx \\
&= -\frac{3\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} \\
&\quad + \frac{\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} \\
&\quad + \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} \\
&\quad + \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} \\
&\quad - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} - \frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f} \\
&\quad + \frac{\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f} \\
&\quad + \frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}
\end{aligned}
$$

output

```
-3*Ci(b/(d*x+c)^(1/3))*sin(a)/f+Ci(b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1
/3))*sin(a-b*f^(1/3)/(-c*f+d*e)^(1/3))/f+Ci((-1)^(1/3)*b*f^(1/3)/(-c*f+d*e
)^(1/3)-b/(d*x+c)^(1/3))*sin(a+(-1)^(1/3)*b*f^(1/3)/(-c*f+d*e)^(1/3))/f+Ci
((-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1/3))*sin(a-(-1)^(2/3)*b
*f^(1/3)/(-c*f+d*e)^(1/3))/f-3*cos(a)*Si(b/(d*x+c)^(1/3))/f-cos(a+(-1)^(1/
3)*b*f^(1/3)/(-c*f+d*e)^(1/3))*Si((-1)^(1/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)-b/
(d*x+c)^(1/3))/f+cos(a-b*f^(1/3)/(-c*f+d*e)^(1/3))*Si(b*f^(1/3)/(-c*f+d*e)
^(1/3)+b/(d*x+c)^(1/3))/f+cos(a-(-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3))*Si(
(-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1/3))/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 25.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.39

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$$

$$= \frac{i\left(\left(-3\,\text{ExpIntegralEi}\left(-\frac{ib}{\sqrt[3]{c+dx}}\right)+\text{RootSum}\left[de-cf+f\#1^3\&,e^{-\frac{ib}{\#1}}\,\text{ExpIntegralEi}\left(-ib\left(\frac{1}{\sqrt[3]{c+}}\right.\right.\right.\right.\right.}{}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```

output
```
((I/2)*((-3*ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] + RootSum[d*e - c*f +
f*#1^3 & , ExpIntegralEi[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1)
 & ])*(Cos[a] - I*Sin[a]) + (3*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - Root
Sum[d*e - c*f + f*#1^3 & , E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(-1/3
) - #1^(-1))] & ])*(Cos[a] + I*Sin[a])))/f
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \left(\frac{d\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}-\frac{d(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{f(c+dx)^{2/3}\left(f+\frac{de-cf}{c+dx}\right)}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(-\frac{d\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{f}b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f}-\frac{d\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{}{\sqrt[3]{c}}\right.}{3f}\right.}{}$$

input
```
Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```

output
```
(-3*((d*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/f - (d*CosIntegral[(b*f^(1/
3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - (b*f^(1/3))/(d*e - c*f)
^(1/3)])/(3*f) - (d*CosIntegral[((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) -
 b/(c + d*x)^(1/3)]*Sin[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/(3*
f) - (d*CosIntegral[((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)
^(1/3)]*Sin[a - ((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/(3*f) + (d*Cos[
a]*SinIntegral[b/(c + d*x)^(1/3)])/f + (d*Cos[a + ((-1)^(1/3)*b*f^(1/3))/(
d*e - c*f)^(1/3)]*SinIntegral[((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b
/(c + d*x)^(1/3)])/(3*f) - (d*Cos[a - (b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIn
tegral[(b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)])/(3*f) - (d*Cos[
a - ((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[((-1)^(2/3)*b*f^
(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)])/(3*f)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.36

method	result
derivativedivides	$-3b^3\Bigg(\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\cos(a)+\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{f\,b^3}-\frac{\sum_{\_R1=\mathrm{RootOf}\left((cf-de)\_Z^3+(-3cfa+3dea)\_Z^2+\left(3a^2cf-3a^2\right.\right.}}{}$
default	$-3b^3\Bigg(\frac{\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\cos(a)+\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)}{f\,b^3}-\frac{\sum_{\_R1=\mathrm{RootOf}\left((cf-de)\_Z^3+(-3cfa+3dea)\_Z^2+\left(3a^2cf-3a^2\right.\right.}}{}$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-3*b^3*(1/f/b^3*(Si(b/(d*x+c)^(1/3))*cos(a)+Ci(b/(d*x+c)^(1/3))*sin(a))-1/
3/f/b^3*sum(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)
*sin(_R1),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a
^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.26

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = \text{Too large to display}$$

input

```
integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

output

```
1/2*(I*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x -
 sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3
)*(I*sqrt(3) + 1) - I*a) - I*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*
e - c*f))^(1/3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*
b^3*f/(d*e - c*f))^(1/3)*(I*sqrt(3) + 1) + I*a) + I*Ei(1/2*(-2*I*(d*x + c)
^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/
(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) - I*a) - I*
Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3
)*(I*d*x + I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d*e - c*f))^(1/3)*(-I*s
qrt(3) + 1) + I*a) - I*Ei((I*(d*x + c)^(2/3)*b + (-I*b^3*f/(d*e - c*f))^(1
/3)*(d*x + c))/(d*x + c))*e^(I*a - (-I*b^3*f/(d*e - c*f))^(1/3)) + I*Ei((-
I*(d*x + c)^(2/3)*b + (I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^
(-I*a - (I*b^3*f/(d*e - c*f))^(1/3)) - 6*cos_integral(b/(d*x + c)^(1/3))*s
in(a) - 6*cos(a)*sin_integral(b/(d*x + c)^(1/3)))/f
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e),x)
```

output

```
Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))/(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx=\int \frac{\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{fx+e}dx$$

input

```
int(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x)
```

output

```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))/(e + f*x),x)
```

3.221 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 566

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

$$-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

$$+\frac{\sqrt[3]{-1}bd\cos\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

$$+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)}$$

$$-\frac{bd\sin\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

$$-\frac{(-1)^{2/3}bd\sin\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

$$-\frac{\sqrt[3]{-1}bd\sin\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}$$

output
```
-1/3*b*d*cos(a+b*f^(1/3)/(c*f-d*e)^(1/3))*Ci(b*f^(1/3)/(c*f-d*e)^(1/3)-b/(
d*x+c)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(2/3)*b*d*cos(a+(-1)^(2/3)*
b*f^(1/3)/(c*f-d*e)^(1/3))*Ci((-1)^(2/3)*b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+
c)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)+1/3*(-1)^(1/3)*b*d*cos(a-(-1)^(1/3)*b*f^
(1/3)/(c*f-d*e)^(1/3))*Ci((-1)^(1/3)*b*f^(1/3)/(c*f-d*e)^(1/3)+b/(d*x+c)^(
1/3))/f^(2/3)/(c*f-d*e)^(4/3)+(d*x+c)*sin(a+b/(d*x+c)^(1/3))/(-c*f+d*e)/(f
*x+e)-1/3*b*d*sin(a+b*f^(1/3)/(c*f-d*e)^(1/3))*Si(b*f^(1/3)/(c*f-d*e)^(1/3
)-b/(d*x+c)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(2/3)*b*d*sin(a+(-1)^(
2/3)*b*f^(1/3)/(c*f-d*e)^(1/3))*Si((-1)^(2/3)*b*f^(1/3)/(c*f-d*e)^(1/3)-b/
(d*x+c)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(1/3)*b*d*sin(a-(-1)^(1/3)
*b*f^(1/3)/(c*f-d*e)^(1/3))*Si((-1)^(1/3)*b*f^(1/3)/(c*f-d*e)^(1/3)+b/(d*x
+c)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 1.61 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.55

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$$

$$= \frac{(\cos(a)+i\sin(a))\left(bd(e+fx)\text{RootSum}\left[de-cf+f\#1^3\&,\frac{\text{ExpIntegralEi}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)-e^{\frac{ib}{\#1}}\,\text{ExpIntegralEi}\left(ib\right.}{\#1}\right.\right.}{}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

output
```
((Cos[a] + I*Sin[a])*(b*d*(e + f*x)*RootSum[d*e - c*f + f*#1^3 & , (ExpInt
egralEi[(I*b)/(c + d*x)^(1/3)] - E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)
^(-1/3) - #1^(-1))])/#1 & ] + (c + d*x)*((3*I)*f*Cos[b/(c + d*x)^(1/3)] -
3*f*Sin[b/(c + d*x)^(1/3)])) + I*(-3*c*f - 3*d*f*x + b*d*(e + f*x)*RootSum
[d*e - c*f + f*#1^3 & , (ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] - ExpInte
gralEi[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1))/#1 & ]*((-I)*Cos
[b/(c + d*x)^(1/3)] + Sin[b/(c + d*x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)]
 - I*Sin[a + b/(c + d*x)^(1/3)]))/(6*f*(-(d*e) + c*f)*(e + f*x))
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3912, 27, 3822, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \frac{d^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{2/3}\left(f+\frac{d\left(e-\frac{cf}{d}\right)}{c+dx}\right)^2}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{27}$$

$$-3d\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{2/3}\left(f+\frac{de-cf}{c+dx}\right)^2}d\frac{1}{\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3822}$$

$$-3d\left(\frac{b\int \frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{f+\frac{de-cf}{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{3(de-cf)}-\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3(de-cf)\left(\frac{de-cf}{c+dx}+f\right)}\right)$$

$$\Big\downarrow \text{3815}$$

$$-3d\left(\frac{b\int\left(\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}\left(\sqrt[3]{f}-\frac{\sqrt[3]{cf-de}}{\sqrt[3]{c+dx}}\right)}+\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}\left(\sqrt[3]{f}+\frac{\sqrt[3]{-1}\sqrt[3]{cf-de}}{\sqrt[3]{c+dx}}\right)}+\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}\left(\sqrt[3]{f}-\frac{(-1)^{2/3}\sqrt[3]{cf-de}}{\sqrt[3]{c+dx}}\right)}\right)d\frac{1}{\sqrt[3]{c+}}}{3(de-cf)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-3d\left(\frac{b\left(-\frac{\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}\sqrt[3]{cf-de}}-\frac{(-1)^{2/3}\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b}{\sqrt[3]{cf-}}\right.}{3f^{2/3}\sqrt[3]{cf-de}}\right.}{}\right.$$

input
```
Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

output
```
-3*d*(-1/3*Sin[a + b/(c + d*x)^(1/3)]/((d*e - c*f)*(f + (d*e - c*f)/(c + d
*x))) + (b*(-1/3*(Cos[a + (b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[(b
*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(f^(2/3)*(-(d*e) + c*
f)^(1/3)) - ((-1)^(2/3)*Cos[a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3
)]*CosIntegral[((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(
1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(1/3)) + ((-1)^(1/3)*Cos[a - ((-1)^(1/3)*
b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e
) + c*f)^(1/3) + b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(1/3)) - (S
in[a + (b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(-(d*e) +
 c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(1/3)) - ((-1)
^(2/3)*Sin[a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[((
-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)
*(-(d*e) + c*f)^(1/3)) - ((-1)^(1/3)*Sin[a - ((-1)^(1/3)*b*f^(1/3))/(-(d*e
) + c*f)^(1/3)]*SinIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) +
b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(1/3))))/(3*(d*e - c*f)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 3822
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
 x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (
IntegerQ[n] || GtQ[e, 0])
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.48 (sec) , antiderivative size = 1554, normalized size of antiderivative = 2.75

method	result	size
derivativedivides	Expression too large to display	1554
default	Expression too large to display	1554

input
```
int(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-3*d*b^3*(a^2*(sin(a+b/(d*x+c)^(1/3))*(1/3/f/b^3*(a+b/(d*x+c)^(1/3))-1/3*a
/f/b^3)/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a^2*d*e*(a+b/(d*x
+c)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+c)^(1/3))^2-c*f
*(a+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)-2/9/f/b^3*sum(1/(_
R1^2*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c
)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*
f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d
*e-f*b^3))-1/9/f/b^3*sum(1/(-_RR1*c*f+_RR1*d*e+a*c*f-a*d*e)*(Si(-b/(d*x+c)
^(1/3)+_RR1-a)*sin(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf
((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a
^3*d*e-f*b^3)))+sin(a+b/(d*x+c)^(1/3))*(-2/3*a/f/b^3*(a+b/(d*x+c)^(1/3))^2
+2/3*a^2/f/b^3*(a+b/(d*x+c)^(1/3)))/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c
)^(1/3))+3*a^2*d*e*(a+b/(d*x+c)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d
*e*(a+b/(d*x+c)^(1/3))^2-c*f*(a+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))
^3+f*b^3)+2/9*a/f/b^3*sum((_R1+a)/(_R1^2*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a
*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(
1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3
*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3))+2/9*a/f/b^3*sum(_RR1/(-_RR1
*c*f+_RR1*d*e+a*c*f-a*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*sin(_RR1)+Ci(b/(d*
x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.41

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/12*((I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*
e))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sq
rt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(
I*sqrt(3) + 1) - I*a) + (-I*b^3*f/(d*e - c*f))^(1/3)*(I*d*f*x + I*d*e - sq
rt(3)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e - c*f)
)^(1/3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d
*e - c*f))^(1/3)*(I*sqrt(3) + 1) + I*a) + (I*b^3*f/(d*e - c*f))^(1/3)*(-I*
d*f*x - I*d*e - sqrt(3)*(d*f*x + d*e))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I
*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c))*e^
(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) - I*a) + (-I*b^3*f/(d*e
- c*f))^(1/3)*(I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x +
 c)^(2/3)*b - (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) +
c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) + I*a)
 - 2*(-I*b^3*f/(d*e - c*f))^(1/3)*(I*d*f*x + I*d*e)*Ei((I*(d*x + c)^(2/3)*
b + (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(I*a - (-I*b^3*f/
(d*e - c*f))^(1/3)) - 2*(I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x - I*d*e)*Ei(
(-I*(d*x + c)^(2/3)*b + (I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*
e^(-I*a - (I*b^3*f/(d*e - c*f))^(1/3)) - 12*(d*f*x + c*f)*sin((a*d*x + a*c
 + (d*x + c)^(2/3)*b)/(d*x + c)))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x
)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin \left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(e+f\,x)^2}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))/(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{f^2x^2+2efx+e^2}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x)
```

output
```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))/(e**2 + 2*e*f*x + f**2*
x**2),x)
```

3.222 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 630

$$
\begin{aligned}
&\int (e+fx)^2 \sin \left(a \right.\\
&\left.+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}\\
&-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}\\
&+\frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}\\
&+\frac{2bf^2(c+dx)^{7/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{21d^3}\\
&+\frac{b^3f(de-cf)\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}\\
&-\frac{16b^{9/2}f^2\sqrt{2\pi}\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{315d^3}\\
&+\frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\cos(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^3}\\
&+\frac{2b^{3/2}(de-cf)^2\sqrt{2\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^3}\\
&+\frac{16b^{9/2}f^2\sqrt{2\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{315d^3}\\
&+\frac{16b^4f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}\\
&-\frac{b^2f(de-cf)(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}\\
&+\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}\\
&-\frac{4b^2f^2(c+dx)^{5/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{105d^3}\\
&+\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}\\
&+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{3d^3}-\frac{b^3f(de-cf)\sin(a)\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}
\end{aligned}
$$

output
```
2*b*(-c*f+d*e)^2*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d^3-8/315*b^3*f^2*(d
*x+c)*cos(a+b/(d*x+c)^(2/3))/d^3+1/2*b*f*(-c*f+d*e)*(d*x+c)^(4/3)*cos(a+b/
(d*x+c)^(2/3))/d^3+2/21*b*f^2*(d*x+c)^(7/3)*cos(a+b/(d*x+c)^(2/3))/d^3+1/2
*b^3*f*(-c*f+d*e)*cos(a)*Ci(b/(d*x+c)^(2/3))/d^3-16/315*b^(9/2)*f^2*2^(1/2
)*Pi^(1/2)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))/d^3+2*b
^(3/2)*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)/(d*x+c)^(1/3))/d^3+2*b^(3/2)*(-c*f+d*e)^2*2^(1/2)*Pi^(1/2)*FresnelC(b^
(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d^3+16/315*b^(9/2)*f^2*2^(1/2
)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d^3+16/
315*b^4*f^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/d^3-1/2*b^2*f*(-c*f+d*e)*
(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(2/3))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x
+c)^(2/3))/d^3-4/105*b^2*f^2*(d*x+c)^(5/3)*sin(a+b/(d*x+c)^(2/3))/d^3+f*(-
c*f+d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))/d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d
*x+c)^(2/3))/d^3-1/2*b^3*f*(-c*f+d*e)*sin(a)*Si(b/(d*x+c)^(2/3))/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.97

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{ie^{-ia}\left(e^{-\frac{ib}{(c+dx)^{2/3}}}\sqrt[3]{c+dx}\left(32b^4f^2+16ib^3f^2(c+dx)^{2/3}+3b^2f\sqrt[3]{c+dx}(-105de+97\right.\right.}{}$$

input
```
Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

output

```
((I/1260)*(((c + d*x)^(1/3)*(32*b^4*f^2 + (16*I)*b^3*f^2*(c + d*x)^(2/3) +
 3*b^2*f*(c + d*x)^(1/3)*(-105*d*e + 97*c*f - 8*d*f*x) - (15*I)*b*(84*d^2*
e^2 + 21*d*e*f*(-7*c + d*x) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(
c + d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x
^2))))/E^((I*b)/(c + d*x)^(2/3)) - E^(I*(2*a + b/(c + d*x)^(2/3)))*(c + d*
x)^(1/3)*(32*b^4*f^2 - (16*I)*b^3*f^2*(c + d*x)^(2/3) + 3*b^2*f*(c + d*x)^
(1/3)*(-105*d*e + 97*c*f - 8*d*f*x) + (15*I)*b*(84*d^2*e^2 + 21*d*e*f*(-7*
c + d*x) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c + d*x)^(2/3)*(c^2
*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))) + 4*(-1)^(1/4
)*b^(3/2)*E^((2*I)*a)*((315*I)*d^2*e^2 - (630*I)*c*d*e*f + (8*b^3 + (315*I
)*c^2)*f^2)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b])/(c + d*x)^(1/3)] - 4*(-1)^(
1/4)*b^(3/2)*(315*d^2*e^2 - 630*c*d*e*f + ((8*I)*b^3 + 315*c^2)*f^2)*Sqrt[
Pi]*Erfi[((-1)^(3/4)*Sqrt[b])/(c + d*x)^(1/3)] + (315*I)*b^3*f*(-(d*e) + c
*f)*ExpIntegralEi[((-I)*b)/(c + d*x)^(2/3)] + (315*I)*b^3*E^((2*I)*a)*f*(-
(d*e) + c*f)*ExpIntegralEi[(I*b)/(c + d*x)^(2/3)]))/(d^3*E^(I*a))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 599, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{3\int \left(f^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)(c+dx)^{8/3}+2f(de-cf)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)(c+dx)^{5/3}+(de-cf)^2\sin\left(a+\frac{b}{(c+dx)}\right.\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{2}{3}\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)^2\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\frac{2}{3}\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)^2\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)-\frac{16}{945}\sqrt{}\right.}{}$$

input
```
Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

output
```
(3*((2*b*(d*e - c*f)^2*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/3 - (8*
b^3*f^2*(c + d*x)*Cos[a + b/(c + d*x)^(2/3)])/945 + (b*f*(d*e - c*f)*(c +
d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/6 + (2*b*f^2*(c + d*x)^(7/3)*Cos[a
+ b/(c + d*x)^(2/3)])/63 + (b^3*f*(d*e - c*f)*Cos[a]*CosIntegral[b/(c + d*
x)^(2/3)])/6 - (16*b^(9/2)*f^2*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi])/(c + d*x)^(1/3)])/945 + (2*b^(3/2)*(d*e - c*f)^2*Sqrt[2*Pi]*Cos[a]*Fr
esnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/3 + (2*b^(3/2)*(d*e - c*f)^2
*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/3 + (16
*b^(9/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin
[a])/945 + (16*b^4*f^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)])/945 - (
b^2*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])/6 + ((d*e -
c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/3 - (4*b^2*f^2*(c + d*x)^(5/3
)*Sin[a + b/(c + d*x)^(2/3)])/315 + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(
c + d*x)^(2/3)])/3 + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^(2/3)])/9 - (b^3
*f*(d*e - c*f)*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/6))/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.67

method	result
derivativedivides	$(cf-de)^2(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2(cf-de)^2 b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{}}{\sqrt{\pi}\,(}\right.\right.\right.$
default	$(cf-de)^2(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2(cf-de)^2 b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{}}{\sqrt{\pi}\,(}\right.\right.\right.$
parts	Expression too large to display

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

output

```
3/d^3*(1/3*(c*f-d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*(c*f-d*e)^2*b*(-
(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/
2)/Pi^(1/2)/(d*x+c)^(1/3))))-1/3*f*(c*f-d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/
3))+2/3*f*(c*f-d*e)*b*(-1/4*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1
/2*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-
1/2*sin(a)*Si(b/(d*x+c)^(2/3)))))+1/9*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(2/3))
-2/9*f^2*b*(-1/7*(d*x+c)^(7/3)*cos(a+b/(d*x+c)^(2/3))-2/7*b*(-1/5*(d*x+c)^
(5/3)*sin(a+b/(d*x+c)^(2/3))+2/5*b*(-1/3*(d*x+c)*cos(a+b/(d*x+c)^(2/3))-2/
3*b*(-(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a
)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))-sin(a)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)))))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.73

$$\int (e+fx)^2 \sin\left(a + \frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{315\,(b^3def - b^3cf^2)\cos(a)\,\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) - 4\sqrt{2}(8\,\pi b^4 f^2\cos(a) - 315\,\pi(bd^2e^2 - 2\,bc}{}$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

output

```
1/630*(315*(b^3*d*e*f - b^3*c*f^2)*cos(a)*cos_integral(b/(d*x + c)^(2/3))
- 4*sqrt(2)*(8*pi*b^4*f^2*cos(a) - 315*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2
*f^2)*sin(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) +
 4*sqrt(2)*(8*pi*b^4*f^2*sin(a) + 315*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*
f^2)*cos(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) -
315*(b^3*d*e*f - b^3*c*f^2)*sin(a)*sin_integral(b/(d*x + c)^(2/3)) - (16*b
^3*d*f^2*x + 16*b^3*c*f^2 - 15*(4*b*d^2*f^2*x^2 + 84*b*d^2*e^2 - 147*b*c*d
*e*f + 67*b*c^2*f^2 + (21*b*d^2*e*f - 13*b*c*d*f^2)*x)*(d*x + c)^(1/3))*co
s((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)) + (210*d^3*f^2*x^3 + 630*d^
3*e*f*x^2 + 32*(d*x + c)^(1/3)*b^4*f^2 + 630*d^3*e^2*x + 630*c*d^2*e^2 - 6
30*c^2*d*e*f + 210*c^3*f^2 - 3*(8*b^2*d*f^2*x + 105*b^2*d*e*f - 97*b^2*c*f
^2)*(d*x + c)^(2/3))*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^3
```

Sympy [F]

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) \, dx = \int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

input
```
integrate((f*x+e)**2*sin(a+b/(d*x+c)**(2/3)),x)
```

output
```
Integral((e + f*x)**2*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 1260, normalized size of antiderivative = 2.00

$$\int (e+fx)^2 \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

output

```
1/1260*(630*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*
cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqr
t((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I
+ 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf
(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I
*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3
))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*e
^2/((d*x + c)^(1/3)*b) - 1260*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x
 + c)^(-4/3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(
d*x + c)^(4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x +
 c)^(2/3)) + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I
- 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sq
rt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-
I*b/(d*x + c)^(2/3))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt(
(d*x + c)^(4/3))*c*e*f/((d*x + c)^(1/3)*b*d) + 630*sqrt(2)*(2*sqrt(2)*(d*x
 + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x +
c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c
)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x +
c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*
cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I +...
```

Giac [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

input

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

output

```
integrate((f*x + e)^2*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)\,{\left(e+f\,x\right)}^2\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))*(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\left(\int \sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)dx\right)e^2+\left(\int \sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)x^2dx\right)f^2+2\left(\int \sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)xdx\right)ef$$

input
```
int((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3)),x)*e**2 + int(sin(((c +
 d*x)**(2/3)*a + b)/(c + d*x)**(2/3))*x**2,x)*f**2 + 2*int(sin(((c + d*x)*
*(2/3)*a + b)/(c + d*x)**(2/3))*x,x)*e*f
```

3.223 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 318

$$\begin{aligned}\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx &= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} \\ &+ \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{b^3f\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} \\ &+ \frac{2b^{3/2}(de-cf)\sqrt{2\pi}\cos(a)\,\mathrm{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2} \\ &+ \frac{2b^{3/2}(de-cf)\sqrt{2\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d^2} \\ &- \frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} \\ &+ \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} - \frac{b^3f\sin(a)\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}\end{aligned}$$

output
```
2*b*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d^2+1/4*b*f*(d*x+c)^(4
/3)*cos(a+b/(d*x+c)^(2/3))/d^2+1/4*b^3*f*cos(a)*Ci(b/(d*x+c)^(2/3))/d^2+2*
b^(3/2)*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/
2)/(d*x+c)^(1/3))/d^2+2*b^(3/2)*(-c*f+d*e)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/
2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d^2-1/4*b^2*f*(d*x+c)^(2/3)*sin(
a+b/(d*x+c)^(2/3))/d^2+(-c*f+d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d^2+1/2*f
*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))/d^2-1/4*b^3*f*sin(a)*Si(b/(d*x+c)^(2/3))
/d^2
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.19

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{8bde\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)-7bcf\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+bdfx\sqrt[3]{c+dx}\,\mathrm{c}}{}$$

input
```
Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output
```
(8*b*d*e*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] - 7*b*c*f*(c + d*x)^(1
/3)*Cos[a + b/(c + d*x)^(2/3)] + b*d*f*x*(c + d*x)^(1/3)*Cos[a + b/(c + d*
x)^(2/3)] + b^3*f*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)] + 8*b^(3/2)*(d*e -
 c*f)*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 8
*b^(3/2)*d*e*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin
[a] - 8*b^(3/2)*c*f*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/
3)]*Sin[a] + 4*c*d*e*Sin[a + b/(c + d*x)^(2/3)] - 2*c^2*f*Sin[a + b/(c + d
*x)^(2/3)] + 4*d^2*e*x*Sin[a + b/(c + d*x)^(2/3)] + 2*d^2*f*x^2*Sin[a + b/
(c + d*x)^(2/3)] - b^2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] - b^3*
f*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$$

$$\Big\downarrow \ 3914$$

$$\frac{3\int \left(f\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)(c+dx)^{5/3}+(de-cf)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{d^2}$$

$$\Big\downarrow \ 2009$$

$$3\left(\frac{2}{3}\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\frac{2}{3}\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\frac{1}{12}b^3f$$

input

```
Int[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output

```
(3*((2*b*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/3 + (b*f*
(c + d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/12 + (b^3*f*Cos[a]*CosIntegral
[b/(c + d*x)^(2/3)])/12 + (2*b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fresnel
S[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/3 + (2*b^(3/2)*(d*e - c*f)*Sqrt[2
*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/3 - (b^2*f*(c
 + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])/12 + ((d*e - c*f)*(c + d*x)*Sin[a
 + b/(c + d*x)^(2/3)])/3 + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^(2/3)])/6 -
(b^3*f*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/12))/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2(cf-de)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\operatorname{FresnelS}\left(\right.\right.\right.$
default	$-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2(cf-de)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\operatorname{FresnelS}\left(\right.\right.\right.$
parts	Expression too large to display

input
```
int((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d^2*(-1/3*(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))+2/3*(c*f-d*e)*b*(-(d*
x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*Fresnel
S(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/
Pi^(1/2)/(d*x+c)^(1/3))))+1/6*f*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))-1/3*f*b*(
-1/4*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d*x+c)^(2/3)*sin(a+
b/(d*x+c)^(2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*sin(a)*Si(b/(d*x+c)
^(2/3))))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.78

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{b^3 f\cos(a)\,\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) - b^3 f\sin(a)\,\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) + 8\sqrt{2}\pi(bde-bcf)\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\big(}{}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
1/4*(b^3*f*cos(a)*cos_integral(b/(d*x + c)^(2/3)) - b^3*f*sin(a)*sin_integ
ral(b/(d*x + c)^(2/3)) + 8*sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*cos(a)*fr
esnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) + 8*sqrt(2)*pi*(b*d*e - b*c*
f)*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3))*sin(a) + (b*
d*f*x + 8*b*d*e - 7*b*c*f)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(1
/3)*b)/(d*x + c)) + (2*d^2*f*x^2 + 4*d^2*e*x - (d*x + c)^(2/3)*b^2*f + 4*c
*d*e - 2*c^2*f)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^2
```

Sympy [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)**(2/3)),x)
```

output
```
Integral((e + f*x)*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.84

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
1/8*(4*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos((
(d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*
x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*
sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt
(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d
*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) -
 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*e/((d*
x + c)^(1/3)*b) - 4*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/
3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(
4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3))
 + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(
pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(er
f(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x +
 c)^(2/3))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^
(4/3))*c*f/((d*x + c)^(1/3)*b*d) + (((Ei(I*b/(d*x + c)^(2/3)) + Ei(-I*b/(d
*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*Ei(-I*b/(d*x + c)^
(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^(2/3)*a + b)/(d*x
 + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin(((d*x + c)^(2/3
)*a + b)/(d*x + c)^(2/3)))*f/d)/d
```

Giac [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

input
```
integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(e+f\,x)\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))*(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\left(\int \sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)dx\right)e\\+\left(\int \sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)xdx\right)f$$

input
```
int((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3)),x)*e + int(sin(((c + d*
x)**(2/3)*a + b)/(c + d*x)**(2/3))*x,x)*f
```

3.224 $\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\begin{aligned}\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx &= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} \\ &+ \frac{2b^{3/2}\sqrt{2\pi}\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d} \\ &+ \frac{2b^{3/2}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}\end{aligned}$$

output
```
2*b*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+2*b^(3/2)*2^(1/2)*Pi^(1/2)*cos(
a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))/d+2*b^(3/2)*2^(1/2)*Pi
^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d+(d*x+c)*s
in(a+b/(d*x+c)^(2/3))/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx = \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right) + 2b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right) + 2b^{3/2}\sqrt{2\pi}\,\mathrm{F}}{d}$$

input

```
Integrate[Sin[a + b/(c + d*x)^(2/3)],x]
```

output

```
(2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Cos
[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 2*b^(3/2)*Sqrt[2*Pi]*
FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d
*x)^(2/3)] + d*x*Sin[a + b/(c + d*x)^(2/3)])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3844, 3890, 3868, 3869, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a + \frac{b}{(c+dx)^{2/3}}\right) dx$$

$$\Big\downarrow \text{ 3844}$$

$$\frac{3\int (c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{ 3890}$$

$$-\frac{3\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{4/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{3\left(\frac{2}{3}b\int \frac{\cos\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{3(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{3\left(\frac{2}{3}b\left(-2b\int \sin\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{3(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3834}$$

$$-\frac{3\left(\frac{2}{3}b\left(-2b\left(\sin(a)\int \cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int \sin\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}\right)-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right.\right.}{d}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{3\left(\frac{2}{3}b\left(-2b\left(\sin(a)\int \cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)-\sin\right.}{d}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{3\left(\frac{2}{3}b\left(-2b\left(\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b(c+dx\right.}{3(c+dx)}\right.}{d}$$

input

```
Int[Sin[a + b/(c + d*x)^(2/3)],x]
```

output

```
(-3*((2*b*(-(Cos[a + b*(c + d*x)^(2/3)]/(c + d*x)^(1/3)) - 2*b*((Sqrt[Pi/2
]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/Sqrt[b] + (Sqrt[P
i/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/Sqrt[b])))/3 -
 Sin[a + b*(c + d*x)^(2/3)]/(3*(c + d*x))))/d
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3844
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Simp[k/f   Subst[Int[x^(k - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3890
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2b\Big(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\Big(\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)^{\frac{1}{3}}}\right)+\sin(a)\,\mathrm{F}}{d}$
default	$\frac{(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2b\Big(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\Big(\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)^{\frac{1}{3}}}\right)+\sin(a)\,\mathrm{F}}{d}$

input
```
int(sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d*(1/3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*x+c)^(1/3)*cos(a+b/(d*x
+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))
)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \sin\Big(a+\frac{b}{(c+dx)^{2/3}}\Big)\,dx=\frac{2\sqrt{2}\pi b\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)+2\sqrt{2}\pi b\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)+2(dx+c)^{\frac{1}{3}}b\cos\Big(}{d}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
(2*sqrt(2)*pi*b*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)
^(1/3)) + 2*sqrt(2)*pi*b*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x +
c)^(1/3))*sin(a) + 2*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/3)*
b)/(d*x + c)) + (d*x + c)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))
)/d
```

Sympy [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)dx$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3)),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

$$\int \sin\Bigg(a+\frac{b}{(c+dx)^{2/3}}\Bigg)\,dx=\frac{\sqrt{2}\Bigg(2\sqrt{2}(dx+c)^{\frac{2}{3}}\sqrt{\frac{1}{(dx+c)^{\frac{4}{3}}}}b^2\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)+\sqrt{2}(dx+c)^{\frac{4}{3}}\sqrt{\frac{1}{(dx+c)^{\frac{4}{3}}}}b\sin\Bigg(\frac{(dx+c}{(dx+}}{}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos(((d*
x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x +
 c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sqr
t(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I
*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x
+ c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1)
)*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))/((d*x + c
)^(1/3)*b*d)
```

Giac [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3)),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3)), x)
```

Reduce [F]

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3)),x)
```

3.225 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 63.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}dx$$

input
```
Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{e+fx}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(2/3))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 42.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{{(c+d\,x)}^{2/3}}\right)}{e+f\,x}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))/(e + f*x),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

output
```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/(e + f*x),x)
```

3.226 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2},x\right)$$

output
```
Defer(Int)(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 58.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

output
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}dx$$

$$\Big\downarrow 3918$$

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}dx$$

input
```
Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f^2*x^2 + 2*e*f
*x + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 21.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e+fx)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e)**2,x)
```

output
```
Integral(sin(a + b/(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 43.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(e+f\,x)^2}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))/(e + f*x)^2,x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{f^2x^2+2efx+e^2}dx$$

input

```
int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

output

```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/(e**2 + 2*e*f*x + f**2*
x**2),x)
```

3.227 $\int (ce+dex)^{4/3} \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 289

$$\begin{aligned}\int (ce&+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}}\\&-\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d}\\&+\frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}\\&-\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\&+\frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d}\\&-\frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d}\\&+\frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\end{aligned}$$

output
```
2160*e*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^7/d/(d*x+c)^(1/3)-1080*e
*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d+90*e*(d*x+c)
*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*e*(d*x+c)^(5/3)*(e*(d*x+
c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b/d+2160*e*(e*(d*x+c))^(1/3)*sin(a+b*(d*x
+c)^(1/3))/b^6/d-360*e*(d*x+c)^(2/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1/
3))/b^4/d+18*e*(d*x+c)^(4/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/
d
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.78

$$\int (ce+dex)^{4/3}\sin\Bigl(a+b\sqrt[3]{c+dx}\Bigr)\,dx = \frac{3(e(c+dx))^{4/3}\Bigl(-\cos\bigl(b\sqrt[3]{c+dx}\bigr)\bigl(\bigl(-720+360b^2(c+dx)^{2/3}-30b^4(c+dx)^{4/3}+b^6(}{}$$

input
```
Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(3*(e*(c + d*x))^(4/3)*(-(Cos[b*(c + d*x)^(1/3)]*((-720 + 360*b^2*(c + d*x
)^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Cos[a] - 6*b*(120*(c +
 d*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Sin[a])) + (6*b*(120
*(c + d*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Cos[a] + (-720
+ 360*b^2*(c + d*x)^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Sin[
a])*Sin[b*(c + d*x)^(1/3)]))/(b^7*d*(c + d*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.81, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {3912, 30, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

$$\Big\downarrow \text{ 3912}$$

$$\frac{3\int (c+dx)^{2/3}(e(c+dx))^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{ 30}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\int (c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\int (c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\int (c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\int (c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{5\int -(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}+\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\int (c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\int (c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\int (c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right)}{b}-\right.}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\int (c+dx)\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right)}{b}\right.}{d\sqrt[3]{c+dx}}$$

↓ 3777

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\left(\frac{3\int -(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}+\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\right.}{b}\right.}{b}\right.}{d\sqrt[3]{c+dx}}$$

↓ 25

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{\left(6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\int(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-(c\right.}{b}\right.\right.}{b}\right.}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\int(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-(c}{b}}{b}}{d\sqrt[3]{c+dx}}$$

$\downarrow$ 3777

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\int\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right.}{b}\right.}{b}\right.}{b}\right.}{b}\right.}{d\sqrt[3]{c+dx}}$$

↓ 3042

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\int\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+}}{b}\right.}{b}\right.}{b}\right.}{b}\right.}{b}\right.}{d\sqrt[3]{c+dx}}$$

↓ 3777

$$3e\sqrt[3]{e(c+dx)} \left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b} - \frac{5\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b} - \frac{3\left(\frac{2\left(\frac{\int -\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b} + \frac{\sqrt[3]{c+}}{}\right.}{b}\right.}{}\right.}{b}\right.}{}\right.}{b}\right.$$

$$\Big\downarrow \text{ 25}$$

$$3\left(\frac{2\left(\frac{\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\int\sin\left(a+b\sqrt[3]{\right.}{}\right.}{b}\right.$$

$$4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\right.$$

$$\frac{5\left(\right.}{b}$$

$$6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\right.$$

$$3e\sqrt[3]{e(c+dx)}\left(\frac{\left(\right.}{b}\right.$$

↓ 3042

$$2\left(\frac{\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\int\sin\left(a+b\sqrt[3]{\right.}{}$$

$$3\,\frac{\quad}{b}$$

$$4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\quad}{}\right.$$

$$5\,\frac{\quad}{b}$$

$$6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\quad}{}\right.$$

$$3e\sqrt[3]{e(c+dx)}\left(\frac{\quad}{b}\right.$$

↓ 3118

$$3e\sqrt[3]{e(c+dx)}\left(\frac{6\left(\frac{(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{5\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\left(\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2}+\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{}\right.}{b}\right.}{b}\right.}{b}\right.}{b}\right.}{b}\right.}{b}\right.$$

input

```
Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output

```
(3*e*(e*(c + d*x))^(1/3)*(-(((c + d*x)^2*Cos[a + b*(c + d*x)^(1/3)])/b) +
(6*(((c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/b - (5*(-(((c + d*x)^(4/3
)*Cos[a + b*(c + d*x)^(1/3)])/b) + (4*(((c + d*x)*Sin[a + b*(c + d*x)^(1/3
)])/b - (3*(-(((c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/b) + (2*(Cos[a
+ b*(c + d*x)^(1/3)]/b^2 + ((c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/b)
)/b))/b))/b))/b))/b))/(d*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30

```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+b(dx+c)^{\frac{1}{3}}\right) dx$$

input

```
int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

output

```
int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.81

$$\int (ce+dex)^{4/3} \sin\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{3\left(\left(30\,b^4d^2ex^2+60\,b^4cdex+30\,b^4c^2e-(b^6d^2ex^2+2\,b^6cdex+(b^6c^2-720)e)(dx+c)^{\frac{2}{3}}\right.\right.}{}$$

input

```
integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output

```
3*((30*b^4*d^2*e*x^2 + 60*b^4*c*d*e*x + 30*b^4*c^2*e - (b^6*d^2*e*x^2 + 2*
b^6*c*d*e*x + (b^6*c^2 - 720)*e)*(d*x + c)^(2/3) - 360*(b^2*d*e*x + b^2*c*
e)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*cos((d*x + c)^(1/3)*b + a) + 6*(12
0*b*d*e*x + 120*b*c*e - 20*(b^3*d*e*x + b^3*c*e)*(d*x + c)^(2/3) + (b^5*d^
2*e*x^2 + 2*b^5*c*d*e*x + b^5*c^2*e)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*
sin((d*x + c)^(1/3)*b + a))/(b^7*d^2*x + b^7*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.61

$$\int (ce+dex)^{4/3}\sin\Big(a+b\sqrt[3]{c+dx}\Big)\,dx=\frac{3\left(3\left(\left(\Gamma\left(6,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(6,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(6,i\,(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(6,-i\,(dx\right.\right.\right.\right.}{}$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
3/2*(3*((gamma(6, I*b*conjugate((d*x + c)^(1/3))) + gamma(6, -I*b*conjugat
e((d*x + c)^(1/3))) + gamma(6, I*(d*x + c)^(1/3)*b) + gamma(6, -I*(d*x + c
)^(1/3)*b))*cos(a) + (-I*gamma(6, I*b*conjugate((d*x + c)^(1/3))) + I*gamm
a(6, -I*b*conjugate((d*x + c)^(1/3))) - I*gamma(6, I*(d*x + c)^(1/3)*b) +
I*gamma(6, -I*(d*x + c)^(1/3)*b))*sin(a))*e - 2*(b^6*d^2*e*x^2 + 2*b^6*c*d
*e*x + b^6*c^2*e)*cos((d*x + c)^(1/3)*b + a))*e^(1/3)/(b^7*d)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.66

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(\frac{c^2e^2\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-2\,c\left(\frac{\left(b^3ce^4-(dex+ce)b^3e^3+6\,(dex+ce)^{\frac{1}{3}}be^3|e|^{\frac{2}{3}}\right)\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^4e^2|e|^{\frac{2}{3}}}+\frac{3\left((dex+ce)^{\frac{2}{3}}b^2e^2\right.}{}\right.\right.}{}$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
-3*(c^2*e^2*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b*abs(e)^(2
/3)) - 2*c*((b^3*c*e^4 - (d*e*x + c*e)*b^3*e^3 + 6*(d*e*x + c*e)^(1/3)*b*e
^3*abs(e)^(2/3))*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^
2*abs(e)^(2/3)) + 3*((d*e*x + c*e)^(2/3)*b^2*e^2*abs(e)^(4/3) - 2*e^4)*sin
((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^2*abs(e)^(2/3))) + (
(b^6*c^2*e^7 - 2*(d*e*x + c*e)*b^6*c*e^6 + (d*e*x + c*e)^2*b^6*e^5 + 12*(d
*e*x + c*e)^(1/3)*b^4*c*e^6*abs(e)^(2/3) - 30*(d*e*x + c*e)^(4/3)*b^4*e^5*
abs(e)^(2/3) + 360*(d*e*x + c*e)^(2/3)*b^2*e^5*abs(e)^(4/3) - 720*e^7)*cos
((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^7*e^4*abs(e)^(2/3)) + 6*
((d*e*x + c*e)^(2/3)*b^5*c*e^5*abs(e)^(4/3) - (d*e*x + c*e)^(5/3)*b^5*e^4*
abs(e)^(4/3) - 2*b^3*c*e^7 + 20*(d*e*x + c*e)*b^3*e^6 - 120*(d*e*x + c*e)^
(1/3)*b*e^6*abs(e)^(2/3))*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e
)/(b^7*e^4*abs(e)^(2/3)))/e)/d
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{4/3}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.95

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3e^{\frac{4}{3}}\left(-360(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^2+30(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^4c+\right.}{}$$

input
```
int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

output
```
(3*e**(1/3)*e*( - 360*(c + d*x)**(2/3)*cos((c + d*x)**(1/3)*b + a)*b**2 +
30*(c + d*x)**(1/3)*cos((c + d*x)**(1/3)*b + a)*b**4*c + 30*(c + d*x)**(1/
3)*cos((c + d*x)**(1/3)*b + a)*b**4*d*x - cos((c + d*x)**(1/3)*b + a)*b**6
*c**2 - 2*cos((c + d*x)**(1/3)*b + a)*b**6*c*d*x - cos((c + d*x)**(1/3)*b
+ a)*b**6*d**2*x**2 + 720*cos((c + d*x)**(1/3)*b + a) + 6*(c + d*x)**(2/3)
*sin((c + d*x)**(1/3)*b + a)*b**5*c + 6*(c + d*x)**(2/3)*sin((c + d*x)**(1
/3)*b + a)*b**5*d*x + 720*(c + d*x)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b -
 120*sin((c + d*x)**(1/3)*b + a)*b**3*c - 120*sin((c + d*x)**(1/3)*b + a)*
b**3*d*x))/(b**7*d)
```

3.228 $\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 202

$$\begin{aligned}\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}\\ &- \frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}\\ &- \frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\ &- \frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}\\ &+ \frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\end{aligned}$$

output
```
36*(e*(d*x+c))^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-72*(e*(d*x+c))^(2/3)*cos
(a+b*(d*x+c)^(1/3))/b^5/d/(d*x+c)^(2/3)-3*(d*x+c)^(2/3)*(e*(d*x+c))^(2/3)*
cos(a+b*(d*x+c)^(1/3))/b/d-72*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^4
/d/(d*x+c)^(1/3)+12*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(1/3))
/b^2/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.55

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$
$$-\frac{3(e(c+dx))^{2/3}\Big(\left(24-12b^2(c+dx)^{2/3}+b^4(c+dx)^{4/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-4b\Big(-6\sqrt[3]{c+dx}+b^2(c+}{b^5d(c+dx)^{2/3}}$$

input
```
Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-3*(e*(c + d*x))^(2/3)*((24 - 12*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(4/3
))*Cos[a + b*(c + d*x)^(1/3)] - 4*b*(-6*(c + d*x)^(1/3) + b^2*(c + d*x))*S
in[a + b*(c + d*x)^(1/3)]))/(b^5*d*(c + d*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {3912, 30, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3\int (c+dx)^{2/3}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{30}$$

$$\frac{3(e(c+dx))^{2/3}\int (c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(e(c+dx))^{2/3}\int (c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \ 3777$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\int (c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\int (c+dx)\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \ 3777$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{3\int -(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}+\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \ 25$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\int (c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\int (c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\int \sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right)}{b}\right.}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\int \sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right.}{b}\right.}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3(e(c+dx))^{2/3}\Bigg(\frac{4\Bigg(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\Bigg(\frac{2\left(\frac{\int -\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}+\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)^{2}}{}}{b}}{b}}{d(c+dx)^{2/3}}$$

$\downarrow$ 25

$$\frac{3(e(c+dx))^{2/3}\Bigg(\frac{4\Bigg(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\Bigg(\frac{2\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\int\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^{2/3}}{}}{b}}{b}}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{\int \sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)^{2/3}}{}\right.}{b}\right.}{b}\right.}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{4\left(\frac{(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{3\left(\frac{2\left(\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2}+\frac{\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c}\right.}{b}\right.}{b}\right.}{b}\right.}{d(c+dx)^{2/3}}$$

input
```
Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(3*(e*(c + d*x))^(2/3)*(-(((c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/b)
+ (4*(((c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/b - (3*(-(((c + d*x)^(2/3)*Co
s[a + b*(c + d*x)^(1/3)])/b) + (2*(Cos[a + b*(c + d*x)^(1/3)]/b^2 + ((c +
d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/b))/b))/b))/b))/(d*(c + d*x)^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int (dex + ce)^{\frac{2}{3}} \sin\left(a + b(dx + c)^{\frac{1}{3}}\right) dx$$

input
```
int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

output
```
int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

$$\int (ce+dex)^{2/3}\sin (a +b\sqrt[3]{c+dx})\, dx = \frac{3((12\,b^2dx+12\,b^2c-(b^4dx+b^4c)(dx+c)^{\frac{2}{3}}-24\,(dx+c)^{\frac{1}{3}})(dex+ce)^{\frac{2}{3}}\cos((dx+c)}{b^5d^2x}$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
3*((12*b^2*d*x + 12*b^2*c - (b^4*d*x + b^4*c)*(d*x + c)^(2/3) - 24*(d*x +
c)^(1/3))*(d*e*x + c*e)^(2/3)*cos((d*x + c)^(1/3)*b + a) - 4*(d*e*x + c*e)
^(2/3)*(6*(d*x + c)^(2/3)*b - (b^3*d*x + b^3*c)*(d*x + c)^(1/3))*sin((d*x
+ c)^(1/3)*b + a))/(b^5*d^2*x + b^5*c*d)
```

Sympy [F]

$$\int (ce+dex)^{2/3}\sin \left(a+b\sqrt[3]{c+dx}\right)\, dx = \int (e(c+dx))^{\frac{2}{3}}\sin \left(a+b\sqrt[3]{c+dx}\right) dx$$

input
```
integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int (ce+dex)^{2/3}\sin \left(a+b\sqrt[3]{c+dx}\right)\, dx = -\frac{3((b^4dx+b^4c)(dx+c)^{\frac{1}{3}}e^{\frac{2}{3}}\cos ((dx+c)^{\frac{1}{3}}b+a)+(3(\Gamma(3, i\,b\overline{(dx+c)^{\frac{1}{3}}})+\Gamma(3, -i\,b\overline{(dx+c)^{\frac{1}{3}}})+}{}$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-3*((b^4*d*x + b^4*c)*(d*x + c)^(1/3)*e^(2/3)*cos((d*x + c)^(1/3)*b + a) +
 (3*(gamma(3, I*b*conjugate((d*x + c)^(1/3))) + gamma(3, -I*b*conjugate((d
*x + c)^(1/3))) + gamma(3, I*(d*x + c)^(1/3)*b) + gamma(3, -I*(d*x + c)^(1
/3)*b))*cos(a) - 4*(b^3*d*x + b^3*c)*sin((d*x + c)^(1/3)*b + a) - 3*(I*gam
ma(3, I*b*conjugate((d*x + c)^(1/3))) - I*gamma(3, -I*b*conjugate((d*x + c
)^(1/3))) + I*gamma(3, I*(d*x + c)^(1/3)*b) - I*gamma(3, -I*(d*x + c)^(1/3
)*b))*sin(a))*e^(2/3))/(b^5*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.42

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(c\left(\frac{(dex+ce)^{\frac{1}{3}}e\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-\frac{e^2\sin\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^2|e|^{\frac{4}{3}}}\right)-\frac{\left((dex+ce)^{\frac{1}{3}}b^4ce^4|e|^{\frac{2}{3}}-(dex+ce)^{\frac{4}{3}}b^4e^3|e|^{\frac{2}{3}}+12\,(dex+ce)\right.}{b^5e^4}\right.}{d}$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
-3*(c*((d*e*x + c*e)^(1/3)*e*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3)
)/e)/(b*abs(e)^(2/3)) - e^2*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))
/e)/(b^2*abs(e)^(4/3))) - (((d*e*x + c*e)^(1/3)*b^4*c*e^4*abs(e)^(2/3) - (
d*e*x + c*e)^(4/3)*b^4*e^3*abs(e)^(2/3) + 12*(d*e*x + c*e)^(2/3)*b^2*e^3*a
bs(e)^(4/3) - 24*e^5)*abs(e)^(2/3)*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)
^(2/3))/e)/(b^5*e^4) - (b^3*c*e^5 - 4*(d*e*x + c*e)*b^3*e^4 + 24*(d*e*x +
c*e)^(1/3)*b*e^4*abs(e)^(2/3))*abs(e)^(2/3)*sin((a*e + (d*e*x + c*e)^(1/3)
*b*abs(e)^(2/3))/e)/(b^5*e^4))/e)/d
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(c\,e+d\,e\,x)^{2/3}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(2/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{3e^{\frac{2}{3}}\left(12(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^{2}-(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^{4}c-(dx+$$

input
```
int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

output
```
(3*e**(2/3)*(12*(c + d*x)**(2/3)*cos((c + d*x)**(1/3)*b + a)*b**2 - (c + d
*x)**(1/3)*cos((c + d*x)**(1/3)*b + a)*b**4*c - (c + d*x)**(1/3)*cos((c +
d*x)**(1/3)*b + a)*b**4*d*x - 24*cos((c + d*x)**(1/3)*b + a) - 24*(c + d*x
)**(1/3)*sin((c + d*x)**(1/3)*b + a)*b + 4*sin((c + d*x)**(1/3)*b + a)*b**
3*c + 4*sin((c + d*x)**(1/3)*b + a)*b**3*d*x))/(b**5*d)
```

3.229 $\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

output

```
18*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*(d*x+c)^(2/3)*(e*(d*x+
c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b/d-18*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^
(1/3))/b^4/d/(d*x+c)^(1/3)+9*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+
c)^(1/3))/b^2/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=$$
$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\left(-6b\sqrt[3]{c+dx}+b^3(c+dx)\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-3\left(-2+b^2(c+dx)^{2/3}\right)\sin\Big(a+b\sqrt[3]{}}{b^4d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-3*(e*(c + d*x))^(1/3)*((-6*b*(c + d*x)^(1/3) + b^3*(c + d*x))*Cos[a + b*
(c + d*x)^(1/3)] - 3*(-2 + b^2*(c + d*x)^(2/3))*Sin[a + b*(c + d*x)^(1/3)]
))/(b^4*d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.407$, Rules used = {3912, 30, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$\Big\downarrow \text{3912}$$
$$\frac{3\int (c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d}$$
$$\Big\downarrow \text{30}$$
$$\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{c+dx}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\int (c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\int (c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\left(\frac{2\int -\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}+\frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}-\frac{(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\left(\frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\int \sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\left(\frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\int \sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}-\frac{(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\left(\frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\left(\frac{\int\cos\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)}{}\right.}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\left(\frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\left(\frac{\int\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right)}{b}-\frac{(c+}{}\right.}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{3\left(\frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\left(\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2}-\frac{\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)\cos\left(a+b\sqrt[3]{c\right.}{b}\right.}{d\sqrt[3]{c+dx}}$$

input
```
Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

output
```
(3*(e*(c + d*x))^(1/3)*(-(((c + d*x)*Cos[a + b*(c + d*x)^(1/3)])/b) + (3*(
((c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/b - (2*(-(((c + d*x)^(1/3)*Co
s[a + b*(c + d*x)^(1/3)])/b) + Sin[a + b*(c + d*x)^(1/3)]/b^2))/b))/b))/(d
*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)dx$$

input
```
int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

output
```
int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(\left(6\,bdx+6\,bc-(b^3dx+b^3c)(dx+c)^{\frac{2}{3}}\right)(dex+ce)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+3\,(dex+ce)^{\frac{1}{3}}\left((b^2dx+b^2\right.\right.}{b^4d^2x+b^4cd}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
3*((6*b*d*x + 6*b*c - (b^3*d*x + b^3*c)*(d*x + c)^(2/3))*(d*e*x + c*e)^(1/
3)*cos((d*x + c)^(1/3)*b + a) + 3*(d*e*x + c*e)^(1/3)*((b^2*d*x + b^2*c)*(
d*x + c)^(1/3) - 2*(d*x + c)^(2/3))*sin((d*x + c)^(1/3)*b + a))/(b^4*d^2*x
 + b^4*c*d)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int \sqrt[3]{e\,(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

input
```
integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(4\left(b^3dx+b^3c\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-3\left(-i\,\Gamma\left(3,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(3,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(3,\right.\right.\right.}{}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-3/4*(4*(b^3*d*x + b^3*c)*cos((d*x + c)^(1/3)*b + a) - 3*(-I*gamma(3, I*b*
conjugate((d*x + c)^(1/3))) + I*gamma(3, -I*b*conjugate((d*x + c)^(1/3)))
- I*gamma(3, I*(d*x + c)^(1/3)*b) + I*gamma(3, -I*(d*x + c)^(1/3)*b))*cos(
a) + 3*(gamma(3, I*b*conjugate((d*x + c)^(1/3))) + gamma(3, -I*b*conjugate
((d*x + c)^(1/3))) + gamma(3, I*(d*x + c)^(1/3)*b) + gamma(3, -I*(d*x + c)
^(1/3)*b))*sin(a))*e^(1/3)/(b^4*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.20

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(\frac{ce\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-\frac{\frac{\left(b^3ce^4-(dex+ce)b^3e^3+6\,(dex+ce)^{\frac{1}{3}}be^3|e|^{\frac{2}{3}}\right)\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^4e^2|e|^{\frac{2}{3}}}+\frac{3\left((dex+ce)^{\frac{2}{3}}b^2e^2|e|^{\frac{4}{3}}-2\,e^4\right)\mathrm{si}}{b^4e^2|e|}}{e}\right.}{d}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output

```
-3*(c*e*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b*abs(e)^(2/3))
 - ((b^3*c*e^4 - (d*e*x + c*e)*b^3*e^3 + 6*(d*e*x + c*e)^(1/3)*b*e^3*abs(e
)^(2/3))*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^2*abs(e)
^(2/3)) + 3*((d*e*x + c*e)^(2/3)*b^2*e^2*abs(e)^(4/3) - 2*e^4)*sin((a*e +
(d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b^4*e^2*abs(e)^(2/3)))/e)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(c\,e+d\,e\,x)^{1/3}\,dx$$

input

```
int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3),x)
```

output

```
int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3e^{\frac{1}{3}}\left(6(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b-\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^{3}c-\cos\left((dx+c)^{\frac{1}{3}}b+a\right)b^{3}dx+3(d\right.}{b^{4}d}$$

input

```
int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

output

```
(3*e**(1/3)*(6*(c + d*x)**(1/3)*cos((c + d*x)**(1/3)*b + a)*b - cos((c + d
*x)**(1/3)*b + a)*b**3*c - cos((c + d*x)**(1/3)*b + a)*b**3*d*x + 3*(c + d
*x)**(2/3)*sin((c + d*x)**(1/3)*b + a)*b**2 - 6*sin((c + d*x)**(1/3)*b + a
)))/(b**4*d)
```

3.230 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}+\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

output

```
-3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d/(e*(d*x+c))^(1/3)+3*(d*x+c)^(1
/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx=\frac{-3b(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)+3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

output
```
(-3*b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(c + d*x)^(1/3)*Sin[a
 + b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3912, 30, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3\int \frac{(c+dx)^{2/3}\sin \left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{e(c+dx)}}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{30}$$

$$\frac{3\sqrt[3]{c+dx}\int \sqrt[3]{c+dx}\sin \left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{c+dx}\int \sqrt[3]{c+dx}\sin \left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{\int \cos \left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}-\frac{\sqrt[3]{c+dx}\cos \left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{\int \sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}-\frac{\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2}-\frac{\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b}\right)}{d\sqrt[3]{e(c+dx)}}$$

input
```
Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(-(((c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/b) + Si
n[a + b*(c + d*x)^(1/3)]/b^2))/(d*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\left((dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}b\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^2d^2ex+b^2cde}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

output
```
-3*((d*e*x + c*e)^(2/3)*(d*x + c)^(2/3)*b*cos((d*x + c)^(1/3)*b + a) - (d*
e*x + c*e)^(2/3)*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a))/(b^2*d^2*e*x
+ b^2*c*d*e)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.52

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx =$$

$$-\frac{3\left(\left(-i\,\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(2,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(2,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\right.}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

output
```
-3/4*((-I*gamma(2, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(2, -I*b*conju
gate((d*x + c)^(1/3))) - I*gamma(2, I*(d*x + c)^(1/3)*b) + I*gamma(2, -I*(
d*x + c)^(1/3)*b))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(1/3))) + ga
mma(2, -I*b*conjugate((d*x + c)^(1/3))) + gamma(2, I*(d*x + c)^(1/3)*b) +
gamma(2, -I*(d*x + c)^(1/3)*b))*sin(a))/(b^2*d*e^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\left(\frac{(dex+ce)^{\frac{1}{3}}e\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b|e|^{\frac{2}{3}}}-\frac{e^{2}\sin\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{b^{2}|e|^{\frac{4}{3}}}\right)}{de}$$

input

```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

output

```
-3*((d*e*x + c*e)^(1/3)*e*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e
)/(b*abs(e)^(2/3)) - e^2*sin((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)
/(b^2*abs(e)^(4/3)))/(d*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{1/3}}\,dx$$

input

```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3),x)
```

output

```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.54

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{-3(dx+c)^{\frac{1}{3}}\cos \left((dx+c)^{\frac{1}{3}}b+a\right)b+3\sin \left((dx+c)^{\frac{1}{3}}b+a\right)}{e^{\frac{1}{3}}b^2d}$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
(3*( - (c + d*x)**(1/3)*cos((c + d*x)**(1/3)*b + a)*b + sin((c + d*x)**(1/
3)*b + a)))/(e**(1/3)*b**2*d)
```

3.231 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

output
```
-3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3912, 30, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3\int \frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e(c+dx))^{2/3}}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{30}$$

$$\frac{3(c+dx)^{2/3}\int \sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(c+dx)^{2/3}\int \sin\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

input
```
Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\,(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{bd^2ex+bcde}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

output
```
-3*(d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*cos((d*x + c)^(1/3)*b + a)/(b*d^2*e
*x + b*c*d*e)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(e\,(c+dx)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{bde^{\frac{2}{3}}}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

output
```
-3*cos((d*x + c)^(1/3)*b + a)/(b*d*e^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=-\frac{3\cos\left(\frac{ae+(dex+ce)^{\frac{1}{3}}b|e|^{\frac{2}{3}}}{e}\right)}{bd|e|^{\frac{2}{3}}}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

output
```
-3*cos((a*e + (d*e*x + c*e)^(1/3)*b*abs(e)^(2/3))/e)/(b*d*abs(e)^(2/3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx=-\frac{3\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{e^{\frac{2}{3}}bd}$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

output
```
( - 3*cos((c + d*x)**(1/3)*b + a))/(e**(2/3)*b*d)
```

3.232 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3b\sqrt[3]{c+dx}\cos(a)\operatorname{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3b\sqrt[3]{c+dx}\sin(a)\operatorname{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

output

```
3*b*(d*x+c)^(1/3)*cos(a)*Ci(b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(1/3)-3*sin(a
+b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(1/3)-3*b*(d*x+c)^(1/3)*sin(a)*Si(b*(d*x
+c)^(1/3))/d/e/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\left(-b\sqrt[3]{c+dx}\cos(a)\operatorname{CosIntegral}\left(b\sqrt[3]{c+dx}\right)+\sin\left(a+b\sqrt[3]{c+dx}\right)+b\sqrt[3]{c+dx}\sin(a)\operatorname{Si}\left(b\sqrt[3]{c+dx}\right)\right)}{de\sqrt[3]{e(c+dx)}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

output
```
(-3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)]) + Sin[a +
b*(c + d*x)^(1/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3
)]))/(d*e*(e*(c + d*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3912, 30, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b \sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3 \int \frac{(c+dx)^{2/3} \sin \left(a+b \sqrt[3]{c+dx}\right)}{(e(c+dx))^{4/3}}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{30}$$

$$\frac{3 \sqrt[3]{c+dx} \int \frac{\sin \left(a+b \sqrt[3]{c+dx}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}}{de \sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3 \sqrt[3]{c+dx} \int \frac{\sin \left(a+b \sqrt[3]{c+dx}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}}{de \sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3778}$$

$$\frac{3\sqrt[3]{c+dx}\left(b\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{3\sqrt[3]{c+dx}\left(b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \textbf{3784}$$

$$\frac{3\sqrt[3]{c+dx}\left(b\left(\cos(a)\int \frac{\cos\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\sin(a)\int \frac{\sin\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{3\sqrt[3]{c+dx}\left(b\left(\cos(a)\int \frac{\sin\left(\sqrt[3]{c+dx}b+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\sin(a)\int \frac{\sin\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \textbf{3780}$$

$$\frac{3\sqrt[3]{c+dx}\left(b\left(\cos(a)\int \frac{\sin\left(\sqrt[3]{c+dx}b+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\sin(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \textbf{3783}$$

$$\frac{3\sqrt[3]{c+dx}\left(b\left(\cos(a)\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)-\sin(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

input
```
Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(-(Sin[a + b*(c + d*x)^(1/3)]/(c + d*x)^(1/3)) + b*(Cos
[a]*CosIntegral[b*(c + d*x)^(1/3)] - Sin[a]*SinIntegral[b*(c + d*x)^(1/3)]
)))/(d*e*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*e^2*x^2 + 2*c
*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(e\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\frac{3\left(\left(\Gamma\left(-1,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1,i\,(dx+c)^{\frac{1}{3}}b\right)+\right.\right.}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

output
```
3/4*((gamma(-1, I*b*conjugate((d*x + c)^(1/3))) + gamma(-1, -I*b*conjugate
((d*x + c)^(1/3))) + gamma(-1, I*(d*x + c)^(1/3)*b) + gamma(-1, -I*(d*x +
c)^(1/3)*b))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(1/3))) + I*ga
mma(-1, -I*b*conjugate((d*x + c)^(1/3))) - I*gamma(-1, I*(d*x + c)^(1/3)*b
) + I*gamma(-1, -I*(d*x + c)^(1/3)*b))*sin(a))*b/(d*e^(4/3))
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{4/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx=\frac{\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dx+c)^{\frac{1}{3}}c+(dx+c)^{\frac{1}{3}}dx}dx}{e^{\frac{4}{3}}}$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin((c + d*x)**(1/3)*b + a)/((c + d*x)**(1/3)*c + (c + d*x)**(1/3)*d*x
),x)/(e**(1/3)*e)
```

3.233 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 175

$$
\begin{aligned}
\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = & -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} \\
& -\frac{3b^2(c+dx)^{2/3}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{2de(e(c+dx))^{2/3}} \\
& -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b^2(c+dx)^{2/3}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}
\end{aligned}
$$

output
```
-3/2*b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b^2*
(d*x+c)^(2/3)*Ci(b*(d*x+c)^(1/3))*sin(a)/d/e/(e*(d*x+c))^(2/3)-3/2*sin(a+b
*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b^2*(d*x+c)^(2/3)*cos(a)*Si(b*(d
*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.66

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx =$$
$$-\frac{3\left(b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)+b^2(c+dx)^{2/3}\,\mathrm{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)+\sin\left(a+b\sqrt[3]{c+dx}\right)\right.}{2de(e(c+dx))^{2/3}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

output
```
(-3*(b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x)^(2/3)*Co
sIntegral[b*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(1/3)] + b^2*(c
+ d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)]))/(2*d*e*(e*(c + d*x))^
(2/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3912, 30, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3\int \frac{(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e(c+dx))^{5/3}}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{30}$$

$$\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{c+dx}d\sqrt[3]{c+dx}}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{c+dx}d\sqrt[3]{c+dx}}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3778$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3778$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(b\int -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 25$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(-b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(-b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3784$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\cos\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}+\cos(a)\int\frac{\sin\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}\right)-\frac{\cos\left(a+b\sqrt[3]{c+\right.}{\sqrt[3]{c+dx}}\right.\right.}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\sin\left(\sqrt[3]{c+dx}b+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}+\cos(a)\int\frac{\sin\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}\right)-\frac{\cos\left(a+b\sqrt[3]{c\right.}{\sqrt[3]{c+}}\right.\right.}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3780}$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\sin\left(\sqrt[3]{c+dx}b+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}+\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{s}{}\right.}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3783}$$

$$\frac{3(c+dx)^{2/3}\left(\frac{1}{2}b\left(-b\left(\sin(a)\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)+\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\right.}{2(c-}\right.}{de(e(c+dx))^{2/3}}$$

input

```
Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

output

```
(3*(c + d*x)^(2/3)*(-1/2*Sin[a + b*(c + d*x)^(1/3)]/(c + d*x)^(2/3) + (b*(
-(Cos[a + b*(c + d*x)^(1/3)]/(c + d*x)^(1/3)) - b*(CosIntegral[b*(c + d*x)
^(1/3)]*Sin[a] + Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])))/2))/(d*e*(e*(c +
 d*x))^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin \left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [F]

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin \left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(1/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*e^2*x^2 + 2*c
*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(e\left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=$$

$$-\frac{3\left(\left(-i\,\Gamma\left(-2,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-2,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-2,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(-2,-i\,(dx+c)^{\frac{1}{3}}\right.\right.\right.}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

output
```
-3/4*((-I*gamma(-2, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(-2, -I*b*con
jugate((d*x + c)^(1/3))) - I*gamma(-2, I*(d*x + c)^(1/3)*b) + I*gamma(-2,
-I*(d*x + c)^(1/3)*b))*cos(a) - (gamma(-2, I*b*conjugate((d*x + c)^(1/3)))
 + gamma(-2, -I*b*conjugate((d*x + c)^(1/3))) + gamma(-2, I*(d*x + c)^(1/3
)*b) + gamma(-2, -I*(d*x + c)^(1/3)*b))*sin(a))*b^2/(d*e^(5/3))
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx=\frac{\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dx+c)^{\frac{2}{3}}c+(dx+c)^{\frac{2}{3}}dx}dx}{e^{\frac{5}{3}}}$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
int(sin((c + d*x)**(1/3)*b + a)/((c + d*x)**(2/3)*c + (c + d*x)**(2/3)*d*x
),x)/(e**(2/3)*e)
```

3.234 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx &= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}\\&+\frac{b^4\sqrt[3]{c+dx}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}\\&+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{b^4\sqrt[3]{c+dx}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}\end{aligned}$$

output
```
1/8*b^3*cos(a+b*(d*x+c)^(1/3))/d/e^2/(e*(d*x+c))^(1/3)-1/4*b*cos(a+b*(d*x+
c)^(1/3))/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)+1/8*b^4*(d*x+c)^(1/3)*Ci(b
*(d*x+c)^(1/3))*sin(a)/d/e^2/(e*(d*x+c))^(1/3)-3/4*sin(a+b*(d*x+c)^(1/3))/
d/e^2/(d*x+c)/(e*(d*x+c))^(1/3)+1/8*b^2*sin(a+b*(d*x+c)^(1/3))/d/e^2/(d*x+
c)^(1/3)/(e*(d*x+c))^(1/3)+1/8*b^4*(d*x+c)^(1/3)*cos(a)*Si(b*(d*x+c)^(1/3)
)/d/e^2/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.69

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \frac{b^3c\cos \left(a+b\sqrt[3]{c+dx}\right)+b^3dx\cos \left(a+b\sqrt[3]{c+dx}\right)-2b\sqrt[3]{c+dx}\cos \left(a+b}{}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

output
```
(b^3*c*Cos[a + b*(c + d*x)^(1/3)] + b^3*d*x*Cos[a + b*(c + d*x)^(1/3)] - 2
*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*CosInt
egral[b*(c + d*x)^(1/3)]*Sin[a] - 6*Sin[a + b*(c + d*x)^(1/3)] + b^2*(c +
d*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*Cos[a]*SinInte
gral[b*(c + d*x)^(1/3)])/(8*d*e*(e*(c + d*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.69, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {3912, 30, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$\frac{3\int \frac{(c+dx)^{2/3}\sin \left(a+b\sqrt[3]{c+dx}\right)}{(e(c+dx))^{7/3}}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{30}$$

$$\frac{3\sqrt[3]{c+dx}\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(c+dx)^{5/3}}d\sqrt[3]{c+dx}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(c+dx)^{5/3}}d\sqrt[3]{c+dx}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 3778$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{(c+dx)^{4/3}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{4(c+dx)^{4/3}}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)}{(c+dx)^{4/3}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{4(c+dx)^{4/3}}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 3778$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(\frac{1}{3}b\int -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{c+dx}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(c+dx)}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{4(c+dx)^{4/3}}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{c+dx}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(c+dx)}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{4(c+dx)^{4/3}}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{c+dx}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(c+dx)}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{4(c+dx)^{4/3}}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \; 3778$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\int\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(c+dx)}\right)-\frac{\sin\left(a+b\sqrt[3]{\right.}}{4(c+dx}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\int\frac{\sin\left(a+b\sqrt[3]{c+dx}+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{3(c+dx)}\right)-\frac{\sin\left(a+b\right.}{4(c}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3778}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(b\int-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)-\frac{\cos\left(a\right.}{}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(-b\int\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)-\frac{\cos\left(a\right.}{}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(-b\int\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{2(c+dx)^{2/3}}\right)-\frac{\cos\left(a\right.}{}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3784}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\cos\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}+\cos(a)\int\frac{\sin\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}\right)-\frac{\cos\left(a-\right.}{\sqrt[3]{}}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\sin\left(\sqrt[3]{c+dx}b+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}+\cos(a)\int\frac{\sin\left(b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}\right)-\frac{\cos}{}\right.\right.\right.\right.\right.}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3780$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\sin\left(\sqrt[3]{c+dx}b+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}+\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)-\frac{\cos\left(a+b\sqrt[3]{c+da}\right.}{\sqrt[3]{c+dx}}\right.\right.\right.\right.}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3783$$

$$\frac{3\sqrt[3]{c+dx}\left(\frac{1}{4}b\left(-\frac{1}{3}b\left(\frac{1}{2}b\left(-b\left(\sin(a)\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)+\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)\right)-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{c+dx}}\right)-\right.\right.\right.}{de^2\sqrt[3]{e(c+dx)}}$$

input
```
Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(-1/4*Sin[a + b*(c + d*x)^(1/3)]/(c + d*x)^(4/3) + (b*(
-1/3*Cos[a + b*(c + d*x)^(1/3)]/(c + d*x) - (b*(-1/2*Sin[a + b*(c + d*x)^(
1/3)]/(c + d*x)^(2/3) + (b*(-(Cos[a + b*(c + d*x)^(1/3)]/(c + d*x)^(1/3))
- b*(CosIntegral[b*(c + d*x)^(1/3)]*Sin[a] + Cos[a]*SinIntegral[b*(c + d*x
)^(1/3)])))/2))/3))/4))/(d*e^2*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{7}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(1/3)*b + a)/(d^3*e^3*x^3 + 3*c
*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\text{Timed out}$$

input
```
integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\frac{3\left(\left(-i\,\Gamma\left(-4,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-4,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-4,i\,(dx+c)\right.\right.\right.}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

output
```
3/4*((-I*gamma(-4, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(-4, -I*b*conj
ugate((d*x + c)^(1/3))) - I*gamma(-4, I*(d*x + c)^(1/3)*b) + I*gamma(-4, -
I*(d*x + c)^(1/3)*b))*cos(a) - (gamma(-4, I*b*conjugate((d*x + c)^(1/3)))
+ gamma(-4, -I*b*conjugate((d*x + c)^(1/3))) + gamma(-4, I*(d*x + c)^(1/3)
*b) + gamma(-4, -I*(d*x + c)^(1/3)*b))*sin(a))*b^4/(d*e^(7/3))
```

Giac [F]

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{\sin \left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx=\frac{\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dx+c)^{\frac{1}{3}}c^2+2(dx+c)^{\frac{1}{3}}cdx+(dx+c)^{\frac{1}{3}}d^2x^2}dx}{e^{\frac{7}{3}}}$$

input

```
int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

output

```
int(sin((c + d*x)**(1/3)*b + a)/((c + d*x)**(1/3)*c**2 + 2*(c + d*x)**(1/3
)*c*d*x + (c + d*x)**(1/3)*d**2*x**2),x)/(e**(1/3)*e**2)
```

3.235 $\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 267

$$\begin{aligned}\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx &= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d}\\ &\quad-\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}\\ &\quad-\frac{45e\sqrt{\pi}\sqrt[3]{e(c+dx)}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}\\ &\quad+\frac{45e\sqrt{\pi}\sqrt[3]{e(c+dx)}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}\\ &\quad+\frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d}\end{aligned}$$

output

```
45/8*e*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b^3/d-3/2*e*(d*x+c)^(4/3)*
(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d-45/16*e*Pi^(1/2)*(e*(d*x+c))^
(1/3)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*2^(1/2)/b^(7
/2)/d/(d*x+c)^(1/3)+45/16*e*Pi^(1/2)*(e*(d*x+c))^(1/3)*FresnelS(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)*2^(1/2)/b^(7/2)/d/(d*x+c)^(1/3)+15/4*
e*(d*x+c)^(2/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx =$$

$$-\frac{3(e(c+dx))^{4/3}\left(15\sqrt{2\pi}\cos(a)\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)-15\sqrt{2\pi}\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)\right.}{16b^{7/2}d(c}$$

input
```
Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(-3*(e*(c + d*x))^(4/3)*(15*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*
(c + d*x)^(1/3)] - 15*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/
3)]*Sin[a] + 2*Sqrt[b]*((c + d*x)^(1/3)*(-15 + 4*b^2*(c + d*x)^(4/3))*Cos[
a + b*(c + d*x)^(2/3)] - 10*b*(c + d*x)*Sin[a + b*(c + d*x)^(2/3)])))/(16*
b^(7/2)*d*(c + d*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3916, 3898, 3896, 3866, 3867, 3866, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)dx$$

$$\Big\downarrow \text{ 3916}$$

$$\frac{\int (e(c+dx))^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{ 3898}$$

$$\frac{e\sqrt[3]{e(c+dx)}\int (c+dx)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{ 3896}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\int (c+dx)^2\sin\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3866}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{5\int (c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}-\frac{(c+dx)^{5/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3867}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\int (c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}\right)}{2b}-\frac{(c+dx)^{5/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3866}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\int\cos\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}-\frac{\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/3}\cos(a}{2b}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3835}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{5\Bigg(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\Bigg(\frac{\cos(a)\int\cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\sin(a)\int\sin\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}-\frac{\sqrt[3]{c+dx}\cos(}{2}}{2b}}{2b}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\cos(a)\int\cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}}{2b}-\sqrt[3]{c+dx}\right.}{2b}\right.}{2b}\right.}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \; 3833$$

$$\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}}{2b}-\sqrt[3]{c+\right.}{2b}\right.}{2b}\right.}{d\sqrt[3]{c+dx}}$$

input
```
Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(3*e*(e*(c + d*x))^(1/3)*(-1/2*((c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(2/3)]
)/b + (5*((-3*(-1/2*((c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/b + ((Sqr
t[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/Sqrt[b] - (Sq
rt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/Sqrt[b])/(2*
b)))/(2*b) + ((c + d*x)*Sin[a + b*(c + d*x)^(2/3)])/(2*b)))/(2*b)))/(d*(c
+ d*x)^(1/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835
```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+b(dx+c)^{\frac{2}{3}}\right) dx$$

input
```
int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{4/3} \sin\left(a+b(c+dx)^{2/3}\right) dx = \int (dex+ce)^{\frac{4}{3}} \sin\left((dx+c)^{\frac{2}{3}}b+a\right) dx$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(4/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.45

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
3/8*(((I*gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) - I*gamma(7/2, I*(d*x
 + c)^(2/3)*b))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, I*b
*conjugate((d*x + c)^(2/3))) + I*gamma(7/2, -I*(d*x + c)^(2/3)*b))*cos(-7/
4*pi + 7/3*arctan2(0, d*x + c)) - (gamma(7/2, -I*b*conjugate((d*x + c)^(2/
3))) + gamma(7/2, I*(d*x + c)^(2/3)*b))*sin(7/4*pi + 7/3*arctan2(0, d*x +
c)) + (gamma(7/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(7/2, -I*(d*x + c
)^(2/3)*b))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((gamma(7/2,
-I*b*conjugate((d*x + c)^(2/3))) + gamma(7/2, I*(d*x + c)^(2/3)*b))*cos(7/
4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, I*b*conjugate((d*x + c)^(2/3
))) + gamma(7/2, -I*(d*x + c)^(2/3)*b))*cos(-7/4*pi + 7/3*arctan2(0, d*x +
 c)) - (-I*gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(7/2, I*(d
*x + c)^(2/3)*b))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*gamma(7/2, I
*b*conjugate((d*x + c)^(2/3))) + I*gamma(7/2, -I*(d*x + c)^(2/3)*b))*sin(-
7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(4/3)
/((d*x + c)^(1/3)*b^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{2/3}\right)\,(c\,e+d\,e\,x)^{4/3}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3), x)
```

Reduce [F]

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{e^{\frac{4}{3}}\left(-3(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)bc+2\left(\int (dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)xdx\right)\right.}{2b^2d}$$

input
```
int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
(e**(1/3)*e*( - 3*(c + d*x)**(2/3)*cos((c + d*x)**(2/3)*b + a)*b*c + 2*int
((c + d*x)**(1/3)*sin((c + d*x)**(2/3)*b + a)*x,x)*b**2*d**2 + 3*sin((c +
d*x)**(2/3)*b + a)*c))/(2*b**2*d)
```

3.236 $\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx &= -\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}\\ &\quad-\frac{9\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}\\ &\quad-\frac{9\sqrt{\pi}(e(c+dx))^{2/3}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}\\ &\quad+\frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}}\end{aligned}$$

output
```
-3/2*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*cos(a+b*(d*x+c)^(2/3))/b/d-9/8*Pi^(1/
2)*(e*(d*x+c))^(2/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3
))*2^(1/2)/b^(5/2)/d/(d*x+c)^(2/3)-9/8*Pi^(1/2)*(e*(d*x+c))^(2/3)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)*2^(1/2)/b^(5/2)/d/(d*x+c)^
(2/3)+9/4*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx =$$

$$-\frac{3(e(c+dx))^{2/3}\left(3\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+3\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)+}{8b^{5/2}d(c+dx)^{2/3}}$$

input
```
Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(-3*(e*(c + d*x))^(2/3)*(3*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(
c + d*x)^(1/3)] + 3*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)
]*Sin[a] + 2*Sqrt[b]*(2*b*(c + d*x)*Cos[a + b*(c + d*x)^(2/3)] - 3*(c + d*
x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(c + d*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3916, 3898, 3896, 3866, 3867, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int (e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{(e(c+dx))^{2/3}\int (c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3896}$$

$$\frac{3(e(c+dx))^{2/3}\int (c+dx)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{d(c+dx)^{2/3}}$$

$$\Big\downarrow 3866$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{3\int (c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}-\frac{(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow 3867$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{3\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{\int \sin\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}\right)}{2b}-\frac{(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow 3834$$

$$\frac{3(e(c+dx))^{2/3}\Bigg(\frac{3\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{\sin(a)\int \cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}+\cos(a)\int \sin\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{2b}\right)}{2b}-(c+dx)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow 3832$$

$$\frac{3(e(c+dx))^{2/3}\Bigg(\frac{3\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{\sin(a)\int \cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}}{2b}\right)}{2b}-(c+}{d(c+dx)^{2/3}}$$

$$\Big\downarrow 3833$$

$$\frac{3(e(c+dx))^{2/3}\left(\frac{3\left(\frac{\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{\frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}}{2b}\right)}{2b}-\right.}{d(c+dx)^{2/3}}$$

input
```
Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(3*(e*(c + d*x))^(2/3)*(-1/2*((c + d*x)*Cos[a + b*(c + d*x)^(2/3)])/b + (3
*(-1/2*((Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/S
qrt[b] + (Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/
Sqrt[b])/b + ((c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)])/(2*b)))/(2*b)))/
(d*(c + d*x)^(2/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int (dex+ce)^{\frac{2}{3}} \sin \left(a+b(dx+c)^{\frac{2}{3}}\right) dx$$

input
```
int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{2/3}\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\int (dex+ce)^{\frac{2}{3}}\sin \left((dx+c)^{\frac{2}{3}}b+a\right)\,dx$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F]

$$\int (ce+dex)^{2/3}\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\int (e(c+dx))^{\frac{2}{3}}\sin \left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

input
```
integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

output
```
Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.87

$$\int (ce+dex)^{2/3}\sin \left(a+b(c+dx)^{2/3}\right)\,dx=\text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
-3/16*(3*(d*x + c)^(2/3)*(((gamma(3/2, -I*b*conjugate((d*x + c)^(2/3))) +
gamma(3/2, I*(d*x + c)^(2/3)*b))*cos(3/4*pi + arctan2(0, d*x + c)) + (gamm
a(3/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(3/2, -I*(d*x + c)^(2/3)*b))
*cos(-3/4*pi + arctan2(0, d*x + c)) + (I*gamma(3/2, -I*b*conjugate((d*x +
c)^(2/3))) - I*gamma(3/2, I*(d*x + c)^(2/3)*b))*sin(3/4*pi + arctan2(0, d*
x + c)) + (I*gamma(3/2, I*b*conjugate((d*x + c)^(2/3))) - I*gamma(3/2, -I*
(d*x + c)^(2/3)*b))*sin(-3/4*pi + arctan2(0, d*x + c)))*cos(a) + ((I*gamma
(3/2, -I*b*conjugate((d*x + c)^(2/3))) - I*gamma(3/2, I*(d*x + c)^(2/3)*b)
)*cos(3/4*pi + arctan2(0, d*x + c)) + (-I*gamma(3/2, I*b*conjugate((d*x +
c)^(2/3))) + I*gamma(3/2, -I*(d*x + c)^(2/3)*b))*cos(-3/4*pi + arctan2(0,
d*x + c)) - (gamma(3/2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(3/2, I*(d
*x + c)^(2/3)*b))*sin(3/4*pi + arctan2(0, d*x + c)) + (gamma(3/2, I*b*conj
ugate((d*x + c)^(2/3))) + gamma(3/2, -I*(d*x + c)^(2/3)*b))*sin(-3/4*pi +
arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(2/3) + 8*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*e^(2/3)*cos((d*x + c)^(2/3)*b + a))/(b^3*d^2
*x + b^3*c*d)
```

Giac [F(-2)]

Exception generated.

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{2/3}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3), x)
```

Reduce [F]

$$\int (ce+dex)^{2/3}\sin \left(a+b(c+dx)^{2/3}\right)\, dx = e^{\frac{2}{3}}\left(\int (dx+c)^{\frac{2}{3}}\sin \left((dx+c)^{\frac{2}{3}} b+a\right) dx\right)$$

input
```
int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
e**(2/3)*int((c + d*x)**(2/3)*sin((c + d*x)**(2/3)*b + a),x)
```

3.237 $\int \sqrt[3]{ce+dex} \sin \left(a+b(c+dx)^{2/3}\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \sqrt[3]{ce+dex} \sin \left(a+b(c+dx)^{2/3}\right) \, dx = -\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)} \cos \left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\sqrt[3]{e(c+dx)} \sin \left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}$$

output

```
-3/2*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+3/2*(e*(d*
x+c))^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \sqrt[3]{ce+dex} \sin \left(a+b(c+dx)^{2/3}\right) \, dx = -\frac{3\sqrt[3]{e(c+dx)}\left(b(c+dx)^{2/3} \cos \left(a+b(c+dx)^{2/3}\right) - \sin \left(a+b(c+dx)^{2/3}\right)\right)}{2b^2d\sqrt[3]{c+dx}}$$

input

```
Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(-3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(2/3)] - Si
n[a + b*(c + d*x)^(2/3)]))/(2*b^2*d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3916, 3862, 3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{\sqrt[3]{e(c+dx)}\int \sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3860}$$

$$\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)^{2/3}}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)d(c+dx)^{2/3}}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{\int \cos\left(a+b(c+dx)^{2/3}\right)d(c+dx)^{2/3}}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{b}\right)}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{\int \sin\left(a+b(c+dx)^{2/3}+\frac{\pi}{2}\right)d(c+dx)^{2/3}}{b}-\frac{(c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{b}\right)}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3\sqrt[3]{e(c+dx)}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{b^2}-\frac{(c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{b}\right)}{2d\sqrt[3]{c+dx}}$$

input
```
Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

output
```
(3*(e*(c + d*x))^(1/3)*(-(((c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(2/3)])/b)
+ Sin[a + b*(c + d*x)^(2/3)]/b^2))/(2*d*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int (dex+ce)^{\frac{1}{3}} \sin\left(a+b(dx+c)^{\frac{2}{3}}\right) dx$$

input
```
int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right) dx = -\frac{3\left((bdx+bc)(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)-(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)\right)}{2\left(b^2d^2x+b^2cd\right)}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
-3/2*((b*d*x + b*c)*(d*e*x + c*e)^(1/3)*(d*x + c)^(1/3)*cos((d*x + c)^(2/3
)*b + a) - (d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*sin((d*x + c)^(2/3)*b + a))
/(b^2*d^2*x + b^2*c*d)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sqrt[3]{e\left(c+dx\right)}\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

input
```
integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

output
```
Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=$$

$$-\frac{3\left(\left(-i\,\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(2,i\,(dx+c)^{\frac{2}{3}}b\right)+i\,\Gamma\left(2,-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\mathrm{co}}{}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
-3/8*((-I*gamma(2, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(2, -I*b*conju
gate((d*x + c)^(2/3))) - I*gamma(2, I*(d*x + c)^(2/3)*b) + I*gamma(2, -I*(
d*x + c)^(2/3)*b))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(2/3))) + ga
mma(2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(2, I*(d*x + c)^(2/3)*b) +
gamma(2, -I*(d*x + c)^(2/3)*b))*sin(a))*e^(1/3)/(b^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\int \sin\left(a+b\,(c+d\,x)^{2/3}\right)\,(c\,e+d\,e\,x)^{1/3}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{3e^{\frac{1}{3}}\left(-(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)b+\sin\left((dx+c)^{\frac{2}{3}}b+a\right)\right)}{2b^2d}$$

input
```
int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

output
```
(3*e**(1/3)*( - (c + d*x)**(2/3)*cos((c + d*x)**(2/3)*b + a)*b + sin((c +
d*x)**(2/3)*b + a)))/(2*b**2*d)
```

3.238 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 44

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

output
```
-3/2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

output
```
(-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3)
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3916, 3862, 3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{\sqrt[3]{c+dx}\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}d(c+dx)}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3860}$$

$$\frac{3\sqrt[3]{c+dx}\int \sin \left(a+b(c+dx)^{2/3}\right)d(c+dx)^{2/3}}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{c+dx}\int \sin \left(a+b(c+dx)^{2/3}\right)d(c+dx)^{2/3}}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{3\sqrt[3]{c+dx}\cos \left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

input
```
Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

output
```
(-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\left(dex+ce\right)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)}{2\left(bd^2ex+bcde\right)}$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

output

```
-3/2*(d*e*x + c*e)^(2/3)*(d*x + c)^(1/3)*cos((d*x + c)^(2/3)*b + a)/(b*d^2
*e*x + b*c*d*e)
```

Sympy [F]

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\sqrt[3]{e\left(c+dx\right)}}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

output

```
Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\cos\left((dx+c)^{\frac{2}{3}}b+a\right)}{2\,bde^{\frac{1}{3}}}$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

output

```
-3/2*cos((d*x + c)^(2/3)*b + a)/(b*d*e^(1/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin \left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx=-\frac{3\cos \left((dx+c)^{\frac{2}{3}}b+a\right)}{2e^{\frac{1}{3}}bd}$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
( - 3*cos((c + d*x)**(2/3)*b + a))/(2*e**(1/3)*b*d)
```

3.239 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}} + \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

output

```
3/2*2^(1/2)*Pi^(1/2)*(d*x+c)^(2/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/3))/b^(1/2)/d/(e*(d*x+c))^(2/3)+3/2*2^(1/2)*Pi^(1/2)*(d*x+c)^
(2/3)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)/b^(1/2)/d/(e
*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\left(\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c}\right.\right.}{\sqrt{b}d(e(c+dx))^{2/3}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

output
```
(3*Sqrt[Pi/2]*(c + d*x)^(2/3)*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x
)^(1/3)] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[b]*
d*(e*(c + d*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3916, 3898, 3864, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e(c+dx))^{2/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{(c+dx)^{2/3}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d(c+dx)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3864}$$

$$\frac{3(c+dx)^{2/3}\int \sin\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \textbf{3834}$$

$$\frac{3(c+dx)^{2/3}\left(\sin(a)\int\cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}+\cos(a)\int\sin\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \textbf{3832}$$

$$\frac{3(c+dx)^{2/3}\left(\sin(a)\int\cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}\right)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \textbf{3833}$$

$$\frac{3(c+dx)^{2/3}\left(\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}\right)}{d(e(c+dx))^{2/3}}$$

input

```
Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

output

```
(3*(c + d*x)^(2/3)*((Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*
x)^(1/3)])/Sqrt[b] + (Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/
3)]*Sin[a])/Sqrt[b]))/(d*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3864

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Simp[2/n   Subst[
Int[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
 n/2 - 1]
```

rule 3898

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

input

```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

output

```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

output

```
integral(sin((d*x + c)^(2/3)*b + a)/(d*e*x + c*e)^(2/3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\left(e\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
```

output

```
Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.66

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx=\text{Too large to display}$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

output

```
3/8*(((-I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1) + I*sq
rt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*cos(1/4*pi + 1/3*arctan2(0, d
*x + c)) + (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) - I
*sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*cos(-1/4*pi + 1/3*arctan2
(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1)
 + sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*sin(1/4*pi + 1/3*arctan2
(0, d*x + c)) - (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1)
+ sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*sin(-1/4*pi + 1/3*arctan
2(0, d*x + c)))*cos(a) + ((sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/
3)))) - 1) + sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*cos(1/4*pi + 1
/3*arctan2(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3
)))) - 1) + sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*cos(-1/4*pi +
1/3*arctan2(0, d*x + c)) + (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^
(2/3)))) - 1) - I*sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*sin(1/4*p
i + 1/3*arctan2(0, d*x + c)) + (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x +
c)^(2/3)))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*sin(-
1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)/((d*x +
 c)^(1/3)*b*d*e^(2/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dx+c)^{\frac{2}{3}}}dx}{e^{\frac{2}{3}}}$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a)/(c + d*x)**(2/3),x)/e**(2/3)
```

3.240 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}}$$

output

```
3*b^(1/2)*2^(1/2)*Pi^(1/2)*(d*x+c)^(1/3)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(1/3)-3*b^(1/2)*2^(1/2)*Pi^(1/2)*(d
*x+c)^(1/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/3))*sin(a)/d/e/(e
*(d*x+c))^(1/3)-3*sin(a+b*(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx =$$

$$-\frac{3\left(-\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\mathrm{s}\right.}{de\sqrt[3]{e(c+dx)}}$$

input

```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

output

```
(-3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/P
i]*(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*FresnelS[Sqrt[b]
*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(2/3)]))/(d*e*(e
*(c + d*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3916, 3898, 3896, 3868, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx$$

$$\Big\downarrow \; 3916$$

$$\frac{\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e(c+dx))^{4/3}}d(c+dx)}{d}$$

$$\Big\downarrow \; 3898$$

$$\frac{\sqrt[3]{c+dx}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{4/3}}d(c+dx)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3896$$

$$\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d\sqrt[3]{c+dx}}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3868$$

$$\frac{3\sqrt[3]{c+dx}\left(2b\int \cos\left(a+b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3835$$

$$\frac{3\sqrt[3]{c+dx}\left(2b\left(\cos(a)\int \cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\sin(a)\int \sin\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3832$$

$$\frac{3\sqrt[3]{c+dx}\left(2b\left(\cos(a)\int \cos\left(b(c+dx)^{2/3}\right)d\sqrt[3]{c+dx}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3833$$

$$\frac{3\sqrt[3]{c+dx}\left(2b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

input

```
Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

output

```
(3*(c + d*x)^(1/3)*(2*b*((Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)])/Sqrt[b] - (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x
)^(1/3)]*Sin[a])/Sqrt[b]) - Sin[a + b*(c + d*x)^(2/3)]/(c + d*x)^(1/3)))/(
d*e*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835
```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin \left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin \left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*e^2*x^2 + 2*c
*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin \left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.26

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

output

```
-3/8*(((-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2, I*
(d*x + c)^(2/3)*b))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (I*gamma(-1/2,
 I*b*conjugate((d*x + c)^(2/3))) - I*gamma(-1/2, -I*(d*x + c)^(2/3)*b))*co
s(-1/4*pi + 1/3*arctan2(0, d*x + c)) - (gamma(-1/2, -I*b*conjugate((d*x +
c)^(2/3))) + gamma(-1/2, I*(d*x + c)^(2/3)*b))*sin(1/4*pi + 1/3*arctan2(0,
 d*x + c)) + (gamma(-1/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(-1/2, -I
*(d*x + c)^(2/3)*b))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*cos(a) + ((ga
mma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(-1/2, I*(d*x + c)^(2/3)
*b))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, I*b*conjugate((d
*x + c)^(2/3))) + gamma(-1/2, -I*(d*x + c)^(2/3)*b))*cos(-1/4*pi + 1/3*arc
tan2(0, d*x + c)) + (-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + I*g
amma(-1/2, I*(d*x + c)^(2/3)*b))*sin(1/4*pi + 1/3*arctan2(0, d*x + c)) + (
-I*gamma(-1/2, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2, -I*(d*x + c
)^(2/3)*b))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)
^(2/3)*b)/((d*x + c)^(1/3)*d*e^(4/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{4/3}} \,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sin \left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx=\frac{\int \frac{\sin \left(\left(dx +c \right)^{\frac{2}{3}} b +a \right)}{\left(dx +c \right)^{\frac{1}{3}} c +\left(dx +c \right)^{\frac{1}{3}} d x}d x}{e^{\frac{4}{3}}}$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a)/((c + d*x)**(1/3)*c + (c + d*x)**(1/3)*d*x
),x)/(e**(1/3)*e)
```

3.241 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 126

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

output
```
3/2*b*(d*x+c)^(2/3)*cos(a)*Ci(b*(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(2/3)-3/2*s
in(a+b*(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b*(d*x+c)^(2/3)*sin(a)*Si(
b*(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx=$$

$$-\frac{3(-b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right)+\sin\left(a+b(c+dx)^{2/3}\right)+b(c+dx)^{2/3}\sin(a)\text{Si}(b(c+}{2de(e(c+dx))^{2/3}}$$

input
```
Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

output
```
(-3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)]) + Sin[a +
b*(c + d*x)^(2/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3
)]))/(2*d*e*(e*(c + d*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3916, 3862, 3860, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e(c+dx))^{5/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{(c+dx)^{2/3}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{5/3}}d(c+dx)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3860}$$

$$\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{4/3}}d(c+dx)^{2/3}}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{4/3}}d(c+dx)^{2/3}}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3778}$$

$$\frac{3(c+dx)^{2/3}\left(b\int \frac{\cos\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3(c+dx)^{2/3}\left(b\int \frac{\sin\left(a+b(c+dx)^{2/3}+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3784$$

$$\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\int \frac{\cos\left(b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}-\sin(a)\int \frac{\sin\left(b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3(c+dx)^{2/3}\Bigg(b\Bigg(\cos(a)\int \frac{\sin\left((c+dx)^{2/3}b+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}-\sin(a)\int \frac{\sin\left(b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}\Bigg)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3780$$

$$\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\int \frac{\sin\left((c+dx)^{2/3}b+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}d(c+dx)^{2/3}-\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3783$$

$$\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\,\text{CosIntegral}\left(b(c+dx)^{2/3}\right)-\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}$$

input

```
Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

output

```
(3*(c + d*x)^(2/3)*(-(Sin[a + b*(c + d*x)^(2/3)]/(c + d*x)^(2/3)) + b*(Cos
[a]*CosIntegral[b*(c + d*x)^(2/3)] - Sin[a]*SinIntegral[b*(c + d*x)^(2/3)]
)))/(2*d*e*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(1/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*e^2*x^2 + 2*c
*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\left(e\left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
```

output
```
Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\bigg(\bigg(\Gamma\bigg(-1, i\,b\overline{(dx+c)^{\frac{2}{3}}}\bigg)+\Gamma\bigg(-1,-i\,b\overline{(dx+c)^{\frac{2}{3}}}\bigg)+\Gamma\bigg(-1,i\,(dx+c)^{\frac{2}{3}}b\bigg)}{}$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

output
```
3/8*((gamma(-1, I*b*conjugate((d*x + c)^(2/3))) + gamma(-1, -I*b*conjugate
((d*x + c)^(2/3))) + gamma(-1, I*(d*x + c)^(2/3)*b) + gamma(-1, -I*(d*x +
c)^(2/3)*b))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(2/3))) + I*ga
mma(-1, -I*b*conjugate((d*x + c)^(2/3))) - I*gamma(-1, I*(d*x + c)^(2/3)*b
) + I*gamma(-1, -I*(d*x + c)^(2/3)*b))*sin(a))*b/(d*e^(5/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

input
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3),x)
```

output
```
int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \frac{\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dx+c)^{\frac{2}{3}}c+(dx+c)^{\frac{2}{3}}dx}dx}{e^{\frac{5}{3}}}$$

input
```
int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
int(sin((c + d*x)**(2/3)*b + a)/((c + d*x)**(2/3)*c + (c + d*x)**(2/3)*d*x
),x)/(e**(2/3)*e)
```

3.242 $\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$\begin{aligned}\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = &-\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}\\ &+\frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d}\\ &-\frac{b^4\sqrt[3]{e(c+dx)}\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{8d\sqrt[3]{c+dx}}\\ &-\frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}\\ &+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d}\\ &-\frac{b^4\sqrt[3]{e(c+dx)}\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}\end{aligned}$$

output
```
-1/8*b^3*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(1/3))/d+1/4*b*(d*x+c)^(2/3)*(e
*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(1/3))/d-1/8*b^4*(e*(d*x+c))^(1/3)*Ci(b/(d
*x+c)^(1/3))*sin(a)/d/(d*x+c)^(1/3)-1/8*b^2*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3
)*sin(a+b/(d*x+c)^(1/3))/d+3/4*(d*x+c)*(e*(d*x+c))^(1/3)*sin(a+b/(d*x+c)^(
1/3))/d-1/8*b^4*(e*(d*x+c))^(1/3)*cos(a)*Si(b/(d*x+c)^(1/3))/d/(d*x+c)^(1/
3)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

$$\int \sqrt[3]{ce+dex}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{\sqrt[3]{e(c+dx)}\left(-2bc\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-2bdx\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

output
```
-1/8*((e*(c + d*x))^(1/3)*(-2*b*c*Cos[a + b/(c + d*x)^(1/3)] - 2*b*d*x*Cos
[a + b/(c + d*x)^(1/3)] + b^3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)] +
 b^4*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] - 6*c*(c + d*x)^(1/3)*Sin[a + b
/(c + d*x)^(1/3)] - 6*d*x*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] + b^2
*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)] + b^4*Cos[a]*SinIntegral[b/(c
+ d*x)^(1/3)]))/(d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {3912, 30, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

$$\Big\downarrow \ 3912$$

$$-\frac{3\int (c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \ 30$$

$$-\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)^{5/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 3778$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{4}b\int (c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{4}(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{4}b\int (c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{4}(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 3778$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{4}b\left(\frac{1}{3}b\int -\left((c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{3}(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\frac{1}{4}(c}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 25$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{4}b\left(-\frac{1}{3}b\int (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{3}(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\frac{1}{4}(c+d}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\int(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{3}(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-\frac{1}{4}(c+d}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\Bigg(\frac{1}{2}b\int(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\Bigg(\frac{1}{2}b\int(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\sin\Bigg(a+\frac{b}{\sqrt[3]{c+dx}}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\Bigg(\frac{1}{2}b\Bigg(b\int-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-}{d\sqrt[3]{c-}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\Bigg(\frac{1}{2}b\Bigg(-b\int\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-}{d\sqrt[3]{c-}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\Bigg(\frac{1}{2}b\Bigg(-b\int\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-}{d\sqrt[3]{c-}}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Bigg(\frac{1}{4}b\Bigg(-\frac{1}{3}b\Bigg(\frac{1}{2}b\Bigg(-b\Bigg(\sin(a)\int\sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int\sqrt[3]{c+dx}\sin\Bigg(\frac{b}{\sqrt[3]{c+}}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\frac{1}{4}b\Big(-\frac{1}{3}b\Big(\frac{1}{2}b\Big(-b\Big(\sin(a)\int\sqrt[3]{c+dx}\sin\Big(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\Big)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int\sqrt[3]{c+dx}\sin\Big(\frac{}{\sqrt[3]{}}}{}$$

$$\Big\downarrow \color{blue}{3780}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\frac{1}{4}b\Big(-\frac{1}{3}b\Big(\frac{1}{2}b\Big(-b\Big(\sin(a)\int\sqrt[3]{c+dx}\sin\Big(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\Big)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\text{Si}\Big(\frac{b}{\sqrt[3]{c+dx}}\Big)\Big)-}{}$$

$$\Big\downarrow \color{blue}{3783}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\frac{1}{4}b\Big(-\frac{1}{3}b\Big(\frac{1}{2}b\Big(-b\Big(\sin(a)\,\text{CosIntegral}\Big(\frac{b}{\sqrt[3]{c+dx}}\Big)+\cos(a)\text{Si}\Big(\frac{b}{\sqrt[3]{c+dx}}\Big)\Big)-\sqrt[3]{c+dx}\cos\Big(a-}{}$$

input

```
Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

output

```
(-3*(e*(c + d*x))^(1/3)*(-1/4*((c + d*x)^(4/3)*Sin[a + b/(c + d*x)^(1/3)])
 + (b*(-1/3*((c + d*x)*Cos[a + b/(c + d*x)^(1/3)]) - (b*(-1/2*((c + d*x)^(
2/3)*Sin[a + b/(c + d*x)^(1/3)]) + (b*(-((c + d*x)^(1/3)*Cos[a + b/(c + d*
x)^(1/3)]) - b*(CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] + Cos[a]*SinIntegral
[b/(c + d*x)^(1/3)])))/2))/3))/4))/(d*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30

```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int (dex + ce)^{\frac{1}{3}} \sin\left(a + \frac{b}{(dx + c)^{\frac{1}{3}}}\right) dx$$

input
```
int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

output
```
int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

Fricas [F]

$$\int \sqrt[3]{ce+dex}\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int (dex+ce)^{\frac{1}{3}}\sin \left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\,dx$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x +
c)), x)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sqrt[3]{e\,(c+dx)}\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

input
```
integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(1/3)),x)
```

output
```
Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{ce+dex}\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=$$
$$-\frac{3\left(\left(-i\,\Gamma\left(-4,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\Gamma\left(-4,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\Gamma\left(-4,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-4,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos\left(a\right.\right.}{4}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-3/4*((-I*gamma(-4, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(-4, -I*b*co
njugate((d*x + c)^(-1/3))) - I*gamma(-4, I*b/(d*x + c)^(1/3)) + I*gamma(-4
, -I*b/(d*x + c)^(1/3)))*cos(a) - (gamma(-4, I*b*conjugate((d*x + c)^(-1/3
))) + gamma(-4, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(-4, I*b/(d*x + c
)^(1/3)) + gamma(-4, -I*b/(d*x + c)^(1/3)))*sin(a))*b^4*e^(1/3)/d
```

Giac [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\,dx$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(1/3)*sin(a + b/(d*x + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)\,(c\,e+d\,e\,x)^{1/3}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx=e^{\frac{1}{3}}\left(\int (dx+c)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)dx\right)$$

input

```
int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

output

```
e**(1/3)*int((c + d*x)**(1/3)*sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3
)),x)
```

3.243 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx &= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} \\ &\quad + \frac{3b^2\sqrt[3]{c+dx}\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{2d\sqrt[3]{e(c+dx)}} \\ &\quad + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} \\ &\quad + \frac{3b^2\sqrt[3]{c+dx}\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}\end{aligned}$$

output
```
3/2*b*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)+3/2*b^2*(d*
x+c)^(1/3)*Ci(b/(d*x+c)^(1/3))*sin(a)/d/(e*(d*x+c))^(1/3)+3/2*(d*x+c)*sin(
a+b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)+3/2*b^2*(d*x+c)^(1/3)*cos(a)*Si(b/(
d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx$$
$$= \frac{3\left(b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^2\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)+c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right.\right.}{2d\sqrt[3]{e(c+dx)}}$$

input

```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

output

```
(3*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*Cos
Integral[b/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)^(1/3)] + d*x*Si
n[a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d
*x)^(1/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3912, 30, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \frac{(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{e(c+dx)}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{30}$$

$$-\frac{3\sqrt[3]{c+dx}\int (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{c+dx}\int (c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{1}{2}b\int (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{1}{2}b\int (c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{2}(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{1}{2}b\left(b\int -\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\frac{1}{2}(c+dx)^{2/3}\right.}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{1}{2}b\left(-b\int \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\frac{1}{2}(c+dx)^{2/3}\right.}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{1}{2}b\left(-b\int \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\frac{1}{2}(c+dx)^{2/3}\right.}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigg(\frac{1}{2}b\Bigg(-b\Bigg(\sin(a)\int\sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+d}}}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigg(\frac{1}{2}b\Bigg(-b\Bigg(\sin(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)d}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigg(\frac{1}{2}b\Bigg(-b\Bigg(\sin(a)\int\sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-\sqrt[3]{c+dx}\cos}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigg(\frac{1}{2}b\Bigg(-b\Bigg(\sin(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)\Bigg)-\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\Bigg)}{d\sqrt[3]{e(c+dx)}}$$

input

```
Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

output

```
(-3*(c + d*x)^(1/3)*(-1/2*((c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)]) + (
b*(-((c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)]) - b*(CosIntegral[b/(c + d
*x)^(1/3)]*Sin[a] + Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])))/2))/(d*(e*(c
+ d*x))^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

output
```
integral(sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*e*x + c*e)^(1
/3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{e\left(c+dx\right)}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx =$$

$$-\frac{3\left((dx+c)^{\frac{1}{3}}\left(\left(-i\,\Gamma\left(-1, i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\Gamma\left(-1, -i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\Gamma\left(-1, \frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-1, -\frac{i\,b}{(dx+c}\right.\right.\right.\right.}{}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

output
```
-3/8*((d*x + c)^(1/3)*((-I*gamma(-1, I*b*conjugate((d*x + c)^(-1/3))) + I*
gamma(-1, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(-1, I*b/(d*x + c)^(1
/3)) + I*gamma(-1, -I*b/(d*x + c)^(1/3)))*cos(a) - (gamma(-1, I*b*conjugat
e((d*x + c)^(-1/3))) + gamma(-1, -I*b*conjugate((d*x + c)^(-1/3))) + gamma
(-1, I*b/(d*x + c)^(1/3)) + gamma(-1, -I*b/(d*x + c)^(1/3)))*sin(a))*b^2 -
 4*(d*x + c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/((d*x + c)^(1/3
)*d*e^(1/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx=\frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{1}{3}}}dx}{e^{\frac{1}{3}}}$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))/(c + d*x)**(1/3),x)/e**
(1/3)
```

3.244 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3b(c+dx)^{2/3}\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

output
```
-3*b*(d*x+c)^(2/3)*cos(a)*Ci(b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(2/3)+3*(d*x+c
)*sin(a+b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(2/3)+3*b*(d*x+c)^(2/3)*sin(a)*Si(b
/(d*x+c)^(1/3))/d/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\frac{3\Big(-b(c+dx)^{2/3}\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+(c+dx)\sin\Big(a+\frac{b}{\sqrt[3]{c+}}}{d(e(c+dx))^{2/3}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

output
```
(3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)]) + (c + d*x)
*Sin[a + b/(c + d*x)^(1/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c +
d*x)^(1/3)]))/(d*(e*(c + d*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3912, 30, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \frac{(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e(c+dx))^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{30}$$

$$-\frac{3(c+dx)^{2/3}\int (c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\int (c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3778$$

$$-\frac{3(c+dx)^{2/3}\left(b\int \sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3042$$

$$-\frac{3(c+dx)^{2/3}\left(b\int \sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3784$$

$$-\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\int \sqrt[3]{c+dx}\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int \sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}\right)\right.}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3042$$

$$-\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\int \sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int \sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+}}\right.\right.}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3780$$

$$-\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\int \sqrt[3]{c+dx}\sin\left(\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\sqrt[3]{c+dx}\sin\left(a\right.\right.}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3783$$

$$-\frac{3(c+dx)^{2/3}\left(b\left(\cos(a)\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)-\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)-\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d(e(c+dx))^{2/3}}$$

input
```
Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*(-((c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)]) + b*(Co
s[a]*CosIntegral[b/(c + d*x)^(1/3)] - Sin[a]*SinIntegral[b/(c + d*x)^(1/3)
])))/(d*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

output
```
integral(sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*e*x + c*e)^(2
/3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\left(e\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx =$$

$$-3\left(\left(\left(\operatorname{Ei}\left(ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\operatorname{Ei}\left(-ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\operatorname{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\operatorname{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos\left(a\right)+\left(i\operatorname{Ei}\left(ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right.\right.\right.\right.$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

output
```
-3/4*(((Ei(I*b*conjugate((d*x + c)^(-1/3))) + Ei(-I*b*conjugate((d*x + c)^
(-1/3))) + Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I
*Ei(I*b*conjugate((d*x + c)^(-1/3))) - I*Ei(-I*b*conjugate((d*x + c)^(-1/3
))) + I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b*e^
(1/3) - 4*(d*x + c)^(1/3)*e^(1/3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3)))/(d*e)
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

output

```
integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3),x)
```

output

```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx=\frac{\int \frac{\sin \left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{2}{3}}}dx}{e^{\frac{2}{3}}}$$

input

```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

output

```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))/(c + d*x)**(2/3),x)/e**
(2/3)
```

3.245 $\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}} \, dx=\frac{3 \sqrt[3]{c+dx} \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde \sqrt[3]{e(c+dx)}}$$

output
```
3*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/b/d/e/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}} \, dx=\frac{3(c+dx)^{4/3} \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bd(e(c+dx))^{4/3}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

output
```
(3*(c + d*x)^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3912, 30, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \frac{(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e(c+dx))^{4/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{30}$$

$$-\frac{3\sqrt[3]{c+dx}\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{c+dx}\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3118}$$

$$\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

input
```
Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

output
```
(3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\,(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)}{bd^2e^2x+bcde^2}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

output
```
3*(d*e*x + c*e)^(2/3)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b
)/(d*x + c))/(b*d^2*e^2*x + b*c*d*e^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = \frac{3\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{bde^{\frac{4}{3}}}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

output
```
3*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(b*d*e^(4/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{4/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx=\frac{3\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{e^{\frac{4}{3}}bd}$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
(3*cos(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3)))/(e**(1/3)*b*d*e)
```

3.246 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

output
```
3*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/b/d/e/(e*(d*x+c))^(2/3)-3*(d*x+c)^(
2/3)*sin(a+b/(d*x+c)^(1/3))/b^2/d/e/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{5/3}\left(\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}-\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}\right)}{d(e(c+dx))^{5/3}}$$

input

```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

output

```
(3*(c + d*x)^(5/3)*(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(1/3)) - Sin[a
 + b/(c + d*x)^(1/3)]/b^2))/(d*(e*(c + d*x))^(5/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3912, 30, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}dx$$

$$\Big\downarrow \text{3912}$$

$$-\frac{3\int \frac{(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e(c+dx))^{5/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \text{30}$$

$$-\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{\int \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{de(e(c+dx))^{2/3}}$$

input
```
Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*(-(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(1/3))) + S
in[a + b/(c + d*x)^(1/3)]/b^2))/(d*e*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\left((dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\sin}{b^2d^2e^2x+b^2cde^2}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

output
```
3*((d*e*x + c*e)^(1/3)*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3
)*b)/(d*x + c)) - (d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*sin((a*d*x + a*c + (
d*x + c)^(2/3)*b)/(d*x + c)))/(b^2*d^2*e^2*x + b^2*c*d*e^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e\,(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx =$$

$$-\frac{3\left(4\,b^2\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-(dx+c)^{\frac{2}{3}}\left(\left(-i\,\Gamma\left(3,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\Gamma\left(3,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\Gamma\left(3,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+\right.\right.\right.}{8}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

output
```
-3/8*(4*b^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - (d*x + c)^(2/3)
*((-I*gamma(3, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(3, -I*b*conjugat
e((d*x + c)^(-1/3))) - I*gamma(3, I*b/(d*x + c)^(1/3)) + I*gamma(3, -I*b/(
d*x + c)^(1/3)))*cos(a) - (gamma(3, I*b*conjugate((d*x + c)^(-1/3))) + gam
ma(3, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(3, I*b/(d*x + c)^(1/3)) +
gamma(3, -I*b/(d*x + c)^(1/3)))*sin(a)))/((d*x + c)^(2/3)*b^2*d*e^(5/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\cos \left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)b-3(dx+c)^{\frac{1}{3}}\sin \left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{e^{\frac{5}{3}}\,(dx+c)^{\frac{1}{3}}\,b^{2}d}$$

input

```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

output

```
(3*(cos(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))*b - (c + d*x)**(1/3)*si
n(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))))/(e**(2/3)*(c + d*x)**(1/3)*
b**2*d*e)
```

3.247 $\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\begin{aligned}\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}} \, dx = & -\frac{18 \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3 d e^2 \sqrt[3]{e(c+dx)}} \\ & + \frac{3 \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b d e^2 (c+dx)^{2/3} \sqrt[3]{e(c+dx)}} - \frac{9 \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2 d e^2 \sqrt[3]{c+dx} \sqrt[3]{e(c+dx)}} \\ & + \frac{18 \sqrt[3]{c+dx} \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4 d e^2 \sqrt[3]{e(c+dx)}}\end{aligned}$$

output

```
-18*cos(a+b/(d*x+c)^(1/3))/b^3/d/e^2/(e*(d*x+c))^(1/3)+3*cos(a+b/(d*x+c)^(
1/3))/b/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)-9*sin(a+b/(d*x+c)^(1/3))/b^2
/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(1/3)+18*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1
/3))/b^4/d/e^2/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{(c+dx)^{2/3}\left(3b\sqrt[3]{c+dx}\left(-6c-6dx+b^2\sqrt[3]{c+dx}\right)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+9(}{b^4d(e(c+dx))^{7/3}}$$

input

```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

output

```
((c + d*x)^(2/3)*(3*b*(c + d*x)^(1/3)*(-6*c - 6*d*x + b^2*(c + d*x)^(1/3))
*Cos[a + b/(c + d*x)^(1/3)] + 9*(c + d*x)*(-b^2 + 2*(c + d*x)^(2/3))*Sin[a
 + b/(c + d*x)^(1/3)]))/(b^4*d*(e*(c + d*x))^(7/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {3912, 30, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}dx$$

$$\Big\downarrow \; 3912$$

$$-\frac{3\int \frac{(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e(c+dx))^{7/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \; 30$$

$$-\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{c+dx}d\frac{1}{\sqrt[3]{c+dx}}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{c+dx}d\frac{1}{\sqrt[3]{c+dx}}}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \ 3777$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\int \frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \ 3777$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\left(\frac{2\int -\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{b}+\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}-\frac{2\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}-\frac{2\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}-\frac{2\left(\frac{\int\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}-\frac{2\left(\frac{\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow 3117$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{3\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}-\frac{2\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

input

```
Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

output

```
(-3*(c + d*x)^(1/3)*(-(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x))) + (3*(Sin
[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(2/3)) - (2*(-(Cos[a + b/(c + d*x)^(1
/3)]/(b*(c + d*x)^(1/3))) + Sin[a + b/(c + d*x)^(1/3)]/b^2))/b))/b))/(d*e^
2*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.93

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\left(\left((dx+c)^{\frac{1}{3}}b^3-6\,bdx-6\,bc\right)(dex+ce)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-3\,(dex\right.}{b^4d^3e^3x^2+2\,b^4cd^2e^3x+}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

output
```
3*(((d*x + c)^(1/3)*b^3 - 6*b*d*x - 6*b*c)*(d*e*x + c*e)^(2/3)*cos((a*d*x
+ a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 3*(d*e*x + c*e)^(2/3)*((d*x + c)^(
2/3)*b^2 - 2*(d*x + c)^(4/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x +
 c)))/(b^4*d^3*e^3*x^2 + 2*b^4*c*d^2*e^3*x + b^4*c^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 1389, normalized size of antiderivative = 8.08

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

output

```
-3/16*(2*(cos(a)^2 + sin(a)^2)*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)) - 2*(b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2*sin(a) + b^4
*sin(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*cos((2*(d*x + c)^(
1/3)*a + b)/(d*x + c)^(1/3)) + 2*(b^4*cos(a)*cos(((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3))^2 + b^4*cos(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)^2)*sin((2*(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + ((((-I*gamma(5, I*b*
conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))
) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*co
s(a)^3 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjug
ate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*
x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d*x + c)^(-1/
3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x
+ c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 - (gamma(5
, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)^(-1
/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))*si
n(a)^3)*d*x + ((-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5,
-I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*ga
mma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)
^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x
 + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*ga...
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx=\frac{-18(dx+c)^{\frac{2}{3}}\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)b+3\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)b^3-9(dx+c)^{\frac{1}{3}}\sin}{e^{\frac{7}{3}}b^4d}$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

output

```
(3*( - 6*(c + d*x)**(2/3)*cos(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))*b
 + cos(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))*b**3 - 3*(c + d*x)**(1/3
)*sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))*b**2 + 6*sin(((c + d*x)**
(1/3)*a + b)/(c + d*x)**(1/3))*c + 6*sin(((c + d*x)**(1/3)*a + b)/(c + d*x
)**(1/3))*d*x - 6*a*c - 6*a*d*x))/(e**(1/3)*b**4*d*e**2*(c + d*x))
```

3.248 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 217

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx &= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} \\ &+ \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} + \frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}} \\ &- \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}\end{aligned}$$

output
```
-36*cos(a+b/(d*x+c)^(1/3))/b^3/d/e^2/(e*(d*x+c))^(2/3)+3*cos(a+b/(d*x+c)^(
1/3))/b/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(2/3)+72*(d*x+c)^(2/3)*cos(a+b/(d*
x+c)^(1/3))/b^5/d/e^2/(e*(d*x+c))^(2/3)-12*sin(a+b/(d*x+c)^(1/3))/b^2/d/e^
2/(d*x+c)^(1/3)/(e*(d*x+c))^(2/3)+72*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/
b^4/d/e^2/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.52

$$\int \frac{\sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx=\frac{(c+dx)^{4/3}\left(3\left(b^4-12b^2(c+dx)^{2/3}+24(c+dx)^{4/3}\right)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+\right.}{b^5d(e(c+dx))^{8/3}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

output
```
((c + d*x)^(4/3)*(3*(b^4 - 12*b^2*(c + d*x)^(2/3) + 24*(c + d*x)^(4/3))*Co
s[a + b/(c + d*x)^(1/3)] + 12*b*(6*c + 6*d*x - b^2*(c + d*x)^(1/3))*Sin[a
+ b/(c + d*x)^(1/3)]))/(b^5*d*(e*(c + d*x))^(8/3))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.80, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {3912, 30, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}dx$$

$$\Big\downarrow \; 3912$$

$$-\frac{3\int \frac{(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e(c+dx))^{8/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d}$$

$$\Big\downarrow \; 30$$

$$-\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{4/3}}d\frac{1}{\sqrt[3]{c+dx}}}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{4/3}}d\frac{1}{\sqrt[3]{c+dx}}}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\int \frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{c+dx}d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)}{c+dx}d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{3\int -\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{b}+\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\left(\frac{2\int\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\left(\frac{2\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}d\frac{1}{\sqrt[3]{c+dx}}}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{4/3}}\right.}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \textcolor{blue}{3777}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\left(\frac{2\left(\frac{\int -\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}+\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}\right)}{b}\right.}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\left(\frac{2\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}-\frac{\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}\right)}{b}-\right.}{de^2(e(c+dx))^{2/3}}$$

$\Big\downarrow$ 3042

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\left(\frac{2\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}-\frac{\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{b}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}\right)}{b}-\right.}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow 3118$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{4\left(\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)}-\frac{3\left(\frac{2\left(\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}+\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}}\right)}{b}-\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b(c+dx)^{2/3}}\right)}{b}\right)}{b}-\frac{\cos\left(a+\frac{}{\sqrt[3]{c}}\right.}{b(c+dx}\right.}{de^2(e(c+dx))^{2/3}}$$

input
```
Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*(-(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(4/3))) + (
4*(Sin[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)) - (3*(-(Cos[a + b/(c + d*x)^(1
/3)]/(b*(c + d*x)^(2/3))) + (2*(Cos[a + b/(c + d*x)^(1/3)]/b^2 + Sin[a + b
/(c + d*x)^(1/3)]/(b*(c + d*x)^(1/3))))/b))/b))/b))/(d*e^2*(e*(c + d*x))^(
2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.83

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{3\left(\left((dx+c)^{\frac{1}{3}}b^4-12\,b^2dx-12\,b^2c+24\,(dx+c)^{\frac{5}{3}}\right)(dex+ce)^{\frac{1}{3}}\cos\left(\frac{adx+ac+}{d}\right.}{b^5d^3e^3x^2}$$

input
```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

output
```
3*(((d*x + c)^(1/3)*b^4 - 12*b^2*d*x - 12*b^2*c + 24*(d*x + c)^(5/3))*(d*e
*x + c*e)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 4*((d*x
 + c)^(2/3)*b^3 - 6*(b*d*x + b*c)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*sin
((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)))/(b^5*d^3*e^3*x^2 + 2*b^5*c*
d^2*e^3*x + b^5*c^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(8/3),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 1943, normalized size of antiderivative = 8.95

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

output

```
-3/20*(2*((cos(a)^2 + sin(a)^2)*b^5*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^
(1/3)) - (b^5*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2*sin(a) + b^5*
sin(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*cos((2*(d*x + c)^(1
/3)*a + b)/(d*x + c)^(1/3)) + (b^5*cos(a)*cos(((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3))^2 + b^5*cos(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2
)*sin((2*(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*(d*x + c)^(1/3)*e^(1/3)
- ((((gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate(
(d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x +
c)^(1/3)))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*ga
mma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3))
 + I*gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (gamma(6, I*b*conju
gate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gam
ma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)
^2 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conju
gate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*
b/(d*x + c)^(1/3)))*sin(a)^3)*d^2*x^2 + 2*((gamma(6, I*b*conjugate((d*x +
c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d
*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I*gamma(6, I
*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/
3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3...
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
```

output

```
integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{8/3}}\,dx$$

input

```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(8/3),x)
```

output

```
int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(8/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx=\frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{2}{3}}c^2+2(dx+c)^{\frac{2}{3}}cdx+(dx+c)^{\frac{2}{3}}d^2x^2}dx}{e^{\frac{8}{3}}}$$

input

```
int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

output

```
int(sin(((c + d*x)**(1/3)*a + b)/(c + d*x)**(1/3))/((c + d*x)**(2/3)*c**2
+ 2*(c + d*x)**(2/3)*c*d*x + (c + d*x)**(2/3)*d**2*x**2),x)/(e**(2/3)*e**2
)
```

3.249 $\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 299

$$\begin{aligned}
\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx &= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
&\quad+\frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
&\quad-\frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+dx)}\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}} \\
&\quad-\frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+dx)}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{35d\sqrt[3]{c+dx}} \\
&\quad-\frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} \\
&\quad+\frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d}
\end{aligned}$$

output
```
-8/35*b^3*e*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+6/35*b*e*(d*x+c)^(4
/3)*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d-8/35*b^(7/2)*e*2^(1/2)*Pi^(
1/2)*(e*(d*x+c))^(1/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1
/3))/d/(d*x+c)^(1/3)-8/35*b^(7/2)*e*2^(1/2)*Pi^(1/2)*(e*(d*x+c))^(1/3)*Fre
snelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d/(d*x+c)^(1/3)-4/35*
b^2*e*(d*x+c)^(2/3)*(e*(d*x+c))^(1/3)*sin(a+b/(d*x+c)^(2/3))/d+3/7*e*(d*x+
c)^2*(e*(d*x+c))^(1/3)*sin(a+b/(d*x+c)^(2/3))/d
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.79

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{(e(c+dx))^{4/3}\left(\frac{\cos\left(\frac{b}{(c+dx)^{2/3}}\right)\left(-8b^3\cos(a)+6b(c+dx)^{4/3}\cos(a)-4b^2(c+dx)^{2/3}\sin(a)+15(c+dx)^2\sin(a)\right)}{c+dx}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output
```
((e*(c + d*x))^(4/3)*((Cos[b/(c + d*x)^(2/3)]*(-8*b^3*Cos[a] + 6*b*(c + d*
x)^(4/3)*Cos[a] - 4*b^2*(c + d*x)^(2/3)*Sin[a] + 15*(c + d*x)^2*Sin[a]))/(
c + d*x) - (8*b^(7/2)*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
+ d*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(c
 + d*x)^(4/3) + ((-4*b^2*(c + d*x)^(2/3)*Cos[a] + 15*(c + d*x)^2*Cos[a] +
8*b^3*Sin[a] - 6*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(2/3)])/(c + d*
x)))/(35*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {3916, 3898, 3896, 3890, 3868, 3869, 3868, 3869, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int (e(c+dx))^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{e\sqrt[3]{e(c+dx)}\int (c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d(c+dx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3896}$$

$$\frac{3e\sqrt[3]{e(c+dx)}\int (c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\sqrt[3]{c+dx}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3890}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{8/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{2}{7}b\int \frac{\cos\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^2}d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{7(c+dx)^{7/3}}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\left(\frac{2}{7}b\left(-\frac{2}{5}b\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{4/3}}d\frac{1}{\sqrt[3]{c+dx}}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{5(c+dx)^{5/3}}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{7(c+dx)^{7/3}}\right)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{2}{7}b\Bigg(-\frac{2}{5}b\Bigg(\frac{2}{3}b\int\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{3(c+dx)}\Bigg)-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{5(c+dx)^{5/3}}\Bigg)-\frac{\sin(a+b(c+}{7(c+dx}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{2}{7}b\Bigg(-\frac{2}{5}b\Bigg(\frac{2}{3}b\Bigg(-2b\int\sin\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\Bigg)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{3(c+dx)}\Bigg)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3834}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{2}{7}b\Bigg(-\frac{2}{5}b\Bigg(\frac{2}{3}b\Bigg(-2b\Bigg(\sin(a)\int\cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int\sin\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+}}}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{2}{7}b\Bigg(-\frac{2}{5}b\Bigg(\frac{2}{3}b\Bigg(-2b\Bigg(\sin(a)\int\cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\Bigg)-}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{3e\sqrt[3]{e(c+dx)}\Bigg(\frac{2}{7}b\Bigg(-\frac{2}{5}b\Bigg(\frac{2}{3}b\Bigg(-2b\Bigg(\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\Bigg)-\frac{\cos(a+b(c}{\sqrt[3]{c+}}}{d\sqrt[3]{c+dx}}$$

input

```
Int[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output

```
(-3*e*(e*(c + d*x))^(1/3)*(-1/7*Sin[a + b*(c + d*x)^(2/3)]/(c + d*x)^(7/3)
 + (2*b*(-1/5*Cos[a + b*(c + d*x)^(2/3)]/(c + d*x)^(5/3) - (2*b*((2*b*(-(C
os[a + b*(c + d*x)^(2/3)]/(c + d*x)^(1/3)) - 2*b*((Sqrt[Pi/2]*Cos[a]*Fresn
elS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/Sqrt[b] + (Sqrt[Pi/2]*FresnelC[
(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/Sqrt[b])))/3 - Sin[a + b*(c
+ d*x)^(2/3)]/(3*(c + d*x))))/5))/7))/(d*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3890
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

input
```
int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{4/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \int (dex+ce)^{\frac{4}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(4/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x +
c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\text{Timed out}$$

input

```
integrate((d*e*x+c*e)**(4/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

output

```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 1120, normalized size of antiderivative = 3.75

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\text{Too large to display}$$

input

```
integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
-3/8*((((-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, -
I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (I*gamma(-7/
2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-7/2, I*b/(d*x + c)^(2/3)))
*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, I*b*conjugate((d*x
+ c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan
2(0, d*x + c)) - (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7
/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) -
 ((gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x +
c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, -I*b*conju
gate((d*x + c)^(-2/3))) + gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi +
7/3*arctan2(0, d*x + c)) - (-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))
) + I*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x +
 c)) - (-I*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-7/2, I
*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*d^2*e
^(4/3)*x^2 + 2*(((-I*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamm
a(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (I*
gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-7/2, I*b/(d*x + c
)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(-7/2, I*b*conjug
ate((d*x + c)^(-2/3))) + gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7
/3*arctan2(0, d*x + c)) - (gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))...
```

Giac [F]

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (dex+ce)^{\frac{4}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

input
```
integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(4/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)\,{\left(c\,e+d\,e\,x\right)}^{4/3}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3), x)
```

Reduce [F]

$$\int (ce+dex)^{4/3}\sin\Bigg(a+\frac{b}{(c+dx)^{2/3}}\Bigg)\,dx=e^{\frac{4}{3}}\Bigg(\Bigg(\Bigg(\int (dx+c)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)xdx\Bigg)d+\Bigg(\int (dx+c)^{\frac{1}{3}}\sin\Bigg(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}$$

input
```
int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
e**(1/3)*e*(int((c + d*x)**(1/3)*sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(
2/3))*x,x)*d + int((c + d*x)**(1/3)*sin(((c + d*x)**(2/3)*a + b)/(c + d*x)
**(2/3)),x)*c)
```

3.250 $\int (ce+dex)^{2/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 262

$$\begin{aligned}
&\int (ce+dex)^{2/3} \sin\Bigg(a \\
&+\frac{b}{(c+dx)^{2/3}}\Bigg)\,dx=\frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} \\
&+\frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}} \\
&-\frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{5d(c+dx)^{2/3}} \\
&-\frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}} \\
&+\frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}
\end{aligned}$$

output
```
2/5*b*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*cos(a+b/(d*x+c)^(2/3))/d+4/5*2^(1/2)
*b^(5/2)*Pi^(1/2)*(e*(d*x+c))^(2/3)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)/(d*x+c)^(1/3))/d/(d*x+c)^(2/3)-4/5*2^(1/2)*b^(5/2)*Pi^(1/2)*(e*(d*x+c))
^(2/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d/(d*x+c)^(
2/3)-4/5*b^2*(e*(d*x+c))^(2/3)*sin(a+b/(d*x+c)^(2/3))/d/(d*x+c)^(1/3)+3/5*
(d*x+c)*(e*(d*x+c))^(2/3)*sin(a+b/(d*x+c)^(2/3))/d
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.87

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{(e(c+dx))^{2/3}\left(2bc\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2bdx\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+4b^{5/2}\sqrt{2\pi}\cos(a)\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output
```
((e*(c + d*x))^(2/3)*(2*b*c*Cos[a + b/(c + d*x)^(2/3)] + 2*b*d*x*Cos[a + b
/(c + d*x)^(2/3)] + 4*b^(5/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)^(1/3)] - 4*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/
(c + d*x)^(1/3)]*Sin[a] - 4*b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)]
 + 3*c*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] + 3*d*x*(c + d*x)^(2/3)*
Sin[a + b/(c + d*x)^(2/3)]))/(5*d*(c + d*x)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3916, 3898, 3896, 3890, 3868, 3869, 3868, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$$

$$\Big\downarrow \text{ 3916}$$

$$\frac{\int (e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{ 3898}$$

$$\frac{(e(c+dx))^{2/3}\int (c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d(c+dx)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3896}$$

$$\frac{3(e(c+dx))^{2/3}\int (c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\sqrt[3]{c+dx}}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3890}$$

$$-\frac{3(e(c+dx))^{2/3}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^2}d\frac{1}{\sqrt[3]{c+dx}}}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3868}$$

$$-\frac{3(e(c+dx))^{2/3}\left(\frac{2}{5}b\int \frac{\cos\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{4/3}}d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{5(c+dx)^{5/3}}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3869}$$

$$-\frac{3(e(c+dx))^{2/3}\left(\frac{2}{5}b\left(-\frac{2}{3}b\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{3(c+dx)}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{5(c+dx)^{5/3}}\right)}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3868}$$

$$-\frac{3(e(c+dx))^{2/3}\left(\frac{2}{5}b\left(-\frac{2}{3}b\left(2b\int \cos\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{3(c+dx)}\right)-\mathrm{si}\right.}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3835}$$

$$-\frac{3(e(c+dx))^{2/3}\left(\frac{2}{5}b\left(-\frac{2}{3}b\left(2b\left(\cos(a)\int \cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int \sin\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}\right)\right.\right.\right.}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{3(e(c+dx))^{2/3}\left(\frac{2}{5}b\left(-\frac{2}{3}b\left(2b\left(\cos(a)\int\cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)-\frac{\sin(a}{3}\right.\right.\right.\right.}{d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{3(e(c+dx))^{2/3}\left(\frac{2}{5}b\left(-\frac{2}{3}b\left(2b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)-\frac{\sin(a+b(c+dx)^{2}}{\sqrt[3]{c+dx}}\right.\right.\right.}{d(c+dx)^{2/3}}$$

input
```
Int[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output
```
(-3*(e*(c + d*x))^(2/3)*(-1/5*Sin[a + b*(c + d*x)^(2/3)]/(c + d*x)^(5/3) +
 (2*b*(-1/3*Cos[a + b*(c + d*x)^(2/3)]/(c + d*x) - (2*b*(2*b*((Sqrt[Pi/2]*
Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/Sqrt[b] - (Sqrt[Pi/
2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/Sqrt[b]) - Sin[a
 + b*(c + d*x)^(2/3)]/(c + d*x)^(1/3)))/3))/5))/(d*(c + d*x)^(2/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835
```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3890
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int (dex+ce)^{\frac{2}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

input
```
int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int (ce+dex)^{2/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \int (dex+ce)^{\frac{2}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x +
c)), x)
```

Sympy [F]

$$\int (ce+dex)^{2/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \int (e(c+dx))^{\frac{2}{3}} \sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

input
```
integrate((d*e*x+c*e)**(2/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

output
```
Integral((e*(c + d*x))**(2/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.86

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \text{Too large to display}$$

input

```
integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

output

```
-3/8*((((-I*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, -
I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (I*gamma(-5/
2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-5/2, I*b/(d*x + c)^(2/3)))
*cos(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, I*b*conjugate((d*x
+ c)^(-2/3))) + gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan
2(0, d*x + c)) - (gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-5
/2, I*b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*cos(a) -
 ((gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-5/2, -I*b/(d*x +
c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, -I*b*conju
gate((d*x + c)^(-2/3))) + gamma(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi +
5/3*arctan2(0, d*x + c)) - (-I*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))
) + I*gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x +
 c)) - (-I*gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2, I
*b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*sin(a))*d*e^(
2/3)*x + (((-I*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-5/2
, -I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (I*gamma(
-5/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-5/2, I*b/(d*x + c)^(2/3
)))*cos(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(-5/2, I*b*conjugate((d
*x + c)^(-2/3))) + gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arc
tan2(0, d*x + c)) - (gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + ga...
```

Giac [F]

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int (dex+ce)^{\frac{2}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)\,dx$$

input
```
integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(2/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(c\,e+d\,e\,x)^{2/3}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3), x)
```

Reduce [F]

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=e^{\frac{2}{3}}\left(\int (dx+c)^{\frac{2}{3}}\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)dx\right)$$

input
```
int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
e**(2/3)*int((c + d*x)**(2/3)*sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3
)),x)
```

3.251 $\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\begin{aligned}\int \sqrt[3]{ce+dex}\sin\Bigg(a&\\ &+\frac{b}{(c+dx)^{2/3}}\Bigg)\,dx=\frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}\\ &+\frac{3b^2\sqrt[3]{e(c+dx)}\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)}{4d\sqrt[3]{c+dx}}\\ &+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}+\frac{3b^2\sqrt[3]{e(c+dx)}\cos(a)\operatorname{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}\end{aligned}$$

output
```
3/4*b*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+3/4*b^2*(e*
(d*x+c))^(1/3)*Ci(b/(d*x+c)^(2/3))*sin(a)/d/(d*x+c)^(1/3)+3/4*(d*x+c)*(e*(
d*x+c))^(1/3)*sin(a+b/(d*x+c)^(2/3))/d+3/4*b^2*(e*(d*x+c))^(1/3)*cos(a)*Si
(b/(d*x+c)^(2/3))/d/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{ce+dex}\sin \Bigg(a+\frac{b}{(c+dx)^{2/3}}\Bigg)\,dx = \frac{3\sqrt[3]{e(c+dx)}\Big(b(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b^2\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)+(c}{4d\sqrt[3]{c+dx}}$$

input

```
Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output

```
(3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)] + b^2
*CosIntegral[b/(c + d*x)^(2/3)]*Sin[a] + (c + d*x)^(4/3)*Sin[a + b/(c + d*
x)^(2/3)] + b^2*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]))/(4*d*(c + d*x)^(1/
3))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3916, 3862, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{\sqrt[3]{e(c+dx)}\int \sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d(c+dx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\int (c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{2}b\int (c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}-\frac{1}{2}(c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{2}b\int (c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}-\frac{1}{2}(c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{2}b\left(b\int -(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}-(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)-\frac{1}{2}(c+d}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{2}b\left(-b\int (c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}-(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)-\frac{1}{2}(c+d}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{2}b\left(-b\int (c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}-(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)-\frac{1}{2}(c+d}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int (c+dx)^{2/3}\cos\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}+\cos(a)\int (c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{}{(c+}\right.\right.\right.}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\frac{1}{2}b\Big(-b\Big(\sin(a)\int(c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}+\cos(a)\int(c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \textcolor{blue}{3780}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\frac{1}{2}b\Big(-b\Big(\sin(a)\int(c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}+\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)\Big)-(c+dx)^{2/3}}{2d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \textcolor{blue}{3783}$$

$$-\frac{3\sqrt[3]{e(c+dx)}\Big(\frac{1}{2}b\Big(-b\Big(\sin(a)\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)+\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)\Big)-(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{c+dx}}$$

input
```
Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

output
```
(-3*(e*(c + d*x))^(1/3)*(-1/2*((c + d*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)])
 + (b*(-((c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)]) - b*(CosIntegral[b/(c
 + d*x)^(2/3)]*Sin[a] + Cos[a]*SinIntegral[b/(c + d*x)^(2/3)])))/2))/(2*d*
(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int (dex+ce)^{\frac{1}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

input
```
int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Fricas [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \int (dex+ce)^{\frac{1}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x +
c)), x)
```

Sympy [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \int \sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

input
```
integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

output
```
Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77

$$\int \sqrt[3]{ce+dex}\sin\left(a + \frac{b}{(c+dx)^{2/3}}\right)dx = \frac{3\left(\left(-i\,\Gamma\left(-2, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right) + i\,\Gamma\left(-2, -i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right) - i\,\Gamma\left(-2, \frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right) + i\,\Gamma\left(-2, -\right.\right.\right.}{}$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

output
```
3/8*((-I*gamma(-2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-2, -I*b*con
jugate((d*x + c)^(-2/3))) - I*gamma(-2, I*b/(d*x + c)^(2/3)) + I*gamma(-2,
 -I*b/(d*x + c)^(2/3)))*cos(a) - (gamma(-2, I*b*conjugate((d*x + c)^(-2/3)
)) + gamma(-2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-2, I*b/(d*x + c)
^(2/3)) + gamma(-2, -I*b/(d*x + c)^(2/3)))*sin(a))*b^2*e^(1/3)/d
```

Giac [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a + \frac{b}{(c+dx)^{2/3}}\right)dx = \int (dex+ce)^{\frac{1}{3}}\sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

input
```
integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(1/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(c\,e+d\,e\,x)^{1/3}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx=e^{\frac{1}{3}}\left(\int (dx+c)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)dx\right)$$

input
```
int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

output
```
e**(1/3)*int((c + d*x)**(1/3)*sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3
)),x)
```

3.252 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3b\sqrt[3]{c+dx}\cos(a)\operatorname{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b\sqrt[3]{c+dx}\sin(a)\operatorname{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

output
```
-3/2*b*(d*x+c)^(1/3)*cos(a)*Ci(b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(1/3)+3/2*(d
*x+c)*sin(a+b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(1/3)+3/2*b*(d*x+c)^(1/3)*sin(a
)*Si(b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{3\left(-b\sqrt[3]{c+dx}\cos(a)\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b\right.}{2d\sqrt[3]{e(c+dx)}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

output
```
(3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)]) + (c + d*x)
*Sin[a + b/(c + d*x)^(2/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c +
d*x)^(2/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3916, 3862, 3860, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{c+dx}}d(c+dx)}{d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{3\sqrt[3]{c+dx}\int (c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt[3]{c+dx}\int (c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{3\sqrt[3]{c+dx}\left(b\int (c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}-(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt[3]{c+dx}\left(b\int (c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}-(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3784}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigl(b\Bigl(\cos(a)\int (c+dx)^{2/3}\cos\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}-\sin(a)\int (c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}\Bigr)-}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigl(b\Bigl(\cos(a)\int (c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}-\sin(a)\int (c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3780}$$

$$-\frac{3\sqrt[3]{c+dx}\Bigl(b\Bigl(\cos(a)\int (c+dx)^{2/3}\sin\left(\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}-\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)\Bigr)-(c+dx)^{2/3}\sin\Bigl(a+}{2d\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3783}$$

$$-\frac{3\sqrt[3]{c+dx}\left(b\left(\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)-\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)\right)-(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{e(c+dx)}}$$

input
```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

output
```
(-3*(c + d*x)^(1/3)*(-((c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)]) + b*(Co
s[a]*CosIntegral[b/(c + d*x)^(2/3)] - Sin[a]*SinIntegral[b/(c + d*x)^(2/3)
])))/(2*d*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

output
```
integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*e*x + c*e)^(1
/3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx=$$

$$-\frac{3\left(\left(\Gamma\left(-1,i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-1,-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-1,\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1,-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(a\right)+\left(-i\right.\right.}{8\,de^{\frac{1}{3}}}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

output
```
-3/8*((gamma(-1, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1, -I*b*conjuga
te((d*x + c)^(-2/3))) + gamma(-1, I*b/(d*x + c)^(2/3)) + gamma(-1, -I*b/(d
*x + c)^(2/3)))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(-2/3))) +
I*gamma(-1, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-1, I*b/(d*x + c)^
(2/3)) + I*gamma(-1, -I*b/(d*x + c)^(2/3)))*sin(a))*b/(d*e^(1/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

output

```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

input

```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3),x)
```

output

```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{(dx+c)^{\frac{1}{3}}}dx}{e^{\frac{1}{3}}}$$

input

```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

output

```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/(c + d*x)**(1/3),x)/e**
(1/3)
```

3.253 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

output

```
-3*b^(1/2)*2^(1/2)*Pi^(1/2)*(d*x+c)^(2/3)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/
Pi^(1/2)/(d*x+c)^(1/3))/d/(e*(d*x+c))^(2/3)+3*b^(1/2)*2^(1/2)*Pi^(1/2)*(d*
x+c)^(2/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/d/(e*(d
*x+c))^(2/3)+3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\left(-\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\,\mathrm{Fres}\right.}{d(e(c+dx))^{2/3}}$$

input

```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

output

```
(3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*FresnelS[(Sqrt[
b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + (c + d*x)*Sin[a + b/(c + d*x)^(2/
3)]))/(d*(e*(c + d*x))^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3916, 3898, 3896, 3840, 3868, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e(c+dx))^{2/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{2/3}}d(c+dx)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3896}$$

$$\frac{3(c+dx)^{2/3}\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\sqrt[3]{c+dx}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 3840}$$

$$-\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(c+dx)^{2/3}}d\frac{1}{\sqrt[3]{c+dx}}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 3868}$$

$$-\frac{3(c+dx)^{2/3}\left(2b\int \cos\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 3835}$$

$$-\frac{3(c+dx)^{2/3}\Bigg(2b\left(\cos(a)\int \cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int \sin\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}\right)-\frac{\sin(a+b(c+a}{\sqrt[3]{c+}}}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 3832}$$

$$-\frac{3(c+dx)^{2/3}\left(2b\left(\cos(a)\int \cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 3833}$$

$$-\frac{3(c+dx)^{2/3}\left(2b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)-\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

input

```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

output

```
(-3*(c + d*x)^(2/3)*(2*b*((Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])
/(c + d*x)^(1/3)])/Sqrt[b] - (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
+ d*x)^(1/3)]*Sin[a])/Sqrt[b]) - Sin[a + b*(c + d*x)^(2/3)]/(c + d*x)^(1/3
)))/(d*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835
```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3840
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[-f^(-1)   Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(
e + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &
& EqQ[n, -2]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}dx$$

input

```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

output

```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

output

```
integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*e*x + c*e)^(2
/3), x)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{2}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
```

output

```
Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.34

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

output

```
-3/8*(d*x + c)^(1/3)*(((-I*gamma(-1/2, I*b*conjugate((d*x + c)^(-2/3))) +
I*gamma(-1/2, -I*b/(d*x + c)^(2/3)))*cos(1/4*pi + 1/3*arctan2(0, d*x + c))
 + (I*gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-1/2, I*b/(d
*x + c)^(2/3)))*cos(-1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, I*b*
conjugate((d*x + c)^(-2/3))) + gamma(-1/2, -I*b/(d*x + c)^(2/3)))*sin(1/4*
pi + 1/3*arctan2(0, d*x + c)) - (gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/
3))) + gamma(-1/2, I*b/(d*x + c)^(2/3)))*sin(-1/4*pi + 1/3*arctan2(0, d*x
+ c)))*cos(a) - ((gamma(-1/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1/
2, -I*b/(d*x + c)^(2/3)))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-
1/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1/2, I*b/(d*x + c)^(2/3)))
*cos(-1/4*pi + 1/3*arctan2(0, d*x + c)) - (-I*gamma(-1/2, I*b*conjugate((d
*x + c)^(-2/3))) + I*gamma(-1/2, -I*b/(d*x + c)^(2/3)))*sin(1/4*pi + 1/3*a
rctan2(0, d*x + c)) - (-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) +
I*gamma(-1/2, I*b/(d*x + c)^(2/3)))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c))
)*sin(a))*sqrt(b/(d*x + c)^(2/3))/(d*e^(2/3))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

output

```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

input

```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3),x)
```

output

```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx=\frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{(dx+c)^{\frac{2}{3}}}dx}{e^{\frac{2}{3}}}$$

input

```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

output

```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/(c + d*x)**(2/3),x)/e**
(2/3)
```

3.254 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{\pi}\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}$$

output

```
-3/2*Pi^(1/2)*(d*x+c)^(1/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c)^(1/3))*2^(1/2)/b^(1/2)/d/e/(e*(d*x+c))^(1/3)-3/2*Pi^(1/2)*(d*x+c)^(1/3)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)/b^(1/2)/d
/e/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{4/3}\left(\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)\right)}{\sqrt{b}d(e(c+dx))^{4/3}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

output
```
(-3*Sqrt[Pi/2]*(c + d*x)^(4/3)*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c +
d*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(Sqr
t[b]*d*(e*(c + d*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3916, 3898, 3864, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e(c+dx))^{4/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{4/3}}d(c+dx)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3864}$$

$$-\frac{3\sqrt[3]{c+dx}\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{\sqrt[3]{c+dx}}}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3834}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\sin(a)\int \cos\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int \sin\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\sin(a)\int \cos\left(\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{\sqrt[3]{c+dx}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}\right)}{de\sqrt[3]{e(c+dx)}}$$

input

```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

output

```
(-3*(c + d*x)^(1/3)*((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c +
 d*x)^(1/3)])/Sqrt[b] + (Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x
)^(1/3)]*Sin[a])/Sqrt[b]))/(d*e*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3864
```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Simp[2/n   Subst[
Int[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
 n/2 - 1]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x +
c))/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(4/3), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.45

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

output

```
3/8*(((-I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))) - 1) + I*sq
rt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/3*arctan2(0,
d*x + c)) + (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3)))) - 1)
- I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/3*arcta
n2(0, d*x + c)) - (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))) -
1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/3*arct
an2(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3))))
- 1) + sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi + 1/3*ar
ctan2(0, d*x + c)))*cos(a) - ((sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^
(-2/3)))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*p
i + 1/3*arctan2(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c
)^(-2/3)))) - 1) + sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4
*pi + 1/3*arctan2(0, d*x + c)) - (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x
+ c)^(-2/3)))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*si
n(1/4*pi + 1/3*arctan2(0, d*x + c)) - (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate
((d*x + c)^(-2/3)))) - 1) - I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1
))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))/((d*x + c)^(1/3)*d*e^(4
/3)*sqrt(b/(d*x + c)^(2/3)))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{4/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx=\frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{(dx+c)^{\frac{1}{3}}c+(dx+c)^{\frac{1}{3}}dx}dx}{e^{\frac{4}{3}}}$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

output
```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/((c + d*x)**(1/3)*c + (
c + d*x)**(1/3)*d*x),x)/(e**(1/3)*e)
```

3.255 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

output
```
3/2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(2/3))/b/d/e/(e*(d*x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3(c+dx)^{5/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bd(e(c+dx))^{5/3}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

output
```
(3*(c + d*x)^(5/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(5/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3916, 3862, 3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e(c+dx))^{5/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{5/3}}d(c+dx)}{de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{3(c+dx)^{2/3}\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3(c+dx)^{2/3}\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{2de(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3118}$$

$$\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

input

```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

output
```
(3*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e*(e*(c + d*x))^(2/3
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\,(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{2\,(bd^2e^2x+bcde^2)}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

output
```
3/2*(d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*cos((a*d*x + a*c + (d*x + c)^(1/3)
*b)/(d*x + c))/(b*d^2*e^2*x + b*c*d*e^2)
```

Sympy [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
```

output
```
Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(5/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3 \cos \left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{2\,bde^{\frac{5}{3}}}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

output
```
3/2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3))/(b*d*e^(5/3))
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \int \frac{\sin \left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx=\frac{3\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{2e^{\frac{5}{3}}bd}$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

output
```
(3*cos(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3)))/(2*e**(2/3)*b*d*e)
```

3.256 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}} - \frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

output
```
3/2*cos(a+b/(d*x+c)^(2/3))/b/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(1/3)-3/2*(d*
x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^2/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = -\frac{3(c+dx)^{5/3}\left(-b\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2b^2d(e(c+dx))^{7/3}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

output

```
(-3*(c + d*x)^(5/3)*(-(b*Cos[a + b/(c + d*x)^(2/3)]) + (c + d*x)^(2/3)*Sin
[a + b/(c + d*x)^(2/3)]))/(2*b^2*d*(e*(c + d*x))^(7/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3916, 3862, 3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e(c+dx))^{7/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3862}$$

$$\frac{\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{7/3}}d(c+dx)}{de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{2/3}}d\frac{1}{(c+dx)^{2/3}}}{2de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{2/3}}d\frac{1}{(c+dx)^{2/3}}}{2de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{\int \cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)d\frac{1}{(c+dx)^{2/3}}}{b}-\frac{\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{b(c+dx)^{2/3}}\right)}{2de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}+\frac{\pi}{2}\right)d\frac{1}{(c+dx)^{2/3}}}{b}-\frac{\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{b(c+dx)^{2/3}}\right)}{2de^2\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{b^2}-\frac{\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{b(c+dx)^{2/3}}\right)}{2de^2\sqrt[3]{e(c+dx)}}$$

input

```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

output

```
(-3*(c + d*x)^(1/3)*(-(Cos[a + b/(c + d*x)^(2/3)]/(b*(c + d*x)^(2/3))) + S
in[a + b/(c + d*x)^(2/3)]/b^2))/(2*d*e^2*(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3862
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Int
egerQ[Simplify[(m + 1)/n]]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\left((dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)-(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}\sin\left(\right.\right.}{2\left(b^2d^3e^3x^2+2\,b^2cd^2e^3x+b^2c^2de^3\right)}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

output
```
3/2*((d*e*x + c*e)^(2/3)*(d*x + c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(1
/3)*b)/(d*x + c)) - (d*e*x + c*e)^(2/3)*(d*x + c)^(4/3)*sin((a*d*x + a*c +
 (d*x + c)^(1/3)*b)/(d*x + c)))/(b^2*d^3*e^3*x^2 + 2*b^2*c*d^2*e^3*x + b^2
*c^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(7/3),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\left(\left(-i\,\Gamma\left(2, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right) + i\,\Gamma\left(2, -i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right) - i\,\Gamma\left(2, \frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right) + i\,\Gamma\left(2, -\right.\right.\right.}{}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

output
```
3/8*((-I*gamma(2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(2, -I*b*conju
gate((d*x + c)^(-2/3))) - I*gamma(2, I*b/(d*x + c)^(2/3)) + I*gamma(2, -I*
b/(d*x + c)^(2/3)))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(-2/3))) +
gamma(2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(2, I*b/(d*x + c)^(2/3))
 + gamma(2, -I*b/(d*x + c)^(2/3)))*sin(a))/(b^2*d*e^(7/3))
```

Giac [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\int \frac{\sin \left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(7/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(7/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx=\frac{\frac{3\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)b}{2}-\frac{3(dx+c)^{\frac{2}{3}}\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{2}}{e^{\frac{7}{3}}\,(dx+c)^{\frac{2}{3}}\,b^{2}d}$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

output
```
(3*(cos(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))*b - (c + d*x)**(2/3)*si
n(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))))/(2*e**(1/3)*(c + d*x)**(2/3
)*b**2*d*e**2)
```

3.257 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 237

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx &= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} \\ &+ \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} \\ &+ \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{4b^{5/2}de^2(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}}\end{aligned}$$

output

```
3/2*cos(a+b/(d*x+c)^(2/3))/b/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(2/3)+9/8*2^(
1/2)*Pi^(1/2)*(d*x+c)^(2/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c)^(1/3))/b^(5/2)/d/e^2/(e*(d*x+c))^(2/3)+9/8*2^(1/2)*Pi^(1/2)*(d*x+c)^(2/
3)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)/b^(5/2)/d/e^2/(
e*(d*x+c))^(2/3)-9/4*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^2/(e*(d*
x+c))^(2/3)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.70

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\frac{(c+dx)^{5/3}\left(9\sqrt{2\pi}(c+dx)\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+9\sqrt{2\pi}(c+dx)\,\text{Fresn}\right.}{8b^{5/}}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

output
```
((c + d*x)^(5/3)*(9*Sqrt[2*Pi]*(c + d*x)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)^(1/3)] + 9*Sqrt[2*Pi]*(c + d*x)*FresnelC[(Sqrt[b]*Sqrt[2/Pi]
)/(c + d*x)^(1/3)]*Sin[a] + 6*Sqrt[b]*(2*b*Cos[a + b/(c + d*x)^(2/3)] - 3*
(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(e*(c + d*x))^(
8/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3916, 3898, 3896, 3890, 3866, 3867, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e(c+dx))^{8/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{8/3}}d(c+dx)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3896}$$

$$\frac{3(c+dx)^{2/3}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^2}d\sqrt[3]{c+dx}}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3890}$$

$$-\frac{3(c+dx)^{2/3}\int (c+dx)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3866}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{3\int (c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}-\frac{(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{3\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}-\frac{\int \sin\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}\right)}{2b}-\frac{(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3834}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{3\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}-\frac{\sin(a)\int \cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}+\cos(a)\int \sin\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}\right)}{2b}-\frac{(c+dx)\cos(a+b(c-}{2b}\right.}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{3\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}-\frac{\sin(a)\int\cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}}{2b}\right)}{2b}-\frac{(c+dx)\cos(a+b(c+d}{2b}\right.}{de^2(e(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{3(c+dx)^{2/3}\left(\frac{3\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}-\frac{\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}}{2b}\right)}{2b}-\frac{(c+dx)\cos(a+b(c+dx)}{2b}\right.}{de^2(e(c+dx))^{2/3}}$$

input
```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*(-1/2*((c + d*x)*Cos[a + b*(c + d*x)^(2/3)])/b + (3*(-
1/2*((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/Sq
rt[b] + (Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])
/Sqrt[b])/b + Sin[a + b*(c + d*x)^(2/3)]/(2*b*(c + d*x)^(1/3))))/(2*b)))/(
d*e^2*(e*(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3890
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

Fricas [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x +
c))/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\text{Timed out}$$

input

```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(8/3),x)
```

output

```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.72

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\text{Too large to display}$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

output

```
3/8*(d*x + c)^(1/3)*(((I*gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))) - I*g
amma(5/2, -I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (
-I*gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(5/2, I*b/(d*x +
c)^(2/3)))*cos(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(5/2, I*b*conjug
ate((d*x + c)^(-2/3))) + gamma(5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/
3*arctan2(0, d*x + c)) - (gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + g
amma(5/2, I*b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*co
s(a) + ((gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(5/2, -I*b/(d*
x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (gamma(5/2, -I*b*co
njugate((d*x + c)^(-2/3))) + gamma(5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi
+ 5/3*arctan2(0, d*x + c)) + (-I*gamma(5/2, I*b*conjugate((d*x + c)^(-2/3)
)) + I*gamma(5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x +
 c)) + (-I*gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(5/2, I*b
/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*sin(a))*e^(1/3)
/((d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)*(b/(d*x + c)^(2/3))^(5/2))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

input

```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
```

output

```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx$$

input

```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(8/3),x)
```

output

```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(8/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx=\frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{(dx+c)^{\frac{2}{3}}c^2+2(dx+c)^{\frac{2}{3}}cdx+(dx+c)^{\frac{2}{3}}d^2x^2}dx}{e^{\frac{8}{3}}}$$

input

```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

output

```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/((c + d*x)**(2/3)*c**2
+ 2*(c + d*x)**(2/3)*c*d*x + (c + d*x)**(2/3)*d**2*x**2),x)/(e**(2/3)*e**2
)
```

3.258 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx &= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} \\ &+ \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} + \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} \\ &- \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}\end{aligned}$$

output

```
-45/8*cos(a+b/(d*x+c)^(2/3))/b^3/d/e^3/(e*(d*x+c))^(1/3)+3/2*cos(a+b/(d*x+
c)^(2/3))/b/d/e^3/(d*x+c)^(4/3)/(e*(d*x+c))^(1/3)+45/16*Pi^(1/2)*(d*x+c)^(
1/3)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)/b^(7/
2)/d/e^3/(e*(d*x+c))^(1/3)-45/16*Pi^(1/2)*(d*x+c)^(1/3)*FresnelS(b^(1/2)*2
^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)/b^(7/2)/d/e^3/(e*(d*x+c))^(1
/3)-15/4*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^3/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.69

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \frac{(e(c+dx))^{2/3}\left(45\sqrt{2\pi}(c+dx)^{5/3}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)-45\sqrt{2\pi}(c+\right.}{}$$

input
```
Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

output
```
((e*(c + d*x))^(2/3)*(45*Sqrt[2*Pi]*(c + d*x)^(5/3)*Cos[a]*FresnelC[(Sqrt[
b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] - 45*Sqrt[2*Pi]*(c + d*x)^(5/3)*FresnelS[(
Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] - 6*Sqrt[b]*((-4*b^2 + 15*(c +
 d*x)^(4/3))*Cos[a + b/(c + d*x)^(2/3)] + 10*b*(c + d*x)^(2/3)*Sin[a + b/(
c + d*x)^(2/3)])))/(16*b^(7/2)*d*e^4*(c + d*x)^(7/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3916, 3898, 3896, 3890, 3866, 3867, 3866, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}dx$$

$$\Big\downarrow \text{3916}$$

$$\frac{\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e(c+dx))^{10/3}}d(c+dx)}{d}$$

$$\Big\downarrow \text{3898}$$

$$\frac{\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{10/3}}d(c+dx)}{de^3\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3896}$$

$$\frac{3\sqrt[3]{c+dx}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(c+dx)^{8/3}}d\sqrt[3]{c+dx}}{de^3\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3890}$$

$$-\frac{3\sqrt[3]{c+dx}\int (c+dx)^2\sin\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{de^3\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3866}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{5\int (c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}-\frac{(c+dx)^{5/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3867}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\int (c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}\right)}{2b}-\frac{(c+dx)^{5/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3866}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\int \cos\left(a+b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{5/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2b}\right.}{de^3\sqrt[3]{e(c+dx)}}$$

$$\Big\downarrow \text{ 3835}$$

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\cos(a)\int\cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\sin(a)\int\sin\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}}{2b}-\frac{\cos\left(a+b(c+dx)^{2/3}\right.}{2b\sqrt[3]{c+dx}}\right.}{2b}\right.}{2b}\right.}{de^3\sqrt[3]{e(c+dx)}}$$

$\Big\downarrow$ 3832

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\cos(a)\int\cos\left(b(c+dx)^{2/3}\right)d\frac{1}{\sqrt[3]{c+dx}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}}{2b}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}\right)}{2b}\right.}{2b}\right.}{de^3\sqrt[3]{e(c+dx)}}$$

$\Big\downarrow$ 3833

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{5\left(\frac{(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)}{2b}-\frac{3\left(\frac{\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{b}}}{2b}-\frac{\cos\left(a+b(c+dx)^{2/3}\right)}{2b\sqrt[3]{c+dx}}\right)}{2b}\right)}{2b}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

input

```
Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

output

```
(-3*(c + d*x)^(1/3)*(-1/2*((c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(2/3)])/b +
 (5*((-3*(-1/2*Cos[a + b*(c + d*x)^(2/3)]/(b*(c + d*x)^(1/3)) + ((Sqrt[Pi/
2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/Sqrt[b] - (Sqrt[
Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/Sqrt[b])/(2*b
)))/(2*b) + ((c + d*x)*Sin[a + b*(c + d*x)^(2/3)])/(2*b)))/(2*b)))/(d*e^3*
(e*(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835
```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3890
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

rule 3896
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a +
 b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
 && IntegerQ[p] && FractionQ[n]
```

rule 3898
```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
 + b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[
p] && FractionQ[n]
```

rule 3916
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/f   Subst[Int[(h*(x/f))^m*(a +
b*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Maple [F]

$$\int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}dx$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

output
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

Fricas [F]

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \int \frac{\sin \left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x +
c))/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c
^4*e^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin \left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(10/3),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.46

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="maxima")
```

output
```
3/8*(((I*gamma(7/2, I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(7/2, -I*b/(
d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, -I
*b*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2, I*b/(d*x + c)^(2/3)))*cos(-
7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, I*b*conjugate((d*x + c)^(-
2/3))) + gamma(7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x
 + c)) - (gamma(7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(7/2, I*b/(d
*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) + ((gamma(7
/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(7/2, -I*b/(d*x + c)^(2/3)))*c
os(7/4*pi + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, -I*b*conjugate((d*x + c
)^(-2/3))) + gamma(7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0,
 d*x + c)) + (-I*gamma(7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2
, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma
(7/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2, I*b/(d*x + c)^(2/3)
))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))/((d^3*e^(10/3)*x^2 + 2*
c*d^2*e^(10/3)*x + c^2*d*e^(10/3))*(d*x + c)^(1/3)*(b/(d*x + c)^(2/3))^(7/
2))
```

Giac [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}\,dx$$

input
```
integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="giac")
```

output
```
integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx=\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{10/3}}\,dx$$

input
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(10/3),x)
```

output
```
int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(10/3), x)
```

Reduce [F]

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx=\frac{\int \frac{\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{(dx+c)^{\frac{1}{3}}c^3+3(dx+c)^{\frac{1}{3}}c^2dx+3(dx+c)^{\frac{1}{3}}c\,d^2x^2+(dx+c)^{\frac{1}{3}}d^3x^3}dx}{e^{\frac{10}{3}}}$$

input
```
int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

output
```
int(sin(((c + d*x)**(2/3)*a + b)/(c + d*x)**(2/3))/((c + d*x)**(1/3)*c**3
+ 3*(c + d*x)**(1/3)*c**2*d*x + 3*(c + d*x)**(1/3)*c*d**2*x**2 + (c + d*x)
**(1/3)*d**3*x**3),x)/(e**(1/3)*e**3)
```

3.259 $\int (ex)^m \sin (a+b(c+dx)^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m \sin (a+b(c+dx)^n) \, dx = \text{Int}((ex)^m \sin (a+b(c+dx)^n), x)$$

output
```
Defer(Int)((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Mathematica [N/A]

Not integrable

Time = 11.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin (a+b(c+dx)^n) \, dx = \int (ex)^m \sin (a+b(c+dx)^n) \, dx$$

input
```
Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
```

output
```
Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx \\ \Big\downarrow \text{3918} \\ \int (ex)^m \sin \left(a+b(c+dx)^n\right)dx \end{array}$$

input
```
Int[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m \sin \left(a+b(dx+c)^n\right)dx$$

input
```
int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

output
```
int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin (a + b(c + dx)^n) \, dx = \int (ex)^m \sin ((dx + c)^n b + a) \, dx$$

input
```
integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Sympy [N/A]

Not integrable

Time = 7.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m \sin (a + b(c + dx)^n) \, dx = \int (ex)^m \sin (a + b(c + dx)^n) \, dx$$

input
```
integrate((e*x)**m*sin(a+b*(d*x+c)**n),x)
```

output
```
Integral((e*x)**m*sin(a + b*(c + d*x)**n), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx = \int (ex)^m \sin \left((dx+c)^n b+a\right) \, dx$$

input
```
integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx = \int (ex)^m \sin \left((dx+c)^n b+a\right) \, dx$$

input
```
integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Mupad [N/A]

Not integrable

Time = 40.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx = \int \sin \left(a+b\,{\left(c+d\,x\right)}^n\right)\,{\left(e\,x\right)}^m \,dx$$

input
```
int(sin(a + b*(c + d*x)^n)*(e*x)^m,x)
```

output
```
int(sin(a + b*(c + d*x)^n)*(e*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (ex)^m \sin \left(a+b(c+dx)^n\right) \, dx = e^m \left(\int x^m \sin \left((dx+c)^n b+a\right) dx\right)$$

input
```
int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

output
```
e**m*int(x**m*sin((c + d*x)**n*b + a),x)
```

3.260 $\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 503

$$
\begin{aligned}
&\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx \\
&= -\frac{ic^3e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n} \\
&\quad+\frac{ic^3e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^4n} \\
&\quad+\frac{3ic^2e^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^4n} \\
&\quad-\frac{3ic^2e^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^4n} \\
&\quad-\frac{3ice^{ia}(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^4n} \\
&\quad+\frac{3ice^{-ia}(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^4n} \\
&\quad+\frac{ie^{ia}(c+dx)^4\left(-ib(c+dx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-ib(c+dx)^n\right)}{2d^4n} \\
&\quad-\frac{ie^{-ia}(c+dx)^4\left(ib(c+dx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},ib(c+dx)^n\right)}{2d^4n}
\end{aligned}
$$

output
```
-1/2*I*c^3*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)
^n)^(1/n))+1/2*I*c^3*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b
*(d*x+c)^n)^(1/n))+3/2*I*c^2*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/
d^4/n/((-I*b*(d*x+c)^n)^(2/n))-3/2*I*c^2*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n
)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(2/n))-3/2*I*c*exp(I*a)*(d*x+c)^3*GAMMA(
3/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^n)^(3/n))+3/2*I*c*(d*x+c)^3*GAMMA
(3/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(3/n))+1/2*I*exp(I*a)*
(d*x+c)^4*GAMMA(4/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^n)^(4/n))-1/2*I*(
d*x+c)^4*GAMMA(4/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(4/n))
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.76

$$\int x^3 \sin\left(a+b(c+dx)^n\right)\,dx =$$

$$-\frac{ie^{-ia}(c+dx)\left(-c^3(ib(c+dx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)+e^{2ia}(-ib(c+dx)^n)^{-4/n}\left(c^3(-ib(c+dx)^n)^{3/n}\right.\right.}{}$$

input
```
Integrate[x^3*Sin[a + b*(c + d*x)^n],x]
```

output
```
((-1/2*I)*(c + d*x)*(-((c^3*Gamma[n^(-1), I*b*(c + d*x)^n])/(I*b*(c + d*x)
^n)^n^(-1)) + (E^((2*I)*a)*(c^3*((-I)*b*(c + d*x)^n)^(3/n)*Gamma[n^(-1), (
-I)*b*(c + d*x)^n] - (c + d*x)*(3*c^2*((-I)*b*(c + d*x)^n)^(2/n)*Gamma[2/n
, (-I)*b*(c + d*x)^n] - (c + d*x)*(3*c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[3
/n, (-I)*b*(c + d*x)^n] - (c + d*x)*Gamma[4/n, (-I)*b*(c + d*x)^n]))))/((-
I)*b*(c + d*x)^n)^(4/n) + ((c + d*x)*(3*c^2*(I*b*(c + d*x)^n)^(2/n)*Gamma[
2/n, I*b*(c + d*x)^n] - (c + d*x)*(3*c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[3/n,
 I*b*(c + d*x)^n] - (c + d*x)*Gamma[4/n, I*b*(c + d*x)^n])))/(I*b*(c + d*x
)^n)^(4/n)))/(d^4*E^(I*a)*n)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \left(-\sin \left(b(c+dx)^n+a\right) c^3+3(c+dx) \sin \left(b(c+dx)^n+a\right) c^2-3(c+dx)^2 \sin \left(b(c+dx)^n+a\right) c+(c+dx)^3 \mathrm{si}\right.}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{ia}c^3(c+dx)(-ib(c+dx)^n)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2n}+\frac{ie^{-ia}c^3(c+dx)(ib(c+dx)^n)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2n}+\frac{3ie^{ia}c^2(c+dx)^2(-ib(c+dx)^n)^{-2/}}{2n}$$

input

```
Int[x^3*Sin[a + b*(c + d*x)^n],x]
```

output

```
(((-1/2*I)*c^3*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(n*((-
I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c^3*(c + d*x)*Gamma[n^(-1), I*b*(c + d*
x)^n])/(E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + (((3*I)/2)*c^2*E^(I*a)*(c +
d*x)^2*Gamma[2/n, (-I)*b*(c + d*x)^n])/(n*((-I)*b*(c + d*x)^n)^(2/n)) - ((
(3*I)/2)*c^2*(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(E^(I*a)*n*(I*b*(c +
 d*x)^n)^(2/n)) - (((3*I)/2)*c*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*b*(c +
d*x)^n])/(n*((-I)*b*(c + d*x)^n)^(3/n)) + (((3*I)/2)*c*(c + d*x)^3*Gamma[3
/n, I*b*(c + d*x)^n])/(E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n)) + ((I/2)*E^(I*a)
*(c + d*x)^4*Gamma[4/n, (-I)*b*(c + d*x)^n])/(n*((-I)*b*(c + d*x)^n)^(4/n)
) - ((I/2)*(c + d*x)^4*Gamma[4/n, I*b*(c + d*x)^n])/(E^(I*a)*n*(I*b*(c + d
*x)^n)^(4/n)))/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^{3} \sin \left(a +b (d x +c)^{n}\right)d x$$

input
```
int(x^3*sin(a+b*(d*x+c)^n),x)
```

output
```
int(x^3*sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx=\int x^{3} \sin \left({\left(d x + c\right)}^{n} b + a\right) \,d x$$

input
```
integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(x^3*sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx=\int x^3 \sin \left(a+b(c+dx)^n\right) dx$$

input
```
integrate(x**3*sin(a+b*(d*x+c)**n),x)
```

output
```
Integral(x**3*sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx=\int x^3 \sin \left((dx+c)^n b+a\right) \, dx$$

input
```
integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx=\int x^3 \sin \left((dx+c)^n b+a\right) \, dx$$

input
```
integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx=\int x^3\,\sin \left(a+b\,{\left(c+d\,x\right)}^n\right) \,d x$$

input
```
int(x^3*sin(a + b*(c + d*x)^n),x)
```

output
```
int(x^3*sin(a + b*(c + d*x)^n), x)
```

Reduce [F]

$$\int x^3 \sin \left(a+b(c+dx)^n\right) \, dx=\int \sin \left(\left(dx+c\right)^{n} b+a\right) x^{3}dx$$

input
```
int(x^3*sin(a+b*(d*x+c)^n),x)
```

output
```
int(sin((c + d*x)**n*b + a)*x**3,x)
```

3.261 $\int x^2 \sin \left(a+b(c+dx)^n\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 369

$$\begin{aligned}\int x^2 \sin \left(a+b(c+dx)^n\right) \, dx = {} & \frac{ic^2e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n} \\ & -\frac{ic^2e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n} \\ & -\frac{ice^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{d^3n} \\ & +\frac{ice^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{d^3n} \\ & +\frac{ie^{ia}(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^3n} \\ & -\frac{ie^{-ia}(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^3n}\end{aligned}$$

output

```
1/2*I*c^2*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^3/n/((-I*b*(d*x+c)^
n)^(1/n))-1/2*I*c^2*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^3/exp(I*a)/n/((I*b*
(d*x+c)^n)^(1/n))-I*c*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^3/n/(
(-I*b*(d*x+c)^n)^(2/n))+I*c*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^3/exp(I*a
)/n/((I*b*(d*x+c)^n)^(2/n))+1/2*I*exp(I*a)*(d*x+c)^3*GAMMA(3/n,-I*b*(d*x+c
)^n)/d^3/n/((-I*b*(d*x+c)^n)^(3/n))-1/2*I*(d*x+c)^3*GAMMA(3/n,I*b*(d*x+c)^
n)/d^3/exp(I*a)/n/((I*b*(d*x+c)^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.78

$$\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx$$
$$=\frac{ie^{-ia}(c+dx)\left(e^{2ia}(-ib(c+dx)^n)^{-3/n}\left(c^2(-ib(c+dx)^n)^{2/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)-(c+dx)\left(2c(-ib(c+\right.\right.\right.}{}$$

input

```
Integrate[x^2*Sin[a + b*(c + d*x)^n],x]
```

output

```
((I/2)*(c + d*x)*((E^((2*I)*a)*(c^2*((-I)*b*(c + d*x)^n)^(2/n)*Gamma[n^(-1
), (-I)*b*(c + d*x)^n] - (c + d*x)*(2*c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[
2/n, (-I)*b*(c + d*x)^n] - (c + d*x)*Gamma[3/n, (-I)*b*(c + d*x)^n])))/((-
I)*b*(c + d*x)^n)^(3/n) + (-(c^2*(I*b*(c + d*x)^n)^(2/n)*Gamma[n^(-1), I*b
*(c + d*x)^n]) + (c + d*x)*(2*c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[2/n, I*b*(c
 + d*x)^n] - (c + d*x)*Gamma[3/n, I*b*(c + d*x)^n]))/(I*b*(c + d*x)^n)^(3/
n)))/(d^3*E^(I*a)*n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx$$
$$\Big\downarrow \; 3914$$
$$\frac{\int \left(\sin\left(b(c+dx)^n+a\right)c^2-2(c+dx)\sin\left(b(c+dx)^n+a\right)c+(c+dx)^2\sin\left(b(c+dx)^n+a\right)\right)d(c+dx)}{d^3}$$
$$\Big\downarrow \; 2009$$

$$\frac{ie^{ia}c^2(c+dx)(-ib(c+dx)^n)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2n} - \frac{ie^{-ia}c^2(c+dx)(ib(c+dx)^n)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2n} + \frac{ie^{ia}(c+dx)^3(-ib(c+dx)^n)^{-3/n}\Gamma(}{2n}$$

input
```
Int[x^2*Sin[a + b*(c + d*x)^n],x]
```

output
```
(((I/2)*c^2*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(n*((-I)*
b*(c + d*x)^n)^n^(-1)) - ((I/2)*c^2*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^
n])/(E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) - (I*c*E^(I*a)*(c + d*x)^2*Gamma[
2/n, (-I)*b*(c + d*x)^n])/(n*((-I)*b*(c + d*x)^n)^(2/n)) + (I*c*(c + d*x)^
2*Gamma[2/n, I*b*(c + d*x)^n])/(E^(I*a)*n*(I*b*(c + d*x)^n)^(2/n)) + ((I/2
)*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*b*(c + d*x)^n])/(n*((-I)*b*(c + d*x)
^n)^(3/n)) - ((I/2)*(c + d*x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(E^(I*a)*n*(I
*b*(c + d*x)^n)^(3/n)))/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \sin\left(a + b(dx + c)^n\right) dx$$

input
```
int(x^2*sin(a+b*(d*x+c)^n),x)
```

output
```
int(x^2*sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx=\int x^2 \sin\left((dx+c)^n b+a\right)\,dx$$

input
```
integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(x^2*sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx=\int x^2 \sin\left(a+b(c+dx)^n\right)dx$$

input
```
integrate(x**2*sin(a+b*(d*x+c)**n),x)
```

output
```
Integral(x**2*sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx=\int x^2 \sin\left((dx+c)^n b+a\right)\,dx$$

input
```
integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int x^2 \sin \left(a+b(c+dx)^n\right) \, dx = \int x^2 \sin \left((dx+c)^n b+a\right) \, dx$$

input
```
integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin \left(a+b(c+dx)^n\right) \, dx = \int x^2 \sin \left(a+b\left(c+d\,x\right)^n\right) \, dx$$

input
```
int(x^2*sin(a + b*(c + d*x)^n),x)
```

output
```
int(x^2*sin(a + b*(c + d*x)^n), x)
```

Reduce [F]

$$\int x^2 \sin \left(a+b(c+dx)^n\right) \, dx = \int \sin \left((dx+c)^n b+a\right) x^2 dx$$

input
```
int(x^2*sin(a+b*(d*x+c)^n),x)
```

output
```
int(sin((c + d*x)**n*b + a)*x**2,x)
```

3.262 $\int x\sin\left(a+b(c+dx)^n\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 243

$$\begin{aligned}\int x\sin\left(a+b(c+dx)^n\right)\,dx=&-\frac{ice^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}\\&+\frac{ice^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^2n}\\&+\frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^2n}\\&-\frac{ie^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^2n}\end{aligned}$$

output
```
-1/2*I*c*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^2/n/((-I*b*(d*x+c)^n
)^(1/n))+1/2*I*c*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^2/exp(I*a)/n/((I*b*(d*
x+c)^n)^(1/n))+1/2*I*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^2/n/((
-I*b*(d*x+c)^n)^(2/n))-1/2*I*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^2/exp(I*
a)/n/((I*b*(d*x+c)^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.79

$$\int x \sin \left(a+b(c+dx)^n\right) \, dx$$
$$= \frac{(c+dx)\left(\left(-ib(c+dx)^n\right)^{-2/n}\left(c\left(-ib(c+dx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)-(c+dx)\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)\right)(-}{}$$

input
```
Integrate[x*Sin[a + b*(c + d*x)^n],x]
```

output
```
((c + d*x)*((((c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*b*(c + d*x)
^n] - (c + d*x)*Gamma[2/n, (-I)*b*(c + d*x)^n])*((-I)*Cos[a] + Sin[a]))/((
-I)*b*(c + d*x)^n)^(2/n) + ((c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), I*b*
(c + d*x)^n] - (c + d*x)*Gamma[2/n, I*b*(c + d*x)^n])*(I*Cos[a] + Sin[a]))
/(I*b*(c + d*x)^n)^(2/n)))/(2*d^2*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin \left(a+b(c+dx)^n\right) \, dx$$
$$\Big\downarrow \; 3914$$
$$\frac{\int \left((c+dx)\sin\left(b(c+dx)^n+a\right)-c\sin\left(b(c+dx)^n+a\right)\right)d(c+dx)}{d^2}$$
$$\Big\downarrow \; 2009$$
$$\frac{\frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)}{2n}-\frac{ie^{ia}c(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2n}+\frac{ie^{-ia}c(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}\right.}{2n}}{d^2}$$

input
```
Int[x*Sin[a + b*(c + d*x)^n],x]
```

output
```
(((-1/2*I)*c*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(n*((-I)
*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n
])/(E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*E^(I*a)*(c + d*x)^2*Gamma
[2/n, (-I)*b*(c + d*x)^n])/(n*((-I)*b*(c + d*x)^n)^(2/n)) - ((I/2)*(c + d*
x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(E^(I*a)*n*(I*b*(c + d*x)^n)^(2/n)))/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x \sin \left(a + b(dx + c)^{n}\right) dx$$

input
```
int(x*sin(a+b*(d*x+c)^n),x)
```

output
```
int(x*sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int x\sin \left(a+b(c+dx)^n\right)\,dx=\int x\sin \left((dx+c)^n b+a\right)\,dx$$

input
```
integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(x*sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int x\sin \left(a+b(c+dx)^n\right)\,dx=\int x\sin \left(a+b(c+dx)^n\right)dx$$

input
```
integrate(x*sin(a+b*(d*x+c)**n),x)
```

output
```
Integral(x*sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int x\sin \left(a+b(c+dx)^n\right)\,dx=\int x\sin \left((dx+c)^n b+a\right)\,dx$$

input
```
integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(x*sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int x\sin\left(a+b(c+dx)^n\right)\,dx=\int x\sin\left((dx+c)^n b+a\right)\,dx$$

input
```
integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(x*sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sin\left(a+b(c+dx)^n\right)\,dx=\int x\,\sin\left(a+b\,(c+d\,x)^n\right)\,dx$$

input
```
int(x*sin(a + b*(c + d*x)^n),x)
```

output
```
int(x*sin(a + b*(c + d*x)^n), x)
```

Reduce [F]

$$\int x\sin\left(a+b(c+dx)^n\right)\,dx=\int \sin\left((dx+c)^n\,b+a\right)x dx$$

input
```
int(x*sin(a+b*(d*x+c)^n),x)
```

output
```
int(sin((c + d*x)**n*b + a)*x,x)
```

3.263 $\int \sin(a+b(c+dx)^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 117

$$\int \sin(a+b(c+dx)^n)\,dx=\frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

output
```
1/2*I*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d/n/((-I*b*(d*x+c)^n)^(1/
n))-1/2*I*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d/exp(I*a)/n/((I*b*(d*x+c)^n)^(
1/n))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \sin(a+b(c+dx)^n)\,dx=-\frac{i(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)(\cos(a)-i\sin(a))}{2dn}+\frac{i(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)(\cos(a)+i\sin(a))}{2dn}$$

input
```
Integrate[Sin[a + b*(c + d*x)^n],x]
```

output
```
((-1/2*I)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n]*(Cos[a] - I*Sin[a]))/(d
*n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c +
d*x)^n]*(Cos[a] + I*Sin[a]))/(d*n*((-I)*b*(c + d*x)^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3846, 2637\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+b(c+dx)^n\right) \, dx$$

$$\Big\downarrow \text{3846}$$

$$\frac{1}{2} i \int e^{-ib(c+dx)^n-ia}dx-\frac{1}{2} i \int e^{ib(c+dx)^n+ia}dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{ie^{ia}(c+dx) \left(-ib(c+dx)^n\right)^{-1/n} \Gamma \left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx) \left(ib(c+dx)^n\right)^{-1/n} \Gamma \left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

input
```
Int[Sin[a + b*(c + d*x)^n],x]
```

output
```
((I/2)*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d*n*((-I)*b*(
c + d*x)^n)^n^(-1)) - ((I/2)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n])/(d*
E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

rule 3846
```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Simp[I/2   I
nt[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Simp[I/2   Int[E^(c*I + d*I*(e +
f*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Maple [F]

$$\int \sin\left(a+b(dx+c)^{n}\right)dx$$

input
```
int(sin(a+b*(d*x+c)^n),x)
```

output
```
int(sin(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int \sin\left(a+b(c+dx)^{n}\right)\,dx=\int \sin\left((dx+c)^{n}b+a\right)\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(sin((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int \sin \left(a+b(c+dx)^n\right) \, dx=\int \sin \left(a+b(c+dx)^{n}\right)dx$$

input
```
integrate(sin(a+b*(d*x+c)**n),x)
```

output
```
Integral(sin(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int \sin \left(a+b(c+dx)^n\right) \, dx=\int \sin \left((dx+c)^{n}b+a\right)\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int \sin \left(a+b(c+dx)^n\right) \, dx=\int \sin \left((dx+c)^{n}b+a\right)\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(a+b(c+dx)^n\right)\,dx=\int \sin\left(a+b\,{\left(c+d\,x\right)}^n\right)\,dx$$

input
```
int(sin(a + b*(c + d*x)^n),x)
```

output
```
int(sin(a + b*(c + d*x)^n), x)
```

Reduce [F]

$$\int \sin\left(a+b(c+dx)^n\right)\,dx=\int \sin\left(\left(dx+c\right)^{n}b+a\right)dx$$

input
```
int(sin(a+b*(d*x+c)^n),x)
```

output
```
int(sin((c + d*x)**n*b + a),x)
```

3.264 $\int \frac{\sin(a+b(c+dx)^n)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sin(a+b(c+dx)^n)}{x} \, dx = \text{Int}\left(\frac{\sin(a+b(c+dx)^n)}{x}, x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^n)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin(a+b(c+dx)^n)}{x} \, dx = \int \frac{\sin(a+b(c+dx)^n)}{x} \, dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^n]/x,x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^n]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin (a+b(c+dx)^n)}{x} \, dx$$

$$\Big\downarrow \; 3918$$

$$\int \frac{\sin (a+b(c+dx)^n)}{x}dx$$

input
```
Int[Sin[a + b*(c + d*x)^n]/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin (a+b(dx+c)^n)}{x}dx$$

input
```
int(sin(a+b*(d*x+c)^n)/x,x)
```

output
```
int(sin(a+b*(d*x+c)^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx=\int \frac{\sin \left((dx+c)^nb+a\right)}{x}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="fricas")
```

output
```
integral(sin((d*x + c)^n*b + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx=\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**n)/x,x)
```

output
```
Integral(sin(a + b*(c + d*x)**n)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx=\int \frac{\sin \left((dx+c)^n b+a\right)}{x}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="maxima")
```

output

```
integrate(sin((d*x + c)^n*b + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx=\int \frac{\sin \left((dx+c)^n b+a\right)}{x}\,dx$$

input

```
integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="giac")
```

output

```
integrate(sin((d*x + c)^n*b + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 40.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^n\right)}{x}\,dx$$

input

```
int(sin(a + b*(c + d*x)^n)/x,x)
```

output

```
int(sin(a + b*(c + d*x)^n)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x}\,dx=\int \frac{\sin \left(\left(dx+c\right)^n b+a\right)}{x}dx$$

input

```
int(sin(a+b*(d*x+c)^n)/x,x)
```

output

```
int(sin((c + d*x)**n*b + a)/x,x)
```

3.265 $\int \frac{\sin(a+b(c+dx)^n)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx = \text{Int}\left(\frac{\sin\left(a+b(c+dx)^n\right)}{x^2},x\right)$$

output
```
Defer(Int)(sin(a+b*(d*x+c)^n)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

input
```
Integrate[Sin[a + b*(c + d*x)^n]/x^2,x]
```

output
```
Integrate[Sin[a + b*(c + d*x)^n]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+b(c+dx)^n)}{x^2} \, dx$$
$$\Big\downarrow \ 3918$$
$$\int \frac{\sin(a+b(c+dx)^n)}{x^2}dx$$

input
```
Int[Sin[a + b*(c + d*x)^n]/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+b(dx+c)^n)}{x^2}dx$$

input
```
int(sin(a+b*(d*x+c)^n)/x^2,x)
```

output
```
int(sin(a+b*(d*x+c)^n)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx=\int \frac{\sin\left((dx+c)^n b+a\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="fricas")
```

output
```
integral(sin((d*x + c)^n*b + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 5.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)**n)/x**2,x)
```

output
```
Integral(sin(a + b*(c + d*x)**n)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x^2}\,dx=\int \frac{\sin \left((dx+c)^n b+a\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="maxima")
```

output
```
integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x^2}\,dx=\int \frac{\sin \left((dx+c)^n b+a\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="giac")
```

output
```
integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 39.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin \left(a+b(c+dx)^n\right)}{x^2}\,dx=\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^n\right)}{x^2}\,dx$$

input
```
int(sin(a + b*(c + d*x)^n)/x^2,x)
```

output
```
int(sin(a + b*(c + d*x)^n)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sin (a+b(c+dx)^n)}{x^2}\,dx=\int \frac{\sin \left((dx+c)^n b+a\right)}{x^2}dx$$

input
```
int(sin(a+b*(d*x+c)^n)/x^2,x)
```

output
```
int(sin((c + d*x)**n*b + a)/x**2,x)
```

3.266 $\int x^3(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 519

$$\begin{aligned}
\int x^3(a+b\sin(c+d(f+gx)^n))\,dx &= \frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{ibe^{-ic}f^3(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad-\frac{3ibe^{-ic}f^2(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^4n}\\
&\quad-\frac{3ibe^{ic}f(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{3ibe^{-ic}f(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^4n}\\
&\quad+\frac{ibe^{ic}(f+gx)^4\left(-id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-id(f+gx)^n\right)}{2g^4n}\\
&\quad-\frac{ibe^{-ic}(f+gx)^4\left(id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},id(f+gx)^n\right)}{2g^4n}
\end{aligned}$$

output
```
1/4*a*x^4-1/2*I*b*exp(I*c)*f^3*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^4/n/((-
I*d*(g*x+f)^n)^(1/n))+1/2*I*b*f^3*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c
)/g^4/n/((I*d*(g*x+f)^n)^(1/n))+3/2*I*b*exp(I*c)*f^2*(g*x+f)^2*GAMMA(2/n,-
I*d*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)^(2/n))-3/2*I*b*f^2*(g*x+f)^2*GAMMA(
2/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*x+f)^n)^(2/n))-3/2*I*b*exp(I*c)
*f*(g*x+f)^3*GAMMA(3/n,-I*d*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)^(3/n))+3/2*
I*b*f*(g*x+f)^3*GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*x+f)^n)^(
3/n))+1/2*I*b*exp(I*c)*(g*x+f)^4*GAMMA(4/n,-I*d*(g*x+f)^n)/g^4/n/((-I*d*(g
*x+f)^n)^(4/n))-1/2*I*b*(g*x+f)^4*GAMMA(4/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/
((I*d*(g*x+f)^n)^(4/n))
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.79

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx = \frac{ax^4}{4}$$

$$-\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-4/n}\left(f^3\left(-id(f+gx)^n\right)^{3/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)-(f+gx)\left(3f^2(-id(f\right.\right.}{}$$

$$+\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-4/n}\left(f^3\left(id(f+gx)^n\right)^{3/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\left(3f^2(id(f+gx)^{}\right.\right.}{}$$

input
```
Integrate[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
(a*x^4)/4 - ((I/2)*b*E^(I*c)*(f + g*x)*(f^3*((-I)*d*(f + g*x)^n)^(3/n)*Gam
ma[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*(3*f^2*((-I)*d*(f + g*x)^n)^(2/
n)*Gamma[2/n, (-I)*d*(f + g*x)^n] - (f + g*x)*(3*f*((-I)*d*(f + g*x)^n)^n^
(-1)*Gamma[3/n, (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[4/n, (-I)*d*(f + g*x
)^n]))))/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) + ((I/2)*b*(f + g*x)*(f^3*(I*d
*(f + g*x)^n)^(3/n)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*(3*f^2*(I*d
*(f + g*x)^n)^(2/n)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g*x)*(3*f*(I*d*(f +
 g*x)^n)^n^(-1)*Gamma[3/n, I*d*(f + g*x)^n] - (f + g*x)*Gamma[4/n, I*d*(f
+ g*x)^n]))))/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int (ax^3+bx^3\sin(c+d(f+gx)^n))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{ibe^{-ic}f^3(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^4n}+\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n}-\frac{3ibe^{-ic}f^2(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^4n}+\frac{ibe^{ic}(f+gx)^4\left(-id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-id(f+gx)^n\right)}{2g^4n}-\frac{3ibe^{ic}f(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{3ibe^{-ic}f(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^4n}-\frac{ibe^{-ic}(f+gx)^4\left(id(f+gx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},id(f+gx)^n\right)}{2g^4n}$$

input
```
Int[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
(a*x^4)/4 - ((I/2)*b*E^(I*c)*f^3*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^
n])/(g^4*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f^3*(f + g*x)*Gamma[n^(
-1), I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^n^(-1)) + (((3*I)/
2)*b*E^(I*c)*f^2*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*
d*(f + g*x)^n)^(2/n)) - (((3*I)/2)*b*f^2*(f + g*x)^2*Gamma[2/n, I*d*(f + g
*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(2/n)) - (((3*I)/2)*b*E^(I*c)*f*(
f + g*x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(3/
n)) + (((3*I)/2)*b*f*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^4
*n*(I*d*(f + g*x)^n)^(3/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^4*Gamma[4/n, (-I)
*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) - ((I/2)*b*(f + g*x)^4
*Gamma[4/n, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^{3}(a+b\sin \left(c+d(gx+f)^{n}\right))\, dx$$

input
```
int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [F]

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(b*x^3*sin((g*x + f)^n*d + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\int x^3(a+b\sin(c+d(f+gx)^n))\,dx$$

input
```
integrate(x**3*(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Integral(x**3*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [F]

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + b*integrate(x^3*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\int x^3\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)\,dx$$

input
```
int(x^3*(a + b*sin(c + d*(f + g*x)^n)),x)
```

output
```
int(x^3*(a + b*sin(c + d*(f + g*x)^n)), x)
```

Reduce [F]

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx=\left(\int\sin\left((gx+f)^n d+c\right)x^3dx\right)b+\frac{a\,x^4}{4}$$

input
```
int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
(4*int(sin((f + g*x)**n*d + c)*x**3,x)*b + a*x**4)/4
```

3.267 $\int x^2(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 383

$$
\begin{aligned}
\int x^2(a+b\sin(c+d(f+gx)^n))\,dx &= \frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n} \\
&\quad -\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^3n} \\
&\quad -\frac{ibe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n} \\
&\quad +\frac{ibe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n} \\
&\quad +\frac{ibe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^3n} \\
&\quad -\frac{ibe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^3n}
\end{aligned}
$$

output
```
1/3*a*x^3+1/2*I*b*exp(I*c)*f^2*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^3/n/((-
I*d*(g*x+f)^n)^(1/n))-1/2*I*b*f^2*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c
)/g^3/n/((I*d*(g*x+f)^n)^(1/n))-I*b*exp(I*c)*f*(g*x+f)^2*GAMMA(2/n,-I*d*(g
*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(2/n))+I*b*f*(g*x+f)^2*GAMMA(2/n,I*d*(g*x
+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(2/n))+1/2*I*b*exp(I*c)*(g*x+f)^3*G
AMMA(3/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(3/n))-1/2*I*b*(g*x+f)^3*
GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.82

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\frac{ax^3}{3}$$
$$+\frac{ibe^{ic}(f+gx)\,(-id(f+gx)^n)^{-3/n}\Big(f^2(-id(f+gx)^n)^{2/n}\,\Gamma\big(\tfrac{1}{n},-id(f+gx)^n\big)-(f+gx)\Big(2f(-id(f+}{2g^3n}$$
$$-\frac{ibe^{-ic}(f+gx)\,(id(f+gx)^n)^{-3/n}\Big(f^2(id(f+gx)^n)^{2/n}\,\Gamma\big(\tfrac{1}{n},id(f+gx)^n\big)-(f+gx)\Big(2f(id(f+gx)^n}{2g^3n}$$

input
```
Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
(a*x^3)/3 + ((I/2)*b*E^(I*c)*(f + g*x)*(f^2*((-I)*d*(f + g*x)^n)^(2/n)*Gam
ma[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*(2*f*((-I)*d*(f + g*x)^n)^n^(-1
)*Gamma[2/n, (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, (-I)*d*(f + g*x)^n
])))/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)*(f^2*(I*d*(f
+ g*x)^n)^(2/n)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*(2*f*(I*d*(f +
g*x)^n)^n^(-1)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, I*d*(f +
 g*x)^n])))/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^2+bx^2\sin(c+d(f+gx)^n)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^3n}+\frac{ibe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n}+\frac{ibe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n}-\frac{ibe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{2g^3n}$$

input

```
Int[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
(a*x^3)/3 + ((I/2)*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^
n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*f^2*(f + g*x)*Gamma[n^(
-1), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^(-1)) - (I*b*E^(
I*c)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x
)^n)^(2/n)) + (I*b*f*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3
*n*(I*d*(f + g*x)^n)^(2/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)
*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)^3
*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^2(a+b\sin(c+d(gx+f)^n))\,dx$$

input
```
int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int (b\sin((gx+f)^n d+c)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(b*x^2*sin((g*x + f)^n*d + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\sin\left(c+d(f+gx)^n\right))\,dx=\int x^2(a+b\sin\left(c+d(f+gx)^n\right))\,dx$$

input
```
integrate(x**2*(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Integral(x**2*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [F]

$$\int x^2(a+b\sin\left(c+d(f+gx)^n\right))\,dx=\int\left(b\sin\left((gx+f)^nd+c\right)+a\right)x^2\,dx$$

input
```
integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x^2(a+b\sin\left(c+d(f+gx)^n\right))\,dx=\int\left(b\sin\left((gx+f)^nd+c\right)+a\right)x^2\,dx$$

input
```
integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\int x^2\,(a+b\,\sin\left(c+d\,{(f+g\,x)}^n\right))\,dx$$

input
```
int(x^2*(a + b*sin(c + d*(f + g*x)^n)),x)
```

output
```
int(x^2*(a + b*sin(c + d*(f + g*x)^n)), x)
```

Reduce [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))\,dx=\left(\int \sin\left((gx+f)^n d+c\right)x^2dx\right)b+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
(3*int(sin((f + g*x)**n*d + c)*x**2,x)*b + a*x**3)/3
```

3.268 $\int x(a+b\sin (c+d(f+gx)^n)) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 255

$$
\begin{aligned}
&\int x(a+b\sin (c+d(f+gx)^n)) \, dx \\
&= \frac{ax^2}{2} - \frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n} \\
&\quad + \frac{ibe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^2n} \\
&\quad + \frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n} \\
&\quad - \frac{ibe^{-ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^2n}
\end{aligned}
$$

output

```
1/2*a*x^2-1/2*I*b*exp(I*c)*f*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^2/n/((-I*
d*(g*x+f)^n)^(1/n))+1/2*I*b*f*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g^
2/n/((I*d*(g*x+f)^n)^(1/n))+1/2*I*b*exp(I*c)*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x
+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))-1/2*I*b*(g*x+f)^2*GAMMA(2/n,I*d*(g*x
+f)^n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.84

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\frac{ax^2}{2}$$

$$+\frac{b(f+gx)\left(-id(f+gx)^n\right)^{-2/n}\Big(f\left(-id(f+gx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)-(f+gx)\Gamma\left(\frac{2}{n},-id(f+gx)^n\right.}{2g^2n}$$

$$+\frac{b(f+gx)\left(id(f+gx)^n\right)^{-2/n}\Big(f\left(id(f+gx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\Gamma\left(\frac{2}{n},id(f+gx)^n\right)\Big)(i\,\mathrm{co}}{2g^2n}$$

input
```
Integrate[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
(a*x^2)/2 + (b*(f + g*x)*(f*((-I)*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (-I)
*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n])*((-I)*Cos[c] +
 Sin[c]))/(2*g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (b*(f + g*x)*(f*(I*d*(f +
 g*x)^n)^n^(-1)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, I*d*
(f + g*x)^n])*(I*Cos[c] + Sin[c]))/(2*g^2*n*(I*d*(f + g*x)^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int (ax+bx\sin(c+d(f+gx)^n))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n}+\frac{ibe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2g^2n}-\frac{ibe^{-ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{2g^2n}$$

input
```
Int[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
(a*x^2)/2 - ((I/2)*b*E^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n]
)/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f*(f + g*x)*Gamma[n^(-1),
 I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*E^(
I*c)*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^
n)^(2/n)) - ((I/2)*b*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2
*n*(I*d*(f + g*x)^n)^(2/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x(a+b\sin\left(c+d(gx+f)^n\right))\,dx$$

input
```
int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(b*x*sin((g*x + f)^n*d + c) + a*x, x)
```

Sympy [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int x(a+b\sin(c+d(f+gx)^n))\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Integral(x*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int(b\sin((gx+f)^nd+c)+a)x\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int (b\sin((gx+f)^n d+c)+a)x\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\int x\,(a+b\,\sin(c+d\,(f+g\,x)^n))\,dx$$

input
```
int(x*(a + b*sin(c + d*(f + g*x)^n)),x)
```

output
```
int(x*(a + b*sin(c + d*(f + g*x)^n)), x)
```

Reduce [F]

$$\int x(a+b\sin(c+d(f+gx)^n))\,dx=\left(\int \sin((gx+f)^n\,d+c)\,xdx\right)b+\frac{a\,x^2}{2}$$

input
```
int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
(2*int(sin((f + g*x)**n*d + c)*x,x)*b + a*x**2)/2
```

3.269 $\int (a+b\sin (c+d(f+gx)^n))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 122

$$\begin{aligned}\int (a+b\sin (c+d(f+gx)^n))\,dx &= ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}\\ &\quad -\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}\end{aligned}$$

output

```
a*x+1/2*I*b*exp(I*c)*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g/n/((-I*d*(g*x+f)^
n)^(1/n))-1/2*I*b*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g/n/((I*d*(g*x
+f)^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\begin{aligned}\int (a+b\sin (c+d(f+gx)^n))\,dx &= ax-\frac{ib(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)(\cos(c)-i\sin(c))}{2gn}\\ &\quad +\frac{ib(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))}{2gn}\end{aligned}$$

input
```
Integrate[a + b*Sin[c + d*(f + g*x)^n],x]
```

output
```
a*x - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*Sin[c]
))/(g*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*(f + g*x)*Gamma[n^(-1), (-I)*
d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/(g*n*((-I)*d*(f + g*x)^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+d(f+gx)^n))\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}$$

input
```
Int[a + b*Sin[c + d*(f + g*x)^n],x]
```

output
```
a*x + ((I/2)*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g*n*(
(-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*
x)^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int (a + b\sin (c + d(gx + f)^n))\,dx$$

input
```
int(a+b*sin(c+d*(g*x+f)^n),x)
```

output
```
int(a+b*sin(c+d*(g*x+f)^n),x)
```

Fricas [F]

$$\int (a + b\sin (c + d(f + gx)^n))\;dx = \int b\sin ((gx + f)^n d + c) + a\,dx$$

input
```
integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="fricas")
```

output
```
integral(b*sin((g*x + f)^n*d + c) + a, x)
```

Sympy [F]

$$\int (a + b\sin (c + d(f + gx)^n))\;dx = \int (a + b\sin (c + d(f + gx)^n))\;dx$$

input
```
integrate(a+b*sin(c+d*(g*x+f)**n),x)
```

output
```
Integral(a + b*sin(c + d*(f + g*x)**n), x)
```

Maxima [F]

$$\int (a+b\sin(c+d(f+gx)^n))\,dx=\int b\sin\left((gx+f)^n d+c\right)+a\,dx$$

input
```
integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="maxima")
```

output
```
a*x + b*integrate(sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int (a+b\sin(c+d(f+gx)^n))\,dx=\int b\sin\left((gx+f)^n d+c\right)+a\,dx$$

input
```
integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="giac")
```

output
```
integrate(b*sin((g*x + f)^n*d + c) + a, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+d(f+gx)^n))\,dx=\int a+b\,\sin\left(c+d\,(f+g\,x)^n\right)\,dx$$

input
```
int(a + b*sin(c + d*(f + g*x)^n),x)
```

output
```
int(a + b*sin(c + d*(f + g*x)^n), x)
```

Reduce [F]

$$\int (a+b\sin(c+d(f+gx)^n))\,dx=\left(\int \sin\left((gx+f)^n d+c\right)dx\right)b+ax$$

input

```
int(a+b*sin(c+d*(g*x+f)^n),x)
```

output

```
int(sin((f + g*x)**n*d + c),x)*b + a*x
```

3.270 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx = \text{Int}\left(\frac{a+b\sin\left(c+d(f+gx)^n\right)}{x},x\right)$$

output
```
Defer(Int)((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx = \int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
```

output
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\int \frac{\sin\left(d(f+gx)^n+c\right)}{x}dx+a\log(x)$$

input
```
Int[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin\left(c+d(gx+f)^{n}\right)}{x}dx$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

output
```
int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^{n}\right)}{x}\,dx=\int \frac{b\sin\left((gx+f)^{n}d+c\right)+a}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="fricas")
```

output
```
integral((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sin\left(c+d(f+gx)^{n}\right)}{x}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^{n}\right)}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)**n))/x,x)
```

output
```
Integral((a + b*sin(c + d*(f + g*x)**n))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\int \frac{b\sin\left((gx+f)^n d+c\right)+a}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="maxima")
```

output
```
b*integrate(sin((g*x + f)^n*d + c)/x, x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\int \frac{b\sin\left((gx+f)^n d+c\right)+a}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 39.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\int \frac{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}{x}\,dx$$

input
```
int((a + b*sin(c + d*(f + g*x)^n))/x,x)
```

output
```
int((a + b*sin(c + d*(f + g*x)^n))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx=\left(\int \frac{\sin\left(\left(gx+f\right)^n d+c\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

output
```
int(sin((f + g*x)**n*d + c)/x,x)*b + log(x)*a
```

3.271 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\operatorname{Int}\left(\frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2},x\right)$$

output
```
Defer(Int)((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]
```

output
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^2}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\int \frac{\sin\left(d(f+gx)^n+c\right)}{x^2}dx-\frac{a}{x}$$

input
```
Int[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin\left(c+d(gx+f)^{n}\right)}{x^{2}}dx$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

output
```
int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin\left(c+d(f+gx)^{n}\right)}{x^{2}}\,dx=\int \frac{b\sin\left((gx+f)^{n}d+c\right)+a}{x^{2}}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="fricas")
```

output
```
integral((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 19.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sin\left(c+d(f+gx)^{n}\right)}{x^{2}}\,dx=\int \frac{a+b\sin\left(c+d(f+gx)^{n}\right)}{x^{2}}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)**n))/x**2,x)
```

output
```
Integral((a + b*sin(c + d*(f + g*x)**n))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx=\int \frac{b\sin\left((gx+f)^n d+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="maxima")
```

output
```
b*integrate(sin((g*x + f)^n*d + c)/x^2, x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx=\int \frac{b\sin\left((gx+f)^n d+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 39.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx=\int \frac{a+b\,\sin\left(c+d\,(f+g\,x)^n\right)}{x^2}\,dx$$

input
```
int((a + b*sin(c + d*(f + g*x)^n))/x^2,x)
```

output
```
int((a + b*sin(c + d*(f + g*x)^n))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx=\frac{\left(\int \frac{\sin\left((gx+f)^n d+c\right)}{x^2}dx\right)bx-a}{x}$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

output
```
(int(sin((f + g*x)**n*d + c)/x**2,x)*b*x - a)/x
```

3.272 $\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 856

$$\begin{aligned}
&\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx \\
&= \frac{(2a^2+b^2)\,f^2x}{2g^2} - \frac{(2a^2+b^2)\,f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)\,(f+gx)^3}{6g^3} \\
&+ \frac{iabe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^3n} \\
&- \frac{iabe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^3n} \\
&- \frac{2iabe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2iabe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n} \\
&- \frac{2^{-1-\frac{2}{n}}b^2e^{2ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^3n} \\
&- \frac{2^{-1-\frac{2}{n}}b^2e^{-2ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{iabe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)}{g^3n} \\
&- \frac{iabe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-2id(f+gx)^n\right)}{g^3n} \\
&+ \frac{2^{-2-\frac{3}{n}}b^2e^{-2ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},2id(f+gx)^n\right)}{g^3n}
\end{aligned}$$

output
```
1/2*(2*a^2+b^2)*f^2*x/g^2-1/2*(2*a^2+b^2)*f*(g*x+f)^2/g^3+1/6*(2*a^2+b^2)*
(g*x+f)^3/g^3+I*a*b*exp(I*c)*f^2*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^3/n/(
(-I*d*(g*x+f)^n)^(1/n))-I*a*b*f^2*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c
)/g^3/n/((I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*exp(2*I*c)*f^2*(g*x+f)*GAMM
A(1/n,-2*I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*f^2*
(g*x+f)*GAMMA(1/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(1/n)
)-2*I*a*b*exp(I*c)*f*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x
+f)^n)^(2/n))+2*I*a*b*f*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/
((I*d*(g*x+f)^n)^(2/n))-2^(-1-2/n)*b^2*exp(2*I*c)*f*(g*x+f)^2*GAMMA(2/n,-2
*I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(2/n))-2^(-1-2/n)*b^2*f*(g*x+f)^2*
GAMMA(2/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(2/n))+I*a*b*
exp(I*c)*(g*x+f)^3*GAMMA(3/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(3/n)
)-I*a*b*(g*x+f)^3*GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)
^(3/n))+2^(-2-3/n)*b^2*exp(2*I*c)*(g*x+f)^3*GAMMA(3/n,-2*I*d*(g*x+f)^n)/g^
3/n/((-I*d*(g*x+f)^n)^(3/n))+2^(-2-3/n)*b^2*(g*x+f)^3*GAMMA(3/n,2*I*d*(g*x
+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.76

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$$

$$=\frac{(2a^2+b^2)\,g^3nx^3+6iabe^{ic}(f+gx)\,(-id(f+gx)^n)^{-3/n}\Big(f^2(-id(f+gx)^n)^{2/n}\,\Gamma\big(\tfrac{1}{n},-id(f+gx)^n\big)-(f}{}$$

input
```
Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
((2*a^2 + b^2)*g^3*n*x^3 + ((6*I)*a*b*E^(I*c)*(f + g*x)*(f^2*((-I)*d*(f +
g*x)^n)^(2/n)*Gamma[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*(2*f*((-I)*d*(
f + g*x)^n)^n^(-1)*Gamma[2/n, (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, (
-I)*d*(f + g*x)^n])))/((-I)*d*(f + g*x)^n)^(3/n) - ((6*I)*a*b*(f + g*x)*(f
^2*(I*d*(f + g*x)^n)^(2/n)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*(2*f
*(I*d*(f + g*x)^n)^n^(-1)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g*x)*Gamma[3/
n, I*d*(f + g*x)^n])))/(E^(I*c)*(I*d*(f + g*x)^n)^(3/n)) + (3*b^2*E^((2*I)
*c)*(f + g*x)*(4^n^(-1)*f^2*((-I)*d*(f + g*x)^n)^(2/n)*Gamma[n^(-1), (-2*I
)*d*(f + g*x)^n] - (f + g*x)*(2^(1 + n^(-1))*f*((-I)*d*(f + g*x)^n)^n^(-1)
*Gamma[2/n, (-2*I)*d*(f + g*x)^n] - (f + g*x)*Gamma[3/n, (-2*I)*d*(f + g*x
)^n])))/(2^((3 + n)/n)*((-I)*d*(f + g*x)^n)^(3/n)) + (3*b^2*(f + g*x)*(4^n
^(-1)*f^2*(I*d*(f + g*x)^n)^(2/n)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n] - (f
+ g*x)*(2^(1 + n^(-1))*f*(I*d*(f + g*x)^n)^n^(-1)*Gamma[2/n, (2*I)*d*(f +
g*x)^n] - (f + g*x)*Gamma[3/n, (2*I)*d*(f + g*x)^n])))/(2^((3 + n)/n)*E^((
2*I)*c)*(I*d*(f + g*x)^n)^(3/n)))/(6*g^3*n)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 819, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{\int \Big(f^2(a+b\sin(d(f+gx)^n+c))^2+(f+gx)^2(a+b\sin(d(f+gx)^n+c))^2-2f(f+gx)(a+b\sin(d(f+gx)^n}{g^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{iabe^{ic}(f+gx)^3\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)(-id(f+gx)^n)^{-3/n}}{n}+\frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\Gamma\left(\frac{3}{n},-2id(f+gx)^n\right)(-id(f+gx)^n)^{-3/n}}{n}-\frac{2iabe^{ic}f(f+gx)^2\Gamma(}{}$$

input

```
Int[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output

```
(((2*a^2 + b^2)*f^2*(f + g*x))/2 - ((2*a^2 + b^2)*f*(f + g*x)^2)/2 + ((2*a
^2 + b^2)*(f + g*x)^3)/6 + (I*a*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1), (-I)
*d*(f + g*x)^n])/(n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*f^2*(f + g*x)*Ga
mma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c)*n*(I*d*(f + g*x)^n)^n^(-1)) + (2^(-
2 - n^(-1))*b^2*E^((2*I)*c)*f^2*(f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)
^n])/(n*((-I)*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2*f^2*(f + g*x)*
Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*n*(I*d*(f + g*x)^n)^n^(-1
)) - ((2*I)*a*b*E^(I*c)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(n*(
(-I)*d*(f + g*x)^n)^(2/n)) + ((2*I)*a*b*f*(f + g*x)^2*Gamma[2/n, I*d*(f +
g*x)^n])/(E^(I*c)*n*(I*d*(f + g*x)^n)^(2/n)) - (2^(-1 - 2/n)*b^2*E^((2*I)*
c)*f*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n])/(n*((-I)*d*(f + g*x)^n)
^(2/n)) - (2^(-1 - 2/n)*b^2*f*(f + g*x)^2*Gamma[2/n, (2*I)*d*(f + g*x)^n])
/(E^((2*I)*c)*n*(I*d*(f + g*x)^n)^(2/n)) + (I*a*b*E^(I*c)*(f + g*x)^3*Gamm
a[3/n, (-I)*d*(f + g*x)^n])/(n*((-I)*d*(f + g*x)^n)^(3/n)) - (I*a*b*(f + g
*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*n*(I*d*(f + g*x)^n)^(3/n)) + (
2^(-2 - 3/n)*b^2*E^((2*I)*c)*(f + g*x)^3*Gamma[3/n, (-2*I)*d*(f + g*x)^n])
/(n*((-I)*d*(f + g*x)^n)^(3/n)) + (2^(-2 - 3/n)*b^2*(f + g*x)^3*Gamma[3/n,
 (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*n*(I*d*(f + g*x)^n)^(3/n)))/g^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2(a+b\sin(c+d(gx+f)^n))^2dx$$

input
```
int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int(b\sin((gx+f)^n d+c)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-b^2*x^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*x^2*sin((g*x + f)^n*d +
 c) + (a^2 + b^2)*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$$

input
```
integrate(x**2*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Integral(x**2*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Maxima [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int(b\sin((gx+f)^nd+c)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*b^2*x^3 - 1/2*b^2*integrate(x^2*cos(2*(g*x + f)^n*d + 2*
c), x) + 2*a*b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int(b\sin((gx+f)^nd+c)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx=\int x^2\,(a+b\,\sin(c+d\,(f+g\,x)^n))^2\,dx$$

input
```
int(x^2*(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

output
```
int(x^2*(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

Reduce [F]

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx = \left(\int \sin\left((gx+f)^n d+c\right)^2 x^2dx\right)b^2 + 2\left(\int \sin\left((gx+f)^n d+c\right)x^2dx\right)ab + \frac{a^2x^3}{3}$$

input

```
int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output

```
(3*int(sin((f + g*x)**n*d + c)**2*x**2,x)*b**2 + 6*int(sin((f + g*x)**n*d
+ c)*x**2,x)*a*b + a**2*x**3)/3
```

3.273 $\int x(a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 556

$$
\begin{aligned}
&\int x(a+b\sin(c+d(f+gx)^n))^2\,dx \\
&= -\frac{(2a^2+b^2)\,fx}{2g}+\frac{(2a^2+b^2)\,(f+gx)^2}{4g^2} \\
&\quad-\frac{iabe^{ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^2n} \\
&\quad+\frac{iabe^{-ic}f(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{g^2n} \\
&\quad-\frac{2^{-2-\frac{1}{n}}b^2e^{2ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^2n} \\
&\quad-\frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}f(f+gx)\,(id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{g^2n} \\
&\quad+\frac{iabe^{ic}(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n} \\
&\quad-\frac{iabe^{-ic}(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},id(f+gx)^n\right)}{g^2n} \\
&\quad+\frac{4^{-1-\frac{1}{n}}b^2e^{2ic}(f+gx)^2\,(-id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^2n} \\
&\quad+\frac{4^{-1-\frac{1}{n}}b^2e^{-2ic}(f+gx)^2\,(id(f+gx)^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2id(f+gx)^n\right)}{g^2n}
\end{aligned}
$$

output

```
-1/2*(2*a^2+b^2)*f*x/g+1/4*(2*a^2+b^2)*(g*x+f)^2/g^2-I*a*b*exp(I*c)*f*(g*x
+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(1/n))+I*a*b*f*(g*x+
f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(1/n))-2^(-2-1
/n)*b^2*exp(2*I*c)*f*(g*x+f)*GAMMA(1/n,-2*I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x
+f)^n)^(1/n))-2^(-2-1/n)*b^2*f*(g*x+f)*GAMMA(1/n,2*I*d*(g*x+f)^n)/exp(2*I*
c)/g^2/n/((I*d*(g*x+f)^n)^(1/n))+I*a*b*exp(I*c)*(g*x+f)^2*GAMMA(2/n,-I*d*(
g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))-I*a*b*(g*x+f)^2*GAMMA(2/n,I*d*(g*
x+f)^n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n))+4^(-1-1/n)*b^2*exp(2*I*c)*(
g*x+f)^2*GAMMA(2/n,-2*I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))+4^(-1-
1/n)*b^2*(g*x+f)^2*GAMMA(2/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^2/n/((I*d*(g*x+
f)^n)^(2/n))
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.78

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx$$
$$=\frac{(2a^2+b^2)\,g^2nx^2-4^{-1/n}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-2/n}\left(2^{\frac{1}{n}}f(-id(f+gx)^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)\right.}{}$$

input

```
Integrate[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output

```
((2*a^2 + b^2)*g^2*n*x^2 - (b^2*E^((2*I)*c)*(f + g*x)*(2^n^(-1)*f*((-I)*d*
(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n] - (f + g*x)*Gamma[
2/n, (-2*I)*d*(f + g*x)^n]))/(4^n^(-1)*((-I)*d*(f + g*x)^n)^(2/n)) - (b^2*
(f + g*x)*(2^n^(-1)*f*(I*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (2*I)*d*(f +
g*x)^n] - (f + g*x)*Gamma[2/n, (2*I)*d*(f + g*x)^n]))/(4^n^(-1)*E^((2*I)*c
)*(I*d*(f + g*x)^n)^(2/n)) + (4*a*b*(f + g*x)*(f*((-I)*d*(f + g*x)^n)^n^(-
1)*Gamma[n^(-1), (-I)*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, (-I)*d*(f + g*
x)^n])*((-I)*Cos[c] + Sin[c]))/((-I)*d*(f + g*x)^n)^(2/n) + (4*a*b*(f + g*
x)*(f*(I*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*
Gamma[2/n, I*d*(f + g*x)^n])*(I*Cos[c] + Sin[c]))/(I*d*(f + g*x)^n)^(2/n))
/(4*g^2*n)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx$$

$$\Big\downarrow \ 3914$$

$$\frac{\int \left((f+gx)(a+b\sin(d(f+gx)^n+c))^2-f(a+b\sin(d(f+gx)^n+c))^2\right)d(f+gx)}{g^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(2a^2+b^2\right)(f+gx)^2-\frac{1}{2}f\left(2a^2+b^2\right)(f+gx)+\frac{iabe^{ic}(f+gx)^2(-id(f+gx)^n)^{-2/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)}{n}-iabe^{ic}f(f+gx)(-id($$

input
```
Int[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
(-1/2*((2*a^2 + b^2)*f*(f + g*x)) + ((2*a^2 + b^2)*(f + g*x)^2)/4 - (I*a*b
*E^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(n*((-I)*d*(f + g*
x)^n)^n^(-1)) + (I*a*b*f*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c
)*n*(I*d*(f + g*x)^n)^n^(-1)) - (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f*(f + g*
x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(n*((-I)*d*(f + g*x)^n)^n^(-1)) -
(2^(-2 - n^(-1))*b^2*f*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/(E^((
2*I)*c)*n*(I*d*(f + g*x)^n)^n^(-1)) + (I*a*b*E^(I*c)*(f + g*x)^2*Gamma[2/n
, (-I)*d*(f + g*x)^n])/(n*((-I)*d*(f + g*x)^n)^(2/n)) - (I*a*b*(f + g*x)^2
*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*n*(I*d*(f + g*x)^n)^(2/n)) + (4^(-1
 - n^(-1))*b^2*E^((2*I)*c)*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n])/(
n*((-I)*d*(f + g*x)^n)^(2/n)) + (4^(-1 - n^(-1))*b^2*(f + g*x)^2*Gamma[2/n
, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*n*(I*d*(f + g*x)^n)^(2/n)))/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x(a+b\sin\left(c+d(gx+f)^{n}\right))^{2}dx$$

input
```
int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [F]

$$\int x(a+b\sin\left(c+d(f+gx)^{n}\right))^{2}\,dx=\int\left(b\sin\left((gx+f)^{n}d+c\right)+a\right)^{2}x\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-b^2*x*cos((g*x + f)^n*d + c)^2 + 2*a*b*x*sin((g*x + f)^n*d + c)
+ (a^2 + b^2)*x, x)
```

Sympy [F]

$$\int x(a+b\sin (c+d(f+gx)^n))^2\,dx=\int x(a+b\sin (c+d(f+gx)^n))^2\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Integral(x*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Maxima [F]

$$\int x(a+b\sin (c+d(f+gx)^n))^2\,dx=\int (b\sin ((gx+f)^n d+c)+a)^2 x\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/4*b^2*x^2 - 1/2*b^2*integrate(x*cos(2*(g*x + f)^n*d + 2*c)
, x) + 2*a*b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int x(a+b\sin (c+d(f+gx)^n))^2\,dx=\int (b\sin ((gx+f)^n d+c)+a)^2 x\,dx$$

input
```
integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx=\int x\,(a+b\,\sin\left(c+d\,{(f+g\,x)}^n\right))^2\,dx$$

input
```
int(x*(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

output
```
int(x*(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

Reduce [F]

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx=\left(\int \sin\left((gx+f)^n d+c\right)^2 x dx\right)b^2 +2\left(\int \sin\left((gx+f)^n d+c\right) x dx\right)ab+\frac{a^2x^2}{2}$$

input
```
int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
(2*int(sin((f + g*x)**n*d + c)**2*x,x)*b**2 + 4*int(sin((f + g*x)**n*d + c
)*x,x)*a*b + a**2*x**2)/2
```

3.274 $\int (a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 261

$$\begin{aligned}
&\int (a+b\sin(c+d(f+gx)^n))^2\,dx \\
&=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn} \\
&\quad-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{gn} \\
&\quad+\frac{2^{-2-\frac{1}{n}}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{gn} \\
&\quad+\frac{2^{-2-\frac{1}{n}}b^2e^{-2ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{gn}
\end{aligned}$$

output
```
1/2*(2*a^2+b^2)*x+I*a*b*exp(I*c)*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g/n/((-
I*d*(g*x+f)^n)^(1/n))-I*a*b*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g/n/
((I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*exp(2*I*c)*(g*x+f)*GAMMA(1/n,-2*I*d
*(g*x+f)^n)/g/n/((-I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*(g*x+f)*GAMMA(1/n,
2*I*d*(g*x+f)^n)/exp(2*I*c)/g/n/((I*d*(g*x+f)^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.95

$$\int (a+b\sin (c+d(f+gx)^n))^2\, dx$$

$$= \frac{2(2a^2+b^2)\, gnx + 2^{-1/n}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}, -2id(f+gx)^n\right) + 2^{-1/n}b^2e^{-2ic}(f+gx)}{}$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
(2*(2*a^2 + b^2)*g*n*x + (b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d
*(f + g*x)^n])/(2^n^(-1)*((-I)*d*(f + g*x)^n)^n^(-1)) + (b^2*(f + g*x)*Gam
ma[n^(-1), (2*I)*d*(f + g*x)^n])/(2^n^(-1)*E^((2*I)*c)*(I*d*(f + g*x)^n)^n
^(-1)) - ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*S
in[c]))/(I*d*(f + g*x)^n)^n^(-1) + ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), (-I)
*d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g*x)^n)^n^(-1))/(4*g*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin (c+d(f+gx)^n))^2\, dx$$

$$\Big\downarrow \text{3848}$$

$$\int \left(a^2+2ab\sin (c+d(f+gx)^n)-\frac{1}{2}b^2\cos (2c+2d(f+gx)^n)+\frac{b^2}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x(2a^2+b^2)+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{gn}+\frac{b^2e^{2ic}2^{-\frac{1}{n}-2}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)}{gn}+\frac{b^2e^{-2ic}2^{-\frac{1}{n}-2}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)}{gn}$$

input
```
Int[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
((2*a^2 + b^2)*x)/2 + (I*a*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g
*x)^n])/(g*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*(f + g*x)*Gamma[n^(-1),
 I*d*(f + g*x)^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1
))*b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g*n*((-
I)*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2*(f + g*x)*Gamma[n^(-1), (
2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3848
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Maple [F]

$$\int \left(a+b\sin\left(c+d(gx+f)^n\right)\right)^2dx$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [F]

$$\int (a+b\sin (c+d(f+gx)^n))^2\,dx=\int (b\sin ((gx+f)^n d+c)+a)^2\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-b^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*sin((g*x + f)^n*d + c) + a^
2 + b^2, x)
```

Sympy [F]

$$\int (a+b\sin (c+d(f+gx)^n))^2\,dx=\int (a+b\sin (c+d(f+gx)^n))^2\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Integral((a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Maxima [F]

$$\int (a+b\sin (c+d(f+gx)^n))^2\,dx=\int (b\sin ((gx+f)^n d+c)+a)^2\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/2*b^2*x - 1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c), x) + 2*a
*b*integrate(sin((g*x + f)^n*d + c), x)
```

Giac [F]

$$\int (a+b\sin (c+d(f+gx)^n))^2\,dx=\int (b\sin ((gx+f)^nd+c)+a)^2\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sin (c+d(f+gx)^n))^2\,dx=\int \left(a+b\,\sin \left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2\,dx$$

input
```
int((a + b*sin(c + d*(f + g*x)^n))^2,x)
```

output
```
int((a + b*sin(c + d*(f + g*x)^n))^2, x)
```

Reduce [F]

$$\begin{aligned}\int (a+b\sin (c+d(f+gx)^n))^2\,dx=&\left(\int \sin \left(\left(gx+f\right)^n d+c\right)^2dx\right)b^2\\&+2\left(\int \sin \left(\left(gx+f\right)^n d+c\right)dx\right)ab+a^2x\end{aligned}$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(sin((f + g*x)**n*d + c)**2,x)*b**2 + 2*int(sin((f + g*x)**n*d + c),x)*
a*b + a**2*x
```

3.275 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx = \text{Int}\left(\frac{(a+b\sin(c+d(f+gx)^n))^2}{x},x\right)$$

output
```
Defer(Int)((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 5.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
```

output
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}dx$$

input
```
Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sin(c+d(gx+f)^n))^2}{x}dx$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

output
```
int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx = \int \frac{(b\sin(((gx+f)^n d+c)+a)^2}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="fricas")
```

output
```
integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a
^2 - b^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 18.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)**n))**2/x,x)
```

output
```
Integral((a + b*sin(c + d*(f + g*x)**n))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(b\sin((gx+f)^nd+c)+a)^2}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="maxima")
```

output
```
-1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c)/x, x) + 2*a*b*integrate(sin(
(g*x + f)^n*d + c)/x, x) + a^2*log(x) + 1/2*b^2*log(x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(b\sin((gx+f)^nd+c)+a)^2}{x}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 39.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(a+b\,\sin(c+d\,(f+g\,x)^n))^2}{x}\,dx$$

input
```
int((a + b*sin(c + d*(f + g*x)^n))^2/x,x)
```

output
```
int((a + b*sin(c + d*(f + g*x)^n))^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 200.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx=\int \frac{(a+b\sin((gx+f)^n d+c))^2}{x}dx$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

output
```
int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

3.276 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \text{Int}\left(\frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2},x\right)$$

output
```
Defer(Int)((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]
```

output
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}dx$$

input

```
Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3918

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sin(c+d(gx+f)^n))^2}{x^2}dx$$

input

```
int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

output
```
int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{(a+b\sin{(c+d(f+gx)^n)})^2}{x^2}\,dx = \int \frac{(b\sin{((gx+f)^n d+c)}+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a
^2 - b^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 59.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sin{(c+d(f+gx)^n)})^2}{x^2}\,dx = \int \frac{(a+b\sin{(c+d(f+gx)^n)})^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)**n))**2/x**2,x)
```

output
```
Integral((a + b*sin(c + d*(f + g*x)**n))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(b\sin((gx+f)^n d+c)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="maxima")
```

output
```
-a^2/x - 1/2*(b^2*x*integrate(cos(2*(g*x + f)^n*d + 2*c)/x^2, x) - 4*a*b*x
*integrate(sin((g*x + f)^n*d + c)/x^2, x) + b^2)/x
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(b\sin((gx+f)^n d+c)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 39.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \int \frac{(a+b\,\sin(c+d\,(f+g\,x)^n))^2}{x^2}\,dx$$

input
```
int((a + b*sin(c + d*(f + g*x)^n))^2/x^2,x)
```

output
```
int((a + b*sin(c + d*(f + g*x)^n))^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2}\,dx = \frac{\left(\int \frac{\sin((gx+f)^n d+c)^2}{x^2}dx\right)b^2x+2\left(\int \frac{\sin((gx+f)^n d+c)}{x^2}dx\right)abx-a^2}{x}$$

input
```
int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

output
```
(int(sin((f + g*x)**n*d + c)**2/x**2,x)*b**2*x + 2*int(sin((f + g*x)**n*d
+ c)/x**2,x)*a*b*x - a**2)/x
```

3.277 $\int \frac{x^2}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\operatorname{Int}\left(\frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

output
```
Defer(Int)(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

input
```
Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)} \, dx$$

$$\Big\downarrow \ 3918$$

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}dx$$

input
```
Int[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}dx$$

input
```
int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin \left(c+d(f+gx)^n\right)}\,dx=\int \frac{x^2}{b\sin \left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\sin \left(c+d(f+gx)^n\right)}\,dx=\text{Timed out}$$

input
```
integrate(x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin \left(c+d(f+gx)^n\right)}\,dx=\int \frac{x^2}{b\sin \left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin(c+d(f+gx)^n)}\,dx=\int \frac{x^2}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a+b\sin(c+d(f+gx)^n)}\,dx=\int \frac{x^2}{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}\,dx$$

input
```
int(x^2/(a + b*sin(c + d*(f + g*x)^n)),x)
```

output
```
int(x^2/(a + b*sin(c + d*(f + g*x)^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx = \frac{-3\left(\int \frac{\sin\left((gx+f)^n d+c\right)x^2}{a+b\sin\left((gx+f)^n d+c\right)}dx\right)b+x^3}{3a}$$

input

```
int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output

```
( - 3*int((sin((f + g*x)**n*d + c)*x**2)/(sin((f + g*x)**n*d + c)*b + a),x
)*b + x**3)/(3*a)
```

3.278 $\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx = \text{Int}\left(\frac{x}{a+b\sin(c+d(f+gx)^n)},x\right)$$

output
```
Defer(Int)(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx = \int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx$$

input
```
Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}dx$$

input
```
Int[x/(a + b*Sin[c + d*(f + g*x)^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}dx$$

input
```
int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 103.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x}{a+b\sin\left(c+d\left(f+gx\right)^n\right)}\,dx$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Integral(x/(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{x}{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}\,dx$$

input
```
int(x/(a + b*sin(c + d*(f + g*x)^n)),x)
```

output
```
int(x/(a + b*sin(c + d*(f + g*x)^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx = \frac{-2\left(\int \frac{\sin((gx+f)^n d+c)x}{a+b\sin((gx+f)^n d+c)}dx\right)b+x^2}{2a}$$

input
```
int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
( - 2*int((sin((f + g*x)**n*d + c)*x)/(sin((f + g*x)**n*d + c)*b + a),x)*b
 + x**2)/(2*a)
```

3.279 $\int \frac{1}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\text{Int}\left(\frac{1}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]
```

output
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3850\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$
$$\Big\downarrow \text{3850}$$
$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}dx$$

input
```
Int[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}dx$$

input
```
int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 43.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\sin\left(c+d\left(f+gx\right)^{n}\right)}\,dx$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Integral(1/(a + b*sin(c + d*(f + g*x)**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{b\sin\left((gx+f)^n d+c\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 40.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}\,dx$$

input
```
int(1/(a + b*sin(c + d*(f + g*x)^n)),x)
```

output
```
int(1/(a + b*sin(c + d*(f + g*x)^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\frac{-\left(\int \frac{\sin\left((gx+f)^n d+c\right)}{a+b\sin\left((gx+f)^n d+c\right)}dx\right)b+x}{a}$$

input
```
int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
( - int(sin((f + g*x)**n*d + c)/(sin((f + g*x)**n*d + c)*b + a),x)*b + x)/
a
```

3.280 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx = \int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

output
```
Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b \sin (c+d(f+gx)^n))} \, dx$$

$$\Big\downarrow \ 3918$$

$$\int \frac{1}{x (a+b \sin (c+d(f+gx)^n))}dx$$

input
```
Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x (a+b \sin (c+d (gx+f)^n))}dx$$

input
```
int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{(b\sin\left((gx+f)^nd+c\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*sin((g*x + f)^n*d + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 104.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d\left(f+gx\right)^n\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Integral(1/(x*(a + b*sin(c + d*(f + g*x)**n))), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{(b\sin\left((gx+f)^n d+c\right)+a)x}\,dx$$

input

```
integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output

```
integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{(b\sin\left((gx+f)^n d+c\right)+a)x}\,dx$$

input

```
integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output

```
integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 40.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}\,dx$$

input

```
int(1/(x*(a + b*sin(c + d*(f + g*x)^n))),x)
```

output
```
int(1/(x*(a + b*sin(c + d*(f + g*x)^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^{n}\right)\right)}\,dx=\frac{-\left(\int \frac{\sin\left((gx+f)^{n}d+c\right)}{\sin\left((gx+f)^{n}d+c\right)bx+ax}dx\right)b+\log(x)}{a}$$

input
```
int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
( - int(sin((f + g*x)**n*d + c)/(sin((f + g*x)**n*d + c)*b*x + a*x),x)*b +
 log(x))/a
```

3.281 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

output
```
Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}dx$$

input
```
Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)}dx$$

input
```
int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*sin((g*x + f)^n*d + c) + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))}\,dx=\int \frac{1}{(b\sin((gx+f)^nd+c)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

output
```
integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 39.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))}\,dx=\int \frac{1}{x^2\ (a+b\ \sin(c+d\,(f+g\,x)^n))}\,dx$$

input
```
int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))),x)
```

output
```
int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^2 \left(a+b\sin \left(c+d(f+gx)^n\right)\right)}\, dx = \frac{-\left(\int \frac{\sin\left((gx+f)^n d+c\right)}{\sin\left((gx+f)^n d+c\right) b\, x^2+a\, x^2}dx\right) bx-1}{ax}$$

input
```
int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

output
```
( - (int(sin((f + g*x)**n*d + c)/(sin((f + g*x)**n*d + c)*b*x**2 + a*x**2)
,x)*b*x + 1))/(a*x)
```

3.282 $\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \mathrm{Int}\left(\frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

output
```
Defer(Int)(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \$\mathrm{Aborted}$$

input
```
Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

$$\Big\downarrow \text{ 3918}$$

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}dx$$

input

```
Int[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3918

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\sin(c+d(gx+f)^n))^2}dx$$

input
```
int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x^2}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-x^2/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c)
 - a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 1509, normalized size of antiderivative = 68.59

$$\int \frac{x^2}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output
```
(2*(a*b*g*x^3 + a*b*f*x^2)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d +
c) + 2*(a*b*g*x^3 + a*b*f*x^2)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(
g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f
)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*c
os((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*
g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*
x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)
*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*
g*n)*cos(2*(g*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*g*n*x^2
*cos((g*x + f)^n*d + c)^2 + 2*(g*x + f)^n*a^2*d*g*n*x^2*sin((g*x + f)^n*d
+ c)^2 + (g*x + f)^n*a*b*d*g*n*x^2*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a
*b*d*g*n*x^2*sin((g*x + f)^n*d + c) + (2*a*b*f*x - (a*b*g*n - 3*a*b*g)*x^2
)*cos((g*x + f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) - (2*a*b*f*x - (a*b*g
*n - 3*a*b*g)*x^2)*cos((g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*g*n*x^2*cos
((g*x + f)^n*d + c) - 2*b^2*f*x + (b^2*g*n - 3*b^2*g)*x^2 - (2*a*b*f*x - (
a*b*g*n - 3*a*b*g)*x^2)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c)
)/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4
 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)
*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + ...
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x^2}{(b\sin\left((gx+f)^n d+c\right)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate(x^2/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 39.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x^2}{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}\,dx$$

input
```
int(x^2/(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

output
```
int(x^2/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20518, normalized size of antiderivative = 932.64

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Too large to display}$$

input
```
int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output

```
( - 4*cos((f + g*x)**n*d + c)*b**2*g**3*n**3*x**3 - 24*cos((f + g*x)**n*d
+ c)*b**2*g**3*n**2*x**3 - 44*cos((f + g*x)**n*d + c)*b**2*g**3*n*x**3 - 2
4*cos((f + g*x)**n*d + c)*b**2*g**3*x**3 + 12*(f + g*x)**n*sin((f + g*x)**
n*d + c)*b**2*d*f**3 - 12*(f + g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*f**2
*g*n*x + 6*(f + g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*f*g**2*n**2*x**2 +
6*(f + g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*f*g**2*n*x**2 - 2*(f + g*x)*
*n*sin((f + g*x)**n*d + c)*b**2*d*g**3*n**3*x**3 - 6*(f + g*x)**n*sin((f +
 g*x)**n*d + c)*b**2*d*g**3*n**2*x**3 - 4*(f + g*x)**n*sin((f + g*x)**n*d
+ c)*b**2*d*g**3*n*x**3 + 12*(f + g*x)**n*a*b*d*f**3 - 12*(f + g*x)**n*a*b
*d*f**2*g*n*x + 6*(f + g*x)**n*a*b*d*f*g**2*n**2*x**2 + 6*(f + g*x)**n*a*b
*d*f*g**2*n*x**2 - 2*(f + g*x)**n*a*b*d*g**3*n**3*x**3 - 6*(f + g*x)**n*a*
b*d*g**3*n**2*x**3 - 4*(f + g*x)**n*a*b*d*g**3*n*x**3 + 24*int(x**3/(tan((
(f + g*x)**n*d + c)/2)**4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**2*g*x
 + 4*tan(((f + g*x)**n*d + c)/2)**3*a*b*f + 4*tan(((f + g*x)**n*d + c)/2)*
*3*a*b*g*x + 2*tan(((f + g*x)**n*d + c)/2)**2*a**2*f + 2*tan(((f + g*x)**n
*d + c)/2)**2*a**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)**2*b**2*f + 4*tan((
(f + g*x)**n*d + c)/2)**2*b**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)*a*b*f +
 4*tan(((f + g*x)**n*d + c)/2)*a*b*g*x + a**2*f + a**2*g*x),x)*sin((f + g*
x)**n*d + c)*a*b**3*g**4*n**3 + 144*int(x**3/(tan(((f + g*x)**n*d + c)/2)*
*4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**2*g*x + 4*tan(((f + g*x)*...
```

3.283 $\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Int}\left(\frac{x}{(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

output
```
Defer(Int)(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}dx$$

$$\Big\downarrow \; 3918$$

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}dx$$

input
```
Int[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sin(c+d\,(gx+f)^n))^2}dx$$

input
```
int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{x}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-x/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
 a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Timed out}$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 1476, normalized size of antiderivative = 73.80

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

input

```
integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output

```
(2*(a*b*g*x^2 + a*b*f*x)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
 + 2*(a*b*g*x^2 + a*b*f*x)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x
+ f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*
d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((
g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n
*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*
sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x +
f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*
x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)
*cos(2*(g*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*g*n*x*cos((
g*x + f)^n*d + c)^2 + 2*(g*x + f)^n*a^2*d*g*n*x*sin((g*x + f)^n*d + c)^2 +
 (g*x + f)^n*a*b*d*g*n*x*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*g*n*x
*sin((g*x + f)^n*d + c) + (a*b*f - (a*b*g*n - 2*a*b*g)*x)*cos((g*x + f)^n*
d + c))*cos(2*(g*x + f)^n*d + 2*c) - (a*b*f - (a*b*g*n - 2*a*b*g)*x)*cos((
g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*g*n*x*cos((g*x + f)^n*d + c) - b^2*
f + (b^2*g*n - 2*b^2*g)*x - (a*b*f - (a*b*g*n - 2*a*b*g)*x)*sin((g*x + f)^
n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*c
os(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x
 + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d +
 c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(...
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

input
```
integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x}{\left(a+b\,\sin\left(c+d\,(f+g\,x)^n\right)\right)^2}\,dx$$

input
```
int(x/(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

output
```
int(x/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 15251, normalized size of antiderivative = 762.55

$$\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \text{Too large to display}$$

input
```
int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output

```
( - 4*cos((f + g*x)**n*d + c)*b**2*g**2*n**2*x**2 - 12*cos((f + g*x)**n*d
+ c)*b**2*g**2*n*x**2 - 8*cos((f + g*x)**n*d + c)*b**2*g**2*x**2 - 4*(f +
g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*f**2 + 4*(f + g*x)**n*sin((f + g*x)
**n*d + c)*b**2*d*f*g*n*x - 2*(f + g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*
g**2*n**2*x**2 - 2*(f + g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*g**2*n*x**2
 - 4*(f + g*x)**n*a*b*d*f**2 + 4*(f + g*x)**n*a*b*d*f*g*n*x - 2*(f + g*x)*
*n*a*b*d*g**2*n**2*x**2 - 2*(f + g*x)**n*a*b*d*g**2*n*x**2 + 16*int(x**2/(
tan(((f + g*x)**n*d + c)/2)**4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**
2*g*x + 4*tan(((f + g*x)**n*d + c)/2)**3*a*b*f + 4*tan(((f + g*x)**n*d + c
)/2)**3*a*b*g*x + 2*tan(((f + g*x)**n*d + c)/2)**2*a**2*f + 2*tan(((f + g*
x)**n*d + c)/2)**2*a**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)**2*b**2*f + 4*
tan(((f + g*x)**n*d + c)/2)**2*b**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)*a*
b*f + 4*tan(((f + g*x)**n*d + c)/2)*a*b*g*x + a**2*f + a**2*g*x),x)*sin((f
 + g*x)**n*d + c)*a*b**3*g**3*n**2 + 48*int(x**2/(tan(((f + g*x)**n*d + c)
/2)**4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**2*g*x + 4*tan(((f + g*x)
**n*d + c)/2)**3*a*b*f + 4*tan(((f + g*x)**n*d + c)/2)**3*a*b*g*x + 2*tan(
((f + g*x)**n*d + c)/2)**2*a**2*f + 2*tan(((f + g*x)**n*d + c)/2)**2*a**2*
g*x + 4*tan(((f + g*x)**n*d + c)/2)**2*b**2*f + 4*tan(((f + g*x)**n*d + c)
/2)**2*b**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)*a*b*f + 4*tan(((f + g*x)**
n*d + c)/2)*a*b*g*x + a**2*f + a**2*g*x),x)*sin((f + g*x)**n*d + c)*a*b...
```

3.284 $\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Int}\left(\frac{1}{(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

input
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]
```

output
```
Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3850}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

$$\Big\downarrow \; 3850$$

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}dx$$

input
```
Int[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sin(c+d\,(gx+f)^n))^2}dx$$

input
```
int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
 a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 1403, normalized size of antiderivative = 77.94

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{(b\sin((gx+f)^n d+c)+a)^2}\,dx$$

input

```
integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output

```
(2*(a*b*g*x + a*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c) + 2
*(a*b*g*x + a*b*f)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x + f)^n*d
*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*co
s((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)
^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*s
in(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x
 + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d +
 c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n
*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(
g*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*n*cos((g*x + f)^n*d
 + c)^2 + 2*(g*x + f)^n*a^2*d*n*sin((g*x + f)^n*d + c)^2 + (g*x + f)^n*a*b
*d*n*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*n*sin((g*x + f)^n*d + c)
- (a*b*n - a*b)*cos((g*x + f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) + (a*b*
n - a*b)*cos((g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*n*cos((g*x + f)^n*d +
 c) + b^2*n - b^2 + (a*b*n - a*b)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^
n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*n*cos(2*(g*x + f)^n*d + 2*c)^2
+ 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b -
a*b^3)*(g*x + f)^n*d*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) +
 (a^2*b^2 - b^4)*(g*x + f)^n*d*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a
^2*b^2)*(g*x + f)^n*d*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g...
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{(b\sin((gx+f)^nd+c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate((b*sin((g*x + f)^n*d + c) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 40.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\int \frac{1}{(a+b\,\sin(c+d\,(f+g\,x)^n))^2}\,dx$$

input
```
int(1/(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

output
```
int(1/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 9964, normalized size of antiderivative = 553.56

$$\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output

```
( - 4*cos((f + g*x)**n*d + c)*b**2*g*n*x - 4*cos((f + g*x)**n*d + c)*b**2*
g*x + 2*(f + g*x)**n*sin((f + g*x)**n*d + c)*b**2*d*f - 2*(f + g*x)**n*sin
((f + g*x)**n*d + c)*b**2*d*g*n*x + 2*(f + g*x)**n*a*b*d*f - 2*(f + g*x)**
n*a*b*d*g*n*x + 12*int(tan(((f + g*x)**n*d + c)/2)**2/(tan(((f + g*x)**n*d
 + c)/2)**4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**2*g*x + 4*tan(((f +
 g*x)**n*d + c)/2)**3*a*b*f + 4*tan(((f + g*x)**n*d + c)/2)**3*a*b*g*x + 2
*tan(((f + g*x)**n*d + c)/2)**2*a**2*f + 2*tan(((f + g*x)**n*d + c)/2)**2*
a**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)**2*b**2*f + 4*tan(((f + g*x)**n*d
 + c)/2)**2*b**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)*a*b*f + 4*tan(((f + g
*x)**n*d + c)/2)*a*b*g*x + a**2*f + a**2*g*x),x)*sin((f + g*x)**n*d + c)*a
*b**3*f*g*n + 12*int(tan(((f + g*x)**n*d + c)/2)**2/(tan(((f + g*x)**n*d +
 c)/2)**4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**2*g*x + 4*tan(((f + g
*x)**n*d + c)/2)**3*a*b*f + 4*tan(((f + g*x)**n*d + c)/2)**3*a*b*g*x + 2*t
an(((f + g*x)**n*d + c)/2)**2*a**2*f + 2*tan(((f + g*x)**n*d + c)/2)**2*a*
*2*g*x + 4*tan(((f + g*x)**n*d + c)/2)**2*b**2*f + 4*tan(((f + g*x)**n*d +
 c)/2)**2*b**2*g*x + 4*tan(((f + g*x)**n*d + c)/2)*a*b*f + 4*tan(((f + g*x
)**n*d + c)/2)*a*b*g*x + a**2*f + a**2*g*x),x)*sin((f + g*x)**n*d + c)*a*b
**3*f*g + 12*int(tan(((f + g*x)**n*d + c)/2)**2/(tan(((f + g*x)**n*d + c)/
2)**4*a**2*f + tan(((f + g*x)**n*d + c)/2)**4*a**2*g*x + 4*tan(((f + g*x)*
*n*d + c)/2)**3*a*b*f + 4*tan(((f + g*x)**n*d + c)/2)**3*a*b*g*x + 2*ta...
```

3.285 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))^2}\,dx=\operatorname{Int}\left(\frac{1}{x(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

output
```
Defer(Int)(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3918\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx \Big\downarrow 3918$$

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}dx$$

input
```
Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\sin\left(c+d\left(gx+f\right)^{n}\right))^{2}}dx$$

input
```
int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x*cos((g*x + f)^n*d + c)^2 - 2*a*b*x*sin((g*x + f)^n*d +
c) - (a^2 + b^2)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 27.05 (sec) , antiderivative size = 5041, normalized size of antiderivative = 229.14

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2x}\,dx$$

input

```
integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output

```
(2*(a^3*b*g*x + a^3*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
 - 2*(b^4*g*x*sin(2*c) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^3*b
- a*b^3)*g*x + (a^3*b - a*b^3)*f + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))
*cos(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*
sin(c))*cos((g*x + f)^n*d) - (a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*sin(2
*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c)
)*sin((g*x + f)^n*d))*cos((g*x + f)^n*d + c) + 4*((a^3*b - a*b^3)*g*x*cos(
c) + (a^3*b - a*b^3)*f*cos(c))*cos((g*x + f)^n*d) + ((g*x + f)^n*a^4*b^2*d
*g*n*x*cos(2*(g*x + f)^n*d + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x*sin(2*(g
*x + f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n
*x*cos(2*(g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6
 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)^2
 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x*sin(2*(g*x + f)^n
*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x
+ f)^n*d) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a
^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*
a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2
- 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g*n*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*si
n(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*cos((g*x +
 f)^n*d) - (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x*cos(2*c) - 2*((a^3*b^3 -...
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 40.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{x\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/(x*(a + b*sin(c + d*(f + g*x)^n))^2),x)
```

output
```
int(1/(x*(a + b*sin(c + d*(f + g*x)^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 6.18

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$
$$=\frac{-\left(\int \frac{\sin\left((gx+f)^n d+c\right)^2}{\sin\left((gx+f)^n d+c\right)^2 b^2 x+2\sin\left((gx+f)^n d+c\right)abx+a^2 x}dx\right)b^2-2\left(\int \frac{\sin\left((gx+f)^n d+c\right)}{\sin\left((gx+f)^n d+c\right)^2 b^2 x+2\sin\left((gx+f)^n d+c\right)abx+a^2 x}dx\right)ab}{a^2}$$

input
```
int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
( - int(sin((f + g*x)**n*d + c)**2/(sin((f + g*x)**n*d + c)**2*b**2*x + 2*
sin((f + g*x)**n*d + c)*a*b*x + a**2*x),x)*b**2 - 2*int(sin((f + g*x)**n*d
 + c)/(sin((f + g*x)**n*d + c)**2*b**2*x + 2*sin((f + g*x)**n*d + c)*a*b*x
 + a**2*x),x)*a*b + log(x))/a**2
```

3.286 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))^2} \, dx$$

$$\Big\downarrow \text{ 3918}$$

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))^2}dx$$

input
```
Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+b\sin(c+d(gx+f)^n))^2}dx$$

input
```
int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output
```
int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^nd+c\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

output
```
integral(-1/(b^2*x^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*x^2*sin((g*x + f)^n*
d + c) - (a^2 + b^2)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 34.92 (sec) , antiderivative size = 5369, normalized size of antiderivative = 244.05

$$\int \frac{1}{x^2 \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left((gx+f)^n d+c\right)+a\right)^2 x^2}\,dx$$

input

```
integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

output

```
(2*(a^3*b*g*x + a^3*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
 - 2*(b^4*g*x*sin(2*c) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^3*b
- a*b^3)*g*x + (a^3*b - a*b^3)*f + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))
*cos(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*
sin(c))*cos((g*x + f)^n*d) - (a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*sin(2
*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c)
)*sin((g*x + f)^n*d))*cos((g*x + f)^n*d + c) + 4*((a^3*b - a*b^3)*g*x*cos(
c) + (a^3*b - a*b^3)*f*cos(c))*cos((g*x + f)^n*d) + ((g*x + f)^n*a^4*b^2*d
*g*n*x^2*cos(2*(g*x + f)^n*d + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x^2*sin(
2*(g*x + f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d
*g*n*x^2*cos(2*(g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2
+ (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f
)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*sin(2*(
g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos(
c)*sin((g*x + f)^n*d) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d)^2 +
 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*si
n(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g*n*x^2 - 2*(2*((a^3*b^3
- a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*
d*g*n*x^2*cos((g*x + f)^n*d) - (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x^2*co...
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{1}{(b\sin((gx+f)^n d+c)+a)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 40.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{1}{x^2\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))^2),x)
```

output
```
int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 6.91

$$\int \frac{1}{x^2 (a+b\sin(c+d(f+gx)^n))^2}\,dx$$
$$= \frac{-\left(\int \frac{\sin((gx+f)^n d+c)^2}{\sin((gx+f)^n d+c)^2 b^2x^2+2\sin((gx+f)^n d+c)ab\,x^2+a^2x^2}dx\right) b^2x - 2\bigg(\int \frac{\sin((gx+f)^n d+c)}{\sin((gx+f)^n d+c)^2 b^2x^2+2\sin((gx+f)^n d+c)ab\,x^2+a^2x^2}}{a^2x}$$

input

```
int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

output

```
( - int(sin((f + g*x)**n*d + c)**2/(sin((f + g*x)**n*d + c)**2*b**2*x**2 +
 2*sin((f + g*x)**n*d + c)*a*b*x**2 + a**2*x**2),x)*b**2*x - 2*int(sin((f
+ g*x)**n*d + c)/(sin((f + g*x)**n*d + c)**2*b**2*x**2 + 2*sin((f + g*x)**
n*d + c)*a*b*x**2 + a**2*x**2),x)*a*b*x - 1)/(a**2*x)
```

3.287 $\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \text{Int}\left((ex)^m (a+b\sin(c+d(f+gx)^n))^p, x\right)$$

output

```
Defer(Int)((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx = \int (ex)^m (a+b\sin(c+d(f+gx)^n))^p \, dx$$

input

```
Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
```

output

```
Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+b\sin (c+d(f+gx)^n))^p\, dx$$
$$\Big\downarrow \text{ 3918}$$
$$\int (ex)^m (a+b\sin (c+d(f+gx)^n))^p dx$$

input
```
Int[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\sin (c+d(gx+f)^n))^p dx$$

input
```
int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

output
```
int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a+b\sin (c+d(f+gx)^n))^p \, dx = \int (ex)^m (b\sin ((gx+f)^n d+c)+a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (a+b\sin (c+d(f+gx)^n))^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b*sin(c+d*(g*x+f)**n))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m (a+b\sin (c+d(f+gx)^n))^p \, dx = \int (ex)^m (b\sin ((gx+f)^n d+c)+a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 150.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m\,(a+b\sin\left(c+d(f+gx)^n\right))^p\ dx=\int (ex)^m\left(b\sin\left((gx+f)^nd+c\right)+a\right)^p dx$$

input
```
integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 40.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^m\,(a+b\sin\left(c+d(f+gx)^n\right))^p\ dx=\int (e\,x)^m\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^p\,dx$$

input
```
int((e*x)^m*(a + b*sin(c + d*(f + g*x)^n))^p,x)
```

output
```
int((e*x)^m*(a + b*sin(c + d*(f + g*x)^n))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 302, normalized size of antiderivative = 12.58

$$\int (ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p\,dx$$
$$= \frac{e^m\left(x^m(a+b\sin\left((gx+f)^n d+c\right))^p x - \left(\int \frac{x^m(gx+f)^n(a+b\sin((gx+f)^n d+c))^p\cos((gx+f)^n d}{\sin((gx+f)^n d+c)bfm+\sin((gx+f)^n d+c)bf+\sin((gx+f)^n d+c)bgmx+\sin((gx+f)}\right.\right.}{}$$

input
```
int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

output
```
(e**m*(x**m*(sin((f + g*x)**n*d + c)*b + a)**p*x - int((x**m*(f + g*x)**n*
(sin((f + g*x)**n*d + c)*b + a)**p*cos((f + g*x)**n*d + c)*x)/(sin((f + g*
x)**n*d + c)*b*f*m + sin((f + g*x)**n*d + c)*b*f + sin((f + g*x)**n*d + c)
*b*g*m*x + sin((f + g*x)**n*d + c)*b*g*x + a*f*m + a*f + a*g*m*x + a*g*x),
x)*b*d*g*m*n*p - int((x**m*(f + g*x)**n*(sin((f + g*x)**n*d + c)*b + a)**p
*cos((f + g*x)**n*d + c)*x)/(sin((f + g*x)**n*d + c)*b*f*m + sin((f + g*x)
**n*d + c)*b*f + sin((f + g*x)**n*d + c)*b*g*m*x + sin((f + g*x)**n*d + c)
*b*g*x + a*f*m + a*f + a*g*m*x + a*g*x),x)*b*d*g*n*p))/(m + 1)
```

3.288 $\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$
\begin{aligned}
\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) dx = {} & ae^2x+aefx^2+\frac{1}{3}af^2x^3 \\
& +bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right) \\
& -bde^2\cos(c)\,\mathrm{CosIntegral}\left(\frac{d}{x}\right) \\
& +\frac{1}{6}bd^3f^2\cos(c)\,\mathrm{CosIntegral}\left(\frac{d}{x}\right) \\
& +bd^2ef\,\mathrm{CosIntegral}\left(\frac{d}{x}\right)\sin(c) \\
& +be^2x\sin\left(c+\frac{d}{x}\right)-\frac{1}{6}bd^2f^2x\sin\left(c+\frac{d}{x}\right) \\
& +befx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right) \\
& +bd^2ef\cos(c)\mathrm{Si}\left(\frac{d}{x}\right)+bde^2\sin(c)\mathrm{Si}\left(\frac{d}{x}\right) \\
& -\frac{1}{6}bd^3f^2\sin(c)\mathrm{Si}\left(\frac{d}{x}\right)
\end{aligned}
$$

output
```
a*e^2*x+a*e*f*x^2+1/3*a*f^2*x^3+b*d*e*f*x*cos(c+d/x)+1/6*b*d*f^2*x^2*cos(c
+d/x)-b*d*e^2*cos(c)*Ci(d/x)+1/6*b*d^3*f^2*cos(c)*Ci(d/x)+b*d^2*e*f*Ci(d/x
)*sin(c)+b*e^2*x*sin(c+d/x)-1/6*b*d^2*f^2*x*sin(c+d/x)+b*e*f*x^2*sin(c+d/x
)+1/3*b*f^2*x^3*sin(c+d/x)+b*d^2*e*f*cos(c)*Si(d/x)+b*d*e^2*sin(c)*Si(d/x)
-1/6*b*d^3*f^2*sin(c)*Si(d/x)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\frac{1}{6}\left(bd\,\text{CosIntegral}\left(\frac{d}{x}\right)\left(\left(-6e^2+d^2f^2\right)\cos(c)+6def\sin(c)\right)+x\left(2a\left(3e^2+3efx+f^2x^2\right)+bdf(6e+fx)\cos\left(c+\frac{d}{x}\right)+b\left(6e^2+6efx-f^2\left(d^2-2x^2\right)\right)\sin\left(c+\frac{d}{x}\right)\right)-bd\left(-6def\cos(c)+\left(-6e^2+d^2f^2\right)\sin(c)\right)\text{Si}\left(\frac{d}{x}\right)\right)$$

input
```
Integrate[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

output
```
(b*d*CosIntegral[d/x]*((-6*e^2 + d^2*f^2)*Cos[c] + 6*d*e*f*Sin[c]) + x*(2*
a*(3*e^2 + 3*e*f*x + f^2*x^2) + b*d*f*(6*e + f*x)*Cos[c + d/x] + b*(6*e^2
+ 6*e*f*x - f^2*(d^2 - 2*x^2))*Sin[c + d/x]) - b*d*(-6*d*e*f*Cos[c] + (-6*
e^2 + d^2*f^2)*Sin[c])*SinIntegral[d/x])/6
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \left(f^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)x^4+2ef\left(a+b\sin\left(c+\frac{d}{x}\right)\right)x^3+e^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)x^2\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+aefx^2+\frac{1}{3}af^2x^3+\frac{1}{6}bd^3f^2\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+bd^2ef\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-bde^2\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)-\frac{1}{6}bd^3f^2\sin(c)\text{Si}\left(\frac{d}{x}\right)+bd^2ef\cos(c)\text{Si}\left(\frac{d}{x}\right)-\frac{1}{6}bd^2f^2x\sin\left(c+\frac{d}{x}\right)+bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right)+be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)$$

input

```
Int[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

output

```
a*e^2*x + a*e*f*x^2 + (a*f^2*x^3)/3 + b*d*e*f*x*Cos[c + d/x] + (b*d*f^2*x^
2*Cos[c + d/x])/6 - b*d*e^2*Cos[c]*CosIntegral[d/x] + (b*d^3*f^2*Cos[c]*Co
sIntegral[d/x])/6 + b*d^2*e*f*CosIntegral[d/x]*Sin[c] + b*e^2*x*Sin[c + d/
x] - (b*d^2*f^2*x*Sin[c + d/x])/6 + b*e*f*x^2*Sin[c + d/x] + (b*f^2*x^3*Si
n[c + d/x])/3 + b*d^2*e*f*Cos[c]*SinIntegral[d/x] + b*d*e^2*Sin[c]*SinInte
gral[d/x] - (b*d^3*f^2*Sin[c]*SinIntegral[d/x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

method	result
parts	$\frac{a(fx+e)^3}{3f}-bd\Bigg(e^2\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\Bigg)+2def\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}$
derivativedivides	$-d\Bigg(-\frac{a\,f^2x^3}{3d}-\frac{aef\,x^2}{d}-\frac{a\,e^2x}{d}+b\,d^2f^2\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x^3}{3d^3}-\frac{\cos\left(c+\frac{d}{x}\right)x^2}{6d^2}+\frac{\sin\left(c+\frac{d}{x}\right)x}{6d}+\frac{\operatorname{Si}\left(\frac{d}{x}\right)\sin}{6}$
default	$-d\Bigg(-\frac{a\,f^2x^3}{3d}-\frac{aef\,x^2}{d}-\frac{a\,e^2x}{d}+b\,d^2f^2\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x^3}{3d^3}-\frac{\cos\left(c+\frac{d}{x}\right)x^2}{6d^2}+\frac{\sin\left(c+\frac{d}{x}\right)x}{6d}+\frac{\operatorname{Si}\left(\frac{d}{x}\right)\sin}{6}$
risch	$a\,e^2x+\frac{a\,f^2x^3}{3}+aef\,x^2+\frac{bd\,e^2\mathrm{e}^{-ic}\operatorname{expIntegral}_1\left(\frac{id}{x}\right)}{2}-\frac{b\,d^3f^2\mathrm{e}^{-ic}\operatorname{expIntegral}_1\left(\frac{id}{x}\right)}{12}-\frac{ib\,d^2ef\,\mathrm{e}^{-ic}\operatorname{exp}}{2}$

input
```
int((f*x+e)^2*(a+b*sin(c+d/x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(f*x+e)^3/f-b*d*(e^2*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
+2*d*e*f*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/
2*Ci(d/x)*sin(c))+d^2*f^2*(-1/3*sin(c+d/x)/d^3*x^3-1/6*cos(c+d/x)/d^2*x^2+
1/6*sin(c+d/x)/d*x+1/6*Si(d/x)*sin(c)-1/6*Ci(d/x)*cos(c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx\\
&=\frac{1}{3}af^2x^3+aefx^2+ae^2x+\frac{1}{6}\left(6\,bd^2ef\,\mathrm{Si}\left(\frac{d}{x}\right)+\left(bd^3f^2-6\,bde^2\right)\mathrm{Ci}\left(\frac{d}{x}\right)\right)\cos\left(c\right)\\
&+\frac{1}{6}\left(bdf^2x^2+6\,bdefx\right)\cos\left(\frac{cx+d}{x}\right)\\
&+\frac{1}{6}\left(6\,bd^2ef\,\mathrm{Ci}\left(\frac{d}{x}\right)-\left(bd^3f^2-6\,bde^2\right)\mathrm{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right)\\
&+\frac{1}{6}\left(2\,bf^2x^3+6\,befx^2-\left(bd^2f^2-6\,be^2\right)x\right)\sin\left(\frac{cx+d}{x}\right)
\end{aligned}$$

input
```
integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 + a*e^2*x + 1/6*(6*b*d^2*e*f*sin_integral(d/x) +
 (b*d^3*f^2 - 6*b*d*e^2)*cos_integral(d/x))*cos(c) + 1/6*(b*d*f^2*x^2 + 6*
b*d*e*f*x)*cos((c*x + d)/x) + 1/6*(6*b*d^2*e*f*cos_integral(d/x) - (b*d^3*
f^2 - 6*b*d*e^2)*sin_integral(d/x))*sin(c) + 1/6*(2*b*f^2*x^3 + 6*b*e*f*x^
2 - (b*d^2*f^2 - 6*b*e^2)*x)*sin((c*x + d)/x)
```

Sympy [F]

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)(e+fx)^2\,dx$$

input
```
integrate((f*x+e)**2*(a+b*sin(c+d/x)),x)
```

output
```
Integral((a + b*sin(c + d/x))*(e + f*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.15

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx=\frac{1}{3}af^2x^3+aefx^2$$
$$-\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right.$$
$$+\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c)+\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d^2+2\,dx\cos\left(\frac{cx+d}{x}\right.\right.$$
$$+\frac{1}{12}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c)+\left(i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d^3+2\,dx^2\cos\left(\frac{cx+d}{x}\right.\right.$$
$$+ae^2x$$

input
```
integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei
(I*d/x) + I*Ei(-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e^2 + 1/2*(((-
I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2
+ 2*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x + d)/x))*b*e*f + 1/12*(((Ei(I*d/
x) + Ei(-I*d/x))*cos(c) + (I*Ei(I*d/x) - I*Ei(-I*d/x))*sin(c))*d^3 + 2*d*x
^2*cos((c*x + d)/x) - 2*(d^2*x - 2*x^3)*sin((c*x + d)/x))*b*f^2 + a*e^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. 2(212) = 424.

Time = 0.16 (sec) , antiderivative size = 1263, normalized size of antiderivative = 5.64

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="giac")
```

output

```
1/6*(b*c^3*d^4*f^2*cos(c)*cos_integral(-c + (c*x + d)/x) + b*c^3*d^4*f^2*s
in(c)*sin_integral(c - (c*x + d)/x) - 3*(c*x + d)*b*c^2*d^4*f^2*cos(c)*cos
_integral(-c + (c*x + d)/x)/x + 6*b*c^3*d^3*e*f*cos_integral(-c + (c*x + d
)/x)*sin(c) - 6*b*c^3*d^3*e*f*cos(c)*sin_integral(c - (c*x + d)/x) - 3*(c*
x + d)*b*c^2*d^4*f^2*sin(c)*sin_integral(c - (c*x + d)/x)/x - 6*b*c^3*d^2*
e^2*cos(c)*cos_integral(-c + (c*x + d)/x) + 3*(c*x + d)^2*b*c*d^4*f^2*cos(
c)*cos_integral(-c + (c*x + d)/x)/x^2 - 18*(c*x + d)*b*c^2*d^3*e*f*cos_int
egral(-c + (c*x + d)/x)*sin(c)/x + b*c^2*d^4*f^2*sin((c*x + d)/x) + 18*(c*
x + d)*b*c^2*d^3*e*f*cos(c)*sin_integral(c - (c*x + d)/x)/x - 6*b*c^3*d^2*
e^2*sin(c)*sin_integral(c - (c*x + d)/x) + 3*(c*x + d)^2*b*c*d^4*f^2*sin(c
)*sin_integral(c - (c*x + d)/x)/x^2 - 6*b*c^2*d^3*e*f*cos((c*x + d)/x) + b
*c*d^4*f^2*cos((c*x + d)/x) - (c*x + d)^3*b*d^4*f^2*cos(c)*cos_integral(-c
 + (c*x + d)/x)/x^3 + 18*(c*x + d)*b*c^2*d^2*e^2*cos(c)*cos_integral(-c +
(c*x + d)/x)/x + 18*(c*x + d)^2*b*c*d^3*e*f*cos_integral(-c + (c*x + d)/x)
*sin(c)/x^2 - 2*(c*x + d)*b*c*d^4*f^2*sin((c*x + d)/x)/x - 18*(c*x + d)^2*
b*c*d^3*e*f*cos(c)*sin_integral(c - (c*x + d)/x)/x^2 - (c*x + d)^3*b*d^4*f
^2*sin(c)*sin_integral(c - (c*x + d)/x)/x^3 + 18*(c*x + d)*b*c^2*d^2*e^2*s
in(c)*sin_integral(c - (c*x + d)/x)/x + 12*(c*x + d)*b*c*d^3*e*f*cos((c*x
+ d)/x)/x - (c*x + d)*b*d^4*f^2*cos((c*x + d)/x)/x - 18*(c*x + d)^2*b*c*d^
2*e^2*cos(c)*cos_integral(-c + (c*x + d)/x)/x^2 - 6*(c*x + d)^3*b*d^3*e...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int (e+f\,x)^2\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

input

```
int((e + f*x)^2*(a + b*sin(c + d/x)),x)
```

output

```
int((e + f*x)^2*(a + b*sin(c + d/x)), x)
```

Reduce [F]

$$\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) \, dx = \text{Too large to display}$$

input

```
int((f*x+e)^2*(a+b*sin(c+d/x)),x)
```

output

```
(cos((c*x + d)/x)*tan((c*x + d)/(2*x))**2*b*d*f**2*x**2 + cos((c*x + d)/x)
*b*d*f**2*x**2 - 12*int(tan((c*x + d)/(2*x))/(tan((c*x + d)/(2*x))**2*x +
x),x)*tan((c*x + d)/(2*x))**2*b*d**2*e*f - 12*int(tan((c*x + d)/(2*x))/(ta
n((c*x + d)/(2*x))**2*x + x),x)*b*d**2*e*f - 2*int(1/(tan((c*x + d)/(2*x))
**2*x + x),x)*tan((c*x + d)/(2*x))**2*b*d**3*f**2 + 12*int(1/(tan((c*x + d
)/(2*x))**2*x + x),x)*tan((c*x + d)/(2*x))**2*b*d*e**2 - 2*int(1/(tan((c*x
 + d)/(2*x))**2*x + x),x)*b*d**3*f**2 + 12*int(1/(tan((c*x + d)/(2*x))**2*
x + x),x)*b*d*e**2 + log(x)*tan((c*x + d)/(2*x))**2*b*d**3*f**2 - 6*log(x)
*tan((c*x + d)/(2*x))**2*b*d*e**2 + log(x)*b*d**3*f**2 - 6*log(x)*b*d*e**2
 - sin((c*x + d)/x)*tan((c*x + d)/(2*x))**2*b*d**2*f**2*x + 6*sin((c*x + d
)/x)*tan((c*x + d)/(2*x))**2*b*e**2*x + 6*sin((c*x + d)/x)*tan((c*x + d)/(
2*x))**2*b*e*f*x**2 + 2*sin((c*x + d)/x)*tan((c*x + d)/(2*x))**2*b*f**2*x*
*3 - sin((c*x + d)/x)*b*d**2*f**2*x + 6*sin((c*x + d)/x)*b*e**2*x + 6*sin(
(c*x + d)/x)*b*e*f*x**2 + 2*sin((c*x + d)/x)*b*f**2*x**3 + 6*tan((c*x + d)
/(2*x))**2*a*e**2*x + 6*tan((c*x + d)/(2*x))**2*a*e*f*x**2 + 2*tan((c*x +
d)/(2*x))**2*a*f**2*x**3 - 6*tan((c*x + d)/(2*x))**2*b*d*e*f*x + 6*a*e**2*
x + 6*a*e*f*x**2 + 2*a*f**2*x**3 + 6*b*d*e*f*x)/(6*(tan((c*x + d)/(2*x))**
2 + 1))
```

3.289 $\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx &= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)\\&\quad-bde\cos(c)\,\mathrm{CosIntegral}\left(\frac{d}{x}\right)\\&\quad+\frac{1}{2}bd^2f\,\mathrm{CosIntegral}\left(\frac{d}{x}\right)\sin(c)\\&\quad+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)\\&\quad+\frac{1}{2}bd^2f\cos(c)\mathrm{Si}\left(\frac{d}{x}\right)+bde\sin(c)\mathrm{Si}\left(\frac{d}{x}\right)\end{aligned}$$

output
```
a*e*x+1/2*a*f*x^2+1/2*b*d*f*x*cos(c+d/x)-b*d*e*cos(c)*Ci(d/x)+1/2*b*d^2*f*
Ci(d/x)*sin(c)+b*e*x*sin(c+d/x)+1/2*b*f*x^2*sin(c+d/x)+1/2*b*d^2*f*cos(c)*
Si(d/x)+b*d*e*sin(c)*Si(d/x)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\frac{1}{2}\left(bdfx\cos\left(c+\frac{d}{x}\right)+bd\,\text{CosIntegral}\left(\frac{d}{x}\right)(-2e\cos(c)+df\sin(c))+x(2e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)+bd(df\cos(c)+2e\sin(c))\text{Si}\left(\frac{d}{x}\right)\right)$$

input

```
Integrate[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

output

```
(b*d*f*x*Cos[c + d/x] + b*d*CosIntegral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) +
x*(2*e + f*x)*(a + b*Sin[c + d/x]) + b*d*(d*f*Cos[c] + 2*e*Sin[c])*SinInte
gral[d/x])/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \left(f\left(a+b\sin\left(c+\frac{d}{x}\right)\right)x^3+e\left(a+b\sin\left(c+\frac{d}{x}\right)\right)x^2\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$aex+\frac{1}{2}afx^2+\frac{1}{2}bd^2f\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)-bde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+bde\sin(c)\text{Si}\left(\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)$$

input
```
Int[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

output
```
a*e*x + (a*f*x^2)/2 + (b*d*f*x*Cos[c + d/x])/2 - b*d*e*Cos[c]*CosIntegral[
d/x] + (b*d^2*f*CosIntegral[d/x]*Sin[c])/2 + b*e*x*Sin[c + d/x] + (b*f*x^2
*Sin[c + d/x])/2 + (b*d^2*f*Cos[c]*SinIntegral[d/x])/2 + b*d*e*Sin[c]*SinI
ntegral[d/x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

method	result
parts	$a\left(\frac{1}{2}f\,x^2+ex\right)-bd\Bigg(e\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\Bigg)+df\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)}{2d^2}$
derivativedivides	$-d\Bigg(-\frac{af\,x^2}{2d}-\frac{aex}{d}+bfd\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\operatorname{Si}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)}{2}\Bigg)+be\Bigg(-$
default	$-d\Bigg(-\frac{af\,x^2}{2d}-\frac{aex}{d}+bfd\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\operatorname{Si}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)}{2}\Bigg)+be\Bigg(-$
risch	$aex+\frac{af\,x^2}{2}+\frac{bde\,\mathrm{e}^{-ic}\operatorname{expIntegral}_1\left(\frac{id}{x}\right)}{2}-\frac{ib\,d^2f\,\mathrm{e}^{-ic}\operatorname{expIntegral}_1\left(\frac{id}{x}\right)}{4}+\frac{bde\,\mathrm{e}^{ic}\operatorname{expIntegral}_1\left(-\frac{id}{x}\right)}{2}+$

input
```
int((f*x+e)*(a+b*sin(c+d/x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+e*x)-b*d*(e*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))+d
*f*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/2*Ci(d
/x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx &= \frac{1}{2}\,bdfx\cos\left(\frac{cx+d}{x}\right)+\frac{1}{2}\,afx^2+aex\\&\quad+\frac{1}{2}\left(bd^2f\operatorname{Si}\left(\frac{d}{x}\right)-2\,bde\operatorname{Ci}\left(\frac{d}{x}\right)\right)\cos\left(c\right)\\&\quad+\frac{1}{2}\left(bd^2f\operatorname{Ci}\left(\frac{d}{x}\right)+2\,bde\operatorname{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right)\\&\quad+\frac{1}{2}\left(bfx^2+2\,bex\right)\sin\left(\frac{cx+d}{x}\right)\end{aligned}$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
1/2*b*d*f*x*cos((c*x + d)/x) + 1/2*a*f*x^2 + a*e*x + 1/2*(b*d^2*f*sin_inte
gral(d/x) - 2*b*d*e*cos_integral(d/x))*cos(c) + 1/2*(b*d^2*f*cos_integral(
d/x) + 2*b*d*e*sin_integral(d/x))*sin(c) + 1/2*(b*f*x^2 + 2*b*e*x)*sin((c*
x + d)/x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x)),x)
```

output
```
Integral((a + b*sin(c + d/x))*(e + f*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx &= \frac{1}{2}afx^2\\ &-\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right.\\ &+\frac{1}{4}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)+\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d^2+2\,dx\cos\left(\frac{cx+d}{x}\right.\right.\\ &+aex\end{aligned}$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output
```
1/2*a*f*x^2 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei
(-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e + 1/4*(((-I*Ei(I*d/x) + I*
Ei(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x
 + d)/x) + 2*x^2*sin((c*x + d)/x))*b*f + a*e*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.41

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

$$=\frac{bc^2d^3f\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)\sin(c)-bc^2d^3f\cos(c)\,\mathrm{Si}\left(c-\frac{cx+d}{x}\right)-2\,bc^2d^2e\cos(c)\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)-\frac{2\,(cx+d)bcd}{}}{}$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="giac")
```

output
```
1/2*(b*c^2*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c) - b*c^2*d^3*f*cos(c
)*sin_integral(c - (c*x + d)/x) - 2*b*c^2*d^2*e*cos(c)*cos_integral(-c + (
c*x + d)/x) - 2*(c*x + d)*b*c*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/
x + 2*(c*x + d)*b*c*d^3*f*cos(c)*sin_integral(c - (c*x + d)/x)/x - 2*b*c^2
*d^2*e*sin(c)*sin_integral(c - (c*x + d)/x) - b*c*d^3*f*cos((c*x + d)/x) +
 4*(c*x + d)*b*c*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x)/x + (c*x + d)
^2*b*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x^2 - (c*x + d)^2*b*d^3*f
*cos(c)*sin_integral(c - (c*x + d)/x)/x^2 + 4*(c*x + d)*b*c*d^2*e*sin(c)*s
in_integral(c - (c*x + d)/x)/x + (c*x + d)*b*d^3*f*cos((c*x + d)/x)/x - 2*
(c*x + d)^2*b*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x)/x^2 - 2*b*c*d^2*
e*sin((c*x + d)/x) + b*d^3*f*sin((c*x + d)/x) - 2*(c*x + d)^2*b*d^2*e*sin(
c)*sin_integral(c - (c*x + d)/x)/x^2 - 2*a*c*d^2*e + a*d^3*f + 2*(c*x + d)
*b*d^2*e*sin((c*x + d)/x)/x + 2*(c*x + d)*a*d^2*e/x)/((c^2 - 2*(c*x + d)*c
/x + (c*x + d)^2/x^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int (e+f\,x)\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

input
```
int((e + f*x)*(a + b*sin(c + d/x)),x)
```

output

```
int((e + f*x)*(a + b*sin(c + d/x)), x)
```

Reduce [F]

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

$$= \frac{-2\left(\int\frac{\tan\left(\frac{cx+d}{2x}\right)}{\tan\left(\frac{cx+d}{2x}\right)^2x+x}dx\right)\tan\left(\frac{cx+d}{2x}\right)^2bd^2f-2\left(\int\frac{\tan\left(\frac{cx+d}{2x}\right)}{\tan\left(\frac{cx+d}{2x}\right)^2x+x}dx\right)bd^2f+4\left(\int\frac{1}{\tan\left(\frac{cx+d}{2x}\right)^2x+x}dx\right)\tan\,(}{}$$

input

```
int((f*x+e)*(a+b*sin(c+d/x)),x)
```

output

```
( - 2*int(tan((c*x + d)/(2*x))/(tan((c*x + d)/(2*x))**2*x + x),x)*tan((c*x
 + d)/(2*x))**2*b*d**2*f - 2*int(tan((c*x + d)/(2*x))/(tan((c*x + d)/(2*x)
)**2*x + x),x)*b*d**2*f + 4*int(1/(tan((c*x + d)/(2*x))**2*x + x),x)*tan((
c*x + d)/(2*x))**2*b*d*e + 4*int(1/(tan((c*x + d)/(2*x))**2*x + x),x)*b*d*
e - 2*log(x)*tan((c*x + d)/(2*x))**2*b*d*e - 2*log(x)*b*d*e + 2*sin((c*x +
 d)/x)*tan((c*x + d)/(2*x))**2*b*e*x + sin((c*x + d)/x)*tan((c*x + d)/(2*x
))**2*b*f*x**2 + 2*sin((c*x + d)/x)*b*e*x + sin((c*x + d)/x)*b*f*x**2 + 2*
tan((c*x + d)/(2*x))**2*a*e*x + tan((c*x + d)/(2*x))**2*a*f*x**2 - tan((c*
x + d)/(2*x))**2*b*d*f*x + 2*a*e*x + a*f*x**2 + b*d*f*x)/(2*(tan((c*x + d)
/(2*x))**2 + 1))
```

3.290 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=ax-bd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)+bd\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

output
```
a*x-b*d*cos(c)*Ci(d/x)+b*x*sin(c+d/x)+b*d*sin(c)*Si(d/x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=ax+bx\cos\left(\frac{d}{x}\right)\sin(c)+bx\cos(c)\sin\left(\frac{d}{x}\right)-bd\left(\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)-\sin(c)\text{Si}\left(\frac{d}{x}\right)\right)$$

input
```
Integrate[a + b*Sin[c + d/x],x]
```

output
```
a*x + b*x*Cos[d/x]*Sin[c] + b*x*Cos[c]*Sin[d/x] - b*d*(Cos[c]*CosIntegral[
d/x] - Sin[c]*SinIntegral[d/x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ax-bd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+bd\sin(c)\text{Si}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)$$

input
```
Int[a + b*Sin[c + d/x],x]
```

output
```
a*x - b*d*Cos[c]*CosIntegral[d/x] + b*x*Sin[c + d/x] + b*d*Sin[c]*SinInteg
ral[d/x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

method	result
default	$ax-bd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)$
parts	$ax-bd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)$
derivativedivides	$-d\left(-\frac{ax}{d}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)\right)$
risch	$ax-\frac{i\pi\ \operatorname{csgn}\left(\frac{d}{x}\right)\mathrm{e}^{-ic}bd}{2}+i\operatorname{Si}\left(\frac{d}{x}\right)\mathrm{e}^{-ic}bd+\frac{\mathrm{e}^{-ic}\operatorname{expIntegral}_{1}\left(-\frac{id}{x}\right)bd}{2}+\frac{\mathrm{e}^{ic}\operatorname{expIntegral}_{1}\left(-\frac{id}{x}\right)bd}{2}+b$

input
```
int(a+b*sin(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
a*x-b*d*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=-bd\cos\left(c\right)\operatorname{Ci}\left(\frac{d}{x}\right)+bd\sin\left(c\right)\operatorname{Si}\left(\frac{d}{x}\right)+bx\sin\left(\frac{cx+d}{x}\right)+ax$$

input
```
integrate(a+b*sin(c+d/x),x, algorithm="fricas")
```

output
```
-b*d*cos(c)*cos_integral(d/x) + b*d*sin(c)*sin_integral(d/x) + b*x*sin((c*
x + d)/x) + a*x
```

Sympy [F]

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

input
```
integrate(a+b*sin(c+d/x),x)
```

output
```
Integral(a + b*sin(c + d/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = -\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right. \\ +ax$$

input
```
integrate(a+b*sin(c+d/x),x, algorithm="maxima")
```

output
```
-1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin
(c))*d - 2*x*sin((c*x + d)/x))*b + a*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(38) = 76.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.61

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx = ax \\ -\frac{\left(cd^2\cos\left(c\right)\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)+cd^2\sin\left(c\right)\mathrm{Si}\left(c-\frac{cx+d}{x}\right)-\frac{(cx+d)d^2\cos(c)\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)}{x}-\frac{(cx+d)d^2\sin(c)\,\mathrm{Si}\left(c-\frac{cx\cdots}{x}\right)}{x}\right.}{\left(c-\frac{cx+d}{x}\right)d}$$

input
```
integrate(a+b*sin(c+d/x),x, algorithm="giac")
```

output
```
a*x - (c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x) + c*d^2*sin(c)*sin_inte
gral(c - (c*x + d)/x) - (c*x + d)*d^2*cos(c)*cos_integral(-c + (c*x + d)/x
)/x - (c*x + d)*d^2*sin(c)*sin_integral(c - (c*x + d)/x)/x + d^2*sin((c*x
+ d)/x))*b/((c - (c*x + d)/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\int a+b\,\sin\left(c+\frac{d}{x}\right)\,dx$$

input
```
int(a + b*sin(c + d/x),x)
```

output
```
int(a + b*sin(c + d/x), x)
```

Reduce [F]

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx=\left(\int \sin\left(\frac{cx+d}{x}\right)dx\right)b+ax$$

input
```
int(a+b*sin(c+d/x),x)
```

output
```
int(sin((c*x + d)/x),x)*b + a*x
```

3.291 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\begin{aligned}\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx &= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f}\\&\quad+\frac{b\,\text{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)\sin\left(c-\frac{df}{e}\right)}{f}\\&\quad+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{f}-\frac{b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}\end{aligned}$$

output
```
a*ln(f+e/x)/f+a*ln(x)/f-b*Ci(d/x)*sin(c)/f+b*Ci(d*f/e+d/x)*sin(c-d*f/e)/f+
b*cos(c-d*f/e)*Si(d*f/e+d/x)/f-b*cos(c)*Si(d/x)/f
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$$

$$= \frac{a\log(e+fx)-b\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c)+b\,\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)+b\cos\left(c-\frac{df}{e}\right)\text{Si}\Big(d\Big(\frac{f}{e}}{f}$$

input
```
Integrate[(a + b*Sin[c + d/x])/(e + f*x),x]
```

output
```
(a*Log[e + f*x] - b*CosIntegral[d/x]*Sin[c] + b*CosIntegral[d*(f/e + x^(-1
))]*Sin[c - (d*f)/e] + b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] -
b*Cos[c]*SinIntegral[d/x])/f
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{3912, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \left(\frac{x\left(a+b\sin\left(c+\frac{d}{x}\right)\right)}{f}-\frac{e\left(a+b\sin\left(c+\frac{d}{x}\right)\right)}{f\left(\frac{e}{x}+f\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log\left(\frac{e}{x}+f\right)}{f}-\frac{a\log\left(\frac{1}{x}\right)}{f}+\frac{b\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{f}-\frac{b\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{f}-\frac{b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}$$

input
```
Int[(a + b*Sin[c + d/x])/(e + f*x),x]
```

output
```
(a*Log[f + e/x])/f - (a*Log[x^(-1)])/f - (b*CosIntegral[d/x]*Sin[c])/f + (
b*CosIntegral[(d*f)/e + d/x]*Sin[c - (d*f)/e])/f + (b*Cos[c - (d*f)/e]*Sin
Integral[(d*f)/e + d/x])/f - (b*Cos[c]*SinIntegral[d/x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

method	result
parts	$\frac{a\ln(fx+e)}{f}-bd\left(\frac{\operatorname{Si}\left(\frac{d}{x}\right)\cos(c)+\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)}{df}-\frac{e\left(\frac{\operatorname{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}-\frac{\operatorname{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin}{e}\right.}{df}\right.$
risch	$\frac{ib\operatorname{expIntegral}_1\left(\frac{id}{x}\right)\mathrm{e}^{-ic}}{2f}-\frac{ib\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\operatorname{expIntegral}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2f}-\frac{ib\operatorname{expIntegral}_1\left(-\frac{id}{x}\right)\mathrm{e}^{ic}}{2f}+$
derivativedivides	$-d\left(\frac{a\ln\left(\frac{d}{x}\right)}{fd}-\frac{a\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}-\frac{be\left(-\frac{\operatorname{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}-\frac{\operatorname{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin}{e}\right.}{fd}\right.$
default	$-d\left(\frac{a\ln\left(\frac{d}{x}\right)}{fd}-\frac{a\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}-\frac{be\left(-\frac{\operatorname{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}-\frac{\operatorname{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin}{e}\right.}{fd}\right.$

input
```
int((a+b*sin(c+d/x))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
a/f*ln(f*x+e)-b*d*(1/d/f*(Si(d/x)*cos(c)+Ci(d/x)*sin(c))-e/d/f*(Si(d/x+c+(
-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/
e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx = -\frac{b\,\mathrm{Ci}\left(\frac{d}{x}\right)\sin\left(c\right)+b\,\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)\sin\left(-\frac{ce-df}{e}\right)+b\cos\left(c\right)\mathrm{Si}\left(\frac{d}{x}\right)-b\cos\left(-\frac{ce-df}{e}\right)\mathrm{Si}\left(\frac{dfx+de}{ex}\right)-a\log\left(fx+\right.}{f}$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="fricas")
```

output
```
-(b*cos_integral(d/x)*sin(c) + b*cos_integral((d*f*x + d*e)/(e*x))*sin(-(c
*e - d*f)/e) + b*cos(c)*sin_integral(d/x) - b*cos(-(c*e - d*f)/e)*sin_inte
gral((d*f*x + d*e)/(e*x)) - a*log(f*x + e))/f
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx = \int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e),x)
```

output
```
Integral((a + b*sin(c + d/x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx = \int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{fx+e}\,dx$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="maxima")
```

output
```
b*(integrate(1/2*sin((c*x + d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e
)*sin((c*x + d)/x)^2), x) + integrate(1/2*sin((c*x + d)/x)/(f*x + e), x))
+ a*log(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.61

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx =$$

$$-\frac{bd\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)\sin\left(c\right)-bd\,\mathrm{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)\sin\left(\frac{ce-df}{e}\right)-bd\cos\left(c\right)\mathrm{Si}\left(c-\frac{cx+d}{x}\right)+bd\cos\left(\frac{ce-df}{e}\right.}{df}$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="giac")
```

output
```
-(b*d*cos_integral(-c + (c*x + d)/x)*sin(c) - b*d*cos_integral(-(c*e - d*f
 - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) - b*d*cos(c)*sin_integral(c - (c*x
 + d)/x) + b*d*cos((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)
/e) - a*d*log(c*e - d*f - (c*x + d)*e/x) + a*d*log(c - (c*x + d)/x))/(d*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx = \int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{e+f\,x}\,dx$$

input
```
int((a + b*sin(c + d/x))/(e + f*x),x)
```

output
```
int((a + b*sin(c + d/x))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx=\frac{\left(\int\frac{\sin\left(\frac{cx+d}{x}\right)}{fx+e}dx\right)bf+\log(fx+e)\,a}{f}$$

input

```
int((a+b*sin(c+d/x))/(f*x+e),x)
```

output

```
(int(sin((c*x + d)/x)/(e + f*x),x)*b*f + log(e + f*x)*a)/f
```

3.292 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx=\frac{a}{e\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}+\frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^2}$$

output
```
a/e/(f+e/x)-b*d*cos(c-d*f/e)*Ci(d*f/e+d/x)/e^2+b*sin(c+d/x)/e/(f+e/x)+b*d*
sin(c-d*f/e)*Si(d*f/e+d/x)/e^2
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx=\frac{-bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+\frac{e\left(-ae+bfx\sin\left(c+\frac{d}{x}\right)\right)}{f(e+fx)}+bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}$$

input
```
Integrate[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

output
```
(-(b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]) + (e*(-(a*e) + b*f*
x*Sin[c + d/x]))/(f*(e + f*x)) + b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e +
 x^(-1))])/e^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3912, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(\frac{e}{x}+f\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(\frac{e}{x}+f\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3798}$$

$$-\int \left(\frac{a}{\left(\frac{e}{x}+f\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{\left(\frac{e}{x}+f\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a}{e\left(\frac{e}{x}+f\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^2}+\frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

input
```
Int[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

output
```
a/(e*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[(d*f)/e + d/x])/e^2 +
(b*Sin[c + d/x])/(e*(f + e/x)) + (b*d*Sin[c - (d*f)/e]*SinIntegral[(d*f)/e
 + d/x])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

method	result
parts	$-\frac{a}{f(fx+e)}-bd\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{\frac{\mathrm{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}}{e}$
derivativedivides	$-d\Bigg(-\frac{a}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+b\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\mathrm{Ci}\left(\frac{d}{x}+c+\right.}{e}$
default	$-d\Bigg(-\frac{a}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+b\Bigg(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\mathrm{Ci}\left(\frac{d}{x}+c+\right.}{e}$
risch	$-\frac{a}{f(fx+e)}+\frac{bd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2e^2}+\frac{bd\,\mathrm{e}^{\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\left(-\frac{id}{x}-ic-\frac{-ice+ifd}{e}\right)}{2e^2}+$

input
```
int((a+b*sin(c+d/x))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/f/(f*x+e)-b*d*(-sin(c+d/x)/(-c*e+d*f+e*(c+d/x))/e+(Si(d/x+c+(-c*e+d*f)/
e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e)/e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

$$\int\frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$$
$$=\frac{befx\sin\left(\frac{cx+d}{x}\right)-ae^2-\left(bdf^2x+bdef\right)\cos\left(-\frac{ce-df}{e}\right)\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)-\left(bdf^2x+bdef\right)\sin\left(-\frac{ce-df}{e}\right)\mathrm{Si}\left(\frac{dfx+d}{ex}\right.}{e^2f^2x+e^3f}$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
(b*e*f*x*sin((c*x + d)/x) - a*e^2 - (b*d*f^2*x + b*d*e*f)*cos(-(c*e - d*f)
/e)*cos_integral((d*f*x + d*e)/(e*x)) - (b*d*f^2*x + b*d*e*f)*sin(-(c*e -
d*f)/e)*sin_integral((d*f*x + d*e)/(e*x)))/(e^2*f^2*x + e^3*f)
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx = \int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(e+fx\right)^2}\,dx$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)**2,x)
```

output
```
Integral((a + b*sin(c + d/x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx = \int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
b*(integrate(1/2*sin((c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) + integrat
e(1/2*sin((c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*cos((c*x + d)/x)^2 + (f^
2*x^2 + 2*e*f*x + e^2)*sin((c*x + d)/x)^2), x)) - a/(f^2*x + e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(96) = 192$.

Time = 0.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.53

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx =$$

$$-\frac{bcd^2e\cos\left(\frac{ce-df}{e}\right)\operatorname{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)-bd^3f\cos\left(\frac{ce-df}{e}\right)\operatorname{Ci}\left(-\frac{ce-df-\frac{(cx+d)e}{x}}{e}\right)+bcd^2e\sin\left(\frac{ce-df}{e}\right)\operatorname{Si}\Bigg(}{}$$

input

```
integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="giac")
```

output

```
-(b*c*d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e
) - b*d^3*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e
) + b*c*d^2*e*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/
e) - b*d^3*f*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e
) - (c*x + d)*b*d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x +
 d)*e/x)/e)/x - (c*x + d)*b*d^2*e*sin((c*e - d*f)/e)*sin_integral((c*e - d
*f - (c*x + d)*e/x)/e)/x + b*d^2*e*sin((c*x + d)/x) + a*d^2*e)/((c*e^3 - d
*e^2*f - (c*x + d)*e^3/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx=\int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

input

```
int((a + b*sin(c + d/x))/(e + f*x)^2,x)
```

output

```
int((a + b*sin(c + d/x))/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx=\frac{\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{f^2x^2+2efx+e^2}dx\right)b\,e^2+\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{f^2x^2+2efx+e^2}dx\right)befx+ax}{e\,(fx+e)}$$

input

```
int((a+b*sin(c+d/x))/(f*x+e)^2,x)
```

output

```
(int(sin((c*x + d)/x)/(e**2 + 2*e*f*x + f**2*x**2),x)*b*e**2 + int(sin((c*
x + d)/x)/(e**2 + 2*e*f*x + f**2*x**2),x)*b*e*f*x + a*x)/(e*(e + f*x))
```

3.293 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 237

$$\begin{aligned}\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = &-\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}\\&-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^3}\\&-\frac{bd^2f\,\text{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)\sin\left(c-\frac{df}{e}\right)}{2e^4}\\&-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}\\&-\frac{bd^2f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{2e^4}+\frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^3}\end{aligned}$$

output

```
-1/2*a*f/e^2/(f+e/x)^2+a/e^2/(f+e/x)-1/2*b*d*f*cos(c+d/x)/e^3/(f+e/x)-b*d*
cos(c-d*f/e)*Ci(d*f/e+d/x)/e^3-1/2*b*d^2*f*Ci(d*f/e+d/x)*sin(c-d*f/e)/e^4-
1/2*b*f*sin(c+d/x)/e^2/(f+e/x)^2+b*sin(c+d/x)/e^2/(f+e/x)-1/2*b*d^2*f*cos(
c-d*f/e)*Si(d*f/e+d/x)/e^4+b*d*sin(c-d*f/e)*Si(d*f/e+d/x)/e^3
```

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.64

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx =$$
$$-\frac{bd\,\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\left(2e\cos\left(c-\frac{df}{e}\right)+df\sin\left(c-\frac{df}{e}\right)\right)+\frac{e\left(ae^3+bdf^2x(e+fx)\cos\left(c+\frac{d}{x}\right)-befx(2e+fx)\sin\left(\right.\right.}{f(e+fx)^2}}{2e^4}$$

input
```
Integrate[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

output
```
-1/2*(b*d*CosIntegral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c
- (d*f)/e]) + (e*(a*e^3 + b*d*f^2*x*(e + f*x)*Cos[c + d/x] - b*e*f*x*(2*e
+ f*x)*Sin[c + d/x]))/(f*(e + f*x)^2) + b*d*(d*f*Cos[c - (d*f)/e] - 2*e*Si
n[c - (d*f)/e])*SinIntegral[d*(f/e + x^(-1))])/e^4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}dx$$
$$\Big\downarrow \text{3912}$$
$$-\int \left(\frac{a+b\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)^2}-\frac{f\left(a+b\sin\left(c+\frac{d}{x}\right)\right)}{e\left(\frac{e}{x}+f\right)^3}\right)d\frac{1}{x}$$
$$\Big\downarrow \text{2009}$$

$$\frac{a}{e^2\left(\frac{e}{x}+f\right)}-\frac{af}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{bd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{2e^4}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^3}-\frac{bd^2f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{2e^4}+\frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^3}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(\frac{e}{x}+f\right)}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(\frac{e}{x}+f\right)^2}$$

input
```
Int[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

output
```
-1/2*(a*f)/(e^2*(f + e/x)^2) + a/(e^2*(f + e/x)) - (b*d*f*Cos[c + d/x])/(2
*e^3*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[(d*f)/e + d/x])/e^3 -
(b*d^2*f*CosIntegral[(d*f)/e + d/x]*Sin[c - (d*f)/e])/(2*e^4) - (b*f*Sin[c
 + d/x])/(2*e^2*(f + e/x)^2) + (b*Sin[c + d/x])/(e^2*(f + e/x)) - (b*d^2*f
*Cos[c - (d*f)/e]*SinIntegral[(d*f)/e + d/x])/(2*e^4) + (b*d*Sin[c - (d*f)
/e]*SinIntegral[(d*f)/e + d/x])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{a}{2f(fx+e)^2}+\frac{ib\,d^2\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)f}{4e^4}+\frac{bd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\Big(\frac{id}{x}+ic-\frac{i(ce-df)}{e}}{2e^3}$
parts	$-\frac{a}{2f(fx+e)^2}-bd\Bigg(\frac{-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{\frac{\mathrm{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)}{e}}{e}}{e}+$
derivativedivides	$-d\Bigg(-\frac{a}{e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}-\frac{(ce-df)a}{2e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^2}+\frac{b\Big(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)}{e}}{}}{e}$
default	$-d\Bigg(-\frac{a}{e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}-\frac{(ce-df)a}{2e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^2}+\frac{b\Big(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)}{e}}{}}{e}$

input
```
int((a+b*sin(c+d/x))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/f/(f*x+e)^2+1/4*I*b*d^2/e^4*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c
*e-d*f)/e)*f+1/2*b*d/e^3*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*e-d*f)/e)
-1/4*I*b*d^2*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-(-I*c*e+I*f*d)/e)/e^4*f+1/
2*b*d*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-(-I*c*e+I*f*d)/e)/e^3+1/4*I*b/e^3
*x*(6*I*d^3*e*f^3*x^2+6*I*d^3*e^2*f^2*x+2*I*d^3*f^4*x^3+2*I*d^3*e^3*f)/(f*
x+e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*cos((c*x+d)/x)-1/4*b/e^2*x*(-2*d^
2*f^3*x^3-8*d^2*e*f^2*x^2-10*d^2*e^2*f*x-4*d^2*e^3)/(f*x+e)^2/(d^2*f^2*x^2
+2*d^2*e*f*x+d^2*e^2)*sin((c*x+d)/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.38

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx =$$

$$-\frac{ae^4+\left(2\left(bdef^3x^2+2\,bde^2f^2x+bde^3f\right)\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)+\left(bd^2f^4x^2+2\,bd^2ef^3x+bd^2e^2f^2\right)\mathrm{Si}\left(\frac{dfx+de}{ex}\right)\right)\mathrm{co}}{}$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*e^4 + (2*(b*d*e*f^3*x^2 + 2*b*d*e^2*f^2*x + b*d*e^3*f)*cos_integra
l((d*f*x + d*e)/(e*x)) + (b*d^2*f^4*x^2 + 2*b*d^2*e*f^3*x + b*d^2*e^2*f^2)
*sin_integral((d*f*x + d*e)/(e*x)))*cos(-(c*e - d*f)/e) + (b*d*e*f^3*x^2 +
 b*d*e^2*f^2*x)*cos((c*x + d)/x) - ((b*d^2*f^4*x^2 + 2*b*d^2*e*f^3*x + b*d
^2*e^2*f^2)*cos_integral((d*f*x + d*e)/(e*x)) - 2*(b*d*e*f^3*x^2 + 2*b*d*e
^2*f^2*x + b*d*e^3*f)*sin_integral((d*f*x + d*e)/(e*x)))*sin(-(c*e - d*f)/
e) - (b*e^2*f^2*x^2 + 2*b*e^3*f*x)*sin((c*x + d)/x))/(e^4*f^3*x^2 + 2*e^5*
f^2*x + e^6*f)
```

Sympy [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(e+fx\right)^3}\,dx$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)**3,x)
```

output
```
Integral((a + b*sin(c + d/x))/(e + f*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{\left(fx+e\right)^3}\,dx$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
b*(integrate(1/2*sin((c*x + d)/x)/(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3
), x) + integrate(1/2*sin((c*x + d)/x)/((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x
 + e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*sin
((c*x + d)/x)^2), x)) - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(227) = 454$.

Time = 0.16 (sec) , antiderivative size = 1501, normalized size of antiderivative = 6.33

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/2*(b*c^2*d^3*e^2*f*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*
e - d*f)/e) - 2*b*c*d^4*e*f^2*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)
*sin((c*e - d*f)/e) + b*d^5*f^3*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/
e)*sin((c*e - d*f)/e) - b*c^2*d^3*e^2*f*cos((c*e - d*f)/e)*sin_integral((c
*e - d*f - (c*x + d)*e/x)/e) + 2*b*c*d^4*e*f^2*cos((c*e - d*f)/e)*sin_inte
gral((c*e - d*f - (c*x + d)*e/x)/e) - b*d^5*f^3*cos((c*e - d*f)/e)*sin_int
egral((c*e - d*f - (c*x + d)*e/x)/e) + 2*b*c^2*d^2*e^3*cos((c*e - d*f)/e)*
cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) - 4*b*c*d^3*e^2*f*cos((c*e -
d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) + 2*b*d^4*e*f^2*cos((
c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) - 2*(c*x + d)*b
*c*d^3*e^2*f*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/
e)/x + 2*(c*x + d)*b*d^4*e*f^2*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e
)*sin((c*e - d*f)/e)/x + 2*(c*x + d)*b*c*d^3*e^2*f*cos((c*e - d*f)/e)*sin_
integral((c*e - d*f - (c*x + d)*e/x)/e)/x - 2*(c*x + d)*b*d^4*e*f^2*cos((c
*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/x + 2*b*c^2*d^2*e
^3*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 4*b*c*
d^3*e^2*f*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) +
 2*b*d^4*e*f^2*sin((c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)
/e) - b*c*d^3*e^2*f*cos((c*x + d)/x) + b*d^4*e*f^2*cos((c*x + d)/x) - 4*(c
*x + d)*b*c*d^2*e^3*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx=\int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{{\left(e+f\,x\right)}^3}\,dx$$

input
```
int((a + b*sin(c + d/x))/(e + f*x)^3,x)
```

output
```
int((a + b*sin(c + d/x))/(e + f*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx$$
$$=\frac{2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{f^3x^3+3e\,f^2x^2+3e^2fx+e^3}dx\right)b\,e^2f+4\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{f^3x^3+3e\,f^2x^2+3e^2fx+e^3}dx\right)be\,f^2x+2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{f^3x^3+3e\,f^2x^2+3e^2fx+e^3}dx\right.}{2f\left(f^2x^2+2efx+e^2\right)}$$

input
```
int((a+b*sin(c+d/x))/(f*x+e)^3,x)
```

output
```
(2*int(sin((c*x + d)/x)/(e**3 + 3*e**2*f*x + 3*e*f**2*x**2 + f**3*x**3),x)
*b*e**2*f + 4*int(sin((c*x + d)/x)/(e**3 + 3*e**2*f*x + 3*e*f**2*x**2 + f*
*3*x**3),x)*b*e*f**2*x + 2*int(sin((c*x + d)/x)/(e**3 + 3*e**2*f*x + 3*e*f
**2*x**2 + f**3*x**3),x)*b*f**3*x**2 - a)/(2*f*(e**2 + 2*e*f*x + f**2*x**2
))
```

3.294 $\int (e+fx) \left(a+b \sin \left(c+\frac{d}{x}\right)\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int (e+fx) \left(a+b \sin \left(c+\frac{d}{x}\right)\right)^2 \, dx = {} & a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right) \\ & -2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) \\ & -b^2d^2f\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right) \\ & +abd^2f\,\text{CosIntegral}\left(\frac{d}{x}\right)\sin(c) \\ & -b^2de\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c) \\ & +2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin\left(c+\frac{d}{x}\right) \\ & +b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right) \\ & +b^2ex\sin^2\left(c+\frac{d}{x}\right)+\frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right) \\ & +abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+2abde\sin(c)\text{Si}\left(\frac{d}{x}\right) \\ & -b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)\end{aligned}$$

output
```
a^2*e*x+1/2*a^2*f*x^2+a*b*d*f*x*cos(c+d/x)-2*a*b*d*e*cos(c)*Ci(d/x)-b^2*d^
2*f*cos(2*c)*Ci(2*d/x)+a*b*d^2*f*Ci(d/x)*sin(c)-b^2*d*e*Ci(2*d/x)*sin(2*c)
+2*a*b*e*x*sin(c+d/x)+a*b*f*x^2*sin(c+d/x)+b^2*d*f*x*cos(c+d/x)*sin(c+d/x)
+b^2*e*x*sin(c+d/x)^2+1/2*b^2*f*x^2*sin(c+d/x)^2+a*b*d^2*f*cos(c)*Si(d/x)+
2*a*b*d*e*sin(c)*Si(d/x)-b^2*d*e*cos(2*c)*Si(2*d/x)+b^2*d^2*f*sin(2*c)*Si(
2*d/x)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99

$$\begin{aligned}\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \frac{1}{4}\Bigg(&4a^2ex+2b^2ex+2a^2fx^2+b^2fx^2\\ &+4abdfx\cos\left(c+\frac{d}{x}\right)-2b^2ex\cos\left(2\left(c+\frac{d}{x}\right)\right)\\ &-b^2fx^2\cos\left(2\left(c+\frac{d}{x}\right)\right)\\ &+4abd\,\text{CosIntegral}\left(\frac{d}{x}\right)(-2e\cos(c)\\ &+df\sin(c))\\ &-4b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)(df\cos(2c)\\ &+e\sin(2c))+8abex\sin\left(c+\frac{d}{x}\right)\\ &+4abfx^2\sin\left(c+\frac{d}{x}\right)+2b^2dfx\sin\left(2\left(c+\frac{d}{x}\right)\right)\\ &+4abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+8abde\sin(c)\text{Si}\left(\frac{d}{x}\right)\\ &-4b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)\\ &+4b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)\Bigg)\end{aligned}$$

input
```
Integrate[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

output
```
(4*a^2*e*x + 2*b^2*e*x + 2*a^2*f*x^2 + b^2*f*x^2 + 4*a*b*d*f*x*Cos[c + d/x
] - 2*b^2*e*x*Cos[2*(c + d/x)] - b^2*f*x^2*Cos[2*(c + d/x)] + 4*a*b*d*CosI
ntegral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) - 4*b^2*d*CosIntegral[(2*d)/x]*(d*
f*Cos[2*c] + e*Sin[2*c]) + 8*a*b*e*x*Sin[c + d/x] + 4*a*b*f*x^2*Sin[c + d/
x] + 2*b^2*d*f*x*Sin[2*(c + d/x)] + 4*a*b*d^2*f*Cos[c]*SinIntegral[d/x] +
8*a*b*d*e*Sin[c]*SinIntegral[d/x] - 4*b^2*d*e*Cos[2*c]*SinIntegral[(2*d)/x
] + 4*b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x])/4
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \left(f\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2x^3+e\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2x^2\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$a^2ex+\frac{1}{2}a^2fx^2+abd^2f\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)-2abde\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+2abde\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin\left(c+\frac{d}{x}\right)+abdfx\cos\left(c+\frac{d}{x}\right)-b^2d^2f\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2de\sin(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)+b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)-b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2ex\sin^2\left(c+\frac{d}{x}\right)+\frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right)+b^2dfx\sin\left(c+\frac{d}{x}\right)\cos\left(c+\frac{d}{x}\right)$$

input
```
Int[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

output
```
a^2*e*x + (a^2*f*x^2)/2 + a*b*d*f*x*Cos[c + d/x] - 2*a*b*d*e*Cos[c]*CosInt
egral[d/x] - b^2*d^2*f*Cos[2*c]*CosIntegral[(2*d)/x] + a*b*d^2*f*CosIntegr
al[d/x]*Sin[c] - b^2*d*e*CosIntegral[(2*d)/x]*Sin[2*c] + 2*a*b*e*x*Sin[c +
 d/x] + a*b*f*x^2*Sin[c + d/x] + b^2*d*f*x*Cos[c + d/x]*Sin[c + d/x] + b^2
*e*x*Sin[c + d/x]^2 + (b^2*f*x^2*Sin[c + d/x]^2)/2 + a*b*d^2*f*Cos[c]*SinI
ntegral[d/x] + 2*a*b*d*e*Sin[c]*SinIntegral[d/x] - b^2*d*e*Cos[2*c]*SinInt
egral[(2*d)/x] + b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.98

method	result
parts	$a^{2} \left(\frac{1}{2} f \,x^{2}+e x \right)-b^{2} d \left(-\frac{e x}{2 d}-\frac{e \left(-\frac{2 \cos \left(\frac{2 d}{x}+2 c \right) x}{d}-4 \,\operatorname{Si}\left(\frac{2 d}{x}\right) \cos \left(2 c \right)-4 \,\operatorname{Ci}\left(\frac{2 d}{x}\right) \sin \left(2 c \right)\right)}{4}-\frac{f \,x^{2}}{4 d}-$
derivativedivides	$-d \left(-\frac{a^{2} f \,x^{2}}{2 d}-\frac{a^{2} e x}{d}+2 f b d a \left(-\frac{\sin \left(c +\frac{d}{x}\right) x^{2}}{2 d^{2}}-\frac{\cos \left(c +\frac{d}{x}\right) x}{2 d}-\frac{\operatorname{Si}\left(\frac{d}{x}\right) \cos \left(c \right)}{2}-\frac{\operatorname{Ci}\left(\frac{d}{x}\right) \sin \left(c \right)}{2}\right)+$
default	$-d \left(-\frac{a^{2} f \,x^{2}}{2 d}-\frac{a^{2} e x}{d}+2 f b d a \left(-\frac{\sin \left(c +\frac{d}{x}\right) x^{2}}{2 d^{2}}-\frac{\cos \left(c +\frac{d}{x}\right) x}{2 d}-\frac{\operatorname{Si}\left(\frac{d}{x}\right) \cos \left(c \right)}{2}-\frac{\operatorname{Ci}\left(\frac{d}{x}\right) \sin \left(c \right)}{2}\right)+$
risch	$a^{2} e x +\frac{a^{2} f \,x^{2}}{2}+a b d e \,{\mathrm e}^{-i c} \operatorname{expIntegral}_{1}\left(\frac{i d}{x}\right)+\frac{i {\mathrm e}^{-2 i c} \operatorname{expIntegral}_{1}\left(\frac{2 i d}{x}\right) b^{2} d e}{2}+\frac{b^{2} e x}{2}+\frac{b^{2} f \,x^{2}}{4}+$

input
```
int((f*x+e)*(a+b*sin(c+d/x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*f*x^2+e*x)-b^2*d*(-1/2*e*x/d-1/4*e*(-2*cos(2*d/x+2*c)/d*x-4*Si(2*
d/x)*cos(2*c)-4*Ci(2*d/x)*sin(2*c))-1/4/d*f*x^2-1/4*d*f*(-cos(2*d/x+2*c)/d
^2*x^2+2*sin(2*d/x+2*c)/d*x+4*Si(2*d/x)*sin(2*c)-4*Ci(2*d/x)*cos(2*c)))-2*
a*b*d*(e*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))+d*f*(-1/2*sin(c+d
/x)/d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/2*Ci(d/x)*sin(c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx
&= abdfx\cos\left(\frac{cx+d}{x}\right)+\frac{1}{2}\left(a^2+b^2\right)fx^2+\left(a^2+b^2\right)ex \\
&\quad -\frac{1}{2}\left(b^2fx^2+2\,b^2ex\right)\cos\left(\frac{cx+d}{x}\right)^2-\left(b^2d^2f\operatorname{Ci}\left(\frac{2\,d}{x}\right)+b^2de\operatorname{Si}\left(\frac{2\,d}{x}\right)\right)\cos\left(2\,c\right) \\
&\quad +\left(abd^2f\operatorname{Si}\left(\frac{d}{x}\right)-2\,abde\operatorname{Ci}\left(\frac{d}{x}\right)\right)\cos\left(c\right) \\
&\quad +\left(b^2d^2f\operatorname{Si}\left(\frac{2\,d}{x}\right)-b^2de\operatorname{Ci}\left(\frac{2\,d}{x}\right)\right)\sin\left(2\,c\right) \\
&\quad +\left(abd^2f\operatorname{Ci}\left(\frac{d}{x}\right)+2\,abde\operatorname{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right) \\
&\quad +\left(b^2dfx\cos\left(\frac{cx+d}{x}\right)+abfx^2+2\,abex\right)\sin\left(\frac{cx+d}{x}\right)
\end{aligned}
$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output
```
a*b*d*f*x*cos((c*x + d)/x) + 1/2*(a^2 + b^2)*f*x^2 + (a^2 + b^2)*e*x - 1/2
*(b^2*f*x^2 + 2*b^2*e*x)*cos((c*x + d)/x)^2 - (b^2*d^2*f*cos_integral(2*d/
x) + b^2*d*e*sin_integral(2*d/x))*cos(2*c) + (a*b*d^2*f*sin_integral(d/x)
- 2*a*b*d*e*cos_integral(d/x))*cos(c) + (b^2*d^2*f*sin_integral(2*d/x) - b
^2*d*e*cos_integral(2*d/x))*sin(2*c) + (a*b*d^2*f*cos_integral(d/x) + 2*a*
b*d*e*sin_integral(d/x))*sin(c) + (b^2*d*f*x*cos((c*x + d)/x) + a*b*f*x^2
+ 2*a*b*e*x)*sin((c*x + d)/x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x))**2,x)
```

output
```
Integral((a + b*sin(c + d/x))**2*(e + f*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=\frac{1}{2}\,a^2fx^2$$
$$-\Bigg(\Bigg(\Bigg(\operatorname{Ei}\left(\frac{i\,d}{x}\right)+\operatorname{Ei}\left(-\frac{i\,d}{x}\right)\Bigg)\cos\left(c\right)-\Bigg(-i\operatorname{Ei}\left(\frac{i\,d}{x}\right)+i\operatorname{Ei}\left(-\frac{i\,d}{x}\right)\Bigg)\sin\left(c\right)\Bigg)d-2\,x\sin\left(\frac{cx+d}{x}\right)\Bigg)$$
$$-\frac{1}{2}\Bigg(\Bigg(\Bigg(-i\operatorname{Ei}\left(\frac{2i\,d}{x}\right)+i\operatorname{Ei}\left(-\frac{2i\,d}{x}\right)\Bigg)\cos\left(2\,c\right)+\Bigg(\operatorname{Ei}\left(\frac{2i\,d}{x}\right)+\operatorname{Ei}\left(-\frac{2i\,d}{x}\right)\Bigg)\sin\left(2\,c\right)\Bigg)d+x\cos\Bigg(\frac{2}{}$$
$$+\frac{1}{2}\Bigg(\Bigg(\Bigg(-i\operatorname{Ei}\left(\frac{i\,d}{x}\right)+i\operatorname{Ei}\left(-\frac{i\,d}{x}\right)\Bigg)\cos\left(c\right)+\Bigg(\operatorname{Ei}\left(\frac{i\,d}{x}\right)+\operatorname{Ei}\left(-\frac{i\,d}{x}\right)\Bigg)\sin\left(c\right)\Bigg)d^2+2\,dx\cos\Bigg(\frac{cx+d}{x}$$
$$-\frac{1}{4}\Bigg(2\Bigg(\Bigg(\operatorname{Ei}\left(\frac{2i\,d}{x}\right)+\operatorname{Ei}\left(-\frac{2i\,d}{x}\right)\Bigg)\cos\left(2\,c\right)+\Bigg(i\operatorname{Ei}\left(\frac{2i\,d}{x}\right)-i\operatorname{Ei}\left(-\frac{2i\,d}{x}\right)\Bigg)\sin\left(2\,c\right)\Bigg)d^2+x^2\cos\Bigg($$
$$+a^2ex$$

input
```
integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output

```
1/2*a^2*f*x^2 - (((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-
I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*a*b*e - 1/2*(((-I*Ei(2*I*d/x) +
I*Ei(-2*I*d/x))*cos(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*co
s(2*(c*x + d)/x) - x)*b^2*e + 1/2*(((-I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) +
 (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x + d)/x) + 2*x^2*sin
((c*x + d)/x))*a*b*f - 1/4*(2*((Ei(2*I*d/x) + Ei(-2*I*d/x))*cos(2*c) + (I*
Ei(2*I*d/x) - I*Ei(-2*I*d/x))*sin(2*c))*d^2 + x^2*cos(2*(c*x + d)/x) - 2*d
*x*sin(2*(c*x + d)/x) - x^2)*b^2*f + a^2*e*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(250) = 500$.

Time = 0.14 (sec) , antiderivative size = 1125, normalized size of antiderivative = 4.43

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
-1/4*(4*b^2*c^2*d^3*f*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x) - 4*a*b*
c^2*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c) + 4*b^2*c^2*d^3*f*sin(2*c)
*sin_integral(2*c - 2*(c*x + d)/x) + 4*a*b*c^2*d^3*f*cos(c)*sin_integral(c
 - (c*x + d)/x) + 8*a*b*c^2*d^2*e*cos(c)*cos_integral(-c + (c*x + d)/x) -
8*(c*x + d)*b^2*c*d^3*f*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x)/x + 4*
b^2*c^2*d^2*e*cos_integral(-2*c + 2*(c*x + d)/x)*sin(2*c) + 8*(c*x + d)*a*
b*c*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x - 4*b^2*c^2*d^2*e*cos(2*
c)*sin_integral(2*c - 2*(c*x + d)/x) - 8*(c*x + d)*b^2*c*d^3*f*sin(2*c)*si
n_integral(2*c - 2*(c*x + d)/x)/x - 8*(c*x + d)*a*b*c*d^3*f*cos(c)*sin_int
egral(c - (c*x + d)/x)/x + 8*a*b*c^2*d^2*e*sin(c)*sin_integral(c - (c*x +
d)/x) + 4*a*b*c*d^3*f*cos((c*x + d)/x) - 16*(c*x + d)*a*b*c*d^2*e*cos(c)*c
os_integral(-c + (c*x + d)/x)/x + 4*(c*x + d)^2*b^2*d^3*f*cos(2*c)*cos_int
egral(-2*c + 2*(c*x + d)/x)/x^2 - 8*(c*x + d)*b^2*c*d^2*e*cos_integral(-2*
c + 2*(c*x + d)/x)*sin(2*c)/x - 4*(c*x + d)^2*a*b*d^3*f*cos_integral(-c +
(c*x + d)/x)*sin(c)/x^2 + 2*b^2*c*d^3*f*sin(2*(c*x + d)/x) + 8*(c*x + d)*b
^2*c*d^2*e*cos(2*c)*sin_integral(2*c - 2*(c*x + d)/x)/x + 4*(c*x + d)^2*b^
2*d^3*f*sin(2*c)*sin_integral(2*c - 2*(c*x + d)/x)/x^2 + 4*(c*x + d)^2*a*b
*d^3*f*cos(c)*sin_integral(c - (c*x + d)/x)/x^2 - 16*(c*x + d)*a*b*c*d^2*e
*sin(c)*sin_integral(c - (c*x + d)/x)/x - 2*b^2*c*d^2*e*cos(2*(c*x + d)/x)
 + b^2*d^3*f*cos(2*(c*x + d)/x) - 4*(c*x + d)*a*b*d^3*f*cos((c*x + d)/x...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \int (e+f\,x)\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2 dx$$

input
```
int((e + f*x)*(a + b*sin(c + d/x))^2,x)
```

output
```
int((e + f*x)*(a + b*sin(c + d/x))^2, x)
```

Reduce [F]

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=\text{Too large to display}$$

input

```
int((f*x+e)*(a+b*sin(c+d/x))^2,x)
```

output

```
( - 2*cos((c*x + d)/x)*sin((c*x + d)/x)*tan((c*x + d)/(2*x))**4*b**2*d*f*x
 - 4*cos((c*x + d)/x)*sin((c*x + d)/x)*tan((c*x + d)/(2*x))**2*b**2*d*f*x
- 2*cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*d*f*x + 6*cos((c*x + d)/x)*tan(
(c*x + d)/(2*x))**4*a*b*d*f*x + 8*cos((c*x + d)/x)*tan((c*x + d)/(2*x))**4
*b**2*f*x**2 + 12*cos((c*x + d)/x)*tan((c*x + d)/(2*x))**2*a*b*d*f*x + 16*
cos((c*x + d)/x)*tan((c*x + d)/(2*x))**2*b**2*f*x**2 + 6*cos((c*x + d)/x)*
a*b*d*f*x + 8*cos((c*x + d)/x)*b**2*f*x**2 + 6*int(sin((c*x + d)/x)**2,x)*
tan((c*x + d)/(2*x))**4*b**2*e + 12*int(sin((c*x + d)/x)**2,x)*tan((c*x +
d)/(2*x))**2*b**2*e + 6*int(sin((c*x + d)/x)**2,x)*b**2*e - 6*int(sin((c*x
 + d)/x)/x,x)*tan((c*x + d)/(2*x))**4*a*b*d**2*f - 12*int(sin((c*x + d)/x)
/x,x)*tan((c*x + d)/(2*x))**2*a*b*d**2*f - 6*int(sin((c*x + d)/x)/x,x)*a*b
*d**2*f + 32*int(tan((c*x + d)/(2*x))/(tan((c*x + d)/(2*x))**4 + 2*tan((c*
x + d)/(2*x))**2 + 1),x)*tan((c*x + d)/(2*x))**4*b**2*d*f + 64*int(tan((c*
x + d)/(2*x))/(tan((c*x + d)/(2*x))**4 + 2*tan((c*x + d)/(2*x))**2 + 1),x)
*tan((c*x + d)/(2*x))**2*b**2*d*f + 32*int(tan((c*x + d)/(2*x))/(tan((c*x
+ d)/(2*x))**4 + 2*tan((c*x + d)/(2*x))**2 + 1),x)*b**2*d*f - 16*int(1/(ta
n((c*x + d)/(2*x))**4*x + 2*tan((c*x + d)/(2*x))**2*x + x),x)*tan((c*x + d
)/(2*x))**4*b**2*d**2*f - 32*int(1/(tan((c*x + d)/(2*x))**4*x + 2*tan((c*x
 + d)/(2*x))**2*x + x),x)*tan((c*x + d)/(2*x))**2*b**2*d**2*f - 16*int(1/(
tan((c*x + d)/(2*x))**4*x + 2*tan((c*x + d)/(2*x))**2*x + x),x)*b**2*d*...
```

3.295 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 94

$$\begin{aligned}\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx &= a^2x-2abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)\\&\quad-b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c)\\&\quad+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)\\&\quad+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)\end{aligned}$$

output
```
a^2*x-2*a*b*d*cos(c)*Ci(d/x)-b^2*d*Ci(2*d/x)*sin(2*c)+2*a*b*x*sin(c+d/x)+b
^2*x*sin(c+d/x)^2+2*a*b*d*sin(c)*Si(d/x)-b^2*d*cos(2*c)*Si(2*d/x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \frac{1}{2}\left(2a^2x+b^2x-b^2x\cos\left(2\left(c+\frac{d}{x}\right)\right) - 4abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right) - 2b^2d\,\text{CosIntegral}\left(\frac{2d}{x}\right)\sin(2c)+4abx\sin\left(c+\frac{d}{x}\right) + 4abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-2b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)\right)$$

input
```
Integrate[(a + b*Sin[c + d/x])^2,x]
```

output
```
(2*a^2*x + b^2*x - b^2*x*Cos[2*(c + d/x)] - 4*a*b*d*Cos[c]*CosIntegral[d/x
] - 2*b^2*d*CosIntegral[(2*d)/x]*Sin[2*c] + 4*a*b*x*Sin[c + d/x] + 4*a*b*d
*Sin[c]*SinIntegral[d/x] - 2*b^2*d*Cos[2*c]*SinIntegral[(2*d)/x])/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3842, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx$$

$$\Big\downarrow \text{3842}$$

$$-\int x^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{3798}$$

$$-\int\left(a^2x^2+b^2\sin^2\left(c+\frac{d}{x}\right)x^2+2ab\sin\left(c+\frac{d}{x}\right)x^2\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$a^2x-2abd\cos(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)-b^2d\sin(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)$$

input
```
Int[(a + b*Sin[c + d/x])^2,x]
```

output
```
a^2*x - 2*a*b*d*Cos[c]*CosIntegral[d/x] - b^2*d*CosIntegral[(2*d)/x]*Sin[2
*c] + 2*a*b*x*Sin[c + d/x] + b^2*x*Sin[c + d/x]^2 + 2*a*b*d*Sin[c]*SinInte
gral[d/x] - b^2*d*Cos[2*c]*SinIntegral[(2*d)/x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

method	result
parts	$a^2x-b^2d\left(-\frac{x}{2d}+\frac{\cos\left(\frac{2d}{x}+2c\right)x}{2d}+\operatorname{Si}\left(\frac{2d}{x}\right)\cos\left(2c\right)+\operatorname{Ci}\left(\frac{2d}{x}\right)\sin\left(2c\right)\right)-2abd\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{d}\right.$
derivativedivides	$-d\left(-\frac{a^2x}{d}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)-\frac{b^2x}{2d}-\frac{b^2\left(-\frac{2\cos\left(\frac{2d}{x}+2c\right)}{d}\right.}{}\right.$
default	$-d\left(-\frac{a^2x}{d}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\operatorname{Si}\left(\frac{d}{x}\right)\sin\left(c\right)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos\left(c\right)\right)-\frac{b^2x}{2d}-\frac{b^2\left(-\frac{2\cos\left(\frac{2d}{x}+2c\right)}{d}\right.}{}\right.$
risch	$\frac{\pi\ \mathrm{csgn}\left(\frac{d}{x}\right)\mathrm{e}^{-2ic}b^2d}{2}-\operatorname{Si}\left(\frac{2d}{x}\right)\mathrm{e}^{-2ic}b^2d+\frac{i\mathrm{e}^{-2ic}\ \mathrm{expIntegral}_1\left(-\frac{2id}{x}\right)b^2d}{2}-\frac{id\,b^2\ \mathrm{expIntegral}_1\left(-\frac{2id}{x}\right)\mathrm{e}^{2ic}}{2}-$

input
```
int((a+b*sin(c+d/x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2*d*(-1/2*x/d+1/2*cos(2*d/x+2*c)/d*x+Si(2*d/x)*cos(2*c)+Ci(2*d/x)*
sin(2*c))-2*a*b*d*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\begin{aligned}\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx=&-b^2x\cos\left(\frac{cx+d}{x}\right)^2-2\,abd\cos\left(c\right)\operatorname{Ci}\left(\frac{d}{x}\right)\\&-b^2d\operatorname{Ci}\left(\frac{2\,d}{x}\right)\sin\left(2\,c\right)-b^2d\cos\left(2\,c\right)\operatorname{Si}\left(\frac{2\,d}{x}\right)\\&+2\,abd\sin\left(c\right)\operatorname{Si}\left(\frac{d}{x}\right)+2\,abx\sin\left(\frac{cx+d}{x}\right)+\left(a^2+b^2\right)x\end{aligned}$$

input
```
integrate((a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output
```
-b^2*x*cos((c*x + d)/x)^2 - 2*a*b*d*cos(c)*cos_integral(d/x) - b^2*d*cos_i
ntegral(2*d/x)*sin(2*c) - b^2*d*cos(2*c)*sin_integral(2*d/x) + 2*a*b*d*sin
(c)*sin_integral(d/x) + 2*a*b*x*sin((c*x + d)/x) + (a^2 + b^2)*x
```

Sympy [F]

$$\int \left(a+b\sin \left(c+\frac{d}{x}\right)\right)^2\,dx=\int \left(a+b\sin \left(c+\frac{d}{x}\right)\right)^2\,dx$$

input
```
integrate((a+b*sin(c+d/x))**2,x)
```

output
```
Integral((a + b*sin(c + d/x))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.46

$$\int \left(a+b\sin \left(c+\frac{d}{x}\right)\right)^2\,dx=$$
$$-\Bigg(\bigg(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\bigg)$$
$$-\frac{1}{2}\Bigg(\bigg(\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\cos\left(2\,c\right)+\left(\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,d}{x}\right)\right)\sin\left(2\,c\right)\right)d+x\cos\Bigg($$
$$+a^2x$$

input
```
integrate((a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output
```
-(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(c))
*d - 2*x*sin((c*x + d)/x))*a*b - 1/2*(((-I*Ei(2*I*d/x) + I*Ei(-2*I*d/x))*c
os(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*cos(2*(c*x + d)/x)
- x)*b^2 + a^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(94) = 188$.

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.24

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx =$$

$$-\,4\,abcd^2\cos(c)\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)+2\,b^2cd^2\,\mathrm{Ci}\left(-2\,c+\frac{2\,(cx+d)}{x}\right)\sin(2\,c)-2\,b^2cd^2\cos(2\,c)\,\mathrm{Si}\Bigl(2\,c-\frac{2\,(cx+d}{x}$$

input
```
integrate((a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
-1/2*(4*a*b*c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x) + 2*b^2*c*d^2*cos_
integral(-2*c + 2*(c*x + d)/x)*sin(2*c) - 2*b^2*c*d^2*cos(2*c)*sin_integra
l(2*c - 2*(c*x + d)/x) + 4*a*b*c*d^2*sin(c)*sin_integral(c - (c*x + d)/x)
- 4*(c*x + d)*a*b*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)/x - 2*(c*x + d
)*b^2*d^2*cos_integral(-2*c + 2*(c*x + d)/x)*sin(2*c)/x + 2*(c*x + d)*b^2*
d^2*cos(2*c)*sin_integral(2*c - 2*(c*x + d)/x)/x - 4*(c*x + d)*a*b*d^2*sin
(c)*sin_integral(c - (c*x + d)/x)/x - b^2*d^2*cos(2*(c*x + d)/x) + 4*a*b*d
^2*sin((c*x + d)/x) + 2*a^2*d^2 + b^2*d^2)/((c - (c*x + d)/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \int \left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2 dx$$

input
```
int((a + b*sin(c + d/x))^2,x)
```

output
```
int((a + b*sin(c + d/x))^2, x)
```

Reduce [F]

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = \left(\int \sin\left(\frac{cx+d}{x}\right)^2 dx\right) b^2 + 2\left(\int \sin\left(\frac{cx+d}{x}\right) dx\right) ab + a^2x$$

input
```
int((a+b*sin(c+d/x))^2,x)
```

output
```
int(sin((c*x + d)/x)**2,x)*b**2 + 2*int(sin((c*x + d)/x),x)*a*b + a**2*x
```

3.296 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 259

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = &-\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\operatorname{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{2f} \\ &+\frac{b^2\cos(2c)\operatorname{CosIntegral}\left(\frac{2d}{x}\right)}{2f} \\ &+\frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a^2\log(x)}{f} \\ &+\frac{b^2\log(x)}{2f}-\frac{2ab\operatorname{CosIntegral}\left(\frac{d}{x}\right)\sin(c)}{f} \\ &+\frac{2ab\operatorname{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)\sin\left(c-\frac{df}{e}\right)}{f} \\ &+\frac{2ab\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{f} \\ &+\frac{b^2\sin\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{2f} \\ &-\frac{2ab\cos(c)\operatorname{Si}\left(\frac{d}{x}\right)}{f}-\frac{b^2\sin(2c)\operatorname{Si}\left(\frac{2d}{x}\right)}{2f}\end{aligned}$$

output
```
-1/2*b^2*cos(2*c-2*d*f/e)*Ci(2*d*(f+e/x)/e)/f+1/2*b^2*cos(2*c)*Ci(2*d/x)/f
+a^2*ln(f+e/x)/f+1/2*b^2*ln(f+e/x)/f+a^2*ln(x)/f+1/2*b^2*ln(x)/f-2*a*b*Ci(
d/x)*sin(c)/f+2*a*b*Ci(d*f/e+d/x)*sin(c-d*f/e)/f+2*a*b*cos(c-d*f/e)*Si(d*f
/e+d/x)/f+1/2*b^2*sin(2*c-2*d*f/e)*Si(2*d*(f+e/x)/e)/f-2*a*b*cos(c)*Si(d/x
)/f-1/2*b^2*sin(2*c)*Si(2*d/x)/f
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$
$$= \frac{-b^2\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+b^2\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)+2a^2\log(e+fx)+b^2\log(e+}{}$$

input
```
Integrate[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

output
```
(-(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))]) + b^2*Cos[2*c
]*CosIntegral[(2*d)/x] + 2*a^2*Log[e + f*x] + b^2*Log[e + f*x] - 4*a*b*Cos
Integral[d/x]*Sin[c] + 4*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e
] + 4*a*b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + b^2*Sin[2*c - (
2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] - 4*a*b*Cos[c]*SinIntegral[d/x]
- b^2*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \left(\frac{x\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{f}-\frac{e\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{f\left(\frac{e}{x}+f\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log\left(\frac{e}{x}+f\right)}{f}-\frac{a^2\log\left(\frac{1}{x}\right)}{f}+\frac{2ab\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{f}-\frac{2ab\sin(c)\,\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{2ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{f}-\frac{2ab\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}-\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{2f}+\frac{b^2\cos(2c)\,\text{CosIntegral}\left(\frac{2d}{x}\right)}{2f}+\frac{b^2\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{2f}-\frac{b^2\sin(2c)\text{Si}\left(\frac{2d}{x}\right)}{2f}+\frac{b^2\log\left(\frac{e}{x}+f\right)}{2f}-\frac{b^2\log\left(\frac{1}{x}\right)}{2f}$$

input
```
Int[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

output
```
-1/2*(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[(2*d*f)/e + (2*d)/x])/f + (b^2*
Cos[2*c]*CosIntegral[(2*d)/x])/(2*f) + (a^2*Log[f + e/x])/f + (b^2*Log[f +
 e/x])/(2*f) - (a^2*Log[x^(-1)])/f - (b^2*Log[x^(-1)])/(2*f) - (2*a*b*CosI
ntegral[d/x]*Sin[c])/f + (2*a*b*CosIntegral[(d*f)/e + d/x]*Sin[c - (d*f)/e
])/f + (2*a*b*Cos[c - (d*f)/e]*SinIntegral[(d*f)/e + d/x])/f + (b^2*Sin[2*
c - (2*d*f)/e]*SinIntegral[(2*d*f)/e + (2*d)/x])/(2*f) - (2*a*b*Cos[c]*Sin
Integral[d/x])/f - (b^2*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.17

method	result
parts	$\frac{\ln(fx+e)a^2}{f} - \frac{b^2\ln\left(\frac{d}{x}\right)}{2f} + \frac{b^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{2f} + \frac{b^2\cos(2c)\operatorname{Ci}\left(\frac{2d}{x}\right)}{2f} - \frac{b^2\sin(2c)\operatorname{Si}\left(\frac{2d}{x}\right)}{2f} - b^2\operatorname{Si}\left(\frac{2d}{x}+2\right.$
risch	$\frac{iab\,\mathrm{expIntegral}_1\left(\frac{id}{x}\right)\mathrm{e}^{-ic}}{f} - \frac{iab\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{f} + \frac{\ln(fx+e)a^2}{f} + \frac{\ln(fx+e)b^2}{2f} +$
derivativedivides	$-d\left(\frac{a^2\ln\left(\frac{d}{x}\right)}{fd} - \frac{a^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd} + \frac{2ab\left(\operatorname{Si}\left(\frac{d}{x}\right)\cos(c)+\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)\right)}{fd} - 2abe\left(-\frac{\operatorname{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right.}{e}\right.\right.$
default	$-d\left(\frac{a^2\ln\left(\frac{d}{x}\right)}{fd} - \frac{a^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd} + \frac{2ab\left(\operatorname{Si}\left(\frac{d}{x}\right)\cos(c)+\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)\right)}{fd} - 2abe\left(-\frac{\operatorname{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right.}{e}\right.\right.$

input
```
int((a+b*sin(c+d/x))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
ln(f*x+e)/f*a^2-1/2*b^2/f*ln(d/x)+1/2*b^2/f*ln(-c*e+d*f+e*(c+d/x))+1/2*b^2
*cos(2*c)*Ci(2*d/x)/f-1/2*b^2*sin(2*c)*Si(2*d/x)/f-1/2*b^2/f*Si(2*d/x+2*c+
2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)-1/2*b^2/f*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)
*cos(2*(-c*e+d*f)/e)-2*a*b*d*(1/d/f*(Si(d/x)*cos(c)+Ci(d/x)*sin(c))-e/d/f*
(Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*e+d*f)/e)*sin((-c
*e+d*f)/e)/e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$
$$= \frac{b^2\cos(2c)\operatorname{Ci}\left(\frac{2d}{x}\right) - b^2\cos\left(-\frac{2(ce-df)}{e}\right)\operatorname{Ci}\left(\frac{2(dfx+de)}{ex}\right) - 4ab\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c) - 4ab\operatorname{Ci}\left(\frac{dfx+de}{ex}\right)\sin\left(-\frac{ce-df}{e}\right.}{}$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="fricas")
```

output
```
1/2*(b^2*cos(2*c)*cos_integral(2*d/x) - b^2*cos(-2*(c*e - d*f)/e)*cos_inte
gral(2*(d*f*x + d*e)/(e*x)) - 4*a*b*cos_integral(d/x)*sin(c) - 4*a*b*cos_i
ntegral((d*f*x + d*e)/(e*x))*sin(-(c*e - d*f)/e) - b^2*sin(2*c)*sin_integr
al(2*d/x) - 4*a*b*cos(c)*sin_integral(d/x) - b^2*sin(-2*(c*e - d*f)/e)*sin
_integral(2*(d*f*x + d*e)/(e*x)) + 4*a*b*cos(-(c*e - d*f)/e)*sin_integral(
(d*f*x + d*e)/(e*x)) + (2*a^2 + b^2)*log(f*x + e))/f
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx=\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$

input
```
integrate((a+b*sin(c+d/x))**2/(f*x+e),x)
```

output
```
Integral((a + b*sin(c + d/x))**2/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx=\int \frac{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}{fx+e}\,dx$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f - 1/2*(2*b^2*f*integrate(1/4*cos(2*(c*x + d)/x)/((f*x +
 e)*cos(2*(c*x + d)/x)^2 + (f*x + e)*sin(2*(c*x + d)/x)^2), x) + 2*b^2*f*i
ntegrate(1/4*cos(2*(c*x + d)/x)/(f*x + e), x) - 2*a*b*f*integrate(sin((c*x
 + d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e)*sin((c*x + d)/x)^2), x)
 - 2*a*b*f*integrate(sin((c*x + d)/x)/(f*x + e), x) - b^2*log(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$$

$$= \frac{b^2d\cos(2c)\,\mathrm{Ci}\left(-2c+\frac{2(cx+d)}{x}\right)-b^2d\cos\left(\frac{2(ce-df)}{e}\right)\mathrm{Ci}\left(-\frac{2\left(ce-df-\frac{(cx+d)e}{x}\right)}{e}\right)-4abd\,\mathrm{Ci}\left(-c+\frac{cx+d}{x}\right)\sin}{}$$

input

```
integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="giac")
```

output

```
1/2*(b^2*d*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x) - b^2*d*cos(2*(c*e
- d*f)/e)*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*d*cos_int
egral(-c + (c*x + d)/x)*sin(c) + 4*a*b*d*cos_integral(-(c*e - d*f - (c*x +
 d)*e/x)/e)*sin((c*e - d*f)/e) + b^2*d*sin(2*c)*sin_integral(2*c - 2*(c*x
+ d)/x) + 4*a*b*d*cos(c)*sin_integral(c - (c*x + d)/x) - b^2*d*sin(2*(c*e
- d*f)/e)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*d*cos((c*e
 - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) + 2*a^2*d*log(c*e -
 d*f - (c*x + d)*e/x) + b^2*d*log(c*e - d*f - (c*x + d)*e/x) - 2*a^2*d*log
(c - (c*x + d)/x) - b^2*d*log(c - (c*x + d)/x))/(d*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = \int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{e+f\,x}\,dx$$

input

```
int((a + b*sin(c + d/x))^2/(e + f*x),x)
```

output

```
int((a + b*sin(c + d/x))^2/(e + f*x), x)
```

Reduce [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = \frac{\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)^2}{fx+e}dx\right)b^2f+2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{fx+e}dx\right)abf+\log(fx+e)\,a^2}{f}$$

input
```
int((a+b*sin(c+d/x))^2/(f*x+e),x)
```

output
```
(int(sin((c*x + d)/x)**2/(e + f*x),x)*b**2*f + 2*int(sin((c*x + d)/x)/(e +
 f*x),x)*a*b*f + log(e + f*x)*a**2)/f
```

3.297 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 199

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx &= \frac{a^2}{e\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^2}\\ &\quad-\frac{b^2d\operatorname{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^2}\\ &\quad+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}\\ &\quad+\frac{2abd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^2}\\ &\quad-\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}\end{aligned}$$

output

```
a^2/e/(f+e/x)-2*a*b*d*cos(c-d*f/e)*Ci(d*f/e+d/x)/e^2-b^2*d*Ci(2*d*(f+e/x)/
e)*sin(2*c-2*d*f/e)/e^2+2*a*b*sin(c+d/x)/e/(f+e/x)+b^2*sin(c+d/x)^2/e/(f+e
/x)+2*a*b*d*sin(c-d*f/e)*Si(d*f/e+d/x)/e^2-b^2*d*cos(2*c-2*d*f/e)*Si(2*d*(
f+e/x)/e)/e^2
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx =$$
$$-\frac{2a^2e^2+b^2e^2+b^2efx\cos\left(2\left(c+\frac{d}{x}\right)\right)+4abdf(e+fx)\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+2b^2df(e}{}$$

input
```
Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

output
```
-1/2*(2*a^2*e^2 + b^2*e^2 + b^2*e*f*x*Cos[2*(c + d/x)] + 4*a*b*d*f*(e + f*
x)*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))] + 2*b^2*d*f*(e + f*x)*Co
sIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e] - 4*a*b*e*f*x*Sin[c + d
/x] - 4*a*b*d*e*f*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - 4*a*b*d
*f^2*x*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 2*b^2*d*e*f*Cos[2*
c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 2*b^2*d*f^2*x*Cos[2*c - (
2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(e^2*f*(e + f*x))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3912, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx$$
$$\Big\downarrow \text{ 3912}$$
$$-\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(\frac{e}{x}+f\right)^2}d\frac{1}{x}$$
$$\Big\downarrow \text{ 3042}$$

$$-\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(\frac{e}{x}+f\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{3798}$$

$$-\int \left(\frac{a^2}{\left(\frac{e}{x}+f\right)^2}+\frac{2b\sin\left(c+\frac{d}{x}\right)a}{\left(\frac{e}{x}+f\right)^2}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{\left(\frac{e}{x}+f\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2}{e\left(\frac{e}{x}+f\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^2}+\frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}-\frac{b^2d\sin\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{e^2}-\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{e^2}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

input
```
Int[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

output
```
a^2/(e*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[(d*f)/e + d/x])/
e^2 - (b^2*d*CosIntegral[(2*d*f)/e + (2*d)/x]*Sin[2*c - (2*d*f)/e])/e^2 +
(2*a*b*Sin[c + d/x])/(e*(f + e/x)) + (b^2*Sin[c + d/x]^2)/(e*(f + e/x)) +
(2*a*b*d*Sin[c - (d*f)/e]*SinIntegral[(d*f)/e + d/x])/e^2 - (b^2*d*Cos[2*c
 - (2*d*f)/e]*SinIntegral[(2*d*f)/e + (2*d)/x])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.47

method	result
parts	$-\frac{a^{2}}{f\left(fx+e\right)}-b^{2}d\left(-\frac{1}{2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{\cos\left(\frac{2d}{x}+2c\right)}{2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{2\,\mathrm{Si}\left(\frac{2d}{x}+2c+\frac{-2ce+2df}{e}\right)\cos\left(\frac{-2ce+2}{e}\right.}{e}\right.$
derivativedivides	$-d\left(-\frac{a^{2}}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+\right.}{}}{e}\right.\right.$
default	$-d\left(-\frac{a^{2}}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{Si}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\mathrm{Ci}\left(\frac{d}{x}+\right.}{}}{e}\right.\right.$
risch	$\frac{abd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_{1}\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{e^{2}}-\frac{a^{2}}{f\left(fx+e\right)}-\frac{b^{2}}{2f\left(fx+e\right)}+\frac{id\,b^{2}\mathrm{e}^{-\frac{2i(ce-df)}{e}}\,\mathrm{expIntegral}_{1}\left(\frac{2id}{x}+\right.}{2e^{2}}$

input
```
int((a+b*sin(c+d/x))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/f/(f*x+e)*a^2-b^2*d*(-1/2/(-c*e+d*f+e*(c+d/x))/e+1/2*cos(2*d/x+2*c)/(-c
*e+d*f+e*(c+d/x))/e+1/2*(2*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e
)/e-2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e)/e)-2*a*b*d*(-sin
(c+d/x)/(-c*e+d*f+e*(c+d/x))/e+(Si(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e
+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e)/e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx =$$

$$-\frac{2\,b^2efx\cos\left(\frac{cx+d}{x}\right)^2 - 4\,abefx\sin\left(\frac{cx+d}{x}\right) - b^2efx + (2\,a^2+b^2)e^2 + 4\,(abdf^2x + abdef)\cos\left(-\frac{ce-df}{e}\right)\mathrm{C}}{}$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*e*f*x*cos((c*x + d)/x)^2 - 4*a*b*e*f*x*sin((c*x + d)/x) - b^2*
e*f*x + (2*a^2 + b^2)*e^2 + 4*(a*b*d*f^2*x + a*b*d*e*f)*cos(-(c*e - d*f)/e
)*cos_integral((d*f*x + d*e)/(e*x)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*cos_inte
gral(2*(d*f*x + d*e)/(e*x))*sin(-2*(c*e - d*f)/e) + 2*(b^2*d*f^2*x + b^2*d
*e*f)*cos(-2*(c*e - d*f)/e)*sin_integral(2*(d*f*x + d*e)/(e*x)) + 4*(a*b*d
*f^2*x + a*b*d*e*f)*sin(-(c*e - d*f)/e)*sin_integral((d*f*x + d*e)/(e*x)))
/(e^2*f^2*x + e^3*f)
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(e+fx\right)^2}\,dx$$

input

```
integrate((a+b*sin(c+d/x))**2/(f*x+e)**2,x)
```

output

```
Integral((a + b*sin(c + d/x))**2/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

input

```
integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="maxima")
```

output

```
-a^2/(f^2*x + e*f) - 1/2*(b^2 + 2*(b^2*f^2*x + b^2*e*f)*integrate(1/4*cos(
2*(c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) + 2*(b^2*f^2*x + b^2*e*f)*int
egrate(1/4*cos(2*(c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*cos(2*(c*x + d)/x
)^2 + (f^2*x^2 + 2*e*f*x + e^2)*sin(2*(c*x + d)/x)^2), x) - 2*(a*b*f^2*x +
 a*b*e*f)*integrate(sin((c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) - 2*(a*
b*f^2*x + a*b*e*f)*integrate(sin((c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*c
os((c*x + d)/x)^2 + (f^2*x^2 + 2*e*f*x + e^2)*sin((c*x + d)/x)^2), x))/(f^
2*x + e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(199) = 398$.

Time = 0.13 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.45

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*(4*a*b*c*d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d
)*e/x)/e) - 4*a*b*d^3*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x
 + d)*e/x)/e) + 2*b^2*c*d^2*e*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/
e)*sin(2*(c*e - d*f)/e) - 2*b^2*d^3*f*cos_integral(-2*(c*e - d*f - (c*x +
d)*e/x)/e)*sin(2*(c*e - d*f)/e) - 2*b^2*c*d^2*e*cos(2*(c*e - d*f)/e)*sin_i
ntegral(2*(c*e - d*f - (c*x + d)*e/x)/e) + 2*b^2*d^3*f*cos(2*(c*e - d*f)/e
)*sin_integral(2*(c*e - d*f - (c*x + d)*e/x)/e) + 4*a*b*c*d^2*e*sin((c*e -
 d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*d^3*f*sin((c*
e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 4*(c*x + d)*a*b*
d^2*e*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)/x -
2*(c*x + d)*b^2*d^2*e*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)*sin(2
*(c*e - d*f)/e)/x + 2*(c*x + d)*b^2*d^2*e*cos(2*(c*e - d*f)/e)*sin_integra
l(2*(c*e - d*f - (c*x + d)*e/x)/e)/x - 4*(c*x + d)*a*b*d^2*e*sin((c*e - d*
f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e)/x - b^2*d^2*e*cos(2*(c*x
 + d)/x) + 4*a*b*d^2*e*sin((c*x + d)/x) + 2*a^2*d^2*e + b^2*d^2*e)/((c*e^3
 - d*e^2*f - (c*x + d)*e^3/x)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{{\left(e+f\,x\right)}^2}\,dx$$

input
```
int((a + b*sin(c + d/x))^2/(e + f*x)^2,x)
```

output
```
int((a + b*sin(c + d/x))^2/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = \text{too large to display}$$

input

```
int((a+b*sin(c+d/x))^2/(f*x+e)^2,x)
```

output

```
( - 4*cos((c*x + d)/x)*sin((c*x + d)/x)*a*b*d**3*e**2*f**3*x - 2*cos((c*x
+ d)/x)*sin((c*x + d)/x)*a*b*d*e**4*f*x - 2*cos((c*x + d)/x)*sin((c*x + d)
/x)*a*b*d*e**3*f**2*x**2 - 5*cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*d**2*e
**3*f**2*x + 3*cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*d**2*e**2*f**3*x**2
- 4*cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*e**5*x - 4*cos((c*x + d)/x)*sin
((c*x + d)/x)*b**2*e**4*f*x**2 + 4*cos((c*x + d)/x)*a*b*d**2*e**3*f**2*x -
 4*cos((c*x + d)/x)*a*b*d**2*e**2*f**3*x**2 + 8*cos((c*x + d)/x)*b**2*d*e*
*4*f*x - 8*cos((c*x + d)/x)*b**2*d*e**3*f**2*x**2 + 24*int(tan((c*x + d)/(
2*x))**2/(tan((c*x + d)/(2*x))**4*e**2*x**2 + 2*tan((c*x + d)/(2*x))**4*e*
f*x**3 + tan((c*x + d)/(2*x))**4*f**2*x**4 + 2*tan((c*x + d)/(2*x))**2*e**
2*x**2 + 4*tan((c*x + d)/(2*x))**2*e*f*x**3 + 2*tan((c*x + d)/(2*x))**2*f*
*2*x**4 + e**2*x**2 + 2*e*f*x**3 + f**2*x**4),x)*b**2*d**3*e**5*f**2*x + 2
4*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*e**2*x**2 + 2*tan((
c*x + d)/(2*x))**4*e*f*x**3 + tan((c*x + d)/(2*x))**4*f**2*x**4 + 2*tan((c
*x + d)/(2*x))**2*e**2*x**2 + 4*tan((c*x + d)/(2*x))**2*e*f*x**3 + 2*tan((
c*x + d)/(2*x))**2*f**2*x**4 + e**2*x**2 + 2*e*f*x**3 + f**2*x**4),x)*b**2
*d**3*e**4*f**3*x**2 - 16*int(tan((c*x + d)/(2*x))/(tan((c*x + d)/(2*x))**
4*e**2*x**2 + 2*tan((c*x + d)/(2*x))**4*e*f*x**3 + tan((c*x + d)/(2*x))**4
*f**2*x**4 + 2*tan((c*x + d)/(2*x))**2*e**2*x**2 + 4*tan((c*x + d)/(2*x))*
*2*e*f*x**3 + 2*tan((c*x + d)/(2*x))**2*f**2*x**4 + e**2*x**2 + 2*e*f*x...
```

3.298 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 478

$$\begin{aligned}\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = & -\frac{a^2 f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)} \\
& -\frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^3} \\
& +\frac{b^2d^2f\cos\left(2c-\frac{2df}{e}\right)\operatorname{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^4} \\
& -\frac{b^2d\operatorname{CosIntegral}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^3} \\
& -\frac{abd^2f\operatorname{CosIntegral}\left(\frac{df}{e}+\frac{d}{x}\right)\sin\left(c-\frac{df}{e}\right)}{e^4} \\
& -\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)} \\
& -\frac{b^2df\cos\left(c+\frac{d}{x}\right)\sin\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{b^2f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2} \\
& +\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{abd^2f\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^4} \\
& +\frac{2abd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{df}{e}+\frac{d}{x}\right)}{e^3} \\
& -\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3} \\
& -\frac{b^2d^2f\sin\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{e^4}\end{aligned}$$

output

```
-1/2*a^2*f/e^2/(f+e/x)^2+a^2/e^2/(f+e/x)-a*b*d*f*cos(c+d/x)/e^3/(f+e/x)-2*
a*b*d*cos(c-d*f/e)*Ci(d*f/e+d/x)/e^3+b^2*d^2*f*cos(2*c-2*d*f/e)*Ci(2*d*(f+
e/x)/e)/e^4-b^2*d*Ci(2*d*(f+e/x)/e)*sin(2*c-2*d*f/e)/e^3-a*b*d^2*f*Ci(d*f/
e+d/x)*sin(c-d*f/e)/e^4-a*b*f*sin(c+d/x)/e^2/(f+e/x)^2+2*a*b*sin(c+d/x)/e^
2/(f+e/x)-b^2*d*f*cos(c+d/x)*sin(c+d/x)/e^3/(f+e/x)-1/2*b^2*f*sin(c+d/x)^2
/e^2/(f+e/x)^2+b^2*sin(c+d/x)^2/e^2/(f+e/x)-a*b*d^2*f*cos(c-d*f/e)*Si(d*f/
e+d/x)/e^4+2*a*b*d*sin(c-d*f/e)*Si(d*f/e+d/x)/e^3-b^2*d*cos(2*c-2*d*f/e)*S
i(2*d*(f+e/x)/e)/e^3-b^2*d^2*f*sin(2*c-2*d*f/e)*Si(2*d*(f+e/x)/e)/e^4
```

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.55

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

output

```
-1/4*(2*a^2*e^4 + b^2*e^4 + 4*a*b*d*e^2*f^2*x*Cos[c + d/x] + 4*a*b*d*e*f^3
*x^2*Cos[c + d/x] + 2*b^2*e^3*f*x*Cos[2*(c + d/x)] + b^2*e^2*f^2*x^2*Cos[2
*(c + d/x)] - 4*b^2*d*f*(e + f*x)^2*CosIntegral[2*d*(f/e + x^(-1))]*(d*f*C
os[2*c - (2*d*f)/e] - e*Sin[2*c - (2*d*f)/e]) + 4*a*b*d*f*(e + f*x)^2*CosI
ntegral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c - (d*f)/e]) -
8*a*b*e^3*f*x*Sin[c + d/x] - 4*a*b*e^2*f^2*x^2*Sin[c + d/x] + 2*b^2*d*e^2*
f^2*x*Sin[2*(c + d/x)] + 2*b^2*d*e*f^3*x^2*Sin[2*(c + d/x)] + 4*a*b*d^2*e^
2*f^2*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 8*a*b*d^2*e*f^3*x*C
os[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 4*a*b*d^2*f^4*x^2*Cos[c -
(d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - 8*a*b*d*e^3*f*Sin[c - (d*f)/e]*Si
nIntegral[d*(f/e + x^(-1))] - 16*a*b*d*e^2*f^2*x*Sin[c - (d*f)/e]*SinInteg
ral[d*(f/e + x^(-1))] - 8*a*b*d*e*f^3*x^2*Sin[c - (d*f)/e]*SinIntegral[d*(
f/e + x^(-1))] + 4*b^2*d*e^3*f*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e +
 x^(-1))] + 8*b^2*d*e^2*f^2*x*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e +
x^(-1))] + 4*b^2*d*e*f^3*x^2*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x
^(-1))] + 4*b^2*d^2*e^2*f^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^
(-1))] + 8*b^2*d^2*e*f^3*x*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(
-1))] + 4*b^2*d^2*f^4*x^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-
1))])/(e^4*f*(e + f*x)^2)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3912, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx$$

$$\Big\downarrow \text{3912}$$

$$-\int \left(\frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e\left(\frac{e}{x}+f\right)^2}-\frac{f\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e\left(\frac{e}{x}+f\right)^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\begin{gathered}\frac{a^2}{e^2\left(\frac{e}{x}+f\right)}-\frac{a^2 f}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{abd^2 f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^4}-\\ \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^3}-\frac{abd^2 f\cos\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^4}+\\ \frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{fd}{e}+\frac{d}{x}\right)}{e^3}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(\frac{e}{x}+f\right)}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)^2}+\\ \frac{b^2d^2 f\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{e^4}-\frac{b^2 d\sin\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{e^3}-\\ \frac{b^2d^2 f\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{e^4}-\frac{b^2 d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(\frac{2fd}{e}+\frac{2d}{x}\right)}{e^3}-\\ \frac{b^2 df\sin\left(c+\frac{d}{x}\right)\cos\left(c+\frac{d}{x}\right)}{e^3\left(\frac{e}{x}+f\right)}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{b^2 f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(\frac{e}{x}+f\right)^2}\end{gathered}$$

input

```
Int[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

output

```
-1/2*(a^2*f)/(e^2*(f + e/x)^2) + a^2/(e^2*(f + e/x)) - (a*b*d*f*Cos[c + d/
x])/(e^3*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[(d*f)/e + d/x]
)/e^3 + (b^2*d^2*f*Cos[2*c - (2*d*f)/e]*CosIntegral[(2*d*f)/e + (2*d)/x])/
e^4 - (b^2*d*CosIntegral[(2*d*f)/e + (2*d)/x]*Sin[2*c - (2*d*f)/e])/e^3 -
(a*b*d^2*f*CosIntegral[(d*f)/e + d/x]*Sin[c - (d*f)/e])/e^4 - (a*b*f*Sin[c
 + d/x])/(e^2*(f + e/x)^2) + (2*a*b*Sin[c + d/x])/(e^2*(f + e/x)) - (b^2*d
*f*Cos[c + d/x]*Sin[c + d/x])/(e^3*(f + e/x)) - (b^2*f*Sin[c + d/x]^2)/(2*
e^2*(f + e/x)^2) + (b^2*Sin[c + d/x]^2)/(e^2*(f + e/x)) - (a*b*d^2*f*Cos[c
 - (d*f)/e]*SinIntegral[(d*f)/e + d/x])/e^4 + (2*a*b*d*Sin[c - (d*f)/e]*Si
nIntegral[(d*f)/e + d/x])/e^3 - (b^2*d*Cos[2*c - (2*d*f)/e]*SinIntegral[(2
*d*f)/e + (2*d)/x])/e^3 - (b^2*d^2*f*Sin[2*c - (2*d*f)/e]*SinIntegral[(2*d
*f)/e + (2*d)/x])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.81

method	result
risch	$\frac{iab\,d^2\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)f}{2e^4}+\frac{abd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}_1\left(\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{e^3}-\frac{a^2}{2f(fx+e}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((a+b*sin(c+d/x))^2/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*a*b*d^2/e^4*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*e-d*f)/e)*f+a*b*
d/e^3*exp(-I*(c*e-d*f)/e)*Ei(1,I*d/x+I*c-I*(c*e-d*f)/e)-1/2/f/(f*x+e)^2*a^
2-1/4/f*b^2/(f*x+e)^2-1/2*d^2*b^2/e^4*exp(-2*I*(c*e-d*f)/e)*Ei(1,2*I*d/x+2
*I*c-2*I*(c*e-d*f)/e)*f+1/2*I*d*b^2/e^3*exp(-2*I*(c*e-d*f)/e)*Ei(1,2*I*d/x
+2*I*c-2*I*(c*e-d*f)/e)-1/2*d^2*b^2*exp(2*I*(c*e-d*f)/e)*Ei(1,-2*I*d/x-2*I
*c-2*(-I*c*e+I*f*d)/e)/e^4*f-1/2*I*d*b^2*exp(2*I*(c*e-d*f)/e)*Ei(1,-2*I*d/
x-2*I*c-2*(-I*c*e+I*f*d)/e)/e^3-1/2*I*a*b*d^2*exp(I*(c*e-d*f)/e)*Ei(1,-I*d
/x-I*c-(-I*c*e+I*f*d)/e)/e^4*f+a*b*d*exp(I*(c*e-d*f)/e)*Ei(1,-I*d/x-I*c-(-
I*c*e+I*f*d)/e)/e^3+1/2*I*a*b/e^3*x*(6*I*d^3*e*f^3*x^2+6*I*d^3*e^2*f^2*x+2
*I*d^3*f^4*x^3+2*I*d^3*e^3*f)/(f*x+e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*
cos((c*x+d)/x)-1/2*a*b/e^2*x*(-2*d^2*f^3*x^3-8*d^2*e*f^2*x^2-10*d^2*e^2*f*
x-4*d^2*e^3)/(f*x+e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*sin((c*x+d)/x)+1/
8*b^2*x/e^2*(-2*d^2*f^3*x^3-8*d^2*e*f^2*x^2-10*d^2*e^2*f*x-4*d^2*e^3)/(f*x
+e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*cos(2*(c*x+d)/x)+1/8*I*b^2*x/e^3*(
4*I*d^3*f^4*x^3+4*I*d^3*e^3*f+12*I*d^3*e*f^3*x^2+12*I*d^3*e^2*f^2*x)/(f*x+
e)^2/(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2)*sin(2*(c*x+d)/x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/4*(b^2*e^2*f^2*x^2 + 2*b^2*e^3*f*x - (2*a^2 + b^2)*e^4 - 2*(b^2*e^2*f^2*
x^2 + 2*b^2*e^3*f*x)*cos((c*x + d)/x)^2 - 4*(2*(a*b*d*e*f^3*x^2 + 2*a*b*d*
e^2*f^2*x + a*b*d*e^3*f)*cos_integral((d*f*x + d*e)/(e*x)) + (a*b*d^2*f^4*
x^2 + 2*a*b*d^2*e*f^3*x + a*b*d^2*e^2*f^2)*sin_integral((d*f*x + d*e)/(e*x
)))*cos(-(c*e - d*f)/e) + 4*((b^2*d^2*f^4*x^2 + 2*b^2*d^2*e*f^3*x + b^2*d^
2*e^2*f^2)*cos_integral(2*(d*f*x + d*e)/(e*x)) - (b^2*d*e*f^3*x^2 + 2*b^2*
d*e^2*f^2*x + b^2*d*e^3*f)*sin_integral(2*(d*f*x + d*e)/(e*x)))*cos(-2*(c*
e - d*f)/e) - 4*(a*b*d*e*f^3*x^2 + a*b*d*e^2*f^2*x)*cos((c*x + d)/x) + 4*(
(a*b*d^2*f^4*x^2 + 2*a*b*d^2*e*f^3*x + a*b*d^2*e^2*f^2)*cos_integral((d*f*
x + d*e)/(e*x)) - 2*(a*b*d*e*f^3*x^2 + 2*a*b*d*e^2*f^2*x + a*b*d*e^3*f)*si
n_integral((d*f*x + d*e)/(e*x)))*sin(-(c*e - d*f)/e) + 4*((b^2*d*e*f^3*x^2
 + 2*b^2*d*e^2*f^2*x + b^2*d*e^3*f)*cos_integral(2*(d*f*x + d*e)/(e*x)) +
(b^2*d^2*f^4*x^2 + 2*b^2*d^2*e*f^3*x + b^2*d^2*e^2*f^2)*sin_integral(2*(d*
f*x + d*e)/(e*x)))*sin(-2*(c*e - d*f)/e) + 4*(a*b*e^2*f^2*x^2 + 2*a*b*e^3*
f*x - (b^2*d*e*f^3*x^2 + b^2*d*e^2*f^2*x)*cos((c*x + d)/x))*sin((c*x + d)/
x))/(e^4*f^3*x^2 + 2*e^5*f^2*x + e^6*f)
```

Sympy [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx=\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(e+fx\right)^3}\,dx$$

input
```
integrate((a+b*sin(c+d/x))**2/(f*x+e)**3,x)
```

output
```
Integral((a + b*sin(c + d/x))**2/(e + f*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx=\int \frac{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}{\left(fx+e\right)^3}\,dx$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="maxima")
```

output
```
-1/2*a^2/(f^3*x^2 + 2*e*f^2*x + e^2*f) - 1/4*(b^2 + 4*(b^2*f^3*x^2 + 2*b^2
*e*f^2*x + b^2*e^2*f)*integrate(1/4*cos(2*(c*x + d)/x)/(f^3*x^3 + 3*e*f^2*
x^2 + 3*e^2*f*x + e^3), x) + 4*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*i
ntegrate(1/4*cos(2*(c*x + d)/x)/((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)
*cos(2*(c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*sin(2*(c
*x + d)/x)^2), x) - 4*(a*b*f^3*x^2 + 2*a*b*e*f^2*x + a*b*e^2*f)*integrate(
sin((c*x + d)/x)/(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3), x) - 4*(a*b*f^
3*x^2 + 2*a*b*e*f^2*x + a*b*e^2*f)*integrate(sin((c*x + d)/x)/((f^3*x^3 +
3*e*f^2*x^2 + 3*e^2*f*x + e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2
 + 3*e^2*f*x + e^3)*sin((c*x + d)/x)^2), x))/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3078 vs. $2(474) = 948$.

Time = 0.19 (sec) , antiderivative size = 3078, normalized size of antiderivative = 6.44

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="giac")
```

output

```
1/4*(4*b^2*c^2*d^3*e^2*f*cos(2*(c*e - d*f)/e)*cos_integral(-2*(c*e - d*f -
 (c*x + d)*e/x)/e) - 8*b^2*c*d^4*e*f^2*cos(2*(c*e - d*f)/e)*cos_integral(-
2*(c*e - d*f - (c*x + d)*e/x)/e) + 4*b^2*d^5*f^3*cos(2*(c*e - d*f)/e)*cos_
integral(-2*(c*e - d*f - (c*x + d)*e/x)/e) - 4*a*b*c^2*d^3*e^2*f*cos_integ
ral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) + 8*a*b*c*d^4*e*f^2
*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) - 4*a*b*d
^5*f^3*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e)*sin((c*e - d*f)/e) + 4
*b^2*c^2*d^3*e^2*f*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*e - d*f - (c*x +
 d)*e/x)/e) - 8*b^2*c*d^4*e*f^2*sin(2*(c*e - d*f)/e)*sin_integral(2*(c*e -
 d*f - (c*x + d)*e/x)/e) + 4*b^2*d^5*f^3*sin(2*(c*e - d*f)/e)*sin_integral
(2*(c*e - d*f - (c*x + d)*e/x)/e) + 4*a*b*c^2*d^3*e^2*f*cos((c*e - d*f)/e)
*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 8*a*b*c*d^4*e*f^2*cos((c*e
- d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) + 4*a*b*d^5*f^3*cos(
(c*e - d*f)/e)*sin_integral((c*e - d*f - (c*x + d)*e/x)/e) - 8*a*b*c^2*d^2
*e^3*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/x)/e) + 16*
a*b*c*d^3*e^2*f*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x + d)*e/
x)/e) - 8*a*b*d^4*e*f^2*cos((c*e - d*f)/e)*cos_integral(-(c*e - d*f - (c*x
 + d)*e/x)/e) - 8*(c*x + d)*b^2*c*d^3*e^2*f*cos(2*(c*e - d*f)/e)*cos_integ
ral(-2*(c*e - d*f - (c*x + d)*e/x)/e)/x + 8*(c*x + d)*b^2*d^4*e*f^2*cos(2*
(c*e - d*f)/e)*cos_integral(-2*(c*e - d*f - (c*x + d)*e/x)/e)/x - 4*b^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx=\int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(e+f\,x\right)^3}\,dx$$

input

```
int((a + b*sin(c + d/x))^2/(e + f*x)^3,x)
```

output

```
int((a + b*sin(c + d/x))^2/(e + f*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = \text{too large to display}$$

input

```
int((a+b*sin(c+d/x))^2/(f*x+e)^3,x)
```

output

```
( - 2*cos((c*x + d)/x)*sin((c*x + d)/x)*a*b*d**3*e**4*f**3*x + cos((c*x +
d)/x)*sin((c*x + d)/x)*a*b*d*e**6*f*x + 2*cos((c*x + d)/x)*sin((c*x + d)/x
)*a*b*d*e**5*f**2*x**2 + cos((c*x + d)/x)*sin((c*x + d)/x)*a*b*d*e**4*f**3
*x**3 - 2*cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*d**4*e**3*f**4*x - 2*cos(
(c*x + d)/x)*sin((c*x + d)/x)*b**2*d**4*e**2*f**5*x**2 - 5*cos((c*x + d)/x
)*sin((c*x + d)/x)*b**2*d**2*e**5*f**2*x - 10*cos((c*x + d)/x)*sin((c*x +
d)/x)*b**2*d**2*e**4*f**3*x**2 - cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*d*
*2*e**3*f**4*x**3 - 6*cos((c*x + d)/x)*sin((c*x + d)/x)*b**2*e**7*x - 12*c
os((c*x + d)/x)*sin((c*x + d)/x)*b**2*e**6*f*x**2 - 6*cos((c*x + d)/x)*sin
((c*x + d)/x)*b**2*e**5*f**2*x**3 + 4*cos((c*x + d)/x)*a*b*d**2*e**5*f**2*
x - 4*cos((c*x + d)/x)*a*b*d**2*e**4*f**3*x**2 - 8*cos((c*x + d)/x)*b**2*d
**3*e**2*f**5*x**3 - 8*cos((c*x + d)/x)*b**2*d*e**6*f*x + 8*cos((c*x + d)/
x)*b**2*d*e**5*f**2*x**2 - 8*cos((c*x + d)/x)*b**2*d*e**4*f**3*x**3 - 16*i
nt(tan((c*x + d)/(2*x))/(tan((c*x + d)/(2*x))**4*e**3*x**2 + 3*tan((c*x +
d)/(2*x))**4*e**2*f*x**3 + 3*tan((c*x + d)/(2*x))**4*e*f**2*x**4 + tan((c*
x + d)/(2*x))**4*f**3*x**5 + 2*tan((c*x + d)/(2*x))**2*e**3*x**2 + 6*tan((
c*x + d)/(2*x))**2*e**2*f*x**3 + 6*tan((c*x + d)/(2*x))**2*e*f**2*x**4 + 2
*tan((c*x + d)/(2*x))**2*f**3*x**5 + e**3*x**2 + 3*e**2*f*x**3 + 3*e*f**2*
x**4 + f**3*x**5),x)*a*b*d**3*e**8*f**2*x - 32*int(tan((c*x + d)/(2*x))/(t
an((c*x + d)/(2*x))**4*e**3*x**2 + 3*tan((c*x + d)/(2*x))**4*e**2*f*x**...
```

3.299 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\operatorname{Int}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

output
```
Defer(Int)((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

output
```
Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

$$\Big\downarrow \ 3918$$

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(fx+e\right)^{2}}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

output
```
int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+f\,x)^2}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
int((e + f*x)^2/(a + b*sin(c + d/x)),x)
```

output
```
int((e + f*x)^2/(a + b*sin(c + d/x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 6.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \frac{-3\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)b\,e^2-3\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)x^2}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)b\,f^2-6\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)x}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)bef+3e^2x+3ef\,x^2+f^2}{3a}$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

output
```
( - 3*int(sin((c*x + d)/x)/(sin((c*x + d)/x)*b + a),x)*b*e**2 - 3*int((sin
((c*x + d)/x)*x**2)/(sin((c*x + d)/x)*b + a),x)*b*f**2 - 6*int((sin((c*x +
 d)/x)*x)/(sin((c*x + d)/x)*b + a),x)*b*e*f + 3*e**2*x + 3*e*f*x**2 + f**2
*x**3)/(3*a)
```

3.300 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\operatorname{Int}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

output
```
Defer(Int)((f*x+e)/(a+b*sin(c+d/x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

output
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

$$\Big\downarrow \; 3918$$

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
int((f*x+e)/(a+b*sin(c+d/x)),x)
```

output
```
int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 167.72 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x)
```

output
```
Integral((e + f*x)/(a + b*sin(c + d/x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 39.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+f\,x}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
int((e + f*x)/(a + b*sin(c + d/x)),x)
```

output
```
int((e + f*x)/(a + b*sin(c + d/x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \frac{-2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)be-2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)x}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)bf+2ex+f\,x^2}{2a}$$

input
```
int((f*x+e)/(a+b*sin(c+d/x)),x)
```

output
```
( - 2*int(sin((c*x + d)/x)/(sin((c*x + d)/x)*b + a),x)*b*e - 2*int((sin((c
*x + d)/x)*x)/(sin((c*x + d)/x)*b + a),x)*b*f + 2*e*x + f*x**2)/(2*a)
```

3.301 $\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \text{Int}\left(\frac{1}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(c+d/x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
Integrate[(a + b*Sin[c + d/x])^(-1),x]
```

output
```
Integrate[(a + b*Sin[c + d/x])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3850}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

$$\Big\downarrow \; 3850$$

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
Int[(a + b*Sin[c + d/x])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
int(1/(a+b*sin(c+d/x)),x)
```

output
```
int(1/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
integral(1/(b*sin((c*x + d)/x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 27.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(a+b*sin(c+d/x)),x)
```

output
```
Integral(1/(a + b*sin(c + d/x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate(1/(a+b*sin(c+d/x)),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 40.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
int(1/(a + b*sin(c + d/x)),x)
```

output
```
int(1/(a + b*sin(c + d/x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\frac{-\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)b+x}{a}$$

input
```
int(1/(a+b*sin(c+d/x)),x)
```

output
```
( - int(sin((c*x + d)/x)/(sin((c*x + d)/x)*b + a),x)*b + x)/a
```

3.302 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Int}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

output
```
Defer(Int)((f*x+e)/(a+b*sin(c+d/x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

output
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
int((f*x+e)/(a+b*sin(c+d/x)),x)
```

output
```
int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 166.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x)
```

output
```
Integral((e + f*x)/(a + b*sin(c + d/x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+f\,x}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
int((e + f*x)/(a + b*sin(c + d/x)),x)
```

output
```
int((e + f*x)/(a + b*sin(c + d/x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \frac{-2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)be-2\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)x}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)bf+2ex+f\,x^2}{2a}$$

input
```
int((f*x+e)/(a+b*sin(c+d/x)),x)
```

output
```
( - 2*int(sin((c*x + d)/x)/(sin((c*x + d)/x)*b + a),x)*b*e - 2*int((sin((c
*x + d)/x)*x)/(sin((c*x + d)/x)*b + a),x)*b*f + 2*e*x + f*x**2)/(2*a)
```

3.303 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\text{Int}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

output
```
Defer(Int)((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

input
```
Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

output
```
Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{a+b\sin\left(c+\frac{d}{x}\right)}dx$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

output
```
int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input

```
integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

output

```
integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

input

```
integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
```

output

```
integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+f\,x)^2}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

input

```
int((e + f*x)^2/(a + b*sin(c + d/x)),x)
```

output
```
int((e + f*x)^2/(a + b*sin(c + d/x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 6.09

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$
$$= \frac{-3\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)b\,e^2-3\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)x^2}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)b\,f^2-6\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)x}{\sin\left(\frac{cx+d}{x}\right)b+a}dx\right)bef+3e^2x+3ef\,x^2+f}{3a}$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

output
```
( - 3*int(sin((c*x + d)/x)/(sin((c*x + d)/x)*b + a),x)*b*e**2 - 3*int((sin
((c*x + d)/x)*x**2)/(sin((c*x + d)/x)*b + a),x)*b*f**2 - 6*int((sin((c*x +
 d)/x)*x)/(sin((c*x + d)/x)*b + a),x)*b*e*f + 3*e**2*x + 3*e*f*x**2 + f**2
*x**3)/(3*a)
```

3.304 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

$$\Big\downarrow \text{3918}$$

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

output
```
int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output
```
integral(-(f^2*x^2 + 2*e*f*x + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c
*x + d)/x) - a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 25.51 (sec) , antiderivative size = 1281, normalized size of antiderivative = 58.23

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output
```
-(2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos(2*(c*x + d)/x)*cos((c*
x + d)/x) + 2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos((c*x + d)/x)
 + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x
+ d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2
*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^
2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b
 - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*inte
grate(-2*(2*(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*cos((c*x + d)/x)^2
 + 2*(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*sin((c*x + d)/x)^2 + (2*(
2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) - (a*b*d*f^2*x
^2 + 2*a*b*d*e*f*x + a*b*d*e^2)*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + 2*(
2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) + (4*b^2*f^2*x
^3 + 6*b^2*e*f*x^2 + 2*b^2*e^2*x + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*
e^2)*cos((c*x + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*sin(
(c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*
e^2)*sin((c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 -
a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2
*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)
*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 -
b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*c...
```

Giac [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(e+f\,x)^2}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
int((e + f*x)^2/(a + b*sin(c + d/x))^2,x)
```

output
```
int((e + f*x)^2/(a + b*sin(c + d/x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 8294, normalized size of antiderivative = 377.00

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

output

```
( - cos((c*x + d)/x)*a**3*b*d*f**2*x**2 + 2*cos((c*x + d)/x)*a**2*b**2*d**
2*f**2*x - 12*cos((c*x + d)/x)*a**2*b**2*e**2*x + 6*cos((c*x + d)/x)*a**2*
b**2*e*f*x**2 + 2*cos((c*x + d)/x)*a**2*b**2*f**2*x**3 - 36*cos((c*x + d)/
x)*a*b**3*d*e*f*x + 3*cos((c*x + d)/x)*a*b**3*d*f**2*x**2 - 18*cos((c*x +
d)/x)*b**4*d**2*f**2*x + 2*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x
))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2
*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x
+ a**2*x),x)*sin((c*x + d)/x)*a**4*b**2*d**3*f**2 - 12*int(tan((c*x + d)/(
2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x
 + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4
*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**4*b**2*d*e**2
 - 36*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan(
(c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x
+ d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*sin((c*x
 + d)/x)*a**3*b**3*d**2*e*f - 18*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d
)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*
x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*
a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**2*b**4*d**3*f**2 + 2*int(tan((c*x +
 d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*
a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b*...
```

3.305 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size $= 20$, antiderivative size $= 20$

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Mathematica [N/A]

Not integrable

Time $= 18.93$ (sec) , antiderivative size $= 22$, normalized size of antiderivative $= 1.10$

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

output
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$
$$\Big\downarrow \text{3918}$$
$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

output
```
int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output
```
integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 1103, normalized size of antiderivative = 55.15

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input

```
integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output

```
-(2*(a*b*f*x^3 + a*b*e*x^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f
*x^3 + a*b*e*x^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)
^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x
 + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(
a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x)
 + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2
- b^4)*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c
*x + d)/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2
+ 2*a*b*e*x)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*co
s(2*(c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*cos((c*x + d)/x) + (3*b^2*f*x
^2 + 2*b^2*e*x + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2
*a*b*e*x)*sin((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin
((c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)
*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x +
d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((
c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d -
 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x +
 d)/x)), x) + 2*(b^2*f*x^3 + b^2*e*x^2 + (a*b*f*x^3 + a*b*e*x^2)*sin((c*x
+ d)/x))*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(
a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)...
```

Giac [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 40.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+f\,x}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
int((e + f*x)/(a + b*sin(c + d/x))^2,x)
```

output
```
int((e + f*x)/(a + b*sin(c + d/x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 3980, normalized size of antiderivative = 199.00

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

output

```
( - 8*cos((c*x + d)/x)*a*b**2*e*x + 2*cos((c*x + d)/x)*a*b**2*f*x**2 - 12*
cos((c*x + d)/x)*b**3*d*f*x - 8*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)
/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x
))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a
*b*x + a**2*x),x)*sin((c*x + d)/x)*a**3*b**2*d*e - 12*int(tan((c*x + d)/(2
*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x
+ 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*
tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**2*b**3*d**2*f
- 8*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c
*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x +
d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*a**4*b*d*e
 - 12*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan(
(c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x
+ d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*a**3*b**
2*d**2*f - 4*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2 + 4
*tan((c*x + d)/(2*x))**3*a*b + 2*tan((c*x + d)/(2*x))**2*a**2 + 4*tan((c*x
 + d)/(2*x))**2*b**2 + 4*tan((c*x + d)/(2*x))*a*b + a**2),x)*sin((c*x + d)
/x)*a**3*b**2*d*f + 24*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**
4*a**2 + 4*tan((c*x + d)/(2*x))**3*a*b + 2*tan((c*x + d)/(2*x))**2*a**2 +
4*tan((c*x + d)/(2*x))**2*b**2 + 4*tan((c*x + d)/(2*x))*a*b + a**2),x)*...
```

3.306 $\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(a+b*sin(c+d/x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
Integrate[(a + b*Sin[c + d/x])^(-2),x]
```

output
```
Integrate[(a + b*Sin[c + d/x])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3850}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

$$\Big\downarrow \color{blue}{3850}$$

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
Int[(a + b*Sin[c + d/x])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3850
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Unintegrable[(a + b*Sin[c + d*(e + f*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, f, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input

```
int(1/(a+b*sin(c+d/x))^2,x)
```

output

```
int(1/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.36

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input

```
integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output

```
integral(-1/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2 - b^2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \text{Timed out}$$

input

```
integrate(1/(a+b*sin(c+d/x))**2,x)
```

output

```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 974, normalized size of antiderivative = 69.57

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input

```
integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output

```
-(2*a*b*x^2*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*a*b*x^2*cos((c*x + d)/
x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*
x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a
^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x
)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3
*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*in
tegrate(-2*(2*a^2*d*cos((c*x + d)/x)^2 + 2*a^2*d*sin((c*x + d)/x)^2 + 2*a*
b*x*cos((c*x + d)/x) + a*b*d*sin((c*x + d)/x) + (2*a*b*x*cos((c*x + d)/x)
- a*b*d*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + (a*b*d*cos((c*x + d)/x) + 2
*a*b*x*sin((c*x + d)/x) + 2*b^2*x)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*
cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b -
 a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c
*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*
d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x +
 d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(a*b*x^2*sin((c*x
+ d)/x) + b^2*x^2)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/
x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c
*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4
*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/
x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2...
```

Giac [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
integrate((b*sin(c + d/x) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 40.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\ \sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
int(1/(a + b*sin(c + d/x))^2,x)
```

output
```
int(1/(a + b*sin(c + d/x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 1167, normalized size of antiderivative = 83.36

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*sin(c+d/x))^2,x)
```

output

```
( - 4*cos((c*x + d)/x)*b**2*x - 2*int(cos((c*x + d)/x)/(sin((c*x + d)/x)**
2*b**2*x + 2*sin((c*x + d)/x)*a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**2*b**
2*d + 4*int(cos((c*x + d)/x)/(sin((c*x + d)/x)**2*b**2*x + 2*sin((c*x + d)
/x)*a*b*x + a**2*x),x)*sin((c*x + d)/x)*b**4*d - 2*int(cos((c*x + d)/x)/(s
in((c*x + d)/x)**2*b**2*x + 2*sin((c*x + d)/x)*a*b*x + a**2*x),x)*a**3*b*d
 + 4*int(cos((c*x + d)/x)/(sin((c*x + d)/x)**2*b**2*x + 2*sin((c*x + d)/x)
*a*b*x + a**2*x),x)*a*b**3*d + 4*int(cos((c*x + d)/x)/(sin((c*x + d)/x)**2
*b**2 + 2*sin((c*x + d)/x)*a*b + a**2),x)*sin((c*x + d)/x)*a*b**3 + 4*int(
cos((c*x + d)/x)/(sin((c*x + d)/x)**2*b**2 + 2*sin((c*x + d)/x)*a*b + a**2
),x)*a**2*b**2 + int(sin((c*x + d)/x)**2/(sin((c*x + d)/x)**2*b**2 + 2*sin
((c*x + d)/x)*a*b + a**2),x)*sin((c*x + d)/x)*a*b**3 + int(sin((c*x + d)/x
)**2/(sin((c*x + d)/x)**2*b**2 + 2*sin((c*x + d)/x)*a*b + a**2),x)*a**2*b*
*2 + 4*int(sin((c*x + d)/x)/(sin((c*x + d)/x)**2*b**2*x + 2*sin((c*x + d)/
x)*a*b*x + a**2*x),x)*sin((c*x + d)/x)*a*b**3*d + 4*int(sin((c*x + d)/x)/(
sin((c*x + d)/x)**2*b**2*x + 2*sin((c*x + d)/x)*a*b*x + a**2*x),x)*a**2*b*
*2*d + 4*int(sin((c*x + d)/x)/(sin((c*x + d)/x)**2*b**2 + 2*sin((c*x + d)/
x)*a*b + a**2),x)*sin((c*x + d)/x)*b**4 + 4*int(sin((c*x + d)/x)/(sin((c*x
 + d)/x)**2*b**2 + 2*sin((c*x + d)/x)*a*b + a**2),x)*a*b**3 + 4*int((cos((
c*x + d)/x)*sin((c*x + d)/x))/(sin((c*x + d)/x)**2*b**2 + 2*sin((c*x + d)/
x)*a*b + a**2),x)*sin((c*x + d)/x)*b**4 + 4*int((cos((c*x + d)/x)*sin((...
```

3.307 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

output
```
Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

$$\Big\downarrow \; 3918$$

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{fx+e}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

output
```
int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output
```
integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 1103, normalized size of antiderivative = 55.15

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input

```
integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output

```
-(2*(a*b*f*x^3 + a*b*e*x^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f
*x^3 + a*b*e*x^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)
^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x
 + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(
a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x)
 + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2
- b^4)*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c
*x + d)/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2
+ 2*a*b*e*x)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*co
s(2*(c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*cos((c*x + d)/x) + (3*b^2*f*x
^2 + 2*b^2*e*x + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2
*a*b*e*x)*sin((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin
((c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)
*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x +
d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((
c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d -
 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x +
 d)/x)), x) + 2*(b^2*f*x^3 + b^2*e*x^2 + (a*b*f*x^3 + a*b*e*x^2)*sin((c*x
+ d)/x))*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(
a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)...
```

Giac [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+f\,x}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
int((e + f*x)/(a + b*sin(c + d/x))^2,x)
```

output
```
int((e + f*x)/(a + b*sin(c + d/x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 3980, normalized size of antiderivative = 199.00

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

output

```
( - 8*cos((c*x + d)/x)*a*b**2*e*x + 2*cos((c*x + d)/x)*a*b**2*f*x**2 - 12*
cos((c*x + d)/x)*b**3*d*f*x - 8*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)
/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x
))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a
*b*x + a**2*x),x)*sin((c*x + d)/x)*a**3*b**2*d*e - 12*int(tan((c*x + d)/(2
*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x
+ 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*
tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**2*b**3*d**2*f
- 8*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c
*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x +
d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*a**4*b*d*e
 - 12*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan(
(c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x
+ d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*a**3*b**
2*d**2*f - 4*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2 + 4
*tan((c*x + d)/(2*x))**3*a*b + 2*tan((c*x + d)/(2*x))**2*a**2 + 4*tan((c*x
 + d)/(2*x))**2*b**2 + 4*tan((c*x + d)/(2*x))*a*b + a**2),x)*sin((c*x + d)
/x)*a**3*b**2*d*f + 24*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**
4*a**2 + 4*tan((c*x + d)/(2*x))**3*a*b + 2*tan((c*x + d)/(2*x))**2*a**2 +
4*tan((c*x + d)/(2*x))**2*b**2 + 4*tan((c*x + d)/(2*x))*a*b + a**2),x)*...
```

3.308 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$
$$\Big\downarrow \; 3918$$
$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(fx+e)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}dx$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

output
```
int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

output
```
integral(-(f^2*x^2 + 2*e*f*x + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c
*x + d)/x) - a^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 24.27 (sec) , antiderivative size = 1281, normalized size of antiderivative = 58.23

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

output
```
-(2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos(2*(c*x + d)/x)*cos((c*
x + d)/x) + 2*(a*b*f^2*x^4 + 2*a*b*e*f*x^3 + a*b*e^2*x^2)*cos((c*x + d)/x)
 + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x
+ d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2
*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^
2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b
 - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*inte
grate(-2*(2*(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*cos((c*x + d)/x)^2
 + 2*(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2)*sin((c*x + d)/x)^2 + (2*(
2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) - (a*b*d*f^2*x
^2 + 2*a*b*d*e*f*x + a*b*d*e^2)*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + 2*(
2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*cos((c*x + d)/x) + (4*b^2*f^2*x
^3 + 6*b^2*e*f*x^2 + 2*b^2*e^2*x + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*
e^2)*cos((c*x + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*e*f*x^2 + a*b*e^2*x)*sin(
(c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f^2*x^2 + 2*a*b*d*e*f*x + a*b*d*
e^2)*sin((c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 -
a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2
*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)
*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 -
b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*c...
```

Giac [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

input
```
integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(e+f\,x)^2}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

input
```
int((e + f*x)^2/(a + b*sin(c + d/x))^2,x)
```

output
```
int((e + f*x)^2/(a + b*sin(c + d/x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 8294, normalized size of antiderivative = 377.00

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

output

```
( - cos((c*x + d)/x)*a**3*b*d*f**2*x**2 + 2*cos((c*x + d)/x)*a**2*b**2*d**
2*f**2*x - 12*cos((c*x + d)/x)*a**2*b**2*e**2*x + 6*cos((c*x + d)/x)*a**2*
b**2*e*f*x**2 + 2*cos((c*x + d)/x)*a**2*b**2*f**2*x**3 - 36*cos((c*x + d)/
x)*a*b**3*d*e*f*x + 3*cos((c*x + d)/x)*a*b**3*d*f**2*x**2 - 18*cos((c*x +
d)/x)*b**4*d**2*f**2*x + 2*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x
))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2
*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x
+ a**2*x),x)*sin((c*x + d)/x)*a**4*b**2*d**3*f**2 - 12*int(tan((c*x + d)/(
2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x
 + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4
*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**4*b**2*d*e**2
 - 36*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan(
(c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x
+ d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*a*b*x + a**2*x),x)*sin((c*x
 + d)/x)*a**3*b**3*d**2*e*f - 18*int(tan((c*x + d)/(2*x))**2/(tan((c*x + d
)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*a*b*x + 2*tan((c*x + d)/(2*
x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b**2*x + 4*tan((c*x + d)/(2*x))*
a*b*x + a**2*x),x)*sin((c*x + d)/x)*a**2*b**4*d**3*f**2 + 2*int(tan((c*x +
 d)/(2*x))**2/(tan((c*x + d)/(2*x))**4*a**2*x + 4*tan((c*x + d)/(2*x))**3*
a*b*x + 2*tan((c*x + d)/(2*x))**2*a**2*x + 4*tan((c*x + d)/(2*x))**2*b*...
```

3.309 $\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\text{Int}\left((e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p,x\right)$$

output
```
Defer(Int)((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3918}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p dx$$

$$\Big\downarrow \text{3918}$$

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p dx$$

input
```
Int[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3918
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Unintegrable[(g + h*x)^m*(a + b*Sin[c
+ d*(e + f*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (fx+e)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p dx$$

input
```
int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

output
```
int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (fx+e)^m\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p\,dx$$

input
```
integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)^m*(b*sin((c*x + d)/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*sin(c+d/x))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (fx+e)^m\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p\,dx$$

input
```
integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p dx = \int (fx+e)^m \left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p dx$$

input
```
integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 40.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p dx = \int (e+f\,x)^m \left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^p dx$$

input
```
int((e + f*x)^m*(a + b*sin(c + d/x))^p,x)
```

output
```
int((e + f*x)^m*(a + b*sin(c + d/x))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 836, normalized size of antiderivative = 38.00

$$\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx = \text{Too large to display}$$

input

```
int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

output

```
((e + f*x)**m*(sin((c*x + d)/x)*b + a)**p*e + (e + f*x)**m*(sin((c*x + d)/
x)*b + a)**p*f*x + int(((e + f*x)**m*(sin((c*x + d)/x)*b + a)**p*cos((c*x
+ d)/x))/(sin((c*x + d)/x)*b*e*m*x**2 + sin((c*x + d)/x)*b*e*x**2 + sin((c
*x + d)/x)*b*f*m*x**3 + sin((c*x + d)/x)*b*f*x**3 + a*e*m*x**2 + a*e*x**2
+ a*f*m*x**3 + a*f*x**3),x)*b*d*e**2*m*p + int(((e + f*x)**m*(sin((c*x + d
)/x)*b + a)**p*cos((c*x + d)/x))/(sin((c*x + d)/x)*b*e*m*x**2 + sin((c*x +
 d)/x)*b*e*x**2 + sin((c*x + d)/x)*b*f*m*x**3 + sin((c*x + d)/x)*b*f*x**3
+ a*e*m*x**2 + a*e*x**2 + a*f*m*x**3 + a*f*x**3),x)*b*d*e**2*p + 2*int(((e
 + f*x)**m*(sin((c*x + d)/x)*b + a)**p*cos((c*x + d)/x))/(sin((c*x + d)/x)
*b*e*m*x + sin((c*x + d)/x)*b*e*x + sin((c*x + d)/x)*b*f*m*x**2 + sin((c*x
 + d)/x)*b*f*x**2 + a*e*m*x + a*e*x + a*f*m*x**2 + a*f*x**2),x)*b*d*e*f*m*
p + 2*int(((e + f*x)**m*(sin((c*x + d)/x)*b + a)**p*cos((c*x + d)/x))/(sin
((c*x + d)/x)*b*e*m*x + sin((c*x + d)/x)*b*e*x + sin((c*x + d)/x)*b*f*m*x*
*2 + sin((c*x + d)/x)*b*f*x**2 + a*e*m*x + a*e*x + a*f*m*x**2 + a*f*x**2),
x)*b*d*e*f*p + int(((e + f*x)**m*(sin((c*x + d)/x)*b + a)**p*cos((c*x + d)
/x))/(sin((c*x + d)/x)*b*e*m + sin((c*x + d)/x)*b*e + sin((c*x + d)/x)*b*f
*m*x + sin((c*x + d)/x)*b*f*x + a*e*m + a*e + a*f*m*x + a*f*x),x)*b*d*f**2
*m*p + int(((e + f*x)**m*(sin((c*x + d)/x)*b + a)**p*cos((c*x + d)/x))/(si
n((c*x + d)/x)*b*e*m + sin((c*x + d)/x)*b*e + sin((c*x + d)/x)*b*f*m*x + s
in((c*x + d)/x)*b*f*x + a*e*m + a*e + a*f*m*x + a*f*x),x)*b*d*f**2*p)/(...
```

3.310 $\int x^m \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(1+m,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\Gamma(1+m,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

output
```
-1/2*exp(I*a)*x^m*csc(b*x+a)*GAMMA(1+m,-I*b*x)*(c*sin(b*x+a)^3)^(1/3)/b/((
-I*b*x)^m)-1/2*x^m*csc(b*x+a)*GAMMA(1+m,I*b*x)*(c*sin(b*x+a)^3)^(1/3)/b/ex
p(I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{e^{-ia}x^m(b^2x^2)^{-m}\csc(a+bx)\left(e^{2ia}(ibx)^m\Gamma(1+m,-ibx)+(-ibx)^m\Gamma(1+m,ibx)\right)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

input
```
Integrate[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
-1/2*(x^m*Csc[a + b*x]*(E^((2*I)*a)*(I*b*x)^m*Gamma[1 + m, (-I)*b*x] + ((-
I)*b*x)^m*Gamma[1 + m, I*b*x])*(c*Sin[a + b*x]^3)^(1/3))/(b*E^(I*a)*(b^2*x
^2)^m)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7271, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt[3]{c \sin^3(a+bx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x^m \sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x^m \sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}i\int e^{-i(a+bx)}x^m dx-\frac{1}{2}i\int e^{i(a+bx)}x^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+1,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+1,ibx)}{2b}\right)$$

input
```
Int[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
Csc[a + b*x]*(-1/2*(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) -
 (x^m*Gamma[1 + m, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m))*(c*Sin[a + b*x]^3)^(1/
3)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^m \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}dx$$

input
```
int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

output
```
int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left(e^{(-m\log(i\,b)-i\,a)}\Gamma(m+1,i\,bx)+e^{(-m\log(-i\,b)+i\,a)}\Gamma(m+1,-i\,bx)\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)}{2\,b\sin\left(bx+a\right)}$$

input
```
integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*g
amma(m + 1, -I*b*x))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b*sin(b
*x + a))
```

Sympy [F]

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \int x^m \sqrt[3]{c\sin^3\left(a+bx\right)}\,dx$$

input
```
integrate(x**m*(c*sin(b*x+a)**3)**(1/3),x)
```

output
```
Integral(x**m*(c*sin(a + b*x)**3)**(1/3), x)
```

Maxima [F]

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Giac [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx)}\,dx = \int \left(c \sin{(bx+a)}^3\right)^{\frac{1}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt[3]{c \sin^3(a+bx)}\,dx = \int x^m \left(c \sin{(a+b\,x)}^3\right)^{1/3} dx$$

input
```
int(x^m*(c*sin(a + b*x)^3)^(1/3),x)
```

output
```
int(x^m*(c*sin(a + b*x)^3)^(1/3), x)
```

Reduce [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx)}\,dx = c^{\frac{1}{3}} \left(\int x^m \sin{(bx+a)}\,dx\right)$$

input
```
int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

output
```
c**(1/3)*int(x**m*sin(a + b*x),x)
```

3.311 $\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\begin{aligned}\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx = &-\frac{6\sqrt[3]{c \sin^3(a+bx)}}{b^4}+\frac{3x^2\sqrt[3]{c \sin^3(a+bx)}}{b^2}\\ &+\frac{6x\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b^3}\\ &-\frac{x^3\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b}\end{aligned}$$

output

```
-6*(c*sin(b*x+a)^3)^(1/3)/b^4+3*x^2*(c*sin(b*x+a)^3)^(1/3)/b^2+6*x*cot(b*x
+a)*(c*sin(b*x+a)^3)^(1/3)/b^3-x^3*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx = -\frac{(6-3b^2x^2+bx(-6+b^2x^2)\cot(a+bx))\sqrt[3]{c \sin^3(a+bx)}}{b^4}$$

input

```
Integrate[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
-(((6 - 3*b^2*x^2 + b*x*(-6 + b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1
/3))/b^4)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {7271, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x^3\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x^3\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\int x^2\cos(a+bx)dx}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\int x^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\left(\frac{2\int -x\sin(a+bx)dx}{b}+\frac{x^2\sin(a+bx)}{b}\right)}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{25}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\left(\frac{x^2\sin(a+bx)}{b}-\frac{2\int x\sin(a+bx)dx}{b}\right)}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\left(\frac{x^2\sin(a+bx)}{b}-\frac{2\int x\sin(a+bx)dx}{b}\right)}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\left(\frac{x^2\sin(a+bx)}{b}-\frac{2\left(\frac{\int\cos(a+bx)dx}{b}-\frac{x\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\left(\frac{x^2\sin(a+bx)}{b}-\frac{2\left(\frac{\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{x\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{3\left(\frac{x^2\sin(a+bx)}{b}-\frac{2\left(\frac{\sin(a+bx)}{b^2}-\frac{x\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{x^3\cos(a+bx)}{b}\right)$$

input

```
Int[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

output

```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(-((x^3*Cos[a + b*x])/b) + (3*((x^2*
Sin[a + b*x])/b - (2*(-((x*Cos[a + b*x])/b) + Sin[a + b*x]/b^2))/b))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{i(b^3x^3+3ix^2b^2-6bx-6i)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b^4\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}(b^3x^3-3ix^2b^2-6bx+}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b^4}$

input
```
int(x^3*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^4*(b^3*x^3+3*I*x^2*b^2-6*b*x-6*I)/(exp(2*I*(b*x+a))-1)*(I*c*exp(-
3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp
(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)/(exp(2*I*(b*x+a))-1)*(b^3*x^3
-3*I*x^2*b^2-6*b*x+6*I)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left(\left(b^3x^3-6\,bx\right)\cos\left(bx+a\right)-3\left(b^2x^2-2\right)\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{b^4\sin\left(bx+a\right)}$$

input
```
integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-((b^3*x^3 - 6*b*x)*cos(b*x + a) - 3*(b^2*x^2 - 2)*sin(b*x + a))*(-(c*cos(
b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b^4*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} \frac{x^4\sqrt[3]{c\sin^3(a)}}{4} \\ 0 \\ -\frac{x^3\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)}+\frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{6x\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b^3\sin(a+bx)}-\frac{6\sqrt[3]{c\sin^3(a+bx}}{b^4} \end{cases}$$

input
```
integrate(x**3*(c*sin(b*x+a)**3)**(1/3),x)
```

output
```
Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
 -b*x + pi)), (-x**3*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*
x)) + 3*x**2*(c*sin(a + b*x)**3)**(1/3)/b**2 + 6*x*(c*sin(a + b*x)**3)**(1
/3)*cos(a + b*x)/(b**3*sin(a + b*x)) - 6*(c*sin(a + b*x)**3)**(1/3)/b**4,
True))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.52

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx$$

$$= \frac{3\left((bx+a)\cos(bx+a)-\sin(bx+a)\right)a^2c^{\frac{1}{3}} - 3\left(\left((bx+a)^2-2\right)\cos(bx+a)-2\left(bx+a\right)\sin(bx+a)\right)a}{2\,b^4}$$

input
```
integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
1/2*(3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2*c^(1/3) - 3*(((b*x + a)
^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c^(1/3) + 4*a^3*c^(1/3)
/(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1) + (((b*x + a)^3 - 6*b*x - 6*a)*
cos(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*c^(1/3))/b^4
```

Giac [F]

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^3\,dx$$

input
```
integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 40.91 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx$$
$$= \frac{2^{1/3}\left(c\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}\left(3\,b^2\,x^2-12\sin\left(a+b\,x\right)^2+6\,b\,x\,\sin\left(2\,a+2\,b\,x\right)+3\,b^2\,x^2\right.}{4\,b^4\sin\left(a+b\,x\right)^2}$$

input
```
int(x^3*(c*sin(a + b*x)^3)^(1/3),x)
```

output
```
(2^(1/3)*(c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3)*(3*b^2*x^2 - 12*sin
(a + b*x)^2 + 6*b*x*sin(2*a + 2*b*x) + 3*b^2*x^2*(2*sin(a + b*x)^2 - 1) -
b^3*x^3*sin(2*a + 2*b*x)))/(4*b^4*sin(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx$$
$$= \frac{c^{\frac{1}{3}}\left(-\cos\left(bx+a\right)b^3x^3+6\cos\left(bx+a\right)bx+3\sin\left(bx+a\right)b^2x^2-6\sin\left(bx+a\right)\right)}{b^4}$$

input
```
int(x^3*(c*sin(b*x+a)^3)^(1/3),x)
```

output
```
(c**(1/3)*( - cos(a + b*x)*b**3*x**3 + 6*cos(a + b*x)*b*x + 3*sin(a + b*x)
*b**2*x**2 - 6*sin(a + b*x)))/b**4
```

3.312 $\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{2x\sqrt[3]{c \sin^3(a+bx)}}{b^2} + \frac{2\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b^3} - \frac{x^2\cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b}$$

output
```
2*x*(c*sin(b*x+a)^3)^(1/3)/b^2+2*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b^3-x^2
*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{(2bx + (2-b^2x^2)\cot(a+bx))\sqrt[3]{c \sin^3(a+bx)}}{b^3}$$

input
```
Integrate[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
((2*b*x + (2 - b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {7271, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x^2\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x^2\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{2\int x\cos(a+bx)dx}{b}-\frac{x^2\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{2\int x\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{x^2\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{2\left(\frac{\int -\sin(a+bx)dx}{b}+\frac{x\sin(a+bx)}{b}\right)}{b}-\frac{x^2\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{2\left(\frac{x\sin(a+bx)}{b}-\frac{\int \sin(a+bx)dx}{b}\right)}{b}-\frac{x^2\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{2\left(\frac{x\sin(a+bx)}{b}-\frac{\int \sin(a+bx)dx}{b}\right)}{b}-\frac{x^2\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3118}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{2\left(\frac{\cos(a+bx)}{b^2}+\frac{x\sin(a+bx)}{b}\right)}{b}-\frac{x^2\cos(a+bx)}{b}\right)$$

input
```
Int[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(-((x^2*Cos[a + b*x])/b) + (2*(Cos[a
 + b*x]/b^2 + (x*Sin[a + b*x])/b))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

method	result	size
risch	$-\frac{i(x^2b^2+2ibx-2)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b^3\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\right)^{\frac{1}{3}}(x^2b^2-2ibx-2)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b^3}$	133

input

```
int(x^2*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output

```
-1/2*I/b^3*(x^2*b^2+2*I*b*x-2)/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3*I*(b*x+a))
*(exp(2*I*(b*x+a))-1)^3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-3*I*(b*x+a
))*(exp(2*I*(b*x+a))-1)^3)^(1/3)/(exp(2*I*(b*x+a))-1)*(x^2*b^2-2*I*b*x-2)/
b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2\sqrt[3]{c\sin^3(a+bx)}\,dx$$
$$=\frac{(2\,bx\sin(bx+a)-(b^2x^2-2)\cos(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{b^3\sin(bx+a)}$$

input

```
integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

output

```
(2*b*x*sin(b*x + a) - (b^2*x^2 - 2)*cos(b*x + a))*(-(c*cos(b*x + a)^2 - c)
*sin(b*x + a))^(1/3)/(b^3*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int x^2\sqrt[3]{c\sin^3(a+bx)}\,dx$$
$$= \begin{cases} \frac{x^3\sqrt[3]{c\sin^3{(a)}}}{3} & \text{for } b=0 \\ 0 & \text{for } a=-bx \vee a= \\ -\frac{x^2\sqrt[3]{c\sin^3{(a+bx)}}\cos{(a+bx)}}{b\sin{(a+bx)}} + \frac{2x\sqrt[3]{c\sin^3{(a+bx)}}}{b^2} + \frac{2\sqrt[3]{c\sin^3{(a+bx)}}\cos{(a+bx)}}{b^3\sin{(a+bx)}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(c*sin(b*x+a)**3)**(1/3),x)
```

output
```
Piecewise((x**3*(c*sin(a)**3)**(1/3)/3, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
 -b*x + pi)), (-x**2*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*
x)) + 2*x*(c*sin(a + b*x)**3)**(1/3)/b**2 + 2*(c*sin(a + b*x)**3)**(1/3)*c
os(a + b*x)/(b**3*sin(a + b*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int x^2\sqrt[3]{c\sin^3(a+bx)}\,dx =$$
$$-\frac{2\left((bx+a)\cos{(bx+a)}-\sin{(bx+a)}\right)ac^{\frac{1}{3}}-\left(\left((bx+a)^2-2\right)\cos{(bx+a)}-2\,(bx+a)\sin{(bx+a)}\right)c}{2\,b^3}$$

input
```
integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
-1/2*(2*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c^(1/3) - (((b*x + a)^2
- 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^(1/3) + 4*a^2*c^(1/3)/(sin
(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^3
```

Giac [F]

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx = \int \left(c \sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^2\,dx$$

input
```
integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 40.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx =$$
$$-\frac{\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}\,\Big(\sin\left(2\,a+2\,b\,x\right)+b\,x-\frac{b^2\,x^2\,\sin(2\,a+2\,b\,x)}{2}-b\,x\,\cos\left(2\,a+2\,b\right.}{b^3\,\left(\cos\left(2\,a+2\,b\,x\right)-1\right)}$$

input
```
int(x^2*(c*sin(a + b*x)^3)^(1/3),x)
```

output
```
-((2*c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3)*(sin(2*a + 2*b*x) + b*x
- (b^2*x^2*sin(2*a + 2*b*x))/2 - b*x*cos(2*a + 2*b*x)))/(b^3*(cos(2*a + 2*
b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{c^{\frac{1}{3}}\left(-\cos\left(bx+a\right)b^2x^2+2\cos\left(bx+a\right)+2\sin\left(bx+a\right)bx\right)}{b^3}$$

input
```
int(x^2*(c*sin(b*x+a)^3)^(1/3),x)
```

output
```
(c**(1/3)*( - cos(a + b*x)*b**2*x**2 + 2*cos(a + b*x) + 2*sin(a + b*x)*b*x
))/b**3
```

3.313 $\int x \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{\sqrt[3]{c \sin^3(a+bx)}}{b^2} - \frac{x \cot(a+bx) \sqrt[3]{c \sin^3(a+bx)}}{b}$$

output
```
(c*sin(b*x+a)^3)^(1/3)/b^2-x*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int x \sqrt[3]{c \sin^3(a+bx)}\,dx = \frac{(1 - bx \cot(a+bx)) \sqrt[3]{c \sin^3(a+bx)}}{b^2}$$

input
```
Integrate[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
((1 - b*x*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {7271, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[3]{c\sin^3(a+bx)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int x\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{\int\cos(a+bx)dx}{b}-\frac{x\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{x\cos(a+bx)}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{\sin(a+bx)}{b^2}-\frac{x\cos(a+bx)}{b}\right)$$

input
```
Int[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(-((x*Cos[a + b*x])/b) + Sin[a + b*x
]/b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60

method	result	size
risch	$-\frac{i(bx+i)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b^{2}\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}(bx-i)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b^{2}}$	117

input
```
int(x*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^2*(b*x+I)/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b
*x+a))-1)^3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*
(b*x+a))-1)^3)^(1/3)/(exp(2*I*(b*x+a))-1)*(b*x-I)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{(bx\cos(bx+a)-\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{b^2\sin(bx+a)}$$

input
```
integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-(b*x*cos(b*x + a) - sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^
(1/3)/(b^2*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} \frac{x^2\sqrt[3]{c\sin^3(a)}}{2} & \text{for } b=0 \\ 0 & \text{for } a=-bx \vee a=-bx+\pi \\ -\frac{x\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)}+\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(c*sin(b*x+a)**3)**(1/3),x)
```

output
```
Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
 -b*x + pi)), (-x*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x))
 + (c*sin(a + b*x)**3)**(1/3)/b**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx=\frac{((bx+a)\cos(bx+a)-\sin(bx+a))c^{\frac{1}{3}}+\frac{4\,ac^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1}}{2\,b^2}$$

input
```
integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
1/2*(((b*x + a)*cos(b*x + a) - sin(b*x + a))*c^(1/3) + 4*a*c^(1/3)/(sin(b*
x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^2
```

Giac [F]

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx=\int\left(c\sin(bx+a)^3\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)*x, x)
```

Mupad [B] (verification not implemented)

Time = 40.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx=\frac{\left(\frac{\sin(a+b\,x)^2}{2}-\frac{b\,x\,\sin(2\,a+2\,b\,x)}{4}\right)\,\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}}{b^2\,\sin\left(a+b\,x\right)^2}$$

input
```
int(x*(c*sin(a + b*x)^3)^(1/3),x)
```

output
```
((sin(a + b*x)^2/2 - (b*x*sin(2*a + 2*b*x))/4)*(2*c*(3*sin(a + b*x) - sin(
3*a + 3*b*x)))^(1/3))/(b^2*sin(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx=\frac{c^{\frac{1}{3}}\left(-\cos\left(bx+a\right)bx+\sin\left(bx+a\right)\right)}{b^2}$$

input
```
int(x*(c*sin(b*x+a)^3)^(1/3),x)
```

output
```
(c**(1/3)*( - cos(a + b*x)*b*x + sin(a + b*x)))/b**2
```

3.314 $\int \sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx=-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

output
```
-cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx=-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^(1/3),x]
```

output
```
-((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt[3]{c\sin(a+bx)^3}dx$$

$$\Big\downarrow \; 3686$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int \sin(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int \sin(a+bx)dx$$

$$\Big\downarrow \; 3118$$

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

input

```
Int[(c*Sin[a + b*x]^3)^(1/3),x]
```

output

```
-((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.20

method	result	size
risch	$-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}\mathrm{e}^{2i(bx+a)}}{2b\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)b}$	105

input
```
int((c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b/(exp(2*I*(b*x+a))-1)*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^
3)^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1
)^3)^(1/3)/(exp(2*I*(b*x+a))-1)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}\cos\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)*cos(b*x + a)/(b*sin(b*x + a)
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = \begin{cases} x\sqrt[3]{c\sin^3\left(a\right)} & \text{for}\ b=0 \\ 0 & \text{for}\ a=-bx \vee a=-bx+\pi \\ -\frac{\sqrt[3]{c\sin^3\left(a+bx\right)}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((c*sin(b*x+a)**3)**(1/3),x)
```

output
```
Piecewise((x*(c*sin(a)**3)**(1/3), Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a, -b*x
 + pi)), (-(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{2\,c^{\frac{1}{3}}}{b\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
-2*c^(1/3)/(b*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))
```

Giac [F]

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 41.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}}{4\,b\sin\left(a+b\,x\right)^2}$$

input
```
int((c*sin(a + b*x)^3)^(1/3),x)
```

output
```
-(sin(2*a + 2*b*x)*(2*c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3))/(4*b*s
in(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt[3]{c\sin^3(a+bx)}\,dx = -\frac{c^{\frac{1}{3}}\cos(bx+a)}{b}$$

input

```
int((c*sin(b*x+a)^3)^(1/3),x)
```

output

```
( - c**(1/3)*cos(a + b*x))/b
```

3.315 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \text{CosIntegral}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,\text{Si}(bx)$$

output

```
Ci(b*x)*csc(b*x+a)*sin(a)*(c*sin(b*x+a)^3)^(1/3)+cos(a)*csc(b*x+a)*(c*sin(
b*x+a)^3)^(1/3)*Si(b*x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}(\text{CosIntegral}(bx)\sin(a)+\cos(a)\text{Si}(bx))$$

input

```
Integrate[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

output
```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(CosIntegral[b*x]*Sin[a] + Cos[a]*Si
nIntegral[b*x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7271, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int \frac{\sin(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int \frac{\sin(a+bx)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\sin(a)\int \frac{\cos(bx)}{x}dx+\cos(a)\int \frac{\sin(bx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\sin(a)\int \frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx+\cos(a)\int \frac{\sin(bx)}{x}dx\right)$$

$$\Big\downarrow \text{3780}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\sin(a)\int \frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx+\cos(a)\text{Si}(bx)\right)$$

$$\Big\downarrow \text{3783}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}(\sin(a)\operatorname{CosIntegral}(bx)+\cos(a)\text{Si}(bx))$$

input
```
Int[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

output
```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(CosIntegral[b*x]*Sin[a] + Cos[a]*Si
nIntegral[b*x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.93

method	result	si
risch	$-\frac{\left(i c\,{\mathrm e}^{-3 i \left(b x +a \right)} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{3}\right)^{\frac{1}{3}} \left(i {\mathrm e}^{i b x} \pi \,\operatorname{csgn}\left(b x \right)-2 i {\mathrm e}^{i b x} \operatorname{Si}\left(b x \right)+\operatorname{expIntegral}_{1}\left(-i b x \right) {\mathrm e}^{i \left(b x +2 a \right)}-{\mathrm e}^{i b x} \operatorname{expIntegral}_{1}\left(-i b x \right)\right)}{2 \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)}$	1

input
```
int((c*sin(b*x+a)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*(I*exp(I*b*x)*Pi
*csgn(b*x)-2*I*exp(I*b*x)*Si(b*x)+Ei(1,-I*b*x)*exp(I*(b*x+2*a))-exp(I*b*x)
*Ei(1,-I*b*x))/(exp(2*I*(b*x+a))-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x}\,dx = \frac{\left(-i\,\mathrm{Ei}\left(i\,bx\right)e^{\left(i\,a\right)}+i\,\mathrm{Ei}\left(-i\,bx\right)e^{\left(-i\,a\right)}\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{2\,\sin\left(bx+a\right)}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="fricas")
```

output
```
1/2*(-I*Ei(I*b*x)*e^(I*a) + I*Ei(-I*b*x)*e^(-I*a))*(-(c*cos(b*x + a)^2 - c
)*sin(b*x + a))^(1/3)/sin(b*x + a)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \int \frac{\sqrt[3]{c\sin^3{(a+bx)}}}{x}\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**(1/3)/x,x)
```

output
```
Integral((c*sin(a + b*x)**3)**(1/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx$$
$$= \frac{1}{4}\left((i\,E_1(i\,bx) - i\,E_1(-i\,bx))\cos{(a)} + (E_1(i\,bx) + E_1(-i\,bx))\sin{(a)}\right)c^{\frac{1}{3}}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="maxima")
```

output
```
1/4*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x))*cos(a) + (
exp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x))*sin(a))*c^(1/3)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \int \frac{\left(c\sin{(bx+a)}^3\right)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx=\int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{1/3}}{x}\,dx$$

input
```
int((c*sin(a + b*x)^3)^(1/3)/x,x)
```

output
```
int((c*sin(a + b*x)^3)^(1/3)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx=c^{\frac{1}{3}}\left(\int \frac{\sin\left(bx+a\right)}{x}dx\right)$$

input
```
int((c*sin(b*x+a)^3)^(1/3)/x,x)
```

output
```
c**(1/3)*int(sin(a + b*x)/x,x)
```

3.316 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = &-\frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\\ &+ b\cos(a)\,\mathrm{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\\ &- b\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\,\mathrm{Si}(bx)\end{aligned}$$

output
```
-(c*sin(b*x+a)^3)^(1/3)/x+b*cos(a)*Ci(b*x)*csc(b*x+a)*(c*sin(b*x+a)^3)^(1/
3)-b*csc(b*x+a)*sin(a)*(c*sin(b*x+a)^3)^(1/3)*Si(b*x)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx\\ &= \frac{\sqrt[3]{c\sin^3(a+bx)}(-1+bx\cos(a)\,\mathrm{CosIntegral}(bx)\csc(a+bx)-bx\csc(a+bx)\sin(a)\mathrm{Si}(bx))}{x}\end{aligned}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

output
```
((c*Sin[a + b*x]^3)^(1/3)*(-1 + b*x*Cos[a]*CosIntegral[b*x]*Csc[a + b*x] -
 b*x*Csc[a + b*x]*Sin[a]*SinIntegral[b*x]))/x
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {7271, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int \frac{\sin(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\int \frac{\sin(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b\int \frac{\cos(a+bx)}{x}dx-\frac{\sin(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{x}dx-\frac{\sin(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b\left(\cos(a)\int \frac{\cos(bx)}{x}dx-\sin(a)\int \frac{\sin(bx)}{x}dx\right)-\frac{\sin(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b\left(\cos(a)\int\frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx-\sin(a)\int\frac{\sin(bx)}{x}dx\right)-\frac{\sin(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3780}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b\left(\cos(a)\int\frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx-\sin(a)\text{Si}(bx)\right)-\frac{\sin(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3783}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b(\cos(a)\,\text{CosIntegral}(bx)-\sin(a)\text{Si}(bx))-\frac{\sin(a+bx)}{x}\right)$$

input

```
Int[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

output

```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(-(Sin[a + b*x]/x) + b*(Cos[a]*CosIntegral[b*x] - Sin[a]*SinIntegral[b*x]))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 3778

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

rule 3780

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}\left(-\operatorname{expIntegral}_{1}(-ibx)\mathrm{e}^{i(bx+2a)}bx-\mathrm{e}^{ibx}\operatorname{expIntegral}_{1}(ibx)bx+i\mathrm{e}^{2i(bx+a)}-i\right)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)x}$	102

input
```
int((c*sin(b*x+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*(-Ei(1,-I*b*x)*
exp(I*(b*x+2*a))*b*x-exp(I*b*x)*Ei(1,I*b*x)*b*x+I*exp(2*I*(b*x+a))-I)/(exp
(2*I*(b*x+a))-1)/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$$
$$=\frac{\left(bx\mathrm{Ei}(i\,bx)\,e^{(i\,a)}+bx\mathrm{Ei}(-i\,bx)\,e^{(-i\,a)}-2\,\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{2\,x\sin\left(bx+a\right)}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*x*Ei(I*b*x)*e^(I*a) + b*x*Ei(-I*b*x)*e^(-I*a) - 2*sin(b*x + a))*(-(
c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(x*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \int \frac{\sqrt[3]{c\sin^3{(a+bx)}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**(1/3)/x**2,x)
```

output
```
Integral((c*sin(a + b*x)**3)**(1/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$$
$$= \frac{\left(\left(\left(\sqrt{3}-i\right)E_2(i\,bx)+\left(\sqrt{3}+i\right)E_2(-i\,bx)\right)\cos{(a)}^3+\left(\left(\sqrt{3}-i\right)E_2(i\,bx)+\left(\sqrt{3}+i\right)E_2(-i\,bx)\right)\cos{(a)}\,\mathrm{si}}{}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="maxima")
```

output
```
1/8*(((sqrt(3) - I)*exp_integral_e(2, I*b*x) + (sqrt(3) + I)*exp_integral_
e(2, -I*b*x))*cos(a)^3 + ((sqrt(3) - I)*exp_integral_e(2, I*b*x) + (sqrt(3
) + I)*exp_integral_e(2, -I*b*x))*cos(a)*sin(a)^2 + ((-I*sqrt(3) - 1)*exp_
integral_e(2, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*sin(a)^3
 - ((sqrt(3) + I)*exp_integral_e(2, I*b*x) + (sqrt(3) - I)*exp_integral_e(
2, -I*b*x))*cos(a) + (((-I*sqrt(3) - 1)*exp_integral_e(2, I*b*x) + (I*sqrt
(3) - 1)*exp_integral_e(2, -I*b*x))*cos(a)^2 + (I*sqrt(3) - 1)*exp_integra
l_e(2, I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*sin(a))*b*c^(1
/3)/(a*cos(a)^2 + a*sin(a)^2 - (b*x + a)*(cos(a)^2 + sin(a)^2))
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{1/3}}{x^2}\,dx$$

input
```
int((c*sin(a + b*x)^3)^(1/3)/x^2,x)
```

output
```
int((c*sin(a + b*x)^3)^(1/3)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = c^{\frac{1}{3}}\left(\int \frac{\sin(bx+a)}{x^2}dx\right)$$

input

```
int((c*sin(b*x+a)^3)^(1/3)/x^2,x)
```

output

```
c**(1/3)*int(sin(a + b*x)/x**2,x)
```

3.317 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 116

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = &-\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2}-\frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x}\\&-\frac{1}{2}b^2\,\text{CosIntegral}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\\&-\frac{1}{2}b^2\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\text{Si}(bx)\end{aligned}$$

output
```
-1/2*(c*sin(b*x+a)^3)^(1/3)/x^2-1/2*b*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/x-
1/2*b^2*Ci(b*x)*csc(b*x+a)*sin(a)*(c*sin(b*x+a)^3)^(1/3)-1/2*b^2*cos(a)*cs
c(b*x+a)*(c*sin(b*x+a)^3)^(1/3)*Si(b*x)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = -\frac{\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}(bx\cos(a+bx)+b^2x^2\,\text{CosIntegral}(bx)\sin(a)+\sin(a+bx)+b^2x^2\cos(a)S}{2x^2}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

output
```
-1/2*(Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(b*x*Cos[a + b*x] + b^2*x^2*Co
sIntegral[b*x]*Sin[a] + Sin[a + b*x] + b^2*x^2*Cos[a]*SinIntegral[b*x]))/x
^2
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {7271, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x^3} \, dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} \int \frac{\sin(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} \int \frac{\sin(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} \left(\frac{1}{2}b\int \frac{\cos(a+bx)}{x^2}dx - \frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} \left(\frac{1}{2}b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{x^2}dx - \frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} \left(\frac{1}{2}b\left(b\int -\frac{\sin(a+bx)}{x}dx - \frac{\cos(a+bx)}{x}\right) - \frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}b\left(-b\int\frac{\sin(a+bx)}{x}dx-\frac{\cos(a+bx)}{x}\right)-\frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}b\left(-b\int\frac{\sin(a+bx)}{x}dx-\frac{\cos(a+bx)}{x}\right)-\frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\cos(bx)}{x}dx+\cos(a)\int\frac{\sin(bx)}{x}dx\right)-\frac{\cos(a+bx)}{x}\right)-\frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx+\cos(a)\int\frac{\sin(bx)}{x}dx\right)-\frac{\cos(a+bx)}{x}\right)-\frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3780}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}b\left(-b\left(\sin(a)\int\frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx+\cos(a)\text{Si}(bx)\right)-\frac{\cos(a+bx)}{x}\right)-\frac{\sin(a+bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3783}$$

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(\frac{1}{2}b\left(-b(\sin(a)\,\text{CosIntegral}(bx)+\cos(a)\text{Si}(bx))-\frac{\cos(a+bx)}{x}\right)-\frac{\sin(a+bx)}{2x^2}\right)$$

input
```
Int[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

output
```
Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(-1/2*Sin[a + b*x]/x^2 + (b*(-(Cos[a
 + b*x]/x) - b*(CosIntegral[b*x]*Sin[a] + Cos[a]*SinIntegral[b*x])))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{1}{3}}\left(\mathrm{e}^{ibx}\,\mathrm{expIntegral}_{1}(ibx)x^{2}b^{2}-\mathrm{expIntegral}_{1}(-ibx)\mathrm{e}^{i(bx+2a)}x^{2}b^{2}+i\mathrm{e}^{2i(bx+a)}xb+ibx+\mathrm{e}^{2i(bx+a)}\right.}{4\left(\mathrm{e}^{2i(bx+a)}-1\right)x^{2}}$

input
```
int((c*sin(b*x+a)^3)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(1/3)*(exp(I*b*x)*Ei(1
,I*b*x)*x^2*b^2-Ei(1,-I*b*x)*exp(I*(b*x+2*a))*x^2*b^2+I*exp(2*I*(b*x+a))*x
*b+I*b*x+exp(2*I*(b*x+a))-1)/(exp(2*I*(b*x+a))-1)/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$$

$$= \frac{\left(i\,b^2x^2\mathrm{Ei}(i\,bx)\,e^{(i\,a)}-i\,b^2x^2\mathrm{Ei}(-i\,bx)\,e^{(-i\,a)}-2\,bx\cos\left(bx+a\right)-2\,\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\right.}{4\,x^2\sin\left(bx+a\right)}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

output
```
1/4*(I*b^2*x^2*Ei(I*b*x)*e^(I*a) - I*b^2*x^2*Ei(-I*b*x)*e^(-I*a) - 2*b*x*c
os(b*x + a) - 2*sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)
/(x^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = \int \frac{\sqrt[3]{c\sin^3{(a+bx)}}}{x^3}\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**(1/3)/x**3,x)
```

output
```
Integral((c*sin(a + b*x)**3)**(1/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx =$$
$$-\frac{(((\sqrt{3}-i)E_3(i\,bx)+(\sqrt{3}+i)E_3(-i\,bx))\cos{(a)}^3+((\sqrt{3}-i)E_3(i\,bx)+(\sqrt{3}+i)E_3(-i\,bx))\cos{(a}}{}$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

output
```
-1/8*(((sqrt(3) - I)*exp_integral_e(3, I*b*x) + (sqrt(3) + I)*exp_integral
_e(3, -I*b*x))*cos(a)^3 + ((sqrt(3) - I)*exp_integral_e(3, I*b*x) + (sqrt(
3) + I)*exp_integral_e(3, -I*b*x))*cos(a)*sin(a)^2 + ((-I*sqrt(3) - 1)*exp
_integral_e(3, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*sin(a)^
3 - ((sqrt(3) + I)*exp_integral_e(3, I*b*x) + (sqrt(3) - I)*exp_integral_e
(3, -I*b*x))*cos(a) + (((-I*sqrt(3) - 1)*exp_integral_e(3, I*b*x) + (I*sqr
t(3) - 1)*exp_integral_e(3, -I*b*x))*cos(a)^2 + (I*sqrt(3) - 1)*exp_integr
al_e(3, I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*sin(a))*b^2*c
^(1/3)/(a^2*cos(a)^2 + a^2*sin(a)^2 + (b*x + a)^2*(cos(a)^2 + sin(a)^2) -
2*(a*cos(a)^2 + a*sin(a)^2)*(b*x + a))
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{1/3}}{x^3}\,dx$$

input
```
int((c*sin(a + b*x)^3)^(1/3)/x^3,x)
```

output
```
int((c*sin(a + b*x)^3)^(1/3)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = -\frac{c^{\frac{1}{3}}\left(\cos\left(bx+a\right)bx+\left(\int\frac{\sin(bx+a)}{x}dx\right)b^2x^2+\sin\left(bx+a\right)\right)}{2x^2}$$

input
```
int((c*sin(b*x+a)^3)^(1/3)/x^3,x)
```

output
```
( - c**(1/3)*(cos(a + b*x)*b*x + int(sin(a + b*x)/x,x)*b**2*x**2 + sin(a +
 b*x)))/(2*x**2)
```

3.318 $\int x^m \sqrt[3]{c \sin^3\left(a+bx^2\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x^m \sqrt[3]{c \sin^3\left(a+bx^2\right)}\,dx = \frac{1}{4} i e^{ia} x^{1+m} \left(-ibx^2\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right) \Gamma\left(\frac{1+m}{2}, -ibx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} - \frac{1}{4} i e^{-ia} x^{1+m} \left(ibx^2\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right) \Gamma\left(\frac{1+m}{2}, ibx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)}$$

output

```
1/4*I*exp(I*a)*x^(1+m)*(-I*b*x^2)^(-1/2-1/2*m)*csc(b*x^2+a)*GAMMA(1/2+1/2*
m,-I*b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)-1/4*I*x^(1+m)*(I*b*x^2)^(-1/2-1/2*m)*
csc(b*x^2+a)*GAMMA(1/2+1/2*m,I*b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \frac{1}{4} i x^{1+m} \left(b^2x^4\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right)\left(-\left(-ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2}, ibx^2\right)(\cos(a)-i\sin(a)) + \left(ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-ibx^2\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

input
```
Integrate[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
(I/4)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]*(-(((-I)*b*x^2)^((1
+ m)/2)*Gamma[(1 + m)/2, I*b*x^2]*(Cos[a] - I*Sin[a])) + (I*b*x^2)^((1 + m
)/2)*Gamma[(1 + m)/2, (-I)*b*x^2]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^2]^3
)^(1/3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3870, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int x^m\sin\left(bx^2+a\right)dx$$

$$\Big\downarrow \text{3870}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{1}{2}i\int e^{-ibx^2-ia}x^m dx-\frac{1}{2}i\int e^{ibx^2+ia}x^m dx\right)$$

$$\Big\downarrow \ 2648$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{1}{4}ie^{ia}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\Gamma\left(\frac{m+1}{2},-ibx^2\right)-\frac{1}{4}ie^{-ia}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m-1)}\Gamma\Bigg(\right.$$

input

```
Int[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output

```
Csc[a + b*x^2]*((I/4)*E^(I*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Gamma[(1
 + m)/2, (-I)*b*x^2] - ((I/4)*x^(1 + m)*(I*b*x^2)^((-1 - m)/2)*Gamma[(1 +
m)/2, I*b*x^2])/E^(I*a))*(c*Sin[a + b*x^2]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3870

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^{m}\left(c\sin\left(b\,x^{2}+a\right)^{3}\right)^{\frac{1}{3}}dx$$

input
```
int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

output
```
int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int x^m\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx =$$

$$-\frac{\left(e^{\left(-\frac{1}{2}\,(m-1)\log(i\,b)-i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},i\,bx^2\right)+e^{\left(-\frac{1}{2}\,(m-1)\log(-i\,b)+i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-i\,bx^2\right)\right)\Big(-\Big(c\cos\left(bx^2+a\right)}{4\,b\sin\left(bx^2+a\right)}$$

input
```
integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(e^(-1/2*(m - 1)*log(I*b) - I*a)*gamma(1/2*m + 1/2, I*b*x^2) + e^(-1/
2*(m - 1)*log(-I*b) + I*a)*gamma(1/2*m + 1/2, -I*b*x^2))*(-(c*cos(b*x^2 +
a)^2 - c)*sin(b*x^2 + a))^(1/3)/(b*sin(b*x^2 + a))
```

Sympy [F]

$$\int x^m\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\int x^m\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

input
```
integrate(x**m*(c*sin(b*x**2+a)**3)**(1/3),x)
```

output
```
Integral(x**m*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Maxima [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \int \left(c \sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Giac [F]

$$\int x^m \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \int \left(c \sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \int x^m \left(c \sin\left(b\,x^2+a\right)^3\right)^{1/3}\,dx$$

input
```
int(x^m*(c*sin(a + b*x^2)^3)^(1/3),x)
```

output
```
int(x^m*(c*sin(a + b*x^2)^3)^(1/3), x)
```

Reduce [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=c^{\frac{1}{3}}\left(\int x^m \sin\left(b\,x^2+a\right)dx\right)$$

input
```
int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

output
```
c**(1/3)*int(x**m*sin(a + b*x**2),x)
```

3.319 $\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{2b^2} - \frac{x^2 \cot(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)}}{2b}$$

output
```
1/2*(c*sin(b*x^2+a)^3)^(1/3)/b^2-1/2*x^2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(
1/3)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = -\frac{(-1+bx^2 \cot(a+bx^2)) \sqrt[3]{c \sin^3(a+bx^2)}}{2b^2}$$

input
```
Integrate[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
-1/2*((-1 + b*x^2*Cot[a + b*x^2])*(c*Sin[a + b*x^2]^3)^(1/3))/b^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7271, 3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{c \sin^3\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} \int x^3 \sin\left(bx^2+a\right)dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2} \csc\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} \int x^2 \sin\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \csc\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} \int x^2 \sin\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2} \csc\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} \left(\frac{\int \cos\left(bx^2+a\right)dx^2}{b}-\frac{x^2 \cos\left(a+bx^2\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \csc\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} \left(\frac{\int \sin\left(bx^2+a+\frac{\pi}{2}\right)dx^2}{b}-\frac{x^2 \cos\left(a+bx^2\right)}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2} \csc\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} \left(\frac{\sin\left(a+bx^2\right)}{b^2}-\frac{x^2 \cos\left(a+bx^2\right)}{b}\right)$$

input

```
Int[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output

```
(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(-((x^2*Cos[a + b*x^2])/b) + Si
n[a + b*x^2]/b^2))/2
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

method	result	size
risch	$-\frac{i(b x^2+i)\left(i c\,\mathrm{e}^{-3 i \left(b x^2+a\right)}\left(\mathrm{e}^{2 i \left(b x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{2 i \left(b x^2+a\right)}}{4 b^2\left(\mathrm{e}^{2 i \left(b x^2+a\right)}-1\right)}-\frac{i\left(i c\,\mathrm{e}^{-3 i \left(b x^2+a\right)}\left(\mathrm{e}^{2 i \left(b x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}(b x^2-i)}{4\left(\mathrm{e}^{2 i \left(b x^2+a\right)}-1\right) b^2}$	135

input
```
int(x^3*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/b^2*(b*x^2+I)/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(
2*I*(b*x^2+a))-1)^3)^(1/3)*exp(2*I*(b*x^2+a))-1/4*I*(I*c*exp(-3*I*(b*x^2+a
))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))-1)*(b*x^2-I)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int x^3 \sqrt[3]{c \sin^3\left(a+b x^2\right)}\, dx$$
$$= -\frac{\left(b x^2 \cos\left(b x^2+a\right)-\sin\left(b x^2+a\right)\right)\left(-\left(c \cos\left(b x^2+a\right)^2-c\right)\sin\left(b x^2+a\right)\right)^{\frac{1}{3}}}{2\, b^2 \sin\left(b x^2+a\right)}$$

input
```
integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*si
n(b*x^2 + a))^(1/3)/(b^2*sin(b*x^2 + a))
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \begin{cases} \frac{x^4 \sqrt[3]{c \sin^3(a)}}{4} & \text{for } b=0 \\ 0 & \text{for } a=-bx^2 \vee a=-bx^2+\pi \\ -\frac{x^2 \sqrt[3]{c \sin^3(a+bx^2)} \cos(a+bx^2)}{2b \sin(a+bx^2)} + \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{2b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(c*sin(b*x**2+a)**3)**(1/3),x)
```

output
```
Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x**2) | Eq
(a, -b*x**2 + pi)), (-x**2*(c*sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(
2*b*sin(a + b*x**2)) + (c*sin(a + b*x**2)**3)**(1/3)/(2*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx = \frac{(bx^2 \cos(bx^2+a) - \sin(bx^2+a))c^{\frac{1}{3}}}{4\,b^2}$$

input
```
integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
1/4*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*c^(1/3)/b^2
```

Giac [F]

$$\int x^3\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 41.58 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int x^3\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$
$$=\frac{\left(\frac{\sin\left(b\,x^2+a\right)^2}{4}-\frac{b\,x^2\,\sin\left(2\,b\,x^2+2\,a\right)}{8}\right)\,\left(-2\,c\,\left(\sin\left(3\,b\,x^2+3\,a\right)-3\,\sin\left(b\,x^2+a\right)\right)\right)^{1/3}}{b^2\sin\left(b\,x^2+a\right)^2}$$

input
```
int(x^3*(c*sin(a + b*x^2)^3)^(1/3),x)
```

output
```
((sin(a + b*x^2)^2/4 - (b*x^2*sin(2*a + 2*b*x^2))/8)*(-2*c*(sin(3*a + 3*b*
x^2) - 3*sin(a + b*x^2)))^(1/3))/(b^2*sin(a + b*x^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int x^3\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\frac{c^{\frac{1}{3}}\left(-\cos\left(b\,x^2+a\right)b\,x^2+\sin\left(b\,x^2+a\right)\right)}{2b^2}$$

input
```
int(x^3*(c*sin(b*x^2+a)^3)^(1/3),x)
```

output
```
(c**(1/3)*( - cos(a + b*x**2)*b*x**2 + sin(a + b*x**2)))/(2*b**2)
```

3.320 $\int x^2 \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\begin{aligned}
\int x^2 \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx
&= -\frac{x \cot\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)}}{2b} \\
&\quad + \frac{\sqrt{\frac{\pi}{2}} \cos(a) \csc\left(a+bx^2\right) \operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)}}{2b^{3/2}} \\
&\quad - \frac{\sqrt{\frac{\pi}{2}} \csc\left(a+bx^2\right) \operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) \sin(a) \sqrt[3]{c \sin^3\left(a+bx^2\right)}}{2b^{3/2}}
\end{aligned}$$

output

```
-1/2*x*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)/b+1/4*2^(1/2)*Pi^(1/2)*cos(a)
*csc(b*x^2+a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*(c*sin(b*x^2+a)^3)^(1/3
)/b^(3/2)-1/4*2^(1/2)*Pi^(1/2)*csc(b*x^2+a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*x)*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int x^2 \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx =$$
$$-\frac{\csc\left(a+bx^2\right)\left(2\sqrt{b}x\cos\left(a+bx^2\right)-\sqrt{2\pi}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\sqrt{2\pi}\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(\right.}{4b^{3/2}}$$

input
```
Integrate[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
-1/4*(Csc[a + b*x^2]*(2*Sqrt[b]*x*Cos[a + b*x^2] - Sqrt[2*Pi]*Cos[a]*Fresn
elC[Sqrt[b]*Sqrt[2/Pi]*x] + Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[
a])*(c*Sin[a + b*x^2]^3)^(1/3))/b^(3/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7271, 3866, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

$$\Big\downarrow \; 7271$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int x^2\sin\left(bx^2+a\right)dx$$

$$\Big\downarrow \; 3866$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{\int\cos\left(bx^2+a\right)dx}{2b}-\frac{x\cos\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow \; 3835$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{\cos(a)\int\cos\left(bx^2\right)dx-\sin(a)\int\sin\left(bx^2\right)dx}{2b}-\frac{x\cos\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow 3832$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{\cos(a)\int\cos\left(bx^2\right)dx-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}}{2b}-\frac{x\cos\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow 3833$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}}{2b}-\frac{x\cos\left(a+bx^2\right)}{2b}\right)$$

input

```
Int[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output

```
Csc[a + b*x^2]*(-1/2*(x*Cos[a + b*x^2])/b + ((Sqrt[Pi/2]*Cos[a]*FresnelC[S
qrt[b]*Sqrt[2/Pi]*x])/Sqrt[b] - (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]
*Sin[a])/Sqrt[b])/(2*b))*(c*Sin[a + b*x^2]^3)^(1/3)
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.55

method	result
risch	$\frac{\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a \right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1\right)^{3}\right)^{\frac{1}{3}} \left(-\frac{i x\,\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}}{2 b}+\frac{i \sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-i b}\,x \right) \mathrm{e}^{i \left(b\,x^{2}+2 a \right)}}{4 b \sqrt{-i b}}\right)}{2\,\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-2}-\frac{i x \left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a \right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1 \right.\right.}{4 b \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1\right)}$

input
```
int(x^2*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)
^3)^(1/3)*(-1/2*I/b*x*exp(2*I*(b*x^2+a))+1/4*I/b*Pi^(1/2)/(-I*b)^(1/2)*erf
((-I*b)^(1/2)*x)*exp(I*(b*x^2+2*a)))-1/4*I*x/b/(exp(2*I*(b*x^2+a))-1)*(I*c
*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)+1/8*I*(I*c*exp(-3*I*(
b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x
^2)/b*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int x^2\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx =$$

$$-\frac{\left(4\,bx\cos\left(bx^2+a\right)-\sqrt{2}\left(\pi e^{(i\,a)}+\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\left(i\,\pi e^{(i\,a)}-i\,\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\;}\right.\right.}{8\,b^2\sin\left(bx^2+a\right)}$$

input
```
integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cos(b*x^2 + a) - sqrt(2)*(pi*e^(I*a) + pi*e^(-I*a))*sqrt(b/pi)
*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(2)*(I*pi*e^(I*a) - I*pi*e^(-I*a)
)*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)))*(-(c*cos(b*x^2 + a)^2 - c)
*sin(b*x^2 + a))^(1/3)/(b^2*sin(b*x^2 + a))
```

Sympy [F]

$$\int x^2\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\int x^2\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

input
```
integrate(x**2*(c*sin(b*x**2+a)**3)**(1/3),x)
```

output
```
Integral(x**2*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \frac{8\,b^2c^{\frac{1}{3}}x\cos(bx^2+a)+\sqrt{2}\sqrt{\pi}\Big(((i-1)\cos(a)+(i+1)\sin(a))\,\mathrm{erf}\left(\sqrt{i\,b}x\right)+(-(i+1)\cos(a)-(i-}{32\,b^3}$$

input
```
integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
1/32*(8*b^2*c^(1/3)*x*cos(b*x^2 + a) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) +
 (I + 1)*sin(a))*erf(sqrt(I*b)*x) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf
(sqrt(-I*b)*x))*b^(3/2)*c^(1/3))/b^3
```

Giac [F]

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}} x^2\,dx$$

input
```
integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^2)}\,dx = \int x^2 \left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}\,dx$$

input
```
int(x^2*(c*sin(a + b*x^2)^3)^(1/3),x)
```

output
```
int(x^2*(c*sin(a + b*x^2)^3)^(1/3), x)
```

Reduce [F]

$$\int x^2 \sqrt[3]{c\sin^3\left(a+bx^2\right)}\, dx = c^{\frac{1}{3}}\left(\int \sin\left(b\,x^2+a\right) x^2 dx\right)$$

input
```
int(x^2*(c*sin(b*x^2+a)^3)^(1/3),x)
```

output
```
c**(1/3)*int(sin(a + b*x**2)*x**2,x)
```

3.321 $\int x \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int x \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx = -\frac{\cot\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)}}{2b}$$

output
```
-1/2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx = -\frac{\cot\left(a+bx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)}}{2b}$$

input
```
Integrate[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
-1/2*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7266, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sqrt[3]{c \sin^3\left(a+b x^2\right)} \, dx$$

$$\Big\downarrow \ 7266$$

$$\frac{1}{2} \int \sqrt[3]{c \sin^3\left(b x^2+a\right)} dx^2$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2} \int \sqrt[3]{c \sin\left(b x^2+a\right)^3} dx^2$$

$$\Big\downarrow \ 3686$$

$$\frac{1}{2} \csc\left(a+b x^2\right) \sqrt[3]{c \sin^3\left(a+b x^2\right)} \int \sin\left(b x^2+a\right) dx^2$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2} \csc\left(a+b x^2\right) \sqrt[3]{c \sin^3\left(a+b x^2\right)} \int \sin\left(b x^2+a\right) dx^2$$

$$\Big\downarrow \ 3118$$

$$-\frac{\cot\left(a+b x^2\right) \sqrt[3]{c \sin^3\left(a+b x^2\right)}}{2 b}$$

input
```
Int[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
-1/2*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.84

method	result	size
risch	$-\frac{i\left(ic\,{\mathrm e}^{-3i\left(b\,x^{2}+a\right)}\left({\mathrm e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{1}{3}}{\mathrm e}^{2i\left(b\,x^{2}+a\right)}}{4b\left({\mathrm e}^{2i\left(b\,x^{2}+a\right)}-1\right)}-\frac{i\left(ic\,{\mathrm e}^{-3i\left(b\,x^{2}+a\right)}\left({\mathrm e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{1}{3}}}{4b\left({\mathrm e}^{2i\left(b\,x^{2}+a\right)}-1\right)}$	119

input
```
int(x*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/b/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a
))-1)^3)^(1/3)*exp(2*I*(b*x^2+a))-1/4*I/b/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(
-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=-\frac{\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}\cos\left(bx^2+a\right)}{2\,b\sin\left(bx^2+a\right)}$$

input
```
integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)*cos(b*x^2 + a)/(b*si
n(b*x^2 + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(27) = 54.

Time = 0.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\begin{cases}\frac{x^2\sqrt[3]{c\sin^3\left(a\right)}}{2} & \text{for } b=0\\ 0 & \text{for } a=-bx^2\vee a=-bx^2+\pi\\ -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}\cos\left(a+bx^2\right)}{2b\sin\left(a+bx^2\right)} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(c*sin(b*x**2+a)**3)**(1/3),x)
```

output
```
Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x**2) | Eq
(a, -b*x**2 + pi)), (-(c*sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(2*b*s
in(a + b*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx=\frac{c^{\frac{1}{3}}\cos(bx^2+a)}{4\,b}$$

input
```
integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
1/4*c^(1/3)*cos(b*x^2 + a)/b
```

Giac [F]

$$\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx=\int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)*x, x)
```

Mupad [B] (verification not implemented)

Time = 39.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx$$
$$=-\frac{\sin\left(2\,b\,x^2+2\,a\right)\,\left(-2\,c\,\left(\sin\left(3\,b\,x^2+3\,a\right)-3\,\sin\left(b\,x^2+a\right)\right)\right)^{1/3}}{8\,b\sin\left(b\,x^2+a\right)^2}$$

input
```
int(x*(c*sin(a + b*x^2)^3)^(1/3),x)
```

output
```
-(sin(2*a + 2*b*x^2)*(-2*c*(sin(3*a + 3*b*x^2) - 3*sin(a + b*x^2)))^(1/3))
/(8*b*sin(a + b*x^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=-\frac{c^{\frac{1}{3}}\cos\left(b\,x^2+a\right)}{2b}$$

input

```
int(x*(c*sin(b*x^2+a)^3)^(1/3),x)
```

output

```
( - c**(1/3)*cos(a + b*x**2))/(2*b)
```

3.322 $\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\begin{aligned}\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx &= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}\\ &\quad+\frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}\end{aligned}$$

output

```
1/2*2^(1/2)*Pi^(1/2)*cos(a)*csc(b*x^2+a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*x)*(c*sin(b*x^2+a)^3)^(1/3)/b^(1/2)+1/2*2^(1/2)*Pi^(1/2)*csc(b*x^2+a)*Fre
snelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)\left(\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}$$

input
```
Integrate[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
(Sqrt[Pi/2]*Csc[a + b*x^2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Fresne
lC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7271, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int \sin\left(bx^2+a\right)dx$$

$$\Big\downarrow \text{3834}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sin(a)\int \cos\left(bx^2\right)dx+\cos(a)\int \sin\left(bx^2\right)dx\right)$$

$$\Big\downarrow \text{3832}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sin(a)\int \cos\left(bx^2\right)dx+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}\right)$$

$$\Big\downarrow \text{3833}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}\right)$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

output
```
Csc[a + b*x^2]*((Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/Sqrt[b]
 + (Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/Sqrt[b])*(c*Sin[a +
b*x^2]^3)^(1/3)
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-ib}\,x\right)\sqrt{\pi}\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{i\left(b\,x^2+2a\right)}}{4\sqrt{-ib}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)}-\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{1}{3}}\mathrm{e}^{ib\,x^2}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{ib}\right.}{4\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)\sqrt{ib}}$

input
```
int((c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf((-I*b)^(1/2)*x)/(-I*b)^(1/2)*Pi^(1/2)/(exp(2*I*(b*x^2+a))-1)*(I*c*
exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*exp(I*(b*x^2+2*a))-1/4
*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+
a))-1)*exp(I*b*x^2)*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$
$$=\frac{\left(\sqrt{2}\left(-i\,\pi e^{(i\,a)}+i\,\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\left(\pi e^{(i\,a)}+\pi e^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\right)\left(-\left(c\cos\left(bx^2+\right.\right.\right.}{4\,b\sin\left(bx^2+a\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(-I*pi*e^(I*a) + I*pi*e^(-I*a))*sqrt(b/pi)*fresnel_cos(sqrt(2
)*x*sqrt(b/pi)) + sqrt(2)*(pi*e^(I*a) + pi*e^(-I*a))*sqrt(b/pi)*fresnel_si
n(sqrt(2)*x*sqrt(b/pi)))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/
(b*sin(b*x^2 + a))
```

Sympy [F]

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(1/3),x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(1/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$
$$= \frac{\sqrt{2}\sqrt{\pi}\Big(\big(-(i+1)\cos(a)+(i-1)\sin(a)\big)\operatorname{erf}\big(\sqrt{i}\,bx\big)+\big((i-1)\cos(a)-(i+1)\sin(a)\big)\operatorname{erf}\big(\sqrt{-i}\,bx\big)}{16\sqrt{b}}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(I*b)*x)
 + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*c^(1/3)/sqrt(b)
```

Giac [F]

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=\int {\left(c\,\sin\left(b\,x^2+a\right)^3\right)}^{1/3}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(1/3),x)
```

output
```
int((c*sin(a + b*x^2)^3)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx=c^{\frac{1}{3}}\left(\int \sin\left(b\,x^2+a\right)dx\right)$$

input
```
int((c*sin(b*x^2+a)^3)^(1/3),x)
```

output
```
c**(1/3)*int(sin(a + b*x**2),x)
```

3.323 $\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\begin{aligned}\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x}\,dx &= \frac{1}{2}\,\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)} \\ &\quad+\frac{1}{2}\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\text{Si}\left(bx^2\right)\end{aligned}$$

output
```
1/2*Ci(b*x^2)*csc(b*x^2+a)*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)+1/2*cos(a)*csc(
b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)*Si(b*x^2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\begin{aligned}\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x}\,dx &= \frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\text{CosIntegral}\left(bx^2\right)\sin(a)\right. \\ &\quad\left.+\cos(a)\text{Si}\left(bx^2\right)\right)\end{aligned}$$

input
```
Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

output
```
(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(CosIntegral[b*x^2]*Sin[a] + Co
s[a]*SinIntegral[b*x^2]))/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7271, 3858, 3856, 3857}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int \frac{\sin\left(bx^2+a\right)}{x}dx$$

$$\Big\downarrow \text{3858}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sin(a)\int \frac{\cos\left(bx^2\right)}{x}dx+\cos(a)\int \frac{\sin\left(bx^2\right)}{x}dx\right)$$

$$\Big\downarrow \text{3856}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sin(a)\int \frac{\cos\left(bx^2\right)}{x}dx+\frac{1}{2}\cos(a)\text{Si}\left(bx^2\right)\right)$$

$$\Big\downarrow \text{3857}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\frac{1}{2}\sin(a)\,\text{CosIntegral}\left(bx^2\right)+\frac{1}{2}\cos(a)\text{Si}\left(bx^2\right)\right)$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

output
```
Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*((CosIntegral[b*x^2]*Sin[a])/2 +
 (Cos[a]*SinIntegral[b*x^2])/2)
```

Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3857
```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3858
```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Sin[c]   Int[Cos[d*
x^n]/x, x], x] + Simp[Cos[c]   Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n},
 x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

method	result
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{1}{3}}\left(i\mathrm{e}^{ib\,x^{2}}\pi\ \mathrm{csgn}\left(b\,x^{2}\right)-2i\mathrm{e}^{ib\,x^{2}}\ \mathrm{Si}\left(b\,x^{2}\right)+\mathrm{expIntegral}_{1}\left(-ib\,x^{2}\right)\mathrm{e}^{i\left(b\,x^{2}+2a\right)}-\mathrm{e}^{ib\,x^{2}}\ \mathrm{expInt}}{4\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)}$

input
```
int((c*sin(b*x^2+a)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*(I*exp(I*b*x
^2)*Pi*csgn(b*x^2)-2*I*exp(I*b*x^2)*Si(b*x^2)+Ei(1,-I*b*x^2)*exp(I*(b*x^2+
2*a))-exp(I*b*x^2)*Ei(1,-I*b*x^2))/(exp(2*I*(b*x^2+a))-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx = \frac{\left(-i\,\mathrm{Ei}(i\,bx^2)\,e^{(i\,a)} + i\,\mathrm{Ei}(-i\,bx^2)\,e^{(-i\,a)}\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{4\sin\left(bx^2+a\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="fricas")
```

output
```
1/4*(-I*Ei(I*b*x^2)*e^(I*a) + I*Ei(-I*b*x^2)*e^(-I*a))*(-(c*cos(b*x^2 + a)
^2 - c)*sin(b*x^2 + a))^(1/3)/sin(b*x^2 + a)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(1/3)/x,x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(1/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx$$
$$=\frac{1}{8}\left(\left(i\operatorname{Ei}\left(i\,bx^2\right)-i\operatorname{Ei}\left(-i\,bx^2\right)\right)\cos\left(a\right)-\left(\operatorname{Ei}\left(i\,bx^2\right)+\operatorname{Ei}\left(-i\,bx^2\right)\right)\sin\left(a\right)\right)c^{\frac{1}{3}}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="maxima")
```

output
```
1/8*((I*Ei(I*b*x^2) - I*Ei(-I*b*x^2))*cos(a) - (Ei(I*b*x^2) + Ei(-I*b*x^2)
)*sin(a))*c^(1/3)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx=\int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx=\int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}}{x}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(1/3)/x,x)
```

output
```
int((c*sin(a + b*x^2)^3)^(1/3)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx = c^{\frac{1}{3}}\left(\int \frac{\sin\left(b\,x^2+a\right)}{x}dx\right)$$

input
```
int((c*sin(b*x^2+a)^3)^(1/3)/x,x)
```

output
```
c**(1/3)*int(sin(a + b*x**2)/x,x)
```

3.324 $\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^2}\,dx = -\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x} + \sqrt{b}\sqrt{2\pi}\cos(a)\csc(a+bx^2)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3(a+bx^2)} - \sqrt{b}\sqrt{2\pi}\csc(a+bx^2)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\sqrt[3]{c\sin^3(a+bx^2)}$$

output
```
-(c*sin(b*x^2+a)^3)^(1/3)/x+b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a)*csc(b*x^2+a)*F
resnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*(c*sin(b*x^2+a)^3)^(1/3)-b^(1/2)*2^(1/
2)*Pi^(1/2)*csc(b*x^2+a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*x)*sin(a)*(c*si
n(b*x^2+a)^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$
$$=\frac{\left(-1+\sqrt{b}\sqrt{2\pi}x\cos(a)\csc\left(a+bx^2\right)\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{b}\sqrt{2\pi}x\csc\left(a+bx^2\right)\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\right.}{x}$$

input
```
Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

output
```
((-1 + Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/
Pi]*x] - Sqrt[b]*Sqrt[2*Pi]*x*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x
]*Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/x
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7271, 3868, 3835, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$
$$\Big\downarrow \text{7271}$$
$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int \frac{\sin\left(bx^2+a\right)}{x^2}dx$$
$$\Big\downarrow \text{3868}$$
$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(2b\int \cos\left(bx^2+a\right)dx-\frac{\sin\left(a+bx^2\right)}{x}\right)$$
$$\Big\downarrow \text{3835}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(2b\left(\cos(a)\int\cos\left(bx^2\right)dx-\sin(a)\int\sin\left(bx^2\right)dx\right)-\frac{\sin\left(a+bx^2\right)}{x}\right)$$

$$\Big\downarrow \text{3832}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(2b\left(\cos(a)\int\cos\left(bx^2\right)dx-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}\right)-\frac{\sin\left(a+bx^2\right)}{x}\right.$$

$$\Big\downarrow \text{3833}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(2b\left(\frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}\right)-\sin\right.$$

input

```
Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

output

```
Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(2*b*((Sqrt[Pi/2]*Cos[a]*Fresnel
C[Sqrt[b]*Sqrt[2/Pi]*x])/Sqrt[b] - (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]
*x]*Sin[a])/Sqrt[b]) - Sin[a + b*x^2]/x)
```

Defintions of rubi rules used

rule 3832

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3835

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Cos[c]   In
t[Cos[d*(e + f*x)^2], x], x] - Simp[Sin[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3868

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.72

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{1}{3}}\left(-\frac{\mathrm{e}^{2i\left(b\,x^{2}+a\right)}}{x}+\frac{ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-ib}\,x\right)\mathrm{e}^{i\left(b\,x^{2}+2a\right)}}{\sqrt{-ib}}\right)}{2\,\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-2}+\frac{\Bigl(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}}{2x\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)}$

input

```
int((c*sin(b*x^2+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)
^3)^(1/3)*(-1/x*exp(2*I*(b*x^2+a))+I*b*Pi^(1/2)/(-I*b)^(1/2)*erf((-I*b)^(1
/2)*x)*exp(I*(b*x^2+2*a)))+1/2/x/(exp(2*I*(b*x^2+a))-1)*(I*c*exp(-3*I*(b*x
^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)+1/2*I*(I*c*exp(-3*I*(b*x^2+a))*(exp
(2*I*(b*x^2+a))-1)^3)^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*b*Pi^(1/2)
/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$
$$= \frac{\left(\sqrt{2}\left(\pi xe^{(i\,a)}+\pi xe^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\left(i\,\pi xe^{(i\,a)}-i\,\pi xe^{(-i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-2\,\sin\left(bx^2+\right.}{2\,x\sin\left(bx^2+a\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*(pi*x*e^(I*a) + pi*x*e^(-I*a))*sqrt(b/pi)*fresnel_cos(sqrt(2)
*x*sqrt(b/pi)) + sqrt(2)*(I*pi*x*e^(I*a) - I*pi*x*e^(-I*a))*sqrt(b/pi)*fre
snel_sin(sqrt(2)*x*sqrt(b/pi)) - 2*sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 -
 c)*sin(b*x^2 + a))^(1/3)/(x*sin(b*x^2 + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(1/3)/x**2,x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(1/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$
$$= \frac{\sqrt{bx^2}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,bx^2\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, -i\,bx^2\right)\right)\cos\left(a\right)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,bx^2\right)-(i-1)\right.}{16\,x}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I + 1)*sqrt(2)*
gamma(-1/2, -I*b*x^2))*cos(a) + ((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I
 - 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a))*c^(1/3)/x
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx=\int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}}{x^2}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(1/3)/x^2,x)
```

output
```
int((c*sin(a + b*x^2)^3)^(1/3)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx=c^{\frac{1}{3}}\left(\int \frac{\sin\left(b\,x^2+a\right)}{x^2}dx\right)$$

input
```
int((c*sin(b*x^2+a)^3)^(1/3)/x^2,x)
```

output
```
c**(1/3)*int(sin(a + b*x**2)/x**2,x)
```

3.325 $\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = &-\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}\\ &+\frac{1}{2}b\cos(a)\operatorname{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\\ &-\frac{1}{2}b\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\operatorname{Si}\left(bx^2\right)\end{aligned}$$

output

```
-1/2*(c*sin(b*x^2+a)^3)^(1/3)/x^2+1/2*b*cos(a)*Ci(b*x^2)*csc(b*x^2+a)*(c*s
in(b*x^2+a)^3)^(1/3)-1/2*b*csc(b*x^2+a)*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)*Si
(b*x^2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = -\frac{\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(-bx^2\cos(a)\operatorname{CosIntegral}\left(bx^2\right)+\sin\left(a+bx^2\right)+bx^2\sin(a)\operatorname{Si}\left(bx^2\right)\right)}{2x^2}$$

input
```
Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

output
```
-1/2*(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(-(b*x^2*Cos[a]*CosIntegra
l[b*x^2]) + Sin[a + b*x^2] + b*x^2*Sin[a]*SinIntegral[b*x^2]))/x^2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {7271, 3860, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int \frac{\sin\left(bx^2+a\right)}{x^3}dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int \frac{\sin\left(bx^2+a\right)}{x^4}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\int \frac{\sin\left(bx^2+a\right)}{x^4}dx^2$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(b\int \frac{\cos\left(bx^2+a\right)}{x^2}dx^2-\frac{\sin\left(a+bx^2\right)}{x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(b\int \frac{\sin\left(bx^2+a+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(a+bx^2\right)}{x^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(b\left(\cos(a)\int\frac{\cos\left(bx^2\right)}{x^2}dx^2-\sin(a)\int\frac{\sin\left(bx^2\right)}{x^2}dx^2\right)-\frac{\sin\left(a+bx^2\right)}{x^2}\right)$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(b\left(\cos(a)\int\frac{\sin\left(bx^2+\frac{\pi}{2}\right)}{x^2}dx^2-\sin(a)\int\frac{\sin\left(bx^2\right)}{x^2}dx^2\right)-\frac{\sin\left(a+bx^2\right)}{x^2}\right)$$

$$\Big\downarrow 3780$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(b\left(\cos(a)\int\frac{\sin\left(bx^2+\frac{\pi}{2}\right)}{x^2}dx^2-\sin(a)\text{Si}\left(bx^2\right)\right)-\frac{\sin\left(a+bx^2\right)}{x^2}\right)$$

$$\Big\downarrow 3783$$

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(b\left(\cos(a)\,\text{CosIntegral}\left(bx^2\right)-\sin(a)\text{Si}\left(bx^2\right)\right)-\frac{\sin\left(a+bx^2\right)}{x^2}\right)$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

output
```
(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(-(Sin[a + b*x^2]/x^2) + b*(Cos
[a]*CosIntegral[b*x^2] - Sin[a]*SinIntegral[b*x^2])))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{1}{3}}\left(i\,\mathrm{expIntegral}_{1}\left(-ib\,x^{2}\right)b\,x^{2}\mathrm{e}^{i\left(b\,x^{2}+2a\right)}+i\mathrm{e}^{ib\,x^{2}}b\ \mathrm{expIntegral}_{1}\left(ib\,x^{2}\right)x^{2}+\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-\right.}{4\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)x^{2}}$

input
```
int((c*sin(b*x^2+a)^3)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(1/3)*(I*Ei(1,-I*b
*x^2)*b*x^2*exp(I*(b*x^2+2*a))+I*exp(I*b*x^2)*b*Ei(1,I*b*x^2)*x^2+exp(2*I*
(b*x^2+a))-1)/(exp(2*I*(b*x^2+a))-1)/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = \frac{\left(bx^2\mathrm{Ei}\left(i\,bx^2\right)e^{(i\,a)}+bx^2\mathrm{Ei}\left(-i\,bx^2\right)e^{(-i\,a)}-2\,\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{4\,x^2\sin\left(bx^2+a\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

output
```
1/4*(b*x^2*Ei(I*b*x^2)*e^(I*a) + b*x^2*Ei(-I*b*x^2)*e^(-I*a) - 2*sin(b*x^2
 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(x^2*sin(b*x^2 + a
))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(1/3)/x**3,x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(1/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx =$$
$$-\frac{1}{8}\left(\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos(a)-\left(i\,\Gamma\left(-1,i\,bx^2\right)-i\,\Gamma\left(-1,-i\,bx^2\right)\right)\sin(a)\right)bc^{\frac{1}{3}}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

output
```
-1/8*((gamma(-1, I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) - (I*gamma(-1, I*b
*x^2) - I*gamma(-1, -I*b*x^2))*sin(a))*b*c^(1/3)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^3}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}}{x^3}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(1/3)/x^3,x)
```

output
```
int((c*sin(a + b*x^2)^3)^(1/3)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx = c^{\frac{1}{3}}\left(\int \frac{\sin\left(b\,x^2+a\right)}{x^3}dx\right)$$

input
```
int((c*sin(b*x^2+a)^3)^(1/3)/x^3,x)
```

output
```
c**(1/3)*int(sin(a + b*x**2)/x**3,x)
```

3.326 $\int x^m \sqrt[3]{c\sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\begin{aligned}\int x^m \sqrt[3]{c\sin^3(a+bx^n)}\,dx &= \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} \\ &\quad - \frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\,\Gamma\left(\frac{1+m}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}\end{aligned}$$

output
```
1/2*I*exp(I*a)*x^(1+m)*csc(a+b*x^n)*GAMMA((1+m)/n,-I*b*x^n)*(c*sin(a+b*x^n
)^3)^(1/3)/n/((-I*b*x^n)^((1+m)/n))-1/2*I*x^(1+m)*csc(a+b*x^n)*GAMMA((1+m)
/n,I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int x^m \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \frac{ix^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\csc(a+bx^n)\Big(-(-ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1+m}{n}}\Gamma\big(\frac{1+m}{n},-i}{2n}$$

input
```
Integrate[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
((I/2)*x^(1 + m)*Csc[a + b*x^n]*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/
n, I*b*x^n]*(Cos[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n,
(-I)*b*x^n]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*
n))^((1 + m)/n))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt[3]{c \sin^3\left(a+b x^n\right)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+b x^n\right) \sqrt[3]{c \sin^3\left(a+b x^n\right)} \int x^m \sin\left(b x^n+a\right)dx$$

$$\Big\downarrow \text{3904}$$

$$\csc\left(a+b x^n\right) \sqrt[3]{c \sin^3\left(a+b x^n\right)} \left(\frac{1}{2} i \int e^{-i b x^n-i a} x^m dx-\frac{1}{2} i \int e^{i b x^n+i a} x^m dx\right)$$

$$\Big\downarrow \text{2648}$$

$$\csc\left(a+b x^n\right) \sqrt[3]{c \sin^3\left(a+b x^n\right)} \left(\frac{i e^{i a} x^{m+1} \left(-i b x^n\right)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n},-i b x^n\right)}{2 n}-\frac{i e^{-i a} x^{m+1} \left(i b x^n\right)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n},i b x^n\right)}{2 n}\right)$$

input
```
Int[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*
((-I)*b*x^n)^((1 + m)/n)) - ((I/2)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E
^(I*a)*n*(I*b*x^n)^((1 + m)/n)))*(c*Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^m \left(c \sin\left(a+b\,x^n\right)^3\right)^{\frac{1}{3}}dx$$

input
```
int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^m, x)
```

Sympy [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx = \int x^m \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

input
```
integrate(x**m*(c*sin(a+b*x**n)**3)**(1/3),x)
```

output
```
Integral(x**m*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Giac [F]

$$\int x^m \sqrt[3]{c \sin^3\left(a+b x^n\right)}\, dx = \int \left(c \sin\left(b x^n + a\right)^3\right)^{\frac{1}{3}} x^m\, dx$$

input
```
integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt[3]{c \sin^3\left(a+b x^n\right)}\, dx = \int x^m \left(c \sin\left(a+b\, x^n\right)^3\right)^{1/3} dx$$

input
```
int(x^m*(c*sin(a + b*x^n)^3)^(1/3),x)
```

output
```
int(x^m*(c*sin(a + b*x^n)^3)^(1/3), x)
```

Reduce [F]

$$\int x^m \sqrt[3]{c \sin^3\left(a+b x^n\right)}\, dx = c^{\frac{1}{3}} \left(\int x^m \sin\left(x^n b + a\right) dx\right)$$

input
```
int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
c**(1/3)*int(x**m*sin(x**n*b + a),x)
```

3.327 $\int x^3 \sqrt[3]{c \sin^3\left(a+bx^n\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\begin{aligned}\int x^3 \sqrt[3]{c \sin^3\left(a+bx^n\right)}\,dx &= \frac{ie^{ia}x^4\left(-ibx^n\right)^{-4/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{4}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}\\ &\quad-\frac{ie^{-ia}x^4\left(ibx^n\right)^{-4/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{4}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}\end{aligned}$$

output
```
1/2*I*exp(I*a)*x^4*csc(a+b*x^n)*GAMMA(4/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/
3)/n/((-I*b*x^n)^(4/n))-1/2*I*x^4*csc(a+b*x^n)*GAMMA(4/n,I*b*x^n)*(c*sin(a
+b*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(4/n))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int x^3 \sqrt[3]{c \sin^3\left(a+bx^n\right)}\,dx$$

$$= \frac{ix^4\left(b^2x^{2n}\right)^{-4/n}\csc\left(a+bx^n\right)\Big(-\left(-ibx^n\right)^{4/n}\Gamma\left(\frac{4}{n},ibx^n\right)\left(\cos(a)-i\sin(a)\right)+\left(ibx^n\right)^{4/n}\Gamma\left(\frac{4}{n},-ibx^n\right)(\cos(a}{2n}$$

input

```
Integrate[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output

```
((I/2)*x^4*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(4/n)*Gamma[4/n, I*b*x^n]*(Cos[a
] - I*Sin[a])) + (I*b*x^n)^(4/n)*Gamma[4/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]
))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(4/n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx^n)\sqrt[3]{c \sin^3(a+bx^n)}\int x^3 \sin(bx^n+a)dx$$

$$\Big\downarrow \text{3904}$$

$$\csc(a+bx^n)\sqrt[3]{c \sin^3(a+bx^n)}\left(\frac{1}{2}i\int e^{-ibx^n-ia}x^3dx-\frac{1}{2}i\int e^{ibx^n+ia}x^3dx\right)$$

$$\Big\downarrow \text{2648}$$

$$\csc(a+bx^n)\left(\frac{ie^{ia}x^4(-ibx^n)^{-4/n}\Gamma\left(\frac{4}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^4(ibx^n)^{-4/n}\Gamma\left(\frac{4}{n},ibx^n\right)}{2n}\right)\sqrt[3]{c \sin^3(a+bx^n)}$$

input

```
Int[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output

```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*x^4*Gamma[4/n, (-I)*b*x^n])/(n*((-I)*b*x^n)
^(4/n)) - ((I/2)*x^4*Gamma[4/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^(4/n)))*(c*
Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^3\left(c\sin\left(a+b\,x^n\right)^3\right)^{\frac{1}{3}}dx$$

input
```
int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x^3\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^3, x)
```

Sympy [F]

$$\int x^3 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int x^3 \sqrt[3]{c\sin^3(a+bx^n)}\,dx$$

input
```
integrate(x**3*(c*sin(a+b*x**n)**3)**(1/3),x)
```

output
```
Integral(x**3*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x^3 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}} x^3\,dx$$

input
```
integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Giac [F]

$$\int x^3 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}} x^3\,dx$$

input
```
integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \int x^3 \left(c \sin\left(a+b\,x^n\right)^3\right)^{1/3} dx$$

input
```
int(x^3*(c*sin(a + b*x^n)^3)^(1/3),x)
```

output
```
int(x^3*(c*sin(a + b*x^n)^3)^(1/3), x)
```

Reduce [F]

$$\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = c^{\frac{1}{3}} \left(\int \sin\left(x^n b+a\right) x^3 dx\right)$$

input
```
int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
c**(1/3)*int(sin(x**n*b + a)*x**3,x)
```

3.328 $\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\begin{aligned}\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx &= \frac{ie^{ia}x^3(-ibx^n)^{-3/n}\csc(a+bx^n)\,\Gamma\left(\frac{3}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}\\ &\quad-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\csc(a+bx^n)\,\Gamma\left(\frac{3}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}\end{aligned}$$

output
```
1/2*I*exp(I*a)*x^3*csc(a+b*x^n)*GAMMA(3/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/
3)/n/((-I*b*x^n)^(3/n))-1/2*I*x^3*csc(a+b*x^n)*GAMMA(3/n,I*b*x^n)*(c*sin(a
+b*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ix^3(b^2x^{2n})^{-3/n}\csc(a+bx^n)\Big(-(-ibx^n)^{3/n}\,\Gamma\left(\frac{3}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{3/n}\,\Gamma\left(\frac{3}{n},-ibx^n\right)(\cos(a}{2n}$$

input
```
Integrate[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
((I/2)*x^3*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(3/n)*Gamma[3/n, I*b*x^n]*(Cos[a
] - I*Sin[a])) + (I*b*x^n)^(3/n)*Gamma[3/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]
))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(3/n))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[3]{c \sin^3\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c \sin^3\left(a+bx^n\right)}\int x^2 \sin\left(bx^n+a\right)dx$$

$$\Big\downarrow \text{3904}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c \sin^3\left(a+bx^n\right)}\left(\frac{1}{2}i\int e^{-ibx^n-ia}x^2dx-\frac{1}{2}i\int e^{ibx^n+ia}x^2dx\right)$$

$$\Big\downarrow \text{2648}$$

$$\csc\left(a+bx^n\right)\left(\frac{ie^{ia}x^3(-ibx^n)^{-3/n}\Gamma\left(\frac{3}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\Gamma\left(\frac{3}{n},ibx^n\right)}{2n}\right)\sqrt[3]{c \sin^3\left(a+bx^n\right)}$$

input
```
Int[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*x^3*Gamma[3/n, (-I)*b*x^n])/(n*((-I)*b*x^n)
^(3/n)) - ((I/2)*x^3*Gamma[3/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^(3/n)))*(c*
Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^{2}\left(c \sin \left(a +b \,x^{n}\right)^{3}\right)^{\frac{1}{3}}d x$$

input
```
int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x^2 \sqrt[3]{c \sin^3\left(a+bx^n\right)}\,dx=\int \left(c \sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}} x^{2}\,dx$$

input
```
integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^2, x)
```

Sympy [F]

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\, dx = \int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\, dx$$

input
```
integrate(x**2*(c*sin(a+b*x**n)**3)**(1/3),x)
```

output
```
Integral(x**2*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\, dx = \int \left(c \sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^2\, dx$$

input
```
integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\, dx = \int \left(c \sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}} x^2\, dx$$

input
```
integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int x^2 \left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3} dx$$

input
```
int(x^2*(c*sin(a + b*x^n)^3)^(1/3),x)
```

output
```
int(x^2*(c*sin(a + b*x^n)^3)^(1/3), x)
```

Reduce [F]

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = c^{\frac{1}{3}} \left(\int \sin\left(x^n b+a\right) x^2 dx\right)$$

input
```
int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
c**(1/3)*int(sin(x**n*b + a)*x**2,x)
```

3.329 $\int x \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\begin{aligned}\int x \sqrt[3]{c \sin^3(a+bx^n)}\,dx &= \frac{ie^{ia}x^2(-ibx^n)^{-2/n}\csc(a+bx^n)\,\Gamma\left(\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} \\ &\quad - \frac{ie^{-ia}x^2(ibx^n)^{-2/n}\csc(a+bx^n)\,\Gamma\left(\frac{2}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}\end{aligned}$$

output
```
1/2*I*exp(I*a)*x^2*csc(a+b*x^n)*GAMMA(2/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/
3)/n/((-I*b*x^n)^(2/n))-1/2*I*x^2*csc(a+b*x^n)*GAMMA(2/n,I*b*x^n)*(c*sin(a
+b*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int x \sqrt[3]{c \sin^3(a+bx^n)}\,dx = \frac{ix^2(b^2x^{2n})^{-2/n}\csc(a+bx^n)\left(-(-ibx^n)^{2/n}\Gamma\left(\frac{2}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{2/n}\Gamma\left(\frac{2}{n},-ibx^n\right)(\cos(a\right.}{2n}$$

input
```
Integrate[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
((I/2)*x^2*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(2/n)*Gamma[2/n, I*b*x^n]*(Cos[a
] - I*Sin[a])) + (I*b*x^n)^(2/n)*Gamma[2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]
))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(2/n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7271, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sqrt[3]{c \sin^3\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c \sin^3\left(a+bx^n\right)}\int x \sin\left(bx^n+a\right)dx$$

$$\Big\downarrow \text{3904}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c \sin^3\left(a+bx^n\right)}\left(\frac{1}{2}i\int e^{-ibx^n-ia}xdx-\frac{1}{2}i\int e^{ibx^n+ia}xdx\right)$$

$$\Big\downarrow \text{2648}$$

$$\csc\left(a+bx^n\right)\left(\frac{ie^{ia}x^2\left(-ibx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^2\left(ibx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},ibx^n\right)}{2n}\right)\sqrt[3]{c \sin^3\left(a+bx^n\right)}$$

input
```
Int[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*x^2*Gamma[2/n, (-I)*b*x^n])/(n*((-I)*b*x^n)
^(2/n)) - ((I/2)*x^2*Gamma[2/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^(2/n)))*(c*
Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x\left(c\sin\left(a+b\,x^{n}\right)^{3}\right)^{\frac{1}{3}}dx$$

input
```
int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int x\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}\,dx=\int\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x, x)
```

Sympy [F]

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx=\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx$$

input
```
integrate(x*(c*sin(a+b*x**n)**3)**(1/3),x)
```

output
```
Integral(x*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Giac [F]

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\int x\,{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{1/3}\,dx$$

input
```
int(x*(c*sin(a + b*x^n)^3)^(1/3),x)
```

output
```
int(x*(c*sin(a + b*x^n)^3)^(1/3), x)
```

Reduce [F]

$$\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=c^{\frac{1}{3}}\left(\int\sin\left(x^nb+a\right)x dx\right)$$

input
```
int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
c**(1/3)*int(sin(x**n*b + a)*x,x)
```

3.330 $\int \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\frac{ie^{ia}x(-ibx^n)^{-1/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}$$

output

```
1/2*I*exp(I*a)*x*csc(a+b*x^n)*GAMMA(1/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)
/n/((-I*b*x^n)^(1/n))-1/2*I*x*csc(a+b*x^n)*GAMMA(1/n,I*b*x^n)*(c*sin(a+b*x
^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx=\frac{ix(b^2x^{2n})^{-1/n}\csc\left(a+bx^n\right)\left(-(-ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)(\cos(a)+}{2n}$$

input

```
Integrate[(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
((I/2)*x*Csc[a + b*x^n]*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos
[a] - I*Sin[a])) + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*
Sin[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^n^(-1))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {7271, 3846, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c\sin^3\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\int \sin\left(bx^n+a\right)dx$$

$$\Big\downarrow \text{3846}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\left(\frac{1}{2}i\int e^{-ibx^n-ia}dx-\frac{1}{2}i\int e^{ibx^n+ia}dx\right)$$

$$\Big\downarrow \text{2637}$$

$$\csc\left(a+bx^n\right)\left(\frac{ie^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(1/3),x]
```

output
```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n
)^n^(-1)) - ((I/2)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1)))
*(c*Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

rule 3846
```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Simp[I/2   I
nt[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Simp[I/2   Int[E^(c*I + d*I*(e +
f*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \left(c \sin \left(a +b \,x^{n}\right)^{3}\right)^{\frac{1}{3}}d x$$

input
```
int((c*sin(a+b*x^n)^3)^(1/3),x)
```

output
```
int((c*sin(a+b*x^n)^3)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{c \sin^3\left(a+b x^n\right)} \, dx=\int { \left(c \sin \left(b x^{n} + a\right)^{3}\right)^{\frac{1}{3}} \,d x }$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \sqrt[3]{c\sin^3(a+bx^n)}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(1/3),x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \int \left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}\,dx$$

input

```
int((c*sin(a + b*x^n)^3)^(1/3),x)
```

output

```
int((c*sin(a + b*x^n)^3)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = c^{\frac{1}{3}}\left(\int \sin\left(x^n b+a\right)dx\right)$$

input

```
int((c*sin(a+b*x^n)^3)^(1/3),x)
```

output

```
c**(1/3)*int(sin(x**n*b + a),x)
```

3.331 $\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x}\,dx=\frac{\operatorname{CosIntegral}\left(bx^n\right)\csc\left(a+bx^n\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{n}+\frac{\cos(a)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\operatorname{Si}\left(bx^n\right)}{n}$$

output
```
Ci(b*x^n)*csc(a+b*x^n)*sin(a)*(c*sin(a+b*x^n)^3)^(1/3)/n+cos(a)*csc(a+b*x^
n)*(c*sin(a+b*x^n)^3)^(1/3)*Si(b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x}\,dx=\frac{\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\left(\operatorname{CosIntegral}\left(bx^n\right)\sin(a)+\cos(a)\operatorname{Si}\left(bx^n\right)\right)}{n}$$

input
```
Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

output
```
(Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*(CosIntegral[b*x^n]*Sin[a] + Co
s[a]*SinIntegral[b*x^n]))/n
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7271, 3858, 3856, 3857}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\int \frac{\sin(bx^n+a)}{x}dx$$

$$\Big\downarrow \text{3858}$$

$$\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left(\sin(a)\int \frac{\cos(bx^n)}{x}dx+\cos(a)\int \frac{\sin(bx^n)}{x}dx\right)$$

$$\Big\downarrow \text{3856}$$

$$\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left(\sin(a)\int \frac{\cos(bx^n)}{x}dx+\frac{\cos(a)\text{Si}(bx^n)}{n}\right)$$

$$\Big\downarrow \text{3857}$$

$$\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left(\frac{\sin(a)\,\text{CosIntegral}(bx^n)}{n}+\frac{\cos(a)\text{Si}(bx^n)}{n}\right)$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

output
```
Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*((CosIntegral[b*x^n]*Sin[a])/n +
 (Cos[a]*SinIntegral[b*x^n])/n)
```

Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3857
```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3858
```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Sin[c]   Int[Cos[d*
x^n]/x, x], x] + Simp[Cos[c]   Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n},
 x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i(a+b\,x^{n})}\left(\mathrm{e}^{2i(a+b\,x^{n})}-1\right)^{3}\right)^{\frac{1}{3}}\left(i\mathrm{e}^{ib\,x^{n}}\pi\ \mathrm{csgn}(b\,x^{n})-2i\mathrm{e}^{ib\,x^{n}}\ \mathrm{Si}(b\,x^{n})+\mathrm{expIntegral}_{1}(-ib\,x^{n})\mathrm{e}^{i(b\,x^{n}+2a)}-\mathrm{e}^{ib\,x^{n}}\ \mathrm{expInte}\right.}{2n\left(\mathrm{e}^{2i(a+b\,x^{n})}-1\right)}$

input
```
int((c*sin(a+b*x^n)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*c*exp(-3*I*(a+b*x^n))*(exp(2*I*(a+b*x^n))-1)^3)^(1/3)*(I*exp(I*b*x
^n)*Pi*csgn(b*x^n)-2*I*exp(I*b*x^n)*Si(b*x^n)+Ei(1,-I*b*x^n)*exp(I*(b*x^n+
2*a))-exp(I*b*x^n)*Ei(1,-I*b*x^n))/n/(exp(2*I*(a+b*x^n))-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x}\,dx = \frac{\left(-i\,\mathrm{Ei}(i\,bx^n)\,e^{(i\,a)}+i\,\mathrm{Ei}(-i\,bx^n)\,e^{(-i\,a)}\right)\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{1}{3}}}{2\,n\sin\left(bx^n+a\right)}$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="fricas")
```

output
```
1/2*(-I*Ei(I*b*x^n)*e^(I*a) + I*Ei(-I*b*x^n)*e^(-I*a))*(-(c*cos(b*x^n + a)
^2 - c)*sin(b*x^n + a))^(1/3)/(n*sin(b*x^n + a))
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x}\,dx = \int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(1/3)/x,x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(1/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx$$
$$= \frac{\left(\left((\sqrt{3}+i)\operatorname{Ei}(i\,bx^n) - (\sqrt{3}+i)\operatorname{Ei}(-i\,bx^n) - (\sqrt{3}-i)\operatorname{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right) + (\sqrt{3}-i)\operatorname{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right.\right.}{}$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="maxima")
```

output
```
1/8*(((sqrt(3) + I)*Ei(I*b*x^n) - (sqrt(3) + I)*Ei(-I*b*x^n) - (sqrt(3) -
I)*Ei(I*b*e^(n*conjugate(log(x)))) + (sqrt(3) - I)*Ei(-I*b*e^(n*conjugate(
log(x)))))*cos(a) - ((-I*sqrt(3) + 1)*Ei(I*b*x^n) + (-I*sqrt(3) + 1)*Ei(-I
*b*x^n) + (I*sqrt(3) + 1)*Ei(I*b*e^(n*conjugate(log(x)))) + (I*sqrt(3) + 1
)*Ei(-I*b*e^(n*conjugate(log(x)))))*sin(a))*c^(1/3)/n
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx=\int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}}{x}\,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(1/3)/x,x)
```

output
```
int((c*sin(a + b*x^n)^3)^(1/3)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx=c^{\frac{1}{3}}\left(\int \frac{\sin\left(x^n b+a\right)}{x}dx\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(1/3)/x,x)
```

output
```
c**(1/3)*int(sin(x**n*b + a)/x,x)
```

3.332 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx} - \frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}$$

output
```
1/2*I*exp(I*a)*(-I*b*x^n)^(1/n)*csc(a+b*x^n)*GAMMA(-1/n,-I*b*x^n)*(c*sin(a
+b*x^n)^3)^(1/3)/n/x-1/2*I*(I*b*x^n)^(1/n)*csc(a+b*x^n)*GAMMA(-1/n,I*b*x^n
)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/x
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx$$

$$= \frac{i\csc(a+bx^n)\left(-(ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(-ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)}{2nx}$$

input
```
Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

output
```
((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^n^(-1)*Gamma[-n^(-1), I*b*x^n]*(Cos[a]
- I*Sin[a])) + ((-I)*b*x^n)^n^(-1)*Gamma[-n^(-1), (-I)*b*x^n]*(Cos[a] + I*
Sin[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x^2}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\int \frac{\sin\left(bx^n+a\right)}{x^2}dx$$

$$\Big\downarrow \text{3904}$$

$$\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\left(\frac{1}{2}i\int \frac{e^{-ibx^n-ia}}{x^2}dx-\frac{1}{2}i\int \frac{e^{ibx^n+ia}}{x^2}dx\right)$$

$$\Big\downarrow \text{2648}$$

$$\csc\left(a+bx^n\right)\left(\frac{ie^{ia}\left(-ibx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ibx^n\right)}{2nx}-\frac{ie^{-ia}\left(ibx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ibx^n\right)}{2nx}\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

output
```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*((-I)*b*x^n)^n^(-1)*Gamma[-n^(-1), (-I)*b*x
^n])/(n*x) - ((I/2)*(I*b*x^n)^n^(-1)*Gamma[-n^(-1), I*b*x^n])/(E^(I*a)*n*x
))*(c*Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \frac{\left(c\sin\left(a+b\,x^{n}\right)^{3}\right)^{\frac{1}{3}}}{x^{2}}dx$$

input
```
int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

output
```
int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x^{2}}\,dx=\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}}{x^{2}}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(1/3)/x**2,x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(1/3)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x^2}\,dx=\int \frac{{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{1/3}}{x^2} \,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(1/3)/x^2,x)
```

output
```
int((c*sin(a + b*x^n)^3)^(1/3)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{x^2}\,dx=c^{\frac{1}{3}}\left(\int \frac{\sin\left(x^n b+a\right)}{x^2}d x\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

output
```
c**(1/3)*int(sin(x**n*b + a)/x**2,x)
```

3.333 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \frac{ie^{ia}(-ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2} - \frac{ie^{-ia}(ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}$$

output

```
1/2*I*exp(I*a)*(-I*b*x^n)^(2/n)*csc(a+b*x^n)*GAMMA(-2/n,-I*b*x^n)*(c*sin(a
+b*x^n)^3)^(1/3)/n/x^2-1/2*I*(I*b*x^n)^(2/n)*csc(a+b*x^n)*GAMMA(-2/n,I*b*x
^n)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/x^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx$$
$$= \frac{i\csc(a+bx^n)\,\Big(-(ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},ibx^n\right)(\cos(a)-i\sin(a))+(-ibx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-ibx^n\right)(\cos(a)+i\sin(a}{2nx^2}$$

input
```
Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

output
```
((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^(2/n)*Gamma[-2/n, I*b*x^n]*(Cos[a] - I*
Sin[a])) + ((-I)*b*x^n)^(2/n)*Gamma[-2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c \sin^3\left(a+b x^n\right)}}{x^3} \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$\csc\left(a+b x^n\right) \sqrt[3]{c \sin^3\left(a+b x^n\right)} \int \frac{\sin\left(b x^n+a\right)}{x^3}dx$$

$$\Big\downarrow \text{ 3904}$$

$$\csc\left(a+b x^n\right) \sqrt[3]{c \sin^3\left(a+b x^n\right)} \left(\frac{1}{2} i \int \frac{e^{-i b x^n-i a}}{x^3}dx-\frac{1}{2} i \int \frac{e^{i b x^n+i a}}{x^3}dx\right)$$

$$\Big\downarrow \text{ 2648}$$

$$\csc\left(a+b x^n\right) \left(\frac{i e^{i a} \left(-i b x^n\right)^{2/n} \Gamma\left(-\frac{2}{n},-i b x^n\right)}{2 n x^2}-\frac{i e^{-i a} \left(i b x^n\right)^{2/n} \Gamma\left(-\frac{2}{n},i b x^n\right)}{2 n x^2}\right) \sqrt[3]{c \sin^3\left(a+b x^n\right)}$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

output
```
Csc[a + b*x^n]*(((I/2)*E^(I*a)*((-I)*b*x^n)^(2/n)*Gamma[-2/n, (-I)*b*x^n])
/(n*x^2) - ((I/2)*(I*b*x^n)^(2/n)*Gamma[-2/n, I*b*x^n])/(E^(I*a)*n*x^2))*(
c*Sin[a + b*x^n]^3)^(1/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \frac{(c\sin(a+b\,x^n)^3)^{\frac{1}{3}}}{x^3}dx$$

input
```
int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

output
```
int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{(c\sin(bx^n+a)^3)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="fricas")
```

output

```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx$$

input

```
integrate((c*sin(a+b*x**n)**3)**(1/3)/x**3,x)
```

output

```
Integral((c*sin(a + b*x**n)**3)**(1/3)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

input

```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="maxima")
```

output

```
integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

input

```
integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="giac")
```

output

```
integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}}{x^3}\,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(1/3)/x^3,x)
```

output
```
int((c*sin(a + b*x^n)^3)^(1/3)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = c^{\frac{1}{3}}\left(\int \frac{\sin(x^n b+a)}{x^3}dx\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

output
```
c**(1/3)*int(sin(x**n*b + a)/x**3,x)
```

3.334 $\int x^m(c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$
\begin{aligned}
\int x^m\left(c\sin^3(a+bx)\right)^{2/3}\,dx &= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} \\
&+ \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b} \\
&- \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}
\end{aligned}
$$

output

```
x^(1+m)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)/(2+2*m)+I*2^(-3-m)*exp(2*I*a)*
x^m*csc(b*x+a)^2*GAMMA(1+m,-2*I*b*x)*(c*sin(b*x+a)^3)^(2/3)/b/((-I*b*x)^m)
-I*2^(-3-m)*x^m*csc(b*x+a)^2*GAMMA(1+m,2*I*b*x)*(c*sin(b*x+a)^3)^(2/3)/b/e
xp(2*I*a)/((I*b*x)^m)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int x^m(c\sin^3(a+bx))^{2/3}\,dx = \frac{2^{-3-m}x^m(b^2x^2)^{-m}\csc^2(a+bx)\,(2^{2+m}bx(b^2x^2)^m - i(1+m)(-ibx)^m\Gamma(1+m,2ibx)(\cos(a) -}{b(1+m}$$

input
```
Integrate[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
(2^(-3 - m)*x^m*Csc[a + b*x]^2*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I
)*b*x)^m*Gamma[1 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)
^m*Gamma[1 + m, (-2*I)*b*x]*(Cos[a] + I*Sin[a])^2)*(c*Sin[a + b*x]^3)^(2/3
))/(b*(1 + m)*(b^2*x^2)^m)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7271, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(c\sin^3(a+bx))^{2/3}\,dx$$

$$\Big\downarrow 7271$$

$$\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}\int x^m\sin^2(a+bx)dx$$

$$\Big\downarrow 3042$$

$$\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}\int x^m\sin(a+bx)^2dx$$

$$\Big\downarrow 3793$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+ bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\Gamma(m+1,-2ibx)}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\Gamma(m+1,2ibx)}{b}+\right.$$

input

```
Int[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

output

```
Csc[a + b*x]^2*(x^(1 + m)/(2*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamm
a[1 + m, (-2*I)*b*x])/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Gamma[1 + m, (2
*I)*b*x])/(b*E^((2*I)*a)*(I*b*x)^m))*(c*Sin[a + b*x]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^m \left(c \sin \left(bx+a\right)^3\right)^{\frac{2}{3}} dx$$

input
```
int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

output
```
int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int x^m \left(c \sin^3(a+bx)\right)^{2/3} dx =$$

$$-\frac{(4\,bxx^m - (i\,m+i)e^{(-m\log(2i\,b)-2i\,a)}\Gamma(m+1, 2i\,bx) - (-i\,m-i)e^{(-m\log(-2i\,b)+2i\,a)}\Gamma(m+1, -2i\,bx))\,(-(}{8\left((bm+b)\cos\left(bx+a\right)^2 - bm - b\right)}$$

input
```
integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*x^m - (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x
) - (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))*(-(c*cos
(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/((b*m + b)*cos(b*x + a)^2 - b*m - b)
```

Sympy [F]

$$\int x^m \left(c \sin^3(a+bx)\right)^{2/3} dx = \int x^m \left(c \sin^3\left(a+bx\right)\right)^{\frac{2}{3}} dx$$

input
```
integrate(x**m*(c*sin(b*x+a)**3)**(2/3),x)
```

output
```
Integral(x**m*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [F]

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
1/4*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))*c
^(2/3)/(m + 1)
```

Giac [F]

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x^m \left(c\sin\left(a+b\,x\right)^3\right)^{2/3}\,dx$$

input
```
int(x^m*(c*sin(a + b*x)^3)^(2/3),x)
```

output
```
int(x^m*(c*sin(a + b*x)^3)^(2/3), x)
```

Reduce [F]

$$\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx = c^{\frac{2}{3}}\left(\int x^m \sin\left(bx+a\right)^2 dx\right)$$

input

```
int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

output

```
c**(2/3)*int(x**m*sin(a + b*x)**2,x)
```

3.335 $\int x^3(c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 165

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx=-\frac{3(c\sin^3(a+bx))^{2/3}}{8b^4}+\frac{3x^2(c\sin^3(a+bx))^{2/3}}{4b^2}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}-\frac{3x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{8b^2}+\frac{1}{8}x^4\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

output

```
-3/8*(c*sin(b*x+a)^3)^(2/3)/b^4+3/4*x^2*(c*sin(b*x+a)^3)^(2/3)/b^2+3/4*x*c
ot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b^3-1/2*x^3*cot(b*x+a)*(c*sin(b*x+a)^3)^(
2/3)/b-3/8*x^2*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)/b^2+1/8*x^4*csc(b*x+a)^
2*(c*sin(b*x+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48

$$\int x^3 \left(c \sin^3(a+bx)\right)^{2/3} \, dx = \frac{\csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} \left(2b^4x^4 + (3-6b^2x^2)\cos(2(a+bx)) + (6bx-4b^3x^3)\sin(2(a\right.}{16b^4}$$

input
```
Integrate[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*b^4*x^4 + (3 - 6*b^2*x^2)*Cos[
2*(a + b*x)] + (6*b*x - 4*b^3*x^3)*Sin[2*(a + b*x)]))/(16*b^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {7271, 3042, 3792, 15, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c \sin^3(a+bx)\right)^{2/3} \, dx$$

$$\Big\downarrow \; 7271$$

$$\csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} \int x^3 \sin^2(a+bx) dx$$

$$\Big\downarrow \; 3042$$

$$\csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} \int x^3 \sin(a+bx)^2 dx$$

$$\Big\downarrow \; 3792$$

$$\csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} \left(-\frac{3\int x \sin^2(a+bx) dx}{2b^2} + \frac{\int x^3 dx}{2} + \frac{3x^2 \sin^2(a+bx)}{4b^2} - \frac{x^3 \sin(a+bx)\cos(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{15}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{3\int x\sin^2(a+bx)dx}{2b^2}+\frac{3x^2\sin^2(a+bx)}{4b^2}-\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^4}{8}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{3\int x\sin(a+bx)^2dx}{2b^2}+\frac{3x^2\sin^2(a+bx)}{4b^2}-\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^4}{8}\right)$$

$$\Big\downarrow \text{3791}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{3\left(\frac{\int xdx}{2}+\frac{\sin^2(a+bx)}{4b^2}-\frac{x\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3x^2\sin^2(a+bx)}{4b^2}-\frac{x^3\sin(a+bx)\cos(a}{2b}\right.$$

$$\Big\downarrow \text{15}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{3x^2\sin^2(a+bx)}{4b^2}-\frac{3\left(\frac{\sin^2(a+bx)}{4b^2}-\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)}{2b^2}-\frac{x^3\sin(a+bx)\cos(a+b}{2b}\right.$$

input
```
Int[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(x^4/8 - (x^3*Cos[a + b*x]*Sin[a +
 b*x])/(2*b) + (3*x^2*Sin[a + b*x]^2)/(4*b^2) - (3*(x^2/4 - (x*Cos[a + b*x
]*Sin[a + b*x])/(2*b) + Sin[a + b*x]^2/(4*b^2)))/(2*b^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{x^{4} \left(i c\,{\mathrm e}^{-3 i \left(b x +a \right)} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{3}\right)^{\frac{2}{3}} {\mathrm e}^{2 i \left(b x +a \right)}}{8 \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{2}}-\frac{i \left(4 b^{3} x^{3}+6 i x^{2} b^{2}-6 b x -3 i\right) \left(i c\,{\mathrm e}^{-3 i \left(b x +a \right)} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{3}\right)^{\frac{2}{3}} {\mathrm e}^{4 i \left(b x +a \right)}}{32 b^{4} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{2}}+$

input

```
int(x^3*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output

```
-1/8*x^4/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1
)^3)^(2/3)*exp(2*I*(b*x+a))-1/32*I/b^4*(4*b^3*x^3+6*I*x^2*b^2-6*b*x-3*I)/(
exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)
*exp(4*I*(b*x+a))+1/32*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2
/3)/(exp(2*I*(b*x+a))-1)^2*(4*b^3*x^3-6*I*b^2*x^2-6*b*x+3*I)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.67

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{\left(2\,b^4x^4+6\,b^2x^2-6\,(2\,b^2x^2-1)\cos{(bx+a)}^2-4\,(2\,b^3x^3-3\,bx)\cos{(bx+a)}\sin{(bx+a)}-3\right)\left(-\left(c\cos\left(\right.\right.\right.}{16\,\left(b^4\cos{(bx+a)}^2-b^4\right)}$$

input
```
integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/16*(2*b^4*x^4 + 6*b^2*x^2 - 6*(2*b^2*x^2 - 1)*cos(b*x + a)^2 - 4*(2*b^3
*x^3 - 3*b*x)*cos(b*x + a)*sin(b*x + a) - 3)*(-(c*cos(b*x + a)^2 - c)*sin(
b*x + a))^(2/3)/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F]

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x^3\left(c\sin^3{(a+bx)}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x**3*(c*sin(b*x+a)**3)**(2/3),x)
```

output
```
Integral(x**3*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(141) = 282$.

Time = 0.16 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{32\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a^3+6\left(2\,(bx+a)^2-2\,(bx+a)\sin\left(2\,bx+2\right.\right.}{}$$

input
```
integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/32*(32*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(
b*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)
/(2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1
)^4 + 1))*a^3 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*
x + 2*a))*a^2*c^(2/3) - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) -
3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*c^(2/3) + (2*(b*x + a)^4 - 3*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*
b*x + 2*a))*c^(2/3))/b^4
```

Giac [F]

$$\int x^3 \left(c \sin^3(a+bx)\right)^{2/3} \, dx = \int \left(c \sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x^3 \, dx$$

input
```
integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c \sin^3(a+bx)\right)^{2/3} \, dx = \int x^3 \left(c \sin\left(a+b\,x\right)^3\right)^{2/3} \, dx$$

input
```
int(x^3*(c*sin(a + b*x)^3)^(2/3),x)
```

output
```
int(x^3*(c*sin(a + b*x)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int x^3 \left(c \sin^3(a + bx)\right)^{2/3} \, dx = \frac{c^{\frac{2}{3}} \left(-4 \cos(bx+a) \sin(bx+a) b^3 x^3 + 6 \cos(bx+a) \sin(bx+a) bx + 6 \sin(bx+a)^2 b^2 x^2 - \right.}{8b^4}$$

input

```
int(x^3*(c*sin(b*x+a)^3)^(2/3),x)
```

output

```
(c**(2/3)*( - 4*cos(a + b*x)*sin(a + b*x)*b**3*x**3 + 6*cos(a + b*x)*sin(a
 + b*x)*b*x + 6*sin(a + b*x)**2*b**2*x**2 - 3*sin(a + b*x)**2 + b**4*x**4
- 3*b**2*x**2 + 6))/(8*b**4)
```

3.336 $\int x^2(c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}+\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}-\frac{x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{1}{6}x^3\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

output
```
1/2*x*(c*sin(b*x+a)^3)^(2/3)/b^2+1/4*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b^3
-1/2*x^2*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b-1/4*x*csc(b*x+a)^2*(c*sin(b*x
+a)^3)^(2/3)/b^2+1/6*x^3*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.50

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(4b^3x^3-6bx\cos(2(a+bx))+(3-6b^2x^2)\sin(2(a+bx))\right)}{24b^3}$$

input
```
Integrate[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x
)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)]))/(24*b^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {7271, 3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int x^2\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int x^2\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{\int \sin^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{15}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{\int \sin^2(a+bx)dx}{2b^2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{\int \sin(a+bx)^2dx}{2b^2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)$$

$$\Big\downarrow 3115$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}}{2b^2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)$$

$$\Big\downarrow 24$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{x\sin^2(a+bx)}{2b^2}-\frac{\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)$$

input

```
Int[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

output

```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(x^3/6 - (x^2*Cos[a + b*x]*Sin[a +
 b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2) - (x/2 - (Cos[a + b*x]*Sin[a + b
*x])/(2*b))/(2*b^2))
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{x^{3} \left(i c \,{\mathrm e}^{-3 i \left(b x +a \right)} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{3}\right)^{\frac{2}{3}} {\mathrm e}^{2 i \left(b x +a \right)}}{6 \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{2}}-\frac{i \left(2 x^{2} b^{2}+2 i b x -1\right) \left(i c \,{\mathrm e}^{-3 i \left(b x +a \right)} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{3}\right)^{\frac{2}{3}} {\mathrm e}^{4 i \left(b x +a \right)}}{16 b^{3} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{2}}+$ $i\left(i c \,{\mathrm e}^{-3 i (b x}\right.$

input

```
int(x^2*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output

```
-1/6*x^3/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1
)^3)^(2/3)*exp(2*I*(b*x+a))-1/16*I/b^3*(2*x^2*b^2+2*I*b*x-1)/(exp(2*I*(b*x
+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*exp(4*I*(b*
x+a))+1/16*I*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/(exp(2*I
*(b*x+a))-1)^2*(2*x^2*b^2-2*I*b*x-1)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int x^2 \left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{\left(2\,b^3x^3-6\,bx\cos\left(bx+a\right)^2-3\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,bx\right)\bigl(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\bigl(}{12\left(b^3\cos\left(bx+a\right)^2-b^3\right)}$$

input
```
integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*s
in(b*x + a) + 3*b*x)*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^3*cos
(b*x + a)^2 - b^3)
```

Sympy [F]

$$\int x^2 \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int x^2 \left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x**2*(c*sin(b*x+a)**3)**(2/3),x)
```

output
```
Integral(x**2*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.58

$$\int x^2 \bigl(c\sin^3(a+bx)\bigr)^{2/3}\,dx = \frac{48\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a^2+6\bigl(2\left(bx+a\right)^2-2\left(bx+a\right)\mathrm{s}}{}$$

input
```
integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
1/48*(48*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/
(2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)
^4 + 1))*a^2 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x
 + 2*a))*a*c^(2/3) - (4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*
(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c^(2/3))/b^3
```

Giac [F]

$$\int x^2 \left(c \sin^3(a+bx)\right)^{2/3} \, dx = \int \left(c \sin\left(bx+a\right)^3\right)^{\frac{2}{3}} x^2 \, dx$$

input
```
integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c \sin^3(a+bx)\right)^{2/3} \, dx = \int x^2 \left(c \sin\left(a+b\,x\right)^3\right)^{2/3} dx$$

input
```
int(x^2*(c*sin(a + b*x)^3)^(2/3),x)
```

output
```
int(x^2*(c*sin(a + b*x)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{c^{\frac{2}{3}}\left(-6\cos(bx+a)\sin(bx+a)\,b^2x^2+3\cos(bx+a)\sin(bx+a)+6\sin(bx+a)^2\,bx+9a+\right.}{12b^3}$$

input

```
int(x^2*(c*sin(b*x+a)^3)^(2/3),x)
```

output

```
(c**(2/3)*( - 6*cos(a + b*x)*sin(a + b*x)*b**2*x**2 + 3*cos(a + b*x)*sin(a
 + b*x) + 6*sin(a + b*x)**2*b*x + 9*a + 2*b**3*x**3 - 3*b*x))/(12*b**3)
```

3.337 $\int x(c\sin^3(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

output
```
1/4*(c*sin(b*x+a)^3)^(2/3)/b^2-1/2*x*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b+1
/4*x^2*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=-\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(\cos(2(a+bx))+2bx(-bx+\sin(2(a+bx))))}{8b^2}$$

input
```
Integrate[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
-1/8*(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(Cos[2*(a + b*x)] + 2*b*x*(-
(b*x) + Sin[2*(a + b*x)])))/b^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7271, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int x\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int x\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3791}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{\int xdx}{2}+\frac{\sin^2(a+bx)}{4b^2}-\frac{x\sin(a+bx)\cos(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{15}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{\sin^2(a+bx)}{4b^2}-\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)$$

input
```
Int[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(x^2/4 - (x*Cos[a + b*x]*Sin[a + b
*x])/(2*b) + Sin[a + b*x]^2/(4*b^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{x^{2}\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{2i(bx+a)}}{4\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}-\frac{i(2bx+i)\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{4i(bx+a)}}{16b^{2}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}+\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(b}{16\left(\mathrm{e}^{2i(bx+}}$

input
```
int(x*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1
)^3)^(2/3)*exp(2*I*(b*x+a))-1/16*I/b^2*(2*b*x+I)/(exp(2*I*(b*x+a))-1)^2*(I
*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*exp(4*I*(b*x+a))+1/16*I
*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/(exp(2*I*(b*x+a))-1)
^2*(2*b*x-I)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$
$$-\frac{\left(2\,b^2x^2-4\,bx\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,\cos\left(bx+a\right)^2+1\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{8\left(b^2\cos\left(bx+a\right)^2-b^2\right)}$$

input
```
integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/8*(2*b^2*x^2 - 4*b*x*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 1)*
(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int x\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x*(c*sin(b*x+a)**3)**(2/3),x)
```

output
```
Integral(x*(c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.05

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx =$$

$$-\frac{16\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a+\left(2\,(bx+a)^2-2\,(bx+a)\sin\left(2\,bx+2\,a\right)-\right.}{16\,b^2}$$

input
```
integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/16*(16*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(
b*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)
/(2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1
)^4 + 1))*a + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
2*a))*c^(2/3))/b^2
```

Giac [F]

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int x\,{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^{2/3}\,dx$$

input
```
int(x*(c*sin(a + b*x)^3)^(2/3),x)
```

output
```
int(x*(c*sin(a + b*x)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.52

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{c^{\frac{2}{3}}\left(-2\cos\left(bx+a\right)\sin\left(bx+a\right)bx+\sin\left(bx+a\right)^2+b^2x^2-2\right)}{4b^2}$$

input
```
int(x*(c*sin(b*x+a)^3)^(2/3),x)
```

output
```
(c**(2/3)*( - 2*cos(a + b*x)*sin(a + b*x)*b*x + sin(a + b*x)**2 + b**2*x**
2 - 2))/(4*b**2)
```

3.338 $\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

output
```
-1/2*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b+1/2*x*csc(b*x+a)^2*(c*sin(b*x+a)^
3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(2(a+bx)-\sin(2(a+bx)))}{4b}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^(2/3),x]
```

output
```
(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*(a + b*x) - Sin[2*(a + b*x)]))
/(4*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(c\sin(a+bx)^3\right)^{2/3}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \sin^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \sin(a+bx)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 24}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)$$

input

```
Int[(c*Sin[a + b*x]^3)^(2/3),x]
```

output

```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(x/2 - (Cos[a + b*x]*Sin[a + b*x])
/(2*b))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.87

method	result
risch	$-\frac{x\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{2i(bx+a)}}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}-\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{4i(bx+a)}}{8b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}+\frac{i\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}}{8\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}b}$

input
```
int((c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^
3)^(2/3)*exp(2*I*(b*x+a))-1/8*I/b/(exp(2*I*(b*x+a))-1)^2*(I*c*exp(-3*I*(b*
x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*exp(4*I*(b*x+a))+1/8*I*(I*c*exp(-3*I*(
b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)/(exp(2*I*(b*x+a))-1)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = -\frac{\left(bx-\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*(b*x - cos(b*x + a)*sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x +
a))^(2/3)/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \int \left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**(2/3),x)
```

output
```
Integral((c*sin(a + b*x)**3)**(2/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \frac{c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right) - \frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1} - \frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4} + 1}}{b}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*x + a)/(
cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2*sin(b*
x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4 + 1))/
b
```

Giac [F]

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int \left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx=\int \left(c\sin\left(a+b\,x\right)^3\right)^{2/3}\,dx$$

input
```
int((c*sin(a + b*x)^3)^(2/3),x)
```

output
```
int((c*sin(a + b*x)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.47

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx=\frac{c^{\frac{2}{3}}\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+bx\right)}{2b}$$

input
```
int((c*sin(b*x+a)^3)^(2/3),x)
```

output

```
(c**(2/3)*( - cos(a + b*x)*sin(a + b*x) + b*x))/(2*b)
```

3.339 $\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx = -\frac{1}{2}\cos(2a)\,\mathrm{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} + \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3} + \frac{1}{2}\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\mathrm{Si}(2bx)$$

output
```
-1/2*cos(2*a)*Ci(2*b*x)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)+1/2*csc(b*x+a)
^2*ln(x)*(c*sin(b*x+a)^3)^(2/3)+1/2*csc(b*x+a)^2*sin(2*a)*(c*sin(b*x+a)^3)
^(2/3)*Si(2*b*x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx = \frac{1}{2}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(-\cos(2a)\operatorname{CosIntegral}(2bx)+\log(x)+\sin(2a)\operatorname{Si}(2bx))$$

input
```
Integrate[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

output
```
(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x]) +
 Log[x] + Sin[2*a]*SinIntegral[2*b*x]))/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7271, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \frac{\sin^2(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \frac{\sin(a+bx)^2}{x}dx$$

$$\Big\downarrow \text{3793}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \left(\frac{1}{2x}-\frac{\cos(2a+2bx)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\frac{1}{2}\cos(2a)\,\mathrm{CosIntegral}(2bx)+\frac{1}{2}\sin(2a)\mathrm{Si}(2bx)+\frac{\log(x)}{2}\right)$$

input

```
Int[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

output

```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1/2*(Cos[2*a]*CosIntegral[2*b*x]
) + Log[x]/2 + (Sin[2*a]*SinIntegral[2*b*x])/2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\left(-i\mathrm{e}^{2ibx}\pi\ \mathrm{csgn}(bx)+2i\mathrm{e}^{2ibx}\ \mathrm{Si}(2bx)+2\ln(x)\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2ibx}\ \mathrm{expIntegral}_1(-2ibx)+\mathrm{expIntegra}\right.}{4\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}$

input
```
int((c*sin(b*x+a)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*(-I*exp(2*I*b*x)
*Pi*csgn(b*x)+2*I*exp(2*I*b*x)*Si(2*b*x)+2*ln(x)*exp(2*I*(b*x+a))+exp(2*I*
b*x)*Ei(1,-2*I*b*x)+Ei(1,-2*I*b*x)*exp(2*I*(b*x+2*a)))/(exp(2*I*(b*x+a))-1
)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \frac{(\mathrm{Ei}(2i\,bx)\,e^{(2i\,a)} + \mathrm{Ei}(-2i\,bx)\,e^{(-2i\,a)} - 2\,\log(x))\,(-(c\cos(bx+a)^2 - c)\sin(bx}{4\,(\cos(bx+a)^2 - 1)}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="fricas")
```

output
```
1/4*(Ei(2*I*b*x)*e^(2*I*a) + Ei(-2*I*b*x)*e^(-2*I*a) - 2*log(x))*(-(c*cos(
b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(cos(b*x + a)^2 - 1)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \int \frac{(c\sin^3(a+bx))^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**(2/3)/x,x)
```

output
```
Integral((c*sin(a + b*x)**3)**(2/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx =$$
$$-\frac{1}{8}\left((E_1(2i\,bx)+E_1(-2i\,bx))\cos(2\,a)+(-i\,E_1(2i\,bx)+i\,E_1(-2i\,bx))\sin(2\,a)+2\,\log(bx)\right)c^{\frac{2}{3}}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="maxima")
```

output
```
-1/8*((exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a)
+ (-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))*sin(2*a)
 + 2*log(b*x))*c^(2/3)
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \int \frac{\left(c\sin(bx+a)^3\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x}\,dx = \int \frac{\left(c\sin(a+b\,x)^3\right)^{2/3}}{x}\,dx$$

input
```
int((c*sin(a + b*x)^3)^(2/3)/x,x)
```

output
```
int((c*sin(a + b*x)^3)^(2/3)/x, x)
```

Reduce [F]

$$\int \frac{\left(c \sin^3(a+bx)\right)^{2/3}}{x}\,dx = c^{\frac{2}{3}}\left(\int \frac{\sin\left(bx+a\right)^2}{x}dx\right)$$

input
```
int((c*sin(b*x+a)^3)^(2/3)/x,x)
```

output
```
c**(2/3)*int(sin(a + b*x)**2/x,x)
```

3.340 $\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx &= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\\ &\quad + b\,\text{CosIntegral}(2bx)\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\\ &\quad + b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\text{Si}(2bx)\end{aligned}$$

output

```
-(c*sin(b*x+a)^3)^(2/3)/x+b*Ci(2*b*x)*csc(b*x+a)^2*sin(2*a)*(c*sin(b*x+a)^
3)^(2/3)+b*cos(2*a)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)*Si(2*b*x)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx = \frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-1+\cos(2(a+bx))+2bx\,\text{CosIntegral}(2bx)\,\text{si}\right.}{2x}$$

input

```
Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

output
```
(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 2*b*x*Co
sIntegral[2*b*x]*Sin[2*a] + 2*b*x*Cos[2*a]*SinIntegral[2*b*x]))/(2*x)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7271, 3042, 3794, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \frac{\sin^2(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \frac{\sin(a+bx)^2}{x^2}dx$$

$$\Big\downarrow \text{3794}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(2b\int \frac{\sin(2a+2bx)}{2x}dx-\frac{\sin^2(a+bx)}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(b\int \frac{\sin(2a+2bx)}{x}dx-\frac{\sin^2(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(b\int \frac{\sin(2a+2bx)}{x}dx-\frac{\sin^2(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(b\left(\sin(2a)\int \frac{\cos(2bx)}{x}dx+\cos(2a)\int \frac{\sin(2bx)}{x}dx\right)-\frac{\sin^2(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(b\left(\sin(2a)\int\frac{\sin\left(2bx+\frac{\pi}{2}\right)}{x}dx+\cos(2a)\int\frac{\sin(2bx)}{x}dx\right)-\frac{\sin^2(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3780}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(b\left(\sin(2a)\int\frac{\sin\left(2bx+\frac{\pi}{2}\right)}{x}dx+\cos(2a)\text{Si}(2bx)\right)-\frac{\sin^2(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3783}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(b(\sin(2a)\,\text{CosIntegral}(2bx)+\cos(2a)\text{Si}(2bx))-\frac{\sin^2(a+bx)}{x}\right)$$

input

```
Int[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

output

```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-(Sin[a + b*x]^2/x) + b*(CosInteg
ral[2*b*x]*Sin[2*a] + Cos[2*a]*SinIntegral[2*b*x]))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\right)^{\frac{2}{3}}\left(-2i\mathrm{e}^{2i(bx+2a)}\operatorname{expIntegral}_{1}(-2ibx)bx+2i\mathrm{e}^{2ibx}\operatorname{expIntegral}_{1}(2ibx)bx-\mathrm{e}^{4i(bx+a)}-1+2\,\mathrm{e}^{2i(bx+a)}\right)}{4\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}x}$

input
```
int((c*sin(b*x+a)^3)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*(-2*I*exp(2*I*(b*
x+2*a))*Ei(1,-2*I*b*x)*b*x+2*I*exp(2*I*b*x)*Ei(1,2*I*b*x)*b*x-exp(4*I*(b*x
+a))-1+2*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1)^2/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \frac{(i\,bx\mathrm{Ei}(2i\,bx)\,e^{(2i\,a)} - i\,bx\mathrm{Ei}(-2i\,bx)\,e^{(-2i\,a)} - 2\,\cos(bx+a)^2 + 2)\,(-(c\cos(bx}{2\,(x\cos(bx+a)^2 - x)}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

output
```
1/2*(I*b*x*Ei(2*I*b*x)*e^(2*I*a) - I*b*x*Ei(-2*I*b*x)*e^(-2*I*a) - 2*cos(b
*x + a)^2 + 2)*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(x*cos(b*x + a
)^2 - x)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^2}\,dx = \int \frac{(c\sin^3(a+bx))^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x+a)**3)**(2/3)/x**2,x)
```

output
```
Integral((c*sin(a + b*x)**3)**(2/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.08

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx = \frac{(((-i\sqrt{3}+1)E_2(2i\,bx)+(i\sqrt{3}+1)E_2(-2i\,bx))\cos(2\,a)^3-((\sqrt{3}+i)E_2(2i}{}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

output
```
1/16*(((-I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_i
ntegral_e(2, -2*I*b*x))*cos(2*a)^3 - ((sqrt(3) + I)*exp_integral_e(2, 2*I*
b*x) + (sqrt(3) - I)*exp_integral_e(2, -2*I*b*x))*sin(2*a)^3 + (((-I*sqrt(
3) + 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_integral_e(2, -2*
I*b*x))*cos(2*a) - 4)*sin(2*a)^2 + ((I*sqrt(3) + 1)*exp_integral_e(2, 2*I*
b*x) + (-I*sqrt(3) + 1)*exp_integral_e(2, -2*I*b*x))*cos(2*a) - 4*cos(2*a)
^2 - (((sqrt(3) + I)*exp_integral_e(2, 2*I*b*x) + (sqrt(3) - I)*exp_integr
al_e(2, -2*I*b*x))*cos(2*a)^2 - (sqrt(3) - I)*exp_integral_e(2, 2*I*b*x) -
 (sqrt(3) + I)*exp_integral_e(2, -2*I*b*x))*sin(2*a))*b*c^(2/3)/(a*cos(2*a
)^2 + a*sin(2*a)^2 - (b*x + a)*(cos(2*a)^2 + sin(2*a)^2))
```

Giac [F]

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}}{x^2}\,dx$$

input
```
int((c*sin(a + b*x)^3)^(2/3)/x^2,x)
```

output
```
int((c*sin(a + b*x)^3)^(2/3)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx = \frac{c^{\frac{2}{3}}\left(\left(\int \frac{\sin(bx+a)^2}{x^2}dx\right)x-2\left(\int \frac{1}{x^2}dx\right)x-2\right)}{x}$$

input
```
int((c*sin(b*x+a)^3)^(2/3)/x^2,x)
```

output
```
(c**(2/3)*(int(sin(a + b*x)**2/x**2,x)*x - 2*int(1/x**2,x)*x - 2))/x
```

3.341 $\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x}$$

$$+b^2\cos(2a)\,\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - b^2\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}\,\text{Si}(2b$$

output
```
-1/2*(c*sin(b*x+a)^3)^(2/3)/x^2-b*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/x+b^2*
cos(2*a)*Ci(2*b*x)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)-b^2*csc(b*x+a)^2*si
n(2*a)*(c*sin(b*x+a)^3)^(2/3)*Si(2*b*x)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-1+\cos(2(a+bx))+4b^2x^2\cos(2a)\,\text{CosInteg}\right.}{4x^2}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

output
```
(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 4*b^2*x^
2*Cos[2*a]*CosIntegral[2*b*x] - 2*b*x*Sin[2*(a + b*x)] - 4*b^2*x^2*Sin[2*a
]*SinIntegral[2*b*x]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {7271, 3042, 3795, 14, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \frac{\sin^2(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\int \frac{\sin(a+bx)^2}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-2b^2\int \frac{\sin^2(a+bx)}{x}dx+b^2\int \frac{1}{x}dx-\frac{\sin^2(a+bx)}{2x^2}-\frac{b\sin(a+bx)\cos(a+bx)}{x}\right)$$

$$\Big\downarrow \text{14}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-2b^2\int \frac{\sin^2(a+bx)}{x}dx-\frac{\sin^2(a+bx)}{2x^2}-\frac{b\sin(a+bx)\cos(a+bx)}{x}+b^2\log(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-2b^2\int \frac{\sin(a+bx)^2}{x}dx-\frac{\sin^2(a+bx)}{2x^2}-\frac{b\sin(a+bx)\cos(a+bx)}{x}+b^2\log(x)\right)$$

$$\Big\downarrow \text{3793}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-2b^2\int\left(\frac{1}{2x}-\frac{\cos(2a+2bx)}{2x}\right)dx-\frac{\sin^2(a+bx)}{2x^2}-\frac{b\sin(a+bx)\cos(a+bx)}{x}+b^2\log($$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-2b^2\left(-\frac{1}{2}\cos(2a)\operatorname{CosIntegral}(2bx)+\frac{1}{2}\sin(2a)\operatorname{Si}(2bx)+\frac{\log(x)}{2}\right)-\frac{\sin^2(a+bx)}{2x^2}-$$

input
```
Int[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

output
```
Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(b^2*Log[x] - (b*Cos[a + b*x]*Sin[
a + b*x])/x - Sin[a + b*x]^2/(2*x^2) - 2*b^2*(-1/2*(Cos[2*a]*CosIntegral[2
*b*x]) + Log[x]/2 + (Sin[2*a]*SinIntegral[2*b*x])/2))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

method	result
risch	$\frac{\left(i c\,\mathrm{e}^{-3 i (b x+a)} \left(\mathrm{e}^{2 i (b x+a)}-1\right)^{3}\right)^{\frac{2}{3}} \left(4 \,\mathrm{expIntegral}_{1}(-2 i b x) \mathrm{e}^{2 i (b x+2 a)} x^{2} b^{2}+4 \,\mathrm{e}^{2 i b x} \,\mathrm{expIntegral}_{1}(2 i b x) x^{2} b^{2}-2 i \mathrm{e}^{4 i (b x+a)} x b+2 i b x+2\right.}{8 \left(\mathrm{e}^{2 i (b x+a)}-1\right)^{2} x^{2}}$

input
```
int((c*sin(b*x+a)^3)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(I*c*exp(-3*I*(b*x+a))*(exp(2*I*(b*x+a))-1)^3)^(2/3)*(4*Ei(1,-2*I*b*x)
*exp(2*I*(b*x+2*a))*x^2*b^2+4*exp(2*I*b*x)*Ei(1,2*I*b*x)*x^2*b^2-2*I*exp(4
*I*(b*x+a))*x*b+2*I*b*x+2*exp(2*I*(b*x+a))-exp(4*I*(b*x+a))-1)/(exp(2*I*(b
*x+a))-1)^2/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx =$$

$$-\frac{\left(b^2x^2\mathrm{Ei}(2i\,bx)\,e^{(2i\,a)}+b^2x^2\mathrm{Ei}(-2i\,bx)\,e^{(-2i\,a)}-2\,bx\cos\left(bx+a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)^2-1\right)\left(-(c\,\mathrm{co}\right.}{2\left(x^2\cos\left(bx+a\right)^2-x^2\right)}$$

input

```
integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

output

```
-1/2*(b^2*x^2*Ei(2*I*b*x)*e^(2*I*a) + b^2*x^2*Ei(-2*I*b*x)*e^(-2*I*a) - 2*
b*x*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 - 1)*(-(c*cos(b*x + a)^2 -
c)*sin(b*x + a))^(2/3)/(x^2*cos(b*x + a)^2 - x^2)
```

Sympy [F]

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

input

```
integrate((c*sin(b*x+a)**3)**(2/3)/x**3,x)
```

output

```
Integral((c*sin(a + b*x)**3)**(2/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.49

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx =$$

$$-\frac{\left(\left(\left(-i\sqrt{3}+1\right)E_3(2i\,bx)+\left(i\sqrt{3}+1\right)E_3(-2i\,bx)\right)\cos\left(2\,a\right)^3-\left(\left(\sqrt{3}+i\right)E_3(2i\,bx)+\left(\sqrt{3}-i\right)E_3(-2i\,bx}{}$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

output
```
-1/16*(((-I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)*exp_
integral_e(3, -2*I*b*x))*cos(2*a)^3 - ((sqrt(3) + I)*exp_integral_e(3, 2*I
*b*x) + (sqrt(3) - I)*exp_integral_e(3, -2*I*b*x))*sin(2*a)^3 + (((-I*sqrt
(3) + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)*exp_integral_e(3, -2
*I*b*x))*cos(2*a) - 2)*sin(2*a)^2 + ((I*sqrt(3) + 1)*exp_integral_e(3, 2*I
*b*x) + (-I*sqrt(3) + 1)*exp_integral_e(3, -2*I*b*x))*cos(2*a) - 2*cos(2*a
)^2 - (((sqrt(3) + I)*exp_integral_e(3, 2*I*b*x) + (sqrt(3) - I)*exp_integ
ral_e(3, -2*I*b*x))*cos(2*a)^2 - (sqrt(3) - I)*exp_integral_e(3, 2*I*b*x)
- (sqrt(3) + I)*exp_integral_e(3, -2*I*b*x))*sin(2*a))*b^2*c^(2/3)/(a^2*co
s(2*a)^2 + a^2*sin(2*a)^2 + (b*x + a)^2*(cos(2*a)^2 + sin(2*a)^2) - 2*(a*c
os(2*a)^2 + a*sin(2*a)^2)*(b*x + a))
```

Giac [F]

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{2/3}}{x^3}\,dx$$

input

```
int((c*sin(a + b*x)^3)^(2/3)/x^3,x)
```

output

```
int((c*sin(a + b*x)^3)^(2/3)/x^3, x)
```

Reduce [F]

$$\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx = \text{Too large to display}$$

input

```
int((c*sin(b*x+a)^3)^(2/3)/x^3,x)
```

output

```
(c**(2/3)*(2*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**4*b*x + 4*cos(a +
 b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b*x + 2*cos(a + b*x)*sin(a + b*x)*b
*x - 8*cos(a + b*x)*tan((a + b*x)/2)**4 - 16*cos(a + b*x)*tan((a + b*x)/2)
**2 - 8*cos(a + b*x) - 48*int(tan((a + b*x)/2)**2/(tan((a + b*x)/2)**4*x +
 2*tan((a + b*x)/2)**2*x + x),x)*tan((a + b*x)/2)**4*b**2*x**2 - 96*int(ta
n((a + b*x)/2)**2/(tan((a + b*x)/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)
*tan((a + b*x)/2)**2*b**2*x**2 - 48*int(tan((a + b*x)/2)**2/(tan((a + b*x)
/2)**4*x + 2*tan((a + b*x)/2)**2*x + x),x)*b**2*x**2 + 6*log(x)*tan((a + b
*x)/2)**4*b**2*x**2 + 12*log(x)*tan((a + b*x)/2)**2*b**2*x**2 + 6*log(x)*b
**2*x**2 + sin(a + b*x)**2*tan((a + b*x)/2)**4 + 2*sin(a + b*x)**2*tan((a
+ b*x)/2)**2 + sin(a + b*x)**2 + 8*sin(a + b*x)*tan((a + b*x)/2)**4*b*x +
16*sin(a + b*x)*tan((a + b*x)/2)**2*b*x + 8*sin(a + b*x)*b*x - 8*tan((a +
b*x)/2)**4 - 16*tan((a + b*x)/2)**2 - 32*tan((a + b*x)/2)*b*x + 8))/(6*x**
2*(tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1))
```

3.342 $\int x^m(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int x^m\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)}$$
$$+2^{-\frac{7}{2}-\frac{m}{2}}e^{2ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},-2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$
$$+2^{-\frac{7}{2}-\frac{m}{2}}e^{-2ia}x^{1+m}\left(ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

output

```
x^(1+m)*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/(2+2*m)+2^(-7/2-1/2*m)*exp
(2*I*a)*x^(1+m)*(-I*b*x^2)^(-1/2-1/2*m)*csc(b*x^2+a)^2*GAMMA(1/2+1/2*m,-2*
I*b*x^2)*(c*sin(b*x^2+a)^3)^(2/3)+2^(-7/2-1/2*m)*x^(1+m)*(I*b*x^2)^(-1/2-1
/2*m)*csc(b*x^2+a)^2*GAMMA(1/2+1/2*m,2*I*b*x^2)*(c*sin(b*x^2+a)^3)^(2/3)/e
xp(2*I*a)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int x^m \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx = \frac{2^{\frac{1}{2}(-7-m)} x^{1+m} \left(b^2x^4\right)^{\frac{1}{2}(-1-m)} \csc^2\left(a+bx^2\right) \left(2^{\frac{5+m}{2}} \left(b^2x^4\right)^{\frac{1+m}{2}} + (1+m)\left(-ibx^2\right)^{\frac{1+m}{2}} \Gamma(\frac{1+m}{2}, 2}{}$$

input

```
Integrate[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output

```
(2^((-7 - m)/2)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]^2*(2^((5 +
 m)/2)*(b^2*x^4)^((1 + m)/2) + (1 + m)*((-I)*b*x^2)^((1 + m)/2)*Gamma[(1 +
 m)/2, (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) + (1 + m)*(I*b*x^2)^((1 + m)/2
)*Gamma[(1 + m)/2, (-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a]))*(c*Sin[a + b*x^2
]^3)^(2/3))/(1 + m)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx$$

$$\Big\downarrow \ 7271$$

$$\csc^2\left(a+bx^2\right) \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} \int x^m \sin^2\left(bx^2+a\right) dx$$

$$\Big\downarrow \ 3884$$

$$\csc^2\left(a+bx^2\right) \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} \int \left(\frac{x^m}{2} - \frac{1}{2} x^m \cos\left(2bx^2+2a\right)\right) dx$$

$$\Big\downarrow \ 2009$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(e^{2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\Gamma\left(\frac{m+1}{2},-2ibx^2\right)+e^{-2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\right.$$

input
```
Int[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
Csc[a + b*x^2]^2*(x^(1 + m)/(2*(1 + m)) + 2^(-7/2 - m/2)*E^((2*I)*a)*x^(1
+ m)*((-I)*b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*b*x^2] + (2^(-7/2 -
 m/2)*x^(1 + m)*(I*b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (2*I)*b*x^2])/E^((
2*I)*a))*(c*Sin[a + b*x^2]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^{m}\left(c\sin\left(b\,x^{2}+a\right)^{3}\right)^{\frac{2}{3}}dx$$

input
```
int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

output
```
int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx =$$

$$-\frac{(8\,bxx^m-(i\,m+i)e^{\left(-\frac{1}{2}\,(m-1)\log(2i\,b)-2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2i\,bx^2\right)-(-i\,m-i)e^{\left(-\frac{1}{2}\,(m-1)\log(-2i\,b)+2i\,a\right)}\Gamma(\frac{1}{2}\,m+}{16\left((bm+b)\cos\left(bx^2+a\right)^2-bm-b\right)}$$

input
```
integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/16*(8*b*x*x^m - (I*m + I)*e^(-1/2*(m - 1)*log(2*I*b) - 2*I*a)*gamma(1/2
*m + 1/2, 2*I*b*x^2) - (-I*m - I)*e^(-1/2*(m - 1)*log(-2*I*b) + 2*I*a)*gam
ma(1/2*m + 1/2, -2*I*b*x^2))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2
/3)/((b*m + b)*cos(b*x^2 + a)^2 - b*m - b)
```

Sympy [F]

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = \int x^m\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x**m*(c*sin(b*x**2+a)**3)**(2/3),x)
```

output
```
Integral(x**m*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [F]

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = \int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

input
```
integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^2 + 2*a), x))*c^(2/3)/(m + 1
)
```

Giac [F]

$$\int x^m \left(c \sin^3\left(a+b x^2\right)\right)^{2/3} \, dx = \int \left(c \sin\left(b x^2+a\right)^3\right)^{\frac{2}{3}} x^m \, dx$$

input
```
integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(c \sin^3\left(a+b x^2\right)\right)^{2/3} \, dx = \int x^m \left(c \sin\left(b\,x^2+a\right)^3\right)^{2/3} \, dx$$

input
```
int(x^m*(c*sin(a + b*x^2)^3)^(2/3),x)
```

output
```
int(x^m*(c*sin(a + b*x^2)^3)^(2/3), x)
```

Reduce [F]

$$\int x^m \left(c \sin^3\left(a+b x^2\right)\right)^{2/3} \, dx = \frac{c^{\frac{2}{3}} \Big(2 x^m x - 2\left(\int x^m dx\right) m - 2\left(\int x^m dx\right) + \left(\int x^m \sin\left(b\,x^2+a\right)^2 dx\right) m + \int x^m \sin\left(b\,x^2+a}{m+1}$$

input
```
int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

output

```
(c**(2/3)*(2*x***m*x - 2*int(x**m,x)*m - 2*int(x**m,x) + int(x**m*sin(a + b
*x***2)**2,x)*m + int(x**m*sin(a + b*x**2)**2,x)))/(m + 1)
```

3.343 $\int x^3(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{8}x^4\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

output
```
1/8*(c*sin(b*x^2+a)^3)^(2/3)/b^2-1/4*x^2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(
2/3)/b+1/8*x^4*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=-\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\cos\left(2(a+bx^2)\right)+2bx^2(-bx^2+\sin\left(2(a+bx^2)\right))\right)}{16b^2}$$

input
```
Integrate[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
-1/16*(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(Cos[2*(a + b*x^2)] + 2
*b*x^2*(-(b*x^2) + Sin[2*(a + b*x^2)])))/b^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7271, 3860, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int x^3\sin^2\left(bx^2+a\right)dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int x^2\sin^2\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int x^2\sin\left(bx^2+a\right)^2dx^2$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\frac{\int x^2dx^2}{2}+\frac{\sin^2\left(a+bx^2\right)}{4b^2}-\frac{x^2\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\frac{\sin^2\left(a+bx^2\right)}{4b^2}-\frac{x^2\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}+\frac{x^4}{4}\right)$$

input
```
Int[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output

```
(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(x^4/4 - (x^2*Cos[a + b*x^2]*
Sin[a + b*x^2])/(2*b) + Sin[a + b*x^2]^2/(4*b^2)))/2
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3860

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{x^{4}\left(ic\,e^{-3i\left(b\,x^{2}+a\right)}\left(e^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}}e^{2i\left(b\,x^{2}+a\right)}}{8\left(e^{2i\left(b\,x^{2}+a\right)}-1\right)^{2}}-\frac{i\left(2b\,x^{2}+i\right)\left(ic\,e^{-3i\left(b\,x^{2}+a\right)}\left(e^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}}e^{4i\left(b\,x^{2}+a\right)}}{32b^{2}\left(e^{2i\left(b\,x^{2}+a\right)}-1\right)^{2}}+$

input
```
int(x^3*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x^4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2
+a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))-1/32*I/b^2*(2*b*x^2+I)/(exp(2*I*(b*x^2
+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(4*I
*(b*x^2+a))+1/32*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3
)/(exp(2*I*(b*x^2+a))-1)^2*(2*b*x^2-I)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int x^{3}\left(c\sin^{3}\left(a+bx^{2}\right)\right)^{2/3}\,dx =$$

$$-\frac{\left(2\,b^{2}x^{4}-4\,bx^{2}\cos\left(bx^{2}+a\right)\sin\left(bx^{2}+a\right)-2\,\cos\left(bx^{2}+a\right)^{2}+1\right)\left(-\left(c\cos\left(bx^{2}+a\right)^{2}-c\right)\sin\left(bx^{2}+a\right)\right.}{16\left(b^{2}\cos\left(bx^{2}+a\right)^{2}-b^{2}\right)}$$

input
```
integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/16*(2*b^2*x^4 - 4*b*x^2*cos(b*x^2 + a)*sin(b*x^2 + a) - 2*cos(b*x^2 + a
)^2 + 1)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 +
 a)^2 - b^2)
```

Sympy [F]

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x**3*(c*sin(b*x**2+a)**3)**(2/3),x)
```

output
```
Integral(x**3*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=-\frac{\left(2\,b^2x^4-2\,bx^2\sin\left(2\,bx^2+2\,a\right)-\cos\left(2\,bx^2+2\,a\right)\right)c^{\frac{2}{3}}}{32\,b^2}$$

input
```
integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/32*(2*b^2*x^4 - 2*b*x^2*sin(2*b*x^2 + 2*a) - cos(2*b*x^2 + 2*a))*c^(2/3
)/b^2
```

Giac [F]

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

input
```
integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x^3\,\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

input
```
int(x^3*(c*sin(a + b*x^2)^3)^(2/3),x)
```

output
```
int(x^3*(c*sin(a + b*x^2)^3)^(2/3), x)
```

Reduce [F]

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int \sin\left(b\,x^2+a\right)^2 x^3dx\right)$$

input
```
int(x^3*(c*sin(b*x^2+a)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(a + b*x**2)**2*x**3,x)
```

3.344 $\int x^2(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int x^2(c\sin^3(a+bx^2))^{2/3}\,dx=\frac{1}{6}x^3\csc^2(a+bx^2)\,(c\sin^3(a+bx^2))^{2/3}+\frac{\sqrt{\pi}\cos(2a)\csc^2(a+bx^2)\,\mathrm{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)(c\sin^3(a+bx^2))^{2/3}}{16b^{3/2}}+\frac{\sqrt{\pi}\csc^2(a+bx^2)\,\mathrm{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\,(c\sin^3(a+bx^2))^{2/3}}{16b^{3/2}}-\frac{x\csc^2(a+bx^2)\,(c\sin^3(a+bx^2))^{2/3}\sin(2a+2bx^2)}{8b}$$

output
```
1/6*x^3*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)+1/16*Pi^(1/2)*cos(2*a)*csc
(b*x^2+a)^2*FresnelS(2*b^(1/2)*x/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)/b^(3/2
)+1/16*Pi^(1/2)*csc(b*x^2+a)^2*FresnelC(2*b^(1/2)*x/Pi^(1/2))*sin(2*a)*(c*
sin(b*x^2+a)^3)^(2/3)/b^(3/2)-1/8*x*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3
)*sin(2*b*x^2+2*a)/b
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.58

$$\int x^2 \left(c \sin^3 \left(a\right.\right.$$

$$\left.\left.+bx^2\right)\right)^{2/3} dx = \frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(3\sqrt{\pi}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+3\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\mathrm{s}\right.}{48b^{3/2}}$$

input
```
Integrate[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(3*Sqrt[Pi]*Cos[2*a]*FresnelS
[(2*Sqrt[b]*x)/Sqrt[Pi]] + 3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin
[2*a] + 2*Sqrt[b]*x*(4*b*x^2 - 3*Sin[2*(a + b*x^2)])))/(48*b^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int x^2\sin^2\left(bx^2+a\right)dx$$

$$\Big\downarrow \text{3884}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2bx^2+2a\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\frac{\sqrt{\pi}\sin(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{x\sin\left(2a+\right.}{8b}\right.$$

input
```
Int[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(x^3/6 + (Sqrt[Pi]*Cos[2*a]*Fr
esnelS[(2*Sqrt[b]*x)/Sqrt[Pi]])/(16*b^(3/2)) + (Sqrt[Pi]*FresnelC[(2*Sqrt[
b]*x)/Sqrt[Pi]]*Sin[2*a])/(16*b^(3/2)) - (x*Sin[2*a + 2*b*x^2])/(8*b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.58

method	result
risch	$\frac{ix\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}}}{16b\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{2}}-\frac{i\left(ic\,\mathrm{e}^{-3i\left(b\,x^{2}+a\right)}\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}}\mathrm{e}^{2ib\,x^{2}}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{64\left(\mathrm{e}^{2i\left(b\,x^{2}+a\right)}-1\right)^{2}b\sqrt{ib}}+$

input
```
int(x^2*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/16*I*x/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x
^2+a))-1)^3)^(2/3)-1/64*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^
3)^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)/b*Pi^(1/2)*2^(1/2)/(I*b)^
(1/2)*erf(2^(1/2)*(I*b)^(1/2)*x)+1/4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*
I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(-1/4*I*x/b*exp(4*I*(b*x^2+a)
)+1/8*I/b*Pi^(1/2)/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a
)))-1/6*x^3/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*
x^2+a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx =$$

$$-\frac{\left(16\,b^2x^3-24\,bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)+3\,\left(-i\,\pi e^{(2i\,a)}+i\,\pi e^{(-2i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+3\,\left(\pi e^{(2i\,a)}+\pi\right.}{96\,\left(b^2\cos\left(bx^2+a\right)^2-b^2\right)}$$

input
```
integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/96*(16*b^2*x^3 - 24*b*x*cos(b*x^2 + a)*sin(b*x^2 + a) + 3*(-I*pi*e^(2*I
*a) + I*pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi)) + 3*(pi*e^(2
*I*a) + pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_sin(2*x*sqrt(b/pi)))*(-(c*cos(b*
x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a)^2 - b^2)
```

Sympy [F]

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x**2*(c*sin(b*x**2+a)**3)**(2/3),x)
```

output
```
Integral(x**2*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.51

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=$$

$$-\frac{3\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left((i+1)\cos\left(2a\right)-(i-1)\sin\left(2a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}x\right)+\left(-(i-1)\cos\left(2a\right)+(i+1)\sin\left(2a\right)\right)}{768\,b^3}$$

input
```
integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/768*(3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*
erf(sqrt(2*I*b)*x) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*
b)*x))*b^(3/2)*c^(2/3) + 16*(4*b^3*x^3 - 3*b^2*x*sin(2*b*x^2 + 2*a))*c^(2/
3))/b^3
```

Giac [F]

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}x^2\,dx$$

input
```
integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x^2\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

input
```
int(x^2*(c*sin(a + b*x^2)^3)^(2/3),x)
```

output
```
int(x^2*(c*sin(a + b*x^2)^3)^(2/3), x)
```

Reduce [F]

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int\sin\left(b\,x^2+a\right)^2x^2dx\right)$$

input
```
int(x^2*(c*sin(b*x^2+a)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(a + b*x**2)**2*x**2,x)
```

3.345 $\int x(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=-\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

output

```
-1/4*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(2/3)/b+1/4*x^2*csc(b*x^2+a)^2*(c*sin
(b*x^2+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2\left(a+bx^2\right)-\sin\left(2\left(a+bx^2\right)\right)\right)}{8b}$$

input

```
Integrate[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(2*(a + b*x^2) - Sin[2*(a + b
*x^2)]))/(8*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7266, 3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \left(c\sin^3\left(bx^2+a\right)\right)^{2/3}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \left(c\sin\left(bx^2+a\right)^3\right)^{2/3}dx^2$$

$$\Big\downarrow \text{3686}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \sin^2\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \sin\left(bx^2+a\right)^2dx^2$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\frac{\int 1dx^2}{2}-\frac{\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\frac{x^2}{2}-\frac{\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}\right)$$

input
```
Int[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(x^2/2 - (Cos[a + b*x^2]*Sin[
a + b*x^2])/(2*b)))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{x^{2}\left(i c\,{\mathrm e}^{-3 i \left(b \,x^{2}+a \right)} \left({\mathrm e}^{2 i \left(b \,x^{2}+a \right)}-1\right)^{3}\right)^{\frac{2}{3}} {\mathrm e}^{2 i \left(b \,x^{2}+a \right)}}{4 \left({\mathrm e}^{2 i \left(b \,x^{2}+a \right)}-1\right)^{2}}-\frac{i \left(i c\,{\mathrm e}^{-3 i \left(b \,x^{2}+a \right)} \left({\mathrm e}^{2 i \left(b \,x^{2}+a \right)}-1\right)^{3}\right)^{\frac{2}{3}} {\mathrm e}^{4 i \left(b \,x^{2}+a \right)}}{16 b \left({\mathrm e}^{2 i \left(b \,x^{2}+a \right)}-1\right)^{2}}+\frac{i \left(i c\,{\mathrm e}^{-3 i (b}}{16}$

input
```
int(x*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2
+a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))-1/16*I/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c
*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(4*I*(b*x^2+a))+1/
16*I/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a
))-1)^3)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = -\frac{\left(bx^2-\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{4\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

input
```
integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(b*x^2 - cos(b*x^2 + a)*sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*si
n(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F]

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(x*(c*sin(b*x**2+a)**3)**(2/3),x)
```

output
```
Integral(x*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=-\frac{(2\,bx^2-\sin\left(2\,bx^2+2\,a\right))c^{\frac{2}{3}}}{16\,b}$$

input
```
integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/16*(2*b*x^2 - sin(2*b*x^2 + 2*a))*c^(2/3)/b
```

Giac [F]

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int x\,{\left(c\,{\sin\left(b\,x^2+a\right)}^3\right)}^{2/3}\,dx$$

input
```
int(x*(c*sin(a + b*x^2)^3)^(2/3),x)
```

output
```
int(x*(c*sin(a + b*x^2)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\frac{c^{\frac{2}{3}}\left(-\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)+b\,x^2\right)}{4b}$$

input
```
int(x*(c*sin(b*x^2+a)^3)^(2/3),x)
```

output
```
(c**(2/3)*( - cos(a + b*x**2)*sin(a + b*x**2) + b*x**2))/(4*b)
```

3.346 $\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx=\frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{\sqrt{\pi}\csc^2\left(a+bx^2\right)\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}$$

output

```
1/2*x*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)-1/4*Pi^(1/2)*cos(2*a)*csc(b*
x^2+a)^2*FresnelC(2*b^(1/2)*x/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)/b^(1/2)+1
/4*Pi^(1/2)*csc(b*x^2+a)^2*FresnelS(2*b^(1/2)*x/Pi^(1/2))*sin(2*a)*(c*sin(
b*x^2+a)^3)^(2/3)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.63

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = \frac{\csc^2\left(a+bx^2\right)\left(2\sqrt{b}x-\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\right)(c\sin}{4\sqrt{b}}$$

input

```
Integrate[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output

```
(Csc[a + b*x^2]^2*(2*Sqrt[b]*x - Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/
Sqrt[Pi]] + Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a +
 b*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {7271, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \sin^2\left(bx^2+a\right)dx$$

$$\Big\downarrow \text{3838}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(2bx^2+2a\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\frac{\sqrt{\pi}\cos(2a)\,\mathrm{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}+\frac{\sqrt{\pi}\sin(2a)\,\mathrm{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}+\frac{x}{2}\right)$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

output
```
Csc[a + b*x^2]^2*(x/2 - (Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]
])/(4*Sqrt[b]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(4*S
qrt[b]))*(c*Sin[a + b*x^2]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.51

method	result
risch	$\frac{\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)^{\frac{2}{3}}\mathrm{e}^{2ib\,x^2}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{16\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2\sqrt{ib}}+\frac{\mathrm{erf}\left(\sqrt{-2ib}\,x\right)\sqrt{\pi}\left(ic\,\mathrm{e}^{-3i\left(b\,x^2+a\right)}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\right)}{8\sqrt{-2ib}\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2}$

input
```
int((c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/16*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)/(exp(2*I*(b*
x^2+a))-1)^2*exp(2*I*b*x^2)*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*erf(2^(1/2)*(I*b)
^(1/2)*x)+1/8*erf((-2*I*b)^(1/2)*x)/(-2*I*b)^(1/2)*Pi^(1/2)/(exp(2*I*(b*x^
2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(2*
I*(b*x^2+2*a))-1/2*x/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(ex
p(2*I*(b*x^2+a))-1)^3)^(2/3)*exp(2*I*(b*x^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = \frac{\left(\left(\pi e^{(2ia)}+\pi e^{(-2ia)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2x\sqrt{\frac{b}{\pi}}\right)+\left(i\pi e^{(2ia)}-i\pi e^{(-2ia)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2x\sqrt{\frac{b}{\pi}}\right)-4bx\right)\bigl(-\bigl(}{8\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

output
```
1/8*((pi*e^(2*I*a) + pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))
 + (I*pi*e^(2*I*a) - I*pi*e^(-2*I*a))*sqrt(b/pi)*fresnel_sin(2*x*sqrt(b/pi
)) - 4*b*x)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b*cos(b*x^2
+ a)^2 - b)
```

Sympy [F]

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int \left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(2/3),x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(2/3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=$$

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(((i-1)\cos(2a)+(i+1)\sin(2a))\operatorname{erf}\left(\sqrt{2i\,b}x\right)+(-(i+1)\cos(2a)-(i-1)\sin(2a))\operatorname{erf}}{64\,b^2}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/64*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf
(sqrt(2*I*b)*x) + (-(I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(-2*I*b)*
x))*b^(3/2)*c^(2/3) + 16*b^2*c^(2/3)*x)/b^2
```

Giac [F]

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=\int \left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(2/3),x)
```

output
```
int((c*sin(a + b*x^2)^3)^(2/3), x)
```

Reduce [F]

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int \sin\left(b\,x^2+a\right)^2dx\right)$$

input
```
int((c*sin(b*x^2+a)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(a + b*x**2)**2,x)
```

3.347 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\begin{aligned}\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx &= \\ &-\frac{1}{4}\cos(2a)\operatorname{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} \\ &+\frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} \\ &+\frac{1}{4}\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\operatorname{Si}\left(2bx^2\right)\end{aligned}$$

output

```
-1/4*cos(2*a)*Ci(2*b*x^2)*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)+1/2*csc(
b*x^2+a)^2*ln(x)*(c*sin(b*x^2+a)^3)^(2/3)+1/4*csc(b*x^2+a)^2*sin(2*a)*(c*s
in(b*x^2+a)^3)^(2/3)*Si(2*b*x^2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx=\frac{1}{4}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\cos(2a)\,\text{CosIntegral}\left(2bx^2\right)+2\log(x)+\sin(2a)\text{Si}\left(2bx^2\right)\right)$$

input
```
Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

output
```
(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x
^2]) + 2*Log[x] + Sin[2*a]*SinIntegral[2*b*x^2]))/4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \frac{\sin^2\left(bx^2+a\right)}{x}dx$$

$$\Big\downarrow \text{3884}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \left(\frac{1}{2x}-\frac{\cos\left(2bx^2+2a\right)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\frac{1}{4}\cos(2a)\,\text{CosIntegral}\left(2bx^2\right)+\frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)+\frac{\log(x)}{2}\right)$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

output
```
Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-1/4*(Cos[2*a]*CosIntegral[2*
b*x^2]) + Log[x]/2 + (Sin[2*a]*SinIntegral[2*b*x^2])/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

method	result
risch	$\frac{\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a\right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}} \left(i \mathrm{e}^{2 i b\,x^{2}} \pi \,\mathrm{csgn}\left(b\,x^{2}\right)-2 i \mathrm{e}^{2 i b\,x^{2}} \mathrm{Si}\left(2 b\,x^{2}\right)-\mathrm{expIntegral}_{1}\left(-2 i b\,x^{2}\right) \mathrm{e}^{2 i \left(b\,x^{2}+2 a\right)}-4 \ln\left(x\right) \mathrm{e}^{2 i}\right.}{8 \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{2}}$

input
```
int((c*sin(b*x^2+a)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(I*exp(2*I*b*
x^2)*Pi*csgn(b*x^2)-2*I*exp(2*I*b*x^2)*Si(2*b*x^2)-Ei(1,-2*I*b*x^2)*exp(2*
I*(b*x^2+2*a))-4*ln(x)*exp(2*I*(b*x^2+a))-exp(2*I*b*x^2)*Ei(1,-2*I*b*x^2))
/(exp(2*I*(b*x^2+a))-1)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = \frac{\left(\mathrm{Ei}\left(2i\,bx^2\right)e^{(2i\,a)}+\mathrm{Ei}\left(-2i\,bx^2\right)e^{(-2i\,a)}-4\,\log\left(x\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)}{8\left(\cos\left(bx^2+a\right)^2-1\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(2*I*b*x^2)*e^(2*I*a) + Ei(-2*I*b*x^2)*e^(-2*I*a) - 4*log(x))*(-(c*
cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(cos(b*x^2 + a)^2 - 1)
```

Sympy [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = \int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(2/3)/x,x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(2/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = \frac{1}{16}\left(\left(\mathrm{Ei}\left(2i\,bx^2\right)+\mathrm{Ei}\left(-2i\,bx^2\right)\right)\cos\left(2\,a\right)-\left(-i\,\mathrm{Ei}\left(2i\,bx^2\right)+i\,\mathrm{Ei}\left(-2i\,bx^2\right)\right)\sin$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="maxima")
```

output
```
1/16*((Ei(2*I*b*x^2) + Ei(-2*I*b*x^2))*cos(2*a) - (-I*Ei(2*I*b*x^2) + I*Ei
(-2*I*b*x^2))*sin(2*a) - 4*log(x))*c^(2/3)
```

Giac [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(2/3)/x,x)
```

output
```
int((c*sin(a + b*x^2)^3)^(2/3)/x, x)
```

Reduce [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = c^{\frac{2}{3}}\left(\int \frac{\sin\left(b\,x^2+a\right)^2}{x}dx\right)$$

input

```
int((c*sin(b*x^2+a)^3)^(2/3)/x,x)
```

output

```
c**(2/3)*int(sin(a + b*x**2)**2/x,x)
```

3.348 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 132

$$\begin{aligned}\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx &= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} \\ &\quad+\sqrt{b}\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} \\ &\quad+\sqrt{b}\sqrt{\pi}\csc^2\left(a+bx^2\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\end{aligned}$$

output

```
-(c*sin(b*x^2+a)^3)^(2/3)/x+b^(1/2)*Pi^(1/2)*cos(2*a)*csc(b*x^2+a)^2*Fresn
elS(2*b^(1/2)*x/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)+b^(1/2)*Pi^(1/2)*csc(b*
x^2+a)^2*FresnelC(2*b^(1/2)*x/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.81

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \frac{\csc^2\left(a+bx^2\right)\left(-1+\cos\left(2(a+bx^2)\right)+2\sqrt{b}\sqrt{\pi}x\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+2}{2x}$$

input
```
Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

output
```
(Csc[a + b*x^2]^2*(-1 + Cos[2*(a + b*x^2)] + 2*Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]
*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] + 2*Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[
b]*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b*x^2]^3)^(2/3))/(2*x)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {7271, 3874, 5084, 3854, 3834, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \frac{\sin^2\left(bx^2+a\right)}{x^2}dx$$

$$\Big\downarrow \text{3874}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(4b\int \cos\left(bx^2+a\right)\sin\left(bx^2+a\right)dx-\frac{\sin^2\left(a+bx^2\right)}{x}\right)$$

$$\Big\downarrow \text{5084}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2b\int \sin\left(2\left(bx^2+a\right)\right)dx-\frac{\sin^2\left(a+bx^2\right)}{x}\right)$$

$$\Big\downarrow \color{blue}{3854}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2b\int \sin\left(2bx^2+2a\right)dx-\frac{\sin^2\left(a+bx^2\right)}{x}\right)$$

$$\Big\downarrow \color{blue}{3834}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2b\left(\sin(2a)\int \cos\left(2bx^2\right)dx+\cos(2a)\int \sin\left(2bx^2\right)dx\right)-\frac{\sin^2\left(a+bx^2\right)}{x}\right)$$

$$\Big\downarrow \color{blue}{3832}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2b\left(\sin(2a)\int \cos\left(2bx^2\right)dx+\frac{\sqrt{\pi}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{2\sqrt{b}}\right)-\frac{\sin^2\left(a+b\right.}{x}\right.$$

$$\Big\downarrow \color{blue}{3833}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2b\left(\frac{\sqrt{\pi}\sin(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{2\sqrt{b}}+\frac{\sqrt{\pi}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{2\sqrt{b}}\right)-\sin^2\left(\right.\right.$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

output
```
Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(2*b*((Sqrt[Pi]*Cos[2*a]*Fresn
elS[(2*Sqrt[b]*x)/Sqrt[Pi]])/(2*Sqrt[b]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x
)/Sqrt[Pi]]*Sin[2*a])/(2*Sqrt[b])) - Sin[a + b*x^2]^2/x)
```

Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3834
```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[Sin[c]   In
t[Cos[d*(e + f*x)^2], x], x] + Simp[Cos[c]   Int[Sin[d*(e + f*x)^2], x], x]
 /; FreeQ[{c, d, e, f}, x]
```

rule 3854
```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

rule 3874
```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[Sin[a + b*x^
n]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[
m + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

rule 5084
```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Sin[
2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.28

method	result
risch	$-\frac{\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a \right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1\right)^{3}\right)^{\frac{2}{3}}}{4 x \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1\right)^{2}}-\frac{i \left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a \right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1\right)^{3}\right)^{\frac{2}{3}} \mathrm{e}^{2 i b\,x^{2}} b \sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{i b}\,x \right)}{4 \left(\mathrm{e}^{2 i \left(b\,x^{2}+a \right)}-1\right)^{2} \sqrt{i b}}+$

input
```
int((c*sin(b*x^2+a)^3)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a
))-1)^3)^(2/3)-1/4*I*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2
/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*b*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)
*erf(2^(1/2)*(I*b)^(1/2)*x)+1/4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*
x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(-1/x*exp(4*I*(b*x^2+a))+2*I*b*Pi^
(1/2)/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))+1/2/x/(ex
p(2*I*(b*x^2+a))-1)^2*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(
2/3)*exp(2*I*(b*x^2+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \frac{\left(\left(i\,\pi x e^{(2i\,a)} - i\,\pi x e^{(-2i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right) - \left(\pi x e^{(2i\,a)} + \pi x e^{(-2i\,a)}\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x\right.\right.}{2\left(x\cos\left(bx^2+\right.\right.}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

output
```
1/2*((I*pi*x*e^(2*I*a) - I*pi*x*e^(-2*I*a))*sqrt(b/pi)*fresnel_cos(2*x*sqr
t(b/pi)) - (pi*x*e^(2*I*a) + pi*x*e^(-2*I*a))*sqrt(b/pi)*fresnel_sin(2*x*s
qrt(b/pi)) - 2*cos(b*x^2 + a)^2 + 2)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2
+ a))^(2/3)/(x*cos(b*x^2 + a)^2 - x)
```

Sympy [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(2/3)/x**2,x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(2/3)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \frac{\sqrt{2}\sqrt{bx^2}\left(\left(-(i+1)\ \sqrt{2}\Gamma\left(-\frac{1}{2},2i\,bx^2\right)+(i-1)\ \sqrt{2}\Gamma\left(-\frac{1}{2},-2i\,bx^2\right)\right)\cos\left(2\,a\right)+}{32}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

output
```
1/32*(sqrt(2)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) + (I -
 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*cos(2*a) + ((I - 1)*sqrt(2)*gamma(-1/
2, 2*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*sin(2*a))*c^(2/3)
 + 8*c^(2/3))/x
```

Giac [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x^2}\,dx$$

input
```
int((c*sin(a + b*x^2)^3)^(2/3)/x^2,x)
```

output
```
int((c*sin(a + b*x^2)^3)^(2/3)/x^2, x)
```

Reduce [F]

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^2}\,dx = c^{\frac{2}{3}}\left(\int \frac{\sin(b\,x^2+a)^2}{x^2}dx\right)$$

input
```
int((c*sin(b*x^2+a)^3)^(2/3)/x^2,x)
```

output
```
c**(2/3)*int(sin(a + b*x**2)**2/x**2,x)
```

3.349 $\int \frac{\left(c\sin^3(a+bx^2)\right)^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{\left(c\sin^3(a+bx^2)\right)^{2/3}}{x^3}\,dx = -\frac{\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{4x^2}$$
$$+\frac{\cos\left(2(a+bx^2)\right)\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{4x^2}$$
$$+\frac{1}{2}b\,\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\sin(2a)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right.\right.$$

output
```
-1/4*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/x^2+1/4*cos(2*b*x^2+2*a)*csc(
b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/x^2+1/2*b*Ci(2*b*x^2)*csc(b*x^2+a)^2*s
in(2*a)*(c*sin(b*x^2+a)^3)^(2/3)+1/2*b*cos(2*a)*csc(b*x^2+a)^2*(c*sin(b*x^
2+a)^3)^(2/3)*Si(2*b*x^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-1+\cos\left(2(a+bx^2)\right)+2bx^2\,\mathrm{CosIntegral}\left(\right.\right.}{4x^2}$$

input

```
Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

output

```
(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-1 + Cos[2*(a + b*x^2)] + 2*
b*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + 2*b*x^2*Cos[2*a]*SinIntegral[2*b*x^2
]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \frac{\sin^2\left(bx^2+a\right)}{x^3}dx$$

$$\Big\downarrow \text{3884}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\int \left(\frac{1}{2x^3}-\frac{\cos\left(2bx^2+2a\right)}{2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\frac{1}{2}b\sin(2a)\,\mathrm{CosIntegral}\left(2bx^2\right)+\frac{1}{2}b\cos(2a)\mathrm{Si}\left(2bx^2\right)+\frac{\cos\left(2(a+bx^2)\right)}{4x^2}-\right.$$

input
```
Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

output
```
Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-1/4*1/x^2 + Cos[2*(a + b*x^2
)]/(4*x^2) + (b*CosIntegral[2*b*x^2]*Sin[2*a])/2 + (b*Cos[2*a]*SinIntegral
[2*b*x^2])/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3884
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(i c\,\mathrm{e}^{-3 i \left(b\,x^{2}+a\right)} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{3}\right)^{\frac{2}{3}} \left(2 i\mathrm{e}^{2 i b\,x^{2}} b\,\operatorname{expIntegral}_{1}\left(2 i b\,x^{2}\right) x^{2}-2 i b\,\operatorname{expIntegral}_{1}\left(-2 i b\,x^{2}\right) \mathrm{e}^{2 i \left(b\,x^{2}+2 a\right)} x^{2}+2\,\mathrm{e}^{2 i (b\,x^{2}}}{8 x^{2} \left(\mathrm{e}^{2 i \left(b\,x^{2}+a\right)}-1\right)^{2}}$

input
```
int((c*sin(b*x^2+a)^3)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(I*c*exp(-3*I*(b*x^2+a))*(exp(2*I*(b*x^2+a))-1)^3)^(2/3)*(2*I*exp(2*I*
b*x^2)*b*Ei(1,2*I*b*x^2)*x^2-2*I*b*Ei(1,-2*I*b*x^2)*exp(2*I*(b*x^2+2*a))*x
^2+2*exp(2*I*(b*x^2+a))-exp(4*I*(b*x^2+a))-1)/x^2/(exp(2*I*(b*x^2+a))-1)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \frac{\Big(i\,bx^2\mathrm{Ei}(2i\,bx^2)\,e^{(2i\,a)} - i\,bx^2\mathrm{Ei}(-2i\,bx^2)\,e^{(-2i\,a)} - 2\,\cos\left(bx^2+a\right)^2 + 2\Big)\Big(-\Big(c}{4\left(x^2\cos\left(bx^2+a\right)^2 - x^2\right)}$$

input
```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

output
```
1/4*(I*b*x^2*Ei(2*I*b*x^2)*e^(2*I*a) - I*b*x^2*Ei(-2*I*b*x^2)*e^(-2*I*a) -
 2*cos(b*x^2 + a)^2 + 2)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/
(x^2*cos(b*x^2 + a)^2 - x^2)
```

Sympy [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(b*x**2+a)**3)**(2/3)/x**3,x)
```

output
```
Integral((c*sin(a + b*x**2)**3)**(2/3)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^3}\,dx = -\frac{(((i\,\Gamma(-1,2i\,bx^2)-i\,\Gamma(-1,-2i\,bx^2))\cos(2\,a)+(\Gamma(-1,2i\,bx^2)+\Gamma(-1,-2i\,bx^2))\sin(2\,a))bx^2-1)c^{\frac{2}{3}}}{8\,x^2}$$

input

```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

output

```
-1/8*(((I*gamma(-1, 2*I*b*x^2) - I*gamma(-1, -2*I*b*x^2))*cos(2*a) + (gamm
a(-1, 2*I*b*x^2) + gamma(-1, -2*I*b*x^2))*sin(2*a))*b*x^2 - 1)*c^(2/3)/x^2
```

Giac [F]

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

input

```
integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

output

```
integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin^3(a+bx^2))^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x^3}\,dx$$

input

```
int((c*sin(a + b*x^2)^3)^(2/3)/x^3,x)
```

output
```
int((c*sin(a + b*x^2)^3)^(2/3)/x^3, x)
```

Reduce [F]

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx=\frac{c^{\frac{2}{3}}\left(\left(\int \frac{\sin\left(b\,x^2+a\right)^2}{x^3}dx\right)x^2-2\left(\int \frac{1}{x^3}dx\right)x^2-1\right)}{x^2}$$

input
```
int((c*sin(b*x^2+a)^3)^(2/3)/x^3,x)
```

output
```
(c**(2/3)*(int(sin(a + b*x**2)**2/x**3,x)*x**2 - 2*int(1/x**3,x)*x**2 - 1)
)/x**2
```

3.350 $\int x^m(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 217

$$\begin{aligned}\int x^m\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx={}&\frac{x^{1+m}\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2(1+m)}\\&+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc^2\left(a+bx^n\right)\Gamma\left(\frac{1+m}{n},-2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}\\&+\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\csc^2\left(a+bx^n\right)\Gamma\left(\frac{1+m}{n},2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}\end{aligned}$$

output

```
x^(1+m)*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/(2+2*m)+exp(2*I*a)*x^(1+m)
*csc(a+b*x^n)^2*GAMMA((1+m)/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/(2^((1+
m+2*n)/n))/n/((-I*b*x^n)^((1+m)/n))+x^(1+m)*csc(a+b*x^n)^2*GAMMA((1+m)/n,2
*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/(2^((1+m+2*n)/n))/exp(2*I*a)/n/((I*b*x^
n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int x^m \left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\csc^2(a+bx^n)\Big(2^{\frac{1+m+n}{n}}e^{2ia}n(b^2x^{2n})^{\frac{1+m}{n}}+e^{4ia}(1+m)\,(ibx^n}{(1+m)n}$$

input

```
Integrate[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output

```
(x^(1 + m)*Csc[a + b*x^n]^2*(2^((1 + m + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n))
^((1 + m)/n) + E^((4*I)*a)*(1 + m)*(I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n,
(-2*I)*b*x^n] + (1 + m)*((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (2*I)*b*
x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*(1 + m)
*n*(b^2*x^(2*n))^((1 + m)/n))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\left(c\sin^3(a+bx^n)\right)^{2/3}dx$$

$$\Big\downarrow \ 7271$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int x^m\sin^2(bx^n+a)\,dx$$

$$\Big\downarrow \ 3906$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2bx^n+2a)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\left(\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ib$$

input
```
Int[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
Csc[a + b*x^n]^2*(x^(1 + m)/(2*(1 + m)) + (E^((2*I)*a)*x^(1 + m)*Gamma[(1
+ m)/n, (-2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) +
(x^(1 + m)*Gamma[(1 + m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)
*n*(I*b*x^n)^((1 + m)/n)))*(c*Sin[a + b*x^n]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^m\left(c\sin\left(a+b\,x^n\right)^3\right)^{\frac{2}{3}}dx$$

input
```
int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

input
```
integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(c*sin(a+b*x**n)**3)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

input
```
integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))*c^(2/3)/(m + 1
)
```

Giac [F]

$$\int x^m\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

input
```
integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int x^m\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

input
```
int(x^m*(c*sin(a + b*x^n)^3)^(2/3),x)
```

output
```
int(x^m*(c*sin(a + b*x^n)^3)^(2/3), x)
```

Reduce [F]

$$\int x^m\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int x^m\sin\left(x^n b+a\right)^2dx\right)$$

input
```
int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
c**(2/3)*int(x**m*sin(x**n*b + a)**2,x)
```

3.351 $\int x^3(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{4^{-1-\frac{2}{n}}e^{2ia}x^4(-ibx^n)^{-4/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{4}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} + \frac{4^{-1-\frac{2}{n}}e^{-2ia}x^4(ibx^n)^{-4/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{4}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

output
```
1/8*x^4*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+4^(-1-2/n)*exp(2*I*a)*x^4*
csc(a+b*x^n)^2*GAMMA(4/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n
)^(4/n))+4^(-1-2/n)*x^4*csc(a+b*x^n)^2*GAMMA(4/n,2*I*b*x^n)*(c*sin(a+b*x^n
)^3)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(4/n))
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.86

$$\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{2^{-3-\frac{4}{n}}e^{-2ia}x^4\left(b^2x^{2n}\right)^{-4/n}\csc^2(a+bx^n)\left(16^{\frac{1}{n}}e^{2ia}n\left(b^2x^{2n}\right)^{4/n}+2e^{4ia}(ibx^n)^{4/n}\Gamma\left(\frac{4}{n},-2ibx^n\right)\right.}{n}$$

input
```
Integrate[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
(2^(-3 - 4/n)*x^4*Csc[a + b*x^n]^2*(16^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^
(4/n) + 2*E^((4*I)*a)*(I*b*x^n)^(4/n)*Gamma[4/n, (-2*I)*b*x^n] + 2*((-I)*b
*x^n)^(4/n)*Gamma[4/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)
*a)*n*(b^2*x^(2*n))^(4/n))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int x^3\sin^2(bx^n+a)\,dx$$

$$\Big\downarrow \text{3906}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int\left(\frac{x^3}{2}-\frac{1}{2}x^3\cos(2bx^n+2a)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+bx^n)\left(\frac{e^{2ia}4^{-\frac{2}{n}-1}x^4(-ibx^n)^{-4/n}\,\Gamma\left(\frac{4}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}4^{-\frac{2}{n}-1}x^4(ibx^n)^{-4/n}\,\Gamma\left(\frac{4}{n},2ibx^n\right)}{n}+\frac{x^4}{8}\right)(c\sin^3($$

input
```
Int[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
Csc[a + b*x^n]^2*(x^4/8 + (4^(-1 - 2/n)*E^((2*I)*a)*x^4*Gamma[4/n, (-2*I)*
b*x^n])/(n*((-I)*b*x^n)^(4/n)) + (4^(-1 - 2/n)*x^4*Gamma[4/n, (2*I)*b*x^n]
)/(E^((2*I)*a)*n*(I*b*x^n)^(4/n)))*(c*Sin[a + b*x^n]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^3\left(c\sin\left(a+b\,x^n\right)^3\right)^{\frac{2}{3}}dx$$

input
```
int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

input
```
integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(c*sin(a+b*x**n)**3)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

input
```
integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/16*(x^4 - 4*integrate(x^3*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Giac [F]

$$\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

input
```
integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int x^3\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

input
```
int(x^3*(c*sin(a + b*x^n)^3)^(2/3),x)
```

output
```
int(x^3*(c*sin(a + b*x^n)^3)^(2/3), x)
```

Reduce [F]

$$\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int\sin\left(x^nb+a\right)^2x^3dx\right)$$

input
```
int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2*x**3,x)
```

3.352 $\int x^2(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx = \frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{2^{-2-\frac{3}{n}}e^{2ia}x^3(-ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n} + \frac{2^{-2-\frac{3}{n}}e^{-2ia}x^3(ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

output

```
1/6*x^3*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+2^(-2-3/n)*exp(2*I*a)*x^3*
csc(a+b*x^n)^2*GAMMA(3/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n
)^(3/n))+2^(-2-3/n)*x^3*csc(a+b*x^n)^2*GAMMA(3/n,2*I*b*x^n)*(c*sin(a+b*x^n
)^3)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{2^{-2-\frac{3}{n}}e^{-2ia}x^3(b^2x^{2n})^{-3/n}\csc^2(a+bx^n)\Big(2^{\frac{3+n}{n}}e^{2ia}n(b^2x^{2n})^{3/n}+3e^{4ia}(ibx^n)^{3/n}\Gamma\big(\frac{3}{n},-2ibx^n}{3n}$$

input
```
Integrate[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
(2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*(2^((3 + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*
n))^(3/n) + 3*E^((4*I)*a)*(I*b*x^n)^(3/n)*Gamma[3/n, (-2*I)*b*x^n] + 3*((-
I)*b*x^n)^(3/n)*Gamma[3/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(3*E^
((2*I)*a)*n*(b^2*x^(2*n))^(3/n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int x^2\sin^2(bx^n+a)dx$$

$$\Big\downarrow \text{3906}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos(2bx^n+2a)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+bx^n)\left(\frac{e^{2ia}2^{-\frac{3}{n}-2}x^3(-ibx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{3}{n}-2}x^3(ibx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2ibx^n\right)}{n}+\frac{x^3}{6}\right)(c\sin^3($$

input

```
Int[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output

```
Csc[a + b*x^n]^2*(x^3/6 + (2^(-2 - 3/n)*E^((2*I)*a)*x^3*Gamma[3/n, (-2*I)*
b*x^n])/(n*((-I)*b*x^n)^(3/n)) + (2^(-2 - 3/n)*x^3*Gamma[3/n, (2*I)*b*x^n]
)/(E^((2*I)*a)*n*(I*b*x^n)^(3/n)))*(c*Sin[a + b*x^n]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^{2}\left(c\sin\left(a+b\,x^{n}\right)^{3}\right)^{\frac{2}{3}}dx$$

input

```
int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output

```
int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^2\,dx$$

input
```
integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(c*sin(a+b*x**n)**3)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^2\,dx$$

input
```
integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/12*(x^3 - 3*integrate(x^2*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Giac [F]

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^2\,dx$$

input
```
integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int x^2\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

input
```
int(x^2*(c*sin(a + b*x^n)^3)^(2/3),x)
```

output
```
int(x^2*(c*sin(a + b*x^n)^3)^(2/3), x)
```

Reduce [F]

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int \sin\left(x^nb+a\right)^2x^2dx\right)$$

input
```
int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2*x**2,x)
```

3.353 $\int x(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 188

$$\begin{aligned}\int x(c\sin^3(a+bx^n))^{2/3}\,dx &= \frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\\ &+\frac{4^{-1-\frac{1}{n}}e^{2ia}x^2(-ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}\\ &+\frac{4^{-1-\frac{1}{n}}e^{-2ia}x^2(ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}\end{aligned}$$

output

```
1/4*x^2*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+4^(-1-1/n)*exp(2*I*a)*x^2*
csc(a+b*x^n)^2*GAMMA(2/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n
)^(2/n))+4^(-1-1/n)*x^2*csc(a+b*x^n)^2*GAMMA(2/n,2*I*b*x^n)*(c*sin(a+b*x^n
)^3)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

$$\int x\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{4^{-\frac{1+n}{n}}e^{-2ia}x^2\left(b^2x^{2n}\right)^{-2/n}\csc^2(a+bx^n)\left(4^{\frac{1}{n}}e^{2ia}n\left(b^2x^{2n}\right)^{2/n}+e^{4ia}(ibx^n)^{2/n}\Gamma\left(\frac{2}{n},-2ibx^n\right)+\right.}{n}$$

input
```
Integrate[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
(x^2*Csc[a + b*x^n]^2*(4^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(2/n) + E^((4*
I)*a)*(I*b*x^n)^(2/n)*Gamma[2/n, (-2*I)*b*x^n] + ((-I)*b*x^n)^(2/n)*Gamma[
2/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4^((1 + n)/n)*E^((2*I)*a)*
n*(b^2*x^(2*n))^(2/n))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int x\sin^2(bx^n+a)\,dx$$

$$\Big\downarrow \text{3906}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int\left(\frac{x}{2}-\frac{1}{2}x\cos(2bx^n+2a)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+bx^n)\left(\frac{e^{2ia}4^{-\frac{1}{n}-1}x^2(-ibx^n)^{-2/n}\Gamma\left(\frac{2}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}4^{-\frac{1}{n}-1}x^2(ibx^n)^{-2/n}\Gamma\left(\frac{2}{n},2ibx^n\right)}{n}+\frac{x^2}{4}\right)(c\sin^3($$

input
```
Int[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
Csc[a + b*x^n]^2*(x^2/4 + (4^(-1 - n^(-1))*E^((2*I)*a)*x^2*Gamma[2/n, (-2*
I)*b*x^n])/(n*((-I)*b*x^n)^(2/n)) + (4^(-1 - n^(-1))*x^2*Gamma[2/n, (2*I)*
b*x^n])/(E^((2*I)*a)*n*(I*b*x^n)^(2/n)))*(c*Sin[a + b*x^n]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x\left(c\sin\left(a+b\,x^{n}\right)^{3}\right)^{\frac{2}{3}}dx$$

input
```
int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int x \left(c \sin^3(a+b x^n)\right)^{2/3} dx = \int \left(c \sin\left(b x^n + a\right)^3\right)^{\frac{2}{3}} x \, dx$$

input
```
integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x, x)
```

Sympy [F]

$$\int x \left(c \sin^3(a+b x^n)\right)^{2/3} dx = \int x \left(c \sin^3\left(a + b x^n\right)\right)^{\frac{2}{3}} dx$$

input
```
integrate(x*(c*sin(a+b*x**n)**3)**(2/3),x)
```

output
```
Integral(x*(c*sin(a + b*x**n)**3)**(2/3), x)
```

Maxima [F]

$$\int x \left(c \sin^3(a+b x^n)\right)^{2/3} dx = \int \left(c \sin\left(b x^n + a\right)^3\right)^{\frac{2}{3}} x \, dx$$

input
```
integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/8*(x^2 - 2*integrate(x*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Giac [F]

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int x\,\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

input
```
int(x*(c*sin(a + b*x^n)^3)^(2/3),x)
```

output
```
int(x*(c*sin(a + b*x^n)^3)^(2/3), x)
```

Reduce [F]

$$\int x\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int \sin\left(x^n b+a\right)^2xdx\right)$$

input
```
int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2*x,x)
```

3.354 $\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 178

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3} + \frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\csc^2\left(a+bx^n\right)\Gamma\left(\frac{1}{n},-2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n} + \frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\csc^2\left(a+bx^n\right)\Gamma\left(\frac{1}{n},2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}$$

output

```
1/2*x*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+2^(-2-1/n)*exp(2*I*a)*x*csc(
a+b*x^n)^2*GAMMA(1/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^(1
/n))+2^(-2-1/n)*x*csc(a+b*x^n)^2*GAMMA(1/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(
2/3)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \left(c\sin^3(a+bx^n)\right)^{2/3}\,dx=\frac{2^{-2-\frac{1}{n}}e^{-2ia}x\left(b^2x^{2n}\right)^{-1/n}\csc^2(a+bx^n)\left(2^{1+\frac{1}{n}}e^{2ia}n\left(b^2x^{2n}\right)^{\frac{1}{n}}+e^{4ia}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)+(\right.}{n}$$

input
```
Integrate[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
(2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*(2^(1 + n^(-1))*E^((2*I)*a)*n*(b^2*x^(
2*n))^n^(-1) + E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-2*I)*b*x^n] +
((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3)
)/(E^((2*I)*a)*n*(b^2*x^(2*n))^n^(-1))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7271, 3848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^3(a+bx^n)\right)^{2/3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int \sin^2(bx^n+a)dx$$

$$\Big\downarrow \text{3848}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int \left(\frac{1}{2}-\frac{1}{2}\cos(2bx^n+2a)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+bx^n)\left(\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}\right)\left(c\sin^3(a+\right.$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

output
```
Csc[a + b*x^n]^2*(x/2 + (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I
)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)
*b*x^n])/(E^((2*I)*a)*n*(I*b*x^n)^n^(-1)))*(c*Sin[a + b*x^n]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3848
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \left(c\sin\left(a+b\,x^{n}\right)^{3}\right)^{\frac{2}{3}}dx$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
int((c*sin(a+b*x^n)^3)^(2/3),x)
```

Fricas [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3), x)
```

Sympy [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(2/3),x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(2/3), x)
```

Maxima [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

output
```
-1/4*c^(2/3)*(x - integrate(cos(2*b*x^n + 2*a), x))
```

Giac [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=\int \left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}\,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(2/3),x)
```

output
```
int((c*sin(a + b*x^n)^3)^(2/3), x)
```

Reduce [F]

$$\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx=c^{\frac{2}{3}}\left(\int \sin\left(x^n b+a\right)^2dx\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3),x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2,x)
```

3.355 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx = -\frac{\cos(2a)\,\text{CosIntegral}(2bx^n)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2n} + \frac{1}{2}\csc^2(a+bx^n)\log(x)\left(c\sin^3(a+bx^n)\right)^{2/3} + \frac{\csc^2(a+bx^n)\sin(2a)\left(c\sin^3(a+bx^n)\right)^{2/3}\text{Si}(2bx^n)}{2n}$$

output

```
-1/2*cos(2*a)*Ci(2*b*x^n)*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/n+1/2*cs
c(a+b*x^n)^2*ln(x)*(c*sin(a+b*x^n)^3)^(2/3)+1/2*csc(a+b*x^n)^2*sin(2*a)*(c
*sin(a+b*x^n)^3)^(2/3)*Si(2*b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x}\,dx=\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\left(-\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)+n\log(x)+\right.}{2n}$$

input
```
Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

output
```
(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x
^n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n]))/(2*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\int \frac{\sin^2\left(bx^n+a\right)}{x}dx$$

$$\Big\downarrow \text{3906}$$

$$\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\int \left(\frac{1}{2x}-\frac{\cos\left(2bx^n+2a\right)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\left(-\frac{\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)}{2n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}+\frac{\log(x)}{2}\right)$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

output
```
Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)*(-1/2*(Cos[2*a]*CosIntegral[2*
b*x^n])/n + Log[x]/2 + (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\left(i c\,\mathrm{e}^{-3 i (a+b\,x^{n})} \left(\mathrm{e}^{2 i (a+b\,x^{n})}-1\right)^{3}\right)^{\frac{2}{3}} \big(-i \mathrm{e}^{2 i b\,x^{n}} \pi \;\mathrm{csgn}(b\,x^{n})+2 i \mathrm{e}^{2 i b\,x^{n}} \;\mathrm{Si}(2 b\,x^{n})+2 \ln(x) \mathrm{e}^{2 i (a+b\,x^{n})} n+\mathrm{e}^{2 i b\,x^{n}} \;\mathrm{expIntegral}_{1}(-}{4 \left(\mathrm{e}^{2 i (a+b\,x^{n})}-1\right)^{2} n}$

input
```
int((c*sin(a+b*x^n)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(I*c*exp(-3*I*(a+b*x^n))*(exp(2*I*(a+b*x^n))-1)^3)^(2/3)*(-I*exp(2*I*
b*x^n)*Pi*csgn(b*x^n)+2*I*exp(2*I*b*x^n)*Si(2*b*x^n)+2*ln(x)*exp(2*I*(a+b*
x^n))*n+exp(2*I*b*x^n)*Ei(1,-2*I*b*x^n)+Ei(1,-2*I*b*x^n)*exp(2*I*(b*x^n+2*
a)))/(exp(2*I*(a+b*x^n))-1)^2/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \frac{(\mathrm{Ei}(2i\,bx^n)\,e^{(2i\,a)} + \mathrm{Ei}(-2i\,bx^n)\,e^{(-2i\,a)} - 2\,n\log(x))\,(-(c\cos(bx^n+a)^2 - c)}{4\,(n\cos(bx^n+a)^2 - n)}$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="fricas")
```

output
```
1/4*(Ei(2*I*b*x^n)*e^(2*I*a) + Ei(-2*I*b*x^n)*e^(-2*I*a) - 2*n*log(x))*(-(
c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/(n*cos(b*x^n + a)^2 - n)
```

Sympy [F]

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \int \frac{(c\sin^3(a+bx^n))^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(2/3)/x,x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(2/3)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x}\,dx = \frac{(((i\sqrt{3}+1)\mathrm{Ei}(2i\,bx^n) + (i\sqrt{3}+1)\mathrm{Ei}(-2i\,bx^n) + (-i\sqrt{3}+1)\mathrm{Ei}(2i\,be^{(n\log}}{}$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="maxima")
```

output
```
1/16*(((I*sqrt(3) + 1)*Ei(2*I*b*x^n) + (I*sqrt(3) + 1)*Ei(-2*I*b*x^n) + (-
I*sqrt(3) + 1)*Ei(2*I*b*e^(n*conjugate(log(x)))) + (-I*sqrt(3) + 1)*Ei(-2*
I*b*e^(n*conjugate(log(x)))))*cos(2*a) - 4*n*log(x) - ((sqrt(3) - I)*Ei(2*
I*b*x^n) - (sqrt(3) - I)*Ei(-2*I*b*x^n) - (sqrt(3) + I)*Ei(2*I*b*e^(n*conj
ugate(log(x)))) + (sqrt(3) + I)*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*
a))*c^(2/3)/n
```

Giac [F]

$$\int \frac{\left(c\sin^{3}\left(a+bx^{n}\right)\right)^{2/3}}{x}\,dx=\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^{3}\left(a+bx^{n}\right)\right)^{2/3}}{x}\,dx=\int \frac{\left(c\sin\left(a+b\,x^{n}\right)^{3}\right)^{2/3}}{x}\,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(2/3)/x,x)
```

output
```
int((c*sin(a + b*x^n)^3)^(2/3)/x, x)
```

Reduce [F]

$$\int \frac{\left(c \sin^3\left(a+b x^n\right)\right)^{2/3}}{x} \, dx = c^{\frac{2}{3}} \left(\int \frac{\sin\left(x^n b+a\right)^2}{x}dx\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3)/x,x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2/x,x)
```

3.356 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^2}\,dx = {} & -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x} \\ & + \frac{2^{-2+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\frac{1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx} \\ & + \frac{2^{-2+\frac{1}{n}}e^{-2ia}(ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\frac{1}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx}\end{aligned}$$

output
```
-1/2*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/x+2^(-2+1/n)*exp(2*I*a)*(-I*b
*x^n)^(1/n)*csc(a+b*x^n)^2*GAMMA(-1/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)
/n/x+2^(-2+1/n)*(I*b*x^n)^(1/n)*csc(a+b*x^n)^2*GAMMA(-1/n,2*I*b*x^n)*(c*si
n(a+b*x^n)^3)^(2/3)/exp(2*I*a)/n/x
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = \frac{e^{-2ia}\csc^2\left(a+bx^n\right)\left(-2e^{2ia}n+2^{\frac{1}{n}}e^{4ia}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2ibx^n\right)+2^{\frac{1}{n}}(ibx^n)^{\frac{1}{n}}}{4nx}$$

input

```
Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

output

```
(Csc[a + b*x^n]^2*(-2*E^((2*I)*a)*n + 2^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^n^
(-1)*Gamma[-n^(-1), (-2*I)*b*x^n] + 2^n^(-1)*(I*b*x^n)^n^(-1)*Gamma[-n^(-1
), (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\int \frac{\sin^2\left(bx^n+a\right)}{x^2}dx$$

$$\Big\downarrow \text{3906}$$

$$\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\int \left(\frac{1}{2x^2}-\frac{\cos\left(2bx^n+2a\right)}{2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2\left(a+bx^n\right)\left(\frac{e^{2ia}2^{\frac{1}{n}-2}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2ibx^n\right)}{nx}+\frac{e^{-2ia}2^{\frac{1}{n}-2}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2ibx^n\right)}{nx}-\frac{1}{2x}\right)\left(c\sin^3\left(a+bx^n\right)\right)$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

output
```
Csc[a + b*x^n]^2*(-1/2*1/x + (2^(-2 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(
-1)*Gamma[-n^(-1), (-2*I)*b*x^n])/(n*x) + (2^(-2 + n^(-1))*(I*b*x^n)^n^(-1
)*Gamma[-n^(-1), (2*I)*b*x^n])/(E^((2*I)*a)*n*x))*(c*Sin[a + b*x^n]^3)^(2/
3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \frac{\left(c \sin \left(a+b\,x^{n}\right)^{3}\right)^{\frac{2}{3}}}{x^{2}}dx$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

output
```
int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(2/3)/x**2,x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(2/3)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="maxima")
```

output
```
1/4*(x*integrate(cos(2*b*x^n + 2*a)/x^2, x) + 1)*c^(2/3)/x
```

Giac [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = \int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}}{x^2}\,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(2/3)/x^2,x)
```

output
```
int((c*sin(a + b*x^n)^3)^(2/3)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^2}\,dx = c^{\frac{2}{3}}\left(\int \frac{\sin\left(x^n b+a\right)^2}{x^2}dx\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2/x**2,x)
```

3.357 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx &= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4x^2}\\ &+ \frac{4^{-1+\frac{1}{n}}e^{2ia}(-ibx^n)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx^2}\\ &+ \frac{4^{-1+\frac{1}{n}}e^{-2ia}(ibx^n)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{nx^2}\end{aligned}$$

output

```
-1/4*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/x^2+4^(-1+1/n)*exp(2*I*a)*(-I
*b*x^n)^(2/n)*csc(a+b*x^n)^2*GAMMA(-2/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/
3)/n/x^2+4^(-1+1/n)*(I*b*x^n)^(2/n)*csc(a+b*x^n)^2*GAMMA(-2/n,2*I*b*x^n)*(
c*sin(a+b*x^n)^3)^(2/3)/exp(2*I*a)/n/x^2
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx = \frac{e^{-2ia}\csc^2(a+bx^n)\Big(-e^{2ia}n + 4^{\frac{1}{n}}e^{4ia}(-ibx^n)^{2/n}\,\Gamma\big(-\frac{2}{n},-2ibx^n\big) + 4^{\frac{1}{n}}(ibx^n)^{2/}}{4nx^2}$$

input
```
Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

output
```
(Csc[a + b*x^n]^2*(-(E^((2*I)*a)*n) + 4^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^(2
/n)*Gamma[-2/n, (-2*I)*b*x^n] + 4^n^(-1)*(I*b*x^n)^(2/n)*Gamma[-2/n, (2*I)
*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 3906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int \frac{\sin^2(bx^n+a)}{x^3}dx$$

$$\Big\downarrow \text{3906}$$

$$\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\int \left(\frac{1}{2x^3}-\frac{\cos(2bx^n+2a)}{2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\csc^2(a+bx^n)\left(\frac{e^{2ia}4^{\frac{1}{n}-1}(-ibx^n)^{2/n}\,\Gamma\big(-\frac{2}{n},-2ibx^n\big)}{nx^2}+\frac{e^{-2ia}4^{\frac{1}{n}-1}(ibx^n)^{2/n}\,\Gamma\big(-\frac{2}{n},2ibx^n\big)}{nx^2}-\frac{1}{4x^2}\right)(c\sin^3(a+b$$

input
```
Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

output
```
Csc[a + b*x^n]^2*(-1/4*1/x^2 + (4^(-1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^(
2/n)*Gamma[-2/n, (-2*I)*b*x^n])/(n*x^2) + (4^(-1 + n^(-1))*(I*b*x^n)^(2/n)
*Gamma[-2/n, (2*I)*b*x^n])/(E^((2*I)*a)*n*x^2))*(c*Sin[a + b*x^n]^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3906
```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \frac{\left(c \sin \left(a +b \,x^{n}\right)^{3}\right)^{\frac{2}{3}}}{x^{3}}d x$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

output
```
int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

Fricas [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx=\int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx=\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(a+b*x**n)**3)**(2/3)/x**3,x)
```

output
```
Integral((c*sin(a + b*x**n)**3)**(2/3)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx=\int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="maxima")
```

output
```
1/8*(2*x^2*integrate(cos(2*b*x^n + 2*a)/x^3, x) + 1)*c^(2/3)/x^2
```

Giac [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x^n + a)^3)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = \int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}}{x^3}\,dx$$

input
```
int((c*sin(a + b*x^n)^3)^(2/3)/x^3,x)
```

output
```
int((c*sin(a + b*x^n)^3)^(2/3)/x^3, x)
```

Reduce [F]

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx = c^{\frac{2}{3}}\left(\int \frac{\sin\left(x^n b+a\right)^2}{x^3}dx\right)$$

input
```
int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

output
```
c**(2/3)*int(sin(x**n*b + a)**2/x**3,x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file